Abstract

In this paper, stepping over a zero height obstacle with minimal actuation is studied for a limit cycle walker modeled as a double inverted pendulum. The obstacle position is estimated by stereo vision. Actuation is realized by a constant torque per step on the hip and a push-off collinear to the trailing leg. Stepping over the obstacle must be accomplished with the obstacle position exactly on a predefined position in between the legs with the final state right after push-off being equal to the initial state. Thus, at least two steps must be taken to perform this task, such that the first step is used to make sure the relative position of the obstacle is correct.

In the best case scenario, the obstacle is exactly in between the legs during a nominal walk. In that case, actuation does not have to be adjusted with respect to the nominal actuation. In the worst case scenario the obstacle is exactly at a stepping position. In that case, a translation of the step positions is needed. For stepping over in two steps this is not possible; this is only possible for a small range around the best case scenario. For stepping over in three steps this is possible when actuation is applied according to the optimization results presented.

1 Introduction

Over the last few years several limit cycle walkers have been developed with a very human-like way of walking [1]. A limit cycle walker is a walking robot based on the limit cycle walking paradigm which is defined as [1]:

\textit{Limit Cycle Walking is a nominally periodic sequence of steps that is stable as whole but not locally stable at every instant in time.}

Exactly controlling the motion of the individual body parts is not necessary as long as it moves in a region of state space for which it converges to its desired nominal path; its limit cycle. Therefore, limit cycle walkers are characterized by their low power consumption and human-like way of walking [1].

Many researchers are convinced that the limit cycle walking paradigm, which exploits the natural dynamics of walking, is very promising for the control of humanoid robots, in terms of speed, disturbance rejection, efficiency and versatility [1]. TUlip, developed at the Delft Biorobotics Laboratory\(^1\), is such a limit cycle walker; see figure 1. In order to enhance TUlip’s autonomy it has recently been equipped with a stereo camera system. This enables TUlip to detect obstacles in its walking trail so that it can step over them. The focus of this research is the proper adjustment of step lengths for a limit cycle walker in order to step over an obstacle in its walking trail. Similar research has been conducted on vision enhanced humanoid robots that use a Zero Moment Point (ZMP) based algorithm [2] for generating a stable walking trajectory [3] [4] [5].

The research question is;

\textit{What is the optimal way for a 2D limit cycle walker to step over an obstacle with minimal torque?}

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2.1 Model

Theories on walking are often initially explored by studying simple models like the model used here; the double inverted pendulum \[1\] \[6\]. This model relies on the limit cycle walking principle and thus can be called a limit cycle walker. It therefore is a very suitable model for this study.

A schematic of the model is shown in figure 2. Two links are connected at their ends with a joint representing the hip. The other ends are the contact points with the ground. Sideways motion is not taken into account by constraining the pendulum to the sagittal plane. The step length is controlled by a torque \( T \) in the hip and a push-off impulse on the trailing leg adding an amount of energy \( E_{po} \). When walking on a flat floor, \( T \), \( E_{po} \) or both should be non-zero in order to realize a limit cycle. If both are zero, the energy loss due to impact is not compensated for and eventually it will fall over. The push-off impulse is modeled as instantaneous and is discussed in 2.1.1. The impact with the ground is modeled as fully inelastic. Collision between the feet and obstacles not considered. Therefore, the only obstacles that are taken into account are holes in the ground. Stepping on these holes is not allowed. Due to the fact that foot scuffing is inevitable for this model, several criteria have to be met in order to qualify crossing the floor plane as an impact. These criteria are treated

In order to step over an obstacle, some steps between obstacle detection and reaching the final position in front of the object, have to be adjusted (with respect to the nominal steps) to make sure the robot doesn’t hit the obstacle. How to adjust the steps is done by optimization of a double inverted pendulum in simulation. In this optimization, the maximal actuation inputs are minimized.

In this part the optimization and simulation on an inverted double pendulum that is used to study walking behavior will be discussed; in particular, the adjustment of step lengths. In 2.1 the model will be described. The nominal limit cycle is discussed in 2.2. The optimization problem is discussed in 2.3. The results are discussed in 2.4 and in 2.5 the results are explained by theory using a simple model.

2 Step Behavior

In order to step over an obstacle, the stepping positions have to be adjusted such that one foot reaches a desired position in front of the obstacle and the other foot a desired position behind the obstacle while the robot remains dynamically stable. These adjustments can be accomplished by properly applying torques on the hip joint and addition of energy by means of an impulse on the trailing leg. The more distant the obstacle, the easier it should be to adjust the steps properly in order to step over it, as the average step adjustment decreases due to an increasing number of steps. Thus, in case an obstacle is nearby, higher actuation is needed in order to step over it.

