Cruise Performance Optimization of the Airbus A320 through Flap Morphing

Martin Orlita
CRUISE PERFORMANCE OPTIMIZATION OF THE AIRBUS A320 THROUGH FLAP MORPHING

by

Martin Orlita

in partial fulfillment of the requirements for the degree of

Master of Science
in Aerospace Engineering

at the Delft University of Technology,
to be defended publicly on Tuesday August 23, 2016 at 9:30AM.

Thesis registration number: 084#16#MT#FPP
Supervisor: Dr.ir. R. Vos, TU Delft
Thesis committee: Prof.dr.ir. L.L.M. Veldhuis, TU Delft
Dr.ir. F.E.J. Schrijer, TU Delft
Ir. D.P. Jansen, Fokker Aerostructures

An electronic version of this thesis is available at http://repository.tudelft.nl/.
The undersigned hereby certify that they have read and recommend to the Faculty of Aerospace Engineering for acceptance a thesis entitled “Cruise Performance Optimization of the Airbus A320 through Flap Morphing” by Martin Orlita in partial fulfillment of the requirements for the degree of Master of Science.

Dated: August 23, 2016

Head of department:  
Prof.dr.ir. L.L.M. Veldhuis

Supervisor:  
Dr.ir. R. Vos

Reader:  
Dr.ir. EFJ. Schrijer

Reader:  
Ir. D.P. Jansen
Approximately one year ago I had almost all courses finished and my internship done, visiting my former professors in search of a master thesis topic. While I did enjoy specialized courses on aerodynamics, performance and optimization, it is not often one has the opportunity to combine almost the entire worth of one’s Master track in pursue of a complex piece of work.

Perhaps it was luck that while approaching the faculty elevator one day I met Leo Veldhuis and just out of curiosity asked him if he had an interesting topic to work on, which is how I got to know about the design efforts at the Fokker Aerostructures. At first the flap morphing sounded also like a narrow topic, but as I started digging in the literature I became overflown with ideas and connections to what I had learned before. By the time I met the people at Fokker I had a plan so big it would perhaps suit better a PhD research. I remember presenting them an enormous diagram of the individual components of my thesis coming together in a multi-level multi-objective optimization including cruise and high-lift, with large amount of variables and designer inputs. They simply commented that the plan was ambitious and I started to work.

Piece by piece I realized that the world is not ideal and things do not always go as planned. Out of my huge diagram, blocks were slowly cut off, connections broken and in the end only a small piece remained. Although this sounds depressing, it took me some time to realize this was one of the best things that could happen. Finally after years of education I received the lecture I was missing. I have learned not to lose track of the underlying task and that a negative answer has at times no lesser value than a positive one. Thanks to the struggle I have learned great deal about aerodynamics compared to what I knew before, enjoyed countless conversations both on and off the topic and had the chance to see a bit of the engineering industry world which made me want even more to participate in it in the future.

The greatest acknowledgement must go to my family for their love, limitless support and endurance concerning my studies. For providing the interesting research topic, demonstrations and always a pleasant stay at the Fokker Aerostructures I would like to thank Daniel Jansen, Ad Bastiaansen and their co-workers. For council, supervision and friendly debates I thank my main supervisor Roelof Vos and the head of the department Leo Veldhuis. Last but not least, my thanks go to my best friends, flatmates and colleagues at Kamertje 1 on the 6th floor who managed to stop me at the right times to keep an enjoyable balance between work and relax during the past year.

Martin Orlita
Delft, August 2016
In the era of increasing aviation traffic the conditions are right to promote design of ambitious concepts. At Fokker Aerostructures attention is drawn to smooth in-flight shape morphing to produce a structurally functional Variable Camber Trailing Edge Flap (VCTEF). The deployment mechanism would fit into the flap, not limiting other functionality such as Fowler motion, while at the same time allowing small camber variations during cruise. This is based on the assumption that such morphing will bring performance improvements which are commercially interesting.

The main goal of this research was therefore to predict these performance benefits and thus the applicability for a specific case of the Airbus A320 aircraft in cruise flight. This aircraft is large enough to accommodate the technology; it is operated in great numbers and cruise is the most fuel demanding part of its mission. Since the concept is in the development phase the further task is to determine the morphing design setup which performs best. The amount of morphing is driven by a circular reference function, which is added to the base geometry at any desired streamwise cut of the wing by manipulation of the airfoil coordinates as seen on the cover. The design is specified by the points on the airfoil upper surface where the morphing begins and ends, boundaries of the morphing region where upper surface bending is allowed.

As also found in other literature it is shown that morphing can bring drag reduction for a section, wing and the complete aircraft. This varies throughout the cruise, which is translated to more sophisticated performance indicators for comparison and evaluation of the benefits. The first indicator is the increase of range over the design mission for the given aircraft. The second and third are the fuel savings which can either be obtained by increasing the cruise end weight, or by decreasing the cruise beginning weight, both by the amount of the saved fuel while keeping the aircraft range constant.

In order to evaluate these indicators, the Breguet range equation is used in a discretized form, utilizing an interpolated lift-to-drag ratio determined by aerodynamic analysis at 7 cruise points. This was done using both the 2D solver MSES and a quasi-3D tool Q3D developed at TU Delft comprising of MSES and AVL vortex lattice solver. For the analysis a complete A320 model is required, which was not available and was created from the known performance data and partially assumed geometry. The unknown wing geometry was optimized with respect to the mid-cruise drag simulating an already efficient aircraft, as suggested by literature. Other model components were the horizontal stabilizer, fuselage and center of gravity position allowing trim at the reference cruise points and obtaining the lift requirements for the wing and a representative section.

Under these lift requirements the 2D and 3D analyses were performed at individual cruise points to obtained improved lift-to-drag ratios which could be then used to evaluate the range improvement. It was found that with morphing in 2D the drag reduction can amount up to 9% at the beginning of cruise but parabolically decreases towards mid cruise after which it remains below 0.5%. This is primarily due to manipulation of the shockwave and the boundary layer at the given lift requirements, which is most dominant at high cruise lift coefficients. Since the induced drag was found not affected by the assumed morphing, such improvements are further scaled down when evaluated for the entire wing and even further from the aircraft point of view, resulting in a range improvement in order of 20km and fuel savings of below 0.5% of trip fuel. A sensitivity analysis on the design variables has shown that these performance benefits have small sensitivity to the size of the morphing region and that a very aft located regions are the most beneficial, suggesting that a small tab at the trailing edge might be a better and easier solution.

In view of these results the smooth morphing concept is deemed not applicable for the cruise of short range aircraft such as A320. However, given the parabolic behaviour of the drag improvements, larger potential can be expected for long range aircraft, which is the main resulting recommendation of the conducted research. Furthermore it cannot be excluded that other regimes could benefit more from the morphing concept, such as high-lift, which would probably require wind-tunnel testing, as discussed in the final Appendix of this work.
# Contents

Summary vii  
List of Figures xi  
List of Tables xiii  
Nomenclature xv  

## 1 Introduction  
1.1 Background and definition of morphing .......................... 2  
1.2 Study case description ........................................... 5  
1.3 Problem statement and research goals ............................ 6  

## 2 Methodology  
2.1 Aerodynamic analysis tools ......................................... 8  
2.1.1 MSES 2D solver .................................................. 8  
2.1.2 Quasi-3D approach .............................................. 11  
2.2 Performance indicators evaluation .................................. 15  
2.3 Optimization strategies ............................................. 19  
2.3.1 Multi-variable built-in algorithms ............................... 19  
2.3.2 Single-variable developed algorithm ............................ 20  

## 3 Morphing of airfoil sections  
3.1 Trailing edge morphing implementation ........................... 24  
3.2 2D section morphing phenomena .................................... 28  

## 4 Model of the complete aircraft  
4.1 Reference wing geometry ............................................. 34  
4.1.1 Class-Shape transformation .................................... 34  
4.1.2 Lifting surface geometry generation ........................... 35  
4.1.3 Tuning of the wing model ...................................... 39  
4.2 Aircraft components participating in trim ....................... 42  
4.2.1 Horizontal stabilizer model ................................... 42  
4.2.2 Addition of the fuselage ...................................... 45  
4.2.3 Center of gravity position .................................... 46  
4.3 Trim procedure description ........................................ 48  
4.4 Reference aircraft performance ..................................... 51  

## 5 Morphing performance evaluation  
5.1 2D morphing of reference effective section ..................... 56  
5.2 3D morphing of wing ............................................... 58  
5.3 2D analysis on morphing variables ................................ 61  

## 6 Conclusions and Recommendations  
6.1 Summary of partial conclusions .................................... 63  
6.2 Discussion on applicability of the morphing concept ............ 64  
6.3 Recommendations on further research .............................. 65  

Bibliography 67  
A History of morphing aircraft ......................................... 71  
B Airbus A320 aircraft data ............................................ 73  
C MSES settings, validations and examples ........................... 79
<table>
<thead>
<tr>
<th>Content</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>D Q3D validation case</td>
<td>83</td>
</tr>
<tr>
<td>E Method to determine aerodynamic center shift due to fuselage</td>
<td>85</td>
</tr>
<tr>
<td>F Reference aircraft trim results</td>
<td>87</td>
</tr>
<tr>
<td>G High lift investigation on morphing single slotted flap</td>
<td>89</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

1.1 Adaptive Dropped Hinge Flap mechanism on Airbus A350 XWB in high lift configuration . . . . 3
1.2 Cruise improvements due to morphing according to other literature . . . . . . . . . . . . . . . . 4
1.3 Demonstration of morphed upper flap surface concept by Fokker Aerostructures . . . . . . 5

2.1 MSES convergence studies on section drag and moment coefficient . . . . . . . . . . . . . . . . . 9
2.2 SC(3)-0712(B) airfoil drag polar validation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 9
2.3 MSES initial grid morphing region refinement . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10
2.4 Upper surface grid refinement study . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10
2.5 Wing lifting surface parameters for strip method derivation . . . . . . . . . . . . . . . . . . . . . 12
2.6 Simple sweep theory explanation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 13
2.7 Definition of forces and angles for iteration on effective section angle of attack . . . . . . . . . 13
2.8 Typical specific range distribution over cruise mission . . . . . . . . . . . . . . . . . . . . . . . . 15
2.9 Example of interpolation and extrapolation techniques . . . . . . . . . . . . . . . . . . . . . . . . 16
2.10 Design mission legs for method of fuel fractions . . . . . . . . . . . . . . . . . . . . . . . . . . . . 16
2.11 Test case for 2D drag morphing optimization . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 20
2.12 Example of a discontinuous single-variable function optimization . . . . . . . . . . . . . . . . . 21

3.1 Airfoil morphing region delimitation and surface vectors definition . . . . . . . . . . . . . . . . . 24
3.2 Reference morphing function and morphing displacements mapping . . . . . . . . . . . . . . . . 24
3.3 Applying morphing displacements to the airfoil upper surface . . . . . . . . . . . . . . . . . . . . 25
3.4 Result of the morphing procedure on airfoil section . . . . . . . . . . . . . . . . . . . . . . . . . . . 26
3.5 Morphed airfoil between 0.665-0.685 of chord length with deflection $\delta_m = 5^\circ$ . . . . . . 27
3.6 Morphed airfoil between 0.80-0.95 of chord length with deflection $\delta_m = 10^\circ$ . . . . . . . . 27
3.7 Morphed airfoil between 0.97-0.98 of chord length with deflection $\delta_m = 50^\circ$ . . . . . . . . 27
3.8 Morphed airfoil between 0.80-0.95 of chord length with deflection $\delta_m = -10^\circ$ . . . . . . . 27
3.9 Drag polars of RAE2822 airfoil at different Mach numbers and morphing deflections . . . . . . . 28
3.10 Effective angle of attack and moment coefficient of RAE2822 airfoil with morphing . . . . . . . 29
3.11 Morphing deflection effect on drag components of RAE282 airfoil at high lift coefficient . . . . . 29
3.12 Morphing deflection effect on pressure distribution of RAE2822 airfoil at high lift coefficient . . 30
3.13 Morphing deflection effect on drag components of RAE2822 airfoil at low lift coefficient . . . . 31
3.14 Morphing deflection effect on pressure distribution of RAE2822 airfoil at low lift coefficient . . . 31

4.1 Example of re-meshing done on SC(3)-0712(B) airfoil coordinates . . . . . . . . . . . . . . . . . . 35
4.2 Wing defining airfoils and interpolated sections . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 35
4.3 Distributions of thickness-to-chord ratio and twist . . . . . . . . . . . . . . . . . . . . . . . . . . . 36
4.4 Example of single airfoil z-scaling from 10% to 15% thickness . . . . . . . . . . . . . . . . . . . . 36
4.5 Swept section extraction at 60% spanwise position, perpendicular to local half-chord line . . . . 37
4.6 Swept section extraction at 60% spanwise position . . . . . . . . . . . . . . . . . . . . . . . . . . . 37
4.7 Swept section extraction at 11.6% spanwise position . . . . . . . . . . . . . . . . . . . . . . . . . 37
4.8 A320 wing geometry from the literature . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 38
4.9 A320 modeled wing geometry . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 38
4.10 Wing geometry generation Class-Object diagram . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 38
4.11 A320 outboard flap mid-span coordinates extraction from CATIA . . . . . . . . . . . . . . . . . 39
4.12 Flap mid-span geometry and RAE2822 geometry defining initial outboard airfoil design . . . . . 39
4.13 Demonstration of section camber manipulation by pivoting of reference surfaces . . . . . . . . . 40
4.14 Reference wing optimization results at 58% span streamwise section . . . . . . . . . . . . . . . . 40
4.15 Reference wing optimization objective and constraints convergence . . . . . . . . . . . . . . . . 41
4.16 Reference wing polar diagrams . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 41
4.17 Horizontal stabilizer defining airfoils and interpolated sections . . . . . . . . . . . . . . . . . . . 42
4.18 Horizontal stabilizer polar diagrams ................................................. 43
4.19 Horizontal stabilizer lift curve slope and downwash ................................................. 44
4.20 Relation of free-stream and effective horizontal stabilizer forces ................. 44
4.21 Fuselage drag coefficients for an assumed angle of attack range ................. 46
4.22 Center of gravity positions during mission for varying maximum take-off weight position ................. 47
4.23 Diagram of forces and moments included in aircraft trim ................................................. 48
4.24 Trimmed cruise points results on wing polar diagrams ................................................. 51
4.25 Trimmed cruise points results on horizontal stabilizer polar diagrams ................. 51
4.26 Effect of increasing wing pitching moment on the re-Trimmed drag values at beginning of cruise ................. 52
4.27 Polar diagrams of aircraft components combinations ................................................. 52
4.28 Cruise lift-to-drag ratio function of the reference wing and the complete aircraft ................................................. 53
4.29 Wing lift distributions at individual cruise points ................................................. 53
4.30 Sectional lift coefficient requirements at 60% spanwise position ................................................. 54
5.1 2D morphing optimization at end of cruise and beginning of cruise point ................................................. 56
5.2 2D morphing optimization summary for all cruise points at the design altitude of 37000ft ................................................. 56
5.3 2D morphing optimization summary for all cruise points at altitude of 36000ft ................................................. 57
5.4 2D morphing optimization summary for all cruise points at altitude of 38000ft ................................................. 57
5.5 Wing morphing optimization summary ................................................. 58
5.6 Optimized wing morphing deflections ................................................. 58
5.7 Wing optimization individual drag components outputs ................................................. 59
5.8 Mid-cruise non-morphed and morphed lift distributions ................................................. 59
5.9 Comparison of wing induced drag and wing profile drag ................................................. 59
5.10 Wing morphing effects on aircraft lift-to-drag ratio ................................................. 60
5.11 Sensitivity of range improvement to morphing design variables ................................................. 61
5.12 Sensitivity of fuel savings to morphing design variables ................................................. 61
5.13 Sensitivity of control derivative to morphing design variables ................................................. 62
5.14 Sensitivity of control derivative to the morphing region position ................................................. 62
B.1 A320 wing planform scheme ................................................. 73
B.2 A320 overall planform scheme ................................................. 74
B.3 D-ATRA aircraft general data ................................................. 74
B.4 Wing planform visualization over schematic and photography ................................................. 75
B.5 A320 horizontal stabilizer planform photo comparison ................................................. 77
B.6 A320 horizontal stabilizer photo - view bottom ................................................. 77
B.7 A320 horizontal stabilizer photo - top view ................................................. 77
B.8 A320 fuel arm as function of fuel mass ................................................. 78
B.9 A320 balance computer ................................................. 78
C.1 MSES validation of RAE2822 Cp distribution from official MSES website ................................................. 80
C.2 MSES validation of RAE2822 Cp distribution with applied settings ................................................. 80
C.3 MSES example of separated flow and grid refinement ................................................. 81
C.4 MSES example of separated flow and pressure distribution ................................................. 81
D.1 Q3D validation of Fokker F100 wing shockwave position ................................................. 83
D.2 Q3D validation of Fokker F100 wing total drag coefficient ................................................. 84
D.3 Q3D validation of Fokker F100 wing drag coefficient components ................................................. 84
E.1 Fuselage planform areas definition ................................................. 85
E.2 Upwash gradient determination for front fuselage areas ................................................. 85
E.3 Fuselage planform sections extraction in Solidworks ................................................. 86
E.4 Fuselage planform sections data ................................................. 86
F.1 Standard deviation on lift-to-drag ratio of aircraft divided by lift-to-drag ratio of effective section ................................................. 88
G.1 Flap geometry B-spline control points ................................................. 89
G.2 RAE2822 airfoil split into multiple elements ................................................. 90
G.3 Two element configuration lift curves for various Reynolds numbers, at $M = 0.13$ ........................................ 90
G.4 RAE2822 airfoil split in multiple elements .................................................. 90
G.5 Pressure distributions on two element airfoil for several angles of attack ................. 91
G.6 Pressure distributions on two element airfoil at flap deflections $\delta_f = 30^\circ$ and $\delta_f = 25^\circ$ .............. 91
G.7 Slat effect on multi-element airfoil ................................................................. 92

LIST OF TABLES

2.1 Data used for cruise points derivation ............................................................... 17
2.2 Resulting cruise data for performance evaluation .............................................. 18
4.1 Parameters used for downwash gradient calculation .......................................... 43
4.2 Parameters used for fuselage drag coefficient calculation ................................. 45
6.1 Comparison of A320 design mission parameters to other aircraft ....................... 64
B.1 Basic wing data list ......................................................................................... 76
B.2 Basic horizontal stabilizer data list ................................................................. 76
C.1 Example of grid settings in gridpar.* file ......................................................... 79
C.2 Example of run settings in mses.* file ............................................................ 79
C.3 Example of domain settings in blade.* file ...................................................... 80
\( \vec{1} \) \hspace{1cm} \text{vector of ones with dimension required to complete the transformation of coordinates} [-]

\( \Delta C_{Dw} \) \hspace{1cm} \text{wing drag improvement with respect to a reference base geometry drag value} [%]

\( \Delta C_{Lh} \) \hspace{1cm} \text{convergence criterion for the horizontal stabilizer (inner) trim iteration} [-]

\( \Delta C_{Lw} \) \hspace{1cm} \text{convergence criterion for the main (outer) trim iteration} [-]

\( \Delta F \) \hspace{1cm} \text{fuel savings on the design mission in terms of trip fuel} [%]

\( \Delta F_1 \) \hspace{1cm} \text{fuel savings on the design mission by morphing with variable end of cruise aircraft mass in terms of trip fuel} [%]

\( \Delta F_2 \) \hspace{1cm} \text{fuel savings on the design mission by morphing with variable beginning of cruise aircraft mass in terms of trip fuel} [%]

\( \Delta R \) \hspace{1cm} \text{range improvement over the design mission} [km]

\( \Delta X_{crit} \) \hspace{1cm} \text{developed optimization algorithm design value tolerance}

\( \Delta p \) \hspace{1cm} \text{forward offset of refined grid region on the airfoil upper surface} [-]

\( \Delta c_{d,eff} \) \hspace{1cm} \text{effective section drag improvement with respect to a reference base geometry drag value} [%]

\( \Delta \epsilon \) \hspace{1cm} \text{aft offset of refined grid region on the airfoil upper surface} [-]

\( \Delta x_{MC} \) \hspace{1cm} \text{shift of aerodynamic center by the addition of a fuselage to the wing} [%MAC]

\( \Delta z_{LE_l} \) \hspace{1cm} \text{interpolated lower surface differences between the re-meshed geometry and the reference geometry of the RAE2822 airfoil leading edge} [-]

\( \Delta z_{LE_u} \) \hspace{1cm} \text{interpolated upper surface differences between the re-meshed geometry and the reference geometry of the RAE2822 airfoil leading edge} [-]

\( \Delta z_{TE} \) \hspace{1cm} \text{interpolated differences on both surfaces between the re-meshed geometry and the reference A320 flap geometry} [-]

\( \Lambda_{0.25} \) \hspace{1cm} \text{lifting surface quarter-chord sweep angle} [°]

\( \Lambda_{0.5} \) \hspace{1cm} \text{lifting surface half-chord sweep angle} [°]

\( \alpha \) \hspace{1cm} \text{angle of attack measured between the free-stream and a lifting surface reference} [°]

\( \alpha_{CW=0} \) \hspace{1cm} \text{wing zero-lift angle of attack} [°]

\( \alpha_{eff} \) \hspace{1cm} \text{section effective angle of attack} [°]

\( \alpha_i \) \hspace{1cm} \text{section induced angle of attack} [°]

\( \delta_m \) \hspace{1cm} \text{angular deflection of the trailing edge due to morphing} [°]

\( \delta_{m_i} \) \hspace{1cm} \text{inboard flap morphing deflection} [°]

\( \delta_{m_o} \) \hspace{1cm} \text{outboard flap morphing deflection} [°]

\( \epsilon \) \hspace{1cm} \text{local angle of incidence (twist) of streamwise section on a lifting surface} [°]

\( \epsilon_h \) \hspace{1cm} \text{downwash angle at the horizontal stabilizer position} [°]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_f )</td>
<td>ratio of cylinder drag coefficients of finite and infinite lengths</td>
<td></td>
</tr>
<tr>
<td>( \eta_h )</td>
<td>ratio of dynamic pressure at the horizontal stabilizer and the free-stream</td>
<td></td>
</tr>
<tr>
<td>( \lambda )</td>
<td>wing taper ratio</td>
<td></td>
</tr>
<tr>
<td>( \mu )</td>
<td>dynamic viscosity of air at cruising altitude</td>
<td>kg/m/s</td>
</tr>
<tr>
<td>( \psi )</td>
<td>normalized x-coordinate position on an airfoil</td>
<td></td>
</tr>
<tr>
<td>( \rho_\infty )</td>
<td>free-stream air density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>( \theta_i, \phi_i )</td>
<td>angles between the morphed and base geometry points used for calculation</td>
<td>°</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>re-meshed normalized z-coordinate recovered from CST transformation method</td>
<td></td>
</tr>
<tr>
<td>( \zeta_{TE} )</td>
<td>normalized trailing edge z-coordinate denoting position (thickness) added</td>
<td></td>
</tr>
<tr>
<td>( A )</td>
<td>wing aspect ratio</td>
<td></td>
</tr>
<tr>
<td>( AC )</td>
<td>wing aerodynamic center</td>
<td></td>
</tr>
<tr>
<td>( A_l )</td>
<td>lower surface Class-Shape Transformation coefficients assembled in a vector</td>
<td></td>
</tr>
<tr>
<td>( A_u )</td>
<td>upper surface Class-Shape Transformation coefficients assembled in a vector</td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td>Bernstein polynomial terms assembled in a vector</td>
<td></td>
</tr>
<tr>
<td>( b )</td>
<td>span of a lifting surface</td>
<td>m</td>
</tr>
<tr>
<td>( c_\perp )</td>
<td>chord length along the direction perpendicular to the local half-chord line</td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>local streamwise oriented chord on a lifting surface</td>
<td>m</td>
</tr>
<tr>
<td>( c_{dp} )</td>
<td>profile drag coefficient of a section in perpendicular to the half-chord</td>
<td></td>
</tr>
<tr>
<td>( c_{dpf} )</td>
<td>(profile) friction drag coefficient of a section in perpendicular to the</td>
<td></td>
</tr>
<tr>
<td>( c_{dpp} )</td>
<td>(profile) pressure drag coefficient of a section in perpendicular to the</td>
<td></td>
</tr>
<tr>
<td>( C_D )</td>
<td>total drag coefficient of the aircraft</td>
<td></td>
</tr>
<tr>
<td>( c_d )</td>
<td>drag coefficient of a section parallel to the free-stream</td>
<td></td>
</tr>
<tr>
<td>( C_{D_0} )</td>
<td>component zero lift drag coefficient</td>
<td></td>
</tr>
<tr>
<td>( C_{dc} )</td>
<td>steady state cross-flow drag coefficient</td>
<td></td>
</tr>
<tr>
<td>( c_{de} )</td>
<td>effective section drag coefficient</td>
<td></td>
</tr>
<tr>
<td>( c_{d_{ep}m} )</td>
<td>effective section pressure drag coefficient improved by morphing</td>
<td></td>
</tr>
<tr>
<td>( c_{d_{ep}} )</td>
<td>effective section pressure drag coefficient</td>
<td></td>
</tr>
<tr>
<td>( C_{DF} )</td>
<td>fuselage drag coefficient</td>
<td></td>
</tr>
<tr>
<td>( C_{D_f 0} )</td>
<td>lift-independent fuselage drag coefficient</td>
<td></td>
</tr>
<tr>
<td>( C_{D_f L} )</td>
<td>lift-dependent fuselage drag coefficient</td>
<td></td>
</tr>
<tr>
<td>( C_{D_h} )</td>
<td>horizontal stabilizer drag coefficient</td>
<td></td>
</tr>
</tbody>
</table>
$C_{D_{\text{eff}}}$ effective drag coefficient of the horizontal stabilizer

$C_{D_{\text{iw}}}$ wing induced drag coefficient

$C_{D_{\text{m}}}$ aircraft drag coefficient improved by morphing

$C_{D_{\text{N}}}$ drag coefficient of components which are assumed non-lift-contributing (lift independent)

$C_{D_{\text{nm}}}$ drag coefficient of components with assumed negligible influence from morphing

$c_{d_{p}}$ streamwise section profile drag coefficient

$c_{d_{pf}}$ streamwise section (profile) friction drag coefficient

$C_{D_{\text{ppw}}}$ wing profile friction drag coefficient

$c_{d_{pp}}$ streamwise section (profile) pressure drag coefficient

$C_{D_{\text{ppw}}}$ wing profile pressure drag coefficient

$C_{D_{\text{ppw}}^{\text{m}}}$ wing profile pressure drag coefficient improved by morphing

$C_{D_{\text{pw}}}$ wing profile drag coefficient

$C_{D_{w}}$ wing drag coefficient

$C_{f}$ component turbulent mean skin friction coefficient

$C_{f_{\text{fs}}}$ fuselage turbulent mean skin friction coefficient

$C_{G_{CR}}$ center of gravity at a given cruise point

$C_{G_{\text{mean}}}$ mean center of gravity

$C_{L_{\text{uw}}}$ constant approximated wing lift curve slope [1/\text{rad}]

$c_{L_{\perp}}$ local lift coefficient required from a section perpendicular to local half-chord line

$C_{L}$ aircraft lift coefficient

$c_{l}$ lift coefficient of a section parallel to the free-stream

$C_{L_{uw}M}$ wing lift-curve slope at cruise Mach number [1/\text{rad}]

$C_{L_{uw}M=0}$ wing lift-curve slope at low subsonic Mach number [1/\text{rad}]

$c_{\text{left}}$ effective section lift coefficient

$C_{L_{h}}$ lift coefficient of the horizontal stabilizer

$C_{L_{h_{\text{eff}}}}$ effective lift coefficient of the horizontal stabilizer

$C_{L_{h_{\text{res}}}}$ horizontal stabilizer lift coefficient resulting from effective lift and drag during inner trim iteration

$C_{L_{w}}$ wing lift coefficient including the contribution of the fuselage

$C_{L_{\text{uw,new}}}$ update of the required wing lift coefficient during the main trim iteration

$c_{m}$ pitching moment coefficient of a streamwise section about its aerodynamic center

$c_{\text{MAC}}$ wing mean aerodynamic chord [m]

