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HIGH LIFT BOUNDARY LAYER CONTROL

by

ir. H. Tennekes

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translation of: "Draagkrachtsverhoging door Grenslaagafzuiging", an article in the 3rd "Blue Book" commemorating the 15th anniversary of the association of aeronautical students in the Netherlands "Leonardo da Vinci"; dated October 1960

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Summary

Boundary layer control may be used to prevent separation of the boundary layer at the suction side of airfoils at large angles of attack. In this way the maximum lift coefficient can be increased. At a prescribed value of the lift coefficient the power needed by the suction pump depends on the distribution of the suction velocity over the surface of the airfoil. The calculation methods for the required suction distribution as presented by Schlichting-Pechau and by Raspet are discussed. Actual calculations for the airfoil NACA 747-A-315 at \( c_L = 2.35 \) and \( R_c = 3 \times 10^6 \) indicate that a suction distribution according to Schlichting-Pechau is preferable if a small suction volume flow is desired. On the other hand, the calculation method presented by Raspet gives a markedly lower value of the effective drag coefficient. A modification of Raspet's calculation scheme appears promising if, due to requirements of sound wing construction, the suction has to be started rather far from the pressure peak at the leading edge of the airfoil. These conclusions however, are based on approximative calculations; they are valid only insofar as the calculation methods may be trusted.
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Notation

N.B.: for the symbols not shown in this list is referred to fig. 1. The quantities are nondimensional, except where the dimension is indicated by square brackets.

\[ c_d \] airfoil drag coefficient
\[ c_{deff} \] effective drag coefficient
\[ c_f \] local skin friction coefficient
\[ c_l \] airfoil lift coefficient
\[ C_p \] pressure loss coefficient, eq'n (2)
\[ C_Q \] suction volume coefficient, eq'n (3)
\[ d \] airfoil drag, per unit span \( \text{kg/m} \)
\[ G \] "natural growth rate" of boundary layer, eq'n (10)
\[ H \] boundary layer form parameter
\[ P \] power of suction pump, per unit span \( \text{kg/sec} \)
\[ p \] static pressure on surface of airfoil \( \text{kg/m}^2 \)
\[ P_{int} \] static pressure in plenum chamber \( \text{kg/m}^2 \)
\[ P_\infty \] static pressure of free stream \( \text{kg/m}^2 \)
\[ P_t \] total pressure of free stream \( \text{kg/m}^2 \)
\[ \Delta p \] \( P_t - P_{int} \) \( \text{kg/m}^2 \)
\[ Q \] suction volume flow, per unit span \( \text{m}^2/\text{sec} \)
\[ R_c \] \( U_\infty c/\nu \)
\[ R_\theta \] \( U_\theta \theta/\nu \), see also fig. 3 \( \text{m/sec} \)
\[ U \] velocity parallel to surface within boundary layer \( \text{m/sec} \)
\[ U_o \] local velocity at edge of boundary layer \( \text{m/sec} \)
\[ U_\infty \] free stream velocity \( \text{m/sec} \)
\[ -v_o \] suction velocity \( \text{m/sec} \)
\[ \theta \] momentum thickness of boundary layer \( \text{m} \)
\[ \nu \] kinematic viscosity of air \( \text{m}^2/\text{sec} \)
\[ \rho \] density of air \( \text{kg/sec}^2 \)
\[ \eta_p \] efficiency of suction pump
\[ \eta_T \] efficiency of propulsion system
1. Introduction

1.1. General remarks

Two distinctly different applications of boundary layer suction are possible if the performance of aircraft is to be improved. In cruising flight, at small angles of incidence, suction of the laminar boundary layer can prevent transition of the boundary layer. The obtained "laminarization" of the boundary layer produces a large reduction of wing drag \[1\]. Boundary layer suction can be applied also to prevent boundary layer separation at large angles of incidence \[2\], \[3\]. This increases the maximum lift coefficient of aircraft wings. This result may be used to give a reduction in the runway length required for the take-off and landing of aircraft \[4\].

The discussion given in this paper is restricted to some aerodynamic aspects of high lift boundary layer control; special attention will be paid to the problems associated with the choice of a suitable distribution of the suction velocity.

1.2. The nature of the problem

The discussion of boundary layer control at large angles of incidence, as presented in this paper, is restricted to the effects of suction on turbulent boundary layers. This simplification is meaningful only for the flow around airfoils whose maximum lift coefficient is limited by trailing edge separation of the turbulent boundary layer. This is generally valid for rather thick airfoils, or for airfoils on which laminar boundary layer separation is artificially prevented. In other words, it is assumed that boundary layer separation near the pressure peak at the leading edge of the airfoil (which usually is due to the "explosion" of the laminar separation bubble) does not occur.

