Robust scheduling in an uncertain environment

M. Wilson

Robust scheduling in an uncertain environment

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Preface

With great satisfaction, and, admittedly, more than a little relief at being finished, I am writing this preface to my thesis. It has been a long and at times difficult journey resulting in something I am very proud of. I've sometimes heard the finishing of a thesis or the defence of one being referred to as giving birth to one's baby. Being a young father, I can certainly see some of the parallels. But the differences are many, not in the least the fact that delivering a thesis is (fortunately!) physically much less demanding than delivering a baby. Another striking difference is that the delivery of this thesis marks the end of an academic journey, whereas the true journey only begins after one's child is born.

Many people assisted me during this journey, for which I am grateful. I want to start by thanking Arjan van Gemund, the supervisor for my master's thesis, who started me on this journey by steering me in the direction of a vacancy for a PhD student at the Algorithmics group. This is where I met Cees Witteveen, to which I owe many thanks. Cees was always available for advice, and we had many in-depth meetings discussing papers and chapters in progress, which benefited greatly from his unwavering attention to detail. I also want to thank Tomas, who joined in supervising me as co-promotor, at a slightly later stage. I enjoyed our long discussions on the experimental analysis of algorithm and on the proper design of experiments. I may have sighed at the extra experimental work his critical eye caused, but at the end it is clear that the results I have obtained are much improved. I also enjoyed our discussions on teaching, lecturing and education in general.

The other colleagues in the Algorithmics group made my time as PhD student very pleasant. I want to thank my office mates over the years: Pieter, Renze, Adriaan and Jeroen, for giving me lots of survival tips for beginning PhD students, some of which I've followed with good results, others which I've ignored to my own detriment; Joris, for his happy moods and his love for his beautifully ugly car, but also for ensuring that "our room" was ranked at the top of the indoor karting competition; Gleb, for the interesting discussions on language, and for all the odd discussions on Dutch culture and life in general; and of course Simon, more or less my "successor" for this research project, with which I've had lots of very interesting and deeply technical discussions on scheduling topics, after each

of which the whiteboard looked even more arcane, but also on operating systems, desktop user interfaces, and typesetting software. And of course I want to thank all the other group members, Chetan, Hans, Léon, Marijn, Mathijs, Matthijs, Peter, Shruti, Sicco, Yinqian, for all the engaging conversations that took place during lunch or over coffee. I especially want to thank Peter for all the extended coffee breaks, over which we discussed science, academia, life after academia, and life in general. And of course I want to thank Ronald, Simon and Jonathan for working with me on their Master thesis, contributing to my research greatly, and Erik, Jan, Erwin and Wilco for their work on the scheduling demonstrator tool for their bachelor project.

I also am indebted the many people at NedTrain who helped me in my research: Leo van Dongen for heading the research program as board member of NedTrain, but also Bob Huisman for his day-to-day leadership in the research program, for the many insightful discussions we had on my research, his useful feedback on my writing, and his social insight into some of the people I interacted with. The atmosphere among the group of people doing research at NedTrain was very warm and cooperative, and I want to thank Joachim and Jorge, the other PhD students in the group, as well as all the Master students which worked at NedTrain, for the many enjoyable lunch walks we took, and for the broad ranging discussions on all the differing fields of research that were presented in our little group. The ideas and suggestions by Fred van Houten, Geert-Jan van Houtum, Leo Kroon, and Rob Basten, prompted by my presentations during the meetings of the steering group committee, also helped me in my research. My work with Pim Op 't Land and Mariëlle ten Have on data analysis of the on-board diagnostic system of the VIRM train series gave me interesting insight into the workings of the maintenance procedures at NedTrain. I enjoyed being able to visit some of the NedTrain workshops, and I want to thank the engineers I met there there, for being patient enough to explain their work to another one of those 'theoretical guys' visiting them.

I am glad to have friends that did not grew too bored with me when I talked about what occupied my mind, and I'm also glad for all the times when they succeeded in getting my mind otherwise involved, be it by going to concerts, having dinner, playing a game, or just by having a good talk. I want to thank my parents as well, for supporting me all this time, and for bearing with me during the explanations of what, exactly, I was doing all this time. And of course, I want to express my gratitude to Simone, for her love, for always being at my side and for believing in me at all times: even when I was in doubt myself you never lost your trust in me. And, I want to thank Emma, our dear daughter, for bringing joy in my life, and for teaching me lots of new things about flexibility in planning.

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Introduction 1

In this chapter, an introduction will be given to the research presented in this thesis. The research is part of the rolling stock life cycle logistics applied research and development program funded by Dutch railway industry companies. We will start this introduction with a bit of historical overview, to illustrate some of the reasons why such a program is needed in what seems to be a mature and well-developed industry. After this, we will discuss the general structure and operation of the railway organizations in the Netherlands. Having knowledge of the context in which the research is performed is important, because this context has a significant influence on the direction of the research, and on the choices which are made.

After discussing the railway organizations, the research program of which this research forms a part will be described, along with the problems and context inspiring the research. These problems will then be used to formulate a set of research goals that are used to guide the research in this thesis.

1.1 A bit of historical context

Before turning to an explanation and motivation of the exact research presented in this thesis, we start by taking a step back, to examine some of the historical aspects of railway organizations, in general, and applied to the Netherlands in particular. We do this to answer a question the reader might have about the *why* of research on railway asset maintenance operations: what is the necessity of performing this research, besides the obvious one of wanting to continually improve upon the state of the art?

This is a valid question, since there is a long and rich history of railway operations. The earliest known evidence of railway operations dates back to the 6th century BC in ancient Greece, in the form of a 6 km stretch of railway used to transport boats in Corinth (Lewis, 2001). Starting in the 16th century, narrow-gauge railways started to see a lot of use in the mining industry (Agricola, 1556). But the real breakthrough came in the form of the steam engine. Early steam engines used low-pressure steam, which made them large, bulky, and unsuitable for non-stationary use. The beginning of the 19th century saw the

invention of high-pressure steam engines in Great Britain by Richard Trevithick, among others (Kirby et al., 1956, p. 175–177). This led to a rapidly developing railway industry: in the 1950s more than 7 000 miles of track had been laid in Britain alone (Wolmar, 2009, p. 99).

The railway sector is a mature industry, in part due to its age, and this would indeed suggest that the problem of maintaining railway assets has been studied extensively. But if we examine the more recent history of, in this case, Dutch railway operations, an important change in operations is currently occurring. In the past, the Dutch national railways developed rolling stock in very close cooperation with Dutch industry. If we restrict ourselves to train sets, this already started with the development of the first electrical train set in 1924 (Mat'24, 259 coaches produced in total), and continued with the development of the first streamliner train sets (the diesel-electrical Mat'34, as well as the electrical Mat'35, Mat'36 and Mat'40, totalling 508 coaches). For all of these train sets, the Dutch railways collaborated closely with the Dutch companies of Werkspoor, Allan and Beijnes in both design and construction (van Gestel et al., 1992, 1989).

This trend continued after the war, which saw a lot of engines and train sets being confiscated, not to be returned, or outright destroyed. The new train series Mat'46 was developed to replenish the supply of rolling stock (418 coaches total), together with an effort to electrify a major part of the Dutch railway network. Again, Werkspoor, and to a lesser extent, Allan and Beijes, were involved. Both 1954 and 1964 saw the introduction of newly designed train sets (Mat'54, 428 coaches, and Mat'64, 616 coaches), with Werkspoor as the main supplier. Due to capacity problems, part of the production at Werkspoor was outsourced to Düwag (sold to Siemens in 1989), in Germany. In 1972, Werkspoor ceased its activity in the construction of railway stock, and its activities were taken over by Talbot (taken over by Bombardier in 1995) in Germany (van Gestel et al., 1997a,b).

More recent train sets developed in close cooperation with the Dutch railways are the series SGM (Talbot, 1975, 240 coaches), ICM (Talbot, 1977, 482 coaches), DD-AR/DDZ (Talbot, 1991, 308 coaches) and V-IRM (Talbot/Bombardier, 1994 and later, 872 coaches). Note that all these series, from Mat'24 all the way to V-IRM, were developed in cooperation with, and exclusively for, the Dutch railway. The year 2009 therefore marked an important change, with the introduction of the SLT (Bombardier/Siemens, 648 coaches). At first glance, this seems to be a train set developed by familiar companies: Bombardier acquired Talbot, which was the company to take over the activities of Werkspoor in '72, and Siemens acquired Düwag, which was tasked with producing a part of the Mat'64 train series. A key difference however is that the SLT series is an "off the shelf" train set, so to speak: it is based, in large part, on the German train set BR 425, developed by Siemens and Bombardier (van Gestel et al., 1997a, RailWiki (DDAR, VIRM, SLT)).

This change in acquiring rolling stock—from actively cooperating with industry in the specification of requirements and design, towards a role as buyer of an existing design, albeit with adaptations—also brings important changes towards maintenance operations (van Dongen, 2011). An evident one is the gaining of technical knowledge required to perform maintenance. Cooperating

in the development of a train set leads to a large build-up of technical knowledge, whereas, when buying an existing design, one has to rely on the manufacturer of that design to impart technical knowledge on maintenance engineers. A somewhat more subtle point is the interaction between train design and maintenance capabilities: the capabilities of the maintenance organization can have influence on design choices. Familiarity with certain procedures will be reflected in design choices, which will ease the complexity of maintenance operations.

Compounding the situation above is the increase in technical complexity of trains. Due to technical progress, trains are computerized more and more, and contain highly complex electrical systems for functions like traction, braking and current conversion and control—systems that used to be largely mechanical in nature in the past. In addition, more and more systems of a facilitating nature are present in a train: HVAC systems, passenger information systems, on-board internet.

In light of the trends of buying existing train designs, and the increasingly complex technical nature of these systems, it is evident that an applied research and development program focused on the changing maintenance operations surrounding these trains has merit, for more than the usual reason of a continuous improvement upon the current state of the art—there is a clear change in the environment in which maintenance operations take place.

1.2 Railway organizations

Another important part of the environment of a maintenance organization is the system of railway organizations around it. To gain a better understanding of how NedTrain operates, we look at how the system of railway organizations in the Netherlands is structured. In Figure 1.1, the major railway organizations in the Netherlands are depicted, together with their relationships.

It is important to keep in mind that this organizational structure has been subject to some large changes throughout history. As in many European countries, the railway industry initially consisted of several different pioneering companies. Gradually, these companies consolidated through a series of mergers and take-overs, leading to the establishment of the *Nederlandse Spoorwegen* (NS, the Dutch Railways), starting as a rate harmonization collective, which transformed to an interest group in 1917, and to a state-owned company in 1938. This process started to reverse again in the period between 1995 and 2002, when the European trend of privatization emerged, ending in the current situation.

In this situation, the railway infrastructure is organizationally separated from the companies using it. All infrastructure is owned by ProRail, which is still a state-owned company. ProRail is responsible for ensuring railway safety, managing the railway timetables for all combined traffic, and for infrastructure maintenance and construction. Actual maintenance and construction work is not performed by ProRail itself, but by several subcontractors working under directions of ProRail. Examples of subcontractors used are Strukton, BAM and VolkerRail.

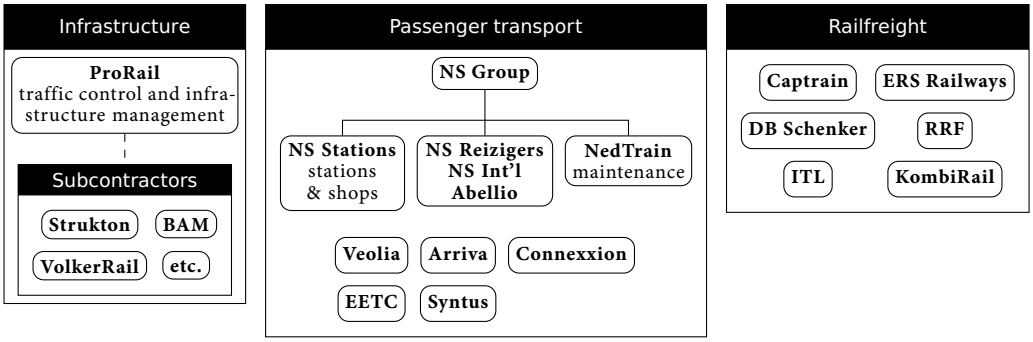


Figure 1.1
Railway organizations in the Netherlands.

NedTrain is a part of NS Group, which is the largest railway operator in the Netherlands. NS Group can be decomposed in three important parts: railway station exploitation, train maintenance and of course passenger transportation. The railway station exploitation is performed by NS Stations. This division is tasked with the design and construction of new railway stations, the renovation and modernization of existing railway stations, and with the managing of, for example, shop concessions in the larger railway stations.

The majority of passenger trains running on the Dutch railway network are operated by NS Reizigers (NSR). Other NS Group subsidiaries active in this field are NS International, which operates international and high speed lines, and Abellio, which operates some railways in the United Kingdom and in Germany. Other passenger train operators active in the Netherlands are Veolia Transport, Arriva (part of Deutsche Bahn), Connexxion, Syntus (partially owned by NS) and Euro-Express Treincharter (EETC, operating several seasonal international trains). Other users of the railway network are the various railfreight operators, such as Captrain (owned by SNCF, the national railway company of France), DB Schenker Rail (owned by Deutsche Bahn, of which the former NS Cargo is now part), ERS Railways, and Rotterdam Rail Feeding.

The other company in this group is NedTrain, which performs rolling stock maintenance. NedTrain is currently contracted to perform maintenance for the rolling stock used by NSR and NS International. The fleet used by these two divisions has a size of around 3000 passenger carriages, of which some 250 are under some form of maintenance at any one time. These maintenance activities are scheduled with a time horizon of roughly two weeks, in which somewhere between 20 and 30 trains are serviced in a single workshop, with approximately 40 tasks per train, resulting in a schedule containing between 800 and 1200 tasks.

Maintenance operations are spread out over 37 locations throughout the Netherlands (see Figure 1.2 for a map). A distinction can be made between service locations and workshop locations; both perform different types of maintenance.

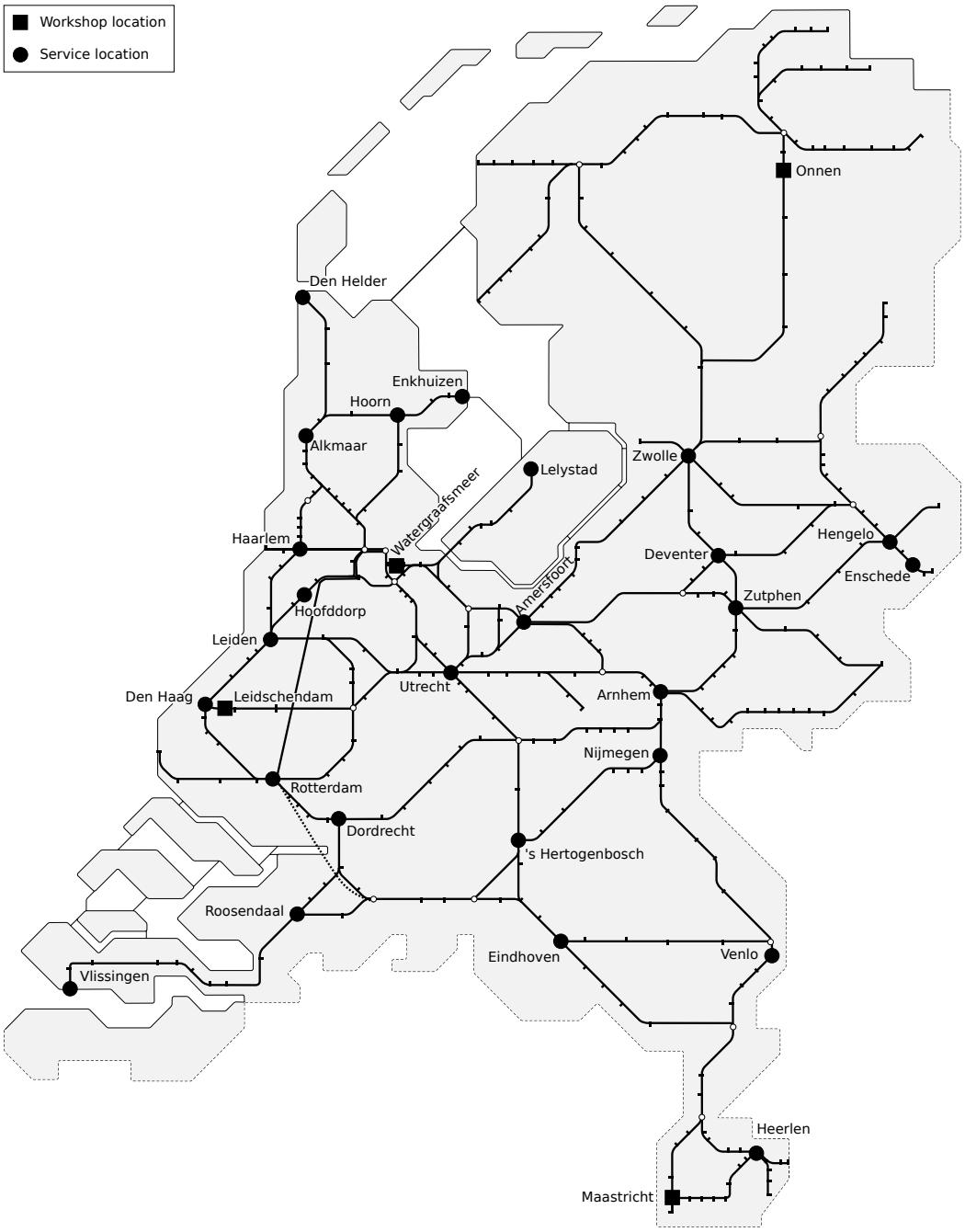


Figure 1.2
A map of the Netherlands, showing all (passenger) railway connections. All service and maintenance locations of NedTrain are marked on the map.

