DEVELOPMENTS IN MATHEMATICAL MODELS
OF HUMAN PILOT BEHAVIOUR

by

O. H. Gerlach

DELFt - THE NETHERLANDS

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# Contents

1. Introduction  
   2. Engineering models  
      2.1. Introduction  
      2.2. The cross-over model  
      2.3. The optimal control model  
      2.4. Remarks on engineering models  
3. Physiological and psychological concepts  
   3.1. Introduction  
   3.2. Data processing and decision making  
      3.2.1. Characteristic features  
      3.2.2. Internal models  
   3.3. Actuating the aircraft's controls  
      3.3.1. The neuromuscular/manipulator system  
      3.3.2. Standard output motions patterns  
4. A biomorphic model  
   4.1. Introduction  
   4.2. The sampling process  
   4.3. The Observation Model  
   4.4. The Response Model  
   4.5. The Control Law  
5. Relations with pilot workload  
6. Concluding remarks  
7. References  
   Figures  

<table>
<thead>
<tr>
<th>Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2. Engineering models</td>
<td>2</td>
</tr>
<tr>
<td>2.1. Introduction</td>
<td>2</td>
</tr>
<tr>
<td>2.2. The cross-over model</td>
<td>2</td>
</tr>
<tr>
<td>2.3. The optimal control model</td>
<td>3</td>
</tr>
<tr>
<td>2.4. Remarks on engineering models</td>
<td>4</td>
</tr>
<tr>
<td>3. Physiological and psychological concepts</td>
<td>5</td>
</tr>
<tr>
<td>3.1. Introduction</td>
<td>5</td>
</tr>
<tr>
<td>3.2. Data processing and decision making</td>
<td>5</td>
</tr>
<tr>
<td>3.2.1. Characteristic features</td>
<td>5</td>
</tr>
<tr>
<td>3.2.2. Internal models</td>
<td>6</td>
</tr>
<tr>
<td>3.3. Actuating the aircraft's controls</td>
<td>8</td>
</tr>
<tr>
<td>3.3.1. The neuromuscular/manipulator system</td>
<td>8</td>
</tr>
<tr>
<td>3.3.2. Standard output motions patterns</td>
<td>8</td>
</tr>
<tr>
<td>4. A biomorphic model</td>
<td>9</td>
</tr>
<tr>
<td>4.1. Introduction</td>
<td>9</td>
</tr>
<tr>
<td>4.2. The sampling process</td>
<td>10</td>
</tr>
<tr>
<td>4.3. The Observation Model</td>
<td>12</td>
</tr>
<tr>
<td>4.4. The Response Model</td>
<td>14</td>
</tr>
<tr>
<td>4.5. The Control Law</td>
<td>14</td>
</tr>
<tr>
<td>5. Relations with pilot workload</td>
<td>15</td>
</tr>
<tr>
<td>6. Concluding remarks</td>
<td>17</td>
</tr>
<tr>
<td>7. References</td>
<td>17</td>
</tr>
<tr>
<td>Figures</td>
<td>20</td>
</tr>
</tbody>
</table>
1. Introduction

Presenting the Twentieth Lanchester Memorial Lecture certainly is an honourable and pleasant experience. The subject of my Lecture has several ties with the memory of Frederic William Lanchester.

By way of introduction, I suggest you consider for a moment four aircraft as dissimilar as a small general aviation aircraft, a large transport such as the Airbus, a glider, and finally a fighter aircraft such as the Tornado. As regards handling in the air of these widely differing flying machines, their most significant common quality is perhaps, that they can all be flown by a human pilot.

In the definition of the desirable handling qualities of these aircraft, one might - in complete innocence - expect to find a description of such desirable qualities in terms of the behaviour, capabilities and limitations of the human pilot, which one would, of course, expect to be independent of class or category of aircraft.

As we all know, the actual situation is somewhat different. We have different sets of rules for each different class of aircraft. These rules are expressed in terms of the characteristics thereof, although implicitly they lean heavily on the capabilities and limitations of the human pilot. In this situation one cannot help wondering, whether more fundamental answers to the perennial question of what constitute good handling qualities, could not be given with the aid of more explicit studies of the behaviour of the human pilot, his capabilities and his limitations. Quite naturally in an engineering community such as ours, the results of such studies would be expressed in terms of mathematical models of the human pilot. Precisely the development of such mathematical models is the subject of this paper.

There is more than one connection between this subject and the work of Lanchester, whose name and work we honour tonight. Lanchester's contributions to the field of aircraft stability and control are well known. They have been described in the Memorial Lecture (1) two years ago. Perhaps not so well known is the fact, that Lanchester had more than a passing interest in human perception, as exemplified by his paper on Discontinuities in the Normal Field of Vision (2), published in 1934. Undoubtedly the most fascinating aspect of the works of Lanchester is the extremely wide range of human activities covered. In modern jargon, Lanchester's interests were truly multi-disciplinary. On a more limited scale, the present paper will draw from a number of different disciplines which had Lanchester's interests and this is a further link with his work.

The history of mathematical models of human controllers goes back some thirty years, when Tustin (3) published the results of his pioneering experiments. It was the time when control theory was rapidly developing. Right from the beginning of this development the control of an aircraft was clearly recognized as an excellent example of a closed-loop control situation. Mathematical description of the system required a mathematical model of all elements in the loop, including the pilot.

Since those early days, mathematical models of the behaviour of a human operator in a closed-loop control system have been much improved. They describe a very narrow, highly specialized area of human activities. Even within this small area, four distinct pilot activities can still be discerned:

1. sensing the data required to control the aircraft,
2. processing the sensed data in the nervous system and the brain,
3. decision making in the brain, based on the processed data,
4. actuating the aircraft's controls by implementing the decisions, through the
actions of the brain, nervous system, muscles and skeletal bones.

In the past, pilot models tended to concentrate on the more overt actions of the pilot, in particular on the actuation of the controls. The present and future emphasis in modelling the pilot's activities will increasingly be laid on predominantly mental aspects, such as flight monitoring and decision making. For this reason this paper pays much attention to the physiological and psychological parts of the pilot's task, viz. processing of sensed data and decision making.