Part 2 deals with the optimal stepping positions when walking towards an obstacle in order to step over it. In order to make sure the limit cycle walker does not step on the obstacle a safety range should be taken into account. This is based on the accuracy of the camera system and the accuracy of foot placement. These are treated in part 3. The conclusions are presented in 4.

The TUlip limit cycle walker from the Delft Biorobotics Laboratory is the motivation for this study and simulation parameters are based on this robot. For example, for the depth estimation of an object, only the implementation on TUlip will be studied.

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STEP BEHAVIOR 2.1 Model

Figure 2: Schematic of a double pendulum. The angles and angular velocities are positive in counterclockwise direction with the zero leg angles along the vertical. The step length is controlled by the hip torque \( T \) and a push-off impulse on, and collinear with, the rear leg adding an amount of energy \( E_{po} \). In the sub-picture the direction of the torque in shown on the individual legs. \( c \) is the distance of the CoM with respect to the hip. The double pendulum is constrained to the sagittal plane, disabling sideways motion. Only forward walking is allowed (x-direction).

The parameters of the model are given in table 1. For clarification of some of the parameters, see figure 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity</td>
<td>( g )</td>
<td>9.81</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>Leg Length</td>
<td>( l )</td>
<td>0.55</td>
<td>m</td>
</tr>
<tr>
<td>Leg Mass</td>
<td>( m )</td>
<td>3</td>
<td>kg</td>
</tr>
<tr>
<td>Leg Inertia</td>
<td>( I )</td>
<td>0.075</td>
<td>kgm(^2)</td>
</tr>
<tr>
<td>CoM Offset</td>
<td>( c )</td>
<td>0.1</td>
<td>m</td>
</tr>
</tbody>
</table>

2.1.1 Mathematics of Model

The horizontal and vertical coordinate of the hip are \( x_h \) and \( y_h \), respectively. Together with the leg angles they are the independent generalized coordinates of the system:

\[
q = \begin{bmatrix} x_h & y_h & \phi_1 & \phi_2 \end{bmatrix}^T
\]

The state vector is

\[
S = [q^T \ \dot{q}^T]^T
\]

At \( t = 0 \), the x-position of the front leg is zero with both legs on the ground as in figure 2. The accelerations of the system are determined by

\[
\ddot{M} \ddot{q} = \ddot{f}
\]

with \( \ddot{M} \) and \( \ddot{f} \) the generalized mass matrix and the generalized force vector, respectively. They are determined by

\[
\ddot{M} = T^T MT
\]

and

\[
\ddot{f} = T^T [f_g - Mh]
\]

\( M \) and \( f_g \) are the mass and gravity force terms from the unconstrained Newton-Euler equations (and is thus related to 6 coordinates). The matrix \( T \) transfers the independent generalized coordinates \( q \) in the positions and angles of the center of mass (CoM) \( x \). The vector \( h \) represents the convective accelerations.

In order to make sure there is a static contact point between the leg and ground, constraints have to be added in horizontal and vertical direction. For each leg, these are defined by

\[
g_y = y_h - l \cos \phi
\]

and

\[
g_x = x_h - l \sin \phi - x_f
\]

\( x_f \) is the x-coordinate of the impact position of the foot. If the second derivatives of equations 6 and 7 are expressed in terms of \( D \) and \( D_2 \), they can be combined with equation 3 resulting in the total system of equation that solves the generalized accelerations and foot contact forces \( F_c \):

\[
\begin{bmatrix} \dddot{M} & D^T \\ D & 0 \end{bmatrix} \begin{bmatrix} \dddot{q} \\ F_c \end{bmatrix} = \begin{bmatrix} \dddot{f} \\ D_2 \dddot{q}^2 \end{bmatrix}
\]

The torques can be added to the elements related to the angles in \( \dddot{f} \) in equation 8. The system of equation 8 has to be evaluated and solved for the relevant stance leg. The integration is done using a Runge-Kutta 4 integration. Every integration step the hip
coordinates and velocities are recalculated using the angles and angular velocities in order to negate the accumulation of numerical errors in the constraint equations.