- At a *service location* ('servicebedrijf' (SB) in Dutch) day-to-day maintenance is performed. Such maintenance consists of cleaning, safety checks and minor repairs, which are usually performed overnight in an single shift of eight hours. Service locations are marked with small circles in Figure 1.2.

- At a *workshop location* ('onderhoudsbedrijf' (OB) in Dutch) periodic maintenance is performed. Roughly every three months, a train undergoes a mostly fixed set of preventive and corrective maintenance actions. The usual duration of such maintenance is approximately three days. Trains that break down in service and cannot be repaired at location are also sent to service locations. In Figure 1.2, workshop locations are marked with small squares.

  Workshop locations are also used to perform refurbishments. Such refurbishments are only executed once or twice in the lifetime of a train, to significantly extend its usable life.

The core task of NedTrain is guaranteeing fleet availability in the broadest sense of the word. Firstly, this means that trains should have as few breakdowns during service as possible. To ensure this, NedTrain has to focus on the quality of their maintenance work. The time at which maintenance is performed is important as well: a train should not be left running too long without maintenance, since this raises the risk of a breakdown, lowering availability. On the other hand, frequent maintenance has a negative impact on fleet availability as well. A second aspect impacting availability is the number of trains undergoing maintenance at the same time, and the length of the maintenance visits. If maintenance can be performed quickly, fewer trains will be out of service for maintenance simultaneously, increasing availability for passenger transport. Such an increase might mean that fewer trains are needed to guarantee a certain passenger transport capacity. But, note that short maintenance times are only worthwhile if they can be relied on: otherwise, the schedule of NSR will be disrupted, or insufficient passenger transport capacity will be available, resulting in fines for NedTrain.

Maintenance activities are largely scheduled by hand. This currently is a feasible approach, since (scheduled) maintenance takes place only every three months, and it is possible to schedule these activities with a lot of slack. In such a three-monthly visit, a lot of tasks are clustered in a single visit. Durations of the individual tasks that need to be performed in these visits are uncertain, but the number of tasks is large, which reduces the variance. Combined with the allowed slack in scheduling, this results in a feasible mode of operation. If this slack is not enough, as a last resort it is sometimes possible to omit certain maintenance activities.

However, there is a push for higher efficiency, caused by various desires in the organization with respect to maintenance operations:

- There is a desire to fragment maintenance visits into multiple, smaller, visits, which can then be scheduled in off-peak periods, so that more rolling stock is available at peak periods.

- There is a desire to increase the efficiency of maintenance operations by reducing slack, so that a train in the workshop has less idle time.

- And lastly, there is pressure to avoid the last resort option of omitting activities.

To be able to support these wishes, it is important to have automated systems that can support the scheduling of maintenance activities.

1.3 Research program at NedTrain

The previous two sections outline some of the changes NedTrain is facing with regard to its maintenance operations. The acquisition procedure of rolling stock has changed—from a cooperation between operator and constructor, where knowledge was transferred in both directions, towards a buyer/seller-relationship in which knowledge is shared only reluctantly. This change introduces new challenges in the field of maintenance: the structure, tooling or knowledge of a certain maintenance organization might be ill-suited for the maintenance procedures as devised by the constructor. The scheduling of maintenance activities is also faced with new challenges due to the changing environment: the desire to split the large maintenance visits into multiple smaller visits increases the importance of having accurate and flexible schedules.

In an effort to address these problems NedTrain has initiated a broad research effort. The goal of the research program is to enable and support NedTrain in being a leading rolling stock maintenance company (Huisman, 2009). The direction is governed by the key performance indicators for the fleet:

- total cost of ownership, or life cycle cost;
- availability of sufficient transport capacity;
- reliability of transport; and
- quality of transport.

The program is divided into three separate studies, which are closely interrelated. The division is made using the various time scales of the processes involved in the procuring, supporting and maintaining of rolling stock, and separate studies are performed on strategic, tactical and operational aspects:

strategic At the strategic level, the decisions with the longest time scales are made. These are decisions on the buying of rolling stock and their specification—typically, rolling stock lasts about three to four decades, so that decisions on this level can have a long-lasting impact on performance levels. Another long-term strategic decision concerns the location of maintenance workshops and the available equipment.

tactical With a time scale of about one to five years—between the strategic

and operational level—the tactical level concerns the allocation and management of spare parts, human resources and maintenance tasks. With the introduction of new rolling stock comes the option of contracting out some of the maintenance work to the manufacturer. To evaluate the use of such an option, decision makers need to have a good understanding of how such changes in the supply chain influence the key performance indicators.

operational The individual jobs where the actual maintenance work on the rolling stock is performed lie on the operational level. The scheduling of these jobs is highly dynamic, owing to the dominant policy of condition-based maintenance: parts are only repaired or replaced once they are defective or once their condition has deteriorated below a certain threshold. This creates unforeseen fluctuations in workload and spare parts requirements, which are currently dealt with by having sufficient buffer stock, and by having a fair amount of slack in the maintenance schedule.

The strategic aspects are investigated at the University of Twente, where research is performed on assessing the supportability, in terms of maintenance and logistics, when acquiring new rolling stock (Parada Puig et al., 2011), and on the decision to repair certain components of the train in the workshop itself, or to replace them, and repair them in another more specialized shop. Tactical aspects are investigated at the Technical University of Eindhoven, in the form of a framework for spare parts planning and control (Arts, 2013).

This thesis will treat the last part, the operational aspects of maintenance. First, an overview is given on the current situation and practices, since these (partially) dictate some of the constraints on any approach used to solve the problem.

1.3.1 Operational aspects of maintenance

In the current situation, the maintenance workload for each workplace is set in advance in cooperation with NSR, and depends on the schedule of the rolling stock. NSR decomposes its schedule into three different levels:

rolling stock unit At the lowest level are the schedules of the rolling stock units themselves ('materieeleenheid'). Each such unit is (for practical purposes) indivisible, and uniquely identified by a number.

path A path, sometimes (confusingly) called train, is the smallest indivisible task a unit can carry out. A *regular path* connects two major nodes in the railway network together and is used for the actual transport of passengers. There are also *maintenance paths*, these lead from and to maintenance facilities. Each path is to be executed by a rolling stock unit of a specified type. Note also that only one unit is to be assigned to each path. If passenger flow dictates higher capacity than a single unit can provide, the schedule contains additional paths departing and arriving at the same time in the same locations.

circulation The circulation ('omloop') is at the top of the scheduling structure. They connect paths together, such that they can be used to allocate the rolling stock units. Two paths can be connected to each other in a circulation if the arrival station of one path is identical to the departure station of the other, and if the arrival time precedes the departure time. Each circulation has a length of roughly three months, and has a maintenance path at the start and at the end, such that timely maintenance is automatically performed.

As an example, take the 1723 train; this is the 07:08 intercity service departing at The Hague Central Station, arriving at Enschede at 09:32. In the train schedule, this train is connected to some other train, departing from Enschede after 09:32. Such a pair of trains (t_1, t_2) is called a (*regular*) *transition* (Maróti and Kroon, 2007): both t_1 and t_2 are planned to be carried out by the same rolling stock unit, and t_2 is the earliest task after t_1. In this fashion, a chain of trains is formed into a circulation. The rolling stock type of these trains (and hence of the circulation) is ICM ('Intercity-materiaal'). The 4011 is an example of such a unit, and could be assigned to execute the train services in this circulation. Since the usual length of a circulation is about three months, ending in a visit to a maintenance location, each rolling stock unit visits a maintenance location every three months if the schedule is executed without any disturbances.

In practice however, this rarely happens. Delays cause disturbances, which then might cause material units to be switched to a different circulation. Therefore, it happens often that a unit that needs maintenance has no maintenance path scheduled in its circulation at that time. Additionally, when breakdowns occur, a unit might need to undergo extra maintenance at the earliest possible opportunity. To schedule maintenance visits, the circulation is modified with *interchanges*. For this, a small number of regular transitions n is considered, of the form (t_i, t_i') with $1 \leq i \leq n$, where the arrival stations of all trains t_i is the same, such that the time intervals between arrival and departure of all trains overlap. Two train units i and j participating in these transitions can be interchanged, by removing their regular transitions and adding two custom transitions (t_i, t_j') and (t_j, t_i'). By extending this idea to more than two trains, and by applying it in more than one location, interchanges can be added to divert rolling stock units needing maintenance to a maintenance path in time. An example is depicted in Figure 1.3: Some paths (the thick lines) on (parts of) three circulations (the thin lines) are displayed. The rolling stock unit assigned to the top left path has developed a defect, and custom interchanges (the arrows) are added, to move the unit to the path on the lower right, which leads to a maintenance location. It must be noted that this approach only works for certain non-fatal defects, since the rolling stock unit must still be capable of executing the passenger service of the path leading towards the interchange to the maintenance path.

While the intricacies of how a rolling stock unit arrives at the maintenance location are not the main focus of this research, it is important to keep some of the details in mind. They impose some additional constraints on how the problem is solved. Predictions on when units will visit a maintenance location

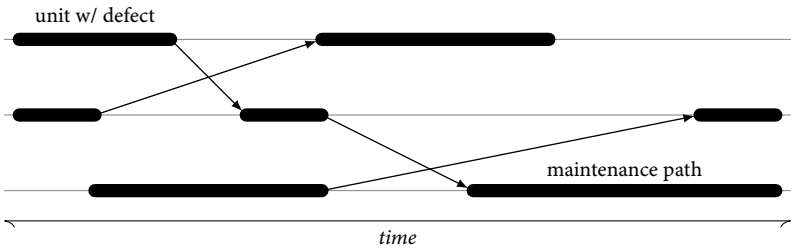


Figure 1.3 An example of the interchange method, where one rolling stock unit is routed to a maintenance path (adapted from Maróti and Kroon, 2007).

are based on the circulation they are currently running on. This applies to unscheduled maintenance work as well (EBK, 'extra binnenkomst' in Dutch), and in this sense, unscheduled maintenance is scheduled as well, but only at a very late moment. The circulation method also highlights the importance of meeting the deadlines: if the maintenance is not finished in time, the unit cannot depart the maintenance location to resume its service. If maintenance is not ready in time, an interchange can be performed by switching the unit scheduled for departure with an equivalent unit of the same time which might be finished earlier than planned.

1.3.2 Scheduling of maintenance

The actual scheduling of maintenance jobs is currently a labor-intensive task. The base schedule is the same for every week. It contains arrival and departure time of rolling stock units of a certain type, dictated by the circulations which pass through a maintenance location. Using the type of the rolling stock unit, a generic set of tasks is included for each unit. Roughly two weeks in advance, the exact unit which will arrive in a slot is known: around this time a unit needing maintenance is routed to a circulation sending it to a maintenance location in one or two weeks time, using the interchange method described above. Additionally, it is 'flagged'; this means that this unit should not partake in any interchanges, since this would disturb its path towards the maintenance facility. A disturbance this late could cause difficulty adjusting the path back using interchanges, to prevent this from happening the flagging method is used.

If problems have been reported for this unit, or if part of the maintenance of last time was not completed, some maintenance tasks are known in advance. The rest of the tasks to be performed is usually not known in advance. This might seem odd, since there is obviously a maintenance plan for each rolling stock series describing which tasks need to be performed at which interval. The difficulty however is that the tasks in this plan are often loosely defined in terms of either time or resource requirements. The time requirements might change with the amount of resources allocated to the tasks, or with the type of resources allocated. Time requirements might also be location-dependent: at one workshop, there might be more experience with some task, leading to lower time requirements. But the biggest problem is that most maintenance tasks are condition-based, and the condition of the rolling stock unit is generally

	No uncertainty	Uncertainty
No human recovery	Smooth shop *optimize*	Stress shop *support reactive scheduling*
Human recovery	Social shop *schedule as advice*	Sociotechnical shop *schedule as framework*

Table 1.1
A classification of scheduling situation based on *uncertainty* and *human recovery* (Wiers, 1997)

not known in advance. These tasks generally have the form of an inspection, followed by some (conditional) corrective action. Consequently, this leads to uncertainty with respect to the time needed to execute the task, and uncertainty with respect to the resources required, in terms of personnel, equipment and often parts as well. Collectively, the issues described above can be referred to as *task uncertainty*, and form one part of the scheduling problem.

Schedulers currently allocate workplace resources manually (i.e., tracks, possibly of special type, and personnel of different types) to the various (high-level) tasks in the base schedule. Any information on the actual tasks to be performed is of course used, but often, little is known and the schedulers have to rely on their own expertise to know how many maintenance engineers are needed to ensure a task is completed on time. Problems are compounded by the fact that the spare capacity that is theoretically available (per the base schedule dictated by the train paths) is usually fully used by unexpected extra arrivals. The schedulers must also contend with unexpected capacity changes, for example due to personnel taking sick leave. These issues form the second part of the scheduling problem, *resource uncertainty*. Note that the task uncertainty also contains an uncertainty with respect to resources. The difference is that the resource uncertainty caused by task uncertainty pertains to the resource *usage*, as opposed to resource *availability*.

1.3.3 A classification of scheduling situations

With the knowledge of some of the practical aspects of maintenance scheduling in the NedTrain workshop, we now turn to the work of Wiers (1997), in which the interaction between (automated) scheduling systems and human scheduling is analyzed. Wiers uses the dimensions of *uncertainty* and *human recovery* are used to distinguish between four stereotypical scheduling situations. Here, uncertainty refers to *task uncertainty* as discussed above: incomplete information about the tasks that need to be performed. Human recovery refers to the *ability* of the teams executing the schedule in the workshop to compensate for this uncertainty, by utilizing flexibility present in the schedule. Wiers makes a distinction between the *ability* of operators to correct for uncertainty (human recovery) and the *authority* they have to do so (autonomy).

In Table 1.1, the four stereotypical scheduling situations as described by Wiers (1997) are shown. The names of these situations refer to the distribution of autonomy between the (central) scheduler, and the teams executing that schedule

in the workshop. Each of the possible distributions has certain consequences for the type of scheduling information system employed.

In the case of NedTrain, there is both a high uncertainty in the tasks to be executed, as well as the opportunity for human recovery: given the autonomy, the teams executing maintenance tasks are often able to adjust the schedule based on information about the actual situation in the workshop. The sociotechnical shop is a good fit for this scheduling situation. In such a shop, schedules are used as framework for execution. They are not executed exactly, so expensive optimization of the schedule is to be discouraged.

1.3.4 The future of maintenance operations: some requirements

The system as described in this section of the introduction is currently in operation at NedTrain, and the research program of which this thesis forms a part is intended to provide methods for future improvement in maintenance operations. A reason for having a research program devoted to this improvement is the change from in-house participation in the design phase of new train sets to the procurement of train sets based on existing designs, as outlined in Section 1.1. The impact on the operational aspect of maintenance operations is expressed in higher task uncertainty. Separately, another important change to the operational aspect of maintenance operations currently being discussed at NedTrain is *off-peak maintenance*. Instead of taking a train out of revenue service for two days every three months, it might be more advantageous to take the train out of service for several shorter periods during those three months. If these periods are restricted to fall within off-peak times, this would lead to more trains being available at peak hours. This, in turn, would enable NSR to get by with fewer total trains, which could be a substantial cost saving.