2. Engineering models
2.1. Introduction

In the past, many efforts have been made to describe pilot's behaviour by means of mathematical models of varying complexity. In order to set the scene for later discussions, this Chapter briefly presents two of the better known engineering pilot models which are used on a wide scale, namely:

1. the cross-over model,
2. the optimal control model.

Essential for these models is the empirically established fact (3, 4), that the human operator behaves like an essentially linear element if he is the controller in a closed-loop control system as shown in Fig. 1. In common with most other engineering models, the two models to be discussed here, are strictly valid solely under certain restrictive conditions:

1. the forcing function, \( i \) in Fig. 1, is assumed to be stochastic, or at least to appear stochastic to the human operator,
2. the controlled element in the closed-loop - i.e. the aircraft - is assumed to be linear,
3. full operator's attention is assumed in the performing of a continuous compensatory tracking task.

The latter condition implies that the operator is presented only with the error signal, \( e \) in Fig. 1. He tries to compensate or null the error through suitable manipulation of the controlled variable, \( c \) in Fig. 1. As will be seen later, in experiments it is not always easy to fulfil the requirement of attracting and holding the operator's full attention. In many cases it leads to a forcing function of rather higher bandwidth than would occur in actual flight.

2.2. The cross-over model

This model has been developed by McRuer, Krendel, Elkind and several others (5, 6). It presents the pilot's behaviour in the frequency domain, in the form of a describing function. The pilot is assumed to be the controller in a time-invariant, single-display, single-control system, as shown in Fig. 1. The describing function relates the pilot's output, \( c \), to his visually observed input, \( e \). There are several excellent descriptions of this model in the literature (7, 8, 4, 5).

Under the various restrictive conditions summed up in 2.1., the cross-over model
briefly summarized in Fig. 2 appears to be a very acceptable approximation of the open-loop characteristics of the control system in the frequency range around the cross-over frequency, \( \omega_c \), where the open-loop gain \( |H(j\omega_c)| = 1 \). The cross-over model of the human pilot is complete only, if two additions are made to the model.

In the first place, the so-called "verbal adjustment rules" are needed to select suitable quantitative values for the parameters \( K_p, \tau_t, \tau_d, \tau_i, \tau_n \) and \( \omega_c \) in the model. It is assumed that the pilot, through previous experience as well as through practice in the actual control situation, adjusts his behaviour in the closed-loop. The adjustment rules are intended to describe his behaviour in this adjusted situation. Since these rules can be found at numerous places in the literature (7, 8), they are not repeated here.

The second necessary addition to the above model takes the form of data on the so-called pilot's remnant, i.e. that part of the pilot's output not linearly correlated with his input signal. From experiments it has been found, that the remnant can be described as an uncorrelated stochastic signal, having a continuous and reasonably smooth spectrum (7, 8, 9).

As stated, e.g. in (7), the cross-over model has been applied in many pilot-vehicle system analyses and also in pilot-vehicle-display system analyses, both in design and in simulation problems. The model has been extended to multi-display, multi-control situations (7, 10, 11), based on classical multi-loop control theory. As such, the use of the extended model relies heavily on judgements concerning the closed-loop system structure.

2.3. The optimal control model

This model, attributable mainly to Baron, Kleinman and Levison (12, 13), is applicable under the restrictions already mentioned in 2.1. The optimal control model, however, is of more recent origin than the cross-over model. It has been formulated in the time-domain, by means of the statevariable notation. As a consequence, the model is, in principle, suited to describe multi-display, multi-control situations. The matrix notation employed makes calculations on the digital computer rather straightforward (14). It has been stated (12), that the basic assumption underlying the optimal control model is, that the well-motivated, well-trained human operator behaves in a near optimal manner, subject to his inherent limitations and constraints and his control task. As it appears, the difficulty in using the model often lies in defining the optimum and these inherent limitations and constraints.

The blockdiagram of the model given in Fig. 3 shows, that the model contains a number of distinct, mathematically formulated elements: a time-delay, a Kalman-Bucy filter, a predictor to compensate for the time-delay, an optimal control law, the neuromuscular dynamics, and finally the observation noise- and motor noise-parameters. Extensive descriptions of the optimal control model can be found in the literature already mentioned. Data have also been given on the selection of the various parameters in the model. The correct selection of the weighting factors in the cost function, which is to be minimised to attain the optimum, and especially the selection of the noise levels, are important to match the model to experimentally determined pilot describing functions.

In the literature a number of successful applications of the model can be found (12, 15). In addition, several attempts have been made to extend the optimal control
model from the basic single-display, single-control situation to multi-display visual observations (16), to multi-model observations (17, 18), to include the human information processing and decision-making processes (19, 20), and finally to describe various aspects of task-interference and pilot workload (21, 22). It should be noted, however, that several of these extensions of the optimal control model still need much further substantiation.

2.4. Remarks on engineering models

Engineering models have been useful and continue to be so. They have found many successful applications, as can be judged from the cited references.

It is the unquestionable merit of the investigators who developed the cross-over model, that they defined, experimentally verified, and subsequently applied a number of basic notions related to the performance of the human operator in closed-loop control situations. An excellent summary of this work is presented in (7). Most further related work, also that performed by different schools of researchers, builds on many of these same notions.

Both models discussed in this Chapter, however, have their limitations. If the cross-over model is to be used as a design tool, the fact that the model has been applied mainly to single-display, single-control compensatory tracking tasks sometimes is a severe restriction. Applying the verbal adjustment rules in more complicated situations is not always as straightforward a process as might be desirable. Using the cross-over model as a diagnostic tool requires identification of the model parameters in the given control situation. To this end a test signal has to be employed having a bandwidth much higher than, and in many cases quite different from, the input signal occurring in actual life. This may badly impair the realism of the entire test.