The criteria for a heel strike are:

1. Sign of swing foot height has changed with respect to previous integration step
2. Stance lag has passed vertical position
3. Swing foot height is negative
4. Leg angles have opposite sign

If all criteria are met, heel strike has occurred between the previous and the current integration step. The time of impact is estimated by a 3rd order polynomial, given in equation 9.

\[
y = a_y + b_y t + c_y t^2 + d_y t^3
\]

with \( t = 0 \) at the beginning of the interpolation. Using this equation and its derivative, equation 10 can be constructed:

\[
\begin{bmatrix}
0 & 0 & 0 & 1 \\
\Delta t^3 & \Delta t^2 & \Delta t & 1 \\
0 & 0 & 1 & 0 \\
3\Delta t^2 & 2\Delta t & 1 & 0
\end{bmatrix}
\begin{bmatrix}
d_y \\
y_1 \\
y_2 \\
a_y
\end{bmatrix}
= \begin{bmatrix}
y_1 \\
y_2 \\
a_y
\end{bmatrix}
\]

(10)

with \( \Delta t \) the integration step time, \( y_1 \) the foot height right before impact (at \( t = 0 \)) and \( y_2 \) the foot height right after impact (at \( t = \Delta t \)). Substituting the solution from 10 in equation 9 and solving for zero height gives three values for the time of impact; one of which must be between 0 and \( \Delta t \). If not, the integration time must be decreased.

The state at impact is also determined by a 3rd order polynomial. The interpolated state is given in equation 11.

\[
q = a_q + b_q T + c_q T^2 + d_q T^3
\]

with \( T = \frac{t}{\Delta t} \) normalized time. For \( T = 0 \), it can be seen that \( q_0 = a_q \) and \( \dot{q}_0 = b_q \). For \( T = 1 \), it can be seen that \( q_1 = q_0 + q + c_q + d_q \) and \( \dot{q}_1 = q_0 + 2c_q + 3d_q \). Solving for \( c_q \) and \( d_q \) gives \( c_q = 3q_1 - 3q_0 - 2q_0 - q_1 \), \( d_q = -2q_1 + q_0 + q_0 + q_1 \). Now equation 11 can be rewritten as

\[
q = q_0 (1 - 3T^2 + 2T^3) + \Delta t \dot{q}_0 (T - 2T^2 + T^3) + q_1 (3T^2 - 2T^3) + \Delta t \ddot{q}_1 (-T^2 + T^3)
\]

(12)

A similar approach can be used for the interpolation of \( \dot{q} \).

Impact is modeled as fully inelastic. If equation 8 is integrated over the duration of impact for the limit of this integration to zero, this results in equation 13.

\[
\begin{bmatrix}
M & D^T \\
D & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q}^+ \\
F_e
\end{bmatrix}
= \begin{bmatrix}
\dot{M} \ddot{q}^- \\
0
\end{bmatrix}
\]

(13)

The push-off is modeled as an impulse collinear to the longitudinal axis of the leg. The inputs to the model related to push-off are in terms of energy. However, the added energy is a function of both the impulse as well as the inter-leg angle. In order to find the impulse that results in an addition of the desired amount of energy, an iteration is used until the energy of the system after push-off is within \( 1 \cdot 10^{-12} J \) of the energy of the system before push-off plus the desired push-off energy. The impulse components can be added to the terms related to the generalized coordinates representing the hip position on the right side of equation 13.

### 2.2 Nominal limit cycle

In this part, the nominal limit cycle is determined from which stepping over is initiated. For the nominal limit cycle the actuation is chosen, in terms of \( T \) and \( E_{po} \), for which the sensitivity of the system to state variations right after push-off is as small as possible. Here it will be explained in detail how and why this type of actuation is chosen for the nominal limit cycle.

First, a limit cycle for a double inverted pendulum, with given hip torque \( T \) and a push-off energy \( E_{po} \), is determined [7] [8]. What we are looking for is an initial state that leads to a final state that equals the mirrored initial state. For the analysis of a limit cycle, the initial state can be redefined as

\[
S_0 = \begin{bmatrix}
\phi_0 & \phi_{1,0} & \phi_{2,0}
\end{bmatrix}^T
\]

(14)
since the position is irrelevant and the angles are opposite in sign with the same magnitude. \( \phi_0 \) is the inter-leg angle, and \( \phi_{1,0} \) and \( \phi_{2,0} \) the angular velocities. \( \phi_0 = \phi_{1,0} - \phi_{2,0} \), see figure 2. The last symbol subscript always indicates the state (initial state is 0 and final state is 1) and the other one, if any, indicates the leg. The values in (14) can be guessed; it suffices when the values lead to a valid step. The final state

\[
S_1 = [\phi_1 \ \dot{\phi}_{1,1} \ \dot{\phi}_{2,1}]^T
\]  

(15)
is a function of the initial state with the torque and push-off energy as given parameters and is determined in simulation:

\[
S_1 = F(S_0)
\]  

(16)

Doing the same one-step simulation three more times, but now for each simulation with a small perturbation on one of the three initial parameters from equation (14) provides all the information needed to construct the Jacobian matrix.