For this concept to be able to work, tight and quick scheduling is of the essence. The schedules need to be *tight*, because the off-peak period is short. To make good use of such a short period for maintenance, there should be little slack in the schedule. Scheduling should also be *quick*: if disturbances cause a different set of trains to be at the workshop, it must be possible to adapt the maintenance schedule for that day quickly.

Current scheduling systems employed at NedTrain are not quick and flexible enough to adequately deal with the added requirements of off-peak maintenance operations. Assimilating new trains into the maintenance schedule is a time-consuming process, which is currently made to work by utilizing the fact that there is a lot of slack available to correct any deviations from the assumed work load of each train. Hence, there is need for a software system based on smart algorithms supporting the schedulers in a workshop. The system should be able to quickly form a work plan based on given information. The scheduler should be able to modify the resulting work plan, to take additional, implicit, preferences and constraints into account. A complicating factor is the potentially large processing time caused by the size and complexity of the problem. A maintenance schedule usually covers two weeks of maintenance work, in which between 20 and 30 trains are scheduled. Per train about 40 tasks are executed,

using various resources, leading to a complex schedule containing between 800 and 1200 tasks.

An important requirement is that the work plan should be *flexible*, in the sense that there is room for adaptation—it is unreasonable to assume that exact task information is present before commencing work on a train. When this information becomes available, for example if the need for extra work becomes apparent after an inspection has been performed on a certain component of the train, we want to be able to integrate this task into the existing work plan, without altering it. However, if an extra task, such as discussed above, or even an extra train, needs to be added to an existing work plan, it is clear that the desire for the plan to be unaltered will often be an unreasonable assumption. This leads us to the requirement of *robustness*: if the flexibility we have available is insufficient to integrate the extra tasks, we want the necessary adaptations to the work plan to be as minimal as possible.

A last requirement is that the execution of the work plan by independently working teams should be facilitated. Different teams operate in the same workplace concurrently—each team might be performing maintenance operations on a different train. The requirements of flexibility and robustness come into play here as well. A team desires a certain measure of independence in executing its part of the schedule, flexibility in the schedule is a property which can facilitate this. Robustness, to counter the effects of the uncertainty in the schedule, is especially important in a setting with multiple teams, as it is undesirable for delays in one team to affect the other teams.

1.4 Research goals

The problems above sketch a dynamic environment in which a schedule continuously evolves as new information comes in. Currently, the schedules contain a fair amount of slack to cope with uncertain information and changing circumstances. Trains visit a workplace for maintenance for a longer period than strictly necessary, which results in a sizeable number of trains being at a workshop at any one time on which no maintenance work is performed. Workplace utilization is therefore sub-optimal, but more importantly, fewer trains are available to transport passengers. To satisfy a given transportation demand, more rolling stock units are required. If, by reducing slack in the maintenance schedule, even one train less has to be bought, this represents a substantial saving. In some cases the available slack is still not enough to deal with an incident. In this case, a workshop can opt to omit certain tasks. It depends on the type of task and on safety regulations if this is possible. Schedulers start at a high level, with a baseline schedule based on the schedule used by NSR. As more information becomes available, this schedule is updated and refined, and resources are allocated to the tasks to be performed.

While this approach was satisfactory in the past, there is a demand to increase the efficiency of the scheduling process. Shorter maintenance times lead to higher availability of the rolling stock, which in turn allows a smaller fleet to

serve the same demand. As noted above, the cost saving of having a smaller fleet is significant. In the same vein, pilot studies are being conducted in which the current, long, maintenance visit is replaced by several smaller visits to the workshops—timed in such a way to avoid the rush hour, in which fleet availability demand is at its highest.

The scheduling process such as currently employed at NedTrain is based on the requirement of having a fairly large amount of slack in the schedule to be able to adapt to unforeseen circumstances. Additionally, it is a relatively labor-intensive procedure, which involves a lot of manual work. Such a process is well-suited if the tasks to be scheduled are of a static nature, but even in the current situation this is not the case. The timing of the depot visits is sometimes uncertain, but also the exact set of tasks resulting from a visit can be uncertain: inspection tasks can result in additional work. Frequent unexpected visits caused by breakdowns create additional uncertainty. The changing requirements to the scheduling process will increase the load even more.

This leads us to the first research goal, on the scheduling process itself:

RG1 The scheduling process employed in the NedTrain workshops relies on a large amount of slack to keep unexpected events under control. In addition, it is labor-intensive and inflexible. In what way can this process be at least partially automated? Note that automation alone is not sufficient: the process of constructing and adapting a schedule has to be quick enough to make it part of an interactive process, and it has to be flexible, to be able to adapt to the incomplete and changing nature of the environment.

Automation of at least part of the scheduling process is an obvious solution to decrease the labor-intensive nature of the process. This in itself can be a part of increasing the flexibility of the scheduling process: if the process to generate a schedule is quick, and if a user is able to influence the generation process by asserting different preferences, a set of different alternative schedules can be created. This also coincides with the idea that not all of the preferences of what constitutes a "good schedule" can be encoded in a formal way—some form of user intervention will always be necessary.

Having a portfolio of automatically generated schedules gives a form of flexibility: if unexpected events occur, switching to a different schedule might still result in meeting imposed deadlines, even if the slack in the schedule is to be reduced. But this seems a rather roundabout way of solving the problem: flexibility in the process of *creating* schedules itself is important, but another element of improvement is the flexibility in the schedules itself. If a schedule is very rigid, any change in one of the tasks necessitates a restart of the scheduling process, to partially or completely adapt to the new situation. In contrast, if the schedule is flexible as well, adaptation to such a new situation could be much easier.

RG2 Schedules, by their nature, prescribe a certain way to execute a set of tasks, giving rise to a form of rigidity. To be able to adapt to uncertain events it is

desirable to reduce this rigidity.

a) As a primary goal, we want to have a schedule which prescribes task execution in a *flexible* way, such that we can adapt to changing events without needing to revise the schedule, and without having to resort to using large amounts of slack.

b) As a secondary goal, we want the resulting schedule to be *robust*: if the available flexibility is not enough to be able to adapt to some events, we want the unavoidable revisions we have to make to continue executing the needed tasks to be as minor as possible.

A final challenge is the interaction between uncertainty and the structure in which the schedule is to be executed. Multiple independent teams in the workplace cooperate to execute a single schedule. Some amount of independent control for the teams operating in a workshop is desirable, especially in light of the sociotechnical shop scheduling framework as discussed in Section 1.3.3. A form of dependence between the teams is inevitable however, as they make use of shared resources to execute their tasks. This leads to situations where uncertainties in the schedule cause problems in one team to affect the other teams as well.

RG3 The need to use shared resources to complete tasks, combined with uncertainties in the schedule negatively affects the independence of the teams in the workshop. Our goal is to construct schedules such that there is more isolation between the teams, in terms of disturbances in the execution of the schedule in one team not affecting the other teams.

Summarizing the goals above, we want to devise a method with which schedules can be created, based on a *flexible process* (RG1), where the resulting schedules are both *flexible*, in the sense that they can easily be adapted (RG2a), and *robust*, in the sense that they are resilient to the effects of uncertain events (RG2b). Additionally, we want to improve the isolation between independent teams in terms of disturbances—a disturbance in one team should have no effect on other teams (RG3).

1.5 Outline and contributions

In Chapter 2, we will start by examining existing research in light of the research goals introduced above. It is shown that the scheduling problem at NedTrain can be mapped to the Resource-Constrained Project Scheduling Problem (RCPSP), a well-studied scheduling problem. Several methods for solving the RCPSP are described, and as it turns out, the Precedence Constraint Posting method (PCP), which uses the Simple Temporal Network (STN) as a flexible way of representing a set of solutions to an RCPSP instance, seems an especially good fit in light of the

first problem statement. Nevertheless, at the end of the chapter, several research questions are formulated in response to the other research goals. These research questions will serve as a basis for the following chapters.

In Chapter 3 a start is made in answering the second research goal, by examining how we can transform our intuitive notion of flexibility into a method to formally measure the flexibility of an STN. It is shown that existing methods do not take the full effect of dependencies between tasks into account, resulting in an over-estimation of the available flexibility. We present a new method to measure flexibility that does not suffer from this problem. This method also gives a partial answer to the last research goal, on isolating teams jointly executing a schedule—it turns out that our method to measure flexibility also results in an optimally decoupled schedule, which is a schedule in which constraints between tasks belonging to different teams have been replaced by constraints within the team itself, without losing flexibility. This work has been published in Wilson et al. (2013a) and has received a Distinguished Paper Award at the International Joint Conference on Artificial Intelligent, a major conference in the computer science community.

Chapter 4 examines the other half of the second research goal, on how to ensure that flexible schedules are robust as well. We examine different ways of using the available flexibility in an STN, and analyze the effect this has on the performance in terms of the robustness of the schedule. It is shown that distributing the flexibility in different ways can have a significant effect on performance. In particular, it is shown that sacrificing some total flexibility to improve the equality of distribution has a positive effect. This work has been published in Wilson et al. (2013b).

Chapter 5 looks at the second research goal in a slightly different way. The idea of being flexible in how a schedule prescribes the execution of tasks has so far been interpreted as being flexible in the temporal aspect of execution. Here, we examine if it is possible to also be flexible in the sequential aspect of execution. We show that it is possible to postpone some of the ordering decisions of creating a schedule to the time of execution, creating a new strategy of being flexible in the prescription of schedule execution. It is shown that this interpretation of flexibility also leads to schedules that are robust, as described by the second research goal. This work has been published in Wilson et al. (2012).

Finally, Chapter 6 summarizes the results presented in this thesis. The research goals introduced in this chapter are analyzed again, using the information and results from the preceding chapters. We conclude that the contributions made by this thesis constitute a theoretical framework on which a practical solution to these problems can be built. Nevertheless, while giving answers to the questions raised in this introduction, many more questions can still be raised. Therefore, we briefly discuss some of the possible avenues of future research prompted by some of those questions.

Existing work

2

In the previous chapter, the scheduling problem present at NedTrain was introduced, and we formulated three problem questions. In this chapter we will investigate existing scheduling research, with a focus on research that might help in answering these questions. Since there is a very large body of research with scheduling as the subject, we will start with formalizing the scheduling problem such as it occurs at the NedTrain workshops. It turns out that this problem maps very well to the Resource-Constrained Project Scheduling Problem, often abbreviated as RCPSP. Therefore, it is solution methods and algorithms for the RCPSP towards which we will turn next. We will pay special attention to Simple Temporal Networks (STNs), as these can be used to represent RCPSP solutions in a flexible way. Lastly, we will review the material discussed in the light of the problem questions presented in the previous chapter, and research questions will be formulated to address unsolved problems.

2.1 Resource-constrained project scheduling

Let us first consider the basic scheduling problem at NedTrain. It consists of a set of trains arriving and departing at a given time, with a set of maintenance tasks to be performed on each of those trains. Some ordering constraints might be present on these tasks, such that, for example, a cleaning task is carried out last, and to complete a task one or more resources may be required, such as engineers or special tools. The aim is to find a time assignment to these tasks such that all tasks are completed before the train departs, any ordering constraints are respected, and the capacity of the available resources is never exceeded.

As already mentioned in the introduction, the Resource-Constrained Project Scheduling Problem (RCPSP, see Hartmann and Briskorn (2010) for a good overview of the basic problem and a lot of common extensions and modifications to it) closely resembles this problem. Additionally, it is a widely-studied problem, both in operations research and in other research disciplines. The basic RCPSP considers a single project to be scheduled. The basic RCPSP consists of the following elements:

tasks A set T of *n tasks* to be executed,[1] where each task $t_i \in T$ has an associated *length* $l_i \in \mathbb{R}$. Each of these tasks maps to a maintenance operation to be carried out, in the case of NedTrain. The lengths will be an approximation in this case, since it is not always known in advance if a repair is needed, or how long the repair will take. Note that we will restrict ourselves to using deterministic task lengths.

precedences A precedence relation $\prec$ inducing a partial order on T; the intuitive interpretation of $t_i \prec t_j$ being that t_i has to be finished before t_j can start. The precedence relation is used to enforce certain orderings on maintenance tasks: for example, one task might specify the removal of a certain component, such that another component becomes accessible on which the second tasks has to be performed.

resources A set R of m renewable resources, where each resource $r_k \in R$ has integer capacity $\text{cap}(r_k) \geq 1$. These resources are used to model personnel and special tools, both with limited availability, with which the maintenance tasks are to be performed.

In some formalisms a distinction is made between *renewable* and *consumable* resources. Renewable resources become available again after a task using them has completed, and can be used to model, for example, engineers and tools. Consumable resources are lost upon completion of a task, and can be used to model materials that are used in the construction of some artefact. In our work, we assume such materials are always readily available in a warehouse, and as such we will not include them in the model—we will be dealing with renewable resources only.

resource consumption Lastly, the tasks need to be connected to the resources they require: for each $r_k \in R$ and $t_i \in T$, $\text{req}(t_i, r_k) \in \mathbb{N}$ specifies the amount that task t_i requires of resource r_k to be executed. If this number is zero, this particular resource is not required.

In addition to the elements of the basic RCPSP, a *release time* $s_i \in \mathbb{R}$, and a *due time* $d_i \in \mathbb{R}$ are associated with each task t_i, which are used to model, for example, the time at which the train enters and leaves the workshop. To model multiple trains, it can at times be convenient to use the Resource-Constrained Multi-Project Scheduling Problem (RCMPSP), which is in essence a group of RCPSPs, whereby the set R of resources is shared among all projects. Each of the projects then represents the maintenance work for a single train, and the individual projects have release and due times that represent the times at which the trains enter and leave the workshop.

A more formal definition of the scheduling problem at NedTrain can now be formulated as follows:

[1] Some scheduling formalisms allow the *pre-emption* of a task, meaning that the execution of a task can be stopped when it is in progress, and resumed at a later time, sometimes at the cost of slightly longer total execution time for that task. We will however assume that pre-emption is not possible—once a task has been started, it can not be stopped before its completion time.

Definition 2.1 An instance of the resource-constrained multi-project scheduling problem with release and due times can be specified using a tuple $\langle T, \prec, R, l, s, d, \text{req} \rangle$, where $T = \{t_1, \ldots, t_n\}$ is a set containing n tasks, $\prec$ a precedence relationship inducing a partial order over the tasks in t, R a set of k resources r_k, with $\text{cap}(r_k) \geq 1$ specifying the capacity of that resource. The function $l : T \to \mathbb{R}$ defines the length of each task $t_i \in T$, and the functions $s : T \to \mathbb{R}$ and $d : T \to \mathbb{R}$ define release times and due times for the tasks. Lastly, $\text{req} : (T, R) \to \mathbb{N}$ specifies the amount each task uses of the available resources in R.

Example 2.1 Consider the (fictional, and very short) maintenance visit of train 8604. This train will arrive in the workshop on Monday morning, at 08:00, and needs to return to revenue service at 12:00, after the tasks that are described below are performed:

- An exchange of the air compressor, due to air pressures being out of tolerance. Besides an engineer, a winch is required to do the heavy lifting. Roughly one hour of work is required.
- A routine check of the brake pad thickness, which takes 45 minutes if two engineers work together.
- A verification of the proper operation of the ATB system ("automatische treinbeïnvloeding", a train protection system), which is to be performed by an engineer by using specialized ATB testing equipment on the train. This check is expected to take around 30 minutes.

This can be modeled as an instance of the RCPSP, with a set $T = \{t_1, t_2, t_3\}$ of three tasks, for, respectively, the compressor exchange, the brake check, and the ATB verification. If temporal information is specified in minutes, relative to Monday morning 08:00, we have $s_1 = s_2 = s_3 = 0$, and $d_1 = d_2 = d_3 = 240$, to model release and due times such that the tasks take place when the train is in the workshop. We set $l_1 = 60, l_2 = 45$ and $l_3 = 30$ to model the lengths.

