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Application of a Greedy Algorithm to Military Aircraft Fleet Retirements

Jeffrey Newcamp[1], Wim Verhagen[1], Heiko Udluft[1], Richard Curran[1]

ABSTRACT: This article presents a retirement analysis model for aircraft fleets. By employing a greedy algorithm, the presented solution is capable of identifying individually weak assets in a fleet of aircraft with inhomogeneous historical utilization. The model forecasts future retirement scenarios employing user-defined decision periods, informed by a cost function, a utility function and demographic inputs to the model. The model satisfies first-order necessary conditions and uses cost minimization, utility maximization or a combination of the 2 as the objective function. This study creates a methodology for applying a greedy algorithm to a military fleet retirement scenario and then uses the United States Air Force A-10 Thunderbolt II fleet for model validation. It is shown that this methodology provides fleet managers with valid retirement options and shows that early retirement decisions substantially impact future fleet cost and utility.

KEYWORDS: Aircraft retirement, Fleet manager, Aircraft cost, Retirement model.

INTRODUCTION

Military aircraft fleet managers are responsible for providing strategic capability to their owning command. Thus, aircraft are based around the globe to perform various roles under a variety of operating conditions. As these individual aircraft are flown over time, each one develops a historical utilization profile that is related to its fatigue life expended (Molent *et al.* 2012). When a fleet of individual assets nears projected end-of-life, it is imperative that the fleet manager plan for retirement so that operational demand can be satisfied. Retirement planning varies greatly across military services and within service fleets (Garcia 2001; AFSB 2011). It can be proactive and data-driven but at times it has been reactionary, driven by changing budgetary conditions or critical aircraft failures. As the average age of aircraft fleets is increasing, retirement planning tools and methodology are necessary to aid fleet managers through the retirement decision process (Carpenter and White 2001).

The objective of this research was to develop a tool to provide fleet managers with a list of aircraft serial numbers that should be considered for retirement, sorted by precedence and timing. This tool is called the Fleet and Aircraft Retirement Model (FARM). It provides a list of aircraft indicating which one should be retired first and when this should happen. To improve the applicability of the tool, its interface is simplistic, the greedy algorithm implementation is clear and the inputs are accepted in a variety of formats. FARM was built for the spectrum of fleet managers including those who seek to minimize lifecycle cost, to maximize aircraft utility and to maximize the fleet's utility to cost ratio. The methodology also supports a fleet manager who wishes to use his own objective function that might be based on a variety of weighted metrics.

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Prior to discussing retirement, a fleet manager must understand the fleet's demands and historical utilization (Jin and Kite-Powell 2000). A previous study analyzed this opportunity using operational data from the United States Air Force (USAF) A-10 Thunderbolt II fleet (Newcamp 2016). The next step in retirement thinking is to develop replacement policy for a fleet utilizing the operation research methodologies contained in the study of replacement theory (Peters 1956).

Unfortunately, current fleet retirement schemes are primarily based, after an initial objective screening, on subjective means because economic life calculations are exceedingly complex (Tang 2013; Lincoln and Melliere 1999; Unger 2008). For example, the USAF gathers maintenance and logistics experts to decide which aircraft can get retired; however, the decision is very complex, and the decision-makers lack suitable tools (Marx 2016). Aircraft can be identified for retirement based on flight hours, repairs that limit usability, limit exceedances, corrosion, owning unit capabilities, among many other factors. While the bulk of replacement theory literature discusses the replacement of current (defender) assets with more modern (challenger) assets, this study ignores the latter because their acquisition does not directly hasten defender retirements (Robbert *et al.* 2013). Also, the authors treat military aircraft as parallel assets that independently contribute to supply (Stuivenberg *et al.* 2013), which allows for the specificity of individual serial numbers in the fleet.

Military aircraft fleet's assets do not continually operate at maximum capacity. Since retirement schedules depend on utilization, a fleet manager may alter utilization patterns leading to a more optimal retirement schedule. Testing various retirement schedules with an objective tool is necessary to quantify the net present value of each scheme. This paper contributes with a methodology that answers this need and enables fleet managers to make utilization decisions now that will affect future fleet statuses.

The novel contribution of the FARM methodology is the use of individual serial number utilization histories and cost data as a basis for future year predictions. Traditional replacement models have used fleet-wide utilization averaging or ignored asset utilization altogether, which has led to non-optimal solutions (Hartman 1999). To overcome the limitation of basing forecasts on outdated information, fleet managers can periodically use FARM to update their fleet retirement forecasts, including updated cost and utility data for each iteration. This approach also allows fleet managers to alter their utilization levels across a fleet to optimize their retirement scheduling.

The remainder of this article will discuss the methodology employed in the FARM software. The background section contains relevant literature on asset retirement plus a discussion of capital asset replacement theory. In the methodology section, the greedy algorithm approach to the retirement problem and the mathematical formulation for FARM are described. Then the results section shows data from a simulation run using FARM for a virtual fleet. The discussion section highlights the usefulness of a serial number specific retirement tool and shows validation of FARM using the real USAF A-10 fleet. Lastly, the conclusions section emphasizes the major findings from this study.