\[
J_F = \frac{\partial \left( \phi_{1,1}, \dot{\phi}_{1,1}, \dot{\phi}_{2,1} \right)}{\partial \left( \phi_0, \phi_1, \phi_2 \right)} = \begin{bmatrix}
\frac{\partial \phi_1}{\partial \phi_0} & \frac{\partial \phi_1}{\partial \phi_1,0} & \frac{\partial \phi_1}{\partial \phi_2,0} \\
\frac{\partial \phi_{1,1}}{\partial \phi_0} & \frac{\partial \phi_{1,1}}{\partial \phi_1,0} & \frac{\partial \phi_{1,1}}{\partial \phi_2,0} \\
\frac{\partial \phi_{2,1}}{\partial \phi_0} & \frac{\partial \phi_{2,1}}{\partial \phi_1,0} & \frac{\partial \phi_{2,1}}{\partial \phi_2,0}
\end{bmatrix}
\]  

(17)

E.g. the first column of equation (17) is

\[
S_1^* - S_1 \approx \frac{\Delta S_1}{\Delta \phi_0}
\]  

(18)

with \( S_1^* \) the changed final state as a result of a small perturbation of \( \Delta \phi_0 \):

\[
S_1^* = F(S_0 + [\Delta \phi \ 0 \ 0]^T)
\]  

(19)
The change in initial state that results in a zero state change (per step) can be estimated by the linearized equation

\[
S_1 + J_F \Delta S_0 \approx S_0 + \Delta S_0
\]  

(20)
Solving for \( \Delta S_0 \) gives

\[
\Delta S_0 = \left[I_3 - J_F\right]^{-1} (S_1 - S_0)
\]  

(21)
where \( I_3 \) is the three by three unity matrix. Adding \( \Delta S_0 \) to the initial state and repeating this several times will result in a limit cycle if a stable limit cycle exists for the given inputs. The number of iterations depends on the initial state used, on perturbation size and eigenvalues.

When a limit cycle is found its sensitivity can be determined. This is done by determining the eigenvalues of the limit cycle from the Jacobian \( J_F \). The smaller the maximal absolute eigenvalue is, the lower the sensitivity is.

In figure 3 some characteristics from limit cycles are plotted as a function of the push-off energy \( E_{po} \) with zero hip torque. It can be seen that the step length, cost of transport and velocity increase for increasing push-off energy. The maximal absolute eigenvalue has a minimum at \( E_{po} = 0.113J \).

![Figure 3: Properties of limit cycles of a double inverted pendulum at zero torque as a function of the push-off energy](image)

When a limit cycle is found its sensitivity can be determined. This is done by determining the eigenvalues of the limit cycle from the Jacobian \( J_F \). The smaller the maximal absolute eigenvalue is, the lower the sensitivity is.

In figure 3 some characteristics from limit cycles are plotted as a function of the push-off energy \( E_{po} \) with zero hip torque. It can be seen that the step length, cost of transport and velocity increase for increasing push-off energy. However, at zero push-off energy the maximal absolute eigenvalue exceeds one. Therefore, at zero push-off energy, no stable limit cycle exists. The lowest maximal absolute eigenvalue is 0.28 at a push-off energy of 0.113J, which corresponds to the most stable limit cycle. The maximal absolute eigenvalues were also determined for non-zero torque cases. However, for the non-zero torque case, the maximal eigenvalue is higher than for the zero torque case.
2.3 Optimization problem

Next, an optimization for the double inverted pendulum stepping over an obstacle will be discussed for which the maximum actuation is minimized. In order to step over an obstacle the step positions have to be adjusted such that the obstacle isn’t stepped on. The desired way of stepping over the hole is defined by four constraints:

\[
\begin{align*}
    x_{so,1} - x_d &= 0 \quad (22) \\
    x_{so,2} - x_{so,1} - l_{ns} &= 0 \quad (23) \\
    \dot{\phi}_{0,l} - \dot{\phi}_{so,l} &= 0 \quad (24) \\
    \dot{\phi}_{0,t} - \dot{\phi}_{so,t} &= 0 \quad (25)
\end{align*}
\]

with \(x_{so,1}\) and \(x_{so,2}\) the foot x-coordinate in front and behind the obstacle, respectively, \(x_d\) the desired foot x-coordinate before stepping over (which depends on the desired position of the obstacle between the legs and is halfway in the optimization) and \(l_{ns}\) the nominal step length. \(\dot{\phi}\) is the angular velocity with the subscript indicating initial state (0), step-over state (so), trailing leg (t) and leading leg (l); see figure 4.

