Proceedings of Meetings on Acoustics

Volume 19, 2013 http://acousticalsociety.org/

ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013

Biomedical Acoustics
Session 1aBA: Ultrasound Tomography

1aBA2. Quantitative ultrasound tomography

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Tomogrophic ultrasound imaging is gaining popularity in breast cancer detection. Reconstructing the acoustic properties of a breast from the ultrasound measurements is stated as a nonlinear inverse problem, which is usually solved by linearized methods because of computational efficiency. However, linearization of the problem reduces the quality of the reconstruction. To improve the accuracy, we developed and tested a three-dimensional nonlinear inversion method that allow for three-dimensional reconstruction of the breast in terms of speed of sound. The method, referred to as contrast source inversion (CSI), uses an integral equation formulation to describe the inverse acoustic scattering problem. The resulting integral equation is solved to reconstruct the unknown contrast (speed-of-sound profile of the breast). The contrast and contrast sources (the product of the contrast with the total field) are iteratively updated by minimizing a cost functional using conjugate gradient directions. In this study, we tested the CSI method on synthetic data retrieved form full-wave simulations for a realistic three-dimensional cancerous breast model. Results show that the CSI method outer performs other conventional methods as it yields speed-of-sound reconstructions that are akin to the model. This shows that the approach offers a contribution to the detection of breast cancer.

Published by the Acoustical Society of America through the American Institute of Physics

Received 22 Jan 2013; published 2 Jun 2013

INTRODUCTION

Breast imaging is an important method to screen for and to diagnose breast cancer in asymptomatic patients. In many cases, these images are obtained using X-rays or MRI-scans. Currently, ultrasound is gaining interest due to its success in detecting tumors in dense breasts. Unfortunately, hand-held ultrasound scanning systems are severely operator dependent and cause therefore difficulties in producing repeatable accurate measurements. To overcome the limitations of the hand-held systems, research is conducted towards fully-automated three-dimensional ultrasound breast scanning systems [1]. An additional advantage is that the data acquired do allow for advanced imaging algorithms. Taking advantage of the tomographic measurements, several imaging algorithms are currently investigated, varying from inverse Radon transforms similar to the one used for CT-scans [2] up to Born inversion methods. However, these ray based or linearized methods will have problems in obtaining accurate speed of sound of profiles. Consequently, full non-linear inversion methods [3] are currently being developed to overcome the limitations of the approximation based methods. To show the advances of these non-linear inversion methods, five different imaging methods have been tested on the same synthetic tomographic measurement data set. This data set is obtained by solving the forward problem using a full wave method [4, 5].

THEORY

To ease the comparison between the various imaging methods, only media showing a spatially varying speed of sound are considered, and both attenuation and a spatially varying volume density of mass are neglected. Consequently, the propagation and scattering of acoustic wave fields in inhomogeneous media may be described via the scalar wave equation, which equals

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})}\frac{\partial^2 p(\vec{x},t)}{\partial t^2} = -S(\vec{x},t)\ , \tag{1}$$

where $p(\vec{x},t)$ is the acoustic pressure field, $c(\vec{x})$ is the speed of sound, $S(\vec{x},t)$ is the primary source generating the acoustic wave field and where $\vec{x}$ and t represent the space and time coordinate, respectively. In the frequency domain, equation (1) yields the Helmholtz equation for inhomogeneous media, i.e.

$$\nabla^2 \hat{p}(\vec{x}) + \frac{\omega^2}{c^2(\vec{x})}\hat{p}(\vec{x}) = -\hat{S}(\vec{x})\ , \tag{2}$$

where ω is the angular frequency and where the caret symbol $\hat{}$ is used to emphasize that a given quantity is defined in the angular frequency domain. In order to model the propagation of the acoustic wave field in inhomogeneous media it may be advantageous to recast the Helmholtz equation of equation (2) into an integral equation which equals

$$\hat{p}(\vec{x}) = \hat{p}^{\mathrm{inc}}(\vec{x}) + \omega^2 \int_{\vec{x}'\in\mathbb{D}} \hat{G}(\vec{x}-\vec{x}')\chi(\vec{x}')\hat{p}(\vec{x}')\mathrm{dV}(\vec{x}')\ , \tag{3}$$

