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Turbulence schematization for stone stability assessment

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Turbulence schematization for stone stability assessment

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CLIENT: Ministry of Transport, Public Works and Water Management
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TITLE: Turbulence schematization for stone stability assessment

ABSTRACT:

This study is part of the joint-venture research project on design of loose-element flexible revetments between the Ministry of Transport, Public Works and Water Management, Road and Hydraulic Engineering Division and WL|Delft Hydraulics.

This study deals with potential methods for schematization of turbulence relevant for flexible revetment design. This is especially important in highly turbulent flows, such as occur near hydraulic structures. At first, a state-of-the-art review is given of current engineering practice. After that a detailed analysis is given of the effect of turbulence on the destabilizing forces that act on individual particles. Pressure fluctuations are considered to be the major phenomenon for transferring the turbulent fluctuations throughout the entire flow towards forces at the particles. A comparative model is suggested that relates shear stress fluctuations in highly complex flows to shear stress fluctuations in simple (uniform) flow via the respective pressure fluctuations. Furthermore, an expression is derived for the weighing function for determination of the pressure fluctuations out of the flow field (including turbulence), such as e.g. can be computed with numerical k-ε models.

This concept still need to be validated, a striking resemblance has been obtained with measurements on the correlation between wall pressures and turbulent velocity: the prediction as well as the measurements indicated a maximum correlation at about 1/6 of the stone diameter above the stones.

REFERENCES: Joint-venture agreement between the Ministry of Transport, Public Works and Watermanagement, Road and Hydraulic Engineering Division and WL|Delft Hydraulics, contract number DWW-1382

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# Appendices

A Model for wall pressure in turbulent shear flows dedicated to estimating stability of coarse-grained non-cohesive bed materials
I Introduction and summary of findings

1.1 Review of research program

Within the framework of a joint research co-operation project on stone stability, between the Ministry of Transport, Public Works and Water Management Department (DWW) and WL DELFT HYDRAULICS under contract number DWW-1382, the stability of rockfill material supporting the stability of structures is investigated via desk studies, experimental and numerical methods. An important trigger for this research is the increasing capability of computational models, with increasing accuracy for determination of the detailed flow patterns near structures. Prediction of stone stability through computational modelling, however, requires a stone stability concept which allows for the detailed flow parameters (local flow characteristics, turbulence). Preferable the prediction capability should be extended beyond a certain (subjective) stability criterion, e.g. the Shields for uniform flow, towards quantitative prediction of the mobility.

In complicated cases and for calibration purposes, stone stability will still need to be assessed by scale model investigations. As an intermediate stage, however, measurement of the detailed flow patterns in scale models or in the field might suffice. The stone stability concept should therefore also connect to flow measurements.

As far as detailed flow prediction is concerned, emphasis needs to be laid on prediction of the increased turbulence levels downstream of structures.

For the present research project the following tentative phasing according to the structural appearance is envisaged:

- 1998
  bed protections near structures that cause abrupt vertical changes from the bottom upwards (e.g. sills, steps, stilling basin, bottom gates)
- 1999
  bed protections near structures that cause abrupt vertical changes from the waterlevel downwards (e.g. undershot gates, culverts)
- 2000
  river training structures (e.g. spurs, guide bunds, bed and bank fixations)
- 2001
  fairway stabilisation under ship-induced water motion

The research program for 1998 is agreed upon, still leaving the possibility of adaptations. The programs for the subsequent years are subject to major change.

Within the research for 1998 into the stability of bed protection near structures that cause abrupt changes from the bottom upwards, the following phasing is anticipated:

- literature inventory (projectnumber Q2395.10)
- probabilistic analysis (projectnumber Q2395.20)
- turbulence schematization (projectnumber Q2395.30)
- computation with k-ε-model (has been done (partly) beyond the present project (Q2369, 1998))
- validation tests (projectnumber Q2395.40)
- analysis and evaluation (projectnumber Q2395.50)
The present report deals with the subject of turbulence schematization (Q2395.30).

The study has been carried out by Mr Gijs J.C.M. Hoffmans (advisor at the Public Works and Water Management Department of the Ministry of Transport), Mr Rob E. Uittenbogaard (turbulence specialist at WL | DELFT HYDRAULICS) and Mr Gert Jan Akkerman (scour protection specialist and project team leader at WL | DELFT HYDRAULICS). Mr Pieter K. Klok was team leader on behalf of the Public Works and Water Management Department of the Ministry of Transport. Mr Rob Bootj (Delft Technical University) contributed in discussions about forces on particles and turbulence. Finally, Mr Leo C. van Rijn provided quality assurance for WL | DELFT HYDRAULICS.

1.2 Study approach and report set-up

Turbulence schematization is primarily of importance for numerical flow computations, which in turn is of importance for revetment stability predictions. The present state-of-the-art numerical models usually focus on the reproduction of the major flow characteristics, but not necessarily near the bed. An example is k-ε models in which the turbulent kinetic energy k is related to the time-averaged shear stress velocity u* (see equation (2.12) in this report). This implies that near stagnation points turbulence energy is computed as zero, whereas in reality high turbulence levels may occur, e.g., downstream of a backward-facing step. Attention will be paid to solve this problem (Section 3.2). To this, in Section 3.2 pressure and shear stress predictions from k-ε models for general situations (also valid for recirculation zones) near the bed are derived. Details on the turbulence concepts are given in Appendix A. Introductory remarks about turbulence in normal flows (e.g., rivers) are made in Section 3.1. The implications of the proposed turbulence concept, indicated “comparative model”, for k-ε models is treated in Section 3.3

Prior to the above, a state-of-the-art review is given of stability formulae that take into account turbulence (Section 2), starting from: forces on particles (Section 2.1), stability assessment in non-uniform flow (Section 2.2) and a practical approach to predict turbulence downstream of structures (Section 2.3).

The main conclusions and recommendations are summarised in Section 3.2 and Section 3.3.

References of the main report are presented after Chapter 3. For convenience sake, references of the Appendix A are included in the Appendix itself. A summary of the major findings is given in the section hereafter.

It must be remarked that due to the different approaches and contributions in this report the notation used can be inconsistent. Within the present scope of this study such an inconsistency has been accepted. Hence no overall notation list has been presented; instead, the notation used has been explained in their respective sections.

1.3 Summary of findings

The present report deals with pioneering research on the schematization of turbulence for inclusion in stability formulae of stone revetments. The possibility to include turbulence is promoted by the present generation of computation flow models, e.g. k-ε models.

Stability formulae must be able to cope with turbulence; in the present report the concept of Grass (1970), eq. (2.7) in Section 2.2, has been taken as a basis for design. This concept has a similar structure as the Shields criterion which, in adapted forms, has been used extensively in engineering
practice for non-uniform flows. However, in Grass' concept the variance of loading and strength is explicitly incorporated.

Even in natural channels and rivers, in near-uniform flow conditions, turbulence has a complex nature; an introduction is presented in Section 3.1.

A substantial part of this study has been included in Section 3.2 and Appendix A. Herein a.o. a post-processing model, following k-ε flow computations, has been elaborated that can serve as a pre-processor for Grass' concept. To this, a conceptual model has been proposed, eq. (3.2.1) based on the point-of-view that turbulence-induced pressure fluctuations are the "key" to dislocation of particles. Based on this, a comparative model has been proposed, eq. (3.2.3), which has been substantiated by theoretical principles in Appendix A. This model links wall pressure fluctuations in complex flows to the shear stress velocity in complex flows, relative to uniform flow conditions. The quantification of the turbulence induced wall pressure fluctuations is shown in detail in Appendix A and leads to expression (3.2.6a). This model encompasses the idea that pressure fluctuations depend on the mean flow and turbulence along the entire water depth rather than just on the wall shear stress.

Full justification of this approach cannot be met within the limited scope of this project and recommendations have been given for model improvements and validation (end of Section 3.2). However, striking results are gained from theoretical derivations. An example is the weight function on the pressure sources and their contributions to the wall pressure which yields maximum correlation between wall pressures and turbulent velocity at about 18% of the diameter above the stones. This corresponds remarkably well with the experimental data from (Xingkui and Fontijn, 1988). In addition, for a simple turbulent boundary layer flow, the model estimates the correct ratio between rms wall pressure and wall-shear stress.

Section 3.3 presents a discussion on the implications of the findings for the research strategy, with emphasis on the lacking detailed "mechanical" model (indicated "M&M model") that is to describe the detailed mechanical fluid-particle interactions as well as the particle-particle interactions. Up to now rather enclosed formulations are available only (indicated MF's) that use the overall force balance and in which closure parameters are involved that are to be determined from calibration. The latter stability models can use various input parameters that can be scaled down for uniform ("simple") flow by the shear stress velocity u-. As a consequence, calibration for non-uniform flow should be done over a wide range of conditions, as then the parameters do not scale down with u- anymore.

In Section 3.3 it is indicated that the drag force at an element downstream of a backward facing step follows a similar pattern in downstream direction as the turbulent kinetic energy (TKE) and the rms value of wall shear stress, scaled over the upstream kinetic energy (C_r').

In this analysis the drag force pattern has been taken from measurements by Xingkui and Fontijn (1998).

In Section 3.4 recommendations are being made on application of the turbulence concept above to numerical model (DELTFT-3D-FLOW and CFX), with emphasis on k-ε modelling. However, closure problems can be avoided by applying a Reynolds Stress Model code (RSM); such computations will however be very demanding. LES, at the other hand, can be attractive as regards the importance of slower fluctuations on stone stability.

The design of a dedicated post-processor for DELFT-3D-FLOW and CFX is recommended, based on the concept presented in the present study.
2 Stone stability formulae and turbulence, state-of-the-art

As an illustration of recent attempts to incorporate turbulence in practical design of flexible revetments, a detailed elaboration on this item is summarised below, supplemented with an introduction on turbulence downstream of structures. However, it should be stressed that certain assumptions need further proof and calibration.

2.1 Forces on particles

When considering the equilibrium of one single particle against flow attack, all eroding forces \( F \), viz. drag- and liftforce and skin friction force, can be expressed by the following relation (Booij, 1998):

\[
F = c_F A \rho_w u^2
\]  

(2.1)

with \( c_F = \) coefficient and \( A \) is the effective projected area of the particle. These loads on a particle are counterbalanced by the strength, i.e. the submerged weight \( W \):

\[
W = V(\rho_s - \rho_w)g
\]  

(2.2)

with \( V = \) volume of the particle.

In fact, the forces \( F \) and \( W \) cannot be equated as the turning moments upon instability of a particle have to be taken into account. Sometimes this is done however (Booij, 1998) as the distances to the turning point are proportional to the particle diameter \( D \) for both forces, but merely equating \( F \) and \( W \) is not quite correct.

Taking the turning moment balance and replacing for \( A: c_A D^2 \) and for \( W: c_W D^3 \), with \( c_A \) and \( c_W \) being particle-dependent coefficients, the well-known Shields stability parameter \( \Psi \) is obtained:

\[
\Psi = \frac{u^2}{\Delta g D} = \frac{c_W}{c_F c_A} c_T
\]  

(2.3)

where \( c_T \) is the residual coefficient for the turning moment balance. The values of the coefficients are highly dependent on particle shape, orientation and protrusion from the surrounding particles and cannot be determined accurately. For a larger number of particles, however, the individual effects are averaged out to some extent and a qualitative or even quantitative mobility level can be assessed from experiments on the initiation of motion.

Many researchers, like Schukking (1972), established design curves for \( \Psi \), as a function of the mobility rate. For natural mixtures of particles and for man-made stone protection layers, these curves can only be indicative, due to the spatial variation of particle sizes, shape, position and orientation. Standard bed protections usually are narrowly graded; from systematic experiments Breusers (1965) it follows that taking for \( D \) the nominal diameter \( D_n \) of the protection material, the effects of shape variation and gradation is practically absent. \( D_n \) is defined as:
\[ D_s = \sqrt{\frac{M_{50}}{\rho_s}} \] (2.4)

with \( M_{50} \) = mass (kg) exceeded by 50% of the number of particles.

It should be noted that \( \Psi \) can be written in the form of shear stress \( \tau \) as well:

\[ \Psi = \frac{\tau}{\Delta \rho_u g D} \] (2.5)

This shear stress accounts for the total effect of all relevant forces per \( m^2 \) bed surface, i.e. drag- and lift forces and skin friction force.

This approach works quite well in practice for approximate uniform flow conditions.

It will be clear that normal turbulent fluctuations in the forces \( F \) under uniform conditions are implicitly incorporated in the value of \( \Psi \).

The consistency of the values of \( \Psi \) can be attributed to the fact that the variancy of bed shear stress \( \tau_u \), defined as the ratio of the standard deviation of the instantaneous bed shear stress \( \sigma \), and the time averaged bed shear stress \( \tau \) probably is (nearly) constant for uniform flow (= 0.4, as follows from the work of Grass (1970)).