BACKGROUND

LITERATURE REVIEW

A military aircraft fleet retirement methodology must connect the domains of replacement theory, capital asset economics and military operational analysis. Relevant studies concerning asset replacement include Jones *et al.* (1991), Rajagopalan (1998) and Bethuyne (1998) and the thorough treatment of capital equipment replacement in Jardine and Tsang (2013). While insights can be gained from other domains, 2 considerations are important to aircraft replacements. First, aircraft lifecycles and planning/construction timelines are much greater than some other asset categories. Second, upgrades and overhauls significantly alter the capability and lifetime projection (Tang 2013).

Tang (2013), in a study on replacement schedules, discussed a time-space network approach for helicopters. The study concluded that cost parameters like fixed and variable operating costs can be simplified for benefit of the model's approach. The author assumed all helicopters were homogenous regardless of age and utilization history and excluded variable staff costs from the model. The present research advances this assumption by accommodating variable staff costs in the variable cost function and allows an inhomogeneous fleet input. Hartman's complementary study on replacement schedules showed that these are highly dependent on asset utilization through time (Hartman 2004). Hartman's integer programming method used a cost-minimization technique for asset replacement over a finite horizon (Hartman 1999). His paper suggested that future research should address fleet management and fleet sizing options.

Jin and Kite-Powell (2000) relied on system utilization and replacement decisions to meet the demands of a profit-maximizing manager. The authors looked at operating cost trends and the cost of replacement as factors for the retirement decision for ships. The primary contribution of Jin and Kite-Powell (2000) is the conclusion that an asset should be retired if its net benefit in a fleet is less than the salvage value.

Evans (1989) studied ship replacement theory basing his approach on costs rather than profits and concluding that replacement should occur when it becomes cheaper to purchase a replacement than to continue operating an aging system. The paper has many similarities to aircraft fleet replacement study, mainly that replacement should only be affected by costs in real terms. Additionally, this author posited that replacement decisions should focus on the existing fleet and not on the costs or capabilities of the replacement assets. The present study uses the same approach, suggesting that retirement is based on the current operating costs of the fleet. Since ship replacement requires years for contracting, construction and testing, ships are more similar to aircraft than assets in the motor vehicle, farm machinery and locomotive industries. As Evans posited, ships are often replaced with like replacements. However, aircraft are commonly replaced with newer assets with greater capability (Boness and Schwartz 1969).

Malcomson (1979) determined replacement rules for capital equipment and concluded that an iterative approach was the most efficient. Like in this paper, Malcomson also assumed that the replacement trigger point must be when the operating cost of aging assets is greater than operating new equipment. Further, the author noted that finite answers to the replacement problem are more desirable than approximate answers, and given modern computing power, finite solutions are attainable at very low cost.

Landry (2000) analyzed multiple courses of action for maintaining the aging fleet of Canadian CF188 (F-18) and CP140 (P-3) aircraft. His study treated the problem as a business case analysis with the aim of providing a fleet manager with objective data for a retirement decision. His Airframe Life Extension Program (ALEX) software used fatigue test control point data to forecast early retirement dates.

Lu and Anderson-Cook (2015) concluded that future reliability estimations can be improved when assets of the same age are not treated homogeneously, but are rather based on historical usage. The authors used an automobile example to illustrate that 2 cars of the same age do not possess the same reliability. Understanding mileage and usage conditions can improve maintenance and replacement decisions, just as understanding aircraft demographics can improve retirement decisions.

REPLACEMENT THEORY

Replacement theory is a decision-making process from operations research dealing with substitute system selection conducted by an agent. For a group of assets, the formulation becomes a parallel replacement problem. If the goal is to minimize lifecycle cost, replacement theory can help to determine a capital asset's optimum life. As capital assets age, increasing maintenance costs and reduced utility draw attention to the necessity for replacement (Bethuyne 1998; Lu and Anderson-Cook 2015). Retiring assets is half of the parallel replacement puzzle and the subject of this research. It is assumed that the selection of replacement equipment occurs outside the scope of this methodology.

Generally, new equipment with better capability replaces older equipment (Nair and Hopp 1992). For aircraft, replacement theory might suggest 2 courses of action: upgrades/overhauls or retirement. As Landry's research concluded, the crux is deciding whether it is more fiscally responsible to upgrade aircraft structure or to replace the aircraft altogether (Landry 2000). This paper only addresses the retirement course of action, which is termed the replacement model. It is believed that providing a fleet manager with the best replacement model will yield the most sensible economic replacement policy.

A parallel replacement problem, by its nature, addresses a set of assets. Unlike the single asset case, assets under consideration for parallel replacement can have their utilization levels adjusted to prolong or accelerate deterioration (Bethuyne 1998). This can be an invaluable approach for fleet managers trying to meet operational requirements or retirement mandates.

METHODOLOGY

FRAMING THE PROBLEM

To determine the optimal aircraft to retire at a point in the future, managers could use previous aircraft information as the best predictor for residual aircraft life (Hsu *et al*. 2011; Hawkes and White III 2007). However, analyzing the current fleet and each smaller fleet size was not computationally feasible for fleet sizes greater than approximately 15 assets, so a greedy algorithm was implemented. Calculating every permutation was not necessary since a greedy algorithm provides the same global optimum if the problem is appropriately bounded and local optima are avoided through logic (Cormen *et al.* 2009). This model consisted of a fleet of n aircraft with each subsequent fleet size, $n - 1$, dependent on the previous reduction. This methodology was grounded in the assumption that a fleet manager desiring to retire 2 or more aircraft would always choose the worst asset to retire at each iteration. Therefore, all smaller fleet size problems became $n - 1$ easier until $n - (n - 1)$, when the single remaining aircraft was the least desirable option. This iterative approach resulted in a Pareto front of fleet cost, fleet utility or the ratio of fleet utility to cost. Changing from a minimization model to a maximization model, a second Pareto front could be found. The space between the Pareto fronts indicates the relative goodness or inferiority of retirement choices.