These four constraints define the position in front of the obstacle when stepping over, the step-length over the obstacle and angular velocities right after the last contact release in front of the obstacle.

The adjustment of steps may obviously not destabilize the robot. This constrained nonlinear multivariate optimization problem will be solved using a gradient search (fmincon in Matlab). It is constrained in order to make sure that step positions in front and after the obstacle are fixed and that the angular velocities of the legs right after push-off after the final impact equal the initial ones (with mirrored legs, if necessary). This is to make sure the initial limit cycle is obtained again (with an accuracy of \(10^{-3}\) (m or rad/s)). “Multivariate” relates to the variables to be optimized (one torques and one push-off energy per step).

Optimizations are performed over a range of the obstacle distance. In the optimization, the maximal torque \(T\) and push-off energy \(E_{po}\) are minimized. For an optimization of \(n\) steps the range of the initial obstacle distance from the leading leg is \([\left( (n-1)l_{ns}, nl_{ns}\right)\] with \(l_{ns}\) the nominal step length. E.g., for three steps the optimizations are performed for an obstacle distance from 2 to 3 times the nominal step length. This corresponds to a foot position in front of the obstacle of 1.5 to 2.5 nominal step lengths. Optimizations are performed for a number of distances within this range. The incremental distance (i.e. the change in obstacle distance between two optimizations) influences the optimization results of the total range due to the presence of local minima and the highly non-linear behavior of the double pendulum. This is shown in figure 5. The solution converges for an increasing number of optimization distances. Above 256 distances, increasing the number of distances hardly influences the solution.

2.4 Results

By definition it is not possible to step over the obstacle as defined in 2.3 in one step for obstacle distances other than half the nominal step length; in that case, the first step must be used to make sure that the distance from foot to obstacle is correct. Thus, the position constraint (equation (22) mentioned in 2.3) can not be satisfied when it steps over the obstacle in
Two steps

For the two step case, the system has four inputs: two torques and two impulses, see figure 6. It could be possible to meet the four constraints as long as the columns of the Jacobian, of the vector containing the constraint functions with respect to the inputs, are linearly independent and the necessary impulses are non-negative. This Jacobian can be determined by increasing the input by a small amount and determining what change of the constraints per unit of change of the input. This should be done for each of the inputs independently. The number of rows of this Jacobian equals the number of constraints and the number of columns equals the number of inputs.

In figure 7 some results of the optimization are shown for stepping over the obstacle in two steps.

There is a unique solution per distance, if any, due to the four constraints and four inputs. Remember that for a positive torque during step one the swing leg is pulled backward (which usually results in energy dissipation), while for a positive torque during step two the swing leg is pushed forward (which usually results in energy addition).
all figures the distance of the last step position before the obstacle with respect to the initial position is used, not the obstacle distance). It turns out that at the positive side of this range (around 1.02 nominal step lengths) the determinant of the above mentioned Jacobian matrix becomes zero.

On the negative side of this range (around 0.84 nominal step lengths) this determinant is not zero. However, the value of the first push-off energy becomes zero at this distance, see figure 7. Push-off energy values below zero are physically not possible and therefore not allowed in the optimization model.

Two things can be concluded here.

1. For this model much care should be taken, when only two steps remain, to make sure the state and distance to the obstacle are in a range for which the determinant of the Jacobian is non-zero.

2. In order to make sure stepping over an obstacle is possible when only two steps remain additional actuation is needed, possibly on new joints, hereby guaranteeing that all constraints can be satisfied.

Three steps

The optimization is also done for the situation where the obstacle must be stepped over in the third step. This case is depicted in figure 8. In this case there are six variables (3 torques and 3 push-off energies) in the optimization with still four constraints. Therefore, this system is under determined. The results are plotted in figure 8.

It can be seen in figure 8 that for a change in foot position in front of the obstacle slightly above two nominal step lengths the step length of the second step hardly changes. This is consistent with the results from the two step case. For an obstacle position up to 2.5 nominal step lengths, the step length of the first two steps is a linear function of the distance. The influence of the distance is stronger for the first step. As expected, the maximal absolute torque graphs are smooth, as they are directly related to the evaluation function.

In figure 8 it can be seen that the torque of step two and the push-off energies of the first two steps show discontinuous behavior. This can be explained by the fact that there are multiple ways to actuate the system while satisfying the constraints that are all evaluated equally by the evaluation function. It is desired to have continuous functions of torque and push-off energy instead of the discontinuous behavior in figure 8. This will be necessary for implementation on a real robot. How to obtain this (piecewise) continuous behavior is discussed in the next paragraph.