The discussion is further restricted to the study of distributed suction through a porous surface (distributed perforation is also accepted). Only two-dimensional flow around airfoils is considered. A suction slit at the leading edge of the porous surface will be allowed; see fig. 1. Aspects of the problem related to flight performance and wing construction are kept out of consideration. However, full attention will be paid to the problem of choosing distributions of the suction intensity that will give (at prescribed airfoil lift coefficient) the lowest possible value of the power required by the suction pump or of the effective drag (to be defined later). The criteria used to estimate the point of separation of the turbulent boundary layer also receive attention. It will turn out that the momentum thickness of the boundary layer is an important, often even decisive parameter.
1.3. The choice of a suction intensity distribution

In the literature two schemes of boundary layer control through distributed suction of airfoils at large angles of incidence are available. In both cases the calculations are based on the momentum integral equation for boundary layers; in both cases equivalent assumptions concerning the skin friction and the form parameter of the boundary layer are used. Hence, a discussion of the relative merits of the two schemes, like the one presented below, is possible and meaningful.

The suction scheme advocated by Schlichting and Pechau [3] incorporates a constant suction velocity, which is applied from the stagnation point to a location rather close behind the pressure peak at the leading edge, over the upper surface of the airfoil. The magnitude of the suction velocity is determined with the criterion that the boundary layer at the trailing edge of the airfoil is allowed to be on the verge of separation; see fig. 2a.

According to the suction scheme proposed by Raspet [2], distributed suction is applied over the upper surface of the airfoil, starting from a point close behind the pressure peak up to the trailing edge. The distribution of the suction intensity is determined in such a way that the Reynolds number based on the momentum thickness of the boundary layer and the local velocity just above the boundary layer is maintained at a constant value; see fig. 2b.

These two schemes are used for a quantitative analysis of high lift boundary layer control as applied to a particular airfoil at prescribed angle of attack. This analysis is presented in chapter 4; the numerical results of a modification of Raspet’s suction scheme are given and discussed also.

2. Equations

2.1. General

The interior of the airfoil (resp. wing section) is assumed to be constructed as a single plenum chamber. The pressure in this chamber \( p_{int} \) has to be less than the lowest static pressure on the porous surface, see fig. 1. It is assumed that the suction pump increases the total head of the sucked air to the total head of the free stream, \( p_t \). This implies that the air can flow out of the exhaust of the suction system at a pressure \( p_\infty \) and a velocity \( U_\infty \). If \( Q \) is the suction flow per unit time and per unit span, the power required by the suction pump is:

\[
P = \frac{1}{\eta_p} \Delta \rho Q = \frac{1}{\eta_p} (p_t - p_{int}) Q,
\]  

(1)
In this equation $\eta_p$ is the efficiency of the suction system.
The pressure loss coefficient $C_p$ and the suction volume coefficient $C_Q$ can be defined now as follows:

$$C_p = \frac{\Delta p}{\frac{1}{2} \rho U^2_{\infty}}$$  \hspace{1cm} (2)

$$C_Q = \frac{Q}{\frac{U}{C \rho U^3_{\infty}}} = \int_{x_i}^{x_f} \left( \frac{Q}{\frac{U}{C \rho U^3_{\infty}}} \right) dx/c$$  \hspace{1cm} (3)

The pump power per unit span can be written as:

$$P = \frac{C_p C_Q}{\eta_p} \cdot \frac{1}{2} \rho U^3_{\infty} C$$  \hspace{1cm} (4)

The power of the propulsion system that is necessary to balance the airfoil drag amounts, per unit span, to:

$$W = \frac{1}{\eta_T} d.U_{\infty} = \frac{1}{\eta_T} \frac{C_d}{\eta_p} \cdot \frac{1}{2} \rho U^3_{\infty} C,$$  \hspace{1cm} (5)

In this equation, $\eta_T$ is the efficiency of the propulsion system of the aircraft.
The total power required per unit span is:

$$P + W = \left\{ \frac{C_p C_Q}{\eta_p} + \frac{C_d}{\eta_T} \right\} \frac{1}{2} \rho U^3_{\infty} C.$$  \hspace{1cm} (6)

The effective drag coefficient $c_{d,\text{eff}}$ is defined as follows:

$$c_{d,\text{eff}} = \frac{\eta_T (P + W)}{\frac{1}{2} \rho U^3_{\infty} C} = \frac{\eta_T}{\eta_p} C_p C_Q + c_{d_p}$$

In the following pages, $\eta_T$ and $\eta_p$ are assumed to have equal values. This assumption is acceptable if the aircraft is fitted with piston engines for propulsion. For jet engines however, $\eta_T$ may be lower than $\eta_p$. If this assumption holds, the effective drag coefficient can be written as:

$$c_{d,\text{eff}} = C_p C_Q + \frac{c_{d_p}}{\eta_p}$$  \hspace{1cm} (7)