where $\hat{p}^{\mathrm{inc}}(\vec{x})$ is the acoustic wave field generated by the primary source $\hat{S}(\vec{x})$ and propagating in the homogeneous background medium with speed of sound c_0, $\hat{G}(\vec{x})$ is the scalar Green's function, and $\chi(\vec{x}) = \frac{1}{c^2(\vec{x})} - \frac{1}{c_0^2}$ is the contrast function defined in the spatial domain $\mathbb{D}$. This integral equation is linear in the pressure field $\hat{p}(\vec{x})$, however the pressure field is non-linear with respect to the contrast function $\chi(\vec{x})$.

Next, two particular cases of in interest are considered. The first case refers to the situation where both the contrast function $\chi(\vec{x})$ and the primary source $\hat{S}(\vec{x})$ are known, but where the total or actual field $\hat{p}(\vec{x})$ is unknown. In literature, this case is referred to as the forward problem. The second case is where the total field $\hat{p}(\vec{x})$ is measured at some particular locations, where the primary source $\hat{S}(\vec{x})$ is known, but where the contrast function $\chi(\vec{x})$ is unknown. Reconstructing the contrasts function from the measured total field is in literature referred to as the inverse problem.

Various methods exist to reconstruct the contrast profile from the measured wave field $\hat{p}(\vec{x})$. Here, five different methods are considered, all applicable to a cylindrical geometry; a cylindrical ring of transducers surrounding an unknown contrast. Although the method is fully applicable to three-dimensional problems, we have limited ourselves to the two-dimensional case.

Ray Approximation

The first method is based on the assumption that the pressure field travels in straight lines, and that the presence of inhomogenities in the speed of sound only results in a shift in travel and hence arrival time. For the two-dimensional case, the forward problem may within this approximation be formulated as a Radon transform which equals

$$P_\beta(\gamma) = \int_{(x,y)=-\infty}^{\infty} \frac{1}{c(x,y)} \delta\left(-x\sin[\beta+\gamma] - y\cos[\beta+\gamma] - R\sin[\gamma]\right) \mathrm{d}x\,\mathrm{d}y \;, \tag{4}$$

with $P_\beta(\gamma)$ the travel time for a wave propagating from the transmitter to the receiver, R the radius of the circle on which the transducer are positioned, β the angle between the y-axis and the transmitter and γ the position of a ray in a fan beam [2, 4]. From here on, equation (4) refers to the Radon transform. Reconstructions of the region of interest may be obtained by computing the inverse Radon transform [2].

Time Domain Migration

Imaging using Time Domain Migration or Synthetic Aperture Focusing Techniques may be adequate in situations where only reflectivity images are needed, i.e. only the boundary of the object needs to be located. An important condition for retrieving useful images is that the presence of multiple scattering is only limited. Within this constraint, the location of a contrast may be determined in a straightforward manner by measuring the time τ needed for a wave to travel from the source located at $\vec{x}^{\mathrm{src}}$ via the boundary of an object to the receiver at $\vec{x}^{\mathrm{rec}}$. Combining all possible source/receiver combinations leads to constructive and destructive interference of the reflected field. Hence, the boundary of the contrast $\chi(\vec{x})$ may be imaged via

$$\chi(\vec{x}) = \sum_{\vec{x}^{\mathrm{src}},\vec{x}^{\mathrm{rec}}} \int p(\vec{x}^{\mathrm{rec}},t)\delta(t-t')\mathrm{d}t, \quad \text{with } t' = \frac{|\vec{x}-\vec{x}^{\mathrm{src}}|}{c_0} + \frac{|\vec{x}-\vec{x}^{\mathrm{rec}}|}{c_0} \;. \tag{5}$$

Born Inversion

Imaging by using integral equation (3) poses an ill-posed problem as the integral equation is non-linear in the contrast $\chi(\vec{x})$. Fortunately, the integral equation may be linearized by replacing the total field $\hat{p}(\vec{x})$ within the integrand with the known incident field $\hat{p}^{\mathrm{inc}}(\vec{x})$. In literature, this approximation is referred to as the Born approximation and the resulting integral equation equals

$$\hat{p}(\vec{x}) = \hat{p}^{\mathrm{inc}}(\vec{x}) + \omega^2 \int_{\vec{x}\in\mathbb{D}} \hat{G}(\vec{x}-\vec{x}')\chi(\vec{x}')\hat{p}^{\mathrm{inc}}(\vec{x}')\mathrm{dV}(\vec{x}') \;. \tag{6}$$

Next, an image may be retrieved by solving the linearized integral equation using a conjugate gradient scheme. Note that the first step of this scheme is quite similar to the SAFT method, and is in this work referred to as backpropagation (BP).