Several resources can be identified in the example: maintenance engineers, a winch to do heavy lifting on the compressor, and ATB test equipment. To model this, we set $r_1 = 2$, to indicate two engineers are available, and $r_2 = r_3 = 1$, to indicate one winch and one piece of test equipment. The resource consumption of the three tasks is modeled by setting $\text{req}(t_1, r_1) = 1$, $\text{req}(t_2, r_1) = 2$ and $\text{req}(t_3, r_1) = 1$ for the engineers, and $\text{req}(t_2, r_2) = 1$ and $\text{req}(t_3, r_3) = 1$ for the winch and test equipment, respectively.

Additionally, since the ATB system is tightly coupled to the brake system, there is a precedence constraint between the brake check and the ATB check: the latter can only be performed after the brake check has been completed, so we have $t_2 \prec t_3$.

The goal of the RCPSP is to find a *schedule* for an instance of the problem. Such a schedule is a function $\sigma : T \to \mathbb{R}$, which assigns start times to all the tasks $t \in T$, which satisfy all the constraints in the problem: release and due times are respected, precedence relationships hold, and resource capacities are not exceeded.

Definition 2.2 A *solution* to an RCMPSP $\langle T, \prec, R, l, s, d, \text{req} \rangle$, also called a *schedule*, is a function $\sigma : T \to \mathbb{R}$ assigning a start time to each task in T, such that all the constraints in the problem are satisfied: For each $t_i \in T, s_i \leq \sigma(t_i) \leq d_i + l_i$, such that release and due times are satisfied; for each tuple $t_i \prec t_j$ we have $\sigma(t_i) + l_i \leq \sigma(t_j)$, such that the precedence constraints are satisfied; and lastly, for any resource r_k at any time point τ, $\sum_{t_i \in \text{active}(\tau,\sigma)} \text{req}(t_i, r_k) \leq \text{cap}(r_k)$, where $\text{active}(\tau, \sigma) = \{t_i : \tau \in [\sigma(t_i), \sigma(t_i) + l_i]\}$, such that the capacity of the available resources is never exceeded.

Example 2.2 Continuing the example started above, we note that the three tasks cannot be executed at the same time, as this would require four engineers, and only two are available. A solution is to start with t_2 as soon as the train enters the workshop, so we set $\sigma(t_2) = 0$. The other two tasks can then be started after t_2 has finished, so we set $\sigma(t_1) = \sigma(t_3) = \sigma(t_2) + l_2 = 45$. Now, σ assigns starting times to all tasks, and we have a complete solution. Note that this solution also satisfies the precedence constraint $t_2 \prec t_3$.

There are many different schedules that can solve a particular instance. Instead of simply treating the RCPSP as a constraint satisfaction problem, in which we aim to find one single solution that satisfies all constraints, the RCPSP is often turned into an optimization problem, in which an objective function is used to find an optimal solution. In nearly all scheduling models, the vector $(\sigma(t_1) + l_1, \ldots, \sigma(t_n) + l_n)$ of *completion times* is used as input for the objective function $\kappa : \mathbb{R}^n \to \mathbb{R}$ that maps these vectors onto a one-dimensional scalar. Many authors enforce a *regularity condition*

$$(\sigma(t_1) + l_1, \ldots, \sigma(t_n) + l_n) \leq (\sigma'(t_1) + l_1, \ldots, \sigma'(t_n) + l_n) \Rightarrow \\ \kappa(\sigma(t_1) + l_1, \ldots, \sigma(t_n) + l_n) \leq \kappa(\sigma'(t_1) + l_1, \ldots, \sigma'(t_n) + l_n) \quad (2.1)$$

on the objective functions used (Bartusch et al., 1988) that states that the objective function must be nondecreasing in the completion times of the tasks, i.e., higher completion times can never result in a lower objective function.[2] Many different objective functions have been used in the literature (Hartmann and Briskorn, 2010), and for the common ones this condition holds:

makespan The most basic and widely encountered objective function optimizes the *makespan* of the solution, which is usually defined as the completion

[2] If *earliness* is to be penalized as well, for example in just-in-time production facilities, the objective function is no longer regular (see Baker and Scudder (1990) for a review of such objective functions).

time of the latest task that is executed:

$$\text{makespan}(\sigma, T) = \max_{t_i \in T}(\sigma(t_i) + l_i)$$

In multi-project problems, this objective function can be modified to optimize the average makespan of the individual projects.

lateness/tardiness If the constraints of the problem can not all be satisfied, a common strategy is to minimize the total *lateness* of the project. First, we define $L_i = \sigma(t_i) + l_i - d_i$ as the lateness of task t_i when executing schedule σ. Then, we can define the lateness of a solution σ as

$$\text{lateness}(\sigma, T) = \sum_{t_i \in T} L_i.$$

The *tardiness* objective is similar, the difference is that it cannot be negative:

$$\text{tardiness}(\sigma, T) = \sum_{t_i \in T} \max(0, L_i).$$

resource-based A different kind of objective looks at the resource usage of the solution, usually combined with makespan minimization. The idea is that the capacity of the resources is optimized as well; the lower the capacity needed to execute a certain solution, the less investment is needed to make these resources available.

The situation at NedTrain can be compared best to the tardiness objective function, with a slight difference: for each train that is delivered late, a fixed penalty is given, which does not depend on the amount of tardiness.

The RCPSP is an NP-complete problem, since it is a generalization of the job shop scheduling problem, for which NP-completeness has been proven (Garey et al., 1976; Blazewicz et al., 1983). Despite this fact, we will still start with a discussion of optimal methods to solve the RCPSP, since the NP-completeness has not withheld numerous researchers to come up with new and innovative optimal solution methods. After this, we will turn to non-optimal methods, using heuristics to arrive at high-quality solutions. Many of these methods however still result in a fixed schedule, which is undesirable in the highly dynamic environment at NedTrain. Therefore, we will also turn to methods using a different, more flexible representation of RCPSP solutions.

Our discussion of RCPSP solution methods will take place in light of the specific criteria of the NedTrain workshop planning problem:

- The first of these is the *performance* of the method, or, to be exact, its ability to arrive at a solution within a reasonable time frame. The size of the problem is an important factor for this criterion: if we are to plan ahead for two weeks, we need to plan maintenance activities for roughly 40 trains. The number of tasks per train is variable, but if we take 10 tasks as average (which is a low estimate), the number of tasks to be planned is 400—a substantial number.

Compounding this problem is the need for interactivity: as already described, it is desirable to let a planner interact with the planning software. For this to work, the method must be able to quickly solve the problem: it is unrealistic to ask the planner to wait more than a couple of minutes.

- The second important criterion is the *flexibility* of the method. Due to the uncertain nature of the tasks to be planned, changes and incidents during execution of the schedule are to be expected. The flexibility requirement represents the desire that the schedule be easily and efficiently adaptable to overcome the consequences of such changes and incidents.

2.2 Mixed Integer Linear Programming solutions

A traditional method to provide exact solutions to the RCPSP is to encode it as a Mixed Integer Linear Programming (MILP) problem, and to use a general MILP solver to find a solution. Many different encodings have been suggested in the literature over the years. Two important and very different encoding types will be discussed below:

- (0–1) time indexed formulations, using binary decision variables x_{it} such that $x_{it} = 1$ if and only if task t_i starts at time t; and
- flow-based formulations, which involve binary decision variables x_{ij} and continuous start-time variables S_i, such that $x_{ij} = 1$ if and only if $S_i + l_i \leq S_j$, meaning that t_i precedes t_j.

Note that whereas the flow-based formulations use continuous time variables, the time indexed formulations are restricted to time points in $\mathbb{N}$. In the context of task scheduling this is usually not a big limitation, since the start times of tasks are usually measured at the scale of minutes. Especially in combination with uncertainty, higher accuracy is not necessary.

Many MILP-formulations are sensitive to the size of the problem horizon. Therefore, a preprocessing step is often used to find the bounds $[\text{est}(t_i), \text{lst}(t_i)]$, whereby $\text{est}(t_i)$ is the earliest start time of t_i, before which t_i can never start, and $\text{lst}(t_i)$ is the latest start time, after which t_i can no longer start. These bounds are used to limit the scope of the search for a solution, and are based on the precedence constraints and release and due times of the original problem. They can be found in polynomial time (Koné et al., 2013).

In the following sections we will give a brief overview of both time-indexed and flow-based methods.

2.2.1 *Time-indexed formulations*

In these formulations, decision variables are used that are indexed by discrete time. The classical formulation is due to Pritsker et al. (1969), in which there is

only one type of decision variable, $x_{i\tau}$, indexed by time and task index; $x_{i\tau} = 1$ if task t_i starts at time τ.

To give an impression of the structure of the constraints involved, we will discuss the constraint used to encode a precedence constraint. First, note that there is only one τ such that $x_{i\tau} = 1$, for a certain i—this is enforced by another constraint. A precedence constraint between t_i and t_j can then be formulated as

$$\sum_{\tau=\text{est}(t_j)}^{\text{lst}(t_j)} \tau x_{j\tau} \geq \sum_{\tau=\text{est}(t_i)}^{\text{lst}(t_i)} \tau x_{i\tau} + l_i \quad \text{if } t_i \prec t_j. \tag{2.2}$$

Since there is only one value for τ for which $\tau x_{j\tau} = 1$, the left summation is in fact equal to the currently selected start time for t_j. Similarly, $\tau x_{i\tau} + l_i$ is equal to the currently selected end time for t_i. Therefore, this constraint ensures that t_j starts at or after the finish of t_i. Other constraints of the RCPSP are encoded using a similar strategy.

The decision variables are indexed by both time and task index, this means that the number of decision variables scales with the size of the temporal horizon. As a result, this formulation is highly sensitive to the (temporal) size of the problem, since larger problems involve larger numbers of decision variables. This problem is partially mitigated by the high quality of the linear-relaxation bounds available (Artigues et al., 2008). In Möhring et al. (2003), a method based on a time-indexed formulation is used to solve well-known benchmark instances of up to 120 tasks. The time horizon size is shown to be the dominant factor in the run time, and not all instances of this size can be solved to optimality. Still, instances with 120 tasks are relatively small compared to the scheduling problems that have to be solved at NedTrain, making this class of methods impractical. Even if instances could be solved within a reasonable time frame, we still must deal with the uncertain nature of the execution environment at NedTrain. An exact solution to the scheduling problem has fixed time assignments, any change would invalidate the schedule, leading to a time-consuming re-computation.

2.2.2 *Flow-based formulations*

Formulations based on resource flows, such as described by Artigues et al. (2003), exploit the fact that, in any feasible solution, any task t_i using some amount of resource r_k transfers some or all of this resource to some other, unique, task t_j following it, when t_i is done executing. In a practical situation such as a maintenance workshop this is also very obvious: a resource such as a maintenance engineer will move from one task to a distinct next task, given by the schedule. The model uses a set of continuous start-time variables S_i for each task, and to model the flow of resources, decision variables f_{ijk} are introduced. They denote how many units of resource r_k are transferred from task t_i to t_j, after t_i has finished executing. The flow of resources originates and terminates in two special source and sink tasks t_0 and t_{n+1}, with zero length, for which we set $\text{req}(t_0, r_k) = \text{req}(t_{n+1}, r_k) = \text{cap}(r_k)$ for all resources.

Again, we will not discuss all the constraints involved, but we will use the precedence constraint as an example. In this case, variables x_{ij} are used:

$$x_{ij} = \begin{cases} 1 & \text{if } t_i \prec t_j \\ 0 & \text{otherwise.} \end{cases} \tag{2.3}$$

These variables are used in the constraint

$$S_j - S_i - Mx_{ij} \geq l_i - M \quad \forall t_i \in T \cup \{t_0\}, \forall t_j \in T \cup \{t_{n+1}\}, \tag{2.4}$$

where M is an arbitrary large integer, usually set to the scheduling horizon size. If $t_i \not\prec t_j$, this reduces to $S_j - S_i \geq l_i - M$, which is always true, given the large value of M. If $t_i \prec t_j$, this reduces to $S_j - S_i \geq l_i$, ensuring that t_j starts at or after the finish of t_i.

Since this formulation uses continuous-time variables for the start times of the tasks, the temporal horizon scaling problem encountered with the previous formulation does not occur. But this formulation also has its problems: it is difficult to find high-quality linear-relaxation (lower) bounds, making it hard to solve for problems that are large in the number of tasks. Applegate and Cook (1991) show some numerical results for job shop problems with 10 tasks and 10 resources (machines); different cuts are discussed, but for all of them there is still a large gap between the lower bound and the optimal value. This formulation is therefore mainly of use for instances with large time horizon, which cannot be solved within a reasonable time frame by time indexed formulations (Koné et al., 2013). Nevertheless, instances with large number of tasks still form a problem for this formulation, and as the NedTrain problems are large in both number of tasks and problem horizon, it is clear that flow-based formulations are unsuitable as well.

2.3 Branch and bound methods

Next, we will discuss branch and bound methods, which, in contrast to encoding the RCPSP as a set of constraints to be satisfied by a general MILP solver, explicitly construct a solution in the form of a schedule. The general form of the branch and bound procedure is that of a search tree: at each point in the search procedure where multiple choices are available, *branches* are created for all of these choices. Using a backtracking procedure, all of these branches are eventually evaluated. The other key part of the algorithm is the *bounding* part: before exploring a new branch, a lower bound is computed on the solution quality that this branch will result in. If the computed bound is higher that the currently known best solution, it is certain that none of the solutions resulting from this branch can improve upon the current solution, and the branch is not explored. The combination of these mechanisms guarantees that an optimal solution will be found (given enough computational power), possibly without exploring all solutions explicitly (Demeulemeester and Herroelen, 2002).

2.3.1 High-level algorithm

A competitive branch and bound procedure for solving the RCPSP was proposed by Demeulemeester and Herroelen (1992), based on the time incrementing depth first search procedure proposed by Christofides et al. (1987). In these procedures, schedules are constructed incrementally: during the execution of the algorithm, a part of the tasks is assigned a starting time. Using the time assigned to the latest task, the set of tasks can be divided into three sets: *finished* tasks, tasks *in progress*, and *unfinished* tasks. In the set of unfinished tasks, certain tasks are said to be *eligible*: these are the tasks for which all predecessors are finished. The start times for the finished tasks are temporary as well, in the sense that these can be delayed at a later time.

The current partial schedule is then extended by selecting a task from the set of eligible tasks. In some cases, there is a unique task, or a unique combination of tasks, which can be scheduled directly. If this is not the case, a *resource conflict* is at the root of this: some combinations of the eligible tasks and tasks in progress result in resource over-subscription. This leads to *branching* in the tree of solutions: a set of tasks is picked to be delayed, by adding precedence constraints between the last scheduled task and this set. This procedure is repeated until all the tasks have been added to the schedule.

In some cases however, not the whole schedule has to be constructed: the algorithm keeps track of a lower bound for the makespan, and if it can be proven that the current branch will not improve on this bound, it can be abandoned. The tree is then traversed upwards, to find a decision point at which alternative choices are still available. The choice made in the abandoned branch is undone, and an alternative choice is made. If we manage to find a complete solution, we check if the upper bound is equal to the lower solution: if this is the case, we have found the optimal solution.

Another method to effectively prune parts of the search space is the application of *dominance rules*. These rules allow us to prove that, if they apply, the current solution is dominated by another solution, and will never lead to an optimal solution.

Demeulemeester and Herroelen (1997) introduces a set of lower bounds and dominance rules that improve the performance markedly, creating a very competitive method for finding optimal solutions. Still, branch and bound methods suffer from the general tractability problems associated with solving an NP-complete problem to optimality as well, and due to the size of the NedTrain problem instances, these methods are again impractical to use in this case.

For these reasons, we will next turn to heuristic methods to solve the RCPSP. Such methods do not purport to find an optimal solution—often, they cannot even guarantee that a solution will be found. Still, in many common cases, these methods are able to find a solution of reasonable quality in a fraction of the time an exact method would need.

2.4 Schedule Generation Schemes

The first heuristic methods we will discuss are the serial and parallel schedule generation schemes (SGS, Hartmann (1999); Kolisch (1996)). After this, we will turn to more complex heuristic methods.