The Jacobian of the four constraint functions with respect to the six inputs (actuation) is $4 \times 6$. Therefore it could be possible to change some of the inputs while the constraint values remain unchanged. This is only the case when these columns of the Jacobian related to these inputs are dependent; the determinant of these columns is thus zero. Now, if the
determinants are evaluated for all combinations of four columns from the Jacobian matrix (a total of 15) it turns out that for three of them it evaluates to zero (for the optimization solution at two nominal step lengths). This means that these actuation inputs can be changed without altering the values of the constraint functions. The three zero determinant cases are shown in figure 9. In all three cases \( T_2, E_{p01} \) and \( E_{p02} \) are present. These are exactly the ones related to the discontinuous graphs in figure 8. It seems that the columns of the Jacobian related to the second torque and the first two push-off energies are linearly dependent (in the range from 1.5-2 nominal step lengths); this is evaluated for several distances.

A similar hypothesis can be stated for distances of the foot position before the obstacle above two nominal steps. The zero determinant of the Jacobian (out of 15) is the one related to the case where \( T_1 \) and \( T_3 \) are omitted. One of the other inputs can then be set as a (linear) function of distance.

The two hypotheses just mentioned are validated by an optimization where \( E_{p01} \) is set as a linear function of distance in the range \([1.5, 2]\) times the nominal step length and \( E_{p02} \) is set constant in the range \([2, 2.5]\) times the nominal step length, see figure 8, see figure 10.

The range of the second step position, from 1.5 to 2.5 nominal steps, can now be divided into several ranges. For each of these ranges the torque and push-off energy for each step can be described by a fit. The results of these fits can then be used for implementation.

Figure 9: The three combinations of four inputs for which the four constraints can not be satisfied.

Based on the conclusion in the previous paragraph I hypothesize that it will be possible to make one of the above mentioned inputs a (linear) function of the desired foot position. This linear input then serves as an extra constraint in the optimization. This will then be compensated for by the other two inputs while satisfying the constraints. Care should me taken that this does not affect the maximal torque and push-off energy.

Next it will be argued why the optimization results are as presented. The solution for the optimal
actuation as a function of the obstacle distance can be partly explained by looking at the energy of the simplest walker [9]. This model consists of two pendula connected at the hip with a single point-mass at this joint and infinitesimal point masses in the feet; see figure 11. Right before impact the hip mass has a velocity $v^-$ due to the rotation of the trailing leg around its contact point with the ground. Right after the fully inelastic impact the hip mass has a velocity $v^+$ due to the rotation of the leading leg around its contact point with the ground.

Figure 11: For the fully inelastic impact the simplest walker model [9] is used. It has a point mass in the hip and infinitesimal point masses in the feet. Right before impact rotation is around the contact point of the trailing leg resulting in a velocity $v^-$. Right after impact rotation is around the contact point of the leading leg resulting in a velocity $v^+$.

The energy of the system before impact $E^-$ is the sum of the potential energy $E_p$ and the kinetic energy $E_k$, see equation (26).

$$E^- = E_p + E_k = mgl \cos(\phi/2) + \frac{1}{2} m(v^-)^2$$

For a fully inelastic impact the speed after impact is the speed before impact times the cosine of the inter-leg angle. Thus, the energy of the system after impact is

$$E^+ = mgl \cos(\phi/2) + \frac{1}{2} m(v^+)^2$$

$$= mgl \cos(\phi/2) + \frac{1}{2} m(v^- \cos(\phi))^2$$

(27)

From equation 27 it can be seen that the energy loss increases quadratically for increasing inter-leg angle. For smaller step-over distances the step lengths decrease, e.g. look at a distance of 1.5 times the nominal step length in figure 10. Due to the smaller steps the resulting energy dissipation at impact decreases. In order to prevent the energy from becoming too high, which may cause the pendula to fall over forward, energy is dissipated by $T_1$ (which is responsible for the decreasing step length in the first place) and $E_{po1}$ is reduced (compared to the nominal push-off). In order to obtain the nominal energy level again, (most of the) energy is added again by $T_3$ and $E_{po3}$. For step over distances bigger than two nominal step lengths an opposite reasoning applies. However, this only explains how the energy of the system is controlled; this should of course return to its initial value. The energy of the model used for the optimization for an obstacle distance of two nominal step lengths is shown in figure 12. For the potential energy the difference is used with respect to its initial value.

Figure 12: Energy of the optimized model for an obstacle distance of two times the nominal step length (corresponding to 1.5 times the nominal step length for step position two). For the potential energy the difference is shown with respect to the initial value.