Contrast Source Inversion

All of the above methods only work if refraction and multiple scattering may be neglected. In addition, only the inverse Radon transform will give speed of sound profiles, whereas the remaining two (SAFT and Born Inversion) will yield images displaying only the boundary of the object. In order to retrieve speed of sound profiles from the measured data, the full non-linear wave equation needs to be solved. A suitable method to reconstruct speed of sound profiles for strongly inhomogeneous media is the Contrast Source Inversion (CSI) method [3]. CSI is an iterative method, where within each update step first a contrast source $\hat{w}(\vec{x}')$ is constructed from the measured data. Next, an update of the unknown total field within the entire spatial domain is computed. Finally, the contrast function $\chi(\vec{x})$ is reconstructed from the contrast source and the updated total field using a direct minimization method. Consequently, with CSI integral equation (3) is formulated as

$$\hat{p}(\vec{x}) = \hat{p}^{\mathrm{inc}}(\vec{x}) + \omega^2 \int_{\vec{x}\in\mathbb{D}} \hat{G}(\vec{x}-\vec{x}')\hat{w}(\vec{x}')\mathrm{dV}(\vec{x}') \, , \tag{7}$$

with contrast source

$$\hat{w}(\vec{x}) = \chi(\vec{x})\hat{p}(\vec{x}). \tag{8}$$

The advantage of this method is that allows for speed of sound reconstruction, while in the presence of noise additional regularization such as total variation may be applied [3].

RESULTS

Prior to testing the five imaging methods, synthetic measurement data is computed using a synthetic breast model constructed from a MRI scan of a cancerous breast model [6]. To ease the comparison between the ray based method and the remaining ones, only the two-dimensional problem is considered. To solve the resulting scalar integral equation, a conjugate gradient solution methods is applied after weakening the integral equation to overcome problems associated with singularities [7].

Figure 1 shows the synthetic breast model. Both the tissue index, based on table 1, and the speed of sound profile is shown.

TABLE 1: Speed of sound of different tissues

Index	Tissue	Speed of sound c [m/s]
1	Tumor	1500
2	Breast tissue	1485
3	Fat	1478
4	Water	1524

For the shown synthetic breast, the forward problem is solved twice to compute the fan-beam measurement data, see figure 2. Once using the Radon transform and once using the full wave method. With the latter one, variations in travel time are computed by comparing the incident field with the total field using cross-correlation techniques. Both Radon transforms are only limited similar with respect to shape and amplitude. Variations between both transforms are fully explainable by the fact that all kind of wave phenomena, such as multiple scattering, interference, refraction and diffraction, are neglected in the ray based Radon transform. The resulting reconstructions are computed using the same inverse Radon transform applied on both data sets.

SAFT, BP, born inversion and CSI have all been applied to the same synthetic measurement data set and the resulting reconstructions are shown in figure 3. Due to the limited temporal

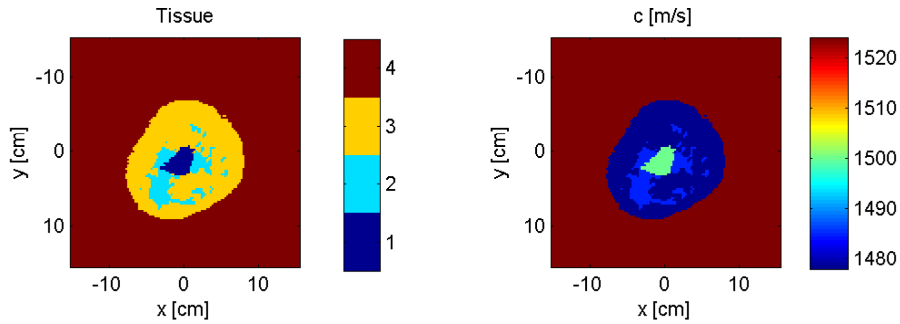

FIGURE 1: Cross sectional image of breast phantom; (left) tissue index and (right) speed of sound parameters are shown in table 1

