WELL TRAJECTORY OPTIMIZATION

Ashish Kumar Loomba

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Abstract

Delineating the well placement and trajectory of production or injection wells is an important step of any field development program. Drilling a well is an expensive process in terms of money, time and effort invested. Being an expensive item, wells must be carefully studied before being drilled. In order to make this process more efficient, this thesis presents a computer-assisted approach of determining the optimal well trajectory using adjoint-based optimization technique.

The algorithm is based on surrounding the injection or production wells with dummy wells; a technique proposed in earlier studies. These dummy wells have a minimal rate of injection or production so as to minimalize their influence on the simulated output. The sum of the gradients of the objective function with respect to the flowrate in each dummy well over the life-time of the reservoir is used to define an improved well trajectory. The whole process is repeated until the maximum net present value is achieved. In order to ensure that the newly optimized well trajectory is drillable, the algorithm restricts the curvature of the trajectory. The optimization algorithm was applied to synthetically generated homogeneous and heterogeneous reservoirs to improve a single well trajectory. The algorithm was also successfully tested to improve multiple well trajectories in the ‘Egg Model’, a three-dimensional heterogeneous channelized reservoir model.

In addition to well trajectory optimization, the thesis also presents an approach to optimize well numbers. This algorithm is based on drilling a dense quasi-well configuration to get an initial knowledge of the reservoir and utilizing this knowledge along with ‘dummy well’ gradients to reduce the well count.

Depending on the reservoir, fluid and economic parameters, both the algorithms display the adeptness to predict optimized type, trajectory and number of wells. Although the optimization results showed a considerable improvement in the net present value of the project, the algorithm can get stuck in a local optima.
Acknowledgements

This project is, undoubtedly, a result of productive contribution of many supportive and generous people. I owe a great many thanks to so many people who helped and supported me during the compilation of this thesis.

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Last but not least, I would like to dedicate this project to the loving memory of my late father, who always stood by my side and encouraged me to strive for excellence.

Ashish Kumar Loomba
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1 Introduction

1.1 General Background

Well number, location and trajectory design is one of the crucial aspects of Field Development Program (FDP). Optimal well trajectories, with a good connectivity between the well and the reservoir, are essential to maximize the objective(s) of the FDP.

Well trajectory design is an arduous task to estimate the best ways to drill and perforate the desired reservoir layers to achieve pre-defined company objective(s). Besides, wells are an expensive part of the development phase and thus, they must be carefully studied and planned. Hence, designing a well trajectory demands an interdisciplinary team effort calling for inputs from geologists, geophysicists, reservoir engineers and drilling engineers. Many parameters affect the well trajectory placement including but not limited to field location (offshore/onshore), location of the rig, cost effectiveness, total drilling length, target zones, dogleg severity, step-out distance, basic well design, and the objectives of maximizing Recovery Factor (RF), economic values and/or production. Based on these parameters, the well trajectory design can either be vertical, deviated or horizontal. Usually, a well trajectory is determined after numerous discussions about the aforementioned cost and efficiency factors.

Since the well locations are usually represented as integer well gridblock indices in reservoir simulators, well path design is treated as a discrete optimization problem.

1.2 Literature Review

This chapter covers a brief overview of the previous work in the field of gradient-based well placement and well trajectory optimization and gradient-free optimization techniques.

1.2.1 Gradient-based Well Placement Optimization

There are two notable methods of well placement optimization: gradient-based and non-gradient-based. It is advantageous to use a gradient-based optimization technique to improve the computational speed albeit, it cannot differentiate between global and local optimal solution. At the same time, it demands writing complicated computer-based codes.

The gradient-based well placement optimization was established by Zandvliet et al. (2008). This paper was based on a conference paper with the same title but a different first author: Handels et al. (2007). The proposed method finds the optimal location of vertical wells using eight ‘pseudo-wells’ surrounding the well whose location has to be optimized. These pseudo-wells produce (or inject) at very low rate compared to their corresponding main production (or injection) well. The adjoint method is used to calculate the gradients of the Net Present Value...
(NPV) over the prescribed lifespan of the reservoir with respect to flowrates in pseudo-wells. This is utilized to calculate the improving direction to achieve better well placement.

Besides demonstrating an approach to determine the optimal well locations using a gradient-based optimization method, the authors also exhibited that production constraints significantly influence the well placement optimization problem. Three different production constraints were run for three different terminal times and the results demonstrated the need to use as realistic production constraints as possible to obtain the best well placement results. Although this adjoint-based optimization technique may get stuck in local optimal solution, the paper presents the significance of using adjoint method to compute improving directions for all wells in only one forward and one backward simulation.

Inspired by the work of Handels et al. (2007), Sarma et al. (2008) presented an efficient well placement method with the direct application of gradients. Contrary to Handels et al. (2007), the gradients of the objective function were calculated with respect to well location indices. Instead of the original discrete-parameter well placement problem, a continuous approximation was defined in order to compute the gradients and thus obtain the optimal well locations with standard gradient-based algorithms and adjoint models. This continuous approximation was achieved and displayed by using a bivariate Gaussian function instead of the Dirac-delta function in the governing equations. Discretizing the equations resulted in modification to the well terms (and approximations to the original well terms) in the mass balance equations. Similar to Handels et al. (2007), the original wells were surrounded by pseudo-wells but the search direction and the step size weren’t limited. The authors confirmed the practical applicability of the approach on a few synthetic optimization problems.

Keeping the production optimization in mind, an alternate method of optimal well placement was proposed by Wang et al. (2007). The algorithm suggested placing an injector well in each gridblock (except the gridblocks containing producer wells) and then optimizing the NPV (objective function in this case). To ensure the elimination of the injector wells during the NPV optimization process, the authors added a drilling well costs term in the NPV calculation function. As the cost of drilling a well detracts from the NPV, elimination of most of the non-optimally located wells becomes inevitable. In order to further decrease the number of wells, the authors also included a bound constraint and hence constrained the problem by postulating a constant total injection rate. This constraint accompanied by a steepest ascent algorithm is used to redistribute the injection rates among the wells remaining at each iteration of the NPV optimization process to maximize NPV over a specified reservoir lifetime. Although the algorithm take numerous iteration steps to converge (only one well can be deleted in each iteration step), it demonstrated consistency and exactitude in two simple 2D reservoir cases.

Improving the work of Wang et al. (2007), Zhang et al. (2010) presented a much more efficient method of finding optimal well placement locations using gradient project method instead of steepest ascent method. The gradient projection method was used to efficiently handle all constraints and eliminate several injectors during a single iteration in the early stages. In total, the developed algorithm by Wang et al. (2007) and Zhang et al. (2010) solved the well
placement problem without knowing a priori the optimal number of injectors, locations and well rates.

After the initial developments in the well optimization placement by Wang et al. (2007) and Zhang et al. (2010), Forouzanfar et al. (2010) proposed a two-stage well placement optimization method based on the methodology introduced in Wang et al. (2007) and Zhang et al. (2010). The first initialization stage in the methodology determines appropriate total production/injection rates for a realistic operational lifetime of a reservoir. The second stage is executed to estimate the optimal number of wells, their optimal locations and flowrates. The authors also present a practical way for imposing nonlinear bound constraints on the bottomhole pressure of each well. This helps in guaranteeing a check on the bottomhole pressure at all times. The authors also exhibited through case studies that the developed two-stage procedure is a computationally efficient method generating significantly higher value of objective function (NPV in this case) compared to single-stage procedure. The algorithm presents a robust way of presenting solution to well placement optimization at a cost of numerous reservoir simulations.

1.2.2 Gradient-based Well Trajectory Optimization

A method to automate the process of optimizing the trajectory of production well was first proposed by Vlemmix et al. (2009). Motivated by the advancements in the adjoint-based well location optimization, the authors extended the method of Handels et al. (2007) to determine an optimal well trajectory in a three-dimensional reservoir model. Since optimal well trajectory is crucial to avoid gas cusping and water coning, the authors verified the method on a thin oil rim reservoir with a relatively large gas cap and aquifer.

The Vlemmix et al. (2009) method is based on surrounding each trajectory point with ‘pseudo-side tracks’. These fictional ‘pseudo-side tracks’ have very small perforations and thus production rates. The reason that this approach was chosen over placing vertical pseudo wells in each gridblock is that the effect of the side tracks on the total well behaviour including lift and well bore friction can be taken into account. The gradients of the objective function with respect to a dimensionless multiplication factor for perforations (interpreted as a ‘pseudo valve representation’ of an Inflow Control Valve) are used to find the improving directions for the trajectory points.

Two of the main disadvantages of the method were restriction of the trajectory movement to only two directions and restriction to optimize the well length. Additionally, the adjoint-based optimization technique may get stuck in a local optimal solution. The method employs drillability/smoothing algorithm to ensure dogleg severity is below the predefined limit. This ensures that the final well trajectory is drillable and realistic.

This approach confirmed the scope for developing an algorithm to predict the well trajectory optimization and significantly improve the overall efficiency during FDP.
1.2.3 Gradient-free Optimization

Gradient-free optimization technique is another widely used method of optimizing well placements and trajectories. Although it is a computationally demanding methodology and numerous reservoir simulations have to be executed, it can in theory, capture the global solution. Several efficient algorithms have been established so far by means of Genetic Algorithm (GA), Artificial Neural Network (ANN), Simulated Annealing, etc. to compute the optimal well placement and trajectory.

Yeten et al. (2002) presented a novel method for the optimization of the type (number of laterals), location and trajectory of nonconventional wells (NCWs). The authors presented a procedure of optimizing NCW using GA. GAs are stochastic search algorithms based on the general principles of Darwinian evolution and hence require numerous simulation runs. So as to moderate the actual number of simulations and improve the efficiency, GAs are used in conjunction with three routines: ANN, hill climber and a near-well upscaling technique. While ANN is used as a proxy to the reservoir simulator, hill climbing procedure enhances the search in the immediate neighbourhood of the solution and near-well upscaling methodology speeds up the finite difference simulation runs. Besides, the authors also successfully attempted to account to the effects of reservoir uncertainty in few cases by including numerous equiprobable geostatistical realizations of the reservoir in the calculations of the objective function.

Another method of optimizing well location was presented by Badru et al. (2003) to maximize the NPV in an FDP. The paper presented the use of Hybrid Genetic Algorithm (HGA) in conjunction with a reservoir simulator. HGA includes GA, polytope algorithm and a kriging proxy. Although the algorithm optimizes the number and location of wells, the produced results were strongly dependent on process variables (completion, recovery process, production/injection rates, project life, etc.). Various field-scale example reservoirs presented in the paper demonstrate its worth of judging locations as good as engineering judgment.