FLEET AND AIRCRAFT RETIREMENT MODEL

FARM uses a greedy algorithm to determine which aircraft in an inhomogeneous fleet should be retired and in what order. For each smaller fleet size, the algorithm chooses the current optimal solution before analyzing the next smaller fleet size. FARM's methodology is outlined in Fig. 1. The multi-year outlook makes retirement decisions using projected asset cost and utility. The model is valid for any initial and final fleet sizes. FARM operates with user inputs (decision periods, minimum/maximum aircraft ages and rate of yearly budget increase) and 3 user functions (fixed cost, variable cost and utility). The fixed cost is distributed evenly across assets while the variable cost and utility are both functions of aircraft age. Costs are modeled as equivalent costflow. Inflation and the effects of various methods for cost reporting were removed from the model by using maintenance man-hours as a proxy in the variable cost calculations. Utility is analogous to aircraft availability, is a number between 0 and 1 and is computed as the number of available days out of 31. However, individual FARM users may alter the format of input functions as necessary.

The methodology underlying FARM is useful for modeling a real fleet of aircraft as well as a virtual fleet of aircraft. Virtual fleet modeling follows the conventions found in literature: aircraft operations and support (O&S) costs are high in the first few years of operation, then decrease sharply as the fleet matures and finally the costs increase at approximately 3% per year of age into the future (Dixon and Project Air Force (U.S.) 2006). Utility begins low for a new aircraft, then quickly peaks, followed by a decrease with age. An example of the cost and utility models used for FARM's development are shown in Fig. 2. Step functions in utility levels and costs that occur due

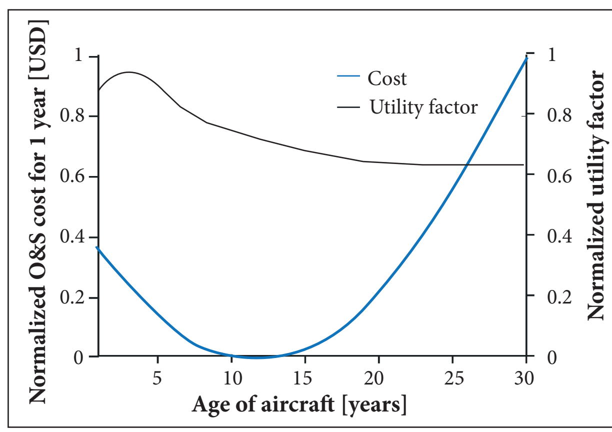

Figure 2. Representation of cost and utility models in FARM.

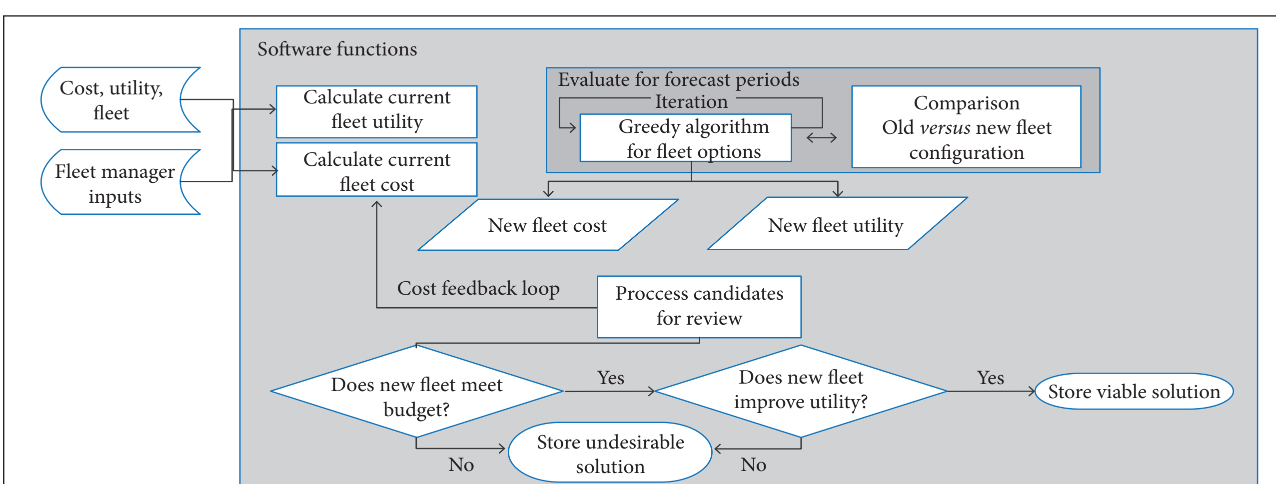

Figure 1. Flow chart for methodology steps.

to major overhaul or repairs were not added to the model. Real fleets were modeled with actual cost and utility functions, which in general were found to follow the published conventions. To forecast future fleet conditions, the most recent cost and utility were extrapolated through time. Otherwise, depending on the age distribution of the fleet, FARM would suggest retiring very young aircraft with high cost and low utility.

