Specialization: Transport Engineering and Logistics

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Title: The impact of routing optimization on the profit of the process for EPS waste collection.

Author: R.O. de Boer

Title (in Dutch): De impact van route optimalisatie op de winst voor het proces van EPS afval inzameling.

Assignment: Master’s thesis

Confidential: no

Initiator (university): Dr. B. Atasoy

Initiator (company): P. Wouters

Supervisor: P. Wouters

Date: May 27, 2019

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Route optimalisation can be a complex task when taking into account a lot of factors. EPS NL is collecting waste EPS to be recycled. This process brings along some specific constraints which aren’t currently available in a general route spatialization problem.

Currently the company is planning their vehicle routes manually. They wonder what the impact on their profits would be if better routes are used.

Your assignment is to determine the characteristics of the EPS routing problem. For this, the different types of vehicles need to be specified as well as their interactions with each other and the nodes to be visited. Then to develop a model to investigate the impact of route optimalisation on the process for EPS waste collection.

The report should comply with the guidelines of the section. Details can be found on the website.

The supervisor,

Dr. B. Atasoy
1. Summary

This report considers the transport processes within EPS NL. EPS NL is a company which specializes in the transport and processing of waste EPS. EPS NL owns several types of vehicles which are used to collect waste EPS at customers and deliver the waste to a recycling plant. In between the collecting and the recycling the waste EPS is processed by EPS NL as well, either in a mobile shredding vehicle or in a shredding plant. Currently the use of vehicles is planned manually but EPS NL wants to increase its profits by improving their routing strategy. The main problem in this report is therefore: What is the impact of route optimisation on the profits for EPS NL?

A literature review on the vehicle routing problem (VRP) has shown a great variety of VRP specialisations. A general conclusion in the literature is that it would be interesting to combine these VRP variants into one, more complex model, to better simulate the real world. Recent research is conducted to combine several VRP variants. This report will also consider a rich model with different combinations of VRP variants to contribute to the literature.

A mathematical model was formulated by combining existing VRP variants into one rich problem. This model includes multiple depots, heterogeneous capacitated vehicles with a planning horizon and selective nodes. The model was successfully verified and a case study was used to validate the model. In this case study, the mathematical model is used to solve a problem which appeared in the real world. The model showed and improvement of 12.8% in profits.

The model was also adapted to answer some managerial questions. The recycling plant is located in the south of the Netherlands while the shredding plant is located in the north of the Netherlands. EPS NL wondered which customers should be serviced by which vehicle type. Because of the nature of the vehicle characteristics, mobile vehicles are more useful for customers near the recycling plant while regular and special vehicles are more useful near the shredding plant. This experiment resulted in an imaginary line at the height of Zwolle which splits the space between the shredding and recycling plant.

Another question was whether a new shredding plant should be opened to serve customers in the west of the Netherlands. This case was tested with a set of customers in the west of the Netherlands. In one set of tests, a shredding plant was added in the west of the Netherlands while in another test there was no extra shredding plant. The results showed an increase in profits when an extra shredding plant was added.

To conclude, literature has shown that additional research in combining VRP variants can help to better simulate real-world problems. Different vehicles and nodes and their characteristics are investigated. A mathematical model was created by combining several VRP variants and this model has shown an improvement in profits. Finally, the model was adapted to answer managerial questions which resulted in an imaginary line between the recycling and shredding plant to divide the different type of vehicles.
Another adapted model showed that profits would increase if an extra shredding plant is added in the west.

Further research is encouraged to focus on heuristics to improve solution speed and make larger problems solvable. Also, the model could further be developed by adding more factors such as multiple trips on a day and a stochastic future demand.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
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<tr>
<td>$B_{ij,l}$</td>
<td>Number of shredded EPS transported from shredding plant $i \in N^s$ to recycling plant $j \in N^r$ on day $l \in L$.</td>
</tr>
<tr>
<td>$D_{i,l}$</td>
<td>Number of pouches at node $i \in N^c$ on Day $l \in L^l$.</td>
</tr>
<tr>
<td>$E_{i,l}$</td>
<td>Number of pouches at node $i \in N^s$ on Day $l \in L$.</td>
</tr>
<tr>
<td>$E'_{i,l}$</td>
<td>Number of shredded EPS at shredding plant $i \in N^s$ on day $l \in L$.</td>
</tr>
<tr>
<td>$F_{ik}$</td>
<td>Vehicle $k \in K$ is assigned to depot $i \in N^d$.</td>
</tr>
<tr>
<td>$R_{i,k,l}$</td>
<td>Total pouches shredding plant $i \in N^s$ received from vehicle $k \in K$ on day $l \in L$.</td>
</tr>
<tr>
<td>$S_{i,l}$</td>
<td>Number of pouches shred by shredding plant $i \in N^s$ on day $l \in L^l$.</td>
</tr>
<tr>
<td>$T_{i,k,l}$</td>
<td>Number of pouches picked up at node $i \in N^c$ by vehicle $k \in K$ on day $l \in L$.</td>
</tr>
<tr>
<td>$U_{k,l}$</td>
<td>Vehicle $k \in K$ is used on day $l \in L$.</td>
</tr>
<tr>
<td>$U_k$</td>
<td>Vehicle $k \in K$ is used at all.</td>
</tr>
<tr>
<td>$V_{i,k,l}$</td>
<td>Node $i \in N^d \cup N^c$ is visited by vehicle $k \in K$ on day $l \in L$.</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Node $i \in N^c$ is visited at all.</td>
</tr>
<tr>
<td>$u_{i,k,l}$</td>
<td>Dummy variable for each $i \in N^c, k \in K, l \in L$ to eliminate subtours</td>
</tr>
<tr>
<td>$x_{i,j,k,l}$</td>
<td>Arc $i, j \in N$ is traversed or not on by vehicle $k \in K$ on day $l \in L$.</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Price of a pouch that is shredded and delivered at the recycling plant</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Price for a pouch that is collected from a customer</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>Investment cost of a mobile vehicle normalized for the planning horizon</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>Investment cost of a regular vehicle normalized for the planning horizon</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>Investment cost of a special vehicle normalized for the planning horizon</td>
</tr>
<tr>
<td>$\lambda_6$</td>
<td>Cost per minute of work</td>
</tr>
<tr>
<td>$\lambda_7$</td>
<td>Cost per driven km</td>
</tr>
<tr>
<td>$DM_{i,1...3}$</td>
<td>Number of vehicles at each depot specified by vehicle type</td>
</tr>
<tr>
<td>$MaxC$</td>
<td>Maximum number of customers in a route.</td>
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## List of Abbreviations

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<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>CVRP</td>
<td>Capacitated Vehicle Routing Problem</td>
</tr>
<tr>
<td>DFJ</td>
<td>Dantzig Fulkerson Johnson</td>
</tr>
<tr>
<td>EPS</td>
<td>Expanded Polystyrene</td>
</tr>
<tr>
<td>FPVRP</td>
<td>Flexible Periodic Vehicle Routing Problem</td>
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<tr>
<td>HFVRP</td>
<td>Heterogeneous Fleet Vehicle Routing Problem</td>
</tr>
<tr>
<td>IRP</td>
<td>Inventory Routing Problem</td>
</tr>
<tr>
<td>MDVRP</td>
<td>Multiple Depot Vehicle Routing Problem</td>
</tr>
<tr>
<td>MFVRP</td>
<td>Mixed Fleet Vehicle Routing Problem</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed Integer Linear Programming</td>
</tr>
<tr>
<td>MTZ</td>
<td>Miller Tucker Zemlin</td>
</tr>
<tr>
<td>PVRP</td>
<td>Periodic Vehicle Routing Problem</td>
</tr>
<tr>
<td>RD</td>
<td>RijksDriehoeken-coordinates</td>
</tr>
<tr>
<td>VRP</td>
<td>Vehicle Routing Problem</td>
</tr>
<tr>
<td>VRPB</td>
<td>Vehicle Routing Problem with Backhauls</td>
</tr>
<tr>
<td>VRPPD</td>
<td>Vehicle Routing Problem with Pickup and Delivery</td>
</tr>
<tr>
<td>VRPTW</td>
<td>Vehicle Routing Problem with Time Windows</td>
</tr>
<tr>
<td>WCVRP</td>
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5 Introduction

Expanded Polystyrene (EPS) is a widely used material which is hard to recycle. A company, EPS Nederland, is pioneering to change the recycling process of EPS waste. If treated correctly, EPS waste can be reused into new EPS products. For this, the waste material must be clean and shredded in small pieces. EPS Nederland has two options to do this. The first option is to bring the waste EPS to a shredding plant where the material will be sorted and then be shredded. The other option is to use a mobile shredding vehicle to collect sort and shred the waste EPS. EPS Nederland has developed and patented such a mobile shredding vehicle which is now still in a prototyping phase. The shredded EPS needs to be transported to a recycling plant which is operated by another company. The methodology in this report can be used to solve a variety of real-world problems but here the case study is on EPS collection.

This report will consider a vehicle routing and planning model which is used for the collection and recycling of waste EPS. First, the problem is described. Next, the material to be transported is discussed followed by the company transporting it. In the transportation process, several nodes need to be visited which are discussed next. Then, the different vehicle types are explained following by a section with a basic model to solve the problem. Finally, the methodology of this research is explained.

5.1 Problem Description

EPS must be collected from customers and finally arrive at a recycling plant in a shredded state. There are several ways this can be achieved. A possible way of doing so is by shredding the EPS on site using a mobile shredding vehicle and transporting it directly to a recycling plant. Another approach is to transport the material to a shredding plant by means of a regular or special vehicle, have the material shred and then use a special vehicle to transport it to the recycling plant.

All these ways to process the material together with the new prototype vehicle make it hard to estimate what the best option is. This leads to the main research question and some sub-questions:

What is the impact of routing optimization on the profit of the process for EPS waste collection?

- What is the relevant literature on vehicle routing problems?
- Which processes play an important role within EPS NL and what are their characteristics?
- How to formulate the processes within EPS NL in a mathematical model?
- How to adapt the model to answer managerial questions?
  - Which customer should be serviced with which vehicle?
  - Should a shredding plant be added in the west to serve the customers in that location?
  - Is there anything else which can be done to increase profits?

5.2 EPS

EPS, also known as Styrofoam™ and Airpop®, is made of polystyrene pearls, these pearls are injected in a mold and expanded by adding steam [1]. This expanding process with steam requires a huge
amount of energy (83 MJ/kg EPS) [2]. Several of its properties, such as shock absorption, lightness, thermal isolation and high moist resistance, make EPS an outstanding material for packaging and isolation [3].

These properties make the material hard to recycle. Due to its lightness, the transport costs per mass are very high and due to its durability, the material won’t degrade in nature. Common end of life stages include landfilling, incinerating, melting, shredding and chemical recycling [4] [5] [6]. If the material is shredded, it can be mixed with virgin material to create new EPS [7]. A life cycle analysis by Marten & Hicks [5] showed recycling more EPS would decrease the environmental impacts because the need for virgin material will be decreased and the required energy for expanding the virgin material will decrease.

**EPS Nederland**

EPS Nederland is a company that is working hard to change the life cycle of EPS, according to their web page:

“EPS Nederland is founded to stimulate and improve the recycling and reuse of EPS as a pure element. Doing so, EPS Nederland actively contributes to the transition towards a circular economy. It is essential to maximize the reusability of products and decrease value destruction.

In cooperation with specialized partners and an inhouse innovative gathering concept, they offer a sustainable chain-solution for waste EPS. This is how waste EPS changes into circular EPS.” [8]

EPS Nederland uses innovative vehicles and a shredding plant to collect waste EPS from customers, shred it into small pieces, and finally, deliver these pieces to a recycling plant. At the recycling plant, which is operated by another company, the small EPS pieces are used to create new EPS products.

### 5.3 Nodes

The vehicles must visit a series of nodes to fulfil their job. Waste EPS is collected at customers and brought to shredding and recycling plants. To answer the research question, these nodes are implemented in a mathematical model.

