Flux-normalized Elastodynamic Wavefield Decomposition using only Particle Velocity Recordings
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SUMMARY

We present a new approach to apply wavefield decomposition, illustrated for an energy flux-normalized elastodynamic case. We start by considering a situation where two horizontal boreholes are closely separated from each other. By recording only the particle velocities at both depth levels (for example with conventional 3-component geophones) and expressing the one-way wavefields at one depth level in terms of the fields at the other depth level, an inverse problem can be formulated and solved. This new approach of multi-depth level (MDL) wavefield decomposition is illustrated with a synthetic 2D finite-difference example, showing correct one-way wavefield retrieval. We then modify the methodology for a special case with a single receiver array just below a free surface, where the problem is naturally constrained by the (Dirichlet) boundary condition at the free-surface. Again, it is shown that correct elastodynamic wavefield decomposition takes place, for both P- and S-waves.

INTRODUCTION

Separation of recorded wavefields into downgoing and upgoing constituents is a technique that is used in many geophysical methods. Decomposed wavefields form the basis for various surface-related multiple elimination and deghosting procedures (e.g. Frijlink et al. (2011), Majdanski et al. (2011)) and for depth imaging using primary and multiple reflections (e.g. Muijs et al. (2007)). Novel methodologies that make use of downhole sensors, such as the virtual source method (Burnstad et al., 2012) and multidimensional deconvolution (Wapenaar et al., 2011) rely heavily on decomposing the seismic wavefield at depth.

The principle of decomposition can be applied to all physical wave phenomena. Here, we focus on the elastodynamic wavefields. Theory tells us that for decomposing elastodynamic wavefields into upgoing and downgoing compressional waves (P-waves) and shear waves (S-waves), it is required to have registered both 3-component particle velocity fields and 3-component tractions at a certain receiver level (Wapenaar et al., 1990). Depending on the setting, certain components can vanish. For land acquisition (with receivers placed at the Earth’s surface), it is well known that the traction tensor is zero due to the Dirichlet boundary condition, such that decomposition can be carried out with 3-component geophones only (Dankbaar, 1985). At the seafloor (for example in marine Ocean-Bottom-Cable acquisition), only the shear tractions vanish, such that 4-component sensors are required (Schalkwijk et al., 2003). When considering a land acquisition setting with receivers placed in a horizontal borehole, all traction and particle velocity components are non-zero. For this case, to carry out a successful elastodynamic wavefield decompositi-
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\[
\begin{pmatrix}
\hat{p}^+ \\
\hat{p}^-
\end{pmatrix} =
\begin{pmatrix}
L^+_d \\
L^-_d
\end{pmatrix}^{-1}
\begin{pmatrix}
-\tau_z \\
\eta
\end{pmatrix}.
\tag{2}
\]

As can be observed in (2), in order to be able to perform the up/down decomposition correctly for the elastodynamic system, all 3 components of both the stress vector as well as the particle velocity vector must be recorded. As already mentioned before, in practice, not all of these field quantities are available. We come up with a new technique for carrying out up/down decomposition by only measuring the particle velocity at two depth levels, closely separated from each other. Let us first illustrate the principle in terms of the governing matrix-vector equations. When we consider two depth levels \(z_d\) and \(z_B\), where \(z_d < z_B\) we can write the decomposed downgoing and upgoing energy flux-normalized wavefields at one depth level in terms of the other, respectively:

\[
\begin{align*}
\hat{p}^+_d &= \hat{W}^+(z_B,z_d)\hat{p}^+_B, \\
\hat{p}^-_d &= \hat{F}^-\left(z_B,z_d\right)\hat{p}^-_B,
\end{align*}
\tag{3}
\]

where the inverse wavefield extrapolation operator \(\hat{F}^-\left(z_B,z_d\right)\) in (4) is closely related to the forward propagator \(\hat{W}^+(z_B,z_d)\) (Wapenaar, 1998) as:

\[
\hat{F}^-\left(z_B,z_d\right) \approx (\hat{W}^+(z_B,z_d))^*.
\tag{5}
\]