For each decision period, FARM outputs the recommended serial numbers to retain for all fleet size options with associated metrics for each option. Fleet managers may use these data to identify their ideal fleet size and makeup. Fleet changes with time can then be evaluated. The limitations of this methodology and associated software model are few but important. The methodology is only valid for 1 mission design series. For example, a mixed fleet of KC-135s and F-15s cannot be evaluated. Second, the methodology does not allow for subjective valuations or weighting factors for the aircraft. Lastly, FARM does not provide a time-sequence of retirement decisions. Rather, FARM forecasts future asset cost and utility to support a retirement decision forecast.

MATHEMATICAL FORMULATION

This section presents the optimization model that the greedy algorithm solves in each of its iterations for a given year of interest. Lastly, the calculation equations and problem constraints are presented.

The decision variables are:

$$X_{ta}^{i} = \begin{cases} 1, operating, \\ 0, not\ operating. \end{cases}$$

$$R_{ta}^{i} = \begin{cases} 1, retired, \\ 0, not\ retired. \end{cases}$$

The objective function (Eq. 1) seeks to maximize:

$$Z = -W_c \int_0^a \int_0^t C_{ta} X_{ta}^{i} dtda + W_u \int_0^a \int_0^t U_{ta} X_{ta}^{i} dtda + \\ + W_r \int_0^a \int_0^t \frac{U_{ta} X_{ta}^{i}}{C_{ta} X_{ta}^{i}} dtda \quad (1)$$

where:

$$W_c, W_u, W_r \in \{0,1\}, W_c + W_u + W_r = 1 \quad (2)$$

a represents the aircraft of interest; t means year of interest; C_{ta} is the cost of aircraft a in year t; X^{i}_{ta} means that aircraft a is operating in year t in iteration i; W_c, W_u, W_r represent weighting — cost, utility, and utility/cost ratio, respectively; U_{ta} is the utility of aircraft a in year t.

The objective function contains 3 terms. The first is the cost calculation, a combination of all fixed and variable costs for operations and sustainment. The second term is the utility calculation, measured as wished by the fleet manager. The third term is the utility per cost ratio, a way to balance the cost associated with changes to utility. It is assumed that only 1 term can be optimized at a time in the model. That is, 1 and only 1 of the weights is equal to 1 each time the optimization model is solved, as shown in Eq. 2. The following equations are required to evaluate the objective function.

The cost of an aircraft a in year t is the integration of aircraft cost from simulation start until the year of interest, assuming that the integration increment is small enough to yield small error (Eq. 3):

$$C_{ta} = \int_0^t C_a dt \quad (3)$$

where C_a is the annualized cost function of aircraft a.

The utility of an aircraft a in year t is the integration of aircraft utility from simulation start until the year of interest, assuming that the integration increment is small enough to yield small error (Eq. 4):

$$U_{ta} = \int_0^t U_a dt \quad (4)$$

where U_a is the annualized utility function of aircraft a.

The equations are subjected to several constraints. The sum of aircraft a in year t must be between the bounds of operational aircraft in year t (Eq. 5):

$$\underline{NA_t} \leq \sum_{a \in A} X_{ta}^{i} \leq \overline{NA_t} \quad (5)$$

where $\underline{NA_t}$ is the minimum number of operational aircraft in year t; A represents the aircraft type, a; $\overline{NA_t}$ is the maximum number of operational aircraft in year t.

The sum of the cost of aircraft a times inventory must be less than or equal to budget in year t (Eq. 6):

$$\sum_{a \in A} C_{ta} X_{ta}^{i} \leq \overline{B_t} \quad (6)$$

where $\overline{B_t}$ is the maximum budget in year t.

The sum of utility of aircraft a times inventory must be greater than or equal to the minimum acceptable utility threshold in year t (Eq. 7):

$$\sum_{a \in A} U_{ta} X_{ta}^{i} \geq \underline{U_t} \quad (7)$$

where $\underline{U_t}$ represents the minimum utility threshold of the fleet in year t.

The opportunity to retire an aircraft a in year t is contingent upon the existence of aircraft a in the fleet in the previous year (Eq. 8):

$$R^i_{ta} \leq X^i{}_{(t-1)a}, \forall\, a \in A \tag{8}$$

where R^i_{ta} means that the aircraft a is retired in year t in iteration i.

The presence of an aircraft a in year t, given the knowledge of previous years of interest and the decision made in year t, is represented in Eq. 9:

$$\left(X^i_{(t-1)a} - R^i_{ta}\right), \forall\, a \in A \tag{9}$$

where, upon initialization, all aircraft are operational (Eq. 10):

$$X^i_{0a} = 1, \forall\, a \in A \tag{10}$$

The fleet size in year t, Eq. 11, is the summation of the operating aircraft:

$$F^i_t = \sum_{a\in A} X^i_{ta} \tag{11}$$

where F^i_{ta} is the fleet size in year t in iteration i and must be 1 smaller at each iteration (Eq. 12):

$$F^i_t = F^{i-1}_t - 1 \tag{12}$$

and the initial fleet size, Eq. 13, is the summation of the operating aircraft in the initial year:

$$F^i_{0a} = \sum_{a\in A} X^i_{0a} \tag{13}$$

RESULTS

This section presents results from the FARM program. A virtual fleet is used for simulation and simplified output plots show representative results. Then, to validate the methodology, A-10 case study FARM results are shown with plots showing detail to the tail number level.