Since the suction pump increases the total head of the sucked air to $p_t$, the so-called "sink drag" is absent. The nondimensional quantity $C_p C_Q$ is called the "pump drag coefficient". The airfoil drag coefficient $c_{d_p}$ is proportional
to the momentum thickness of the boundary layer at the trailing edge of the airfoil. The momentum thickness of a boundary layer is defined in fig. 3; the Reynolds number based on the momentum thickness and the local velocity at the outer edge of the boundary layer \( U_o \) is written as \( \Theta_o \). The airfoil drag coefficient can, according to 5, approximately be written as:

\[
\frac{C_d}{2} = 2 \frac{\Theta_{t+} + \Theta_{t-}}{c} = \frac{2R \Theta_{t+} + 2R \Theta_{t-}}{R_c},
\]

(8)

in which the subscript "t" denotes the trailing edge of the airfoil, in which further the subscripts "a" and "o" denote the high pressure side (lower surface) and the low pressure side (upper surface) of the airfoil, respectively. The second step given in eq'n (8) can be performed since the local velocity at the outer edge of the boundary layer at the trailing edge of the airfoil \( U_{o_t} \) is approximately equal to \( U_\infty \) at least if the boundary layer at the trailing edge is not separated [5].

2.2. Power required by the suction pump

The momentum thickness distribution of the boundary layer on the upper surface of the airfoil is a function of the chosen distribution of the suction velocity. If the distribution of the static pressure on this surface is known, the suction distribution can be chosen arbitrarily through a suitable choice of the porosity of the surface as a function of \( x \) and the pressure \( p_{int} \) in the plenum chamber. Using the momentum integral equation for boundary layers, the development of the boundary layer can be calculated. This equation reads [5]:

\[
-\frac{V_o}{U_\infty} = \frac{1}{2} c f \frac{U_o}{U_\infty} - (H+1) \frac{R_o Q}{R_c U_\infty} \frac{dU_o}{dx/c} - \frac{1}{R_c} \frac{dR_o}{dx/c}.
\]

(9)

The coordinate \( \frac{x}{c} \) should be measured along the surface of the airfoil. However, for airfoils whose thickness is small and which have small camber, \( \frac{x}{c} \) may be measured along the chord line, except in the direct vicinity of the (appreciably curved) leading edge. This is indicated in fig. 1.

To facilitate the interpretation of a number of formulae, the quantity \( G \) is introduced:

\[
G = \frac{1}{2} c f \frac{U_o}{U_\infty} - (H+1) \frac{R_o Q}{R_c U_\infty} \frac{dU_o}{dx/c} ;
\]

(10)
This quantity is a measure for the growth rate of the boundary layer insofar as this is due to the skin friction and the pressure gradient. It is proposed to call it the "natural growth rate", it depends on the suction velocity since suction decreases \( R_{\Omega} \), which is effectful in either of the two terms of which \( \mathcal{G} \) consists. Using eq'ns (3), and (10), the momentum integral equation, eq'n (9), can be written as:

\[
\frac{dC_Q}{dx/c} = -\frac{v_o}{U_{\infty}} = G - \frac{1}{R_c} \frac{dR_{\Omega}}{dx/c}
\]  

(11)

The application of slit suction at the leading edge of the suction surface is foreseen, as indicated also in section 1.2. Slit suction will give a sudden reduction of \( R_{\Omega} \) at the point where it is applied. The relation between the suction volume flow through the slit and the reduction of \( R_{\Omega} \) can be calculated with eq'n (11), neglecting the local contribution of \( \mathcal{G} \) (if is assumed to remain finite) and integrating, to obtain:

\[
\Delta C_Q = -\frac{R_{\Omega}}{R_c} \frac{R_{\Omega_j} - R_{\Omega_i}}{R_c}
\]

In this equation \( R_{\Omega_j} \) relates to the boundary layer just in front of, and \( R_{\Omega_i} \) to the boundary layer just behind the slit.

The pump drag coefficient can now be written as:

\[
C_p C_Q = C_p \left\{ \int_{x_i}^{x_f} \frac{dC_Q}{dx/c} \frac{dx}{c} + \Delta C_Q \right\} = C_p \left\{ \int_{x_i}^{x_f} \frac{R_{\Omega_j} - R_{\Omega_i}}{R_c} dx/c + \int_{x_i}^{x_f} G \frac{dx}{c} \right\}
\]

(12)

in which \( R_{\Omega_f} \) relates to the boundary layer at the trailing edge of the porous surface (which does not need to be the trailing edge of the airfoil).