$C_{M_{AC}}$ pitching moment of a lifting surface about its aerodynamic center

$c_{\text{MAC}_{h}}$ horizontal stabilizer mean aerodynamic chord [m]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{MC}$</td>
<td>aircraft moment coefficient about the position of center of gravity</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$c_{m,ef}$</td>
<td>pitching moment coefficient of an effective section</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$C_{Mh}$</td>
<td>horizontal stabilizer moment coefficient about the horizontal stabilizer aerodynamic center</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$C_{Mw}$</td>
<td>wing moment coefficient about the wing aerodynamic center</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$C^{N_1}_{N_2}$</td>
<td>Class function in the Class-Shape Transformation</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>pressure coefficient expressing the pressure distribution over an airfoil</td>
<td>$[-]$</td>
</tr>
<tr>
<td>CR</td>
<td>subscript denoting a given cruise point</td>
<td></td>
</tr>
<tr>
<td>$C_{Bh}$</td>
<td>resultant aerodynamic force coefficient of the horizontal stabilizer</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$c_T$</td>
<td>thrust specific fuel consumption in kilograms of fuel</td>
<td>[kg/$s/N$]</td>
</tr>
<tr>
<td>$C_{Tp}$</td>
<td>thrust force coefficient</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$c_T(N)$</td>
<td>thrust specific fuel consumption in Newtons of fuel</td>
<td>[N/$s/N$]</td>
</tr>
<tr>
<td>$d_\perp$</td>
<td>drag of a unit section in perpendicular to half-chord line reference frame</td>
<td>[N/m]</td>
</tr>
<tr>
<td>$d_{\perp,p}$</td>
<td>profile drag of a unit section in perpendicular to the half-chord reference frame</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$D$</td>
<td>total drag of the aircraft</td>
<td>[N]</td>
</tr>
<tr>
<td>$d_{\text{eff}}$</td>
<td>effective drag of a unit section with respect to the effective flow velocity</td>
<td>[N/m]</td>
</tr>
<tr>
<td>$d_f$</td>
<td>fuselage diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_h$</td>
<td>horizontal distance of the aerodynamic center of the horizontal stabilizer and the aerodynamic center of the wing</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_i$</td>
<td>displacement of an upper surface point by morphing</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$d_m$</td>
<td>size of morphing region enclosed by $x_b$ and $x_b$, as fraction of local chord</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$dm$</td>
<td>pitching moment of an infinitesimally thin strip on a lifting surface about its aerodynamic center</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$dm_{AC}$</td>
<td>pitching moment of an infinitesimally thin strip on a lifting surface about the aerodynamic center of the entire lifting surface</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$f_m$</td>
<td>reference morphing function</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
<td>[m/$s^2$]</td>
</tr>
<tr>
<td>$G_{\text{crit}}$</td>
<td>developed optimization algorithm objective gradient value tolerance</td>
<td></td>
</tr>
<tr>
<td>$h_{CR}$</td>
<td>horizontal distance between the wing lift vector line of action and the aircraft center of gravity at a given cruise point in terms of wing mean aerodynamic chord</td>
<td>[%MAC]</td>
</tr>
<tr>
<td>$h_h$</td>
<td>vertical distance between the aerodynamic center of the horizontal stabilizer and the aerodynamic center of the wing</td>
<td>[m] or [%MAC]</td>
</tr>
<tr>
<td>$h_i$</td>
<td>distance between the base geometry upper surface point and its preceding morphed point</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$i_{\text{eff}}$</td>
<td>local angle of incidence of the effective section in the perpendicular to half-chord line reference frame</td>
<td>[$^\circ$]</td>
</tr>
<tr>
<td>$J$</td>
<td>general optimization problem objective</td>
<td>$[-]$</td>
</tr>
</tbody>
</table>
\( K \) correlation factor between effective section pressure drag and wing profile pressure drag [-]

\( k \) number of interpolated points along the cruise for range extraction [-]

\( K_A, K_d, K_h \) coefficients in the method for determining downwash at the horizontal stabilizer [-]

\( K_f \) component form factor correcting differences between flat plate and the curved geometry [-]

\( l_l \) lift of a unit section in perpendicular to half-chord line reference frame \([N/m]\)

\( L \) aircraft lift force \([N]\)

\( l_{CR} \) horizontal distance between the mean aerodynamic center of the horizontal stabilizer and the aircraft center of gravity at a given cruise point in terms of wing mean aerodynamic chord \[%MAC\]

\( l_{eff} \) effective lift of a unit section with respect to the effective flow velocity \([N/m]\)

\( l_f \) fuselage length \([m]\)

\( M_L \) Mach number component along the direction perpendicular to the local half-chord line [-]

\( M \) free-stream Mach number [-]

\( m \) aircraft mass \([kg]\)

MAC mean aerodynamic chord

\( m_b \) aircraft mass at the beginning of cruise \([kg]\)

\( m_{CR1} - m_{CR7} \) aircraft masses at the respective cruise points \(CR_1 - CR_7\) \([kg]\)

\( m_{CR} \) aircraft mass at a given cruise point \([kg]\)

\( m_e \) aircraft mass at the end of cruise \([kg]\)

\( m_F \) total fuel mass on board at maximum take-off weight \([kg]\)

\( M_{BL1} - M_{BL7} \) mass fuel fractions for the respective legs \(L1 - L7\) of the design mission [-]

\( m_{Fres} \) reserve fuel mass for the design mission \([kg]\)

\( m_{Fused} \) used fuel mass for the design mission \([kg]\)

\( m_{MTOW} \) aircraft mass at maximum take-off weight \([kg]\)

\( m_{OEW} \) aircraft mass at operating empty weight \([kg]\)

\( m_{P/L} \) payload mass for the design mission \([kg]\)

\( m_{SD} \) aircraft mass at engine shut down \([kg]\)

\( m_{ZFW} \) aircraft mass at zero fuel weight \([kg]\)

\( N \) number of streamwise sections along a half-span of a lifting surface [-]

\( n \) number of effective sections along a half-span of a lifting surface [-]

\( N_1, N_2 \) coefficients defining the class of the approximated geometry shapes in the Class-Shape Transformation [-]

NaN Not-a-Number value returned when solution is unsuccessful

PCHIP Piece-wise Cubic Hermite Interpolating Polynomial
\( q_\infty \) free-stream dynamic pressure \([Pa]\)

Q3D Quasi-Three-Dimensional tool developed at TU Delft

\( q_h \) dynamic pressure of the effective flow at the horizontal stabilizer \([-\] \)

\( R \) range of aircraft driven by trip fuel and aerodynamic characteristics during cruise \([m]\)

\( R_0 \) reference aircraft range \([m]\)

\( Re \) free-stream Reynolds number \([-\] \)

\( Re_{eff} \) effective Reynolds number imposed on an effective section \([-\] \)

\( r_m \) radius of morphing function applied to upper surface \([mm] \) or \([-\] \)

\( r_{mg} \) initial guess of morphing radius in an iteration to arrive at specified morphing deflection \([-\] \)

\( R_{N fus} \) fuselage Reynolds number \([-\] \)

\( r_T \) vertical distance between the line of action of the thrust vector and the aircraft center of gravity at a given cruise point in terms of wing mean aerodynamic chord \([%MAC]\)

\( R_{wf} \) wing-fuselage interference factor \([-\] \)

\( S \) Shape function in the Class-Shape Transformation \([-\] \)

\( S_{b fus} \) fuselage base area (at the end of emphanage) \([m^2]\)

\( S_h \) horizontal stabilizer planform area \([m^2]\)

\( s_i \) distance between the base geometry upper surface point and its preceding point \([-\] \)

\( S_l \) lower surface Shape function in the Class-Shape Transformation \([-\] \)

\( S_{pl fus} \) fuselage planform area \([m^2]\)

SQP Sequential Quadratic Programming optimization algorithm

\( SR \) specific range of aircraft (distance per weight unit of fuel) \([m/N]\)

\( S_{ref} \) component reference area \([-\] \)

\( S_u \) upper surface Shape function in the Class-Shape Transformation \([-\] \)

\( S_w \) aircraft reference (wing) area, including extension inside fuselage (Airbus definition) \([m^2]\)

\( S_{wet} \) component wetted area \([m^2]\)

\( S_{wetfus} \) fuselage wetted area \([m^2]\)

\( t_c \) local thickness to chord ratio of a streamwise section \([\%]\)

\( V_\infty \) free-stream velocity \([m/s]\)

\( V_\perp \) velocity component along the direction perpendicular to the local half-chord line \([m/s]\)

\( V_{eff} \) effective flow velocity as seen by the effective section \([m/s]\)

\( W \) aircraft weight \([N]\)

\( W_b \) aircraft weight at the beginning of cruise \([N]\)

\( W_e \) aircraft weight at the end of cruise \([N]\)

\( x' \) x-axis of mapping coordinate system for the reference morphing function
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>general optimization problem design vector</td>
<td>[-]</td>
</tr>
<tr>
<td>$\overrightarrow{X_0}, \overrightarrow{Z_0}$</td>
<td>array of streamwise section dimensional coordinates</td>
<td>[m]</td>
</tr>
<tr>
<td>$x_b$</td>
<td>chordwise position of morphing region beginning, as fraction of local chord</td>
<td>[-]</td>
</tr>
<tr>
<td>$x_b, z_b$</td>
<td>normalized upper surface interpolation of beginning of morphing region point</td>
<td>[-]</td>
</tr>
<tr>
<td>$x_{CG_{C,R}}$</td>
<td>distance of the aircraft center of gravity from a reference station at a given cruise point in terms of the mean aerodynamic chord</td>
<td>[%MAC]</td>
</tr>
<tr>
<td>$x_{CG_{MTOW}}$</td>
<td>distance of the aircraft center of gravity from a reference station at maximum take-off weight in terms of the mean aerodynamic chord</td>
<td>[%MAC]</td>
</tr>
<tr>
<td>$x_{dm}$</td>
<td>morphing region chordwise mid-position, as fraction of local chord</td>
<td>[-]</td>
</tr>
<tr>
<td>$x_e$</td>
<td>chordwise positions of morphing region end, as fraction of local chord</td>
<td>[-]</td>
</tr>
<tr>
<td>$x_e, z_e$</td>
<td>normalized upper surface interpolation of end of morphing region point</td>
<td>[-]</td>
</tr>
<tr>
<td>$m_F{CR}$</td>
<td>fuel mass at a given cruise point</td>
<td>[kg]</td>
</tr>
<tr>
<td>$x_F{CR}$</td>
<td>distance of the fuel center of mass from a reference station at a given cruise point in terms of the mean aerodynamic chord</td>
<td>[%MAC]</td>
</tr>
<tr>
<td>$x_F{MTOW}$</td>
<td>distance of the fuel center of mass from a reference station at maximum take-off weight in terms of the mean aerodynamic chord</td>
<td>[%MAC]</td>
</tr>
<tr>
<td>$x_i, z_i$</td>
<td>normalized base geometry upper surface airfoil coordinates of a point within the morphing region</td>
<td>[-]</td>
</tr>
<tr>
<td>$x_{m_i}, z_{m_i}$</td>
<td>normalized morphed geometry upper surface airfoil coordinates of a point within the morphing region</td>
<td>[-]</td>
</tr>
<tr>
<td>$\overrightarrow{x_m}, \overrightarrow{z_m}$</td>
<td>array of trailing edge points outside of morphing region but affected by morphing</td>
<td>[-]</td>
</tr>
<tr>
<td>$x_{qc}$</td>
<td>distance of the aerodynamic center of a local section on a lifting surface to the aerodynamic center of the entire lifting surface</td>
<td>[m]</td>
</tr>
<tr>
<td>$\overrightarrow{X_t}, \overrightarrow{Z_t}$</td>
<td>array of streamwise section dimensional coordinates rotated to the required local twist</td>
<td>[m]</td>
</tr>
<tr>
<td>$\overrightarrow{x}, \overrightarrow{z}$</td>
<td>array of base geometry trailing edge points outside of the morphing region</td>
<td>[-]</td>
</tr>
<tr>
<td>$x_{ZFW}$</td>
<td>distance of the aircraft center of gravity from a reference station at zero-fuel weight in terms of the mean aerodynamic chord</td>
<td>[%MAC]</td>
</tr>
<tr>
<td>$y$</td>
<td>spanwise position on a lifting surface</td>
<td>[m]</td>
</tr>
<tr>
<td>$y_{fus}$</td>
<td>y-position of wing-fuselage junction from the aircraft center-line (fuselage radius)</td>
<td>[m]</td>
</tr>
<tr>
<td>$z'$</td>
<td>z-axis of mapping coordinate system for the reference morphing function</td>
<td></td>
</tr>
</tbody>
</table>
With exponential trends in aviation traffic over the last decades, ambitious plans are being set on innovation and emissions reduction, such as the Horizon 2020 or Flightpath 2050 by the European Commission’s Advisory Council for Aeronautics Research in Europe [1]. The recent research in aviation is therefore yet again focused towards improving the fuel efficiency and reducing the direct operating costs in order to approach the requirements for future air transport. A major area of concern remains the aerodynamic shaping of aircraft, increasingly supported by computational fluid dynamics [2].

The clean outer geometry of the modern transport aircraft is usually well optimized for the mid-cruise design point [3, 4]. In off-design conditions the performance of a fixed geometry aircraft is generally non-optimal and some of these conditions require by now standard use of piece-wise rigid geometry changes. An example would be the deployment of conventional high-lift devices, which can serve to enlarge the flight envelope and obtain feasible landing speeds or provide a more optimum lift-to-drag ratio during climb. As the cruise is the most fuel demanding part of a typical transport aircraft mission, opportunities are also sought to improve the transonic cruise performance under given design mission requirements. Broad studies such as the review by Barbarino et.al [5], indicate a potential for improvements with smooth in-cruise geometry changes.

One such promising concept is the variation of section camber, within this thesis investigated in the form of Variable Camber Trailing Edge Flaps (VCTEF). Design efforts are being made at the Fokker Aerostructures to provide a structurally functional morphing flap which allows such in-flight modification of wing sections. To better understand the effects of applying it to a large scale aircraft, an aerodynamic and performance analysis is to be made to arrive at optimum set of design variables and estimate the performance benefits of the concept. The main objective of this thesis work is to determine the applicability of flap upper surface morphing on Airbus A320, intended as a retrofit to the large fleet of aircraft in service. Furthermore the study should indicate the performance benefits of variable camber wings to other short-medium range aircraft.

Within this introduction chapter first a background and motivation for geometry morphing is given in section 1.1. The basic features of the concept by Fokker Aerostructures are explained in section 1.2. The problem statement with assumptions definition and the research goals of the thesis are set in section 1.3.

The thesis report is then organized as follows. First, the methodology overview is given and the applied tools are presented in chapter 2. Adaptation of the morphing concept and an insight of its effect on 2D flow are shown in chapter 3. The 2D and 3D model of the A320 aircraft are discussed in chapter 4, which gives basis for the trailing edge morphing performance benefits evaluation in chapter 5 with the final conclusions on this research made in the chapter 6.
1.1. BACKGROUND AND DEFINITION OF MORPHING

Even before the first powered flight by the Wright brothers it was understood that lift depends on the shape of the lifting surfaces, the attitude and velocity relative to the oncoming flow, and the conditions of the air. The basic equation of lift (1.1) is a formalization of the so-called Buckingham π-theorem, dating as early as 1878 [6].

For the purposes of this thesis, quasi-steady states are assumed and therefore the total aircraft lift $L$ is required to match the actual weight of the aircraft $W$ at any point of cruising flight.

$$L = W = \frac{1}{2} \rho \infty V^2 L(C_{L}, Re, M, geometry) \frac{1}{2} \rho \infty S_{W}(planform)$$

Although being a simplification, this equation demonstrates the dependency of lift on velocity $V$ with a second power, whereas the density $\rho$, the reference area $S$ and the lift coefficient $C_L$ contribute linearly. In real flight however, the conditions in this equation vary significantly. The dependency of the lift coefficient on angle of attack $\alpha$, wing geometry (airfoils and planform) and flow parameters (Reynolds number $Re$ and Mach number $M$) is not straightforward. Moreover, if at a given weight the aircraft is to fly at a range of speeds or similarly for a given speed at varying weights (e.g., due to fuel consumption), it follows that something has to be done about the remaining parameters to keep the equation (1.1) true.

On an aircraft with a fixed geometry wing, the assumed quasi-steady straight level cruising flight is maintained by adjustment of the aircraft angle of attack. Trimming is applied to balance the aircraft which involves applying either center of gravity change and/or the use of control/trim surfaces. There are however limits to what a certain fixed geometry can handle and where in the envelope it performs best, for example due to flow separation, buffet, drag divergence, etc.

Another means of balancing the equation of lift is by in-flight changes of the aircraft shape, which includes also morphing. According to Barbarino [5], this term has no strict definition across the literature and can in some cases include for example the commonly used deployment of conventional piece-wise rigid devices such as leading and trailing edge flaps where each element has constant camber. A further example of in-flight modifications with piece-wise rigid geometry is the variable sweep wing with pivoting hinge. However, for the case of the current thesis, let the term morphing be reserved for the case when:

Region of aircraft outer geometry is undergoing by intention a conformal deformation from initial state without geometric discontinuities within or adjacent to the morphed region [7].

According to Barbarino [5], the types of in-flight geometry modifications found in the literature can be divided into three major groups: planform alteration (involving span, chord and sweep changes), out-of-plane transformation (twist, dihedral/gull and spanwise bending) and airfoil adjustment (camber and thickness distribution). The topic of morphing VCTEF belongs to this last group with the upper surface of the flap undergoing a smooth deformation and altering the flow over the wing.

A historical note should be made that on early aircraft, the use of morphing was not uncommon. Wright brothers used out-of-plane twist morphing on their Flyer aircraft in order to produce asymmetric spanwise lift distribution over the wing for roll control [5, 8, 9]. Unlike with the contemporary hinged control devices actuation, the pilot controlled wires which directly pulled on the wing tips and resulted in wing warping. Such concept was simple and effective for that time, but was soon replaced with piece-wise rigid variable geometry [10]. In 1910 Henry Farman introduced ailerons, from which by using similar devices with symmetrical deflection the plain trailing edge flaps were developed and fitted to Fairey Hamble Baby aircraft in 1916 [11].

A reasonable explanation to this change is given by Sanders [7]. The performance of early aircraft engines was just enough to make flight possible. With growth of the industry the increasing demand on travel speed, payload and range led together with improvement in engine power to larger flight envelopes. Suppression of associated negative effects required a stiffer structure to allow for safe design of aircraft. These included aeroelastic instabilities as well as stable but unpredictable static aeroelastic deformations which affect the wing characteristics. With limited computational power it was an uneasy task to tailor the difference between jig shape and in-flight shape during design. True conformal morphing still poses a structural challenge, requiring on one hand flexibility to allow wanted deformations and on the other hand stiffness to maintain certain aerodynamic shape without collapsing [12]. Until composite materials were brought in practice, this was difficult to achieve at higher dynamic pressures without significant structural weight penalties. Another issue with conventional metal alloys is the fatigue, which is especially critical for cyclic morphing [13].

Weight penalties of the morphing and even piece-wise rigid in-flight modification concepts were only rarely overcome by benefits in such way that the aircraft had serious production potential, with several exceptions presented in Appendix A. It should be noted that majority of the morphing technology was applied
to either small aircraft, uninhabited aerial vehicles or military aircraft during the periods of good funding in research (approximately at and before the World War II and during the cold war). The use of morphing for full-scale transport aircraft is very rare, example of it being the experiments with Lockheed L-1011 during the fuel crisis [9, 14]. Technology of wing trailing edge morphing from FlexSys Inc. is only recently being tested by NASA on a Gulfstream GIII aircraft [15].

Even considering the variable camber with piece-wise rigid devices the high speed deployment was limited to combat military aircraft which benefit from improved maneuverability provided that the actuation is fast. Usually simple leading and trailing flaps are used for this, examples being the P-51 Mustang, F-86 Sabre Jet, F-100 Super Sabre, F-4 Phantom or F-16 Fighting Falcon [11, 16]. Until recent years however, the design of transport aircraft was not aimed at deployment during transonic cruise. Unauthorized attempts to do so occurred in history with disastrous consequences. According to National Transportation Safety Board [11] on 4th April 1979 a crew of Boeing B727 deployed trailing edge flaps to 2 degrees which were on this aircraft deflected simultaneously with leading edge slats. Upon retraction one of the slats jammed under the high dynamic pressure and caused an uncontrollable dive from 36000ft to 5000ft in 63 seconds. Modern examples of aircraft that have been certified to use the piece-wise rigid high-lift devices during cruise are the Airbus A350 and Boeing B787 [13, 17]. The Adaptive Dropped Hinge Flap (ADHF) in figure 1.1 uses flap together with spoiler deflection which is capable of a seamless trailing edge camber variation in cruise.

Some lessons can be learned from other research projects concerning the effects of morphing VCTEF although the results vary significantly depending on the aircraft, the study case (morphing concept), and the flight conditions. Research lead by Grumman [18] in the late 2000's aimed at Airbus A340-300 aircraft has shown that in transonic cruise the shock-wave position and strength can be largely manipulated by the airfoil trailing edge shaping, which gives potential for wave drag reduction. Concerning the smoothness of the contour, wind tunnel measurements within the Smart Wing project for F-18 fighter [7] at lower speeds have shown that conformal deflection lacks the sharp upper surface peak (present at plain flap deflection) and demonstrates a rather smooth pressure bump more aft of the hinge line (beginning of the smooth morphing). On the other hand a review by Szodruch [11] mentions that the X-29 demonstrator using a discrete camber variation has shown surprisingly low penalties of non-smooth upper surface in transonic cruise both in wind-tunnel and flight tests. Furthermore, the project NEW is mentioned by Szodruch which identified the general effects of camber variation at the trailing edge to be:

- rotation of the drag polar
- shift of minimum drag point and maximum lift-to-drag point to a higher lift coefficient
- reduction of leading edge pressure peaks due to aft loading of the airfoil
- increase in the airfoil pitching moment

The net effect of the above statements for a typical 1990’s transonic cruise aircraft can be observed on the lift-to-drag ratio $L/D$ during cruise in figure 1.2, which is found in research by Austin [19] and the FlexSys company [20]. However, this figure is not taken as representative, since clearly a large improvement of 3% is claimed at the mid-cruise design point, which this research will assume the aircraft to be already optimized for, as suggested in literature [8, 9, 11]. During a literature study phase a lack of details was found on exact conditions and evaluation methods towards the entire cruise performance to be able to draw conclusions specifically for an A320 aircraft.
Due to the fact that changing section camber was shown to have effect on the overall lift, drag and also the pitching moment of the wing, for practical application the aircraft must be re-trimmed. The deployment of either conventional or conformal high-lift devices during cruise in combination with re-trimming could be used to arrive at a performance optimum at a given cruise condition that is particularly interesting for modern aviation - that of reduced cruise drag. Integrated performance throughout the entire cruise can then be translated into reduced direct operating costs [21, 22]. According to an estimation provided by Gilyard [14], if 1 unit of fuel is saved for a typical transport aircraft at Maximum Fuel Weight (MFW), it is possible to add 3 units of payload, each worth up to 30 times the cost of fuel. Reduction of 1% in fuel consumption can then typically save over $12,000,000 each year on top of helping the environmental issue, showing that the motivation is relatively high.
1.2. STUDY CASE DESCRIPTION

In this section the morphing flap concept by Fokker Aerostructures is introduced. However, the exact specifics and internal structure in the study case provided by the company are confidential. Only the outer geometry is therefore described here and on the level that is already retrievable from other literature. The concept involves morphing of a small flexible chordwise segment of the flap upper surface denoted as the “morphing region”, followed by a rigid trailing edge wedge, requiring a discontinuity on the lower surface. The mechanism is fully internal to the flap and would still allow Fowler motion in the high lift case.

Several parameters have to be chosen (optimized for), as defined in figure 1.3, which will be in the further text denoted as the “morphing design variables”. The segment of the upper surface where the morphing takes place is bounded by a morphing beginning $x_b$ and a morphing end $x_e$, expressed relative to a local streamwise section chord. Furthermore, a morphing function is defined on the morphing region. For purposes of this research the function is assumed circular, expressed through a morphing radius $r_m$. Later in the research it became clear that another way of morphing region definition is advantageous with setting the value $d_m$ of the morphing region size and the morphing mid-position $x_{d_m}$, from which the $x_b$ and $x_e$ are equidistant ($\pm \frac{d_m}{2}$). Furthermore, for performance evaluation it is more convenient to work with an angular deflection $\delta_m$ instead of specifying the morphing radius, which goes to infinity close to the base (non-morphed) geometry. The angular deflection together with the morphing region size and position fully define the morphed contour and result in a certain morphing radius, which for the described concept must be above 300 \text{mm} to comply with the allowable stresses in the upper surface skin.

In figure 1.3, the projections of the $x_b$ and $x_e$ on the non-morphed upper surface (magenta projection points) are situated behind the spoiler tip which coincides with the point where the flap emerges from the cove (blue projection point). This is one of the design constraints, since morphing the flap earlier would result in a sharp kink on the upper surface unless the spoiler is allowed to droop. On the A320 aircraft, only a small range of morphing mid-positions would then be allowed. However, for aerodynamic analysis it is possible and desirable to assume the flap and the main wing to be a joint element to which the morphing is applied. This constraint on the morphing region position is therefore dropped for comparison of different possibilities to vary section camber and to understand the underlying aerodynamic phenomena. The allowable locations of the morphing region beginning will be only limited beyond the spoiler hinge line, which was estimated at approximately 65-67\% of chord for an A320 outboard wing.

When the morphing trailing edge concept is brought to a full 3D wing, adjustments have to be made to deal with the wing taper. In the provided concept the bending of the upper surface is that of a thin sheet material with large in-plane stiffness. Therefore two options are available, respecting the taper of the flap itself and the way it is deployed on A320 aircraft. Since the inboard flap has along its span an approximately constant absolute chord (no taper), the deflection is cylindrical, using a constant absolute size of morphing region $d_m$ together with spanwise constant inbound morphing deflection $\delta_{m_i}$. The outboard flap has a constant relative chord along the span (taper matching the outboard wing) and therefore the morphing deflection is conical. This is modelled by adjusting $d_m$ according to the taper again with a spanwise constant outbound deflection $\delta_{m_o}$. Such resulting morphing distribution is demonstrated on the cover of this thesis report where the magenta lines illustrate the morphing regions along the span. The morphing could be also applied on the aileron surface for a full span flow control.
1.3. **PROBLEM STATEMENT AND RESEARCH GOALS**

Using a model of the presented concept, the main objective of interest to the Fokker Aerostructures and the addition of this research to the scientific knowledge is to determine whether a smooth morphing of section camber at the trailing edge is an effective method of flow control during transonic cruise of a short-to-medium range aircraft. To measure this, performance indicators need to be established to enable comparison of various forms of the concept implementation in terms of their added value. As pointed out by Bolonkin and Gilyard \[9, 14\], from operational point of view and while keeping other parameters constant the reduction of drag by morphing can be used to either:

- increase the range over the design mission
- save fuel on the design mission
- fly the design mission at a different Mach number
- change the cruise altitude

To limit the scope of the study only the first two interpretations of performance benefits will be evaluated, i.e. the range improvement $\Delta R$ and the fuel savings $\Delta F$, denoted as the performance indicators. As a reference for performance benefits evaluation the design cruise mission of A320 will be taken into account. The reason why a design mission was selected is to aim at a critical assessment of the concept. According to E.Roux \[23\] and Jane’s All the World’s Aircraft \[24\], the design mission of an A320 is to transport 150 passengers (equivalent to 15 tons of payload) over a mission range of 4800km. This will be assumed at a constant altitude of 37000ft and with a constant Mach number of 0.78. An even more critical point of view would be a continuous cruise climb or the stepped cruise. However, on a short-medium range mission this scenario is less likely to be approved by the air traffic control \[4\], bringing additional complexity to the research problem.

In the mid-cruise condition of such mission the performance is assumed to be at an optimum and theoretically no morphing should bring significant improvement. Anywhere else during the cruise the conditions are off-design and some performance improvements can be expected. For their evaluation the aircraft geometry will be analysed in terms of its morphed and base (non-morphed) aerodynamic characteristics at several cruise points from beginning to end of cruise, using interpolation to save computational time. The answer to the main research question: "Is it beneficial to apply camber morphing on A320?" is based on the result of the main research goal, i.e. measuring how much fuel/range improvements can one expect on the design mission. In order to do that the following sub-goals are proposed:

- create an aircraft model with known and assumed data for A320, compatible with analysis methods
- tune the unknowns in the model to obtain a near mid-cruise optimum reference set-up
- calculate requirements for each cruise point
- optimize section morphing variables using improvements over each cruise point
- evaluate range and fuel improvements by interpolation from cruise point performance

Since a full 3D evaluation is time consuming, the morphing design (variables $x_b$ and $x_e$) shall be based on 2D analysis. Therefore the given lift requirements on the entire (3D) aircraft must be translated to section requirements at each cruise point at a chosen reference section. The improvements in section performance are then translated back to the 3D aircraft performance as a prediction of the morphing variables setup effect.
As was mentioned in the introduction, both 2D and 3D analysis are used to complete the research goal within a reasonable computational effort. The purpose of applying 2D analysis shall be to design the morphing concept variables, whereas the 3D analysis serves to create the reference model of the aircraft and obtain the lift requirements for several selected cruise points conditions. A relation is then made between the 2D and 3D analysis in order to compare the performance benefits of the morphing flap concept setups in terms of the selected performance indicators - the range improvement $\Delta R$ and the fuel savings $\Delta F$.

These performance indicators are top level and require performance evaluation which is based on aerodynamic analysis and is capable of putting the results of such analysis into perspective of the entire cruise flight of the aircraft. Although a concrete A320 aircraft is chosen and an attempt is made to match the reference calculations to the known performance data, the performance benefits evaluation is a matter of relative values. The key role in this process has the optimization which is used to generate these relative performance improvements and assemble them into the resulting top level indicators.

