A coupled numerical solution to the shallow water-Hirano model

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1. Introduction

Modelling the morphodynamic evolution of rivers due to flow-driven sediment transport requires the adoption of a flow model, such as the shallow water equations, and a conservation equation for sediment mass (the Exner equation). When dealing with mixtures of grain sizes, one can account for the interaction among sediment transport, bed level variation and development of bed stratigraphy using a grainsize-specific form of the sediment conservation equation. Hirano (1971) was the first to develop such a continuity model. He introduced a sediment exchange layer (the "active layer") providing a source of sediments to be entrained in the flow and regulating the exchange with the substrate layer located underneath. The active layer model requires a closure relation for the (time varying) active layer thickness. The classical approach is to assume for it a constant value, which deeply affects the celerity of sorting waves (Ribberink, 1987). A preferable approach, however, is to link its value to physical properties of the river bed, such as some reference sediment diameter in the plane bed case or the dune height in the bedform-dominated case (e.g. Ribberink (1987)).

2. A coupled mathematical formulation

Classical numerical models for hydro-morphodynamic problems are uncoupled: the hydrodynamic part is solved before the morphodynamic part, under the assumption that the morphodynamic part evolves at much lower pace, e.g. Blom (2008). This however is not necessarily true even when dealing with one single sediment fraction (Lyn and Altinakar, 2002): a Froude trans-critical region exists ($0.8 \leq F_r \leq 1.2$) where the two parts of the problem evolve on comparable time scales. In this work we focus on the additional waves introduced by the equations of the active layer model. Our hypothesis is that (at least in some flow and transport conditions) they may propagate at a speed comparable to that of the hydrodynamic waves, which implies that the interactions between the hydrodynamic and morphodynamic part take place on short time scales. This claim motivates the need for a coupled formulation of the considered system of partial differential equations (PDEs), in which all the equations are treated simultaneously: this is the main goal of this work.

3. Numerical solution and results

The non-conservative system arising from the coupled shallow water-Hirano model is numerically solved by Finite Volume path-conservative schemes (Dal Maso et al., 1995), allowing to restore conservation within a non-conservative formulation. Due to the complex nature of the problem at hand we adopt a centred solution technique (the PRICE-C scheme of Canestrelli et al. (2009)) where the insertion of an upwind bias (Stecca et al., 2012) and extension to second order help to improve accuracy. The final form of the system strongly depends on the chosen model for the active layer thickness. We use

- a constant value,
- a model based on a characteristic but time varying grain size of the bed surface for plane bed cases,
- a model based on a time varying bedform height for bedform-dominated conditions,
- an exponential lag model for bedform height.

By performing a characteristic analysis and comparing the coupled solution with competing uncoupled solutions we assess in which flow and transport conditions the coupling plays an important role: we find it to be important in a wider region than the above mentioned Froude transcritical region.

4. Conclusions

We present a novel coupled mathematical formulation of the shallow water-Hirano model with various closure relations for the time-varying active layer thickness. The coupled model results in a non-conservative system of PDEs which is solved numerically by path-conservative schemes. In contrast to the commonly-adopted uncoupled approach, the proposed formulation improves taking into account the time scale of sorting processes, which may evolve at comparable speed as the hydrodynamics.

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