2.4.1 *Serial SGS*

The serial SGS is a greedy algorithm, which incrementally constructs a solution σ in a single pass, by considering a single activity at a time and adding it to the current solution. An outline of the SGS is shown in Algorithm 2.1. The algorithm works by considering the set D of all the *eligible* tasks (line 1), defined as

$$D = \{t_i : \sigma(t_i) \text{ is undefined} \wedge \forall t_j \prec t_i \in C[\sigma(t_j) \text{ is defined}]\}, \tag{2.5}$$

the set of tasks for which all predecessors have been scheduled. Using a priority heuristic, the eligible tasks are ranked, and the task with the highest value is chosen for execution (line 2). The algorithm assigns the earliest time point to this task at which all predecessors have completed execution (line 9) and all resources required to execute this task are available (line 12, where $S_\tau = \{t_i : \sigma(t_i) \leq \tau < \sigma(t_i) + l_i\}$ is the set of tasks scheduled to be executing at time τ). This procedure is repeated, until there are no more eligible tasks in D. At this point, all tasks are assigned a start time and the solution is complete.

The serial SGS can be implemented to work in $O(n^2m)$ time, where n is the number of tasks and m the number of resources: we do not need to consider all time points, it is enough to merely consider those points at which a task starts or ends.

2.4.2 *Parallel SGS*

The parallel SGS also constructs a schedule σ greedily, but instead of considering a single task at a time, it considers as many tasks as possible at a time point, before advancing it. The set of eligible tasks for the parallel SGS is defined as

$$D_\tau = \{t_i : \sigma(t_i) \text{ undefined} \wedge \forall t_j \prec t_i \in C[\sigma(t_j) \text{ is defined} \wedge \sigma(t_j) + l_j < \tau]\}, \tag{2.6}$$

and contains all tasks for which the predecessors have been scheduled, and finished, at time point τ.

The outline is shown in Algorithm 2.2. The parallel nature is evident in line 4: every task in the eligible set is checked for feasibility, in the order specified by the heuristic, and all tasks that are feasible at time τ are added (line 15). Again, since we only need to consider the starting and ending time points of the tasks, the algorithm has a time complexity of $O(n^2m)$.

2.4.3 *Priority rules*

The performance of both SGS approaches depends highly on the priority rule used to pick the task to schedule next. Many different rules have been proposed in literature; a few of the more important ones are discussed below.

Algorithm 2.1: Serial schedule generation

```
1  while D ≠ ∅ do
2      t_i = h(D)
3      τ = −1
4      repeat
5          τ ← τ + 1
6          prec-feasible ← TRUE
7          res-feasible ← FALSE
8          forall the {t_j : t_j ≺ t_i} do
9              if σ(t_j) + l_j >= τ then prec-feasible ← FALSE
10         end
11         for τ ≤ τ' ≤ τ + l_i do
12             if ∃r_k[(∑_{t_j∈S_τ'} req(t_j, r_k)) + req(t_i, r_k) > cap(r_k)] then
               res-feasible ← FALSE
13         end
14     until prec-feasible = TRUE ∧ res-feasible = TRUE
15     σ(t_i) ← τ
16 end
```

Algorithm 2.2: Parallel schedule generation

```
1  τ = 0
2  while ∃t_i[σ(t_i) is undefined] do
3      D'_τ ← D_τ
4      while D'_τ ≠ ∅ do
5          t_i = h(D'_τ)
6          D'_τ ← D'_τ\{t_i}
7          prec-feasible ← TRUE
8          res-feasible ← FALSE
9          forall the {t_j : t_j ≺ t_i} do
10             if σ(t_j) + l_j >= τ then prec-feasible ← FALSE
11         end
12         for τ ≤ τ' ≤ τ + l_i do
13             if ∃r_k[(∑_{t_j∈S_τ'} req(t_j, r_k)) + req(t_i, r_k) > cap(r_k)] then
               res-feasible ← FALSE
14         end
15         if prec-feasible ∧ res-feasible then σ(t_i) ← τ
16     end
17     τ ← τ + 1
18 end
```

random Tasks are selected at random, each eligible task has an equal chance of being picked. This strategy is often used for comparison purposes, to see how much is gained by using a certain heuristic.

latest start time Alvarez-Valdes and Tamarit (1989) propose to calculate $\mathrm{lst}(t)$ for each task, which is the latest starting time for t, given the temporal constraints, such that t and all its successors can still finish in time, assuming unlimited resource capacity. If the heuristic needs to select a task to schedule, it selects task t such that $\mathrm{lst}(t)$ is minimized. The rationale is that this task needs to be executed most urgently, to be able to finish in time.

most total successors In the same paper, a different rule is proposed as well, which selects the task with the largest number of total successors to be executed first. A slightly different intuition of urgency holds for this task: since this task has many successors, many tasks need this task to be finished to become eligible.

Randomization is often employed in combination with the priority rules described above. Tie breaking is one reason: often, multiple tasks may have the same priority, and a choice needs to be made. More importantly, randomization may be employed to improve performance significantly for certain pathological cases, in which the rule persistently chooses exactly the wrong task.

It is clear from the complexity of the basic algorithms that both schemes scale well, compared to the exact solution methods discussed earlier. It is expected that problem sizes common for the NedTrain case are easily solvable with these methods. This seems to solve the questions raised by the first problem statement at the end of the first chapter. One of the SGS schemes can be used to quickly generate a set of solutions to a scheduling problem, reducing the labor-intensive character of the scheduling process. The solution itself however is still in the form of fixed starting times assigned to each task. In light of the second problem statement, this is undesirable: such a fixed schedule would be invalidated quickly in the highly dynamic environment of a maintenance workshop. We will therefore discuss a method yielding a more flexible solution form next.

2.5 Precedence constraint posting

As discussed above, it is desirable to have a method to solve the RCPSP that is based on heuristics, and that results in a solution in a flexible form. One such family of methods is based on *precedence constraint posting* (PCP). These methods can be seen as a *transformation* of the RCPSP into a much simpler problem, from which in turn we can efficiently derive multiple fixed-time solutions. This is exactly in line with the idea of the second problem statement: it is a flexible way to prescribe task execution. Within the bounds of the simplified problem, it is easy to generate different fixed-time schedules, which can be used to adapt to changing circumstances, without completely generating a new solution.

The basic idea underlying PCP is to make a distinction between *resource* constraints and *temporal* constraints. As it turns out, a problem having only (non-disjunctive) temporal constraints is relatively easy to solve: finding a fixed-time schedule for such a problem can be done in polynomial time (Dechter et al., 1991; Planken, 2013). The method of constraint posting therefore tries to add a set of precedence constraints that replace the resource constraints: if these precedence constraints are satisfied, the resource constraints will be satisfied as well. The problem is thus transformed from one with resource and temporal constraints to a problem with only temporal constraints. From the latter, we can easily extract multiple fixed-time schedules, as will be discussed later. But, the transformation is not symmetrical, owing to the NP-complete nature of the original problem. While any solution to the transformed problem is also a solution to the original problem, the reverse does not hold.

2.5.1 Finding resource conflicts

The first step in the algorithm is *finding* resource conflicts. We will start with a high-level discussion of profile-based approaches (Cesta et al., 1998, 2000). These methods focus on the *resource profile* of the problem: the resource use over time by the tasks in the problem is computed, and based on the capacity of each of the resources *peaks* are identified that violate the resource constraints. Precedence constraints are then added to the problem based on these peaks, in an attempt to flatten them such that the resource capacity is no longer exceeded.

Profile-based methods

For a fixed time solution, computing the resource profile is conceptually very simple: we know the exact start time for each task, and its length, so we can determine which tasks are executing at each point in time. Combining this information with the resource usage per task gives the total resource usage at a certain time point. But in this case we do not have a fixed time schedule available, as the whole point is to use a more flexible representation.

To still be able to construct a resource profile we make a distinction between two tasks *necessarily* or *possibly* overlap. In the first case, the two tasks always execute at the same time, for *every* possible fixed time schedule we can derive. In the second case, there is at least *one* fixed time schedule where the two tasks overlap. Based on necessarily and possibly overlapping tasks, we can now compute a *lower* and *upper* bound for the resource profile at each time point. The lower bound is based on all the tasks that necessarily execute at a time point—the resource usage can never be lower than this. Correspondingly, the upper bound is computed based on all the tasks that possibly overlap (Cesta et al., 1998).

Using the resource profile, we can identify *peaks*: time points at which a certain set of tasks together consume more resources than available. It is at these points that we need to add precedence constraints between tasks: by doing this, we spread the execution of the tasks, lowering the contention for a resource. A peak is said to be *unresolvable* if the lower bound resource profile of a resource is higher than the available capacity of that resource. In this case, there are no fixed

time solutions we can generate that satisfy the resource capacity constraints at this point in time. This means that no solution can be found, and we can terminate the search. Note that a solution might still exist: we are using a heuristic to arrive at a solution, and the heuristic does not always make the right choices.

If the lower bound is equal to, or lower than the resource capacity, we know that we can derive at least one feasible fixed-time solution. If the upper bound exceeds the resource capacity, some solutions will still exceed the resource capacity. In the ideal situation, the upper bound resource profile never exceeds the allowed capacity: this means that *all* solutions we can generate are resource-feasible.

Cesta et al. (1998) uses a pairwise method to compute the resource profiles, which is very quick but over-estimates the resource consumption; this results in an over-constrained solution.

Example 2.3 Consider three tasks t_a, t_b and t_c, all making use of a shared resource with capacity two. If t_a and t_b can execute concurrently, and t_a and t_c as well, a pair-wise method might conclude that the upper bound on the resource use equals three, leading to the need for a temporal constraint. This might not be needed, since there might be a constraint such that $t_b \prec t_c$.

Minimal critical sets

To overcome this over-estimation, a different strategy, based on *minimal critical sets* can be used. The idea behind this method is that we want to find a *critical set*, that is, a set of tasks (executing at a certain time point) that together exceeds the resource capacity. If, in addition, this set is *minimal*, it means that by removing just one task from this set, this set is no longer critical, meaning that the resource conflict caused by these tasks is solved. Removing a task is done in this case by adding a precedence constraint such that this task no longer executes at the chosen time point. If no (minimal) critical sets are present, all the fixed time schedules we can generate are guaranteed to be resource-feasible.

Finding minimal critical sets however is a difficult problem, as the number of minimal critical sets is in general exponential in the size of the problem. Nonetheless, several good approximation algorithms have been proposed (Lombardi and Milano, 2009b,a; Laborie and Ghallab, 1995).

2.5.2 *Solving resource conflicts*

Above, methods have been described allowing us to find the tasks involved in a resource conflict. The next step in the algorithm, from which its name is derived, is solving that conflict by adding (*posting*) a precedence constraint—hence the name, precedence constraint posting. In some of the possible variants we first need to select two tasks between which we are going to post the constraint. For example, in a profile-based method we select a contention peak containing multiple (more than two) tasks. To flatten this peak, two tasks need to be selected. Cheng and Smith (1996) proposes to look at the available flexibility, and to select the pair of tasks for which the flexibility is *lowest*: the intuition is that we have to

make a choice when it is still possible—if choices are made at other points in the schedule, the flexibility for this choice might decrease even further, limiting our options.

If a pair of tasks has been selected, the next decision is the direction of the precedence constraint to add. In some cases, our decision is forced: one of the directions might be temporally infeasible. In other cases, both options are still open. In this case, a similar reasoning is used as above (Cheng and Smith, 1996; Cesta et al., 1998): in order to preserve as many future choices as possible, we select the ordering that removes as little flexibility from the schedule as possible.

2.5.3 *Esta and chaining*

An interesting variation on the profile-based method is proposed by Cesta et al. (1998): instead of computing resource profile bounds to represent all possible fixed-time schedules we can generate, the resource profile for a single fixed-time assignment is computed, namely that of the earliest start-time assignment (hence the name, ESTA). The advantage is that the resource profile computation is very quick and straight-forward, and as such we quickly arrive at a feasible solution. The downside is that only a single fixed-time schedule is guaranteed to be resource-feasible.

Cesta et al. (1998) proposes, and Policella et al. (2007) extends *chaining*, a method to convert such a schedule into a flexible one, in which we can again generate multiple fixed-time schedules from a single solution. The basic idea of chaining is that a single fixed-time solution is taken, and all precedence constraints are removed, and different precedence constraints are added. The key is in adding the right precedence constraints: chaining looks at the resource flow between the tasks and adds precedence constraints that encode the transfer of a resource between one task and the next. For each resource, a *chain* of tasks is constructed in this way, ensuring that the capacity of the resource can never be exceeded.

2.6 Simple temporal networks

The previous section introduced precedence constraint posting as a method to transform the complex RCPSP into a simpler problem without resource constraints, representing a subset of the solution space, from which several fixed-time solutions can be derived efficiently. This section will discuss the formalism most commonly used, the Simple Temporal Network (STN). Note that many authors also refer to the Simple Temporal Problem (STP)—however, as Planken (2013) points out there is no single Simple Temporal Problem: instead, several different temporal queries can be formulated.

Recall that the idea of PCP is to remove all resource contention in an RCPSP instance, resulting in an STN, which is a purely temporal representation, in which resources do not play a role. Such an STN can be seen as a *flexible* solution to the original RCPSP instance: from it, various solution assignments σ can be retrieved

very efficiently. This intermediate step avoids re-solving the RCPSP instance every time an incident occurs during execution of the computed solution.

2.6.1 *The STN formalism*

The Simple Temporal Network (STN) was introduced by Dechter et al. (1991), and has become a commonly used temporal problem representation, with the advantage that solutions can be obtained efficiently. It consists of a pair $S = (\mathcal{X}, \mathcal{C})$, where $\mathcal{X} = X_0, X_1, \ldots, X_n$ is a set of time point variables, and $\mathcal{C}$ is a set of constraints. Each constraint C_{ij} may only contain a single interval, of the form $X_j - X_i \leq c_{ij}$. Like in the RCPSP, the goal is to find a *schedule* for the events in $\mathcal{X}$, that is, a function $\sigma : \mathcal{X} \to \mathbb{R}$ that assigns a non-negative value to every time point in $\mathcal{X}$, such that all constraints in $\mathcal{C}$ are satisfied. Unlike the RCPSP, resource usage is not considered, i.e., there are no concurrency constraints other than those specified in $\mathcal{C}$. If a schedule does not exist, the STN is said to be *inconsistent*, and *consistent* otherwise. Note that there might be many different schedules satisfying the STN, as such an STN can be seen to encode a family of schedules. This is a desirable property in the case of NedTrain, since this can be used to adapt to changing circumstances during execution. In order to be able to express absolute time constraints, the time-point X_0, also denoted by z, is used. It represents a fixed reference point on the time line, usually having the value 0.

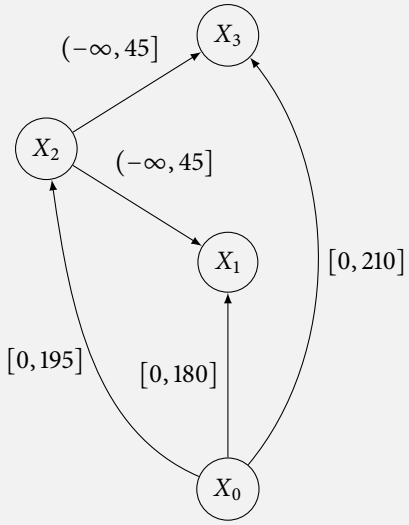
Example 2.4 As an example, consider the three tasks introduced in Example 2.1. The start time of each task is modeled using time point variables X_1, X_2, X_3 for t_1, t_2, t_3. The release and due times are modeled using constraints relative to X_0. For the release time, we add $X_0 - X_i \leq 0$ for $i \in \{1, 2, 3\}$. For the due time, the task lengths have to be considered as well. Recall that we have $l_1 = 60$, $l_2 = 45$ and $l_3 = 30$, and due times of $d_i = 240$ for $i \in \{1, 2, 3\}$. This yields the constraints $X_1 - X_0 \leq 180$, $X_2 - X_0 \leq 195$ and $X_3 - X_0 \leq 210$.

Next, we encode the precedence constraints from the problem. In Example 2.1, the constraint $t_2 \prec t_3$ is formulated explicitly. Additionally, in Example 2.2, we have that "the other two tasks can then be started after t_2 has finished", so this implies $t_2 \prec t_1$ as well. The first constraint is encoded as $X_3 - X_2 \leq 45$, and the second constraint as $X_1 - X_2 \leq 45$, again taking into account the task length $l_2 = 45$.

2.6.2 *Graph representation*

A STN can be represented as a directed weighted *constraint graph*, this is the origin of the "network" in STN. Each node in this graph represents a time point variable, and an edge $X_i \to X_j$ with weight c_{ij} represents a constraint $X_j - X_j \leq c_{ij}$. For simplicity, the edges $X_i \to X_j$ and $X_j \to X_i$ are often combined into a single edge, labeled with an interval, to represent the constraint $X_j - X_i \in [-c_{ji}, c_{ij}]$.