### 2.2 Stone stability assessment

Adaptations can be made to the Shields concept to incorporate higher turbulence levels in case of non-uniform flow. In engineering applications this usually has been done up to now by introducing a load multiplication factor \( k \) at the flow velocity (from which \( u^* \) is calculated) or at the average bed shear stress. This factor can be combined with local flow distortion effects (e.g. when taking the vertically-averaged flow velocity as a reference in case of non-logarithmic flow profiles).

A more fundamental approach is to incorporate the variations of the bed shear stress in the stability formulae, e.g. via the turbulence intensity near the bed:

\[ \Psi = \frac{\tau}{\Delta \rho_u g D} \left( \frac{1 + 3 \tau_u}{1.45} \right)^2, \] (2.6)

where the reference ("background") relative turbulence intensity near the bed \( \tau_u \) is assumed to be 0.15 and \( \tau = \) time-averaged bed shear stress. A drawback of this formula is that e.g. in recirculation zones the forces may become nil (by multiplication with \( \tau \)), whereas in practice the turbulent fluctuations may require a heavy stone protection (instead, the formula indicates that no protection would be required there).

More basically, the shear stress fluctuations can be introduced separately, together with the variation in resistive shear stress (due to strength variation). This avoids the problems in recirculation zones.

This method was adopted by Grass (1970):
\[ \Delta D = \frac{\tau + \gamma \sigma_r}{\Psi \rho_w g (1 - \alpha_c \gamma)} \]  

(2.7)

with:

- \( \gamma \) = factor which determines the intersection point of the probability distributions of variation of instantaneous shear stress and resistive shear stress variation
- \( \sigma_r \) = standard deviation of instantaneous shear stress
- \( \Psi_G \) = adapted Shields parameter
- \( \alpha_c \) = standard deviation of the instantaneous resistive bed shear stress

For uniform flow over a fine sand bed Grass found that a value for \( \gamma \) of 0.625 meets the Shields criterion (Rouse curve). Taking a value of 0.3 for \( \alpha_c \), he thus found that:

\[ \Psi_G = 1.54 \Psi \]

From this the following relation is obtained:

\[ \Delta D = \frac{2.5 \sigma_r}{\Psi \rho_w g} \]  

(2.8)

In this relation only the standard deviation of the shear stress is incorporated; this originates from the assumptions from uniform flow.

Equation (2.8) can also be written in terms of turbulent kinetic energy near the bed \( k_{0b} \) via:

\[ \sigma_r = \alpha_0 \sqrt{c_m \rho k_{0b}} \]  

(2.9)

with, \( \alpha_0 = \sigma_r / \tau \) ( = 0.4 for uniform flow) and \( c_m (=0.09) \) is a constant used in \( k-\varepsilon \) models.

In how far this relation is able to cover non-uniform flow situations has still to be substantiated. However, calibration of this type of formula was done with experimental data for a backward facing step (Hoffmans et al., 1998). The calibrated formula was obtained after switching to the depth-averaged flow velocity \( U \) and the depth-averaged turbulence intensity \( r_0 \). In addition, as an intermediate step, the above mentioned turbulent kinetic energy near the bed \( k_{0b} \) was considered. The calibrated formula reads (Hoffmans et al, 1998):

\[ \Delta D = \alpha \frac{(r_0 U)^2}{\Psi g} \quad \text{with} \quad \alpha = 0.7 \]  

(2.10)

This equation shows reasonable results for \( \alpha = 0.55 \) in stead of 0.7. This deviation can be attributed a.o. to the uniform flow assumptions that underlay some coefficients in this approach. Thus the following simple formula can be suggested:

\[ \Delta D = 0.55 \frac{(r_0 U)^2}{\Psi g} \]  

(2.11)

Note: it should be checked in how far the value 0.55 is specific for the geometry of the upstream structure and the associated flow pattern, so calibration to more data sets is required.
Another approach has been proposed by Escarameia et al. (1992), derived from near-bed flow velocities and near-bed turbulence intensities measured for high-turbulence flow during stability experiments. Their formula reads:

$$\Delta D = f(r_{0.1}) \frac{u_{0.1}^{2}}{2 \ g}$$

(2.12)

with $r_{0.1} =$ relative turbulence intensity at 0.1 of the depth above the bed and $u_{0.1} =$ flow velocity at 0.1 of the depth above the bed. The value of the function $f(r_{0.1})$ was determined from model tests:

$$f(r_{0.1}) = 0.36 \quad \text{for} \quad r_{0.1} < 0.10$$

and

$$f(r_{0.1}) = 1.23r_{0.1} - 0.87 \quad \text{for} \quad r_{0.1} > 0.10$$

A difference with the formula of Hoffmans et al. (1998) is that the formula of Escarameia et al. suggests that the strength parameter ($\Delta D$) increases linearly with increasing turbulence level, whereas in the present study $\Delta D$ is assumed to increase progressively with the turbulence intensity. However, it must be noted that the differences are marginal for $r_{0} < 0.5$.

A more thorough validation of both approaches should be done prior to advise on which of the formulae should be applied for design purposes.

The most fundamental and direct research on the effect of highly turbulent flow on stones, known thusfar to the projectteam, has been carried out by Xingkui and Fontijn (1988) at the Delft University of Technology. In this research the instantaneous drag and lift force on an individual bed element (being part of a stone cover layer) downstream of a backward-facing step has been measured together with the flow characteristics near the element with LDA. The element was angular to represent a natural riprap stone. The distance to the step was varied. Interesting findings are:

- the flow field proved to be consistent with the measurements of Nezu and Nagakawa (1987);
- an indication of the best correlation between the time-averaged flow velocity and average value of drag force was at 0.15 D above the top of the stones; it should be remarked that the dataset for determination of this level was limited, i.e. based on levels of 0.09 D, 0.15 D and 0.30 D. In addition, the maximum correlation function shown in their Figure 19 is rather flat;
- the ratio of time-averaged lift and drag forces well outside of the reattaching flow (in the downstream section far from the step) was 0.42;
- fluctuations in drag and lift forces are in the same order of magnitude as the average values at larger distances from the step;
- correlation between lift force and longitudinal average flow velocity was generally poor;
- spectral energy in force and flow fluctuations is concentrated in low-frequencies (90% lower than 3 Hz).

A limitation to this study is the measurement of forces at only one stone out of a field of stones, so absolute figures should be considered as approximate. The analysis does not seem to be fully utilising the potentials hidden in the measured data, especially to mention the correlation of drag and liftforces (histogram of lift over drag force fluctuations to be considered against the internal angle of friction); and correlation of lift and drag force and $u(z))$. A more in-depth analysis probably will be very useful when the dataset is still available.

Nevertheless, the findings as have been reported by Xingkui and Fontijn are very interesting and form a contribution to fill the gap between turbulence predictions and stone stability response.
The study above also contributes to a practical choice of the reference height above the stones: the best correlation for the drag force proves to be at 0.15 D (for the lift force this applies only at longer distances from the step). For practical reasons during earlier studies (EKOR, 1982) the reference level was taken to be 2.5 k_s with k_s being the equivalent sand roughness (≈ 2D for stone cover layers). At this level, the disturbance of the individual stones was absent and, consequently, the flow attack was more simple to determine. However, one can argue that the approach flow condition at that level is not specific for an individual stone, so at a microscale dealing with the balance of forces on an individual stone this level is not appropriate.

At the other hand, the closer the level near the stone surface, the more specific, but also the more erratic the flow condition will be (due to the individual distortion of the flow by the stone under consideration and the adjoining stones. The appropriate reference level for flow attack will, therefore, be dependent on the scale at which the stability is considered (micro or meso scale).

### 2.3 Practical approach to representation of turbulence downstream of structure

In the previous Section is was shown that with (2.11) a bed protection downstream of a backward facing step can be designed. In this formula, the vertically averaged turbulence intensity r_0 has to be specified. In the section below a prediction method for r_0 has been presented based on the work of Hoffmans (1993c).

Structures and abrupt bottom profile changes cause major disturbances of the natural flow field and, hence, major disturbance of the undisturbed turbulence field. E.g. downstream of a backward facing step a mixing layer is generated with high turbulence levels and subsequent flow reattachment at some distance downstream of the step. The typical flow regions are indicated in the Figure 2.1 for flow downstream of a sill. These zones are: mixing layer, recirculation zone, relaxation zone and new wall boundary layer. There is a stagnation point where the centre of the mixing layer touches the bed: the reattachment point.

![Figure 2.1 Typical flow regions downstream of a structure (from Hoffmans & Verheij, 1997)](image)

A comparable flow pattern occurs in scour holes and downstream of dunes. Turbulence fluctuations in the approach flow are probably (mainly) overruled by the turbulence associated with the mixing layer and reattachment flow when considering the flow pattern near the bed. Typical for the recirculation zone downstream of steps/dunes/scour holes is the small (and at the stagnation point zero) average flow velocities and extremely high turbulence levels. In k-ε computer models this zone poses difficulties, as the near-bed turbulent kinetic energy k_{0,b} can only be computed from the average value of u*, according to:
\[ k_{0,b} = \frac{u_*^2}{\sqrt{C_\mu}} \]

(2.13)

with \( C_\mu \) = constant for k-\( \epsilon \) models = 0.09 (e.g. Rodi, 1980).

In view of (2.13), locally \( k_{0,b} \) can become zero while high turbulence may still require a heavy stone protection. This typical flow field has been clearly demonstrated by Van Mierlo et al. (1988) for flow over sand dunes, Figure 2.2. This figure shows that the turbulence energy in the centre of the mixing layer increases up to reattachment whereafter it decreases gradually.

![Figure 2.2 Turbulent kinetic energy at sand dunes](image)

In fact the turbulent kinetic energy and, hence, the effective shear stresses are not determined by the local flow conditions only: turbulence is transferred over longer distances towards the stagnation zone via pressure fluctuations \( p' \) and this forms the starting point of improving the input to various mobility descriptors, see Section 3.2 and Appendix A. The latter study demonstrates that (2.13) is not significant as its excluded from the presented model. Nevertheless, (2.13) could be replaced by a more appropriate boundary condition such as a Neumann (flux) condition but for the present purpose its relevance is doubted.

Hoffmans performed extensive research work in the context of mathematical modelling of local-scour holes downstream of structures (1992, 1993a, 1993b and 1993c). He extended his studies to flow downstream of backward facing steps and flow downstream of sand dunes. Special emphasis was put on the estimate of the eddy viscosity \( v_t \) for the typical flow regions as shown in Figure 2.2.

Based on his theoretical analysis of decay of turbulence energy in the relaxation zone (1992) he proposes a vertically-averaged relative turbulence intensity \( \tau_0 \) downstream of a step (e.g. backward facing step/ sill) as a function of the step height \( D \) and the waterdepth \( h \) (1993c):
\[ r_v = \sqrt{C_k \left(1 - \frac{D}{h}\right)^2 \left(\frac{x - 6D}{6.67h} + 1\right)^{1.09}} + \frac{1.45g}{C^2} \] (2.13)

being valid for values of \( x > 6D \) (downstream of reattachment). For backward facing steps the value of \( C_k \) approximately equals 0.0225, whereas for sills with gentle slopes \( C_k \) will be smaller and dependent on the slope angle.
3 Quantifying turbulence

In the present study the possibility for a better quantification of turbulence in relation with coarse bed material stability is investigated. As an introduction, general aspects of turbulence in open channel flow is outlined hereafter.

3.1 General aspects of turbulence in open channels

Time and length scales

The structure of turbulence in flows has been extensively studied over many years and still is a major topic of hydraulic research. Nowadays research is speeding up by advanced measurement techniques (ADV, ADCP). However due to its complexity, turbulence in various circumstances still cannot be described completely. Studies are mainly based on small scale experiments in straight open channels, but interesting analyses of field measurements have been reported recently (Barua et al., 1998; Sukhodolov, 1998).

In natural flow conditions, e.g. in alluvial rivers, turbulence has been observed to cover a wide range of scales. Boundary turbulence, i.e. originating from the presence of the bed, typically has a eddy size $\lambda$ (m) in between the dissipation scale (Kolmogorov scale: $\nu/u^*$) and the water depth $h$.

The corresponding period $T_r$ (index $\nu$ for indicating eddies with a predominant vertical structure) can be found by applying Taylor’s hypothesis: $T_r = \frac{c \lambda}{U}$, with $c$ = coefficient and $U$ = (free) flow velocity. The following range of values for $T_r$ can be observed (Yalin, 1992):

$$\frac{c_1 \left( \frac{U}{u^*} \right)}{U} \leq T_r \leq \frac{c_2 h}{U}$$

(3.1.1)

For stone stability the Kolmogorov scale is not of importance as the eddies are too small. The value of $c_2$ may vary between 3 and 7 with an average of 6 (Yalin, 1992).

