Optimisation of vehicle-track interaction at railway crossings

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Optimisation of vehicle-track interaction at railway crossings

Chang Wan
Optimisation of vehicle-track interaction at railway crossings

Proefschrift

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus Prof. ir. K.C.A.M. Luyben,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen op
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door

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I dedicate this dissertation to my beloved families.

致我最親愛的家人
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Chang Wan
Jan 2016, Trondheim
Summary

Railway crossings are important operational elements in railway infrastructures. The discontinuity in the rail geometry at the crossings causes high impact loads to occur at crossing noses. These dynamic forces can cause severe damage to the crossings. In the Netherlands approximately 300 railway crossings are systematically replaced each year at a cost of € 6.4 Mio.

The goal of this research is to develop a methodology that optimises the vehicle-track interaction at railway crossings by tuning the crossing design. This will reduce damage to the crossings and finally increase the availability of the railway network.

The optimisation in the traditional railway system design is carried out in a primitive way by modifying the design parameters, mostly based on the designer’s experience, and repeated numerical analyses.

In the methodology developed here, the crossing design improvement is performed systematically by using numerical optimisation techniques to solve problems with multiple criteria, design variables, and constraints.

The assessment of the crossing design is based on dynamic analyses of the train-track behaviour using the multibody simulation method and the finite element method, which in the design improvement process were coupled with the optimisation technique.

Moreover, the developed methodology is extended to solve optimisation problems with parametric uncertainties, such as track irregularities. For this purpose, the probabilistic formulation of the problem and the robust optimisation method are used.

To demonstrate the effectiveness of the proposed methodology, it is applied to the optimisation of a turnout crossing with the angle of 1:15 or 1:9. The optimisation focuses on the reduction of damage to crossing, especially that by rolling contact fatigue (RCF), from impact forces. In order to achieve this reduction, the geometry of the crossing and the vertical elasticity of the turnout are tuned, as experimental and numerical results showed that the crossing performance is the most sensitive to these parameters. From an implementation perspective, the optimisations of the crossing geometry and the elasticity are considered separately. The optimisation of the crossing geometry is solved in both deterministic and probabilistic formulations. The probabilistic approach supports the robust design of the crossing. The performed optimisations are briefly described below.
Optimisation of crossing geometry

The main results and conclusions of this study are summarised below.

- The vertical distance between the top of the wing rail and nose rail at the transition point is the key factor determining the vehicle-track interactions at a railway crossing.

- A practical and efficient way to tune only the railhead of the nose rail was suggested. Two methods for representing the cross-sectional shape of the nose rail were proposed, namely by using semi-ellipses and using B-splines.

- Contact pressure and energy dissipation in the wheel-rail interface (reflecting RCF and wear) were used in the optimisation criteria. The solution of this two-criteria problem was obtained as a Pareto set of the compromised solutions representing a trade-off between the reductions of RCF and wear.

- A robust optimum solution for the problem was obtained that considered the design tolerances and variations in the wheel profile and track alignment.

- The optimised crossing geometry can be implemented in new crossings during the manufacturing process, and in existing crossings during the grinding/welding maintenance.

Optimisation of track elasticity

In the optimisation of the vertical track elasticity, the stiffness and damping properties of the rail pads and under sleeper pads (USPs) were tuned. The high-frequency forces on the rail, chosen as indicators of the level of RCF damage, were to be minimised. The reduction of (low-frequency) forces on the sleepers and ballast, responsible for the degradation of the overall crossing geometry, was also taken into account during the optimisation process.

Results show that the dynamic forces can be reduced significantly by applying:

- Softer rail pads under the crossing. Combined with the USPs, the dynamic forces, particularly the forces acting on the sleepers and ballast, can be further reduced.

- Rail pads with different properties. By applying relatively less soft elements before and after the crossing nose than those used under the crossing nose, the dynamic forces can be reduced most effectively.

Finally, this study provides guidance for both the production of new crossings and the maintenance of existing crossings.
Samenvatting

Spoorwissels vormen een belangrijke element in de spoorinfrastructuur en beïnvloeden daardoor ook de algemene prestaties van het spoorvervoer. Discontinuïteiten in de geometrie van wissels leiden tot hoge impactkrachten in het puntstuk. Deze dynamische krachten kunnen resulteren in ernstige schade aan de wissels. Elk jaar worden ongeveer 300 wissels systematisch vervangen met een budget van € 6.4 miljoen.

Het doel van dit onderzoek is om een ontwerpmethode te ontwikkelen die de interactie tussen trein en spoor op locatie van wissels verbeterd en het ontwerp van laatstgenoemde optimaliseert. Dit leidt tot verminderde schade aan wissels en verhoogt de beschikbaarheid van het spoor.

In het huidige ontwerpproces is de optimalisatieprocedure een primitieve aanpak door repeterend ontwerpparameters te wijzigen (veelal gebaseerd op de ervaring van de ontwerper) gevolgd door een numerieke analyse.

In dit onderzoek is een unieke ontwerpmethode ontwikkeld waarbij het ontwerp op systematische wijze wordt geoptimaliseerd, gebruikmakend van numerieke optimaliseringstechnieken met meerdere criteria, ontwerpvariabelen en nevenvoorwaarden.

De voorgestelde ontwerpmethode is gebaseerd op de dynamische analyse van wiel-spoor interactie. Deze analyse maakt gebruik van een multibody simulatie en de eindige-elementen methode. In het optimalisatieproces zijn ze beiden aan het optimalisatietechniek gekoppeld.

Daarnaast is de ontwikkelde ontwerpmethode uitgebreid door onzekerheden (bijv. oneffenheden in het spoor) op te nemen in het optimalisatieproces. Dit is bereikt door het optimalisatieprobleem probabilistisch te formuleren en de robuuste optimalisatiemethode toe te passen.

Om de effectiviteit van de voorgestelde ontwerpmethode aan te tonen is een wissel met de hoekverhouding 1:15 of 1:9 geoptimaliseerd. De focus van deze optimalisatie is de reductie van schade aan het puntstuk van het wissel (met name de “rolling contact fatigue” (RCF)) als gevolg van impactkrachten. Om dit te bereiken worden de geometrie van het puntstuk en de verticale elasticiteit gevarieerd. De keuze voor deze twee variabelen is gebaseerd op experimentele metingen en numerieke simulaties waarbij is geconstateerd dat deze variabelen de prestaties van het wissel het meest significant beïnvloeden. De optimalisatie van de puntstukgeometrie en de verticale elasticiteit van het wissel zijn onafhankelijk beschouwd in de voorgestelde ontwerpmethode. De optimalisatieprocedure van de puntstukgeometrie is gebaseerd op zowel deterministische als probabilistische formuleringen. Laatstgenoemde resulteert in een robuust ontwerp van het wissel. De uitgevoerde optimalisaties zijn hieronder kort beschreven.
Optimalisatie van puntstukgeometrie

De belangrijkste resultaten en conclusies van deze studie zijn hieronder samengevat.

- De verticale afstand tussen de top van de vleugel en het puntstuk in het transitiepunt is de essentiële factor in trein-spoor interactie bij een wissel.

- Een praktische en efficiënte manier om alleen de spoorstaafkop van het puntstuk aan te passen is voorgesteld. Om de dwarsdoorsnede van het puntstuk te beschrijven is zowel het gebruik van semi-ellipsen en B-splines voorgesteld.

- Contactdruk en energiedissipatie tussen wiel en rails (een indicatie RCF en slijtage respectievelijk) zijn gebruikt als optimalisatiecriteria. De verkregen oplossing is een Pareto set van de optimale ontwerpen die de afweging tussen reductie van RCF en slijtage weergeven.

- Een robuust optimum is gevonden met inachtneming van de ontwerptoleranties en variaties in wielprofiel en spooruitlijning.

- De geoptimaliseerde wisselgeometrie kan zowel worden geïmplementeerd tijdens het productieproces alsmede tijdens het uitvoeren van onderhoud door middel van slijpen danwel oplassen op bestaande wissels.

Optimalisatie van spoorelasticiteit

Om de verticale elasticiteit van het spoor te optimaliseren zijn de stijfheids- en dempingseigenschappen van de onderlegplaten en dwarsliggeronderlegplaten gevarieerd. De hoogfrequente krachten op de rails zijn gekozen als de indicatoren voor de mate van RCF en zijn geminimaliseerd. De reductie van (laagfrequente) krachten op de dwarsliggers en ballast, verantwoordelijk voor de degradatie van de wisselgeometrie, is ook meegenomen in het optimalisatieproces.

Resultaten tonen aan dat de dynamische krachten significant gereduceerd kunnen worden door:

- Zachtere railpads onder het wissel te plaatsen. In combinatie met dwarsliggeronderlegplaten kunnen dynamische krachten verder gereduceerd worden, met name op de dwarsliggers en ballast.

- Railpads met aangepaste stijfheden te gebruiken. Door gebruik te maken van minder zachte elementen voor en na het puntstuk, vergeleken met onder het puntstuk, kunnen dynamische krachten het meest effectief gereduceerd worden.

Ten slotte, deze studie verstrekt een handvat voor een verbeterd ontwerpproces voor nieuwe wissels en voor onderhoud aan bestaande wissels.
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<td>DOE</td>
<td>design of experiment</td>
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<tr>
<td>FE</td>
<td>Finite element</td>
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<tr>
<td>MAM</td>
<td>multi-point approximation method</td>
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<tr>
<td>MBS</td>
<td>multi-body simulation or multi-body system</td>
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<tr>
<td>RCF</td>
<td>rolling contact fatigue</td>
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<tr>
<td>RSD</td>
<td>relative standard deviation</td>
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<tr>
<td>SD</td>
<td>standard deviation</td>
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<tr>
<td>S&amp;C</td>
<td>switches and crossings</td>
</tr>
<tr>
<td>TP</td>
<td>Tip point (nose point)</td>
</tr>
<tr>
<td>USPs</td>
<td>under sleeper pads</td>
</tr>
<tr>
<td>Y/Q</td>
<td>ratio between the lateral force and the vertical force</td>
</tr>
<tr>
<td>2-D</td>
<td>two-dimensional</td>
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<tr>
<td>3-D</td>
<td>three-dimensional</td>
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Thesis Contents

This thesis consists of an extended summary and the following appended papers:

Paper A

Paper B

Paper C

Paper D

Paper E
Part I

EXTENDED SUMMARY
1 Introduction

1.1 Background

1.1.1 Railway turnouts

Turnouts (also called switches, points, switches and crossings, or S&C) are the components of a railway network that guide trains from one track to another. A turnout is comprised of switch and crossing panels, connected by a closure panel \([1,2]\) (Figure 1.1).

![Figure 1.1 Schematic illustration of a railway turnout and components (figure reproduced based on [3]): 1) switch rail, 2) stock rail, 3) switching machine, 4) switch toe, 5) stock front joint, 6) closure rail, 7) the crossing, 8) nose rail, 9) wing rail, 10) check rail, 11) the V-shape, 12) running rail.](image-url)

A switch panel is the section of a turnout used to alter the direction of motion of passing trains. This panel consists of two switch rails, two stock rails, and switching machines (or point machines), as shown in Figure 1.1. The switch rails are movable, whereas the stock rails forming the two outer rails are fixed. The switching machine moves and locks the switch rails into the correct position. By moving the switch toe to either the straight or diverging stock rail, a train can be directed to approach either a straight or diverging path.

In the crossing panel the two tracks intersect at one level, in which the running rails disconnect at the intersection point (Figure 1.1) to enable the wheel of the train to pass the two crossed rails. To bridge this gap, wing rails are mounted on each side of the intersecting portion of the two running rails. This intersecting portion is the so-called nose rail, or crossing nose. The nose and wing rails form the intersecting inner rails of the crossing panel, which is called the crossing, or frog. Outside of the crossing, check rails are fixed next to the outer running rails, guiding the opposite train wheel so that it does not strike the crossing nose. Trains approaching from the narrow end of the nose rail (the nose point) towards the running rail perform facing movements; trailing movements follow the opposite path.
Part I: Extended Summary

The closure panel is the portion of the turnout between the switch panel and the crossing panel.

**Target turnouts in this study**

In the current research, the studied turnouts are common single turnouts with the following properties:

- A straight track forms the through route (also called the main route) and a curved track shapes the diverging route, as shown in Figure 1.2;

![Figure 1.2 Standard railway turnout and travelling routes (from Paper B).](image)

- Fixed-point crossing;
- A crossing angle;

![Figure 1.3 The crossing angle 1:α.](image)

- The crossing angle, measured between the centre line of the through route and the tangent to the centre line at the rear of the diverging route, is approximately the angle of the V-shape as shown in Figure 1.3. The angle is generally described as 1:α, with common values of 1:7, 1:9, 1:12, 1:14, 1:15, and 1:20 [1]. In this dissertation, crossings with different angles (1:9, 1:12, and 1:15) were studied.
- The steel grade of the crossing rails is assumed to be identical to that of the plain track; and
• No rail inclination exists in the turnout.

1.1.2 Kinematics of wheel passing over crossing

Figure 1.1-Figure 1.2 show that the rails in the crossing are discontinuous. The running surface of the rail is disrupted between the V-shape and the wing rails. When a wheel passes through the crossing it encounters a discontinuity in rail geometry. Several stages can be recognised, which are shown in Figure 1.4:

• Stage 1: The wheel travels on the wing rail approaching the crossing nose (Figure 1.4a-b). The contact between the wheel and rail is generally at a single-point.

• Stage 2: The rail touches the crossing nose and the wheel load is transported from the wing rail to the crossing nose (Figure 1.4c-d). At this transition stage the wheel contacts both the wing and nose rails.

• Stage 3: The wheel leaves the wing rail and only rolls over the nose rail followed by the through rail (Figure 1.4e-f).

Figure 1.4 Process of wheel passing through a crossing.

The wheel motion is disturbed because of the discontinuity of the crossing, forming an excitation source in the train-track interaction at the crossings. From measurements on instrumented turnouts [4], the impact force can be twice the static wheel load, which is a considerable source of nose rail damage. The smoother the wheel transition from the wing rail to the crossing nose, the lower the dynamic amplification of the wheel/rail contact forces.

1.1.3 Damage to crossings

At the transition from the wing rail to the crossing nose the wheel/rail contact point experiences a jump on the wheel tread, which causes an impact force (P1) with a high-frequency content (Figure 1.5). The cyclic impact force that results in high contact pressure is a source of plastic deformation on the crossing nose. Once all dislocations in the crystalline matrix of the material in the affected area reach the ratchetting regime, cracks occur and propagate. The growth rate of the whole process increases with the formation of cracks. This process is known as rolling contact fatigue (RCF) damage to the crossing nose. It is thus
common that a turnout crossing, the nose rail in particular suffers from both RCF damage and plastic deformation at the location of frequent dynamic impact (see Figure 1.6), in which plastic deformation is more severe for newly installed crossings [5]. The damage initiated from high frequency forces has a significant effect on the life span of the turnout structure, producing local damage to the wheel-rail contact surface and transmitting noise and vibrations to the environment.

Figure 1.5 Typical wheel force behaviour in presence of short wave rail irregularities [6].

The dynamic wheel forces acting on the rail also contain low-frequency components, the so-called P2 forces (Figure 1.5). These forces influence the supporting structures of the railway, such as sleepers and ballast beds. Therefore, they are responsible for the degradation of the overall turnout geometry. In turn, the overall turnout geometry also influences the dynamic wheel forces [7]. The deterioration of turnout geometry amplifies the deterioration rate of the
local geometry of the crossing point, which results in increasing P1 forces and thereby further accelerating rail degradation.

Figure 1.7 Wear of crossing (together with RCF damage).

A second type of damage, called wear, occurs on both the wheel and the rail because of the relative velocity difference between the two contacting bodies, where both adhesion and sliding mechanisms of contact are active. An example of wear damage on the nose rail is shown in Figure 1.7. Wear on the nose and wing rails causes severe deviations from the nominal profiles of turnout crossings and wheels; the resulting dynamic effects can range from affecting ride comfort to derailment.

1.2 Research problem, goal, and methods

1.2.1 Research problem

Turnouts have been described as “the hungry asset” [8] because of their high maintenance cost. The Dutch railway network has 7195 turnouts in its 7033 km of railway track (in the year 2012). According to the track investigation [9] performed by ProRail (the Dutch railway infrastructure management agency), urgent turnout crossing replacements have become a serious problem: the average number of urgent frog replacements is 2 per week and approximately 100 per year. Additionally, each year there are approximately 300 crossings systematically replaced. The annual replacement budget is €6.4 mln for railway crossings. Moreover, 6% of train delays in 2010 occurred because of railway turnouts (30 times), which was responsible for 55% of the total disruption time. The effect of a broken crossing is 28% of the mentioned 55% disruption time. In the last decade, RCF damage to normal railway tracks has significantly decreased (more than 50% less) [9], whereas this reduction was not noticeable at turnouts. By contrast, there has even been an increase of RCF damages to railway turnouts in previous years due to lack of maintenance at turnouts.

The identical results have been reported by other railway administrations. In Switzerland, approximately 25% of the budget for the maintenance and renewal of railway tracks – more than one billion Swiss Franks – was used for the switches (points) and crossings [10]. In the
UK, less than 5% of track miles consist of switches and crossings, but more than 23% percent of the renewal budget was spent on these components in 2013 [11]. Turnout failure statistics based on information in different databases at Banverker (the Swedish Rail Administration) [12] concluded that turnouts account for 50% of registered inspection remarks and 21% of the operation disturbances. The failure or disturbance of a single turnout leads to an indirect cost in the form of a traffic shutdown. S&Cs contribute to approximately 13% of the maintenance budget for Banverker.

Obviously, performance of turnouts should be improved in order to reduce the maintenance costs and extend the life span of turnout crossings.

1.2.2 Research goal

The purpose of this research is to develop a methodology to improve the performance of railway crossings by tuning the crossing design. Assessment criteria for the crossing performance are focused on reductions of the RCF damage, plastic deformation, and wear (Section 1.1.3) caused by wheel transitions at crossings.

Numerical and experimental studies performed at TU Delft [7,13,14,15] have shown that rail geometry and vertical track elasticity significantly affect the dynamic behaviour of railway crossings. Therefore, improvements in the crossing performance are achieved here by varying the cross-sectional shape of the nose and wing rails, as well as by tuning the elastic properties of the crossing track structure. The work towards optimising the elastic track properties presented here is a continuation of the work performed in TU Delft [13].

From a practical point of view, the optimisations of the rail geometry and of the vertical track elasticity of the crossing were considered separately, because the adjustment of the crossing elasticity would be difficult to achieve in existing turnouts due to constructional limitations, such as the lack of space for added rail pads. The improved crossing geometry, however, can be implemented in existing turnouts during maintenance processes, as well as in new turnouts during the manufacturing process.

1.2.3 Research methods

The number of studies on dynamic vehicle-turnout interactions has increased in the last two decades. However, only a few publications on improving train-turnout interactions are available. Early crossing improvements were achieved through predefined variations of the crossing design, as described in [5,7,13,16,17]. A numerical optimisation of crossing performance was recently performed in [18], although the crossing performance evaluation therein was not based on dynamic analyses of the vehicle-turnout interaction.

Therefore, in this study, the numerical optimisation method combined with the dynamic analyses of vehicle-turnout interactions was used to improve the performance of the turnout crossing. Improvements to the dynamic crossing behaviour were formulated and solved as
numerical optimisation problems, wherein the values of the objective and constraint functions were obtained from dynamic simulations. In addition, field measurements were performed to assess train-turnout interactions and validate the numerical models developed in this study before they were used in the design optimisations.

As mentioned above, the proposed methodology for the improvement of the turnout performance combines developed numerical models for the analysis of vehicle-turnout interactions, numerical optimisation techniques, and field measurements, which are briefly outlined below.

**Parameterisation of the crossing geometry**

Proper representation of the crossing geometry is necessary for the success of the optimisation. Two methods of representing the cross-sectional shape of the railhead of the nose rail were developed here, namely one using semi-ellipse and the other using B-spline approximations. The methods were implemented in the MATLAB environment.

**Dynamic analysis of vehicle-turnout interaction**

Two models were developed here, one using the finite element (FE) method and the other using the multi-body system (MBS) method. Dynamic vehicle-turnout interactions were investigated only for the main-facing direction and the switch panel was excluded in the turnout model.

- **Finite Element (FE) model developed in DARTS_NL software from TU Delft**
  - This is a robust and efficient 2-D model for analysis of train-track interactions. The vehicle model is simplified as mass-spring elements and half of the track is modelled by assuming a symmetric track. Wheel-rail contact is represented by the Hertz-spring. The main advantage of this model is the possibility to model up to three elastic layers in a track structure (e.g. rail pads, under sleeper pads (USPs), and ballast), while the elastic properties of each layer can be defined non-homogeneously along the track.

- **Multi-body System (MBS) model developed in VI-Rail (Chapter 2)**
  - This 3-D model is advanced at realising a complete railway vehicle and track combination, in which the realistic track layout and wheel/rail profiles can be used. The wheel-rail contact is solved based on Hertz’s theory and modified Kalker’s FASTSIM algorithm [19].

**Numerical optimisation tool**

The multi-point approximation method (MAM) is chosen as the general optimisation technique for the turnout crossing design improvement. In this dissertation, the multi-criteria formulation was used to account for the multiple damage formats introduced in Section 1.1.3. Moreover,
the robust optimisation approach is proposed here to account for uncertainties relating to the loading conditions, track alignments, and design tolerance.

**Field measurements**

Two types of measurements were performed in this research: 1) dynamic acceleration measurements of the nose rail and 2) crossing geometry measurements, in the form of track layout and rail profiles along the crossing.

**1.3 Thesis outline**

An outline of the thesis is shown in Figure 1.8.

Chapter 1 provides an introduction to the research topic of this thesis. Chapters 2-4 describe the research methods used in this study; in which

- Chapter 2 presents the numerical models for analysis of the dynamic vehicle-turnout interaction.
- Chapter 3 focuses on the experimental analysis and
- Chapter 4 introduces the numerical optimisation technique.
1 Introduction

Combining the three methods, two approaches proposed to improve the crossing performance by optimising the crossing geometry and the vertical track elasticity, are demonstrated in Chapters 5 and 6, respectively.

Chapter 7 concludes the thesis with the main findings and limitations in the study, and recommendations to crossing designs and future research directions.
2 Simulation of the dynamic vehicle-turnout interaction

For a complex system, such as a turnout crossing with varying geometry and structure, the development of realistic models requires significant effort. In the specific case of turnout crossings, it is necessary to consider the dynamic behaviours of both the track and vehicle, because the behaviour of a crossing is sensitive to both the track features and vehicle characteristics.

When addressing the problem of modelling a train-turnout interaction, specific considerations are necessary to model the main source of excitation: spatially varying geometry. Rail profile variations in the relatively short length of the crossing cause sudden transfers of contact points between the wheel and rail surfaces, which result in impacts, the primary source of damage to turnout crossings. In addition, a ‘drop-rise’ mechanism is produced by the vertical motion of the wheel running over the crossing panel, because of the transition of the wheel from the wing rail to the crossing nose [20]; this generates high wheel-rail contact forces because of the increased inertial effects of the vehicle. Spatial variations in the rail profiles along the turnout cause changes in the wheel-rail contact by both the lateral wheel-rail displacement and the longitudinal position of the wheel along the track.

An investigation of the dynamic vehicle-turnout interaction using numerical models generally demands a qualitative analysis, which tends to engage in a much more dialectic process between the research questions and observed data. In such cases, the major characteristics of the vehicle-turnout system, such as rail geometry variation, must be considered. The necessity of other details for modelling vehicles and tracks depends on the specific investigative purposes. An overview of vehicle-track responses in the frequency domain was previously provided in [21]. The criteria for vehicle-turnout interaction models were outlined in an extensive study of turnout models under the European joint railway research project INNOTRACK [22]:

- They are mechanical models that produce engineering quantities derived from the fundamental principles of mechanics and dynamics;
- They include a representation of all three system components: the vehicle, turnout track, and wheel-rail contact;
- They include a level of detail corresponding to at least one of the three system components.
Following these requirements, two models for the analysis of vehicle-turnout interactions were developed.

### 2.1 2-D FE model: DARTS_NL

The analysis of the dynamic track-vehicle interaction that includes track flexibility can be computationally expensive. To reduce the computational effort, the modelling in DARTS_NL is restricted to two dimensions (the vertical and longitudinal directions, as shown in Figure 2.1) while using linear material property elements. DART_NL has been successfully used for various railway applications such as optimisation of a slab track [23-24] identification of dynamic properties of track components [25], assessment of various high-speed track structures [26], effect of elastic track elements [13], and optimisation of vertical track elasticity (Paper B).

![](Figure 2.1 DARTS_NL model of vehicle and ballast track (from [13] and Papers A, B).

The DARTS_NL model uses track symmetry, which is valid for a conventional track structure in which the rail, rail support, and vertical rail geometry on both sides of the track are identical. However, for a turnout, these geometries differ on the inner and outer sides of the crossing panel (e.g. a normal rail on the outer side and crossing on the inner side of the modelled track). Therefore, by assuming a symmetric track in the turnout model, some dynamic effects such as the rolling of the vehicle were omitted; such a model better represents the main direction of a symmetric three-way turnout, a structure not considered in this study. For the relative and qualitative comparison of various design changes, however, it is acceptable to leave these effects out of consideration. The main elements of the DARTS_NL model are described below.

#### 2.1.1 Vehicle model

Figure 2.2 shows a basic railway vehicle model implemented in DARTS_NL. Each vehicle is represented by a mass-spring system that consists of four wheels, two bogies and a car body, which are modelled as rigid bodies and connected to each other by the primary and secondary
suspensions (Kelvin elements). The basic vehicles can be combined in a series so that a whole train can be simulated.

![Vehicle model](image1)

Figure 2.2 Vehicle model [27].

2.1.2 Track model

The inner part of the crossing panel related to the through rail (Figure 2.3 dashed area) is modelled using the DARTS_NL software.

![Modelled rail](image2)

Figure 2.3 A common single turnout (from Paper A).

**Rail**

The cross-sectional shape of the inner rail varies along the turnout. In the numerical model, the rail profile in the crossing part, which combines the wing and nose rails, was defined using various cross-sectional data obtained from either the corresponding manufacturing drawings or direct measurement at the field sites. Outside of the crossing, the normal rail profile (e.g. 54E1) was used in the numerical model.

**Rail support**

The lengths of the sleepers used in a turnout are not identical (Figure 2.3), and therefore, the vertical support stiffness of the through rail is not homogeneous. In order to consider this aspect, sleepers with variable equivalent lengths were used in the numerical model. The rail support per sleeper was then calculated based on one-half of the supporting area of each sleeper.
**Rail geometry**

Only the vertical rail geometry along the crossing can be considered in the DARTS_NL model, thus simplification of the 3-D rail geometry is required. In the DARTS_NL model, the rail geometry was modelled by the longitudinal rail surface profile along the crossing, representing the trajectory of the wheel rolling over the rail.

![Diagram of rail geometry](image)

**Figure 2.4** Determination of vertical rail geometry of turnout in DARTS_NL model (from Paper A): (a) visual image of crossing and theoretical (as manufactured, $\alpha$ is the crossing angle) height profile of crossing, a simplified method in [13] applied in absence of measured geometry in (b), (b) measured vertical distance between the top of wing rails and the nose rail along the crossing (crossing angle 1:12, the nose point is the 0 point), and (c) rail geometry in DARTS_NL model (crossing angle 1:12, the nose point is at 36.0 m).

Because a direct approach for measuring the wheel trajectory is not always available, an alternative method, using the simulated wheel trajectory from the corresponding turnout of a 3-D model, can be used. The application of such a method can be found in Paper A, in which the vertical wheel trajectory from the 3-D MBS package VI-Rail was used as the rail geometry in the DARTS_NL model to compare the dynamic responses of both models. Another method of modelling the vertical wheel trajectory relies on visual images of the crossing, which provide information on the transition zone, as shown in Figure 2.4a. This method (originally proposed in [13]) was extended in this study using the measured vertical distance between the top of
the wing rails and the nose rail along the crossing (Figure 2.4b). By this image-based data, the
rail geometry in the crossing before and after the transition zone follows the wing rail and the
theoretical crossing nose profile, respectively. In the transition zone (zone b in Figure 2.4), the
linear vertical rail geometry was assumed.

2.1.3 Wheel-rail contact

The wheel-rail contact is modelled using the non-linear Hertzian spring with the stiffness [28]

\[ K' = \frac{6E'P\sqrt{R_wR_r}}{4(1-v^2)} \]  

(2.1)

where

- \( P \) is the static wheel load
- \( R_w \) and \( R_r \) are the radii of the wheel and rail profile (the lateral cross-section)
- \( E \) is the Young’s modulus of the wheel and rail material
- \( v \) is the Poisson coefficient.

A moving vehicle generates a load caused by the roughness of the rail (longitudinal rail surface profile). Because of this roughness the distance between the wheel axis and the rail surface changes and the so-called Hertz spring applies a load at both the wheel axis and the rail surface as shown in Figure 2.5. Therefore, the magnitude of the load is directly dependent on the shape of the rail surface along the track.

![Figure 2.5 Wheel-rail contact on rough rail surface [27.]](image)

2.1.4 Validation of the DARTS_NL model

The accuracy of the vehicle-turnout model was investigated in the work presented in Paper A, in which the dynamic behaviour of the crossing during the passage of passenger trains in the Dutch railway network was studied on three instrumented turnouts of varied crossing angles, service states, and load conditions. The dynamic accelerations of the nose rail from the measurements and simulations in DARTS_NL were compared, with equivalence between the simulated and measured results. The results of this validation demonstrated that the effect of the crossing geometry on the dynamic response of the turnout was accurately observed in the 2-D model.
2.2 3-D MBS dynamic model: VI-Rail

The commercial software VI-Rail is a specialised package for simulating the dynamic vehicle-track interaction based on the industry standard multi-body dynamics code MSC Adams [29]. The local contact geometry (location of contact point on wheel and rail, contact angle, size of the contact patch etc.), contact forces, energy dissipation and wheel/wheelset displacements w.r.t the rails etc. as functions of the position along the turnout during the passage of each wheel, are outputs of the dynamic simulations.

Realisation of the complete railway vehicle and track combination is essential in the multi-body modelling. The complete railway system in VI-Rail consists of a vehicle model (either a single wagon or a whole train), a track model, and wheel-rail contact elements. These models are first built separately and are assembled afterwards to obtain a complete vehicle-track system.

![Figure 2.6 Typical vehicle-track model in VI-Rail (reproduced from [30]).](image)

2.2.1 Vehicle model

In VI-Rail the vehicle model contains all necessary information (for the chosen level of discretisation) about the vehicle design. A typical model of the vehicle is composed of the car body, the front bogie and the rear bogie, in which the car body, the bogie frames, as well as the wheelsets are treated as rigid bodies and are defined by their mass-inertia characteristics (mass, moments of inertia, and the position of the gravity centre). Each rigid body has six degrees of freedom: three translations and three rotations. All bodies are connected by spring-damping elements, representing the primary and secondary suspensions. A typical vehicle model (together with track) is shown in Figure 2.6.
2.2.2 Track model

**Track structure**

In the VI-Rail simulation system, the ‘moving track’ (Figure 2.6) and *FlexTrack* (Figure 2.7) structure models can be used. Track elasticity can be defined by adjusting the properties of the rail pads and ballast, which are represented as flexible connections of sleepers to the rail and sleepers to the ground by linear spring-damper elements in the vertical and lateral directions.

![Diagram of track structure](image)

Figure 2.7 Schematic illustration of track structure of *FlexTrack* model (ballasted): (a) the mechanical representation of the structure and (b) 3-D view of track model in VI-Rail GUI window.

In the ‘moving track’ model, rails and sleepers are modelled as rigid bodies moving together with the wheel load; the track elasticity is independent of location along the crossing. In the *FlexTrack* model, the rails which are discretely supported are modelled as either Adams beams or FE beams integrated with MSC.NASTRAN. Track elasticity in the *FlexTrack* model can be defined depending on the longitudinal location along the track by assigning various properties to the spring-damper elements at different locations.

Presumably, the *FlexTrack* model represents vehicle-turnout interactions more accurately than the ‘moving track’ model does; however, this model is much more time consuming. Therefore, an indirect method was used to validate the ‘moving track’ model, as introduced in Section 2.2.4 by comparison with the results from the *FlexTrack* model. Based on the similarity of the results from the two models, the ‘moving track’ model was utilised in the parametric study of wheel transition behaviour at crossings (*Paper D*) and crossing geometry optimisation (*Papers C, E*), in which repeated simulations were required.
Track geometry

The nominal track geometry, or layout, is defined by the length of straight track, the length and curvature of the curves and transition curves in the track, track cant, track gauge, rail inclination, and the nominal rail profile. To represent the geometry at the crossing panel, various rail profiles were used along the track. The applied rail profiles were either obtained from data measured at instrumented crossings (Figure 2.8) or mathematically produced (Figure 2.9). With the varying rail profiles, the wing rails, the nose rail, and the crossing angle were all defined. Additionally, the guiding rails were modelled.

![Figure 2.8 Measured rail profiles along instrumented crossing—examples for profiles at 150 mm to 650 mm from nose point (from Paper C).](image)

![Figure 2.9 Mathematical approximation of rail profiles (from Paper C).](image)

In reality, actual track geometry deviates from the designed one. Deviations or irregularities can occur due to manufacturing faults, excessive dynamic loads, wear, or due to the deterioration of the substructure. These deviations can be introduced in the VI-Rail track model either analytically or directly using measured track irregularities. Papers C, D, and E report the applications of differently shaped analytical irregularities, such as sinusoidal, ramping, and dipping irregularities, each representing deviations from the nominal geometry. These irregularities could be introduced to either side of the track in both lateral and vertical directions. Cant deficiencies and track gauge variations can also be defined in the VI-Rail model.
2.2.3 Wheel-rail contact

The vehicle model and the track model are connected through the wheel/rail contact model [29]. To model the wheel-rail contact at turnout crossings, the contact element that uses actual wheel and rail profiles to compute the contact kinematics at each calculation step was applied, in which the shape of a contact patch was computed by segmenting the contact area into stripes. The calculation was performed online (in ‘real time’) during the dynamic simulation.

The normal contact forces are calculated using the Hertz theory for a given ellipse with undeformed penetration. The creep forces are computed based on the modification of Kalker’s FASTSIM algorithm developed at TU-Berlin [31-32]. Additionally, the Polach method [33] and user-defined calculation algorithms can be implemented in calculation of creep forces.

Once the model of a railway system is completed, the dynamic behaviour of railway vehicles on the track can be investigated.

2.2.4 Validation of the VI-Rail model

Both the FlexTrack and ‘moving track’ models were validated (Paper C). The FlexTrack model was validated through measurements of an instrumented turnout crossing. Because the dynamic responses of the crossing obtained from measurements were not available for the ‘moving track’ model, the FlexTrack model validated by measurements was used as the reference to validate the ‘moving track’ model. The contact pressure and contact forces from both the ‘moving track’ and FlexTrack models were consistent with one another.

2.3 Estimation of damage to turnout crossings

In the current study, the RCF damage to a crossing was estimated based on an engineering model [34] that indicates surface-initiated RCF defects and subsurface-initiated fatigue as formulated below:

\[
F_{I_{\text{surf}}} = \mu \frac{2\pi abk}{3F_z} \tag{2.2}
\]

\[
F_{I_{\text{sub}}} = \frac{F_z}{4\pi ab} \left(1 + \mu^2\right) + a_{\text{dy}} \sigma_{\text{yres}} \tag{2.3}
\]

where \(F_{I_{\text{surf}}}\) and \(F_{I_{\text{sub}}}\) are the surface-initiated and subsurface-initiated RCF damage indices, respectively; \(\mu\) is the traction coefficient, defined by the ratio between the tangential force and the normal force during the wheel-rail interaction; \(a\) and \(b\) are the equivalent semi-axes of the Hertzian contact patch; \(F_z\) is the normal force; and \(k\) is the yield stress in pure shear. If \(F_{I_{\text{surf}}} > 0\) is satisfied, the yield strength in shear of the material is exceeded; surface fatigue damage can be predicted to occur. Higher magnitudes of \(F_{I_{\text{surf}}}\) indicate that the yield strength
is farther below the acting forces; thus, surface-initiated RCF develops faster (without considering wear). \( a_{ov} \) is a material parameter evaluated by the fatigue limits in alternating shear and rotating bending, and \( \sigma_{h, res} \) is the hydrostatic component of the residual stress, with a positive value under tension. Subsurface-initiated RCF is predicted to occur whenever the inequality \( F_{l, sub} > \sigma_{f, e} \) is fulfilled, where \( \sigma_{f, e} \) is the fatigue limit in shear. Similar to the \( F_{l, surf} \), a high magnitude of \( F_{l, sub} \) indicates a high level of subsurface-initiated RCF damage.

According to the above model (equations 2.2-2.3), the normal contact force and normal contact pressure are defining elements for the RCF damage. In Papers A and B, the vertical (normal) contact force \( P_1 \) (Section 1.1.3) from the dynamic analysis of wheel-rail interaction is used as the damage indicator of RCF, considering that the contact area is assumed to be constant in the DARTS_NL model. However, in Papers C, D, and E, RCF damage is estimated by the accumulated contact pressure, which largely depends on the maximum contact pressure, during the wheel-rail interaction. The contact pressure at each contact point is calculated as

\[
S = \frac{3F_n}{2A}
\]

where \( F_n \) is the normal contact force and \( A \) is the area of the contact patch.