Example 2.5 Continuing the previous example, if the STN instance described is transformed into its equivalent graph representation, we obtain the following graph:



As there is no constraint limiting $X_2 - X_3$ and $X_2 - X_1$, these weights have been set to $-\infty$.

The graph representation is very helpful in finding constraints implied by the set $\mathcal{C}$ of constraints, interpreting the weights c_{ij} on the arcs as lengths of the path from X_i to X_j. If $\mathcal{C}$ contains the constraints $X_j - X_i \leq c_{ij}$ and $X_k - X_j \leq c_{jk}$, then there exists a path from X_i via X_j to X_k having a length $c_{ij} + c_{jk}$. This means $X_k - X_i \leq c_{ij} + c_{jk}$ is an implied constraint, even though $\mathcal{C}$ may not have contained a constraint limiting $X_k - X_i$.

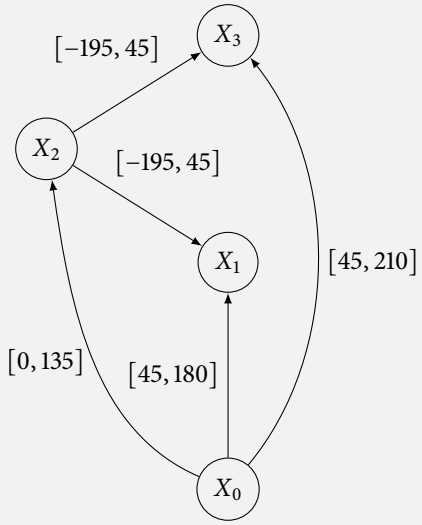
Example 2.6 We return to the example described above, to demonstrate implied constraints. The distance $X_2 - X_3$ is not limited by any constraint, however, we have

$$X_2 - X_0 \leq 195 \wedge X_0 - X_3 \leq 0,$$

which implies

$$(X_2 - X_0) + (X_0 - X_3) \leq 195 \Leftrightarrow X_2 - X_3 \leq 195.$$

In similar fashion, other constraints can be derived for the example. This procedure is also called *constraint tightening*. If all constraints are tightened as much as possible, we obtain the so-called *minimal* STN:



2.6.3 *Algorithms*

Exploiting the parallel between path lengths and constraints, an obvious procedure to find the tightest constraints, and thus the minimal STN, is to use an algorithm that computes the shortest paths in a graph, such as Floyd-Warshall (Floyd (1962), see Algorithm 2.3). This algorithm runs in $\Theta(n^3)$, and the tightest constraints it finds are represented as the elements of the $(n+1)\times(n+1)$ *distance matrix* D_S, containing for every pair of time-point variables X_i and X_j the length of the shortest path in the distance graph between X_i and X_j.Using this minimal network, a solution can be extracted using any time point variable as starting point.

Other algorithms exists, which are more efficient (but more complex). One such algorithm is Snowball (Planken et al., 2011), which runs in $O(n^2 w_d)$, where w_d is the graph width induced by a vertex ordering d.

Theorem 2.1 (Dechter et al., 1991) An STN is consistent if, and only if, its associated graph representation does not contain any negative cycles.

Proof. Assume the STN contains a negative cycle $C = X_1, \ldots, X_k = X_1$. The sum of the inequalities represented by the edges in C yields $X_1 - X_1 < 0$, which can never be satisfied.

If there is no negative cycle, the shortest path between any pair of nodes is well-defined. For any pair of nodes X_i and X_j, the shortest paths satisfy the equation

$$d_{0j} \leq d_{0i} + c_{ij}, \tag{2.7}$$

Algorithm 2.3: Floyd-Warshall's all-pairs shortest-path algorithm

Input: An STP $S = (\mathcal{X}, \mathcal{C})$
Output: A distance matrix D_S

```
1  Initialize all entries in (n+1) × (n+1) matrix D_S to ∞
2  for i := 0 to n do
3  |  D_S[i, i] = 0
4  end
5  foreach X_j − X_i ≤ c_ij ∈ C do
6  |  D_S[i, j] = c_ij
7  end
8  for k := 0 to n do
9  |  for i := 0 to n do
10 |  |  for j := 0 to n do
11 |  |  |  D_S[i, j] := min(D_S[i, j], D_S[i, k] + D_S[k, j])
12 |  |  end
13 |  end
14 end
```

which can be rewritten as

$$d_{0j} - d_{0i} \leq c_{ij}. \tag{2.8}$$

From this, it can be seen that assigning $X_j = d_{0j}$ and $X_i = d_{0i}$ satisfies the constraint $X_j - X_i \leq c_{ij}$. Note that these are the entries $D_S[0, i]$ and $D_S[0, j]$ in the distance matrix D_S. Therefore, the tuple

$$X = (d_{01}, \ldots, d_{0n}) \tag{2.9}$$

is a solution to the entire STN. Since $X_i - X_0 \leq d_{0j}$, this tuple contains the *latest starting times* $\mathrm{lst}(X_i)$ for all time point variables $X_i \in \mathcal{X}$ (assuming $X_0 = 0$). Similarly, it can be shown that the tuple

$$X = (-d_{10}, \ldots, -d_{n0}) \tag{2.10}$$

is a solution containing the *earliest starting times* $\mathrm{est}(X_i)$ for all time point variables. □

If we are only interested in the paths from a single source, for example to check the consistency of an STN instance, a more efficient procedure is to use the Bellman-Ford algorithm (see Algorithm 2.4), which computes the shortest paths to all nodes from a single source. This algorithm runs in $\Theta(nm)$ time (with $n = |\mathcal{X}|$ and $m = |\mathcal{C}|$) and can be used to obtain an earliest start time solution by first running this algorithm, and then using Equation 2.9. In the loop starting at line 5, the distance matrix is updated at most $|\mathcal{X}|$ times, since a path can never be

Algorithm 2.4: Bellman-Ford's single-source shortest-path algorithm

Input: An STN $S = (\mathcal{X}, \mathcal{C})$
Output: A distance list D, relative to X_0, or INCONSISTENT

```
1  for i := 0 to n do
2  |  D[i] := ∞
3  end
4  D[X_0] := 0
5  repeat |X| times
6  |  foreach X_j − X_i ≤ c_ij ∈ C do
7  |  |  D[X_j] = min(D[X_j], D[X_i] + c_ij)
8  |  end
9  end
10 foreach X_j − X_i ≤ c_ij ∈ C do
11 |  if D[X_j] > D[X_i] + c_ij then return INCONSISTENT
12 end
13 return D
```

longer than the total number of nodes in the graph. In line 11, an inconsistency can be detected. If, after $|\mathcal{X}|$ updates, a shorter distance is still possible, this can only be caused by a negative cycle.

The following theorem plays an important role in the *dispatching* of an STN: it describes how we can construct one of the many fixed-time schedules represented by the STN, by picking a start time for one time point variable satisfying the shortest path constraints, and then extending this partial solution variable by variable, until all time point variables have been assigned a value.

Theorem 2.2 (Dechter et al., 1991) Let $S = (\mathcal{X}, \mathcal{C})$ be a consistent STN. Any instantiation of a subset $\mathcal{X}_k \subset \mathcal{X}$ of k variables ($1 \leq k < n$) that satisfies all the shortest path constraints applicable to $\mathcal{X}_k$ is extensible to any other variable.

Proof. Our proof will be by induction on $|\mathcal{X}_k| = k$.

For $k = 1$, $\mathcal{X}_1$ consists of a single variable X_i, instantiated to v_i. We need to show that we can find an assignment $X_j = v_j$ for any other variable X_j that satisfies the shortest path constraint between X_i and X_j. The value v_j must satisfy

$$-d_{ji} \leq v_j - v_i \leq d_{ij}. \tag{2.11}$$

We know that $d_{ji} + d_{ij} \geq 0$, since a consistent STN does not have any negative cycles, hence we can conclude that there exists a value v_j that satisfies Equation 2.11.

Now, assume the theorem holds for $\mathcal{X}_k$. We need to show that it also holds for $\mathcal{X}_{k+1}$. Without loss of generality, let $\mathcal{X}_k = \{X_1, \ldots, X_k\}$, and let $\{X_i = v_i : 1 \leq i \leq k\}$ be an assignment that satisfies the shortest path constraints among the variables in $\mathcal{X}_k$. Let $X_{k+1} \notin \mathcal{X}_k$. Now we need to find a value $X_{k+1} = v_{k+1}$

that satisfies the shortest path constraints between X_{k+1} and all variables in $\mathcal{X}_k$:

$$-d_{k+1,i} \leq v_{k+1} - v_i \leq d_{i,k+1} \tag{2.12}$$

for $i = 1, \ldots, k$, or,

$$v_{k+1} \leq \min\{v_i + d_{i,k+1} : 1 \leq i \leq k\}, \tag{2.13}$$

$$v_{k+1} \geq \max\{v_i - d_{k+1,i} : 1 \leq i \leq k\}. \tag{2.14}$$

Now, suppose that the minimum over the set above is attained at i_0, and the maximum is attained at j_0. Then, we know that v_{k+1} must satisfy

$$v_{j_0} - d_{k+1,i_0} \leq v_{k+1} \leq v_{i_0} + d_{i_0,k+1}. \tag{2.15}$$

Since, again, we know that v_{i_0} and v_{j_0} satisfy the constraint between X_{i_0} and X_{j_0}, we have

$$v_{j_0} - v_{i_0} \leq d_{i_0,j_0}. \tag{2.16}$$

Since d represents a *shortest* path constraint, we also have

$$d_{i_0,j_0} \leq d_{i_0,k+1} + d_{k+1,j_0}. \tag{2.17}$$

Combining Equations 2.16 and 2.17, we get

$$v_{j_0} - d_{k+1,i_0} \leq v_{i_0} + d_{i_0,k+1}. \tag{2.18}$$

Therefore, we can conclude that there exists at least one value v_{k+1} that satisfies Equation 2.15. □

The following corollary can be directly derived from this theorem, and contains two important properties that hold for STNs and their schedules:

Corollary 2.1 Let $S = (\mathcal{X}, \mathcal{C})$ be an STN and D_S its distance matrix. For $i = 1, \ldots, n$, let $\text{lst}(X_i) = D_S[0, i]$ and $\text{est}(X_i) = -D_S[i, 0]$. Then, for every schedule σ for S, and every $X \in \mathcal{X}$, it holds that $\sigma(X) \in [\text{est}(X), \text{lst}(X)]$. Moreover, given any $X \in \mathcal{X}$ and $v \in [\text{est}(X), \text{lst}(X)]$, there exists a schedule σ for S such that $\sigma(X) = v$.

This shows the flexible nature of an STN: regardless of *which* value $v \in [\text{est}(X), \text{lst}(X)]$ we pick for X, we are guaranteed to be able to find a schedule in which X starts at the selected time. So, in this sense, the STN formalism fits well with the second problem statement: we can quickly derive fixed-time solutions from an STN, making it a flexible method to prescribe task execution. Combined with precedence constraint posting, we are able to flexibly prescribe the execution of an RCPSP instance.

2.7 Multi-agent scheduling

Proposition 2.1 shows that multiple fixed-time schedules can be constructed for a single STN, but the process by which these schedules are constructed is centralized. This is the source of the third problem statement: often, and especially in the case of NedTrain, a schedule has to be executed by multiple teams. This does not combine well with the need for central control over the schedule execution: we would rather have a form of decentralized execution.

To map the model of the STN to the idea of decentralized execution, we can distribute the tasks in $\mathcal{X}$ over a set of m agents $A_1, \ldots, A_m$. Each agent A_i is responsible for the execution of a subset $\mathcal{X}_i \subseteq \mathcal{X}$ of events, where these subsets are mutually disjoint, i.e., $\mathcal{X}_i \cap \mathcal{X}_j = \varnothing$ when $i \neq j$. If the schedule we want to execute is fixed, i.e., a function $\sigma : \mathcal{X} \to \mathbb{R}$, this poses no particular problem. But in the case of NedTrain, it is desirable for the teams executing their schedule to have some form of flexibility: due to the uncertainty present in the tasks it is desirable if a team has at least partial control over the exact start times of the tasks it needs to execute. This is a more difficult problem, because here, we want agent A_i to determine $\sigma_i : \mathcal{X}_i \to \mathbb{R}$ for the STN[3] $S_i = (\mathcal{X}_i \cup \{z\}, \mathcal{C}_i)$, derived from the original STN $S = (\mathcal{X}, \mathcal{C})$ solving the RCPSP instance, where $\mathcal{X}_i$ is the subset of time point variables assigned to A_i, and $\mathcal{C}_i = \mathcal{X}_i^2 \cap \mathcal{C}$ is the set of constraints from $\mathcal{C}$ restricted to the time point variables in $\mathcal{X}_i$. This problem is known as the *Temporal Decoupling Problem* (Hunsberger, 2002).

Definition 2.3 Let $S = (\mathcal{X}, \mathcal{C})$ be a consistent STN. Suppose that $\mathcal{X} - \{z\} = \{\mathcal{X}_i\}_{i=1}^m$ is partitioned in m subsets $\mathcal{X}_i$. Then the *temporal decoupling problem* is to find m STNs $S_i = (\mathcal{X}_i \cup \{z\}, \mathcal{C}_i)$ such that, whenever $\sigma_1, \ldots, \sigma_m$ are independently constructed schedules for the individual STNs $S_1, \ldots, S_m$, respectively, their merge $\sigma = \bigcup_{i=1}^m \sigma_i$ is a schedule for the original STN S.

The problem is that, while we want the agents to be able to independently pick σ_i, we also want the merge $\sigma = \bigcup_{i=1}^m \sigma_i$ of the individual schedules to be a valid total schedule for the original S. If we take an *inter-agent constraint*, i.e., a constraint $X_j - X_i \leq c_{ij}$ for which $X_i \in \mathcal{X}_a$ and $X_j \in \mathcal{X}_b$ with $a \neq b$, this constraint is considered by neither A_a nor A_b in determining σ_a and σ_b, it might very well be that $\sigma_b(X_i) - \sigma_a(X_j) > c_{ij}$. Hunsberger (2002) proposes an iterative procedure, using the distance matrix D_S of the original problem S. In this procedure, each inter-agent constraint $X_j - X_i \leq D_S[i, j]$ is made obsolete by tightening the *intra-agent* constraints $X_j - z \leq D_S[0, j]$ and $z - X_i \leq D_S[i, 0]$. These tightenings consist in selecting values $\delta_{i,1}$ and $\delta_{i,2}$ such that

- $X_j - z \leq \delta_{i,1} \leq D_S[0, j]$,
- $z - X_i \leq \delta_{i,2} \leq D_S[i, 0]$, and

[3] Note that we consider the *execution* of a flexible solution here. Hence, it suffices to only consider the STN decoupling problem.

- $\delta_{i,1} + \delta_{i,2} \leq D_S[i, j]$.

If we add the new constraints $X_j - z \leq \delta_{i,1} \leq D_S[0, j]$ and $z - X_i \leq \delta_{i,2} \leq D_S[i, 0]$ to S while making sure that the resulting system remains consistent, the effect of adding these constraints is that an inter-agent constraint $X_j - X_i = (X_j - z) + (z - X_i) \leq \delta_{i,1} + \delta_{i,2} \leq D_S[i, j]$ is now *implied* by the new, tighter, intra-agent constraints. Hence, the inter-agent constraint can be removed from the updated STN. Applying this procedure for every non-implied inter-agent constraint ensures that all inter-agent constraints are implied by intra-agent constraints and the resulting system is a temporal decoupling.

2.8 Conclusions and research questions

In this chapter, the exact scheduling problem as faced by NedTrain has been formalized, and a few of the many approaches proposed during the years have been discussed. It is clear from the nature of the problem that exact solution methods are not feasible. This is reflected in the sizes of the problems solved by such methods: they are much smaller than the problems NedTrain are faced with. We are thus forced to turn to heuristic methods to solve this problem. Classical methods such as the serial and parallel schedule generation schemes are able to solve problems of the scale NedTrain requires, but they fail to offer the flexibility required during the execution of the schedule, since they give fixed execution times for each task.

Simple temporal networks offer an elegant way to represent this flexibility, while avoiding the complexity of the original RCPSP. Constraint posting methods form the bridge between these two worlds: they transform the RCPSP problem into an STN, which can then be used to execute the tasks in a flexible way. The STN offers a quick way to adapt the execution, since new solutions can be computed quickly.