In open channels also large horizontal eddies do develop with a vertical axis. Their typical time scale $T_h$ can be expressed in terms of the channel width $B$ (Yalin 1992):

$$T_h = \frac{c_3 B}{U}$$

(3.1.2)

To give an impression of this time scale: Yokoshi (1967) reported a value for $c_3$ of 9 from measurements in a river.

Barua et al. (1998) analysed ADCP measurements in the Brahmaputra-Jamuna River in Bangladesh and reported recurrence intervals of turbulence ranging from 11.5 s (high-frequency vertical eddies) to about 18 minutes (low-frequency horizontal eddies). To estimate the turbulence intensity a 900 s averaging time should be used at minimum! The value of the relative turbulence intensity in the wall region was observed to vary from about 0.11 to 0.23. Yet, Sukhodolov et al. conclude that basic concepts developed for laboratory open channel flows are valid as well for more complex flows that occur in weakly three-dimensional natural streams.
From turbulence measurements during construction of the storm surge barrier in the Eastern Scheldt, considerable turbulence was found for periods higher than 10 s (Flokstra, 1988). This type of turbulence, dominated by large horizontal eddies, gave rise to increased flow attack at the bed due to increased quasi-static loading (EKOR, 1982).

**Turbulence intensity**

In straight open channels turbulence is generated by the bed roughness. This turbulence can be indicated as “background turbulence”. A major indication of the turbulence level is by the introduction of the turbulence intensity $\sigma_{t}$ of the velocity fluctuations in $x$-direction, defined as the root-mean-square value of the fluctuations. Accordingly, the relative turbulence intensity $r$ is defined as $\sigma_{t}/U$, with $U$ = flow velocity. From Nezu (1977) and introducing the Chezy- coefficient $C$ for the bed roughness, the vertically-averaged relative turbulence intensity $r_{0}$ can be expressed as being a function of $C$:

$$r_{0} = \sqrt{\frac{1.45g}{C^2}} \tag{3.1.3}$$

with $g$ the acceleration of gravity. Generally, for open channels a value for $r_{0}$ of 0.10 is taken. This corresponds with a $C$ value of 37.7 m\(^{0.5}\)/s, which -in order of magnitude- can be considered as an average figure for the bed roughness in open channels.

Likewise, the relative turbulence intensity very close to the bed $r_{b}$ can be expressed for uniform flow as:

$$r_{b} = \sqrt{\frac{3.70g}{C^2}} \tag{3.1.4}$$

at a height of $z^{+}$ of 70, with:

$$z^{+} = \frac{z\nu}{\nu} \tag{3.1.5}$$

It must be stated however, that this height is very close to the bed, i.e. generally 1 mm or less; this is difficult to interpret when dealing with stone protections.

Introducing for $C$ the value of 37.7 m\(^{0.5}\)/s, with (3.4) a value for $r_{b}$ of 0.16 is obtained. This is close to the value of 0.15 which is generally taken as an average value of $r_{b}$ . Moreover, there is a tendency that $r_{b}$ will be lower when the reference level is taken at a more practical higher level.

### 3.2 Mean flow and turbulence properties contributing to forces on coarse-grained mobile bed material

**Introduction**

The title of this section excludes bed material of such a small size that friction and drag due to groundwater flow affects the mobility of non-cohesive bed material.
Kalinske (1947) considered theoretically the force balance on a single bed element and used probability distributions of wall pressure and near-wall velocity for estimating the conditions of initial motion of non-cohesive bed material.

H.A. Einstein (1949) presents one of the earliest experimental investigation towards forces, rather than stresses, on bed material. Based on observed pressure differences, he reports the observed lift and drag forces exerted by a turbulent flow on a single spherical ball for 50% protruding a flat bed. He observed that the turbulence-induced fluctuations in forces dominate the mean forces.

Grass (1970) designed an estimator for bed mobility based on the mean wall-shear stress and its variance, as explained in Section 2.2. He proposed using the mean wall-shear stress as input to his model and that input description forms the essential problem of this section. There are important flow conditions, with locally zero wall-shear stress, but with notable erosion (Van Mierlo et al., 1988). Nevertheless, potential success of Grass’ concept in k-ε models demands that the input to his concept should be modified to include flows near stagnation points. The latter extension is based on the argument that if the local flow and pressure fluctuations determine the mobility of bed material. Preferably this correction to the Grass model should be derived by means of post-processing Reynolds-averaged numerical simulations using e.g. the k-ε turbulence model or through observations in the field or in the laboratory. This section presents such a pre-processor for Grass’ model.

To that purpose, first a conceptual model is proposed on which this pre-processor is founded and then this section presents theoretically the mean flow and turbulence properties that in essence determine the forces on a single element of mobile bed material.

The theoretical foundation for the latter is derived from extensive theoretical as well as experimental studies on pressure fluctuations on walls. Such pressure fluctuations may excite mechanical vibrations which are of importance in many problems of mechanical engineering and sound generation. Flowcs Williams (1969) presents an overview of modelling concepts for turbulence-induced pressure fluctuations and the prominent spectral model is given in (Chase, 1987).

The following conceptual mechanical model summarises the concepts on instability of bed material and its relation to turbulence-induced wall pressure:

**Conceptual mechanical model**

*Wall pressure fluctuations at nominal wave lengths of twice the typical size of mobile bed material create the largest pressure differences between subsequent bed elements. These pressure differences induce the largest pressure-related lift force on a single element by the flow induced in between and below the upper bed elements (see Figure 3.2.1). In addition, the local velocity induces additional lift forces and is even essential for the drag forces. The local velocity fluctuations are considered to be driven by pressure gradients through:*  

$$\rho \frac{\partial u'}{\partial t} + \rho u' \cdot \nabla u' + \nabla p' = 0$$  

(3.2.1)

so that near-wall pressure fluctuations are considered as the principle source of forces on sufficiently coarse-grained bed material.
Figure 3.2.1 Optimal wave length of turbulence-induced pressure for creating lift forces on bed material.

For the purpose of improving the input to Grass' model no more details about lift and drag forces are required in this conceptual model. The model that is presented in this section is designed to be a comparative model. By comparative model we imply that it serves as estimator about correcting the input to Grass' model

\[ \Delta D = \frac{\tau + \gamma \sigma_r}{\Psi G \rho_s g (1 - \alpha_r \gamma)} \]  

(3.2.2)

using his simple flow condition as reference. Our proposal is to copy the mean bed-shear stress \( \tau \) from a flow simulation with vertical resolution but to supply Grass' expression directly with an estimate for \( \sigma_r \), the standard deviation of the bed shear stress. Particularly, the comparative model corrects the input to Grass' model (1970) based on the conversion

\[ \sigma_{r, \text{complex flow}} = \frac{|p'|_{\text{complex flow}}}{|p'|_{\text{simple flow}}} \sigma_{r, \text{simple flow}} \]  

(3.2.3)

with \( |p'|_{\text{simple flow}} \) the rms of wall pressure of a turbulent flow in a uniform channel, as investigated by Grass.

The essence of the presented model is predicting in (3.2.3) the ratio in rms wall pressure for different flow conditions or geometries. The presented comparative model, however, has the capability of becoming an absolute predictor of rms wall pressure as well as for the induced forces on single bed element but that development is beyond the scope of the present study. It should be noted that even if such a quantitative model would be available then still the major problem is the lack of suitable mechanical-mobility model determining the probability of mobility of a single bed element.

The comparative model
As announced previously, the turbulence-induced pressure fluctuations $p'$ form the cornerstone of the comparative model. Appendix A presents the mathematical details as well as references to experimental substantiation, although additional theoretical, experimental as well as numerical verification is recommended.

For an incompressible flow without density stratification and neglecting Coriolis forces, the general equation for $p'$ reads

$$\nabla^2 p' = -\rho \frac{\partial^2 U_j u'_i + U_i u'_j + u'_i u'_j - \langle u'_i u'_j \rangle}{\partial x_i x_j} ; \quad i, j = x, y, z \quad ,$$

(3.2.4)

with mean-flow variables in capitals, turbulence fluctuations are $(..)'$ and have zero mean value, the brackets $<...>$ indicate ensemble averaging which equals time averaging for stationary turbulence. Here turbulent flows are considered with dominant vertical flow structure as well as where for the relative turbulence level $|u'|/\bar{U} < 1$ holds, with the overbar in $\bar{U}$ indicating depth-averaging of velocity $U$. In these cases, the product of turbulence velocity components is neglected against the remaining terms in (3.2.3) yielding

$$\nabla^2 p' = -2\rho \left( \frac{\partial U \partial w'}{\partial z \partial x} + \frac{\partial V \partial w'}{\partial z \partial y} \right) ,$$

(3.2.5)

with $(U, V)$ the mean orthogonal horizontal velocity components and $w'$ is the vertical turbulence velocity component. The RHS of (3.2.5) shows that the pressure fluctuations are due to pressure sources which may be distributed in the entire fluid and these sources are stimulated by the presence of mean shear rates. The solution of (3.2.5) shows that the contributions of the pressure sources to the wall pressure depends on their respective wave length as well as to the distance between source and the wall point under consideration. The importance of these contributions is expressed by a so-called weight function $P(z)$ with distance $z$ to the wall. The conceptual mechanical model states that wall pressure fluctuations with wave length

$$\lambda_p = 2d_n \quad ,$$

(3.2.6)

parallel to the wall and with $d_n$ the typical size of the bed material, are the most prominent candidates for inducing motion of bed material. Solving (3.2.5) and using (3.2.6) yields the weight function $P(z)$ of Figure 3.2.2. This function is maximal at about $18\%$ of $d_n$ and this is the level where maximal correlation between wall pressure and turbulent velocity is expected. This conclusion appears to be supported by experiments of Xingkui and Fontijn (1998) who correlate observed forces on mobile bed material to turbulent velocity fluctuations above the bed.

Despite the dominance of (3.2.6), turbulent motions at other lengths scales contribute also to the variance of wall pressure, Figure 3.2.3 present $P(z)$ for turbulent motions with 10 metre streamwise wave length in 5 metre deep water with a logarithmic velocity profile depending on the roughness length which is related to $d_n$ in the usual manner, for details see Appendix A. The example of Figure 3.2.3 is dedicated to flows with e.g. wakes or mixing layer remote from the bed and containing large horizontal structures such as Von Kármán vortex streets.

In view of Figures 3.2.2 and 3.2.3, the variance of wall pressure is a weighted depth integral over all significant turbulence contributions at their respective horizontal length scales. The term significant refers to those turbulence-induced wall pressure fluctuations with wave length
exceeding the size $d_n$ of bed material because the force due to pressure fluctuations with wave lengths shorter than $d_n$ are zero after integrating the pressure over the surface of a single bed element.

Appendix A presents an in-depth derivation of the variance of wall pressure as a function of depth-dependent turbulence and mean-flow properties. Further, that derivation is simplified by substantiated closures about the mutual ratio of turbulence-velocity variances as well as length scales. These closures yield an expression suitable for post-processing some simulation of Reynolds averaged flow using the k-ε turbulence model. The general form of this expression reads

$$|p|^2 = \rho^2 \int_0^h \left( U'^2(x) + V'^2(x) \right) W(z; h, d_n, k^\frac{3}{2} / \varepsilon) k(z) dz,$$

(3.2.6a)

with $k(z)$ the turbulent kinetic energy at distance $z$ above the bed and water depth $h$. Function $W$ has no physical dimensions and its parameters, appearing after the semicolon, are the required input as function of level of the bed at the horizontal point of interest. Notice that the geometry- and flow-dependent ratio $k^\frac{3}{2} / \varepsilon$, representing a turbulence length scale, enters $W$.

The term $\sqrt{U'^2(z) + V'^2(z)}$ is the magnitude of the mean velocity parallel to the bed and at level $z$ above the bed. The formal expression (3.2.6a) shows that the pressure fluctuations depend on mean flow and turbulence properties along the entire water depth rather than just on the wall-shear stress as assumed by Grass (1970). Figure 3.2.4 presents $P(z)$ defined by

$$P(z) = \left[ U'^2(z) + V'^2(z) \right] W(z; h, d_n, k^\frac{3}{2} / \varepsilon)$$

(3.2.6b)

for a logarithmic velocity profile with parabolic mixing length distribution as investigated by Grass.

The application of (3.2.6a) to the flow profile investigated by Grass as well as to the complex flow simulated with the k-ε turbulence model yields the ratio (3.2.2) for adapting the input of Grass’ model to that complex flow. The proposed pre-processor to Grass’ model should use the full mathematical equation underlying (3.2.6a) and this pre-processor computes the ratio in rms wall pressure appearing in (3.2.3) rather than the magnitude of wall pressure.

