THE COLLEGE OF AERONAUTICS
CRANFIELD

SUPERCRISSON COMBUSTION STUDIES

I. DESIGN, CONSTRUCTION AND PERFORMANCE
OF A HIGH-ENTHALPY FACILITY

by

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SUMMARY

The alternative methods of providing a high-enthalpy stream of air for supersonic combustion studies are discussed, and the reasons given for the decision to design and construct a pebble-bed heater at Cranfield.

Much of the report is given over to the design of the facility and necessary instrumentation. The limiting mass-flow and temperature operating conditions are outlined, and details of a typical supersonic combustion test-section are included.

Operating experience and problems are discussed and the calculated performance of the facility is compared with that actually obtained.
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1. Introduction

The advantages of employing supersonic combustion as the mode of heat release in hypersonic propulsive systems have been much publicized in recent years, and it is generally recognised that the scramjet (supersonic combustion ramjet) is the only feasible air-breathing propulsive system for high flight Mach numbers.

Since the scramjet becomes clearly superior to other air-breathing systems in the flight speed range of Mach number 8 to 10, it is assumed that this will be the order of flight velocity of any first generation transport employing supersonic combustion. Hence, assuming optimum diffusion of the free-stream air, the Mach number at entry to the combustor would be 3.0 to 3.5, although combustion at lower values is obviously of great interest since the scramjet is likely to be in operation from the upper limit of the first stage system (e.g. turbomachinery), up to optimum conditions. Similarly it can be shown that for the likely operating altitude (100,000 feet to 140,000 feet), the static pressure within the combustor will be of the order of atmospheric.

From the above it is possible to specify the range of test conditions required, in order that conditions within the scramjet combustor be realistically simulated. Briefly, these test conditions indicate the need for combustor inlet air with the following properties,

(a) Mach number from 2.0 to 3.5
(b) Static pressure of about 15 to 30 p.s.i.abs.
(c) Static temperature greater than that required for the auto-ignition of the fuel (e.g. 580°C for hydrogen).

The combination of the above properties indicates a need for a supply of air at a stagnation temperature of at least 1800°K, and if possible several hundred degrees higher, and at a stagnation pressure of at least 120 p.s.i.abs. To produce test-air at these stagnation temperatures and pressures, there are four main types of installation possible:

(a) Arc-heater
(b) Shock-tube
(c) Vitiated-air system
(d) Regenerative heat exchanger.

At first sight the use of an arc-heater for producing high-enthalpy air is very attractive, for example, the process is clean and easily controlled. Unfortunately, the existing power lines at Cranfield would not carry the added burden.

In considering the shock-tube, one is faced with a choice between transient and steady-state testing conditions, that is, between running times of milliseconds or minutes. In view of the type of research programme envisaged and of the excellent research facilities available in the Propulsion Department at Cranfield (for example the compressed-air supply), a steady-state type of facility was decided upon. This would obviously set an upper limit on the temperature to which the supply air could be raised, a limit not imposed when a shock-tube is employed, but in this type of problem where heat
transfer and mechanical integrity will play so large a part, the view of reference (1) is supported.

In the vitiated-air system, fuel is added to the air-stream and ignited, and the heat so released raises the enthalpy of the total flow. Oxygen is later added to the stream to replace that consumed during combustion. The resulting high-enthalpy gas stream can then be said to be a mixture of air and combustion products (for example, water vapour). In this particular application where there are so many unknowns, it was thought that the introduction of further complexities, such as the determination of the thermodynamic properties of the mixture, would make the use of the vitiated-air system undesirable. The final choice therefore was for a regenerative heat exchanger of the pebble-bed type.

2. Pebble-bed Heater

2.1 General Description

The layout of this type of regenerative heat exchanger is shown in Figure 1. As can be seen, the facility consists of a pressure vessel lined with several grades of refractory materials and with a bed of randomly packed refractory pebbles at its centre, as illustrated by Figures 2a and 2b.