An important consideration is the *amount* of flexibility present in such an STN. Intuition suggests that this flexibility plays an important role in the ability of a schedule to adapt to changing events, but how we can compare different STNs, and how we can quantify this ability, remains unclear. This gives rise to our first research question:

RQ1 What criteria encapsulate our intuitive notion of the concept of *flexibility*, as applied to solutions of scheduling problems, and can we devise a method to measure flexibility, conforming to these criteria?

Having defined what flexibility is, the next question that arises is how to actually make use of said flexibility. It is natural to assume that, given a certain solution, there are multiple ways of distributing the available flexibility. Given the goal of attaining a robust schedule, i.e., a schedule in which a disturbance in one task has low impact on the rest of the schedule, we can ask which of the possible distributions yields the highest robustness:

RQ2 Given a schedule with a certain potential flexibility, in what ways can this flexibility be used best to achieve a schedule with high *robustness*?

So far, our discussion on flexibility has focused on the temporal aspect of schedules. If we include the sequential aspect of schedules, it might be possible to increase the flexibility, leading to higher levels of robustness:

RQ3 In what way can the concept of temporal flexibility of a schedule be extended to include some *sequential flexibility*?

Another important aspect of the scheduling process at NedTrain is that maintenance is performed on multiple trains in parallel, by independent teams. This can be abstractly represented as a multi-agent scheduling problem. An important concern here is the independence of the teams—ensuring their independence can be mapped to decoupling in multi-agent systems. The natural concern here is keeping teams independent in the face of uncertain events:

RQ4 How can our earlier work on ensuring robustness of entire schedules be applied to the *decoupling* of multi-agent schedules, in which we strive to decouple in such a way that disturbances in the schedule of one agent minimally affect the schedules of the other agents?

Measuring flexibility

3

The first problem statement from Chapter 1 describes the need for a flexible automated process that solves the scheduling problem at NedTrain. In Chapter 2 it is shown that the RCPSP formalism matches well with the scheduling problem at NedTrain, and precedence constraint posting (Cesta et al., 1998; Lombardi and Milano, 2009b) is a method matching the requirement of a flexible process: it solves the RCPSP heuristically, meaning that it is quick enough to be used interactively, and the result has the form of a Simple Temporal Network (STN). From this STN, multiple fixed-time schedules can be derived very efficiently, as described in the previous chapter—this shows that the requirement of having a process resulting in a flexible schedule is satisfied.

It is intuitively clear that such an STN can be called *flexible*, since it can be used to encode a *range* of possible schedules. But how flexible is a given STN exactly? And how can two STNs be compared with each other in terms of flexibility? These questions are summarized in the first research question, which will be answered in this chapter by exploring what criteria encapsulate this intuitive notion of flexibility, and by determining the flexibility of an STN, as representation of both intermediate and final schedules in the precedence constraint posting process. One strategy, which seems very straightforward, is to compare the range of possible schedules. But this only serves to replace our question by a different one, namely how exactly we can determine this range. Ideally, we would want to have a *metric* with which we can measure the flexibility of an STN. Such a metric can then be used to evaluate different STN solutions produced by precedence constraint posting algorithms, or even as a heuristic, to guide the process of precedence constraint posting.

There are several existing metrics attempting to do this, but we will show that they all overestimate the flexibility contained in a certain schedule, by not fully taking into account dependencies among tasks. A new method is presented here, which does take dependencies into account more completely, yielding a flexibility metric that better conforms to the intuitive properties such a metric should have.

3.1 The need for flexibility

The primary reason for wanting to have a flexible schedule is the limited *preservability* of a fixed-time schedule. Start times of tasks are determined off-line, before execution starts, which means that unforeseen disturbances during the execution can prevent certain tasks from starting at their predetermined time. Delaying the start of these tasks is not always the best option, since this will cause the delay to spread to other parts of the schedule, which can ultimately lead to a violation of the due time of certain tasks. A single disturbance could therefore be a reason to modify or completely recompute the schedule. However, there might not be enough time to compute a new schedule each time a disturbance occurs. Additionally, commitments might have been made already on the basis of the old schedule, which might have to be broken if the new schedule takes effect.

Some techniques that try to incorporate flexibility in their schedule already exist (see Policella et al., 2004, 2005). They ensure flexibility in the off-line phase of schedule construction, but a transformation is still needed to ensure efficient execution (Smith, 2003). To avoid such a time-critical transformation, we propose the construction of an *interval schedule*, an assignment of time intervals to tasks, such that we are free to choose a specific starting time out of the interval for a task, without running the risk of violating any of the imposed scheduling constraints. If this is possible, this interval corresponds to the intuitive concept of flexibility we have with respect to scheduling this task.

Using time intervals in lieu of fixed time assignments has been proposed before: if a scheduling formalism is *decomposable*, we can attain flexibility by assigning an interval of time points to a task (see Dechter et al., 1991; Dechter, 2003; Pollack and Tsamardinos, 2005; Policella et al., 2004). A scheduling problem is decomposable if, for any partial schedule satisfying the constraints, there exists a complete schedule extending that partial schedule, satisfying all constraints. Note that this property is useful in the case of working with interval schedules: If there is an interval I_t from which all values are valid starting times for a task t, satisfying the constraints, we are sure that we can extend this partial schedule to a complete one. As we can start from *any* task t, it has often be proposed that these intervals are representative of the flexibility of the system. While intuitively appealing, this idea is not sufficient to construct flexible schedules, since decomposability does not imply that these intervals are *independent*. Decomposability allows us to determine the flexibility of two tasks in isolation, but it does not allow us to determine the flexibility of the *combination* of these two tasks.

A different reason to investigate alternatives to fixed-time schedules is the presence of events in the problem that are not under control of the agent executing the schedule (see Vidal and Fargier, 1999; Pollack and Tsamardinos, 2005). For such events, we might only know the interval in which they will take place, not the exact time. Even having an exact schedule can only be a partial assignment of time points, in such a way that the schedule does not violate the imposed constraints, for *any* realization of the events not under control of the executing agents. This can be seen as flexibility required by an external party, and

in this case it also seems like a natural solution to use time intervals to represent the events not under our control.

3.2 Flexibility of an STN

To start our discussion on flexibility, we begin by examining what our intuitive notion of this concept might translate to, in the context of the STN. Intuitively, the flexibility of an STN $S = (\mathcal{X}, \mathcal{C})$ refers to the amount of freedom we have in assigning values to the time point variables in $\mathcal{X}$. Earlier, we defined a fixed schedule σ for an STN as an assignment $\sigma : \mathcal{X} \to \mathbb{R}$ satisfying all the constraints in $\mathcal{C}$. Such a schedule, however, is just an arbitrary fixed assignment among a large set of possible alternative assignments also satisfying the constraints. Therefore a single fixed schedule does not offer any indication of the flexibility we have in assigning values to the variables of the STN it has been derived from. We are thus interested in other types of solutions, which do give an indication of the flexibility.

3.2.1 *Using earliest and latest starting times*

If we want flexibility to indicate our freedom of choice, then one option for defining the flexibility for scheduling event X is to use the difference $\mathrm{lst}(X) - \mathrm{est}(X) = \mathrm{flex}_N(X)$. The total flexibility of an STN S then could be defined as

$$\mathrm{flex}_N(S) = \sum_{X \in \mathcal{X}} \mathrm{flex}_N(X). \tag{3.1}$$

Obviously, $\mathrm{flex}_N(S)$ can be computed in $O(n^3)$ time. We also know that $\mathrm{flex}_N(X)$ gives an exact measure of the flexibility of event X, since for every choice $v \in [\mathrm{est}(X), \mathrm{lst}(X)]$, there exists a schedule σ for S such that $\sigma(X) = v$ (see Proposition 2.1). However, although for every individual event, $\mathrm{flex}_N(X)$ offers a precise measure of its flexibility, using the sum $\mathrm{flex}_N(S)$ of these flexibilities has a serious disadvantage, due to *dependencies* that might exist between starting times of events. To see this, consider the following example:

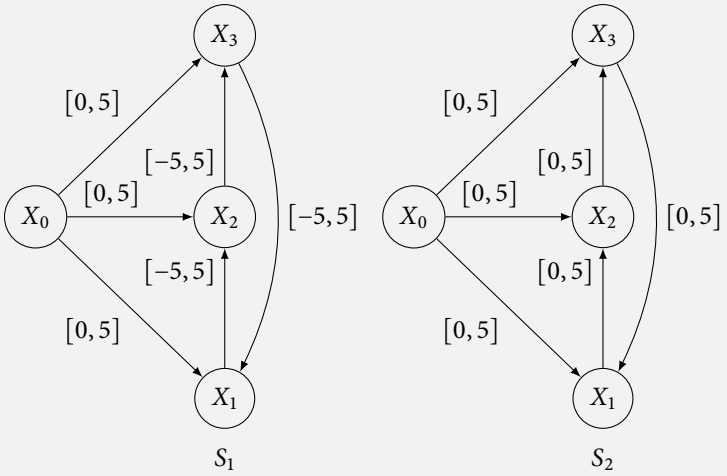
Example 3.1 Consider the STNs S_1 and S_2 shown on the next page, where S_1 specifies the *concurrent* execution of $k = 3$ events within 5 time units, while S_2 specifies the *sequential* execution of $k = 3$ consecutive events, i.e., X_2 should not occur before X_1, while X_3 should not occur before X_2 and all should occur within 5 time units.

Note that both in S_1 and in S_2, for each $i \in \{1, 2, 3\}$, $\mathrm{est}(X_i) = 0$ and $\mathrm{lst}(X_i) = 5$. Hence,

$$\mathrm{flex}_N(S_1) = 3 \times 5 = 15 = \mathrm{flex}_N(S_2). \tag{3.2}$$

So, according to the measure flex_N, S_1 and S_2 have the same flexibility, which, of course, is counterintuitive: For each event X_i in S_1 we can determine its

starting time between 0 and 5 independently from the other events. So, the total flexibility, expressed as the amount of choice we have in selecting a starting time for an event equals $5 \times 3 = 15$.



The choice for the starting time of an event X_i in S_2, however, may influence the choice of the starting time of the next event X_{i+1}. More precisely, if v_1, v_2 and v_3 are the latest starting times for the events X_1, X_2 and X_3, respectively, in S_2, it is clear that

$$0 \leq v_1 \leq v_2 \leq v_3 = 5. \tag{3.3}$$

Hence, the flexibility of X_1 is $v_1 - 0$, the flexibility of X_2 is $v_2 - v_1$ and the flexibility of X_3 is $5 - v_2$. So, the total flexibility of S_2 equals

$$\text{flex}(S_2) = v_1 + (v_2 - v_1) + (5 - v_2) = 5. \tag{3.4}$$

Therefore, the flexibility of S_2 should come out as only $\frac{1}{3}$ of the flexibility of S_1. Generalizing S_1 and S_2 to systems containing k concurrent and sequential events, respectively, we see that the freedom to schedule in S_1 equals $\text{flex}(S_1) = 5 \times k$, while the flexibility of S_2 is independent of k and remains $\text{flex}(S_2) = 5$. Hence, if k goes to infinity, the ratio of the real flexibilities of S_1 and S_2 goes to infinity:

$$\lim_{k \to \infty} \frac{\text{flex}(S_1)}{\text{flex}(S_2)} = \lim_{k \to \infty} \frac{5 \times k}{5} = \infty. \tag{3.5}$$

while their ratio according to flex_N is exactly 1:

$$\lim_{k \to \infty} \frac{\text{flex}_N(S_1)}{\text{flex}_N(S_2)} = \frac{5 \times k}{5 \times k} = 1. \tag{3.6}$$

We conclude that the flex_N metric seriously overestimates the flexibility of STNs similar to S_2.

Using upper and lower bounds for every pair of events
One reason flex_N fails is that it is not able to deal with *dependencies* between timed events. To overcome this disadvantage, we have to incorporate information about the dependencies between two events. This was proposed in the definition of flexibility by Hunsberger (2002) and others (Policella et al., 2005, 2007). Hunsberger defined his flexibility metric, which we will denote by $\text{flex}_H(S)$, by taking into account not only the apparent flexibility $\text{lst}(X) - \text{est}(X)$ of every event X, but also the flexibility between pairs of events X_i and X_j. Since the distance matrix D_S contains the entries $D_S[i,j]$ and $D_S[j,i]$ such that

$$-D_S[j,i] \leq X_j - X_i \leq D_S[i,j] \tag{3.7}$$

are the strongest constraints implied by $\mathcal{C}$ w.r.t. the temporal difference $X_j - X_i$, we can define the flexibility associated with this difference as the length of the corresponding interval:[1]

$$\text{flex}_H(X_i, X_j) = D_S[i,j] + D_S[j,i]. \tag{3.8}$$

Now the flexibility of the system S can be defined by taking $\text{flex}_N(S)$ and adding to it the sum of all these flexibilities for every distinct pair of time-points,[2] i.e.,

$$\text{flex}_H(S) = \text{flex}_N(S) + \sum_{i=1}^{n} \sum_{j>i}^{n} \text{flex}_H(X_i, X_j). \tag{3.9}$$

If there is a dependency between two events X_i and X_j, this will result in a lower value of $D_S[i,j] + D_S[j,i]$. Hence, dependencies among events will have an influence on flex_H, allowing us to account for its impact on flexibility. It turns out this metric is indeed an improvement, compared to the naive metric flex_N. But, the next example shows that flex_H also has serious shortcomings with respect to dependencies.

Example 3.2 Consider again the STNs S_1 and S_2, as described in the previous example. According to flex_H, the flexibility of S_1 equals

$$\text{flex}(S_1) = 3 \times 5 + 3 \times 10 = 45, \tag{3.10}$$

while the flexibility of S_2 equals

$$\text{flex}(S_2) = 3 \times 5 + 3 \times 5 = 30. \tag{3.11}$$

So, according to the Hunsberger metric flex_H, S_2 has a flexibility equal to $\frac{2}{3}$ of the flexibility of S_1. This seems to be an improvement with respect to

[1] Although Hunsberger defined the inverse of flexibility, i.e., *rigidity* of an STN, that is not relevant for our discussion at hand, since we assume a finite time horizon for an STN: this avoids the problem of infinitely sized intervals.

[2] Actually, Hunsberger took the square root of the sum of the squares of these flexibility measures per pair, divided by $(n \times (n-1)/2)$, the total number of different pairs of time-points.

the previous flexibility metric flex_N assigning the same flexibility to S_1 and S_2. However, generalizing S_1 and S_2 to systems containing k concurrent and sequential events, respectively, we observe that

$$\text{flex}_H(S_1) = 5 \times k + \binom{k}{2} \times 10 = 5 \times k^2, \tag{3.12}$$

while

$$\text{flex}_H(S_2) = 5 \times k + \binom{k}{2} \times 5 = 5 \times \frac{k^2 - k}{2}. \tag{3.13}$$

This means that for larger values of k, the ratio between $\text{flex}_H(S_1)$ and $\text{flex}_H(S_2)$ converges to 2:

$$\lim_{k \to \infty} \frac{\text{flex}_H(S_1)}{\text{flex}_H(S_2)} = \lim_{k \to \infty} \frac{5 \times k^2}{5 \times \frac{k^2 - k}{2}} = 2. \tag{3.14}$$

The ratio of the real flexibilities of S_1 and S_2 (see Equation 3.5 in the previous example) goes to *infinity* if k goes to infinity. We conclude that this metric is also not able to capture the dependencies between the flexibilities of events in a satisfactory way.

3.2.2 *Rationality postulates: towards a suitable flexibility metric*

Having demonstrated the shortcomings of flex_N and flex_H, we will now analyze these in more detail. We will do this by stating some simple *rationality postulates* (Alchourrón et al., 1985), characterizing these metrics. Using this characterization, an additional "independence" postulate is introduced, which represents the idea that the flexibility of one event should be independent of all other events. Having characterized the properties a metric should have to conform to the intuitive notion of flexibility, a suitable flexibility metric flex is derived, and we show that it can be computed in an efficient way.

Rationality postulates for flex_N

We take flex_N as main representative of the two metrics discussed above. The metric is defined as the sum of intervals, where each such interval $[a_X, b_X]$ indicates the flexibility of an event X. Flexibility is interpreted as the ability to pick a starting time for the event, which means that for every choice $v \in [a_X, b_X]$ as starting time for X, there is a schedule σ for S such that $\sigma(X) = v$. Obviously, there are many such intervals $[a_t, b_t]$ we can choose, including trivial choices having $a_X = b_X$. Since our aim is to determine the *maximum* available flexibility for a certain STN, we want the sum of these interval sizes to be maximal.

