Extracting dominant three-dimensional coherent structures from time-resolved planar PIV measurements in the wakes of cylindrical bodies

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ABSTRACT
The present study investigates two techniques for phase averaging time resolved two-dimensional PIV measurements for the purpose of extracting dominant three-dimensional wake characteristics. The first one is the classical phase averaging which uses a reference wake velocity signal measured via Laser Doppler Velocimetry (LDV). The second technique involves obtaining the phase information in each measurement plane using Proper Orthogonal Decomposition (POD) of PIV data and then establishing the relative phase between the planes utilizing the reference LDV signal. These techniques are applied to the results of numerical simulations and experiments on the flow past a circular cylinder immersed in a uniform shear flow. Numerical simulations are completed for \( \text{Re}_D = 100 \) and 300, and experiments are completed for \( \text{Re}_D = 2100 \). The phase-averaged results show that both techniques are able to reconstruct the oblique shedding of dominant spanwise vortices. For the basic case of laminar shedding at \( \text{Re}_D = 100 \), the techniques perform equally well. However, the new POD-based approach is superior when there are temporal variations in the three-dimensional wake topology present in the uniform cylinder wake for \( \text{Re}_D = 300 \) and 2100.

1. Introduction

Unsteady, three-dimensional flows over various geometries are encountered in a wide range of engineering applications. Even flows over canonical, two-dimensional geometries subjected to uniform, steady free-stream conditions exhibit significant spatial variations and unsteadiness. For example, at Reynolds numbers greater than about 190, the wake of a uniform cylinder in cross-flow features periodic shedding of spanwise vortices that develop substantial three-dimensional deformations associated with the formation of smaller scale streamwise vortex filaments [1]. To analyze such flows, capturing spatial and temporal variations in the flow field is essential. In experiments, direct quantitative measurements of three-dimensional flows have been made possible with recent developments in tomographic PIV, three-dimensional PTV, multi-plane PIV, and holography techniques [2]. However, compared to more conventional measurement methods, these techniques are still quite expensive due to high capital costs and computational resources. Moreover, their application is often restricted because of specific requirements for spatial resolution, temporal resolution, the size of the flow domain of interest, as well as facility-specific limitations. Thus, at present, the majority of experimental research laboratories rely on such conventional techniques as 2D PIV.

The aim of this investigation is to reconstruct the essential features of periodic three-dimensional wake structures using time resolved two-dimensional PIV measurements in multiple planes. Three-dimensional flow topology can be studied with two dimensional measurements via phase averaging, which requires a reference signal for resolving the phase variation of the dominant wake structures. Previous experimental studies have successfully utilized Laser Doppler Velocimetry (LDV), hot-wire, surface pressure, and other measurements as reference signals to phase average PIV results (e.g., [3]-[5]). In turbulent flows, signal processing is required to extract a reliable phase signal [4].

An alternative technique of phase averaging involves obtaining a reference signal from the POD of the 2D PIV data, as shown by [6]-[8]. In this case, no external reference signal is required to phase average the flow in a single two-dimensional plane. Perrin et al. [7] successfully computed the phase-averaged flow field in the wake of a uniform circular cylinder in a single two-dimensional plane using POD. Their results show that an improvement over classical phase averaging techniques is obtained using POD.

In the present study, it is proposed to use a POD-based phase extraction for reconstructing phase-averaged three-dimensional flow topology from PIV measurements performed at multiple planes. The results obtained with this new methodology are compared to the reconstructions acquired based on a reference LDV signal. The feasibility of the proposed approach is evaluated using numerical simulations and experimental results for a uniform circular cylinder in a uniform shear flow.
2. Method of Investigation

In the present study, methods of extracting three-dimensional flow topology with two-dimensional velocity measurements are first validated through numerical modeling, and then tested in a controlled experiment. The following sections describe the computational modeling, experimental setup, and methodology of data processing.