The optimal control model has so far seen application mainly as a diagnostic tool. This may be due in part to the fact already mentioned, that it appears to be not always easy to define a priori the optimum cost function, which the human operator is assumed to minimize. The selection of the levels of remnant power often appears a matter of judicious selection a posteriori, to match experimentally obtained data.

A rather severe limitation of both engineering models lies in the fact that they lack a direct connection with the pilot's subjective opinion of the workload involved in a given control situation. In the end it is this expert opinion, which decides on the acceptability, or otherwise, of an aspect of the aircraft's handling qualities. There exists, as yet, no generally applicable method to derive the pilot's workload from the parameters of the engineering model. There are serious doubts whether such a method will ever be developed.

An explanation of this very real shortcoming of most existing engineering models is considered to lie in the fact, that such models portray primarily the control aspect of the human operator's activities, thereby ignoring or bypassing the equally important mental activities of data processing and decision making (23). It seems, therefore, that there is a need for new developments, explicitly combining into a single mathematical model the internal mental processes of data handling and decision making going on in the human brain, with the more overt control activities previously discussed.

In this paper the view is held, that such new developments should lean heavily on the vast amount of relevant knowledge available in physiology and experimental psychology. In doing so, it may be well to consider the admonition of Rasmussen (24):
"When entering a study of human performance in real-life tasks, one rapidly finds oneself rushing in where angles fear to tread. It turns out to be a truly interdisciplinary study for which an accepted frame of reference has not yet been established, and iteration between rather general hypothesis, test of methods, and detailed analysis is necessary."

3. Physiological and psychological concepts
3.1. Introduction

Before a further developed mathematical model of the human pilot can be described, it is necessary to discuss some concepts to be incorporated in such a model. Emphasis in the following is, therefore, on certain physiological and psychological aspects of human pilot behaviour. Four categories of closely related activities of the pilot have been distinguished in Chapter 1. Of these four, the processing of sensed data, the decision making in the brain, and the actuation of the aircraft's controls are considered below.

It will be clear from the outset, that in this field the discussion cannot aim for completeness. Only a few topics of the very many possible can be mentioned, and attention is concentrated on those having a direct bearing on mathematically modelling the human pilot. A discussion on the fascinating subject of how the pilot senses with his visual, tactile and proprioceptive, and vestibular sensors the multitude of data he uses to control the aircraft, has to be omitted altogether. Several surveys on this subject are, however, available in the literature (25, 26, 27, 28).

The discussions in this Chapter are centered around Fig. 4. This figure depicts the relations between the human operator and his environment in a way not unlike several authors have done before (29, 30, 31, 32). It pretends to be a very crude description of what goes on in the pilot's sensors, brain and muscles when controlling an aircraft. In order to keep the picture as simple as possible, many simplifications have been implied in the figure. The environment with which the pilot interacts, i.e. the aircraft and its dynamic behaviour, is depicted in an abstract form at the bottom of the figure.

The pilot interacts through two different interfaces with his aircraft environment. On the left side of the figure is indicated, that he senses the aircraft's behaviour through his various sensors (visual, tactile and proprioceptive, vestibular). In the language of the control engineer this is the input side of the pilot. The right side of the figure shows, that the second interface between pilot and aircraft is formed by the effectors, i.e. hands and feet, with which he moves the manipulators, primarily the stick and pedals. Thereby he actuates the controls of the aircraft, in order to influence the aircraft's motions according to his will. This is, of course, the pilot's output side. In a highly schematic way the figure shows how the flow of sensed data is processed in the nervous system and brain, and how it is ultimately transformed into output commands, resulting in the desired movements of the manipulators and finally of the aircraft itself.

3.2. Data processing and decision making
3.2.1. Characteristic features

A first impression of what happens with the visually sensed data may be derived
from the fact that the retina in the eye contains some 120 to 130 million individual photoreceptors, the well-known rods and cones, whereas only about 1 million separate nerve fibers leave the eye through the optic nerve, see Fig. 5. The inevitable conclusion is, that in the various layers of nerve cells also contained in the retina, as shown in Fig. 6, a significant reduction of the data flow originating from the individual photoreceptors has already taken place.

The psychophysical theory of space perception presented by J.J. Gibson, see e.g. (33, 34, 26, 32), discusses the subsequent data processing taking place in the brain. Gibson noted, that the data about the world around us, as sensed by the very many individual sensing elements in the various sensing organs of the human, contain a large redundancy. In the handling of these redundant data in the brain, the key effect is, that the redundancy is removed step by step through a process of successive data reduction, obtained through increasing selectivity of the brain cells. This is precisely what has been indicated schematically in the left hand side of Fig. 4. In this process of data reduction the most essential elements of what we sense are emphasized.

As an example it can be mentioned, that physiological experiments (35, 36, 26) have shown, that in the visual cortex of the brain, certain cells are exclusively sensitive to certain elements or features in the visual image projected on the retina. It has been found, for instance, that some brain cells respond only to a band of light having a particular orientation, sweeping over the visual image on the retina. This experimental fact and other similar evidence fortify the theory, that from the multitude of sensed data only the most relevant parts are retained and passed on. Less essential parts are ignored and do not reach the higher centers of the Central Nervous System. In more general terms, one might say, that we do not perceive in a neutral way. We extract those data which we expect to be most relevant to our interaction with the outside world. Through this process of data reduction, the human operator is able to filter from the enormous amount of sensed data about the world around him, merely those data he needs to perform his task.

For a pilot controlling an aircraft these data are derived mainly from the visual sense, but also from the vestibular, the auditory, the tactile and the proprioceptive senses. To a certain extent the decision between essential features to be passed on and non-essential features to be dropped, seems to be an individualistic one, depending on criteria which are applied largely subconsciously.

An illustration of this latter point is the fact, that some pilots are far more sensitive to imperfections in the motion qualities of a flight simulator than others. Apparently the more sensitive pilots attach greater importance to vestibular motion cues. Usually they are not at all conscious of their subjective weighting of the various cues their brain processes.