In deriving (3.2.6), Appendix A presents various intermediate expressions such as one for the correlation:

$$\langle p'(x,y,z=0,t)u'(x,y,z,t) \rangle$$

(3.2.7)

which is an observable quantity in laboratory channel flows and also in the field. Consequently, the comparative model has various options for experimental validation as well as validation by Direct Numerical Simulation of turbulent flows at all length scales, see e.g. (Schumann, 1975) and (Moin and Kim, 1982).
Discussion

In obtaining (3.2.6a), first an equation is derived for the variance of wall pressure, formulated as a function of spatial as well as temporal spectral properties of turbulence velocity. Because such spectra are not predicted by numerical simulations based on Reynolds-averaged equations, various closures are introduced for arriving at the form (3.2.6a), which is suitable for input from the k-ε model. These closures involve the ratio of the variance of turbulence velocity components relative to the turbulent kinetic energy k of the k-ε model. Other closures connect the so-called transversal and longitudinal integral lengths of the velocity components to the local length scale \( k^{1/2} / \epsilon \) as well as distance z to the bed. These closures are based on extensive observations in boundary layer flows but depend on complex flow conditions. The essence, however, are the mutual ratios between the various velocity components rather than to turbulent kinetic energy and to a lesser extent this argument holds also for the mutual ratios of length scales, relative to the local water depth. Therefore, the model is defined to be a *comparative estimator* rather than an *absolute estimator* of wall-pressure variance. Notice that even if the model of Appendix A would become a good estimator for the magnitude of variance of wall pressure then still the complicated problem remains of implementing this estimator directly into a mechanical-mobility model. It appears that no such detailed mechanical-mobility model is available and the connection to Grass’ model is proposed instead.

Summary

The model of Grass (1970) describes the instability of non-cohesive bed material. This model has been calibrated against detailed observations made on mobility of bed material in a turbulent flow in a uniform channel with a plane horizontal bed. The calibration requires input in terms of the so-called bed-shear stress velocity \( u_* \). The latter input represents several complicated near-bed as well as far field turbulence properties that, fortunately, appear to scale to \( u_* \) in case of simple channel flow over a flat horizontal bed. The problem of this section concerns the direct application of Grass’ model to complex flows behind structures where locally \( u_* = 0 \) holds in stagnation points but where still significant scour is observed.

Rather than adapting Grass’ model intrinsically, we propose a conversion of the turbulence property \( u_* \) of his flow to a representative one for complex flows e.g. including flow-stagnation points on the bed. For a proper design of such a converter, a conceptual model is required. Here a conceptual mechanical model is proposed in which pressure fluctuations at and near the bed are regarded to affect directly, or indirectly through inducing velocity fluctuations, the stability of non-cohesive coarse-grained bed material.

This model concept yields as converter the formula

\[
\sigma_{r, \text{complex flow}} = \frac{\sigma_{r, \text{complex flow}}}{\sigma_{r, \text{simple flow}}} \frac{P_{\text{complex flow}}}{P_{\text{simple flow}}} \quad (3.2.3)
\]

where the subscript *simple flow* refers to the flow investigated by Grass. The converter is the ratio in rms bed pressure between the simple flow and the complex flow. Therefore the converter is defined as a *comparative model* i.e. only ratios in rms pressure are of importance rather than prediction of the magnitude proper.
The **comparative model** is derived in Appendix A and it is an integral over the depth-dependent mean-flow velocity, turbulent kinetic energy $k$ and a mixing length derived from the $k$-$\epsilon$ model. The latter properties follow from 2DV or 3D simulations. By post-processing the simulation’s output, the pressure ratio in (3.2.3) is derived easily. The result is the left-hand side of (3.2.3) which is the adapted input for the standard deviation $\sigma_f$ of bed-shear stress in Grass’ model; the mean bed-shear stress ($\tau$) still follows directly from the flow simulation.

The intermediate steps in the **comparative model** allow for experimental validation. A particular verification is in good agreement with the observed correlation (Xingkui and Fontijn, 1988) between forces on a single bed element and the turbulence velocity above it.

**Recommendations**

**Model improvements**

1. Some of the a priori deficiencies of the comparative model can be relieved by implementing analytical models for spectra of turbulence velocity thereby avoiding closures using integral length scales.

2. Further, closures for the flow and geometry-dependent ratios between the variances of the three turbulence velocity components could be discarded by simulating the Reynolds-averaged flow equations with one of the many versions of Reynolds Stress Model closure (RSM), such as implemented in several commercially available codes dedicated to industrial flows conditions; this subject is presented in Section 3.3.

**Model validation**

1. A desk study of available literature on pressure fluctuations in various flow conditions forms the first and relatively simple step towards validation of the model for pressure variance.

2. The intermediate steps made for arriving at (3.2.6a) can be checked by spatial as well as spectral analysis of the correlation between wall pressure and turbulence velocity fluctuations above the bed.

3. In the laboratory the relevant properties are observable by wall-mounted pressure sensors or pressure-sensitive foils, and LDA observations of turbulence velocity using an LDA for observing one, two or three velocity components.

4. In the field, a pressure sensor can be mounted in the bed and commercially available ADCP equipment can record the turbulence velocity at intermediate-large scales and as a function of level above the pressure sensor. Particularly, ADCP’s based on the coherent detection of sound pulses have the smallest measuring volumes.

5. The state-of-the-art of Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS) is such that these numerical methods reliably resolve the largest (LES) or all eddy motions (DNS). Therefore, LES and DNS are recommended for detailed validation of the spectral properties and correlation’s that appear in the intermediate stages of deriving (3.2.6a).

In Section 3.4 recommendations are made on the implementation and application of the **comparative model**.

### 3.3 Discussion and synthesis of research strategies

A serious omission in this study is the availability of a detailed Mechanical and Mobility model, abbreviated hereafter as M&M model, which should identify objectively the required input of hydrodynamic/turbulence properties. Instead several rather enclosed formulations for the state of mobility of bed material are formulated, we abbreviate these functions as Mobility Functions,
MF’s. With *enclosed* we imply here that overall force balances are used which neglect details of the mechanical-dynamically fluid-particle as well as particle-particle interactions. The lacking part of the MF’s is represented by closure parameters which are determined by calibration. The consequences are exemplified as follows. Without an M&M model one could select the required hydrodynamic input to MF’s from

\[ \frac{\overline{U^2}}{}; u_*^2; \left| (u'w')^2 - |u'w'|^2 \right|^k; \frac{|x|^2}{x}; \frac{|x|^2 + |v|^2}{x}; k; \frac{|p|}{p} \text{ etc.} \]  

(3.3.1)

Except for \( u_* \), all other candidates for hydrodynamic input to MF’s could be defined at or near the bed or after some depth-averaging operation. The point, of course, is that for simple channel flow, such as most frequently investigated for calibrating the MF’s, one of these flow parameters can be used to represent all others, possibly after adding some weak log-dependence on bed roughness.

The weak point of the closures in the MF’s, therefore, shows up only when significant deviations in the nature of the turbulence as well as the particle size or particle-mass or fluid density are applied. In this respect, notice that in the experiments by which Grass (1970) calibrated his MF, the particle-mass density was not varied, the particle size was of typical sand and varied just a factor two and, moreover, the nature of the turbulence remained unaltered, being a logarithmic boundary-layer flow. A similar lack of variability is in the unpublished experiments of Xingkui and Fontijn (1988) who measured forces on a single stone of about 33 mm size in a equally rough bed behind a backward facing step in laboratory facility with maximal 300 mm water depth.

Thus the proof-of-the-pudding of Grass’s MF, or of other MF’s, is testing them in significantly different flows, geometry’s as well as particle sizes. Notice that the research plan, of which this report forms part, aims at abrupt changes in the flow geometry, such as occurs near sills, stilling basins, semi-submerged gates, culverts, etceteras. Consequently, although not independently, also turbulence properties may vary strongly.

Steps on this route were taken by Hoffmans (1993c) and Hoffmans and Booij, (1993a, 1993b). Their work demonstrates the necessity of correcting the hydrodynamic input to the MF’s. The most prominent example concerns input to MF’s based on the bed-shear velocity \( u_* \) : this concept fails in and near a stagnation point on the bed.

At least the following two routes of attack for solving these and other inconsistencies in just the hydrodynamic input could be pursued:

- empirically, i.e. without guidance by an M&M model, re-define the hydrodynamic input to MF’s or,
- theoretically, using at least concepts of an M&M model, derive the proper conversion of hydrodynamic input to MF’s that were calibrated for simple boundary-layer flows.

Notice that in both approaches, the MF’s remain unaltered i.e. the implied assumption is that, despite their weak concepts, the MF’s are applicable locally at each point on the bed. Intuitively, the latter assumption appears valid if mean-flow properties and bed geometry vary slowly in comparison to the typical size of the bed material. The latter proposition is used in this study and it holds until either a detailed M&M model is available or if testing “all” options of changing the input to the MF’s shows the failure of using MF’s that were tuned on simple boundary-layer flows.

The route of the first strategy (point 1 above) would be to search empirically for the proper hydrodynamic input to the MF’s. Incidentally, in case of designing rules for engineering
applications, the turbulence properties obtained through numerical flow simulations using the k-ε turbulence model, or collected from observations, are summarised in analytical formulations. Notice that despite the application of involved numerical flow simulations or observations, this engineering approach still is empirical if no M&M model guides the selection of flow variables.

On the other hand, the complexity of the problem is such that the success of a purely theoretical approach in the second strategy (point 2 above) is not guaranteed either.

For clarity, we acknowledge that both strategies are scientifically sub-optimal choices. The most fundamental approach being the design and testing of a full M&M model. The latter generic strategy, however, is regarded as a too lengthy route in the context of the research plans and their objectives of our project.

Instead, a dual approach is pursued in this study, using intermediate theoretical arguments (second point), either available or to be formulated yet, for guiding the empirical search (first point). In other words, during the progress of theoretical approach some of its findings are condensed such as to guide the empirical approach.

A promising demonstration of this dual approach is the following. Emmerling (1973) observed with much detail the momentary distribution of wall pressure, due to a logarithmic boundary-layer flow over a smooth wall. He detected wall pressure by an closely packed array of 17 pressure sensors of 2.5 mm diameter as well as using two-dimensional (optical) observation of wall pressure sensed on rigid membranes. Emmerling’s observations show the passage of streaks in pressure similar to those of velocity.

Further, pressure fluctuations propagate rapidly, at about 1,500 m/s in water, the perturbations induced by turbulent motions to e.g. a wall whereas the flow advects or diffuses these perturbations much slower. We therefore regard the pressure signal as primary source for transmitting remote flow perturbations to a wall.

A simple concept from an M&M model shows that not all wave lengths in the near-bed TKE are responsible for forces on bed elements. Basic arguments about integrating stresses over a bed element into a single force and moment vector on that particle show that wall pressure fluctuations, affecting bed mobility, should have horizontal length scales of at least twice the typical size of the bed material. Mathematically, the size \( \phi \) of the bed material acts as truncator in wave number space. Practically, the larger the bed material the smaller the relevant wall-pressure fluctuations, which we define by \( |p|_{|\phi} \). Pressure fluctuations are responsible for lift forces, pressure gradients accelerate the flow and this flow acceleration creates virtual mass forces and, last but not least, the flow around bed material creates drag and additional lift forces.

Section 3.2 and Appendix A present our incomplete theoretical investigation, based on pressure as primary transmitter. This theory concludes that velocity fluctuations near the bed and with wave length of twice the typical size of bed material determine most of the pressure fluctuations on the bed (Figure 3.2.2). The latter is confirmed indirectly by unpublished experiments (Xingkui and Fcntijn, 1988) involving correlation’s between fluid velocity and forces, rather than pressure, on a single bed element. If the energy in all wave lengths exceeding twice the typical size of bed material is collected then the position of the maximum in the weighting function that converts turbulent kinetic energy (TKE) into the wall pressure, is more remote from the bed, see Figure 3.2.4.

H.A. Einstein (1949) and Kalinske (1947) show that rms turbulence properties are essential for mobility of the bed material. Guided by the previous theory for wall pressure, a first rough empirical step is comparing the observed TKE (integrated over all wave lengths) near the bed
with the rms of lift and drag forces on a bed element, see Figure 3.2.5. To this figure is added the observation of the ratio, defined as $C'_f$, between rms of wall-shear stress in streamwise direction and the kinetic energy of the mean flow before the step (Adams and Johnston, 1988). Notice that $C'_f$ is the non-dimensional form for the third turbulence parameter in (1). The comparison between profiles of $C'_f$ or TKE and the variances of forces on bed material promises a more generic, i.e. flow-and geometry-independent, input to MF's but with additional corrections of which one is discussed below.