**Depot**

Vehicles are stationed at depots. Each vehicle must start its journey from a depot and at the end of the journey it must return to the depot it came from. The depots are only used to store vehicles when they aren’t in use. Since there aren’t a lot of vehicles currently, the capacity of the depots is not limited by the model.

**Customer**

Customers produce waste EPS pouches. They pay the company when vehicles collect the pouches from them. The customers have limited storage with a known threshold.
Shredding Plant

Shredding plants can be used to shred the waste EPS in small pieces. Special and regular vehicles can deliver the pouches to shredding plants. The shredded material can then be transported to the Recycling plant.

Because the shredding plant is over-dimensioned, considering the currently available customers, there is no limit on the amount of EPS which can be temporarily stored at the shredding plant. Also, for the same reason, the amount that can be processed on a given day is unlimited. If the model size would increase significantly these factors might become a limiting constraint so they should be added, or the model isn't valid anymore.

Recycling Plant

The final stage of the EPS is the recycling plant, where the material is used to create new EPS, but this is outside of the scope of the routing model. The recycling plant is operated by another company and is currently being adjusted to increase capacity. For these reasons, both the storage and handling capacity of the recycling plant are assumed unlimited.

5.4 Vehicles

At EPS Nederland, three types of vehicles are used. A mobile shredding vehicle, a large truck with a special inline bag and regular vehicles. As with the nodes, these vehicle types are implemented in a mathematical model to answer the research questions. The specifications of these vehicles are described below.

Mobile

The mobile shredding vehicle is a patented [9] prototype which is still under development. It is used to shred material when collected from the customers. The vehicle drives to the customer where it is parked. Inside the vehicle, there is a shredding machine and a dedicated storage compartment for the storage of the shredded EPS. The vehicle contains seats for two employees. They collect the waste EPS pouches and sort the material they contain. Suitable fragments of EPS go onto a conveyor belt and the rest is put aside. By shredding the EPS on site, the bulk density is increased, therefore more material can be stored in the compartment.

When two employees are working in the vehicle, on average, 20 pouches can be processed each hour. The storage compartment can hold up to 110 pouches. The mobile vehicle always starts its journey at a vehicle depot, then visits customers and finally delivers the shredded EPS at the recycling plant. A schematic of this journey is shown in Figure 1.
Regular

A regular vehicle can only be used to store EPS pouches, it is used to collect pouches from customers and transport them to a shredding plant.

A regular vehicle can store up to 50 EPS pouches. On average it takes about 25 seconds to load a pouch into the vehicle. A regular vehicle can only be used to collect pouches from customers. From the vehicle depot, it goes to visit customers, and at the end of the journey it delivers the collected pouches to the shredding plant and then returns to the vehicle depot, see Figure 2.

Special

The special vehicle is fitted with a retractable antistatic inline bag. Shredded EPS pieces can be stored inside the bag and when retracted, the vehicle can be used to store EPS pouches. The shredded pieces and the pouches must not be stored in the same compartment since the static energy would cause small EPS beads to stick to the pouches and create a mess. The main use of this vehicle is to transport
the shredded EPS from the shredding plant to the recycling plan. On its way back from the recycling plant to the shredding plant, it can collect EPS pouches.

The special vehicle can store up to 100 EPS pouches and the equivalent of 200 EPS pouches shredded into pieces. On average, loading pouches into the vehicle takes 30 seconds per pouch. Filling the vehicle with shredded EPS takes about an hour. A special vehicle can have several different purposes and different routes, the vehicle always starts and ends its journey at a vehicle depot. In Figure 3 the vehicle is used to collect Pouches from customers and then delivers these pouches to the shredding plant after which it returns to the vehicle depot. In Figure 4, the vehicle drives to the shredding plant where it loads shredded EPS which is then transported to the recycling plant. After unloading, the vehicle returns to the vehicle depot. In Figure 5, the vehicle also transports EPS from the shredding plant to the recycling plant but instead of returning to the depot afterward, it collects pouches from customers. The vehicle then delivers these pouches to the shredding plant and finally returns to the vehicle depot.
A large part of the research questions can be answered by the vehicle routing problem. In 1959, Dantzig & Ramser [10] first described a vehicle routing problem. This was, in fact, a generalization of the traveling salesman problem as described by Flood [11] in 1956 and covers the use of multiple vehicles.
instead of one. These problems often are much more complex, and include for example time windows [12], multiple depots [13], vehicle capacity [14] and due dates [15]. Kulkarni & Bhave [16] present some integer programming formulations on these subjects.

5.6 Methodology

In this paper, a mathematical model is formulated to simulate the routing of the vehicles for EPS NL. The model is verified and validated. Several experiments of different sizes are then run with the model. Two objective functions are considered, one that maximizes profit and one that minimizes energy waste. The results can be used to answer the research questions.

5.7 Summary

EPS is a white foa-like material which costs a lot of energy to produce and is hard to recycle as waste. One solution provided by EPS Nederland is to shred the material into small pieces and feed those pieces back into the production process to reduce the energy that is required. EPS Nederland uses several types of vehicles to collect waste EPS, and they want a model to optimize their routing strategy. Since this is a complex model, finding a solution will cost a lot of time. Preliminary research has shown that the subtour elimination formulations are accountable for most of the constraints and solving time. The goal of this research is to generate a reliable model for the planning and routing of vehicles. The main question that arises is what the best way is to eliminate subtours.
The collection of EPS waste is a typical waste collection VRP with some of its own characteristics. The goal of the VRP also known as CVRP is to visit some number of nodes with a set of vehicles while reducing the total amount of distance driven by the vehicles. There are many types of VRP's for a recent overview see Braekers et al. [17]. Braekers et al. indicate some commonly researched VRP variants namely: Heterogeneous Fleet VRP (HFVRP) also knows as the Mixed Fleet VRP (MFVRP), the VRP with Time Windows (VRPTW), the VRP with Pickup and Delivery (VRPPD), VRP with Backhauls (VRPB), the Multi-Depot VRP (MDVRP) and the Periodic VRP (PVRP). Breakers et al. found that some research combines some of these variants into rich vehicle routing problems to simulate real-life aspects. They finally conclude that these that there is still a large gap with the real-life cases and state that VRP's with even richer characteristics could be investigated.

**EPS and Waste Collection**

“Waste Collection VRPs (WCVRPs) differ from traditional VRP due to the number of characteristics and constraints that complicate the routing and require special handling” [18]. In the waste collection VRP or WCVRP typically waste is collected from customers and disposed at a disposal site. The vehicles come start at a depot and end their journey at this same depot again. The collection of EPS differs from the standard WCVRP, as the material needs to be processed before it can be disposed of at the recycling plant. If only mobile vehicles are used, this problem is a typical WCVRP but instead, the fleet of vehicles is heterogeneous and the other type of vehicles need to visit a shredding plant, which is comparable to a transhipment facility, with the difference that the material needs some time to be processed. To simulate the routing of the vehicles, a combination of the existing variants must be investigated to create a realistic model. One such study which combines several problems into a rich stochastic problem is conducted by Markov et all. [19]. They consider a rich waste collection problem among others and they provide a non-linear rich model to solve a variety of problems.

**Heterogenous Fleet VRP (HFVRP)**

The HFVRP isn’t very different from the standard VRP. In the HFVRP, there are several types of vehicles. Bula et al. [20] modelled a routing problem for the transportation of hazardous materials. They came up with an algorithm based on a Multi Start Variable Neighbourhood Search which resulted in equal or better solutions compared to a Mixed Integer Linear Programming (MILP) formulation. Leung et al. [21] considered the two-dimensional loading and routing of heterogeneous vehicles. This work is different in the fact that it focuses more on the loading of different vehicle types, there are six metaheuristics used to optimally load the vehicles. Their work is proven to obtain good solutions and they state that future research could be done on the inclusion of time windows. Penna et al. [22] developed a simply structured algorithm for the HFVRP which was proven to be competitive with other known algorithms. They also suppose that future work could include more VRP variants.
VRP with Time Windows (VRPTW)

In the VRPTW, customers can only be serviced within specified time windows. For an overview of exact, heuristic and metaheuristic methods on the VRPTW see [23]. Keskin and Çatay [24] combined the VRPTW with electrical vehicles. They created a MILP as well as a metaheuristic and showed that the heuristic was faster than solving the MILP with CPLEX. They suggest that further research could include heterogeneous vehicles and they are working on extensions to the model themselves. In [25], Sivaramkumar et al. demonstrate the importance of total time balance compared to route balance. They found that in the VRPTW total time balance is much more important to balance the workload of the different vehicles. Sivaramkumar et al. also propose that future research could develop more advanced algorithms to reach 100% balance.

VRP with Pickup and Delivery (VRPPD)

In the VRPPD, the vehicles need to deliver or pickup something at the customers. Ballesteros Silva and Escobar Zuluaga [26] made an overview of recent papers on the VRPPD, one of their conclusions is that most of the work is focused on heuristics to solve the problem rather than exact methods. Yanik et al. [27] developed a hybrid algorithm for the VRPPD with multiple pickups and single deliveries which is quite like the problem this research is facing, they also include time windows in their model. They demonstrate the applicability of their algorithm and suggest further research to expand the problem.

VRP with Backhauls (VRPB)

In [28], Koç and Laporte made an overview of recent literature about the VRPB. In the VRPB, there are customers that need deliveries and customers that need pickups. The deliveries need to be dispatched from a depot and the pickups need to be delivered at a depot. In their literature study, Koç and Laporte make a general conclusion that “there is room for a very significant research effort on models and solution methods for VRPBs”.

Recent literature on VRPBs focus on metaheuristics such as Tabu Search Approach [29], Two Level VNS [30] and Large Neighbourhood Search [31], they all found some improvements for specific types when compared to exact methods for the general VRPB.

Multi-Depot VRP (MDVRP)

In the MDVRP there are multiple depots from where the vehicles can start and end their journeys. Montoya et al. [32] conducted a literature survey on the multi-depot vehicle routing problem (MDVRP). They state that this topic is much less researched than the general VRP and found that early works focus more on exact solutions while later works were more towards heuristics. Montoya et al. Also found that there were very few papers considering objective functions other than reducing the total driven distance and indicate the gap between real-world multi-objective functions. Lahyani et al. [33], wrote a paper in which they model a more complex MDVRP with heterogeneous vehicles in five different ways by proposing new inequalities. The result of these new inequalities led to faster solving speeds and better bounds.
Periodic VRP (PVRP)

The periodic VRP considers a planning horizon ahead. Archetti et al. [34] introduce a model for a Flexible Periodic Vehicle Routing Problem (FPVRP). They consider a set of customers which need to be serviced within a specified planning horizon. They presented a worst-case scenario and showed that the FPVRP has some advantages to a more generalized version of the problem such as the Periodic Vehicle Routing Problem (PVRP) and the Inventory Routing Problem (IRP). Kurz and Zäpfel [35] developed a heuristics-based solution method to solve the generalized PVRP, a case study showed the applicability to their research. They state that further research could extend their model with time windows or compare their model with other heuristics and multi-objective optimization problems.

6.2 Subtour Elimination

Generally, the formulated problems will generate subtours to prevent this, subtour elimination constraints must be added. There are two main subtour elimination formulations, the Dantzig, Fulkerson and Johnson (DFJ) and the Miller, Tucker and Zemlin (MTZ) subtour elimination constraints.

Dantzig Fulkerson and Johnson (DFJ)

Dantzig et al. [36] formulated the DFJ subtour elimination constraints. These constraints impose that any subset of nodes has at least two arcs moving in or out of the subset. If this is not the case, the subset contains a subtour. This method requires $2^n$ constraints (where $n$ is the number of nodes). To reduce the number of constraints an iterative process could be used. At the start of this process not all DFJ constraints are added, merely a small portion. The model is then solved and if necessary, more constraints are added. The iterative process solves the model and adds more constraints as needed.