To evaluate FARM, this discussion uses a simulated aircraft fleet of size, n = 100, over a period of 5 years with cost and utility data similar to those represented in Fig. 2. Aircraft ages were drawn from a uniform distribution. Budget was set at the current budget plus a 1% yearly budget increase to mimic the defense budgeting process. Minimum acceptable utility was set to 45% of the existing utility. Three objective functions are used: cost minimization, utility maximization and utility per cost maximization.

Figure 3 shows simplified simulation cost results for a sample fleet in year 5 for fleet size options from 1:n. The 2 lines represent the feasible solutions, which include only those results meeting budget and utility requirements. The bottom curve represents the cost-minimization solutions. These solutions show the cost of the fleet for n aircraft, n – 1 aircraft, etc. The top curve shows fleet cost for cost maximization or worst case retirement choices made for each fleet size. The vertical gap between the curves is the cost delta that can be saved by making the cost-minimization serial number retirement decisions. The curves are cutoff at the both ends, caused by budget and utility constraints.

Figure 4 is an expanded view of a small portion of the lines in Fig. 3. This expanded view shows that the lines in Fig. 3 are composed of many discreet points. At each fleet size, n, FARM calculates all of the possible options. These are shown in Fig. 4 between the most expensive and the least expensive options. Knowing the range of options is useful because it is not always practical for a fleet manager to retire the optimum aircraft.

Figure 5 shows the simplified simulation results for the same scenario, but with a utility-centered management focus.

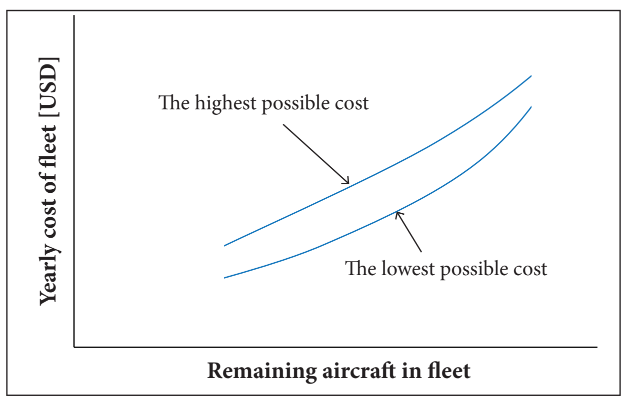

Figure 3. High and low cost choices for fleet of various fleet size options.

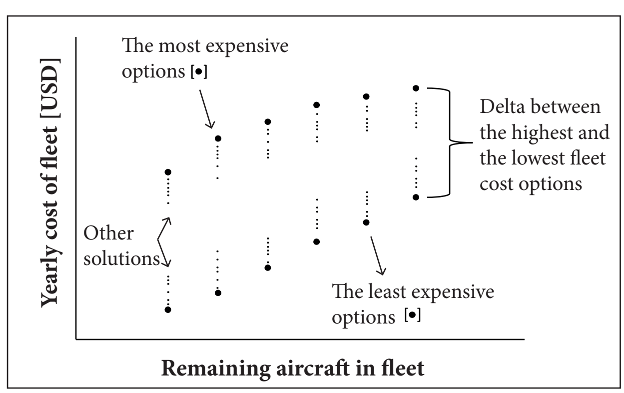

Figure 4. Expanded view of cost options showing all solutions.

These results inform the fleet manager which serial numbers to retire if the fleet goal was to maximize the utility factor, which for this scenario is the sum of aircraft days available per month for the existing fleet. The expanded view shows that, for each fleet size, there are $n - 1$ utility outcomes. The shapes of the curves shown in Fig. 3 to Fig. 5 are the manifestation of the cost and utility input data.

The curves in Fig. 6 show the Pareto fronts for the utility per cost ratio calculations for the sample fleet. As aircraft are retired from the fleet (right to left), the curves diverge, showing that a fleet manager can make poor retirement decisions that impact the fleet's utility per cost ratio. As the fleet size shrinks, the shape of the Pareto curves shifts which is due to the fixed cost distribution function. Maintaining a constant fixed cost distribution function but varying the fleet retirement scenarios always results in local maxima (optimality condition). This result is valuable to fleet managers because it recommends a minimum practical fleet sizing solution. For example, this simulation shows a maximum utility per cost ratio that can be achieved for a fleet size of 30 aircraft.

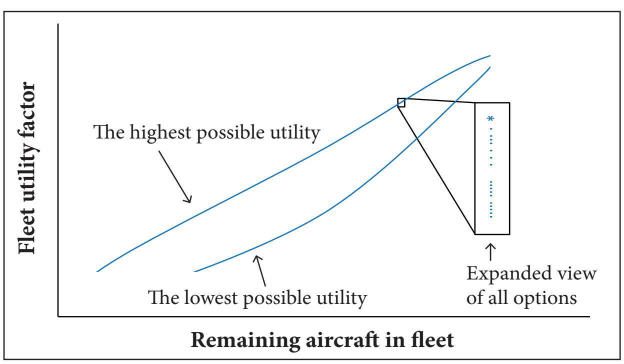

Figure 5. High and low utility choices for various fleet size options with expanded view of all possibilities.