2.1 Computational Modeling

The finite-volume code, ANSYS CFX 13.0, was employed for numerical simulations. The computational domain is shown in Fig. 1, with the extent and placement of the domain boundaries specified. The orientation of the coordinate system used for data presentation is depicted in Fig. 1, and its origin is located on the cylinder axis at the mid-span of the domain. A free-slip boundary condition is applied to the left, right, top and bottom surfaces of the domain and a no-slip condition is applied at the model surface. A linear velocity profile, \((U, V, W) = (U_0 + 0.01U_0z/D, 0, 0)\), is applied at the inlet of the domain, while a constant (zero) pressure is applied at the outlet of the domain. Two free stream velocities \((U_0)\) were investigated, so that the Reynolds number based on cylinder diameter \((Re_D)\) varied from 100 to 110 and from 300 to 330 along the length of the cylinder. For brevity, these two cases will be henceforth characterized by \(Re_D = 100\) and \(Re_D = 300\), respectively.

The structured O-grid type mesh shown in Fig. 1 was refined based on mesh independence studies performed for a uniform circular cylinder with a span of 6D by Morton & Yarusevych [9]. Details of the meshes employed for each of the two numerical simulations are provided in Table 1. In the present work, the depth of the domain was extended to 10D in order to minimize the effects of the top and bottom free-slip wall boundary conditions on the bulk of the computational domain. The time step for each simulation was selected so that the CFL number was less than 1 in the entire computational domain. All numerical data presented were obtained following the onset of a quasi-steady vortex shedding in the wake of the cylinder. The acquired numerical data contained over twenty-five vortex shedding cycles for \(Re_D = 100\), while a smaller data set of eight vortex shedding cycles was acquired for \(Re_D = 300\).

The measurement of a reference wake velocity signal with an LDV probe was simulated by exporting the streamwise velocity component, \(U\), from a fixed location downstream of the cylinder, \((x/D, y/D, z/D) = (5, 0.75, 3)\), for each time step. The acquisition of time-resolved, two-dimensional PIV data was simulated by exporting \(U\) and \(V\) wake velocity data from five planes at \(z/D = -1.0, -1.5, -2.0, -2.5,\) and \(-3.0\). For each plane, the velocity data were extracted to match the resolution of experimental PIV measurements described in the next section. In particular, the data were interpolated from the original curvilinear grid (Fig. 1) into rectangular coordinates and averaged within simulated interrogation windows. Thus, from each of the two numerical simulations, the data extracted for analysis emulated the simultaneous experimental acquisition of a reference signal at a fixed location and planar velocity data at five different planes.

Figure 1 Computational domain.
2.2 Experimental Setup

Experiments were carried out in a water flume at the University of Waterloo. The uniform cylinder model (Fig. 2), with an aspect ratio of approximately 17, was mounted between circular endplates and placed in a region of the flow with a turbulence intensity of less than 1%. A small vertical free-stream velocity gradient of about 3% was imposed to produce oblique vortex shedding. The Reynolds number based on the free-stream flow speed at $z/D = 0$ was $Re_D = 2100$. To aid in the validation of the experimental results, flow visualization was completed using hydrogen bubble technique. In particular, a small 0.085 mm diameter stainless steel wire was mounted approximately 0.7 $D$ upstream of the cylinder model along its entire span. The wire was offset from the $y/D = 0$ plane, so that hydrogen bubbles traced vortices only on one side of the wake. Hydrogen bubbles were illuminated by producing conical laser beam generated by a two-watt continuous wave Argon-Ion laser and two cylindrical lenses. Flow visualization images were captured with a high-speed Photron camera operating at 100 Hz, which is more than one hundred times the expected shedding frequency of the uniform cylinder.

A reference wake velocity signal was measured using a single-component Measurement Science Equipment miniLDV system. The flow was seeded with hollow glass spheres with a mean diameter of 10µm. Flow seeding was controlled so that mean data acquisition rates were greater than 50 Hz.

Time-resolved planar velocity measurements were completed with a LaVision PIV system. The measurement planes were illuminated using a pulsed Nd:YLF Photonics laser at 527nm. Particle images were recorded at a rate of 100 Hz using a high-speed 1024x1024 pixels Photron camera and processed using DaVis 8 image processing software. Interrogation windows of 16x16 pixels were selected to ensure the displacement of particles in each window was approximately constant, and the mean number of particles in each window was approximately equal to ten [10]. A 50% overlap between adjacent interrogation windows produced a vector spacing of about 0.1 $D$.