Miller Tucker and Zemlin (MTZ)

Miller et al. [37] also introduced a set of subtour elimination constraint which later became known as the MTZ formulation. They add a variable which defines the order of a node in a route for a given vehicle. They then add a constraint which imposes that if a vehicle travels from node $i$ to node $j$, the order of $j$ is higher than that of $i$. Instead of using only the ordering of the nodes, the capacity of the vehicle and the amount taken or delivered at a node can be used as explained by Kulkarni and Bhave [16]. Kara et al. [38] observed a notational mistake by Kulkarni and Bhave which prohibited some legal solutions. Desrochers & Laporte [39] also improved to MTZ subtour elimination constraints to include various types of vehicle routing problems. There is also an option to use time constraints instead to get to the same goal, this is a variation on the MTZ constraints as explained by Cordeau [40].

6.3 Summary

The research mentioned before varies in complexity, there are a lot of different models to solve problems which are very similar, an overview of the research is shown in Table 1. As mentioned by Montoya et al. [32], research on VRP’s with multiple depots is not common. Braeckers et al. [17] also indicated that more research should be done on rich combination of the VRP variants. A common suggestion of the other authors is that their work can be extended to include more of the VRP variants.
to better simulate a real-world case. Markov et al. [19] have developed a rich model which encapsulates a lot of real-world problems. However, their solution is non-linear. Table 1 shows an overview of the covered topics in the literature. It shows that recent literature is more focussed on heuristics rather than exact models. Furthermore, it can be seen that most of the research only covers a small portion of real-world problems. To my knowledge, there is no research done on a VRP with multiple depots, heterogeneous capacitated vehicles with a planning horizon and selective nodes which is a linear model. That is what this research tries to contribute to the literature.

Table 1 Overview of covered topics in literature

<table>
<thead>
<tr>
<th></th>
<th>MILP</th>
<th>Heuristics</th>
<th>HMAAP</th>
<th>VRPTW</th>
<th>VAP</th>
<th>VPRP</th>
<th>MODVRP</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Markov et al. [19]</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
<td></td>
<td>Stochastic, non-linear</td>
</tr>
<tr>
<td>Bula et al. [20]</td>
<td>•</td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Hazardous materials</td>
</tr>
<tr>
<td>Leung et al. [21]</td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Two-dimensional loading</td>
</tr>
<tr>
<td>Penna et al. [22]</td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Keskin and Catay [24]</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Electrical vehicles</td>
</tr>
<tr>
<td>Sivaramkumar et al. [25]</td>
<td>•</td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total time balance</td>
</tr>
<tr>
<td>Yanik et al. [27]</td>
<td>•</td>
<td>•</td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td>Hybrid algorithm</td>
</tr>
<tr>
<td>Chávez et al. [29]</td>
<td>•</td>
<td></td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td>Tabu Search Approach</td>
</tr>
<tr>
<td>Wassan et al. [30]</td>
<td>•</td>
<td></td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td>Two Level VNS</td>
</tr>
<tr>
<td>Domínguez et al. [31]</td>
<td>•</td>
<td></td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td>Large Neighbourhood Search</td>
</tr>
<tr>
<td>Lahyani et al. [33]</td>
<td>•</td>
<td>•</td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td>Complex</td>
</tr>
<tr>
<td>Archetti et al. [34]</td>
<td>•</td>
<td></td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td>Flexible</td>
</tr>
<tr>
<td>Kurz and Zäpfel [35]</td>
<td>•</td>
<td></td>
<td></td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This Thesis</td>
<td>•</td>
<td></td>
<td>•</td>
<td>•</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 6 shows the flow of EPS material. Customers that produce EPS waste, shredding plants that shred the EPS waste, so it can be recycled, recycling plants which can use the recycled EPS and depots at which vehicles can be stored. There are three types of vehicles. Mobile shredding vehicles [9] that can shred the EPS at a customer location and deliver the shredded EPS directly to the recycling plant. Regular vehicles that can collect EPS from customers and deliver it to the shredding plants. And special vehicles that are equipped to deliver the small shredded pieces from a shredding plant to a recycling plant, these vehicles can also collect EPS at customers. The vehicles all have a limited capacity depending on the vehicle type. A working day consists of eight hours, in this time the vehicle must do all its movements including driving from and to the vehicle depot. At all times the EPS is collected at customers and eventually delivered at recycling plants.

Let $a$ denote the number of depots, $b$ the number of shredding plants, $c$ the number of recycling plants and $d$ the number of customers to be served. The problem may be defined on a complete directed graph $G = (N, A)$ that consists of the set of nodes $N = N^d \cup N^s \cup N^r \cup N^c$, where $N^d = \{1 \ldots a\}, N^s = \{a+1 \ldots a+b\}, N^r = \{a+b+1 \ldots a+b+c\}, N^c = \{a+b+c+1 \ldots a+b+c+d\}$ and the set of arcs $A = \{(i,j) : i,j \in N, i \neq j\}$. Subsets $N^d, N^s, N^r$ and $N^c$ represent the depots, shredding plants, recycling plants and customers respectively. Let $K = K^m \cup K^n \cup K^s$ be the set of vehicles where $K^m = \{1 \ldots e\}, K^r = \{e+1 \ldots e+f\}$ and $K^s = \{e+f+1 \ldots e+f+g\}$. Subsets $K^m, K^r$ and $K^s$ represent the mobile shredding vehicles, the regular vehicles and the special vehicles respectively. And finally let $L_L = L_0 \cup L$ be the set of days where $L_0 = \{0\}$ and $L = \{1 \ldots h\}$. Subsets $L_0$ and $L$ represent the initiation day and the working days respectively.

$\delta_{i,j}$ and $\tau_{i,j}$ represent the distance and respectively time, associated with traversing arc $(i,j) \in A$. $\tau_{i}^j$ represents the stop time at node $i \in N$. For each node $i \in N^c$ there is an associated $P_{i,j}$ and $D_{i,j}^{\text{thres}}$ which represent the production on day $l \in L$ and the maximum number of pouches which may be stored.
respectively. With each vehicle $k \in K$, there is an associated load time per pouch $\tau_k^l$ and a capacity $C_k$.

Let $MaxC$ denote the maximum number of customers in a route. $DM_{i,1,3}$ represents a depot matrix which specifies how many of each vehicle type are available at the specific depot. Table 2 gives an overview of the used parameters in the model and Table 3 gives an overview of the decision variables used in the mathematical model.

### Table 2: Specification of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>Price of a pouch that is shredded and delivered at the recycling plant</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Price for a pouch that is collected from a customer</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>Cost per minute of work</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>Cost per driven km</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>Cost to shred one pouch</td>
</tr>
<tr>
<td>$MaxC$</td>
<td>Maximum number of customers in a route</td>
</tr>
<tr>
<td>$DM_{i,1,3}$</td>
<td>Number of vehicles at each depot specified by vehicle type</td>
</tr>
</tbody>
</table>

### Table 3: Decision variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{i,j,k,l}$</td>
<td>Arc $i, j \in N$ is traversed or not on by vehicle $k \in K$ on day $l \in L$</td>
</tr>
<tr>
<td>$D_{i,l}$</td>
<td>Number of pouches at node $i \in N^c$ on Day $l \in L^t$</td>
</tr>
<tr>
<td>$E_{i,l}$</td>
<td>Number of pouches at node $i \in N^s$ on Day $l \in L$</td>
</tr>
<tr>
<td>$T_{i,k,l}$</td>
<td>Number of pouches picked up at node $i \in N^c$ by vehicle $k \in K$ on day $l \in L$</td>
</tr>
<tr>
<td>$u_{i,k,l}$</td>
<td>Dummy variable for each $i \in N^c, k \in K, l \in L$ to eliminate subtours</td>
</tr>
<tr>
<td>$R_{i,k,l}$</td>
<td>Total pouches shredding plant $i \in N^s$ received from vehicle $k \in K$ on day $l \in L$</td>
</tr>
<tr>
<td>$S_{i,l}$</td>
<td>Number of pouches shred by shredding plant $i \in N^s$ on day $l \in L^t$</td>
</tr>
<tr>
<td>$B_{i,j,l}$</td>
<td>Number of shredded EPS transported from shredding plant $i \in N^s$ to recycling plant $j \in N^r$ on day $l \in L$</td>
</tr>
<tr>
<td>$E_{i,l}$</td>
<td>Number of shredded EPS at shredding plant $i \in N^s$ on day $l \in L$</td>
</tr>
</tbody>
</table>

### Objective

Maximize:

$$
\max \lambda_1 \sum_{i \in N^c} \sum_{k \in K} \sum_{l \in L^t} T_{i,k,l} + \lambda_2 \sum_{i \in N^s} \sum_{l \in L} B_{i,l} + \lambda_2 \sum_{i \in N^s} \sum_{k \in K} \sum_{l \in L} T_{i,k,l} - \lambda_3 \sum_{i \in N^c} \sum_{k \in K} \sum_{l \in L^t} T_{i,k,l} \tau_k^l + \sum_{i \in N^s} \sum_{j \in N^r} \sum_{l \in L} \tau_{i,j} + \sum_{i \in N^s} \sum_{j \in N^r} \sum_{l \in L} \tau_k^l \delta_{i,j} - \lambda_4 \sum_{i \in N^s} \sum_{l \in L^t} S_{i,l} - \lambda_5 \sum_{i \in N^s} \sum_{l \in L^t} S_{i,l}
$$

(1)
The objective maximizes profits by summing the total amount of collected pouches and delivered shredded material and subtracting the costs. The costs consist of the cost for labour, the cost for each driven kilometre and the cost to shred pouches at a shredding plant.

Subject to

\[ D_{i,l} = D_{i,l-1} - \sum_{k \in K} T_{i,l,k} + P_{i,l} \quad \forall i \in N^c, l \in L \]  
(2)

\[ D_{i,l} \leq D_{i}^{\text{wires}} \quad \forall i \in N^c, l \in L \]  
(3)

\[ \sum_{i \in N^c} T_{i,k,l} \leq C_k \quad \forall k \in K, l \in L \]  
(4)

\[ T_{i,k,l} \leq C_k \sum_{j \in N^c} x_{i,j,k,l} \quad \forall i \in N^c, k \in K, l \in L \]  
(5)

\[ x_{i,l,k,l} = 0 \quad \forall i \in N, k \in K, l \in L \]  
(6)

\[ \sum_{i \in N^c \cup N^d, i \neq j} x_{i,j,k,l} = \sum_{i \in N} x_{i,j,k,l} \quad \forall j \in N^c, k \in K^m \cup K^r, l \in L \]  
(7)

\[ \sum_{i \in N^c \cup N^d, i \neq j} x_{i,j,k,l} = \sum_{i \in N} x_{i,j,k,l} \quad \forall j \in N^c, k \in K^s, l \in L \]  
(8)

\[ \sum_{j \in N^c \cup N^d, j \neq l} x_{i,j,k,l} = \sum_{j \in N} x_{i,j,k,l} \quad \forall i \in N^c, k \in K^m, l \in L \]  
(9)

\[ \sum_{j \in N^c \cup N^d, j \neq l} x_{i,j,k,l} = \sum_{j \in N} x_{i,j,k,l} \quad \forall i \in N^c, k \in K^r \cup K^s, l \in L \]  
(10)

\[ \sum_{i \in N^c} x_{i,j,k,l} = \sum_{j \in N^c \cup N^d} x_{j,i,k,l} \quad \forall j \in N^c, k \in K^s, l \in L \]  
(11)

\[ x_{i,l,k,l} = 0 \quad \forall i \in N^c, j \in N^c, k \in K^s, l \in L \]  
(12)

\[ \sum_{i \in N^c} \sum_{j \in N^c} x_{i,j,k,l} + \sum_{i \in N^c} \sum_{j \in N^d} x_{i,j,k,l} \quad \forall k \in K^s, l \in L \]  
(13)

\[ \sum_{i,j \in N} x_{i,j,k,l} = (\text{MaxC} + 4) \sum_{i \in N^c \cup N^d} \sum_{j \in N^c} x_{i,j,k,l} \quad \forall k \in K^s, l \in L \]  
(14)

\[ \sum_{i \in N \setminus \{k\}} x_{i,j,k,l} \quad \forall j \in N, k \in K, l \in L \]  
(15)