The ceramic elements are heated by means of the kerosine burner shown mounted in the roof of the heater. The combustion products from this burner are blown down through the pebble-bed and are then exhausted to atmosphere. The heating cycle usually lasts three to four hours until the required temperature distribution is obtained. At this point the kerosine burner is closed off, the exhaust valve closed and test-air, supplied from the compressor house at the required pressure, is passed upward through the bed. This test-air, heated by the pebbles during its passage through the bed, is then discharged through the appropriate test-section.

2.2 Choice of Ceramic Material

It is obviously desirable for the air entering the test-section to be at the highest possible stagnation temperature, since it is the available stagnation temperature which is likely to limit the operating Mach number at the test-section. With this consideration in mind this facility was originally envisaged as utilising zirconia elements. The maximum operating temperature of zirconia is in the region of 2500°C. Unfortunately, it was found that the cost of zirconia bricks and pebbles for the size of facility contemplated, would be prohibitive.

The obvious alternative to zirconia was alumina, for although alumina has a maximum operating temperature of only 1800°C, the cost of producing the lining and pebbles in alumina was found to be only about one-third of the cost in zirconia.

Several manufacturers were approached for their estimate of cost and recommendations with regard to the design of a pebble-bed heater. It soon
became apparent that Messrs. Norton Abrasive Ltd., has considerably more experience to draw upon in view of their customers in the U.S.A.

3. **Heat-exchanger Design**

3.1 **Size of Facility**

In considering the size of facility required, it is necessary to take into account both the mass flow of air which is to be heated and the temperature to which it must be raised. The limit in air mass-flow which can be heated in any particular facility is reached when the aerodynamic lift force acting on each pebble exceeds the pebble weight and the pebbles "lift-off". This limiting mass flow of air can be calculated in the manner indicated in Appendix 1, but it is obvious that all other things being equal, the limiting mass will be proportional to the bed area.

The temperature to which the air is raised during its passage through the bed is obviously a function of the residence time within the bed (i.e. bed depth), and the physical properties of the bed such as the pebble diameter. This consideration is dealt with in the section on heat-transfer, Appendix 2.

The bed dimensions decided upon from economic as well as performance considerations, were a diameter of 30 inches and a bed depth of 90 inches. This volume gave a pebble mass of 2 tons. The bed diameter was later modified to 28 inches when it was discovered that a considerable saving in cost could be made by accepting this dimension. This was due to the fact that moulds for the brick lining of a 28 inch diameter bed were already in existence. Once this inner dimension was fixed, calculations for the heat transfer away from the bed in a radial direction could be made, and requirements for the outer layers of refractory, and for the shell dimensions fell naturally into place.

A pebble diameter of 0.5 inches was finally decided upon from all of the considerations listed above. Obviously, improved heat transfer could be obtained with a smaller diameter pebble but possibly only at the expense of mechanical integrity. The pebbles were purchased from the Aluminium Company of America (ALCOA) as supplied in this country by Hydronyl Ltd. The Alcoa pebbles were considerably cheaper than those supplied by any other manufacturer.

3.2 **Refractory Lining**

The insulating lining is made up of individual units in three layers as shown in Figure 1. The inner layer is constructed of Norton RA 5190 high density alumina in small interlocking shapes. The lower half of this layer is self-supporting from the floor and the upper half is supported from the steel ring welded inside the pressure vessel. Thus the lower elements do not have to support all of the weight of the roof lining. The RA 5190 is backed by a layer of Norton RA 4058 bubble insulating alumina, and behind the RA 4058 is a layer of low-temperature firebrick.
Allowance for the appreciable thermal expansion of the lining is left between the third layer and the steel shell, and this space is filled with a resilient cement Detrick MW 711.

Figure 3 gives an overall impression of the size and layout of the facility.

3.3 Support Grate

The support for the pebbles is shown in Figure 4. This consists of 1" diameter Incaloy D.S. round bars supported by four Incaloy D.S. support bars bolted together in the configuration shown. This total assembly is mounted into a support ring which in turn is set in a cast base of Norton 33-1 castable cement, and through which the load is transmitted to the pressure vessel. Stress calculations made on the Incaloy support bars indicate that they will withstand the load at temperatures up to 1000°C. More importantly, creep calculations indicate that the bars have a life expectancy of up to 10,000 hours operation providing that their temperature does not exceed 900°C.