The LDV and PIV measurements were completed simultaneously (Fig. 2b), with the LDV probe positioned at a fixed location ($x/D = 2.5, y/D = 0.75, z/D = 3$) above the measurement PIV planes located at $z/D = -1.0, -1.5, -2.0, -2.5,$ and $-3.0$. Since the miniLDV system operated at a wavelength of 658nm, a corresponding red filter was used for the PIV camera. The LDV and PIV systems were triggered for simultaneous acquisition using trigger signals from a LaVision high speed controller. Thus, five uncorrelated simultaneous LDV and PIV data sets were acquired.

2.3 Data Processing and Analysis

The aim of the analysis of numerical and experimental results is to reconstruct the dominant three-dimensional coherent structures in the wake region for $-3.0 \leq z/D \leq -1.0$ using the five sets of planar velocity data and the single point velocity reference signal. The following two phase-averaging methods are considered in the present work. The LDV-based method relies solely on the reference LDV signal to phase average the PIV data in each plane, and reconstruct the three-dimensional flow topology. The POD-based method uses a local reference signals signal obtained for each local PIV plane via POD analysis, and the LDV reference signal. The local reference signal is used to phase average planar velocity data, and the LDV reference is used to estimate the phase variation between the measurement planes.

In the LDV-based method, for a given PIV plane, post processing of the LDV signal was used to estimate the phase of vortex shedding. Figures 3a and 3b show representative plots of reference signal segments obtained numerically and experimentally, respectively. In order to mitigate local turbulent fluctuations in the velocity signal which result in irregular phase variations, post processing was employed. Specifically, the reference signal was reconstructed using one hundred Fourier modes pertaining to a frequency-band of 0.2$f_s$ centered at the vortex shedding frequency ($f_s$). A Hilbert transform was performed on the reconstructed signals to determine the phase angle. The corresponding phase plots are shown in Figs. 3a and 3b and illustrate the effectiveness of the employed signal conditioning for removing significant irregularities from the phase signal. Similar signal processing methodologies have been used for obtaining a phase reference in turbulent flows with hot-wire and surface pressure measurements (e.g., [4]-[5]). Using the phase reference, PIV snapshots in each plane are assigned a distinct phase angle and sorted. Twenty four phase angle bins are used, with a bin width of five degrees and a bin separation of fifteen degrees. Note that PIV snapshots associated with phase angles outside of the selected bins are not used in the analysis. Within each bin, PIV data are averaged to give a phase-
averaged representation of vortex shedding in a given plane. Then, the phase-averaged planar fields are meshed using a cubic spline interpolation in order to reconstruct the three-dimensional phase-averaged wake topology.

In the POD-based method, a local phase reference signal is obtained in each plane from POD analysis of the planar velocity data, similar to the methodology used by Perrin et al. [7]. In particular, POD is performed on planar velocity measurements, and the phase angle ($\theta$) of vortex shedding in the local PIV plane computed via Equation 1:

$$\theta = \arctan \left[ \frac{\lambda_1 a_1(t)}{\lambda_2 a_2(t)} \right]$$

where $\lambda_1$ and $\lambda_2$ are the first two eigenvalues obtained from the POD, and $a_1$ and $a_2$ are the first two temporal POD coefficients. Figs. 4a and 4b depict segments of POD coefficient $a_1$ for the numerical data at $Re_D = 100$ and experimental data at $Re_D = 2100$, respectively. In contrast with the turbulent wake velocity measurements obtained at a single point (Fig. 3b), the periodic fluctuations in the temporal POD coefficient (Fig. 4b) are not contaminated by turbulent fluctuations, which eliminates the need for signal processing prior to phase extraction. Based on the phase reference obtained from Equation 1, PIV snapshots are assigned a distinct phase and binned, as discussed earlier. Following this, phase-averaged planar fields are obtained. It must be stressed that, inherent to the energy-based phase decomposition, a given phase angle corresponds to the same instant in the shedding cycle across all planes. Thus, in order to resolve any possible phase variations across the span, such as that expected in the selected test case of a cylinder in shear flow, an external reference signal is required. Here, the relative phase shift between the planar fields is obtained by comparing the LDV reference signal to the local temporal coefficient reference signal. The mean difference in the phase angle between these two reference signals is used to adjust the phase of planar PIV data. Once all five planes have been adjusted, the planes are meshed using a cubic spline interpolation in order to reconstruct the three-dimensional phase-averaged wake topology.