\[ u_{i,k,l} - u_{j,k,l} + \text{MaxC} \cdot x_{i,j,k,l} + (\text{MaxC} - 2) \cdot x_{i,j,k,l} \leq \text{MaxC} - 1 \quad \forall l, j \in N^c : i \neq j, k \in K, l \in L \]  
(16)
\[
0 \leq u_{i,k,l} \leq \text{Max}C \\
\sum_{i \in \mathbb{N}^d} \sum_{j \in \mathbb{N}^2} x_{i,j,k,l} \leq 1 \quad \forall k \in K, l \in L (17) \\
\sum_{i \in \mathbb{N}^2} \sum_{j \in \mathbb{N}^d} x_{i,j,k,l} \cdot (\text{Max}C + 2) = \sum_{i,j \in \mathbb{N}^2} x_{i,j,k,l} \quad k \in K^r, l \in L (19) \\
\sum_{i \in \mathbb{N}^2} \sum_{j \in \mathbb{N}^d} x_{i,j,k,l} \cdot (\text{Max}C + 2) = \sum_{i,j \in \mathbb{N}^2} x_{i,j,k,l} \quad k \in K^r, l \in L (20) \\
\sum_{i \in \mathbb{N}^2} T_{i,k,l}^{1} + \sum_{i,j \in \mathbb{N}^2} x_{i,j,k,l}^{1} + T_{i,j}^{1} \leq 480 \quad \forall j \in \mathbb{N}, k \in K, l \in L (21) \\
R_{i,k,l} \leq c_k \sum_{j \in \mathbb{N}^d} x_{i,j,k,l} \quad \forall i \in \mathbb{N}^2, k \in K \cup K^x, l \in L (22) \\
R_{i,k,l} \geq 0 \quad \forall i \in \mathbb{N}^2, k \in K \cup K^x, l \in L (23) \\
R_{i,k,l} \leq \sum_{i \in \mathbb{N}^2} T_{i,k,l}^{1} \quad \forall i \in \mathbb{N}^2, k \in K \cup K^x, l \in L (24) \\
R_{i,k,l} \geq \sum_{i \in \mathbb{N}^2} T_{i,k,l}^{1} - c_k \left(1 - \sum_{j \in \mathbb{N}^d} x_{i,j,k,l}\right) \quad \forall i \in \mathbb{N}^2, k \in K \cup K^x, l \in L (25) \\
E_{i,l} = E_{i,l-1} + \sum_{k \in K \cup K^x} R_{i,k,l} - S_{i,l-1} \quad \forall i \in \mathbb{N}^2, l \in L (26) \\
E'_{i,l} = E'_{i,l-1} + S_{i,l-1} - \sum_{j \in K^r} B_{i,j,l} \quad \forall i \in \mathbb{N}^2, l \in L (27) \\
\sum_{k \in K^x} x_{i,j,k,l} \geq \frac{B_{i,j,l}}{\beta \cdot c_k} \quad \forall i \in \mathbb{N}^2, j \in \mathbb{N}^r, k \in K^x, l \in L (28) \\
S_{i,l} \leq E_{i,l} \quad \forall i \in \mathbb{N}^2, l \in L (29) \\
\sum_{j \in \mathbb{N}^r} B_{i,j,l} \leq E'_{i,l} \quad \forall i \in \mathbb{N}^2, l \in L (30) \\
\sum_{(k \in K_1 \cup L)} T_{i,k,l} \geq D_{i,0} \quad \forall i \in \mathbb{N}^c (31)
\]
\[ \sum_{k \in K^m} X_{i,j,k,l} \leq DM_{i,1} \quad \forall i \in N^d, j \in N, l \in L \] (32)

\[ \sum_{k \in K^r} X_{i,j,k,l} \leq DM_{i,2} \quad \forall i \in N^d, j \in N, l \in L \] (33)

\[ \sum_{k \in K^s} X_{i,j,k,l} \leq DM_{i,3} \quad \forall i \in N^d, j \in N, l \in L \] (34)

Constraints (2) defines the number of pouches at the customer nodes. Constraints (3) impose that the number of pouches at the nodes stay below the threshold if that node is used in the model. Constraints (4) impose that the sum of loaded pouches does not exceed the vehicles capacity. Constraints (5) impose that if a vehicle retrieves pouches from a node then that node is visited. Constraints (6),(7),(8),(9),(10),(11),(12),(13),(14) and (15) are flow constraints which force or permit some flows. Constraints (6) prohibit the use of an arc that returns to itself. Constraints (7) impose that mobile and regular vehicles can only reach customers from another customer or a depot. Constraints (8) impose that special vehicles can only reach customers from another customer, depot or a recycling plant. Constraints (9) impose that mobile shredding vehicles which leave a customer can only go to another customer or a recycling plant. Constraints (10) impose that regular and special vehicles that leave a customer can only go to another customer or a shredding plant. Constraints (11) impose that if a vehicle moves from a shredding plant to a recycling plant then it must have come from a depot or a customer towards that shredding plant. Constraints (12) prohibit special vehicles to move from a shredding plant to a customer. Constraints (13) impose that special vehicles leave a shredding plant every time it arrives there from a customer and when it transports something from the shredding plant directly to the recycler. Constraints (14) impose that if a special vehicle is used on a given day, it must go back to a depot. Constraints (15) impose that if a node is visited, it is also left. Constraints (16) and (17) are MTZ subtour elimination constraints. MTZ constraints are chosen over DFJ constraints to reduce the amount of constraints as explained by van Tol et al. (2016) [41]. Constraints (18) assure that each vehicle leaves the depot not more than once on a given day. Constraints (19) and (20) ensure that if a vehicles is used on a day, it will visit a recycling plant (19) or shredding plant (20) depending on the vehicle type, and then return to the depot. Constraints (21) make sure that a working day isn’t longer than 8 hours. Constraints (22),(23),(24) and (25) impose that if a regular or special vehicle’s route goes through a shredding plant then all its collected pouches are unloaded at that shredding plant. Constraints (26) impose that the number of pouches at a shredding plant on a given day is equal to the amount on the day before plus the pouches delivered at that plant minus the pouches processed on the day before. Constraints (27) impose that the number of processed pouches at a recycling plant on a given day is equal to the number of processed pouches on that day plus the number of processed pouches on the day before minus the number of pouches transported to the recycling plant. Constraints
impose that the amount of special vehicles transporting shredded EPS from a shredding plant to a recycling plant is enough to cover the total transportation from the shredding plant to the recycling plant. Constraints (29) impose that the number of EPS pouches shredded on a given day is less or equal to the number of pouches available on that day. Constraints (30) impose that the amount of shredded EPS transported from a shredding plant to a recycling plant is less or equal to the amount of available shredded EPS at the shredding plant. Constraints (31) impose that the total amount of pouches that is picked up satisfies at least the amount that the customers has ordered. Constraints (32),(33) and (34) limit the amount of vehicles available at a depot.
7.2 Data
To create a realistic set of data to experiment with, historical data is analysed. This data runs from May 2018 to September 2018 and only customers that have been visited more than three times are taken into account. This left 12 customers of which the anonymised data is shown in Table 4. The average values and the standard deviation is computed and used to generate test data.

<table>
<thead>
<tr>
<th>Customer</th>
<th>Total amount of pouches</th>
<th>Amount of times this customer was visited</th>
<th>Average number of pouches taken</th>
<th>Maximum amount taken on one day</th>
<th>Daily production</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>426</td>
<td>12</td>
<td>36,82</td>
<td>66</td>
<td>14,20</td>
</tr>
<tr>
<td>2</td>
<td>211</td>
<td>6</td>
<td>35,17</td>
<td>66</td>
<td>7,03</td>
</tr>
<tr>
<td>3</td>
<td>195</td>
<td>4</td>
<td>48,75</td>
<td>60</td>
<td>6,50</td>
</tr>
<tr>
<td>4</td>
<td>450</td>
<td>11</td>
<td>40,91</td>
<td>65</td>
<td>15,00</td>
</tr>
<tr>
<td>5</td>
<td>440</td>
<td>15</td>
<td>29,29</td>
<td>36</td>
<td>14,67</td>
</tr>
<tr>
<td>6</td>
<td>194</td>
<td>6</td>
<td>32,33</td>
<td>48</td>
<td>6,47</td>
</tr>
<tr>
<td>7</td>
<td>1543</td>
<td>22</td>
<td>70,14</td>
<td>142</td>
<td>51,43</td>
</tr>
<tr>
<td>8</td>
<td>129</td>
<td>3</td>
<td>43,00</td>
<td>60</td>
<td>4,30</td>
</tr>
<tr>
<td>9</td>
<td>160</td>
<td>6</td>
<td>26,67</td>
<td>35</td>
<td>5,33</td>
</tr>
<tr>
<td>10</td>
<td>391</td>
<td>10</td>
<td>39,10</td>
<td>65</td>
<td>13,03</td>
</tr>
<tr>
<td>11</td>
<td>262</td>
<td>8</td>
<td>33,71</td>
<td>60</td>
<td>8,73</td>
</tr>
<tr>
<td>12</td>
<td>1243</td>
<td>25</td>
<td>49,46</td>
<td>101</td>
<td>41,43</td>
</tr>
<tr>
<td>Average</td>
<td>470,33</td>
<td>10,67</td>
<td>40,44</td>
<td>67,00</td>
<td>15,68</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>450,54</td>
<td>6,95</td>
<td>11,70</td>
<td>29,05</td>
<td>15,02</td>
</tr>
</tbody>
</table>

7.3 Model Verification
To verify the model some more extreme cases are evaluated. These cases are described below. They all have the same set of variables to start with. There are 12 customers of which 5 are required to be visited, the rest is optional. Of all three vehicle types, one vehicle is available. Vehicle 1 is a mobile shredding vehicle stationed at a depot near the recycling plant. Vehicle 2 and 3 represent a regular and a special vehicle respectively, both are stationed near the shredding plant.

Case 1 No mobile vehicles
In this case, there are no mobile vehicles, this means that only regular and special vehicles can be used which need to come from the depot near the shredding plant and use the shredding plant for the processing of the material. This should lead to a longer driving distance and thus less profits.
Case 2 Pouch production
Customers are now assumed to produce pouches each day. Only the pouches available at day one need to be taken, the produced pouches are optional. In this case, more pouches may be taken from optional nodes, this would increase the driven distance, but simultaneously increase the total profit.

Case 3 Nothing to collect
The number of pouches at customers is set to zero. The production is set to zero as well. This should lead to no traffic and a profit of 0.

Case 4 No vehicles available
In this test case, there are no vehicles available. It is expected that the solver cannot solve the model.

Case 5 Less vehicle capacity
The capacity of each vehicle is now only 25% of the original capacity, this means that a lot more vehicles are required to collect all the pouches and thus the total driven distance will increase.

Case 6 Visit all 12 customers
Here, all customers have several pouches at the start, but they don’t produce anything. Now all the pouches must be collected. Since all pouches must now be collected instead of it being an option to collect extra pouches the profits will decrease, and the driven distance will increase. Also, the number of collected pouches will increase.

Case 7 Customers in the north
In this case, all the customers are located in the north of the Netherlands, near the shredding plant. Now they can be easily serviced by regular and special vehicles. Though the special vehicles still must travel a large distance to the recycling plant. Compared to test case one the profits should increase since the regular and special vehicles can collect the pouches more efficiently, however since the mobile vehicles play a part now and they must drive a longer distance, the total driven distance cannot be determined up front.
7.4 Verification Results

<table>
<thead>
<tr>
<th>Case</th>
<th>Total driven distance [km]</th>
<th>Number of pouches taken</th>
<th>Profit</th>
<th>Gap [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Case in practice</td>
<td>692</td>
<td>325</td>
<td>712.3</td>
<td>0</td>
</tr>
<tr>
<td>Real case model solution</td>
<td>564</td>
<td>325</td>
<td>803.2</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1442</td>
<td>325</td>
<td>231.2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2317</td>
<td>1069</td>
<td>2355.8</td>
<td>4.67</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>No Solution</td>
<td>No Solution</td>
<td>No Solution</td>
<td>No Solution</td>
</tr>
<tr>
<td>5</td>
<td>3648</td>
<td>325</td>
<td>-1188</td>
<td>32.63</td>
</tr>
<tr>
<td>6</td>
<td>1267</td>
<td>426</td>
<td>685.2</td>
<td>42.44</td>
</tr>
<tr>
<td>7</td>
<td>1804</td>
<td>325</td>
<td>595.5</td>
<td>0</td>
</tr>
</tbody>
</table>

The results of the test cases are shown in Table 5. All the test cases behave as expected. For the first test case, Figure 7 shows the routing of all days in one figure. The right side of Figure 7 is zoomed in and shows that no vehicles come from the vehicle depot for mobile vehicles.
For the second case with the production of pouches, the total number of pouches at a customer at any given moment stays below the threshold. This can be seen in Figure 8.
Test case 3 and 4 are self-explanatory, in case 3 there is nothing to collect and thus the results are all equal to zero. In test case 4 there was no solution thus also nothing to be shown.