The normal contact force and normal contact pressure can also be used as indicators of plastic deformation.

Wear is assessed by the energy dissipation method. In this method, the amount of energy dissipated in the contact patch, known as \( T_\gamma \) (T-Gamma) and based on the wear prediction work in [34], is calculated by the sum of the product of creep forces (\( \gamma_\) \) and creepages (\( \gamma \)) for the longitudinal, lateral and spin components, as expressed in the following equation

\[
T_\gamma(t) = |F_x(t)\gamma_x(t)| + |F_y(t)\gamma_y(t)| + |M(t)\psi(t)|
\]

where \( F_x \) and \( F_y \) are the creep forces in the longitudinal and lateral directions, respectively; \( \gamma_x \) and \( \gamma_y \) are the corresponding creepages; \( M \) is the normal bending moment and \( \psi \) is the spin creepage. In this research, the vehicle-turnout interaction of the main (straight) route was considered, along which the spin was relatively small. Therefore, the third component of equation 2.5 can be ignored. The following simplified \( T_\gamma \) expression is applied in the dissertation:

\[
T_\gamma(t) = |F_x(t)\gamma_x(t)| + |F_y(t)\gamma_y(t)|
\]
3 Experimental analysis of vehicle-turnout interaction

In addition to numerical simulations, experimental analysis is necessary to understand the mechanisms of the dynamic behaviour of turnout crossings and the effects of the parameters that may influence the behaviour of a given crossing. Experimental studies are frequently used in research on railway turnouts.

In [35], vertical geometry variations in railway turnouts that were exposed to different operating conditions were measured; the results demonstrated that, often, some vertical elevation exists in the track geometry as the train approaches the mid-section of turnouts on straight main tracks. The geometrical degradation of turnouts at the crossing sections of a Swedish heavy-haul railway was analysed experimentally in [36-37], which provided a better understanding of the degradation rate and helped to define the optimal maintenance thresholds for the planning process. In [14] and [15], experimental measurements were performed at different turnouts in the Netherlands; the effects of the turnout structure, rail geometry, and loading situations such as vehicle type, axle load, wheel wear state, and travelling speed were studied. The measurements indicated that the dynamic impact on the nose rail was influenced by all these factors and was particularly sensitive to the rail geometry.

In addition to its history of utility, experimental analysis is also reliable and direct in validating both theoretical assumptions and numerical models. Such analyses of the dynamic vehicle-turnout interaction can be found, for instance, in [38-40], in which numerical simulations and numerical models were validated by field measurements. Experiments and field tests are useful in judging whether a numerically obtained design or approach is optimal, as illustrated in [40]. In that work, a modified turnout geometry, intended to provide a low-cost means to increase the speed of diverging routes through a turnout, was installed and tested to determine whether the expected improvements were achieved.

In this research, several field measurements were performed on instrumented common single turnouts with 54E1 rail in the Dutch railway network. Measurements included crossing geometries and dynamic accelerations of the nose rail.

3.1 Crossing geometry measurements

To obtain the turnout rail geometry, varying rail profiles, track gauge, and superelevation along the crossing panel were measured; the track gauge and superelevation were assessed at
multiple locations along the crossing using a digital track gauge and superelevation measuring device. Two different methods were used to measure local crossing geometry in the study.

In Paper C, a laser-based profile-measurement device, Calipri (Figure 3.1), was used to measure actual rail profiles along the crossing, whereas in Paper A, a simplified method of measuring the crossing geometry was applied by estimating the vertical wheel trajectory during the passage of a wheel over the crossing. This simplified measurement contained information on the start-and-end locations of the transition zone, the location of the transition point, and the vertical distance between the top of the wing and nose rails along the crossing (Figure 2.4), as introduced in Section 2.1.2: Rail geometry.

![Figure 3.1 Measurement of rail profiles of a turnout crossing.](image)

### 3.2 Dynamic acceleration measurements

The dynamic behaviour of the crossing was investigated by measuring the nose rail accelerations using the ESAH-M device (Figure 3.2). In the measurements, the acceleration sensor was attached to the nose rail at the location where the major impacts occurred. The locations of the maximum accelerations on the crossing nose, which presumably corresponded to where the wheel hits the rail, were shown as a wheel contact distribution histogram (Figure 3.2c). Based on the histogram of the impact locations, the most probable area for fatigue damage (fatigue area) on the surface of the crossing nose could be determined. These dynamic responses of the crossing were recorded when a train passed through.
Figure 3.2 Measurement of dynamic acceleration: (a) Instrumented crossing nose, (b) acceleration, and (c) fatigue area measurement data (from Paper C).

3.3 Experimental analysis results

In the experiments reported in Paper A, measurements were obtained from several turnouts with crossing angles of 1:12 and 1:15 and service stages of good condition and newly repaired. The results showed that the dynamic behaviours of the crossings differed between the turnouts. Among turnouts of the same type, the geometry of the crossings significantly affected the dynamic behaviour. In addition, the DARTS_NL model (Section 2.1) was validated against the measurements in Paper A.

In the study described in Paper C, measurements were obtained for the same turnout before and after maintenance by grinding and welding on the crossing (Figure 3.3). The response of the turnout before maintenance differed significantly from that after the grinding of the crossing rail. After maintenance, the dynamic impacts were spread along the crossing nose, rather than becoming converged in a narrow area. Moreover, after the welding and grinding maintenance, the average acceleration of the nose rail caused by passing wheels was reduced by approximately 50% relative to the acceleration value before the grinding (Figure 3.4). These measurements were used to validate the VI-Rail model.

Both field measurements showed that the dynamic behaviour of the crossing was sensitive to the rail geometry. Based on these observations, further studies to optimise the crossing geometry were performed as reported in Papers C, D, and E.

Figure 3.3 Distribution of maximum wheel forces along the crossing nose (from Paper C): (a) before grinding and (b) after grinding.
Numerical optimisation method

The numerical and experimental results discussed in the previous chapters have shown that the dynamic vehicle-turnout interaction is very sensitive to the rail geometry and the track elasticity in the crossing. To improve the crossing design, the numerical optimisation method was applied, allowing for systematic search of the optimum crossing design rather than performing limited number of tests by changing design parameters manually. In this chapter, the optimisation method used in this research is described.

4.1 Optimisation problem

The optimisation problem can be stated in a general form as:

Minimise

\[ F_0(x), \quad x \in \mathbb{R}^n \]  

subject to

\[ F_j(x) \leq 1, \quad j = 1, \ldots, M \]  

and

\[ A_i \leq x_i \leq B_i, \quad i = 1, \ldots, N \]  

where \( x \ (x = [x_1, \ldots, x_n]^T) \) is the vector of design variables, which in a mechanical design problem represent various parameters such as geometry, material, stiffness and damping properties. \( A_i \) and \( B_i \) are the side limits defining the lower and upper bounds of the \( i \)-th design variable, respectively. \( F_0 \) is the objective function and \( F_j, j = 1, \ldots, M \) is the constraint function.
For some problems the constraints may not exist, example of such problems for optimisation of railway turnouts can be found in [18, 42].

**Single-objective versus multi-objective problem**

Based on the number of objective functions, an optimisation problem can be classified as a single-objective or a multi-objective optimisation problem. In the optimisations of a turnout crossing, the optimal decisions usually need to be made in the presence of trade-offs between two or more objectives. A typical method to solve a multi-objective problem is to convert it to a single-optimum one. This can be done by either using a combined weighted sum of the objective functions or by selecting one objective as the objective function while using the other objectives as constraints. The approach of weighted-sum objective functions was used in both Paper B and Paper C, whereas in Paper E the approach of treating some objectives as constraints was used.

**Deterministic versus probabilistic optimisation**

Based on the nature of input parameters of the problem an optimisation problem can be formulated either as a deterministic or as a probabilistic optimisation problem. In deterministic optimisation problem, it is assumed that all input parameters are deterministic, whereas in probabilistic optimisation problems, some or all of the parameters are uncertain. In a probabilistic design problem, these uncertainties are incorporated into the optimisation model. Namely, the optimisation process considers uncertainty or variability in the evaluation of the objective and constraint functions.

Dynamic vehicle-turnout interaction is affected by many parameters most of which are uncertain [43]. Therefore, optimisation of the turnout crossings should be defined as a probabilistic design problem. To reduce the effect of uncertainty, this dissertation associates the following characteristics with crossing optimisations on main railway lines:

- trains passing through the crossing are of the same type
- trains travel in the main-facing direction
- the travelling velocity is the typical speed of Dutch intercity trains: approximately 140 km/h.
- the wheels are in good condition, either newly installed or regularly re-profiled (once every three month in the Netherlands)
- 54E1 common turnouts with the identical crossing angle (1:9 in Paper B, 1:15 in Papers C and E)

Under these assumptions, many uncertainties in the crossing optimisation problems are eliminated. In Papers B and C, the crossing design improvement was formulated and solved as deterministic optimisation problems.
Generally, a deterministic optimal solution is unstable and can fail (become unfeasible) under slight changes to the design variables. To address uncertainty and variability in the crossing optimisation problem, a robust optimisation method was suggested and applied to optimise crossing geometry in Paper E.

4.2 Multipoint approximation method

For the optimisation of vehicle-turnout interaction at railway crossings, optimisation tools should be able to solve a complex constrained nonlinear programming problem. Moreover, the computational efficiency should be particularly taken into account.

The Multipoint Approximation Method (MAM) [44-47] is used in this dissertation. MAM was specifically developed for problems where time consuming simulations and numerically noisy functions are involved. Instead of the original problem (4.1)-(4.3) a succession of simpler approximated sub-problems similar to the original one, which is formulated using approximation functions, is to be solved. Thus, the original problem is replaced with the approximated one that reads

Minimise

\[ \tilde{F}_0^k(x), \quad x \in \mathbb{R}^n \]  

subject to

\[ F_j^k(x) \leq 1, \quad j = 1, \ldots, M \]  

and

\[ A^k_i \leq x_i \leq B^k_i, \quad A^k_i \geq A_i, \quad B^k_i \geq B_i, \quad i = 1, \ldots, N \]  

where the superscript \( k \) is the number of the iteration step; \( \tilde{F} \) is the approximation of the original function \( F \); \( A^k \) and \( B^k \) are the move limits that define the range of applicability of the approximations. The solution of the problem at the \( k \)-th step \( x_k \) is chosen as the starting point for the \( (k+1) \)-th step, then the optimisation problem (4.4) - (4.6) reformulated with the new approximation functions \( \tilde{F}_{j}^{k+1}(x) \leq 1, (j = 1, \ldots, M) \) and move limits \( A^{k+1} \) and \( B^{k+1} \) is to be solved. The process is repeated until the convergence criteria are satisfied.

The whole optimisation process is controlled by the change of move limits. In each iteration step the approximation functions, the original functions, and the current solution of design variables are checked, through which the change of move limits in each step is determined. The iteration process will be terminated if the approximations are good, none of the move limits is active, and the search sub-region is small enough. More details can be found in [47].
The use of the approximations in the MAM usually results in reduction of the total computational efforts compared to the conventional methods in solving nonlinear programming problems. Different from the global optimisation methods, to assure a global optimum the optimisation should be solved with different initial points (valid for majority of the structural optimisation methods).

5 Optimisation of crossing geometry

Proper rail geometry is essential for improving vehicle-track dynamics at turnout crossings. From the field measurements presented in Papers A, C and [15], the dynamic behaviour of a crossing is sensitive to rail geometry. Although many factors can directly or indirectly influence the wheel-rail contact, grinding/welding that allows the alteration of the rail geometry of an existing crossing remains a primary maintenance approach for railway crossings. Recent studies on the optimisation of crossing geometry can be found in [17] and [18]. The former publication used a manual selection for the optimised geometry, while the latter approached optimisation numerically but without considering the dynamic components of the vehicle-turnout interaction. In contrast to these studies, here, the crossing geometry for a crossing angle of 1:15 was optimised using the optimisation method MAM (Section 4.2) while the dynamic responses were obtained using the VI-Rail ‘moving track’ model (Section 2.2).

5.1 Challenges and solutions

The following are the primary challenges in optimising the crossing geometry:

- **Parameterisation of crossing geometry**
  Parameterisation of the crossing geometry is necessary for successful optimisation. On one hand, the parameterisation should realistically represent the crossing geometry. On the other hand, the number of design parameters should be small enough to guarantee a reasonable computational time of the optimisation. Furthermore, the new geometry should be easy to implement. Therefore, two different methods were proposed to parameterise the crossing geometry by approximating the nose railhead using semi-ellipses and B-splines, respectively. This permits the usage of no more than four design variables while producing a realistic crossing geometry.

- **Design assessment based on dynamic vehicle-turnout interaction**
  Design assessments based on the crossing design response is essential during optimisation. Discontinuity in the crossing geometry is one of the main sources for the
5 Optimisation of crossing geometry

Dynamic excitation of high wheel-rail contact forces; therefore, dynamic effects must be included in the crossing response evaluation. The wheel-rail contact is also sensitive to the dynamic motion of the wheel. To properly simulate the wheel-rail interaction at the crossing, the yaw motion and rolling of the wheelset along the longitudinal track centre should be considered. The inclusion of dynamic analyses of the vehicle-track interaction in the optimisation is challenging because the analyses can be time-consuming; however, faster simulation models may fail to represent real-time wheel-rail contact accurately during a dynamic vehicle-track interaction. To address this problem of balance in the study, the validated VI-Rail ‘moving track’ model (Section 2.2) was used to simulate the dynamic vehicle-turnout interaction, in which the wheel-rail contact was analysed in time. Therefore, the evaluation of the geometry design was based on the dynamic behaviour of the crossing.

- **Balance between the accuracy of the solution and its computational cost**

By using the dynamic analysis of the vehicle-turnout interaction in the response calculations during optimisation, the process of optimisation may become extremely time-consuming or even prohibitive if a single simulation requires considerable computation time. A balance between the accuracy of the dynamic simulations and computational cost of the optimisation should be achieved. This challenge is addressed in the study as well.

The ‘moving track’ model simulations are relatively fast, requiring approximately two minutes to simulate the passage of a vehicle over a crossing with a full travelling distance of approximately 45 m (15 m before and 30 m after the nose point) at the sampling frequency of 5000 Hz. Meanwhile, the ‘moving track’ model is accurate enough to capture the main responses of the crossing during the dynamic vehicle-track interaction, as it has been validated through measurements and simulation results from a more complex model (Section 2.2.4).

In addition, the numerical optimisation method MAM used to solve the optimisation of crossing geometry has special features that reduce the overall computation costs of the process (Section 4.2).

### 5.2 Parameterisation of crossing geometry

#### 5.2.1 Method 1: Parameterisation of crossing geometry using semi-ellipse

The parameterisation of the crossing geometry is based on the drawings (Figure 5.1) used for controlling the common 54E1 crossing geometry in the Dutch railway, wherein four cross-
sections (I-IV) along the crossing (crossing angle $1: \alpha$) with distance 0, 10$\alpha$, 20$\alpha$, and 70$\alpha$ mm from the nose point (tip point, TP) were selected as control cross-sections.

Cross-section I represents the nose point, which is not involved in the wheel/rail contact. Cross-section IV, which has the same width as the normal rail (70 mm), is regarded as the end of the nose rail. As a simplification, cross-sections I and IV were fixed. To reduce the complexity of the problem, the wing rail profiles obtained from the current manufacturing process were not changed. Therefore, the two control cross-sections II and III were chosen as tuning cross-sections in defining the crossing geometry, wherein the nose rail above 14 mm below the top of the normal rail was simplified as semi-ellipses. Using this approach, the cross-sections II and III can be adjusted by varying the semi-axes $a$ (lateral semi-axis) and $b$ (vertical semi-axis), that is, four parameters in total to parameterise the crossing geometry. It should be noted that, the variation of the semi-axis $a$ can result in the change of the track gauge.

5.2.2 Method 2: Parameterisation of crossing geometry using B-spline

To achieve more practical results when varying the crossing rail shape, a series of discussions with the manufacturers and infrastructure specialists from Dutch railway took place. The feedback obtained from these discussions concerning the practical aspects of the crossing design suggested that:
• The height of the nose rail should be monotonically increased;
• The change of the rail geometry along the crossing should preferably be linear for both the wing rails and the nose rail;
• The track gauge should not be changed, i.e., the nominal thickness of the nose rail, which is in accordance with the standard track gauge, may not be changed.

Based on these suggestions, an alternative parameterisation of the crossing geometry was proposed by tuning the longitudinal height profile of the nose rail and the wing rail, and adjusting the cross-sectional shape of the nose rail. A parametric study was performed in Paper D, wherein it was concluded that the major effective parameters of the longitudinal height profile of the crossing is the vertical distance between the top of the wing rail and the nose rail, which can be tuned either through adjusting the height of the nose rail or the height of the wing rail. For the cross-sectional shape of the nose rail (with fixed height), the gauge corner part has major effect on the wheel-rail interaction.

Figure 5.2 Longitudinal height profile of the crossing nose (from Papers D, E).

Figure 5.2 shows the parameterisation of the longitudinal height profile of the nose rail. An extra control cross-section V was suggested in addition to the original four control cross-sections (I-IV, Figure 5.1) because RCF damage is often observed after cross-section III. Additionally, it was assumed that impact will not occur before cross-section II. Thus cross-sections III and V were selected as the tuning cross-sections. Figure 5.3 shows the parameterisation of the cross-sectional shape of the nose rail defined by a B-spline representation using six control points, in which control point $P_0$ defines the height of the nose rail centre from the longitudinal height profile of the nose rail (Figure 5.3). Therefore, three parameters were selected in defining crossing geometry in the optimisation (Paper E) with two parameters defining the longitudinal height profile ($h_1$ and $h_2$ in Figure 5.2) and one
parameter defining the cross-sectional shape of the nose rail (location of $P_3$ in Figure 5.3) as described in Section 5.5.1.

![Figure 5.3 B-spline representation of nose rail profile (from Paper E).](image)

### 5.3 Goal of geometry optimisation

The goal of the optimisation is to reduce wear and impact-related damage such as RCF and plastic deformation at the crossing. This reduction is to be achieved by minimising the energy dissipation (equation 2.6) and the damage indicators of normal contact pressure (equation 2.4).

Depending on the geometry parameterisation method and the type of optimisation problem, three optimisation problems were formulated and solved, as presented in Sections 5.4-5.6.

### 5.4 Optimisation problem I: deterministic optimisation using semi-ellipse

This approach is documented in Paper C. As a starting step of the methodology for optimising the crossing geometry, the variation of wheel profiles is excluded in the optimisation problem.

#### 5.4.1 Design variables

The crossing geometry was parameterised using Method 1, the ellipse. Therefore, four design variables were adopted in the optimisation problem: $x = [a_1, b_1, a_2, b_2]$ representing the semi-axis of the control cross-sections II and III. The crossing design from the current manufacturing process was used as the reference crossing geometry, $x_{\text{ref}} = [4, 4, 9, 9]$. 
5.4.2 Objective function

The optimisation problem was formulated using a weighted-sum objective function to minimise both the contact pressure ($S$) and energy dissipation ($W$).

\[
F_0(x) = w_1 \frac{\bar{S}(x)}{\bar{S}^*} + w_2 \frac{\bar{W}(x)}{\bar{W}^*} \rightarrow \min
\]

(5.1)

where $\bar{S}(x)$ and $\bar{W}(x)$ are the accumulated contact pressure and energy dissipation using the Kreisselmeier-Steinhauser function (KS function) [49]; $\bar{S}^*$ and $\bar{W}^*$ are the corresponding values for the reference crossing.

5.4.3 Constraints

- The geometrical constraints to avoid a zigzag shape in both the lateral and the longitudinal directions;
- Constraint to avoid derailment risk;

A danger zone of wheel-rail contact was assumed as 45 mm away from the nominal centre (where the wheel radius equalled to the nominal wheel radius) of the wheel towards the flange back side, as shown in Figure 5.4. Wheel-rail contact in the danger zone was indicated as a wheel climbing over the rail, which would consequently result in derailment. Therefore, wheel-rail contact should not occur in the danger zone. In exception, the wheel flange back contact with the guard rail was not recognised as a derailment risk.

![Figure 5.4 'Danger zone' of wheel-rail contact on wheel (from Papers C, E).](image)

- Constraint on minimum nose rail width at the impact location (nose rail should be wider than 15 mm at the impact location);
- Constraint on normal contact force (the maximum contact force should not be higher than the reference crossing).
5.4.4 Optimisation results

The optimisation problem was solved by applying different weight coefficients. A trade-off between reduction of the contact pressure and the wear index was obtained as shown in Figure 5.5 and Figure 5.6, in which Figure 5.5 shows the Pareto frontier of the two-criteria optimisation problem and Figure 5.6 shows the distribution of the normal contact pressure and wear index along the crossing.

![Pareto frontier diagram](image1)

Figure 5.5 Compromised solutions for different weight coefficients (from Paper C): normalised contact pressure ($S$) w.r.t normalised wear index ($W$). The weight coefficient pair ($[w_1, w_2]$) is shown next to the corresponding point.

![Dynamic response graphs](image2)

Figure 5.6 Dynamic responses of optimised crossings (from Paper C): (a) normal contact pressure and (b) wear index at the crossing. opt_1: reduction of contact pressure weights more, opt_2: reduction of wear weights more.
5 Optimisation of crossing geometry

Figure 5.7 Design variables corresponding to the optimum designs (from Paper C). min_pressure ([w₁,w₂]=[1,0]): the optimisation with minimum normal contact pressure, min_wear ([w₁,w₂]=[0,1]): the optimisation with minimum wear index).

The design variables of the optimum nose shape are shown in Figure 5.7. From this figure it can be seen that the nose rail height at the beginning (cross-section II) was significantly increased and the height difference between cross-sections II and III was reduced from 5 mm to less than 1 mm. The width of the nose rail at the beginning was also increased. A comparison of the nose rail shape between the optimised design obtained with [w₁,w₂]=[0.5,0.5] and the reference design is shown in Figure 5.8.

The robustness of the optimum solutions was briefly analysed by applying irregularities to the track. The results showed that the optimum designs were robust under various track conditions.

5.4.5 Conclusions

- The dynamic performance of vehicle-turnout interaction can be improved by tuning the nose rail shape.
- Crossing geometry can be parameterised by defining four control cross-sections along the crossing: two are fixed and the other two are variable. The nose rail shape of the two variable control cross-sections can be simplified by semi-ellipse.
• After optimisation the height of the nose rail at the beginning of the crossing nose was considerably increased, which resulted in a reduction of impact force. The width of the nose rail at the beginning of the crossing was also increased.

• There is a trade-off between the reduction of RCF-related damage and wear damage to the crossing.

5.5 Optimisation problem II: deterministic optimisation using B-spline

This study was documented in Paper E. Two wheel profiles were considered in the optimisation problem: the standard s1002 profile and the HIT profile used for regular re-profiling wheels of VIRM passenger train in the Netherlands. The dynamic vehicle-track interaction was analysed for these two wheel profiles. Moreover, to account for the effect of track alignment, two irregularities (irr_1 and irr_2 as shown in Figure 5.9) were introduced in the track alignment in addition to the ideal track alignment. The feasibility of the design was assessed for all three track alignments.

5.5.1 Design variables

The crossing geometry was parameterised through Method 2, the B-spline. Three design variables were used in the optimisation problem: \( \mathbf{x} = [h, t, \lambda] \) with \( h_1 = h \), \( h_2 = h + t \) (Figure 5.2); where \( \lambda \) defines the position of \( P_3 \) \( (\lambda = \frac{|PP_3|}{|PM|} \; (\lambda \leq 1)) \) (Figure 5.3). The design vector of the reference crossing was \( \mathbf{x}_{ref} = [9, 2, 1/3] \).

5.5.2 Objective function

The optimisation problem was formulated based on the objective function of problem I (equation 5.2). However, the vector of the weight coefficients was fixed as \( [w_1, w_2] = [1.0, 0.0] \). Namely, only the contact pressure was to be minimised. In contrast to the problem I, the variation of wheel profiles was included in the objective function:

\[
F_0(\mathbf{x}) = w_1 \sum_{i=1}^{d} \alpha_i \frac{S_i(x)}{S_i^*} + w_2 \sum_{i=1}^{d} \alpha_i \frac{W_i(x)}{W_i^*} \rightarrow \min, \tag{5.2}
\]

where \( d \) is the total number of wheel profiles evaluated in the optimisation problem (which was 2). \( \alpha_i \) is the weight coefficient for the \( i \)th wheel profile, \( \sum_{i=1}^{d} \alpha_i = 1 \). Considering that the HIT wheel profile (2nd wheel) is more commonly used than the standard s1002 wheel profile (1st wheel), the response of the HIT wheel was treated with the higher weight. The weight
coefficient pair \([a_1, a_2]=[0.25, 0.75]\) was applied. \(S_r\) and \(W_r\) are the contact pressure and wear number on the crossing, which were expressed as a Kresselmeier-Steinhauser function [49].

![Figure 5.9 Track irregularities considered in crossing design: (a) vertical and (b) lateral deviation of track alignment.](image)

### 5.5.3 Constraints

- The geometrical constraints to obtain a monotonically increased longitudinal height profile and smooth-convex cross-sectional shape of the nose rail;
- Constraint on the transition location of the wheel;
  
  The wheel-rail contact should not occur in zone A or zone C (Figure 5.10). Zone A is defined as 0–250 mm from TP on the nose rail and zone C began 650 mm away from TP on the wing rail. The impact location on the nose rail is thus restricted between 250 and 650 mm from TP.

![Figure 5.10 Transition area of a crossing—example of through route (from Paper E).](image)

- Constraint to avoid derailment risk (identical to optimisation problem I).

Note that the constraints of transition location and derailment risk were investigated based on the dynamic response of the crossing under the ideal track alignment and two irregular
track alignments shown in Figure 5.9. A design was regarded as feasible if it was feasible in all track alignment situations.

5.5.4 Optimisation results

The optimum design was \([h, t, \lambda] = [7.030, 3.186, 0.155]\). The optimum and reference designs are compared in Figure 5.11 and Figure 5.12, demonstrating that the nose rail is lower in the optimal design than it is in the reference design. The cross-sectional shape of the nose rail also changes by the outward movement of the gauge corner.

Figure 5.11 Longitudinal height profile of nose rail (from Paper E). ref: reference design, opt: optimum solution from optimisation problem II.

Figure 5.12 Changing nose rail shape at location 300, 600 and 900 mm from TP (from Paper E). ref: reference design, opt: optimum solution from optimisation problem II.
Notably, in optimisation problem I, the nose rail is significantly higher in the design minimizing only the contact pressure (min_pressure in Figure 5.7), as shown in Figure 5.13. To understand the difference in results, the contact pressure along the crossing for the s1002 wheel profile and ideal track alignment is shown in Figure 5.14. In this design, the impact occurs at approximately 500 mm from TP (the location of maximum contact pressure), where the vertical distance between the top of the wing rail and nose rail was 4.82 mm. However, in the solution from optimisation problem I, the impact occurred approximately 205 mm from TP, and the vertical distance between the top of the wing rail and nose rail was 4.63 mm, close to the optimal design obtained here in problem II. This result indicates that the vertical distance between the top of the wing rail and the nose rail at the transition zone, particularly the transition point, is a key factor determining the dynamic behaviour of a crossing. Therefore,
the number of design parameters in the optimisation problem can be reduced to two by considering only the longitudinal height profile of the nose rail.

As the reduction of wear was excluded from the optimisation problem, an increase of wear number was observed, as shown in Figure 5.15.

**5.5.5 Conclusions**

- The dynamic performance of vehicle-turnout interaction was improved by tuning the nose rail shape, approximated using B-splines, and by considering variations in the wheel profiles and track alignments.

- The vertical distance between the top of the wing rail and nose rail at the transition zone, particularly the transition point, is a determining factor for the dynamic behaviour of a crossing. This indicates that the design parameters can be reduced to two by considering only the longitudinal height profile of the nose rail.

- Constraints on the location of the wheel transition over the crossing can influence the optimised solution of the crossing geometry design. By shifting the desired impact location, the optimal solution of the crossing geometry can be changed.

- Compared with the current manufacturing design, the optimal solution for the nose rail has a lower height profile and a wider, flatter cross-sectional shape.

- The reduction of contact pressure can result in increased wear.
5 Optimisation of crossing geometry

5.6 Optimisation problem III: robust optimisation using B-spline

This study is presented in Paper E. It is based on the deterministic optimisation using B-spline approximations of the nose rail (optimisation problem II). However, the robust optimisation method [50-53] is utilised to account for uncertainties in the design variables.

5.6.1 Considering design vector uncertainty

A three-level, full-factorial design of experiments (DOE) was applied to the sample design variable variations at a given tolerance. Table 5.1 shows the parameters and level settings of the DOE, in which the variation of the longitudinal height profile of the nose rail is simplified as a combined parameter to reduce the sampling size. A relatively large tolerance (±0.1 mm) in the longitudinal height profile \((x_2)\) was considered in the level settings, which corresponded to the tolerance considered during the manual grinding/welding maintenance of crossings. In total, there were \(3^2 = 9\) design variances or design vector variances.

Table 5.1 Parameters and level settings in the full factorial design (from Paper E).

<table>
<thead>
<tr>
<th>Factors</th>
<th>Parameter description</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_1)</td>
<td>Variation of cross-sectional shape of the nose rail w.r.t the design parameter (\lambda): (\Delta \lambda)</td>
<td>-0.01</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>(x_2)</td>
<td>Variation of longitudinal height profile of the nose rail w.r.t the design parameters ([h, t]): ([\Delta h, \Delta t])</td>
<td>([-0.1, -0.1])</td>
<td>([0, 0])</td>
<td>([0.1, 0.1])</td>
</tr>
</tbody>
</table>

5.6.2 Formulation of the robust optimisation problem

The design variables, constraints, and objective function from optimisation problem II were used for the robust optimisation problem as well. All constraint and objective functions should address design-vector uncertainty, as introduced in Section 5.6.1. Moreover, in a robust optimisation problem, minimisation of both the average objective value and variance of the objective value is required. Here, the minimisation of the objective value variance was treated as a problem constraint. Thus the problem was formulated as:

\[
\bar{F}_0^{\text{rob}}(x, \xi) = \frac{\mu(F_0(x, \xi))}{\hat{\mu}}, \quad \rightarrow \min
\]  \hspace{1cm} (5.3)

\[
g_{\text{rob}}(x, \xi) = \frac{\sigma(F_0(x, \xi))}{0.05} \leq 1
\]  \hspace{1cm} (5.4)

where \(\bar{F}_0^{\text{rob}}(x, \xi)\) is the objective of the robust optimisation problem; \(\xi\) is design vector uncertainty (Section 5.6.1); \(F_0(x, \xi)\) is the objective function from equation 5.2 considering uncertainty; \(\mu(F_0(x, \xi))\) is the average of the objective value under all design variances; \(\hat{\mu}\) is
the average of the objective value for the reference design; $\sigma(F_0(x, \xi))$ is the relative standard deviation (RSD) of the objective value; $g_{\omega}(x, \xi)$ is the constraint on the objective value variance, i.e., the stochastic response of the optimum solution was assured with the confidence level of more than 90%.

5.6.3 Comparison between the deterministic and robust optimisation results

Figure 5.16 and Figure 5.17 show the optimised height profile and cross-sectional shape of the nose rail, respectively. Both optimised solutions have reduced heights and wider, flatter cross-sectional shapes relative to those of the reference design.

Figure 5.16 Longitudinal height profile of nose rail (from Paper E). ref: reference design, opt1: optimum solution from optimisation problem II, opt2: optimum solution from optimisation problem III.

Figure 5.17 Changing nose rail shape at location 300, 600 and 900 mm from TP (from Paper E). ref: reference design, opt1: optimum solution from optimisation problem II, opt2: optimum solution from optimisation problem III.
5 Optimisation of crossing geometry

Table 5.2 Comparison of design robustness between deterministic and robust optimisation.

<table>
<thead>
<tr>
<th>Optimal solution</th>
<th>Objective value without design variance</th>
<th>Average of objective value</th>
<th>RSD of objective value</th>
<th>Feasibility under design variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>deterministic optimisation</td>
<td>0.738</td>
<td>0.888</td>
<td>0.2603</td>
<td>2 variances are unfeasible</td>
</tr>
<tr>
<td>robust optimisation</td>
<td>0.743</td>
<td>0.748</td>
<td>0.0323</td>
<td>all variances are feasible</td>
</tr>
</tbody>
</table>

The robustness of the optimal solution was analysed for both the deterministic and robust optimisation problems. A summary of the robustness analysis is shown in Table 5.2, which indicates that, although the solution from the robust optimisation had a slightly higher objective value without considering design variances, it was robust. However, the solution from the deterministic optimisation was not robust, and it also became unfeasible under slight variations in the design vector.

Table 5.3 Variations of the vehicle-turnout system.

<table>
<thead>
<tr>
<th>Variations</th>
<th>Wheel profile</th>
<th>Track alignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>s1002</td>
<td>ideal alignment</td>
</tr>
<tr>
<td>A2</td>
<td>HIT</td>
<td>ideal alignment</td>
</tr>
<tr>
<td>B1</td>
<td>s1002</td>
<td>with vertical irregularity</td>
</tr>
<tr>
<td>B2</td>
<td>HIT</td>
<td>with vertical irregularity</td>
</tr>
<tr>
<td>C1</td>
<td>s1002</td>
<td>with lateral irregularity</td>
</tr>
<tr>
<td>C2</td>
<td>HIT</td>
<td>with lateral irregularity</td>
</tr>
</tbody>
</table>

Figure 5.18 Accumulative contact pressure (from Paper E). The groups A1-C2 are defined according to the variation of wheel profiles and the track alignment as listed in Table 5.3.
A comparison of the contact pressures in the robust optimal solution and the reference design under all design variances is shown in Figure 5.18. A reduced contact pressure was obtained under variations in wheel profile and track alignment.

### 5.6.4 Conclusions

- The robust optimisation provided a solution with a less optimistic objective value than the corresponding deterministic optimisation solution; however, the solution had a robust response under reasonable design variances (based on realistic design tolerances). The deterministic optimum solution did not display robust performance, because it failed under slight changes to the design variables.
- Improved wheel transition behaviour under variable wheel profiles and track alignments was obtained through the robust optimisation of the crossing geometry.
- Compared to the current manufacturing design, the optimal solution from the robust optimisation resulted in a nose rail with a lower height profile and a wider, flatter cross-sectional shape.

### 5.7 Main conclusions

The numerical optimisation approach for improving crossing performance based on the dynamic vehicle-turnout interaction was proposed, with the goal of reducing damage to the crossing due to impact forces. The optimisation problem was solved in both deterministic and probabilistic formulations. The main results and conclusions of the crossing geometry optimisation study are summarised below.

- The vertical distance between the top of the wing rail and nose rail in the transition point is the key factor determining the train-track interaction at a railway crossing.
- A practical and efficient way to tune only the railhead of the nose rail was suggested. Two methods for representing the cross-sectional shape of the nose rail were proposed, namely by using semi-ellipses and using B-splines. The number of effective parameters for adjusting the crossing geometry was not more than four in total.
- Contact pressure and the energy dissipation in the wheel-rail interface (reflecting RCF and wear) were used in the optimisation criteria. Solution of this two-criteria problem was obtained as a Pareto set of the compromised solutions representing a trade-off between the reduction of RCF and of wear.
- A robust optimum solution of problem was obtained through robust optimisation approach, accounting for the design tolerances and the variation of the wheel profile and the track
alignmet. The deterministic approach for the same design problem, however, did not display a robust performance.

The optimised crossing geometry can be implemented in the new crossings during the manufacturing process as well as in the existing crossings through welding/grinding maintenance.

6 Optimisation of vertical track elasticity

Elastic elements, such as soft rail pads, USPs, light sleepers, and ballast mats, are used along the railway track to tune the overall track elasticity and improve the transmission of force between different layers. The application of such elastic elements has been efficient in protecting the track structure, as it reduces vibration and improves load transmission [54-57].

In crossings, the advantage of applying elastic elements is more significant in terms of attenuating the dynamic impact and resulting vibrations. The study in [39] showed that a significant reduction of rail vibrations, particularly in soil, could be achieved by increasing the turnout resilience by inserting ballast mats and USPs. The influences of the rail pads, USPs, and ballast mats on the dynamic forces acting on the crossing nose and supporting structures were investigated numerically in [13]; the forces on the rail, sleepers, and ballast bed were significantly reduced using these elastic elements. Field measurements and numerical simulations of turnouts with different rail pad stiffness values were presented in [58], which demonstrated that the wheel-rail contact forces on the crossing could be reduced by using more resilient rail pads.

To improve the vehicle-track interaction at crossings, some studies, although limited in scope, have been performed towards optimising the track elasticity at the crossing [13,5,16]. These optimisations were performed by assessing a limited number of predefined designs. The optimum solutions obtained were thus dependent on the experience or assumptions of each of the authors.