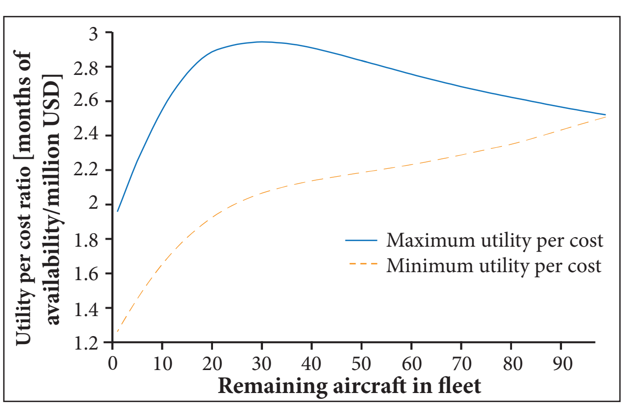

Figure 6. High and low utility per cost of Pareto fronts for various valid fleet size options.

A-10 CASE STUDY

A realistic retirement scenario for the USAF A-10 fleet (2016 active fleet) sought to reduce the fleet size to simulate the closure of a base. Right-censored A-10 data were provided by the USAF and were used as demographic data for FARM. Maintenance man-hour data were provided for each active tail number for each month for fiscal years 1995 to 2015 (66,172 total observations). Figure 7 shows 2 different percentile categories for the distribution of man-hours and the median line of the aircraft in the set. For example, the median number of maintenance man-hours for a 14 year-old A-10 was approximately 100 h per month. The dashed line is a 3% growth prediction, which validates the relationship between aircraft age and maintenance burden for agile aircraft investigated by Dixon and the Project Air Force (U.S.) (2006). The A-10 maintenance man-hour data increased at a rate of approximately 3% per year. A 1-way ANOVA confirmed this age effect (factor: aircraft age; dependent variable: maintenance man-hours; p-factor = 0.014). A 159 USD labor cost rate derived from USAF depot cost data was applied to the man-hour data for illustrative purposes in the case study. Fixed cost and variable cost values were derived from the USAF's Total Ownership Cost Tool (Robbert *et al.* 2013).

The USAF also provided mission capable rates as a utility measure for use in FARM simulations. These data were recorded monthly for each active tail number for the years 2009 – 2015 (2,792 observations). The mission capable rate was a reasonable utility metric to use for the A-10 because it is a function of failure frequency, which represents asset reliability (Balaban *et al.* 2000). The mission capable rate data did fluctuate in response to funding changes, upgrades and operational conditions. During the data collection period, for example, the A-10 fleet underwent a system life extension program that altered the

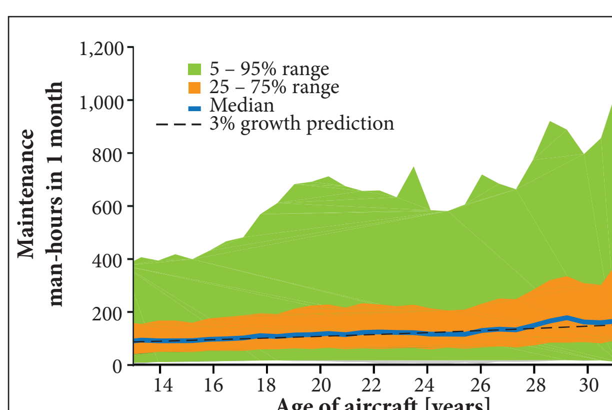

Figure 7. A-10 maintenance man-hours historical look.

mission capable rates of the fleet. These fluctuations in the data were useful for testing the software.

The data from maintenance man-hour (cost) and mission capable rate (utility) were input functions to FARM. Given that information, simulations were run to determine which aircraft would be chosen for retirement. For the active fleet of 349 A-10 aircraft, FARM produced the cost minimization output (Fig. 8) and the utility maximization output (Fig. 9) for the decision period of 5 years. Although not shown here, the accompanying outputs list the serial numbers that should be retired for each desired end-strength fleet size.

The cost-minimization objective function results (Fig. 3 and Fig. 8) exhibit different shapes. This is due to the variance in the cost data inputs ($\sigma_{A\text{-}10} > \sigma_{\text{model}}$) and emphasizes the potential advantage to this method's approach in identifying weak assets in a capital equipment fleet. Also, the expanded view in Fig. 9 highlights the inhomogeneity of utility factors in the actual A-10 fleet. The groupings of solutions occur in the expanded view result because the utility input data possess groups of aircraft with low factors, probably due to major corrective maintenance on some serial numbers during the data collection period. Fleet managers must be aware that a low utility factor may be the result of corrective maintenance or upgrades, which may make an asset less desirable in the interim but more desirable in the future. FARM allows managers to cater the utility function to reflect this, and recently improved aircraft are not identified for retirement.