According to the theory of Gibson, the internal representation of the outside world in our brain is thus based on a collection of characteristic features, or cues. One can distinguish the visual characteristic features and also the aforementioned vestibular cues, which are an important part of the well-known, but rarely defined motion cues.

3.2.2. Internal models

The use of characteristic features or cues to reduce the redundancy of the sensed data is a first and highly important step in the data handling process going on in the brain. But it is by no means the final one. It has been stated by many physiologists
and psychologists (26, 30, 32), that perception cannot be separated from memory. If we have in our memory an expectancy or estimate of what we are going to perceive next, we can and will use this expectancy. We do this, in order to obtain a further reduction in the data flow needed to remain aware of the world around us. The uncertainty about the surrounding world, which we have to bring down to an acceptably low level through our observations, is drastically reduced by employing these expected values. In nearly all day to day situations such an expectancy or estimate exists.

A pilot flying an aircraft also knows, or rather expects, in a general sense what will happen next. This expectancy stems from his general flying experience and from the flight plan he made before taking off. On a more limited time scale, the pilot expects certain motions from the aircraft in the next few moments, because of his previous observations of the aircraft's motion and because he remembers the subsequently applied control deflections.

From this example it follows, that the expectancy or estimate of what we will observe in the very near future, is obtained from knowledge residing in our memory. It can generally be said to be due to the internal representation in our mind of the world around us. This internal representation is commonly called our "internal model". Obviously, the term "internal model" is used here in an abstract sense, rather than in a pictorial sense. Observing the continuously changing world around us can thus be described as entailing the construction and regularly updating of a predictive internal model, using various parts of memory to incorporate past experience.

The idea of internal models is not at all new. It was proposed as early as 1943 by Craik (37). The concept is gaining more and more acceptance, see (38, 39, 40, 41, 42, 24, 32).

In the more particular case of a trained pilot flying an aircraft fully familiar to him, an internal model can be said to exist in his brain for each of the sensory channels. These internal models provide a prediction or estimate for each characteristic feature the pilot is paying attention to. The various partial models combined make up the total internal model mentioned above. The internal model is thus - subconsciously - employed to predict the impending motion of the aircraft.

Observations in the form of observed characteristic features serve to update this predicting internal model. Each subsequent observation updates the internal model anew. Under suitable conditions, the observation of very few, but well selected, characteristic features may suffice to realign the internal model, and to keep it going for a while, without an immediate need for further sensory data processing. In this way the utmost reduction in the data flow in the brain is achieved.

To amplify on the above, it should be noted, that for the pilot to observe his own control actions - through characteristic features perceived by the tactile and proprioceptive senses - is quite essential. They help to provide him with a sufficiently accurate estimate of the aircraft's motions in the near future.

Returning once more to Fig. 4, it will be seen, that in the middle bottom part of the figure the receptors partly overlap with the effectors. This has been done intentionally, to indicate clearly, that some of the sensors, namely the tactile and some of the proprioceptive sensors, are located right inside the effectors. In this way these sensors provide the highly essential, direct feedback to the pilot's nervous system of the actual manipulator deflection: applied to the aircraft.

Vestibular motion cues are used in the pilot's brain for the same purpose. In the absence of motions directly due to external disturbances, the vestibular motion cues may also help to improve knowledge about the most recent control actions, in order to
provide a more accurate prediction of the subsequent motion of the aircraft.

3.3. Actuating the aircraft's controls
3.3.1. The neuromuscular/manipulator system

The neuromuscular system is the interface between the pilot's brain and the aircraft's manipulator, i.e. the control stick, the wheel, or the side-arm controller. The input to the system is generated in the brain. The output signals are both the control force exerted on the manipulator and the manipulator deflection. Because of the close interactions between the characteristics of the neuromuscular system and those of the manipulator, a description of the neuromuscular system has to include the manipulator as well.

Mathematical models of the neuromuscular/manipulator system have been presented, e.g. in (23, 43). If known or estimated quantitative values of the various constants (44, 45) are substituted in such a model, it turns out, that this system should generally have a bandwidth of more than about 50 rad/sec. Carefully executed measurements on two widely differing neuromuscular/manipulator systems, reported in (46), showed in both cases a bandwidth of only 10-11 rad/sec. This fact clearly indicates, that the bandwidth of the system is not determined by the masses, spring constants and dampings of limb and manipulator alone, although these play a certain role.

It appears, that the bandwidth or speed of response of the neuromuscular system is restricted rather severely in higher centers of the brain. According to (30) and others, this activity takes place in the part of the brain called the cerebellum, where an important part of our control of limb movements resides. In terms of a mathematical model, the mechanism accomplishing this fact might be called a shaping network or, more specifically, a low-pass filter.

3.3.2. Standard output motions patterns

In further discussing the way in which the pilot effectuates the desired motions of the aircraft's manipulator, another psychological aspect has to be mentioned, apart from the various physiological items discussed in the previous paragraph.

It is of course well known, that highly skilled and heavily practised, complex patterns of limb movements play a major role in most manual systems, including manual control systems. Such situations comprise, for instance, playing a musical instrument, or practising a physical sport, or - indeed - controlling a car or an aircraft.

To perform any of these skills the human operator has built up a large repertory of skilled movement patterns of sequences, which he can evoke when needed (39, 41, 30, 32). Typical examples are: striking a golf ball, returning a tennis ball, playing the violin, or even writing one's signature.

The human operator response programs incorporate such motion patterns without the necessity for consciously planning in detail the individual movements that make up each pattern. As a consequence, a selection of output motion patterns made at a sufficiently high level in the brain, need not involve specification of the details of execution at the muscular level. This implies, that at a sufficiently high level in the brain the pilot's entire response pattern may be represented by just a few elements characteristic for that response pattern. These elements have been called the motor features (41) or characteristic output features. At the pilot's output side they are the equivalents to the previously discussed characteristic (input) features, or cues at the pilot's input side. In the control engineer's jargon the relation
between the input features and the output features may be described as the control law.