3.4 Pressure Vessel

The pressure vessel was designed to encompass the pebble-bed and the three layers of refractory brickwork with a necessary allowance for thermal expansion. Although the heater was to be used at a pressure of 165 p.s.i.abs. for the early supersonic combustion tests, the steel shell was designed to withstand a pressure of 265 p.s.i.abs. which should be the available air supply at a later date. The pressure vessel was constructed to B.S.1500 1958 Class 1, and the steel used in the construction was to B.S.1501-151, Grade C. Heat transfer calculations indicated that the shell would operate at 450°F, which value is well within the 700°F limit specified. From Figure 1 it can be seen that a steel support ring is welded to the inside of the shell to support the roof bricks.

The large support ring required to hold the Incaloy grate bars was transported to the shell manufacturers (Messrs. A.J. Riley Ltd. of Batley) and was located within the vessel before it was completed.

3.5 Heater Burner

Reference (2) describes a regenerative heat exchanger and a heater-burner which utilizes propane as a heating fuel. Propane is obviously a very good fuel for this type of use, as it is easily ignited and easily controlled. Unfortunately, propane is expensive to burn in the quantities required by the above facility. Added to this, there is an existing kerosine supply to each of the test cells in the Propulsion Department. Thus it was decided to construct a kerosine/air burner capable of heating the mass of pebbles and lining. This burner was based upon the 'Shell' toroidal burner design as shown in reference (3). With this type of burner a proportion of the inlet air is passed outside the burner head with an imparted swirl. The remaining air is passed through the burner head and is used to atomize the fuel. Ideally, the resulting flame should be intense and toroidal in shape, but in practice it was found that the machining and location of the atomizing slit was
very critical, much more so than in the original kerosine/oxygen toroidal burner. A mock-up of the combustion space in the pebble-bed heater was built and extensive tests carried out on burner adjustments and flow rates. The usual kerosine flow-rate is within the range 12 to 18 gallons per hour.

The burner design currently being used is illustrated by Figure 5. The whole construction is of brass which is, of course, heavily cooled since it is left within the pebble-bed heater at all times.

The kerosine burner is ignited by means of a spark-ignited propane torch which is withdrawn after a kerosine flame is initiated.

3.6 Exhaust Cooling

Towards the end of the heating-up cycle the hot gases leaving through the outlet at the bottom of the unit are at a temperature near to 900°C. However, as the control valves used in the existing air-lines will not tolerate temperatures higher than 200°C, it is obvious that the exhaust gases must be cooled. This is brought about by means of cooling water sprays inside the exhaust pipe. Cooling water is forced at 150 p.s.i., through two fuel atomizers which produce two conical sheets of cooling spray, one pointing upstream, and the other downstream. The exhaust gas temperature is monitored by means of two thermocouples, one as a probe in mid-stream and the other as a surface plug in the wall of the exhaust pipe.

4. Instrumentation

The temperatures of the pebbles, brickwork and grate support are monitored by means of the five platinum/platinum-13% rhodium and twenty-one chromel/alumel thermocouples, located in the positions indicated by Figure 6. Two chromel/alumel thermocouples are welded to the outside of the steel shell and two further chromel/alumel thermocouples are used to indicate the exhaust duct temperature during the heating-up cycle.

The platinum thermocouples are cemented into holes, drilled ultrasonically, in alumina pebbles. It is expected that these thermocouples will fail after a few runs but it is hoped that sufficient operating experience will be gained during this period. After the thermocouples have failed, an indication of the operating temperature can still be obtained from the Land optical pyrometer which measures the pebble surface temperature and which is located as shown in Figure 6.

The optical pyrometer and thermocouple readings are all recorded by means of a Honeywell multi-point potentiometric chart recorder. The full cycle of this recorder takes approximately 2½ minutes to complete, but this period is quite reasonable in the context of the several hours taken to heat up the facility.