The purpose of this chapter is to present an overview of the used methods and tools. First the aerodynamic tools are discussed in section 2.1 which are applied to generate results at individual cruise points. Section 2.2 then explains in greater detail the performance evaluation process, which requires the optimization techniques discussed in section 2.3.
2.1. AERODYNAMIC ANALYSIS TOOLS

The basic requirement on the aerodynamic solvers applied to a problem is that the most relevant phenomena of the problem are captured by the used computational methods. However any model, especially ones with lower fidelity, will have errors coming from inability to completely capture the real flow. Transonic flight investigated in this work is complex and therefore compromises have to be made so that the solution time is not too long, since many evaluations will be required as will be seen further.

The aim is therefore to avoid using the computationally expensive full 3D calculations or directly computing the viscous effects. Also, an assumption is made that for examination of the morphing flap concept some phenomena can be omitted if they are not purposely affected by the morphing itself and/or would require an expensive higher order analysis method. Example of such phenomena are the root and tip effects on swept wings [3]. Another omitted effect would be the boundary layer cross-flow, which will not be modeled directly apart from forcing an early transition of boundary layer in 2D analysis which normally occurs within few percents of chord for wings of moderate sweep [25]. Finally, all interference effects will also be dropped from the model, such as between wing and fuselage body.

The overall scheme used in this thesis work is inspired by the article of Reckzeh [21], which presents the "Chain of Methods" in Computational Fluid Dynamics (CFD) used at Airbus, there applied to an A380 aircraft wing design in a multidisciplinary environment. Similar to what was proposed in the introduction, Airbus uses 2D calculations based on viscous-inviscid interaction which are complemented with quasi-3D calculations for assessment of the wing performance, making the process fast and allowing runs with many variations on design variables especially when computational resources are limited. The adopted 2D and 3D tools are described in sections 2.1.1 and 2.1.2, respectively.

2.1.1. MSES 2D SOLVER

There exist several methods for solving 2D flow that could be used in the proposed setup. To assure consistency between 2D and 3D analysis in the project, the same solver will be applied for 2D analysis as will be used in the quasi-3D method explained in section 2.1.2.

The most important factors for selecting a suitable solver were the availability for this project, the ability to capture shock-waves and mild separation in transonic cruise and most importantly the computational speed for the large number of evaluations as will become clear in the quasi-3D method. Shockwaves are highly probable for transonic aircraft and in this project their strength and associated entropy gradients could vary significantly. For this reason according to Iollo [26] the Euler solvers are preferred to the potential methods since they offer a good balance between accuracy and speed of evaluation. The Euler solver MSES developed at MIT by Drela [27] was selected based on experience of several other research projects, for instance Steenhuisen and Tooren [28], or Milhoen [18]. These were done on morphing involving similar flight conditions of transonic Mach numbers above 0.7 and a lift coefficient $c_{l_{ref}}$ of around 0.7. The need for these conditions will be shown in chapter 4 on the reference wing.

MSES is a FORTRAN written viscous-inviscid interaction 2D solver which makes use of a streamlined grid. This allows reduction to one dimension for the momentum balance and some parts of the drag estimation are then simplified to determining momentum loss within the stream-tubes, sampled at the exit-plane of the domain. The inviscid flow momentum loss represents the wave drag, whereas the momentum loss within the boundary layer and wake flow is denoted as the viscous drag. The normal and tangential forces on the surface are evaluated by line integrals and result in the lift force, friction force and pitching moment [29]. The total drag is the sum of the wave and viscous drag (only partly comprised of friction drag), whereas the standard pressure drag component is evaluated indirectly by subtracting friction drag from the total drag, which is deemed more accurate compared to direct evaluation from surface forces.

The initial mesh of MSES is automatically generated based on inviscid potential method but after that it is solved by iteration throughout the convergence to the final flow solution. Furthermore, the viscous effects are included through numerical integration of integral momentum and energy equation using Green's lag equation for dissipation in turbulent flow. The inviscid Euler and viscous integral boundary layer equations are strongly coupled, solved together in a Newton scheme with fast convergence [30]. All the above mentioned features however translate in reduced robustness, meaning that the solver at times does not converge [28]. However, if a solution is obtained it is consistent and as with other Euler solvers relatively grid independent. The same holds for the domain size. MSES is also capable of handling multi-element airfoils and being based on Euler equations it is versatile for great range of conditions including high-altitude, transonic or low speed flight [31]. One of the features of this solver is the ability to include a lift coefficient requirement into the solution iteration and easily arrive at the drag for a given lift, which is convenient for this research.
To test the solver settings used in this thesis, the validation case of MSES with RAE2822 airfoil was reproduced from an article by Giles and Drela [32] for $M = 0.726$, $Re = 6.5e6$ and required $c_l = 0.743$. A convergence study with respect to the number of grid points on the airfoil is shown in figure 2.1. It can be said that by using more than 200 points the results will not change due to grid resolution significantly. Compared to the experimental results for this airfoil, drag coefficient $c_d = 0.0127$ and moment coefficient $c_m = -0.095$, the MSES determined values have around 9% and 13% error, respectively. These absolute values are not large and can be caused by the differences between wind-tunnel measurements and the free-stream conditions assumed for the calculation. The average computational time for one such evaluation was measured to be 2.08 seconds on an Intel® Core™ i7-3612QM mobile processor. The pressure coefficient $c_p$ distribution has also been successfully reproduced from another validation case of MSES C.1 of Appendix C for a case with a moderate shockwave including the increased boundary layer thickness behind the shock, as seen in figure C.2 where the same settings are used as for the remainder of this work.

The above figures show that for a single lift coefficient the errors are acceptable. However, a proof needs to be made that the drag polar diagram is valid throughout the expected range of lift coefficients and therefore a comparison is made in figure 2.2 to the NASA cryogenic wind tunnel measurements available in a report by Johnson [33] for the SC(3)-0712(B) airfoil at $M = 0.78$, $Re = 15e6$ and a fixed transition at 5% chord.

This airfoil was chosen for its significant amount of aft camber and similar thickness to the A320 outer wing airfoils according to Obert [3]. The airfoil shape is demonstrated later in section 4.1.1 where a technique is presented for obtaining a fine geometry description. It can be observed in figure 2.2 that there is a slight but rather constant over-prediction of drag at high Reynolds numbers and high cruise lift coefficients.
Various settings are possible within MSES through its grid and run settings files gridpar.* and mses.*, respectively. These are explained in Appendix C. Since the morphing will take place on a limited range within airfoil chord, surface grid refinement should be in order to capture the morphed curvature which could be at low grid resolution replaced with a single kink by the automatic grid generation. Example of grid refinement for the demonstration of the Fokker concept presented in section 1.2 is shown in figure 2.3.

The upper surface of the morphing region including certain offsets $x_b - \Delta_b$ and $x_e + \Delta_e$ was set to have the ratio between the local density and the average density over the surface of 2, as a result of a convergence study on the grid refinement done in figure 2.4. The choice of the density ratio not larger than 2 is due to the fact that a finite number of points are available for MSES of version 3.11. It was observed that increasing the density inside a certain region draws the points from other regions, for example at the leading edge or near the 50% of chord, where a shockwave modelling is important. The offsets $\Delta_b = 0.01$ and $\Delta_e = 0.05$ have been added to enlarge possible small morphing regions behind which separation could be likely. An example of such separated flow is given in figures C.3 and C.4 of Appendix C.

MSES user guide [29] makes several further recommendations. During convergence run it is possible to have the grid being automatically coarsened or refined from the original number of points (if they are odd number). For transonic cases with shockwave present it is beneficial to make two runs, first with a coarsening which promotes faster shockwave position stabilization, by leaving out every other grid node. The first run does not need to be fully converged, but it was observed that 40 iterations are enough to provide a good initial solution on the mesh and the shockwave. The second run is set with refining of the grid back to the original number of points and the average number iterations in this run to a successful convergence was found to be below 20. As a safety margin both runs are set with 40 iterations.
Another measure taken to improve the convergence of the tool are the perturbations of the initial conditions. As was mentioned before, MSES has at times a numerically based divergent behavior which has no physical basis. However solutions which did converge with the settings coming from the converge study were found to be consistent with other solutions (functions of flow parameters are smooth). A chance exists that a very slight change to the set of input variables can cause convergence again. Without the need to change flow conditions. As was mentioned before, MSES has at times a numerically based divergent behavior which has no physical basis. However solutions which did converge with the settings coming from the converge study were found to be consistent with other solutions (functions of flow parameters are smooth). A chance exists that a very slight change to the set of input variables can cause convergence again. Without the need to change flow parameters such as $M$, $Re$ or the $c_l$ requirement, it is possible to change the initial grid angle in the gridpar.* file, essentially providing the solver with different initial solution. The in-flight effective angles of attack in cruise were found to be in between $0^\circ - 5^\circ$ and therefore the initial grid angle of $0^\circ$ is increased by $1.5^\circ$ each time the solver fails to converge. The limit of attempts was set to 5, since it was observed that there is relatively little chance of convergence far from the final solution. Such perturbation approach was shown successful in promoting convergence at wide variety of sections and at more than 95% of evaluations and it is therefore used in the quasi-3D approach for each effective section solution as described in section 2.1.2.

### 2.1.2. Quasi-3D Approach

This section introduces in general the quasi-3D methods and then goes into detail of their application within the Q3D tool developed at the TU Delft by Mariens, Elham and Tooren [34]. For the use in the transonic cruise morphing flap evaluation only the Q3D input handling had to be adjusted to accept the morphed wing and the cooperation with the MSES tool including the perturbation approach as described in section 2.1.1.

The field of quasi-3D methods is a very favorable option for design with many variables and studied cases, especially in Multidisciplinary Design Optimization (MDO) as in the article by Reckzeh [21]. Most of the quasi-3D methods are based on the principle that the rather expensive evaluation of profile drag is left upon the 2D analysis, which gives opportunity to use a relatively low order 3D method for determining the (3D) induced drag component and generating section requirements for the 2D tool. Conversion from 2D data obtained for multiple sections is usually done using a strip method with integration over the lifting surface span. It makes sense that the 2D solver must be able to predict the profile drag at least to some extent better to gain any advantage over using simply the low order 3D method.

Multiple combinations are available depending on the fidelity of the tools used in both analyses. The fastest quasi-3D approach would start with the lifting-line theory based 3D solver in combination with a panel method based 2D solver with iteration on the effective body shape due to the boundary layer thickness. The 3D solvers can be also based on panel methods, such as the in-house modified VSAERO solver used in Airbus [21], but usually not beyond that since the larger evaluation time then brings down the advantage of obtaining fast solution. The 2D solvers can be upgraded from potential (panel method) based to Euler equations based solver computed on a true grid domain instead of a vector field. In the article by Reckzeh also the use of 2D Navier-Stokes solvers is mentioned for high-lift cases. Within this thesis work the Euler based MSES tool is selected as described in section 2.1.1.

The Q3D solver developed under the Faculty of Aerospace Engineering of TU Delft is used as the basis for the evaluation of the A320 lifting surfaces aerodynamic properties, as explained in section 2.2. The main interest are the lift-drag and lift-moment polar diagrams, evaluated at lift coefficients required from the lifting surface at multiple cruise points. The overall principle of operation of the solver at a given lift coefficient is explained on the following example of the wing lifting surface illustrated in figure 2.5.

The wing drag coefficient $C_{D_{pw}}$ is split into two main components, the wing induced drag $C_{D_{iw}}$ and the wing profile drag $C_{D_{pw}}$, as seen in equation 2.1. The induced drag is evaluated by a vortex-lattice method code AVL, using the far-field Trefftz plane analysis. The profile drag is assumed to be a result of the distributed section profile drag $c_{dp}$ along the lifting surface span. This is in turn a function of the local friction drag $c_{dpf}$ and pressure drag $c_{dpp}$, as mentioned in section 2.1.1, both depending on the section lift $c_l$ distribution along the span through the results of the section analysis.

\[ C_{D_{iw}} = \frac{C_{D_{iw}} + C_{D_{iw}} \cdot c_{dp}}{f(c_{dpf}, c_{dpp})} \quad (2.1) \]

The wing profile drag is calculated as an integral over the section profile drag distribution $c_{dp}$ seen in equation 2.2, with $S_{w}$ being the reference wing planform area, $b/2$ the wing halfspan and $c$ the local streamwise oriented chord, function of spanwise position $y$. For the current approach where the $c_{dp}$ is sampled discretely along the span, the equation takes its summation form in 2.3, where averaged quantities are computed from an array of $c_{dp}$ results.

\[ C_{D_{pw}} = \frac{2}{S_{w}} \int_{y_{ fus}}^{b/2} c_{dp}(y) \cdot c \cdot dy \quad (2.2) \]
Similar to the wing profile drag coefficient, the wing pitching moment coefficient \( C_{M_{AC}} \) about its aerodynamic center can be obtained with reference to figure 2.5. In equation 2.4 the dimensional moment contribution \( dm_{AC} \) of a streamwise aligned element with an infinitesimally small width \( dy \) is twofold. First, there is the pitching moment \( dm \) about its aerodynamic center. Second, its lift \( dl \) acts through an arm \( x_{qc} \) creating an additional moment. An integration is taken over the wing to arrive at expression in equation 2.5, with \( c_{MAC} \) being the wing mean aerodynamic chord. A discretization is done in equation 2.6, based on the \( c_{l} \) and \( c_{m} \) distributions.

\[
C_{D_{pw}} = 2 \sum_{i=1}^{n-1} \left( \frac{c_{dp}(i)c^{(i)} + c_{dp}(i+1)c^{(i+1)}}{2} \cdot \Delta y \right)
\]  
\( (2.3) \)

\[
C_{M_{AC}} = \frac{2}{S_{w}} \sum_{i=1}^{n-1} \left( \frac{c_{m}(i)c^{(i)}(i)+c_{m}(i+1)c^{(i+1)}(i+1)}{2} + c_{l}^{(i)}c^{(i)}x_{qc}(i)+c_{l}^{(i+1)}c^{(i+1)}(i+1)x_{qc}(i+1) \right) \cdot \Delta y
\]  
\( (2.6) \)

The streamwise oriented section profile drag coefficient \( c_{dp} \) and section moment coefficient \( c_{m} \) figuring in equations 2.3 and 2.6 are obtained according to the simple sweep theory, as discussed by Mariens [35]. For this procedure the lifting surface geometry generator must be capable of extracting effective sections, which are perpendicular to a line along the wing of a constant relative chord position. An example of such effective section is shown as the red chord line \( c_{\perp} \) in figure 2.6. The line with respect to which these sections are perpendicular is selected at the half-chord of the streamwise sections \( c \) which is a likely position for a shockwave, as it is also advised for the best transonic drag prediction in a report by NASA [36].

From the same figure the relations can be made between the local streamwise oriented characteristics \((V_{\infty}, M_{\infty}, c_{l})\) and the characteristics perpendicular to half-chord line \((V_{\perp}, M_{\perp}, c_{l\perp})\), as shown in equations 2.7.

Note that the velocity vector \( V_{\perp} \) lies in the x-y plane where x-axis is aligned with the free-stream velocity \( V_{\infty} \).
A distinction must be made between the effective lift and drag coefficients \( (c_{l_{\text{eff}}}, c_{d_{\text{eff}}}) \) in the 2D analysis and their projections \( (c_{l_{\perp}}, c_{d_{\perp}}) \) as explained with use of unity forces decomposition in figure 2.7. The 2D solver which solves for flow properties at a given \( c_{l_{\text{eff}}} \) will have as an output the corresponding effective angle of attack \( \alpha_{\text{eff}} \) at which this \( c_{l_{\text{eff}}} \) is achieved and the effective drag coefficient \( c_{d_{\text{eff}}} \). However, the incidence \( i_{\text{eff}} \) of the section with respect to the x-y plane is given by the sum of aircraft (wing) angle of attack \( \alpha \) and the local twist along the span \( \epsilon \), in the form corrected for half chord sweep \( i_{\text{eff}} = (\alpha + \epsilon) \cos \Lambda_{0.5} \). The difference between the incidence angle of the section on the wing and this effective angle of attack is denoted as the induced angle of attack \( \alpha_{i} \). The induced angle of attack will tilt the effective forces of the section and therefore the net lift will be different from the section requirement imposed on the \( c_{l_{\perp}} \). An iteration is therefore in order, such that the solver output of the \( \alpha_{\text{eff}} \) matches through the values of \( \alpha_{i} \) and the required \( c_{l_{\perp}} \) and the actual incidence of the section. Equations 2.8 can be derived directly from figure 2.7 in order to relate the effective and the perpendicular to half-chord properties, assuming the airfoil chord is constant in both frames.

\[
\begin{align*}
V_{\perp} &= V_{\infty} \cos(\Lambda_{0.5}) \\
M_{\perp} &= M \cos(\Lambda_{0.5}) \\
c_{l_{\perp}} &= \frac{c_{l_{\text{eff}}}}{\cos(\Lambda_{0.5})}
\end{align*}
\] (2.7)
Since for a given cruising altitude the air density and dynamic viscosity are constant irrespective of the reference frame, the effective Reynolds number is simply the local streamwise strip Reynolds number $Re(y)$ multiplied by the ratio of effective and free-stream velocities and chords as seen in equation 2.9.

$$Re_{\text{eff}} = Re(y) \frac{V_{\text{eff}} c_\perp}{V_\infty c} = Re(y) \frac{\cos(\Lambda_{0.5}) c_\perp}{\cos(\alpha_i)/c} \quad (2.9)$$

The effective lift coefficient is derived from equations 2.8. Since effective and perpendicular to half-chord line characteristics are based on different velocities, the derivation in equation 2.10 of $c_{\text{eff}}$ must start from the unity airfoil forces parallelograms in figure 2.7 to arrive at the final form in equation 2.11.

$$l_i = l_{\text{eff}} \cos(\alpha_i) = \frac{1}{2} \rho V_{\text{eff}}^2 c_{\text{eff}} \cos(\alpha_i) - \frac{1}{2} \rho V_{\text{eff}}^2 c_{\text{eff}} \sin(\alpha_i)$$

$$\cos^2(\alpha_i) c_{\text{eff}} = c_{\text{eff}} \cos(\alpha_i) - c_{\text{eff}} \sin(\alpha_i) \quad (2.10)$$

$$c_{\text{eff}} = c_\perp \cos(\alpha_i) + c_{\text{eff}} \tan(\alpha_i) \quad (2.11)$$

After initially setting $\alpha_i = 0$ and obtaining the properties $V_{\perp}, M_{\perp}$ and $c_\perp$ from equations 2.7, each iteration step on the effective angle of attack then takes the following form:

- obtain required effective lift coefficient $c_{\text{eff}}$ by equation 2.11
- determine $V_{\text{eff}}, Re_{\text{eff}}$ and $M_{\text{eff}}$ by equations 2.8
- evaluate the 2D solver for required $c_{\text{eff}}$ at $V_{\text{eff}}, Re_{\text{eff}}$ and $M_{\text{eff}}$ to obtain $\alpha_{\text{eff}}$ and $c_{\text{eff}}$
- update the induced angle of attack as $\alpha_i = i_{\text{eff}} - \alpha_{\text{eff}}$

After arriving at the converged effective angle of attack, similar derivation as in equations 2.10 can be made for relating the resulting effective drag coefficient obtained from 2D solver back to the perpendicular to half-chord line frame. However, the projection of effective lift into the perpendicular to half-chord line reference frame is not necessary, since resulting "induced" drag is already accounted for from the results of the 3D solver Trefftz plane analysis. Therefore, the derivation is simplified in equations 2.12 and the profile drag coefficient $c_{d,\perp}$ in the perpendicular to half-chord line frame is then expressed in its friction and pressure drag components by equations 2.13 and 2.14, respectively.

$$d_{\perp \text{p}} = \frac{1}{2} \rho V_{\text{eff}}^2 c_{\text{eff}} \cos(\alpha_i)$$

$$d_{\perp \text{f}} = \frac{1}{2} \rho V_{\text{eff}}^2 c_{\text{eff}} \cos(\alpha_i) \quad (2.12)$$

$$c_{d,\perp} = \frac{d_{\perp \text{f}}}{\cos(\alpha_i)} \quad (2.13)$$

$$c_{d,\perp} = \frac{d_{\perp \text{f}}}{\cos(\alpha_i)} \quad (2.14)$$

It was shown by NASA [36] that for the best drag prediction the friction drag determined along the effective chord is comparable with its streamwise magnitude, whereas the pressure drag is scaled with a third power of cosine of the sweep angle of the half-chord line $\Lambda_{0.5}$. The expression for $c_{d,\text{p}}$ then becomes the equation 2.15.

$$c_{d,\text{p}} = c_{d,\perp \text{f}} + c_{d,\perp \text{p}} \cos^3(\Lambda_{0.5}) = c_{d,\perp \text{f}} + c_{d,\text{pp}} \quad (2.15)$$

Since the effective moment coefficient $c_{m,\text{eff}}$ based on the result after iteration is oriented along the same axis in the perpendicular to half-chord line frame, its component $c_m$ acting about $y$-axis is given simply by equation 2.16.

$$c_m = c_{m,\text{eff}} \cos(\Lambda_{0.5}) \quad (2.16)$$

The distribution of section profile drag coefficients $c_d$ and the moment coefficients $c_m$ evaluated through the 2D solver iterations at multiple $y$-positions can be now used to determine the wing profile drag $C_{D_p}$ and the pitching moment about the aerodynamic center $C_{M_C}$ by discretized equations 2.3 and 2.6, respectively. With addition of the wing induced drag $C_{D_i}$ from AVL, the total wing drag $C_{D_w}$ is obtained. Validation of the quasi-3D tool done by Mariens is shown in the Appendix D. The same procedure as done for the wing can also be done for any other lifting surface, for example the horizontal stabilizer.
2.2. PERFORMANCE INDICATORS EVALUATION

The chosen performance indicators, range improvement $\Delta R$ and fuel savings $\Delta F$, are a result of evaluating aircraft range $R$ and comparing it to the reference range for the design mission $R_0$. The calculated range itself is function of the specific range $SR$ which determines at any cruise point the distance that can be travelled per unit weight of fuel. A typical distribution of the specific range is seen in figure 2.8 as a function of the aircraft cruise weight $W$ which is getting lower throughout the cruise due to the consumption of fuel.

Standard expression used for aircraft range calculation is the Breguet equation [4]. There are multiple possibilities how to arrive at its simplified analytic forms based on assumptions taken, however for numerical handling with own calculated specific range the equation must be used in its basic non-integrated form. Expressed as given by Roskam [37] this basic form is shown in equation 2.17, with $W_b$ being the aircraft weight at the beginning of cruise and $W_e$ the aircraft weight at the end of the cruise.

$$R = \int_{W_b}^{W_e} SR dW = \int_{W_b}^{W_e} \frac{V_{\infty}}{F} dW$$ (2.17)

The specific range is a function of the flight velocity $V_{\infty}$ and the fuel consumption $F$. Since the cruise flight is assumed quasi-steady, the thrust is always adjusted accordingly to balance the drag. A simple model of the fuel consumption can be adopted by setting $F = c_T(N)/D$, where $c_T(N)$ is the thrust specific fuel consumption (in unit of $[N/s]$) and it is assumed constant for the design mission. This results in the need of finding the aircraft drag along the cruise with the actual weight being the requirement, which conveniently fits the way the quasi-3D solver operates. However, the Q3D solver gives a results in form of a drag coefficients for the lifting surfaces. With the model of fuel consumption and assuming the lift is equal to weight the Breguet range equation on which the performance evaluation is based takes form of equation 2.18. As a side remark a frequently used first order approximation involves assuming a constant L/D ratio which results in equation 2.19.

$$R = \int_{W_b}^{W_e} \frac{V_{\infty}}{c_T(N)C_D} dW = \int_{W_b}^{W_e} \frac{V_{\infty}}{c_T(N)C_D} \frac{C_L}{C_D} dW$$ (2.18)

$$R = \frac{V_{\infty}}{c_T(N)C_D} \frac{C_L}{C_D} \ln \left( \frac{W_b}{W_e} \right)$$ (2.19)

A discretized form of equation 2.18 is necessary for numerical calculations, which is shown in the equation 2.20. Here the aircraft weight was converted to mass in kilograms for easier manipulation with known aircraft data and at the same time the specific fuel consumption is expressed in kilogram dependent unit found for the basic version of A320 engine (CFM56-5A1) in the publication by Roux [23] to be $c_T = 1.688 \times 10^{-5} \frac{kg}{N}$. This requires an extra factor of the gravitational acceleration $g$ in the denominator.

$$R = \sum_{i=1}^{k-1} \frac{V_{\infty}}{c_T g} \left( \frac{C_L}{C_D m} (i+1) + \frac{C_L}{C_D m} (i) \right) \Delta m / 2$$ (2.20)

The discretized form will have better precision the larger the amount of cruise points are taken into account. However, evaluations of cruise points using viscous analysis on multiple sections are costly. An easy solution is to evaluate only some sufficient number of points to define a smooth interpolation, from which a finer resolution of cruise points can be determined. If the cruise is sampled at an odd number of equidistant points (with respect to aircraft mass) the important mid-cruise condition will also be included. Furthermore,
the polar diagrams are often approximated by parabolic functions [6], for which a bare minimum of 3 points are necessary. To increase the possibility of capturing at least some non-parabolic behaviour 7 cruise points are used in this work.

The fine resolution can be obtained by using a Piece-wise Cubic Hermite Interpolating Polynomial (PCHIP) in Matlab and refining the obtained data to a large number of points. A total number of $k = 50$ interpolated cruise points enter the range evaluation. The advantage of PCHIP compared to other interpolations such as splines, is that it is shape preserving, i.e. there are no overshoots, undershoots or oscillations around the data points. Example of interpolated data using different interpolation techniques is shown in figure 2.9, which is an illustration of what could be a $C_L/C_D$ function over cruise. Concerning morphing, an overshoot in interpolated optimized drag function could for instance create a drag increase in-between two evaluated cruise points although the morphing optimization will by logic perform always at least as well as the original geometry which is included by zero morphing deflections. Another disadvantage of polynomial approximation is the inability to extrapolate data well. The PCHIP is still better than spline extrapolation, however the linear extrapolation was found the most reliable, especially for geometry extrapolation in chapter 4.

![Figure 2.9: Example of interpolation and extrapolation techniques](image)

As seen from equation 2.20, for the range to be evaluated it is required to determine the aircraft mass $m$ and the corresponding $C_L$ and $C_D$ values at 7 cruise points denoted further by subscripts $CR_1 - CR_7$, such that $k$ interpolated data points can be created of the factor $\frac{C_L}{C_D}m$. Therefore, the first step is to obtain the aircraft mass and the lift coefficients corresponding to those points for the design mission specified in section 1.2 and illustrated in figure 2.10, with $L_1 - L_7$ being the legs of the mission from engine startup to shut down including the cruise leg $L_5$. Such detailed data on A320 were not found in the literature and the aircraft operators usually have a software used for flight planning, which was not available for this thesis work. Fortunately, the method of Fuel Fractions can be followed by the design book of Roskam [38], since the aircraft does fit the typical jet transport category statistics. Then only basic data are needed which can be found for an A320 [23, 24]. The input data for the fuel fractions method are summarized in table 2.1, where the mission legs $L_1 - L_7$ are explained through their respective fuel fractions $M_{ff_{L1}} - M_{ff_{L7}}$.