To continue the study presented in [13], a numerical optimisation approach was proposed to further improve the crossing performance by tuning the mechanical properties of the rail pads and/or USPs (Paper B), in which the numerical optimisation method MAM (Section 4.2) was used to obtain the optimal solution.
6.1 Overview of track elasticity optimisation at crossings

6.1.1 Challenges and solutions

In optimising track elasticity at crossings, the main challenges are the following:

- **Modelling a flexible track with variable track elasticity**
  To tune the track elasticity at a crossing, the crossing is assumed to be independent from other sections of the track; the track elasticity can differ between the crossing and other track components. Thus, track elasticity must be defined based on the location along the track. In this study, the DARTS_NL model (Section 2.1) was used to simulate the vehicle-track interaction at crossings. This model defines track elasticity as a function of the location along the track by assigning the elastic properties of each element at each track layer.

- **Automatically renewing the vehicle-track model when track elasticity changes**
  For a numerical optimisation, constraints and objective functions must be calculated automatically for renewed design vectors. This is a considerable challenge in the evaluation of the dynamic response of a turnout crossing. For most available MBS packages, including VI-Rail, in which both the vehicle and track can be modelled and assembled, the flexible track model and assembly of the vehicle-track model must be updated manually when the elastic track properties are re-assigned. In this case, manual updating is both time-consuming and inconvenient, because many simulations may be required to obtain an optimal solution for a given crossing.

  The DARTS_NL model used in this study allows modelling and simulation by means of a script file, which combines all commands of the complete modelling process that can be edited freely. Thus, by editing the commands of the track elasticity definition in the script file, the model is automatically renewed. The editing parameters are defined by the design vector of the optimisation problem.

- **Balance between the accuracy of a dynamic simulation and the computational cost of optimisation**
  A reasonable balance between accuracy and computational cost was achieved in this study. The DARTS_NL model used is fast, requiring between one to two minutes to simulate a vehicle passing over a crossing with an entire travelling distance of approximately 50 m (28 m before and 22 m after the nose point, element length 0.1 m, calculation step 0.001s). Moreover, the definition of various track elasticities requires no extra model-initiation time compared to defining homogenous track elasticity. Secondly, the DARTS_NL model has been demonstrated to represent with accuracy the
dynamic vehicle-track interaction at crossings via comparing measured results at different turnouts in the Dutch railway network with the simulation (Paper A). Thirdly, the numerical optimisation method MAM (Section 4.2), advanced in reducing computational cost during simulation, was used to solve the track elasticity optimisation problem.

6.1.2 Goal of track elasticity optimisation

The goal of optimising the track elasticity at turnout crossings was to reduce the impact-related damage (RCF and plastic deformation) at the crossing. Based on the theory introduced in Section 2.3, the normal contact force (P1) was used as an indicator for RCF and plastic deformation. Moreover, the forces on the ballast bed and sleepers, which are responsible for geometric degradations and thus the amplification of the P1 forces, were to be reduced in addition to the P1 forces.

6.2 Optimisation problem

The analysed turnout was a 54E1 1:9 common single turnout, of which the dynamic vehicle-turnout interaction was modelled using the DARTS_NL package (Section 2.1). The crossing part is shown in Figure 6.1. To investigate the effect of track elasticity with respect to the longitudinal location along the crossing, the optimisation problem was solved by considering both homogeneous and non-homogeneous track elasticity along the crossing (considering the crossing as a whole and as different sections) individually:

- **The crossing as a whole**
  The elastic properties of the track layers were identical along the crossing.

- **The crossing divided into 11 sections according to the sleepers**
  The elastic properties of the track layers at each sleeper were defined independently.

- **The crossing divided into 3 sections**
  The crossing was divided into three sections, namely the front, middle (transition zone), and rear, as shown in Figure 6.2.

![Figure 6.1 The crossing (from Paper B).](image)
Part I: Extended Summary

Figure 6.2 Crossing divided as three parts (from Paper B): F—front part, M—middle part (transition part), and R—rear part.

- The crossing divided into 2 sections
  The crossing was divided into the sensitive section (transition section) and the remainder, as shown in Figure 6.2.

6.2.1 Design variables
The vertical stiffness and damping properties of the rail pads and/or USPs along the crossing were used as design variables, as described in Table 6.1. The number of design variables depended on the consideration of the crossing as a whole or as different independent sections, and whether the USPs were applied along the crossing.

Table 6.1 Design variables (from Paper B).

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>stiffness of rail pads</td>
<td>(MN/m)</td>
<td>20</td>
<td>3032</td>
</tr>
<tr>
<td>damping of rail pads</td>
<td>(kNs/m)</td>
<td>29</td>
<td>500</td>
</tr>
<tr>
<td>stiffness of USP</td>
<td>(MN/m³)</td>
<td>30</td>
<td>300</td>
</tr>
<tr>
<td>damping of USP</td>
<td>(kNs/m³)</td>
<td>10</td>
<td>300</td>
</tr>
</tbody>
</table>

6.2.2 Objective function
By tuning the stiffness and damping properties of the rail pads and USPs, the performance of the crossing design was assessed using the following criteria and responses of the train-track system:

- Damage to the crossing nose estimated by the maximum dynamic normal contact force (P1 force).
- Damage on the sleepers estimated by the maximum force acting on the sleepers ($F_{sl}$).
- Damage to the ballast bed estimated by the maximum force acting on the ballast bed ($F_{bl}$).

Therefore the dynamic forces acting on the rails, sleepers, and the ballast bed need to be minimised to obtain the optimum design.
6 Optimisation of vertical track elasticity

\[ F_0(X) \equiv W_{p1} \frac{P1}{P1^*} + W_{s1} \frac{F_{s1}}{F_{s1}^*} + W_{s2} \frac{F_{s2}}{F_{s2}^*} \rightarrow \min \]  

(6.1)

where \( W_{p1}, W_{s1} \), and \( W_{s2} \) are the weight coefficients \((W_{p1} + W_{s1} + W_{s2} = 1)\). * means the response of the reference crossing. The P1 force has a major effect on the damage to the crossing nose and thus it was weighted more in the objective function \([W_{p1}, W_{s1}, W_{s2}] = [0.80, 0.15, 0.05]\) or \([W_{p1}, W_{s1}, W_{s2}] = [1.00, 0, 0]\) when only the P1 force was to be minimised.

Table 6.2 Comparison of the reference design and the optimised design.

<table>
<thead>
<tr>
<th>Var.</th>
<th>( K_{pad} ) (MN/m)</th>
<th>( C_{pad} ) (kN/m/s)</th>
<th>( K_{usp} ) (MN/m^3)</th>
<th>( C_{usp} ) (kN/m/s^3)</th>
<th>P1</th>
<th>F_{s1}</th>
<th>F_{s2}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>M</td>
<td>R</td>
<td>F</td>
<td>M</td>
<td>R</td>
<td>F</td>
</tr>
<tr>
<td>ref</td>
<td>3032</td>
<td>3032</td>
<td>3032</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>-</td>
</tr>
<tr>
<td>opt</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>93</td>
<td>29</td>
<td>93</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>195</td>
</tr>
</tbody>
</table>

6.2.3 Constraints

The main constraints of the problem were imposed on the maximum displacement of the rail and the sleepers and the maximum compressive deflection of the rail pads.

6.3 Optimisation results

The results of the optimisation showed that the dynamic forces in the turnout could be reduced by increasing the vertical elasticity of the track structure. Table 6.2 lists the results of the reference design (ref) and one of the optimal design (opt), in which the crossing was considered as the sensitive section (M) and the remainder (F and R) as shown in Figure 6.2.

6.4 Main conclusions

In the optimisation of the vertical track elasticity, the stiffness and damping properties of the rail pads and under sleeper pads (USPs) were tuned. The high-frequency forces on the rail, chosen as indicators of the level of RCF damage, were to be minimised. The reduction of (low-frequency) forces on the sleepers and ballast that are responsible for the degradation of the overall crossing geometry was also considered.

Results show that the dynamic forces can be reduced significantly by applying:

- Softer rail pads under the crossing. Combined with the USPs, the dynamic forces, particularly the forces acting on the sleepers and ballast, can be further reduced.
- Rail pads with different properties. By applying relatively less soft elements before and after the crossing nose than those used under the crossing nose, the dynamic forces can be reduced most effectively.

7 Concluding remarks

7.1 Main conclusions

This study was proposed to develop a methodology to optimise vehicle-track interactions at railway crossings by tuning the crossing design. Numerical and experimental studies have shown that the rail geometry and the track elasticity significantly affect the dynamic behaviour of the crossing. Therefore, this research focused on optimising the crossing geometry and track elasticity.

In order to account for multiple criteria and constraints in the design optimisation, the numerical optimisation technique was applied to solve the optimisation problems. The usage of numerical optimisation method allows for systematic search of the optimum solution instead of manual search through predefined variations of design parameters as it is used in the traditional design optimisation of railway systems.

Moreover, the numerical analysis of the dynamic vehicle-turnout interaction was coupled with the optimisation technique for automated use in the optimisation process. Therefore, the design assessment during the optimisation process was based on the dynamic behaviour of the crossing.

Additionally, the proposed methodology was extended to solve the optimisation problem of railway crossing with uncertainties, such as design tolerances, track irregularities, wheel profiles, resulting in a robust optimum design.

The merits of the developed methodology and the design improvements obtained using the methodology are summarised below.

7.1.1 Methodology for improvement of crossing performance

The proposed methodology for the design improvement of railway crossings combines modern optimisation techniques and dynamic analyses of vehicle-turnout interactions. This methodology comprehensively considers

- **Multiple assessment criteria in the design optimisation.** Reductions of RCF and wear were considered the objectives in tuning the crossing geometry; the minimisation of the P1 force and the forces on the sleepers and the ballast was considered in tuning the vertical track elasticity of the crossing.
7 Concluding remarks

• **Uncertainties in the vehicle-turnout system.** Two typical wheel profiles and various track alignment conditions were considered in the optimisation of crossing geometry. The methodology could easily be extended to include more variations of the vehicle-turnout system, such as more wheel profiles, track conditions, velocities, and axle loads. Moreover, the robust optimisation approach was demonstrated in this research to accommodate the design tolerance of the crossing geometry.

• **Realistic parameterisation of crossing designs with small amount of design parameters.** Effective design parameters were identified for a realistic representation of the crossing design. The number of design parameters was reduced to three when tuning the nose rail geometry using B-spline representations of the cross-sectional shape in optimising the crossing geometry.

• **Balance between the accuracy of the solution and its computational cost.** The MBS VI-Rail model and the DARTS_NL model for fast simulations of dynamic vehicle-turnout interactions were developed to optimise the crossing geometry and the track elasticity, respectively. Both models were validated by field measurements on instrumented crossings. The numerical optimisation method MAM, used in solving the crossing optimisation problems, includes features that reduce the overall computation costs of the optimisation process.

7.1.2 Improvement of crossing performance via two approaches

**Crossing geometry optimisation**
Practical and efficient tuning of the crossing geometry can be achieved by tuning only the railhead of the nose rail. Two methods were proposed for representing the cross-sectional shape of the nose rail, approximating the railhead by semi-ellipses and by B-splines, respectively. Both methods required no more than four design parameters.

By including RCF and wear in the optimisation criteria, the optimised crossing geometry reduces RCF and wear, and a Pareto-front was obtained expressing the trade-off between the reduction of RCF and that of wear.

Additionally, the vertical distance between the top of the wing rail and nose rail at the transition zone, particularly the transition point, was identified as the determining factor for the dynamic behaviour of a crossing. By shifting the desired impact location, the optimal solution of the crossing geometry can be changed.

A new crossing geometry can be implemented in both new crossings during the manufacturing process and existing crossings by grinding/welding maintenance operations.
Track elasticity optimisation

The vertical track elasticity can be tuned by adjusting the stiffness and damping properties of the rail pads and/or USPs. Both the homogeneous and non-homogeneous elasticity along the crossing were considered in the optimisation problems.

The dynamic forces on the crossing track can be significantly reduced by using softer rail pads. With the application of USPs, the dynamic forces, particularly the low-frequency components acting on the sleepers and ballast, can be further reduced. By applying non-homogeneous track elasticity along the crossing, that is, using less soft elements before and after the crossing nose than those used under the crossing nose, the dynamic forces can be reduced most effectively.

The adjustment of the track elasticity can be practically performed during the installation of new crossings.

7.2 Limitations in the study

The proposed methodology has some limitations:

Lack of validation of the optimised crossing designs against field measurements
During the period of the research, the optimised crossing designs were not tested at field sites; the improvement of the designs has not been proven by field measurements.

Simplification of the crossing model
To allow for fast simulations of dynamic vehicle-turnout interactions, the numerical models used were somewhat simplified. The VI-Rail model used in crossing geometry optimisations was the ‘moving track’ model, wherein the elasticity was assumed to be independent of the longitudinal location along the track. The DARTS_NL model assumes a symmetric track, in which both sides of the track were modelled as crossing inner rails. Thus, the rolling of the vehicle was omitted. Moreover, DARTS_NL is a 2-D FE model, limiting the study to the vertical and longitudinal directions without considering the lateral disturbances in the system. Both models were used for the analysis of the dynamic crossing performance within a limited frequency (up to 500 Hz); therefore, the higher-frequency components of the dynamic response were not accounted for.

Specification of the target vehicle-turnout system
To reduce the number of uncertain parameters in the vehicle-turnout system, this study focused on a specific combination of crossings and trains; and it considered the passage of trains only in the main-facing direction. Therefore, the optimum solutions from the current study must be adjusted for use under different conditions.
7.3 Recommendations

7.3.1 Recommendations for crossing designs

The study provides guidance for improving the designs of crossings during both manufacturing and maintenance procedures.

Firstly, the vertical distance between the top of the wing rail and the nose rail is the most important factor for tuning the wheel transition behaviour at the crossing; this behaviour determines both the magnitude and location of the impact forces on the track. Therefore, to improve the existing crossing geometry, the longitudinal height of the nose rail should be carefully adjusted.

Secondly, crossings should be equipped with softer rail pads combined with USPs. Moreover, the track elasticity should vary along the crossing to achieve most significant reduction of dynamic forces.

Thirdly, design optimisations of crossings should accommodate the various vehicles, wheel profiles, and loading conditions that exist on the line under investigation; reasonable design tolerances should be used in implementations of such designs through manufacturing and maintenance.

7.3.2 Recommendations for future research directions

Based on the limitations in this study, future research directions are recommended here.

**Validation of optimised crossing design**

A railway crossing with optimised geometry or track elasticity should be tested in the field. In the test, the dynamic performances of crossings with the original and optimised geometry (or track elasticity) should be examined to evaluate the degree of achievement of the expected improvements. If improvement is not notably measured in the optimised crossings, further investigations are required to clarify the problem. Once the optimal design is validated against field measurements, it can be used in the manufacture and the maintenance to implement new crossing design.

**Improvement of the numerical models of vehicle-turnout interaction**

A more realistic model for the fast simulation of vehicle-turnout interaction must be developed to better represent the wheel-rail interactions at railway crossings. In particular, this requires enhancements of the track and contact models.

**Consideration of more uncertain parameters**

In order to increase the robustness of the crossing design, increased uncertain parameters in the vehicle-turnout interaction, as compared to those used in the presented study, should be considered. This would extend the optimisation method to crossings on railway lines with
mixed types of trains under different travelling velocities, axle loads, and wheel profiles (with different worn stages, if available).

**Application of the optimisation approach in the switch panel**
The switch panel is excluded in the current study; further research can be performed by extending the proposed methodology to the switch panel.
References


References


[33] Polach O. A fast wheel-rail force calculation computer code. 16th IAVSD Symposium. 30 August-02 September 1999, Pretoria, South Africa.


References


Part II

APPENDED PAPERS
Analysis of train/turnout vertical interaction using a fast numerical model and validation of that model

Chang Wan, Valeri Markine, Ivan Shevtsov


Abstract

A two-dimensional (2-D) finite element model has been developed for simulation and analysis of train/turnout vertical dynamic interactions at a common crossing. The model has been validated through field measurements and simulation results obtained using a three-dimensional (3D) multi-body system (MBS) model. The dynamic behaviour of three turnouts on the Dutch railway network was simulated using the 2D model. The simulation results were compared with measured data collected from the instrumented crossings containing corresponding turnouts. It was observed that the values of the vertical acceleration of the crossing nose obtained from the simulations were in good agreement with the measured values. The 2D model was verified by adapting the more intricate 3D MBS model established in the VI-Rail software. The vertical geometry of the rail used in the 2D model was obtained using the wheel trajectory from the 3D model. It was observed that the dynamic wheel forces in the two models were close to one another. From these results it was concluded that the 2D model is able to simulate train/turnout interactions with a good accuracy and thus it can be used instead of complex time-consuming numerical simulations.
1 Introduction

Railway turnouts (switches and crossings) are essential components of the railway infrastructure and provide flexibility in the system by enabling railway vehicles to switch between tracks at a railway junction. Due to the varying rail geometry and changes in track flexibility, high contact forces arise during wheel passage over the switch toe and the crossing, resulting in local damage of the contact surfaces as well as transmitting noise and vibration to the environment. According to Markine et al. [1], the impact load acting on the crossing nose can be several times higher than the static wheel load. Therefore, the train/track interaction is of high importance in the design of turnouts and the maintenance of railway systems.

When analysing the train/turnout interaction, special attention should be paid to the discontinuity in the rail geometry which contributes to the major rail excitations. Several numerical procedures have been proposed that have focused on modelling the continuously changing track geometry, such as a rail with various cross-sections [2] and sleepers varying in length along the crossing [1,3], to account for the complex geometric and elasticity-related effects on the dynamic performance of a turnout.

Various three-dimensional (3D) simulation methods have been proposed recently to investigate the dynamic behaviour of a turnout. In Andersson and Dahlberg [4], the dynamic interaction between a railway vehicle and a turnout was simulated using the commercial multi-body system (MBS) software GENSYS and a home-developed package DIFF3D. The simulation results from the two models were compared with each other and verified through full-scale field tests [5]. Sun et al. [6] used the dynamic simulation package VAMPIRE to determine the wheel impact forces when a train passed over a turnout. The flexible track and real-time wheel/rail contact models implemented in NUCARS software have been used to evaluate the design of a tread bearing diamond crossing [7]. The MBS simulation package SIMPACK has been used to calculate a vehicle’s dynamic behaviour when passing over a turnout [2,8]. A mathematical model that accounted for the effects of the variation in rail profile along the track and the local variation of the track flexibility was proposed by Alfi and Bruni [9]. They used the proposed model to simulate the train/turnout interaction and compared the obtained results with experimental data. All the discussed numerical models can be used for dynamic analysis of the train/turnout interaction. However, a high computational effort is required when using the 3D simulation methods, especially when the turnout is modelled as a flexible track.

Recently, numerical optimisation methods have been used to improve the dynamic performance of the turnout by tuning the parameters of the train/turnout system [10,11]. These studies have clearly demonstrated that numerical optimisation has significant potential to improve the dynamic properties of the track system. Usually, in real-life optimisation problems multiple numerical analyses are involved, and the total computational cost of the
optimisation often becomes prohibitive when implementing complicated numerical methods. Therefore, less time-consuming approaches are preferable in the numerical optimisation process.

The computer package DARTS_NL (TU Delft) for fast dynamic vehicle/track analysis is used in this paper in order to create a turnout model that is computationally less expensive than existing approaches. To validate the two-dimensional (2D) numerical model developed in this package, field measurements are performed on several instrumented turnouts to acquire the dynamic responses of the crossings to a train passage. Additionally, a 3D MBS model of a turnout developed in the VI-Rail software is used to validate the 2D model. The 2D numerical model of the dynamic train/turnout interaction is described in the next section. The comparison of the measured and simulated results is presented in the section 'Model validation using field measurements'. The section 'Validation through VI-Rail' presents the validation case based on the 3D model of a standard right-hand turnout built in the VI-Rail software. A comparison of the results from the two models is also presented in this section. Finally, conclusions are drawn.

2 Numerical model of train-turnout interaction

2.1 Train-turnout interaction in the 2D model

The 2D software DARTS_NL was developed for the dynamic analysis of railway track/train interactions. In order to reduce the computational costs, the modelling procedure is restricted to two dimensions (vertical and longitudinal) and linear material property elements are considered. The modelling approach is based on the coupling of a finite element model of the track and a multi-body model of the moving train. Figure A-1 is a schematic diagram of the vehicle/track model [12].

The railway turnout structure is more complicated than a normal rail track (see Figure A-2), and is modelled using a variable rail cross-section and sleepers that vary in length, based on the corresponding design drawings. The detailed modelling approach can be found in Markine et al. [1].

It should be noted that DARTS_NL uses the symmetry properties displayed by the track in the vertical plane; thus, only half of the track is represented in the model. However, for a turnout, the rail, rail support and vertical rail geometry on the inner and outer side are not the same (a normal rail on the outer side and the crossing on the inner side). Therefore, some dynamic effects such as rolling of the vehicle are ignored, which is acceptable for relative and qualitative analyses. Since the inner part, including the crossing nose, is more sensitive to damage it needs to be considered in order to obtain a representative model of the train/turnout interaction. In order to analyse the dynamic response of the crossing, the inside half of the through rail of the turnout is modelled (Figure A-2).
The discussed points were applied by Markine et al. [1] to create a model that was used for a comparison of turnout designs with various vertical stiffness levels and a qualitative analysis of the nature and frequency of the impact forces on the components. From the experimental results obtained on an instrumented turnout it was observed that the vertical wheel forces (measured vertical rail accelerations) dominate the wheel/rail interactions [13].

![Model of ballast track structure and railway vehicle.](image)

Figure A-1 Model of ballast track structure and railway vehicle.

![Common single turnout.](image)

Figure A-2 Common single turnout ([1]).

To assess the accuracy and reliability of the 2D model, passenger trains negotiating turnouts with varying crossing angles, rail geometry and service condition are considered in this paper. The results are compared with corresponding field measurements.

Table A-1 Parameters of reference turnout mode ([14]).

<table>
<thead>
<tr>
<th>Track components</th>
<th>Type</th>
<th>E [N/m²]</th>
<th>Poisson coeff.</th>
<th>Stiffness [MN/m]</th>
<th>Damping [kNs/m]</th>
<th>Mass [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>S4 E1</td>
<td>2.1e11</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>7850</td>
</tr>
<tr>
<td>Rail pad</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3032 (EVA)</td>
<td>29</td>
<td>-</td>
</tr>
<tr>
<td>Sleeper</td>
<td>B70</td>
<td>3.85e10</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
<td>2400</td>
</tr>
<tr>
<td>Ballast</td>
<td>Quality ‘good’</td>
<td>-</td>
<td>120</td>
<td>-</td>
<td>48</td>
<td>-</td>
</tr>
</tbody>
</table>

The turnouts were modelled based on the standard designs. For a 1:12 turnout, the total length is 54 m with a 7 m common crossing, whereas a 1:15 turnout has a total length of 72 m with a 9 m crossing. The turnout structure was modelled using a series of alternating hard
and soft layers, representing the rail, rail pads, sleepers and ballast bed. Relatively hard rail pads were used in the model to represent the connections between the rail and sleepers. The mechanical properties of the turnout models are listed in Table A-1; these are parameters of a real turnout used in the Netherlands. Most of these parameters were obtained from the track component manufacturers, the rail pad as well as the ballast bed properties used here can also be found in Esveld [14]. The same data were also used in Markine et al.[1].

2.2 The vertical rail geometry of the turnout

The moving vehicle generates a dynamic impact because of the roughness of the rail surface (Figure A-1). Therefore, an appropriate representation of the track geometry is essential for realistic simulation of the train/track interaction. In the 2D model the vertical rail geometry (e.g. the rail surface and track settlements etc.), can be defined either as a periodic function or as a data profile derived from measurements.

Since only the vertical rail geometry is considered in DARTS_NL, some simplification of the 3D rail geometry is required. The simplification approach used in this paper to obtain the vertical rail geometry in the crossing point is now briefly described. The following three stages can be recognised as a wheel passes through the crossing.

1. The wheel travels on the wing rail when approaching the crossing nose as shown in Figure A-3a. The contact between wheel and rail is at a single point.

2. The rail touches the crossing nose and the wheel load is transferred from the wing rail to the crossing nose (Figure A-3b and Figure A-3c). In this transition stage the wheel contacts both the wing rail and the nose.

3. The wheel leaves the wing rail and rolls only over the crossing nose followed by the through rail (Figure A-3d).

![Figure A-3 The stages of a wheel passing through a crossing ([1]).](image)

The geometrical properties of the wheel and rail (wing rail and crossing nose) determine the wheel transition in the crossing. In Markine et al. [1] the vertical rail geometry of the turnouts was created based on the theoretical profile (as manufactured) and the visual image of the turnout which provides information on the transition zone (Figure A-4a). Using this method, the rail geometry in the crossing before and after the transition zone follows, respectively, the wing rail and the theoretical crossing nose profile based on the engineering
drawing (Figure A-4a). In the transition zone (zone b in Figure A-4a) and after the transition zone (zone c in Figure A-4a) a linear vertical rail geometry is assumed.

To increase the accuracy of the model, in this study, the vertical rail geometry along the crossing was obtained by manually measuring the vertical distance from the top of the wing rail to the crossing nose) as shown in Figure A-4b. The rail geometry of the turnout was then obtained by combining the profile obtained using the visual image (Figure A-4a) and the measured geometry data (Figure A-4b). The height of the vertical dip (Hd in Figure A-4c) and the rail geometry after the transition zone (zone c) were thus defined by the measured data (Figure A-4b). It should be noted that in contrast with the simplified method presented in Markine et al. [1] the rail geometry after the transition zone (zone c in Figure A-4c) is not necessarily linear.

Figure A-4 Determination of the vertical rail geometry of turnout in the model: (a) simplified method presented in Markine et al. [1], (b) measured geometry of the crossing nose (the nose point is the 0 point), (c) rail geometry in the model.

It should be noted that, on account of the 2D restriction and the simplifications used in the model, the wheel is assumed to move through the turnout completely guided by the given rail geometry, that is, the wheel trajectory is the same as the vertical rail geometry. From the 3D dynamic simulations it was found that the real wheel trajectory changes if the wheel load
or the speed of the train changes [15]. For the same turnout, the higher the travelling speed, then the farther away is the transition zone from the tip of the crossing nose. Additionally, the shape of the trajectory also differs according to the travelling speed [15]. Since the train velocity is implicitly taken into account in the simplified rail geometry [1], the obtained geometry should be used in the simulations with the typical velocity of the trains passing the particular turnout. This fact will be used later (the section ‘Setup of field measurement’) in the selection of the representative train measurements for the model validation.

3 Model validation using field measurements

Two 1:12 turnouts and one 1:15 turnout have been measured and modelled as described in the previous section. Thus, three validation cases characterised by different crossing angles and different load conditions of the turnouts are investigated in this paper. The results from the numerical simulations are compared with the corresponding field measurements.

3.1 Setup of field measurement

The measured turnouts were 54 E1 turnouts. An ESAH-M device was used to record the rail acceleration signal and the train speed. The 3D acceleration sensor was mounted on the crossing as shown in Figure A-5. It is expected that the major dynamic impacts occur around that point. The location of the sensor depends on the train’s direction of travel.

![Figure A-5 Setup for measurement of accelerations at the crossing nose.](image)

The speed measurement device, consisting of two inductive sensors, was installed on the rail before the nose point. The distance between the two inductive sensors was 1 m. The signals from the train speed and acceleration sensors were synchronised.

It should be noted that the measurements were performed under normal operational conditions. Data was collected for two or three trains passing each turnout. From these data the wheels with rail accelerations close to the average maximum acceleration (the bad wheels producing extremely high accelerations were excluded), with clearly defined impact peak and corresponding to the typical velocity for the particular turnout (see the section ‘The vertical rail geometry of the turnout’) were chosen. Then one of these measurements for each turnout was used in the validation case.
To allow valid comparison with the numerical simulations, the data from the measurements sampled at 10 kHz were low-pass filtered with a cut-off frequency of 500 Hz. Figure A-6 presents an example of the measurement data before and after filtering.

![Figure A-6 Measured vertical acceleration signal of the crossing nose: (a) raw and (b) filtered data.](image)

3.2 Comparison of measured and simulated results

1:12 turnouts

The first case used for validation refers to the 54 E1 1:12 turnouts on a ballast bed (turnout A and turnout B). The two turnouts were located close to a train station, which means that the train velocity was not constant due to acceleration and braking of the trains.

![Figure A-7 Measured and theoretical rail geometry of 1:12 turnouts.](image)

At the location of the crossing nose, the railway wheel encounters a discontinuity in the rail geometry, which is responsible for the shape and amplitude of the acceleration of the crossing nose. The vertical distance from the top of the wing rail to the top of the crossing nose was
measured every 12 cm along the crossing nose to allow the vertical rail geometry in the numerical model to be adjusted. The measured and theoretical vertical rail geometry is depicted in Figure A-7, from which it can be seen that due to wear the geometry of both turnout crossings slightly differs from the theoretical one. The rail surface of turnout A was more damaged than turnout B.

Based on the image of the turnout and the measured rail geometry, the vertical geometry of the two turnouts was obtained using the procedure discussed in the section ‘The vertical rail geometry of the turnout’ (Figure A-4). Figure A-8 and Figure A-9 show the vertical geometry of the turnouts, for the case where the nose point is located at a distance of 36 m from the beginning of the model.

![Transition zone (9.8 cm)](image)

Figure A-8 Vertical rail geometry of turnout A in the model: (a) image of the turnout, (b) vertical geometry in the model.

![Transition zone (18.0 cm)](image)

Figure A-9 Vertical rail geometry of turnout B in the model: (a) image of the turnout, (b) vertical geometry in the model.

The measured time histories of the vertical acceleration of the crossing nose during a train passage at 88 km/h on turnout A and at 87 km/h on turnout B were compared with the 2D simulation results. The measured train on turnout A has four four-axle coaches (static wheel load 78.4 kN) with 20 m between the centre of the front and the rear bogie. The same type of
train was also measured in the turnout B case. For the sake of clarity and simplicity, a comparison was performed only for one bogie of a normal wagon passing through the turnout over a time span of 0.3 s (Figure A-10 and Figure A-11). For the simulated bogie, the distance between wheelsets is 2.75m (for a motorised bogie) and 2.5m (for a normal bogie) for turnouts A and B.

![Figure A-10 Turnout A—vertical acceleration at the crossing nose during the passage of a whole bogie (88 km/h): (a) field measurement, (b) simulation.](image)

![Figure A-11 Turnout B—vertical acceleration at the crossing nose during the passage of a whole bogie (87 km/h): (a) field measurement, (b) simulation.](image)

Figure A-10 and Figure A-11 report the measured and simulated vertical acceleration of the crossing nose of turnout A and turnout B, respectively. In both measurements (Figure A-10a and Figure A-11a) the impact produced by the wheel passage is clearly visible, although less predominant as compared with the random effect arising from rail/wheel irregularities in the
case of turnout B (Figure A-11a). This could be due to the better geometric condition of turnout B, which is newly installed. The computed vertical acceleration signal (Figure A-10b and Figure A-11b) was slightly different from the measured response. In the simulation the extra vibrations arising from rail/wheel irregularities were not presented due to lack of information about the actual geometry of the wheel profile and the railhead along the turnout. Nevertheless, in the simulated results the major impact has a similar shape to the measurement for both turnouts. Moreover, when looking at the extreme amplitudes of the acceleration, it can be seen that they are in good agreement with the measurements (Table A-2).

Table A-2 Comparison of the amplitude of the vertical acceleration at the crossing nose of 1:12 turnouts.

<table>
<thead>
<tr>
<th>Maximum amplitude</th>
<th>Turnout A</th>
<th>Turnout B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>Measurement (M) (m/s²)</td>
<td>Simulation (S) (m/s²)</td>
</tr>
<tr>
<td></td>
<td>214.05</td>
<td>286.66</td>
</tr>
<tr>
<td></td>
<td>-238.30</td>
<td>-204.50</td>
</tr>
</tbody>
</table>

It should be noted that the train model used in the simulations of 1:12 turnouts slightly differs from the actual trains. The numerical model was based on data for a VIRM train, whereas the measurements were taken for DDAR trains. In fact the trains are quite similar. Both trains have double-deck car bodies and the bogies are of the same type. Therefore, it was decided to use the VIRM model in the numerical simulations. If the details of the DDAR trains were known, then the model could be adjusted which would improve the numerical results.

Comparing the dynamic behaviour of the two turnouts, it should be noted that both the shape and the amplitude of the acceleration signal are different for turnouts A and B, although the two turnouts were identical except for the local geometry of the crossing. The maximum acceleration of turnout A is almost 1.5 times that of turnout B in the measurement and more than 1.5 times that of turnout B in the simulations (Table A-2). This is mainly attributed to the more damaged rail surface of turnout A as compared with turnout B, which demonstrates the importance of the rail geometry in the train/turnout interaction and it can be effectively accounted for in the numerical model.

1:15 turnout

The second comparison also deals with the 54 E1 turnout on a ballast bed (turnout C) but for a crossing angle of 1:15. The measurements were performed when trains were passing the turnout in the main facing direction.

As in the 1:12 turnouts case, the vertical rail geometry was measured for turnout C, and it also reveals certain difference as compared with the theoretically designed turnout (Figure A-
The vertical rail geometry for the turnout model was obtained based on the image of turnout C and the measured rail geometry data, it is presented in Figure A-13.

![Figure A-12 Measured rail geometry of 1:15 turnout (turnout C).](image)

![Figure A-13 Vertical rail geometry of turnout C in the model: (a) image of the turnout, (b) vertical geometry in the model.](image)

Figure A-14 depicts the time histories of the measured and simulated vertical accelerations of the crossing nose during a train passage at 130 km/h over turnout C. The measured train has six four-axle coaches with a static wheel load of 78.4 kN. A comparison was performed for the passage of one bogie over a time span of 0.3 s. It can be seen in Figure A-14 that the result of the simulation is very close to the measurement both in shape and in amplitude (Table A-3), which indicates that the main behaviour of the turnout has been captured by the model.

It should be mentioned that the crossing nose of turnout C has been repaired, and therefore the measured rail geometry differs from the theoretical one as shown in Figure A-12. Here the vertical geometry of the rail in the crossing after the transition zone is not linearly changing as in the case of the measured geometry of the previous 1:12 turnouts (almost linear). Based on the measurement, a bilinear geometry after the transition zone was used in the numerical
simulations (Figure A-13). By doing this, more accurate results were obtained than when using only the theoretical geometry [1]. This again indicates that the effect of the rail geometry on the turnout response can be captured with the 2D model.

![Figure A-14](image)

Figure A-14 Turnout C—vertical acceleration at the crossing nose during the passage of a whole bogie (130 km/h): (a) field measurement, (b) simulation.

<table>
<thead>
<tr>
<th>Turnout C</th>
<th>Maximum amplitude</th>
<th>Measurement (m/s²)</th>
<th>Simulation (m/s²)</th>
<th>M vs. S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>234.0274</td>
<td>254.7000</td>
<td>1:1.085</td>
<td></td>
</tr>
<tr>
<td>Negative</td>
<td>-233.1764</td>
<td>-215.0900</td>
<td>1:0.922</td>
<td></td>
</tr>
</tbody>
</table>

### 4 Validation through VI-rail

The commercial software VI-Rail was developed as a specialised environment for railway virtual prototyping and is based on the industry standard multi-body dynamics code MSC Adams.16 This program was used in this work to validate the proposed 2D model. The same turnout was simulated using both VI-Rail and the 2D model. The vertical wheel trajectory derived from VI-Rail was used in the 2D model. The numerical results of both models are compared and discussed in the following sections.

#### 4.1 Train-turnout model

In this study, a ballasted track with a single right-hand turnout was modelled. The elastic parameters of the track were taken from Table A-1 and the structure of the track is shown in Figure A-15. The considered crossing angle was 1:12 and a 60 E1 rail was used outside the crossing. The main direction (straight track) of the turnout with a total length of 100m was considered in the model (two rails were modelled), and the crossing part, which is the main
focus of the simulation, was simplified to be 2m in length (from 47 to 49m with the nose point at 48.28 m). At the crossing part, a guiding rail with a length of 2m along the left rail was modelled, and continuously changing rail profiles were used at the right track to represent the crossing. These profiles were distributed every 15 and 16mm along the crossing, resulting in a total of 60 profiles to account for the continuous change of the rail geometry.

![Figure A-15 Schematic of the flexible track structure (ballasted) [16].](image)

![Figure A-16 Changing rail profiles in the crossing with schematic transition area.](image)

Table A-4 Parameters of the train model.