Emerick et al. (2009) proposed a robust algorithm of simultaneously optimizing the number, placement and trajectory of production/injection wells. To deal with well placement constraints, the developed software uses Genocop III – Genetic Algorithm for Numerical Optimization of Constrained Problems. The software is capable of dealing with realistic reservoirs with arbitrary well trajectories, complex model grids and linear and non-linear well placement constraints. Although initializing population randomly or using engineer’s proposed base case are two possible ways of assigning initial population for the GA, the paper advocates the idea of starting the optimization process from the initial guess of engineers to achieve the “improved engineer’s base case” which, in general, is the best possible solution to the problem. Based on the study of a synthetic case, the authors also concluded that the reservoir quality maps can be used for complex cases (or time-restricted scenarios) to define the initial well locations followed by optimization of type and number of wells. Full-field reservoir models were tested using the developed software to demonstrate the reliability of the algorithm.

Lee et al. (2009) presented a paper on designing economically optimized wells by GA. The authors proposed using GA with a node-based configuration to get robust and more realistic well-designs considering location, trajectories and interwell interference. The developed
algorithm is capable of designing both vertical and horizontal wells (with several kick-off points) while improving the objective of the FDP. It was also duly presented that the interwell interference yields a comparatively lesser NPV than a non-interwell interference scenario.

Beckner et al. (1995) proposed a cost-effective method to optimize the well schedule and placement plan for the FDPs using simulated annealing algorithm in conjunction with a reservoir simulator and an economic analysis module. Although the paper does not describe an algorithm to optimize the well location, the field-scale example reservoir presented in the paper demonstrated differing responses of well positions and scheduling for areal variations in reservoir properties and/or well costs and complete times. Moreover, a non-uniform well spacing is optimal for development to avoid effects on well productivity from well interference.

1.3 Outline of the thesis

Motivated by the proposal of Vlemmix et al. (2009), the thesis intends to further extend his proposal of using a computer-assisted process of optimizing the well trajectory. In order to do so, the thesis establishes a new method to assist the determination of the optimal well trajectory with the aid of adjoint-based technique. The algorithm has been developed to predict improved well trajectories in three-dimensional reservoirs.

Given the initial well number, location and perforations, the first part of the thesis aims at maximizing the objective function by improving the well trajectory. To maintain practicality, it has been ensured that the well trajectories are drillable by bringing dogleg severity into play. In order to better understand the method, the report also discusses the results obtained by testing the algorithm on the ‘Egg Model’.

Chapter 2 covers a detailed description of the algorithm and the various concepts used to build it. In the first section, the basic assumptions and an overview of the optimization process have been presented. This is further broken up into smaller sub-sections on the objective function, the concept of dummy wells and their assistance in finding the improving direction and revised position. This is followed by discussion on multi-point trajectories and their involvement in rotating and modifying the well types. The dogleg severity, controlling the drillability of the well, has also been discussed, followed by a small section on termination/convergence algorithm.

Chapter 3 presents the results of optimizing the well trajectories in a homogeneous and heterogeneous synthetic models. The results help in understanding the behaviour of the well trajectory under the provided reservoir models. The optimization procedure was also successfully tested on the ‘Egg Model’ reservoir. Multiple trajectories were optimized, producing encouraging results.

Chapter 4 forms the second part of the thesis where a new multi-stage method of optimizing the number of wells and their placements is proposed. The method exercises the concept of dummy wells to generate gradients and reduce the well count. The methodology of the proposed algorithm is followed by three examples.
To conclude, Chapter 5 summarizes the observations and the scope of improving the computer-assisted process of well number, placement and trajectory prediction.
2 Methodology

2.1 Formulation

This chapter introduces the concept of optimizing the trajectory of any well using the gradient method. Adjoint-based optimization (Appendix B) has been implemented to obtain the best location for the well(s). To keep things simple, this chapter presumes that the production well location and trajectory is known while the optimal trajectory of an injector well needs to be determined in order to optimize the objective function. The procedure stays the same for optimizing the production wells and one can easily figure out the analogies. The following subsections describe the principal assumptions (section 2.1.1) and the overview (section 2.1.2) of the whole optimization process.

2.1.1 Assumptions

Following are the key assumptions:

- Fluid Properties: Two-phase fluid (oil-water).
- Rock Properties: Incompressible.
- Producer wells: Operated on prescribed Bottom-hole Pressure (BHP).
- Injector wells: Operated on prescribed rate while ensuring that the injection pressure does not exceed a predefined maximum injection pressure. If the injection pressure exceeds the maximum allowed pressure, the injector starts operating on prescribed maximum BHP.
- Injection rate: Pre-defined.
- Well Index: Projection well index has been used (Appendix A).

2.1.2 Overview

This section gives an overview of the process of optimizing well trajectory. As illustrated by Fig. 2.1, the whole process is started by initializing the reservoir parameters (reservoir geometry, rock and fluid property, etc.) and the simulation parameters (number of simulations, etc.). The algorithm helps the user to define the number of producers and perforated gridblocks, radius of the wellbore, reservoir simulation steps, initial reservoir pressure and phase saturation. Besides, kick-off point can also be honoured by fixing the heel position as per the requirement of the user.
Once the parameterization has been completed, an initial well trajectory is obtained from the user and dummy wells are introduced. These dummy wells (Section 2.3) are used to calculate an improving direction (Section 2.4) by calculating the change in the NPV for a unit change in injection rate. In order to calculate the gradients, a forward reservoir simulation and a backward adjoint simulation are performed. The gradients are then used to compute and define a new trajectory position using the maximum gradient. If the new trajectory position is not drillable, a smoothing step is taken. If the NPV is increased, the optimal trajectory is displayed; otherwise, the trajectory of the well is updated.
adjoint simulation are run. The maximum gradient values are used to ascertain the improving direction. It must be noted that the minimum gradient values are used to define the improving direction for the producers since the flowrate is conventionally negative in magnitude. Although the improving direction points toward the best location for the corresponding step, it is necessary to make sure that the new trajectory is drillable. This is guaranteed by using the dogleg severity (DLS) as drillability constraint. The whole process is reiterated until a trajectory with highest objective function is reached or the termination/convergence criteria (Section 2.7) is applicable.

2.2 Objective function

As mentioned earlier, the main goal of the thesis is to find the best trajectory for a well so that the objective of the FDP is maximized. The objective function can be either economic value of the project (NPV) or RF. In the developed algorithm, the NPV of the project has been considered as the objective:

\[ J = NPV. \]

In a further section of this chapter, the objective function has been expanded and rewritten as a function of the controllable inputs and output variables to give a better idea.

2.3 Concept of Dummy Well

In section 2.1.2, dummy wells were introduced as an important component of the whole process. This chapter explains dummy wells, their implementation and purpose.

![Diagram of dummy wells surrounding a horizontal injector well](image-url)

**Figure 2.2:** Four dummy wells (red) surrounding two consecutive nodes of the main horizontal injector (dashed blue) well in a two-dimensional reservoir
In order to find the optimal location for a trajectory, Vlemmix et al. (2009) surrounded the main production well by an imaginary pseudo-side track to all adjacent gridblocks in each direction. This helped the authors to move the trajectory points (based on the gradients) and hence move the whole trajectory towards the optimal solution. Quite similar to their approach, this thesis presents a method for optimizing the trajectory of a well using dummy wells. A dummy well is nothing but a replica of the original trajectory segment in six directions, i.e. $\pm x, \pm y$ and $\pm z$ as shown in Fig. 2.2. Please note that two more dummy wells are created in a three-dimensional reservoir. These imaginary dummy wells are placed at a certain distance parallel to the main trajectory well segments. This approach is advantageous to optimize both the producer and injector well trajectory in a three dimensional environment.

The dummy wells inject (or produce) at a negligible rate and hence hardly affect the overall flow behaviour of the reservoir. The beauty of using the dummy well is that it helps to find the improving direction (Section 2.4) which in turn helps to change the well trajectory in such a way that the objective function is improved. These efficient dummy wells also assist in changing the direction of the trajectory as explained in the next section.

### 2.4 Improving Direction and Revised Position

After accepting an initial well trajectory from the user, the algorithm intends to delineate a new well trajectory so that a higher NPV can be achieved. As the name suggests, an improving direction guides the well in the direction of the highest objective function value. Once the dummy wells are generated for each well segment, the sum of the gradients of the objective function with respect to the flowrate in each dummy well (parallel to a particular trajectory segment) over the life-time of the reservoir is calculated (Appendix B):

$$
g = \sum_{k=1}^{K} \frac{\partial \mathcal{J}}{\partial u_{d,k}}.
$$

Here $g$ denotes the summed gradient, $u_{d,k}$ denotes the flowrate $u$ in dummy well $d$ at time-step $k$ and $K$ denotes the total number of simulation time steps. The computed gradients are used to figure out the improving direction. The highest summed gradient value and the corresponding dummy wells are nominated (for each individual direction). These selected dummy wells and their positions are then used to define the new and improved location of the well trajectory with the help of a weighing factor $\beta$:

$$
x_i^{i+1} = \beta p_i^j + (1 - \beta)x_i^j,
$$

where $x$ denotes the position vector of the $i^{th}$ trajectory point, $j$ denotes the iteration and the weight factor $\beta$ defines the step size towards the improving direction. A weight factor of one would imply that the new well trajectory is same as the revised position $p$. 

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Figure 2.3: An overview of the computation of the revised position for all the well segments
The revised position vector of the $i^{th}$ trajectory point is represented by $p_i^r$:

$$
p_i^r = x_i^r + \frac{\sum_{n=1}^{N_d} |g'_{i,n}| (s_i^r - x_i^r)}{\sum_{n=1}^{N_d} |g'_{i,n}|},
$$

where $N_d$ signifies the number of dummy wells taken into account to evaluate the revised position $p$ and $s$ represents the position vector of the dummy well. Fig. 2.3 illustrates an overview of the revised position computation.

As discussed earlier, six dummy wells are generated for each trajectory segment. Since the dummy wells are present in $\pm$ directions, only three of those point in the direction of improvement. Considering the improvement of injector well trajectory, the maximum gradient always indicates an improved location. The summed gradients may or may not be positive depending on the prescribed flowrate. If the flowrate is higher than the optimized flowrate, negative gradients are observed. Similarly, a flowrate lower than the optimized flowrate yields positive gradients. Hence, the absolute value of the gradient is taken in account to assure that the revised position is unaffected by the positivity/negativity of the gradients.