3 Safety Range

In order to make sure the limit cycle walker does not step on the obstacle, appropriate safety ranges
on both sides of the obstacle have to be determined. This will be done in this part. This problem is twofold; first, the error of the horizontal distance between the object and the camera has to be estimated. For TUlip, the distance estimation is realized by a stereo camera system. Second, the variation in foot placement has to be determined. In 3.1 the obstacle position estimation will be treated. In 3.2 the foot placement variation of the limit cycle walker will be discussed.

3.1 Object Position Estimation

In order for a limit cycle walker to be able to deal with an obstacle in its walking trail the distance to it has to be estimated as well as the error. Due to the fact that 2D obstacles are considered (at floor level), the distance has to be estimated with respect to the feet as these are the only parts of the robot that are in contact with the floor. This distance is estimated using stereo vision.

In figure 13 the schematic of a limit cycle walker together with an obstacle is shown. The horizontal component of the depth, the distance, from the camera to the obstacle is:

$$x_{co} = \sqrt{D_{co}^2 - y_{co}^2}$$

(28)

For the height of the camera $y_{co}$, the height while standing still is used. The depth between obstacle and camera $D_{co}$ is estimated by TUlip by measuring the disparity [10]. The disparity $d$ is the difference in projection positions in both camera images and is:

$$d = U_r + U_l = \frac{f}{D_{co}} b (29)$$

This is represented in figure 14, which is based on the pinhole camera model [11]. Here, $f$ is the focal length of both cameras and $b$ is the baseline between the cameras. In both camera images the obstacle is identified by color segmentation after which a bounding box is attached to the region with the color corresponding to the obstacle. A bounding box is the minimal set of $x$ by $y$ pixels which contains all pixels that are identified as being part of the obstacle based on its perceived color. When the images are rectified [11], the disparity can be approached by the difference in horizontal position of the bounding boxes in the images. This is illustrated in figure 15. The depth can be found by solving 29 for $D_{co}$:

$$D_{co} = f \left(1 + \frac{b}{d}\right)$$

(30)

The maximal disparity quantization error $\epsilon_d$ is the length related to the disparity of half a pixel to pixel distance;

$$\epsilon_d = \frac{1}{2} \frac{W_{ia}}{R_x}$$

(31)

with $W_{ia}$ the width of the image area of the camera and $R_x$ the resolution related to this width. The maximal disparity quantization error $\epsilon_{ds}$ for a stereo camera is $2\epsilon_d$; the length related to the disparity of a pixel to pixel distance;

$$\epsilon_{ds} = \frac{W_{ia}}{R_x}$$

(32)

Using equation 30 and 32, this results in a maximal depth estimation quantization error of

$$\epsilon_{D_{co}} = f \left(1 + \frac{b}{d}\right) - f \left(1 + \frac{b}{d + \epsilon_{ds}}\right)$$

$$= f \left(\frac{b \epsilon_{ds}}{d^2 + \epsilon_{ds}}\right)$$

(33)
3.2 Foot Placement Variation

Due to the fact that very little control effort is used on limit cycle walkers they are quite sensitive to disturbances. This can be seen in the variation of step lengths due to the floor height variation. In order to determine the influence of the floor height variation on the step length variation simulations are performed on a double inverted pendulum walking over an imperfect floor. This is done for several maximal floor height variations. The same model is used as described in 2.1 with the parameters from table 1.

The cumulative distribution function of step lengths for a maximal random floor height variation of 0.0005m is shown in figure 16. For each simulated range of random variations about 450 steps are simulated. For each step the floor has a random offset (either positive or negative) with respect to the perfectly flat and horizontal case. The results from all these simulations are shown in figure 17 in terms of

This is an optimal maximal error for the given hardware and situation. It depends linearly on absolute distance and resolution. Factors such as obstacle shape and reflectance, lighting, and motion may cause additional errors.

3.2 Foot Placement Variation

In order to get an idea of the magnitude of the maximal error of the estimated horizontal obstacle position with respect to the foot, equation 33 is evaluated for the TUlip limit cycle walker. TUlip is equipped with a CMOS camera module from MicroJet Technology [12]. The maximal error for the stereo camera distance estimation due to quantization is determined at a horizontal camera-object distance of 0.68 meter, which is the horizontal distance from camera to obstacle at the beginning of the final three steps. With the camera height being 1.2m the camera obstacle distance is 1.38m. According to [12] and equation 31 the maximal stereo disparity quantization error is 

\[ \epsilon_{ds} = \frac{3.26mm}{1028} = 3.19 \times 10^{-3} \text{mm}. \]

The parameters \( f \) and \( b \) are 4.5mm and 7.2cm, respectively. Using equation 30, the disparity is \( d = 2.4 \times 10^{-4} \text{m}. \) According to equation 33 the maximal depth estimation quantization error is \( \epsilon_{Dco} = 1.8 \text{cm}. \) Using equation 28 this gives a distance estimation error of about \( \epsilon_{Dx} = 0.9 \text{cm}. \)

This is an optimal maximal error for the given hardware and situation. It depends linearly on absolute distance and resolution. Factors such as obstacle shape and reflectance, lighting, and motion may cause additional errors.