For test case 5 the vehicle capacity is most critical but Figure 9 shows that the vehicles aren't filled above their capacity.
In test case 6, all the customers needed to be visited and all their pouches needed to be collected. Figure 10 shows that all customers have some pouches on the first day and on the final day, all pouches have been collected.
Finally, test case 7 was about handling customers in the north, near the shredding plant. The routing results in Figure 11 show that the model can give a solution to serve customers in the north as well.

Test case 5 and 6 have a large gap between the best-found solution and the upper bound. This is because these cases are harder to solve. For case 5 there was almost no space left in the vehicles since their capacity had decreased dramatically. Therefore, the trade-off between serving the demand and the capacity usage becomes very intricate. For case 6, all customers had to be visited and all pouches had to be collected. Which drove the model to its limits.
The solutions from the model all show that the expected results are indeed correct. Also, there haven’t been any abnormalities. Therefore, it is highly likely that the model is correct and can be used for experiments.

7.5 Model Validation

To validate that the model is suitable for the job, a real-world case is used. Week 39 of 2018 is used as a reference case. In this week, 7 of the 12 customers were visited of which one customer was visited twice. Only one mobile shredding vehicle was used to collect the pouches. The amount of collected pouches, which can be seen in Table 6, is set as a start amount for the week and it is assumed that there is no production.

Table 6 Amount of pouches collected at each customer

<table>
<thead>
<tr>
<th>Customer</th>
<th>Amount of pouches collected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>44</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>26</td>
</tr>
<tr>
<td>12</td>
<td>138</td>
</tr>
</tbody>
</table>

In this case, a total distance of 692km was driven and 325 pouches were taken, which led to a total profit of 712.3. The driven route is depicted in Figure 12.
To validate that the model comes with a similar answer, the goal must be the same as used for the real world case. Which is to try to get the maximum amount of pouches without bothering over the costs for driving and labour. This resulted in the following route, shown in

![Figure 13 Model routing solution](image)

### 7.6 Data points for experiments

For the next experiments, a good set of data points was needed. At first, I had the idea to take the centre of each municipality of the Netherlands as a customer node. For this a dataset from "het Kadaster" was acquired. A plot of this data is shown in Figure 14. The map looks quite okay, but the coordinates are not in the common lat-lon coordinates but in Rijksdriehoek (RD) -coordinates.
For this reason, the data had to be transformed. First, the RD-coordinates were transformed to Bessel coordinates from which they could be converted to World Geodetic System (WGS) coordinates. The new set of data points can now be displayed on a map of the Netherlands and is shown in Figure 15.

Figure 14 Map of each municipality in the Netherlands (in RD coordinates)
When I was deciding on how to define the centre of each municipality, I found that this centre might not always be a reachable place resulting in unrealistic traveling times because there are no good roads leading to the coordinates. Thus, instead of the centre, the capital city of each municipality was used as a customer node, there are always many roads leading to a capital city. To get the coordinates of each capital city, a piece of MATLAB code was created to search for the name of each place on Bing Maps and extract the lat and lon coordinates. The results were correct most of the times, only a handful
of cities had to be adjusted manually, for example where a city name existed multiple times. The result
is shown in Figure 16. Where the capital cities are plotted on the map with the municipality outlines.

Figure 16 Map of each municipality and its capital city in the Netherlands

1 The markers may seem off place, this is because the center of the marker indicates the exact
coordinates, not the bottom tip.
With the valid model, we can now check what the impact of optimized routing is on the profits for EPS waste collection by optimizing the real world case, used for validating the model. This time, the full objective function is active. Which means that the costs are taken into account as well. This led to an improved solution.

The same case as in the validation was passed to the solver, which gave the optimal solution as seen in Figure 13. The total driven distance was 564 km, 325 pouches were collected and the total profit was 803.2. This means an improvement of 12.8% on profits when solved to optimality.

![Optimised routing solution](image)

8.2 Between Shredder and Recycler in Blocks

One of the questions from the company was which type of vehicle should be sent to which customer location. Some experiments were done to find the answer to this question. For the experiment, an area between the shredding plant and the recycling plant is analysed. This area is approximately 80 km wide and 185 km high. This area is divided into five parts named A through E as can be seen in Figure 18. For each area the experiment is repeated with four different vehicle sets.
Vehicle set one represents the current fleet for EPS NL. The second set was created to see what would happen if the prototype mobile shredding vehicle was left out and replaced with an extra normal and special vehicle. The third vehicle set only contains mobile shredding vehicles, in order to see how they would behave if the normal and special vehicle were traded in for mobile shredding vehicles. Finally, a fourth set of vehicles containing only special vehicles was added to the test, this set is interesting since the capacity to send EPS to the recycling plant outweighs the amount of EPS pouches that can be collected. This leads to a set of 20 experiments shown in Table 7.
All the experiments were run for 30 minutes. Most of the solutions show a clear difference between the different settings and the gap between the solution and the bound is small enough to draw some conclusions.
<table>
<thead>
<tr>
<th></th>
<th>Driven distance</th>
<th>Pouches taken</th>
<th>Solution</th>
<th>Bound</th>
<th>Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2,8094e+03</td>
<td>922</td>
<td>1465,6885</td>
<td>1599,2870</td>
<td>9,12%</td>
</tr>
<tr>
<td>A2</td>
<td>2,3198e+03</td>
<td>1100</td>
<td>2256,2192</td>
<td>2459,1516</td>
<td>8,99%</td>
</tr>
<tr>
<td>A3</td>
<td>3,7938e+03</td>
<td>596</td>
<td>-16,9356</td>
<td>1928,8119</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>2,8868e+03</td>
<td>1202</td>
<td>2412,4405</td>
<td>2608,4749</td>
<td>8,13%</td>
</tr>
<tr>
<td>B1</td>
<td>2,7278e+03</td>
<td>1072</td>
<td>2048,1603</td>
<td>2130,3960</td>
<td>4,02%</td>
</tr>
<tr>
<td>B2</td>
<td>2,3092e+03</td>
<td>1099</td>
<td>2277,7192</td>
<td>2431,7262</td>
<td>6,76%</td>
</tr>
<tr>
<td>B3</td>
<td>2,6839e+03</td>
<td>803</td>
<td>1354,9923</td>
<td>2280,5424</td>
<td>68,31%</td>
</tr>
<tr>
<td>B4</td>
<td>2,8120e+03</td>
<td>1204</td>
<td>2487,3758</td>
<td>2642,2427</td>
<td>6,23%</td>
</tr>
<tr>
<td>C1</td>
<td>2,4303e+03</td>
<td>1034</td>
<td>2098,0555</td>
<td>2383,7932</td>
<td>13,62%</td>
</tr>
<tr>
<td>C2</td>
<td>2,7204e+03</td>
<td>1092</td>
<td>2041,1313</td>
<td>2101,7586</td>
<td>2,97%</td>
</tr>
<tr>
<td>C3</td>
<td>1,8142e+03</td>
<td>884</td>
<td>2065,9282</td>
<td>2601,8143</td>
<td>25,94%</td>
</tr>
<tr>
<td>C4</td>
<td>3,3265e+03</td>
<td>1205</td>
<td>2208,1423</td>
<td>2338,3199</td>
<td>5,90%</td>
</tr>
<tr>
<td>D1</td>
<td>2,3006e+03</td>
<td>1042</td>
<td>2234,7957</td>
<td>2385,1640</td>
<td>6,73%</td>
</tr>
<tr>
<td>D2</td>
<td>3,1681e+03</td>
<td>1092</td>
<td>1796,0277</td>
<td>1857,9791</td>
<td>3,45%</td>
</tr>
<tr>
<td>D3</td>
<td>1,4990e+03</td>
<td>979</td>
<td>2621,7265</td>
<td>2741,2000</td>
<td>4,56%</td>
</tr>
<tr>
<td>D4</td>
<td>4,3443e+03</td>
<td>1205</td>
<td>1968,2998</td>
<td>2079,1175</td>
<td>5,63%</td>
</tr>
<tr>
<td>E1</td>
<td>2,6294e+03</td>
<td>1052</td>
<td>2117,7902</td>
<td>2273,5835</td>
<td>7,36%</td>
</tr>
<tr>
<td>E2</td>
<td>3,8352e+03</td>
<td>1084</td>
<td>1451,1688</td>
<td>1608,1792</td>
<td>10,82%</td>
</tr>
<tr>
<td>E3</td>
<td>1,3589e+03</td>
<td>992</td>
<td>2753,6143</td>
<td>2919,7511</td>
<td>6,03%</td>
</tr>
<tr>
<td>E4</td>
<td>4,3567e+03</td>
<td>1204</td>
<td>1645,1943</td>
<td>1780,4916</td>
<td>8,22%</td>
</tr>
</tbody>
</table>
Table 8 and Figure 19, show the results of the experiments. Both the best found solution and the upper bound are shown. The best found solution exists for sure, and even though an even better solution might exist, it will definitely not be better than the upper bound. So we can conclude that if the upper bound of one set is below the best found solution of another set, the second set will always be better. For the experiments with a large gap above 50% the experiments were tried with a longer run time of two hours, however, this would not decrease the gap significantly as can be observed in Figure 20 and Figure 21.
Figure 20 Statistics for rerun of experiment A3

Figure 21 Statistics for rerun of experiment A3
As seen in Figure 22, the best way to serve people in the north part of the Netherlands is to use vehicle set A2 or A4, there is only a small difference. The choice between the two should be made based on other factors that are not included in the model.

Figure 22 Results of the experiments in area A
A little bit more south, the vehicle sets come more close to each other as shown in Figure 23. The best set is clearly B4, however, for this, the company need to invest in three extra special vehicles. Normal vehicles are already available and set 2 is also a good choice and is definitely an improvement over the current set of vehicles. Using only mobile vehicles might also be better than the current set of vehicles, however the results don't show a conclusion on that.

Figure 23 Results of the experiments in area B
The area in the middle is shown in Figure 24. This area is quite interesting, as the results can't give a clear conclusion on which vehicle set is best to use because there is a lot of overlap. The set with two normal and two special vehicles however seems to be the worst solution. The current situation is already a good solution, using only mobile vehicles or only special vehicles could prove to be better, however they could also be worse.

Figure 24 Results of the experiments in area C
Moving even more south, a clear distinction can be made in Figure 25. Using special vehicles isn’t beneficial here since they have to drive back to the north again. The current set of vehicles, D1, already works better than using special vehicles in set D2 and D4. However, investing in an extra mobile shredding vehicle will improve the profits compared to the current set of vehicles.
Finally, the results of area E are shown in Figure 26. It is clear that the set of mobile shredding vehicles shows the best results. Second best is the set of vehicles currently in use. It would be unwise to invest in special and normal vehicles to use in this area since they show the worst results.

Looking at all the areas, it can be observed that the different vehicle sets show different results in each region. In the top regions, the special vehicle’s show the best results, they could be supported by normal vehicles since they are already owned by the company and only decrease profits by a small amount. In the middle area, all vehicle sets operate more or less the same. Any set of vehicles will operate quite well. More to the bottom areas, the mobile shredding vehicles show better results.