<table>
<thead>
<tr>
<th></th>
<th>Profile</th>
<th>S1002</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wheel</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radius</td>
<td>0.46 m</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>1813 kg</td>
<td></td>
</tr>
<tr>
<td>I_XX</td>
<td>1120 kg m^2</td>
<td></td>
</tr>
<tr>
<td>I_YY</td>
<td>1120 kg m^2</td>
<td></td>
</tr>
<tr>
<td>I_ZZ</td>
<td>112 kg m^2</td>
<td></td>
</tr>
<tr>
<td>Wheelbase</td>
<td>2.56 m</td>
<td></td>
</tr>
<tr>
<td><strong>Wheelset</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>2615 kg</td>
<td></td>
</tr>
<tr>
<td>I_XX</td>
<td>1722 kg m^2</td>
<td></td>
</tr>
<tr>
<td>I_YY</td>
<td>1476 kg m^2</td>
<td></td>
</tr>
<tr>
<td>I_ZZ</td>
<td>3067 kg m^2</td>
<td></td>
</tr>
<tr>
<td><strong>Bogie</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>1613 kg</td>
<td></td>
</tr>
<tr>
<td>I_XX</td>
<td>1234 kg m^2</td>
<td></td>
</tr>
<tr>
<td>I_YY</td>
<td>1476 kg m^2</td>
<td></td>
</tr>
<tr>
<td>I_ZZ</td>
<td>3067 kg m^2</td>
<td></td>
</tr>
<tr>
<td><strong>Primary Suspension</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K_x</td>
<td>3.139×10^7 N/m</td>
<td></td>
</tr>
<tr>
<td>K_y</td>
<td>3.884×10^6 N/m</td>
<td></td>
</tr>
<tr>
<td>K_z</td>
<td>1.220×10^6 N/m</td>
<td></td>
</tr>
<tr>
<td><strong>Secondary Suspension</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K_x</td>
<td>1.6×10^5 N/m</td>
<td></td>
</tr>
<tr>
<td>K_y</td>
<td>1.6×10^5 N/m</td>
<td></td>
</tr>
<tr>
<td>K_z</td>
<td>4.3×10^5 N/m</td>
<td></td>
</tr>
</tbody>
</table>
Figure A-16 presents a part of the rail profiles along the crossing in the model. The rail supporting structure of the turnout was modelled using both the rigid and the flexible model available in VI-Rail.

A passenger train with one wagon was used to analyse the train/turnout interaction. The parameters of the train model are listed in Table A-4.

The same train and turnout was modelled using DARTS_NL. The total length of the model was approximately 57 m (the nose point was at 36.0 m) and the element length in the model was equal to 0.1 m. The vertical wheel trajectory from VI-Rail was used as the vertical rail geometry of the corresponding 2D model.

4.2 Results of the validation

In order to account for the dynamic effect, a train with a speed of 10, 20 and 40 m/s was considered in the validation study. In each case, the wheel trajectory for rigid and flexible tracks was obtained separately using VI-Rail as shown in Figure A-17. The integration time was 4, 2 and 1 s for a train velocity of 10, 20 and 40 m/s, respectively. It should be noted that the wing rail was raised before the crossing nose. The following three stages can be recognised as the wheel passes the crossing (Figure A-17).

(1) The wheel is travelling on the wing rail approaching the crossing nose, when it is close to the nose the wheel starts to climb and follows the rising wing rail.

(2) The nose rail rises gradually and the wheel is transported from the wing rail to the crossing nose. At the transition point the wheel is in contact with both the wing rail and the nose, whereas the tread contact is solely on the wing rail.

(3) The wheel leaves the wing rail, creating a discontinuity in the contact area and a vertical drop of the wheel. The wheel then only rolls along the crossing nose followed by the through rail.
It can be observed from Figure A-17 that the wheel leaves the wing rail earlier and produces larger vertical gap under lower velocities. However, for relatively higher velocities (40 m/s) the wheel can ‘fly’ over the crossing. Furthermore, on the rigid track the wheel jumps are more obvious than on the flexible track due to the hardness of the rail, especially for the 10 and 40 m/s cases. However, the difference in wheel trajectory becomes much less for a velocity of 20 m/s. This can be explained by noting that for this crossing the typical velocity, which yields the optimum contact trajectory and largely reduces the contact forces, is around 80 km/h (about 22 m/s). The farther away the train velocity is from the typical velocity, the larger the gap between the contact trajectory and the optimum one and the higher are the dynamic wheel/rail forces. In light of this point, the velocity of 20 m/s can be regarded as the best travelling velocity among the simulations which results in better wheel/rail contact and results in a smaller difference between the values for rigid and flexible tracks.

Moreover, the vertical wheel trajectory of the VI-Rail model at the speed of 20 m/s and the vertical rail geometry of the 1:12 turnouts in the section ‘Comparison of measured and simulated results’ (see Figure A-8 and Figure A-9) have similar shapes despite the effect of the rising wing rail in the VI-Rail turnout model, which means the simplified method in defining the rail geometry in the 2D model is reliable.

The dynamic wheel forces of the two models were compared, focusing on the front wheel of the first wheelset of the wagon. The vertical dynamic wheel force calculated in the 2D model is shown in Figure A-18. From this figure it can be observed that the dynamic wheel force is always higher when adopting the wheel trajectory from the VI-Rail rigid track model as the vertical rail geometry in the 2D model rather than from the VI-Rail flexible model.
However, only a small difference was found for the speed of 20 m/s. This is mainly ascribed to the wheel trajectory as shown in Figure A-17.

Figure A-19 presents the vertical contact force along the crossing in the VI-Rail model with flexible track. Table A-5 lists the results in terms of maximum wheel force (P1) in the 2D model and the maximum vertical creep force in VI-Rail model. It is clear from this table that the results of the 2D model are close to the results of the 3D model with the flexible track, no matter which trajectory from the VI-Rail model (with rigid or flexible track) is used in the 2D model.

Figure A-18 Dynamic vertical wheel force calculated in the 2D model using wheel trajectory from VI-Rail model: (a) with rigid track, (b) with flexible track.
Figure A-19 Dynamic wheel force calculated in the VI-Rail model.

It should be noted that in the 2D model the turnout is always flexible; the rigid and flexible tracks used in the DARTS_NL means using the vertical wheel trajectory from the train/turnout interactions with rigid and flexible tracks in the VI-Rail as the rail geometry, respectively.

Table A-5 Comparison of the dynamic wheel force between the two models.

<table>
<thead>
<tr>
<th>Dynamic wheel force P1 (kN)</th>
<th>V=10 m/s</th>
<th>V=20 m/s</th>
<th>V=40 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VI-rail</td>
<td>DARTS_NL</td>
<td>VI-rail</td>
</tr>
<tr>
<td>Rigid track</td>
<td>622</td>
<td>215</td>
<td>485</td>
</tr>
<tr>
<td>Flexible track</td>
<td>187</td>
<td>168</td>
<td>182</td>
</tr>
</tbody>
</table>

Additionally, these results indicate that a higher velocity of the vehicle does not always result in a higher dynamic wheel force. In both models the dynamic wheel force is much higher when the train velocity is relatively low (10 m/s) whereas the lowest wheel forces are found when the velocity is 20 m/s (except for the flexible track case in the VI-Rail model). This is because the varying rail geometry plays a crucial role in the dynamic train/turnout interaction. As shown in Figure A-17 the wheel/rail contact occurs earlier under lower velocities when the vertical gap is larger, which in turn results in higher dynamic wheel forces.

It should also be noted that the use of the 2D model can save a considerable amount of computational effort as compared with the complicated 3D numerical models. For instance, for the simulation of a train passing the turnout over a 2 s period at a speed of 20 m/s the computational cost of the 2D model is 1-2 min whereas 2-3 h is required for the VI-Rail case when the detailed flexible track model is used. Therefore, the 2D model (possibly in combination with the VI-Rail model with the simplified rigid track) can be used in the numerical optimisation of the train/turnout interaction.
5 Conclusion and remarks

The 2D finite element model DARTS_NL that is used for simulation of train/turnout interactions has been validated through the field measurements and the 3D MBS model VI-Rail.

Three instrumented turnouts from the Dutch railway network varying in crossing angle, service state and load condition have been used as the validation cases. The method of modelling the rail geometry based on the visual image of the turnout introduced in Markine et al.[1] has been further developed. The use of measured rail properties combined with visual images of the crossing has been proposed in order to obtain a simplified representation of the rail geometry. The simulation results are in good agreement with measured data. The main behaviour of the turnouts has been correctly captured; in particular the shape and extreme amplitudes of the vertical acceleration of the crossing nose are close to the measured values. The simulation results have demonstrated that the effect of the vertical track geometry on the dynamic response of the turnout can be effectively accounted for in the 2D model.

Furthermore, the train/track interaction on a standard right-hand turnout has been simulated both in DARTS_NL and VI-Rail, and the vertical wheel trajectory from the 3D model is used as the rail geometry of the 2D model. A comparison of the dynamic wheel forces has been performed for various travelling velocities, which shows that the dynamic wheel forces (P1) from the 2D model are close to VI-Rail model with flexible track. Thus the 2D model (which is very fast) can effectively be used instead of complex (time-consuming) 3D models.

From these results it can be concluded that the DARTS_NL model is able to simulate train/turnout interactions with a high accuracy. The next step will be to include optimisation of the turnout using the proposed model.

References


Optimisation of the elastic track properties of turnout crossings

Chang Wan, Valeri Markine, Ivan Shevtsov


Abstract

Rail pads and under sleeper pads (USPs) are resilient elements inserted between the rail and the sleeper, and between the sleeper and the ballast, respectively. They improve the elastic properties of the track’s superstructure. In this paper, the approach of estimating the performance of a turnout by using the dynamic forces acting on the crossing as indicators of the extent of crossing nose damage is improved by tuning the stiffness and damping of the rail pads and USPs using a numerical optimisation method. In the optimisation problem, the dynamic forces acting on rails, sleepers and the ballast bed, which should be minimised, are considered in the objective function. Constraints are imposed on the displacements of the structural elements of the turnout crossing. The combined multi-objective optimisation problem is solved using the multipoint approximation method. The results of the optimisation show that application of softer rail pads combined with USPs can significantly reduce the dynamic forces acting on the rails, sleepers and ballast. Moreover, the track elasticity should be varied along the crossing.
1 Introduction

Railway turnouts are an important element of a railway’s infrastructure; they enable trains to be guided from one track to another at a railway junction as shown in Figure B-1.

In this figure it can be clearly seen that at the location of the crossing nose the vertical level of the inner rail is discontinuous. Due to this discontinuity, the crossing (and especially the crossing nose) experiences high impact loads generated by the wheels of passing trains. These forces create various types of damage to the railway turnout. Statistical evidence shows that failures in turnouts can cause major operational disturbances in a railway network.

The present paper only considers damage to the crossing nose. At the transition from the wing rail to the crossing nose the wheel/rail contact point experiences a jump on the wheel tread, which causes the high frequency impact force (P1). Since these dynamic forces are cyclic or repetitive, once all dislocations in the crystalline matrix of the material in the affected area reach the ratchetting regime, cracks are initiated and start to propagate. Once the cracks have been formed, the growth rate of the whole process increases. This reveals itself as rolling contact fatigue (RCF) damage of the crossing nose (Figure B-2). This type of rail damage has a significant impact on the life span of the turnout structure.
To reduce the negative effect of the dynamic wheel loads, elastic elements such as rail pads and under sleeper pads (USPs) have been introduced in turnout designs in order to reduce deterioration of the turnout structure.

The rail pads are inserted between the sleepers and the rails and provide electrical insulation between the rails and increase the elasticity of the superstructure. They reduce vibration and impact transmission from rail to sleeper by providing resilience and impact attenuation, which results in greater passenger comfort and less wear on the superstructure components and rolling stock. In addition they prevent rail/sleeper abrasion, and crushing of the sleeper under the rail foot. Fermér and Nielsen [1] performed full-scale experiments on a moving train. The influence of soft and stiff rail pads on the wheel/rail contact force, the sleeper-end acceleration and the railhead acceleration were studied. The soft rail pads were found to result in lower sleeper-end accelerations and higher railhead accelerations than the stiff rail pads. Egana et al.[2] found that the use of soft rail pads reduced corrugation growth and eliminated one of the wavelengths developed when using stiff pads.

A USP is a resilient pad that is attached to the bottom of the sleeper to provide an intermediate elastic layer between the sleepers and ballast. The installation of a USP leads to a decrease in the resonance frequency of the track and thus a reduction in the levels of vibration transmission from sleepers to the ballast bed and track substructure. As a result, the degradation of ballast material can be reduced, which is especially beneficial for places where access for tamping is difficult, such as a turnout crossing. A literature survey on the influence of USPs on the track response was presented in Bolmsvik [3]. The effect of a USP on the dynamic train/track interaction for a tangent track was investigated in Johansson et al. [4] where it was concluded that a USP could influence the stress level experienced by the ballast, although no direct relation between the use of a USP and the dynamic forces on the rails was found.

The discussed research provides a basic understanding of the mechanism of using rail pads and USPs in railway tracks. However, a relatively limited number of studies on the elastic properties of turnouts are available. Bruni et al.[5] found that a significant reduction of rail and especially of soil vibration levels can be achieved by increasing turnout resilience through the insertion of ballast mats and USPs. The influence of more resilient rail pads, USPs and ballast mats on the dynamic forces acting on the crossing nose and the supporting structure was investigated numerically in Markine et al.[6] wherein a set of combinations of rail pads and USPs with different elastic properties were analysed and the best performance of the turnout was achieved by using more resilient rail pads in combination with USPs.

This paper uses a numerical optimisation approach to extend the study Markine et al.[6] on the relation between the track’s elasticity and the dynamic performance of the turnout. An attempt has been made in this study to improve the turnout design by optimising the elastic properties of the track structure with the aim of reducing the impact-related damage on the
crossing nose. The numerical models, optimisation problem and results are presented and discussed in the following sections.

2 Numerical simulation of the vehicle-turnout interaction

The dynamic performance of a turnout is investigated here using a finite element model implemented in DARTS_NL software. The software has been successfully used for various railway applications such as optimisation of a slab track [7-8], identification of the dynamic properties of track components [9] and assessment of various high-speed track structures [10]. The turnout model developed in DARTS_NL has recently been validated through field measurements performed on the Dutch railway network [11]. The results of the validation showed that the calculated vertical accelerations of the nose rail due to the passing trains were in good agreement with the field measurements.

In order to reduce the required computational effort, the DARTS_NL model is restricted to two dimensions (the vertical and longitudinal directions) and to have linear material behaviour. A typical model of a railway vehicle on ballast track is shown in Figure B-3. The dynamic analysis is performed in the time domain following the concept of the displacement method [12-13]. It should be noted that DARTS_NL uses the symmetry of a track in the vertical plane and therefore only half of the track (containing one rail) is represented in the model.

![Figure B-3 Model of a railway vehicle on classical track](6).

The contact forces between wheel and rail are calculated using a nonlinear Hertzian spring with a stiffness that can be calculated using [14]:

$$K_n = \sqrt{\frac{6E^2P}{4(1-\nu^2)^2}} \sqrt{R_wR_r}$$

(1)

where \( P \) is the static wheel load; \( R_w \) and \( R_r \) are the radii of the wheel and rail profile (the lateral cross-section); \( E \) is the Young modulus of the wheel and rail material; \( \nu \) is the Poisson coefficient.
The motion of a wheel over the crossing is a complex phenomenon and depends on the geometrical properties of the wheel and rail (wing rail and crossing nose). In the proposed model, only the vertical level of the rail is considered. The irregularities in the vertical level of the rail can be defined as either a periodic function or as a numerical data profile obtained from measurements. In this paper the wheel/rail geometry in the crossing is simplified using the method suggested in Markine et al.[6] which was based on visual images of a crossing and is thus applicable for already installed and operational turnouts. Outside the crossing, no
rail irregularities are considered. The vertical rail geometry used in the numerical simulations consists of the global (crossing panel) and local (crossing nose) profiles as shown in Figure B-4(a). This rail geometry is in a good agreement with the measured wheel trajectory (passing in the main direction) for new and worn EH 60-500-1:12 crossings (different from the one used here) as shown in Figure B-4(b). More details on the turnout models and the DARTS_NL software can be found in Markine et al.[6].

In the present study a common single 54E1 turnout with a crossing angle of 1:9 is considered. The total length of the model is approximately 50m and the length of one finite element used for the rail is 0.1 m. Due to the simplicity of the model and the fact that a low speed is required for the divergent route, only the through route is considered here. The numerical model represents only the inner part (crossing side) of the turnout, where the cross-sectional shape of the rail varies along the crossing according to schematic drawings of the track (Figure B-5). Table B-1 lists the cross-sectional properties of the rail along the crossing used in the model. Outside the crossing, a UIC54 rail is used. Moreover, the vertical stiffness properties of the support structure of the rail are adjusted by varying the length of the sleepers, based on engineering drawings of the turnout.

Table B-1 Cross-sectional property of the rail.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional area (mm²)</td>
<td>69</td>
<td>69</td>
<td>140</td>
<td>130</td>
<td>277</td>
<td>332</td>
<td>383</td>
<td>255</td>
<td>237</td>
<td>69</td>
<td>69</td>
</tr>
<tr>
<td>Moment of inertia (mm⁴)</td>
<td>2346</td>
<td>2346</td>
<td>4677</td>
<td>4206</td>
<td>5919</td>
<td>7106</td>
<td>9227</td>
<td>8269</td>
<td>7461</td>
<td>2346</td>
<td>2346</td>
</tr>
</tbody>
</table>

Two models of the turnout were developed, namely:
- turnout on ballast bed (Figure B-6 (a)).
- turnout on ballast bed with USPs (Figure B-6(b)).

An ICE locomotive with a static wheel load of approximately 82 kN passing over the turnout in the main-facing direction at a speed of 140 km/h was considered in the simulations. In each simulation the integration time was 0.61 s and the integration step was 0.001 s.
As a reference design, a 1:9 turnout used in the Dutch railway network is considered. It should be noted that the rail pads used in the crossing in this design are relatively stiff and are primarily used for protection of the sleepers. The mechanical properties of the reference turnout are listed in Table B-2 [6].

<table>
<thead>
<tr>
<th>Track components</th>
<th>Type</th>
<th>Young’s modulus [N/m²]</th>
<th>Poisson coefficient</th>
<th>Stiffness [MN/m]</th>
<th>Damping [kNs/m]</th>
<th>Mass [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>UIC-54</td>
<td>2.10E+11</td>
<td>0.3</td>
<td>−</td>
<td>−</td>
<td>7850</td>
</tr>
<tr>
<td>Rail pad</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>3032 (EVA)</td>
<td>29</td>
<td>−</td>
</tr>
<tr>
<td>Sleeper</td>
<td>B70</td>
<td>3.85E+10</td>
<td>0.2</td>
<td>−</td>
<td>−</td>
<td>2400</td>
</tr>
<tr>
<td>Ballast</td>
<td>Quality ‘good’</td>
<td>−</td>
<td>−</td>
<td>120</td>
<td>48</td>
<td>−</td>
</tr>
</tbody>
</table>

### 3 Assessment criteria of the turnout performance

Each dynamic wheel load acting on the rail in the crossing can be characterised by its high- and low frequency components, i.e. the so-called P1 and P2 forces that are typical observed in the dynamic behaviour of a wheel in the presence of short-wave irregularities [16]. The P1 and P2 forces act in different frequency regions, as a result, their effect on the track structure depends on the frequency range of the track components.

The damage to the crossing nose usually originates from the high-frequency P1 forces caused by a contact point jump on the wheel tread at the transition from the wing rail to the crossing nose. The cyclic high frequency impact loads P1 cause highly localised plastic deformations and work-hardening in the rail until the material reaches the ratchetting regime that contains crack initiation and propagation resulting in RCF defects. The P2 forces, which act in a relatively lower frequency range, influence the supporting structures such as sleepers and ballast bed. Therefore, they are considered to be responsible for the degradation of the overall track geometry. As shown in Markine and Steenbergen [17], the overall switch geometry also influences the dynamic wheel forces. The degradation of the track geometry amplifies the deteriorated geometry of the crossing point, which in turn results in increasing the P1 forces and further speeding up the rail degradation process. Therefore, to reduce damage on the crossing nose, both the high-frequency and low-frequency forces on the track should be reduced.

The performance of each crossing design was assessed using the following criteria and responses of the train/track system [6].

1. Damage on the crossing nose estimated by the maximum dynamic wheel force (P1 force).
2. Damage on the sleepers estimated by the maximum force acting on the sleepers (Fₜₙ).
3. Damage on the ballast bed estimated by the maximum force acting on the ballast bed ($F_{bl}$).

It should be noted that the above-mentioned responses were analysed only for the crossing area where the changes in the turnout design have been applied (locations correspond to 11 sleepers in Figure B-5).

4 Optimisation problem

The improvement of the turnout performance is formulated here as an optimisation problem solved using a numerical optimisation method. The optimisation problem can be stated in a general form as

Minimise

$$F_0(x) \rightarrow \min, \quad x \in \mathbb{R}^n$$

subject to

$$F_j(x) \leq 1, \quad j = 1, \ldots, M$$

and

$$A_i \leq x_i \leq B_i, \quad i = 1, \ldots, N$$

where $x$ ($x = [x_1, \ldots, x_n]^T$) is the vector of design variables, which in a mechanical design problem can represent various parameters such as geometry, material, stiffness and damping properties. $A_i$ and $B_i$ are the side limits, which define the lower and upper bounds of the $i$-th design variable. $F_0$ is the objective function and $F_j$, $j = 1, \ldots, M$ are the constraint functions.

5 Objective function

According to the assessment criteria of the turnout performance formulated in the previous section, the dynamic forces acting on the rails, sleepers and the ballast should be minimised in order to obtain the optimum turnout. In order to use the formulation of the optimisation problem in the general form (2)-(4) the multi-objective (three objectives) optimisation problem is transformed into a single-objective problem using the weighted combined objective function

$$F_0(X) \equiv W_{P1} \frac{P1}{P1 + W_{F_{sl}} \frac{F_{sl}}{F_{bl}} + W_{F_{bl}} \frac{F_{bl}}{F_{sl}}} \rightarrow \min$$

(5)

Where $W_{P1}$, $W_{F_{sl}}$, and $W_{F_{bl}}$ are the weight coefficients ($W_{P1} + W_{F_{sl}} + W_{F_{bl}} = 1$). The P1 force has a major effect on the damage level on the crossing nose. In this study, reduction of that kind of
damage is of great importance. Therefore, a relatively high value was assigned to the weight coefficient \( W_{P1} \) (0.8). The reduction of the forces acting on the ballast, which can slow down degradation of the overall turnout crossing geometry, has a major effect on degradation of the overall turnout geometry; however, it only indirectly affect the damage on the crossing nose. Therefore the weight coefficient \( W_{f_b} = 0.15 \), which is smaller than \( W_{P1} \) was chosen. A weight coefficient of 0.05 was assigned to \( W_{f_s} \). It should be noted that different values of the weight coefficients could be chosen if a different order of importance of the objectives is desired.

The values of \( P1^* \), \( f_{bl}^* \) , and \( f_{sl}^* \) listed in Table B-3 are the reference values of the maximum forces acting on the rail, ballast and sleepers, respectively, they were obtained from the dynamic analysis with the reference turnout (Table B-2). They were used as the normalising factors in the definition of the objective function (5).

**Table B-3 Dynamic responses of the reference turnout.**

<table>
<thead>
<tr>
<th>Variant</th>
<th>( P1 ) [kN]</th>
<th>( f_{bl} ) [kN]</th>
<th>( f_{sl} ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>v00</td>
<td>257</td>
<td>104.55</td>
<td>100.55</td>
</tr>
</tbody>
</table>

### 5.1 Design variables

Markine et al.[6] showed that rail pads and USPs could significantly reduce the dynamic forces acting on a turnout. Ballast mats and light sleepers also have positive effects. The former provides vibration protection and slows down ballast degradation and the latter has a similar effect on the dynamic forces as does a USP. Markine et al.[6] also observed that ballast mats had almost no influence on the dynamic wheel forces acting on the turnout. Therefore, ballast mats are not considered in this paper. Additionally, the use of light sleepers in a turnout design is not a preferred option, due to the fact that it affects the stability of the whole track structure.

Therefore, the material properties (stiffness and damping) of the rail pads and USPs were chosen to vary the vertical elasticity of the turnout. Generally, there is a correlation between the damping and stiffness properties, such as in the commonly used classic Rayleigh damping model

\[
C = \mu M + \lambda K
\]  

where \( C \) is the damping matrix, \( M \) is the mass matrix and \( K \) is the stiffness matrix of the material. \( \mu \) and \( \lambda \) are the mass proportional coefficient and stiffness proportional coefficient, respectively, which depend on the physical/chemical composition of the material. The current study aims to numerically determine the mechanical properties of the elastic pads, the results of these simulations can subsequently aid the selection of the corresponding material.
Therefore, in the current optimisation studies the damping and stiffness properties are considered as independent design variables (Table B-4).

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Units</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>stiffness of rail pads ($X_1$)</td>
<td>(MN/m)</td>
<td>20</td>
<td>3032</td>
</tr>
<tr>
<td>damping of rail pads ($X_2$)</td>
<td>(kNs/m)</td>
<td>29</td>
<td>500</td>
</tr>
<tr>
<td>stiffness of USP ($X_3$)</td>
<td>(MN/m$^3$)</td>
<td>30</td>
<td>300</td>
</tr>
<tr>
<td>damping of USP ($X_4$)</td>
<td>(kNs/m$^3$)</td>
<td>10</td>
<td>300</td>
</tr>
</tbody>
</table>

It should be noted that the variation of the elastic pads is considered only under the crossing (Figure B-5). The parameters of the numerical models other than the design variables are the same as the ones in the reference turnout (Table B-2).

### 5.2 Constraints

In order to satisfy the requirements of safety and stability of the train/track system constraints are imposed on the following responses.

1. The maximum deflection of the rail ($U_r$), defined by the node displacement of the rail element due to the passage of wheels.
2. The maximum displacements of sleepers ($U_{sl}$), defined by the node displacement of the sleeper element.
3. The maximum compressive deflection of rail pads ($D_p$), defined by the difference in the deformation between the rail and the sleeper.

Moreover, two additional constraints are included in the problem:

1. the maximum bending moments of rail ($M_r$).
2. the maximum shear forces of rail ($Sh_r$).

On one hand, application of these additional constraints can avoid excessive metal fatigue effects in the rails. On the other hand, since the optimum solution of a constrained optimisation problem usually belongs to the boundary of the feasible design space, adding additional constraints reduces the search space and usually improves convergence of the optimisation.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response</td>
<td>$U_r$ (mm)</td>
<td>$U_{sl}$ (mm)</td>
<td>$D_p$ (mm)</td>
<td>$M_r$ (MN·m)</td>
<td>$Sh_r$ (Kn)</td>
</tr>
<tr>
<td>Critical value ($^*$)</td>
<td>2</td>
<td>1.5</td>
<td>1.5</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Constraint function</td>
<td>$U_r &lt; U_r^*$</td>
<td>$U_{sl} &lt; U_{sl}^*$</td>
<td>$D_p &lt; D_p^*$</td>
<td>$M_r &lt; M_r^*$</td>
<td>$Sh_r &lt; Sh_r^*$</td>
</tr>
</tbody>
</table>
Table B-5 lists all the constraints in the optimisation problems. It should be noted that the critical values of the responses are obtained based on the structural strength of the track components using certain safety factors (i.e. the actual limits of these quantities should not be exceeded). The response quantities used in the constraint functions are calculated only for the crossing part of the turnout (Figure B-5).

5.3 Optimisation method

The problem (2)-(4) was solved using the multipoint approximation method (MAM) that was specifically developed for problems where time-consuming simulations and numerically noisy functions are involved [18-19].

Instead of the original problem (2)-(4) a succession of simpler approximated sub-problems similar to the original one, which is formulated using the approximation functions, is to be solved. Then the original problem is replaced with the approximated one that reads as

Minimise

$$\tilde{F}_0^k(\mathbf{x}) \to \min, \quad \mathbf{x} \in \mathbb{R}^N$$

subject to

$$\tilde{F}_j^k(\mathbf{x}) \leq 1, \quad j = 1, \ldots, M$$

And

$$A_i^k \leq x_i \leq B_i^k, \quad A_i^k \geq A_i, \quad B_i^k \geq B_i, \quad i = 1, \ldots, N$$

where the superscript $k$ is the number of the iteration step, $\tilde{F}$ is the approximation of the original function $F$, $A_i^k$ and $B_i^k$ are move limits that define the range of applicability of the approximations.

Since the functions (7) and (8) are chosen to be simple and computationally inexpensive, any conventional optimisation method can be used to solve the problem (7)-(9). The solution of the problem at the $k$-th step $\mathbf{x}_k$ is chosen as starting point for the $(k+1)$-th step, then the optimisation problem (7)-(9) reformulated with the new approximation functions $\tilde{F}_j^{k+1}(\mathbf{x}) \leq 1, \quad (j = 1, \ldots, M)$ and move limits $A_i^{k+1}$ and $B_i^{k+1}$ is to be solved. The process is repeated until the convergence criteria are satisfied.

In the MAM method, each approximation is defined as a function of the design variables $\mathbf{x}$ and tuning parameters $\mathbf{a}$ (for brevity the indices $k$ and $j$ will be omitted). To determine the components of the vector $\mathbf{a}$ the following weighted least-squares minimisation problem is to be solved.

Find vector $\mathbf{a}$ that minimises:
where: $F(x_p)$ is the value of the original function evaluated at the point $x_p$, and $\rho$ is the total number of such points; $w^{(0)}_\rho$ is a weight factor that characterises the relative contribution of the information about the original function at the point $x_p$. The approximation function, which has to depend on the same design variables as the original function, should be simple enough to be used in numerous repeated calculations and should not contain any significant level of numerical noise in order to avoid convergence problems in the optimisation process. In this study, the approximation functions are defined in a multiplicative form

$$\tilde{F}(x) = a_b \prod_{i=1}^{N} (x_i)^{a_i}$$

The whole optimisation process is controlled by the change of move limits. In each iteration step the approximation functions, the original functions and the current solution of design variables are checked, and the change of move limits in each step are determined as follows.

1. The approximations are not sufficiently adequate to represent the original ones in the current optimum point: this indicates that the search sub-region is larger than the range of applicability of the current approximations. The move limits (9) are changed to reduce the size of the search sub-region.

2. The approximations are good and the optimum point is an internal point of the search sub-region: this is considered as an approximation of the solution of the original optimisation problem (2)-(4). In this case the search sub-region is reduced.

3. The approximations are good and the current optimum point belongs to the boundary of the search sub-region (at least one of the move limits is active): the size of the sub-region is not changed in the next iteration.

4. The approximations are good and the current optimum point is not internal. Moreover, no oscillation is in the search sub-region: if the search points move in the same direction, this means the search sub-region is too small. In this case the size of the sub-region is enlarged; if the search points do not move in the same direction the size of the sub-region is kept the same.

5. The approximations are good and the current optimum point is not internal. Moreover, oscillations are in the search sub-region. In this case the size of the sub-region is reduced.

The iteration process is terminated if the approximations are good, none of the move limits is active and the search sub-region is small enough. More details on the MAM method can be found in the literature [18-19].
The MAM performs well with problems in the presence of numerical noise. The use of the approximations prevents it from converging to one of the local optima in such problems. Also, the use of the approximations in the MAM usually results in reduction of the total computational effort of the optimisation as compared with the conventional methods of non-linear optimisation. Different from the global optimisation methods, in order to ensure a global optimum the optimisation should be solved with different initial points (valid for majority of the structural optimisation methods).

6 Numerical results

In order to determine if the extra elasticity should be applied, two types of turnout designs have been considered:

- turnout with rail pads (Figure B-6(a)).
- turnout with rail pads and USPs (Figure B-6(b)).

6.1 Optimisation with homogeneous track elasticity along the crossing

One of the research aims in this paper is to obtain a better understanding of the relation between the dynamic performance of the turnout and application of the additional elastic elements. Therefore, a number of optimisation problems were formulated as listed in Table B-6.

<table>
<thead>
<tr>
<th>Var.</th>
<th>USP</th>
<th>Design variables</th>
<th>Weight coefficient in objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$X_1$ $X_2$ $X_3$ $X_4$</td>
<td>$W_{p}$ $W_{fx}$ $W_{fz}$</td>
</tr>
<tr>
<td>v01</td>
<td>-</td>
<td>√ √ - -</td>
<td>0.80 0.15 0.05</td>
</tr>
<tr>
<td>v02</td>
<td>-</td>
<td>√ √ - -</td>
<td>1.00 0.00 0.00</td>
</tr>
<tr>
<td>v03</td>
<td>√</td>
<td>√ √ √ -</td>
<td>0.80 0.15 0.05</td>
</tr>
<tr>
<td>v04</td>
<td>√</td>
<td>√ √ √ -</td>
<td>1.00 0.00 0.00</td>
</tr>
<tr>
<td>v05</td>
<td>√</td>
<td>√ √ √ √</td>
<td>0.80 0.15 0.05</td>
</tr>
<tr>
<td>v06</td>
<td>√</td>
<td>√ √ √ √</td>
<td>1.00 0.00 0.00</td>
</tr>
</tbody>
</table>

The optimisation problems v01 and v02 consider variation of the rail pads. In the problems v03 to v06 the properties of both rail pads and USPs are varied, whereas in the problems v03 and v04 the damping of the USPs remains constant (48 kNs/ m$^3$).

The numerical results of the optimisations are collected in Table B-7. These results consist of the values of the optimised elastic properties of the rail pads and/or USPs, the values of the objective function and the normalised constraints. Table B-7 shows that the optimised stiffness and damping properties of the rail pads reach their lower bounds, which indicates that the rail...
pads should be softer in order to reduce the damage on the crossing nose. Deformations of rail/rail pads and the bending moment in the rail increase as the rail pads become softer. It can be observed from the results that in most of the optimisations the constraints related to the bending moments in the rails, the rail displacements and the deformations of the rail pads become active (Table B-7), which means that these responses are decisive in determining the optimum elastic properties of a crossing.

Table B-7 Results of optimisations: design variables and objectives.

<table>
<thead>
<tr>
<th>Var.</th>
<th>K_{pad} (MN/m)</th>
<th>C_{pad} (kNS/m)</th>
<th>K_{usp} (MN/m³)</th>
<th>C_{usp} (kNS/m³)</th>
<th>U_r/U_r^*</th>
<th>U_{sl}/U_{sl}^*</th>
<th>D_p/D_p^*</th>
<th>M_r/M_r^*</th>
<th>Sh_r/Sh_r^*</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>v01</td>
<td>20</td>
<td>29</td>
<td>-</td>
<td>-</td>
<td>0.37</td>
<td>0.49</td>
<td>0.02</td>
<td>0.78</td>
<td>0.74</td>
<td>0.7068</td>
</tr>
<tr>
<td>v02</td>
<td>20</td>
<td>29</td>
<td>-</td>
<td>-</td>
<td>0.88</td>
<td>0.28</td>
<td>0.93</td>
<td>0.93</td>
<td>0.70</td>
<td>0.7554</td>
</tr>
<tr>
<td>v03</td>
<td>20</td>
<td>29</td>
<td>99</td>
<td>48</td>
<td>1.00</td>
<td>0.51</td>
<td>0.92</td>
<td>0.95</td>
<td>0.71</td>
<td>0.6992</td>
</tr>
<tr>
<td>v04</td>
<td>20</td>
<td>29</td>
<td>300</td>
<td>48</td>
<td>0.92</td>
<td>0.37</td>
<td>0.93</td>
<td>0.93</td>
<td>0.70</td>
<td>0.7594</td>
</tr>
<tr>
<td>v05</td>
<td>21</td>
<td>29</td>
<td>83</td>
<td>300</td>
<td>1.00</td>
<td>0.52</td>
<td>0.89</td>
<td>0.95</td>
<td>0.70</td>
<td>0.6889</td>
</tr>
<tr>
<td>v06</td>
<td>20</td>
<td>29</td>
<td>133</td>
<td>300</td>
<td>0.97</td>
<td>0.43</td>
<td>0.92</td>
<td>0.95</td>
<td>0.70</td>
<td>0.7591</td>
</tr>
</tbody>
</table>

In the optimisations v01 and v02, in which only the elastic properties of the rail pads are varied, the same results are obtained despite the different sets of the weight coefficients. This can be explained by the fact that rail pads have a significant influence on the high-frequency dynamic forces (such as the P1 force) but only a very limited influence on the low-frequency forces (such as F_{bl} and F_{sl}). The dynamic responses of the optimised crossing are shown in Table B-8, from which it can be observed that the minimum dynamic force P1 (194 kN) is achieved by solely applying the soft rail pads (v01 and v02). The forces on the ballast and the sleeper are also significantly reduced; however, they are higher than in the other variants (v03 to v06).

Table B-8 Results of optimisations: dynamic performance of the turnout.

<table>
<thead>
<tr>
<th>Var.</th>
<th>P1 (kN)</th>
<th>F_{bl} (kN)</th>
<th>F_{sl} (kN)</th>
<th>U_r (mm)</th>
<th>U_{sl} (mm)</th>
<th>D_p (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>v00</td>
<td>257</td>
<td>104.55</td>
<td>100.55</td>
<td>0.73</td>
<td>0.73</td>
<td>0.03</td>
</tr>
<tr>
<td>v01/v02</td>
<td>194</td>
<td>61.36</td>
<td>29.06</td>
<td>1.76</td>
<td>0.42</td>
<td>1.39</td>
</tr>
<tr>
<td>v03</td>
<td>196</td>
<td>52.08</td>
<td>28.88</td>
<td>2.00</td>
<td>0.77</td>
<td>1.38</td>
</tr>
<tr>
<td>v04</td>
<td>195</td>
<td>57.97</td>
<td>29.07</td>
<td>1.84</td>
<td>0.55</td>
<td>1.39</td>
</tr>
<tr>
<td>v05</td>
<td>195</td>
<td>46.23</td>
<td>28.75</td>
<td>2.00</td>
<td>0.78</td>
<td>1.33</td>
</tr>
<tr>
<td>v06</td>
<td>195</td>
<td>49.56</td>
<td>28.77</td>
<td>1.93</td>
<td>0.65</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Comparing crossings with and without USPs (v03 to v06 versus v01 and v02), it can be observed that the application of the USPs leads to a significant reduction in the forces acting on the sleepers and ballast, while slightly increasing the P1 force. Additionally, results from v03 and v04 show that the optimum stiffness of the USPs in v03 differs from the one in v04 (99 versus 300MN/m³). Similar results are obtained in the optimisations v05 and v06 (83
versus 133MN/m³), in which the damping of the USPs was included as a design variable. From these results it can be concluded that a stiffer USP has a positive effect on the reduction of the P1 force (v04 and v06) whereas a softer USP results in a larger reduction of the forces on the ballast as well as the forces on the sleepers (v03 and v05). These results are fully in agreement with the conclusions of the study of Markine et al. [6] who showed that the use of a softer USP results in lower forces on the ballast bed and sleeper (F₁₁ and F₁₂) but slightly increases the forces between the wheel and rail (P1 force) as compared with a harder USP.