Here, the asterisk (*) denotes complex conjugation. The approximation sign is applied because this equation is not valid for the evanescent wavefield. The forward wavefield extrapolation operator \(\hat{W}^+(z_B,z_d)\), extrapolates the downgoing (+) wavefield downwards, from depth level \(z_d\) to depth level \(z_B\). The inverse wavefield extrapolation operator \(\hat{F}^-\left(z_B,z_d\right)\), extrapolates the upgoing wavefield (-) downwards from depth level \(z_d\) to depth level \(z_B\). As mentioned, we imagine a field situation where we have obtained only particle velocity recordings, at different depth levels. According to equation (1) we can express the two-way particle velocity fields recorded at depth level \(z_d\) in terms of the one-way up and downgoing fields as

\[
\begin{pmatrix}
L^+_2A \\
L^-_2A
\end{pmatrix}
\begin{pmatrix}
\hat{p}^+_2A \\
\hat{p}^-_2A
\end{pmatrix} = \eta_A
\tag{6}
\]

and for the recordings at depth level \(z_B\)

\[
\begin{pmatrix}
L^+_2B \\
L^-_2B
\end{pmatrix}
\begin{pmatrix}
\hat{p}^+_2B \\
\hat{p}^-_2B
\end{pmatrix} = \eta_B.
\tag{7}
\]

If more recordings at more than two depth levels are available, this procedure can be extended for all possible depth levels. Using (3) and (4), we can express the one-way wavefields for depth level \(z_B\) also in terms of the one-way wavefields for \(z_d\),

\[
\begin{pmatrix}
L^+_2B \hat{W}^+ \\
L^-_2B \hat{F}^-
\end{pmatrix}
\begin{pmatrix}
\hat{p}^+_2B \\
\hat{p}^-_2B
\end{pmatrix} = \eta_B.
\tag{8}
\]

Combining equations (6) and (8) in terms of the one-way wavefields at depth level \(z_d\), we obtain

\[
\begin{pmatrix}
L^+_2A \hat{W}^+ \\
L^-_2A \hat{F}^-
\end{pmatrix}
\begin{pmatrix}
\hat{p}^+_2A \\
\hat{p}^-_2A
\end{pmatrix} = \eta_A.
\tag{9}
\]

We assume that the medium between depth levels \(z_d\) and \(z_B\) is homogeneous, and that the medium properties around the well are known, and therefore the wavefield extrapolators are known as well. In other words, the medium properties at \(z_d\) are equal to the medium properties at \(z_B\). This is a reasonable assumption, taking into account that the two depth levels are separated over only a small distance. Therefore, we have omitted the subscripts \(A,B\) in the composition matrix. Alternatively, one might be interested to estimate the wavefield extrapolation operators directly from the data. One way of doing this is via direct-field interferometry. For a discussion on interferometric propagator estimation, the reader is referred to Van der Neut et al. (2013). We have now obtained an expression relating the one-way wavefields at depth level \(z_d\) via the composition matrix to the recorded particle velocity components at both depth levels \(z_d\) and \(z_B\). By multiplying both the left- and right-hand sides of (9) with the inverse of the composition matrix, the one-way up- and downgoing P- and S-wavefields at depth level \(z_d\) can be obtained

\[
\begin{pmatrix}
\hat{v}^+_d \\
\hat{v}^-_d
\end{pmatrix} =
\begin{pmatrix}
L^+_2A \hat{W}^+ \\
L^-_2A \hat{F}^-
\end{pmatrix}^{-1}
\begin{pmatrix}
\hat{v}^+_2A \\
\hat{v}^-_2A
\end{pmatrix}.
\tag{10}
\]

In other words, the elastodynamic wavefield system has now been decomposed (for depth level \(z_d\)), by using only particle velocity field recordings at two depth levels. Equation 10 forms the basis of our new approach to wavefield decomposition.