8.3 With and Without a New Shredding Plant in the West

Another question to answer was how to serve potential customers in the west of the Netherlands. Here the customer location would be much denser and there is no shredding plant nor a recycling plant nearby. Should those customers be served by mobile or special vehicles or would it be better to open a new shredding plant in that area? A fictional situation is shown in Figure 27.
Two sets of experiments were run, first without a new shredding plant named experiment F. And in experiment G there is an additional shredding plant and vehicle depot nearby in a place called Bodegraven. Again, as in the previous section, each experiment F and G is conducted with four different vehicle sets. This leads to a total of 8 experiments shown in Table 9, the outcome of these experiments is shown in Table 10.

Table 9. Set of experiments for the west

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1 mobile, 1 regular and 1 special vehicle)</td>
<td>(2 regular and 2 special vehicles)</td>
<td>(2 mobile vehicles)</td>
<td>(4 special shredding vehicles)</td>
</tr>
<tr>
<td>F</td>
<td>F1</td>
<td>F2</td>
<td>F3</td>
<td>F4</td>
</tr>
<tr>
<td>G</td>
<td>G1</td>
<td>G2</td>
<td>G3</td>
<td>G4</td>
</tr>
</tbody>
</table>

Figure 27 Situation overview with a shredding plant in the west
Table 10 Overview of the experiment results for the west

<table>
<thead>
<tr>
<th></th>
<th>Driven distance</th>
<th>Pouches taken</th>
<th>Solution bound</th>
<th>Bound</th>
<th>Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>4.5475e+03</td>
<td>922</td>
<td>1464,5547</td>
<td>1728,3083</td>
<td>18,01%</td>
</tr>
<tr>
<td>F2</td>
<td>4.0096e+03</td>
<td>1084</td>
<td>1326,6143</td>
<td>1486,9441</td>
<td>12,09%</td>
</tr>
<tr>
<td>F3</td>
<td>2.5049e+03</td>
<td>820</td>
<td>1498,3766</td>
<td>2423,8601</td>
<td>61,77%</td>
</tr>
<tr>
<td>F4</td>
<td>4.0275e+03</td>
<td>1087</td>
<td>1453,1210</td>
<td>1648,2890</td>
<td>13,43%</td>
</tr>
<tr>
<td>G1</td>
<td>2.0969e+03</td>
<td>1080</td>
<td>2413,2937</td>
<td>2565,0552</td>
<td>6,29%</td>
</tr>
<tr>
<td>G2</td>
<td>1.6958e+03</td>
<td>1181</td>
<td>2837,7446</td>
<td>2975,2158</td>
<td>4,84%</td>
</tr>
<tr>
<td>G3</td>
<td>2.5169e+03</td>
<td>803</td>
<td>1413,0056</td>
<td>2331,3333</td>
<td>64,99%</td>
</tr>
<tr>
<td>G4</td>
<td>4.7383e+03</td>
<td>1204</td>
<td>2994,8182</td>
<td>3116,1313</td>
<td>4,05%</td>
</tr>
</tbody>
</table>

According to the results shown in Figure 28, when there is no shredding plant nearby, all the different vehicle sets behave more or less the same. Although, vehicle set F3 does have a chance at being a lot better than the other vehicle sets. However, this can’t be confirmed due to the large gap between the best found solution and the upper bound. However, when adding a shredding plant in Bodegraven, the vehicle sets appear to show large improvements. Only set G3 is comparable with F3 which is logical since those mobile vehicles don’t make use of the shredding plant. The other vehicle sets all make (partly) use of the shredding plant. The clear winner is to only use special vehicles, although a combination of special and normal vehicles also shows good performance.
When comparing the number of pouches taken from the customers it becomes clear that the mobile vehicles tend to take fewer pouches than the special vehicles even though their capacity is larger, see Figure 29. Although this can be partly explained by the fact that the special vehicles are supported by regular vehicles, another explanation is that the mobile vehicles are limited by the available working time rather than the vehicle capacity. This is a known fact and currently the mobile vehicles aren’t always completely filled because there is not enough time.

8.4 Other Observations

When comparing the number of pouches taken from the customers it becomes clear that the mobile vehicles tend to take fewer pouches than the special vehicles even though their capacity is larger, see Figure 29. Although this can be partly explained by the fact that the special vehicles are supported by regular vehicles, another explanation is that the mobile vehicles are limited by the available working time rather than the vehicle capacity. This is a known fact and currently the mobile vehicles aren’t always completely filled because there is not enough time.
When comparing the total distance travelled between the first set of experiments, see Figure 30, there is nothing unexpected to be seen. Mobile vehicles travel the longest when serving customers far away from the recycling plant since they only deal with this recycling plant. Normal and special vehicles show the opposite behaviour, they travel the longest distance when far away from the shredding plant. A combination of the different type of vehicles shows more or less the same results for the different areas, this is due to the fact that the vehicles behaviour cancel each other out.

Figure 29 Total number of pouches collected in each experiment

Figure 30 Total traveled distance in experiment A1 to E3
This research was conducted to improve the routing of EPS collection vehicles and to support strategic decision making. Research on literature has shown many VRP variants exist. Some previous research has been conducted to make a combination of these variants. However, these combinations weren’t rich enough to support the model of EPS collection vehicles without sacrificing some key details. Thus, this research continued to focus on creating a mathematical model which is rich enough to capture the specific attributes for the EPS collection vehicles. This model includes:

- Multiple depots
- Heterogeneous vehicles
- Capacitated vehicles
- Planning horizon
- Selective nodes.

Furthermore, this model is rich enough to support strategic decision making. Due to the complexity of the model, only small customer sets could be solved. This means that the model can be solved quickly enough for the current set of customers but in the future when the set is to increase, this model can no longer be solved by a generic mathematical model solver. Therefore future research should focus on creating a more efficient model or to develop heuristics to solve the model faster.

9.1 What is the relevant literature on VRP’s

A literature review has shown that there are a lot of VRP variants to solve different kinds of problems. However, to simulate reality, most of the times one would want to use multiple VRP variants in one problem. This combining of VRP variants is mentioned in a lot of literature as a recommendation for further research. Only recent literature cover this topic of rich combinations of VRP variants. This report also contributes to these rich problems.

9.2 Which processes play an important role within EPS NL and what are their characteristics?

The nodes and vehicle types were discussed and their characteristics were explained. The customers nodes require service from the vehicles and are quite common in literature. The vehicle depots are common starting and end points. The difficulty arises for the shredding and recycling plants, they need to be visited by specific vehicle types in a specific order. Regular vehicles are a common sight in VRP’s. The mobile vehicle can be seen as a commonly used service vehicle. However the special vehicle has multiple options for its trips which requires some extra attention in the model.

9.3 How to formulate the processes within EPS NL in a mathematical model?

The model can be used to generate an efficient route, depending on the planning horizon, number of customers and some other factors the solver will be able to give a near optimal solution in a reasonable amount of time. The comparison with a real-world scenario has shown an improvement of 12.8%. After
making the software more user-friendly, it could be used to generate optimal or close to optimal routes for the collection of EPS waste. Since the demand changes daily, there is no single optimal route.

9.4 How to adapt the model to answer managerial questions?

The experiments conducted with the model were small enough to be solved by the model and contain a realistic set of customers. First, a test was run to determine which customers could better be served by mobile shredding vehicles and which customers could better be served by regular and special vehicles. The result was an imaginary line or area around Zwolle which split the customers between the shredding and the recycling plant. All customers above this line can better be served with regular and special vehicles while the customers below this line can better be serviced by mobile vehicles. However, when a special vehicle takes shredded EPS from the shredding plant to the recycling plant, on its way back, it could also serve customers below the line. This line isn’t a hard rule, however, more a rule of thumb. Actual demand could see a more efficient routing plan with vehicles crossing this line.

The second test shows that adding a shredding plant in the west will definitively be more efficient than when there is no extra shredding plant. If there is an extra shredding plant, the customers should be served with regular and special vehicles, if however, there is no extra shredding plant, the mobile vehicles offer a more efficient route. Of course, the fixed cost for this new shredding plant should also be considered, as this is currently not included in the model.

Investigating the filling rate of the vehicles shows that mobile vehicles are not always completely filled due their time limit. A more efficient mobile shredding process could increase the mobile vehicle’s efficiency. Trying to increase the capacity does not lead to better results. On the other hand, special vehicles are more limited by their capacity, when they serve customers near the shredding plant, they might be available for multiple trips a day.

9.5 Further Research

Future research should focus on heuristics to improve solution speed and make larger problems solvable. One way to do so might be to use machine learning, solving the VRP by the help of machine learning isn’t researched at all but it could be an very interesting topic. The model could further be developed by adding more factors such as multiple trips on a day so that the vehicles can be utilized more. Furthermore, a stochastic future demand could be added to simulate the changes in customer demand, since this is not exactly known up front. The current model doesn’t include these factors. However, if it is to be used to determine the optimal route for all the vehicles, it is expected that these factors play an important role. Future research is also encouraged to adapt the model for different cases, other than EPS waste collection. By doing so the model can be used for other cases to better simulate reality.
References


Vehicle Routing for the Collection of EPS with Mobile Shredding Vehicles.
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Abstract. In times when there is more and more focus on environmental impact, all aspects must be taken into account. Expanded polystyrene (EPS) is easy and cheap to produce and has lots of applications because of its properties. These properties however, also make it a hard material to recycle. This paper focuses on a rich mathematical model for the collection of waste EPS in order to improve the recyclability of the material. A real-world scenario is simulated and the result is a model for routing different types of vehicles which can also be used to make strategic decisions.

1 Introduction

The recycling industry currently sees a growing interest, with environmental impact becoming a huge driver. Until now, the recycling of expanded polystyrene (EPS) has been held back due to inefficiencies. EPS is a white foam material also known as Styrofoam® or Airpop® that is widely used as packaging and insulation material because of its properties [8]. However, these properties also make the material inefficient to transport and hard to recycle. One way of recycling EPS is to shred it into small pieces (20-50mm) and together with virgin material and reuse it in the production of new EPS [7]. This can, for instance, be done with a mobile shredding vehicle which directly shreds the EPS at collection before moving to the next location. Such a system has been proven to work [9]. However, insufficient research is done on the economic feasibility of such a system.

This research compares a mobile shredding vehicle with a stationary recycling plant. For this purpose, a mathematical model has been formulated which describes the routing of mobile shredding vehicles and regular vehicles for the collection and recycling of EPS. The remainder of this paper is organized as follows: First, a literature review is conducted in section 2. Following is the formulation of the mathematical model in section 3. Section 4 presents a case study for the collection of EPS in the Netherlands. Finally, section 5 concludes the results and sets the direction for further research.

2 Existing literature

EPS

EPS is made of polystyrene pearls, these pearls are injected in a mold and expanded by adding steam [1]. This expanding process with steam requires a huge amount of energy (83 MJ/kg EPS) [2]. Several of its properties such as shock absorption, lightness, thermal isolation, and high moist resistance, make EPS an outstanding material for packaging and isolation [8].

These properties also make the material hard to recycle, due to its lightness, the transport costs per mass are very high and due to its durability, the material won’t degrade in nature. Common end of life stages include landfilling, incinerating, melting and shredding, and chemical recycling [4] [5] [6]. The shredded material can be directly mixed with virgin material to create new EPS [7]. A life cycle analysis by Martin & Hicks (2018) showed recycling more EPS would decrease the environmental impacts because the need for virgin material will be decreased and the required energy for expanding the virgin material will decrease [5].

VRP

The collection of EPS waste is a typical waste collection VRP with some of its own characteristics. The goal of the VRP is to visit some number of nodes with a set of vehicles while reducing the total amount of distance driven by the vehicles. There are many types of VRP’s for a recent overview see Braekers et al. [17]. Braekers et al. indicate some commonly researched VRP variants namely: Heterogeneous Fleet VRP (HFVRP) also known as the Mixed Fleet VRP (MVRP), the VRP with Time Windows (VRPTW), the VRP with Pickup and Delivery (VRPPD), VRP with Backhauls (VRFB), the Multi Depot VRP (MDVRP) and the Periodic VRP (PVRP). Braekers et al. found that some research combines a couple of these variants into rich vehicle routing problems to simulate real life aspects. They finally conclude that there is still a large gap with the real-life cases and state that VRP’s with even richer characteristics could be investigated.