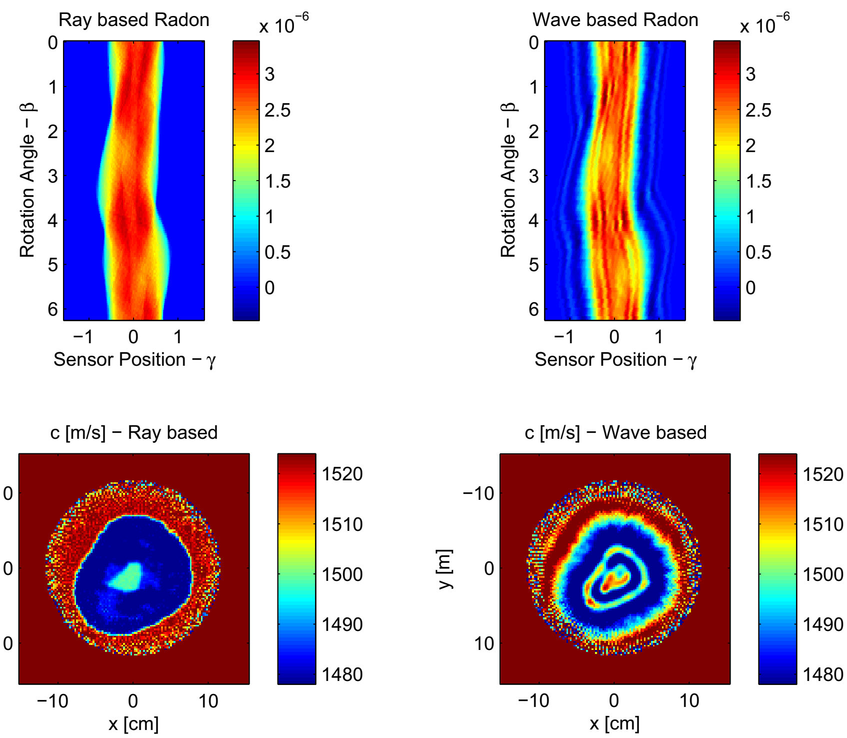

FIGURE 2: Fan-beam Radon transforms computed (left) directly from the speed of sound profile or (right) by cross-correlating the incident and total acoustic pressure fields. The reconstructions are obtained by applying the inverse Radon transforms on the (left) ray and (right) wave based data.

resolution, both the SAFT and BP reconstruction are severely blurred. With Born Inversion, the boundary of the contrast, i.e. both breast and tumor, are clearly localized but with a wrong amplitude. Finally, the CSI method yields both accurate spatial resolution and amplitudes.

DISCUSSION AND CONCLUSION

Five different tomographic reconstruction methods have been tested on the same synthetically measured data set. The data set has been computed using a full wave method to include wave phenomena such as multiple scattering, refraction, diffraction and interference. First, an inverse Radon transform has been tested on the data. The resulting image shows many artifacts which are not present in the image obtained from the same inverse Radon transform