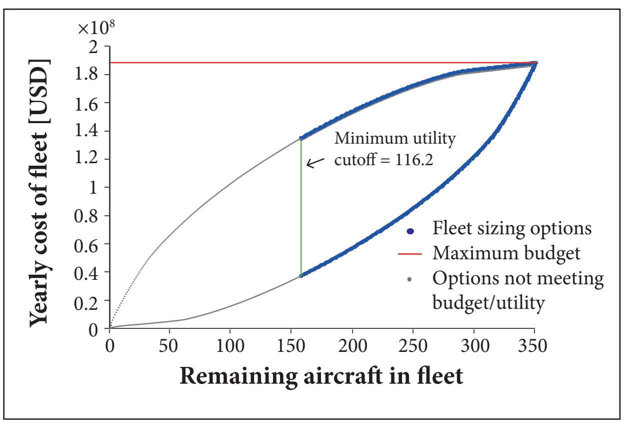

Figure 8. A-10 cost of fleet for various valid fleet size options.

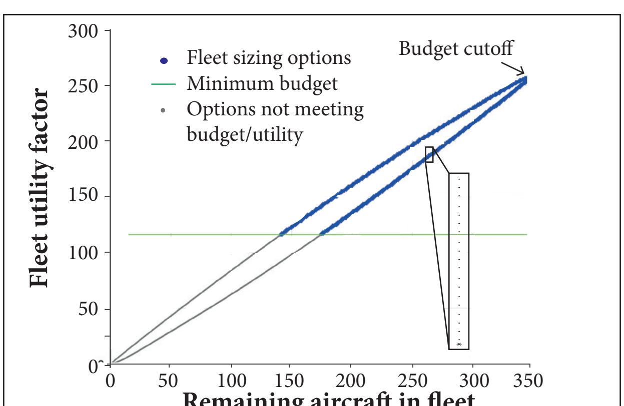

Figure 9. A-10 utility of fleet for various valid fleet size options.

DISCUSSION

FARM experiments revealed several tenets important for retirement policy analysis, namely that the inputs drive the results, uncertainty dramatically reduces the model accuracy and the earlier retirement decisions have the greatest impact on lifetime fleet cost and utility. Further, using the greedy algorithm enabled a computationally fast asset retirement model so that each of these tenets could be explored.

The shapes of the input functions directly impact the results. For example, if aircraft cost linearly increases as a function of age, then the oldest aircraft (the most costly) are indicated by the greedy algorithm for retirement first. However, real fleets exhibit more complex input functions so FARM's value increases as the fleet complexity increases.

Once uncertainty is entered the retirement model framework, a fleet manager must be careful about forecasting aircraft that would be candidates for retirement in future years. In year 1, the retirement suggestion is a direct representation of the initial cost and utility inputs. In the following years, uncertainty in cost and utility forecasts grows, therefore making future year retirement decisions mere predictions, worsening with time. Cost uncertainty is shown in Fig. 10. One facet of this uncertainty is the effect of short production runs. For a wide distribution of aircraft ages, FARM results show a finite solution. As the aircraft production timespan decreases, the retirement prediction confidence decreases. This occurs because the cost differences between individual capital assets decrease, thus making assets less distinguishable, particularly with confidence intervals. Retirement planning should be updated yearly with more recent cost and utility functions to lessen the uncertainty.

FARM shows that it is more important to make the right retirement choices from the start. Retirement policy errors propagate through time, making the initial net present value decision an assumption of future net present value. Retiring an asset with more future potential than a neighboring asset will affect the cost baseline in each subsequent year.

For generic fleets, FARM shows that the costliest aircraft possessing the lowest utility should be retired first. Actual fleet data show that the oldest serial numbers sometimes are not the costliest, least useful aircraft because of usage variation. This is the most basic reason for using a methodology like the one developed for FARM in retirement analysis.

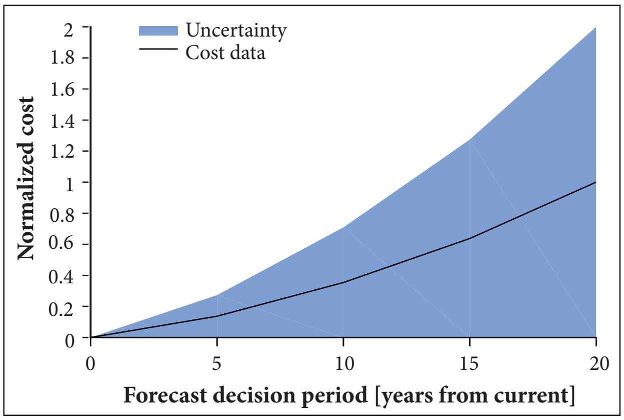

Figure 10. Uncertainty growth for FARM decision periods.

VALIDATION

Sensitivity analysis showed accurate model response to a wide range of reasonable variable and function inputs. FARM calculated fleet retirement options for both very large and very small fleets but the results were most valuable to real-world fleet sizes in the tens to hundreds of aircraft. Computation time for all scenarios described in this article was below 60 s, and the principal component affecting run time was the fleet size. A summary of run times for relevant USAF fleet sizes is shown in Table 1. The model's big *O* notation is: $O(n^2)$.

The model was developed using assumed values from previous studies but was validated using data from the United States Air Force's Logistics, Installations and Mission Support Enterprise View repository. F-16 Fighting Falcon and A-10 Thunderbolt II data validated the general forms of the cost and utility models. One necessary step for validating the model was to catalog and analyze the aircraft serial numbers recommended for retirement to ensure the model accurately identified the weak assets. The model was found to produce repeatable results, recommending the same serial numbers for retirement given static input conditions. Likewise, whether the fleet manager wanted to retire n aircraft or some multiple of n, the sequence of retired serial numbers remained the same.