As failure of the cooling water to the several cooled points would be likely to lead to the failure of the complete rig, care has been taken to indicate the cooling water pressure at each point. Pressure switches have
also been fitted to these points so that a warning light will indicate failure of
the cooling water pressure, and as a further safeguard a Klaxon has been
installed to indicate the failure of the water pressure at any of the cooled
sections.

The heater-burner air supply, both atomizing and secondary, and the
kerosine supply, are all monitored in the usual fashion with the aid of mano­
meters, orifice plates and rotameters.

Air at high pressure for test runs is metered by means of an orifice
plate and the mass-flow is indicated both by manometer and by a transducer
giving a permanent record on a chart recorder.

Some difficulty has been experienced in accurately assessing the temper­
ature of the outlet test-air. At the high temperatures experienced, the cor­
rection for radiation, conduction and velocity for a probe thermocouple is
quite large. Even at the relatively low temperatures ($\approx 1850^\circ$K) produced for
our early experiments, the recommended maximum for the continuous use of
platinum/platinum-13% rhodium has been exceeded with a resulting drift in
calibration. Work is going on in constructing a line-reversal pyrometer for
use when the maximum thermal potential of the facility is realised. A sodium
injector and quartz viewing windows have already been incorporated into the
existing transition section between the heater outlet and the test section. This
pyrometer will utilize a photomultiplier as the sensing device and the final
reading will be taken as the point of balance between the radiation at 5890-6A
from the background continuum provided by a tungsten strip lamp, and the
intensity of radiation from the sodium injected into the stream. Unfortunately
this technique cannot be used for our earlier tests since the air temperature
for these runs is below the lowest level at which the line-reversal system will
operate. In the meantime a further thermocouple pair are to be tried, namely
iridium-40% rhodium/iridium. This thermocouple has a recommended maxi­
mum continuous operating temperature of $2000^\circ$C but is unfortunately very
expensive and very brittle.

5. Operating Precautions

In the last section there is a description of precautions taken to warn of
the failure of the cooling water to the various cooled points. Two further
safeguards against the failure of the facility are described below.

a) Blow-Off Valve

With the water-cooled kerosine burner remaining within the pebble-bed
heater at all times, there is some danger of a rapid rise in heater pressure
if the burner cooling jacket failed and cooling water was suddenly discharged
on to the hot pebbles. As a safety precaution, a blow-off valve has been
installed at the bottom outlet from the bed, and the result of such an
occurrence should simply be that the pebbles would be packed a little closer.
b) Stand-By Air

If the air supply should fail, for any reason, at a time in the heating-up cycle when the thermal storage approached a maximum, there would be a very real danger that the support bar temperature would exceed the maximum allowable, before some of the energy could be blown away. To allow for this possibility, provision has been made for the storage of a quantity of air to be used as blow-down air.

6. Test Section

This particular facility obviously has a number of applications to combustion or high-speed flight research, and the number of useful experiments which require a stream of high-enthalpy air increases from year to year. However, the purpose for which this pebble-bed heater was constructed was to carry out research into supersonic combustion, and the first test section is shown in Figure 7.

This test section is in the form of a Mach 2 convergent-divergent axisymmetric nozzle with a central fuel injector. The nozzle runs choked at a stagnation pressure of 113 p.s.i.a. and discharges at a static pressure of 14.7 p.s.i.a. Fuel, hydrogen or methane, is injected into the hot Mach 2 airstream and ignites spontaneously. Studies are then made of the mixing, ignition and combustion of the mixture.

The problem of cooling the stainless steel nozzle and injector presented some difficulty. In particular, boiling occurred in the central injector until the cooling water pressure was increased to 400 p.s.i. The heat-transfer rate in the region of the throat has been calculated and found to be at the high value of $1.8 \times 10^6$ B.Th.U/s/ft$^2$/hr.

In these early experiments a water-cooled transition section has been used, reducing the exit diameter from 5.5" diameter to the 3" diameter of the test section. Because of the obvious heat loss to this section an alternative transition section is being cast from high grade alumina and will be positioned within the outlet branch of the heater. A simple calculation indicates that this alternative should result in an increase of approximately 50°C in the outlet air temperature.

