Memorandum M-361

Evaluation of the new
ASTM geometry function
for Compact Tension Specimens

by

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ABSTRACT

In this report an evaluation is made of the new geometry function for the ASTM Standard Compact Tension Specimen as described in the ASTM standard E399-78.

It is shown that the old geometry function (ASTM standard E399-74 and earlier) does not differ from the new one by more than 5% in the range $0.29 < a/W < 0.77$.

This implies that old $K_{IC}$-data obtained in that range are still acceptable.

CONTENTS

1. Introduction
2. Results
3. Discussion
4. Conclusion
5. References
6. Tables and figures
1. INTRODUCTION

The stress intensity factor $K_I$ of a standard compact tension specimen can be calculated using the formula:

$$K_I = \frac{P}{B \cdot W} \cdot F(a/W)$$  \hspace{1cm} (1)

where:

$P$ = load at 5% offset
$B$ = specimen thickness
$W$ = specimen width
$a$ = crack length
$F(a/W)$ = geometry function

For the ASTM Standard Compact Tension specimen with $H/W = 0.6$ (see figure 1 for relative dimensions) the expression for the geometry function $F(a/W)$ that has been used for a long time, is:

$$F(a/W) = \left[ 29.6(a/W)^{1/2} - 185.5(a/W)^{3/2} + 655.7(a/W)^{5/2} ight] - 1017(a/W)^{7/2} + 638.9(a/W)^{9/2}$$  \hspace{1cm} (2)

In the ASTM E399 standard the validity of equation (2) is explicitly limited to a crack length ratio $0.45 \leq a/W \leq 0.55$ (ref. 1a).

For fatigue tests where a much wider range of $a/W$ has to be used, this limitation always has been a serious disadvantage. So in the past several attempts were made to show that the validity of eq. (2) could be extended. One of the most recent attempts is that of Duggan et al. who showed that the validity of eq. (2) could be extended to crack length ratios $0.3 \leq a/W \leq 0.7$ (ref. 2).

In 1978 ASTM introduced a new geometry function, derived by Srawley (ref. 3). This function was derived to fit available analytical data in the range $0.2 \leq a/W \leq 1.0$.

The expression of this geometry function is:

$$F(a/W) = \frac{(2 + a/W)}{(1 + a/W)^{3/2}} \left[ 0.886 + 4.64(a/W) - 13.32(a/W)^2 ight. \left. + 14.72(a/W)^3 - 5.6(a/W)^4 \right]$$  \hspace{1cm} (3)
Eq. (3) is now incorporated in the new ASTM standard E399-78 (ref. 1b), with a validity range of $0.2 \leq a/W \leq 1.0$.

In this report a comparison is made between eq. (2) and eq. (3), respectively called the "old" (or $F_1$) and the "new" (or $F_2$) ASTM equation. The main purpose of this comparison is to investigate for which crack range $a/W$ the $K_{IC}$-data obtained with the old equation are now still acceptable.

The calculations were made on a DEC PDP 11/05 computer.

2. RESULTS

The value of both the old ($F_1$) and new ($F_2$) ASTM geometry functions is calculated for a crack length range $a/W$ from 0 to 1.0. The results are given in table 1. In figure 2 the results are graphically presented. As can be seen from table 1 and figure 2 there is a noticeable difference between the two equations for crack length ratios $a/W < 0.25$ and $a/W > 0.80$.

For a crack length ratio approaching 0, $F_1$ goes to the value 0, while $F_2$ reaches the discrete value of 1.772. For a crack length ratio approaching 1, $F_1$ goes to the discrete value of 121.700, whilst $F_2$ goes to infinity.

In table 1 also the relative difference between the two equations $[(F_2-F_1)/F_2 \times 100\%]$ is given. In figure 3 this difference is presented graphically.

(It is sometimes convenient to write eq. (1) as:

$$K_1 = \frac{P}{B \cdot W} \cdot \sqrt{a \cdot F'(a/W)}$$

(1a)

The factor $\frac{P}{B \cdot W}$ then has the dimension of a stress.