![Figure 2](image1.png)  
**Figure 2** Experiment setup: (a) sketch of experimental setup, (b) photo of experimental setup.

![Figure 3](image2.png)  
**Figure 3** LDV-based reference signals and phase variation at (a) $Re_D = 100$ and (b) $Re_D = 2100$. Raw velocity data are shown in red, and filtered data used for phase averaging are shown in black. $t^*$ is the number of vortex shedding periods.
3. Results

In this section, the two phase averaging techniques are applied to planar velocity measurements in the wake of a circular cylinder subjected to a uniform shear flow. First, the techniques are evaluated and compared using results of numerical simulations for $Re_D = 100$ and 300. Following this, experimental results pertaining to $Re_D = 2100$ are considered.

3.1 Oblique vortex shedding in the wake of a uniform circular cylinder for $Re_D = 100$ and 300

Figures 5a and 5b illustrate the instantaneous wake topology of a circular cylinder in a uniform shear flow for $Re_D = 100$ and 300, respectively. In the images, isosurfaces of spanwise vorticity (dark gray) visualize obliquely shed spanwise vortices. For $Re_D = 300$ (Fig. 5b), smaller-scale structures are visualized by isosurfaces of streamwise vorticity (lighter gray). The formation of these structures and their average spanwise wavelength agree with previous investigations [1]. Also shown in Fig. 5 are the locations of the simulated PIV measurement planes (marked by dashed lines) and the simulated LDV measurement volume location (marked by an ‘x’). Note that, due to the imposed boundary conditions, vortex filaments are deformed slightly near the top and bottom boundaries of the computational domain; however, this effect is confined to a narrow region and does not affect vortex filaments away from the boundaries where the measurement planes are located. In the simulations for $Re_D = 100$, the angle of shedding remained
approximately constant (Fig. 5a). In contrast, for $Re_D = 300$, small variations in the shedding angle occurred from cycle to cycle due to spanwise deformations of vortex filaments (Fig. 5b). Also, larger variations in the shedding angle, on the order of $30^\circ$, occurred in the flow. Although such substantial variations may be attributed to the relatively short cylinder aspect ratio employed in the simulations, they serve to emulate comparable variations in the shedding angle observed in the experiment.

The simulated PIV measurements were phase averaged using the LDV-based and POD-based techniques, as outlined in Section 2. Phase-averaged flow images corresponding to $\theta = 0^\circ$ are shown in Figs. 6a-b and 7a-b for $Re_D = 100$ and 300, respectively. Representative instantaneous spanwise vorticity isosurfaces from the corresponding phase bin are shown in Fig. 6c and 7c for comparison. It can be seen that the phase-averaged results obtained with the two techniques successfully capture the oblique nature of the shedding for both Reynolds numbers. For $Re_D = 100$ (Fig. 6), the two techniques produce equivalent results, with minor differences between the phase-averaged and instantaneous

![Figure 6](image_url)

**Figure 6** Isosurfaces of spanwise vorticity constructed from five PIV planes at $Re_D = 100$ for $\theta = 0^\circ$: (a) phase-averaged results for LDV-based technique, (b) phase-averaged results for POD-based technique, and (c) instantaneous flow.
vorticity fields attributed primarily to the discrete size of the phase angle bins. For \( Re_D = 300 \) (Fig. 7), the spanwise deformations of the vortex cores, induced by the streamwise vortices (Fig. 7c) are, as expected, smoothed in the phase-averaged reconstructions. Also, the LDV-based results appear to have a higher rate of vorticity decay compared to both POD-based phase-averaged and instantaneous results for this Reynolds number.