7. Operating Experience

7.1 Performance

Trial runs were begun at relatively low temperatures of approximately 1400°K and for heating-up periods of only about 1 hour. However, as
operating experience was gathered and the facility was seen to function well, the operating temperature and the heating-up times were increased. The full potential of 2070°K has still not been reached, firstly because this is not necessary for our early tests, but chiefly because this temperature cannot be reached without either pre-heating the kerosine burner air or operating for part of the heating-up cycle on oxygen instead of air.

Air outlet temperatures of approximately 1850°K have been attained, and have proved to be sufficient to achieve ignition of hydrogen at the nozzle exit. Figure 8 is an example of the air outlet temperature obtained during a typical run, and of the variation of outlet temperature with running time.

The temperature-time profile shown in Figure 8 is for the case where the bed is heated up from cold. If the pebble-bed was warm at the beginning of the heating-up cycle, the temperature distribution in the pebble matrix would be severely modified, and this would probably result in a reduced outlet air temperature. The temperature gradient shown at the beginning of the test run is due to heat loss by radiation from the top layers of pebbles during the period between the end of the heating-up cycle and the beginning of blow-down.

The running times shown are for an air mass-flow of 1.7 lb/sec, obviously the running time available at any particular temperature would be increased if the mass-flow was decreased. Similarly if a greater mass-flow was passed through the bed (implying an increased operating pressure), then the temperature-time gradient would be steeper.

A comparison of the actual bed temperature distribution and air outlet temperature with the calculated values, shows a reasonable agreement. The difference shown is due to the thermal storage capacity of the brickwork which has not been taken into account in the calculations.

7.2 Problems

The first problem connected with the operation of the facility arose when a local maximum in the shell temperature was observed at a point about mid-way down the shell. Although this maximum did not exceed the allowable temperature for the shell, it was nevertheless thought to be undesirable. It was found to correspond to the position of the steel support ring welded to the inner surface of the vessel and was obviously due to the enhanced conduction of heat through the support ring. To reduce the shell temperature locally, a narrow water cooling jacket was attached to the outside of the heater shell in the region of the support ring. This measure was found to be effective but as an added safeguard, two more thermocouples were spot-welded to the outside of the steel shell.

Our second cause for concern arose when the pebbles "lifted off" at the beginning of one of the full-scale tests. This occurrence was as mystifying as it was drastic since the bed pressure was well below the operating pressure at the time. Upon investigation it was found that water, which had been used to cool the exhaust gases, had been trapped in the outlet pipe. Thus, when the cycle was reversed this water was carried into the bottom of the pebble-bed by the flow of test-air. The resulting sudden expansion of
steam so produced was equivalent to a mass-flow far greater than the maximum allowable. Fortunately the annular throat of the test-section has a dimension less than the pebble diameter, so that pebbles were not blown over the surrounding countryside. Even so, the outlet pipe of the heater was choked by pebbles, and this meant that the test section had to be removed and the pebbles pushed back into the main body of the heater before the test could continue.

The most recent of the problems associated with the operation of the facility still lacks a solution. This takes the form of severe cracking of some of the lining bricks. This phenomena is in no way connected with overheating of the bed, but has all the appearances of thermal shock. The manufacturers are still investigating this occurrence but have stated that it has not taken place in any of the U.S.A. facilities for which they have supplied the ceramics. The bricks shown cracked in Figure 9 are not load-bearing members and have been replaced as they are, for further test runs. If this problem is due to thermal shock it is surprising, since the bricks lower down the bed are subject to greater temperature gradients yet remain undamaged.

8. Concluding Remarks

A regenerative heat exchanger has been designed and constructed, and has been used successfully for research into supersonic combustion.

The facility has been operated at conditions of 1850°K, 120 p.s.i. and at an air mass-flow of 1.7 lb/sec, and there would appear to be no reason why these conditions should not be raised to 2050°K and 250 p.s.i. with an air mass-flow of at least 2 lb/sec when required.