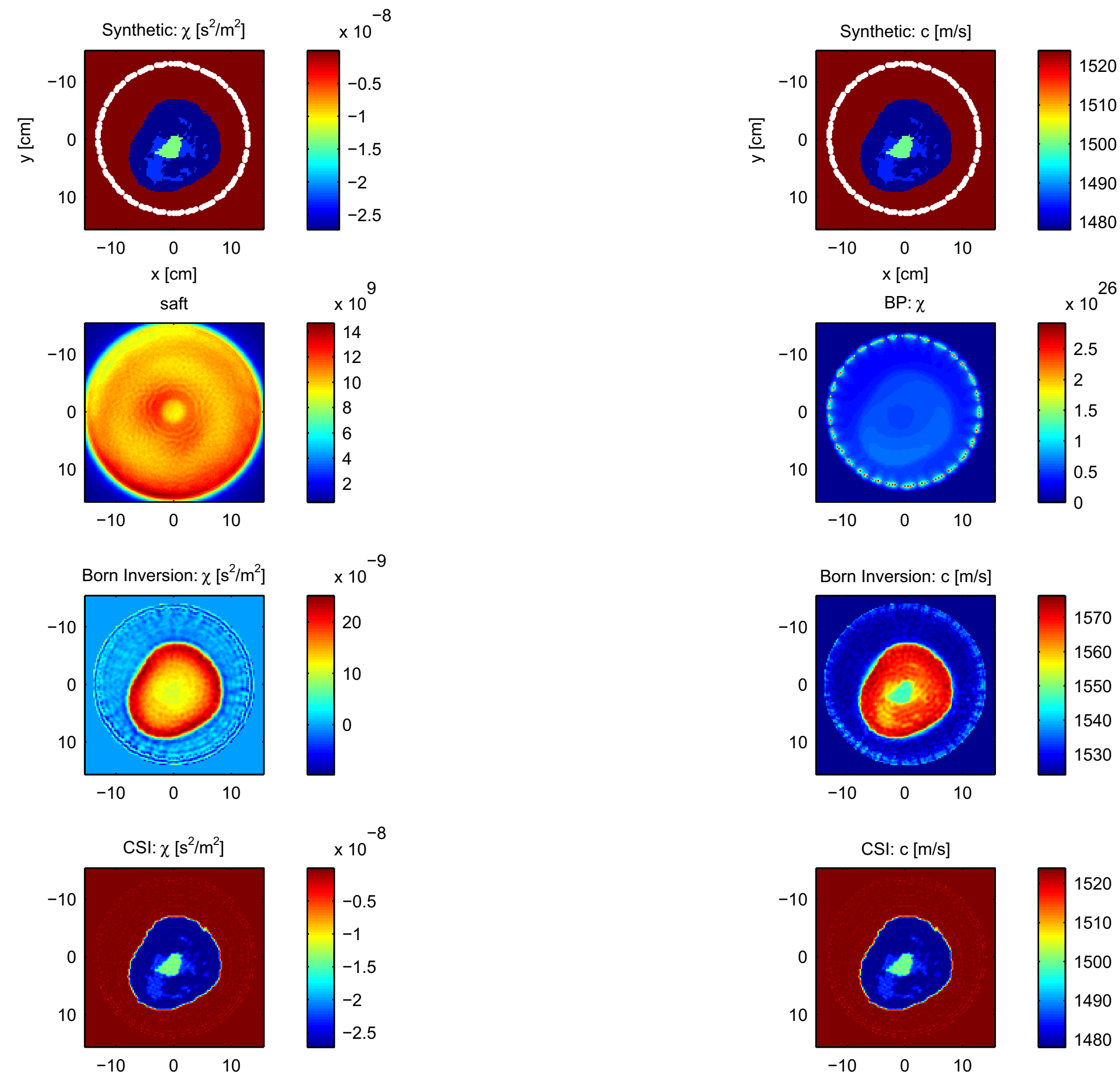

FIGURE 3: Reconstructions of the (left) contrast profiles and (right) speed of sound profiles; from top to bottom the synthetic profile, SAFT & BP, Born inversion, and finally CSI inversion.

applied on a ray based Radon transform data set. This shows the need for a wave based imaging method, although some of the artifacts may be overcome by applying e.g. filtering techniques. Of the remaining four imaging techniques, only one uses time domain data: SAFT. This fast and easy to implement method will only generate reflection images, if applied successfully. With this method, no speed of sound reconstructions can be obtained, and the method requires a fine temporal discretization. With BP and Born Inversion, only the outline of a contrast may be obtained. Note that, in theory BP show (some) similarities with both SAFT and the first step of the Born Inversion scheme. Finally, CSI method has been tested and yields accurate speed of sound profiles with correct amplitudes. In conclusion, of the five imaging methods tested on synthetically measured data, only with CSI quantitative ultrasound imaging is feasible.

ACKNOWLEDGMENTS

This research has been financially supported by the Dutch Technology Foundation STW.

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