![Figure 2.10: Design mission legs for method of fuel fractions](image)

The method of fuel fractions can be used to evaluate the aircraft mass at the beginning of cruise by equation 2.21, with $m_{MTOW}$ being the aircraft mass already after engines start, warm-up and taxiing.

$$m_b = M_{ff_{L1}} \cdot M_{ff_{L4}} \cdot m_{MTOW}$$ (2.21)
2.2. PERFORMANCE INDICATORS EVALUATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\text{MTOW}}$</td>
<td>73500</td>
<td>kg</td>
<td>aircraft mass at maximum take-off weight</td>
</tr>
<tr>
<td>$m_{\text{OEW}}$</td>
<td>40000</td>
<td>kg</td>
<td>aircraft mass at operating empty weight</td>
</tr>
<tr>
<td>$m_{\text{P/L}}$</td>
<td>15000</td>
<td>kg</td>
<td>mass of design mission payload</td>
</tr>
<tr>
<td>$M_{\text{f}1}$</td>
<td>0.990</td>
<td>[-]</td>
<td>fuel fraction - engine start, warm-up</td>
</tr>
<tr>
<td>$M_{\text{f}2}$</td>
<td>0.990</td>
<td>[-]</td>
<td>fuel fraction - taxi</td>
</tr>
<tr>
<td>$M_{\text{f}3}$</td>
<td>0.995</td>
<td>[-]</td>
<td>fuel fraction - take-off</td>
</tr>
<tr>
<td>$M_{\text{f}4}$</td>
<td>0.980</td>
<td>[-]</td>
<td>fuel fraction - climb</td>
</tr>
<tr>
<td>$M_{\text{f}5}$</td>
<td>unknown</td>
<td>[-]</td>
<td>fuel fraction - cruise</td>
</tr>
<tr>
<td>$M_{\text{f}6}$</td>
<td>0.990</td>
<td>[-]</td>
<td>fuel fraction - descent</td>
</tr>
<tr>
<td>$M_{\text{f}7}$</td>
<td>0.992</td>
<td>[-]</td>
<td>fuel fraction - landing, taxi, shut-down</td>
</tr>
<tr>
<td>$S_w$</td>
<td>122.44</td>
<td>m$^2$</td>
<td>reference wing planform area</td>
</tr>
<tr>
<td>$\rho_\infty$</td>
<td>0.3482</td>
<td>kg/m$^3$</td>
<td>air density at cruise altitude</td>
</tr>
<tr>
<td>$V_\infty$</td>
<td>230.2</td>
<td>m/s</td>
<td>flight velocity derived from altitude and Mach number</td>
</tr>
</tbody>
</table>

Tab. 2.1: Data used for cruise points derivation [38],[23],[24]

The aircraft mass at the end of cruise needs to be extracted indirectly along with the unknown fuel fraction for cruise. First, the total fuel mass $m_F$ for the design mission is evaluated from the known data in equation 2.22.

$$m_F = m_{\text{MTOW}} - m_{\text{OEW}} - m_{\text{P/L}}$$  \hspace{1cm} (2.22)

Using the total fuel mass the reserve fuel $m_{\text{Fres}}$ is determined. This would in operation depend on the fuel policy applied, but from the minimum requirements [39] it can be found as 5% of planned trip fuel (contingency fuel) and an additional 30+15 minutes flight at 1500ft altitude (final reserve fuel and minimum additional fuel). Without the prior knowledge of drag at this low altitude the total reserve fuel was simply estimated to be approximately 10% of the fuel used $m_{\text{Fused}}$. Combining this with the fact that $m_F = m_{\text{Fused}} + m_{\text{Fres}}$, the reserve fuel is expressed in equation 2.23.

$$m_{\text{Fres}} = \frac{0.1}{1.1} m_F$$  \hspace{1cm} (2.23)

Using the unknown fuel fraction for cruise $M_{\text{f}5} = \frac{m_e}{m_b}$ and the fuel fractions for descent, landing until shut-down, the aircraft mass at shut-down $m_{\text{SD}}$ is expressed in equation 2.24. The same mass can also be expressed by the sum in equation 2.25.

$$m_{\text{SD}} = M_{\text{f}7} \cdot M_{\text{f}6} \cdot M_{\text{f}5} \cdot m_b = M_{\text{f}7} \cdot M_{\text{f}6} \cdot m_e$$  \hspace{1cm} (2.24)

$$m_{\text{SD}} = m_{\text{OEW}} + m_{\text{P/L}} + m_{\text{Fres}}$$  \hspace{1cm} (2.25)

Assembling equations 2.24, 2.25 and 2.23 the aircraft mass at the end of cruise is found in equation 2.26.

$$m_e = \frac{m_{\text{OEW}} + m_{\text{P/L}} + m_{\text{Fres}}}{M_{\text{f}7} \cdot M_{\text{f}6}}$$  \hspace{1cm} (2.26)

Using the above procedure and further dividing the cruise into 7 cruise points, the consequent aircraft and fuel masses were found in table 2.2 where also the aircraft lift coefficients $C_L$ belonging to the individual cruise points $C_{R1}$ through $C_{R7}$ were found using $\rho_\infty$, $V_\infty$ and $S_w$ by the equation of lift 1.1.

Now that the reference cruise conditions are known the methodology of performance indicators evaluation is revisited. Both the morphed and reference performance are based on evaluation of the discretized equation 2.20, where the aircraft drag throughout the cruise is obtained by aerodynamic analysis and cannot be perfect to the last detail of the aircraft. At the same time the reference aircraft range $R_0$ has to be equal the design mission range of 4800km. This can be managed, if a consistency constant is left in the calculation of aircraft drag, determined once for the reference aircraft model. For the purpose of this work such constant is chosen to be the drag coefficient $C_{D_{N1}}$ associated with any component that is assumed not to bear lift, in other words the "remainder of the aircraft drag" after the other drag models are evaluated. The aircraft drag coefficient is assumed to have the components in shown in equation 2.27, each normalized by the same reference wing area $S_w$, where the lift dependent (or $\alpha$ dependent) quantities are the wing drag coefficient $C_{D_{w}}$, horizontal stabilizer drag coefficient $C_{D_{h}}$ and the fuselage drag coefficient $C_{D_{f}}$. 

2. Methodology

<table>
<thead>
<tr>
<th>Aircraft mass</th>
<th>Fuel mass</th>
<th>Aircraft cruise $C_L$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{MTOW}$</td>
<td>73500</td>
<td>18500</td>
<td>-</td>
</tr>
<tr>
<td>$m_{CR1} = m_b$</td>
<td>71670</td>
<td>16670</td>
<td>0.61665</td>
</tr>
<tr>
<td>$m_{CR2}$</td>
<td>69344</td>
<td>14344</td>
<td>0.59664</td>
</tr>
<tr>
<td>$m_{CR3}$</td>
<td>67019</td>
<td>12019</td>
<td>0.57663</td>
</tr>
<tr>
<td>$m_{CR4}$</td>
<td>64693</td>
<td>9693</td>
<td>0.55662</td>
</tr>
<tr>
<td>$m_{CR5}$</td>
<td>62367</td>
<td>7367</td>
<td>0.53661</td>
</tr>
<tr>
<td>$m_{CR6}$</td>
<td>60042</td>
<td>5042</td>
<td>0.51660</td>
</tr>
<tr>
<td>$m_{CR7} = m_e$</td>
<td>57716</td>
<td>2716</td>
<td>0.49659</td>
</tr>
<tr>
<td>$m_{SD}$</td>
<td>56682</td>
<td>1682</td>
<td>-</td>
</tr>
<tr>
<td>$m_{ZFW}$</td>
<td>55000</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

Tab. 2.2: Resulting cruise data for performance evaluation

$$C_D = C_{D_{w}} + \frac{S_H}{S_w}C_{D_b} + C_{D_f} + C_{D_{NL}}$$

(2.27)

In this manner, first the reference geometry model components have to be evaluated at the selected cruise points, after which equation 2.20 is iteratively evaluated with varying $C_{D_{NL}}$ until the range is equal to the reference range of 4800km. After the model is made consistent, the value of $C_{D_{NL}}$ is fixed for evaluation of all performance indicators.

For the $\Delta R$ indicator the discretized equation 2.20 is affected by morphing at each cruise point. In order to determine the performance benefits of a single morphing concept setup, the morphing deflections need to be optimized with respect to the aircraft drag $C_{D_{im}}$ improved by morphing at each cruise point separately to arrive at an optimum deflection schedule. This drag contains the fixed $C_{D_{NL}}$ and by analysis of the lift dependent components a new function of the factor $\frac{C_L}{C_{D_{im}}}$ is determined and interpolated in the same way as in the reference performance consideration. Therefore the evaluation of the range improvement indicator $\Delta R$ takes form of the equation 2.28.

$$\Delta R = \sum_{i=1}^{k-1} \left[ \frac{V_{\infty}}{c_T} \left( \left( \frac{C_L}{C_{D_{im}} m} \right)^{(i+1)} + \left( \frac{C_L}{C_{D_{im}} m} \right)^{(i)} \right) \frac{\Delta m}{2} \right] - R_0$$

(2.28)

The second chosen performance indicator, the fuel savings $\Delta F$ uses an iterative process where one of the masses defining the trip fuel is made free. The first possibility for a free variable in this iteration is the end of cruise mass $m_e$. The evaluation of such $\Delta F$ in percent of trip fuel is seen in equation 2.29. Another possibility is to let the beginning of cruise mass $m_b$ free, for which the evaluation of $\Delta F$ is given by equation 2.30. Both approaches are explained below.

$$\Delta F_1 = \frac{\Delta m_e}{m_b - m_e} \cdot 100\%$$

(2.29)

$$\Delta F_2 = \frac{\Delta m_e}{m_b - m_e} \cdot 100\%$$

(2.30)

When the optimized morphing schedule results in an improved ratio of $\frac{C_L}{C_{D_{im}} m}$ function, the range with all other parameters fixed would be increased by $\Delta R$. The iteration in the first case is aimed at making the $m_e$ higher by a certain $\Delta m_e$ until the point when $\Delta R = 0$ is reached again, showing the relation between these two performance indicators. Effectively the increase in cruise end-weight not only translates to fuel savings but also in zero fuel weight increase, since the maximum take-off weight is kept constant by assumption. This gives either a possibility to increase the weight of the structure (for example if the morphing mechanism weight becomes larger) or otherwise to increase payload, relating to the claim at the end of section 1.1, that each mass unit of payload is worth far more revenue than cost of same mass unit of fuel, both advantages taken at the same time. In the second case the $m_b$ is lowered until the $\Delta R = 0$ is reached, which effectively reduces the maximum take-off weight of the aircraft without changing the payload.
2.3. Optimization Strategies

As was indicated in the introduction, changing the section camber can lead to drag variations for the same lift coefficient and the preceding section 2.2 addressed that for the performance benefits to be maximum an optimization is in order. It was shown that the range can be improved if the ratio \( \frac{C_L}{C_D} \) can be maximized throughout the mission. Since the lift coefficients \( C_L \) are specified for each cruise point and are in fact a result of the aircraft mass \( m \) at the respective cruise point, the problem reduces to minimizing the drag coefficient \( C_D \) for each of the discretized cruise points. In general such optimization will have the morphing deflection(s) \( \delta_m \), morphing beginning \( x_b \) and morphing end \( x_e \) as the design variables vector \( X \) and the objective \( J \) (depending on whether 2D or 3D analysis is performed) will be the drag coefficient of section, wing or the whole aircraft, as it is formally written in equation 2.31.

\[
\text{min } J(X); \quad J = \{C_D \cup C_{D_w} \cup c_d\}, \quad X = \{\delta_m, x_b, x_e\} \tag{2.31}
\]

As seen in the literature on engineering optimization such as the book of Rao [40], the optimization problem can further be limited by constraints, which can be of two major types. Linear constraints limit the design space directly by a linear combination of the design variables, which can usually be efficiently implemented by the optimization algorithm. On the other hand a typical non-linear constraint requires evaluation of a constraint function which returns a value of how compliant the selected design variables vector is with such constraint. In this work there could be a linear constraint imposed to limit the design space in maximum deflections, however it was observed that in principle small morphing deflections are necessary, far from any impossible scenarios. Furthermore, a constraint of a non-linear kind would be the evaluated morphing radius \( r_m \) for a certain morphing setup and deflection \( \delta_m \). This was explained to be dropped from the analysis for a better understanding of the morphing phenomena and will only be addressed when the analysis of morphing variables effect is done.

2.3.1. Multi-variable built-in algorithms

Optimization within a design space of multiple variables is required if the separate inboard and outboard flap morphing is allowed or when the reference geometry of the aircraft is created as will be explained in section 4.1.3. In both cases an initial design vector can be provided and therefore optimization algorithms which investigate the nearby design space are a good option. Since in such case there is no direct method of providing gradients of the objective function, these algorithms are limited to using either finite differencing or pattern searches. In simple terms, each iteration using the nearby information the algorithm decides on a shift to a new design point where the process repeats, until convergence is found by satisfying the constraints tolerances and the optimality conditions - zero first derivative vector and positive-definite second derivative matrix, denoted as Hessian.

There exists a range of optimization functions in the used Matlab environment. Efficient algorithms can be used for multi-variable continuous functions with possibility of adding linear and non-linear constraints. As it is noted in the Matlab documentation [41], the Sequential Quadratic Programming (SQP) algorithm outperforms every other tested method in terms of efficiency, accuracy, and percentage of successful solutions, over a large number of test problems. Furthermore it can handle cases when the objective evaluation returns a Not-a-Number (NaN) value. The "sqp" is an optional setting of a function "fmincon" in Matlab which implements the SQP algorithm. Some of the solutions in this work with "fmincon" in combination with "sqp" were compared to with solutions by algorithm "patternsearch", which gave similar results but took longer time. In SQP method several steps are taken:

- updating a Hessian matrix, meaning that a positive definite quasi-Newton approximation of a Lagrangian function is calculated, which is done by the finite differencing
- quadratic programming solution, which is a solution of a sub-problem defined by the above approximation to determine the direction of the line search
- line search, where the length of the step to the new design point is determined by a merit function, for which the "fmincon" creates several solutions along the search line

As it is seen the multi-dimensional problem is effectively reduced to a combination of several single-dimensional problems. However, the complexity of such problem increases with size of the design vector (number of dimensions). In combination with a long time for a single evaluation the optimization can quickly become a matter of days of computation.
2.3.2. **Single-variable developed algorithm**

A part of this research is to investigate the effect of the morphing design variables translated to the aircraft performance. An analysis in 2D is used to achieve this goal in which for each given combination of morphing design variables $x_b$ and $x_e$, the optimization design vector reduces to simply the morphing deflection $\delta_m$ with the section drag $c_d$ being the objective. This gives a one-dimensional problem which can be repeated for several other morphing setups and cruise points. The above mentioned optimization algorithms and other methods usually suitable to such problem were tried, for instance the Newton’s optimization method or Golden ratio search. However all had problems with occasional non-convergence of MSES, terminating the optimization due to the regions of the design space where no answer could be obtained. Unlike with the multi-variable optimization the single dimension does not have the freedom to avoid non-converged objective values, unless resorted to global optimization and genetic algorithms requiring longer time.

Therefore an own optimization algorithm was developed, given the availability of parallel computing and the observed parabolic behaviour of the objective function which is illustrated in figure 2.11. This objective function is of the shape $J = 0.00002(X - 8)^2 + 0.008$, simulating section drag coefficient as a function of the trailing edge morphing deflection and is purposely set to return NaN values over certain regions of the design space to serve as a rather extreme test example.

![Fig. 2.11: Test case for 2D drag morphing optimization](image)

The following procedure is adopted to deal with optimization of such discontinuous function. Prior to the procedure the problem must be specified, including bounds on the design variable $X$, which means setting the minimum and maximum morphing deflection in this case. It can be expected that large deflections will have bigger problems with convergence. The solution at the $X = 0$ is taken as the initial point, representing the non-morphed geometry. Therefore an interval around zero (in this case $[-10, 20]$) is selected. It is crucial for the initial point to have a converged solution of the objective (drag coefficient) and therefore the measures are taken as explained in the end of section 2.1.1 to perturb grid parameters if necessary, else the attempt is dropped. For further evaluations the perturbations are no longer necessary and the following sequence of actions is performed in relation to figure 2.12 bearing in mind that a processor with 8 parallel computational threads is used:

- **Initial sampling** (black ticks on the bottom of figure 2.12): the objective is evaluated on a grid of 16 values equally spaced between the bounds of $X$ and the non-morphed case (red circle value)

- **Minimum location**: three minimum objective value deflections are found and their indexes $l$ and $r$ are read, belonging to the smallest $X_{(l)}$ and the largest $X_{(r)}$ of the three deflections, respectively

- **New bounds selection**: the bounds of the search interval are narrowed between the points $X_{(l)}$ and $X_{(r+1)}$, unless these are positioned on the original bounds in which case only $X_{(l)}$ or $X_{(r)}$ is used

- **Re-sampling** (red ticks): a grid of 8 new objective evaluations is created with irrational cosine spacing to prevent repetitive evaluations, the distribution pattern taking form of:
  
  $X_{dist} = X_{(l-1)} + (X_{(r+1)} - X_{(l-1)}) \cdot \left[ \frac{1+\cos(\frac{2\pi}{2})}{2}, \frac{1+\cos(\frac{4\pi}{2})}{2}, \frac{1+\cos(\frac{6\pi}{2})}{2}, \ldots, \frac{1+\cos(\frac{8\pi}{2})}{2}, \frac{1+\cos(\frac{9\pi}{2})}{2} \right]$

- **Repetition of minimum location, bounds selection and re-sampling** (ticks of green, etc), until the convergence criteria described further are met
Depending on whether the minimum is located between the bounds or on one of the bounds, two combinations of convergence criteria exist. If the minimum is on a bound the algorithm will gradually approach it until the distance measured between the minimum and second minimum point is under a set value $\Delta X_{\text{crit}}$, while one of these two points must be the bound itself. If the minimum is located between the bounds then neither minimum nor second minimum point are on any of the bounds and the optimum is found when the $\Delta X_{\text{crit}}$ is reached and at the same time the gradient $\frac{\Delta J}{\Delta X}$ observed on these two values is smaller than a set value of $G_{\text{crit}}$. In case the algorithm continues receiving only NaN values it is terminated if more than 5 samplings were attempted.

Although this algorithm seems rather brute-forced, in the above test case it has been successful in finding the approximate minimum drag value within 4 evaluations per core (for increased precision the initial sampling takes 2 evaluations). As seen in figure 2.12 on the initial sampling, the initial $X_l$ and $X_r$ are 3 spacings apart as a result of non-convergence interval near the optimum which returned a NaN for one of the samples. It is clear that if at least one of the new 8 re-sampled design values returns an objective the optimizer can continue unless it is unfortunate to be a sharp peak within the narrowed bounds. The reason why the new sampling bounds are always taken as $X_{(l-1)}$ and $X_{(r+1)}$ is for the case when sharp gradients are present and the minimum would be just outside the $[X_l, X_r]$ interval in which case this allows the algorithm to gradually approach it. Convergence on a bound is indicated after reaching $\Delta X_{\text{crit}}$ criterion by a return flag, which is used to set large enough bounds for the problem at hand.
MORPHING OF AIRFOIL SECTIONS

Since the methods used in this work are based on 2D or quasi-3D approach, the first step in the analysis of impact of morphing trailing edge on the aircraft performance is the investigation on section behaviour. The main tasks of this chapter will be to implement the morphing concept by Fokker Aerostructures into a universal method applicable to airfoil sections and then zoom-in on the flow phenomena that can be expected when morphing is applied before the full wing or aircraft model is taken into account.

The tool used in 2D evaluation (MSES) was described in section 2.1.1 and the input to this solver are the airfoil coordinates in the Selig format, as shown in the input file C.3. This fixes also the format of the morphed section and suggests that the morphing could work with simple coordinates as well. The input of the method apart from the non-morphed section are the morphing design variables discussed in section 1.2, defining where the morphing takes place on the airfoil and the amount of deflection from the original shape.

From aerodynamic perspective at a given cruise point condition the section is required to achieve a certain lift coefficient to maintain balance of lift and weight of the aircraft. When using morphing, the section is expected to change its chordwise lift distribution while keeping the section lift coefficient on this required value. As discussed in section 1.1, also the section drag coefficient is expected to change, optimum of which is to be found. It will be assumed that the possible changes in spanwise lift distribution do not change this lift requirement as a first order approximation. In reality, when the morphing is applied to a wing the local section lift coefficient could vary while the overall aircraft lift coefficient could still be maintained. Once the reference aircraft model is created in the following chapters a suitable section along the span is selected for which a relation can be made between the sectional and 3D wing properties.

In this chapter the section 3.1 deals with geometry manipulation to achieve a morphed airfoil from given airfoil coordinates and morphing variables. Section 3.2 gives a preliminary investigation of flow control with morphing at transonic cruise condition for the same setup.
3.1. **TRAILING EDGE MORPHING IMPLEMENTATION**

The provided morphing concept was described briefly in the introduction section 1.2. In order to investigate multiple morphing configurations the implementation in the code must allow selection of any morphing region within designated portion of airfoil and simulating a bending of the upper surface to a certain deflection of the trailing edge using a prescribed morphing function, while at the same time monitoring the morphing radius for shear stress constraints. The designated portion of airfoil is set aft of spoiler hinge line, approximately 67% of local chord on A320.

Morphing shall be applicable to any chosen section along the wing span. Therefore the method has to be fast and universal for wide range of airfoil shapes. Furthermore, it must not affect the non-morphed regions of the airfoil and it must preserve the length of the upper surface as a stiff in plane composite sheet bending would. The following procedure was developed according to these principles.

The input airfoil in coordinate form is loaded and the morphing region delimited by the projections of morphing region beginning $x_b$ and morphing region end $x_e$ onto the upper surface as in figure 3.1. Using a PCHIP interpolation the projected points (magenta color) are included in the upper surface coordinates. In the shown example the morphing region is defined between 93% and 96% of the airfoil chord. The airfoil coordinate points located between the delimiting morphing region points are connected with surface vectors represented by red arrows. The sum of absolute lengths of these surface vectors represents the length of the skin which must be preserved.

These vectors are placed along a straight line over $x'$ axis of a mapping coordinate system in figure 3.2, where the circular reference morphing function $f_m$ with morphing radius $r_m$ is plotted on the $z'$ axis. The function is in the form shown in equation 3.1, with the circle focus point being positioned at a distance $r_m$ below the beginning of morphing in the mapping coordinate system.

![Fig. 3.1: Airfoil morphing region delimitation and surface vectors definition](image1.png)

![Fig. 3.2: Reference morphing function and morphing displacements mapping](image2.png)

$$f_m = \frac{-r_m + \sqrt{r_m^2 - (x' - x_b)^2}}{c}$$  \hspace{1cm} (3.1)

The evaluation of the morphing function at $x'$ positions determined from the lengths of the surface vectors gives the morphing displacements shown as black vectors in figure 3.2. These displacements are paired with each coordinate points in the morphing region in figure 3.1 and can therefore be used to apply the morphing function back to the airfoil coordinates as shown in figure 3.3.
The calculation of the displaced coordinates of each next point \([x_m, z_m]\) from its non-morphed (base) state \([x, z]\) and the already displaced preceding point \([x_{m-1}, z_{m-1}]\) is done using the rule of cosine. The angle \(\phi_i\) in the triangle between these three points can be obtained since all sides are known from the distance \(h_i\) between points \([x, z]\) and \([x_{m-1}, z_{m-1}]\) and the lengths of the surface vector \(s_i\) and displacement vector \(d_i\). Furthermore, let \(\theta_i\) be the elevation angle between the same points. Then the coordinates of point \([x_m, z_m]\) are written using equations 3.2 through 3.5, and by progression from \(x_b\) all the morphed points can thus be calculated.

\[
\begin{align*}
  s_i &= |[x, z] - [x_{i-1}, z_{i-1}]| \\
  d_i &= |[x_m, z_m] - [x, z]| \\
  h_i &= |[x, z] - [x_{m-1}, z_{m-1}]| \\
  \theta_i &= \arctan\left(\frac{z - z_{m-1}}{x - x_{m-1}}\right) \\
  \phi_i &= \arccos\left(\frac{h_i^2 + s_i^2 - d_i^2}{2s_i h_i}\right)
\end{align*}
\]

\[
\begin{align*}
  x_m &= x_{m-1} + s_i \cdot \cos(\phi_i - \theta_i) \\
  z_m &= z_{m-1} - s_i \cdot \sin(\phi_i - \theta_i)
\end{align*}
\]

Once the last point \([x_e, z_e]\) of the morphing region is solved, the trailing edge from this last point onward is adjusted to complete the morphing process. A translation by this point is taken to the new morphed position and a rotation by an angle \(\delta_m\) is done such that a tangency is achieved with the morphed region. The \(\delta_m\) is the actual morphing deflection and it is the difference between the tangent angle from the last two points of the morphed surface and the original tangent angle from the last two points of the base coordinates. The transformation of the remaining trailing edge geometry is shown in equation 3.6.

\[
\begin{bmatrix}
  \mathbf{x}_m \\
  \mathbf{z}_m \\
  \mathbf{1}
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & x_m \\
  0 & 1 & z_m \\
  0 & 0 & 1
\end{bmatrix} \times 
\begin{bmatrix}
  \cos(\delta_m) & \sin(\delta_m) & 0 \\
  -\sin(\delta_m) & \cos(\delta_m) & 0 \\
  0 & 0 & 1
\end{bmatrix} \times 
\begin{bmatrix}
  1 & 0 & -x_e \\
  0 & 1 & -z_e \\
  0 & 0 & 1
\end{bmatrix} \times 
\begin{bmatrix}
  \mathbf{x} \\
  \mathbf{z} \\
  \mathbf{1}
\end{bmatrix}
\]

Fig. 3.3: Applying morphing displacements to the airfoil upper surface
As the lower trailing edge surface undergoes the same transformation an intersection can be found between the base and morphed lower surface geometry. This serves as a corner point where the continuity is made between the morphed and non-morphed surface. Although strictly speaking this point changes with the deflection applied to a single morphing setup it will be simply assumed that an elastic filament is in place allowing adjustment to all possible deflections and that the lower surface is without gap. The result of the morphing procedure demonstration are shown in figure 3.4.

![Figure 3.4: Result of the morphing procedure on airfoil section](image)

In the demonstration of the morphing procedure only few points were used together with an exaggerated deflection. Apart from using airfoil definitions with much higher density of coordinates, prior to morphing procedure call 25 points are interpolated within the morphing region including the \( x_b \) and \( x_e \) projection points in order to provide a smooth geometry.

Furthermore, the deflection in the demonstration was a result of a certain morphing radius \( r_m \) in the morphing function \( f_m \). In case a certain \( \delta_m \) is chosen instead, an iterative search for corresponding radius is done using a quick Matlab optimization algorithm "fminbnd" with an initial estimate given by equation 3.7. Since for a given deflection sign (upward/downward) the function between \( \delta_m \) and \( r_m \) is continuous and smooth, convergence is certain and only a matter of few evaluations. The fminbnd algorithm requires bounds and these are set such that the maximum morphing radius corresponds to a deflection of 0.1° and the minimum to a deflection of 50° using an inverse form of equation 3.7. The minimum deflection bound is necessary otherwise the morphing radius goes to infinity. If a smaller deflection than 0.1° is requested the morphing method is set to simply return the non-morphed geometry. It was tested that morphing of up to 80 airfoil sections on a wing, even with each calling the optimization algorithm for a specified \( \delta_m \), is done in order of 1 second.

\[
r_m = \frac{x_e - x_b}{|\sin(\delta_m)|}
\]  

(3.7)

The morphing procedure presented above allows for a whole range of trailing edge camber modifications, from rather sharp deflections simulating a conventional simple flap or aileron-like control device to a very smooth gradual change in the upper surface geometry. Short region morphing resembling the adaptive dropped hinge flap concept of A350 is represented in figure 3.5, although in reality this concept would increase the chord of the section with shown deflection [17]. A smooth deflection over a large morphing region is shown in 3.6. Another extreme of geometry modification is shown in figure 3.7 approaching a Gurney flap, which is essentially a flat plate attached at the trailing edge perpendicularly to the flow [42].
3.1. TRAILING EDGE MORPHING IMPLEMENTATION

Figure 3.5 shows that deflections upward are also possible. The calculation of the upward-morphed surface is still driven by equations 3.2 through 3.6, however some sign changes have to be implemented since the cosine rule is not sensitive to the sign of the angles. Essentially two versions of the morphing procedure then exist with the choice driven by the sign of the input required morphing deflection $\delta_m$. It can be noticed that the corner point on the lower surface gives a sharp kink with upward deflection of the trailing edge. For this research it is not corrected since only small upward deflections are allowed.
3.2. 2D SECTION MORPHING PHENOMENA

For preliminary investigation of morphing aerodynamic phenomena the airfoil RAE2822 was selected, as it is also one of the validation cases for MSES 2D solver, only smoothly modified with a finite thickness trailing edge as on the A320 aircraft. Using the morphing tool developed in section 3.1 the camber at the trailing edge is varied. For this section the morphing setup is selected such that it complies with A320 spoiler tip line position on the outboard mid-flap position, where only chordwise positions aft of 90% of chord are available. For demonstration the morphing region is therefore fixed between $x_b = 0.92$ and $x_e = 0.95$.

For a greater picture the drag polar diagrams are first inspected at different Mach numbers but with the same Reynolds number $Re = 15e6$, which simulates the in flight condition of effective sections at the mid-flap position at cruise altitude. In figure 3.9 the morphing deflections range from negative to positive including the non-morphed geometry. To indicate the drag improvement possibilities by morphing at each given lift coefficient the differences between the non-morphed case and the best observed morphed case are hatched.

The following observations are made from the above figures:

- parabolic shape of the drag polar is responsible for greater drag differences between the polar diagrams of sections with varying camber at higher lift coefficients compared to lower lift coefficients
- there exists an optimum aerodynamic efficiency point where lift to drag ratio $c_l/c_d$ is maximum which is barely improved by morphing, however the $c_l$ at which this occurs changes with Mach number for a certain non-morphed base geometry
- the achievable drag coefficient improvement varies with $c_l$ coefficient and will depend on where the operating $c_l$ range of the airfoil is located
- if a fixed morphing deflection would be used, for example to improve drag above the maximum $c_l/c_d$ point, this would lead to a drag penalty below the maximum $c_l/c_d$ point and vice versa

As it is seen the performance benefits could vary greatly depending on the exact conditions that the sections are subjected to. The main conclusion drawn from this investigation is that in order to determine the performance benefits of morphing on a specific aircraft these conditions (at cruise points) must be known with their requirements on the range of cruise lift coefficients, and furthermore this range has to be consistent with the aircraft geometry, which has influence on the initial non-morphed drag polar. If the aircraft is already designed well, such range will be close the maximum $c_l/c_d$ point of this polar. However, also the size of the lift coefficients range will play a role on the achievable performance improvement with morphing.