In order to assess the two phase averaging methods, the variation between instantaneous fields and phase-averaged results within each phase bin was analyzed. First, the phase-averaged vorticity field for a given bin is subtracted from each of the instantaneous vorticity fields in that bin. The circulation of each of the resulting residual vorticity fields, \( \Gamma \), is then computed as follows:

\[
\Gamma = \oint_A (\omega - \bar{\omega}) dA
\]  

(2)

**Figure 7** Isosurfaces of spanwise vorticity constructed from five PIV planes at \( Re_D = 300 \) and \( \theta = 0^\circ \): (a) phase-averaged results for LDV-based technique, (b) phase-averaged results for POD-based technique, and (c) instantaneous flow.
where \( \omega \) is the spanwise vorticity of an instantaneous velocity field, \( \bar{\omega} \) is the phase-averaged vorticity for the corresponding phase angle bin, \( A \) is the area of the measurement plane. The normalized standard deviation of the residual circulations, \( \varepsilon \), is then computed for each bin using Equation 3.

\[
\varepsilon = \frac{1}{\bar{\Gamma}} \sqrt{\frac{1}{(N-1)} \sum_{i=1}^{N} (\Gamma_i - \bar{\Gamma})^2} \times 100\% \tag{3}
\]

where \( \bar{\Gamma} \) is the circulation of the phase-averaged vorticity field, \( \Gamma_i \) is the average circulation of all residuals, and \( N \) is the total number of fields in the corresponding phase angle bin. The results of this analysis are illustrated in Table 2 for each planar location at \( \theta = 0^\circ \). For \( Re_D = 100 \), both techniques are associated with a comparably low deviation between the corresponding instantaneous and phase-averaged flow fields. For \( Re_D = 300 \), the discrepancy becomes more substantial, as there are more significant cycle-to-cycle variations between the instantaneous fields. Nevertheless, the POD-based method shows a considerable improvement over the LDV-based method.

The observed improvement in the phase-averaged reconstructions using the POD-based technique at \( Re_D = 300 \) is attributed to two factors affecting the accuracy of phase estimation. As shown by Perrin et al. [7], phase-jitter effects caused by the use of an external reference signal in turbulent flows are virtually eliminated in phase averaging with POD techniques, which is done independently in each measurement plane here. Second, there is an adverse effect due to cycle-to-cycle variations in the phase angle along the cylinder span. Figure 8 shows a plot of the absolute difference between the phase angle obtained from the LDV reference signal (\( \theta_{LDV} \)) and that computed form the POD temporal coefficient signal (\( \theta_{POD} \)) for \( Re_D = 100 \) (Fig. 8a) and \( Re_D = 300 \) (Fig. 8b). The phase difference is defined as follows for a given plane:

\[
\Delta \theta = |\theta_{LDV} - \theta_{POD}| \tag{4}
\]

As shown in Fig. 8a, \( \Delta \theta \) fluctuates within ±2.5° at every z/D location for \( Re_D = 100 \). The difference in mean \( \Delta \theta \) values between two planes relates to the average angle of vortex shedding in the corresponding spanwise region. In contrast to the case of \( Re_D = 100 \), significant temporal changes in \( \Delta \theta \) occur for \( Re_D = 300 \) (Fig 8b), reflecting substantial cycle-to-cycle changes in the orientation of vortex filaments. Under such conditions, the LDV-based technique will accumulate errors by placing some instantaneous velocity fields into incorrect phase bins across the cylinder span. This is mitigated in the POD-based technique by the relative phase adjustment performed for each measurement plane based on the time-averaged phase difference in \( \Delta \theta \) for this plane, leading to the improved phase estimation reflected in Table 2.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Relative variation between instantaneous and phase-averaged vorticity fields for ( Re_D = 100 ) and ( Re_D = 300 ) at ( \theta = 0^\circ ).</th>
</tr>
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<tr>
<td></td>
<td>( \varepsilon ) (%)</td>
</tr>
<tr>
<td></td>
<td>LDV-based Phase Averaging</td>
</tr>
<tr>
<td>z/D</td>
<td>( Re_D = 100 )</td>
</tr>
<tr>
<td>-1</td>
<td>2.9</td>
</tr>
<tr>
<td>-1.5</td>
<td>2.6</td>
</tr>
<tr>
<td>-2</td>
<td>1.9</td>
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<tr>
<td>-2.5</td>
<td>2.9</td>
</tr>
<tr>
<td>-3</td>
<td>1.8</td>
</tr>
</tbody>
</table>