Moreover, from the results of the optimisations v03 to v06, it can be seen that a USP with higher levels of damping can further improve the performance of the crossing (variants v03/v04 versus v05/v06). That is, for a compromise improvement in both the high-frequency forces and the low-frequency forces, a USP with a lower stiffness and higher damping is preferable.

6.2 Optimisation with various track elasticity values along the crossing

A specific optimisation problem is considered in this section in order to further investigate the effect of the vertical elasticity of the track along the crossing on the dynamic forces of the track layers. To perform this investigation the crossing is divided into 11 parts in accordance with the 11 sleepers (Figure B-5). The stiffness and damping of each rail pad and USP are considered to be independent design variables, i.e. a total of 44 design variables are used in the optimisation. Only the multi-objective problem is investigated (W₁ = 0.8, W₂ = 0.15, W₃ = 0.05). Numerical results of the optimisation problem are presented in Table B-9 to Table B-11.

Table B-9 Results of optimisation v100: design variables.

<table>
<thead>
<tr>
<th>Elastic element</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kpad (MN/m³)</td>
<td>630</td>
<td>1244</td>
<td>153</td>
<td>27</td>
<td>20</td>
<td>20</td>
<td>681</td>
<td>20</td>
<td>82</td>
<td>183</td>
<td>315</td>
</tr>
<tr>
<td>Cpad (kNs/m)</td>
<td>129</td>
<td>184</td>
<td>29</td>
<td>29</td>
<td>29</td>
<td>29</td>
<td>152</td>
<td>31</td>
<td>41</td>
<td>169</td>
<td>134</td>
</tr>
<tr>
<td>Kusp (MN/m³³)</td>
<td>191</td>
<td>121</td>
<td>97</td>
<td>300</td>
<td>251</td>
<td>46</td>
<td>31</td>
<td>30</td>
<td>37</td>
<td>30</td>
<td>64</td>
</tr>
<tr>
<td>Cusp (kNs/m³³)</td>
<td>109</td>
<td>117</td>
<td>300</td>
<td>216</td>
<td>275</td>
<td>268</td>
<td>191</td>
<td>10</td>
<td>14</td>
<td>192</td>
<td>205</td>
</tr>
</tbody>
</table>

Note: Elastic elements were numbered at each sleeper from the beginning to the end of the crossing as shown in Figure B-5.

Table B-10 Results of optimisation v100: objective and constraint functions.

<table>
<thead>
<tr>
<th>Var.</th>
<th>Normalised Constraints</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>v100</td>
<td>U₁₀/U₁₀*</td>
<td>U₁₁/U₁₁*</td>
</tr>
</tbody>
</table>

| Objective function | 0.6945 |
The results in Table B-9 reveal that the rail pads under the crossing nose (pads 4 to 6) are very soft, with both the lowest stiffness and lowest damping values, whereas those outside the crossing nose are relatively stiffer especially the pads at the beginning of the crossing (pads 1 and 2). It is interesting that pad 8 is as soft as pad 6 whereas pad 7 is much stiffer than these pads that are beside it. This could be due to the different cross-sectional properties of the rail (Table B-1), with the rail having its highest cross-sectional area and moment of inertia at pad 7. In terms of the USPs, those under the crossing nose are much stiffer (except USP 6). The USPs at the front part of the crossing are softer than those under the nose, but much stiffer than the ones at the rear part of the crossing. Moreover, the damping of the USPs is mostly higher under the crossing nose.

Table B-11 Results of optimisation v100: dynamic performance.

<table>
<thead>
<tr>
<th>Var.</th>
<th>P1 (kN)</th>
<th>Fbl (kN)</th>
<th>Fsl (kN)</th>
<th>Ur (mm)</th>
<th>Usl (mm)</th>
<th>Dp (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>v100</td>
<td>193</td>
<td>52.51</td>
<td>33.96</td>
<td>1.79</td>
<td>1.43</td>
<td>1.32</td>
</tr>
</tbody>
</table>

It can be concluded from these results that in improved designs the rail pads are softer with a lower damping under the crossing nose as compared with the rest of the crossing. The USPs under the crossing nose, on the other hand, is stiffer and has higher levels of damping. The relation between the elastic properties of the USPs and their locations is, however, not clear enough from these results.

6.3 Optimisation for crossing zones

From the results for the problem v100, it can be seen that the optimum track elasticity of the turnout depends on the location of the elastic elements along the crossing. From a practical point of view, however, installation of different rail pads and USPs at each sleeper is undesirable. The results from v100 indicate that the crossing could be approximately divided into three or two parts, i.e. under the crossing nose (no. 4 to 6 in v100), before and after the crossing nose:

- middle (M): the most sensitive area where the wheel passes from the wing rail to the nose rail, i.e. from the sleeper before the nose point to two sleepers afterwards, numbers 4 – 6 in Figure B-5;
- front (F): numbers 1-3 in Figure B-5.
- rear (R): numbers 7-11 in Figure B-5.

In the following optimisations the crossing is divided either in three parts (middle, front and rear) or in two parts (middle and side). In the optimisation results with homogeneous track elasticity (v01 to v06), the effect of rail pads on the turnout performance is more clear than
the effect of the USPs. In order to better understand the effect of the rail pads and USPs, two additional optimisations are considered in this section (Table B-12).

- variable rail pads combined with variable USPs along the crossing (v07 to v10).
- variable rail pads combined with the same USPs along the crossing (v11 to v14).

The results of the optimisation problems are collected in Table B-13 to Table B-15.

Table B-12 Optimisation problems for crossing with 3x2 parts.

<table>
<thead>
<tr>
<th>Var.</th>
<th>Design variables</th>
<th>Weight coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_{pad}$ (MN/m)</td>
<td>$C_{pad}$ (kN/m)</td>
</tr>
<tr>
<td>v07</td>
<td>$X_1$ $X_2$ $X_3$</td>
<td>$X_4$ $X_5$ $X_6$</td>
</tr>
<tr>
<td>v08</td>
<td>$X_1$ $X_2$ $X_3$</td>
<td>$X_4$ $X_5$ $X_6$</td>
</tr>
<tr>
<td>v09</td>
<td>$X_1$ $X_2$</td>
<td>$X_3$ $X_4$</td>
</tr>
<tr>
<td>v10</td>
<td>$X_1$ $X_2$</td>
<td>$X_3$ $X_4$</td>
</tr>
<tr>
<td>v11</td>
<td>$X_1$ $X_2$ $X_3$</td>
<td>$X_4$ $X_5$ $X_6$</td>
</tr>
<tr>
<td>v12</td>
<td>$X_1$ $X_2$ $X_3$</td>
<td>$X_4$ $X_5$ $X_6$</td>
</tr>
<tr>
<td>v13</td>
<td>$X_1$ $X_2$</td>
<td>$X_3$ $X_4$</td>
</tr>
<tr>
<td>v14</td>
<td>$X_1$ $X_2$</td>
<td>$X_3$ $X_4$</td>
</tr>
</tbody>
</table>

Note: $X_i$ means the $i$-th design variable of the problem; it is different from $X_i$ in Table B-4 which indicates the physical meaning of design variables (four types in total).

Table B-13 reveals that in the improved designs the properties of the rail pads and USPs vary along the crossing. The stiffness and damping of the rail pads under the crossing nose (the part M) reach their lower bounds. Outside the crossing nose the damping of the rail pads is higher (v07 to v14). Similar to the other optimisations, the constraints on the bending moment of the rail and deflection of rail and rail pads become active, as shown in Table B-14.

Table B-13 Results of optimisations: design variables.

<table>
<thead>
<tr>
<th>Var.</th>
<th>$K_{pad}$ (MN/m)</th>
<th>$C_{pad}$ (kN/m)</th>
<th>$K_{usp}$ (MN/m$^3$)</th>
<th>$C_{usp}$ (kN/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M F R</td>
<td>M F R</td>
<td>M F R</td>
<td>M F R</td>
</tr>
<tr>
<td>v07</td>
<td>20 20 20</td>
<td>29 225 158</td>
<td>140 272 200</td>
<td>298 147 286</td>
</tr>
<tr>
<td>v08</td>
<td>20 217 46</td>
<td>29 43 208</td>
<td>300 298 30</td>
<td>293 266 58</td>
</tr>
<tr>
<td>v09</td>
<td>20 20</td>
<td>29 93</td>
<td>92 100</td>
<td>293 236</td>
</tr>
<tr>
<td>v10</td>
<td>20 20</td>
<td>29 326</td>
<td>242 250</td>
<td>287 108</td>
</tr>
<tr>
<td>v11</td>
<td>20 29 20</td>
<td>29 117 134</td>
<td>76</td>
<td>200</td>
</tr>
<tr>
<td>v12</td>
<td>20 441 144</td>
<td>29 29 58</td>
<td>300</td>
<td>128</td>
</tr>
<tr>
<td>v13</td>
<td>20 20</td>
<td>29 122</td>
<td>85</td>
<td>300</td>
</tr>
<tr>
<td>v14</td>
<td>20 252</td>
<td>29 52</td>
<td>274</td>
<td>173</td>
</tr>
</tbody>
</table>

The results of the single-objective and multi-objective optimisations are of considerable interest. Table B-13 shows that the rail pads are stiffer outside the transition part (M part) when only the P1 force needs to be minimised (v08, v12 and v14), whereas little difference
in the stiffness of rail pads along the crossing can be found in the case of the multi-objective problems (v07, v09, v11 and v13). As for the USP properties, the stiffness of the USPs (Table B-13) under the crossing nose (M part) in the multi-objective optimisations is much lower and the P1 forces (Table B-15) are slightly higher than in the single-objective optimisations (v07 versus v08, v09 versus v10). In the optimisations with homogeneous USPs, stiffer USPs are achieved when only the P1 force is considered in the formulation of the objective function, although the low-frequency forces ($F_{bl}$ and $F_{sl}$) are higher (v11 versus v12, v13 versus v14). This is also in agreement with the results of Markine et al.,[6] who showed that a softer USP had a positive effect on reducing the dynamic forces on the ballast and sleepers while slightly increasing the impact forces (P1).

Table B-14 Results of optimisations: constraints and objective function.

<table>
<thead>
<tr>
<th>Var.</th>
<th>$U_r/U_r^*$</th>
<th>$U_s/U_s^*$</th>
<th>$D_{p}/D_{p}^*$</th>
<th>$M_r/M_r^*$</th>
<th>$S_{h}/S_{h}^*$</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>v07</td>
<td>0.94</td>
<td>0.42</td>
<td>0.91</td>
<td>0.95</td>
<td>0.69</td>
<td>0.6888</td>
</tr>
<tr>
<td>v08</td>
<td>0.97</td>
<td>0.93</td>
<td>0.87</td>
<td>0.95</td>
<td>0.71</td>
<td>0.7521</td>
</tr>
<tr>
<td>v09</td>
<td>0.99</td>
<td>0.49</td>
<td>0.91</td>
<td>0.95</td>
<td>0.70</td>
<td>0.6867</td>
</tr>
<tr>
<td>v10</td>
<td>0.92</td>
<td>0.37</td>
<td>0.92</td>
<td>0.93</td>
<td>0.68</td>
<td>0.7536</td>
</tr>
<tr>
<td>v11</td>
<td>1.00</td>
<td>0.53</td>
<td>0.89</td>
<td>0.98</td>
<td>0.71</td>
<td>0.6869</td>
</tr>
<tr>
<td>v12</td>
<td>0.77</td>
<td>0.46</td>
<td>0.79</td>
<td>0.98</td>
<td>0.72</td>
<td>0.7548</td>
</tr>
<tr>
<td>v13</td>
<td>1.00</td>
<td>0.5</td>
<td>0.91</td>
<td>0.95</td>
<td>0.70</td>
<td>0.6852</td>
</tr>
<tr>
<td>v14</td>
<td>0.77</td>
<td>0.45</td>
<td>0.79</td>
<td>0.98</td>
<td>0.72</td>
<td>0.7547</td>
</tr>
</tbody>
</table>

Table B-15 Dynamic performance of optimised crossings.

<table>
<thead>
<tr>
<th>Var.</th>
<th>P1 (kN)</th>
<th>$F_{bl}$ (kN)</th>
<th>$F_{sl}$ (kN)</th>
<th>$U_r$ (mm)</th>
<th>$U_s$ (mm)</th>
<th>$D_{p}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>v07</td>
<td>194</td>
<td>49.20</td>
<td>28.47</td>
<td>1.88</td>
<td>0.63</td>
<td>1.37</td>
</tr>
<tr>
<td>v08</td>
<td>193</td>
<td>67.88</td>
<td>28.36</td>
<td>1.93</td>
<td>1.40</td>
<td>1.30</td>
</tr>
<tr>
<td>v09</td>
<td>195</td>
<td>46.38</td>
<td>28.36</td>
<td>1.98</td>
<td>0.73</td>
<td>1.36</td>
</tr>
<tr>
<td>v10</td>
<td>194</td>
<td>54.54</td>
<td>28.86</td>
<td>1.83</td>
<td>0.55</td>
<td>1.38</td>
</tr>
<tr>
<td>v11</td>
<td>195</td>
<td>46.26</td>
<td>27.81</td>
<td>2.00</td>
<td>0.80</td>
<td>1.33</td>
</tr>
<tr>
<td>v12</td>
<td>194</td>
<td>71.19</td>
<td>43.33</td>
<td>1.54</td>
<td>0.69</td>
<td>1.18</td>
</tr>
<tr>
<td>v13</td>
<td>194</td>
<td>45.76</td>
<td>28.31</td>
<td>2.00</td>
<td>0.75</td>
<td>1.36</td>
</tr>
<tr>
<td>v14</td>
<td>194</td>
<td>68.60</td>
<td>41.67</td>
<td>1.53</td>
<td>0.68</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Additionally, the damping properties of the USPs obtained in the single- and multi-objective optimisation problems are different. From Table B-13 it can be observed that higher damping and lower stiffness values of the USPs are obtained in the multi-objective problems that account for the ballast and sleeper forces (v11 versus v12, v13 versus v14). In the problems v07 to v10 this is, however, valid only for the transition (middle) part. The above results indicate that, in terms of reduction of the dynamic forces acting on the crossing, the properties of the USPs at the transition part are more representative of the USP properties of the whole crossing.
It should also be noted that comparing the crossing with variable USPs and homogeneous USPs (v07 versus v11, v08 versus v12, v09 versus v13, v10 versus v14) the differences between the values of the components of the objective functions are very small (Table B-15), however, the difference in the optimised parameters in terms of the USP properties are much larger (Table B-13). This may be due to the fact that the impact force component has a much higher weight than the ballast-related component, while the USPs have almost no effect on the impact force (P1) compared with that of the rail pads.

6.4 Discussion

Based on the obtained results, it can be concluded that in order to improve the turnout performance the rail pads should be softer in the transition (middle) part of the crossing where the wheel transfers from the wing rail to the crossing nose and stiffer in the other parts. The optimisation where each elastic element is treated independently (v100) results in a slightly worse performance of the crossing. This could be due to the fact that a large number of design variables can influence the quality of the approximations, which inevitably reduces the reliability of the optimum solution.

By applying elastic elements with variable properties along the crossing, the P1 force can be slightly reduced as compared with the results with the homogeneous track elastic properties along the crossing (v01 to v06). Also, from the numerical results it can be observed that a better performance of the crossing is obtained in the multi-objective optimisations where the forces on the sleepers and ballast are taken into account.

The obtained damping and stiffness properties of the rail pads can be used to isolate a material with similar properties or in the development of a new material (if a material with properties close to the target ones does not exist).

Table B-16 Dynamic performance of optimised crossings.

<table>
<thead>
<tr>
<th>Var.</th>
<th>Rail Geometry</th>
<th>Velocity (km/h)</th>
<th>Axle load (t)</th>
<th>P1</th>
<th>Fbl (mm)</th>
<th>Fsl (mm)</th>
<th>Ur (mm)</th>
<th>Usl (mm)</th>
<th>Dp (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>v0</td>
<td>geo1</td>
<td>140</td>
<td>17</td>
<td>0.76</td>
<td>0.44</td>
<td>0.28</td>
<td>2.00</td>
<td>0.75</td>
<td>1.36</td>
</tr>
<tr>
<td>v1</td>
<td>geo1</td>
<td>110</td>
<td>17</td>
<td>0.74</td>
<td>0.45</td>
<td>0.27</td>
<td>1.92</td>
<td>0.73</td>
<td>1.33</td>
</tr>
<tr>
<td>v2</td>
<td>geo1</td>
<td>80</td>
<td>17</td>
<td>0.76</td>
<td>0.42</td>
<td>0.33</td>
<td>1.94</td>
<td>0.74</td>
<td>1.36</td>
</tr>
<tr>
<td>v3</td>
<td>geo1</td>
<td>80</td>
<td>25</td>
<td>0.79</td>
<td>0.58</td>
<td>0.45</td>
<td>3.48</td>
<td>1.25</td>
<td>2.26</td>
</tr>
<tr>
<td>v4</td>
<td>geo1</td>
<td>80</td>
<td>20</td>
<td>0.77</td>
<td>0.54</td>
<td>0.42</td>
<td>2.82</td>
<td>1.03</td>
<td>1.88</td>
</tr>
<tr>
<td>v5</td>
<td>geo2</td>
<td>140</td>
<td>17</td>
<td>0.88</td>
<td>0.57</td>
<td>0.41</td>
<td>1.91</td>
<td>0.69</td>
<td>1.21</td>
</tr>
<tr>
<td>v6</td>
<td>geo3</td>
<td>140</td>
<td>17</td>
<td>0.82</td>
<td>0.58</td>
<td>0.31</td>
<td>1.91</td>
<td>0.69</td>
<td>1.21</td>
</tr>
<tr>
<td>v7</td>
<td>geo3</td>
<td>80</td>
<td>25</td>
<td>0.68</td>
<td>0.71</td>
<td>0.67</td>
<td>3.45</td>
<td>1.21</td>
<td>2.23</td>
</tr>
</tbody>
</table>

Note: Axle load 17 t, velocity 140 km/h and geo1 were used in the optimisations; geo2 and geo3 were obtained based on geo1 by varying the length of the transition zone or magnitude of the vertical kink.
In this paper, the turnout design has been improved for one type of vehicle and one particular velocity of this vehicle; this allows the turnout optimisation methodology to be demonstrated in a simple and efficient manner. In case of the Netherlands railway network such an optimisation is sufficient since very few mixed traffic flows are encountered. Moreover, most of the turnouts are used by a certain type of train at a particular speed. For situations where the turnout is used under different operational conditions, such as mixed traffic (with higher axle loads) and various velocities, the reduction of the dynamic forces can still be achieved by applying the obtained elastic elements. However, the level of the force reduction in this case could be different and satisfaction of the constraints cannot be guaranteed.

Some responses of the crossing with the optimised track elasticity (v13) under different train velocities and axle loads are presented in Table B-16. The dynamic forces $P_1$, $F_{bb}$, and $F_{sl}$ in this table are normalised using the corresponding results from turnouts with the reference track elasticity and corresponding operational conditions. It can be seen from this table that the forces are reduced; however, some responses exceed their maximum values.

### 7 Conclusions

In the present study, the performance of a turnout is improved by optimising the vertical stiffness and damping properties of the supporting structures such as rail pads and USPs. The performance of the turnout is assessed in terms of the level of the dynamic forces acting on the rail (crossing nose), sleeper and ballast. The improvement of the turnout performance is formulated as single- (based on the crossing nose forces) and multi-objective optimisation problems. The mechanical properties of the elastic elements are chosen as the design variables. The optimisation problems are solved considering homogeneous as well as variable elastic properties along the crossing.

The results of the optimisations show that the dynamic forces in the turnout can be reduced by increasing the vertical elasticity of the track structure. The following conclusions can be drawn from the presented results.

1. A better performance of the crossing is obtained when the track elasticity is varied along the crossing. The softer rail pads and USPs should be applied under the crossing nose.
2. The softer rail pads can significantly reduce the dynamic forces acting on the crossing nose.
3. Application of stiffer USPs can slightly reduce the high-frequency force ($P_1$) but increase the low-frequency forces acting on the sleepers and the ballast bed.
4. Less-stiff USPs with higher damping can reduce the low-frequency forces caused by the low-frequency component of the wheel force.
5. The bending moments in the rails, rail displacements and deformation of rail pads are
decisive for the choice of the optimum properties.

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Improvement of vehicle-turnout interaction by optimising the shape of crossing nose

Chang Wan, Valeri Markine, Ivan Shevtsov

_Veh Syst Dyn. 2014; 52(11): 1517-1540._

Abstract

Proper rail geometry in the crossing part is essential for reducing damage on the nose rail. To improve the dynamic behaviour of turnout crossings, a numerical optimisation approach to minimise rolling contact fatigue (RCF) damage and wear in the crossing panel by varying the nose rail shape is presented in the paper. The rail geometry is parameterised by defining several control cross-sections along the crossing. The dynamic vehicle–turnout interaction as a function of crossing geometry is analysed using the VI-Rail package. In formulation of the optimisation problem a combined weighted objective function is used consisting of the normal contact pressure and the energy dissipation along the crossing responsible for RCF and wear, respectively. The multi-objective optimisation problem is solved by adapting the multipoint approximation method and a number of compromised solutions have been found for various sets of weight coefficients. Dynamic behaviour of the crossing has been significantly improved after optimisations. Comparing with the reference design, the heights of the nose rail are notably increased in the beginning of the crossing; the nominal thicknesses of the nose rail are also changed. All the optimum designs work well under different track conditions.
1 Introduction

Turnouts are important elements of railway infrastructure that provide flexibility of the system by enabling railway vehicles to be guided from one track to another at a railway junction (Figure C-1). Due to discontinuity in the rail geometry introduced in the crossing nose, turnouts experience high impact loads from passing vehicles, which makes them sensitive to various types of rail damage such as surface cracking and crumbling, shelling, excessive wear, plastic deformations, global fracture, etc. Damage of the crossing nose has become a serious problem of the Dutch railways: currently every week two crossings must be replaced urgently.

The problems on turnouts were the subject of the railway research worldwide recently. The analysis of the dynamic vehicle–turnout interaction can be found, for example, in [1-3], which consists of numerical simulations and validation of numerical models through field measurements. In [4], the effect of track elasticity has been studied, and it was observed that the dynamic forces can be significantly reduced by tuning the vertical track elasticity. Based on the validated model in [3], improvement of turnout performance was achieved by adjusting the vertical track elastic properties of the crossing using the numerical optimisation method [5].

Figure C-1 Crossing panel of turnout and damage of crossing nose.

![Crossing panel of turnout and damage of crossing nose.](image)

Figure C-2 Railway turnout: the switch panel and the crossing panel.

The goal of the presented research is to reduce damage of a crossing nose (Figure C-2), which usually originates from high impact loads, caused by the contact point jump on a wheel tread at the transition from the wing rail to the crossing nose. Such cyclic high-frequency impact loads result in local plastic deformation and work hardening in the rail, until the material reaches the ratcheting regime with crack initiation and propagation. This process manifests
itself as severe rolling contact fatigue (RCF) damage of the crossing nose. The abovementioned process, which is essentially influenced by service conditions (type of traffic, vehicle’s speed, traffic frequency and climatic conditions) and track conditions such as track elasticity [5] and track geometry [3], has significant impact on the life span of the turnout structure.

Recently a number of studies have been performed focusing on the effect of rail geometry on damage of turnout crossings due to dynamic impact. In [6] and [7], a series of measurements were performed in different turnouts in the Netherlands. The measurements consisted of the accelerations of the nose rail and the varying rail profiles (cross-sectional geometry) along the crossing. By doing this the measured performance of the turnout crossing can be then related to the corresponding rail geometry. It was observed from these measurements that the accelerations of the nose rail were very sensitive to the rail geometry. The effect of geometry at the switch panel (Figure C-2) has been studied in [8] and [9] which concentrate on the track gauge and the switch rail shape, respectively. In both methods, the vehicle switch performance has been improved by adjusting the corresponding geometry properties. This paper, on the other hand, focuses on improvement of the rail geometry at the crossing panel by using a numerical optimisation procedure, which follows the study presented in [10]. It is based on the numerical and experimental analysis of the dynamic behaviour of the train-turnout system.

Section 2 presents the experimental investigation of the effect of rail geometry at the crossings through the field tests on a turnout before and after grinding maintenance at the crossing. The modelling of vehicle–turnout interaction is discussed in Section 3. Prior to the optimisation, the rail geometry of the crossing has been parameterised as discussed in Section 4, wherein the effect of the rail geometry at the crossings is investigated by varying the nose rail shape. In Section 5, the optimisation problem is addressed, which aims to reduce the dynamic contact pressure and wear on the crossing by adjusting the nose rail shape. Results from the optimisations and further analysis of optimum designs are presented in Section 6.

2 Experimental investigation of rail geometry effect at the crossings

To investigate the effect of crossing geometry on turnout performance, experimental investigations were performed on a common single turnout (54E1, crossing angle 1:15) in the Dutch railway network, wherein the grinding maintenance was required due to observed damages on the crossing nose. During the maintenance the existed defects were removed, after which additional material was added (welded) and finally the crossing rail was grinded in order to restore the original shape. A set of measurements were implemented on the turnout before and after the grinding maintenance of the crossing. The measurements consist of two major parts: the dynamic behaviour of the crossing and the rail geometry of the turnout.
The dynamic interaction between vehicles and the turnout was analysed experimentally using the instrumented crossing (Figure C-3). The measured data consisted of the 3D accelerations of the crossing nose and locations of the maximum accelerations on the crossing nose caused by each passing wheel. The locations of the maximum accelerations, which presumably correspond to the locations of the impact of the wheel, were presented as the wheel contact distribution histogram (Figure C-3). Based on the histogram of the impact location the most probable area for fatigue damage (fatigue area) on the surface of the crossing nose can be determined. Details about the measurement process can be found in [6] and [7].
The rail geometry was obtained by measuring the rail profiles along the crossing. The interval of adjacent measured rail profiles was 50 mm from around 900 mm before the nose point to 1050 mm after the nose point (or farther). Additionally, the normal rail profiles outside the crossing part have been measured, which were used for numerical modelling of the turnout (see Section 3.2). An example of the changing rail profiles at the crossing is shown in Figure C-4.

The maximum acceleration histogram due to the wheel–rail impact on the nose rail before and after the grinding is shown in Figure C-5. It can be observed that the response of the turnout before the grinding maintenance was considerably different from that of after grinding the crossing rail. Based on such a histogram the most probable area for fatigue damage (fatigue area) on the crossing nose was determined, that is, 0.50–0.60 m before the grinding, while 0.40–0.60 m after the grinding. The wider fatigue area indicates the dynamic impacts were spread along the crossing nose instead of converging on a narrow area. This will ultimately increase the lifespan of the crossing nose. Moreover, the magnitudes of the dynamic impacts have been reduced significantly after grinding as shown in Figure C-6. Here the measurements were for the same type of passenger train with the same speed (138.5 km/h). It was observed that after grinding of the crossing the average acceleration of the nose rail due to the passing wheels was about half of that before the grinding.

The measured data confirmed the numerical results presented in [3] that the crossing nose geometry has significant influence on the dynamic response in the crossing area, which was also observed in [6] and [7].

3 Modelling of vehicle-turnout interaction

For a complex system like the turnout crossing that has changing geometry and changing structure, it is an ambitious task to develop realistic numerical models. The simulations should consider all the major influences such as the dynamic impact and the slip between the wheel and the crossing rail, while still being able to calculate the resulting impact forces and stresses.
on the crossing nose. Since the behaviour of a crossing is sensitive to both the track features and the vehicle characteristics, it is necessary to take into account the dynamic behaviour of both the track and the vehicle. Here the commercial software VI-Rail was used in modelling the dynamic vehicle-track interaction at the crossing. VI-Rail is a specialised environment for railway virtual prototyping based on the industry standard multibody dynamics code MSC Adams [11]. The local contact geometry (location of contact point on wheel and rail, contact angle, size of contact patch, etc.), contact forces, energy dissipation and wheel/wheelset displacements w.r.t. the rails, etc., which are as functions of the position along the turnout during the passage of each wheel, are outputs of the dynamic simulations.

3.1 The vehicle-turnout model

The studied turnout is a standard (right turn) design with curve radius 725 m and crossing angle 1:15. The turnout with the total length of 150 m has been modelled. In the crossing part, a number of changing rail profiles were used in order to represent the complex crossing geometry. The interval between adjacent rail profiles was 40 mm before the nose point. After the nose point smaller intervals were used, which were 15 mm from 0 to 300 mm away from the nose point and 25 mm from 300 to 1050 mm away from the nose point along the crossing. Schematic drawing of the changing rail profiles at the crossing part is shown in Figure C-7. Outside the crossing the nominal rail profile 54E1 without inclination was used. It should be noted that at the crossing area the allowable vertical deviation in the rail geometry of 8 mm over 10 m has been chosen at the crossing side of the track as shown in Figure C-8, where the relative location w.r.t. the nose rail was presented. Such deviation in the vertical rail geometry at the crossing part usually appears due to the fact that generally the turnout frogs are replaced without tamping maintenance of the track. At the opposite side of the crossing, the stock rail along with the check rail was modelled.

The turnout was simulated as the ‘moving track’ model. The elasticity of the track can be defined by adjusting the properties of rail pads and ballast, which are represented as the flexible connection of sleeper/rail and sleeper/ground by linear spring-damper elements in the vertical, lateral and roll directions (Figure C-9). In the ‘moving track’ model, the track elasticity is independent from the location along the crossing. The rails and sleepers are
modelled as rigid bodies moving together with the wheel load. Here relatively stiff rail pads and the ballast in good condition have been used as listed in Table C-1.

![Figure C-8 Vertical track irregularity at the crossing side.](image)

![Figure C-9 Schematic illustration of the track structure (ballasted).](image)

Table C-1 Track properties used in the simulations.

<table>
<thead>
<tr>
<th>Track components</th>
<th>Stiffness [MN/m]</th>
<th>Damping [kNs/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail pad</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>1420</td>
<td>34</td>
</tr>
<tr>
<td>Lateral</td>
<td>280</td>
<td>58</td>
</tr>
<tr>
<td>Roll</td>
<td>360</td>
<td>390</td>
</tr>
<tr>
<td>Ballast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>120</td>
<td>48</td>
</tr>
<tr>
<td>Lateral</td>
<td>120</td>
<td>40</td>
</tr>
<tr>
<td>Roll</td>
<td>130</td>
<td>290</td>
</tr>
</tbody>
</table>

A passenger wagon with double-deck coach and the static wheel load 54.5 kN (unloaded) was modelled based on the Manchester passenger train benchmark [12]. The primary suspension system of this vehicle model was slightly modified in order to correspond to the passenger train operating on the Dutch railway network. Parameters of the vehicle are listed in Table C-2.

To model the wheel–rail contact, four major contact elements are available in the VI-Rail package [11]: the General Contact Element (WRGEN), the Table Contact Element (WRTAB), the Linear Contact Element (WRLIN) and the Quasi-linearized Contact Element (WRQLT). The WRGEN element was used in the simulation due to the fact that WRGEN uses actual wheel/rail profiles to calculate the actual contact kinematics at each interaction step and among all the contact elements only WRGEN allows usage of various rail profiles along the track. WRGEN evaluates the contact line taking into account the effect of the wheel/rail angle of attack (pseudo-3D contact). Unlike WRQLT/WRLIN and WRTAB where the contact patches are
assumed to be circles and ellipse, respectively, in WRGEN the contact patches are calculated based on the undeformed penetration of the wheel/rail profiles and the wheel contact line. The shape of a contact patch is computed by segmenting the contact area into stripes, which is more realistic. Moreover, up to 10 contact patches can be considered for one wheel/rail interaction.

<table>
<thead>
<tr>
<th>Table C-2 Parameters of the vehicle model.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wheel</strong></td>
</tr>
<tr>
<td>Wheel Profile</td>
</tr>
<tr>
<td>Radius (m)</td>
</tr>
<tr>
<td>Mass (Kg)</td>
</tr>
<tr>
<td>$I_{xx}$ (Kg×m²)</td>
</tr>
<tr>
<td>$I_{yy}$ (Kg×m²)</td>
</tr>
<tr>
<td>$I_{zz}$ (Kg×m²)</td>
</tr>
<tr>
<td>Wheelbase (m)</td>
</tr>
<tr>
<td><strong>Wheelset</strong></td>
</tr>
<tr>
<td>Mass (Kg)</td>
</tr>
<tr>
<td>$I_{xx}$ (Kg×m²)</td>
</tr>
<tr>
<td>$I_{yy}$ (Kg×m²)</td>
</tr>
<tr>
<td>$I_{zz}$ (Kg×m²)</td>
</tr>
<tr>
<td><strong>Bogie</strong></td>
</tr>
<tr>
<td>Mass (Kg)</td>
</tr>
<tr>
<td>$I_{xx}$ (Kg×m²)</td>
</tr>
<tr>
<td>$I_{yy}$ (Kg×m²)</td>
</tr>
<tr>
<td>$I_{zz}$ (Kg×m²)</td>
</tr>
<tr>
<td><strong>Primary Suspension</strong></td>
</tr>
<tr>
<td>$K_x$ (kN/m)</td>
</tr>
<tr>
<td>$K_y$ (kN/m)</td>
</tr>
<tr>
<td>$K_z$ (kN/m)</td>
</tr>
<tr>
<td><strong>Secondary Suspension</strong></td>
</tr>
<tr>
<td>$K_x$ (kN/m)</td>
</tr>
<tr>
<td>$K_y$ (kN/m)</td>
</tr>
<tr>
<td>$K_z$ (kN/m)</td>
</tr>
</tbody>
</table>

The normal force in the contact area is computed using the Hertz theory for a given ellipse with undeformed penetration. While in the tangential directions the computation is based on the modification of Kalker’s FASTSIM algorithm developed at TU-Berlin [13].

In the present paper, the vehicle-turnout interaction was investigated only for the vehicle passing in the main-facing direction.

### 3.2 Validation of the turnout model

The VI-Rail ‘moving track’ model has been validated before using it in further study. Due to the limited availability of measurement results wherein only the accelerations of the nose rail were available and the fact that it is impossible to measure the acceleration of the rail in the ‘moving track’ model as a consequence of its simplicity, instead of directly validating the ‘moving track’ model through measurements, an indirect approach was proposed with the help of the more complex model in the VI-Rail package called *FlexTrack*.

The *FlexTrack* model has similar track structure as the ‘moving track’ model as shown in Figure C-9. However, this structure is not moving with the load but is independent from the
load situation. The rails were modelled as finite element beams integrated with MSC. Nastran, and the track elasticity can be defined depending on the location.

Firstly, the *FlexTrack* model in the VI-Rail package was validated by the measurements and then the ‘moving track’ model was validated through the *FlexTrack* model.

**Step 1: Validation of the *FlexTrack* model**

Here the turnout applied in the experimental study in Section 2 was modelled. The measurements obtained before the grinding maintenance were used in the comparison. In Figure C-10 the vertical acceleration of the nose rail due to passage of a single wheel has been compared between the simulations with *FlexTrack* model and the field measurement. The sampling frequencies of the simulation and the measurement were 5000 and 10,000 Hz, respectively. Here the acceleration was compared firstly in the frequency domain and then in the time domain. In the frequency domain analysis, the maximum acceleration and its frequency were obtained by low-pass filtering the results with different cut-off frequencies as shown in Figure C-10(a). It can be observed that the results from the *FlexTrack* model were in good agreement with the measurement below 600 Hz. Figure C-10(b) presents the comparison of the vertical accelerations in the time domain by low-pass filtering the results with cut-off frequency 500 Hz.

The results show that the calculated vertical acceleration of the nose rail was also in good agreement with the measurement in the time domain. It can be concluded that the *FlexTrack* model is validated by the measurements.

![Figure C-10](image)

**Step 2: Validation of the ‘moving track’ model through *FlexTrack* model**

Because the *FlexTrack* model has been validated, it was used as the reference instead of the measurements.
The same vehicle-turnout system from the validation through measurements was modelled also in the ‘moving track’ model. The simulation results from the two models were compared in the frequency range up to 5000 Hz. It was found that the results from the two models were close to each other. Figure C-11 shows the normal contact pressures and the normal contact forces low-pass filtered with cut-off frequency 500 Hz from the two models, wherein the green bar and the red bar at the bottom represent the wing rail part and the nose rail part, respectively. It shows that the normal contact pressure and the normal contact force on the nose rail were in good agreement with each other. This indicates that the ‘moving track’ model is also validated and it is qualified to simulate the vehicle dynamics in further studies.