Further investigation must be made, zooming-in on the behavior at a single lift coefficient to arrive at more concrete reasons why changes in the drag polar can be expected as seen in figure 3.9. Without prior knowledge on the lift coefficient cruise range, for the moment let a relatively high lift coefficient of $c_l = 0.8$ be assumed, where based on the above a large improvement in the drag polars can be expected, simulating possibly the beginning of cruise condition. First the morphing effect is inspected on the effective angle of attack.
α_{eff} and the pitching moment c_m of the airfoil, in all cases at the assumed lift coefficient requirement. The result in 3.10 is as expected according to the theory of wing sections [43], since positive increase in sectional camber results in decrease of the zero-lift angle of attack and an increase in the negative pitching moment. However, a change of slope is observed which takes place at approximately \( \delta_m = 5^\circ \) of morphing deflection for both quantities, and requires further explanation.

\[ \begin{align*}
\alpha_{eff} & \quad \text{(\degree)} \\
c_m & \quad [-] \\
\end{align*} \]

\[ \begin{align*}
\delta_m & \quad \text{(\degree)} \\
\end{align*} \]

**Fig. 3.10:** Effective angle of attack and moment coefficient of RAE2822 airfoil at \( c_l = 0.8 \) and \( M = 0.73 \) as function of \( \delta_m \)

The next step is to decompose the drag into individual components, which is done in figure 3.11. Recalling definition in section 2.1.1 on MSES, the total drag is either composed as the sum of viscous and wave drag, or as the sum of pressure and friction drag. Since the wave drag is strictly reserved for the momentum deficit in the inviscid stream-lines, the viscous drag is composed partly due to the integrated friction on the surface and the remainder due to separation and the thickness of the boundary layer. Let for the current problem this difference between the viscous and friction drag be denoted as the viscous pressure drag. The friction drag varies very little with morphing deflection and therefore the main reason for variation of the viscous drag is this viscous pressure drag component. Both the viscous pressure drag and wave drag are of near parabolic-shape functions when it comes to morphing but have in general different optimum morphing deflections. However, their balance determines the overall total drag minimum. From this minimum drag condition the
drag improvement $\Delta c_d$ is read representing the performance benefit obtainable by morphing. Recall that there is a change in angle of attack function slope at $\delta_m = 5^\circ$ of positive (downward) morphing deflection in figure 3.10. It can be seen from figure 3.11, that this is also near where the minimum total drag is located. In conclusion, at such high off-design lift coefficient required from this airfoil the morphing can bring a drag reduction $\Delta c_d$ at a mild positive (downward) morphing deflection.

To visualize the drag components origins on the airfoil, the pressure coefficient $c_p$ distributions are of interest for the different morphing deflections at the constant required lift coefficient $c_l = 0.8$. The minimum drag deflection corresponds with the dark blue plots in figure 3.12.

Looking first at the upper surface, beyond this minimum drag deflection the shockwave on the upper surface shifts further aft, keeping approximately the same strength (difference in pressure), but increasing the pressure gradient at the rear of the airfoil which promotes separation first at the morphing region and second at the foot of the shockwave with a reattachment just ahead of the morphing region. These separation effects not only increase the viscous pressure drag, but also result in a need for slightly higher angle of attack to compensate for the lift loss and hence the slope change in figure 3.10.

Negative (upward) deflections bring de-cambering to the airfoil and an increase in the overall required angle of attack for the same lift coefficient, resulting in higher pressure peaks at the leading edge of the airfoil and an increase in the shockwave strength. In this case however the shockwave shows less chordwise travel and no separation is observed. A larger boundary layer thickness behind the shock can be deduced as a side effect from the slight increase in viscous drag appearing with negative deflections in figure 3.11, since at the same time separation is not present.

The lower surface shows the largest variations in $c_p$ distribution at the rear of the airfoil near the corner point in the geometry. For positive morphing deflections the aft loading is heavily increased which contributes to lift generation. With negative deflections a sharp low pressure peak is present due to the non-smooth kink. As discussed in section 3.1 such large negative deflections are not expected within the investigated morphing concept, nor should they be necessary based on the observations in figure 3.9.
If the lift coefficient is assumed from the lower spectrum, for example $c_l = 0.5$, the situation changes as shown in figures 3.13 and 3.14. The largest difference is that no significant shockwave is present, unless large negative morphing deflections are applied. It also takes large positive deflections to separate the flow at the morphing region. There is a lack of mechanism which would significantly change the drag coefficient compared to the non-morphed case, and therefore obtainable performance improvements are negligible.

In this section it was shown that the viscous pressure drag and wave drag components are responsible for the drag improvements and that their optimum balance is to be found at each cruise point. Furthermore the optimum deflections are found ahead of any major separation on the airfoil. As a result it is important to first find the operating conditions for 2D sections on the aircraft at individual cruise points, which in turn are dictated by the wing geometry and other lift bearing components under the overall aircraft lift requirements discussed in section 2.2. This requires thorough modelling of the wing in terms of camber.
MODEL OF THE COMPLETE AIRCRAFT

In previous chapters the research problem was stated with a goal to investigate cruising performance benefits of morphing concept on an A320 aircraft. As was seen in chapter 3 on morphing implementation, this leads to the need of first creating the reference aircraft model and then applying morphing deformations to it and recovering morphed geometry data such that analysis of the performance indicators is possible.

It was also claimed that an A320-like aircraft will not fly far from its optimum at the cruise altitude and Mach number given for the design mission, with the optimum design being assumed in the mid-cruise condition. The aircraft modelling has to take this into account by fine-tuning of the model. Without knowing the trimmed values of wing lift this is approached by optimizing the wing to the corresponding lift coefficient of the whole aircraft, with the assumption that the aircraft and wing lift are comparable. The exact geometry A320 was not available for this research, being one of the trade secrets and subject to great competition among aircraft manufacturers. This means that the aircraft model has to be created almost from scratch including features of reverse engineering. The aircraft data that have been gathered from the literature are presented in the Appendix B. Although the task is set to investigate the A320 as close as possible, on the positive note the analysis on the created model simulates an analysis of the morphing concept by Fokker for the general class of short-medium range aircraft.

This chapter first discusses the creation of the wing model of A320 gathered from the known data in section 4.1 according to the requirements of the Q3D solver and including the morphing method. Section 4.2 shows the assumed models for the remaining components such as the horizontal stabilizer, fuselage and the aircraft center of gravity. Assembly of all the components into the full aircraft model is discussed in section in section 4.3 by explanation of the trim procedure. The reference aircraft performance is extracted from evaluated cruise points in section 4.4, by which the model is ready for performance benefits estimation in the following chapter.
4.1. Reference wing geometry

The geometry of lifting surfaces on the aircraft has to be modeled such that it is compatible with the analysis tools selected in chapter 2. Although it may seem unrelated, the main consideration was the Linux operating system, under which the MSES 2D analysis tool uses its full potential not only in terms of computational speed, but also visualization of the results using the X11 graphics library [29]. This library was found only executable from the Linux terminal and was required for proper solver manipulation and grid checking. Unfortunately, this ruled out synchronization with most of the conventional 3D CAD design software such as CATIA or Solidworks which are limited to the other platforms.

Although there exist Knowledge Based Engineering (KBE) platform development efforts under the Faculty of Aerospace Engineering of TU Delft which could facilitate the geometry generation process, such as the yet un-published tool ParaPy, it was opted to code a tailored geometry generator instead based on section 3.1 which complies with the morphing concept and is written purely in the Matlab environment. Not only is this directly compatible with the Q3D tool, but it was also seen that the morphology can rely solely on vector operations in which Matlab excels. Another reason is that once the reference geometry is set, the morphology applied to the wing has to be strictly limited to the morphing region. If the wing shape relied on a single piece 3D loft, perturbation could occur outside the desired morphing region which could steer the analysis process. Splitting the geometry in multiple lofted pieces would bring tangency issues at the boundaries and for the analysis sectional cuts are still necessary which would bring additional merging problems especially if the process is to be fully automated for large number of morphed geometry variations.

Therefore the geometry generator works directly with individual streamwise oriented stations each in the x-z plane. As it was discussed in section 2.1.1 on MSES and during the morphing implementation in section 3.1 it is desirable to have the airfoils defined by a fine resolution of coordinate points for a smoother morphed surface and accuracy of the aerodynamic solvers. This is provided by the CST transformation and re-meshing as described in section 4.1.1. From several such defining airfoils the lifting surface can be created as described in section 4.1.2. The model is further tuned to be optimum at mid-cruise condition in section 4.1.3.

4.1.1. Class-Shape transformation

The Class-Shape transformation(CST) is a convenient method of geometry parametrization described by Kulfan [44]. Its main advantage is the description of large variety of geometry types with very few variables, making it useful for optimization processes. In the two-dimensional form suitable for airfoils it is formally defined by equation 4.1, where \( \zeta = \frac{x}{c} \) and \( \psi = \frac{z}{c} \) are the normalized coordinates and the \( C_{N_1}(\psi) = \psi^{N_1}(1 - \psi)^{N_2} \) and \( S(\psi) \) are the class function and shape function, respectively. The \( \zeta_{TE} \) is the required trailing edge thickness added to the given side of the airfoil.

\[
\zeta(\psi) = C_{N_1}(\psi) \cdot S(\psi) + \psi \cdot \zeta_{TE}
\]  

(4.1)

The class function for the case of typical transport aircraft airfoils with round leading edge and pointed or cusped trailing edge uses the \( N_1 = 0.5 \) and \( N_2 = 1 \), i.e. \( C(\psi) = \sqrt{\psi(1 - \psi)} \). The shape function used in this work is based on Bernstein polynomials, such that for each non-dimensional position \( \psi \) a dot product is made between a vector of the polynomial terms in equation 4.2 and each of the two vectors of so called CST coefficients \( A_u \) and \( A_l \), respectively defining the upper and lower airfoil surface shape function \( S_u \) and \( S_l \), as shown in equation 4.3.

\[
\mathbf{B} = \binom{n}{i} \psi^i (1 - \psi)^{n-1} 
\]  

(4.2)

\[
\begin{align*}
S_u(\psi) &= \mathbf{A_u} \cdot \mathbf{B} \\
S_l(\psi) &= \mathbf{A_l} \cdot \mathbf{B}
\end{align*}
\]  

(4.3)

As it is generally true for the Bernstein polynomials, the order of the resulting curve depends on the length of the vector \( A_u \) or \( A_l \). This is always lower by 1 than the this length, unlike with the B-splines where the order can be lower and each coefficient has a more local effect. However for a better smoothness of the resulting curve Bernstein polynomials based shape function seems better, unless a third refining function is added as described by Voskuilj [45]. It was also found even for the Bernstein based shape function that with increasing order (number of CST coefficients) the surface demonstrates waviness which creates unrealistic pressure distributions. On the other hand, a very low order is unable to capture features such as the aft loading. The 6th order seems to reproduce well the validation cases for MSES solver as demonstrated in Appendix C.
The above described method serves to create an airfoil at defined \( x/c \) positions from known CST coefficients. On the other hand, if an already known airfoil is to be modelled by CST coefficients representation, an optimization can be called with an objective being the sum of absolute sampled differences between the modelled airfoil and the constructed airfoil from these coefficients. For the purposes of this report such process is denoted as re-meshing. An example is given in figure 4.1 for the SC0712 airfoil used in section 2.1.1.

Figure 4.1 shows also the implementation of the finite trailing edge thickness. In the case of this airfoil neither of the trailing edge coordinates are defined at the point \([1,0]\), and therefore both upper and lower surface \( \zeta_{TE} \) must be defined. However, in the remainder of this work the thickness is always added only to the upper surface.

### 4.1.2. LIFTING SURFACE GEOMETRY GENERATION

The following text describes the scheme used in the wing geometry generation process. First the wing data are called from the list contained in table B.1 of Appendix B. These data define the wing planform and were due to discrepancies tuned to fit with planform schematic figure as well as the aircraft photographs for verification, seen in figure B.4 of the Appendix B. The wing data also contain a list of “defining airfoils” which are associated with a vector of defining positions along the wing span and the trailing edge thicknesses estimated from an A320 flap geometry which was the only available geometry part. For a wing these would be the section adjacent to the fuselage, the kink section and the tip section as seen in figure 4.2.

The defining airfoils are loaded from vectors of 7 CST coefficients per side of each airfoil (6th order approximation) and re-meshed into a cosine-distributed airfoil coordinates. As it is seen from the list in table B.1 of Appendix B the kink and the tip airfoils are based on the same airfoil, which means that only 2 airfoils define the wing. Two further sets of data are necessary to arrive at a refined number of sections to define fully the lifting surface geometry, the thickness and twist distributions over the wing span. Although the defining airfoils have from their coordinates already a thickness-to-chord ratio \( t_c \), this is not necessarily linear for the sections between them. The twist \( \epsilon \) on the other hand is not defined in any way by the defining airfoils, since these are normalized and defined with leading edge at \([0,0]\) and trailing edge near \([1,0]\) coordinate. In the book by Obert [3] both distributions can be found for an A320 aircraft and are reproduced in figure 4.3. As demonstrated in figure 4.2 with 46 spanwise stations, the resulting wing lifting surface can have any requested number of sections by the following sequence of actions for any given spanwise position:
• creating a cosine distribution of coordinates of equal resolution as on each defining airfoil
• y-interpolation of airfoil from respective points of the defining airfoils
• z-scaling to the required local $t_c$ obtained by PCHIP-interpolation from thickness distribution
• equiaxial scaling to local chord length obtained by linear interpolation of planform data
• rotation of the section to local twist $\epsilon$ obtained by PCHIP-interpolation from twist distribution

![Fig. 4.3: Distributions of thickness-to-chord ratio (left) and twist (right) [3]](image)

The number of spanwise sections cannot be too small, otherwise the thickness distribution will not be well captured. On the other hand the twist distribution can easily be replaced by two linear segments divided by the kink position. It also has to be noted that after investigating the spanwise lift distribution with Q3D a great sensitivity was observed to how the airfoil sections are scaled to the required thickness distribution in figure 4.3. It was discovered that simply z-scaling the airfoil coordinates by a certain ratio for a highly aft-loaded airfoils produces an increased curvature of the mean camber line which is naturally also scaled. If the two defining airfoils used for root and outboard sections have a different aft-loading this difference gets aggravated by the z-scaling. Therefore the required sections are scaled about their mean camber line as seen in figure 4.4 by adding and subtracting half of the local thickness at each $x/c$ position.

![Fig. 4.4: Example of single airfoil z-scaling from 10% to 15% thickness](image)

According to by Abbott and Doenhoff [43], this kind of scaling was common for the older NACA airfoil families with the exception that the chordwise thickness distribution is not applied in a perpendicular to mean camber line fashion but purely vertically. This is due to the interpolation technique applied to the individual required sections in order to keep the resulting x-distribution of points the same on each of the sections. When it comes to the twist the sections are rotated about their leading edge position by a sequence of transformation matrices applied to the coordinates as in equation 4.4.

$$
\begin{bmatrix}
\tilde{X}_t \\
\tilde{X}_f \\
\tilde{I}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & x_{LE} \\
0 & 1 & z_{LE} \\
0 & 0 & 1
\end{bmatrix} \times
\begin{bmatrix}
\cos(\epsilon) & \sin(\epsilon) & 0 \\
-\sin(\epsilon) & \cos(\epsilon) & 0 \\
0 & 0 & 1
\end{bmatrix} \times
\begin{bmatrix}
1 & 0 & -x_{LE} \\
0 & 1 & -z_{LE} \\
0 & 0 & 1
\end{bmatrix} \times
\begin{bmatrix}
\tilde{X}_0 \\
\tilde{X}_0 \\
\tilde{I}
\end{bmatrix}
$$

(4.4)
Once a sufficient number of streamwise oriented sections along the span are available to define the wing, the extraction of the swept sections (cuts perpendicular to the half-chord line) for the Q3D can be made as illustrated in figure 4.5 at 60% of span. It is made use of the fact that each section has the same chordwise distribution of points, and therefore for each n-th chordwise position an interpolation is made (for both upper and lower surface coordinates) with respect to y-axis at an intersection point between the $c_\perp$ line in figure 2.6 and the line of these equal chordwise positions along the span.

A comparison between streamwise airfoil and a swept cut airfoil resulting from an extraction at 60% of span is shown in figure 4.6. Due to the relatively high sweep and small taper the effective sections on the outboard wing show an increased thickness-to-chord ratio compared to the streamwise sections. However this is not true throughout the span, for instance at the very root the effective section shows a slight decrease in $t_c$, as shown in figure 4.7. This is due to large $t_c$ gradients near the root, with the maximum thickness not being at mid chord and the increased distance between the mid chord and the intersection of swept chord with the extrapolated trailing edge. A linear extrapolation is done at the trailing edge root area to produce smooth airfoils as discussed in section 2.2, since PCHIP and spline extrapolations produced a saw tooth surface due to oscillations outside of the defined data points. The same extrapolation has to be done at the tip of the wing but now for the leading edge part. Since the effective sections are cut perpendicularly to the half chord line they share a common upper and lower surface point at 50% chordwise position of the streamwise section. For the effective sections this position is no longer at 50% of the effective chord since the length before and aft this location are changed due to the combination of wing taper and sweep. For a better comparison in figures 4.6 and 4.7 a shift of the effective airfoils was made so that the corresponding points overlap.

Until this point the airfoils used in demonstrations of the geometry generating process were unknown. Two specific defining airfoils need to be filled in for the wing model to be complete. The root airfoil is assumed a geometry from the description of the A320 wing design in the book of Obert [3] reproduced in figure 4.8. Using this figure the coordinates of the airfoil were extracted using points distribution over guided curves in Solidworks that were positioned to approximately match the airfoil. Saving a separate part containing only these points in the *.igs format allowed converting them into a text file accessible to Matlab. These points were then transformed into CST coefficients of 6th order to arrive at the defining airfoil format. Due to the small size of the figure by Obert only the root airfoil could be extracted. The quality of the root airfoil does not need to be high, since on the real aircraft its shape will be taking into consideration root effects which have been neglected in this work. If the root airfoil is not exactly optimum this will be included in the model error coming from neglecting these effects. If a RAE2822 airfoil is assumed for the outboard sections then under
similar view angle the wing model wire-frame can be seen in figure 4.9. The outboard airfoil will be further processed in the following section 4.1.3.

The lifting surface geometry generator was coded using structured variables in Matlab, however the overall scheme can be explained using a class-object scheme in figure 4.10. The resulting geometry object MorphedWing is an instance of a class LiftingSurface and it is made directly compatible with the Q3D tool analysis working with \( n = 20 \) effective sections, which can be either base (reference model) or morphed for the purposes of chapter 5. These are interpolated along a swept chord \( c_\perp \) perpendicular to the half-chord line by the SweptInterpolationModule, in agreement with figure 2.6. For the effective sections to be well defined, at \( N = 80 \) spanwise positions a BaseStreamwiseSection is created by InterpolationModule from the defining airfoils. The defining airfoils are created by re-meshing from CST coefficients in the CSTModule and for the case of wing lifting surface they have a fine resolution of 221 points per side (upper and lower surface), since the input limit of the MSES tool is 251 points and prior to morphing 25 coordinate points are interpolated on the upper surface in the assumed morphing regions of all stations for a smoother geometry.

Each BaseStreamwiseSection can optionally be morphed by the MorphingModule which contains the morphing method described in section 3.1. For instance at the wing tip the sections beyond the flaps’ span can remain intact. The span of the outboard flap can be extended into the area of the aileron to simulate a synchronous aileron morphing with the outboard flap or a complete replacement of the aileron by the morphing flap technology. This results in a variable number of morphed sections, which is for the moment denoted as 1..* in the scheme.
The described geometry generation allows the Q3D tool to evaluate all the assumed wing drag components, with the streamwise sections being an input to the AVL solver and the swept sections to the MSES solver. Such wing model generation method is therefore compatible with performance indicators evaluation within the complete aircraft model.

4.1.3. Tuning of the Wing Model

In section 4.1.2 the wing geometry generation scheme was described and concluded with the lack of data on the outboard defining airfoil. In this section the CST coefficients of this outboard airfoil will be tuned in order to arrive at an optimum mid-cruise condition wing model. This is achieved through a multidimensional optimization in Matlab, using a gradient based algorithm “fmincon”.

Based on the wing generation scheme the optimization is set up with an objective of the wing drag coefficient at the mid-cruise aircraft lift coefficient value from table 2.2 and with the design variables being the CST coefficients of the outboard airfoil. The optimization algorithm requires an initial design for such coefficients and RAE2822 airfoil can be used for such purpose. A small modification to the airfoil was done prior to this reference wing optimization to change the trailing edge shape such that it can house an A320 flap. A CATIA CAD file was available with the outer surface geometry of the outboard A320 flap, which was used to extract the flap coordinate points at its mid-span, as seen in figure 4.11.

![Fig. 4.11: A320 outboard flap mid-span coordinates extraction from CATIA](image)

The initial design point of the outboard airfoil was achieved using a CST transformation based on the first 50% of RAE2822 airfoil and the flap geometry behind the break points in figure 4.12, which represents the part of the flap that belongs to the outer airfoil surface. The CST transformation uses minimization algorithm “fminunc” in Matlab to minimize error between a remeshed CST airfoil and these two reference geometries. The RAE2822 and the CAD reference surfaces are for the initial design assumed in their original form without any rotation. In figure 4.13 it is further seen how the overall camber can change if the two reference surfaces are rotated around their pivot points which were chosen to be the leading and trailing edge, respectively. This is of crucial importance to morphing, since it can be expected that a morphing optimization will show large camber compensations for any overall non-optimum airfoil camber even at the mid-cruise point.

![Fig. 4.12: Flap mid-span geometry and RAE2822 geometry defining initial outboard airfoil design](image)

Although the outboard airfoil reference optimization is to have as much freedom as possible, the first 10% chord span of the RAE2822 airfoil are still used as constraint in order to maintain at least the leading edge radius of this airfoil. This is due to the observation that often when optimizing for a single transonic flight
Fig. 4.13: Demonstration of section camber manipulation by pivoting of reference surfaces

condition the optimizer chooses a sharp leading edge geometry since it does not consider any poor off-design angle of attack performance. The other constraint in the optimization is the match on the flap geometry. Both constraints are evaluated at the outboard flap mid-span position, which is located at approximately 58% of the wing half span. The pivoting of the RAE2822 and the CAD reference surfaces is done through these optimization constraints. Each time the optimizer feeds a new set of CST coefficients into the objective function evaluation the flap and the leading edge geometries are pivoted such that their furthest upper coordinate from the pivot point coincides with upper surface of the CST re-meshed airfoil. In case of the leading edge constraint this is in the form of inequality 4.5, where the \( \Delta z_{LEu}(i) \) and \( \Delta z_{LEl}(i) \) are the interpolated differences between the re-meshed CST airfoil and the leading edge geometry for the upper and lower surface, respectively. Upper surface differences are with a negative sign to force the CST airfoil above the RAE leading edge. The inequality constraints make it sufficient that the leading edge is at least as thick as on the reference RAE2822 airfoil.

\[
\sum_i -\Delta z_{LEu}(i) < 0 \\
\sum_i \Delta z_{LEl}(i) < 0
\]

(4.5)

For the flap surface there is an equality constraint set in the form of equation 4.6. Again the \( \Delta z_{TE}(i) \) are the interpolated differences between the CST airfoil and the reference flap geometry behind the break points, in this case of both upper and lower surface. Since the flap geometry is known for certain, the trailing edge geometry is forced by this equality constraint to accommodate it the best possible.

\[
\sqrt{\sum_i (\Delta z_{TE}(i))^2} = 0
\]

(4.6)

The result of the reference outboard airfoil optimization is seen in figure 4.14. The final and the initial geometry are relatively close showing that the modified RAE2822 airfoil was not a bad initial guess. The camber at the rear of the airfoil was increased slightly, seen from the optimized surface being above the initial airfoil and the opposite happening on the front of the airfoil. Furthermore the objective and constraints evaluations with the iteration number are provided in figure 4.15. The wing drag in the objective is normalized to the drag of the wing with the initial airfoil and the optimized value has approximately 1.2% lower drag. The constraints tolerance was set to be 0.005 for the flap and 0.001 for the slat surfaces.

Fig. 4.14: Reference wing optimization results at 58% span streamwise section
From the known data for A320, the assumed geometry from the literature and the available CAD geometry the creation of the wing model is now complete. Evaluation of the reference wing polar diagrams using the Q3D tool according to the section 2.1.2 is shown in figure 4.16. The moment polar as integrated by the discretized equation 2.6 is compared with the output of the AVL solver, which predicts larger negative values. However, this can be expected since in AVL the pitching moment prediction can only be evaluated from forces on thin lattices representation copying the mean camber lines, whereas the Euler solution includes apart from the correct surface shape the displacement of the flow by the boundary layer.

Although it was assumed in the reference wing model optimization that the wing lift is equal the weight of the aircraft in the mid-cruise condition, the exact values of wing lift throughout the cruise are yet to be determined after the aircraft is trimmed. For the reference performance of the full aircraft other models need to be included, featured in the following section 4.2.
4.2. Aircraft Components Participating in Trim

The trim of the aircraft gives the final reference values for the conditions in which wing operates which allows a comparison between the morphed and base geometry. To manage this it was opted not to use the trim capabilities present in the AVL solver due to the nature of the solver as described in its manual [46]. The main reason is that AVL includes trim by elevator deflection only, whereas the A320 uses an adjustable incidence horizontal stabilizer. It would only be possible to reproduce this behavior by repetitive calls of the AVL solution with redefined geometry to find at which incidence angle the trim balance is achieved. Furthermore the AVL as a vortex-lattice method is not directly capable of modeling the boundary layer, shockwaves and separation, all of which create the differences in the pitching moment coefficient as seen in figure 4.16.

As it was indicated in section 2.1.2 the polar diagrams of a lifting surface can be extracted by spanwise integration from the sectional data. This suggests that if other component models are included for the A320 aircraft the trim can be managed by an iterative process, matching these components in their aerodynamic characteristics. It is out of the scope of this study to model each and every detail of the A320 aircraft and therefore only several main effects are included. In section 4.2.1 the horizontal stabilizer is modelled as another lifting surface. Section 4.2.2 describes the assumed effects of the fuselage and section 4.2.3 explains the assumed center of gravity (CG) positions during cruise. During the trim procedure in the following section 4.3 the effect of thrust moment is further assumed, however many contributions are neglected, such as: the aerodynamic forces on engine nacelles, inclination of the thrust force, effect of vertical stabilizer drag moment, interference effects, etc.

4.2.1. Horizontal Stabilizer Model

Similar to the wing the horizontal stabilizer is a lifting surface. Therefore it uses the geometry generation scheme developed in the section 4.1.2 with only difference that all morphing related modules are inactive. The horizontal stabilizer definition data list is given in table B.2 of Appendix B, together with the planform photo comparison in figure B.5. Only one defining airfoil is assumed for the entire surface and only 25 spanwise sections are interpolated, as seen in figure 4.17.

The thickness was estimated from photos of the aircraft to be linear between 10% at the root and 8% at the tip and the twist was assumed zero throughout the span. An airfoil was again not available for the A320 aircraft and therefore a Boeing B737 airfoil was used, taken from the book of Obert, extracted point by point in Solidworks and re-meshed using the CST transformation, similarly to the wing root airfoil. Compared to photographs of horizontal stabilizer in figures B.6 and B.7 of Appendix B, the A320 horizontal stabilizer seems to have have a smaller leading edge radius, which however cannot be corrected unless the real geometry is available. However a negative camber was observed as on the B737 airfoil, including a flat plateau on the upper surface. The Q3D analysis of the horizontal stabilizer results in the polar diagrams in figure 4.18.