It is postulated and discussed in detail in (30), that the necessary translation of the characteristic output features coming from the highest centers, into the desired complex motion patterns takes place in the cerebellum part of the brain. It has further been suggested (30), that the patterns of stimulation, which the spinal chord receives from higher levels, such as the motor sensory cortex and the cerebellum in the brain, still do not yield twitches of individual muscles. They still consist of co-ordinated patterns of movement, which may involve a number of muscles. It is here, that the neural shaping network introduced in the previous paragraph comes into play again.

The standard output motion patterns experimentally recorded at the pilot's output are - according to the foregoing - commanded by the Central Nervous System through relatively simple commands, the characteristic output features. These commands cause the neural "shaping network" to generate the desired complex output motions. Via the actions of the muscular/manipulator system these desired limb movements are finally transformed into the actual aircraft's manipulator deflections.

4. A Biomorphic model

4.1. Introduction

Based upon the knowledge presented in the previous Chapter, a mathematical model of the pilot's behaviour will now be discussed. The purpose is not primarily to add yet another model to the many already in existence. The emphasis is rather on an attempt to describe - however crudely - how the data handling as well as the decision making processes going on in the pilot's brain may be modelled in mathematical terms, along with the third component of the pilot's activities, the control process proper on which most of the existing engineering models concentrate.

The aim of the new model is to obtain a closer and more detailed mathematical description of what actually goes on in the living organism. This more life-like model is named here a biomorphic model, to clearly distinguish it form the engineering models previously discussed. The term "biomorphic" has been used before - at least by Serebriakov (31) - in a similar, although not quite identical connotation.

Stated more explicitly, the aim of the biomorphic model is twofold.
1. The model should obviously show a control behaviour, as expressed in the usual terms, such as describing function, cross-over frequency, remnant power etc., which is comparable to the behaviour of actual pilots.
2. Equally important, and perhaps even more so than the requirement regarding the overt control behaviour, is the facility of the biomorphic model to allow conclusions to be drawn about the level of difficulty a well-trained human pilot would experience in controlling the given aircraft configuration to the same level of performance subject to the same level of external disturbances. This facility of the biomorphic model should provide a link with the pilot's subjective opinion and thus with some of the factors making up the pilot workload. The data handling and decision making processes have been explicitly included in the model in an effort to permit this connection with the pilot's workload to be made more satisfactorily, than has been proved possible with the existing engineering models.

A pilot model fulfilling these aims inevitably will be more complex than most of the customary engineering models. Provided the complexity of the biomorphic model
remains well organized, it is felt, that in the present age of powerful electronic computers, simplicity need not necessarily be the foremost virtue of a mathematical model intended to describe certain activities in the brain, the most complex computer-like organism we know.

In attempting to describe the behaviour of the human pilot in a closed-loop control situation, one soon finds, that a relatively complete model consists of various elements or sub-models, which can be developed more or less independently, and which can then be joined to describe the overall behaviour. The optimal control model of Fig. 3 already shows this characteristic.

Whereas basically the closed-loop control situation under consideration is still as shown in Fig. 1, the block labelled Human Operator in this figure can now be further elaborated, as shown in Fig. 7. First of all it is indicated, that a model is needed of the Observation Process, comprising the various senses. Such an observation model would describe the data handling process as well part of the human decision making process along the lines indicated in Chapter 3.

If a pure monitoring task is to be modelled, the Observation Model is also needed. As indicated in Fig. 8, it must then be followed by a model of the final part of the decision making process. A model of the latter type has been described - in relation with the optimal control model - as the so-called Subjective Expected Utility Model (20, 22).

If, on the other hand, a pure control task is to be modelled, a Response Model portraying the pilot's physical control actions is required, see Figs. 7 and 8. The Observation Model and Response Model are linked by another sub-model, essentially describing the Control Law.

If the task to be modelled is a multi-control and/or multi-display task, the complications for the pilot - resulting from the fact that he has to divide his attention - are reflected in the model by the need for an element (indicated in Fig. 8), called the Task Interference Model. Such a model has also been described in connection with the optimal control model (21, 22).

In order to limit the discussion in this paper to a manageable size, rather severe restrictions have to be accepted in the scope of the model to be considered here. The description will be confined to pure control tasks of the single-display, single-control compensatory type, see Fig. 1. This implies that only the human operator model shown in Fig. 7 will be considered. Although the intended application is of course the control of aircraft in actual flight, consideration of the sometimes highly important vestibular observations regrettably has to be omitted as well.

A difference between the usual engineering models and the biomorphic model should be mentioned, before going into some of the details of the elements shown in Fig. 7. For reasons to be explained below, the biomorphic model operates in a sampling way, rather than continuous like most engineering models. Like the optimal control model, and in line with modern control theory, the biomorphic model is described in the time domain.

4.2. The sampling process

Sampling models of pilot control behaviour have appeared more or less regularly throughout these thirty years of study of pilot models. Sheridan and Ferrell (23) give a short account of these developments. Various reasons have prompted several authors to propose sampling models. The main reason was usually the desire to improve the correspondence between human operator response data and the results obtained from
the model (47). In particular it was desired to accommodate experimentally observed discontinuities in the eye motions, viz. the saccadic motions (48), as well as discontinuities noted under certain circumstances in the recordings of the exerted control forces (49).

The reasons for choosing a sampling operation in the present model are partly the same. In particular the existence, noted in Chapter 3, of standard output patterns in the human operator behaviour, points in the direction of intermittent operation, at least at a certain level in the Central Nervous System. At the input side of the human operator, the extraction of characteristic features from sensed data, as discussed in Chapter 3, may also be described more convincingly in a sampling model. There is, however, yet another and perhaps overriding argument. In the cockpit, in actual flight, sampled observation of the various instruments, resulting in more or less discontinuous responses are the rule rather than the exception. Conversely, paying continuous and full attention to a single display for any length of time, appears to be quite an extraordinary situation in actual flight.