In the further optimisation stage (Sections 5 and 6), the turnout was modelled as the ‘moving track’ model. The reason for using the simple track representation is that the linearized track model is sufficient for qualitative analysis of the dynamic impact, which could also be observed in Figure C-11. Moreover, the calculation time for the ‘moving track’ model is within 3 min, while more than 4 h are required for the FlexTrack model in the case of the abovementioned simulation. Reduction of computational cost is highly valued in an optimisation problem that requires a large number of simulations.

4 Parametric study of the rail geometry at crossings

For a common single turnout, the crossing panel consists of two wing rails and the crossing nose at the turnout crossing, at its left and right sides are the check rails and the stock rails, which form the through route and divergent route together with the crossing (Figure C-1). The crossing geometry is extremely complex compared with the normal track since both the nose rail and the wing rails have various profiles along the crossing. As the nose rail suffers from most of the damages, it is allowed to parameterise only the nose rail from the simplicity point of view. The vehicle speed was 136 km/h and the friction coefficient of 0.4 was employed in the parametric study.
4.1 Parameterisation of the crossing geometry

The parameterisation of the crossing geometry is based on the drawings provided by the manufacturers, wherein four control cross-sections (A-D) are located at certain distances along the crossing (Figure C-12). Cross-section A represents the nose point (0 mm), which is not involved in the wheel/rail contact. Cross-section D is located at $70 \times \alpha$ mm (1: $\alpha$ is the crossing angle) from the nose point, the cross-sectional shape is fixed. Therefore, two control cross-sections B and C, which are located at $10 \times \alpha$ and $20 \times \alpha$ mm away from the nose point, respectively, were chosen as variable parameters in defining the crossing geometry. For the variable cross-sections, the railhead part that is above the gauge corner is simplified as semi-ellipse. Using this approach, the cross-sections B and C can be adjusted by varying the semi-axes a (lateral semi-axis) and b (vertical semi-axis), see Figure C-12.

Since the present study focuses on the vehicle-turnout interaction at the main-facing direction, only the wing rail that is involved in the wheel-rail contact was considered in the crossing rail model. Therefore, the rail geometry in the crossing (1:15) was described by one wing rail and the crossing nose. Here the standard track gauge (1435 mm) was used as the nominal track gauge, i.e. the local track gauge was determined by the local irregularities of the track geometry and the shape of the rails. The standard wing rail from the manufacturer was used, where the shape and location of the wing rail were not changed during the parametric study. The simplified crossing rail model (start from the nose point) is shown in Figure C-13.
Besides the control cross-sections, the other rail profiles located along the crossing as described in Section 3.1 were together used as track geometry input. Each of these rail profiles was generated by longitudinally interpolating two adjacent control cross-sections around the targeted location as shown in Figure C-14. To avoid the small irregularities in the rail profiles introduced in the mathematical process, the control cross-sections as well as the interpolated rail profiles were smoothed using a B-spline tool before applying them to the turnout model. Based on the given rail profiles, the 3D surface of the crossing rail was generated in the VI-Rail package by using the third-order interpolating spline functions.

4.2 Parameter analysis

In this section, the effect of the control cross-sections B and C on the performance of the turnout crossing is investigated. The crossing as defined by the manufacturer with $a_1 = b_1 = 4 \text{ mm}$, $a_2 = b_2 = 9 \text{ mm}$ was used as the reference design. The corresponding dynamic behaviour of the turnout was analysed using the VI-Rail ‘moving track’ model, where the normal contact pressure on the crossing as estimation of the RCF damage was calculated as follows:
\[ S(t) = \frac{3 \cdot F_n(t)}{2 \cdot A(t)} \]  

where \( F_n \) is the normal contact force and \( A \) is the size of the contact patch. And the energy dissipation as estimation of wear damage was evaluated using \( T \gamma \) criterion:

\[ W(t) = \left| F_x(t)\gamma_x(t) \right| + \left| F_y(t)\gamma_y(t) \right| \]

where \( F_x, F_y \) are the creep forces in the longitudinal and lateral direction, respectively; \( \gamma_x \) and \( \gamma_y \) are the corresponding creepages. Here the simulations were performed with sampling frequency 5000 Hz and all the results were low-pass filtered with cut-off frequency 500 Hz.

In the parametric study, the cross-sections B and C were varied separately, i.e. when adjusting the objective cross-section, the rest control cross-sections remained the same as in the reference design. The study of cross-section B was performed by varying the parameters \( a_1 \) and \( b_1 \) from 2 to 6 mm with step of 1 mm, while for cross-section C the parameters \( a_2 \) and \( b_2 \) were varied from 7 to 11 mm with step of 1 mm. Thus, in the study of each cross-section, 25 variations were considered.

The maximum normal contact pressure, maximum wear index and the location of impact at the nose rail were calculated for five groups of lateral semi-axis \( a \) as shown in Figure C-15. In each group five simulations with five different vertical semi-axes \( b \) were performed individually. In order to evaluate the sensitivities of the two control cross-sections, the results were scaled at the same level. It can be observed that the maximum normal contact pressure and the wear index were significantly affected by the change of the cross-section C.

With the increase in the lateral semi-axis \( a \), the normal contact pressure on the crossing decreases (Figure C-15 (a)), while the wear index increases (Figure C-15 (b)). This is to say, the wider railhead of the control cross-section C is preferable in terms of reducing the dynamic normal contact pressure, which in turn will alleviate damages on the nose rail derived from dynamic impact. But it will cause more wear damage. When looking at the variation of the vertical semi-axis \( b \), the increase in \( b \) results in enlargement of the contact pressure while it has slightly different influence on the wear index. When the thickness of the nose rail is not sufficient (with \( a \) within 9 mm), the wear index on the crossing increases with the increase in \( b \), as the rail becomes more thicker (\( a \) above 9 mm), the wear index has a slight reduction with \( b \) equals 9 mm and continues increase as \( b \) becomes larger.

Comparing the effect of the parameters \( a \) and \( b \) (Figure C-15 (a) and (b)) it can be observed that the change of \( b \) has higher influence on the dynamic contact pressure and the wear index. Since \( b \) determines the vertical distance between the wing rail and the nose rail, it again conforms the feasibility of the previous work where the crossing geometry has been simplified in the vertical direction, formed by the vertical distance between the top of the wing
rail and the crossing nose [3,5] or the dynamic vertical wheel trajectory along the crossing [3,14].

Figure C-15 Dynamic responses of crossing in parametric study: (a) the maximum normal contact pressure on the crossing, (b) the maximum wear index on the crossing and (c) location of the dynamic impact on the nose rail. Left: with variation of cross-section B; right: with variation of cross-section C.

Moreover, the shape of the nose rail influences the location of the dynamic impact along the crossing. As shown in Figure C-15(c), the impact occurs earlier with the increase in the rail thickness. While as the height of nose rail decreases the impact moves farther from the nose point, where the rail has higher capacity of bearing the dynamic impact and therefore is less susceptible for damage.

The variation of the cross-section B, however, has little influence on the dynamic behaviour of the crossing when compared with the cross-section C. This could be explained by the observation in Figure C-15(c), where it can be found that with the nominal height of the cross-section C (b2 = 9 mm), the impact occurs around 450 mm, which is much farther from the nose point compared with the location of cross-section B. By increasing the height of cross-section C, the impact moves closer to the nose point. Thus, it can be deduced that the variation of the
cross-section B in the case of a higher cross-section C could have larger influence on the dynamic behaviour of the crossing.

The parametric study reveals the significance of the nose rail shape, however, it was limited in adjusting only one control cross-section at each analysis and thus the observations were narrowed to the assigned restrictions. To have an overall evaluation of the parameterised nose rail shape, further study is demanded by releasing both of the control cross-sections B and C.

5 Optimisation problem

The above observations from both the measurements and the parametric study indicate the importance of the crossing geometry and inspired further research to improve the crossing geometry with the aim of reducing the impact damage. This section will focus on improvement of a crossing nose for new turnouts through numerical optimisations.

The friction coefficient between the rail and wheel surface was assumed to be 0.35. The same vehicle model as introduced in the parametric study was used in the optimisation. Simulations were performed for the train travelling with the speed of 140 km/h, which is the typical speed of the trains on this type of turnout passing in the main direction in the Netherlands. The output from the simulations of vehicle-track dynamics was recorded with the sampling frequency 5000 Hz. All results were low-pass filtered with cut-off frequency 500 Hz before calculating the damage criteria. It should be noted that in the present study the dynamic response of the leading wheelset was used in the optimisations.

5.1 Formulation of an optimisation problem

A general optimisation problem can be stated in the form that reads:

Minimise

\[ F_0(x) \rightarrow \min, \quad x \in R^n \]  \hspace{1cm} (3)

subject to

\[ g_j(x) \leq 1, \quad j = 1, \cdots, M \]  \hspace{1cm} (4)

and

\[ A_i \leq x_i \leq B_i, \quad i = 1, \cdots, N \]  \hspace{1cm} (5)

where \( F_0 \) is the objective function; \( g_j \) is the constraint; \( x \) is the vector of design variables; and \( A_i \) and \( B_i \) are the side limits, which define lower and upper bounds of the \( i \)-th design variable.

The components of the vector \( x \), which can represent various parameters in a mechanical design problem, such as geometry, load condition, material properties, etc., can be varied to
improve the design performance. Depending on the problem under consideration, the objective and constraint functions (3)-(5) can describe various structural and dynamic response quantities such as weight, reaction forces, stresses, strain, natural frequencies, displacements, velocities, accelerations, etc. The objective function provides a basis for improvement of the properties or behaviour of the structure.

The multipoint approximation method (MAM) was employed in solving the optimisation problem, wherein the mid-range approximations were used instead of the original functions to reduce the computational cost of the optimisations. MAM does not require the design sensitivity information though it can effectively be taken into account if available. Moreover the optimisation tool can easily be coupled with most of the response analysis software. More details of MAM can be found in [15-17].

5.2 Design variables

Parameterisation of the crossing geometry was based on the process introduced in Section 4.1. The design variables of the optimisation problem are the semi-axes of the control cross-sections B and C (Figure C-12):

\[ \mathbf{x} = [a_1, b_1, a_2, b_2] \]  

(6)

When imposing the constraints on the design variable, the requirement of the flangeway F (Figure C-16) to be larger than the wheel flange width was used (minimum 34 mm), which resulted in the following side limits:

\[
\begin{align*}
2 \leq a_1 &\leq 13 (\text{mm}), \\
2 \leq b_1 &\leq 14 (\text{mm}), \\
2 \leq a_2 &\leq 18 (\text{mm}), \\
2 \leq b_2 &\leq 14 (\text{mm}).
\end{align*}
\]  

(7)

The reference design introduced in Section 4.2 was used during the optimisation as the reference design \( \mathbf{x}_{\text{ref}} = [4, 4, 9, 9] \). During the optimisation neither the shape nor the location of the wing rails with respect to the track centre line were changed with the variation of design parameters. By introducing the variation of the control cross-sections of the nose rail, the track gauge \( T \) as well as the flangeway \( F \) will be varied along the crossing as a function of the location (Figure C-16). The variation of \( T \) and \( F \) was equal at both the through route and the divergent route. For the control sections B and C, the track gauge and flangeway can be calculated as
Here $T_{\text{ref}}$ and $F_{\text{ref}}$ are the reference (standard) track gauge and flangeway, which were taken as 1435 mm and 43 mm, respectively. $a_{\text{ref}}$ is the reference half-axle of the control cross sections (Figure C-12). The track gauge at any location of the crossing can be calculated similar to the control sections B and C (Equations (8) and (9)) by applying the geometry parameters of the interpolated cross-section at that location.

According to the defined design space (7), the track gauge from the nose point (cross-section A) to the end of the nose rail (cross-section D) was allowed to be varied between -9 and 7 mm on each travelling route.

5.3 Objective function

The optimisation problem focuses on reducing the most common damages on the crossings due to dynamic impact. Wear encountered at the wheel-rail interface could also shorten the life span of the crossing and speed up the worn process of wheels, which in turn exacerbates the damage on the crossing. Both RCF and wear are to be reduced in order to improve the turnout behaviour. Therefore, the normal contact pressure and energy dissipation during the wheel passage through the crossing, which characterise RCF damage and wear, respectively, are to be minimised. The contact pressure $S$ was calculated based on Equation (1):

$$S(x,t) = \frac{3 \cdot F(x,t)}{2 \cdot A(x,t)}$$  \hspace{1cm} (10)$$

The wear index $W$ was calculated based on Equation (2):

$$W(x,t) = \left| F_{t,x}(x,t) \gamma_{x}(x,t) \right| + \left| F_{t,y}(x,t) \gamma_{y}(x,t) \right|$$  \hspace{1cm} (11)$$

where $t$ is the time vector in Equation (10) and (11).

The response functions (10) and (11) are time dependent. In order to be used in the optimisation problem, the time dependence should be eliminated. Thus, the cumulative
contact pressure $\bar{S}$ and wear index $\bar{W}$ expressed in the form of the Kreisselmeier-Steinhauser function (KS function) [18] were used as the objectives:

$$\bar{S}(\mathbf{x}) = \frac{1}{\mu} \ln \left[ \sum_{i=1}^{N} e^{\mu S_{i}(\mathbf{x}) / S_{\text{max}}(\mathbf{x})} \right] \cdot S_{\text{max}}(\mathbf{x})$$  \hspace{1cm} (12)

$$\bar{W}(\mathbf{x}) = \frac{1}{\mu} \ln \left[ \sum_{i=1}^{N} e^{\mu W_{i}(\mathbf{x}) / W_{\text{max}}(\mathbf{x})} \right] \cdot W_{\text{max}}(\mathbf{x})$$  \hspace{1cm} (13)

Where:

$$S_{\text{max}}(\mathbf{x}) = \max(S(\mathbf{x}, t_{i})), \quad i = 1, \ldots, N$$

$$W_{\text{max}}(\mathbf{x}) = \max(W(\mathbf{x}, t_{i})), \quad i = 1, \ldots, N$$

Here $N$ is the total number of time points during the simulations. The parameter $\mu$ determines the discrepancy between $\bar{S}$ (or $\bar{W}$) and the most critical value $S_{\text{max}}$ (or $W_{\text{max}}$) of $S(\mathbf{x}, t_{i})$ (or $W(\mathbf{x}, t_{i})$). The KS function is usually used to eliminate the time dependence of the objective/constraint functions. Moreover, it is smooth and differentiable, which is preferable for the use of numerical optimisation methods. Details about the KS function can be found in [18].

It should be mentioned that in the original form of KS function, the function elements (constraints or objectives) in the KS form are scaled values between -1 and 1. Therefore, in Equations (12) and (13), the normal contact pressure and the wear index were normalised by dividing the maximum values so as to satisfy the KS form. To make the KS value more close to the critical value, a $\mu$ with sufficiently large value (usually between 5 and 200) is required [19]. In the present paper, $\mu$ equal to 50 was used in all KS functions.

The combined weighted two-criteria objective function is then formulated as:

$$F_{w}(\mathbf{x}) \equiv W_{1} \frac{\bar{S}(\mathbf{x})}{S^{+}(\mathbf{x})} + W_{2} \frac{\bar{W}(\mathbf{x})}{W^{+}(\mathbf{x})} \rightarrow \min$$  \hspace{1cm} (14)

The normalising factors $S^{+}$ and $W^{+}$ in (14) are the cumulative contact pressure and wear index corresponding to the reference turnout. $w_{1}$ and $w_{2}$ are the weight coefficients ($w_{1} + w_{2} = 1$). In order to explore the space of the compromised solutions the optimisation problem was solved for various combinations of the weight coefficients [$w_{1}$, $w_{2}$].

It should be noted that during the optimisations $S$ and $W$ were checked only at the crossing (exclude the stock rail and rails at the other side of the track) due to the fact that the dominant impact occurs at the crossing when the wheel–rail contact transfers from the wing rail to the nose rail. Moreover, at each time point the normal contact pressure $S$ was calculated based on
the contact point that caused the largest dynamic forces, i.e. only one contact point was investigated per time point.

5.4 Constraints

Several constraints were used in the optimisations. The design variables should follow the restriction \((a_1 \leq a_2\) and \(b_1 \leq b_2\)) to avoid dimples on the crossing rail in both the lateral and vertical directions along the crossing nose.

Figure C-17 shows the profile of the S1002 wheel, when the contact occurs in the area marked in red (danger zone), derailment was supposed to happen. To avoid the risk of derailment, a constraint was imposed on the position of contact point w.r.t. the wheel, i.e. the contact point on the wheel should not occur in the danger zone.

Additionally, the impact on the nose was not allowed to occur when the nose rail was not thick enough to resist the high dynamic force. Based on the observations from the field site the minimum thickness to carry the dynamic force was taken as 15 mm, although it is preferable to use a constraint on the allowed lateral and vertical loading on the nose rail as a function of the nominal thickness if the related information is available.

Figure C-17 Wheel profile S1002 with ‘danger zone’ of contact marked in red.

Moreover, the normal contact force, which was expressed in the KS form in Equation (15), should not be larger than that of the reference turnout as presented in Equation (16):

\[
\bar{F}_s(x) = \frac{1}{\mu} \left[ \sum_{i=1}^{N} C_i F_s(x_i^*) / F_{s_{min}}(x) \right] \cdot F_{s_{min}}(x)
\]

(15)

\[
F_s(x) \leq \bar{F}_s(x)
\]

(16)

where \(F_s(x_i^*) \ i = 1, \cdots, N\) represents the normal contact force of the turnout and \(\bar{F}_s^*\) was the cumulative normal contact force for the reference turnout calculated in the KS form.
6 Results of optimisations

In this section, the optimisation problem has been solved with different sets of weight coefficients. For each set the optimisation was performed by applying multi-initial points, where the corresponding optimum solution of design parameters as well as the pair of objectives was the solution with minimum objective function. Among all the optimisations, the most active constraints, i.e. the driving factors of the optimisation, were the side limit $b_1 \leq b_2$ and the constraints of the nose rail thickness where the dynamic impact occurs.

6.1 Overview of the optimisation solutions

Figure C-18 shows the solutions $[\vec{S}(x), \vec{S}^+(x), \vec{W}(x), \vec{W}^+(x)]$ of the optimisation problem obtained with different weight coefficients. Generally, the higher the weight is, the smaller the corresponding objective component becomes. The method of obtaining the optimum solutions is the so called weighted sum strategy, which converts the multi-objective problem into a scalar problem by constructing a weighted sum $F(x)$ of all the objectives:

$$ F(x) = \sum_{i=1}^{n} w_i f_i(x) $$

where $n$ is the number of objective components. These solutions are assumed to be (close to) global optima, which belong to the Pareto frontier of the multi-objective problem.

From Figure C-18 it can be found that the obtained frontier of Pareto set is not perfectly convex. This can be due to complex problems with multiple local optima. The MAM uses approximations of the constraints and objective function, so the quality of the solution depends on the accuracy of the approximations of the objective and the constraint functions. If the quality is not sufficient it can lead to premature termination in a local optimum.

The design variables of the optimum nose shape obtained with different weight coefficients are shown in Figure C-19, which were quite different from the reference design. The height difference between the cross-sections B and C ($b_1$ and $b_2$) was reduced to less than 2 mm, where $b_1$ was significantly increased to around 9 mm and $b_2$ was slightly increased to around 10 mm, which resulted in the reduction of the impact forces. The thickness of the nose rail, however, was dependent on the weights of the objective function components. Compared with the reference design, the thickness at the cross-section B ($2a_1$) was generally increased varying from around 2×4 mm to 2×7 mm. While at the cross-section C, the thickness ($2a_2$), which varied from around 2×7 mm to around 2×10 mm, can be narrower or thicker than the reference. That is, depending on the desired reduction of the two damage indicators, the thickness of the nose rail at 150 mm from the nose point should be increased with different levels, while at 300 mm the nose rail can be thinner or thicker.
6.2 The dynamic responses of optimised crossings

The dynamic responses of the reference and optimum designs are presented in Figure C-20, where the optimisations were marked in two groups based on the weight coefficient sets: ‘opt_1’ were the solutions with the normal contact pressure weighted (w1) from 1.0 to 0.5 and the other solutions with w1 from 0.4 to 0.0 were marked as ‘opt_2’. As expected, the reduction of the normal contact pressure was more significant in ‘opt_1’ and the wear index was more effectively reduced in ‘opt_2’.

Figure C-20 shows that the contact pressure and the wear index can be more effectively reduced with the cost of increasing each other, which represents the trade-off between reduction of the RCF damage and wear on the crossing. Due to the constraint on the normal
contact force, the increase in the normal contact pressure (compared with the reference design) was avoided.

Due to the fact that the wheel-rail contact on the wing rail still exists after the nose point, it is essential to figure out whether the nose rail or the wing rail carries the dynamic loads. As a complement to the results in Figure C-20, the dynamic responses were zoomed in as shown in Figure C-21, where the starting points of contacts on the nose rail were marked in circles. It can be found that for all the designs (including the reference crossing) the maximum normal contact pressures were located on the nose rail, while the peak wear indexes occurred on the wing rail, which means the nose rail suffered more from the RCF damage, while the wing rail mainly suffered from the wear damage. Because of the increase in the nose rail height, the wheel transferred earlier from the wing rail to the nose rail. As a result, the potential wear damage on the wing rail occurs earlier (in terms of location along the crossing).
In order to better understand the relation between the nose rail shape and the damage indicators, the vertical wheel trajectory on both sides of the track is analysed as shown in Figure C-22. It can be observed that the vertical dip during the wheel transferring from the wing rail to the nose rail became smaller, which was a result of the shortened vertical distance between the wing rail and the nose rail because of the increase in the semi-axes $[b_1, b_2]$ (Figure C-19). On the other side of the track, the wheel trajectory was hardly influenced. This change of wheel motion at the transition area directly contributes to the reduction of the contact force and contact pressure on the crossing.

Figure C-23 Contact location on the wheel (lateral direction): (a) the contact location on the wheel along the crossing and (b) spread of contact along the wheel, left: the crossing side; right: opposite side of the track.
Due to the change of the nose rail thickness, the local track gauge was also changed, which makes it necessary to investigate the risk of interference contact between wheel flange and the crossing nose. Figure C-23 shows the results of contact location on the wheel (lateral direction) on both sides of the track. For all the optimum solutions, no contact between the wheel and the check rail was observed, although the local track gauge has been widened in some designs due to reduction of the nose thickness. In Figure C-23, it can be observed that on the crossing side, when the wheel transfers from the wing rail to the nose rail, the contact jumps from the field tread side of the wheel to the flange root. While on the opposite side of the track, contact occurred only at the wheel tread. These results indicate that for all the optimised designs, there were no contacts between the wheel flange and the nose rail.

6.3 Analysis of the optimum design under various track conditions

In reality, the vehicle-track system is not inclined to be the ideal situation as designed. A good turnout design should be reliable under the variation of vehicle-track system. To this end, the analysis was performed by varying the track condition. The dynamic response of the optimised designs and the reference turnout have been checked for the following five different cases, including the condition used in the optimisation as shown in Figure C-24:

1. Default vehicle-track condition used in the optimisations (vertical dip at the crossing);
2. Ideal track without irregularities in the vertical plane and the horizontal plane;
3. The same as the condition (1), but there was a lateral disturbance on the track right before the wheel entering the crossing. This is to account for the lateral interference of the wheel-rail interaction at the crossing;
4. The same as the condition (1), but the track gauge was widened by 2 mm in a length of 10 m including the crossing area (track gauge 1437 mm).
5. The same as the condition (1), but the track gauge was narrowed by 2 mm in a length of 10 m including the crossing area (track gauge 1433 mm).

It should be noted that, the maximum allowable deviation of the track gauge was employed in the last two cases according to the current criteria [20] used in applications of railway switches and crossing, where the tolerance of track gauge should be less than ±2 mm. This means that the most risky conditions were considered in order to achieve a reliable design.

Each design from the optimisations has been checked under the five vehicle-track conditions individually. The lateral displacements of both the front and the rear wheelsets were examined. It was found that the lateral displacements were all less than 5.5 mm and well damped out under the varying track conditions, which indicates the stability of the vehicle operation on the varying track conditions was achieved with the optimised designs. No safety issue was found being violated during the vehicle-turnout interaction.
Moreover, the risk of interference contact between wheel flange and the crossing nose has been studied. Table C-3 shows the location of the margin of contact point on the flange side (location positive towards the outer side of the wheel, cf. Figure C-17 Wheel profile S1002 with ‘danger zone’ of contact marked in red) of the wheel for each design under each track condition, wherein the evaluation was made for wheels on both sides of the track and the
contact location farthest from the wheel tread field was chosen. Among the five track conditions, the track with lateral disturbance (case 3) drives the contact closest to the wheel flange, followed by the track with narrowed gauge (case 5). On the contrary, contact is less likely to occur at the wheel flange with the wider track gauge (case 4). Additionally, the results in Table C-3 indicate that for all the designs under all these track conditions, there was no contact on the check rail.

![Graphs](image)

Figure C-25 Contact location on the wheel (lateral direction): (a) the crossing side and (b) the opposite side.

The contact zone on the wheels for the most risky track condition of flange-nose rail contact (case 3) has been illustrated in Figure C-25, wherein the marked contact zone covers the contact locations of all the optimisation designs. No flange contact was observed as shown in Figure C-25, which means none of the optimum designs has wheel flange contact, although the contact at the opposite side of the track moves closer to the wheel flange as compared with the default track condition (Figure C-23) due to the narrow track gauge.

The contact pressure and the wear index from the reference design and the optimised design obtained with the weight coefficients $[0.5, 0.5]$ ($\mathbf{X}=[5.267, 9.012, 10.232, 9.752]$) were compared under different track conditions as lists in Table C-4, wherein the results were in the same scale. From the results it can be observed that, on one hand, the maximum amplitudes of both the normal contact pressure and the wear index (except track case 5) at the crossing were reduced compared with the reference design, which means attenuation of damages. On the other hand, similar to the observation in Section 6.2 the potential wear damages on the crossing, which still appeared on the wing rail, occurred earlier as a result of the earlier transition due to increase of the nose rail height.

Additionally, Table C-4 shows that the varying track conditions have different levels of influence on the vehicle performance, especially on the wear index in the present example. An improvement in one track condition may not be valid for other conditions. This means the optimum solution of the turnout design depends on the assigned track condition. Therefore in the real life crossing designs the service condition of the track and the type of traffic on the specific line are determinant factors. Moreover, the assessment of the vehicle dynamics on the optimised design with variation of track conditions is indispensable.
Table C-4 Dynamic response of the crossing under different track conditions.

<table>
<thead>
<tr>
<th>Track</th>
<th>Contact pressure</th>
<th>Wear index</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>(2)</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>(3)</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>(4)</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td>(5)</td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
</tbody>
</table>
7 Conclusion

The effect of rail geometry at railway crossings has been investigated both through the field measurements and the numerical simulations. In the field site it was found that the grinding of the crossing can spread the impact to a wider area and the magnitude of the impact has been significantly reduced. Apart from the field measurements the effect of the crossing nose geometry has been analysed numerically. In the simulations the crossing geometry has been parameterised by defining four control cross-sections from the nose point to the end of the nose along the crossing, wherein the railhead of the nose rail over the gauge corner has been simplified as semi-ellipse. Dynamic simulations in MBS package VI-Rail have shown that the shape of the nose rail has considerable influence on the turnout behaviour and adjustment of nose rail shape is preferable.

A numerical optimisation approach for reducing the damage on the crossing nose by adjusting the nose rail shape is proposed. The optimisation problem is formulated as a weighted multi-objective problem, which has been solved with different sets of weight coefficients. The dynamic behaviour of the crossing has been improved after optimisations. The height of the nose rail at the beginning is considerably increased, which resulted in the reduction of the impact forces. The width of the nose rail in the beginning is also increased. Additionally, the dynamic responses of the optimal designs have been analysed under different track conditions, it shows that all the optimum designs worked well under these track conditions.

The optimisation methodology presented was for determinative traffic parameters. It should be considered as a methodology demonstrator which shows that vehicle dynamics in railways crossings can be improved and damage indicators reduced at the nose rail. As demonstrated by varying the track conditions, the dynamic vehicle-turnout interaction was appreciably influenced. The method can be extended to account for a spread in traffic parameters in each design evaluation as presented in [21], but at the cost of much more time consuming calculations. In the real practice, the parameters utilised in the evaluation should depend on the specific traffic case for that crossing.

References


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Parametric study of wheel transition at railway crossings

Chang Wan, Valeri Markine


**Abstract**

Vehicle-track interaction at the railway crossing is complex due to the discontinuity of the crossing. In this study, the effect of the local crossing geometry, the track alignment and the transvers wheel profiles on the wheel transition behaviour, which is evaluated by the transition point along the crossing (and location of impact), the contact pressure and energy dissipation during the wheel-rail contact, is investigated using the MBS software package VI-Rail. A detailed parametric study of the crossing geometry has been performed, through which the most effective parameters for defining the crossing geometry are identified to be the crossing sectional shape of the nose rail and the vertical distance between the top of the wing rail and the nose rail. Further, the interaction influence of the crossing geometry, the track alignment and the wheel profile are investigated using the design of experiment method with a two-level full factorial design. The longitudinal height profile of the crossing and the wheel profile are found to be the most significant.
1 Introduction

Due to the discontinuity in the rail geometry introduced at the crossing nose, railway crossings experience high impact loads from passing vehicles, which makes them sensitive to several types of damage. The dynamic interaction between a train and a railway turnout depends on a number of parameters related to the nominal design and the current conditions of both systems. A stochastic study in [1] investigated the influence of 14 parameters on the train-track interaction at the switch panel, based on which the main factors were identified, such as the axle load, wheel profile, running direction and friction coefficient. It also showed that the turnout rail profiles have significant influence on the combined dynamic performance of the switches. Because the influence of the axle load and the friction coefficient on the contact forces and the energy dissipation during the wheel/rail contact is relatively clear (i.e., the higher the axle load and the friction coefficient, the higher the dynamic forces and the energy dissipation), they are not included in the current study. The running direction is an important factor, especially for a fixed turnout. Considering the fact that many railway turnouts serve a single running direction (either facing or training route), it is applicable to select a fixed running direction for a turnout in the deterministic serving direction. Moreover, the diverging route is normally associated with much lower travelling speed compared with the main route. Thus, the main factors used here have been narrowed to the turnout geometry and the wheel profiles. The goal of this study is to investigate the effective parameters for defining the turnout geometry, as well as the effect of the track alignment and the wheel profiles on the wheel transition behaviour.

It has been observed from field measurements [2-3] that the dynamic behaviour of the turnout is sensitive to the rail geometry. In [4], the nose rail accelerations and the locations of impact due to the passage of each wheel set were measured before and after the grinding/welding maintenance at a 1:15 single left-hand crossing. It was observed that the magnitude of impact was significantly reduced and the location of impact was shifted—leading to a wider spread of impact on the nose rail.

In [5], the crossing geometry was modified by tuning the height profile of a movable-point crossing, whereas in [4], the height profile and the cross-sectional shape of the nose rail (fixed crossing), which were simplified by a semi-ellipse, were tuned to improve the crossing performance. In the optimisation of the crossing geometry presented in [6], the nose rail profiles and the wing rail profiles were modified. Also the effects of various wheel profiles were considered. However, the wheel-rail contact was determined statically in that study and therefore the dynamic effects were not taken into account. The cross-sectional shape of the wing rails, the cross-sectional shape of the crossing nose, the height profile of the wing rail, and the height profile of the crossing nose were modified separately, and the effects of the cross-sectional shape and the longitudinal height were assumed to be independent.
Here, to analyse the effect of the crossing geometry on the dynamic wheel-rail interaction during the transition of railway wheels over the crossing, a parameterisation method for defining the crossing geometry is proposed by parameterising the longitudinal height profile and the cross-sectional shape of the crossing rails. In the study, the quality of the crossing geometry is evaluated against the wheel transition behaviour obtained from the dynamic simulation of the vehicle-turnout interaction, wherein the wheel/rail contact is calculated online (‘real time’); therefore, the effect of the dynamic vehicle-track interaction is fully included. Moreover, the crossing geometry is treated as an entirety, in which the effect of the cross-sectional shape and the longitudinal height of the crossing rails, as well as the effect of the nose rail and the wing rails, are assumed to be dependent.

The numerical model used in this study and the criteria for the assessment of the crossing performance are presented in Sections 2 and 3. The crossing geometry parametrisation method is described in Section 4, followed by a parametric study of the geometry in Section 5. In Sections 6 and 7, the effects of the track alignment and the cross-sectional shape of the passing wheel profiles on the transition behaviour of the wheel are studied. The interaction influences of the crossing geometry, the track alignment, and the wheel profile are investigated in Section 8 using the design of experiments (DOE) method with a two-level full factorial design. Finally, some conclusions and suggestions are presented in Section 9.

2 Vehicle-turnout model

To simulate the dynamic vehicle-turnout interaction, it is necessary to take into account both the track and the vehicle since the dynamic behaviour of a crossing is sensitive to both the track features and the vehicle characteristics. The multi-body system (MBS) package VI-Rail is here used to model the dynamic vehicle-turnout interaction, wherein a complete vehicle model and a track model with varying geometry are presented.

The vehicle model is composed of the car body, the front bogie, and the rear bogie, wherein the car body, the bogie frames, as well as the wheelsets are treated as rigid bodies and are defined by their mass-inertia characteristics (mass, moments of inertia, and the position of the gravity centre). Each rigid body has six degrees of freedom: three translations and three rotations. All bodies are connected by spring-damping elements, representing the primary and secondary suspensions.

The turnout track is simulated using the ‘moving track’ model. In the track model the rails and sleepers are modelled as rigid bodies moving together with the wheel load. Linear spring-damper elements in the vertical and lateral directions are used to represent the flexible connection of sleepers/rail and sleeper/ground. The ‘moving track’ model is fast in simulating the vehicle-turnout interaction (about 2 min in general) and has been validated based on measured results and simulated results from a more complex (realistic) model [4].
For the wheel-rail contact, the Hertz theory is used for calculating the normal contact force and the creep forces are computed based on the modification of Kalker’s FASTSIM algorithm developed at TU Berlin [7]. Details about modelling of railway vehicle, turnout, and wheel-rail contact have been introduced/discussed in [4].

The studied turnout is a standard right-turn one with a curve radius of 725 m and a crossing angle of 1:15. The through route of the turnout with a total length of 150 m is considered in the model. In the crossing part, a number of changing rail profiles are used to represent the complex crossing geometry, as shown in Figure D-1. To analyse the sensitivity of the wheel motion on the crossing, the rail profiles are densely spread along the crossing. Table D-1 lists the rail profiles used in the crossing model, including the intervals between the adjacent rail profiles along the crossing.

![Crossing geometry](image)

**Table D-1** Rail profiles along the crossing in the MBS model.

<table>
<thead>
<tr>
<th>Location (mm) (distance from TP)</th>
<th>Interval (mm)</th>
<th>Rail profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>~ -875</td>
<td>-</td>
<td>constant rail profile (S4E1)</td>
</tr>
<tr>
<td>-875 ~ -225</td>
<td>50</td>
<td>wing rail</td>
</tr>
<tr>
<td>-225 ~ 1050</td>
<td>15</td>
<td>parameterised nose rail (changing profile) + wing rail</td>
</tr>
<tr>
<td>1050 ~</td>
<td>-</td>
<td>constant rail profile (S4E1)</td>
</tr>
</tbody>
</table>

On the opposite side to the crossing, the stock rail and the check rail are modelled.

A single wagon representing the wagon of the VIRM passenger train (Dutch intercity train) is used in the study, which has a static wheel load of 89 kN. In the present paper, the investigations are only for the vehicle passing in the main-facing direction with a velocity of 140 km/h (typical velocity of VIRM intercity trains in the Netherlands) and a friction coefficient of 0.4. The train starts 15 m before the nose point and ends 30 m after the nose point. A sampling frequency of 5000 Hz is used during the simulation, and all the results are low-pass
filtered with a cut-off frequency of 500 Hz before the evaluation of the transition behaviour. Here the sampling frequency 5000 Hz is chosen according to two bases: 1) no further change of the dynamic response by increasing the sampling frequency; 2) the calculation length should be shorter than the smallest interval between two adjacent rail profiles. While the cut-off frequency is chosen as 500 Hz because the results from the simulation and the measurement are close to each other in a frequency range within 500 Hz [4].

3 Assessment of wheel transition over crossing

The main source of damage at railway crossings is the dynamic impact during the wheel passing through the wing rail and transferring to the nose rail. In the present paper, the contact pressure and the energy dissipation at the wheel-rail contact patch along the crossing, which are applied as the damage indicators of the rolling contact fatigue (RCF) and wear, are used as evaluation criteria for the wheel transition behaviour. The contact pressure is expressed as

$$s = \frac{3F_n}{2A}$$

(1)

where $F_n$ is the normal contact force, and $A$ is the size of the contact patch. The energy dissipation is evaluated using the $\gamma \gamma$ criterion according to Kalker [8]:

$$W = \left| F_x \gamma_x \right| + \left| F_y \gamma_y \right|$$

(2)

where $F_x$ and $F_y$ are the creep forces in the longitudinal and lateral directions, respectively; $\gamma_x$ and $\gamma_y$ are the corresponding creepages.

In addition to the wheel/rail damage indicators, the location of the transition zone defined by the beginning of the wheel-rail contact on the nose rail and the end of the contact on the wing rail is considered here for assessment of the crossing performance. The transition zone related to the impact location is used as the indication of the potential damage location on the crossing.