The ability to estimate the relative phase difference between two adjacent measurement planes in the POD-based technique provides added insight into the flow development. In particular, it allows estimating the average physical angle of vortex filaments (\( \alpha \)) between the two planes. Using the convective velocity of the wake vortices (\( U_c \)) and the local Strouhal number (\( St = f_o D/U_o \)), the streamwise spacing between consecutive wake vortices, \( L \), can be computed as follows.

\[
L = \frac{U_c}{f_o} = \frac{U_c \cdot D}{U_o \cdot St} \tag{5}
\]
Given the spacing between adjacent PIV planes, $\Delta z$, the physical angle of the vortex shedding given by

$$
\alpha = \arctan \left[ \frac{2\pi \Delta z}{(\overline{\Delta \theta_1} - \overline{\Delta \theta_2})L} \right] 
$$

where $\overline{\Delta \theta_1}$ and $\overline{\Delta \theta_2}$ are the time-averaged phase differences for the two selected PIV planes. Estimates of the vortex shedding angle were found to be in agreement with instantaneous flow visualization images (e.g., Fig. 5).

3.2 Oblique vortex shedding in the wake of a uniform circular cylinder at $Re_D = 2100$

The flow past a uniform cylinder in a uniform shear flow is depicted in Fig. 9 with a sequence of hydrogen bubble flow visualization images. Also shown in Fig. 9a are the approximate locations of the PIV measurement planes (marked by dashed lines) and the LDV measurement volume (marked by an ‘x’). In the image sequence, the hydrogen bubbles trace the vortex cores and illustrate that vortex shedding occurs at an angle relative to the cylinder axis, similar to that seen for $Re_D = 100$ and 300. Analysis of flow visualization video records revealed that there are substantial temporal variations in the angle of vortex shedding due to the occurrence of vortex dislocations in the wake.

Figure 8 Phase angle difference between the LDV reference signal and POD temporal coefficient signal at each $z/D$ location. $t^*$ is the number of vortex shedding periods.

Figure 9 Hydrogen bubble flow visualization in the wake of a uniform circular cylinder at $Re_D = 2100$. $t^*$ is the number of vortex shedding periods.
Phase-averaged flow reconstructions for $\theta = 0^\circ$ are shown in Fig. 10. While both techniques are able to capture the oblique nature of the shedding in the vortex formation region, the POD-based technique (Fig. 10b) provides an improved reconstruction of the shed vortical structures in the near wake. Utilizing the same analysis performed for the numerical results, the degree of relative deviation between planar instantaneous and phase-averaged flow fields was assessed for each phase bin. The results are illustrated in Table 3 for $\theta = 0^\circ$. The data suggest that the POD-based technique leads to a significantly lower deviation compared to the LDV-based method, which is attributed to the improved phase estimation. Moreover, for the LDV-based results, the deviation increases substantially as the spanwise distance between the LDV measurement volume and the PIV plane increases. As a consequence, the LDV method leads to a less accurate reconstruction of the wake topology (Fig. 10). Comparing the results in Table 3 and Table 2, it can be seen that both techniques are associated with significantly higher deviations between instantaneous and phase averaged results for $Re_D = 2100$. This is attributed primarily to the following three factors: (i) substantial cycle-to-cycle variations in the strengths of vortices shed from the cylinder in the turbulent shedding regime, which are reflected in the amplitude modulations of the temporal POD coefficient in Fig. 4b and reported to be up to 40% in [11]; (ii) additional temporal variations in the vorticity fields due to the passage of vortex dislocations, associated with vortex splitting and out of plane deformations of vortex filaments [12]; (iii) inherent experimental uncertainties in both PIV and LDV measurements, which are amplified in the computations of vorticity fields. On the other hand, these factors lead to more severe phase-jitter effects and cycle-to-cycle variations in the phase angle at each plane. As the POD-based technique is less susceptible to these adverse effects, more significant differences are observed between the results obtained with the two phase averaging techniques for $Re_D = 2100$ than for the lower Reynolds numbers (cf., Tables 3 and 2).