Figure 14: The optics of a stereo camera system based on the pinhole model. Two cameras perceive an obstacle \( O \) (in red) at a distance \( D_{co} \). Both cameras have a focal length \( f \). The optical axes are parallel and image planes are coplanar. The camera offset \( b \) between the cameras is in \( x \)-direction only (in the figure plane).

Figure 15: A schematic of the disparity of the bounding boxes of two images each obtained from a different camera from the stereo camera. The top two sub-figures represent the left and right camera images while observing the same obstacle. The lower figure shows both bounding boxes in one image. The horizontal distance between the bounding boxes is called the disparity \( d \).
several percentiles, with on the x-axis the maximal floor height variation $FHV_{max}$.

The percentiles are fitted by a linear fit. The equations for the minimal and maximal of these fits are:

1. 5th:
   \[ 0.2259 - 19.8FHV_{max} \]  
   \[ (34) \]

2. 95th:
   \[ 0.2259 + 12.1FHV_{max} \]  
   \[ (35) \]

It can be seen that the percentiles above the median change more for changing floor height variations than the percentiles below the median. This indicates that different safety ranges should be taken into account for both sides of the obstacle; the safety range in front of the obstacle should be smaller.

### 3.3 Integration foot placement & vision

Now, the results from 3.1 and 3.2 will be integrated. That enables us to determine for example what the maximal obstacle length is that can be stepped over. This is done for a confidence interval of 90%.

The fits for the actuation can be used as long as equations (36) and (37) are met.

\[
P_{95,s2} + \epsilon_{Ds} + \frac{l_o}{2} + T_o < \frac{l_{ns}}{2} \quad (36)\]

\[
-P_{5,s3} + \epsilon_{Ds} + \frac{l_o}{2} - T_o < \frac{l_{ns}}{2} \quad (37)\]

These ranges are shown in figure 18. $P_{95,s2}$ and $P_{5,s3}$ are the 95th percentile of step two and the 5th percentile of step three respectively, given as offset with respect to the average (see equations (34) and (35)). $\epsilon_{Ds}$ is the distance estimation quantization error (see 3.1), $l_o$ is the length of the obstacle, $l_{ns}$ is the nominal step length and $T_o$ is the translation of the desired relative obstacle position due to asymmetric step length variation.

With the assumption that the step length variations are independent, the above mentioned percentiles are

\[
P_{95,s2} = \frac{P_{95}}{\sqrt{2}} \quad (38)\]

\[
P_{5,s3} = \frac{P_5}{\sqrt{3}} \quad (39)\]

with $P_{95}$ and $P_5$ the variable parts from equations (34) and (35).

\[
T_o = \frac{|P_{5,s3} + P_{95,s2}|}{2} \quad (40)\]

Substituting equations (38), (39) and (40) in equations (36) and (37) gives the same equations:

\[
10.1FHV_{max} + \epsilon_{Ds} + \frac{l_o}{2} < \frac{l_{ns}}{2} \quad (41)\]
To Figure 18: Ranges when stepping over the obstacle. $P_{95,s2}$ and $P_{5,s3}$ are the 95th percentile of step two and the 5th percentile of step three respectively, given as offset with respect to the average. $l_0$ is the obstacle length and $\epsilon_v$ the maximal quantization error of the distance estimation.

For a maximal floor height variation of 0.5mm equation (41) can be solved for the maximal length of the obstacle: $l_o = 19.8cm$. When an additional safety range is taken into account for the additional errors related to the vision system (due to e.g. bad lighting conditions) $l_o$ will be smaller.

4 Conclusion

The research question was:

**What is the optimal way for a 2D limit cycle walker to step over an obstacle with minimal torque?**

It is shown that, for the model used in this study, stepping over an obstacle is only possible for a small range of the obstacle position with respect to the ideal position: $[-0.16, 0.02]$ times the nominal step length.

For the three steps case, the maximal translation of step positions is at least half a step in both directions. Actuation should be performed according to the fits of the results shown in figure 10.

In order to step over an obstacle, the safety range must be taken into account as defined in 3.3. In practice, more imperfections should be taken into account. This will result in increasing safety ranges and a smaller obstacle that can be stepped over.

References


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