2.3. Effective drag

The effective drag coefficient can be written as (according to eq'n (7) and substituting eq'ns (8) and (12)):

\[
C_{d, eff} = \frac{2}{R_c} \frac{R_{\Omega_t}^+}{R_{\Omega_t}^+} + \frac{2}{R_c} \frac{R_{\Omega_t}^-}{R_{\Omega_t}^-} + \frac{C_p}{R_c} (R_{\Omega_j} - R_{\Omega_f}^+ + C_p \int_{x_i}^{x_f} G \frac{dx}{c})
\]

For the unsucked part of the boundary layer is valid:

\[
\frac{R_{\Omega_j}^-}{R_c} = \frac{R_{\Omega_f}^-}{R_c} + \int_{x_f}^{x_i} G \frac{dx}{c}
\]
which is derived from eq'n (11) through removal of the suction term. The effective drag coefficient can then be written as follows:

$$c_{d_{\text{eff}}} = \frac{1}{R_c} (2R_c Q_t + C_p R_c Q_j) + \frac{2-C_P}{R_c} R_c Q_f +$$

$$+ C_p \int_{x_1}^{x_f} G \frac{dx}{c} + 2 \int_{x_f}^{x_t} G \frac{dx}{c}.$$  \hspace{1cm} (13)

If the distributed suction extends to the trailing edge of the airfoil, eq'n (13) reduces to:

$$c_{d_{\text{eff}}} = \frac{1}{R_c} (2R_c Q_t + C_P R_c Q_j) - \frac{C_P}{R_c} R_c Q_t + C_p \int_{x_1}^{x_t} G \frac{dx}{c}.$$  \hspace{1cm} (14)

3. Qualitative analysis

3.1. Introduction

In this chapter the qualitative effects of several variables on the power required by the suction pump and on the effective drag are discussed in their relation to the desired optimization of boundary layer control systems. The lift coefficient $C_L$ and the Reynolds number $R_c$ are supposed to be fixed. High lift boundary layer control during the landing of aircraft should have a suction distribution which prevents separation with minimum suction pump power, at least if the suction pump is driven by an auxiliary power unit. If the power needed by the suction pump is delivered by the propulsion system of the aircraft, this power is of less importance. In this case, the suction distribution which gives the least value of the suction volume flow is preferable, since this gives the smallest possible size of the ducting etc. High lift boundary layer control designed for the improvement of take-off performance of aircraft on the contrary, has the largest possible efficiency if the effective drag is minimized.

3.2. Suction for minimum pump power

During the landing of aircraft a large magnitude of the airfoil drag is not prohibited, as long as separation is prevented. If the suction pump is driven by an auxiliary power unit (this solution may be chosen in relation to the safety of aircraft operation), it is desired to determine the suction distribution which minimizes the power required by the suction pump. At prescribed values of $C_L$ and $R_c$ this means that the minimum value of $C_p Q$ which still prevents boundary
layer separation should be determined. A study of eq'n (12) will reveal the
effects of a number of variables on $C_p C_Q$. The effects of these variables
(apart from $C_p$) on $C_Q$ (the quantity which should be minimized of the suction
pump is coupled to the propulsion system) is not discussed separately; if it
is desired, these effects can be studied in a manner similar to the one given
below.

3.2.1.: The pressure loss coefficient $C_p$ should be small. At a prescribed
value of $c_\ell$, thus with a given pressure distribution (see fig. 1) the minimum
value of $C_p$ depends only on the location $x_1$ of the leading edge of the porous
surface, since the pressure $p_{int}$ in the plenum chamber has to be less than
the lowest static pressure on the porous surface. If this condition is not met,
outflow would occur. To obtain a small value of $C_p$, $x_1$ should apparently be
taken as large as possible. However, if an appreciable increase in maximum
lift coefficient is required, suction has to be initiated rather close behind
the pressure peak at the leading edge of the airfoil, since an unsucked boundary
layer has a strong tendency to separate in that region at elevated values of
$c_\ell$. The smaller the desired increase in the maximum $c_\ell$, the more backward the
location $x_1$ can be chosen.

3.2.2.: The Reynolds number $R_{\Omega_1}$ should be small. This can be realized by
choosing a small value of $x_1$, see fig. 1. A small value of $x_1$ is anyway required
if a considerable increase in maximum lift coefficient is desired, as indicated
in sub-section 3.2.1.

3.2.3.: The Reynolds number $R_{\Omega_f}$ should be as large as possible. This condition
contradicts the main objective of high lift boundary layer control: prevention
of boundary layer separation. A thick boundary layer separates more easily than
a thin one. Hence, a very large value of $R_{\Omega_f}$, although desirable to obtain a
small value of $C_Q$, cannot be tolerated.