The flow around a horizontal stabilizer differs from that of the free-stream to which the wing and the remainder of the aircraft are assumed to be subjected. The first effect is a decrease in dynamic pressure due to the wake of the wing, which will be assumed lowered by a factor of $\eta_h = q_h/q_\infty = 0.85$ for a low tail jet airliner, according to Torenbeek [47]. The second effect is the downwash $\epsilon_h$, which can be according to Roskam [48] based on a first order approximation in equation 4.7.

$$\epsilon_h = (\alpha - \alpha_{C_{Lw}=0}) \frac{dC_h}{d\alpha}$$  \hspace{1cm} (4.7)

The downwash is mainly caused by the fact that the generated lift force on the wing is a reaction to the net change of momentum of the airflow. In other words the flow past the wing receives a downward component which changes the angle at which it attacks the horizontal stabilizer. The inverse is also true, meaning that if the wing is at zero-lift angle of attack $\alpha_{C_{Lw}=0}$ the downwash is near zero, neglecting influence of the fuselage
4.2. AIRCRAFT COMPONENTS PARTICIPATING IN TRIM

or wing twist and the 3D nature of the flow. Assuming a linear relation between the two, it is only necessary to know the \( \alpha_{C_{L_w}} \) and the derivative \( \frac{d\alpha}{d\alpha} \), which according to Torenbeek is approximately \( \frac{d\alpha}{d\alpha} = 0.4 \) for a low wing jet airliner. A more profound statistical relation is found in the book of Roskam [48], using the wing planform data, estimated position of the horizontal stabilizer from the wing aerodynamic center, and other calculated and evaluated data for the reference wing model which are summarized in the table 4.1. Q3D was used at two different lift coefficients to estimate the lift curve slopes \( C_{L_w,M} \) and \( C_{L_w,M=0} \) by finite differences.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_b )</td>
<td>1.5</td>
<td>m</td>
<td>vertical position of the horizontal stabilizer w.r.t the wing</td>
</tr>
<tr>
<td>( d_b )</td>
<td>17.59</td>
<td>m</td>
<td>horizontal position of the horizontal stabilizer w.r.t the wing</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.246</td>
<td>-</td>
<td>wing taper ratio</td>
</tr>
<tr>
<td>( A )</td>
<td>9.41</td>
<td>-</td>
<td>wing aspect ratio</td>
</tr>
<tr>
<td>( b )</td>
<td>34.1</td>
<td>m</td>
<td>wing span</td>
</tr>
<tr>
<td>( \Lambda_{0.25} )</td>
<td>25</td>
<td>(^\circ)</td>
<td>wing sweep angle at quarter-chord line</td>
</tr>
<tr>
<td>( C_{L_{w,M}} )</td>
<td>0.1089</td>
<td>( rad^{-1} )</td>
<td>wing lift curve slope at cruise Mach number from Q3D</td>
</tr>
<tr>
<td>( C_{L_{w,M=0}} )</td>
<td>0.0817</td>
<td>( rad^{-1} )</td>
<td>wing lift curve slope at low Mach number from Q3D</td>
</tr>
</tbody>
</table>

Tab. 4.1: Parameters used for downwash gradient calculation

The calculation of the downwash derivative is done by first evaluating coefficients \( K_A, K_\lambda \) and \( K_h \) in equations 4.8 and then substituting into the final equation 4.9. As it is seen the value of the downwash gradient \( \frac{d\epsilon_h}{d\alpha} = 0.3857 \) is close to the value predicted by Torenbeek.

\[
K_A = \frac{1}{A} - \frac{1}{1+\sqrt{A}}
\]

\[
K_\lambda = \frac{10-3\lambda}{4}
\]

\[
K_h = \frac{1+\frac{h_b}{2dh}}{a}
\]

\[
\frac{d\epsilon_h}{d\alpha} = 4.44 \left[ K_A K_\lambda K_h \cos(\Lambda_{0.25}) \right]^{1.19} = 0.3857
\]

The value of downwash is variable during cruise and it is necessary to calculate it separately for each cruise point, since it is driven by the angle of attack of the aircraft as seen in equation 4.7. To have a better idea on the approximate values for cruise, the lift curve of the wing is shown in figure 4.19 as evaluated by the Q3D together with the values of the downwash at the horizontal stabilizer. By looking at the table of aircraft lift requirements 2.2, the downwash can be expected in the order of 2° during the cruise.

For performance calculations the effect of downwash is important with respect to the drag of the horizontal stabilizer within the reference frame aligned with the free-stream, since the total drag of the aircraft...
is also in this reference frame and it dictates the thrust required to maintain the steady state in longitudinal
direction, which is related to the fuel consumption. However, the polar diagrams in figure 4.18 are expressed
in the horizontal stabilizer’s effective reference frame which is aligned with the oncoming flow influenced by
the downwash. In figure 4.20 the relation is shown between the free-stream and effective coordinate system
from which equations 4.10 are derived.

\[
C_{L_{h}} = C_{L_{h\text{eff}}} \cos \epsilon_{h} - C_{D_{h\text{eff}}} \sin \epsilon_{h} \\
C_{D_{h}} = C_{L_{h\text{eff}}} \sin \epsilon_{h} + C_{D_{h\text{eff}}} \cos \epsilon_{h}
\] (4.10)

It is clear that the two reference frames are connected through the resultant aerodynamic force coefficient
\(C_{R_{h}}\). For a positive lift force as shown in the figure the drag component in the free-stream direction is higher
than that of the effective drag simply determined by the lift-drag polar. However, if a certain net negative lift
(downforce) would be required on the horizontal stabilizer for aircraft trim then from the free-stream oriented
point of view the net drag of the horizontal stabilizer would be lessened by the effective lift component acting
in the forward direction. For the trim in section 4.3 the consistency of the required horizontal stabilizer lift
force and the effective forces must be therefore found by iteration from the polar diagram of the horizontal
stabilizer.
4.2.2. Addition of the Fuselage

Fuselage addition to the model has several effects that are to be included. Apart from the drag the fuselage contributes to the lift and moment in the wing-body combination. The lift contribution is already indirectly included in the model of the wing itself, since the reference wing area spans also a portion inside the fuselage, as it was given by the book of Roux [23]. This results in an effective decrease of the lift coefficient requirement on the wing, which is assumed to be the amount of the fuselage contribution. The other two effects are modeled based on empirical data from the book of Roskam [48].

The fuselage drag coefficient $C_D_f$ normalized with respect to the reference wing area $S_w$ is assumed to be composed of a lift independent part $C_{Df,0}$ and a lift dependent part $C_{Df,L}$, as seen in equation 4.11. The formulas for the lift independent and lift dependent fuselage drag coefficients are given by equations 4.12 and 4.13, respectively.

$$C_{Df} = C_{Df,0} + C_{Df,L}$$

$$C_{Df,0} = R_{wf} C_{ffus} \left[ 1 + \frac{60}{l_f^3} + 0.0025 \left( \frac{l_f}{d_f} \right) \right] \frac{S_{wet_fus}}{S_w}$$

$$C_{Df,L} = 2 \alpha^2 \frac{S_{ffus}}{S_w} + \eta_f C_{d_l} \alpha^3 \frac{S_{pl_fus}}{S_w}$$

The data gathered from charts in the books of Roskam and Roux required for the drag calculation are shown in table 4.2. The values of the fuselage areas are estimated from measurements of a cylindrical body in Solid-works with the fuselage planform contour matched to a top-view of A320 as also described in the method in Appendix E.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{ffus}$</td>
<td>0.21</td>
<td>$m^2$</td>
<td>fuselage base area</td>
</tr>
<tr>
<td>$S_{pl_fus}$</td>
<td>127</td>
<td>$m^2$</td>
<td>fuselage planform area</td>
</tr>
<tr>
<td>$S_{wet_fus}$</td>
<td>399</td>
<td>$m^2$</td>
<td>fuselage wetted area</td>
</tr>
<tr>
<td>$l_f$</td>
<td>37.57</td>
<td>$m$</td>
<td>fuselage length</td>
</tr>
<tr>
<td>$d_f$</td>
<td>3.95</td>
<td>$m$</td>
<td>fuselage diameter</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.422e-5</td>
<td>$kg/m/s$</td>
<td>dynamic viscosity of air at cruising altitude</td>
</tr>
<tr>
<td>$R_{Nfus}$</td>
<td>2.12e8</td>
<td>-</td>
<td>fuselage Reynolds number given as $R_{Nfus} = \frac{\rho \infty V_{\infty} l_f}{\mu}$</td>
</tr>
<tr>
<td>$R_{wf}$</td>
<td>1</td>
<td>-</td>
<td>wing-fuselage interference factor at $R_{Nfus}$</td>
</tr>
<tr>
<td>$C_{ffus}$</td>
<td>0.0018</td>
<td>-</td>
<td>turbulent mean skin friction coefficient at $R_{Nfus}$</td>
</tr>
<tr>
<td>$\frac{l_f}{d_f}$</td>
<td>9.51</td>
<td>-</td>
<td>fuselage body fineness ratio</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>0.68</td>
<td>-</td>
<td>ratio of cylinder drag coefficients of finite and infinite lengths</td>
</tr>
<tr>
<td>$C_{d_l}$</td>
<td>1.2</td>
<td>-</td>
<td>steady state cross-flow drag coefficient</td>
</tr>
</tbody>
</table>

Tab. 4.2: Parameters used for fuselage drag coefficient calculation

The resulting fuselage drag coefficient for a range of angles of attack between $0 - 5^\circ$ is shown in figure 4.21. It is seen that unless the fuselage attains an angle of attack of more than 4 degrees, the variation of its drag is fairly negligible. This has the implication that using morphing as a means to lower the fuselage incidence is only advantageous if the fuselage is at a large angle of attack in the first place. However, the A320 aircraft is not likely to cruise at large angle of attack of the fuselage, which is assumed to be in the same reference frame as the twist distribution definition in figure 4.3.

At this point a comparison can be made of the equivalent (turbulent) mean skin friction coefficients $C_f$ of the individual aircraft components with data from the book of Obert [3] to verify that no extreme differences are present. From the examples of Boeing B-727 and Fokker 100 aircraft these friction coefficients should be in the order of $C_f = 0.0025 - 0.0028$ for wing, $C_f = 0.0024 - 0.0028$ for horizontal stabilizer and $C_f = 0.0018$ for the fuselage. Values for the current model are obtained from the equation 4.14, where for any given aircraft component $S_{ref}$ is the reference area, $C_{Dh}$ is the zero-lift drag and the $S_{wet}$ is the outer wetted area of the component. The coefficient $K_f$ is the form factor taking into account the influence of super-velocities, thickness and pressure drag of the component. Typical values of the form factor by the book of Obert are for a wing in
the order of $K_f = 1.35$, for a horizontal stabilizer $K_f = 1.25$ and for a fuselage $K_f = 1.05$, but the method of Roskam seems to be using a fuselage $K_f = 1.094$ when evaluated from equation 4.12.

$$C_{D_f}S_{ref} = C_fK_fS_{wet}$$

(4.14)

It is clear from table 4.2 that the fuselage already uses the correct $C_f = 0.0018$. Furthermore the horizontal stabilizer resulted in $C_f = 0.00281$, which is also well comparable. However the wing shows $C_f = 0.0039$, which is higher than anticipated for this component. There are several reasons why this can happen. First is the twist of the wing being non-optimum from the point of view of the quasi-3D method, creating larger drag at the inboard wing even at zero lift condition. This would explain why the horizontal stabilizer does not suffer from this effect because no twist is applied to it. The second is the wetted area which was simply assumed as twice the wetted planform, but in reality the wing will have this area larger because of thickness and the front and aft loading of the airfoil. The zero-lift drag could be corrected by scaling to a lower value. However this is not necessary, since the drag of the non-lift bearing components $C_{D_{NL}}$ will then simply recover the difference when it is iterated such that the aircraft range matches the reference range.

Without wind tunnel measurements or higher fidelity CFD calculations in 3D, the moment of the fuselage is hard to predict. Therefore instead of modelling it directly only the additional moment of the wing-body combination is assumed, as a result of a shift in the aerodynamic center $\Delta x_{AC}$ due to fuselage addition to the wing based on an empirical relation by Roskam as given in equation 4.15;

$$\Delta x_{AC} = -\frac{dM}{d\alpha} \frac{1}{\rho\infty S_w C_{MAC} C_{L_{w}}}

(4.15)

Here the lift curve slope $C_{L_{w}}$ is approximated as a constant, the $C_{MAC}$ is the mean aerodynamic chord of the wing and the moment derivative $\frac{dM}{d\alpha}$ is a function of a so called upwash derivative $\frac{\omega}{d\alpha}$ which is a function of a position along the fuselage. To determine the upwash derivative a method using distribution of the fuselage planform areas is used, further described in Appendix E. As a result of this method the shift in aerodynamic center was calculated to be $\Delta x_{AC} = -21.7\%$MAC. Due to this shift the lift force will create a larger positive pitching moment about the center of gravity of the aircraft, going against the wing negative pitching moment and lessening the demand on the horizontal stabilizer downforce.

4.2.3. CENTER OF GRAVITY POSITION

The following described procedure is applied to deduce the center of gravity positions in cruise for the A320 aircraft from its technical documentation. The main mechanism behind the change of the center of gravity (CG) position is the change in both the mass and moment arm of the fuel as it is depleted from the fuel tanks in a swept wing with dihedral.

For the CG calculations a balance of mass moments on the aircraft can be done where the choice of the reference station is arbitrary, since in a static condition the net moment on a rigid body is zero irrespective of the reference point. Let it therefore be assumed at the position of the leading edge of the wing mean aerodynamic chord (MAC). Apart from the fuel the remainder of the aircraft with passengers and payload can
be assumed to have a constant mass and arm throughout the mission. Let the mass moment of this zero fuel mass be denoted as the term $m_{ZFW} \cdot x_{ZFW}$, where $x_{ZFW}$ is the distance of the center of this mass $m_{ZFW}$ from the reference station. With all masses in kilograms and arms in percents of MAC, the mass moment balance is captured by equation 4.16 for situation at maximum take-off weight (MTOW) as well as by the equation 4.17 for an arbitrary cruise point (CR). In these equations the $x_{CG_{MTOW}} \cdot m_{MTOW}$ is the mass moment of the complete aircraft at maximum take-off weight and the $x_{FM_{MTOW}} \cdot m_{FM_{MTOW}}$ is the mass moment of the total fuel for the design mission.

\[
\begin{align*}
    x_{CG_{MTOW}} \cdot m_{MTOW} &= x_{FM_{MTOW}} \cdot m_{FM_{MTOW}} + m_{ZFW} \cdot x_{ZFW} \\
    x_{CG_{CR}} \cdot m_{CR} &= x_{FM_{CR}} \cdot m_{FM_{CR}} + m_{ZFW} \cdot x_{ZFW}
\end{align*}
\]

(4.16) (4.17)

Substituting from equation 4.16 into equation 4.17 for the unknown term $m_{ZFW} \cdot x_{ZFW}$ the relation can be established for the CG position $x_{CG_{CR}}$ in terms of MAC for any cruise point by equation 4.18.

\[
x_{CG_{CR}} = \frac{x_{FM_{CR}} \cdot m_{FM_{CR}} + x_{CG_{MTOW}} \cdot m_{MTOW} - x_{FM_{MTOW}} \cdot m_{FM_{MTOW}}}{m_{CR}}
\]

(4.18)

In this equation the aircraft mass at maximum take-off weight $m_{MTOW}$ is a known constant and the aircraft masses $m_{CR}$ and fuel masses $m_{FM_{CR}}$ at the individual cruise points are seen in table 2.2. The arms of the fuel $x_{FM_{CR}}$ corresponding to the individual cruise points fuel masses $m_{FM_{CR}}$ can be found in a diagram taken from the Weight and Balance Manual (WBM) of the A-320 aircraft in figure B.8 of Appendix B. The only unknown left is the CG position of the aircraft at maximum take-off weight $x_{CG_{MTOW}}$. This can be assumed a certain value, however the aircraft must comply with the CG limits which were found from the Balance Computer shown in figure B.9 of Appendix B. The most aft allowed CG position is 39% of MAC and from the figure B.8 the most aft CG position for any given configuration occurs when the fuel mass is at 2000kg. This condition must be therefore verified for the assumed $x_{CG_{MTOW}}$. Figure 4.22 shows the CG positions from MTOW, throughout the cruise and at the critical aft condition for a range of $x_{CG_{MTOW}}$.

Fig. 4.22: Center of gravity positions during mission for varying maximum take-off weight position

It can be seen that $x_{CG_{MTOW}} = 32\%$ would result at a CG position of 38% of MAC in the critical condition. However the aircraft can from figure B.9 be also loaded to the most forward limit of $x_{CG_{MTOW}} = 21\%$. A compromise is assumed to be $x_{CG_{MTOW}} = 27\%$, as the CG position for the design mission should allow for various adaptations in configuration of the aircraft both to load it with forward and aft CG positions with respect to the reference value. With $x_{CG_{MTOW}}$ assumed the design mission CG positions are specified as highlighted in figure 4.22.
4.3. TRIM PROCEDURE DESCRIPTION

As a recapitulation of the previous sections the available aircraft model components are: the wing lift-drag and lift-moment polar diagrams, the horizontal stabilizer effective lift-drag and lift-moment polar diagrams, the model of downwash at the horizontal stabilizer, the fuselage drag model, the center of gravity position as a function of the fuel amount and the shift of aerodynamic center of the wing-body combination with respect to the aerodynamic center of the wing itself. In this section the trim procedure is explained as applied to each cruise point separately for the reference cruise model. The main mechanism for obtaining the intermediate iteration results in the trim is the interpolation from the characteristics of the above mentioned aircraft model components.

The longitudinal trim of the aircraft in the x-z plane aligned with the free-stream at a given cruise point is driven by the three steady state equations 4.19 through 4.21, as illustrated in figure 4.23. The total aircraft lift coefficient is set in magnitude equal to the normalized weight of the aircraft in expression \( \frac{m_CR g}{\infty S_w} \), where the \( m_{CR} \) is the weight at the given cruise point, \( q_{\infty} \) is the constant free-stream dynamic pressure and \( S_w \) is the reference wing area. Lengths such as \( h_h \), \( h_{CR} \), \( l_{CR} \) and \( r_T \) explained further are normalized by the wing mean aerodynamic chord \( c_{MAC} \). As mentioned before the \( C_{Lw} \) is assumed as the lift generated by the wing-fuselage combination due to the definition of the wing reference area.

\[
C_L = C_{Lw} + \eta_h \frac{S_h}{S_w} C_{Lh} \tag{4.19}
\]

\[
C_D = C_{Dw} + \eta_h \frac{S_h}{S_w} C_{Dh} + C_{Df} + C_{D_{NL}} \tag{4.20}
\]

\[
C_{MLg} = C_{Mw} + h_{CR} C_{Lw} - \eta_h \frac{S_h}{S_w} l_{CR} C_{Lh} + \eta_h \frac{S_h c_{MAC}}{S_w c_{MAC}} C_{Mh} + r_T C_{TD} + h_h C_{Dh} \tag{4.21}
\]

The position of the center of gravity \( CG_{CR} \) is a variable throughout the cruise and it is determined from the assumed CG positions function in figure 4.22. According Roux [23], the position of the horizontal stabilizer aerodynamic center from the mean of these CG positions \( x_{CG\text{mean}} \) in terms of MAC is given as \( h_{h\text{mean}} = 17.59 \ c_{MAC} \). From these quantities the distance \( h_{CR} \) between the CG and the actual position of the wing lift vector can be determined along with the actual distance \( l_{CR} \) from the CG to the horizontal stabilizer in equations 4.22. Apart from the use in the downwash model the vertical position of the horizontal stabilizer \( h_h \) is neglected in
the trim procedure, since its effect on the moment balance coming from its drag is small compared to other moments on the aircraft.

\[
\begin{align*}
\text{h}_{CR} &= x_{CGCR} - \left( \frac{1}{2} + \Delta x_{MC} \right) \\
\text{l}_{CR} &= l_{h\text{mean}} + x_{CG\text{mean}} - x_{CGCR}
\end{align*}
\] (4.22)

The trim procedure iteration starts by requiring a zero moment about the center of gravity \( C_{MC} = 0 \) in equation 4.21. Two nested iteration loops are employed, where the inner loop makes sure that the net (free-stream oriented) horizontal stabilizer lift force \( C_{Lh} \) required by the moment balance in the outer loop is accomplished by the resultant of the effective lift and effective drag coefficients of the horizontal stabilizer tilted by the influence of the downwash. After assuming initial values for the parameters in equations 4.23, where the \( |\Delta C_{Lw}| < 1 \cdot 10^{-6} \) is the outer loop convergence criterion, the outer iteration begins.

\[
\begin{align*}
C_{Lh} &= 0 \\
C_{Lh\text{eff}} &= 0 \\
C_{Mh} &= C_{Mh}(C_{Lh} = 0) \\
C_{Dh} &= C_{Dh\text{eff}(C_{Lh} = 0)} \\
C_{Lw} &= C_{Lh} - (C_{Mw}(C_{Lw} = C_{L}) + C_{Lh\text{CR}}) / l_{CR} \\
\Delta C_{Lw} &= C_{Lw} - C_{L}
\end{align*}
\] (4.23)

Each inner iteration the following steps are taken:

- interpolation at the lift requirements from previous iteration, \( C_{Lw} \) and \( C_{Lh\text{eff}} \), of the following coefficients from the known polar diagrams: wing drag \( C_{Dw} \) and wing moment \( C_{Mw} \), effective horizontal stabilizer drag \( C_{Dh\text{eff}} \) and (effective) horizontal stabilizer moment \( C_{Mh} \), downwash \( \epsilon_{h} \) and fuselage drag \( C_{Df} \) both obtained from the wing (aircraft) angle of attack \( \alpha \), being a result for the \( C_{Lw} \) requirement
- calculation of the free-stream horizontal stabilizer drag \( C_{Dh} \) coefficient from the effective lift and drag coefficients from previous iteration, \( C_{Lh\text{eff}} \) and \( C_{Dh\text{eff}} \), and approximation of the aircraft drag which is put equal to the thrust force coefficient \( C_{Tf} \) acting on the arm \( r_{f} \) from CG, which completes the moment balance equation 4.21 and allows expression of the net required lift force \( C_{Lh} \) on the horizontal stabilizer as seen in equations 4.24.

\[
\begin{align*}
C_{Dh} &= C_{Dh\text{eff}} \cos \epsilon_{h} + C_{Lh\text{eff}} \sin \epsilon_{h} \\
C_{Tf} &= C_{Dw} + C_{Dh} + C_{Df} \\
C_{Lh} &= \frac{S_{w}}{\eta_{h} S_{0} l_{CR}} \left[ C_{Mw} + r_{f} C_{Tf} + h_{CR} C_{Lw} + C_{Mh} \eta_{h} S_{0} \epsilon_{MAC} \right]
\end{align*}
\] (4.24)

- iteration of the inner loop on the effective horizontal stabilizer lift and drag coefficients with the initial parameters in equations 4.25, where the \( |\Delta C_{Lh}| < 1 \cdot 10^{-6} \) is the inner loop convergence criterion on the difference between the required and the resulting free-stream oriented horizontal stabilizer lift coefficients, \( C_{Lh} \) and \( C_{Lh\text{res}} \), respectively. The equations follow from the figure 4.20 and equations 4.10.

\[
\begin{align*}
C_{Lh\text{eff}} &= C_{Lh} \\
C_{Dh\text{eff}} &= C_{Dh\text{eff}(C_{Lh} = C_{Lh})} \\
C_{Lh\text{res}} &= C_{Lh\text{eff}} \cos \epsilon_{h} - C_{Dh\text{eff}} \sin \epsilon_{h} \\
\Delta C_{Lh} &= C_{Lh\text{res}} - C_{Lh}
\end{align*}
\] (4.25)

Each inner iteration consist of the following steps:

- update the effective stabilizer lift coefficient using the \( \Delta C_{Lh} \) difference: \( C_{Lh\text{eff}} = C_{Lh\text{eff}} - 0.5 \cdot \Delta C_{Lh} \)
- interpolate updated effective horizontal stabilizer drag coefficient \( C_{Dh\text{eff}} \)
- determine the resulting free-stream stabilizer lift coefficient \( C_{Lh\text{res}} = C_{Lh\text{eff}} \cos \epsilon_{h} - C_{Dh\text{eff}} \sin \epsilon_{h} \)
- determine the remaining lift difference to the required value \( \Delta C_{Lh} = C_{Lh\text{res}} - C_{Lh} \)
- repeat until inner loop convergence criterion is met

- continuation of the outer loop: an update of the free-stream oriented horizontal stabilizer drag \( C_{Dh} \) by equation 4.26 and an update of the horizontal stabilizer moment coefficient \( C_{Mh} \) by interpolation from its moment polar diagram at effective \( C_{Lh\text{eff}} \)

\[
C_{Dh} = C_{Dh\text{eff}} \cos \epsilon_{h} + C_{Lh\text{eff}} \sin \epsilon_{h}
\] (4.26)
• calculation of the new required wing lift coefficient $C_{L,w,new}$ satisfying equation 4.19 in z-axis, determination of its difference $\Delta C_{L,w}$ to the $C_{L,w}$ from the previous iteration and an update of this value, seen in equations 4.27.

$$
\begin{align*}
C_{L,w,new} &= C_L - \eta_{h} \frac{S_h}{S_w} C_{L_h} \\
\Delta C_{L,w} &= C_{L,w,new} - C_{L,w} \\
C_{L,w} &= C_{L,w,new}
\end{align*}
$$

(4.27)

• repetition of all steps until outer loop convergence criterion is met

• assembly of the known parts of the aircraft drag $C_D$ in equation 4.20, with exception of the non-lift bearing part $C_{D_{NL}}$

The trim procedure as described above has several model errors. The first comes from the moment due to the thrust of the engines. The thrust $C_{T,D}$ that is acting on the arm $r_T$ estimated as 1 m length cannot be updated by the true aircraft drag coefficient, since during the trim procedure the last component $C_{D_{NL}}$ cannot be known beforehand, and is therefore ignored. The next is the fact that the position of the horizontal stabilizer in the free-stream coordinate system changes with respect to the center of gravity due to the overall angle of attack change of the fuselage, which is not taken into account in the downwash model and neither in the small changes in the arm of the horizontal stabilizer and thus the required lift.

As explained in section 2.2, after the trim procedure is made for each cruise point, the reference aircraft performance can be tuned to the required mission range by iteration on the unknown $C_{D_{NL}}$ such that the mission range $R$ reaches its design value $R_0$. By equation 4.20 the ratio $\frac{C_{L}}{C_{D_{NL}}}$ becomes a known function throughout the cruise and the reference aircraft performance can be determined as given in the following section 4.4, which complies with the A320 known design mission parameters.
4.4. REFERENCE AIRCRAFT PERFORMANCE

Using the iterative trim procedure described in section 4.3, the aircraft model is evaluated at the 7 representative cruise points and the trimmed aerodynamic characteristics are used to make conclusions on the behaviour of the model. In figures 4.24 and 4.25 the wing and horizontal stabilizer trimmed values are shown against the reference model polar diagrams, evaluated by the Q3D.

Fig. 4.24: Trimmed cruise points results on wing polar diagrams

Fig. 4.25: Trimmed cruise points results on horizontal stabilizer polar diagrams

A function is shown in figure 4.25 between the wing moment coefficient $C_{Mw}$ and the required horizontal stabilizer lift force $C_{Lh}$ in the free-stream coordinate system. This function is a solution to the trim procedure and its extrapolation is not correct, but within the cruise range it in principle reflects the equation 4.21, including the non-linearity coming from the function of center of gravity position along the cruise, which has the shape shown in figure 4.22. Due to these changes in CG position the requirement on the horizontal stabilizer downforce is not maximum at the beginning of cruise, but at approximately the third cruise point out of the seven, making the curve reverse in direction. Moreover, the change in the downwash angle throughout the cruise is responsible for the points not following exactly the same path after the reversal.

The trimmed and non-trimmed conditions differ only slightly in terms of wing lift compared to the final trimmed value, which is due to a relatively small horizontal stabilizer negative lift necessary to balance the aircraft. The trimmed values of the horizontal stabilizer lift coefficient are in the free-stream oriented coordinate system, whereas the base polar diagram is in the effective coordinate system. This shows the decrease in the horizontal stabilizer drag, which is due to the downwash tilting the effective horizontal stabilizer lift forward and creating a thrust component.
The results in numerical form are presented in the Appendix F. There the fuselage drag coefficient is seen together with its angle of attack (equal to the wing and aircraft $\alpha$). The mid-cruise value of this angle of attack is approximately $1.8^\circ$, varying between $2.3^\circ$ – $1.2^\circ$. As a result the fuselage drag varies less than $0.8\%$ of its mid-cruise value. As a verification of the trim procedure the moment about the center of gravity $C_{MCG}$ is re-evaluated using equation 4.21 and its absolute value turns out below $2.5 \cdot 10^{-7}$ for all cruise points. At the same time the additions of the wing and horizontal stabilizer lift coefficients (normalized by $S_w$) give the total lift coefficient at each cruise point as required by equation 4.19.

From section 3.2 a downward morphing deflection is expected at the beginning of cruise which produces an increase in the pitching moment. The isolated effect of such pitching moment increase on the drag of individual re-Trimmed model components is shown in figure 4.26. An increase of wing pitching moment results in a negative increase of the horizontal stabilizer lift coefficient requirement which under the downwash value for the given cruise point results in a decrease of the free-stream oriented horizontal stabilizer drag. This from large part compensates the increase of the wing drag coefficient due to its larger lift requirement and therefore the trim drag increase is relatively low. More specifically, even if the wing moment increased by quarter of its original value the resulting aircraft drag increase would be below $0.5\%$ of its reference value.

![Fig. 4.26: Effect of increasing wing pitching moment on the re-trimmed drag values at beginning of cruise](image)

Addition of the trimmed aircraft components to the polar diagram in figure 4.24 is shown on the combinations of lift-drag polar diagrams in figure 4.27. Since further contribution to the lift coefficient is small, the trimmed wing points are practically shifted right with each added component in the diagram.

![Fig. 4.27: Polar diagrams of aircraft components combinations](image)
This has an effect on the maximum lift-to-drag ratio point on the polar curve occurring at higher lift coefficients and therefore getting closer to the mid cruise condition. As it is seen in figure 4.28, the complete aircraft model is relatively consistent with the mid-cruise optimum requirement, given the fact that the variation of the aircraft $L/D$ is only between 15.6 – 16.1. As a verification this is compared with the often made approximation of constant $L/D$ ratio in the Breguet range equation, which was evaluated to be 15.94 by equation 2.19 with use of the design mission range of 4800km, Mach number of 0.78, altitude 37000ft and the fuel consumption of $c_T = 1.688 \cdot 10^{-5}$ kg/s/\text{N}.