Incidentally, notice that $C'_f$ is based on just the streamwise component of the wall-shear stress vector whereas the rms contribution from the transverse velocity and its fluctuating shear stress should be taken into account as well. A similar observational difficulty arises with TKE because the transverse velocity component ($v$) is poorly observable near a wall.

Now we continue with a correction on the hydrodynamic input such as TKE or $C'_f$. For the turbulent boundary-layer flow along a flat bed, Figure 3.2.6 presents the expected decrease of $|\rho|^*_f$, normalised by the full wall-shear stress, with increasing ratio $\phi/H$ between bed material with size $\phi$ and water depth $H$ (notice the log-scale for $\phi/H$). The level where TKE contributes mostly to the rms wall pressure is presented by lower graph in Figure 3.2.6. Incidentally, the latter plot as well the one in Figure 3.2.3 and 3.2.4 demonstrates that changing the boundary condition for TKE at the bed in the k-$\varepsilon$ turbulence model is significant only if that boundary condition affects the simulated turbulence properties far from the wall.

The independence of the scaled parameters in Figure 3.2.6 or water depth is demonstrated by making the same plot for a tenfold different water depth. Notice that the model for Figure 3.2.6 has closures for using input from numerical flow simulation with the k-$\varepsilon$ turbulence model, which falls in the class of so-called high-Reynolds (number) turbulence model. That turbulence model does not take into account limiting effects such as truncation at smallest (Kolmogorov) turbulence length scales or the inclusion of viscous sublayers in case of very small particle sizes. Nevertheless, such corrections could be implemented in our model by using turbulence-energy spectra including viscous effects such as the Pao spectrum, see (Uittenbogaard, 1994) and the original references cited therein.

With respect to Figure 3.2.6, we note that Grass’ experiments are in the range $9.10^{-4} \leq \phi/H \leq 2.10^{-3}$ but those of Xingkui and Fontijn (1988) are at $\phi/H = 0.11$. In view of Figure 2, we expect that the rms forces are about twice as large as observed by Xingkui and Fontijn (1988) for a tenfold larger water depth of 3 metre.

For this study on stability of stones with sizes several orders-of magnitude larger than sand, Figure 3.2.6 indicates that the hydrodynamic input to e.g. Grass’ MF not only needs a conversion to other hydrodynamic/turbulence conditions but also a conversion based on the ratio of the turbulence length scales of the flow and the size of the bed material.

Given the previous considerations and assertions, we recommend to pursue the dual approach of theory guiding empirical research as an optimal strategy for achieving improvements in practical mobility functions for bed material near structures.
3.4 Application of the comparative model in numerical simulations

Codes for application of the model

A significant part of the closures of the comparative model is dedicated to apply the information obtained by flow simulations using the k-ε model. The code DELFT3D-Flow of WEL DELFT HYDRAULICS is equipped with several classes of turbulence models including the k-ε model, see e.g. (Uittenbogaard, et al., 1992). This code solves the horizontal momentum equations in 3D using the hydrostatic pressure assumption with a water surface. For simulating the far field behind structures, while including the complicated geometry of a river or estuary, this code has been extended with the option of including thin barriers that partly protrude the flow from the free surface downwards or from the bed upwards. This work has been performed in collaboration with Kajima Research Institute in Japan. Figure 3.3.1 shows examples of DELFT3D-Flow near vertical structures. For Rijkswaterstaat, this code has been applied for simulating the salt-stratified tidal flow through a partly opened Haringvliet barrier.

For the near field behind structures, the applicability of the hydrostatic pressure assumption in DELFT3D-Flow is doubted and then the flow field is simulated preferably with codes using a pressure solver. For the latter purpose, the commercial code CFX is available at WEL DELFT HYDRAULICS, although it is dedicated to industrial flows conditions without a mobile air-water interface.

Some closures in the comparative model refer to flow- and geometry-dependent variances of the three turbulence velocity components. These closures, however, can be avoided by simulating the Reynolds-averaged flow with one of the versions of Reynolds Stress Model closure (RSM), such as implemented in the code CFX available at WEL DELFT HYDRAULICS.

Experience with various RSM simulations show that these simulations are computationally very demanding because at least eleven stiffly-coupled turbulence equations should be solved in conjunction with the equations for the three momentum components as well as the equation for mean pressure (mass conservation). Large Eddy Simulation (LES), however, are computationally less demanding than RSM solvers. The closures in LES, called subgrid models, are dedicated to the unresolved part of small-scale turbulence only. However, for the stability of rockfill bed material just the intermediate and larger length scales as well as slower fluctuations are of importance and these scales may be simulated directly by LES, without further closure assumptions. Notice that LES could apply directly the conversion (3.2.2) based on simulated wall pressure variations rather than using the comparative model of Appendix A.

Recommendations

For practical applications it is recommended to design a dedicated post-processor for reading the output [U(x,y,z);V(x,y,z);k(x,y,z);ε(x,y,z)] of DELFT3D-Flow and CFX for application of the comparative model as well as Grass' model in an off-line post-processing stage. The proposed post-processor then supplies stability parameters along the bed e.g. presented through 2D plots.

Subsequently, for validating the entire model suite of
• Reynolds-averaged hydrodynamics,
• k-ε turbulence model,
• comparative model and
• Grass' model,
we recommend simulating available scour experiments, such as referred to in (Hoffmans and Booij, 1993). Such an in-depth study aims at determining the weakest or most parameter-sensitive link in this model suite with the intent of recommending further research.
References to the main report


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Normalized weighting function for turbulence inducing maximal wall-pressure fluctuations. Water depth 0.3 and 3 [m]; stone size 33 [mm]; log. vel. profile; maximal weight at $z/\varphi=0.18$; for comparison to exp's of Xingkui and Fontijn (1988).
Weighting function for turbulence inducing wall-pressure fluctuations. Water depth 5 [m]; wave length 10 [m]; stone size 0.3 [m]; maximal weight at \( z/h = 1.6 \).
Normalized weighting function $P(z)$ for turb. kin. energy at level $z$ contributing to variance of bed-pressure.

Water depth $5 \text{ [m]}$; stone size 30 and 300 [mm]; logarithmic velocity profile and parabolic mixing length.

$- d_n = 300 \text{ [mm]}$

$- d_n = 30 \text{ [mm]}$
distance to step relative to water depth (-)

\[
\begin{align*}
\star & \quad = \text{variance of drag force (N/m}^2) \\
\times & \quad = \text{variance of lift force (N/m}^2) \\
\bullet & \quad = \text{TKE near the bed (=} (k / u_0^2) * 10^3\text{), with } u_0 = \text{approach flow velocity (m/s)} \\
+ & \quad = \text{rms of bed shear stress over approach kinetic energy: } C_l^1 \text{ (no: to scale)}
\end{align*}
\]

Figure 3.2.5 : Pattern of drag and lift forces, TKE and rms of bed shear stress against distance with backward facing step
Normalized rms wall pressure as a function of stone size. 
Water depth 0.5 and 5 [m]; logarithmic velocity profile 
and parabolic mixing length.
Figure 3.3.1  Vertical cross-section of flow near structures as implemented in DELFT3D-FLOW
Model for wall pressure in turbulent shear flows dedicated to estimating stability of coarse-grained non-cohesive bed materials

A.1 Formulation of the problem

For the purpose of modelling hydrodynamic forces on walls in free-surface flows, this section introduces some properties of pressure fluctuations induced by internal fluid motion but excluding surface waves in water flow with a free surface as well as the influence of density stratification and the Earth’s rotation.

In view of experiments with observations of turbulence and wall pressure in a closed channel i.e. with a top plat, this appendix is mainly devoted that particular flow case. However, we have available integral expressions for the wall pressure as a function of velocity in case of a turbulence flow with surface waves.

Consider the Navier-Stokes equations for non-stratified flows without the Coriolis force and consider the incompressibility condition:

$$\frac{\partial u}{\partial t} + u \cdot \nabla u + \frac{1}{\rho} \nabla p + \nu \nabla^2 u - g \cdot \nabla = 0 \quad ; \nabla \cdot u = 0.$$  \hspace{1cm} (A.1.1)

Using in (A.1.1) the vector property

$$u \cdot \nabla u = \nabla \left( \frac{1}{2} (u \cdot u) + u \cdot \omega \right) \quad ; \quad \omega = \nabla \times u,$$

where the last expression is the definition of vorticity, yields:

$$\frac{\partial u}{\partial t} + u \cdot \omega + \nabla \left[ \frac{1}{\rho} p - g Z + \frac{1}{2} (u \cdot u) \right] + \nu \nabla^2 u = 0.$$  \hspace{1cm} (A.1.3)

In (A.1.3), $Z$ is the vertical distance between the observation point and some constant pressure level such as the water surface; the contribution of atmospheric pressure to (A.1.3) has been discarded. Taking the divergence of (A.1.3) and using the incompressibility condition $\nabla \cdot u = 0$ yields one of the forms of the Poisson equation for pressure:

$$\nabla^2 \left\{ \frac{1}{\rho} p - g Z + \frac{1}{2} (u \cdot u) \right\} = \nabla \cdot (u \cdot \omega).$$  \hspace{1cm} (A.1.4)

The formal solution of (A.1.4) reads:

$$\left\{ \frac{1}{\rho} p - g Z + \frac{1}{2} (u \cdot u) \right\}_x = \frac{1}{4 \pi} \int \frac{\nabla \cdot (u \cdot \omega)}{|x-r|} dV(r).$$  \hspace{1cm} (A.1.5)
where the volume integral extends over the entire fluid volume. Despite its formal nature, (A.1.5) reveals the following interesting properties:
1. pressure is not necessarily a local property,
2. just rotational motions play a role in the non-local influence on pressure and
3. just rotational motions determine the wall-pressure fluctuations, provided \( u' = 0 \) holds on the wall.

Here and in the following, we exclude the influence of surface waves on pressure i.e. \( Z \) is time independent.

The first conclusion is obvious from the volume integral in (A.1.5) which, in addition, demonstrates that the influence on pressure decays inversely proportional to the distance between pressure source and point of pressure observation.

The second conclusion is due to the essential existence of vorticity in the volume integral. If the flow is irrotational then just the local kinetic energy is essential and Bernoulli’s equation for pressure holds. The force (per unit mass)

\[
T = u' \omega
\]

(A.1.6)

is called Lighthill’s vortex force and its ensemble-average is essentially due to turbulent motions and due to internal waves but the latter are excluded by the neglect of density stratification in (A.1.1). The inverse of the second conclusion is that the irrotational part of turbulent motions do not induce pressure fluctuations elsewhere and Townsend (1976) therefore defines the irrotational part of turbulent motions as the “inactive” part of turbulence.

Finally, the third conclusion is of importance for this study about hydrodynamic forces on walls. The no-slip condition implies zero velocity on the wall and then the fluctuation \( p' \) of wall pressure, defined with zero mean, follows from (A.1.5):

\[
\bar{x} \in \text{wall}: \quad p'_w = \frac{\rho}{4\pi} \int \frac{\nabla \cdot T'(r)}{|x - r|} \, dV(r)
\]

(A.1.7)

showing that wall-pressure fluctuations are weighted averages of the fluctuating part of Lighthill’s vortex force

\[
T = U \ast \omega' + u' \ast \Omega + u' \ast \omega' - u \ast \omega
\]

(A.1.8)

where capital symbols are ensemble-averaged, or time-averaged in steady flows, variables. In terms of order-of-magnitudes where turbulent motions are a fraction of mean properties, (A.1.8) indicates that its last term is the least important contribution to the wall pressure. Not only the fluctuating wall pressure is of importance but also the near-wall velocity as well as its fluctuations exert drag and lift forces on objects protruding the mean bed level. To first order, these near-wall velocity fluctuations follow from

\[
\frac{\partial u'}{\partial t} + U \cdot \nabla u' + \frac{1}{\rho} \nabla p' \approx 0
\]

(A.1.9)

where the local pressure gradient drives the fluctuations. Howe (1992, eq. (A.2.8)) uses the concept (A.1.9) for the purpose of deriving spectra for wall-pressure fluctuations.
The previous general but formal approach suggests the following integral dependency of the rms $|\sigma_r|$ of any hydrodynamic force component on walls:

$$x \in \text{wall} : \quad |\sigma_r| \propto \rho \int W_1(r-x) U(r) dV(r) + \rho \int W_2(r-x) |u'(r)|^2 dV(r) \quad (A.1.10)$$

with $W_{1,2}$ appropriate weighting functions which are expected to decay with increasing distance to the wall point under consideration. Notice that the expression (A.1.10) weights all pressure sources over the entire fluid volume and that the mean velocity, remote from the wall, may play a role which will be investigated in the following sections.