As discussed earlier, the superior performance of the POD-based technique is due to the improved phase estimation for each measurement plane. Figure 11a shows a representative temporal variation of the difference in phase angle between the LDV and POD-based reference signals at $z/D = -1.0$. Compared to the numerical results, there are more substantial fluctuations in the phase difference, and they occur on shorter time scales, i.e., every few vortex shedding cycles. Agreeing with flow visualization results, the observed fluctuations in $\Delta \theta$ reflect cycle-to-cycle changes in the angle of vortex shedding filaments. Since the relative phase difference between the planar PIV measurements is adjusted using the time average of $\Delta \theta$, it is important to ensure that a sufficiently long sample of data is acquired to adequately calculate this parameter. Figure 11b shows a plot of the cumulative average of $\Delta \theta$ for all $z/D$ planes investigated. The results indicate that, for the conditions investigated here, over 50 vortex shedding cycles must be captured in order to properly estimate the average difference in phase and average shedding angle.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Relative variation between instantaneous and phase-averaged vorticity fields for $Re_D = 2100$ at $\theta = 0^\circ$.</th>
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<tbody>
<tr>
<td>$z/D$</td>
<td>LDV-based Phase Averaging</td>
</tr>
<tr>
<td>-1</td>
<td>55.9</td>
</tr>
<tr>
<td>-1.5</td>
<td>64.7</td>
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<tr>
<td>-2</td>
<td>86.1</td>
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<td>-2.5</td>
<td>129.2</td>
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<tr>
<td>-3</td>
<td>192.1</td>
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The results in Fig. 11b suggest that, for planes separated by large spanwise distances, e.g., larger than the distance between planes at $z/D = -1.0$ and -3.0 in Fig. 11b, it is possible for the mean phase difference to exceed $2\pi$ radians. For a mean phase difference greater than $2\pi$, e.g., $\Delta \theta \approx 2\pi + \pi/4$, the phase can be incorrectly interpreted as $\Delta \theta \approx \pi/4$. To avoid this problem, the spanwise spacing between adjacent PIV planes must be kept sufficiently small. If the phase variation along the span is roughly known, Equation (6) can be re-arranged to obtain the minimum required spacing between two PIV planes, $\Delta z$.

4. Conclusions

A new POD-based phase averaging technique has been proposed for extracting dominant three-dimensional wake characteristics based on time-resolved two-dimensional PIV measurements. The technique relies on extracting the phase signal in each measurement plane from the Proper Orthogonal Decomposition (POD) of planar PIV data. The relative phase between planar phase-averaged fields is established using a reference LDV signal. The method has been compared with a classical phase averaging approach that uses a reference wake velocity signal measured at a fixed location with Laser Doppler Velocimetry (LDV). The two techniques have been evaluated using the results of numerical simulations and experiments on the flow past a circular cylinder immersed in a uniform shear flow at $Re_D = 100, 300$, and 2100.

The results show that both techniques are able to reconstruct the oblique vortex shedding patterns equally well in the laminar cylinder wake at $Re_D = 100$. However, when spatial and temporal variations appear in the flow field at higher $Re_D$, the POD-based approach is shown to produce superior results. This is attributed to the ability of the POD-based method to mitigate adverse effects of phase jitter and cycle-to-cycle variation in shedding angle, leading to improved estimation of the phase. The experimental results also reveal that, for adequate estimation of the relative angle between the adjacent PIV planes, a relatively large number of samples is required.

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References