**EXAMPLE**

To illustrate the multi-depth level particle velocity (MDL) decomposition approach, we will apply this method to a synthetic elastodynamic example. We will make use of a 2D elastodynamic finite-difference model (Virieux, 1986), where receivers are being placed at two depth levels \(z_d = 1000\ m\) and \(z_B = 1010\ m\) (see figure 1a) and where it is assumed that the field quantities and medium parameters are independent of the \(y\)-direction. The P-wave and S-wave velocities for the layer in which the receivers are located are 2500 \(m/s\) and 1800 \(m/s\), respectively. The density of the layer is 1500 \(kg/m^3\). The source is a vertical dipole force source with a peak frequency of 20 \(Hz\). For applying the decomposition techniques, use will be made of the 2D versions of the composition matrix \(\hat{L}\) as presented in Wapenaar et al. (2008). Figure 1c represents the original shot records registered at depth level \(z_d\), with the two-way physical wavefield quantities \(\tau_{Ez}, \tau_{EZ}, v_s\) and \(v_E\). Figure 1d shows the decomposition results for \(z_d\) after applying the conventional decomposition, using both recorded traction and particle velocity fields at depth level \(z_d\). Figure 1e shows the decomposition results for \(z_d\) after applying our new, multi-depth level particle velocity recording approach (MDL), using only particle velocity recordings at \(z_d\) and \(z_B\). One can clearly observe the almost identically retrieved one-way wavefields in figures 1d and 1e. In other words, our new MDL approach manages to retrieve the correct one-way wavefields at \(z_d\), both in amplitude and phase, using only particle velocities at \(z_d\) and \(z_B\). To illustrate this further, figure 1b shows a wiggle-trace comparison between the conventionally retrieved one-way wavefields in black and the MDL retrieved fields in red (dashed).
where $z$ level

We can modify the procedure for a special case, moving depth

ARRAY BELOW A FREE-SURFACE

SPECIAL CONFIGURATION WITH A SINGLE SENSOR ARRAY BELOW A FREE-SURFACE

We can modify the procedure for a special case, moving depth level $z_A$ upwards such that it coincides with the Earth’s free-surface. Similar to the basic case described above, we express the up- and downgoing wavefields at $z_A$ in terms of the up- and downgoing wavefields at $z_B$

\[ \tilde{p}_A^+ = \tilde{F}^+(z_A,z_B)\tilde{p}_B^+ \]
\[ \tilde{p}_A^- = \tilde{W}^-(z_A,z_B)\tilde{p}_B^- . \]

where

\[ \tilde{F}^+(z_A,z_B) \approx (\tilde{W}^+(z_A,z_B))^* . \]

We now make use of the fact that all tractions are zero at the free-surface due to the Dirichlet boundary condition. Hence, we do not explicitly need physical receivers at depth level $z_A$. We combine this constraint with the physical recordings of the particle velocity at depth level $z_B$, as

\[ \left( \begin{array}{c} \tilde{L}^+_1 \tilde{F}^+ \tilde{L}^-_2 \\ \tilde{L}^-_2 \tilde{L}^+_2 \end{array} \right) \left( \begin{array}{c} \tilde{p}_B^+ \\ \tilde{p}_B^- \end{array} \right) = \left( \begin{array}{c} -\tilde{e}_{z_A} \\ \tilde{v}_B \end{array} \right) . \]

where $-\tilde{e}_{z_A} = 0$. By multiplying both the left- and right-hand sides of (14) with the inverse of the composition matrix, the one-way up- and downgoing P- and S-wavefields at depth level $z_B$ can be obtained

\[ \left( \begin{array}{c} \tilde{p}_B^+ \\ \tilde{p}_B^- \end{array} \right) = \left( \begin{array}{c} \tilde{L}^+_1 \tilde{L}^-_2 \\ \tilde{L}^-_2 \tilde{L}^+_2 \end{array} \right)^{-1} \left( \begin{array}{c} \tilde{e}_{z_A} \\ \tilde{v}_B \end{array} \right) . \]

In other words, the elastodynamic wavefield system has now been decomposed (for depth level $z_B$), by using only particle velocity recordings at $z_B$, combined with the fact that the tractions at $z_A$ are zero.
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**EXAMPLE**