Further, to understand the kinematic behaviour of the wheel transition, the wheel trajectory in both the vertical and the lateral directions is investigated.

It has to be mentioned that the derailment ratio ($Y/Q$), which is an important aspect of wheel transition performance, is not directly used as indicator of the wheel transition behaviour. Because the check rail largely reduces the risk of derailment at crossings. Moreover, in the dynamic simulations, the wheelset has six degrees of freedom (three in translational directions and three in rotational directions), which allows the wheel to climb over the rail until derailment occur; if derailment happens, the simulation will be terminated before the
vehicle reaches the defined travel length. In this way, the safety of vehicle-turnout interaction is accounted for.

4 The crossing geometry

Apart from the given nominal track gauge and nominal rail profile (for the normal rail), the crossing geometry is defined by the height and the cross-sectional shape of the nose and wing rails along the crossing. Here, in the definition of the crossing geometry, the ProRail (Dutch rail infrastructure provider) drawings 412,778-488,985 [9] describing the common turnout 54E1 are used. These drawings are usually used as a guideline by turnout manufacturers in the Netherlands. According to these drawings (Figure D-2), the geometry is defined by four control cross-sections, which are located at distances of 0, 10\(\alpha\), 20\(\alpha\), and 70\(\alpha\) (the crossing angle is 1:\(\alpha\)) mm from the tip point (TP) of the nose rail (nose point).

In the previous study [4], the control cross-sections of the crossing nose were simplified by ellipses with the semi-axes used as the design parameters, whereas the wing rail geometry remained the same as the corresponding manufacturing design. One disadvantage of such parametrisation is that along with the parameters, the track gauge is varied as well, which is not allowed in practice. Additionally, in [6], the introduced crossing geometry parametrisation can result in an impractical wavy longitudinal height profile of the wing rail.

To achieve more practical results when varying the crossing rail shape, a series of discussions with the manufacturers and infrastructure specialists from ProRail took place. The feedback obtained from these discussions concerning the practical aspects of the crossing design suggested that

(1) the height of the nose rail should be monotonically increased;
(2) the change of the rail geometry along the crossing should preferably be linear for both the wing rails and the nose rail; and
(3) the track gauge should not be changed, that is, the nominal thickness of the nose rail, which is in accordance with the standard track gauge, may not be changed. Based on these suggestions, an alternative parameterisation of the crossing geometry was suggested, which is described below.
4.1 Parameterisation of the nose rail geometry

The same idea as in [4] of using the control cross-sections (Figure D-2) to define a crossing nose was adopted here. The observations from the field site [2] indicate that dynamic impact often occur after the control section III (Figure D-2). Thus, an extra control section (section V) at 40°α mm from TP was suggested in addition to the other four control sections. The nominal thickness of the nose rail is not changed here to preserve the track gauge. The nose rail profiles between the control sections were obtained by linearly interpolating the cross-section of the two adjacent control sections.

**Longitudinal height profile of the nose rail**

The longitudinal height profile of the nose rail is adjusted by tuning the height of the five control sections, where each height segment between two control sections is linear. It is assumed that the wheel impact should not occur before section II, where the nose rail is too thin to sustain the dynamic load. Thus, only the height of the sections III and V is tuned. The longitudinal height profile of the nose rail is shown in Figure D-3. The height of 0 mm is set to 14 mm below the top of the normal rail. When tuning the height profile, the monotonic increase of the profile should be maintained to obtain a realistic design.
Cross-sectional shape of the nose rail

The cross-sectional shape of the nose rail above the zero level (Figure D-3) is described using a parametric curve B-spline. The equation for the $k$th order B-spline with $n+1$ control points $(P_0, P_1, \ldots, P_n)$ is

$$S(u) = \sum_{i=0}^{n} N_{i,k}(u)P_{i}, \quad u \in [u_0, u_{n+1}]$$

(3)

where $(u_0, u_1, \ldots, u_{n+1})$ is the non-decreasing knot vector. The basis function $N_{i,k}$ is a polynomial of degree $k-1$ defined as [10].

$$N_{i,k}(u) = \begin{cases} 1 & \text{if } u_i \leq u \leq u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

(4)

$$N_{i,k}(u) = N_{i,k-1}(u) \left( \frac{u - u_i}{u_{i+k-1} - u_i} \right) + N_{i+1,k-1}(u) \left( \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} \right)$$

The B-spline approximation has been successfully used in optimising the wheel profiles [11,12]. One of the advantages of a B-spline is that each control point affects only $k$ segments of the B-spline among its $n-k+2$ segments, which gives global rather than local control of the shape of the B-spline [13]. Thus, using the locations of the control points as tuning variables can help to avoid unrealistic shape designs. Figure D-4 shows an example of the change of a B-spline ($n=7, k=4$) by moving one of the control points, wherein it can be observed that the curve moves in the same direction with the control point and four of the five segments are influenced.
A cubic B-spline \((k=4)\) with six control points \((n=5)\) is chosen as a representation of the nose rail profile, where the non-decreasing knot vector is \([0 \ 0 \ 0 \ 0.2 \ 0.7 \ 1 \ 1 \ 1 \ 1]\). Figure D-5 shows the B-spline representation of the controlling cross-sections of the crossing nose inspired by the parameterisation method of the wheel profile from [11]-[12] and the parameterisation of the crossing nose introduced in [6]. The 6 control points \(P_i, i=0,...,5\) are defined as follows:

- \(P_0\) and \(P_5\) – fixed; defined by the height profile and thickness of the section.
- \(P_1\) and \(P_4\) – fixed; originally on lines \(P_0P\) and \(P_5P\), respectively. To maintain the flatness of the top centre of the nose rail, \(P_1\) is adjusted to the same vertical level as the control point \(P_0\) in the B-spline representation of the nose rail.
- \(P_2\) – movable; the lateral coordinate of \(P_2\) is the same as the point \(P_2'\) on the line \(P_0P\), where the segment \(P_0P\) has the same length as the segment \(P_5P\), if the length of the segment \(P_0P\) is larger than that of \(P_5P\), otherwise \(P_2'\) is at the location of \(P_5\). \(P_2\) can be
varied along the vertical line. The location is determined by the parameter $\gamma$: the inclination of the segment $P_0P_2$. It should be noted that, if the control point $P_2$ locates at the left side of the control point $P_1$, the order of the two control points will be exchanged in the B-spline function.

- $P_3$ – movable; along the line $PM$. Its location is defined by parameter $\lambda = \frac{|PP_3|}{PM}$.

![Figure D-6 B-spline representation of the measured nose rail profile with a width of 70 mm.](image)

![Figure D-7 B-spline representation of the nose rail geometry: (a) cross-sectional shape at the control sections, and (b) 3-D nose rail geometry.](image)

Therefore among the 6 control points, $P_2$ and $P_3$ are movable to adjust the shape of the nose rail. Parameter $\gamma$, which is responsible for tuning the control point $P_2$ upwards or downwards, adjusts the height of the top middle part of the nose rail profile. Parameter $\lambda$ drives the control point $P_3$ outwards or inwards and thus influences the corner part that forms
more than half of the nose rail profile. An acceptable B-spline representation of the nose rail profile obtained by tuning these control points should be a smooth convex curve.

Reference crossing representation

Figure D-6 shows the B-spline representation of the measured nose rail profile at control section IV, where the width of the crossing nose is 70 mm. From this figure, it can be observed that the B-spline representation is in good correspondence with the real profile. Here, the parameter set \([\gamma, \lambda]\) equal to \([1/25, 1/3]\) is applied.

The reference crossing from the current manufacturing process can be represented using the parameters \([h_1, h_2, \gamma, \lambda]=[9, 11, 1/25, 1/3]\). Figure D-7 (a) shows the cross-sectional shape of the nose rail of the five control cross-sections, and Figure D-7 (b) shows the 3-D geometry of the nose rail.

4.2 Parameterisation of wing rail geometry

The geometry of the wing rails plays an important role in the wheel transition in the crossing as well. In the manufacturing process, the wing rails are produced based on the normal rail. Thus, the cross-sectional shape of the wing rail can be parameterised in a similar way as the nose rail. For the sake of simplicity, only the longitudinal height profile of the wing rail is varied here. The standard wing rail profiles from the manufacturing drawings of 1:15 crossings are used in this study.

Based on the manufacturing process of crossings with raised wing rails, the wing rail is raised before TP. A schematic drawing of the wing rail raised to level \(d\) (above the top of the normal rail) is shown in Figure D-8. To achieve the vertical level \(d\), the wing rail is bent along the location \(L_1\)-\(L_2\) before TP. Starting from \(L_2\), the height of the rail reaches the designed level. Usually, the values of the parameters \(L_1\) and \(L_2\) depend on the height of the wing rail \(d\). Here, the values \(L_1=1225\ \text{mm}\) and \(L_2=225\ \text{mm}\), corresponding to \(d=3\ \text{mm}\) taken from the manufacturer, are used. For simplicity, these values are not changed when adjusting the value of \(d\).

![Figure D-8 Longitudinal height profile of the wing rail.](image-url)
5 Parametric study of crossing geometry

Based on the parameterisation approach presented in Section 4, the nose rail geometry can be tuned by adjusting the height parameters \([h_1, h_2]\) (Figure D-3) and the shape parameters \([\gamma, \lambda]\), whereas the wing rail geometry is defined by the fixed cross-sectional shape and the height parameter \(d\). In the parametric study of the crossing geometry presented below, the ideal track alignment with no irregularities is used in the crossing model during the parametric study of the crossing geometry. The parameters of the initial design (reference design) of the crossing are set as \([h_1, h_2, \gamma, \lambda, d]=[9, 11, 1/25, 1/3, 0]\).

5.1 Cross-sectional shape of the nose rail

In this section, the sensitivity of the movable control points defining the nose rail profile is analysed. Three groups of the B-spline representations of the nose rail profile, which have different base parameter sets, are obtained by varying the parameters \(\gamma\) and \(\lambda\) as listed in Table D-2.

<table>
<thead>
<tr>
<th>Base</th>
<th>Tuning parameters ([\gamma, \lambda]) Varying (\gamma)</th>
<th>Varying (\lambda)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td>([0, 1/3])</td>
<td>([1/15, 1/3])</td>
</tr>
<tr>
<td>Group B</td>
<td>([1/25, 0])</td>
<td>([1/15, 0])</td>
</tr>
<tr>
<td>Group C</td>
<td>([1/25, 1/2])</td>
<td>([0, 1/3])</td>
</tr>
</tbody>
</table>

Figure D-10 shows the B-spline representations of the nose rail with different combinations of \([\gamma, \lambda]\) in accordance with Table D-2. From this figure, the shape of the crossing nose changes significantly by varying parameter \(\lambda\), but it is less affected by varying parameter \(\gamma\).
Figure D-9 B-spline representations of the nose rail by varying parameters \([γ, λ]\) from Table 2: (a) Group A, (b) Group B, and (c) Group C.

A close look at the B-spline presentations is taken by investigating the dynamic responses of the crossing with the crossing geometry defined by the reference height profile (Figure D-2) and the nose rail shapes from Figure D-9. To clarify the wheel-rail interaction on the nose rail and the wing rail, respectively, the results in the rest of the paper are all plotted with the
solid line representing the response for the wing rail and the dotted lines representing the response of the nose rail.

Figure D-10 Contact pressure and energy dissipation of the crossing with the nose rail profile defined by the varying parameters $[\gamma, \lambda]$ from Table D-2: (a) Group A, (b) Group B, and (c) Group C.

Figure D-10 shows the normal contact pressure and the energy dissipation in the wheel-rail contact along the crossing, which are used as the damage indicators of RCF and the wear of the crossing, respectively [4]. Both the contact pressure and the energy dissipation are sensitive to the variation of parameter $\lambda$, whereas little influence on the contact pressure or energy dissipation is observed by tuning parameter $\gamma$. The corner of the nose rail is more sensitive than the top middle part in terms of the effect on the dynamic performance of the crossing. Additionally, it can be observed that the contact pressure and wear are competitive, so that an increase of the pressure leads to a reduction of wear.
Figure D-11 Wheel trajectory under the crossing geometry with the nose rail profile defined by the varying parameters $[\gamma, \lambda]$ from Table 2: (a) Group A, (b) Group B, and (c) Group C.

Moreover, Figure D-11 shows both the vertical and lateral wheel trajectories at the crossing side. A positive displacement of the wheel in this figure in the vertical direction corresponds to motion above the normal rail top, and positive displacement in the lateral direction corresponds to the motion outwards from the track. This figure shows that variation of the parameters $[\gamma, \lambda]$ influences the wheel trajectory. Parameter $\lambda$ is more dominant than $\gamma$, which is of interest in evaluating the geometric effect of the crossing nose. The previous study of the authors [4] showed that variation of the elliptical shape of the nose railhead could influence the vertical wheel trajectory, where both the height profile and cross-sectional shape of the crossing nose were varied. This result, on the other hand, is in accordance with the variation of shape without changing the longitudinal height profile of the nose rail. The wheel trajectory is influenced by the shape of the crossing nose, even without tuning the height profile.
Consequently, the longitudinal location of the transition of the wheel contact from the wing rail to the nose rail changes, which can be observed in Figure D-10 and Figure D-11.

Comparing the results of Groups B and C, by dragging the control point $P_3$ inwards (i.e. direction $P_3\text{−}TP$), as shown in Figure D-9(b) and (c), the transition of the contact from the wing rail to the nose rail moves farther from TP (Figure D-10 (b) and (c) and Figure D-11 (b) and (c)). Consequently, the impact on the crossing defined by the maximum peak value of the contact pressure occurs farther from TP (Figure D-10 (b) and (c), contact pressure) because the inwards movement of control point $P_3$ reduces the height of the rail surface at the same lateral level. Thus, the wheel contacts the crossing nose farther from TP, and the transition point of the wheel is lower (Figure D-11 (b) and (c), vertical wheel trajectory), which results in higher impact. Therefore the contact pressure can be higher, as shown in Figure D-10 (b) and (c), and vice versa.

The lateral wheel motion is also dependent on the shape of the crossing nose. The lateral wheel trajectory shown in Figure D-11 (b) and (c) reveals that the wheel moves to the outer side (the wing rail side) if control point $P_4$ moves inwards, and vice versa. This is because the contact surface on the crossing nose becomes narrower at the same vertical level. Therefore, the wheel does not have support from the crossing nose and moves towards the wing rail side.

5.2 Longitudinal height profile of the nose rail

The height of the crossing nose can be adjusted by tuning the parameters $h_1$ and $h_2$ (Figure D-3). The present study focuses on the overall effect of the height profile of the nose rail but not the detailed effect of each parameter. Hence, a simplified tuning of the height profile of the crossing nose is used here by introducing the parameter $h$, as shown in Figure D-12:

$$\begin{align*}
  h_1 &= h_{1,\text{ref}} + h, \\
  h_2 &= h_{2,\text{ref}} + h 
\end{align*}$$

(5)

where $h$ determines the change of the height of the crossing nose at the two control locations, and $h_{1,\text{ref}}$ and $h_{2,\text{ref}}$ are the heights at the control locations of the reference design: $[h_{1,\text{ref}}, h_{2,\text{ref}}]=[9,11]$. The wing rail and the B-spline parameters of the nose rail shape are the same as in the reference design.

The dynamic vehicle-track interaction at the crossing with $h$ varying from -1 to 1 mm has been analysed, as shown in Figure D-13. The results indicate that by lifting the nose rail, the wheel contacts the nose rail earlier, and the down-outwards movement of the wheel (Figure D-13 (a) and (b)) stops earlier as a result of contacting the nose rail. Due to the change of the transition location on the nose rail, the damage at the crossing indicated by the contact pressure and the energy dissipation are altered in accordance with the transition location. The
earlier transition of the contact to the nose rail causes both higher contact pressure and higher energy dissipation.

![Figure D-12](image)

Figure D-12 Adjustment of longitudinal height profile of the nose rail.

![Figure D-13](image)

Figure D-13 Effect of varying the longitudinal height profile of the nose rail: (a) vertical wheel trajectory, (b) lateral wheel trajectory, (c) contact pressure, and (d) energy dissipation.

5.3 Longitudinal height profile of the wing rail

A set of crossing designs are prepared by adjusting the height of the wing rail, that is, the parameter $d$ in Figure D-8, whereas the crossing nose remains the same as in the reference design.

Figure D-14 shows that the impact point on the nose rail is shifted due to raising the wing rail: the higher the wing rail is, the farther the wheel transfers to the nose rail. That is why, in practice, some crossing designs use raised wing rails to alter the wheel contact on the nose rail, which can also be achieved by decreasing the height of the nose rail (Figure D-13). Figure D-14 shows that a design with a higher wing rail results in smaller lateral displacement of the wheel (i.e. the wheel moves towards the crossing nose side), which is because the stock rail
supporting the wheel on the other side of the wheelset remains the same. The higher the wing rail is, the larger the rolling angle $\theta$ of the wheel set becomes (Figure D-15).

![Figure D-14](image)

Figure D-14 Effect of varying the longitudinal height profile of the wing rail: (a) vertical wheel trajectory, (b) lateral wheel trajectory, (c) contact pressure, and (d) energy dissipation.

Consequently, the peak values of the contact pressure and the energy dissipation on the crossing move further from TP. Additionally, the magnitude of the dynamic response is influenced by varying the height of the wing rail. The results indicate that a smooth transition with reduced impact could be achieved by adjusting the wing rail height.

**5.4 Vertical distance between the wing rail and crossing nose**

The results from Sections 5.2 and 5.3 show that the transition behaviour of the wheel over the crossing can be influenced by either adjusting the longitudinal height profile of the nose rail or the wing rail. In both ways, the vertical distance between the wing rail and the nose rail (VDBWN) changes. What is the effect of the geometry changes when the VDBWN does not change? What if the change of the VDBWN is through different variation levels of the longitudinal height profile of the wing rail and the nose rail? To answer these questions and to
further understand the effect of the height of the wing rail and the nose rail, three different groups of investigations are proposed in Table D-3 by adjusting the longitudinal height profile of both the nose rail and the wing rail, as shown in Figure D-16. The change of VDBWN is calculated as

\[
\Delta(\text{VDBWN}) = d - h
\]

The results of the dynamic interaction between the vehicle and the crossing for three groups of designs are shown in Figure D-17 to Figure D-19, where it can be observed that within all three groups, the impact location does not change for the same VDBWN. The magnitude of the dynamic responses, however, depends on the height of the wing rail. A design with a higher wing rail results in smaller lateral displacement of the wheel, that is, the wheel moves towards the track centre, which can be explained in Figure D-15.

Moreover, the design with a higher wing rail results in higher energy dissipation but lower contact pressure. If the VDBWN does not change what is the reason? Figure D-20(a) shows the nose rail shapes of the designs from Group A, where the shapes for each design correspond to 14 mm below the top of the original wing rail (i.e. the top of the normal rail). By only increasing the height of the nose rail with parameter \( h \) (Figure D-16), the supporting surface becomes narrower and lower at the corner, which is similar to the effect of moving the control point inwards (Section 5.1), and vice versa. Accordingly, the contact pressure and the energy dissipation are influenced with the same trend as moving the control point \( P_3 \) inwards.

Table D-3 Groups of VDBWN designs.

<table>
<thead>
<tr>
<th>Groups</th>
<th>Description</th>
<th>Designs</th>
<th>( \Delta(\text{VDBWN}) ) (mm)</th>
<th>( d ) (mm)</th>
<th>( h ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td>VDBWN does not change</td>
<td>Design 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Design 2</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Design 3</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Group B</td>
<td>VDBWN decreases by 1 mm</td>
<td>Design 1</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Design 2</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>Group C</td>
<td>VDBWN increases by 1 mm</td>
<td>Design 1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Design 2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure D-17 Wheel transition behaviour with $\Delta(VDBWN) = 0$ mm (Group A): (a) vertical wheel displacement, (b) lateral wheel displacement, (c) contact pressure, and (d) energy dissipation.

Figure D-18 Wheel transition behaviour with $\Delta(VDBWN) = -1$ mm (Group B): (a) vertical wheel displacement, (b) lateral wheel displacement, (c) contact pressure, and (d) energy dissipation.
Figure D-19 Wheel transition behaviour with $\Delta(VDBWN) = 1$ mm (Group C): (a) vertical wheel displacement, (b) lateral wheel displacement, (c) contact pressure, and (d) energy dissipation.

To eliminate the effect of the nose rail shape, a test has been performed for Group A by adjusting the zero point of the nose rail height to 14 mm below the top of the corresponding adjusted wing rail. The modified nose rail profiles are shown in Figure D-20(b), and the dynamic response of the wheel transition is shown in Figure D-21. It can be observed in Figure D-21 that there is almost no influence on the contact pressure and energy dissipation along the crossing during the passage of the wheel.

The studies above indicate that the VDBWN is the main reason for the influence on the transition behaviour. By adjusting the VDBWN, the wheel transition behaviour can be improved. Because it is complex to tune both the nose rail and the wing rail, adjusting only the nose rail is applicable from the practical point of view. However, in the case of maintenance, where the wing rail suffers more damage and the nose rail is in relatively good condition, it is preferable to tune the wing rail geometry.
Figure D-20 Comparison of the nose rail profiles with different heights: (a) the nose rail profile above 14 mm below the top of the original wing rail and (b) the nose rail profile above 14 mm below the top of the adjusted wing rail.
Due to the repeated dynamic impact at the crossing, degradation of the track geometry at both the vertical and the lateral directions is commonly observed at railway crossings. Compared with normal track components, the geometry degradation at crossings is more likely to occur, and it grows much faster if proper maintenance is not performed in time. Unfortunately, the maintenance actions at crossings require either special machines (mostly more expensive) or manual treatment, for instance, tamping maintenance. Due to the limited amount of track length at one location and the huge expense of transporting the machines, the cost of tamping the crossings is extremely expensive compared with tamping common track. As a result, the frequency of tamping maintenance is much lower than for other parts of the railway network, although the ideal maintenance frequency should be higher. Moreover, crossings also suffer from many other forms of degradation of the track alignment due to loose fastening systems, broken sleepers, temperature force, or extreme hunting oscillation of the vehicles. Therefore, it is necessary to take into account existing irregularities or degradation of the track geometry to evaluate the crossing design.

In this paper, various track alignments are investigated based on the reference crossing geometry introduced in Section 5:

- reference: ideal track alignment.

Figure D-21 Wheel transition behaviour with VDBWN = 0 mm under adjusted nose rail shape from Figure D-20b: (a) vertical wheel displacement, (b) lateral wheel displacement, (c) contact pressure, and (d) energy dissipation.

6 Effect of track alignment
• irr_1: a vertical deviation in the rail geometry of 8 mm over 10 m at the crossing side (Figure D-22 (a)), which commonly occurs if tamping maintenance at the crossing is not performed in time.

• irr_2: a lateral serpentine deformation of the rail with wavelength 12 m and amplitude 5 mm at the stock rail side of the crossing panel, starting from 16 until 4 m before the nose point (Figure D-22 (b)). This irregularity can happen, for instance, due to damage on the fastening system on one side of the track.

• irr_3: narrow track gauge (1433 mm) starting from 5 m before the nose point until 5 m after the nose point (Figure D-22 (c)), where the nominal track gauge is 1435 mm.

The results of the transition behaviour are shown in Figure D-23. The irregularities of the track alignment influence the wheel kinematics; thus, the location of the transition point is shifted, and the magnitude of the contact pressure and the energy dissipation between the wheel-rail interface are changed. Even without defects on the material or the wheel rail contact surface, the transition behaviour can vary depending on the quality of the track alignment.
Figure D-23 Wheel transition behaviour under varying track alignments: (a) vertical wheel displacement, (b) lateral wheel displacement, (c) contact pressure, and (d) energy dissipation.

7 Effect of wheel profile on the transition behaviour

Because the wheel-rail interface consists of both the rail and the wheel, the effect of the wheels on the vehicle dynamics has been considered as a key element in improving the wheel-rail interface. Research on adjusting the wheel profile to improve the wheel-rail interaction started in the 1970s [14-20]. The worn state of the wheel determines the interval of maintenance and the applicable strategies maintenance, for example, whether the wheel should be replaced, re-profiled, or simply lubricated. For a railway crossing, apart from the rail geometry, the transition behaviour of the wheel is also dependent on the wheel.

Figure D-24 Wheel profiles.

Four different wheels shown in Figure D-24 are considered in studying the effect of wheel profiles: the new s1002 wheel profile, the HIT profile that is used as the standard re-profiling
mode for Dutch passenger trains, a worn root wheel profile (radius of the flange corner is less than 13 mm) and a false flange wheel profile, wherein the latter two were obtained using B-spline representations.

The corresponding transition behaviour of the wheels is shown in Figure D-25. The change of wheel profiles significantly influences the transition behaviour of the wheel when passing through the crossing. With severely worn wheels, such as the worn root wheel and the false flange wheel, the wheel transition point at the crossing is shifted far from the original position. As a result, the potential damage shifts. Moreover, the wheel with a false flange results in a flying wheel over the crossing, that is, after the wheel leaves the wing rail it loses contact with the crossing, flying over a distance and suddenly jumps to the nose rail. Consequently the rail stresses are increased by approximately five times. The worn root wheel also causes high dynamic augments on the rails, as shown in Figure D-25, where the rail stress is increased threefold.

Figure D-25 Wheel transition behaviour under varying wheel profiles: (a) vertical wheel displacement, (b) lateral wheel displacement, (c) contact pressure, and (d) energy dissipation.

The HIT profile is relatively similar to the new s1002 profile; thus, the dynamic responses of the wheels are similar. However, the influence on the transition behaviour is still obvious: the impact occurs earlier with slightly increased rail stress but reduced energy dissipation.

With the exception of newly produced trains, the transverse profiles of railway wheels vary from the ideal new wheel profiles with different levels of deviation. The transverse wheel profiles are essential when analysing the vehicle dynamics, especially at the crossings, where
the rail geometry varies along the track. Therefore, it is necessary to take into account the variation of the wheel profiles to investigate the transition behaviour of the wheel.

8 Interaction influence of the parameters

8.1 Design of experiments

Based on the above study, the design of experiments approach is used to investigate the key, main, and interaction effects on the wheel transition behaviour. DOE is a methodology used to study the influence of two or more independent input variables on the response (output), evaluating which inputs have a significant impact on the output, and what the target level of those inputs should be to achieve a desired result (output). The DOE method has been used to search for an optimal design of rail structures [21] and to collect data points to optimise the railway wheels [22]. In [1], the influence of 14 input parameters on the wheel-rail contact force, wear, and RCF on the switch rail was investigated using the DOE methodology.

This study uses a two-level full factorial design with four input parameters (factors), which measures all the interactions of the different levels of all factors. The following factors are considered:

- The cross-sectional shape of the nose rail as defined by parameter \( \lambda \) (\( \gamma = 1/25 \)).
- The longitudinal height profile of the crossing rails as defined by the VDBWN, which in the present case is described by the parameter \( h \) (\( d=0 \)).
- The quality of track alignment.
- The wheel profile.

The parameters and the level settings in the two-level full factorial design are shown in Table D-4. In total, \( 2^4=16 \) experiments are required.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Parameter description</th>
<th>Low (-)</th>
<th>High (+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>Cross-sectional shape of the nose rail</td>
<td>( \lambda = 1/3 )</td>
<td>( \lambda = 1/2 )</td>
</tr>
<tr>
<td>x2</td>
<td>Longitudinal height profile of the crossing rails</td>
<td>( h = 0 )</td>
<td>( h = 2 )</td>
</tr>
<tr>
<td>x3</td>
<td>Track alignment</td>
<td>no irregularity</td>
<td>lateral irregularity ( ^a )</td>
</tr>
<tr>
<td>x4</td>
<td>Wheel profile</td>
<td>new S1002</td>
<td>HIT profile</td>
</tr>
</tbody>
</table>

\( ^a \): the irregularity \( \text{ir}_L2 \) in Section 6 is used to represent the lateral irregularity on the track

In a two-level DOE study, a factor can be important if it leads to a significant change in the response variable when going from the "-" setting of the factor to the "+" setting of the factor. Unless specified to the contrary, when a factor is claimed to be important, the implication is
that the factor caused a large change in the response. Among all of the factors, a factor setting is best if it results in a typical response that is closest to the desired project goal.

Given the definitions of important and best, the DOE mean plot and the effects plot are applied for determining the important factors and the best settings [23]. The DOE mean plot shows the average response for a given setting ("+" or ") of a factor for each of the studied factors. The effects plot gives a graphical representation of the importance of each factor by showing the absolute effect of each factor. The effect of a factor is defined as the difference between the normalised average response under setting "+" and setting ".". The effect of a factor in the effects plot is demonstrated by the following equations:

\[ Ef_i = |Y_i^+ - Y_i^-| \]

with

\[ Y_i^+ = \frac{av_i^+}{av_i}, \quad Y_i^- = \frac{av_i^-}{av_i} \]

where \( Ef_i \) is the effect of the \( i \)-th factor, \( av_i^+ \) and \( av_i^- \) are the average responses of the \( i \)-th factor for setting "+" and setting ",", and \( av_i \) is the mean response of the \( i \)-th factor.

8.2 Results of DOE

To evaluate the transition behaviour of the crossing, the contact pressure, the energy dissipation and the location of impact on the nose rail are chosen as the outputs (responses) as shown in Appendix D-1. The contact pressure and energy dissipation are calculated as accumulative values using the Kreisselmeier-Steinhauser function; the details of the calculations are documented in [4]. Figure D-26 shows the DOE mean plot and the effects plot for the three responses individually. The average response is normalised by the mean value of the response in all of the DOE mean plots of the study. Based on the given level settings of the parameters, it can be found that

- Factor \( x2 \) (the longitudinal height profile of the crossing rails) has the most significant effect on the contact pressure, whereas the factors \( x3 \) (track alignment) and \( x4 \) (the wheel profile) have little influence on the contact pressure.
- Factors \( x4 \) (the wheel profile) and \( x2 \) (the longitudinal height profile of the crossing rails) have significant influence on the energy dissipation between the wheel-rail interface. Factor \( x1 \) (cross-sectional shape of the nose rail), however, has much less effect on the energy dissipation.
- The location of the impact on the nose rail is mainly determined by factor \( x2 \) (the longitudinal height profile of the crossing). Because the impact location is the most sensitive part of the transition zone, by adjusting the longitudinal height profile of the crossing rails, the potential damage area can be shifted as desired.
Both the contact pressure and the wear index are commonly applied as damage indicators of the railway track; the influence on the multi-response is also studied, where the combined response is defined as a weighted-sum function of the two normalised responses:

$$R = w_1 R_1 / \text{mean}(R_1) + w_2 R_2 / \text{mean}(R_2)$$  \hspace{1cm} (9)

where $R$ is the combined response, $R_1$ and $R_2$ represent the accumulated contact pressure and the accumulated energy dissipation, respectively, which are given the same weights, and $w_1$ and $w_2$ are the weight coefficients (both equal to 0.5). Figure D-27 shows the results of the DOE mean plot and the effects plot for the combined response. The two major factors on the combined response are $x_2$ (the longitudinal height profile of the crossing nose) and $x_4$ (wheel profile). Because of the opposite effect on the contact pressure and the energy dissipation (DOE plots in Figure D-26 (a) and (b)), factor $x_1$ (cross-sectional shape of the nose rail) results in the least influence on the combined response.
9 Concluding remarks and future work

The complex vehicle-track interaction during the transition of the wheel over the railway crossing is studied using the MBS software Vi-Rail, where the transition point along the crossing, the contact pressure, and the energy dissipation in the wheel-rail contact are applied in assessing the transition behaviour. Special attention has been paid to the tuning factors of the dynamic properties of the vehicle-crossing system: the local crossing geometry, the track alignment, and the transverse wheel profiles. These factors have significant influences on the transition behaviour of the wheel.

For the effect of the rail geometry, the shape of the nose rail and the vertical distance between the top of the wing rail and the nose rail have significant influences on the vehicle dynamics during the transition. By changing the height profile of either the nose rail or the wing rail while maintaining the vertical distance between the top of the wing rail and the nose rail, the transition behaviour is slightly affected. On the basis of this study, the most effective parameters defining the crossing geometry have been found.

As with the other components of the track geometry, the track alignment also alters the wheel transition performance over the crossing. Moreover, the wheel profile has considerable effect on the wheel-rail interaction during the transition. Severe worn wheel profiles alter the transition location along the crossing and cause dramatic increases of the contact pressure and energy dissipation in the wheel-rail contact. However, a proper wheel profile, such as the HIT profile, has a positive effect on the transition behaviour of the wheel.

Further, the DOE method with full factorial design was adopted in studying the influence of the crossing geometry, the track alignment and the wheel profile. Although limited by the level settings of the parameter variation, the DOE study shows that the longitudinal height profile of the crossing has the largest effect on the wheel transition behaviour, followed by the wheel profile.

This study provides guidance in the maintenance of crossings and gives useful suggestions for the optimisation of railway crossings.
References


Appendix D-1: Dynamic response of crossing in the DOE study

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<th>Level of inputs</th>
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Robust optimisation of railway crossing geometry

Chang Wan, Valeri Markine, Rolf Dollevoet


**Abstract**

This paper presents a methodology for improving the crossing (frog) geometry through the robust optimisation approach, in which the variability of the design parameters within a prescribed tolerance is included in the optimisation problem. Here, the crossing geometry is defined by parameterising the B-spline represented crosssectional shape and the longitudinal height profile of the nose rail. The dynamic performance of the crossing is evaluated considering the variation of wheel profiles and track alignment. A multipoint approximation method (MAM) is applied in solving the optimisation problem of minimising the contact pressure during the wheel–rail contact and constraining the location of wheel transition at the crossing. To clarify the difference between the robust optimisation and the normal deterministic optimisation approaches, the optimisation problems are solved in both approaches. The results show that the deterministic optimum fails under slight change of the design variables; the robust optimum, however, has improved and robust performance.
1 Introduction

As junctions of railway tracks, turnout crossings are weak parts of the railway network that suffer from high dynamic loads from passing trains. Studies in [1-3] show that the dynamic wheel transition over a crossing is very sensitive to changes in rail geometry and can be improved by adjusting the crossing geometry. Studies concerning numerical optimisation of the crossing geometry have been performed recently [3-4], wherein significant reduction of contact pressure and/or wear in the wheel-rail interface at a crossing has been achieved. These optimisation problems are formulated as deterministic optimisation problems, which tend to push a design towards one or more constraints until the constraints are active. For a design optimisation problem of the crossing geometry, however, uncertainties of the vehicle-track system (e.g., loading conditions and track alignment quality), as well as deviations between the theoretical design and its implementation, are inevitable. If the design variables or some system parameters cannot be achieved exactly, the deterministic optimum (lying on one or more active constraint surfaces or the boundary of the feasible region) will fail to remain feasible.

The decision of a practical optimisation problem is often characterised by the following facts [5]:

- F.1 The optimum solution, even if computed very accurately, may be difficult to implement accurately. Moreover, deviation between a newly implemented design and a design in the service stage is inevitable;
- F.2 Uncertain disturbances, i.e., parameters that influence the evaluated properties of the solution are not deterministic in the real-world;
- F.3 The design must remain feasible for allowable (tolerated) deviations of the optimum solution;
- F.4 Optimal solutions become severely unfeasible in the face of even relatively small changes in the input parameters of the problem.

Therefore, a design optimisation problem should cope with modelling uncertainty/variability and improving or controlling performance variation. Instead of deterministic optimisation, a probabilistic formulation of the optimisation problem, which considers uncertainties, should be used.

Robust optimisation, which is one of the methods for solving a probabilistic design problem [6], is an optimisation theory that addresses optimisation problems in which a certain measure of robustness is sought against uncertainty that can be represented as deterministic variability in the values of the parameters of the problem itself and/or its solution. That is, the optimisation process considers uncertainties in the evaluation of the objective and constraint functions. By itself, the robust optimisation methodology can be applied to every generic
optimisation problem, where one can separate numerical data that can be partly uncertain and are only known to belong to a given uncertainty set from the problem’s structure, which is known in advance and is common for all instances of the uncertainty problem [7]. Being designed to meet some major challenges associated with uncertainty-affected optimisation problems, the major purpose of a robust optimisation is to provide guarantees about the performance of the solution. Applications of robust optimisation in solving engineering problems can be found in many publications such as [8-11].

In the present paper, the robust optimisation approach is proposed in the design of crossing geometry, wherein the wing rails are prescribed to the selected turnout design, while the nose rail is adjusted. Based on the previous parametric study [12] on wheel transition behaviour, three design variables are chosen in the optimisation to tune the longitudinal height profile and the B-spline represented cross-sectional shape of the nose rail. The optimisation is performed with the consideration of varying wheel profiles and various track alignments. To compare with the normal deterministic optimisation approach, the optimisation problems are solved both from the deterministic optimisation and the robust optimisation approaches. Sections 2-4 clarify the criteria of crossing geometry design. The optimisation problem, formulated as deterministic optimisation and robust optimisation problems, is solved and discussed in Section 5 and Section 6, respectively. Concluding remarks are provided in Section 7.