3.2.4.: The "natural growth rate" G of the boundary layer should be as small as
possible. This parameter depends (with a prescribed pressure distribution) mainly
on the value of $R_{\Omega'}$ which in its turn depends on the suction distribution. The
magnitude of $G$ is small when the boundary layer is kept rather thin by suction
(see also sub-section 4.1.3.). Suction through slits is less effective than suction
through a porous surface, since on a large part of the surface in the former case
$R_{\Omega}$ does not have the optimum value, i.e. that which gives the smallest value of $G$. 
3.2.5.: If \( x_f \) is kept small, the contribution of the integral of \( C_p \) to the pump drag coefficient \( C_p Q \) is small. It is possible to choose such a suction distribution between \( x_i \) and \( x_f \) that the boundary layer on the trailing edge of the airfoil is on the verge of separation. This choice has been made by Schlichting and Pechau [3]. The result of their calculations is that the smallest value of \( C_Q \) which satisfies the condition of prevention of separation is obtained if a high suction intensity is applied over a small part of the surface. A further discussion is given in chapter 4.

3.2.6.: The boundary layer at the lower surface of the airfoil is not subjected to suction, since at that side the pressure gradient is favorable, so that the boundary layer cannot separate. At a given pressure distribution the value of \( R_{\theta_v} \) is a constant.

3.3. Suction for minimum effective drag

If suction is applied to increase aircraft performance at take-off, a large value of the airfoil drag is not desirable, since this decreases the possible angle of climb. In these circumstances the value of the effective drag determines the efficiency of the suction installation. If \( C_p \) and \( U \) are prescribed, we have to determine the suction distribution which gives a minimum value of \( c_{d_{\text{eff}}} \). It appears to be probable that we have to choose a suction distribution which extends to the trailing edge of the airfoil, since only in this way \( c_{d_{\text{eff}}} \) can be kept small. In this case, we have to investigate the conditions which minimizes \( c_{d_{\text{eff}}} \), eq'n (14).

The influence of \( c_p \) on the power of the suction pump has been discussed in subsection 3.2.1.; it is similarly desirable to make \( c_p \) small (i.e. to choose \( x_i \) large), if a small value of the effective drag is wanted. Insofar as the effects of \( R_{\theta_v} \) and \( G \) are concerned, the statements made in sub-sections 3.2.2. and 3.2.4. are valid also in this context. The Reynolds number \( R_{\theta_v} \) has to be large (in any case if \( c_p > 2 \) which is generally true). See for this effect also the discussion given in chapter 4. Comparison of the term \(-C_p R_{\theta_v}\) in eq'n (12) with the term \(-(C_p - 2)R_{\theta_v}\) in eq'n (14) indicates that if suction pump power for the prevention of separation, a larger boundary layer thickness can be tolerated than for suction distributions aiming at a small value of the effective drag. This is also intuitively evident because a larger value of the boundary layer thickness increases the airfoil drag coefficient, which decreases the take-off performance,
but not necessarily the landing properties.
The qualitative analysis given in this chapter is illustrated in chapter 4 with a number of numerical results.

3.4. Auxiliary conditions

After the discussion of the effects of some variables on pump power and effective drag we will now point out some of the features associated with the mathematics of the problem. The auxiliary conditions which make the problem amenable to numerical analysis will also be given. The heart of the problem is that the expressions for $C_p$, $C_Q$, and $c_{d_{\text{eff}}}$ have to be minimized, meanwhile satisfying the condition on the prevention of separation. These two parameters depend on the variables discussed in section 3.3. The mathematical problem is related to the calculus of variations, since the integral of the natural growth rate $G$ occurs in eq'ns (12) and (14). Furthermore, the value of $G$ can be determined only if for the skin friction coefficient $c_f$ and the form parameter $H$ numerical expressions are substituted. These expressions have generally the nature of approximations. Approximations like these can be avoided only if we return to the equations of motion for boundary layers as given originally by Prandtl; see [5]. The equations are diffusion equations with non-linear convective terms, in which the suction velocity enters only as a boundary condition. In this form, it is virtually impossible to solve the problem. Hence, notwithstanding the rather poor accuracy of the available expressions for $c_f$ and $H$ which have to be used in conjunction with the momentum integral equations, boundary layer calculations are generally based on the latter; the formulae in the preceding chapter were likewise derived from eq'n (9). The approximate expressions involved are discussed in sub-section 3.4.1.; the assumptions which have to be made to make the problem determinate are discussed in sub-section 3.4.2.

