The Barnett effect refers to the magnetization induced by rotation of a demagnetized ferromagnet. We describe the location and stability of stationary states in rotating nanostructures using the Landau–Lifshitz–Gilbert equation. The conditions for an experimental observation of the Barnett effect in different materials and sample geometries are discussed. © 2009 American Institute of Physics.

\[ m = -\gamma m \times \mathbf{H}_{\text{eff}} + \alpha m \times \dot{m}_{\text{Lat}}, \]  

(2)

where \( \mathbf{H}_{\text{eff}} \) is the effective magnetic field, \( m \) is the unit vector of magnetization, and \( \alpha \) is the dimensionless damping constant. We can separate the dynamics caused by the rotation of the system as a whole from the dynamics in the rotating frame of reference by the transformation \( \mathbf{m} = R(\phi)\mathbf{m}_R \) and \( \mathbf{H}_{\text{eff}} = R(\phi)\mathbf{H}_{\text{eff}}^R \), where \( R(\phi) \) is a unitary matrix describing the rotation by a time-dependent angle \( \phi(t) \) around the axis of rotation and \( \mathbf{m}_R \) (\( \mathbf{H}_{\text{eff}}^R \)) denote the magnetization (effective magnetic field) in the rotating frame of reference. The damping is caused by the magnetization motion relative to the lattice

\[ m \times m_{\text{Lat}} = R(\phi(t)) (m_R \times m_R). \]  

(3)

In the rotating frame of reference, Eq. (2) becomes

\[ \dot{m}_R = m_R \times [-\gamma \mathbf{H}_{\text{eff}}^R + \phi(t)e_z + \alpha \mathbf{m}_R]. \]  

(4)

In this derivation, we have tacitly assumed that the Hamiltonian transforms trivially under rotation, i.e., rotation only generates the gauge Zeeman field [Eq. (1)] in the rotating frame of reference. [Note that if rotation stems from a rotating field\(^{19,10}\) rather than the lattice, we would have to use a different form of damping, viz. \( m = R(\phi)m_R \) in \( m \times m \).] Then the right hand side of Eq. (4) contains an additional term \( \alpha m_{\text{Lat}} \times e_z \) and the stationary states of Eq. (4) depend on the damping constant \( \alpha \).

Following Barnett,\(^{1} \) we are looking for stationary state solutions in the rotating frame of reference, i.e., solutions \( \mathbf{m}_R \) for which \( \mathbf{m}_R = 0 \), at constant angular velocity \( \phi(t) = \omega = \text{const} \). From Eq. (4) it follows that the stationary states obey

\[ 0 = m_R \times (-\gamma \mathbf{H}_{\text{eff}}^R + \omega e_z). \]  

(5)

Here the magnetization in the laboratory frame of reference precesses around the axis of rotation (\( z \) axis) at a fixed angle. We analyze the stability of the stationary states in spherical coordinates, i.e.,

\[ m_R = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \]  

by linearizing the set of equations resulting from Eq. (4) for small deviations \( (\delta \theta, \delta \phi) \) from the equilibrium (rotating-frame) positions \( (\theta_\circ, \phi_\circ) \). When \( \mathbf{H}_{\text{eff}} = 0 \), e.g., in a spherical particle without crystalline anisotropy, the stationary states are
given by $\pm e_z$. Clearly the stationary state at $e_z$ is unstable and $-e_z$ is stable.

For a film with free energy $F=DM_z^2/2$, i.e., $D>0$, it refers to an easy-plane magnetization.

FIG. 1. (Color online) I, II, and III indicate regions in the $(\omega, \gamma M_D)$ plane with stable and unstable stationary states located at $e_z$, respectively (region I), stable stationary states at $\pm e_z$, and unstable stationary states located at a fixed angle $\theta=\arccos(-\omega/\gamma M_D)$ in the upper half plane (region II) and stable stationary states located in the lower half plane and unstable stationary states at $\pm e_z$ (region III).

To summarize, in a system with in-plane magnetization, i.e., $D>0$, the stable stationary states acquire an $z$ component by rotation. The rotation acts like a magnetic field along the magnetic hard axis. Figure 2 shows the $z$ component (component along the axis of rotation) of the magnetization in the stationary state in the $\omega$ versus $\gamma M_D$ plane. In this regime the magnetization displays a hysteresis loop when $\omega$ is cycled. The larger $\gamma M_D$, the slower the transients become.

Limit cycles do not exist, since when $\omega$ is constant, we find for the time-derivative of the free energy $\dot{\gamma}/M_z=-\alpha(m_R)^2$. In other words, the magnetization approaches its stationary state.