If a dummy well pair does not exist in a direction, the existing gradient in that direction is compared with the maximum gradient values in the other two directions. If it is higher than at least one of those, the lone gradient value and its corresponding direction are taken in account to compute the revised position. One must also note that equal gradients in the $\pm$ direction indicate local optimal solution. In short, this approach of taking three maximum gradients of each direction is credible as it does not have a tendency to get stuck except in a local optimum.

To better appreciate the concept, assume that Fig. 2.2 represents a two-dimensional reservoir with an injector and its dummy wells. The four dummy wells provide four summed gradient values (no movement in $\pm z$ direction). Assume the dummy well in the $+y$ and the $+x$ directions have the two highest gradients compared to their counterpart gradients. Hence, the main well trajectory will be moved to the $+y$ and $+x$ direction using the above described equations (taking gradients as weight). Once the revised positions are known, the weight factor $\beta$ is used to compute the final improved trajectory. New dummy wells will be generated for this new trajectory to recalculate the gradients, revised position and hence further improve the trajectory of the well.

Given the vast uncertainties in terms of geology, petrophysics and reservoir properties, it is difficult to delineate the well type (horizontal, vertical or deviated) to produce/inject optimally. To answer this problem, the presented algorithm uses multi-point trajectories to predict the well type and improve the well trajectory. Multi-point trajectories, i.e. using multiple well segments helps to break up a single well segment and therefore, allow multiple dummy wells (instead of just six). The corresponding gradient values help to move the well segment independently in the three-dimensional environment using the concept of revised position.
described in Fig. 2.3. Since the movement of each well segment is independent, the trajectory is free to rotate gradually and change the drilling azimuth and inclination to achieve a higher NPV compared to its initial placement. An example of rotation is illustrated by Fig. 2.4.

![Figure 2.4](image)

**Figure 2.4**: 90° Rotation of an initially horizontal injector (blue) to reach its optimal location (light green) at the centre of the 105 x 105 x 55 m³ reservoir

The optimization algorithm also presents the unique feature of fixing the heel of the well trajectory. By fixing the heel of the trajectory, the gradients only change the position of the toe and hence the inclination and azimuth are modified to reach the best target location.

Once the well trajectory reaches its optimal location, either the trajectory starts bouncing between two trajectory positions or all the dummy wells may produce equal gradient values. Again, it must be noted that the optimum trajectory may not be the globally best solution in some cases. The number of iterations can be decreased by either changing the weight factor $\beta$ to increase the movement in improving direction or constraining the algorithm using one of the termination criteria described in Section 2.7.

### 2.5 Dogleg Severity

It is important to ensure that the proposed well trajectory is drillable. The drilling industry uses Dogleg Severity (DLS) to define the drillability of a well. DLS provides the wellbore curvature between two consecutive directional surveys. Usually, it is expressed in degree/100 feet or degree/30 meter. In this thesis, DLS has been used to improve and predict a realistic trajectory using Jansen (1993) and Vlemmix et al. (2009) method.

The curvature ($C$) is inversely proportional to the radius of curvature ($R$) and measures the curvature of a circular arc:

$$C = \frac{1}{R}$$
The direction vectors of the tangents to the borehole trajectory in the trajectory points are assumed to be initially unknown except for the first point. A ‘marching algorithm’ has been used to compute the curvatures sequentially. The Cartesian coordinates of the trajectory points are known and are given by \( \mathbf{x}_i \) for a trajectory point \( X_i \) (a point located at the \( i^{th} \) location). Using the Cartesian coordinates of the trajectory points, a unit-length direction vector \( \mathbf{c}_i \) along the chord between the trajectory points \( X_i \) and \( X_{i+1} \) is computed first:

\[
\mathbf{c}_i = \frac{\mathbf{x}_{i+1} - \mathbf{x}_i}{|\mathbf{x}_{i+1} - \mathbf{x}_i|}.
\]

Further, it is assumed that the unit-length direction vector \( \mathbf{e}_i \) is equal to \( \mathbf{c}_i \) for \( i = 1 \). In order to use dogleg severity, the radius of curvature is defined between the 2 trajectory points \( X_i \) and \( X_{i+1} \) (see Fig. 2.5) as:

\[
R_i = \frac{|\mathbf{x}_{i+1} - \mathbf{x}_i|}{2 \tan \gamma_i},
\]

where the angle \( \gamma_i \) is given by

\[
\gamma_i = \arccos(\mathbf{e}_i \cdot \mathbf{c}_i).
\]

The next unit-length direction vector \( \mathbf{e}_{i+1} \) at trajectory point \( X_{i+1} \) can be computed as:

\[
\mathbf{e}_{i+1} = 2\mathbf{c}_i \cos \gamma_i - \mathbf{e}_i.
\]

If the computed radius of curvature is less than a pre-defined radius obtained by dogleg severity, an iterative process follows to increase the radius of curvature. This iterative process

**Figure 2.5:** The figure displays trajectory represented as a sequence of circular arcs and the associated variables essential to calculate the radius of curvature using the minimum curvature method.
was defined as smoothing process by Vlemmix et al. (2009). The smoothing process, in general, smoothens all the points except the heel and toe points:

\[
x'_i = (1 - \delta)x_i + \delta \left( \frac{x_{i-1} + x_{i+1}}{2} \right),
\]

where \(\delta\) is a dimensionless weighing factor and \(0 < \delta \leq 1\). The smoothening process is reiterated until a drillable trajectory is obtained as illustrated in Fig. 2.6. The figure shows a non-drillable multi-point trajectory being converted into a drillable trajectory using the algorithm described.

Figure 2.6: Smoothing of the trajectory: A non-drillable trajectory (top) followed by a drillable trajectory (bottom) produced by the discussed algorithm

Figure 2.7 displays a flow chart of the process of checking and if necessary, smoothing the trajectory points to achieve a drillable well trajectory. For the future cases, a DLS of 10°/30 m has been assumed unless stated otherwise. This corresponds to a radius of curvature of approximately 172 m.
2.6 Drilling Cost

As mentioned earlier, the main objective of the thesis is to find the best well trajectory so that the NPV of the project is maximized. The following equation has been used in order to calculate the NPV of the project:

\[
J = \sum_{k=1}^{K} \left[ \sum_{i=1}^{N_{ui}} r_{w,i} \cdot u_{w,i} + \sum_{j=1}^{N_{wi}} \left( r_{w,pr} \cdot |y_{w,j}|_k + r_{w,pr} \cdot |y_{wi,j}|_k \right) \right] \frac{t_k}{(1 + b)^t} \times \Delta t_k + \sum_{i=1}^{N_{wi} + N_{pr}} C_i \cdot L_i,
\]

where,

- \( b \) Discount Rate, 1/year
- \( C_i \) Drilling cost (of well trajectory \( i \)) per unit meter, $/m
- \( k \) Simulation time step, -
Although Appendix B covers the necessary details about the objective function, this section sheds light on the second term augmented to modify the objective function. The second term sums up the cost of drilling the injector well to the desired depth. This term helps in penalizing the trajectories which improve the objective function value by increasing the length of the well trajectory. Generally, this can be observed in the cases where the heel of the well trajectory is fixed. Hence, the second term can be overlooked for the freely-moving trajectory cases. The sole intention of the additional drilling cost term in the objective function formulae is to control the length of the well. Since, the trajectory optimization algorithm does not aim to eliminate any wells, the total cost of drilling till the reservoir zone always stays constant and hence plays a trivial role. In such cases, one can also use the length of the well trajectory within the target reservoir instead of taking the complete well length.

### 2.7 Convergence

The algorithm can be terminated by either of the following five criteria:

1. **Principal criterion:** Lower relative increase of the objective can be used to terminate the algorithm using:

   \[
   \frac{J^j - J^{j-1}}{J^{j-1}} < \varepsilon .
   \]

   Here, \( \varepsilon \) can range as per the requirement of the user. Higher value of \( \varepsilon \) means saving a lot more iterations albeit, it comes with a trade-off with best optimal well trajectory assignment.

2. **Decrement criterion:** The algorithm can also be concluded if the NPV does not increase with each iteration. The maximum number of decrements can be adjusted as per the needs of the user. This criterion ensures that the highest NPV and the equivalent trajectories are displayed as the final output.

3. **Oscillating criterion:** In some cases, it may be observed that the trajectory starts oscillating alternately between two nearly equal solutions. In such case, the oscillating
criterion calculates the maximum objective function and displays the corresponding trajectory.

4. Equal gradients criterion: In some scenarios, it is possible to observe equal gradients of the objective function over the lifespan of the reservoir with respect to the flowrates in the dummy wells. In such scenarios, one can presume the achievement of either a local or global optimal solution. The algorithm is terminated under such a circumstance.

5. Maximum Iterations criterion: The user can define the maximum number of iterations and the algorithm is run till the maximum value of the iteration unless this criterion is interrupted by any of the above mentioned auxiliary criterion.
3 Adjoint-based Well Trajectory Optimization

3.1 Homogeneous Reservoir

This section covers a detailed outcome of the first attempt on optimizing the well trajectory in two and three-dimensional homogeneous reservoirs. In order to understand the behaviour of the algorithm and the nature of the output well trajectory, the base case of both two and three-dimensional reservoirs is followed by few altered cases. Note that the reservoirs represented in the following sections were synthetically generated using reservoir simulation packages; MRST (see Lie, 2014) and ECLIPSE (see Eclipse 2013.1). Following reservoir and fluid parameters (Table 3.1) were initialized for all the cases of homogeneous reservoir. Please note that all the reservoirs have Vertical Transverse Isotropy (VTI), with $k_z = 0.1k_x$ (or $k_y$):

<table>
<thead>
<tr>
<th>Table 3.1 – Reservoir and Fluid Properties</th>
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<tbody>
<tr>
<td><strong>Symbol</strong></td>
</tr>
<tr>
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</tr>
<tr>
<td>$S_{w,0}$</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>$R_{well}$</td>
</tr>
<tr>
<td>$\phi$</td>
</tr>
<tr>
<td>$k$</td>
</tr>
</tbody>
</table>
The two phase relative permeability is depicted in Fig. 3.1.