3.4.1.: The two unknowns in the growth rate $G$, i.e. $c_f$ and $H$, are discussed here. As long as separation of the boundary layer can be prevented, $H$ may be taken as equal to 1.5 without introducing large errors; see [2], [3], and [5]. The local skin friction coefficient $c_f$ depends on the suction velocity: suction increases $c_f$ since the velocity profile of the boundary layer is made more convex. In literature no data on the effect of $C_Q$ on $c_f$ are available, so that we have to use a skin friction law valid for boundary layers without suction. In chapter 4 the expression given by Ludwig and Tillmann will be used [7].
3.4.2.: The prevention of boundary layer separation is the main goal of high lift boundary layer control. A qualitative feature associated with the development of a boundary layer in the vicinity of a separation point is that the Reynolds number \( R_\theta \) increases very rapidly. This feature can be used to determine whether a particular boundary layer with suction will separate or not. Separation of a boundary layer occurs when \( c_f = 0 \), but this condition cannot be applied easily in practice. A suction distribution which is chosen such that \( R_\theta \) is kept constant over the porous surface, appears to be promising in this respect. In most other cases, only a complete calculation of the boundary layer over a given airfoil will show whether the boundary layer will separate or not; this cannot be determined a priori. The criterion for separation of a boundary layer which is used in conjunction with such calculations is that \( H \) passes a certain value \( [3, 5] \). In general, it is impossible to predict the location of the point of separation and a critical value of \( R_\theta \) without previous solution of the boundary layer equations.


4.1. General

In the paper by Schlichting and Pechau \( [3] \) the theory is illustrated by a number of calculations of boundary layer development on the airfoil NACA 747-A-315. This airfoil has been used in the calculations presented here, to be able to make a comparison between the calculation schemes of Raspet \( [2] \) and of Schlichting-Pechau \( [3] \).

4.1.1.: The numerical data given in this chapter refer to the airfoil NACA 747-A-315 at a Reynolds number \( R_C = 3 \times 10^6 \). The angle of incidence is \( 20^\circ \), \( c_\theta = 2.35 \) (if separation can be prevented). For the airfoil without suction, the maximum value of \( c_\theta \) is 1.22 at \( 15^\circ \) incidence.

4.1.2.: The interior of the airfoil is assumed to be a single undivided plenum chamber. The static pressure in the interior, \( p_{\text{int}} \), is taken to be \( (p_u - p_\infty) \) less than the lowest static pressure on the porous surface. For the example considered here, we have, if the suction distribution of Schlichting-Pechau is chosen, \( C_p = 15.4 \) (see fig. 2). If a suction distribution according to Raspet is chosen, \( C_p = 6.3 - 5.0 - 4.2 \) respectively if \( x_{1/c} \) is respectively equal to 0.1 - 0.2 - 0.3. These values of \( C_p \) have been calculated from the data given in \( [3] \).
4.1.3: For the calculations according to the procedure of Raspet, the velocity distribution above the boundary layer at the upper surface of the airfoil is approximated here by a hyperbola which is found in the following manner. If the skin friction law of Ludwig-Tillman \[7\] is used and if \(H = 1.5\) is taken, we can deduce the value of \(R_0\) which (locally) minimizes \(G\). Partial differentiation of eq'n (10) w.r.t. \(R_0\) yields for this minimum:

\[
R_0^{1.268} = 1.264 \cdot 10^{-3} \frac{U_o^2}{U_o^2} \frac{dx/c}{dU_o/U_o}
\]

This expression gives a value of the optimum \(R_0\) that is independent of \(x\), if:

\[
\frac{dU_o/U_o}{dx/c} = - \frac{n}{n-1} \frac{U_o^2}{U_o^2}
\]

This condition is satisfied by a velocity distribution of the main flow according to:

\[
\frac{U_o}{U_o} = \frac{n}{1+(n-1)x/c}.
\]

If, for the airfoil in the particular circumstances considered, the parameter \(n\) is chosen to be equal to 2.68, the resulting curve is a reasonable accurate approximation to the actual velocity distribution, as is indicated in fig. 4. It is noted that the approximation is insufficient at \(x/c \ll 0.05\). Hyperbolae like the one chosen here are expected to give a fair representation of the velocity distribution only at large angles of incidence. The parameter \(n\) is equal to \(c_\parallel + \frac{1}{3}\).