Table 1. Model run times for sample fleet sizes.

Fleet size	Run time (s)*
16	3.2
100	4.2
160	5.5
320	11.3
500	22.5
1,000	95.2
2,000	567.6

*Intel Core 2 Duo, 3 GHz, 16 GB RAM.

To determine model efficacy for an actual retirement scenario, the fiscal year 2013 retirement of 41 A-10s was analyzed. More aircraft were retired during this wave, but this validation effort focused on the 41 aircraft sent to retirement and ignored those aircraft reassigned as maintenance and egress trainers. The decision process to retire the 41 aircraft began in December 2011 and continued until early 2013. The FARM model was fed with cost, utility and demographic data about the fleet in the years preceding and including 2012. Using the utility per cost ratio metric and allowing FARM to choose 41 aircraft for retirement, 19 (46%) FARM choices matched the USAF ones. Using just the cost metric resulted in 17 matches (41%) and just the utility metric resulted in 15 matches (37%). These validation results do not necessarily suggest that the choice of aircraft in the 2013 retirement wave was based on a utility-per-cost metric. The stakeholders involved in the retirement used a risk-based analytical process followed by other metrics and subjective determinations to select aircraft (Thomsen *et al.* 2011).

A second A-10 retirement population was evaluated to test the model. However, the 2011 retirement wave only consisted of 9 serial numbers. Of that group, 7 were reassigned to non-flying duties allowing only 2 serial numbers for model validation. The model would have retired 1 of those 2 aircraft, but the small population size limits the value of the finding. Due to the lack of additional aircraft fleet retirement data, no further validation analyses could be conducted. Retirement decisions are complex, with many subjective factors; but a simple tool that can provide decision-makers with a starting point for choosing serial numbers shows the value of this methodology. In the case of the 2013 retirement wave, FARM would have provided an initial list that was nearly 50% accurate when compared to the final one.

A fleet manager could employ any of the 3 retirement strategies (cost minimization, utility maximization or utility per cost maximization) used in this study. To show validity, each strategy was compared to the others for both the A-10 case study and for a virtual fleet. In each case and as expected, the named strategy outperformed the remaining ones. Figure 11 shows how the 3 strategies for the A-10 fleet compare with each other for the utility-per-cost maximization strategy. The similarity between the utility-per-cost maximization and cost-minimization strategies (Fig. 11) evidences why the 2013 retirement data match well for those 2 strategies.

Other validation plots show greater stratification between the 3 strategies. This shows the value of giving the fleet manager multiple objective function options.

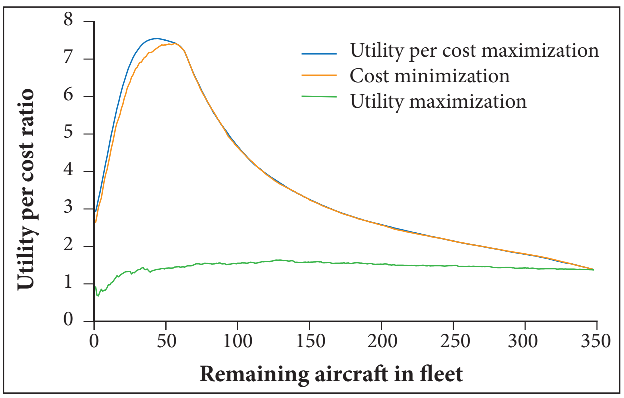

Figure 11. Comparison of retirement strategies for utility per cost ratio.

CONCLUSIONS

This study applied a greedy algorithm to an aircraft fleet retirement decision. It answered the question of which individual aircraft serial numbers should be retired and in what order. The hallmarks of this study were the use of inhomogeneous utilization histories for parallel assets and decision period forecasting. The methodology developed herein showed applicability to a virtual fleet as well as to the current USAF A-10 fleet. It was found that the correlation between usage history and retirement susceptibility could be better understood by fleet managers. The managers can control utilization levels of their assets to prolong or accelerate deterioration, which ultimately impacts the retirement schedule. Because fleet planning is a multi-year forecast, using a tool like FARM to make forecasts and periodically update them is more useful than one with a limited or finite horizon. Since suboptimal early retirement decisions cannot be remedied, a robust retirement policy is necessary.

This methodology can inspire future research in several ways. First, the methods may be extended to similar fields where parallel assets have unique usage histories. Though the objective function may change and the greedy algorithm may not present the globally optimal solution, this approach may fit into other domains. Further, other domains may also wish to study the retirement problem with non-like assets. Second, this methodology did not accommodate decision-makers with complex needs. Only cost minimization, utility maximization and utility-per-cost ratio maximization were considered. An amalgamation of weighted fleet priorities could be applied to this methodology, which can better satisfy some fleet managers. Lastly, future research might expand the scope of this methodology to include multiple aircraft mission designs in the retirement analysis. The F35A Joint Strike Fighter, for example, was designed to replace both the USAF's F-16 and A-10 aircraft. Fleet managers may be interested in evaluating which mission design should be retired first and in what quantities.

AUTHOR'S CONTRIBUTION

Newcamp J and Verhagen W conceived the idea for the study; Udluft H contributed to the methodology section and assisted with code generation; Curran R edited the text and provided the scope for the research. All authors discussed the results and commented on the manuscript.

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