The geometry functions $F_1$ and $F_2$ can then be written as:

$$F_1' = 29.6 - 185.5(a/W) + 655.7(a/W)^2 - 1017(a/W)^3$$

$$+ 636.9(a/W)^4$$

(2a)
\[ F_2' = \frac{(2 + a/W)}{(a/W)^{1/2}} \cdot [0.886 + 4.64(a/W) - 13.32(a/W)^2 + 14.72(a/W)^3 - 5.6(a/W)^4] \] (3a)

The values of \( F_1' \) and \( F_2' \) are given in table 2 and in figure 4.

3. DISCUSSION

As is mentioned before, the new geometry function \( F_2 \) is valid for a crack length ratio of 0.2 to 1.0. If \( F_2 \) is taken as the basis for a comparison it can be seen that the old geometry function \( F_1 \) does not differ more than \( \pm 5\% \) of \( F_2 \) for a crack length ratio \( 0.29 \leq a/W \leq 0.77 \). This means that the validity range of \( F_1 \) as given in the old ASTM standard E399 (0.45 \leq a/W \leq 0.55) was very conservative (assumed that \( F_2 \) is indeed correct in the validity range \( 0.2 \leq a/W \leq 1.0 \)). For a crack length ratio of \( a/W = 0.45 \) the values of \( F_1 \) and \( F_2 \) are almost the same. For crack length ratio smaller than \( a/W = 0.45 \) the value of \( F_1 \) is greater than the value of \( F_2 \) and for crack length ratios larger than \( a/W = 0.45 \) the value of \( F_2 \) is greater than \( F_1 \).

4. CONCLUSIONS

A comparison of the old and the new ASTM geometry functions for compact tension specimens showed that the old one does not differ by more than 5\% in the range \( 0.29 \leq a/W \leq 0.77 \). This implies that old \( K_{IC} \) data obtained in that range are still acceptable.

5. REFERENCES

1. ASTM
   a) E399-74
   b) E399-78

2. Duggan, T.V., Proctor, M.W., Spence, L.J.
3. Srawley, J.E.  Wide range stress intensity factor expressions  
for ASTM E399 standard fracture toughness  
specimens.  

6. TABLES AND FIGURES

\[
F_1 = 29.6 \alpha^{1/2} - 185.5 \alpha^{3/2} + 655.7 \alpha^{5/2} - 1017 \alpha^{7/2} + 638.9 \alpha^{9/2}
\]

\[
F_2 = \frac{2+\alpha}{(1-\alpha)^{3/2}} \cdot (0.886 + 4.64 \alpha - 13.32 \alpha^2 + 14.72 \alpha^3 - 5.6 \alpha^4)
\]

\[\alpha = \frac{a}{W}\]

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<th>\alpha</th>
<th>(F_1) &quot;old&quot;</th>
<th>(F_2) &quot;new&quot;</th>
<th>((F_2-F_1)/F_2) x 100%</th>
<th>(F_1) &quot;old&quot;</th>
<th>(F_2) &quot;new&quot;</th>
<th>((F_2-F_1)/F_2) x 100%</th>
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Table 1: Values of \(F_1\) and \(F_2\).
\[ F'_1 = 29.6 - 185.5 \alpha + 655.7 \alpha^2 - 1017 \alpha^3 + 638.9 \alpha^4 \]

\[ F'_2 = \frac{2+\alpha}{\alpha^4 (1-\alpha)^{3/2}} \cdot (0.886 + 4.64 \alpha - 13.32 \alpha^2 + 14.72 \alpha^3 - 5.6 \alpha^4) \]

\[ \alpha = a/N \]

<table>
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Table 2: Values of \( F'_1 \) and \( F'_2 \).
Figure 1: Relative dimensions of the ASTM Standard Compact Tension Specimen (H=0.6W).
\[ F_1 = 29.6 \alpha \frac{1}{2} - 185.5 \alpha^{3/2} + 655.7 \alpha^{5/2} - 1017 \alpha^{7/2} + 638.9 \alpha^{9/2} \]

\[ F_2 = \frac{2 + \alpha}{(1 - \alpha)^{3/2}} \cdot (0.886 + 4.64 \alpha - 13.32 \alpha^2 + 14.72 \alpha^3 - 5.6 \alpha^4) \]

\[ \alpha = \frac{q}{W} \]

**Figure 2**: Values of \( F_1 \) and \( F_2 \).
Figure 3: Relative difference between $F_1$ and $F_2$. 
Figure 4: Values of $F_1'$ and $F_2'$. 