For steady turbulent flows in channels without significant curvature and without significant cross-sectional changes, the integrals in (A.1.10) could be simplified to

$$x \in \text{wall} : \quad |\sigma_r| \propto \rho u^2 \quad (A.1.11)$$

with $u$ the wall shear-stress velocity, usually derived from the mean near-wall velocity. The previous general approach, however, demonstrates that (A.1.11) is a simplification to (A.1.10). This simplification may break down in case of more complex flows and notably in and near wall-stagnation points having zero wall-shear stress.

The latter conclusion is the problem formulation of this appendix and the aim of this appendix is replacing (A.1.11) by the more general form (A.1.10).

The next section is devoted to specifying the functional form of the most significant contribution to (A.1.10).

### A.2 Simplified equation for wall-pressure fluctuations

Given the problem formulated in the previous introduction, this section deals with deriving an explicit expression for the pressure fluctuations on walls as function of mean and turbulence flow variables. This derivation aims at estimating the most significant contribution to the general formula (A.1.10). Further, this derivation aims at non-uniform turbulent flows where the vertical flow structure dominates but where locally the wall-shear stress velocity may be zero or negligible.

Rather than starting with (A.1.10), another form of the Poisson equation for pressure is taken. The following equation is due to taking the divergence of (A.1.1) directly and considering just its fluctuation part:

$$\nabla^2 p' = -\rho \frac{\partial^2 U_i u'_j + U_j u'_i + u'_i u'_j - \langle u'_i u'_j \rangle}{\partial x_i x_j} ; \quad i, j = x, y, z \quad (A.2.1)$$

with mean-flow variables in capitals, turbulence fluctuations with zero mean have $(..)'$ and $<..>$ implies ensemble averaging of the terms between brackets.

Here turbulent flows are considered with a dominant vertical flow structure as well as with relatively low turbulence levels $|u'|/\bar{U} << 1$, where $\bar{U}$ is the depth-averaged velocity. In these
cases, the product of turbulence velocity components is neglected against the remaining terms in (A.2.1). Validation of this approximation is given at the end of this section.

Further, the x-direction is considered as mean flow direction so that $\partial U / \partial z$ is the single non-zero mean velocity gradient. These assumptions reduce (A.2.1):

$$\frac{1}{\rho} \nabla^2 p' = -2 \frac{\partial U}{\partial z} \frac{\partial w'}{\partial x} + \text{H.O.T.},$$

where the higher-order terms contain the product of turbulent motions as well as the contribution

$$-2 \frac{\partial V}{\partial z} \frac{\partial w'}{\partial y}$$

due to veering of the mean horizontal velocity vector. In the following derivation, the inclusion of (A.2.3) is straightforward but, for maintaining simplicity in solving, it is discarded first.

Of course, the essential point is whether (A.2.2) is indeed a fair approximation to wall-pressure fluctuations. Bull (1967) made detailed observations of wall-pressure fluctuations and presents spatial and temporal correlation's in a turbulent boundary layer along a smooth wall. Bull (1967) confirms that (A.2.2), proposed first by Kraichnan (1957), is indeed valid for explaining the observed patterns and correlation of pressure fluctuations on the wall.

The next section considers solutions to (A.2.2) in terms of Fourier spectra of turbulence.

### A.3 Solution for wall-pressure fluctuations in a turbulent shear flow with water surface

The general Poisson equation (A.2.1) for pressure is expressed now as

$$\nabla^2 p' = Q'(r, t)$$

with $Q'$ representing all RHS terms of (A.2.1) and $r$ is a position vector in the fluid, of course, $Q'=0$ outside the fluid domain.

For a viscous-turbulent flow with a free water surface, the boundary conditions to (A.3.1) read

on rigid walls: $n \cdot \nabla p' = 0$ ; \hspace{1em} at free surface: $p' = 0$ ,

with $n$ the normal to the wall. The first boundary condition is due to the no-slip condition on walls, see e.g. (Kraichnan, 1956). The second boundary condition can be replaced by others in case of a rigid lid (zero normal velocity) or mobile free surface (coupling with surface waves). For the latter we present the well-known dispersion relation for infinitesimal waves

$$\left. \frac{\partial p}{\partial z} \right|_h = -\rho \frac{D w}{D t} = -\rho \frac{D^2 \zeta}{D t^2} = -\frac{1}{g} \frac{D^2 p}{D t^2} \right|_h \quad \text{with} \quad \frac{D \zeta}{D t} = w \quad \text{and} \quad p(h) = \rho g \zeta .$$

The dispersion relation (A.3.2b) couples the vertical pressure gradient to its second-order material derivative at the mean level $z=h$ of the free surface. The condition (A.3.2b) yields the
combined solution of pressure by turbulence and surface waves but in view of this appendix we shall not present it here.

Instead, the following combined spatial as well as temporal Fourier-transform, dedicated to (A.3.1), is considered:

\[
p(k,\omega;z) = \frac{1}{(2\pi)^3} \int \int p(r,t) \exp\{-i(k \cdot r + \omega t)\} \, dr \, dt \quad ; \quad k = (k_x, k_y, 0) \tag{A.3.3}
\]

with \(k_x\) and \(k_y\) the orthogonal components of the horizontal wave number vector \(k\) and \(\omega\) the angular frequency. Notice that here and in the following Fourier-transformed variables have no superscript.

Formally, the Fourier-transform (A.3.3) should be a Fourier-Stieltjes transform for dealing with random variables in an infinitely long fluid volume but this is a superfluous technicality when considering the variance of (A.3.3). A well-known alternative for still using Fourier transforms is assuming a finite box of fluid along the bed with its minimal length exceeding the relevant streamwise turbulence scales with periodic continuation of the turbulence field outside this box.

The application of the transform (A.3.3) to (A.3.1) yields the following ordinary differential equation in terms of distance \(z\) normal to the horizontal bed:

\[
\frac{d^2 p}{dz^2} - k^2 p = Q(k,\omega;z) \quad ; \quad k = |k| = (k_x^2 + k_y^2)^{1/2} \tag{A.3.4a}
\]

with boundary conditions

\[
z = 0: \quad \frac{dp}{dz} = 0 \quad ; \quad z \geq h: \quad p = 0 \tag{A.3.4b}
\]

as derived from (A.3.2a) with water depth \(h\). Equation (A.3.1) is solved by considering the Laplace transform with respect to the vertical \(z\) co-ordinate:

\[
\tilde{p}(s) = \tilde{p}(k,\omega;s) = \int_0^\infty p(k,\omega;z) e^{-sz} \, dz \tag{A.3.5}
\]

Application of (A.3.5) to (A.3.4) and using the bed condition for pressure, yields for the Laplace-transform of \(p\):

\[
\tilde{p}(s) = \frac{\tilde{Q}}{s^2 - k^2} + \frac{s \tilde{p}(0)}{s^2 - k^2}
\]

whence the inverse transform of the Fourier transformed pressure as function of vertical \(z\)-co-ordinate reads:

\[
p(k,\omega;z) = \int_0^z k^{-1}Q(k,\omega;r)\sinh[k(z-r)] \, dr + p(k,\omega;z = 0)\cosh(kz) \tag{A.3.6}
\]
In (A.3.6), \( p(k, \omega; z = 0) \) is the Fourier transform of the wall pressure. The conditions for zero pressure fluctuations at the free surface demands \( p(k, \omega; z = h) = 0 \) and then from (A.3.6) follows for the Fourier transform of the wall pressure:

\[
p(k, \omega; 0) = -k^{-1} \int_0^h Q(k, \omega; r) W[k(h-r)] \, dr \quad ; \quad W[k(h-r)] = \frac{\sinh[k(h-r)]}{\cosh(kh)} . \quad \text{(A.3.7a)}
\]

In (A.3.7a), \( W \) weights the turbulence-induced perturbations in \( Q \) along the water depth and this is the mathematical formulation supporting the assessment (A.1.10). Notice that \( W=0 \) holds for turbulence perturbations at the free surface \( (r=h) \). As explained in connection to (A.3.2b) or the second condition in (A.3.2a), other options for closing (A.3.6) are possible e.g. the condition of zero vertical velocity at \( z=h \) yields \( \sinh(kh) \) rather than \( \cosh(kh) \) in (A.3.7a).

For a better insight, the following approximation to the weighting function in (A.3.7a) is of interest:

\[
\lim_{kh \to \infty} W[k(h-r)] = e^{-kr}, \quad \text{(A.3.7b)}
\]

showing for infinite water depth or, alternatively, sufficiently-small wavelengths compared to water depth, the exponentially-decaying influence of turbulent fluctuations contributing to the wall pressure.

A confirmation of (A.3.6) and (A.3.7a) is found in (Ffowcs Williams, 1982) who formulated the much more involved problem of wall-pressure fluctuations due to compressible turbulent flows. The expression (A.3.7b) follows from (Ffowcs Williams, 1982, eq. 2.18) for subsonic perturbations and particularly for vanishing ratio \( \omega / (ck) \), with \( c \) the speed of sound in the fluid.

The ratio \( \omega / k \) is the phase speed or convective velocity of the perturbation considered. As quantified by Bull (1967), the most energetic pressure perturbation travel at phase speeds near or below the boundary-layer averaged velocity. The latter substantiates G.I. Taylor’s hypothesis of frozen turbulence. Consequently, \( \omega / (ck) \) is negligible for turbulence in river flows and consequently (A.3.7) is substantiated by Ffowcs Williams (1982).

The formulation (A.3.7) is still based on the general form for \( Q \), being the RHS of (A.3.1), but the purpose of this section, however, is deriving a more specific formulation of (A.3.7) dedicated to turbulent shear flows. Therefore, the simplified version (A.2.2) with

\[
Q(x, t) = -2\rho \left. \frac{\partial U}{\partial z} \frac{\partial w}{\partial x} \right|_z
\]

is considered hereafter. The Fourier transform (A.3.3) of this simplified \( Q' \) reads

\[
Q(k, \omega; r) = -2i\rho k_x \frac{dU(r)}{dr} w(k, \omega; r) \quad \text{(A.3.9)}
\]

and it is substituted into (A.3.7a)
\[
\frac{p(k, \omega; 0)}{2 \rho k_z} = \frac{i k}{k_0} \mathcal{W}[k(h-r)] \frac{dU(r)}{dr} w(k, \omega; r) dr .
\] (A.3.10)

Integrating (A.3.10) by parts yields

\[
\frac{p(k, \omega; 0)}{2 \rho} = \frac{i k}{k_0} \int U(r) \left\{ \mathcal{W}[k(h-r)] \frac{dw(k, \omega; r)}{dr} + w(k, \omega; r) \frac{dW[k(h-r)]}{dr} \right\} dr ,
\] (A.3.11)

after using the no-slip boundary condition \( U(0) = 0 \) on the bed as well as \( W = 0 \) at the immobile surface \( r = h \), see (A.3.6). With reference to the second boundary condition in (A.3.2a), we have chosen with \( W = 0 \) at \( r = h \) another boundary condition. Further analysis, not shown in this appendix, shows that the solution is not based on \( \cosh(kh) \)-functions but \( \sinh(kh) \)-functions and hardly affects the quantitative results. Analytical expressions for the pressure due to turbulence and surface waves (mobile free surface) can be derived with (A.3.2b) but these alternatives are not presented here.

The integrand of (A.3.11) can be converted further by using the Fourier transform of the incompressibility condition \( \nabla \cdot \mathbf{u} = 0 \) which yields

\[
\frac{dw(k, \omega; r)}{dr} = ik \cdot \mathbf{u}(k, \omega; r) .
\]

Notice that just the horizontal velocity components appear in the RHS because the wave number vector is defined to be horizontal, see (A.3.3). Substituting the transformed incompressibility condition into (A.3.11) yields:

\[
p(k, \omega; 0) = 2 \rho \frac{k_z}{k} \int U(r) \left\{ i w(k, \omega; r) \frac{dW[k(h-r)]}{dr} - k \cdot \mathbf{u}(k, \omega; r) \mathcal{W}[k(h-r)] \right\} dr .
\] (A.3.12a)

As announced previously, the simple extension of (A.3.12a) using (A.3.2a) for a veering mean horizontal velocity vector \( \mathbf{U}(z) \) reads:

\[
p(k, \omega; 0) = 2 \rho \int \frac{k \cdot U(r)}{k} \left\{ i w(k, \omega; r) \frac{dW[k(h-r)]}{dr} - k \cdot \mathbf{u}(k, \omega; r) \mathcal{W}[k(h-r)] \right\} dr .
\] (A.3.12b)

Using (A.3.12) several important inferences can be made and this is the subject of the next section A.4. Further, from (A.3.12) a fundamental estimator for the rms of wall-pressure fluctuations can be derived and this is the subject of Section A.5.