For the airfoil in consideration the value of \(R_0\) which minimizes \(G\), eq'n (15), is equal to 950 (assuming that eq'n (16) is valid). This value is comparable to the value of \(R_0\) for the boundary layer a short distance behind the pressure peak at the leading edge of the airfoil. Hence, it may be stated quite generally that the boundary layer has to be kept thin if it is desired to keep \(G\) small. This has been indicated already in sub-section 3.2.4. It must be noted that the figure given here is intended only to support the qualitative statement made in 3.2.4. With these restrictions, the hyperbolic velocity distribution may be introduced as a reasonable approximation without suggesting any connections to \(R_0\) or \(G\). Nevertheless, the relation between the hyperbola and the boundary layer at constant \(R_0\) is of some theoretical interest in relation to the study of turbulent equilibrium layers.
4.1.4: The (constant) contribution of the boundary layer at the lower surface of the airfoil to \( c_{dp} \) and \( c_{d_{eff}} \) is not accounted for in the following numerical results. This contribution is not neglected, but it may be left out of account since it is approximately independent of the suction distribution (as long as separation is prevented). For the calculations according to the procedure of Raspet and according to the modification presented in section 4.4, the data on \( R_0 \), given in [3] are used. The hyperbolic velocity distribution, eq'n (16) with \( n \geq 2.68 \), is used also in these calculations. Further, the friction law of Ludwig-Tillmann [7] is used and \( H = 1.5 \). The calculations of Schlichting-Pechau, section 4.2, were carried out using the friction formula of Prandtl [5]. This formula, at the chosen value of \( H \), is equivalent to the formula of Ludwig-Tillmann; the numerical results are hardly affected by the difference.

4.2. The procedure of Schlichting-Pechau

According to the procedure of Schlichting-Pechau [3] suction is applied from the stagnation point at the leading edge of the airfoil to a variable \( x_f \). The suction distribution is uniform: \( v_o \) does not depend on \( x \). The velocity \( v_o \) is chosen in such a way that the boundary layer at the trailing edge of the airfoil is on the verge of separation \(( H = 2 \)\). It is noted that it must be considered risky to allow the boundary layer to come so close to separation: any accidental disturbance of the flow may cause premature separation. The data presented in [3], insofar they relate to the airfoil NACA 747-A-315 in the prescribed conditions, can be summarized as follows:

<table>
<thead>
<tr>
<th>( x_f/c )</th>
<th>( C_q \times 10^3 )</th>
<th>( C_p )</th>
<th>( C_p C_q \times 10^3 )</th>
<th>( c_d \times 10^3 )</th>
<th>( c_{d_{eff}} \times 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.45</td>
<td>15.4</td>
<td>22.3</td>
<td>44</td>
<td>66.3</td>
</tr>
<tr>
<td>0.5</td>
<td>1.85</td>
<td>15.4</td>
<td>28.5</td>
<td>40</td>
<td>68.5</td>
</tr>
<tr>
<td>1.0</td>
<td>2.95</td>
<td>15.4</td>
<td>45.4</td>
<td>36</td>
<td>81.4</td>
</tr>
</tbody>
</table>

4.3. The procedure of Raspet

According to Raspet [2] the suction distribution is chosen so as to obtain a constant value of \( R_0 \) over the porous surface. The perforation extends from the variable \( x_f \) to the trailing edge of the airfoil. Slit suction is not
applied, so that \( R_\Theta = R_\Theta = R_\Theta \). Calculations according to this procedure yield for the example in consideration:

<table>
<thead>
<tr>
<th>( x_i/c )</th>
<th>( R_\Theta )</th>
<th>( C_q \times 10^3 )</th>
<th>( C_p )</th>
<th>( C_p C_q \times 10^3 )</th>
<th>( c_d \times 10^3 )</th>
<th>( c_d \times 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,1</td>
<td>1380</td>
<td>3.23</td>
<td>6.3</td>
<td>20.4</td>
<td>0.9</td>
<td>21.3</td>
</tr>
<tr>
<td>0.2</td>
<td>4900</td>
<td>4.17</td>
<td>5.0</td>
<td>20.8</td>
<td>3.3</td>
<td>24.1</td>
</tr>
<tr>
<td>0.3</td>
<td>8380</td>
<td>4.99</td>
<td>4.2</td>
<td>21.4</td>
<td>5.6</td>
<td>27.0</td>
</tr>
</tbody>
</table>

4.4. A modification of Raspets' procedure

If we apply slit suction at the leading edge of the porous surface and if we keep \( R_\Theta \) constant over the remaining part, we can choose the volume flow through the slit in such a way that the effective drag coefficient is minimized. In other words: the value of the constant \( R_\Theta \) used in the calculation procedure of Raspert can be changed by slit suction; the change can be chosen in such a way that \( c_{d_{eff}} \) attains a minimum value. If the porous surface extends to the trailing edge of the airfoil, if furthermore the hyperbolic velocity distribution according to eq'n (16), the friction law of Ludwieg and Tillmann and \( H = 1.5 \) are used, the expression for \( c_{d_{eff}} \), eq'n (14), can be written as follows:

\[
c_{d_{eff}} = \frac{1}{R_c} \left( 2R_\Theta - R_\Theta \right) + \frac{C_p - 2}{R_c} \cdot \frac{C_p}{R_\Theta} + \ 
\]

\[
+ C_p \left\{ 0.0118 R_\Theta^{-0.268} \frac{n}{n-1} + 2.5 \frac{R_\Theta}{R_c} \right\} \ln \frac{n}{1+(n-1)x_i/c} \tag{17}
\]