Similar to having a wing lift coefficient associated with each cruise point, there can be found corresponding spanwise lift distributions of the wing as evaluated by the AVL solver within the Q3D. These are shown in figure 4.29 and it is observed that the relation between the aircraft lift coefficient and the streamwise sections lift coefficients is linear, especially between the 40-80% of span, as it can be expected from a first order vortex lattice method.

The choice of a representative spanwise section is therefore made at 60% of span as a reference for the morphing 2D optimization. This section is outside any region with sharp lift and geometry gradients. Figure E1 in Appendix F was used to further support the decision, showing the standard deviation throughout the cruise flight on the ratio between the aircraft $L/D$ and effective sections $C_{l_{eff}}/C_{d_{eff}}$. The lowest variations were found between 40 – 80% of half-span, surrounded by regions of higher gradients. The selected section is subject to the streamwise section lift coefficients from the lift distributions as marked in figure 4.29.

As was seen in the simple sweep theory in section 2.1.2 the streamwise lift coefficient is not directly applicable for 2D analysis since an effective sectional analysis is claimed to give the best wing profile drag prediction. Within this procedure the streamwise section lift coefficients $c_l$ are transformed by the local sweep angle to $c_{l_{\perp}}$ in the perpendicular to half-chord reference frame and then an iterative procedure is applied such that
the effective lift and drag coefficients differing by the induced angle of attack give this required $c_{L_{\text{eff}}}$ at each of the investigated sections (20 in case of the wing). The resulting array of the effective lift coefficients at each cruise point is used to interpolate the effective lift coefficient requirement on the 60% span section. Comparison of the cruise requirement on the streamwise lift coefficient and the interpolated effective lift coefficient as a function of the aircraft cruise lift coefficient is shown in figure 4.30.

These coefficients are also summarized in the Appendix F in form of numerical data. It is seen that the effective lift coefficients are in general higher than the streamwise coefficients, but on the other hand the evaluated effective Mach number $M_{\text{eff}} = 0.7238$ and Reynolds number $Re_{\text{eff}} = 14.432 \cdot 10^6$ are lower than the free-stream values and the thickness to chord ratio of such outboard effective section is larger than that of the streamwise section as it was seen in section 4.1.2. This concludes the chapter on the reference aircraft, since the data is now available on both the trimmed 3D lift requirements as well as the 2D effective section lift requirements on the representative section.
Morphing performance evaluation

The development of the simplified A320 aircraft model in chapter 4 served the purpose of obtaining the reference requirements on lift coefficients for the morphed wing and the representative 2D section. In this chapter under the assumption that these requirements are to remain constant at any given cruise point, the wing and effective section are forced by morphing to attain their respective reference lift coefficients seen in Appendix F at different angles of attack than reference and possibly improved drag coefficients due to changes in the pressure distributions.

As it was seen in chapter 3, the decrease in drag at certain lift coefficients can be achieved due to the shift and strength reduction of the shockwave, but it is limited by the separation effects which affect both the lift and drag. An optimization has to be therefore made at every cruise point irrespective of whether 2D section, wing or aircraft morphing effects are investigated, in order to find the best morphing deflection(s). For the most realistic performance evaluation it would be ideal to obtain directly the full aircraft performance changes with morphing, including variations on all the modelled aircraft components. Although such evaluation was attempted, it requires approximating the polar diagrams at each cruise point for every morphing setup, which has shown to be limited by the robustness of the MSES evaluations. This is due to a large sensitivity of the wing profile drag to the number of non-converged effective sections which increases when the geometry is morphed. However, when it comes to the drag of other components than wing, evidence was found in section 4.4 that their contribution to the total drag is relatively constant at least through the cruise range and their combined effect with pitching moment changes not too large. Therefore, the 2D effective section analysis and the wing analysis alone are still relevant to the complete aircraft performance.

In this chapter first the section 5.1 puts into perspective the morphing considerations in 2D by use of the obtained lift requirements from the reference aircraft model at the representative 60% span effective section. Then in section 5.2 a relation is made between the 2D and 3D performance by looking at a specific morphing setup and optimizing the wing morphing cruise schedule. In section 5.3 the final analysis in 2D is done with this assumed relation and varying morphing setup in order to evaluate the performance indicators as a function of the morphing design variables.
5.1. 2D MORPHING OF REFERENCE EFFECTIVE SECTION

For evaluation of the best morphing deflection and corresponding drag improvement at a given lift requirement for a given setup, an optimization algorithm was developed as described in section 2.3 which can deal with discontinuous objective functions coming from occasional non-convergence of MSES. In this section the algorithm is used to determine the possible drag savings at 7 cruise points on the representative effective section in the model representing the A320 aircraft. In relation to the wing generation scheme in section 4.1.2, each time an outboard wing morphing deflection is specified by the optimization algorithm an instance of the class LiftingSurface is created with the requested morphing applied by the MorphingModule and the corresponding effective geometry at 60% span is generated by the SweptInterpolationModule.

The reference effective lift coefficients corresponding to figure 4.30 are used in the following evaluations. An example for single morphing setup (between 0.92 - 0.95/c) is given in figure 5.1 of optimization algorithm applied on the 7th and 1st cruise point, corresponding to the end of cruise and beginning of cruise, respectively. The end of cruise morphing results in exactly zero morphing deflection due to the maximum limit on the morphing radius within the morphing method. Even for the most improved beginning of cruise point the best morphing deflection for the given setup is in order of few degrees.

A summary of all 7 cruise points evaluations is then presented in figure 5.2 for a few selected morphing regions on the airfoil. The first observation is that the reference wing model is indeed consistent with the mid-cruise optimum, since no significant improvement is possible with any of the tested morphing setups, proving that the reference geometry tuning in section 4.1.3 was successful. Relatively small differences occur between the different locations and sizes of the morphing region. Furthermore, the beginning of cruise has the largest potential in terms of the obtainable drag improvement and a very aft positioned morphing region seems to perform the best out of the few selected setups.
5.1. 2D MORPHING OF REFERENCE EFFECTIVE SECTION

From the few investigated morphing setup cases the morphing can bring up to approximately 8-9% of sectional drag improvement on an effective section at the beginning of cruise. This improvement becomes less as the flight progresses and after the mid-cruise the improvements are less than 0.5%. This is however still not a complete information with respect to the aircraft range or fuel savings.

On the other hand a study can be made on the effect of cruise altitude on the drag improvements with the same reference geometry as seen in figures 5.3 and 5.4. Such study involved evaluating the wing and the horizontal stabilizer models polar diagrams at the new altitude and making a separate trim case for the updated cruise points. The drag of the non-lift-contributing components \( C_{D,N} \) was kept constant to the value obtained for the design mission and as a result the aircraft range at the new altitude would be different from the reference range value.

![Graph showing \( \Delta c_{d_{eff}} \)](image)

**Fig. 5.3:** 2D morphing optimization summary for all cruise points at altitude of 36000ft

![Graph showing \( \Delta c_{d_{eff}} \)](image)

**Fig. 5.4:** 2D morphing optimization summary for all cruise points at altitude of 38000ft

Although this study is done at a fixed assumed Mach number and a number of changes are also neglected (such as the fuselage Reynolds number change), it is concluded that the cruise altitude mostly affects the lift requirements on the wing. At a lower altitude where the air is thicker and the true airspeed higher, also the lift coefficients are lower and it can be observed that the obtainable drag improvements within the cruise are less. However, with respect to the original lift coefficient values in figure 5.2 the function of \( \Delta C_d \) remains nearly the same and only appears shifted to the right. The reverse is happening at higher cruising altitude, where due to the shift to higher required lift coefficients the drag improvements are larger. However, flying at higher lift coefficients will also increase the reference (non-morphed) absolute drag values and result in higher fuel consumption. At the same time the performance of the engines may be deteriorating due to reduction in air density and further investigation is out of scope of this work.
5.2. 3D MORPHING OF WING

In this section, an analysis is made of the wing performance with the morphing region fixed between 0.92 − 0.95/c. This was extended to span also the aileron for greater performance benefits and better relation to the 2D analysis. The outboard flap is morphed conically by adjusting the size of the morphing region $d_m$ according to the outboard wing taper. The inboard flap is assumed to have the same morphing region at the kink, but of rectangular shape towards the fuselage, resulting in a cylindrical deflection. Separate inboard and outboard morphing deflections are optimized, respectively denoted as $\delta_m$ and $\delta_{m_o}$.

Having two design variables calls for a multi-dimensional optimization and therefore “fmincon” optimization function in Matlab is employed. The objective is simply the wing drag coefficient, internally solved by the Q3D solver. However the occasional non-convergence of the MSES section analysis within Q3D must be dealt with and therefore any solution which does not have all 20 spanwise sections converged is discarded by replacing the wing drag coefficient with a NaN value. The SQP algorithm is chosen within the “fmincon” function so that the optimization can work with these NaN values. Each cruise point requires its own separate optimization run and the summary of their results is presented in figure 5.5 normalized by the aircraft drag coefficient for further use, with the corresponding morphing deflections shown in figure 5.6.

At mid-cruise the optimum morphing deflections are non-zero. This is due to numerous assumptions made in the reference model. Recalling that the outboard effective section was shown optimum with respect to outboard section morphing, the cause is mainly attributed to the twist distribution and the inboard airfoil shape, which on the real aircraft are tailored to the effects not modelled by the quasi-3D methods. To simulate an optimum mid-cruise condition these deflections are applied to the reference model and the drag coefficient improvements seen in figure 5.5 are already evaluated with respect to this new reference geometry. The results of the optimization follow the trends that were seen in section 5.1, in terms of the most improvements happening at the beginning of cruise and relatively small improvements due to morphing after the mid-cruise is reached. However the entire diagram shows significantly smaller improvements than obtained in effective section investigation.

To find the reason for this, the optimization at the most improved (beginning of cruise) point is observed in greater detail in figure 5.7. The 7 iterations at a fixed $C_{L,w} = 0.620$ towards the corresponding optimized result at this wing lift coefficient in figure 5.5 are shown normalized by the value belonging to the initial iteration. Intermediate results are shown of the wing induced drag $C_{D, iw}$, and the wing profile drag comprising of separately integrated wing pressure drag $C_{D, wp, w}$ and wing friction drag $C_{D, pf, w}$ components. It is seen that once the geometry is optimum at the mid-cruise condition, the morphing optimization at an off-design lift coefficient results in a nearly constant value for the friction drag and even a slightly increased value of the induced drag. In terms of friction drag this was observed also in case of the 2D morphing analysis, the induced drag is to be further discussed. Therefore, the main driving mechanism for drag improvement of the entire wing with morphing remains the pressure drag variation, which is an order of magnitude larger than variations in $C_{D, iw}$ and $C_{D, wp, w}$. The profile drag (sum of friction and pressure drags) constitutes approximately 40% and the induced drag 60% of the wing drag. Furthermore, from the simple sweep theory presented in section 2.1.2 the effective section pressure drag and the pressure drag of the streamwise
section are related by a factor of $\cos^3(\Lambda_0.5)$. Finally, at the beginning of cruise the wing drag is approximately 64% of the total aircraft drag and therefore a rough estimation gives the aircraft drag improvement as

$$\frac{\Delta C_{Dw}}{C_D} = -7.95\% \cdot \cos^3(21^\circ) \cdot 0.4 \cdot 0.64 = -1.66\%,$$

which is only 11% different with respect to the beginning of cruise improvement shown in figure 5.5.

Figure 5.8 shows the spanwise lift distribution as a result of mid-cruise optimization done to simulate the mid-cruise optimum. It can be seen that the twist distribution was not optimal with respect to the Q3D solver, but this is also not necessarily the case for the real aircraft since other design constraints are not known at this point, for instance on the wing bending moment. This justifies the simulated use of the optimum morphing deflections as the new base geometry. It is noted that a difference in deflections between the inboard and outboard wing creates a jump in the spanwise lift distribution. Increasing this difference would result in an increased induced drag due to shedding of vorticity behind the wing, which is the reason for limited freedom in improving the induced drag by inboard and outboard flap morphing. A refinement on the number of morphing surfaces would be necessary as it was done in research on the NASA Generic Transport Model [49] with flexible in-plane filaments, but this is at the moment not feasible in the concept by Fokker. Furthermore, the comparison between the drag components throughout the cruise of the reference aircraft in figure 5.9 shows that the ratio of wing profile drag and the total wing drag is almost constant throughout the cruise.

The conclusion from the above is that to relate the 2D and 3D performance results, the easiest option is to introduce a coefficient between the effective section pressure drag and the pressure drag portion of the wing drag, both responsible for drag improvements. This coefficient $K$ is made a function of the aircraft weight to account for possible differences throughout the cruise. The expression adopted for this coefficient is seen in equation 5.1 using the non-morphed reference aircraft performance results. Importantly, the purpose of this relation between 2D and 3D is not to make an exact absolute match on the performance benefits, but to serve as a comparison method of different morphing designs in section 5.3. In other words based on the
discretized Brequet equation some sort of integration over the weight of the aircraft is required, where the 2D drag improvements are used to compare relative aircraft range changes with morphing setup without the need of 3D evaluation.

\[ K = \frac{C_{D_{ppw}}}{c_{d_{dp}}} \]  

(5.1)

If an assumption is made that the wing lift is to remain according to the individual reference cruise points requirements the evaluation of aircraft range can be made by reformulating the discretized Brequet equation as seen in 5.2.

\[ \Delta R = k - \sum_{i=1}^{k-1} \frac{V_{\infty}}{c T\ g} \left( \frac{C_L}{(C_{D_{ppw}} + C_{D_{nm}}) m} \right)^{(i+1)} + \left( \frac{C_L}{(C_{D_{ppw}} + C_{D_{nm}}) m} \right)^{(i)} \frac{\Delta m}{2} \]  

(5.2)

In this equation \( C_{D_{ppw}} \) is the wing profile pressure drag improvable by morphing and the \( C_{D_{nm}} \) quantity designates all the remaining drag components, combination of which is as a first order approximation assumed not to be influenced by morphing. These are the remaining wing drag, horizontal stabilizer drag, fuselage drag, and the drag of the non-lift bearing components. Under these considerations the 50 points interpolated aircraft \( L/D \) ratio is shown over the design mission in figure 5.10 both for the base geometry and the optimum morphing deflections schedule from figure 5.6.

![Fig. 5.10: Wing morphing effects on aircraft lift-to-drag ratio](image)

Compared to figure 1.2 from the literature the main difference is that in this work effort was made to have a consistent mid-cruise optimum which means that the morphing does not improve this condition. Therefore the optimum function of \( L/D \) is closer to the base function, reducing the overall integrated performance benefit. Using these functions of \( L/D \) ratio the 3D evaluation results in an overall range improvement of \( \Delta R = 20.3 km \). Translated to the fuel savings, when the end of cruise aircraft mass is varied \( \Delta F_1 = 0.39\% \) and when the beginning of cruise aircraft mass is varied \( \Delta F_2 = 0.46\% \) of trip fuel. As discussed in section 2.2 the first scenario of fuel saving enables replacing the fuel with payload, since A320 does not take-off at maximum payload condition for its design mission.

These performance benefits are claimed small for practical application. Furthermore, the most improvements in drag are registered at the beginning of cruise with positive (downward) deflections, which are known to produce additional nose down pitching moment on the wing. From section 4.4 the net effect of individual aircraft components drag variation after trim is still an increase in the aircraft drag. Therefore the already small performance improvements with morphing can be even further reduced. Due to the solver robustness issues this effect was not further investigated. However it is safe to conclude that if the pitching moment increase effect was included the performance benefits would not be larger than those already obtained.
5.3. 2D ANALYSIS ON MORPHING VARIABLES

As mentioned in the introduction the effect of varying the morphing variables specifying the morphing setup on the obtainable performance benefits is investigated using a 2D analysis, mainly because of the computational time. A sensitivity analysis is made in this section of the performance indicators with respect to the morphing region mid-position \( x_{dm} \) and the size of the morphing region \( d_m \).

By using the coefficient \( K \) introduced in section 5.2 it is possible to approximate for each cruise point the morphed wing pressure drag from each morphed effective section pressure drag as \( C_{D_{ppw}} = c_{d_{eff}} p_{m} K \). This allows interpolation of cruise points for the use of the discretized Breguet range equation 5.2. The results of the 2D analysis converted to the aircraft performance indicators are shown in figures 5.11 and 5.12. In these figures each point is determined as an integrated effect using the individual cruise points optimization runs as shown in figure 5.2.

\[ \Delta R = 20 \text{ km} \]
\[ \Delta F_1 = 0.4\% \]

Evaluations of the performance indicators result in the range improvements in order of \( \Delta R = 20 \text{ km} \), equivalent to the fuel savings of \( \Delta F_1 = 0.4\% \) with varying cruise end mass. The varying beginning of cruise mass case was not evaluated. The exact values for the morphing range between \( 0.92 - 0.95/c \) at the kink are read as \( \Delta R = 18.1 \text{ km} \), \( \Delta F_1 = 0.35\% \), which is again 11\% less than those based on the wing morphing performance estimation, since a 2D effective section behaviour over the cruise is not directly translatable to the behaviour of an entire wing. However, the coefficient \( K \) serves only as an integration parameter used to make conclusions on the relative performance comparison of morphing setups. The main cause behind the difference in predicted performance benefits in 2D and 3D is expected due to the root of the wing, since there the relative chord position and size of the morphing region varies.

The red boundary color in the diagrams expresses the constraint on the 300mm radius of the morphing function \( r_m \). It can seen that the optimum design is with the morphing region at the very aft of the airfoil, beyond where the concept by Fokker is realizable by bending of the upper surface. The size of the morphing region does not influence the obtainable performance benefits for the transonic flight of the A320 aircraft. This is a surprising result of such analysis which would hardly be recognized from only using an optimization on the design variables and would be even more obscure if the morphing region was defined by \( x_b \) and \( x_e \) as it happened in the earlier phases of this research. It also indicates that the separation on the morphing region is not of significant importance and that the aft lower surface loading together with shockwave manipulation are the most dominant factors.

The conclusion from this analysis is that the smooth morphing is not required. However it must be noted that the Euler solvers are less sensitive to the computational grid and that a transonic wind-tunnel test of this phenomenon should be made to verify this. Supporting this conclusion are the experiments with X-29 demonstrator [11] where discrete camber changes were applied. Aft-located regions resembling very small tabs or Gurney flaps seem to be capable of the best performance improvements. In research done by Rosemann [42] the Gurney flaps demonstrate effects on transonic drag reduction as shown in this work. For further elaboration on the effects of morphing design a control derivative \( \frac{dc_l}{c_m} \) can be examined in figures 5.13 and 5.14. This derivative denotes the change in lift with morphing deflection at a fixed angle of attack associated with the mid-cruise condition.
These values are compared with empirical sealed plain flap control derivatives as function of the flap chord from the book of Roskam [48], which are corrected for the compressibility by a Prandtl-Glauert factor $\frac{1}{\sqrt{1-M^2_{eff}}}$. Not surprisingly, it can be seen that the effectiveness of the control by morphing has an opposite trend than the obtainable performance benefits, with moving the morphing region aft. An implication of this is that the most drag reducing configuration could not replace the aileron as a full outboard span device. Due to the finite trailing edge thickness and the morphing method adapted in this work at very aft $x_{d_m}$ positions the improvements eventually go to zero, but the drag prediction returns error values due to mesh errors on such extreme geometry, since the lower surface becomes too close to the upper surface approaching a flat plate. On this note the amount of points available in MSES made it unreliable to analyse such configurations.
CONCLUSIONS AND RECOMMENDATIONS

In this research the goal was set to compare the performance of a stock A320 aircraft with that featuring a variable camber morphing flap concept under development at the Fokker Aerostructures. This was implemented as a sectional coordinates transformation simulating a wide range of upper surface bending configurations defined in terms of few morphing design variables. To measure the possible performance improvements and compare the morphing designs, two performance indicators were chosen - the improvement over the design mission range and the equivalent fuel savings if the design mission range was kept constant. It was established that these are a function of the cruise drag and hence the evaluation and optimization of drag was the means to achieve the research goal. The evaluation was based on a quasi-3D approach comprising of AVL and MSES solvers for wing performance and a 2D performance evaluation in MSES solver. The obtained knowledge is summarized in section 6.1. The applicability of the morphing concept to the A320-type of aircraft is then discussed in section 6.2 and the final recommendations are given in section 6.3.

6.1. SUMMARY OF PARTIAL CONCLUSIONS

Several analyses were made towards measuring the potential of the concept and their results and conclusions are presented below. First the RAE2822 airfoil was examined on the effects of morphing. This preliminary morphing analysis has shown that:

- The potential for drag improvement is the largest at lift coefficients above the maximum lift-to-drag ratio point due to the parabolic shape of the polar diagrams.
- The mechanism behind the drag improvement is the manipulation of the shockwave strength and position, constrained by separation on the airfoil.
- Aft loading on the lower surface of the airfoil varies greatly with trailing edge morphing and therefore the airfoil upper surface can be efficiently relieved.
- If at low lift coefficients the mechanism of manipulating the drag is weak (for example a weak shockwave) the obtainable performance benefits are also small.

Since the obtainable performance improvements largely depend on the actual operating conditions of the airfoil, the reference cruise of the A320 had to be modelled. The included models were that of the wing, horizontal stabilizer, fuselage, center of gravity position and downwash which enabled their separate investigation put together in a trim procedure ensuring consistency of the whole aircraft at requested cruise points. With respect to this model it was found that:

- The moment of the wing is largely compensated by the lift force of the wing acting with an offset from the center of gravity
- As a result the trim requirements are small for the A320 and the drag of the horizontal stabilizer is already near a minimum of this lifting surface.

63
• The downwash from the main wing at the horizontal stabilizer tilts its negative lift vector forward and therefore any increased requirement on the downforce results in a decrease of the trim drag from the horizontal stabilizer, which largely compensates the wing trim drag increase.

• Within the short cruise flight of the A320 the changes in the aircraft angle of attack are small and as a result the fuselage drag variation is also negligible.

• Although the Q3D has problems with non-convergence of effective sections in MSES preventing reliable morphed polar drag prediction and re-trim, by neglecting the effect of increased wing pitching moment on the aircraft drag the separate wing morphing improvements can be translated to aircraft performance as an upper bound on the obtainable performance benefits.

Knowing the reference values of lift for both the wing and the representative effective section at 60% span the optimization runs could further be made for each cruise point both in 2D and 3D. The integrated effects of these were used to predict the morphing performance in terms of the chosen performance indicators. The final phase of research had the following results:

• The maximum drag improvement on an effective section is estimated in order of 8-9% depending on the morphing region and only at the beginning of cruise.

• Under parabolic function this drag improvement reduces towards the mid-cruise condition and remains negligible until the end of cruise.

• The drag improvements on the entire wing have similar behaviour as sectional improvements, however they are scaled below 2%, which can be traced to the observation that morphing does not affect the wing induced drag and the friction drag.

• With all other components of the aircraft assumed to have negligible drag variations these drag improvements are even smaller when compared to the overall drag value of the aircraft.

• As a result the integrated range improvement over the design mission is in the order of only 20km and the corresponding fuel improvement on the design mission in order of 0.4% of the trip fuel.

• In the analysis on multiple combinations of morphing region positions and the sizes it is concluded that the best design would be to have the morphing region at the very aft of individual sections and that the size of the morphing region (smoothness) has practically no effect on the obtainable improvements.

Although the optimization of wing drag by morphing in 3D and the associated analysis of the morphing setup in 2D were done, the direct aircraft morphing performance evaluation including re-trimming was not possible due to the convergence issues when approximating wing polar diagrams with morphing applied. However the performance benefits of morphing on such short mission are small and it can be claimed that including the trim effects would not bring dramatic changes to the main conclusion on the applicability of the morphing concept.

6.2. DISCUSSION ON APPLICABILITY OF THE MORPHING CONCEPT

In view of the results obtained in this research the morphing concept of smooth upper surface bending is deemed not applicable to the cruise of A320 or similar range aircraft. The reason is seen in the small variations of the lift requirements and other parameters in cruise, resulting in the aircraft being at all times close to its optimum design. To support this conclusion a comparison of the A320 is made in table 6.1 to the aircraft which are reported to use some form of variable camber in cruise, such as the Airbus A350, Boeing 787 and the recently tested NASA Gulfstream GIII with similar concept by the FlexSyS company.

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>$MCR(M_{MO})$</th>
<th>$m_{MTOW}[kg]$</th>
<th>$m_{P/L}[kg]$</th>
<th>$m_{IOE}[kg]$</th>
<th>$m_{F}[kg]$</th>
<th>$m_{F}/m_{MTOW}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airbus A320</td>
<td>0.78 (0.82)</td>
<td>73500</td>
<td>15000</td>
<td>40000</td>
<td>18500</td>
<td>25.2%</td>
</tr>
<tr>
<td>Airbus A350</td>
<td>0.85 (0.89)</td>
<td>268000</td>
<td>44000</td>
<td>124100</td>
<td>99900</td>
<td>37.2%</td>
</tr>
<tr>
<td>Boeing 787</td>
<td>0.85 (0.90)</td>
<td>227900</td>
<td>25000</td>
<td>117700</td>
<td>85230</td>
<td>37.3%</td>
</tr>
<tr>
<td>Gulfstream GIII</td>
<td>0.77 (0.85)</td>
<td>31600</td>
<td>2130</td>
<td>17826</td>
<td>11660</td>
<td>36.9%</td>
</tr>
</tbody>
</table>

Tab. 6.1: Comparison of A320 design mission parameters to other aircraft [23],[24]
It is seen that these aircraft have much larger ratios between the design mission fuel mass $m_F$ and the maximum take-off weight mass $m_{MTOW}$. This means much larger variations throughout the cruise and could result in larger performance benefits by morphing. Furthermore, most of these aircraft fly at higher Mach numbers which will increase the wave drag variation throughout the cruise and the possibility for a higher drag reduction on that matter as it was seen in the preliminary section analysis concerning morphing. Finally, based on this research it cannot yet be excluded that other flight regimes could better benefit from morphing, such as the high-lift condition, even for the A320 aircraft.

6.3. **Recommendations on further research**

It is admitted, that one of the features lacked in this work is the validation of obtained results. This was due to unavailability of data for the given aircraft and wind tunnel testing. It is proposed that with a transonic wind tunnel available, a demonstrator is built of several morphing setups with different morphing region positions and sizes, which should be compared in order to verify the observed effects.

Furthermore, since this research focused only on the A320, it is logically recommended to explore the applicability of the morphing concept with different aircraft. Due to the nature of the performance improvements it is suggested that aircraft required to fly at a high off-design lift coefficients in transonic cruise are good candidates. To achieve these artificially the morphing could be applied to improve altitude performance or to increase payload carrying capabilities of an aircraft, provided that structural constraints allow it. However, even within design cruise a high off-design lift coefficient can be required when the mission range is long, meaning that long range aircraft would be a good start for future research. Another possibility is with aircraft flying closer to the sound barrier, which can in some cases bring a more significant wave drag and therefore the potential to manipulate shockwave position and strength.

Finally, other flight regimes should be investigated. The originally planned high-lift condition had to be excluded from this thesis due to time constraints and convergence issues with the solver. Only an insight of the achieved results and discussion is given in Appendix G. However, further research should be done in this area, possibly also supported by wind-tunnel measurements.