The performance of the heat exchanger in terms of the temperature-time curve actually obtained, has been compared with the predicted temperature-time distribution (i.e. temperature "droop"). It is seen that the heat exchanger performance obtained in practice is close to the theoretical value for short durations, but that the temperature droop is far less than that predicted for runs of longer duration. This bonus in performance is assumed to be due to the thermal storage capacity of the alumina lining to the pebble-bed.

9. Acknowledgements

Acknowledgement must be made of the considerable assistance given at every stage by Shell Research Ltd., of Thornton, Cheshire, and in particular by Dr. C.G. Haupt of that establishment.
10. **References**


11. APPENDIX 1.

Air Mass-Flow Calculations

As the compressed air supply in the Propulsion Department is more than adequate for the purpose of the proposed supersonic combustion studies, the pressure loss, as such, through the pebble-bed is of no immediate concern. However, the pressure loss per unit depth of the bed is also an indication of the lift forces operating on the pebbles, and therefore limits the air mass-flow which can pass through the bed without pebble "lift-off" occurring.

Reference 4 gives the pressure drop per foot, thickness of a granular bed as:-

\[
p = \frac{48 \rho}{p \text{Re}^{0.15}} \text{ poundals/ft}^2/\text{ft} \quad \text{(1)}
\]

where \( p \) = pressure drop per foot (also written as \( \frac{\Delta p}{L} \))

\( \rho \) = gas density

\( V \) = apparent gas velocity (volume flowing per sec / bed area)

\( d_p \) = pebble diameter

\( \text{Re} \) = Reynolds number based upon pebble diameter.

The above correlation holds for a solids content \( \lambda = 0.6 \), where \( \lambda \) is defined as the packing density / bulk density. For granular materials with irregular shapes this value of 0.6 is found to be representative, but for smooth spheres the bed will settle down to a value of \( \lambda \) approaching that expected for "perfect packing", i.e. \( \lambda = 0.7 \). Reference 4 indicates the use of a correction factor to allow for the change in \( \lambda \). This is taken to be equal to 3.3 for the purpose of these calculations.

Equation (1) is also limited to turbulent conditions, for this configuration this is interpreted as being for flows with Reynolds number greater than 400. The calculated Reynolds number based upon the pebble diameter was found to be 450 for this facility.

Hence

\[
\frac{\Delta p}{L} = \frac{48 W^2 \cdot 3.3}{g \cdot \rho \cdot \left( \frac{\pi D^2}{4} \right)} \frac{\text{lb/ft}^2}{\text{ft}} \quad \text{(2)}
\]
and since \( \text{Re} = \frac{\rho g V dp}{\mu g} \) where \( \mu \) is the dynamic viscosity of the gas.

\[
\Delta p = \frac{7.67 W^{1.85} \mu^{0.15}}{\rho g D^{3.7} dp^{1.15}}
\]  \hspace{2cm} (3)

This method is described in greater detail in Reference 5 which also contains plotted curves of the various power terms.

From a consideration of the pressure drop per unit bed depth, an evaluation of the ratio of the lift force to pebble mass must be made, since it is at the point where these two quantities become equal that fluidization will occur.

For perfect packing it can be shown that the ratio

\[
\frac{\text{lift force}}{\text{pebble mass}} = \frac{\Delta p}{L} \times \frac{1.655}{\rho_p}.
\]  \hspace{2cm} (4)

\[
\therefore \frac{\text{lift force}}{\text{pebble mass}} = \frac{12.7 W^{1.85} \mu^{0.15}}{\rho_p \rho g d^{1.15} D^{3.7}}.
\]  \hspace{2cm} (5)

In practice a value of 0.5 for the lift/mass ratio has been found effective and Figure 10 is a plot of the allowable air mass-flow for varying pressures and temperatures with the value of the lift/mass ratio taken as equal to 0.5.

Reference 6 quotes an alternative correlation by Leva which appears to have a wider range of application than that given above. However, for this particular application mass-flows calculated from both correlations are similar.

In general the various assumptions made all tend to be on the conservative side, which is as it should be, considering the enormous stored energies involved.
12. APPENDIX 2.