EPS and Waste Collection

"Waste Collection VRPs (WCVRPs) differ from traditional VRP due to the number of characteristics and constraints that complicate the routing and require special handling" [18]. In the WCVRP, typically, waste is collected from customers and disposed at a disposal site. The vehicles start at a depot and return to this depot at the end of their journey. The collection of EPS differs from the standard WCVRP in the fact that the material needs to be processed before disposing it at the recycling plant. If only mobile vehicles are used, this problem is a typical WCVRP but
instead, the fleet of vehicles is heterogeneous and the other type of vehicles need to visit a shredding plant which is comparable to a transshipment facility with the difference that the material needs some time to be processed. To simulate the routing of the vehicles, a combination of the existing variants must be investigated to create a realistic model.

**HVFVRP**
The HVFVRP isn’t very different from the standard VRP.

In the HVFVRP there are different types of vehicles. Bula et al. [20] modelled a routing problem for the transportation of hazardous materials. The came up with an algorithm based on a Multi Start Variable Neighbourhood Search which resulted in equal or better solutions compared to a Mixed Integer Linear Programming (MILP) formulation. Leung et al. [21] considered the two-dimensional loading and routing of heterogeneous vehicles. This work is different in the fact that it focuses more on the loading of different vehicle types, there are six metaheuristics used to optimally load the vehicles. Their work is proven to obtain good solutions and they state that future research could be done on the inclusion of time windows. Penna et al. [22] developed a simply structured algorithm for the HVFVRP which was proven to be competitive with other known algorithms. They also suppose that future work could include more VRP variants.

**VRPTW**
In the VRPTW, customers can only be served within specified time windows. For an overview of exact, heuristic and metaheuristic methods on the VRPTW see [23]. Koskin and Çağatay [24] combined the VRPTW with electrical vehicles. They created a MILP as well as a matheuristic and showed that the heuristic was faster than solving the MILP with CPLEX. They suggest that further research could include heterogeneous vehicles and they are working on extensions to the model themselves. In [25], Sivaramakumar et al. demonstrate the importance of total time balance compared to route balance. They found that in the VRPTW the total time balance is much more important to balance the workload of the different vehicles. Sivaramakumar et al. also propose that future research could develop more advanced algorithms to reach 100% balance.

**VRPPD**
In the VRPPD, the vehicles need to deliver or pickup something at the customers. Ballestros Silva and Escobar Zuluaga [26] made an overview of recent papers on the VRPPD, one of their conclusions is that most of the work is focused on heuristics to solve the problem rather than exact methods. Yanik et al. [27] developed a hybrid algorithm for the VRPPD with multiple pickups and single deliveries which is quite like the problem this research is facing, they also include time windows in their model. They demonstrate the applicability of their algorithm and suggest further research to expand the problem.

**VRPB**
In [28], Koç and Laporte made an overview of recent literature about the VRPB. In the VRPB, there are customers who need deliveries and customers that need pickups. The deliveries need to be dispatched from a depot and the pickups need to be delivered at a depot. In their literature study, Koç and Laporte make a general conclusion that “there is room for a very significant research effort on models and solution methods for VRPBs”.

Recent literature on VRPBs focus on metaheuristics such as Tabu Search Approach [29], Two Level VNS [30] and Large Neighbourhood Search [31], they all found some improvements for specific types when compared to exact methods for the general VRPB.

**MDVRP**
The periodic VRP considers a planning horizon ahead. Archetti et al. [34] introduce a model for a Flexible Periodic Vehicle Routing Problem (FPVRP). They consider a set of customers which need to be serviced within a specified planning horizon. They presented a worst-case scenario and showed that the FPVRP has some advantages to a more generalized version of the problem such as the Periodic Vehicle Routing Problem (PVRP) and the Inventory Routing Problem (IRP). Kurz and Zäpfel [35] developed a heuristics-based solution method to solve the generalized PVRP, a case study showed the applicability to their research. They state that further research could extend their model with time windows or compare their model with other heuristics and multi-objective optimization problems.

**Subtour Elimination**
Generally, the formulated problems will generate subtours to prevent this, subtour elimination constraints must be added. There are mainly two subtour elimination formulations, the Dantzig, Fullerson and Johnson (DFJ) and the Miller, Tucker and Zemlin (MTZ) subtour elimination constraints.

**DFJ**
Dantzig et al. [36] formulated the DFJ subtour elimination constraints. These constraints impose that any subset of nodes has at least two arcs moving in or out of the subset. If this is not the case, the subset contains a subtour. This method requires $2^n$ constraints (where $n$ is the number of nodes). To reduce the number of constraints an iterative process could be used. At the start of this process not all DFJ constraints are added but only a small portion. The model is then solved, and if necessary, more constraints are added. The iterative process solves the model and adds more constraints as needed.

**MTZ**
Miller et al. [37] also introduced a set of subtour elimination constraint which later became known as the MTZ formulation. They add a variable which defines the order of a node in a route for a given vehicle. They then add a constraint which imposes that if a vehicle travel from node $i$ to node $j$, the order of $j$ is higher than that of $i$. Instead of using only the ordering of the nodes, the capacity of the vehicle and the amount taken or delivered at a node can be used as explained by Kulkarni and Bhave [10]. Kasa et al. [38] observed a notational mistake by Kulkarni and Bhave.
which prohibited some legal solutions. Desrochers & Laporte [39] also improved to MTZ subtour elimination constraints to include various types of vehicle routing problems. There is also an option to use time constraints instead to get to the same goal. This is a variation on the MTZ constraints as explained by Cordeau [40].

Summary

The research mentioned before varies in complexity. There are a lot of different models to solve problems which are very similar. As mentioned by Montoya et al. [32], research on VRP's with multiple depots is not common. Braeckers et al. [17] also indicated that more research should be done on rich combination of the VRP variants. A common suggestion of the other authors is that their work can be extended to include more of the VRP variants to better simulate a real-world case. To the author's knowledge, no research has been done on a VRP with multiple depots, heterogeneous capacitated vehicles with a planning horizon and selective nodes.

3 Model Formulation

Table 1

<table>
<thead>
<tr>
<th>Boolean decision variables</th>
<th>Integer decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{i,j,l}$</td>
<td>$D_{i,l}$</td>
</tr>
<tr>
<td>Arc $i, j \in N$ is traversed or not on by vehicle $k \in K$ on day $l \in L$</td>
<td>Number of pouches at node $i \in N^k$ on Day $l \in L$</td>
</tr>
<tr>
<td>$E_{i,l}$</td>
<td>$F_{i,l}$</td>
</tr>
<tr>
<td>Number of pouches at node $i \in N^k$ on Day $l \in L$</td>
<td>Number of shredded EPS at shredding plant $i \in N^k$ on day $l \in L$</td>
</tr>
<tr>
<td>$T_{i,k,l}$</td>
<td>$R_{i,k,l}$</td>
</tr>
<tr>
<td>Number of pouches picked up at node $i \in N^k$ by vehicle $k \in K$ on day $l \in L$</td>
<td>Total pouches shredding plant $i \in N^k$ received from vehicle $k \in K$ on day $l \in L$</td>
</tr>
<tr>
<td>$u_{i,k,l}$</td>
<td>$S_{i,l}$</td>
</tr>
<tr>
<td>Dummy variable for each $i \in N^c, k \in K, l \in L$ to eliminate subtours</td>
<td>Number of pouches shred by shredding plant $i \in N^k$ on day $l \in L$</td>
</tr>
<tr>
<td>$B_{i,j,l}$</td>
<td>$F_{i,l}$</td>
</tr>
<tr>
<td>Number of shredded EPS transported from shredding plant $i \in N^k$ to recycling plant $j \in N^c$ on day $l \in L$</td>
<td>Number of shredded EPS at shredding plant $i \in N^k$ on day $l \in L$</td>
</tr>
</tbody>
</table>

The model as seen in Figure 1 will consist of the following. Customers which produce EPS waste, shredding plants that shred the EPS waste, so it can be recycled, recycling plants which can use the recycled EPS and depots at which vehicles can be stored. There are three types of vehicles. Mobile shredding vehicles [9] that can shred the EPS at a customer location and deliver the shredded EPS directly to the recycling plant. Regular vehicles that can collect EPS from customers and deliver it to the shredding plants. And special vehicles that are equipped to deliver the small shredded pieces from a shredding plant to a recycling plant, these vehicles can also collect EPS at customers. The vehicles all have a limited capacity depending on the type. A working day consists of eight hours, in this time the vehicle must do all its movements including driving from and to the vehicle depot.

At all times the EPS is collected at customers and eventually delivered at recycling plants.

![Figure 1 Material flow](image)

Let $a$ denote the number of depots, $b$ the number of shredding plants, $c$ the number of recycling plants and $d$ the number of customers to be served. The problem may be defined on a complete directed graph $G = (N,A)$ that consists of the set of nodes $N = N^d \cup N^c \cup N \cup N^p$, where $N^d = \{1 \ldots a\}, N^c = (a + 1 \ldots a + b)$, $N = (a + b + 1 \ldots a + b + c)$, $N^p = (a + b + c + 1 \ldots a + b + c + d)$ and the set of arcs $A = \{(i,j) : i, j \in N, i \neq j\}$. Subsets $N^d, N^c, N$ and $N^p$ represent the depots, shredding plants, recycling plants and customers respectively. Let $K = K^m \cup K^a \cup K^s$ be the set of vehicles where $K^m = \{1 \ldots e\}, K^a = \{e + 1 \ldots e + f\}$ and $K^s = \{e + f + 1 \ldots e + f + g\}$.

Subsets $K^m, K^a$ and $K^s$ represent the mobile shredding vehicles, the regular vehicles and the special vehicles respectively. And finally let $L_1 = L_2 \cup L$ be the set of days where $L_1 = \{0\}$ and $L = \{1 \ldots h\}$. Subsets $L_2$ and $L$ represent the initiation day and the working days respectively.

$\delta_{i,j}$ and $\tau_i$ represent the distance and respectively time, associated with traversing arc $(i,j) \in A$. $\tau_i$ represents the stop time at node $i \in N$. For each node $i \in N^c$ there is an associated $P_{i,j}$ and $D_{i}^{arc}$ which represent the production on day $l \in L$ and the maximum number of pouches which may be stored respectively. With each vehicle $k \in K$, there is an associated load time per pouch $\tau_i$ and a capacity $C_k$.