When the axis of rotation no longer coincides with the anisotropy axis of the crystal, the rotational symmetry around the axis of rotation is broken. As a consequence, only a finite number of fixed points exist. For an autonomous system on the unit sphere such as the LLG equation with time-independent effective field, it follows from the Poincare index theorem\textsuperscript{11,12} that the number of (un)stable fixed points minus the number of saddles must be equal to two. A magnetic needle along the $y$ axis, i.e., $\mathbf{H}_{\text{eff}}=M_D \mathbf{d} \times (0,0,D) \mathbf{m}$ spun around the $z$ axis exhibits four stationary states when $\omega<\gamma M_D$: $\theta_{1,2}=0$, $\pi$, and $\cos \theta_{3,4}=-\omega/\gamma M_D$, $\cos \phi_{3,4}=0$. If $D>0$, then $\theta_1=0$ is unstable and $\theta_2=\pi$ is a saddle point. In the opposite case, i.e., $D<0$, $(\theta_{1,4}, \phi_{3,4})$ are unstable and $\theta_2=\pi$ is stable, whereas $\theta_1=0$ is a saddle point.

For typical magnetic materials, the critical frequencies to fully rotate the magnetization from in-plane to perpendicular-to-plane orientation are inaccessibly high: $\omega \sim 200$ GHz for Permalloy with $M_s \sim 1000$ emu/cm$^3$ and $D \sim 4\pi$, and $\omega \sim 4$ GHz for a GaMnAs film\textsuperscript{13} with $M_s \sim 15$ emu/cm$^3$ and $D \sim 4\pi$. However, to identify the Barnett effect, it is sufficient to observe small changes in the $z$ component of the magnetization: $M_z=-\omega/\gamma D$. For example, in metals polar magneto-optic Kerr spectroscopy is reported to be sensitive to magnetic moment changes down to $10^{-15}$ emu at a spot diameter of 0.5 $\mu$m.\textsuperscript{14} For a 10-nm-thick Permalloy film ($D \sim 4\pi$) this corresponds to a change
in the magnetization of $M_z \sim 1$ emu/cm$^3$ which is achieved by a rotation frequency of $\omega \approx 200$ MHz. A Kerr angle of $0.3^\circ$ has been measured when the magnetization of GaMnAs is fully aligned perpendicular to the axis of rotation by an external magnetic field. Together with a reported angular resolution in polar Kerr measurements of $\sim 10^{-4}$ this yields a required rotation frequency of a few megahertz. However, since the cubic anisotropy is important in GaMnAs, the above number serves as a lower bound for the frequency estimate. The Barnett effect can be observed at lower spinning rates by choosing a material with small anisotropies. The perpendicular anisotropy in thin magnetic films can be tuned by the layer thickness to cancel the shape anisotropy. Films can be tuned by the layer thickness to cancel the shape anisotropies. The perpendicular anisotropy in thin magnetic films can be tuned by the layer thickness to cancel the shape anisotropies. The perpendicular anisotropy in thin magnetic films can be tuned by the layer thickness to cancel the shape anisotropies.

The Barnett effect can be also used to move domain walls. Consider a wire along the $y$ axis, which contains a transverse Bloch wall in the $xz$ plane. When the wire is rotated around the $z$ axis, the Bloch domain wall moves with a velocity $v = \lambda_z \omega / \alpha$, where $\lambda_z$ is the width of the transverse Bloch domain wall. For $\lambda_z \sim 100$ nm and $\alpha \sim 10^{-2}$, this yields $v \sim (10$ m/s)/($\omega$/MHz).

It might be easier to observe the Barnett effect by vibration rather than rotation, but the mechanical vibration amplitude $\delta \varphi$ then becomes an additional control parameter. The magnetization response is enhanced when the harmonic vibration and ferromagnetic resonance frequencies coincide. At this magnetopolariton mode, a $z$ component of the magnetization is excited in a needle in the $xy$ plane that oscillates around the $z$ axis. Assuming a vibration amplitude $\delta \varphi$ (rad), $M_z$ oscillates with an amplitude $M_z \delta \varphi / 2\alpha$.

In the ideal case of zero anisotropy, only the temperature-induced thermal activation of the magnetization has to be overcome in order to observe a Barnett effect, which sets the lower bound on frequency according to $VM / \omega \approx \gamma k_B T$. For a spherical particle with diameter $d$ and saturation magnetization $M_s$, this yields a minimum frequency of about 500 MHz at $T=1$ K, $M_s = 10$ emu/cm$^3$ and $d=10$ nm. For a 10-nm-thick film with $M_s = 10$ emu/cm$^3$ and area $A=1 \mu m^2$ with compensating form and crystal anisotropies, the required rotation frequency is about 25 kHz.

In conclusion, we discussed the Barnett effect in magnetic nanostructures, which gives a handle to manipulate magnetization by mechanical means. We find that the rotation frequencies necessary to fully switch magnetizations in conventional materials are very high and beyond present experimental possibilities. However, the Barnett effect can be observed via partial magnetization of very soft materials, rotation-induced domain-wall motion, and vibrations close to magnetic resonance frequencies.

This work is part of the research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM), which is financially supported by the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

---

1S. J. Barnett, Phys. Rev. 6, 239 (1915).