Experimental data on display scanning (50) has shown, that the visual sampling occurring in multi-display tasks is never perfectly periodic. The average sample interval differs for each instrument and depends on the flight task. Apart from the sample interval one has to distinguish a dwell-time for each instrument in the display. It is the time during which the pilot fixates foveally on that particular instrument. Like the sample interval, the dwell-time varies about an average value for a given instrument. More complex and higher bandwidth displays require larger dwell-times. The sample interval and the dwell-time also appear to depend very significantly on the individual test subject.

In a sense, the case of a full attention, single-display control task can be considered an extreme of a sampled operation. It is the case where the dwell-time for the single display equals the entire sample interval. If the difficulty of the control task so requires, the sample interval may become as short as the duration of the saccadic eye motions, i.e. some .2 to .3 sec. As Bekey has shown (51), even relatively small fluctuations in the sample interval completely mask the sampling operation from any experimental evidence taken from the human operator in such a full attention, single-display task.

Fig. 9 shows, that a sampling model of the type discussed in more detail below, can indeed produce a control behaviour comparable to the behaviour of actual pilots. In the single-display, compensatory tracking task used as an example, the controlled element was a simple integrator. The random appearing forcing function was the sum of ten sinusoids of 1.5 rad/sec bandwidth, conform to (4). The two test subjects were experienced pilots. Since the control responses (c in Fig. 1) to identical parts of the input signal are reproduced in the figure, the run-to-run variability of pilot K can be seen, as well as the differences with pilot L, who appears to have a slightly different "signature" in his responses. The mathematical model shows a behaviour well comparable to that of both human pilots. At a certain point in the model, the response is made up simply from small incremental steps, and to a much lesser extent from impulses occurring once per sample interval. The model had a sample interval averaging at .25 sec, and varying with a standard deviation of 20%. Analysis of the recorded data from the two test subjects, as well as from the sampling model, gave results as regards the describing function, cross-over frequency and remnant power, which agreed entirely with the experimental data presented in (4).
4.3. The Observation Model

After the preceding general remarks it is now possible to discuss one of the more important sub-models of the human model shown in Fig. 7, the Observation Model. It describes mathematically the sensing of the input signals entering the various human sensors, as well as the subsequent reduction of this multitude of sensed data into a form suitable to base the selection of appropriate output responses on. The data processing as well as part of the decision making process in the brain are thus embodied in the Observation Model.

In order to limit the scope of this presentation, only visual observations shall be described here in any detail. In the case of a single-display compensatory control task, as shown in Fig. 10, the visually observed variable can be thought of as the error in angle of pitch, $\theta$, whereas the control variable going into the controlled element, the aircraft, would be the longitudinal manipulator position, $s_a$.

Pursuing the discussions in Chapter 3, the first step in the observation process is the extraction of the characteristic features. These are the most essential or characteristic elements in the time-history of an observed variable. In the case of visual sampling, the obvious characteristic features are the values of $\theta$ and $\dot{\theta}$ at a particular instant during the dwell-time. By way of definition this instant marks the beginning of the sample interval ($t=0$). Consequently, the sensed characteristic features are $\theta(0)$ and $\dot{\theta}(0)$.

The observed characteristic features, indicated as $\theta(0)$ and $\dot{\theta}(0)$, are obtained in the biomorphic model using an internal model for the visual observations as discussed in Chapter 3. The observed characteristic features serve a twofold purpose as outlined in Fig. 10 and shown in Fig. 10. In the first place, they serve as the basis on which the output responses are selected via the control law. In the second place, the observed characteristic features are needed to regularly up-date the internal model.

The foregoing leads up to a discussion of the way in which the internal model for the visual observations in the human brain is mathematically modelled in the biomorphic model. It is the function of the internal model - both in the actual brain, as well as in the biomorphic model - to provide a continuous prediction or estimate of the visually observed variable and also of its first time derivative, see Fig. 10. It is not yet quite clear how this is actually accomplished in the living brain. The obvious way to obtain such an estimate in a mathematical model, however, is to employ a set of ordinary, linear differential equations.

At the beginning of each sample interval, the differential equations representing the internal model are restarted, using as the initial conditions the visually observed characteristic features, $\theta(0)$ and $\dot{\theta}(0)$, sampled at the beginning of that interval. The solution of the differential equations generated as time proceeds, provides the needed estimates, $\hat{\theta}(t)$ and $\hat{\dot{\theta}}(t)$, as functions of time. At the end of the sample interval the next set of observed characteristic features becomes available to up-date, or rather to restart, the internal model, etc.

An internal model provides a replica of the dynamics of the system to be modelled. In this case the system is the visually observed aircraft motion. Usually the internal model is limited to simpler dynamic characteristics than the actual system. This is a consequence of the fact, that under normal conditions a human observer can visually extract no higher derivative than the first - i.e. the velocity - from a time-varying display. Here, only the error in angle of pitch, $\theta$, and its rate of change, $\dot{\theta}$, can be visually observed. Hence the mathematical internal model for the visual observations can be a differential equation of second order at most.
A significant contribution to the changes in the visually observed error in pitch angle is due to the action of the control variable, the longitudinal manipulator deflection. This control variable is sensed, primarily in the form of the control force, via the tactile and proprioceptive sensors. In the pilot's brain the observed control deflection is used as an input to the visual internal model to estimate the changes in the error in pitch angle. Exactly the same occurs in the mathematical model, as shown in Fig. 10. The exact model by which the varying control deflection is observed in the biomorphic model, using a separate internal model, will not be discussed here.

With the exception of one important aspect, the foregoing describes the operation of the visual internal model. The single remaining item deals in some detail with the manner in which an observation is finally arrived at in the human brain.

From the foregoing it follows that in the Observation Model discussed here, observing a characteristic feature, say $\theta(0)$, requires the simultaneous availability of a sensed value, $\theta(0)$, and an estimate provided by the internal model, $\hat{\theta}(0)$. It is of some consequence to discuss how these two are combined into the resulting observed value, $\hat{\theta}(0)$. The same applies to $\hat{\theta}(0)$, derived from $\theta(0)$ and $\hat{\theta}(0)$. Fig. 11 illustrates this process by further detailing the Decision Element in the Observation Model.