## HISTORY OF MORPHING AIRCRAFT

<table>
<thead>
<tr>
<th>Year</th>
<th>1903</th>
<th>1931</th>
<th>1931</th>
<th>1932</th>
<th>1937</th>
<th>1947</th>
<th>1951</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wright Flyer</td>
<td>Pterodactyl IV</td>
<td>MAK-10</td>
<td>IS-1</td>
<td>LiG-7</td>
<td>MAK-123</td>
<td>X 5</td>
</tr>
<tr>
<td>Twist</td>
<td>Sweep</td>
<td>Span</td>
<td>Bi-to monoplane</td>
<td>Chord</td>
<td>Span</td>
<td>Sweep</td>
<td></td>
</tr>
<tr>
<td>XF10F</td>
<td>F 111</td>
<td>XB 70</td>
<td>Su 17 KG</td>
<td>MiG 23</td>
<td>Su 24</td>
<td>Tu 22 M</td>
<td></td>
</tr>
<tr>
<td>Sweep</td>
<td>Sweep</td>
<td>Span bending</td>
<td>Sweep</td>
<td>Sweep</td>
<td>Sweep</td>
<td>Sweep</td>
<td></td>
</tr>
<tr>
<td>F 14</td>
<td>FS 29</td>
<td>B 1</td>
<td>Tomado</td>
<td>AD 1</td>
<td>Tu 160</td>
<td>AFT/F 111</td>
<td></td>
</tr>
<tr>
<td>Sweep</td>
<td>Span</td>
<td>Sweep</td>
<td>Sweep</td>
<td>Stealing</td>
<td>Sweep</td>
<td>M.A.W.</td>
<td></td>
</tr>
<tr>
<td>FLYRT</td>
<td>MOTHA</td>
<td>AAL</td>
<td>F/A 18</td>
<td>Virginia Tech</td>
<td>Univ. of Florida</td>
<td>Univ. of Florida</td>
<td></td>
</tr>
<tr>
<td>Span</td>
<td>Camber</td>
<td>Pitch</td>
<td>A.A.W.</td>
<td>Span</td>
<td>Twist</td>
<td>Gulf</td>
<td></td>
</tr>
<tr>
<td>MFX 1</td>
<td>Univ. of Florida</td>
<td>Virginia Tech</td>
<td>Univ. of Florida</td>
<td>MFX 2</td>
<td>Dell Univ.</td>
<td>Virginia tech</td>
<td></td>
</tr>
<tr>
<td>Sweep &amp; Span</td>
<td>Sweep</td>
<td>Camber</td>
<td>Folding</td>
<td>Sweep &amp; span</td>
<td>Sweep</td>
<td>Camber</td>
<td></td>
</tr>
</tbody>
</table>
WING PLANFORM VISUALIZATION

Fig. B.1: A320 wing planform scheme [50]
Fuselage planform visualization

![A320 overall planform scheme](image)

**Fig. B.2:** A320 overall planform scheme [51]

### Planform data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing span, $b$</td>
<td>[m]</td>
<td>34.10</td>
</tr>
<tr>
<td>Wing reference area, $A$</td>
<td>[m$^2$]</td>
<td>122.4</td>
</tr>
<tr>
<td>Reference chord, $c_{ref}$</td>
<td>[m]</td>
<td>4.19</td>
</tr>
<tr>
<td>Aspect ratio, $\lambda$</td>
<td>[-]</td>
<td>9.4</td>
</tr>
<tr>
<td>Taper ratio, $\lambda$</td>
<td>[-]</td>
<td>0.246</td>
</tr>
<tr>
<td>$\frac{1}{4}$ chord sweep, $\varphi_{s\frac{1}{4}}$</td>
<td>[°]</td>
<td>25</td>
</tr>
<tr>
<td>Horizontal stabilizer width, $w_{HS}$</td>
<td>[m]</td>
<td>12.45</td>
</tr>
<tr>
<td>Fuselage length, $l_{f}$</td>
<td>[m]</td>
<td>37.57</td>
</tr>
<tr>
<td>Fuselage width, $w_{f}$</td>
<td>[m]</td>
<td>3.95</td>
</tr>
<tr>
<td>Fuselage height, $h_{f}$</td>
<td>[m]</td>
<td>11.76</td>
</tr>
<tr>
<td>Distance between engine axes</td>
<td>[m]</td>
<td>11.51</td>
</tr>
<tr>
<td>Overall engine length, $l_{enl}$</td>
<td>[m]</td>
<td>3.20</td>
</tr>
<tr>
<td>Engine width, $w_{enl}$</td>
<td>[m]</td>
<td>1.68</td>
</tr>
<tr>
<td>Vertical stabilizer height, $h_{VS}$</td>
<td>[m]</td>
<td>5.87</td>
</tr>
</tbody>
</table>

**Fig. B.3:** D-ATRA aircraft general data [52]
WING MODEL PLANFORM MATCH

Fig. B.4: Wing planform visualization over schematic (left) [58] and photography (right) [53]
## DATA USED IN A320 MODEL

<table>
<thead>
<tr>
<th>Data description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>dihedral ( \Lambda )</td>
<td>5.1°</td>
</tr>
<tr>
<td>wing span ( b )</td>
<td>34.1m</td>
</tr>
<tr>
<td>root chord ( c_r )</td>
<td>6.07m</td>
</tr>
<tr>
<td>kink chord ( c_k )</td>
<td>3.76m</td>
</tr>
<tr>
<td>tip chord ( c_t )</td>
<td>1.49m</td>
</tr>
<tr>
<td>tip LE x-position ( x_{LE} )</td>
<td>7.66m</td>
</tr>
<tr>
<td>fuselage radius ( y_f )</td>
<td>0.116/( \frac{b}{2} )</td>
</tr>
<tr>
<td>kink y-position ( y_k )</td>
<td>0.377/( \frac{b}{2} )</td>
</tr>
<tr>
<td>flap end y-position ( y_e )</td>
<td>0.784/( \frac{b}{2} )</td>
</tr>
<tr>
<td>kink flap chord ( c_{f_k} )</td>
<td>0.27/( c_k )</td>
</tr>
<tr>
<td>kink flap clean overlap ( o_{f_k} )</td>
<td>0.5/( c_{f_k} )</td>
</tr>
<tr>
<td>flap end flap clean overlap ( o_{f_e} )</td>
<td>0.7/( c_{f_e} )</td>
</tr>
<tr>
<td>defining airfoils CST coefficients</td>
<td>root CST [ ]</td>
</tr>
<tr>
<td></td>
<td>outboard CST [ ]</td>
</tr>
<tr>
<td></td>
<td>outboard CST [ ]</td>
</tr>
<tr>
<td>defining airfoils y-positions /( \frac{b}{2} )</td>
<td>0.116</td>
</tr>
<tr>
<td></td>
<td>0.377</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
</tr>
<tr>
<td>defining airfoil TE thickness /( c )</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td>0.0033</td>
</tr>
<tr>
<td>defining thickness distribution ( t_c )</td>
<td>from thickness.dat file</td>
</tr>
<tr>
<td>defining twist distribution ( t_c )</td>
<td>from twist.dat file</td>
</tr>
</tbody>
</table>

*Tab. B.1: Basic wing data list*

<table>
<thead>
<tr>
<th>Data description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>dihedral ( \Lambda )</td>
<td>5°</td>
</tr>
<tr>
<td>horizontal stabilizer span ( b )</td>
<td>12.45m</td>
</tr>
<tr>
<td>root chord ( c_r )</td>
<td>3.4m</td>
</tr>
<tr>
<td>tip chord ( c_t )</td>
<td>1.25m</td>
</tr>
<tr>
<td>tip LE x-position ( x_{LE} )</td>
<td>3.77m</td>
</tr>
<tr>
<td>fuselage radius ( y_f )</td>
<td>0.155/( \frac{b}{2} )</td>
</tr>
<tr>
<td>defining airfoils CST coefficients</td>
<td>tail CST [ ]</td>
</tr>
<tr>
<td></td>
<td>tail CST [ ]</td>
</tr>
<tr>
<td>defining airfoils y-positions /( \frac{b}{2} )</td>
<td>0.155</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
</tr>
<tr>
<td>defining airfoil TE thickness /( c )</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>0.002</td>
</tr>
<tr>
<td>defining thickness distribution ( t_c )</td>
<td>from tailthickness.dat file</td>
</tr>
<tr>
<td>defining twist distribution ( t_c )</td>
<td>from tailtwist.dat file</td>
</tr>
</tbody>
</table>

*Tab. B.2: Basic horizontal stabilizer data list*
HORIZONTAL STABILIZER PLANFORM PHOTOGRAPHS

Fig. B.5: A320 horizontal stabilizer planform photo comparison [53]

Fig. B.6: A320 horizontal stabilizer photo - view bottom

Fig. B.7: A320 horizontal stabilizer photo - top view
WEIGHT AND BALANCE DIAGRAMS

Fig. B.8: A320 fuel arm as function of fuel mass [54]

Fig. B.9: A320 balance computer [54]
### MSES SETTINGS, VALIDATIONS AND EXAMPLES

#### Tab. C.1: Example of grid settings in gridpar.* file

<table>
<thead>
<tr>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>251</td>
<td>number of grid points per airfoil side</td>
</tr>
<tr>
<td>0.8</td>
<td>number of points division between multiple elements (arbitrary)</td>
</tr>
<tr>
<td>31</td>
<td>number of grid points in front of airfoil</td>
</tr>
<tr>
<td>31</td>
<td>number of grid points behind the airfoil</td>
</tr>
<tr>
<td>21</td>
<td>number of streamlines above airfoil</td>
</tr>
<tr>
<td>21</td>
<td>number of streamlines below airfoil</td>
</tr>
<tr>
<td>7</td>
<td>number of streamlines between multiple elements (arbitrary)</td>
</tr>
<tr>
<td>1.3</td>
<td>not explained in manual (left default)</td>
</tr>
<tr>
<td>2.5</td>
<td>aspect ration of stagnation point cells (left default)</td>
</tr>
<tr>
<td>0.85</td>
<td>spacing parameter set for more orthogonal grid</td>
</tr>
<tr>
<td>0.5</td>
<td>initial inviscid grid angle of attack</td>
</tr>
<tr>
<td>0.6</td>
<td>leading edge refinement, trailing edge refinement, curvature exponent</td>
</tr>
<tr>
<td>1.00</td>
<td>regions of upper and lower surface grid refinement and refinement factors</td>
</tr>
</tbody>
</table>

#### Tab. C.2: Example of run settings in mses.* file

<table>
<thead>
<tr>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 4 5</td>
<td>global variables: far-field vortex strength, free-stream ( \alpha ), leading edge stagnation point</td>
</tr>
<tr>
<td>3 4 6</td>
<td>global constraints: leading and trailing edge Kutta conditions, drive solution to specified ( C_l )</td>
</tr>
<tr>
<td>0.74 0.71 0.5</td>
<td>Mach number, lift coefficient and free-stream ( \alpha ) (arbitrary)</td>
</tr>
<tr>
<td>3 2</td>
<td>use streamline momentum equation and vortex+source+doublet far-field boundary condition</td>
</tr>
<tr>
<td>1.5e+6 9</td>
<td>Reynolds number and amplification factor for ( e^n ) transition prediction method</td>
</tr>
<tr>
<td>0.05 0.05</td>
<td>upper and lower surface transition trip</td>
</tr>
<tr>
<td>0.97 1</td>
<td>critical Mach number threshold and weight driving artificial dissipation for numerical stability</td>
</tr>
</tbody>
</table>

---

79
### Tab. C.3: Example of domain settings in blade.* file

<table>
<thead>
<tr>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSES A320section</td>
<td>name of run case</td>
</tr>
<tr>
<td>-3.00 4.00 -4.50 5.00</td>
<td>left, right, bottom and upper domain size control</td>
</tr>
<tr>
<td>1.0000 0.0033</td>
<td>upper surface airfoil coordinates</td>
</tr>
<tr>
<td></td>
<td>lower surface airfoil coordinates</td>
</tr>
</tbody>
</table>

**Fig. C.1:** MSES analysis of RAE2822 $C_p$ distribution from official MSES website [55]

**Fig. C.2:** MSES analysis of RAE2822 $C_p$ distribution with applied settings
Fig. C.3: MSES example of separated flow and grid refinement

Fig. C.4: MSES example of separated flow and pressure distribution

MSES AF sweep
Mach = 0.724
Re = 15.127x10^6
A/Fa = -0.868
CL = 0.6337
CD = 0.01172
CM = -0.2045
L/D = 54.04
Ncrit = 9.00
Since the geometry of the Airbus A320 was not available for this work a different aircraft would have to be used for validation of the Q3D tool. On the other hand the authors of the quasi-3D tool Q3D have already conducted a validation for a Fokker F100 wing in an article by Mariens, Elham and Tooren [34], which this Appendix refers to. In this article the already validated F100 model results generated in a full-potential transonic flow solver MATRICS-V are compared with the calculations from the Q3D using the same combination of AVL+MSES solvers as used in this work. Figures D.1, D.2 and D.3 show the obtained shockwave location at Mach number $M = 0.8$, and the total drag comparison and the individual drag components comparison both at at Mach number $M = 0.77$ and Reynolds number $Re = 1.5 \times 10^7$. The average error on the total drag is claimed in the order of 1%. However it can be seen that the under-prediction of the total drag is steadily increasing at higher lift coefficients where it remains within 10%, which is satisfactory for the wing model in this work. Furthermore the figure D.1 shows that the largest discrepancies between the Q3D and MATRICS-V are found at the root of the wing.

![Fig. D.1: Q3D validation of Fokker F100 wing shockwave position [34]](image-url)
Fig. D.2: Q3D validation of Fokker F100 wing total drag coefficient [34]

Fig. D.3: Q3D validation of Fokker F100 wing drag coefficient components [34]
METHOD TO DETERMINE AERODYNAMIC CENTER SHIFT DUE TO FUSELAGE

In the method by Roskam [48] the empirical relation for the shift from the wing aerodynamic center to the aerodynamic center of the wing-fuselage combination is given by equation E.1, which is rewritten using substitution from equation E.2 in the same reference, to arrive at equation E.3.

\[ \Delta x_{AC} = -\frac{dM}{d\alpha} \frac{1}{q_\infty S_w c_{MAC} C_{Lw}} \]  

(E.1)

\[ dM = \frac{q_\infty C_{Lw}}{36.5} 0.08 \sum_{i=1}^{13} \left\{ w_i^2 \frac{d\bar{\epsilon}}{d\alpha} \Delta x_i \right\} \]  

(E.2)

\[ \Delta x_{AC} = -\frac{1}{2.92 S_w c_{MAC}} \sum_{i=1}^{13} \left\{ w_i^2 \frac{d\bar{\epsilon}}{d\alpha} \Delta x_i \right\} \]  

(E.3)

Since the upwash derivative \( \frac{d\bar{\epsilon}}{d\alpha} \) is a function of the position along the fuselage and approximation is made where the fuselage planform is divided into 13 blocks as shown in figure E.1 and a plot is given in figure E.2 where curve 1 is to be used with \( i = 1...4 \), curve 2 for \( i = 5 \) and the equation E.4 for \( i = 6...13 \).

Fig. E.1: Fuselage planform areas definition

![Fuselage planform areas definition](image)

Fig. E.2: Upwash gradient determination for front fuselage areas

![Upwash gradient determination for front fuselage areas](image)

\[ \left( \frac{d\bar{\epsilon}}{d\alpha} \right)_i = \frac{x_i}{x_h} \left( 1 - \frac{d\bar{\epsilon}}{d\alpha} \right) \]  

(E.4)

Based on a schematic figure B.2 from the A320 Airport and Maintenance Planning document [51] the fuselage planform was approximated by an mirrored enclosing curve in Solidworks and further divided into
5 sections with equal spacing in front of the wing root leading edge and 6 sections of equal spacing behind the wing root trailing edge. Two sections were created each of half root chord length for the centroplan as it is required by this method. Figure E.3 shows the Solidworks planform extraction and table E.4 shows the measured parameters.

![Fuselage planform sections extraction in Solidworks](image)

<table>
<thead>
<tr>
<th>Fuselage section</th>
<th>(w_f) [m]</th>
<th>(\frac{d\epsilon}{d\alpha}) [-]</th>
<th>(\Delta x_i) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0296</td>
<td>1.0702</td>
<td>2.580</td>
</tr>
<tr>
<td>2</td>
<td>3.6299</td>
<td>1.1040</td>
<td>2.580</td>
</tr>
<tr>
<td>3</td>
<td>3.9500</td>
<td>1.1777</td>
<td>2.580</td>
</tr>
<tr>
<td>4</td>
<td>3.9500</td>
<td>1.3022</td>
<td>2.580</td>
</tr>
<tr>
<td>5</td>
<td>3.9500</td>
<td>3.5308</td>
<td>2.580</td>
</tr>
<tr>
<td>6</td>
<td>3.9484</td>
<td>0.0818</td>
<td>3.035</td>
</tr>
<tr>
<td>7</td>
<td>3.9484</td>
<td>-0.0060</td>
<td>3.035</td>
</tr>
<tr>
<td>8</td>
<td>3.9469</td>
<td>-0.0050</td>
<td>3.100</td>
</tr>
<tr>
<td>9</td>
<td>3.9469</td>
<td>0.0818</td>
<td>3.100</td>
</tr>
<tr>
<td>10</td>
<td>3.9469</td>
<td>0.1921</td>
<td>3.100</td>
</tr>
<tr>
<td>11</td>
<td>3.4888</td>
<td>0.3124</td>
<td>3.100</td>
</tr>
<tr>
<td>12</td>
<td>2.3519</td>
<td>0.4388</td>
<td>3.100</td>
</tr>
<tr>
<td>13</td>
<td>1.3091</td>
<td>0.5695</td>
<td>3.100</td>
</tr>
</tbody>
</table>

![Fuselage planform sections data](image)

In figure E.3 the sections are numbered left to right and each type of section is shaded at least once as it was measured. Values of \(w_f\) and \(\Delta x_i\) can be simply repeated for the sections with the same dimensions in the table E.4. The blue points denote the guides of the mirrored curve delimiting the fuselage planform. For the sections 6..13 the formula for \(\frac{d\epsilon}{d\alpha}\) contain the unknown local downwash gradient \(\frac{d\epsilon}{d\alpha}\), for which the formula in equation 4.9 was used at the individual mid positions of the fuselage sections, as it is also advised in the described method. Using the data in the table and the equation E.3 the resulting shift of the aerodynamic center is evaluated as \(\Delta x_{AC} = 21.7\%\).
## Reference Aircraft Trim Results

### Cruise Point Data

| Cruise point | Mass [kg] | $C_L$ | $C_{Lw}$ | $\frac{\eta_h}{\delta_f} C_{Lh}$ | $C_{Lh}$ | $c_l$ | $c_{l_{eff}}$ |
|--------------|-----------|-------|---------|---------------------------------|---------|-------|...............|
| 1            | 71670     | 0.6167| 0.6204  | -0.0037                         | -0.0174| 0.6442| 0.7484       |
| 2            | 69344     | 0.5966| 0.6011  | -0.0045                         | -0.0211| 0.6221| 0.7228       |
| 3            | 67019     | 0.5766| 0.5814  | -0.0048                         | -0.0224| 0.5994| 0.6965       |
| 4            | 64693     | 0.5566| 0.5610  | -0.0044                         | -0.0204| 0.5760| 0.6693       |
| 5            | 62367     | 0.5366| 0.5400  | -0.0034                         | -0.0159| 0.5519| 0.6414       |
| 6            | 60042     | 0.5166| 0.5188  | -0.0022                         | -0.0102| 0.5275| 0.6132       |
| 7            | 57716     | 0.4966| 0.4973  | -0.0007                         | -0.0034| 0.5029| 0.5846       |

<table>
<thead>
<tr>
<th>Cruise point</th>
<th>Mass [kg]</th>
<th>$C_D$</th>
<th>$C_{Dw}$</th>
<th>$\frac{\eta_h}{\delta_f} C_{Dh}$</th>
<th>$C_{Dh}$</th>
<th>$C_{Df}$</th>
<th>$C_{DNL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71670</td>
<td>0.0385</td>
<td>0.0246</td>
<td>0.0013</td>
<td>0.0062</td>
<td>0.00644</td>
<td>0.0059</td>
</tr>
<tr>
<td>2</td>
<td>69344</td>
<td>0.0371</td>
<td>0.0233</td>
<td>0.0013</td>
<td>0.0061</td>
<td>0.00642</td>
<td>0.0059</td>
</tr>
<tr>
<td>3</td>
<td>67019</td>
<td>0.0359</td>
<td>0.0220</td>
<td>0.0013</td>
<td>0.0060</td>
<td>0.00641</td>
<td>0.0059</td>
</tr>
<tr>
<td>4</td>
<td>64693</td>
<td>0.0347</td>
<td>0.0209</td>
<td>0.0013</td>
<td>0.0062</td>
<td>0.00640</td>
<td>0.0059</td>
</tr>
<tr>
<td>5</td>
<td>62367</td>
<td>0.0337</td>
<td>0.0198</td>
<td>0.0014</td>
<td>0.0064</td>
<td>0.00639</td>
<td>0.0059</td>
</tr>
<tr>
<td>6</td>
<td>60042</td>
<td>0.0328</td>
<td>0.0188</td>
<td>0.0014</td>
<td>0.0067</td>
<td>0.00639</td>
<td>0.0059</td>
</tr>
<tr>
<td>7</td>
<td>57716</td>
<td>0.0319</td>
<td>0.0178</td>
<td>0.0015</td>
<td>0.0071</td>
<td>0.00638</td>
<td>0.0059</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cruise point</th>
<th>Mass [kg]</th>
<th>$C_{MCG}$</th>
<th>$C_{Mw}$</th>
<th>$\frac{\eta_h}{\delta_f} C_{Mh}$</th>
<th>$C_{Mh}$</th>
<th>$\alpha_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71670</td>
<td>2.4e-07</td>
<td>-0.1777</td>
<td>0.0014</td>
<td>0.0104</td>
<td>2.3430</td>
</tr>
<tr>
<td>2</td>
<td>69344</td>
<td>2.3e-07</td>
<td>-0.1729</td>
<td>0.0014</td>
<td>0.0104</td>
<td>2.1660</td>
</tr>
<tr>
<td>3</td>
<td>67019</td>
<td>2.2e-07</td>
<td>-0.1682</td>
<td>0.0014</td>
<td>0.0104</td>
<td>1.9846</td>
</tr>
<tr>
<td>4</td>
<td>64693</td>
<td>1.6e-07</td>
<td>-0.1635</td>
<td>0.0014</td>
<td>0.0104</td>
<td>1.7966</td>
</tr>
<tr>
<td>5</td>
<td>62367</td>
<td>9.9e-08</td>
<td>-0.1589</td>
<td>0.0014</td>
<td>0.0104</td>
<td>1.6039</td>
</tr>
<tr>
<td>6</td>
<td>60042</td>
<td>4.8e-08</td>
<td>-0.1543</td>
<td>0.0014</td>
<td>0.0104</td>
<td>1.4090</td>
</tr>
<tr>
<td>7</td>
<td>57716</td>
<td>2.1e-09</td>
<td>-0.1495</td>
<td>0.0014</td>
<td>0.0104</td>
<td>1.2119</td>
</tr>
</tbody>
</table>

* Sectional properties, streamwise lift coefficients $c_l$ and effective lift coefficient $c_{l_{eff}}$, are evaluated for station located at 60% of half-span.
Fig. F.1: Standard deviation on lift-to-drag ratio of aircraft divided by lift-to-drag ratio of effective section
HIGH LIFT INVESTIGATION ON MORPHING SINGLE SLOTTED FLAP

This final Appendix is intended to describe the research efforts done on the high-lift flight condition with use of the same flap morphing concept as in the cruise flight. As mentioned in the introduction chapters the concept by Fokker allows Fowler motion of the flap while at the same time morphing of the trailing edge. Only 2D case was considered and the goal remains to find the effect of morphing on the maximum lift coefficient.

From the point of geometry, the 2D section first undergoes the coordinate manipulation as seen in section 3.1, provided that the morphing is strictly limited to regions behind the spoiler tip line. At the same time the nose of the flap is modeled using a 3rd order B-spline with control points positioned such to approximate the A320 flap surface as provided by the Fokker Aerostructures. The B-spline is shown in figure G.1, in comparison with the reference flap geometry. The 7 control points of the B-spline include the breaking points which delimit the flap nose geometry from the original airfoil. They are associated with a knot vector \([0,0,0,0.2,0.4,0.6,0.8,1,1,1]\), which dictates uniform point distribution over the spline. The multiplicity of zeros and ones ensures that the beginning and end point of the B-spline coincide for the given order with the airfoil surface and at the same time the three co-linear control points at the leading edge force it to be tangent to their common x-position, which is given using the outboard flap chord on the A320 wing as 0.73/c [56].

The airfoil is split into individual elements and the corner point in the lower surface of the main wing element is adjusted to avoid acute angles in the geometry, which spoil the MSES initial grid creation. The exact cove shape is not important since MSES does not solve for the separated region which is known to occur in the main wing cove and this is enforced by a double coordinate corner point in the geometry definition according to the MSES manual [29]. Then the Fowler motion and flap deflection transformation is performed followed by iterative z-positioning such that the requested gap is matched. These transformations are driven by similar matrices as seen in equations 4.4 or 3.6. Thus the flap position with respect to the main wing is fully defined and can be directly specified by the flap deflection \(\delta_f\), gap \(G_f\) and overlap \(O_f\), as it is usual in high-lift aerodynamics. A similar procedure can be done to obtain a slat geometry and apply deflection \(\delta_s\), gap \(G_s\) and overlap \(O_s\) to it. Such split airfoil including morphing on the flap trailing edge is shown in figure G.2.

Although the geometry and grid were carefully examined for any problematic areas to the aerodynamic analysis, MSES solver convergence on such case was practically none. Two element airfoils seemed to produce less problems, although the convergence there was also a matter of luck on the grid and run settings.
However, with a slightly different morphing deflection a previously converged case could be without convergence, rendering the MSES solver useless for a successful optimization. Ideally such optimization would have all the above mentioned deflection parameters and the morphing deflection and setup as the input variables and the maximum lift coefficient as the objective.

The solver had poor convergence when asked separately for a range of angles of attack in order to approximate the lift curve from which the maximum lift coefficient could be determined. Another option in this solver is to make use of the routine MPOLAR on a range of angles of attack, which according to the MSES manual should be spaced by 0.1°. This method provided by the MSES package attempts one by one the \( \alpha \) values given in a special file and uses the previously converged states as the next initial solution. When unsuccessful it starts dividing the interval between the asked and previously converged \( \alpha \). This way there is a chance of proceeding closer to the next \( \alpha \) in the list for lift curve approximation. Prior to MPOLAR run, a few low \( \alpha \) separate MSES run cases are attempted to ensure that the routine has an initial solution. Several lift curves are shown in figure G.3 for the two element configuration shown in figure G.4, demonstrating the effect of Reynolds number choice on the obtained results, where the blue line is created by the separate MSES runs and the red curve is then generated by MPOLAR.

![Fig. G.3: Two element configuration lift curves for various Reynolds numbers, at \( M = 0.13 \)](image)

![Fig. G.4: RAE2822 airfoil split in multiple elements, \( \delta_f = 35^\circ, \theta_f = -0.001/c, G_f = 0.015/c \)](image)
Although the maximum on the lift curves occurs at similar values, it is often hard to determine if and when the maximum lift coefficient was really reached. The Reynolds number of an outboard 60% span airfoil during landing is approximately \( Re = 9.5 \cdot 10^6 \), which is in the region where convergence was even more difficult. The conclusion is that the multi-element high-lift analysis in MSES including morphing would be near impossible, since the expected improvements on the maximum lift coefficient and the uncertainty of this coefficient due to non-convergence are too comparable. At least in the low Reynolds number region, a maximum on the lift curve is usually reached and there the flow can be examined looking at series of pressure distributions for increasing angle of attacks in figure G.5 and varying flap deflections in figure G.6.

Fig. G.5: Pressure distributions on two element airfoil for several angles of attack, 
\( \delta_f = 35^\circ, O_f = -0.001/c, G_f = 0.015/c \)

Fig. G.6: Pressure distributions on two element airfoil at deflections \( \delta_f = 30^\circ \) (left) and \( \delta_f = 25^\circ \) (right), 
\( \alpha = 5^\circ, O_f = -0.001/c, G_f = 0.015/c \)
It was verified with numerous other runs that MSES predicts wake bursting to be the reason for reaching a maximum lift coefficient. Wake bursting is a result of a fast pressure recovery in the wake of the main element where the flow slows down rapidly [57]. It has a de-cambering effect on the overall flow around an airfoil, reducing its lift. No separation is directly noticeable by looking at the pressure distributions or surface velocities, but it can be observed from a sudden increase of the wake size and near-zero velocities in the flow field (off-surface separation). Some cases demonstrated that the wake bursting is followed by a leading edge separation as it can be expected for a thin airfoil with small nose radius such as RAE2822. On the contrary, separation on the flap is only a matter of low angles of attack of the entire configuration (measured with respect to the main wing element) and at the same time large flap deflections, as shown in figure G.6. There the flap deflection was reduced twice and compared at an approximately same overall $ \alpha \approx 5^\circ$ to the figure G.5. The separation on the flap is seen reduced with lower deflection.

An explanation of this is given in the extensive high-lift article by Smith [58] where it is denoted as the "slat effect". The upstream element influences the downstream element by the circulation associated with its lift force, which can be modelled as an additional vortex positioned at the downstream element's leading edge, shown in figure G.7. As such vortex induces velocities which vary with inverse of the distance from the vortex singularity point, the strongest downwash felt by the downstream element is at its leading edge. This reduces the local effective angle of attack and therefore also the velocity peak that would otherwise be reached here. Lower velocity at the leading edge means a more gradual pressure recovery, which helps to reduce separation on the downstream element.

As a consequence, the flap is aerodynamically loaded the most when the main wing element produces a low lift coefficient and therefore the separation on the flap is more likely at lower angles of attack or higher airspeeds. When obtaining the highest possible maximum lift coefficient by increasing the flap deflection a situation occurs where an increase in $\alpha$ results in either separation on the main wing or wake bursting, while reducing the angle of attack leads to flap separation. It is therefore necessary to know what limits are imposed on such condition, i.e. if the increase in drag or unsteady effects are allowable or whether there exists an operating range (between flap placard speed and stall speed) where the configuration has to perform without these effects. In such cases reducing flap separation can indirectly dictate the maximum lift coefficient and a reduction of flap deflection for the same lift contribution with use of morphing can have the required benefit.

The recommendation for the future research is to redefine what exactly is to be accomplished in the high-lift condition by morphing. Furthermore the use of a different solver is advised or wind tunnel testing, since the number of high lift design variables is relatively large for an optimization with high order computational methods. These would then still have an uncertain result, since the turbulence models tuned for base geometry might not hold for morphed geometry.