![Image](image_url)

**Figure 3.1: Oil-Water Relative Permeability**

Table 3.2 lists the economic parameters:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_o$</td>
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</tr>
<tr>
<td>$r_{w,pr}$</td>
<td>18.9</td>
<td>$/m^3$</td>
</tr>
<tr>
<td>$r_{w,inj}$</td>
<td>6.3</td>
<td>$/m^3$</td>
</tr>
<tr>
<td>$C_{wi}$</td>
<td>500</td>
<td>$/m$</td>
</tr>
</tbody>
</table>

### 3.1.1 Two-dimensional reservoir

For the first test case, the trajectory of an injector well was optimized in a 21 x 21 x 1 gridblocks horizontal reservoir model with gridblock dimensions of 5 x 5 x 5 m$^3$. Four production wells, fixed at the corners of the reservoir, deplete the reservoir for an uninterrupted period of 1,000 days. Whereas the producers operate under a prescribed pressure of $380 \times 10^5$ Pa, the injector operates under a rate constraint of 10 m$^3$/day while ensuring that injection pressure does not exceed $420 \times 10^5$ Pa (and frack the reservoir). The initial reservoir pressure of 4,000 m deep reservoir was taken as $400 \times 10^5$ Pa. Intuitively, one would expect the injectors to move towards the centre of this designed homogenous reservoir. The reservoir simulation was run for twenty five simulation time steps. It must be noted that gravity plays no role in the single-layered reservoir and hence, can be turned off for the simulation.

In the first scenario, a two-point trajectory was optimized. Since neither of the ends was fixed, the trajectory was permitted to move in any direction. As anticipated, the trajectory moved towards the centre of the reservoir (Fig. 3.2). The NPV was increased by approximately 127% using this method and it takes about 35 iterations to achieve the maximum NPV.
Figure 3.2: NPV graph, pressure map and corresponding trajectory map when the initial trajectory is a horizontal well (gridblocks [23;24]). The bottommost figure illustrates the top view (XY) of the trajectory map.

As discussed earlier, the number of iterations can be decreased by either changing the weight factor/step size $\beta$ to increase the movement in improving direction or constraining the algorithm to stop converging by using any of the termination/convergence criteria (Section 2.7). Keeping the iterations in mind, $\beta$ was increased from 0.5 to 1. This step helped to reduce
the number of iterations significantly (~49% compared to the base case) while moving the well trajectory to its optimal solution at the centre (See Fig. 3.3). The final outcome of the scenario was identical to the base case. Moreover, the whole process can be further improved in terms of iterations by concluding the algorithm with the aid of principal criterion of termination. In that scenario, the algorithm can be stopped at about 17 iterations (depending upon the definition of $\varepsilon$).

**Figure 3.3:** NPV graph and the optimal trajectory of injection well (top-view) when the initial trajectory is a horizontal well (gridblocks [23;24]) and $\beta=1$

### 3.1.2 Three-dimensional reservoir

Before proceeding to the heterogeneous reservoirs, it is vital to take a quick look at the resultant trajectory in a three-dimensional homogeneous reservoir. In order to test the algorithm in a three-dimensional reservoir, four production wells were fixed at the corners of a $21 \times 21 \times 7$ gridblocks reservoir (with gridblock dimensions of $5 \times 5 \times 5$ m$^3$). The lifespan of the match-box shaped reservoir was assumed to be an uninterrupted duration of 1,500 days. Similar to the two-dimensional scenario, the producers were operated under a prescribed pressure of $380 \times 10^5$ Pa and the injector was operated under a rate constraint of $300 \text{ m}^3/\text{day}$ while ensuring that injection pressure does not exceed $420 \times 10^5$ Pa. The initial reservoir pressure of 4,000 m deep reservoir was taken as $400 \times 10^5$ Pa.

The reservoir simulation was run for thirty five simulation time steps. To guarantee a drillable and optimized well trajectory, the maximum allowed DLS was set to $10^9/30$ m. Gravity plays an important role in a three-dimensional reservoir. Depending on the net thickness and the lateral distance between the injector-producer pair, it may strongly affect the sweep efficiency of the reservoir layers at the top. In order to capture and appreciate this effect, all the scenarios assume gravity in the reservoir.

The first case scrutinised the behaviour of the injector well trajectory while it is free to move in any direction since neither of the ends was fixed. In this scenario, a multi-point trajectory was tested (Fig. 3.4). The injector well was initially located vertically with perforations in all the seven layers. Four vertical production wells, placed at the edges of the reservoir, also perforated all the layers at their respective position. The weight factor/step size $\beta$ was defined as unity. As anticipated, the trajectory moved towards the centre of the reservoir. NPV was
increased by approximately 17% using this method and it took 19 iterations to achieve the maximum NPV.
Figure 3.4: NPV graph, pressure map and corresponding trajectory map when the initial trajectory (magenta) is a vertical well perforating all 7 layers. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively.

The base case was repeated with different initial trajectories. As illustrated through Fig. 3.5, the final solution more or less converges to the same position:
Moreover, the base case mentioned above was also re-examined by placing a horizontal injection well initially (Fig. 3.6). Although it took 28 iterations in this case, the final trajectory and the NPV were almost identical to the ones produced with the base case:

**Figure 3.5:** Different initial trajectories converging to same gridblocks to produce comparable results

**Figure 3.6:** Final trajectory (Black) when the initial trajectory (Blue) is a horizontal well

### 3.2 Heterogeneous Reservoir

In this section, a detailed outcome of optimizing the well trajectory in a two and three-dimensional heterogeneous reservoir is discussed. Similar to the previous sections, the base case is followed by few altered cases to understand the algorithm and behaviour of the well trajectory in the heterogeneous environment. The presented reservoirs in the following sections were also synthetically generated using MRST and ECLIPSE. The reservoir, fluid and
economic parameters were initialized with the parametric values in Table 3.1 and 3.2. The oil-water relative permeability curve is depicted in Fig. 3.1.

3.2.1 Two-dimensional reservoir

In order to test the algorithm in a two-dimensional reservoir, four production wells were fixed at the corners of a 21 x 21 x 1 gridblock reservoir with gridblock dimensions of 5 x 5 x 5 m$^3$. The life of the reservoir was assumed to be 1,000 days. The producers were operated under a prescribed pressure of 380 x 10$^5$ Pa and the injector was operated under a rate constraint of 10 m$^3$/day while ensuring that injection pressure does not exceed 420 x 10$^5$ Pa. The initial reservoir pressure of 4,000 m deep reservoir was taken as 400 x 10$^5$ Pa. The reservoir simulation was run for twenty five simulation time steps. Since gravity plays no role in the single-layered reservoir, it was turned off for the simulation.

![Permeability map of single-layered heterogeneous reservoir](image1)

**Figure 3.7**: Permeability map of single-layered heterogeneous reservoir

A freely moving two-point trajectory was tested in this heterogeneous reservoir. As shown in Fig. 3.8, the trajectory reached an optimal location using 21 iterations. A 210% increase in the NPV was observed. The trajectory tried to position itself in such a fashion that the water production was as delayed as possible.

![NPV vs Iteration](image2)

![Final pressure](image3)
Figure 3.8: NPV graph, pressure map and corresponding trajectory map when the initial trajectory is a horizontal well (gridblocks [23;24]). The bottommost figure illustrates the top view (XY) of the trajectory map.

An altered base case with step-size $\beta = 0.5$ suggested that it takes about 40 iterations to reach a similar result in terms of NPV and well trajectory, compared to 21 iterations with $\beta = 1$.

3.2.2 Three-dimensional reservoir

To examine the working of the algorithm in a three-dimensional heterogeneous reservoir, four production wells were fixed at the corners of a 21 x 21 x 7 gridblocks reservoir with gridblock dimensions of $5 \times 5 \times 5$ m$^3$. The initial reservoir pressure of the 3,500 m deep reservoir was assumed to be $413 \times 10^5$ Pa. While the porosity of the reservoir was presumed to be constant, the permeability of each layer was depicted by the permeability map of Fig 3.7.

The reservoir simulation with 64 time steps depletes the ‘match-box’ shaped reservoir for an uninterrupted period of 1,500 days. DLS was assumed to be $10^6/30$ m. Whereas the producers operated under a prescribed pressure of $398 \times 10^5$ Pa, the injector operated under a rate
constraint of 300 m$^3$/day while ensuring that injection pressure does not exceed 435 x$10^5$ Pa (and frac the reservoir). Contrary to the homogeneous reservoir, predicting the optimized well trajectory in a three-dimensional heterogeneous reservoir is a challenging task. To observe the progress of improving well trajectories and study the behaviour of the algorithm, the step size $\beta$ was reduced to 0.5 in this case.

During the first case, a freely moving multi-point trajectory was tested. The trajectory was initially located vertically perforating all the seven layers. The trajectory progressed to an optimal location near the centre of the reservoir to maximize the NPV. As anticipated, the trajectory does not move all the way towards the centre because of the heterogeneity in the reservoir. NPV was increased by approximately 28% using this method and it took 45 iterations to achieve the maximum NPV. The number of iterations can be reduced by either using a bigger step size or using principal termination criterion (Section 2.7).
Figure 3.9: NPV graph, pressure map and corresponding trajectory map when the initial trajectory is a vertical well perforating all 7 layers. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively.
Similar to the homogeneous three-dimensional case, the base case was repeated with different initial trajectories. To ensure equal time-steps and to avoid any convergence issue, the time-steps of the reservoir simulation were increased to 155 and the step size was defined as 0.5. As illustrated by Fig. 3.10, the final solution more or less converges to the same position:
Figure 3.10: Different initial trajectories converging to almost same gridblocks to produce comparable results

Although, the optimized trajectory for the case with initial injector at the ‘south-east’ corner ended up slightly deviated, all the four optimized trajectories ended up within the same gridblocks. The whole process, probably, can be further improved by taking grid refinement process and reducing the step-size near the local/global optimum. The NPV achieved in all the cases was essentially the same while the number of iterations varied.

3.3 Multiple Trajectories

An optimal field development requires optimizing multiple well trajectories. In this section, several well trajectories are simultaneously optimized in homogeneous as well as heterogeneous reservoir. While a ‘match-box’ shaped reservoir was studied for homogeneous case, the ‘Egg Model’ was investigated as a heterogeneous reservoir. Both the homogeneous and heterogeneous reservoir models are synthetic models developed using MRST and ECLIPSE.
3.3.1 Homogeneous reservoir

In this sub-section, multiple trajectories were optimized in a homogeneous environment. The homogeneous reservoir with 41 x 41 x 5 gridblocks was synthetically generated using MRST/ECLIPSE. The listed parameters in Table 3.1 and 3.2 were initialized for the reservoir model. Fig. 3.1 depicts the oil-water relative permeability of this 4,000 m deep reservoir. The initial pressure of this ‘match-box’ shaped reservoir was defined to be 400 x 10^5 Pa. Nine peripheral producers, optimally placed were operating under a prescribed BHP of 380 x 10^5 Pa. Four injection wells, randomly placed, were required to be optimized to improve the NPV of the field. The simulation was performed for an unbroken period of 1,500 days. The to-be-optimized injectors were operating under a rate constraint of 180 m^3/day while ensuring that injection pressure does not exceed 420 x 10^5 Pa (and frack the reservoir).