Heat-Transfer Calculations

Two types of heat-transfer calculation were necessary for the design of this high-enthalpy facility; (a) structural, and (b) performance. The first set of calculations were of the orthodox conductivity type, and were necessary for an estimate of the thickness of insulation to be made, in order that the working temperature of the steel shell could be maintained within reasonable bounds. This form of calculation is straightforward and need not be recorded here. It is interesting to note however that the shell temperature did not reach the calculated value. This is most probably due to steady-state heat-transfer conditions being assumed.

The second type of heat-transfer calculation required was necessary for deriving the rate of heat-transfer from the pebble matrix to the air flowing, and hence for calculating values of the outlet air temperature as a function of the blow-down time. It is possible to treat this problem in terms of finite-difference relationships as in Reference 5. However, this method is laborious and a reliable approximation known as the Schumann method is available in Reference 7. Lancashire et al in Reference 8 have used an extension of the Schumann method, together with calculated parameters from Reference 9, in order to determine the heat-transfer coefficient $h$ for particular sets of conditions.

In the design of this pebble-bed heater realistic values of $h$ were assumed and used in the Schumann method to calculate temperature-time variations at certain points down the bed and to determine the temperature-time relationship for the outlet air. By comparing the calculated relationship with that derived experimentally, the accuracy of the assumed heat-transfer coefficient can be determined.

**SCHUMANN METHOD**

This method is based upon several simplifying assumptions:
(a) Homogeneous pebble matrix
(b) Incompressible flow
(c) Constant fluid specific heat
(d) Constant and uniform fluid velocity
(e) Conduction in the radial direction is infinite
(f) Conduction in the direction of flow is negligible
(g) Thermal diffusivity of the matrix is assumed large and hence the only resistance to heat-transfer between pebbles and air will be in the film.

Schumann establishes generalised time and position variables thus:

$$\tau = \frac{h A}{m_B c_{p_B}} \left[ t - \frac{m_f}{m_f} \cdot \frac{x}{L} \right]$$  \hspace{1cm} (6)
and \( Z = \frac{h A}{m_f C_{p_f}} \cdot \frac{x}{L} \) .. (7)

where \( h \) = heat-transfer coefficient  
\( A \) = heat-transfer surface area in bed  
\( m_B \) = mass of pebble matrix  
\( m_f \) = mass of fluid resident in the bed  
\( \dot{m}_f \) = mass-flow rate of fluid  
\( C_{p_B} \) = specific heat of solid  
\( C_{p_f} \) = specific heat of fluid  
\( t \) = time elapsed from beginning of blow-down  
\( x \) = distance from plane of entry  
\( L \) = total bed depth

From consideration of an element of the matrix, an energy balance which includes the contribution due to convective heat-transfer, leads to the following differential equations.

\[
\frac{\partial T_B}{\partial \tau} = T_f - T_B
\]  
\( \text{.. (8)} \)

\[
\frac{\partial T_f}{\partial z} = T_B - T_f
\]  
\( \text{.. (9)} \)

where \( T_B \) is the bed temperature  
and \( T_f \) is the fluid temperature.

The solution of the above equations is quite lengthy but leads to the following expression,

\[
\frac{T_{B_1} - T_{f_2}}{T_{B_1} - T_{f_1}} = e^{-z} + e^{-(z+\tau)} \sum_{s=1}^{\infty} \sum_{r=0}^{s} \frac{z^{s} \tau^{r+s}}{s!(r+s)!}
\]  
\( \text{.. (10)} \)

where the subscripts 1 and 2 indicate conditions at the beginning and end of the time interval respectively.

If \( \theta_1 = T_{B_1} - T_{f_1} \)

and \( \Delta T_f = T_{f_2} - T_{f_1} \)
\[
\frac{T_{B_1} - T_{f_2}}{T_{B_1} - T_{f_1}} = 1 - \frac{\Delta T_f}{\theta_1}
\]

Solutions of the above expression have been calculated for a range of \(Z\) and \(\tau\) values and a figure has been constructed with coordinates of \(\tau\) and \(\frac{\Delta T_f}{\theta_1}\) and with lines of constant \(Z\).