### A.4 Determination of \( z \)-levels of sources of wall-pressure fluctuations

The expression (A.3.12) allows an inference about the \( z \)-levels of what we call the direct as well as indirect sources that induce wall-pressure fluctuations. With an energy cascade process in mind, the indirect sources are larger-scale and slower turbulent motions that drive the smaller-scale and faster direct sources and the latter are considered to invoke the wall-pressure
Fluctuations as well as the near-wall velocity fluctuations that induce motion of mobile bed material.

For the purpose of determination of these $z$-levels, equation (A.3.12a) for a mean horizontal flow in a vertical plane is considered in this section. The direct turbulence source of (A.3.12a) are represented by the terms between the curled brackets. The $z$-level with maximal contribution to the wall-pressure fluctuations in a steady channel flow with a horizontal bed as well as constant cross section is derived by the following steps:
1. estimate the central wave-number magnitude that induce motion of mobile bed material and
2. determine the $z$-level of maximal weight of the direct source of wall pressure fluctuations.

The estimation of the magnitude of the central wave-number of wall-pressure fluctuations that create the largest forces on single elements of mobile bed material must be based on a conceptual mechanical model for the forces on a single bed particle. The following model is adopted.

**Conceptual mechanical model**

Wall pressure fluctuations at nominal wave lengths of twice the typical size of mobile bed material create the largest pressure differences between subsequent bed elements. These pressure differences induce the largest pressure-related lift force on a single element by the flow induced in between and below the upper bed elements (see Figure 3.2.1). In addition, the local velocity induces additional lift forces and is even essential for the drag forces. The local velocity fluctuations are considered to be driven by pressure gradients through:

$$
\rho \frac{\partial u}{\partial t} + \rho \mathbf{u} \cdot \nabla u + \nabla p' \approx 0
$$

so that near-wall pressure fluctuations are considered as the principle source of forces on sufficiently coarse-grained bed material.

Following this conceptual model, the dominant wave number magnitude of wall-pressure fluctuations that contribute to lifting mobile bed material of diameter $d_n$ reads:

$$
k_p = \frac{2\pi}{\lambda_p} \quad \text{with} \quad \lambda_p = 2d_n \quad \text{(A.4.1)}
$$

Given this wave number magnitude, the integrand of (A.3.12a) can be simplified further by noting that for sufficiently deep water:

$$
k h \gg 1: \quad \frac{d W[k(h-r)]}{d r} \approx -k W[k(h-r)] \quad \text{(A.4.2)}
$$

For river flows and tidal flows over sills the condition $k_p h \gg 1$ is easily satisfied and then (A.3.12a) simplifies to:

$$
k h \gg 1: \quad \rho(k, \omega; 0) \approx -2 \rho \frac{k_z}{k} \int_0^h U(r) W[k(h-r)] \left\{ k \cdot \mathbf{u}(k, \omega; r) + i k w(k, \omega; r) \right\} d r \quad \text{(A.4.3)}
$$

This expression is adequate for studying small-scale wall pressure fluctuations in water with depths at least one-order-of-magnitude larger than the bed material size. Notice that the approximation (A.4.2) refers to the convolution of the vertical velocity component while the
convolution of the significantly larger horizontal velocity components is correct for all wave
lengths.

Further, for high-Reynolds-number turbulent flow over a sufficiently rough bed, the mean
velocity profile is logarithmic and it satisfies:

\[
U(r) = \frac{\nu_e}{\kappa} \ln \left( 1 + \frac{r}{z_0} \right), \quad \text{(A.4.4a)}
\]

with appropriate roughness length scale \( z_0 \). Van Rijn (1993) relates the roughness length scale \( z_0 \) to the bed material size by:

\[
z_0 = \frac{k_s}{30} \quad \text{with} \quad k_s = 2d_n \quad \text{(A.4.4b)}
\]

with Nikuradse roughness \( k_s \).

The set \( \langle A.4.1 \rangle \) to \( \langle A.4.4 \rangle \) is closed for deriving the z-level of the most important direct source of
wall-pressure fluctuations in a channel with logarithmic velocity profile. This question is
investigated by determining the maximal value of the weight or the transfer function for velocity
to wall-pressure:

\[
P(r) = U(r)W[k(h-r)] \quad \text{with} \quad W[k(h-r)] = \frac{\sinh[k(h-r)]}{\cosh(kh)} . \quad \text{(A.4.5)}
\]

The conceptual model and the experimental observations about roughness length close the
problem of finding the ratio \( r/d_n \) for maximum of weight:

\[
P(r) \propto \ln \left( 1 + \frac{r}{z_0(d_n)} \right) \frac{\sinh[k_p(d_n)(h-r)]}{\cosh[k_p(d_n)h]} . \quad \text{(A.4.6)}
\]

For sufficiently large ratio \( h/d_n \), between water depth and size of bed material, the maximum of
\( \langle A.4.6 \rangle \) reads:

\[
\max P(r) \text{ at } \frac{r}{d_n} \approx 0.18 . \quad \text{(A.4.7)}
\]

The result \( \langle A.4.7 \rangle \) shows that turbulence fluctuations at a level of about 18% of the bed-material
size contribute maximal to the wall-pressure fluctuations. Figure 3.2.2 presents \( \langle A.4.6 \rangle \) for \( d_n=0.3 \)
m in 5 metre deep water, notice the logarithmic horizontal axis. In addition, Figure 3.2.3 shows
\( \langle A.4.6 \rangle \) for a horizontal wave length of twice the water depth. As noted below \( \langle A.4.3 \rangle \),
irrespective of the wave-length magnitude, Figure 3.2.3 represents correctly the weighting
function of horizontal velocity components which are usually the most dominant ones in horizontal channel flows.

Indirectly, \( \langle A.4.2 \rangle \) and the consequence \( \langle A.4.7 \rangle \) of the conceptual model appears to be validated
by experiments in (Xingkui and Fontijn, 1988) with correlation's between forces on a single bed
element and the turbulent velocity observed at various levels above that element. The
experiments of Xingkui and Fontijn (1988) are devoted directly to forces on mobile bed material.
For a more critical validation of (A.4.3), the correlation between wall pressure and e.g. the streamwise velocity component is proposed. Consider therefore (A.4.3) and multiply it with the complex conjugate (indicated by \(c^\ast\)) of the Fourier transform of \(u'\) while neglecting correlation's between \(u'\) and \(v'\) for unidirectional mean flow:

\[
\frac{p(k,\omega;0)u^{c^\ast}(k,\omega;z)}{2\rho k_x/k} \approx - \int_0^h \left[ \frac{U(r)W[k(h-r)]}{k} \right] \{k_x u(k,\omega;r)u^{c^\ast}(k,\omega;z)\} dr
\]

\[
- i \int_0^h \left[ \frac{U(r)W[k(h-r)]}{k} \right] \{k u^{c^\ast}(k,\omega;z)w(k,\omega;r)\} dr
\]

(A.4.8)

Ensemble averaging the integrands of (A.4.8) would include convolution over correlation's between Fourier amplitudes at different levels \(z\) and \(r\). The usual approach is simplifying this convolution by assuming a very sharp spatial correlation proportional to \(\delta(z-r)\) with Dirac's delta function \(\delta\). In addition, define the auto or cross spectrum between two scalar variables by

\[
G_{ab}(k,\omega;z) \overset{def}{=} \langle a(k,\omega;z) b^{c^\ast}(k,\omega;z) \rangle
\]

(A.4.9)

where the brackets represents ensemble averages or averages of the product between individual complex amplitude spectra. Consequently, (A.4.8) yields the approximation

\[
kh > > 1: \left\langle p(k,\omega;0)u^{c^\ast}(k,\omega;z) \right\rangle \approx -2\rho \frac{k_x}{k} U(z)W[k(h-z)] \{k_x G_{uu}(k,\omega;z) + ik G_{uw}(k,\omega;z)\}
\]

(A.4.10)

which is based on observable properties and thus suitable for experimental validation. This validation is recommended for future laboratory experiments or numerically by so-called Direct Numerical Simulations or possibly Large Eddy Simulations if the scales of interest, such as the size of bed mobile bed material, exceed significantly a representative Kolmogorov length scale.

### A.5 Closures and comparative model for wall-pressure fluctuations

In this section a model is presented for mutually comparing wall-pressure fluctuations induced by flows with different vertical distribution of turbulence properties and mean flow profiles. This model is dedicated to input from 2DV or 3D solutions of the Reynolds-averaged equations and using the \(k-\varepsilon\) model as turbulence closure using an isotropic eddy viscosity for estimating anisotropic components of the Reynolds stress tensor. The derivation of the model, however, demonstrates that other turbulence closures or observations of turbulence velocity properties may serve as input to the model.

The essential remark is that, until further substantiation, the presented model is proposed to be used for mutual comparison rather than as absolute estimator for the variance of wall-pressure fluctuations. This comparative model aims at converting estimators for wall stresses, which were designed for assessing the stability of mobile bed material for turbulent flows in uniform channels such as e.g. (Grass, 1970), to other and more complicated flow conditions that may be simulated numerically.
The formal starting point is (A.3.12b) which is repeated below for convenience:

\[
p(k,\omega;0) = 2\rho \int_0^h \frac{k \cdot U(r)}{k} \left\{ iw(k,\omega;r) \frac{dW[k(h-r)]}{dr} - k \cdot u(k,\omega;r)W[k(h-r)] \right\} dr
\]  
(A.5.1)

and from (A.3.7a): \( W[k(h-r)] = \frac{\sinh[k(h-r)]}{\cosh(\alpha h)} \).

For reducing notational complexity of the integrand, the following functions are defined:

\[
\hat{k} = \frac{k}{k} ; \quad W_r[k(h-r)] = \frac{1}{k} \frac{dW[k(h-r)]}{dr}
\]  
(A.5.2)

thereby reducing the notation of (A.5.1) into

\[
p(k,\omega;0) = -2\rho \int_0^h \hat{k} \cdot U(r) \left\{ \hat{k} \cdot u(k,\omega;r)W[k(h-r)] + i k w(k,\omega;r)W_r[k(h-r)] \right\} dr
\]  
(A.5.3)

From (A.5.3), as well as approximations to be presented, an estimator for the variance of wall pressure is derived. This estimator is the comparative model looked for. Multiplication of (A.5.3) with its complex conjugate yields:

\[
G_{pp}(k,\omega;0) = 4\rho^2 \int_0^h \int_0^h \left\{ \hat{k} \cdot U(r) \hat{k} \cdot U(r') \right\} \left\{ W[k(h-r)]W[k(h-r')] + k \cdot u(k,\omega;r)k \cdot u^*(k,\omega;r') \right\} dr dr' +
+ 4\rho^2 \int_0^h \int_0^h \left\{ \hat{k} \cdot U(r) \hat{k} \cdot U(r') \right\} \left\{ W_r[k(h-r)]W_r[k(h-r')] + k^2 w(k,\omega;r)w^*(k,\omega;r') \right\} dr dr' +
+ \text{cross-spectral contributions}
\]  
(A.5.4)

As indicated by the last remark in (A.5.3), the contributions of cross spectra to the pressure variance are neglected for the time being. It can be shown that the cross-spectral contributions are proportional to quadrature spectra (imaginary part of cross spectra (A.4.9)) of the various velocity correlation's. Usually, the quadrature spectra of velocity correlation's due to turbulence are much smaller than the coherence-spectra (real part of cross spectra (A.4.9)). Quadrature spectra of velocity components, however, become significant for wave-like motions such as free surface waves, of course, but also shear instabilities and vortex shedding behind obstacles. For the latter flow cases the cross-spectral contributions in (A.5.4) may be significant but in view of a first presentation in this appendix we first want to present the main lines of the closure for rms wall pressure.

The double integral along the water depth contains products of Fourier amplitude of the three velocity components at different levels \( r \) and \( r' \). As an intermediate step we introduce formally the spectral integral length scale for a scalar turbulence variable \( \alpha \) in vertical direction through
which converts (A.5.4) approximately into integrals contains autospectra of each turbulence velocity component:

\[
G_{pp}(k,\omega;0) \approx 4 \rho^2 \int_0^h \left[ \mathbf{k} \cdot \mathbf{U}(r) \mathcal{W}[\mathbf{k}(h-r)] \right]^2 \left\{ k_x^2 G_{uu}(k,\omega;\mathbf{r}) \ell_{u,x}(k,\omega;\mathbf{r}) + k_y^2 G_{uv}(k,\omega;\mathbf{r}) \ell_{u,y}(k,\omega;\mathbf{r}) + k_z^2 G_{vw}(k,\omega;\mathbf{r}) \ell_{v,z}(k,\omega;\mathbf{r}) \right\} \, dr + 4 \rho^2 \int_0^h \left[ \mathbf{k} \cdot \mathbf{U}(r) \mathcal{W}[\mathbf{k}(h-r)] \right]^2 \left\{ k_x^2 G_{ww}(k,\omega;\mathbf{r}) \ell_{w,x}(k,\omega;\mathbf{r}) \right\} \, dr
\]

(A.5.6)

The approximation made by converting (A.5.3) into (A.5.6) is due to the assumption of sufficiently-small vertical integral-length scales \( \ell_{a,z}(r) \) so that the weighing functions may be taken constant over the vertical interval in which the integrand in (A.5.5) is significant.