For the first case, freely moving multi-point injector well trajectories were optimized. The injector well trajectories, perforating all the seven layers, relocated to an optimal location near the centre of the reservoir to maximize the NPV by maintaining pressure and improving the sweep efficiency. NPV is increased by approximately 16% (Fig. 3.11) using this method and it took around 40 iterations to achieve the maximum NPV with a step size $\beta = 0.5$. Similar results, with almost half the number of iterations, were observed with an increased step size ( $\beta = 1$). The number of iterations can be further reduced by implementing one of the principal termination/convergence criteria.
Figure 3.11: NPV graph, pressure map and corresponding trajectory map of optimized well trajectories. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively.
3.3.2 Heterogeneous reservoir – ‘Egg Model’

The ‘Egg Model’ is a three-dimensional heterogeneous and a synthetic channelized reservoir model; see Jansen (2013). Four producers and eight injectors have been used to produce this 4,000 m deep reservoir using the conventional recovery method of water-flooding. The model consists of $60 \times 60 \times 7 = 25,200$ gridblocks of which 18,553 gridblocks are active. Each gridblock has a dimension of $8 \times 8 \times 4 \text{ m}^3$. Although porosity has been assumed to be constant 20%, a variable permeability can be observed in all the seven layers of the model. The model assumes no aquifer and/or gas cap. Table 3.3 lists the essential reservoir and fluid properties of the ‘Egg model’.

The field economy is computed by initializing the economic parameters listed in Table 3.2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_w$</td>
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</tr>
<tr>
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<td>[-]</td>
</tr>
<tr>
<td>$S_{w,0}$</td>
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<td>[m]</td>
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<tr>
<td>$\phi$</td>
<td>0.2</td>
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</tr>
</tbody>
</table>
Fig. 3.12 depicts the oil-water relative permeability in this egg shaped reservoir.

The permeability of all the seven layers is depicted in Fig. 3.13. Similar to the previous reservoir models, the ‘Egg model’ also has VTI, with $k_z = 0.1k_x$ (or $k_y$).

Figure 3.12: Oil-Water Relative Permeability
Four producers, operating under a prescribed BHP of $395 \times 10^5$ Pa, recover oil from the reservoir for a continuous period of 3,600 days (or 10 years). Eight injection wells are placed in the reservoir. These injectors operate under a rate constraint of $79.5 \text{ m}^3/\text{day}$ while ensuring that injection pressure does not exceed $420 \times 10^5$ Pa (and fracture the reservoir). If the injection pressure is higher or equal to $420 \times 10^5$ Pa, the well is switched to BHP control to maintain injection at $420 \times 10^5$ Pa.

During the first case, multi-point injector well trajectories were optimized while keeping the heel fixed. Fig. 3.14 exhibits the NPV increment and optimized well trajectories for this scenario. The NPV of the project was incremented by approximately 6% using the algorithm and it took nearly 31 iterations to achieve the maximum NPV. One horizontal, one vertical and 6 deviated wells produce the reservoir optimally if the heel of the injectors are fixed. Interestingly, almost all the injectors try to puncture the lower permeability gridblocks to improve the oil recovery and delay the water breakthrough.

**Figure 3.13:** Permeability map of layers from top to bottom

![Permeability Map of Layer 5 [mD]](image1)

![Permeability Map of Layer 6 [mD]](image2)

![Permeability Map of Layer 7 [mD]](image3)
Figure 3.14: NPV graph, pressure map and corresponding trajectory map of optimized well trajectories. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively.

As illustrated by figure 3.15, more oil was drained by the optimized well trajectories compared to the initial well trajectories. The RF was increased by 1.3%, one of the important reasons that facilitated a $5.7 Million increase in NPV.

Figure 3.15: Saturation Maps obtained at the end of 3,600 days with the initial well trajectories (left) and the optimized well trajectories (right).

During the second case-study, freely-moving multi-point injector well trajectories were optimized. Fig. 3.16 displays the increment in NPV and optimized well trajectories for this scenario. The NPV of the project was incremented by approximately 9% using the algorithm and it took around 28 iterations to achieve the maximum NPV.
Figure 3.16: NPV graph, pressure map and corresponding trajectory map of optimized well trajectories. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively.

Compared to the initial well trajectories, more oil was drained by the optimized well trajectories as revealed by saturation maps (Fig. 3.17). The RF was increased by 2.3%.
Figure 3.17: Saturation Maps obtained at the end of 3,600 days with the initial well trajectories (left) and the optimized well trajectories (right)

So far, only injector well trajectories were optimized. One can always improve both producer and injector well trajectories simultaneously. To get a better impression of this, the next two cases exemplify the simultaneous well placement and trajectory optimization of the mini-pattern. Although mini-patterns are generally not preferred in an FDP, the working of the algorithm can be confirmed by studying the outcomes of such a well configuration.

For the next two cases, the four producers operate under a prescribed BHP of 380 x 10^5 Pa and the eight injection wells operate under a rate constraint of 79.5 m^3/day while ensuring that injection pressure does not exceed 420 x 10^5 Pa.

As discussed above, simultaneous well placement optimization was performed on a mini-well pattern for the third case of this sub-section. The NPV of the project was incremented by a staggering 209% using the algorithm and it took 40 iterations to achieve the maximum NPV (See Fig. 3.18). While the producers tended to be more close to the channels to ensure higher productivity, the injectors were more distant to ensure better sweep and late water-breakthrough. Moreover, the producers also tend to locate themselves in such a way that the water-breakthrough is delayed.
Figure 3.18: NPV graph, pressure map and corresponding trajectory map of optimized well trajectories. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively. The initial locations of the producers and injectors are indicated by green and blue colours, respectively.

As anticipated, more oil was drained by the optimized well trajectories (Fig. 3.19). The RF was increased twofold from 30% to 60%.
Following similar ideology, simultaneous trajectory optimization for both producers and injectors was studied during the fourth case. Fig. 3.20 displays the increment in NPV and optimized well locations for this scenario. The NPV of the project was incremented by an astounding 216% using the algorithm and it took about 49 iterations to achieve the maximum NPV. Intriguingly, three vertical producers transform themselves to horizontal wells, perforating top layers alone. Similar outcome was observed for six vertical injectors, which transfigure themselves to horizontal injectors and perforate the bottom-most layers to maintain reservoir pressure. Furthermore, one observes the heel of ‘PROD 2’ near the toe of ‘PROD 3’. One can argue such an arrangement by stating that the ‘PROD3’ arranges itself perpendicular to ‘PROD2’ to evade early water breakthrough, but at the same time one can even delete a producer based on the outweighing cost of drilling a horizontal well compared to the amount of oil being pumped out.

As observed during well placement, the producers yet again have a tendency to be more proximal to the channels to ensure higher productivity while the injectors were, generally, distal to ensure better sweep and late water-breakthrough.
Figure 3.20: NPV graph, pressure map and corresponding trajectory map of optimized well trajectories. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively. The initial locations of the producers and injectors are indicated by green and blue colours, respectively.
The optimized well trajectories produce more oil as shown by saturation maps in Fig. 3.21. The RF was doubled up from 30% to 61%. Compared to the previous result with well placement, the RF was slightly higher yielding a higher NPV.

Figure 3.21: Saturation Maps (with well trajectories) obtained at the end of 3,600 days with the initial well trajectories (left) and the optimized well trajectories (right)
4 Synergism: Well Number and Placement Optimization

4.1 Introduction

Well trajectory and placement optimization play a crucial role in an FDP. As seen in the earlier chapters, designing an optimal trajectory can significantly improve the NPV by manipulating the production rate and water-breakthrough. Although well trajectory optimization assists in field development, there are numerous other parameters that also need careful investigation and study to nurture an optimal FDP. To name a few, well number, scheduling, flowrates and prescribed BHPs are some of the most important variables that are strongly interlinked with the optimization of the objective function. Given the high cost of drilling a well, especially offshore, there is an acute requirement for optimizing the number of wells.

In this chapter, a new algorithm is proposed to calculate the optimal well numbers. This process is performed in conjunction with well placement optimization and furthermore, can be easily integrated with well trajectory and controls optimization process. Such a synergy is much needed to produce an optimal FDP with minimal iterations.

4.2 Methodology

The approach of multi-stage well number optimization process is based on drilling quasi-wells in a dense five-spot network. Using the ideology of dummy wells, the original wells are overlapped by the insignificantly low rate producing (or injecting) dummy wells and the sum of the gradients of the objective function with respect to the flowrate in each dummy well over the life-time of the reservoir is used to compare and delete the inessential quasi-wells. But, most importantly, the dense well pattern helps in deducing a rough estimate of the field injection rate using the summation of the above-mentioned summed gradients and averaging it over the number of injection wells:

\[
\bar{g}_{field} = \frac{1}{N_{inj}} \sum_{d=1}^{N_{inj}} \sum_{k=1}^{K} \frac{\partial J}{\partial u_{d,k}}.
\]

Here, \( \bar{g}_{field} \) is the averaged summed gradient of the field over the number of injection wells. Using the knowledge of relationship between gradients and their magnitude (see Section 5.1), the averaged summed gradient of the field has been used to guesstimate the field injection rate. Fig. 4.1 illustrates the plot of averaged summed gradient against the field injection rate. An averaged summed gradient of zero indicates a rough field injection rate. Intriguingly, Fig. 4.1 demonstrates resemblance between the three graphs for three fields with completely different geometries and reservoir properties.
This ad hoc method of approximating field injection rate per day uses trivial number of reservoir simulations. Since the optimal field injection rate is strongly dependent on ideal well number and their best trajectory, it is practical to not use several reservoir simulations to get its better estimate during this stage.

Besides providing an estimate of the field injection rate, the scheme is also used to estimate the maximum RF and production/injection data. This information, along with user-defined drilling costs, are then utilized to compute the maximum number of permissible-to-drill wells:

\[
\text{Maximum wells} = \frac{\text{Produced oil cost} - \text{Produced water cost} - \text{Injection cost}}{\text{Estimated cost of each well}}.
\]

The minimum number of injection/production wells can be computed by dividing the maximum field production (or injection) rate by the maximum well production (or injection) rate. Note that including as many Capital Expenditure (CAPEX) details as possible can help to determine the best possible solution.