Hence for a particular set of conditions and predetermined time intervals \(\tau\) and \(Z\) values can be calculated and by interpolation, \(\frac{\Delta T_f}{\theta_1}\) can be determined.

Calculation of the gas outlet temperature involves some advanced knowledge of the temperature distribution in the pebble matrix, or at least the need to make a good guess. For the temperature-time curve given in Figure 8 the matrix temperature distribution was approximately constant at 1850\(^\circ\)K for the top 5'-5" of the bed, with a linear variation from 1850\(^\circ\)K to 350\(^\circ\)K for the bottom 21'-0" of the bed.

For the purpose of the above calculations the pebble-bed is considered to be divided into convenient sections, in this instance 1'-0" deep, and the gas temperature at the inlet and outlet of each section is calculated, up to the point where it emerges from the top of the bed. This process is repeated for time intervals from 1 to 20 minutes.

The calculated temperature-time distribution, determined in the manner outlined above, is compared with the measured distribution and the process repeated for different values of the heat transfer coefficient \(h\), until a reasonable agreement is obtained. Figure 11 shows a comparison of the measured outlet temperature with the calculated distribution based upon a value of 10 CHU's/ft\(^2\)\(^\circ\)C for \(h\). The calculated distribution is not strongly dependent upon \(h\) but the agreement shown in Figure 11 is the closest which could be obtained.

This value of 10 CHU's/ft\(^2\)\(^\circ\)C is considerably less than that determined from the correlation suggested by Reference 8,

\[S_t \, Pr^{2/3} = 0.4 \, Re^{-0.437}\]

where

- \(S_t\) is the Stanton number \(\frac{h}{G \, C_{pg}}\)
- \(Pr\) is the Prandtl number \(\frac{C_{pg} \, \mu_g}{k_g}\)
- \(G\) is the specific mass flow \(\text{lb/ft}^2\text{s}\)
k is the thermal conductivity.

From this correlation a value of 15.8 CHU's/ft$^2$°C is obtained for the conditions under consideration.

The value of 10 CHU's/ft$^2$°C for h is a mean value for the bed and must contain some contribution from the surrounding alumina lining. It is suggested that it is this contribution from the lining which leads to a smaller measured temperature droop over the longer running times, than that which was calculated.
FIGURE 1. LAYOUT OF PEBBLE-BED HEATER
FIGURES 2a and 2b. VIEWS OF THE CERAMIC LINING OF THE PEBBLE-BED HEATER AND OF THE PEBBLE MATRIX
FIGURE 3. GENERAL VIEW OF FACILITY
FIGURE 4. PEBBLE-BED SUPPORT BARS
FIGURE 5. KEROSINE HEATER-BURNER
OPTICAL PYROMETER

NOTE
NQ2 THERMOCOUPLE POSITION AT 90° TO NQ1 AND 3 AND AT SAME LEVEL.

FIGURE 6. POSITION OF THERMOCOUPLES
FIGURE 7. SUPERSONIC COMBUSTION TEST SECTION
TIME REQUIRED TO REACH MAXIMUM OPERATING TEMPERATURE, DEPENDING UPON DELAY BETWEEN HEATING-UP AND BLOW-DOWN CYCLES

OPERATING CONDITIONS:
- PRESSURE 115 p.s.i.
- MASS-FLOW 1.7 lb/s

FIGURE 8. VARIATION OF OUTLET AIR TEMPERATURE WITH BLOW-DOWN TIME
FIGURE 9. FRACTURED ALUMINA ELEMENTS, THOUGHT TO BE DUE TO THERMAL SHOCK
FIGURE 10. MAXIMUM ALLOWABLE AIR MASS FLOW THROUGH BED
FIGURE 11. COMPARISON OF CALCULATED AND MEASURED AIR OUTLET TEMPERATURES

OPERATING CONDITIONS

PRESSURE 113 p.s.i.
MASS-FLOW 1.7 lb/s