Table 2 gives an overview of the used parameters in the model and Table 3 gives an overview of the decision variables used in the mathematical model.
Table 2
Specification of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 )</td>
<td>Price of a pouch that is shredded and delivered at the recycling plant</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>Price for a pouch that is collected from a customer</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>Cost per minute of work</td>
</tr>
<tr>
<td>( \lambda_4 )</td>
<td>Cost per driven km</td>
</tr>
<tr>
<td>( \lambda_5 )</td>
<td>Cost for shredding a pouch at the shredding plant</td>
</tr>
</tbody>
</table>

Objective

\[
\text{maximize } \lambda_1 \sum_{i \in N} \sum_{j \in K} \sum_{k \in L} T_{i,k,l} + \lambda_1 \sum_{i \in N} \sum_{j \in K} \sum_{l \in L} B_{i,j,l} + \lambda_2 \sum_{i \in N} \sum_{j \in K} \sum_{l \in L} T_{i,k,l} \]

\[
- \lambda_3 \left( \sum_{i \in N} \sum_{j \in K} \sum_{k \in L} T_{i,k,l} + \sum_{i \in N} \sum_{j \in K} \sum_{k \in L} T_{i,k,l} + \sum_{i \in N} \sum_{j \in K} \sum_{k \in L} T_{i,k,l} + \lambda_3 \sum_{i \in N} \sum_{j \in K} \sum_{l \in L} T_{i,k,l} \right) \]

\[
- \lambda_4 \sum_{i \in N} \sum_{j \in K} \sum_{k \in L} x_{i,j,k} \beta_{i,j} - \lambda_5 \sum_{i \in N} \sum_{j \in K} \sum_{l \in L} S_{i,j} \quad (1) \]

Subject to

\[
D_{i,l} = D_{i,l} - \sum_{k \in K} T_{i,k,l} + R_{i,l} \quad \forall l \in L \quad (2) \]

\[
D_{i,l} \leq D_{i,l}^{\text{max}} \quad \forall l \in L \quad (3) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in L} T_{i,k,l} \leq C_k \quad \forall k \in K, l \in L \quad (4) \]

\[
T_{i,k,l} \leq C_i \sum_{j \in N} x_{i,j,k,l} \quad \forall i \in N, k \in K, l \in L \quad (5) \]

\[
x_{i,j,k,l} = 0 \quad \forall l \in L \quad (6) \]

\[
\sum_{i \in N} x_{i,j,k,l} = \sum_{l \in L} x_{i,j,k,l} \quad \forall j \in N, k \in K, l \in L \quad (7) \]

\[
\sum_{i \in N} x_{i,j,k,l} = \sum_{l \in L} x_{i,j,k,l} \quad \forall j \in N, k \in K, l \in L \quad (8) \]

\[
\sum_{i \in N} x_{i,j,k,l} = \sum_{l \in L} x_{i,j,k,l} \quad \forall j \in N, k \in K, l \in L \quad (9) \]

\[
\sum_{i \in N} x_{i,j,k,l} = \sum_{i \in N} x_{i,j,k,l} \quad \forall j \in N, k \in K, l \in L \quad (10) \]

\[
x_{i,j,k,l} = 0 \quad \forall k \notin K^* \quad (11) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \geq B_{j,l} \quad \forall j \in N, k \in K^*, l \in L \quad (12) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \leq E_{i,j} \quad \forall i \in N \quad (13) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \leq F_{i,j} \quad \forall i \in N \quad (14) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \leq G_{i,j} \quad \forall i \in N \quad (15) \]

\[
v_{i,j,k,l} - u_{i,j,k,l} + \text{MaxC} \cdot x_{i,j,k,l} + (\text{MaxC} - 2) \cdot x_{i,j,k,l} \leq \text{MaxC} - 1 \quad \forall i \in N, j \in K, k \in L \quad (16) \]

\[
0 \leq u_{i,j,k,l} \leq \text{MaxC} \quad \forall i \in N, j \in K, k \in L \quad (17) \]

\[
\sum_{i \in N} \sum_{j \in K} x_{i,j,k,l} \leq 1 \quad \forall k \in K^*, l \in L \quad (18) \]

\[
\sum_{i \in N} \sum_{j \in K} x_{i,j,k,l} = \sum_{i \in N} x_{i,j,k,l} \quad \forall k \in K^*, l \in L \quad (19) \]

\[
\sum_{i \in N} \sum_{j \in K} x_{i,j,k,l} = \sum_{i \in N} x_{i,j,k,l} \quad \forall k \in K^*, l \in L \quad (20) \]

\[
x_{i,j,k,l} \in \{0, 1\} \quad \forall l \in L \quad (21) \]

\[
R_{i,k,l} \leq C_k \sum_{j \in N} x_{i,j,k,l} \quad \forall k \in K, l \in L \quad (22) \]

\[
R_{i,k,l} \geq 0 \quad \forall k \in K, l \in L \quad (23) \]

\[
R_{i,k,l} \leq \sum_{i \in N} \sum_{j \in K} x_{i,j,k,l} \quad \forall k \in K, l \in L \quad (24) \]

\[
R_{i,k,l} \geq -C_k \left( \sum_{j \in N} x_{i,j,k,l} \right) \quad \forall k \in K^*, l \in L \quad (25) \]

\[
E_{i,j} = E_{i,j} + \sum_{k \in K^*} \sum_{l \in L} R_{i,k,l} \quad \forall i \in N \quad (26) \]

\[
E_{i,j} = E_{i,j} + \sum_{k \in K^*} \sum_{l \in L} S_{i,j} \quad \forall i \in N \quad (27) \]

\[
S_{i,j} \leq E_{i,j} \quad \forall i \in N \quad (29) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \geq B_{j,l} \frac{G_{i,j}}{C_k} \quad \forall j \in N, k \in K^*, l \in L \quad (28) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \leq E_{i,j} \quad \forall i \in N \quad (30) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \leq F_{i,j} \quad \forall i \in N \quad (31) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \leq G_{i,j} \quad \forall i \in N \quad (32) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \leq DM_{i,j} \quad \forall i \in N \quad (33) \]

\[
\sum_{i \in N} \sum_{j \in K} \sum_{k \in K^*} \sum_{l \in L} x_{i,j,k,l} \leq DM_{i,j} \quad \forall i \in N \quad (34) \]
Constraints
Constraints (2) define the number of pouches at the customer nodes. Constraints (3) impose that the number of pouches at the nodes stay below the threshold if that node is used in the model. Constraints (4) impose that the sum of loaded pouches does not exceed the vehicles capacity. Constraints (5) impose that if a vehicle retrieves pouches from a node then that node is visited. Constraints (6),(7),(8),(9),(10),(11),(12),(13),(14) and (15) are flow constraints which force or permit some flows. Constraints (6) prohibit the use of an arc that returns to itself. Constraints (7) impose that mobile and regular vehicles can only reach customers from another customer or a depot. Constraints (8) impose that special vehicles can only reach customers from another customer, depot or a recycling plant. Constraints (9) impose that mobile shredding vehicles which leave a customer can only go to another customer or a shredding plant. Constraints (10) impose that regular and special vehicles that leave a customer can only go to another customer or a shredding plant. Constraints (11) impose that if a vehicle moves from a shredding plant to a recycling plant then it must have come from a depot or a customer towards that shredding plant. Constraints (12) prohibit special vehicles to move from a shredding plant to a customer. Constraints (13) impose that special vehicles leave a shredding plant every time it arrives there from a customer and when it transports something from the shredding plant directly to the recycler. Constraints (14) impose that if a special vehicle is used on a given day, it must go back to a depot. Constraints (15) impose that if a node is visited, it is also visited. Constraints (16) and (17) are MTZ subtour elimination constraints. MTZ constraints are chosen over DFJ constraints to reduce the amount of constraints as explained by van Tol et al. (2016) [41]. Constraints (18) assure that each vehicle leaves the depot not more than once on a given day. Constraints (19) and (20) ensure that if a vehicle is used on a day, it will visit a recycling plant (19) or shredding plant (20) depending on the vehicle type, and then return to the depot. Constraints (21) make sure that a working day isn’t longer than 8 hours. Constraints (22),(23),(24) and (25) impose that if a regular or special vehicle’s route goes through a shredding plant then all its collected pouches are unloaded at that shredding plant. Constraints (26) impose that the number of pouches at a shredding plant on a given day is equal to the amount on the day before plus the pouches delivered at that plant minus the pouches processed on the day before. Constraints (27) impose that the number of processed pouches at a recycling plant on a given day is equal to the number of processed pouches on that day plus the number of processed pouches on the day before minus the number of pouches transported to the recycling plant. Constraints (28) impose that the amount of special vehicles transporting shredded EPS from a shredding plant to a recycling plant is enough to cover the total transportation from the shredding plant to the recycling plant. Constraints (29) impose that the number of EPS pouches shredded on a given day is less or equal to the number of pouches available on that day. Constraints (30) impose that the amount of shredded EPS transported from a shredding plant to a recycling plant is less or equal to the amount of available shredded EPS at the shredding plant. Constraints (31) impose that the total amount of pouches that is picked up satisfies at least the amount that the customer has ordered. Constraints (32),(33) and (34) limit the number of vehicles available at a depot.

4 Experiments
To check the performance of the model, a real life case study is done. In this case study, a company has one of each type of vehicle. The company has 12 customers which are located much closer to the recycling plant than the shredding plant, this is shown in Figure 2. One week of real data was implemented in the model, during this week only 7 customers where actually served. The model was solved by CPLEX on a machine using an Intel Core i7-3610QM 2.3 GHz with 8 GB of RAM. Compared to the real-world data, the solution of the model was 12.8% more profitable.

Besides using the model for route optimization, it can also be used to make strategic decisions. For example, the mobile vehicles are located in the south of the Netherlands near the recycling plant while the regular and special vehicles are located in the north of the Netherlands near the shredding plant. To answer the question of which vehicle can better serve which customer, a set of experiments was done. The area between the shredding and recycling plant was divided into 5 equally sized spaces A through E. For each experiment, 10 customers were simulated in these areas to keep reasonable solving time. Doubling the number of customers would increase the solution time to a state that the used pc would run out of memory before finding a reasonable solution because of the complexity of the model. For each area, four experiments where conducted. In the first experiment, there is one vehicle of each type to represent the company’s current fleet of vehicles. For the second, the mobile shredding vehicle is replaced by an extra normal and special vehicle. The third experiment consisted of two mobile vehicles which replace the normal and special vehicle. Finally, an extreme vehicle set containing four special vehicles is analyzed.

Figure 2 Map of the case study

![Figure 2 Map of the case study](image)

Figure 3 Results of experiment A through E
The CPLEX solver found a solution for each experiment, but some combinations showed a clear advantage over the others. The results are shown in Figure 3. For example, near the shredding plant, the second set of vehicles proved to be
the best however near the recycling plant the third set of vehicles was best. More near the middle, all of the experiments showed equal performance. This leads to a certain division line above which the customers can be better served by regular and special vehicles and below this line, the mobile vehicles are more efficient.

Another strategic decision which was tested was whether to open up a shredding plant in the west to serve customers in that area or not. The four combination sets of vehicles were tested twice, once without a new shredding plant (experiment F) and once with a new shredding plant (experiment G). The results are shown in Figure 4, this clearly shows that without a new shredding plant, the mobile vehicle would be the best choice. However, when a new shredding plant was added, the regular and special vehicles operated much better. Of course the costs of a new shredding plant also needs to be taken into consideration in order to make a final decision.

![Figure 4 Results of experiments F and G](image)

Finally, the details of the different solutions showed some bottlenecks which could be improved. The mobile vehicles were not completely filled most of the times due to the limited available working time. These vehicles could operate more efficiently if they could process the collected packages faster. Furthermore, the regular and special vehicles which filed to the top most times and had plenty of time left to make a second trip on a given day. Currently this is not implemented in the model and this should reduce the number of vehicles needed to serve all customers. Since the vehicles are considered as fixed costs and not taken into account by the model, this wouldn’t change the strategic decision making. However, the routing of the vehicles could change a bit if they are used multiple times a day.

5 Conclusion

In this paper, a routing model for EPS waste collection is discussed. First an extensive literature review was conducted on several types of VRP’s. None of these VRP’s could completely simulate the routing of EPS waste collection vehicles thus a new model was generated with the help of existing models. This new model is a much richer combination of VRP’s than seen in the research from the literature review. However, despite the complexity of the problem, it can be used to solve small problem instances. A case comparison showed an improvement of 12.8% compared to manually routing the vehicles.

Due to the complexity of the model and the large set of possible customers, this model cannot be used together with a general solver to solve large instances which would occur in reality. To solve such rich problems, heuristic algorithms need to be developed for a specific problem. Another possible way to solve such problems would be when computing research reaches a major breakthrough which is much better for solving such problems, for example quantum computing.

Although the model can only be used to solve small instances, this can be enough for strategic decision making. According to the experiments, two strategic decisions where advised. First the model was used to determine the trade-off between using a mobile shredding vehicle or a shredding plant for customers at different locations. Secondly, the experiments show that opening a new shredding plant in the west would be the best way to serve customers in that area. Finally, thanks to the model and the experiments, some bottlenecks could be identified. For example the mobile shredding vehicles never reached their capacity because their working time was over.

Future research should study the applicability of heuristics for such large problems. Complex routing problems might also be solved with the help of machine learning. Also, the model could further be developed by adding more factors such as multiple trips on a day and a stochastic future demand. The current model doesn’t include these factors. However, if it is to be used to determine the optimal route for all the vehicles it is expected that these factors play an important role.

References