During the first step, the knowledge of maximum number of permissible-to-drill wells helps to delete the unnecessary quasi-wells altogether. As discussed earlier, this deletion is based on the relative summed gradient values. If an injector well is deleted, its injection rate is equally distributed amongst the remaining injection wells. This helps to ensure that all the injectors have equal rates and hence, an equal opportunity to perform in terms of flowrate.

It must be noted that dummy wells are not required for the wells where flowrate is defined as the control variable. Furthermore, following the results from the preceding chapter, maximum and minimum gradients are considered best for injectors and producers, respectively.

Once the first iteration is completed, the well placement optimization is carried out to find the optimal locations of the remaining quasi-wells. Generally, this process yields an outcome

![Figure 4.1: Plot of averaged summed gradient over the number of injection wells versus total field injection rate to estimate required field injection rate](image-url)
where several injectors or producers are in close proximity. Clearly, such an arrangement is a surreal configuration and wells in such a radial area need to be carefully examined and deleted in such a way that only the best well survives (based on the summed gradients). One must note that this step is very important from the engineer’s perspective as it provides an opportunity to learn about the important producible and injectable areas of the field.

Usually, this leaves a nominal number of quasi-wells in the reservoir. With the new well configuration, one needs to rerun the reservoir simulation to analyse the worst performing well. Based on the gradients, producer-injector ratio and engineering judgement, the wells can be deleted one by one (or >1 as per engineer’s choice). It is advisable to delete one-by-one to track the effect of removing a quasi-well on the NPV.

The deletion of worst quasi-well may or may not lead to an increase in NPV. If it leads to an increase in NPV, the deletion process is continued. Otherwise, one needs to stop and think of a plausible reason for non-increment in NPV. Such a decrement generally indicates two circumstances. First and foremost, this may happen due to wrong well-type deletion. If this isn’t the case, the well configuration clearly doesn’t support any further deletion and needs to be optimized again to find local (or global) optimum.

Once the placement optimization is done again, one must attempt to delete more wells. If the deletion still doesn’t improve NPV, local (or global) optimum is presumed to be achieved. In the case where deletion increases NPV, one has to repeat the whole process mentioned in the last paragraph to achieve the optimal solution.

Fig. 4.2 depicts the process of multi-stage optimization. Once the local optimum solution is achieved by optimizing well number and placement, trajectory and controls optimization can be applied to further improve the NPV.
Figure 4.2: Overview of the multi-stage well number and placement optimization
4.3 Multiple Trajectories

To elucidate the multi-stage optimization procedure described above, three examples have been presented in this section. Although drilling rig and crew cost, drilling time, production platform and facilities, etc. play a significant role in determining the true NPV, these factors have been ignored for the sake of simplicity. One must also note that all the reservoirs presented in the following sub-sections were synthetically generated using MRST and ECLIPSE.

4.3.1 Homogeneous reservoir

In this sub-section, the multi-stage optimization was performed in a homogeneous reservoir. The reservoir and fluid parameters were initialized using Table 3.1. The reservoir consists of 61 x 61 x 2 gridblocks (with gridblock dimensions of 5 x 5 x 5 m$^3$). The lifespan of the matchbox shaped reservoir was assumed to be 1,500 days.

The 4,000 m deep reservoir was presumed to have an initial reservoir pressure of 400 x 10$^5$ Pa. By means of Fig. 4.1, the field injection rate was estimated to be 276 m$^3$/day. While the producers operated under a prescribed pressure of 380 x 10$^5$ Pa, the injectors were conditioned to operate under an equal flowrate constraint while ensuring that the injection pressure does not exceed 420 x 10$^5$ Pa. Each vertical well was assumed to cost $5 Million or $1,250 for every metre of vertical drilling. All commodity price and cost were initialized using Table 3.2.

Fig. 4.3 depicts the initial and final well placement scenario. The final outcome clearly indicates that only single producer and single injector are sufficient to exploit the reservoir optimally. Compared to the scenario with four producers at corners and an injector at the centre (See Section 3.1.2), the NPV is 87% higher.

![Figure 4.3: Homogeneous Reservoir: Initial well placement (left) and the optimized well placement (right)](image-url)
4.3.2 Heterogeneous reservoir

The second example demonstrates multi-stage optimization in a heterogeneous reservoir. The reservoir and fluid parameters were initialized using Table 3.1. The reservoir consists of 21 x 21 x 3 gridblocks with gridblock dimensions of 10 x 10 x 10 m$^3$. The lifespan of the reservoir was presumed to be 1,500 days. The three heterogeneous layers of the reservoir are identical. Fig. 4.4 presents the permeability map of the first layer.

![Permeability Map of the first layer](image)

**Figure 4.4:** Permeability Map of the first layer

The 3,650 m deep reservoir has an initial reservoir pressure of 413 x 10$^5$ Pa. By means of Fig. 4.1, the field injection rate has been estimated to be 400 m$^3$/day. While the producers were operated under a prescribed pressure of 398 x 10$^5$ Pa, the injectors were conditioned to operate under an equal flowrate constraint while ensuring that the injection pressure does not exceed 430 x 10$^5$ Pa. Each vertical well was assumed to cost $4.6 Million ($1,250 for every metre of vertical drilling). All commodity price and cost were initialized using Table 3.2.

As depicted in Fig. 4.5, single producer and single injector are apt to exploit the reservoir optimally. The optimal scenario obtained after 54 iterations, pumps out 70% of the oil in place to give an NPV of $45 Million.

![Well Placement](image)

**Figure 4.5:** Heterogeneous Reservoir: Initial well placement (left) and the optimized well placement (right)
4.3.3 The ‘Egg Model’

The standard ‘Egg Model’ utilizes 8 injectors and 4 producers to recover oil from the reservoir. The multi-stage optimization was also performed on the standard ‘Egg model’ to produce maximum oil using minimal number of wells.

In order to set up the model, the reservoir and fluid properties listed in Table 3.3 were initialized. By means of Fig. 4.1, the field injection rate was estimated to be 616 m³/day. While the producers operated under a prescribed pressure of 390 x 10⁵ Pa, the injectors were conditioned to operate under an equal flowrate constraint while ensuring that the injection pressure does not exceed 420 x 10⁵ Pa. Each vertical well was assumed to cost $5 Million or $1,250 for every metre of vertical drilling. All commodity price and cost were initialized using Table 3.2.

Fig. 4.6 depicts the initial and final well placement scenario. The final outcome indicates that 2 injectors and a producer are sufficient to achieve a RF of 58%. The NPV was observed to be 133% higher than the NPV of the original scenario.

Whereas the well placement optimization of the original scenario was performed using 21 iterations to produce an NPV of $49 Million, the optimal NPV ($92 Million) of the multi-stage optimization was established using 55 iterations. Although the NPV of the multi-stage optimization process is almost double the well placement optimization, the RF was observed to be 2.4% lesser than the optimized well placement scenario.
Figure 4.6: The ‘Egg Model’: Initial well placement (top) and the optimized well placement (below)
5 Discussion and Conclusion

5.1 Discussion and Future Work

1. Flowrate, NPV and gradients are linked closely. This relationship is depicted in Fig. 5.1 and Fig. 5.2. The flowrates influence the NPV which in turn also affects the sign and magnitude of the gradients.

a) NPV versus injection rate: This relationship is depicted in Fig. 5.1. Lower injection rates imply positive gradients or in other terms, an indication that the reservoir needs more injection to displace oil, maintain pressure and improve the NPV. Higher injection rates, on the other hand, imply negative gradients or in other words, an indication that the reservoir does not need more injection and lowering flowrate will help to achieve a higher NPV.

Figure 5.1: Variation of NPV with change in injection rate of a well

b) NPV versus production rate: This relationship is described in Fig. 5.2. As depicted in Fig. 5.2, lower production rates lead to negative gradients which in turn suggests that the producer is being under-utilized and increasing the production rate would help to achieve a higher NPV. On the contrary, higher production rates imply positive gradients and an indication that lowering the production rate can improve the NPV.
c) Trajectory optimization and gradients: Whereas, the maximum of the gradients always points in the direction of the better injector well trajectory, the minimum gradients point towards the direction of the better production well trajectory. The signs of the gradients do not affect the optimization procedure.

d) Trajectory optimization and flowrate: Flowrate plays a governing role in deciding an optimized well trajectory. One should bear in mind that lower-than-optimized flowrate may yield slightly different optimal well trajectory compared to a higher-than-optimized flowrate.

2. Global Solution: In order to achieve global optimum, the trajectory optimization algorithm can be augmented with non-gradient-based methods like ANN, GA, Simulated Annealing, etc.

3. Flowrate Optimization: As inferred earlier, the NPV obtained by optimizing the well number and/or trajectory is still not optimal and there is an extensive opportunity for improving it based on various parameters, especially flowrate. An optimized flowrate of the perfect number of wells with best trajectories can guarantee an optimal FDP. Flowrate optimization can be pursued once the wells have reached the appropriate location. The project can be improved further by utilizing smart well technology to control the flowrate over the time.

4. Two-point trajectory optimization and dummy wells: Even though the well trajectory can be easily rotated with the help of multi-point trajectory, delineating the type of well with a two-point well trajectory is not feasible. For such a case, well trajectories can be developed by generating dummy wells in more than six directions. Furthermore, a multi-directional dummy well configuration can definitely improve the NPV during the last few iterations of the optimization process.

5. Geological Uncertainty: Imaging sub-surface is a difficult task and hence Robust Optimization (RO) is vital to capture the geological uncertainty. Given the lower
number of iterations, RO can be carried out to understand the best well trajectory under the given uncertainty in the reservoir properties.

6. Production constraint: Production constraints strongly affect the final outcome. So far, BHP constrained producers and flowrate (and/or BHP) constrained injectors have been tested. Enforcing more constraints like average reservoir pressure, water-cut, field production/injection rate etc. may further improve the output.

7. CAPEX: In order to produce best output from the multi-stage optimization process, it is strongly recommended to introduce all important CAPEX values. This should not only include tangible costs like drilling rig and crew cost but also drilling time since early oil production will generate a better gain compared to delay in first oil.

8. Optimizing FDP: The multi-stage algorithm needs to be combined with well scheduling and well controls optimization to develop an optimal FDP. Although well controls optimization seems to be straightforward, optimizing the well schedule using gradients seems to be a challenging job.