Another synthetic example will illustrate the special case scenario, having depth level $z_d$ coincide with the free-surface. Again, a 2D finite-difference elastodynamic model will be used, with receivers placed only at $z_B$, at 50 meters depth. For clarity, we now consider a homogeneous medium in order to be able to recognize clearly all expected decomposed wavefields. The P-wave velocity of the medium is 2000 m/s, the S-wave velocity 1400 m/s and the density reads 1000 kg/m$^3$. A 45 degrees (anti-clockwise) oriented dipole force source with a peak frequency of 20 Hz, buried at 2000 m depth, is considered as the (passive) source (figure 3). The only upgoing fields to be expected, are one upgoing P-wave and one upgoing SV-wave. At the free-surface, P-SV wavefield conversion can occur (Aki and Richards, 1980). Therefore, we expect two downgoing P-wave events (P-P and SV-P) and two downgoing SV-wave events (P-SV and SV-SV). In figure 2a the originally recorded two-way wavefields are presented. Due to the 45 degrees anti-clockwise diagonally oriented force source, the registered wavefields show up as asymmetric hyperbolas. The results of the decomposition are shown in figure 2b. Here, a comparison is displayed between the one-way wavefields obtained via conventional decomposition at depth level $z_B$ in black, and the results obtained by using only particle velocity recordings at $z_B$ combined with the zero free-surface traction constraint (MDL approach) in red (dashed). One can clearly observe that the MDL approach, using now only particle velocity data at one depth level, again retrieves the correct one-way wavefields. The decomposition result gives us indeed only the expected one-way wavefields, i.e. one upgoing P-wave and one upgoing SV-wave, two downgoing P-wave events and two downgoing SV-wave events. The downgoing field can be interpreted as the elastodynamic free-surface ghost of the upgoing field. The proposed algorithm can therefore be used for elastodynamic deghosting. This can be very useful for passive data processing, for instance for passive seismic interferometry (Draganov et al. (2006)).

**DISCUSSION AND CONCLUSIONS**

A comparison between the conventional elastodynamic wavefield decomposition approach and our new multi-depth level (MDL) particle velocity recording wavefield decomposition approach has shown that our new MDL approach leads to correctly retrieved energy flux-normalized one-way wavefields, for both P- and S-waves, using only particle velocity field recordings at two depth levels. For the special case where one of those depth levels coincides with the free-surface, it has been demonstrated that particle velocity recordings at only one depth level, combined with the physical constraint that the tractions are zero at the free-surface, are sufficient to obtain the correct one-way wavefields at that depth level. However, solving the decomposition problem is not always this straightforward. Successful decomposition is dependent on the vertical distance between the arrays and the frequency (Van der Neut et al., 2013). The distance was small enough for the first example, and large enough for the second, special case example. However, for certain distances, notches will occur at certain frequencies overlapping with the data bandwidth. In these cases, additional notch filters are required in order to be able to invert the composition matrices correctly.

**Figure 2:** (a) Original shot records registered at depth level $z_B = 50$ m, with the two-way physical wavefield quantities $\tau_{xz}$, $\tau_{zz}$, $v_x$ and $v_z$. (b) Comparison between the conventional decomposition results in black and the MDL decomposition results in red.

**Figure 3:** Geometry for the 2D, special configuration experiment.

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REFERENCES
Almagro Vidal, C., J. van der Neut, D. Draganov, G. Drijkoningen, and K. Wapenaar, 2011, Retrieval of reflections from ambient-noise field data using illumination diagnosticians: Presented at the 81st Annual International Meeting, SEG.
Berron, C., E. Forgues, M. Jervis, A. Bakulin, and R. Burnst, 2012, Buried sources and receivers in a karsted desert environment: Presented at the 74th Annual International Conference and Exhibition, EAGE.
Burnstad, R., A. Bakulin, M. Jervis, and D. Alexandrov, 2012, Successful imaging of land hydrophone and dual sensor data in a dry desert environment: Presented at the 74th Annual International Conference and Exhibition, EAGE.
Cotton, J., and E. Forgues, 2012, Dual-depth hydrophones for ghost reduction in 4D land monitoring: Presented at the 82nd Annual International Meeting SEG.
Van der Neut, J., A. Bakulin, and D. Alexandrov, 2013, Acoustic wavefield separation using horizontal receiver arrays deployed at multiple depth on land: Presented at the 83rd Annual International Meeting, SEG.