From daily life we know that the human mind needs some time to make a decision. This fact has been confirmed by many quantitative tests in psychology. The more options there are, the longer it takes to arrive at a decision. Formulated more precisely: man has a limited capacity to handle information. The word "information" is used here advisedly in the connotation given by Shannon and Weaver (52), meaning a measure of entropy or uncertainty. The limitation of the human mind in information handling capacity is perhaps one of the most fundamental and far reaching limitations of the human mind.

Using simple terms, a person deciding on the basis of perceived data which of two equally likely events has occurred, handles, in so doing, just one bit. Deciding on one out of four equally probable events means handling two bits, one out of eight implies handling three bits, etc. A more precise formulation of these rather loose statements can, of course, be found elsewhere (52, 53). For simple tasks and given the full attention of the test subject, experimental data indicate a remarkably linear relationship between the number of bits the brain handles and the time it takes to do so. This important empirical fact was discovered as early as 1885 by Merkel, see (23), although described in slightly different terms at the time.

The uncertainty in a variable to be observed, resides before the observation in the difference between the sensed and the estimated value, the so-called estimation error. After the observation, the remaining uncertainty in the observed value lies in the residual observation error. The statement that the human operator has a limited information handling capacity, can now be interpreted as meaning that the operator needs some time to reduce the observation error from the initial value of the estimation error to a smaller residual value. The observation time is assumed to be part of the externally measurable dwell-time, during which the pilot looks foveally at the display.

It is relatively easy to show that the linear relation between the number of bits to be handled and the time required to do so, leads to an exponential decrease to the variance of the observation error from that of the initial estimation error to a final, reduced value in the time during which attention is given to the observation. Fig. 12 shows such a relation. The figure shows, however, a number of exponential curves with varying time constants. This requires a further explanation.
Attention is the key notion in this respect. It must be mentioned, that attention always exists at a certain level of intensity. It can vary from full attention to no attention at all (53).

To a certain extent the human observer can opt where to focus his attention and also, at what level of attention he will do so. Consequently, it is reasonable to assume, that the observation error decrease more rapidly to a small value, if full attention is given to the observation, than if only partial attention is given. The mathematical assumption made in Fig. 12 is, that the time constants of the exponential curves are inversely proportional to their corresponding levels of attention. This statement might of course also be taken as an attempt to mathematically define such an illusive concept as "level of attention".

Finally, from Figs. 11 and 12 it can be seen how the observed values are obtained from the sensed and the estimated values, by allowing some finite observation time to arrive at an acceptably small residual observation error. Implications of this model of a decision process in the human brain on the pilot's mental workload will be referred to again in Chapter 5.

4.4. The Response Model

The second sub-model in the model of the human operator, see Fig. 7, is the Response Model. Pursuing the discussion in Chapter 3, the Response Model describes the actions of the neuromuscular/manipulator system. The determining element in the Response Model is the neural shaping filter, modelled here as a low-pass second order filter, having an undamped natural frequency of about 10 rad/sec and a damping of .7 to .8 of critical damping. Since the muscular/manipulator elements can usually respond much faster, it is the response of the neural shaping filter, that essentially determines the standard output patterns of the biomorphic model discussed in Chapter 3.

When it comes to actually deciding on the general shapes of these output patterns, it should be noted that in many cases the control output time-histories of human operators can be taken as being made up of two basic elements: small incremental steps and pulse-type patterns. In many situations the human operator uses combinations of these two basic types of responses.

Experimental data indicate that the actual mixture of the two basic types of response patterns used in a particular closed-loop control situation strongly depends on the dynamics of the controlled element. The pure integrator of Fig. 9 is controlled almost entirely by small incremental steps, whereas a double integrator would evoke predominantly pulse-type responses. Clearly, individual differences between different operators manifest themselves also in their selection of these standard output patterns, as evidenced in Fig. 9.

The two standard output patterns of the Response Model are, therefore, the responses of the neuromuscular/manipulator system to an incremental step input and to an impulse input signal. As discussed in Chapter 3, the magnitudes of the impulse and the incremental step - A and B respectively - are the two characteristic output features of the Response Model.

4.5. The Control Law

The function of the Control Law - the remaining element in Fig. 7 - is to relate, for each sample interval anew, the magnitudes of the output characteristic features, i.e. the commanded impulse, A, and incremental step, B, to those of the observed
characteristic features, \( \Phi(0) \) and \( \Phi(0) \). Assuming a linear control law, the desired relations are expressed by a 2x2 matrix. The four elements of the matrix are found quantitatively by minimizing a suitably selected cost function, in which a trade-off is made of the precision of control against some reasonable measure of the control effort needed to achieve that precision.

Considering once more the variability between different human operators, one should not expect one single definition of such a cost function to lead to an acceptable control behaviour of the model. On the other hand, it is a stringent requirement, that the cost function one selects should yield - for as wide a range of controlled elements as possible - a behaviour of the model well comparable to that of actual human operators.

5. Relations with pilot workload

In the context of this paper, the ultimate goal in building models of human pilot behaviour is to use these models in the evaluation of aircraft handling qualities. It needs not to be emphasized here, that good handling qualities are essential for efficient flight operations; and of course, they are vital for flight safety. At present the common method of assessing handling qualities still relies heavily on the subjective opinions of experienced test pilots (54). These opinions have proved to be quite reliable, but they are sometimes difficult to use for design purposes.

A pilot's opinion is expressed primarily in the form of a pilot rating. The well-known Cooper-Harper scale (55) is widely used for this purpose, whereas other useful subjective rating scales have been studied e.g. by McDonnell (56, 57). An essential complement to a pilot's subjective rating are his more or less detailed verbal comments, often given in the form of a reply to a prepared questionnaire. Rating and comments combined reflect the pilot's opinion of the total workload imposed on him. They determine the suitability of the aircraft under test for a given mission.

The essential point of this commonly used procedure is, that the pilot's opinion is considered primarily an index for evaluating his workload. Obviously, it is the required workload which determines the acceptability of certain handling qualities.