9. Realistic Reservoir Model: To understand the reliability and the loopholes in the current algorithm, a realistic reservoir needs to be investigated. Moreover, the algorithm can also be tested with different reservoir fluids including aquifer and gas cap.

10. Local grid refinement can further help in improving the well trajectory.

11. Since an engineer’s judgement and experience cannot be ignored, the performance of the trajectory optimization algorithm is much appreciated when the optimization process is started from the engineer’s best estimate of the well trajectory. Similarly, including engineer’s judgement in the multi-stage optimization process can not only help in demarcating the best wells and their placements but also reducing the number of iterations. A more successful field development plan can be generated using this method.

5.2 Conclusion

Several two and three-dimensional reservoir models were examined in the previous chapters which help to appreciate the performance of the algorithms presented in the thesis. Based on the various observations and results obtained for different scenarios, it can be concluded that:

1. Computer assisted algorithm:
   a) Local v/s Global Optimal solution: Even though the optimization algorithm functions pretty well, sometimes the well trajectories may get stuck in a local optimum. The algorithm cannot discriminate between global and local optimal solution. Predominantly for large problems, it is highly likely that the method may converge to a local optimal solution. In order to solve this problem, one can repeat the optimization routine from different starting points to select the best trajectory.
   b) NPV Increment: Even if the trajectory optimization algorithm may fail to find the globally optimal solution, any NPV increment is more than welcome. The cases presented in the previous chapters displayed a wide range of NPV increment under the provided reservoir and production parameters (6 - 216%).
c) Automated process: This study suggests that the computer-assisted field development optimization is feasible. While well number optimization does not need any initial knowledge of wells, well trajectory optimization can be accomplished with either the engineer’s best estimate or random guess.

2. Drilling optimal well trajectory:
   a) Well type: The optimization algorithm assists delineating the well types (vertical, horizontal or deviated). As seen, a vertical well can be easily transformed to a horizontal well and vice-versa.
   b) Drilling costs and well length: The additional term included in the objective function plays a significant role to curb and optimize the well length, especially in the case of fixed heel trajectory where the trajectory may extend to a large extent. Realistic information supplied for the drilling costs can help to accurately curb the costs of drilling while ensuring higher NPV.

3. Multi-stage optimization: The combined effect of optimizing well number and placement altogether demonstrates significantly better results compared to well placement (or trajectory) optimization alone.

4. Weight factor/step size $\beta$: Step size plays a crucial role. Increasing the step size can reduce the number of iterations while reducing it increases the number of iterations.
Nomenclature

\( \beta \)  
Step size/weighing factor, \( 0 < \beta \leq 1 \)

\( b \)  
Discount Rate, \( 1/\text{year} \)

\( c \)  
Compressibility, \( \text{Pa}^{-1} \)

\( \mathbf{c} \)  
Unit-length direction vector along chord between two trajectory points, \( \text{m} \)

\( C \)  
Degree of Curvature, \( \text{rad/m} \)

\( \mathcal{C} \)  
Drilling cost per unit meter, \( \$/\text{m} \)

\( \mathbf{e} \)  
Unit-length direction vector tangent to trajectory, \( \text{m} \)

\( \varepsilon \)  
Constant, -

\( \phi \)  
Porosity, -

\( \gamma \)  
Angle, rad

\( g \)  
Gradient, -

\( \bar{g} \)  
Averaged gradient, -

\( J \)  
Net Present Value, \( \$ \)

\( k \)  
Permeability, \( \text{m}^2 \)

\( k \)  
Simulation time step, -

\( K \)  
Total number of simulation time-steps, -

\( L \)  
Length of the trajectory (inside reservoir), \( \text{m} \)

\( \mu \)  
Viscosity, \( \text{Pa s} \)

\( N \)  
Number, -

\( \bar{p} \)  
Initial pressure (top layer), \( \text{Pa} \)

\( \mathbf{p} \)  
Revised Position, \( \text{m} \)

\( \rho \)  
Density, \( \text{kg/m}^3 \)

\( r \)  
Commodity price per unit volume, \( \$/\text{m}^3 \)

\( R \)  
Radius, \( \text{m} \)

\( \delta \)  
Weighing factor, \( 0 < \delta \leq 1 \)

\( s \)  
Position vector of dummy well, \( \text{m} \)

\( S \)  
Saturation, -

\( \tau \)  
Reference time interval, \( \text{year} \)

\( t \)  
Simulation time, \( \text{day} \)

\( u \)  
Injection rate, \( \text{m}^3/\text{day} \)
\( \mathbf{x} \)  Position Vector, m
\( X \)  Trajectory point, - 
\( y \)  Production rate, m\(^3\)/day

**Subscripts**

0  Initial value 
k  Time step counter 
d  Dummy well 
w  Water 
o  Oil 
i  Trajectory point counter 
\( field \)  Field 
\( pr \)  Production 
\( inj \)  Injection 
r  relative 
\( well \)  Well-bore 
\( wc \)  Connate water/Irreducible water 
\( or \)  Irreducible oil 
\( R \)  Reservoir 
x  x-axis 
y  y-axis 
z  z-axis

**Superscripts**

\( T \)  Transpose 
0  End point saturation 
\( j \)  Iteration counter
## List of Abbreviations

<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
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<tr>
<td>BHP</td>
<td>Bottom-hole Pressure</td>
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<tr>
<td>CAPEX</td>
<td>Capital Expenditure</td>
</tr>
<tr>
<td>DLS</td>
<td>Dogleg Severity</td>
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<tr>
<td>FDP</td>
<td>Field Development Plan</td>
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<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>HGA</td>
<td>Hybrid Genetic Algorithm</td>
</tr>
<tr>
<td>ICV</td>
<td>Inflow Control Valve</td>
</tr>
<tr>
<td>O&amp;G</td>
<td>Oil and Gas</td>
</tr>
<tr>
<td>MRST</td>
<td>MATLAB Reservoir Simulation Toolbox</td>
</tr>
<tr>
<td>NaN</td>
<td>Not a Number</td>
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<tr>
<td>NCW</td>
<td>Nonconventional Well</td>
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<tr>
<td>NPV</td>
<td>Net Present Value</td>
</tr>
<tr>
<td>RF</td>
<td>Recovery Factor</td>
</tr>
<tr>
<td>RO</td>
<td>Robust Optimization</td>
</tr>
<tr>
<td>VTI</td>
<td>Vertical transverse isotropy</td>
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<tr>
<td>WI</td>
<td>Well Index</td>
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Bibliography


Appendix A

Well Index

The reservoir simulation, for this study, was run with the help of MATLAB Reservoir Simulation Toolbox (MRST), an open-source framework for rapid prototyping of new models written in MATLAB. This chapter describes Well Index (WI), a key parameter in the reservoir simulation process.

In reservoir simulation, the well pressure is different from the pressure of the gridblock containing the well. This is due to a large difference in the scale of the well and the gridblock. The well pressure and the flowrate are also closely related to each other. This interlinked relationship is defined by WI coefficient. The well index, \( J_{\text{well}} \) is defined as the ratio of the well flowrate and the difference between the reservoir block and the wellbore pressure:

\[
J_{\text{well}} = \frac{q_{\text{well}}}{p_i - p_{\text{well}}},
\]

where \( q_{\text{well}} \) is the well flowrate, \( p_{\text{well}} \) is the pressure in the wellbore and \( p_i \) corresponds to the pressure in the \( i^{th} \) gridblock of the reservoir. The WI coefficient accounts for the geometric characteristics of the well and the surrounding reservoir properties, as well as any interaction with other wells and boundaries.

Peaceman (1978) explained the interpretation of grid cell pressures using the WI. The main assumption of the Peaceman’s WI model is that it is derived for a vertical well in a uniform Cartesian grid, fully penetrating the gridblock, with single phase radial flow and no interaction with boundaries or other wells. Instead of using the classical Peaceman model, the Projection method (see Shu, 2005) has been used to calculate the WI for the reservoir simulation.

![Figure A.1: Projection of the well trajectory L on the x, y and z-axis of the gridblock](image)

Projection WI model is a modified version of Peaceman’s model. In this method, the well trajectory is projected onto the three orthogonal axis (x, y and z-axis). This is illustrated in Fig. A.1 where a part of the well trajectory is passing through a gridblock. The length of the well
trajectory in the gridblock is $L$ units and it projects $L_x$, $L_y$ and $L_z$ units in the x, y and z-axis, respectively. Using these three projected lengths ($L_x$, $L_y$ and $L_z$) and Peaceman’s equation for WI and equivalent well block radius ($r_0$), partial WI values are calculated for each orthogonal direction. Following are the equations:

$$J_x = \frac{2\pi L_x \sqrt{k_x k_z}}{\ln \left( \frac{r_{o,x}}{r_w} \right) + s}, \quad J_y = \frac{2\pi L_y \sqrt{k_y k_z}}{\ln \left( \frac{r_{o,y}}{r_w} \right) + s}, \quad J_z = \frac{2\pi L_z \sqrt{k_y k_z}}{\ln \left( \frac{r_{o,z}}{r_w} \right) + s}$$

(A.1)

$$r_{o,x} = 0.28 \left( \frac{k_x}{k_z} \right)^{\frac{1}{4}} \left( \frac{\Delta x^2 + \Delta y^2}{\Delta z^2} \right)^{\frac{1}{2}}, \quad r_{o,y} = 0.28 \left( \frac{k_y}{k_z} \right)^{\frac{1}{4}} \left( \frac{\Delta x^2 + \Delta z^2}{\Delta y^2} \right)^{\frac{1}{2}}, \quad r_{o,z} = 0.28 \left( \frac{k_y}{k_z} \right)^{\frac{1}{4}} \left( \frac{\Delta y^2 + \Delta z^2}{\Delta x^2} \right)^{\frac{1}{2}}$$

(A.2)

Finally, the WI for the well segment in the gridblock is calculated by taking the square root of the sum of the squares of these partial well indices:

$$J_{\text{well}} = \sqrt{J_x^2 + J_y^2 + J_z^2}.$$  

(A.3)

In case of segmented wells, with more than one segments within the same grid block, the projected length for the well index is the sum of the projections of all segments in that direction; see Fig. A.2.
Appendix B

Adjoint-based Optimization

The whole chapter has been prepared with significant contributions from Jansen et al. (2008). This chapter introduces the methodology of using adjoint equations in order to perform the gradient-based optimization. Specifically, the section discusses the usage of an adjoint equation to obtain the derivative information for the gradient-based optimization. Similar to Jansen et al. (2009), the focus is laid on life-cycle optimization.