2 Wheel transition at crossings and criteria for improvement

2.1 Transition of wheel over crossing

The main source of damage to railway crossings is the dynamic impact that occurs as a wheel transfers from the wing rail to the nose rail. When impacts occur frequently at the same position, the rail can be easily broken, which is observed as one of the main damage formats of turnout crossings in the Dutch railway network. This is especially dangerous when the supporting patch of the rail/wheel contact at the impact location is small. If the wheel contacts the nose rail too close to the nose point (tip point, TP), wherein the nose rail is very thin, the contact patch will be very small. In this situation, the thin rail sustains significantly high contact pressure and is likely to be broken off. Similarly, the wing rail suffers from high contact pressure when the contact moves to the end of the wheel tread where the contact patch becomes small. Figure E-1 shows the location of numerically simulated contact points on the wheel and the corresponding contact pressure when the train passes in the main-facing direction, wherein it can be observed that the contact pressure becomes very high when the contact moves to the edge of the wheel tread.
Therefore, to prevent severe damage to the rail and wheel, the transition of the wheel-rail contact should be neither too close to TP because the nose rail there is very thin nor too far from TP when the contact between the wheel tread edge and the wing rail is very small. A schematic illustration of the transition zone is shown in Figure E-2. The wheel/rail contacts on the nose rail in zone A or on the wing rail in zone C should be avoided.

Figure E-1 Dynamic response of wing rail and nose rail: (a) location of contact point on wheel, (b) contact pressure.

Figure E-2 Transition area of a crossing—example of through route.
2.2 Criteria of wheel transition improvement

The impact position at the crossing nose, which indicates the location of nose rail damage, is dependent upon the crossing geometry. In [3], it was observed from the field measurements that after grinding/welding maintenance of the crossing, not only was the magnitude of impact significantly reduced but also the location of impact was shifted—leading to wider spread of impacts on the nose rail. A close look at the geometric effect on the transition location has been documented in [12], in which it is shown that both the nose rail shape and the vertical distance between the top of the wing rail and the nose rail have considerable influence on the impact location. This is especially valuable in the grinding/welding maintenance where both the reduction of impact magnitude and stress relief in the observed damaged part are demanded.

To improve the wheel transition behaviour, the study here is aimed at reducing the amplitude of dynamic impact while restraining the location of the impact through optimisation of the crossing geometry.

3 Objective vehicle-turnout system in the study

3.1 Basic vehicle-turnout system

The studied turnout is a standard one (right turn) with a curve radius of 725 m and a crossing angle of 1:15, which is the same model as in [3,12]. In the present paper, the investigations are only for a vehicle passing in the main-facing direction. The vehicle is modelled as a passenger wagon based on the VIRM (Verlengd InterRegio Materieel) passenger train [3,12], with a static wheel load of 89 kN, the travelling speed of 140 km/h (typical speed of VIRM intercity trains in the Netherlands) and friction coefficient of 0.4 (intermediate friction level). The ‘moving track’ model from the multi-body simulation software VI-Rail is used for modelling the dynamic vehicle-turnout interaction, in which a sampling frequency of 5000 Hz is used and the results are low-pass filtered with a cut-off frequency of 500 Hz. Note that, in the ‘moving track’ model, the rails and sleepers are modelled as rigid bodies moving together with wheel load and the track elasticity is independent from the location along the crossing. However, the geometry (track layout) is dependent on the longitudinal location, defined by changing rail profiles, track irregularities, extra rails (guiding rail) along the crossing. Details of the vehicle-turnout model can be found in [3,12].

3.2 Consideration of uncertain parameters

In addition to the specified parameters above, there are still uncertain parameters in the dynamic vehicle-turnout interaction, for instance, the track alignment, the quality of the
fastening system and the deviation of the rail pads/ballast bed. Due to the limitation of the ‘moving-track’ model utilised in the study, which assumes a uniform elastic track property along the track, the variation of parameters w.r.t the track elasticity are not considered (the track elasticity parameters used in the current study can be found in Appendix E-1). The following uncertain parameters are considered during the optimisation.

3.2.1 Wheel profile variation

The wheel profile plays an important role in the transition over the crossing. To investigate the dynamic vehicle-track interaction through numerical simulation, it is requested that the corresponding wheel profiles should be used. In reality, it is unwise to account for each and every wheel profile of all trains that pass through the crossing, not only because of the extremely high computational cost for thousands of simulations under various wheel profiles but also due to the inconvenience of measuring all of the wheel profiles both for different vehicles and for the same vehicles at different service cycles. Moreover, the dramatic increase in computational effort in solving the optimisation problem limits the applicability of considering many wheel profiles. Alternatively, the most representative wheel profiles among the passing trains can be used in the simulations.

In this paper two wheel profiles are selected as the representations of the wheel profiles of the passenger train VIRM in the Netherlands: the standard s1002 wheel profile (a flange height 28 mm) and the HIT wheel profile (close to a worn s1002 wheel profile after 95,000 km rolling distance) with a narrowed flange, as shown in Figure E-3, in which the former is the wheel profile for new wheels and the latter is used as the re-profiling template for VIRM trains in the Netherlands. Because most VIRM trains are either equipped with new wheels (standard s1002) or regularly re-profiled as HIT profiles, it is assumed that these two wheel profiles stand for the average wheel profiles of VIRM trains.

![Figure E-3 Representative wheel profiles of VIRM trains.](image)

3.2.2 Track alignment variation

Track alignment, which has significant influence on the dynamic vehicle-turnout interaction, is one of the most common control items during maintenance and manufacturing design. Here, two different track irregularities are considered: the vertical deviation of rail geometry at the
crossing side, ‘irr_1’ (Figure E-4a), and the lateral deviation in the rail geometry before the crossing, ‘irr_2’ (Figure E-4b). The two artificial irregularities are considered in addition to the ideal track alignment to evaluate the feasibility of the crossing geometry design, i.e., a feasible design should remain feasible under any of the three track alignments.

![Figure E-4](image)

Figure E-4 Track irregularities considered in crossing design: (a) vertical and (b) lateral deviation of track alignment.

By including the above-mentioned uncertainties in the basic vehicle-turnout system, the problem of optimising the crossing geometry can be formulated and solved as presented in the following sections.

### 4 Parameterisation of crossing geometry

According to the manufacturing process of constructed crossings, the nose rail is produced by cutting/grinding the rail segments with a rail profile of the corresponding normal rail, whereas the general way of tuning the wing rail is to adjust its longitudinal height profile through a bending treatment. The results in [12] show that the vertical distance between the top of the wing rail and the nose rail affects the wheel transition behaviour. For simplicity of the problem, only the nose rail will be tuned here.

Definition of the crossing geometry is based on the approach used in [12], wherein the nose rail is defined by five control sections along the nose rail (Figure E-5) with distances of 0, 150, 300, 600, and 1050 mm from TP; the wing rail shape is fixed, using the one from the current manufacturing process. The longitudinal height profile of the nose rail is adjusted by tuning the height of the five control sections, where each height segment between two control sections is linear. It is assumed that the impact does not occur before section II where the nose rail is too thin to sustain the dynamic load. Thus, only the heights of sections III and V are to
be tuned. The longitudinal height profile of the nose rail is shown in Figure E-5, wherein the height of 0 mm is set to 14 mm below the top of the normal rail. When tuning the height profile, the monotonic increase in the profile is maintained to obtain a realistic design. The heights of the control cross-sections III and V are defined by parameters $h_1$ and $t$

$$h_1 = h, \quad h_2 = h + t$$

![Figure E-5 Longitudinal height profile of nose rail.](image)

The shape of the nose rail is defined using a cubic B-spline, in which six control points are used, as shown in Figure E-6. The parametric study in [12] shows that $P_3$ has a major influence on the dynamic transition behaviour of the wheel, whereas the remaining control points are less affective. Therefore, $P_3$ is set as a movable control point along the line PM to adjust the transverse shape of the nose rail. The position of $P_3$ is defined by the parameter $\lambda$: $\lambda = |PP_3/PM|$.

![Figure E-6 B-spline representation of nose rail profile (symmetric).](image)
A positive $\lambda$ means $P_3$ locates inside the segment $PM$, and a negative $\lambda$ indicates that $P_3$ moves to the other side of $P$ (outside the segment $PM$).

Thus, the crossing geometry here is defined by the three parameters: $h$, $t$ and $\lambda$.

5 Deterministic optimisation problem

The multipoint approximation method (MAM) [13-15], which has been applied in the design optimisation of railway tracks [3,16,17], is used to solve the crossing geometry optimisation problem. Firstly, the deterministic optimisation problem is to be solved without considering the tolerance of design variables.

5.1 Formulation of deterministic optimisation problem

A general optimisation problem can be stated in the following form:

Minimise

$$F_0(x) \rightarrow \min, \quad x \in \mathbb{R}^n$$

subject to

$$g_j(x) \leq 1, \quad j = 1, \cdots, M$$

and

$$A_i \leq x_i \leq B_i, \quad i = 1, \cdots, N$$

where $F_0$ is the objective function; $g_j$ is the constraint; $x$ is the vector of design variables; $A_i$ and $B_i$ are the side limits which define the lower and upper bounds of the $i$-th design variable.

5.1.1 Design variables

The design variables of the optimisation problem are the adjusting parameters of the crossing geometry, as introduced in Section 4.

$$x = [h, t, \lambda]$$

with:

$$5 \leq h \leq 14 \text{ (mm)}$$

$$0 \leq t \leq 6 \text{ (mm)}$$

$$-1/5 \leq \lambda \leq 1$$

The reference design is $x_{\text{ref}} = [9, 2, 1/3]$. Dynamic vehicle-track interaction will be analysed for each of the two different wheel profiles shown in Figure E-3.
5.1.2 Objective function

The objective function is formulated based on equation (6), wherein the contact pressure ($S$) and wear number ($W$) are to be minimised:

$$F_0(x) = w_1 \sum_{i=1}^{d} \alpha_i \frac{S_i(x)}{S_{max}(x)} + w_2 \sum_{i=1}^{d} \alpha_i \frac{W_i(x)}{W_{max}(x)} \rightarrow \text{min}_i$$  \hspace{1cm} (6)

where $d$ is the total number of wheel profiles evaluated in the optimisation problem. $\alpha_i$ is the weight coefficient for the $i$th wheel profile, $\sum_{i=1}^{d} \alpha_i = 1$. In the present paper, two different wheel profiles are considered; thus, $d$ equals 2. Considering the fact that the HIT wheel profile (second wheel) is more commonly used than the standard s1002 wheel profile (first wheel), the HIT wheel is given more weight. The weight coefficient pair $[\alpha_1, \alpha_2]=[0.25, 0.75]$ is applied. $S_i$ and $W_i$ are the accumulative contact pressure and wear number on the crossing, respectively, and are expressed in the form of the Kresselmeier-Steinhauser function (KS function) [3].

$$S_i(x) = 1 - \ln \left[ \sum_{i=1}^{T} e^{\mu S(x,t_i)/S_{max}} \right] \cdot S_{max}(x)$$  \hspace{1cm} (7)

$$W_i(x) = 1 - \ln \left[ \sum_{i=1}^{T} e^{\mu W(x,t_i)/W_{max}} \right] \cdot W_{max}(x)$$  \hspace{1cm} (8)

Where:

$$S_{max}(x) = \max(S(x,t_i)), \quad i = 1, \ldots, T$$

$$W_{max}(x) = \max(W(x,t_i)), \quad i = 1, \ldots, T$$

Here, $T$ is the total number of time points during the simulations. The parameter $\mu$ determines the discrepancy between $S$ ($W$) and the most critical value $S_{max}$ ($W_{max}$). In the present paper, $\mu$ equal to 50 is used in all KS functions. A more detailed explanation of the KS formation of the objective can be found in [18-19]. The normalising factors $S^*$ and $W^*$ from equations (7-8) are the cumulative contact pressure and wear number, respectively, of the reference design corresponding to the $i$th wheel profile.

$w_1$ and $w_2$ are the weight coefficients for contact pressure and wear, respectively, $w_1 + w_2 = 1$. For the sake of simplicity, the evaluation of wear number is ignored in this optimisation problem (i.e., $w_2 = 0$). It should be noted that during the optimisation the contact pressure at each time point refers to the largest contact pressure from all of the contact points in the case of multipoint contact.
5.1.3 Constraints

Several constraints for the dynamic response of the wheel transition behaviour are defined in the basic optimisation problem.

**Geometric constraints: g\_geo**

Constraints on the geometric shape of the nose rail are considered in the problem. In addition to the side limits of the design variables, additional restrictions are posed on the longitudinal height profile of the nose rail and the cross-sectional shape of the nose rail.

On the one hand, the height of the nose rail should be monotonically increased between cross-sections I and IV, and the maximum height of the nose rail should not exceed 14 mm. On the other hand, the spline-represented nose rail shape described by a set of points with $y$-$z$ coordinates should be a smooth convex curve (Figure E-6), which requires the curvature of the profile at any segment (defined by three adjacent points) to be no larger than zero. Here, the curvature sign of the $i$-th segment of the nose rail profile for design vector $x$ is defined as

$$cv_i = (p_{i+1} - p_i) \cdot (p_i - p_{i-1}), \quad i = 2, \ldots, n-1$$

(9)

where $p_i$ is the coordinates of the $i$-th point (the first point starts from the top of the nose rail centre) of the nose rail profile and $n$ is the total number of points. The constraint of the profile convexity is expressed as

$$g_{cv} = \sum_{i=2}^{n-1} \max[cv_i(x), 0] \leq 0$$

(10)

**Constraint on impact location: g\_iml**

The transition of passing wheels is restrained in the allowed area as discussed in Section 2, that is, the wheel-rail contact should not occur in zone A and zone C (Figure E-2). In this paper, zone A is defined as 0~250 mm from TP on the nose rail and zone C starts from 650 mm away from TP on the wing rail. The location of impacts on the nose rail is thus restricted between 250 and 650 mm from TP.

**Constraint on derailment: g\_drm**

To avoid derailment, a constraint is imposed on the position of the contact point w.r.t the wheel. The danger zone of wheel-rail contact is assumed as 45 mm away from the nominal centre (where the wheel radius equals to the nominal wheel radius) of the wheel towards the flange back side, as shown in Figure E-7. Wheel-rail contact in the danger zone is indicated as a wheel climbing over the rail, which will consequently result in derailment. Therefore, wheel-
rail contact should not occur in the danger zone. As an exception, the wheel flange back contact on the guard rail is not recognised as a derailment risk.

![Figure E-7 'Danger zone' of wheel-rail contact on wheel.](image)

It should be noted that the constraint $g_{geo}$ is evaluated without dynamic simulations, whereas the constraints $g_{iml}$ and $g_{drm}$ are investigated based on the dynamic response of the crossing under the ideal track alignment and the track alignments irr_1 and irr_2 (Section 3.2.2). The constraints are formulated as

$$
\begin{align*}
    g_{geo} &= g_{geo}(x) \leq 1, \\
    g_{iml} &= \max(g_{iml}(x)) \leq 1, & j=1,2,3 \\
    g_{drm} &= \max(g_{drm}(x)) \leq 1, & j=1,2,3
\end{align*}
$$

Here, $j$ refers to the condition of the track alignment: $j=1$, ideal track alignment; $j=2$, track alignment irr_1; and $j=3$, track alignment irr_2.

Therefore, the deterministic optimisation problem formulated using functions (6) and (11) with the design vector described in (4)-(5) can be solved.

### 5.2 Results of deterministic optimisation problem

Similar to many common optimisation tools, MAM is not a global optimisation method that can guarantee a solution to be the global minimum in a nonlinear programming problem. To obtain a solution as close to the global optimum as possible, the optimisation problem is solved with various initial points, among which the solution (feasible) with the minimum objective value is selected as the optimum.

#### 5.2.1 Optimal solution from deterministic optimisation

The optimum design of the deterministic optimisation problem (deterministic optimum) is obtained as $[h, t, \lambda] = [7.030, 3.186, 0.155]$. A comparison of the optimum design and the reference design is shown in Figure E-8, wherein it can be seen that the nose rail is lower than
the reference design; the cross-sectional shape of the nose rail is changed as well, wherein
the gauge corner has moved outwards.

Figure E-8 Comparison of crossing geometry between deterministic optimum (opt) and reference design (ref): (a) longitudinal height profile of nose rail, (b) changing nose rail shape at locations 300, 600 and 900 mm away from TP.

It is interesting to note that in a previous study [3], the nose rail was raised up for the design with optimisation of only the contact pressure, as shown in Figure E-9. To understand the difference, the contact pressure of the current optimum design is presented in Figure E-10. In this design, the impact occurs at approximately 500 mm away from TP (location of maximum contact pressure), where the vertical distance between the top of the wing rail and the nose rail is 4.82 mm. In the previous study, the impact was located approximately 205 mm away from TP, and the vertical distance between the top of the wing rail and the nose rail was 4.63 mm, which is very close to the optimal design obtained here. This indicates that the vertical distance between the top of the wing rail and the nose rail at the transition zone, especially the transition point, is a key factor that influences the dynamic behaviour of a crossing.

Figure E-9 Longitudinal height profile of crossing nose (ref: reference design, opt: optimal solution from previous study [3]).
The reason why the two optimisations result in designs with different impact locations can be explained by the fact that the current study has a constraint on the transition location (250-650 mm away from TP) and thus the impact location is restricted; however, no constraint of impact location was imposed in the former study. Moreover, the track alignments used for calculating the objective function are different in the two studies: track alignment irr_1 was used in [3], and the ideal track alignment is used here.

To minimise the contact pressure due to impact, the longitudinal height profile of the crossing rails is, however, not the unique parameter that determines the impact location. An optimised solution is obtained as a result of the longitudinal height profile and the cross-sectional shape of the nose rail.

5.2.2 Assessment of deterministic optimal solution under design uncertainties

In the manufacturing process and the grinding/welding maintenance of the crossings, a deviation between the theoretical design and the implementation is inevitable. Thus, a further evaluation of the dynamic crossing performance that accounts for the design uncertainties is important.

A three-level, full-factorial design of experiments (DOE) is applied for sampling the variations of the design variables with a given tolerance. Table E-1 shows the parameters and level settings of the DOE, in which the variation of the longitudinal height profile of the nose rail is simplified as a combined parameter to reduce the sampling size. A relatively large tolerance (±0.1 mm) of the longitudinal height profile (x2) is considered in the level setting, corresponding to the tolerance during the manual grinding/welding maintenance of crossings. In total, there are 3^3=9 design variances, i.e., variances of the design vector, as listed in Table E-2.
Table E-1 Parameters and level settings in the full-factorial design.

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<td>[0, 0]</td>
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Table E-2 Design variances.

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<thead>
<tr>
<th>Design variance</th>
<th>Level of factors</th>
<th>x1</th>
<th>x2</th>
</tr>
</thead>
<tbody>
<tr>
<td>v1</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>v2</td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>v3</td>
<td></td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>v4</td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>v5</td>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>v6</td>
<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>v7</td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>v8</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>v9</td>
<td></td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The feasibility and the robustness of the design as a result of applying the design variances is checked. Here, the constraints on the impact location \( g_{im} \) and on the derailment \( g_{drm} \) that are expressed in equation (11) are used to evaluate the design feasibility. Table E-3 shows the results of the objective value and the feasibility evaluation of the design under each design variance, in which it can be observed that the constraint on derailment is violated under variance v1 and v4; therefore, the design becomes unfeasible. Moreover, the objective values of the unfeasible designs are much higher, which results in increasing the relative standard deviation (RSD) of the sampling objective values.

These results indicate that the optimum design from the deterministic optimisation failed with slight variation of the design vector. Therefore, a probabilistic optimisation approach should be applied to deal with the uncertainty of design parameters in the optimisation problem.
Table E-3 Results of feasibility and robustness analysis (deterministic optimum).

<table>
<thead>
<tr>
<th>Design variance</th>
<th>Objective value</th>
<th>$g_{iml}$</th>
<th>$g_{drm}$</th>
<th>feasibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>v1</td>
<td>1.295</td>
<td>0.8323</td>
<td>1.4123</td>
<td>unfeasible</td>
</tr>
<tr>
<td>v2</td>
<td>0.792</td>
<td>0.8084</td>
<td>0.5294</td>
<td>feasible</td>
</tr>
<tr>
<td>v3</td>
<td>0.777</td>
<td>0.7964</td>
<td>0.5270</td>
<td>feasible</td>
</tr>
<tr>
<td>v4</td>
<td>1.294</td>
<td>0.8323</td>
<td>1.4173</td>
<td>unfeasible</td>
</tr>
<tr>
<td>v5</td>
<td>0.738</td>
<td>0.7964</td>
<td>0.5218</td>
<td>feasible</td>
</tr>
<tr>
<td>v6</td>
<td>0.780</td>
<td>0.7964</td>
<td>0.5243</td>
<td>feasible</td>
</tr>
<tr>
<td>v7</td>
<td>0.794</td>
<td>0.8323</td>
<td>0.5243</td>
<td>feasible</td>
</tr>
<tr>
<td>v8</td>
<td>0.740</td>
<td>0.8203</td>
<td>0.5246</td>
<td>feasible</td>
</tr>
<tr>
<td>v9</td>
<td>0.783</td>
<td>0.7964</td>
<td>0.5218</td>
<td>feasible</td>
</tr>
</tbody>
</table>

\[ \text{RSD of objective value: } 0.260306 \]

*The constraint values of $g_{iml}$ and $g_{drm}$ here are chosen as the maximum values among the sub-constraint functions of $g_{iml}$ and $g_{drm}$, respectively. A constraint value larger than 1 means this constraint is violated.

6 Robust optimisation problem

6.1 Formulation of robust optimisation problem

A robust optimisation approach is formulated based on the deterministic optimisation problem to cope with the deviation of the design in the implementation process.

6.1.1 General formulation of robust optimisation problem

A generic robust optimisation problem can be expressed as in [20]:

Minimise

\[ F_0(\mathbf{x}, \xi), \quad \mathbf{x} \in \mathbb{R}^n, \xi \in \mathbb{R}^k \]  \hspace{1cm} (12)

subject to

\[ g_j(\mathbf{x}, \xi) \leq 1, \quad j = 1, \ldots, M \]  \hspace{1cm} (13)

and

\[ A \mathbf{x}_i \leq \mathbf{b}_i, \quad i = 1, \ldots, N \]  \hspace{1cm} (14)

where the design vector $\mathbf{x}$, constraints $\mathbf{g}(\mathbf{x})$, and the objective function $F_0(\mathbf{x})$ are all of the same as those in the deterministic optimisation problem. The parameter $\xi$ stands for the disturbance vectors or parameter uncertainties, which in this study is the variation of the design vector.
6.1.2 Objective function

Different from reliability-based approaches that focus on the probability of constraint satisfaction or violation [21-22], a robust design method emphasises primarily the level of performance variation, i.e., the sensitivity of the design [6,7,23]. In a robust optimisation problem, a design with maximum/minimum ‘mean on target’ and ‘minimised variance’ under uncertainties is sought for. Basically, the variance of the structural performance can be roughly described by its standard deviation (SD) or RSD. Therefore, a robust optimisation problem is generally formulated as a multi-objective problem:

\[
F^\text{rob}_0(x, \xi) \equiv [\mu(F_0(x, \xi)), \sigma(F_0(x, \xi))] \rightarrow \min
\]  

(15)

\(\mu(F_0(x, \xi))\) is the mean value of the stochastic response \(F_0(x, \xi)\) and \(\sigma(F_0(x, \xi))\) is the variance of the response \(F_0(x, \xi)\). A common approach for solving this problem is to use a weighted-sum objective function \(F^\text{rob}_0(x, \xi)\):

\[
F^\text{rob}_0(x, \xi) = \alpha \frac{\mu(F_0(x, \xi))}{\hat{\mu}} + (1-\alpha) \frac{\sigma(F_0(x, \xi))}{\hat{\sigma}}, \rightarrow \min
\]  

(16)

where \(\alpha \in (0,1)\) is the weight of the mean and \(1-\alpha\) is the weight of the variance of the response; \(\hat{\mu}\) and \(\hat{\sigma}\), which are used as normalising factors, are the mean and the variance of the response of the reference design, respectively.

In this study, the formulation of the robust optimisation problem is adjusted in consideration of the following:

- It is difficult to choose proper weights for the mean and variance of the objective value.
- A weighted-sum formulation has the risk of increasing one of the objectives.

To gain the minimum mean response and to maintain the robustness of the response, a single-objective formulation that considers only the mean response \((\alpha = 1)\), as shown in equation (17). Meanwhile, the relative standard deviation (RSD) of response \(F_0(x, \xi)\) is treated as a constraint \((g_{\text{rsd}})\).

\[
F^\text{rob}_0(x, \xi) = \frac{\mu(F_0(x, \xi))}{\hat{\mu}}, \rightarrow \min
\]  

(17)

\[
g_{\text{rsd}}(x, \xi) = \frac{\sigma(F_0(x, \xi))}{\hat{\sigma}}/0.05 \leq 1
\]  

(18)

That is, the RSD should not be larger than 0.05 so that the stochastic response of the optimum solution can be assured with a confidence level of no less than 90%.
6.1.3 Constraints

A common formulation of the constraint function is shown in equation (13), where all of the constraints are calculated considering variation of the design vector. Moreover, a robust solution should have a minimised SD of the response both for the objective value and for the constraint values. In this study, however, the constraints are treated in an adjusted format.

Firstly, minimisation of the variance of constraints is not required when applying design variances. That is, no extra constraints are imposed on the basic constraints as in the case of the objective function (equation (18)). In this way the focus of robustness is on the objective values.

Secondly, the satisfaction/violation of constraints under design variances is not evaluated for all constraints. Here, the constraints \( g \text{-}\text{iml} \) and \( g \text{-}\text{drm} \) are assessed for each design variance, whereas the constraint \( g \_\text{geo} \) is assessed only for the original design vector at each search point during the optimisation. Therefore, a tolerance of the geometry is allowed if all constraints are satisfied for the original design. The feasibility of a design is driven by the satisfaction of the constraints \( g \_\text{iml} \) and \( g \_\text{drm} \) obtained from the dynamic behaviour of the crossing under design variances. Thus a constraint is defined by the worst response from the design variances, that is, the one most violated or closest to violation among all design variances.

The constraints of the robust optimisation problem are adjusted as

\[
\begin{align*}
    g \_\text{geo} & \equiv g \_\text{geo}(x) \leq 1, \\
    g \_\text{iml} & \equiv \max(g \_\text{iml}(x, \xi)) \leq 1, \\
    g \_\text{drm} & \equiv \max(g \_\text{drm}(x, \xi)) \leq 1, \\
    g \_\text{f} (x, \xi) & = \sigma(F(x, \xi))/0.05 \leq 1. 
\end{align*}
\]  

(19)

Therefore, the robust optimisation problem formulated using functions (17) and (19) with the design vector in equations (4)-(5) can be solved. Now it is in general form equations (12)-(14) and any method for solving nonlinear programming problems can be used.

6.2 Robustness evaluation

It is assumed that a design obtained from the robust optimisation should remain feasible and improved with a meaningful variation of the design vector. To evaluate the robustness of a design, the constraints and objective values must be estimated for all variations of the design vector during optimisation. Consequently, robust optimisation can become prohibitively expensive. Instead of infinite variations, sampling methods can be used to measure the possible parameter variations, through which the size of estimation points can be defined. In this study, the design variances from Table E-1 and Table E-2, which are sampled using the
DOE method, are used to represent the uncertainty of the design vector in the robust optimisation problem.

6.2.1 Modified robustness evaluation approach

In principle, the robustness of each design should be evaluated by assessing the design under all design variances. To reduce the calculation time of the robust optimisation, a computationally cheaper approach is applied by performing a selective robustness evaluation. Once the design becomes unfeasible under one design variance, the robustness analysis of the design is stopped. Therefore, the objective value in equation (17) and constraint values in equation (19) are calculated based on the evaluated design variances in the robustness analysis, which may not include all nine variances listed in Table E-1 and Table E-2.

6.2.2 Extra penalty for unfeasible design without applying design variance

Before applying design variances, the feasibility of the design from the current search point is checked. It is suggested that a design that becomes unfeasible before applying design variances is farther beyond the feasible area than a design that becomes unfeasible after assigning design variances. Because the objective and constraint values are obtained from the evaluated design variances, in the case of a sampling size equal to one, the RSD of the objective value will be zero. Hence, the constraint $g_{nd}$ is always satisfied. Although the design is unfeasible, the severance of infeasibility may not be recognised among other search points that become unfeasible after assigning design variances apart from the original design. To cope with this situation, an extra penalty to the RSD is proposed if the design is unfeasible without applying design variances.

A new RSD is defined based on the violated constraints expressed as

$$\sigma(f_{g}(\mathbf{x})) = 0.05 + \sum_{j=1}^{M} k_{\rho} \cdot \max[g_{j}(\mathbf{x}) - 1, 0]$$

(20)

where $M$ is the total number of constraint functions defined in equation (11); $g_{j}(\mathbf{x}) > 1$ indicates that the constraint is violated; $k_{\rho}$ is the penalty factor for RSD, which, in this study, is taken as $k_{\rho} = 10^2$. Therefore, a high RSD will be obtained, which consequently results in severe violation of the constraint $g_{nd}$.

Results of optimisations

The robust optimisation problem is solved using various initial points, among which the solution (feasible) with minimum objective value is selected as the optimum.
6.3.1 Optimal solution from robust optimisation

The optimum design from the robust optimisation (robust optimum) is obtained as \([h, t, \lambda]=[8.250, 2.300, 0.186]\). A comparison of the reference design and the optimum designs from both the deterministic optimisation and the robust optimisation is shown in Figure E-11, wherein it can be observed that the nose rail of the robust optimum is higher than that of the deterministic optimum and lower than that of the reference design.

![Figure E-11 Comparison of crossing geometry between the optimum solutions (opt1: deterministic optimum, opt2: robust optimum) and reference design (ref): (a) longitudinal height profile of nose rail, (b) changing nose rail shape at locations 300, 600 and 900 mm away from TP.](image)

6.3.2 Robustness of optimum solution

Table E-4 shows the results of the objective value and the feasibility of the design under each design variance considered during the optimisation, indicating that the optimum solution from the robust optimisation is robust and the design remains reliable under all design variances.

Table E-4 Results of feasibility and robustness analysis (robust optimum).

<table>
<thead>
<tr>
<th>Design variance</th>
<th>Objective value</th>
<th>(g_{iml})</th>
<th>(g_{drm})</th>
<th>feasibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>v1</td>
<td>0.716</td>
<td>0.7965</td>
<td>0.5266</td>
<td>feasible</td>
</tr>
<tr>
<td>v2</td>
<td>0.739</td>
<td>0.7965</td>
<td>0.5276</td>
<td>feasible</td>
</tr>
<tr>
<td>v3</td>
<td>0.775</td>
<td>0.7965</td>
<td>0.5292</td>
<td>feasible</td>
</tr>
<tr>
<td>v4</td>
<td>0.719</td>
<td>0.7965</td>
<td>0.5243</td>
<td>feasible</td>
</tr>
<tr>
<td>v5</td>
<td>0.743</td>
<td>0.7965</td>
<td>0.5251</td>
<td>feasible</td>
</tr>
<tr>
<td>v6</td>
<td>0.773</td>
<td>0.7965</td>
<td>0.5271</td>
<td>feasible</td>
</tr>
<tr>
<td>v7</td>
<td>0.732</td>
<td>0.7965</td>
<td>0.5221</td>
<td>feasible</td>
</tr>
<tr>
<td>v8</td>
<td>0.751</td>
<td>0.7965</td>
<td>0.5233</td>
<td>feasible</td>
</tr>
<tr>
<td>v9</td>
<td>0.780</td>
<td>0.7965</td>
<td>0.5251</td>
<td>feasible</td>
</tr>
</tbody>
</table>

RSD of objective value: 0.032300

\(^{b}\)The constraint values of \(g_{iml}\) and \(g_{drm}\) here are chosen as the maximum values among the subconstraint functions of \(g_{iml}\) and \(g_{drm}\), respectively. A constraint value larger than 1 means this constraint is violated.
The results of the dynamic wheel transition behaviour, including the response under each design variance estimated during the robust optimisation, are shown in Figure E-12, wherein the results of the robust optimum and the reference design are marked with ‘opt’ and ‘ref’, respectively. To clarify the results, the comparison is performed in six groups classified by two variations of the wheel profiles and three variations of the track alignment, as listed in Table E-5.

Table E-5 Variations of the vehicle-turnout system.

<table>
<thead>
<tr>
<th>Variations</th>
<th>Wheel profile</th>
<th>Track alignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>s1002</td>
<td>ideal alignment</td>
</tr>
<tr>
<td>A2</td>
<td>HIT</td>
<td>ideal alignment</td>
</tr>
<tr>
<td>B1</td>
<td>s1002</td>
<td>with vertical irregularity irr_1</td>
</tr>
<tr>
<td>B2</td>
<td>HIT</td>
<td>with vertical irregularity irr_1</td>
</tr>
<tr>
<td>C1</td>
<td>s1002</td>
<td>with lateral irregularity irr_2</td>
</tr>
<tr>
<td>C2</td>
<td>HIT</td>
<td>with lateral irregularity irr_2</td>
</tr>
</tbody>
</table>

The core part of the wheel transition zone (zone B in Figure E-2), i.e., from the location of the start of contact on the nose rail to the end of contact on the wing rail, is shown in Figure E-12a. It can be observed that the first contact on the nose rail has been shifted farther from TP compared with the reference design, whereas the location of the last contact on the wing rail has little change.

Figure E-12b shows the impact location defined by the location of the maximum contact pressure, wherein the impact location is altered to be farther from TP, which indicates that the potential damage location moves further away. Because the thickness of the nose rail increases gradually from TP, the farther the impact on the nose rail is, the less the damage to the crossing will be.

Figure E-12c compares the accumulated contact pressure (equation (8)) between the reference design and the robust optimum, in which significant reduction of the contact pressure is recognised. It should be mentioned that the upper limit of the Y-axis has been fixed at 5000 MPa, which covers the results under all of the design variances of the optimum design and most design variances of the reference design. The results of the reference design under some design variances that have much higher contact pressure are not visible in this figure. In other words, the reference design is not robust according to the evaluations in this study. The optimum design, however, has robust behaviour, as shown in Figure E-12c.

The above results show that the design from the robust optimisation has robust and improved performance under the ideal track alignment and the existence of irregularities, both for the nominal s1002 wheel profile and for the HIT profile. Regarding the wheel profiles, the
improvement of the transition behaviour is more significant under the HIT wheel profile which has been weighted more in the objective function. Moreover, it is observed that the wheel transition behaviour differs under various track alignments. Among the three cases of track alignment, the lateral serpentine deformation of the rail (groups C1 and C2) has more negative influence on the wheel transition behaviour, which indicates that the occurrence of this type of track alignment is more risky for crossings.

![Figure E-12 Location of wheel transition](image)

Note that, because the reduction of the energy dissipation at the wheel/rail contact patch is excluded in the robust optimisation problem, an increase in the energy dissipation is obtained as shown in Figure E-12d, which raises the risk of wear damage to the crossing. Therefore, the optimisation problem can be adjusted to take into account the attenuation of wear.

### 7 Conclusions

This paper presents a methodology for improving wheel transition behaviour by tuning the crossing geometry through a robust optimisation approach, in which the variations of wheel profiles and track alignments have been accounted for. The wheel transition behaviour is evaluated based on the contact pressure at the wheel-rail interface and the location of the wheel transition from the wing rail to the nose rail. If the transition locates within the defined safe area, then the lower the contact pressure is, the better the wheel transition behaviour
will be. Compared with the current manufacturing design, the optimum design results in lower height profile and a wider, flatter cross-sectional shape of the nose rail.

To clarify the difference between a normal deterministic optimisation approach and a robust optimisation approach, the problem is solved using both the deterministic and robust optimisation methods. The deterministic optimal solution results in better objective value, although it fails under slight change of the design vector. The optimal solution from the robust optimisation, however, has robust performance with variation of the design vector within a defined tolerance, in which the improvement of wheel transition behaviour is observed for various wheel profiles under various track alignments.

Additionally, the study shows that the vertical distance between the top of the wing rail and the nose rail at the transition area, especially at the impact location, is a key factor influencing the dynamic wheel transition behaviour, which in combination with the cross-sectional shape of the nose rail, determines the impact location and the level of impact.

The proposed methodology can be applied both for new crossing designs and the grinding/welding maintenance of existing crossings. For a more specific application case, the parameters of the vehicle-track system can be tuned accordingly. The uncertainty of the vehicle and/or track condition can also be specified by considering the most potential variations of the system.

References


Appendix E-1: Track properties used in the simulations.

<table>
<thead>
<tr>
<th>Track components</th>
<th>Stiffness [MN/m]</th>
<th>Damping [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail pad</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>1420</td>
<td>34</td>
</tr>
<tr>
<td>Lateral</td>
<td>280</td>
<td>58</td>
</tr>
<tr>
<td>Roll</td>
<td>360</td>
<td>390</td>
</tr>
<tr>
<td>Ballast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>120</td>
<td>48</td>
</tr>
<tr>
<td>Lateral</td>
<td>120</td>
<td>40</td>
</tr>
<tr>
<td>Roll</td>
<td>130</td>
<td>290</td>
</tr>
</tbody>
</table>
List of publications

JOURNAL ARTICLES


CONFERENCE PUBLICATIONS


- Chang Wan, Valeri Markine, Ivan Shevtsov, Rolf Dollevoet. Improvement of train-track interaction in turnouts by optimising the shape of crossing nose. *22nd International...*


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