The next step is integrating all spectral energy of \( G_{pp}(k,\omega;0) \) over all horizontal wave number components as well as over all angular frequencies \( \omega \) that contribute to the mobility of the bed material. The latter remark implies that contributions from sufficiently large \( k \) and \( \omega \) should be excluded. The latter truncation should be substantiated by estimating transfer functions \( H(k,\omega) \) for the response of the velocity of a single bed element to the imposed pressure and velocity field at given wave number magnitude \( k \) and angular frequency \( \omega \). For the time being, we propose a simple truncation for wave number magnitudes exceeding a given wave number \( k_p \) which is expected to be larger than the dominant wave number \( k_p \) given by (A.4.1) which was based on a wavelength of twice the typical size of the mobile bed material under consideration.

The final conversion of (A.5.6) into variance of wall pressure is the integration of (A.5.6) over all wave number \( k \leq k_t \) and all angular frequencies. For given spectral shapes such integration could be performed but it is rather cumbersome and, instead, the following closure is introduced. At each vertical level \( r \), the auto spectra \( G_{aa} \) are assumed to be centred at a dominant (horizontal) wave number and, in addition, weighted by function \( W \) and \( W_t \) based on that dominant wave number. The latter procedure is based on analysis summarised in (Uittenbogaard, 1994. Section 3.21). This analysis shows that for approximating integrals of turbulence spectra over horizontal wave numbers, the dominant wave number should be:

\[
k = \Lambda^{-1}_{a,h}(z) \quad \text{with} \quad \Lambda^2_{a,h}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \alpha'(x,y,z) \alpha'(x+\tau,y+\tau',z) \rangle \, d\tau \, d\tau'
\]

(A.5.7a)

where \( \Lambda_{a,h} \) is the integral length scale related to the spatial correlation, in \( x \)- and \( y \)-direction, of turbulence variable \( \alpha \).

After integration over all wave numbers and angular frequencies, the spectral integral-length scale (A.5.5) is replaced by the following spatial integral length scale for correlation's along the water depth:
\[
\Lambda_{x,t}(z) = \frac{\int_{-\infty}^{\infty} \langle \alpha'(x,y,z) \alpha'(x+\tau,y,z+\tau') \rangle \, d\tau}{\langle \alpha'^2(x,y,z) \rangle} . \tag{A.5.7b}
\]

If \( \alpha = u,v \) then equation (A.5.7b) is the transverse integral length scale but for \( \alpha = w \) it is the longitudinal length scale and conversely for (A.5.7a).

The closures (A.5.7) and the simplification of centring auto-spectra at a single dominant wave number, defined by (A.5.7a), convert (A.5.6) into

\[
\langle \rho^2(0) \rangle = \rho^2 \int_0^h \left[ \frac{W^2[\Lambda_{u,u}(r)(h-r)]}{\Lambda_{u,u}(r)} \langle u^2(r) \rangle \Lambda_{u,z}(r) \right] \, dr + \rho^2 \int_0^h \left[ \frac{W^2[\Lambda_{v,v}(r)(h-r)]}{\Lambda_{v,v}(r)} \langle v^2(r) \rangle \Lambda_{v,z}(r) \right] \, dr
\]

\[
+ \rho^2 \int_0^h \left[ \frac{W^2[\Lambda_{w,w}(r)(h-r)]}{\Lambda_{w,w}(r)} \langle w^2(r) \rangle \Lambda_{w,z}(r) \right] \, dr \tag{A.5.8}
\]

The result (A.5.8) is the comparative model suitable for input from simulations using turbulence closures or from experimental observations. Although it is meant as comparative model, attempts are made for maintaining the correct order-of-magnitudes such as the notion that integrating \( (\mathbf{k} \cdot \mathbf{U}(r))^2 \) over all horizontal wave number components yields \( \frac{1}{2} \left| \mathbf{U}(r) \right|^2 \) approximately while noting that \( \mathbf{k} \) has unit length, see (A.5.2). Similarly, for obtaining (A.5.8), an additional approximate factor \( \frac{1}{2} \) has been applied due to averaging \( k^2 = k^2 \cos \theta \) and \( k^2 = k^2 \sin \theta \) over all angles \( \Theta \) of vector \( \mathbf{k} \) in the horizontal plane. Obviously, deviations are expected due to anisotropy of the horizontal wave number spectra of each turbulence velocity component.

Despite all simplifications made, (A.5.8) still requires more detailed input than can be derived directly from simulations using the k-\( \epsilon \) turbulence model because it predicts mostly the isotropic part of turbulence properties. For using input from the k-\( \epsilon \) model, additional assumptions, based on experimental observations, must be introduced.

Below some provisional closures are presented but we recommend more study based on e.g. (Kaimal et al., 1972) and (Kaimal, 1973).

For avoiding confusion, the symbol \( k \) for Turbulent Kinetic Energy (TKE) is replaced by \( q^2 \) i.e.

\[
q^2(r) = \frac{1}{2} \left\{ \langle u^2((r)) \rangle + \langle v^2((r)) \rangle + \langle w^2((r)) \rangle \right\} \tag{A.5.9a}
\]

The overview (Townsend, 1976) of experiments in the high-Reynolds number turbulent part of boundary flows suggest \( z \)-independent ratios \( c_\alpha \) between velocity variance and TKE:

\[
c_\alpha = \frac{\langle \alpha'^2((r)) \rangle}{q^2(r)} ; \quad \alpha = u,v,w. \tag{A.5.9b}
\]
For boundary-layer flows, the k-\(\varepsilon\) turbulence model is dedicated to modelling the distribution of the eddy viscosity component related to the dominant shear rate. For shallow-water flows this shear rate is \(\partial U / \partial z\), with U in streamwise direction, and the corresponding vertical eddy viscosity profile is parabolic, see e.g. (Nezu and Rodi, 1986). The mixing length closure underlying the k-\(\varepsilon\) model yields for the vertical integral length scales the connection:

\[
\Lambda_{\alpha, z} = c_{\alpha, z} \frac{q^3}{\varepsilon}
\]  

(A.5.9c)

with vanishing vertical length scales near the water surface as well as near the bed. The problem with closures for the horizontal integral length scales \(\Lambda_{\alpha, h}\) is that for the horizontal velocity components these scales do not vanish near the water surface.

Uittenbogaard (1994, p. 3-37) summarises for boundary layer flows:

\[
c_u \approx c_{\mu}^{1/2} ; c_{\mu} = 0.09 ; c_u \approx 1.2 ; c_v \approx 0.6 ; c_w \approx 0.2
\]  

(A.5.9d)

\[
\Lambda_{u, h}(z) \approx 3z ; \Lambda_{v, h}(z) \approx 0.9z ; \Lambda_{w, h}(z) \approx 0.03z
\]  

(A.5.9e)

where the first expression is presented in (Uittenbogaard et al., 1992) with the well-known equilibrium coefficient \(c_u\) used in the k-\(\varepsilon\) turbulence model.

Finally, all previous closures yield the following weighting of TKE over the water depth which is the estimator for the variance of wall pressure applicable in conjunction with the k-\(\varepsilon\) turbulence model:

\[
\langle p^2(0) \rangle \approx \rho^2 \int_0^h P(r) q^2(r) \, dr
\]  

(A.5.10a)

with

\[
P(r) = \left[ P(r) \right] \left[ \frac{W_t \Lambda_{u, z}(r)(h-r)}{\Lambda_{u, h}(r)} + c_u \Lambda_{v, z}(r) \left( \frac{W_t \Lambda_{v, h}(r)(h-r)}{\Lambda_{v, h}(r)} \right) + c_w \Lambda_{w, z}(r) \left( \frac{W_t \Lambda_{w, h}(r)(h-r)}{\Lambda_{w, h}(r)} \right) \right]
\]  

(A.5.10b)

and

\[
W(kh) = H(k - k) \frac{\sinh(k \eta)}{\cosh(k \eta)} ; \quad W_t(k \eta) = H(k - k) \frac{\cosh(k \eta)}{\cosh(k \eta)}
\]  

(A.5.10c)

Figure 3.2.4 presents (A.5.10b) using the logarithmic velocity profile (A.4.4a+b) with corresponding parabolic mixing length 0.4r/(1-r/h) for shallow channel flows with an air-water interface. In addition, for the truncating wave length \(k_i\) in (A.5.10c), the size of the mobile bed material is assumed. This figure clearly demonstrates the influence of turbulence remote from the wall as well as the marginal influence of the size of mobile bed material on the form of \(P(r)\).

For steady boundary layer flow over a rough flat bed, the mean velocity profile satisfies (C.4.4a) and then the dependence on physical dimensions of (A.5.10) is clarified by introducing the scaling with wall shear stress divided by density i.e.
\( 
\langle \cdot \rangle_h = \frac{\langle \cdot \rangle}{u^*} 
\)  

(A.5.11)

This scaling, when applicable, yields the dimensionless variance of wall pressure

\[
|p_w|^2 \xlongequal{\text{def}} \frac{\langle p^2(0) \rangle}{\rho u_*^4} = \frac{1}{0} P_*(\mu) q_*^2(\mu) d\mu \quad \text{with} \quad \mu = \frac{r}{h}
\]

(A.5.12a)

and weighting function \( P_*(\mu) \)

\[
P_*(\mu) = \frac{h [U(\mu)]^2}{u_*^4} \left[ c_w \nabla_{u,z}(\mu) \frac{W_1^{\mu_1}(\mu)}{\Lambda^{\mu_1}_{\mu_2}(\mu)} + c_v \nabla_{v,z}(\mu) \frac{W_2^{\mu_1}(\mu)}{\Lambda^{\mu_1}_{\mu_2}(\mu)} + c_w \nabla_{w,z}(\mu) \frac{W_3^{\mu_1}(\mu)}{\Lambda^{\mu_1}_{\mu_2}(\mu)} \right]
\]

also without physical dimensions. Notice that the integral length scales in (A.5.12b) could be scaled with water depth \( h \). Consequently, for a steady boundary layer flow along a rough flat bed, the rms of pressure fluctuations scale with \( u_*^2 \). Of course, the scaling of the mean velocity profile with \( u_* \) is not applicable at or near a stagnation point which invoked the present modelling strategy.

Although the comparative model (A.5.10) for wall pressure variance is not intended to be absolute, the intriguing question is whether it yields \( |p_w|^2 \) defined as

\[
|p_w|^2 = \left( \frac{1}{0} P_*(\mu) q_*^2(\mu) d\mu \right)^{\frac{1}{2}}
\]

(A.5.13)

at least yields the correct order-of-magnitude with the parameter and length scale settings (A.5.9d+e) for a logarithmic velocity profile in a flow with a water surface. The profile for TKE has been taken as

\[
q^2(z) = u_*^2 \left\{ \frac{z}{h} + 3.3 \left( 1 - \frac{z}{h} \right) \right\}
\]

(A.5.14)

in fair accordance with observations in shallow channel flow (Nezu & Rodi, 1986) as well as with observations in the turbulent boundary layer, including wall pressure recordings, see (Bull, 1967, fig. 4). Numerical integration yields

\[
|p_w|^2 = 2.0 - 3.2
\]

(A.5.15)

with the lowest value for \( d_e = 300 \text{ mm} \) and the largest for \( d_e = 30 \text{ mm} \). The variability of these numbers is due mainly to the magnitude of \( U(z) \) near the bed. This range is in fair agreement with the observations by Bull (1967, fig. 7) who reports \( 2.1 < |p_w|^2 < 2.8 \), with the larger value for the largest boundary-layer Reynolds number. In addition, Bull (1967) collects the range \( 2.1 < |p_w|^2 < 3.8 \) from other sources. Both ranges as well as the uncertainty in the dependence of
\[ |p'| \] on the influence of transducer size as well as Reynolds number suggests not to apply (A.5.13) absolutely but instead to rely on the proposed \textit{comparative model} for converting the variance \( \sigma_{\text{r, simple flow}}^2 \) of bed shear stress to the one for complex flows by

\[
\sigma_{\text{r, complex flow}}^2 = \frac{|p'|_{\text{complex flow}}^2}{|p'|_{\text{simple flow}}^2} \sigma_{\text{r, simple flow}}^2 \tag{A.5.16}
\]

where subscript \textit{simple flow} refers to a high-Reynolds number channel flow with logarithmic velocity profile and the same size of bed material as in the complex flow situation. The LHS of (A.5.16) is the corrected input to the variance in Grass’ model for complex flows including stagnation point(s) on the bed.
References to Appendix A


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