Consider the optimization problem,

$$\max_{u_{1K}} \mathcal{J}(u_{1K}, y_{1K}(u_{1K})),$$

where

$$\mathcal{J}(u_{1K}, y_{1K}(u_{1K})) = \sum_{k=1}^{K} \mathcal{J}_k(u_k, y_k),$$

and

$$\mathcal{J}_k = \left[ \sum_{i=1}^{N_{inj}} r_{w,i} \cdot (u_{w,i})_k + \sum_{j=1}^{N_{pr}} (r_{w,pr} \cdot |y_{w,j}|_k + r_{o,pr} \cdot |y_{o,j}|_k) \right] \Delta t_k + \sum_{i=1}^{N_{inj}+N_{pr}} \mathcal{C}_i \cdot L_i.$$

The aim, here, is to maximize the objective function by computing the optimal control $u_{1K}$. To find the optimal control, it is required to calculate the derivatives of $\mathcal{J}$ with respect to $u_{1K}$. The analysis is started from an implicitly time-discretized version of the system equations:

$$g_k(u_k, x_{k-1}, x_k) = 0 \quad \text{for} \quad k = 1, 2, \ldots, K,$$

where $g$ is a vector-valued non-linear function, $x$ is a vector of state variables (pressures, phase saturations or component accumulations in each gridblock), the subscript $k$ indicates discrete time and $K$ is the total number of simulation time steps. It is also important to define the initial conditions in the entire domain:

$$x_0 = x_0.$$

Implicitly, output equations can be expressed as:

$$j_k(u_k, x_k, y_k) = 0,$$

where $j$ is a vector-valued non-linear function, $y$ is a vector of output variables (Flowing BHP or flowrates). To add further, $y$ may also contain the interpreted results of the field-wide measurements like time-lapse seismic or gravity surveys in some cases.
The indirect dependence of the variation $\delta J_{ik}$ in the objective function on a variation $\delta u_{ik}$ of the input appears to be a restriction in determining the derivatives. This happens because a variation $\delta u_{ik}$ ($i^{th}$ element of vector $u$ at time $k$) does not only directly influence $J$ at time $k$, but also, as follows from recursive application of Eq. (B.4), the states $x_{k,k}$, which in turn, through equation (B.6), influence the outputs $y_{k,k}$ and thus $J$ at later time steps. Therefore, chain rule for differentiation is used to assess the effect of a single variation $\delta u_{ik}$:

$$
\delta J = \left[ \frac{\partial J_k}{\partial u_k} + \sum_{j=k}^K \frac{\partial J_j}{\partial y_j} \left( \frac{\partial y_j}{\partial u_{jk}} + \frac{\partial y_j}{\partial x_j} \frac{\partial x_j}{\partial u_k} \right) \right] \frac{\partial u_k}{\partial u_{ik}},
$$

where $j$ is a dummy variable within the summation. As discussed above, the term $\partial x_j/\partial u_k$ is the only problem in the above equation. Recursive system of discrete-time differential equations Eq. (B.4) needs to be solved to link the state vectors $x_j, j=k+1,...,K$ to the input $u_k$ at time $j=k$. To find the connection, the technique of Lagrange multipliers is used by expressing the system equation (B.4), initial condition Eq. (B.5) and output equation (B.6) as constraints (setting aside any ordinary constraint). This leads to the modified objective function:

$$
\bar{J}(u_{1:k}, x_{0:k}, y_{1:k}, \lambda_{0:k}, \mu_{1:k}) \triangleq \sum_{k=1}^K \left[ J_k(u_k, y_k) + \lambda^T_0 (x_0 - x_0) \delta_{k-1}^T \right] + \lambda^T_k g_k(u_k, y_k) + \mu^T_k j_k(u_k, y_k),
$$

where the constraints have been ‘adjoined’ to $J_k$ with the Lagrange multipliers $\lambda$ and $\mu$. The use of Kronecker delta ensures the inclusion of the initial condition constraint in the summation. A necessary condition for maximizing the modified objective function expressed by Eq. (B.8) is stationarity of the first variation of $\bar{J}$ with respect to all dependent variables, i.e., all first order derivatives should be equal to zero. Following are the first-order necessary conditions for an optimum (a.k.a. extended Euler-Lagrange equations):

$$
\frac{\partial \bar{J}}{\partial u_k} = \frac{\partial J_k}{\partial u_k} + \lambda^T_k \frac{\partial g_k}{\partial u_k} + \mu^T_k \frac{\partial j_k}{\partial u_k} = 0^T,
$$

$$
\frac{\partial \bar{J}}{\partial x_0} = \lambda^T_0 \frac{\partial g_0}{\partial x_0} + \lambda^T_0 = 0^T
$$

$$
\frac{\partial \bar{J}}{\partial x_k} = \lambda^T_{k+1} \frac{\partial g_{k+1}}{\partial x_k} + \lambda^T_k \frac{\partial g_k}{\partial x_k} + \mu^T_k \frac{\partial j_k}{\partial x_k} = 0^T
$$

$$
\frac{\partial \bar{J}}{\partial x_K} = \lambda^T_K \frac{\partial g_K}{\partial x_K} + \mu^T_K \frac{\partial j_K}{\partial x_K} = 0^T
$$

$$
\frac{\partial \bar{J}}{\partial y_k} = \lambda^T_k \frac{\partial g_k}{\partial y_k} + \mu^T_k \frac{\partial j_k}{\partial y_k} = 0^T
$$

$$
\frac{\partial \bar{J}}{\partial y_0} = (x_0 - \bar{x}_0)^T = 0^T
$$
Here, the equations are discussed from the bottom to the top. Equations (B.14), (B.15) and (B.16) are identical to the initial condition Eq. (B.5), system equation (B.4) and output equation (B.6), respectively and thus, automatically satisfied. Equation (B.13) helps to compute the Lagrange multipliers $\mu_{1:k}$. Once $\mu$ is known, equation (B.12) is used to compute multiplier $\lambda_k$ for the final discrete time $K$, and thereafter equation (B.11) to compute the multipliers $\lambda_k$ for $k = K - 1, \ldots, 0$, i.e. backward in time. Next, equation (B.10) represents the effect of changing the initial condition $x_0$ on the value of the objective function, while keeping all other variables fixed. Although unnecessary, it can be used to compute $\lambda_0$. Finally, equation (B.4) signifies the effect of changing the control on the value of the objective function, while keeping all the other variables fixed. For a non-optimal control, this term is not equal to zero, but then its residual is just the modified gradient (i.e., the transposed gradient) that we require to iteratively obtain the optimal control using a gradient-based algorithm.

To conclude, the following algorithm can be used to compute the gradient vectors as part of an iterative gradient-based optimization procedure:

I. Choose the initial control vector $u_{1:K}$.

II. Using equations (B.4), (B.5) and (B.6), compute the states $x_{1:K}$ and outputs $y_{1:K}$.

III. Compute the value of the objective function $J$ with equations (B.2) and (B.3). Continue unless converged.

IV. Compute the Lagrange multipliers $\mu_{1:K}$ and $\lambda_{1:K}$ using equations (B.11), (B.12) and (B.13).

V. Compute the total derivatives (transposed gradients) $\frac{\partial J}{\partial u_{1:k}}$ of the objective function with respect to the controls from the residuals of equation (B.9) according to:

$$\frac{\partial J}{\partial u_k} = \frac{\partial J}{\partial u_k} + \lambda_k^T \frac{\partial g_k}{\partial u_k} + \mu_k^T \frac{\partial f_k}{\partial u_k}, k = 1, 2, \ldots, K.$$  \hspace{1cm} (B.17)

VI. Compute an improved estimate of the control vector $u_{1:K}$, using the derivatives as obtained from equation (B.17) and a gradient-based minimization routine of choice.

VII. Return to step 2.

Apparently, adjoint method is an efficient method of computing the gradient of the objective function. It needs only two simulation runs; applying the discretized PDE in the normal forward mode and subsequently its adjoint in a backward mode. Hence, it is advantageous to use adjoint-method for large models demanding a lot of expensive simulations.

Although it is an efficient means of calculating gradients, the adjoint method has few downsides. The presence of output/path constraints can make the adjoint method inefficient.
This may happen because the gradient information of constraint is required which in turn, means that each new constraint requires an extra Lagrange multiplier and hence, a new adjoint system.

Secondly, it is an arduous task to implement the adjoint method howsoever straightforward it is. Besides, as the algorithm suggests, the complete solution of the forward simulation (all the state vectors at each solution time) must be stored since they are used as inputs for the backward simulation (adjoint model).

In short, the adjoint method comes in handy to compute efficiently and it can save several expensive reservoir simulations compared to gradient-free techniques of maximizing the objective function.
Appendix C

Well Placement Optimization Examples

Numerous other cases were experimented and observed to confirm the consistency of the algorithm. This appendix presents two well placement optimization examples.

As a first case, the tested heterogeneous environment of the ‘Egg Model’ is presented. The reservoir and economic parameters were initialized using Table 3.3 and Table 3.2, respectively. Random locations were selected for all the eight injectors. Note, only injectors were optimized for the case. Fig. C.1 reveals a 70% increment in NPV with only 25 iterations. With an oil production of 0.5 Million m$^3$, the RF has been calculated to be 59%.
Figure C.1: NPV graph, saturation map (of last time step) and corresponding trajectory map of optimized well trajectories. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively.

Even though the RF is high, oil saturation appears to be large in some regions. Unless uneconomical, these zones can be produced by planning addition well(s) during the later phase of the field development.
During the second case, the best locations provided in the standard ‘Egg Model’ were used for all the eight injectors. The number of time steps had to be increased to 138 to overcome simulation convergence issues. Fig. C.3 reveals a 7.7% increment in NPV with only 25 iterations. The RF of the optimized project was almost 60%.
Figure C.3: NPV graph, saturation map (of last time step) and corresponding trajectory map of optimized well trajectories. The bottom 3 illustrations are the top view (XY), front view (XZ) and side view (YZ) of the trajectory map, respectively.

Even though the number of iterations were equal in the two above mentioned cases, the NPV increment and the RF % varied drastically. As seen in the first scenario, the wells are content to be at their local optimal solution (e.g.: Injector 1 and Injector 2). To improve this, one could consider either ‘minimum distance’ between two injectors or defining injectivity index.

All things considered, both the scenarios produced a promising well placement scenario compared to the initial scenario.