There are several definitions of the notion: "pilot's workload". Cooper and Harper (55) define it as: the integrated physical and mental effort required to perform a specified piloting task. Another author, Jahns (58) - in a study on workload - defines workload in a more general way as: the extent to which an operator is occupied by a task. As Howitt (59) has pointed out, workload can be divided in three distinct areas:

1. the immediate workload, i.e. the workload experienced over any particular short period of time, e.g. take-off, descent, landing etc.,
2. the duty-day workload,
3. the long-term load.

Clearly, what interests us here, is the immediate workload. As expressed in the definition given by Cooper and Harper (55), this immediate workload consists of a physical and mental component. Of these two, the physical workload can usually be neglected in present-day aircraft: modern control systems permit the frequent use of trimming devices, resulting in relatively low remaining control forces.

Therefore, the sole factor of concern is the mental workload, describes by Bernotat and Wanner (60) as the processing of information by the human being. This aspect of workload is, of course, also important if the pilot performs the purely mental
task of monitoring, e.g. when keeping an eye on the control over the aircraft exercised by the automatic pilot during an approach. But as Firth (61) has pointed out, there is a lack of knowledge about the nature of mental workload, because of the complexity and covert nature of mental functions such as information processing and decision making.

Delving deeper into this mental workload, three related aspects should be distinguished, according to Jahns (58):

1. input load, 2. operator effort and 3. output performance. The relation between these three aspects is shown very schematically in Fig. 13. Somewhat in line with the discussion in Section 4.3. and (53), the operator attention required to perform a task has been used here as a measure of operator effort or mental workload. One of the obvious qualitative conclusions that can be drawn from this figure is, that the workload of a pilot will be quite different, if he tries to obtain different levels of output performance or control precision from the same aircraft at different input disturbance levels. Also, the assumption of full pilot attention is an evident limitation of present pilot models. This requirement will, of course, be met only very rarely in actual flight, normal operations taking place mostly at much lower levels of attention.

Attempts have been made in the past (62,63), to correlate workload (as expressed by pilot's opinions or subjective ratings) with the parameters of the cross-over model. For a variety of reasons these efforts have not been very successful. Mostly perhaps, because the concept of mental workload needs more model refinement in the areas of data processing and decision making than the cross-over model can offer.

It appears, at present, that pilot models operating in the time domain, such as the optimal control model or the biomorphic model, are more amenable to further development in measuring pilot mental workload. The biomorphic model, discussed at some length in the present paper, seems to be particularly well suited to generate measures of mental workload, because of its sampling type of operation and its explicit modelling of the data handling and decision making processes.

At present, however, there is no generally accepted measure of mental workload. As a consequence, there is also no direct method for continuous, precise measurement of the mental workload - in flight or in a simulator - although several authors, e.g. (59), have been confident that some physical measures can be developed, allowing a reasonable assessment of immediate workload.

From this brief account of the state of affairs it may be clear, that further progress in the area of aircraft handling qualities requires, above all, a better understanding of what constitutes mental workload. The concerted efforts of pilots, psychologists and engineers will be needed to perform experimental as well as theoretical work. The results of the studies would have to be expressed - for the benefit of the humble engineer at any rate - in the form of further developed mathematical models of the pilot's behaviour.

Finally, but very clearly, a sobering word on mathematical pilot models should be said. Several authors (39, 24, 64) have repeatedly stated, that great care is needed in using such pilot models. This applies particularly when extrapolating to new situations, such as for design purposes. Our mathematical techniques are but one method, and sometimes a rather inadequate one, to formulate the results of our research on human behaviour. As Christensen (64) has rightly said: "A model is never the real thing, otherwise it wouldn't be called a model".
6. Concluding remark

The title of this paper on mathematical models of human pilot behaviour intentionally includes the word "development". It emphasizes what has hopefully emerged from the foregoing: this fascinating subject is in full development and much imaginative and dedicated work is still required. It is the sincere hope that this paper may help to stimulate such work, and thus pay a fitting tribute to the memory of Frederic Lanchester, whose works have often had such inspiring effects.

7. References

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**FIG. 1: BLOCK DIAGRAM OF COMPENSATORY MANUAL CONTROL SYSTEM**

**Cross-Over Model**

\[
H(j \omega) = \omega_c \cdot \frac{e^{-j \omega \tau_e}}{j \omega}
\]

\[
H(j \omega) = H_p(j \omega) \cdot H_c(j \omega)
\]

Human Operator linear transfer function

\[
H_p(j \omega) = K_p \cdot \frac{e^{-j \omega \tau}}{1 + j \omega \tau_N} \cdot \frac{1 + j \omega \tau_L}{1 + j \omega \tau_S}
\]

Controlled element linear transfer function

\[
H_c(j \omega)
\]

**FIG. 2: THE CROSS-OVER MODEL**

**FIG. 3: THE OPTIMAL CONTROL MODEL**
FIG. 4: DATA PROCESSING IN THE HUMAN OPERATOR

FIG. 5: SECTION OF THE HUMAN EYE

FIG. 6: SECTION OF THE RETINA
FIG. 7: ELEMENTS OF THE HUMAN OPERATOR MODEL IN A SINGLE-VARIABLE CONTROL TASK.

FIG. 8: ELEMENTS OF HUMAN OPERATOR MODELS.
FIG. 9: RECORDED OUTPUT SIGNALS OF TWO PILOTS AND THE BIOMORPHIC MODEL
FIG. 10: THE BIOMORPHIC MODEL IN A SINGLE-VARIABLE AIRCRAFT CONTROL TASK.

FIG. 11: THE DECISION ELEMENT OF THE VISUAL OBSERVATION MODEL.
FIG. 12: THE TRANSFORMATION PROCESS OF VISUAL OBSERVATIONS.

FIG. 13: QUALITATIVE RELATIONS BETWEEN
INPUT DISTURBANCE LEVEL, HANDLING QUALITIES,
OUTPUT PERFORMANCE AND OPERATOR'S ATTENTION.