Partial differentiation of eq'n (17) w.r.t. \( R_\Theta \) yields the value of \( R_\Theta \) on the porous surface which minimizes \( c_{d_{eff}} \). For the airfoil NACA 747-A-315 in the conditions as specified, the results of this modified procedure are:

<table>
<thead>
<tr>
<th>( x_i/c )</th>
<th>( R_\Theta )</th>
<th>( R_\Theta )</th>
<th>( C_q \times 10^3 )</th>
<th>( C_p )</th>
<th>( C_p C_q \times 10^3 )</th>
<th>( c_d \times 10^3 )</th>
<th>( c_d \times 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1330</td>
<td>1330</td>
<td>3.05</td>
<td>6.3</td>
<td>19.2</td>
<td>0.9</td>
<td>20.1</td>
</tr>
<tr>
<td>0.2</td>
<td>4900</td>
<td>1350</td>
<td>3.86</td>
<td>5.0</td>
<td>19.3</td>
<td>0.9</td>
<td>20.2</td>
</tr>
<tr>
<td>0.3</td>
<td>8380</td>
<td>1400</td>
<td>4.57</td>
<td>4.2</td>
<td>19.2</td>
<td>0.9</td>
<td>20.1</td>
</tr>
</tbody>
</table>
5. Discussion and conclusions

5.1. Comparison of the numerical results presented in section 4.2 through 4.4 indicates that a suction distribution according to Schlichting-Pechau is to be preferred if a low value of \( C_Q \) is desired. The magnitude of \( C_Q \) determines the size of the ducting from the wing to the suction pump. For the landing of aircraft this solution may be chosen, if the suction pump is driven by the propulsion system. The rather high magnitude of \( c_d^p \) may have some advantages during the landing.

If the suction pump is powered by an auxiliary power unit, the suction distribution at which \( C_p C_Q \) attains a minimum should be chosen to obtain the smallest possible power unit. This consideration is valid only if the application of suction during the landing of aircraft is planned; in this case, the schemes of Raspert and Schlichting-Pechau are equivalent, at least if \( x_i \), resp. \( x_f \) have optimum values. It is noted that the results presented by Schlichting-Pechau would have been more favorable if suction had not been applied starting from the stagnation point, but starting from some \( x_i \) directly behind the pressure peak at the leading edge. A suction distribution according to Raspert yields considerably lower values of \( c_d^e f f \), so that this solution should be chosen for application of boundary layer control during the take-off. The effective drag coefficient is smallest if \( x_i/c \) is small. The modification of Raspert's procedure, section 4.4, yields only a slight improvement over Raspert's original procedure if \( x_i/c \) is small; the advantages of this modification become clear if \( x_i/c \) is larger. This feature is of interest if, due to some construction problem, the suction cannot be initiated at a short distance behind the leading edge of the airfoil.

5.2. It should be observed that the conclusions presented here are valid only insofar as the calculations can be considered trustworthy. Suction distributions according to Raspert have proved their reliability \([4]\). However, the boundary layer development has not been checked in detail. The validity of the results given by Schlichting-Pechau has not been verified experimentally.

5.3. Summarizing, it can be stated that concentrated suction close behind the leading edge of the wing has some advantages for application during the landing of aircraft if the suction pump is coupled to the propulsion system. In this way, the airfoil drag however becomes large. If, even during the landing, no large drag is desired, a suction distribution should be chosen which keeps the boundary
layer thin. The power required by the suction pump necessary to obtain this, is approximately equal to the power required to prevent separation if a narrow suction surface is chosen, so that the former may considered to be more efficient than the latter. It is evident that a suction distribution which keeps the boundary layer thin, should be chosen in any case if application during take-off is considered.
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FIG. 1: NOTATION.
a) Suction distribution according to Schlichting-Pechau; \( V_0 \) is constant.

b) Suction distribution according to Raspet; \( R_\theta \) is constant on the porous surface.

**FIG. 2: SUCTION METHODS**

\[
\theta = \int_0^\infty \frac{U}{U_0} \left(1 - \frac{U}{U_0}\right) dy
\]

\[
R_\theta = \frac{\theta U_0}{\gamma}
\]

**FIG. 3: MOMENTUM THICKNESS**
exact potential flow [3]
hyperbolic approximation
airfoil: NACA 747-A-315
$\alpha = 20^\circ$, $C_\alpha = 2.35$

**FIG. 4: VELOCITY DISTRIBUTION ON THE UPPER SURFACE OF THE AIRFOIL**