The postulates below capture the essence of a naive flexibility metric such as flex_N for an STN $S = (\mathcal{X}, \mathcal{C})$. We go one step further, and claim that for *all* metrics flex_n that satisfy these postulates, it holds that $\text{flex}_n = \text{flex}_N$, such that we can say that flex_N is completely characterized by these postulates.

If we let $S = (\mathcal{X}, \mathcal{C})$ be an STN, a naive flexibility metric flex_n is characterized by the following postulates:

F1 The flexibility $\text{flex}_n(X_0)$ of the temporal reference event $X_0 \in \mathcal{X}$ is equal to zero.

Comment The temporal reference point should be fixed.

F2 The flexibility $\text{flex}_n(X)$ of an event $X \in \mathcal{X} - \{X_0\}$ is a nonempty interval $[a, b]$ such that any $v \in [a, b]$ can be chosen as a starting time $\sigma(X) = v$ in some schedule σ for S.

Comment It must be ensured that the intervals are not too large: all points must be valid start times.

F3 Let $|[a, b]|$ denote the size $b - a$ of the interval $[a, b]$. Then,

$$\text{flex}_n(S) = \sum_{X \in \mathcal{X}} |\text{flex}_n(X)| \tag{3.15}$$

is the maximal value that can be obtained for all functions

$$\text{flex} : \mathcal{X} \to \{[m, n] : -\infty < m \leq n < \infty\} \tag{3.16}$$

satisfying postulates **F1**–**F2**.

Comment This postulate ensures that we take a solution yielding maximal interval sizes, since we want to determine the maximal available flexibility. Trivial solutions, like fixed time assignments, also satisfy the first two postulates, and these must be avoided if there are better choices.

Next, we will show that these postulates uniquely characterize flex_N, meaning that for all possible metrics flex_n satisfying **F1**–**F3**, we have $\text{flex}_n = \text{flex}_N$:

Proposition 3.1 Let $S = (\mathcal{X}, \mathcal{C})$ an STN, and let flex_n be a flexibility metric satisfying **F1**–**F3**. Then, $\text{flex}_n = \text{flex}_N$.

Proof. It is clear that flex_N satisfies **F1**–**F3**. Let flex_n be any metric satisfying **F1**–**F3**. In order to satisfy **F2**, observe that for any $X \in \mathcal{X}$, with $\text{flex}_n(X) = [a, b]$, by Proposition 2.1 we must have

$$\text{est}(X) \leq a \leq b \leq \text{lst}(X). \tag{3.17}$$

But that implies, by **F1** and **F3**,

$$\text{flex}_n(S) = \sum_{X \in \mathcal{X}} \text{flex}_n(X) = \sum_{X \in \mathcal{X}} (\text{lst}(X) - \text{est}(X)) = \text{flex}_N(S). \tag{3.18}$$

□

Rationality postulates for flex

As has been shown using the examples in the previous sections, the results given by flex_N are counter-intuitive in the presence of dependencies between events, which results in an over-estimation of the flexibility. The underlying reason is the formulation of **F2**: If we have a single flexibility interval $[a_i, b_i]$ for $X_i \in \mathcal{X}$, we are indeed guaranteed to be able to find a schedule σ for any $v_i \in [a_i, b_i]$, having $\sigma(X_i) = v_i$. This does however no longer hold if we want to make $k > 1$ *simultaneous* choices $v_1, \ldots, v_k$ in the intervals $[a_1, b_1], \ldots, [a_k, b_k]$[3].

A simple postulate **F2′** can be added to the existing set of postulates **F1**–**F3** to remedy this deficiency. We will prove that any flexibility metric satisfying this, and the previous set of postulates, satisfies the (missing) requirement that for any series $(v_1, \ldots, v_n)$ of choices $v_i \in [a_i, b_i] = \text{flex}(X_i)$ we are guaranteed to be able to find a schedule σ for S such that $\sigma(X_i) = v_i$.

The complete set of postulates for a suitable flexibility metric for an STN $S = (\mathcal{X}, \mathcal{C})$, conforming to our intuitive notions of the concept, is thus:

F1 The flexibility $\text{flex}_n(X_0)$ of the temporal reference event $X_0 \in \mathcal{X}$ is equal to zero.

F2 The flexibility $\text{flex}_n(X)$ of an event $X \in \mathcal{X} - \{X_0\}$ is a nonempty interval $[a, b]$ such that any $v \in [a, b]$ can be chosen as a starting time $\sigma(X) = v$ in some schedule σ for S.

F2′ For some $X_i, X_j \in \mathcal{X}$ with $\text{flex}(X_i) = [a_i, b_i]$ and $\text{flex}(X_j) = [a_j, b_j]$, there exists a schedule σ for S such that $\sigma(X_i) = v_i$ and $\sigma(X_j) = v_j$, for every $v_i \in [a_i, b_i]$ and every $v_j \in [a_j, b_j]$.

Comment This implies independence between the flexibility intervals for X_i and X_j: a choice for $\sigma(X_i)$ has no impact on the available choices for $\sigma(X_j)$.

F3 Let $|[a, b]|$ denote the size $b - a$ of the interval $[a, b]$. Then,

$$\text{flex}_n(S) = \sum_{X \in \mathcal{X}} |\text{flex}_n(X)| \tag{3.19}$$

is the maximal value that can be obtained for all functions

$$\text{flex} : \mathcal{X} \to \{[m, n] : -\infty < m \leq n < \infty\} \tag{3.20}$$

satisfying postulates **F1**–**F2′**.

The new postulate **F2′** ensures independence for any *pair* of flexibility intervals. We will now show that this is sufficient to ensure independence of *all* flexibility intervals, in case there are more than two:

[3] We assume, without loss of generality, that $v_0 = 0$ is picked as reference time point.

Proposition 3.2 Let $S = (\mathcal{X}, \mathcal{C})$ be an STN. For any metric flex$'$ satisfying **F1**–**F2$'$** it holds that for every series $(v_0, \ldots, v_n)$ of choices, with $v_i \in \text{flex}'(X_i)$ for $i \in [1, n]$, there exists a schedule σ for S such that $\sigma(X_i) = v_i$.

Proof. Let flex$'$ be a metric satisfying **F1**–**F2$'$**, and let $(v_0, \ldots, v_n)$ be a series of arbitrary choices, with $v_i \in \text{flex}'(X_i)$ for $i \in [1, n]$. Define a function $\sigma : \mathcal{X} \to \mathbb{R}$ by $\sigma(X_i) = v_i \forall X_i \in \mathcal{X}$. Suppose, on the contrary, that σ is not a valid schedule for S. Then, there must exist a constraint $X_j - X_i \leq c_{ij} \in \mathcal{C}$ such that

$$\sigma(X_j) - \sigma(X_i) = v_j - v_i > c_{ij}. \tag{3.21}$$

By Postulate **F2$'$** however, there exists a schedule σ' for S such that $\sigma'(X_i) = v_i$ and $\sigma'(X_j) = v_j$. Therefore,

$$\sigma'(X_j) - \sigma'(X_i) = v_j - v_i \leq c_{ij}, \tag{3.22}$$

and we have derived a contradiction. Therefore, σ must be a schedule for S. □

This shows that the flexibility property, missing in flex_N and flex_H, can be repaired easily, by adding a postulate ensuring independence of the flexibility intervals. Having characterized the essential properties of our new flexibility metric flex, we will now show how to actually *compute* the flexibility $\text{flex}(S)$ of an STN S.

3.2.3 *Computing the flexibility of an STN*

As remarked already, we wish to depart from the idea of having fixed time assignments in a schedule σ for an STN S. Instead, we would like to have an *interval schedule* for S, having the property that we can freely choose a start time value v_i from within the interval for every event X_i, such that the combination of all those values comprises a schedule σ' for S. As a start, we will define this notion of an interval schedule formally:

Definition 3.1 Given an STN $S = (\mathcal{X}, \mathcal{C})$, a set $I_S = \{I_i = [l_i, u_i] : X_i \in \mathcal{X}\}$ of (non-empty) intervals for the time point variables $X_i \in \mathcal{X}$ is an *interval schedule* for S iff, for every $X_i \in \mathcal{X}$ and every $v_i \in [l_i, u_i]$, the assignment σ defined by $\sigma(X_i) = v_i$ is a schedule for S.

Any flexibility metric satisfying the postulates **F1**–**F2$'$** defines an interval schedule $I_s = \{I_i : X_i \in \mathcal{X}\}$ for an STN S by defining the correspondence $I_s = \{\text{flex}(X_i) : X_i \in \mathcal{X}\}$:

Proposition 3.3 Let $S = (\mathcal{X}, \mathcal{C})$ be an STN. The function $\text{flex} : \mathcal{X} \to \{[m, n] : -\infty < m \leq n < \infty\}$ satisfies the postulates **F1**–**F2$'$** iff the set of intervals $I_S = \{\text{flex}(X_i) : X_i \in \mathcal{X}\}$ is an interval schedule for S.

Proof (*if*). Suppose $I_S = \{\text{flex}(X_i) : X_i \in \mathcal{X}\}$ is an interval schedule for S. Then any series of choices $(v_0, \ldots, v_n)$ from the given intervals I_i in the interval schedule implies that the assignment $\sigma(X_i) = v_i$ is a schedule. In particular, this means that $\sigma(X_0) = 0$. Hence, we have $I_0 = [0, 0]$, satisfying **F1**. For all $i \geq 1$, this property also immediately implies that **F2** and **F2′** are satisfied.
(*only-if*). Suppose $\text{flex} : \mathcal{X} \to \{[m, n] : -\infty < m \leq n < \infty\}$ satisfies the postulates **F1**–**F2′**. By Proposition 2.1 and Definition 3.1, it follows that $I_S = \{\text{flex}(X_i) : X_i \in \mathcal{X}\}$ is an interval schedule for S. □

From this proposition it follows that if we want to compute a flexibility metric for an STN $S = (\mathcal{X}, \mathcal{C})$, satisfying **F1**–**F3**, we must be able to

1. *find* an interval schedule $I_S = \{[l_i, u_i] : X_i \in \mathcal{X}\}$ for S, such that
2. the sum $\sum_{X_i \in \mathcal{X}} (u_i - l_i)$ is *maximal*.

The first problem, that of computing an interval schedule, is solved by a transformation of the given STN S, to another STN S'. We will show that any solution to S' can be used to construct an interval schedule for S.

The second problem is solved by realizing that any STN, such as S', can be expressed as a set of linear expressions. A linear programming approach can then be used to solve the problem, and if we set the maximization of the sum of the interval sizes as objective, we are able to efficiently find an interval schedule that is maximal.

Computing interval schedules for an STN
As a start in computing an interval schedule for an STN $S = (\mathcal{X}, \mathcal{C})$, we will state a useful relationship between the bounds of an interval in a schedule I_S for S, and the constraints occurring in $\mathcal{C}$. Intuitively, as a consequence of requiring independent intervals, if there exists a constraint $X_j - X_i \leq c_{ij} \in \mathcal{C}$, then the intervals I_i and I_j should be such that independent choices for $v_i \in I_i$ and $v_j \in I_j$ will not lead to a violation of this constraint. The following proposition states that the independence property is preserved if this constraint is satisfied whenever v_j takes its *maximal* value in I_j, while v_i takes its *minimal* value in I_i:

Proposition 3.4 Let $S = (\mathcal{X}, \mathcal{C})$ be an STN. A set $I_S = \{[l_i, u_i] : X_i \in \mathcal{X}\}$ of intervals for the variables in $\mathcal{X}$ is an interval schedule for S iff, for all $(X_i, X_j) \in \mathcal{X}^2$, $X_j - X_i \leq c_{ij} \in \mathcal{C}$ implies $u_j - l_i \leq c_{ij}$.

Proof (*if*). Suppose the implication holds for all $(X_i, X_j) \in \mathcal{X}^2$. To show that I_S is an interval schedule for S, we choose an arbitrary value $v_i \in [l_i, u_i]$ for every $X_i \in \mathcal{X}$, and we let σ be defined by $\sigma(X_i) = v_i$ for all $X_i \in \mathcal{X}$.

We show that σ is a schedule for S. Suppose, on the contrary, that it is not. Then σ violates some constraint in $\mathcal{C}$, meaning that there exists a constraint

$X_j - X_i \leq c_{ij} \in \mathcal{C}$ such that $\sigma(X_j) - \sigma(X_i) > c_{ij}$. But this implies

$$c_{ij} < v_j - v_i \leq u_j - l_i, \tag{3.23}$$

as $v_i \leq u_j$ and $v_i \geq l_j$.

On the other hand, since $X_j - X_i \leq c_{ij} \in \mathcal{C}$, by the assumed implication, we have

$$u_j - l_j \leq c_{ij}. \tag{3.24}$$

Combining 3.23 and 3.24 we derive a contradiction, so σ cannot violate any of the constraints in $\mathcal{C}$. Hence, σ is a schedule for S.
(*only-if*). Suppose I_S is an interval schedule for S. Take some $X_j - X_i \leq c_{ij}$. By the definition of I_S there is a schedule σ for S such that $\sigma(X_j) = u_j$ and $\sigma(X_i) = l_i$. But then $u_j - l_i \leq c_{ij}$. □

This result is significant, since it allows us to know which constraints the endpoints of an interval schedule I_S have to satisfy for it to be valid. The proposition assumes that these endpoints are given, but they could just as well be *variables* that have to satisfy these constraints. So instead of using the endpoints l_i and u_i for X_i, we specify two variables, X_i^- and X_i^+, representing the lower bound l_i and the upper bound u_i of the interval I_i, respectively.

For each constraint $X_j - X_i \leq c_{ij} \in \mathcal{C}$ in the original problem, using Proposition 3.4, we know we have to add the constraint $X_j^+ - X_i^- \leq c_{ij}$. We also add the constraint $X_i^- - X_i^+ \leq 0$, to ensure the interval bounds are valid. And lastly, to ensure that X_0^+ and X_0^- form a valid reference time point, we add a reference time point X_0', and the constraints $X_0' - X_0^- \leq 0$ and $X_0^+ - X_0' \leq 0$, which ensure that $X_0' = X_0^+ = X_0^-$.

Note that all constraints constructed in this matter have the form of the temporal constraints admitted in an STN. The set of variables and constraints together form another STN S', which we will call the *double* STN. Using Proposition 3.4 we show that schedules for this double STN can be used to construct an interval schedule for the original STN, S:

Proposition 3.5 Let $S = (\mathcal{X}, \mathcal{C})$ be an STN. Consider the *double* STN $S' = (\mathcal{X}', \mathcal{C}')$ derived from S as follows:

$$\mathcal{X}' = \{X_i^+, X_i^- : X_i \in \mathcal{X}\} \cup \{X_0'\} \tag{3.25}$$

$$\begin{aligned}\mathcal{C}' = {} & \{X_j^+ - X_i^- \leq c_{ij} : X_j - X_i \leq c_{ij} \in \mathcal{C}\} \cup \\ & \{X_i^- - X_i^+ \leq 0 : X_i \in \mathcal{X}\} \cup \\ & \{X_0' - X_0^- \leq 0, X_0^+ - X_0' \leq 0\}\end{aligned} \tag{3.26}$$

Then, for every solution σ of S', the set $I_S = \{[\sigma(X_i^-), \sigma(X_i^+)] : X_i \in \mathcal{X}\}$ is an interval schedule for S.

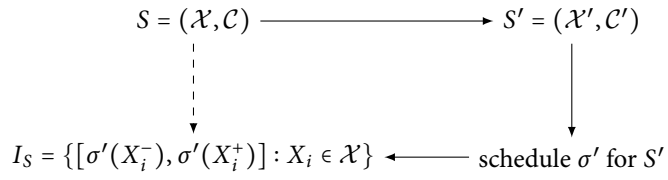


Figure 3.1
Computing an interval schedule I_S for S by using a schedule σ' for a double STN S'.

Proof. First, remember that we always assume a given STN S to be consistent. We will start with showing that S' is consistent, too. If we take an arbitrary schedule σ for S and construct an assignment σ' for S', by letting $\sigma'(X_i^+) = \sigma'(X_i^-) = \sigma(X_i)$ for every $X_i \in \mathcal{X}$, it is obvious that σ' will satisfy all constraints in $\mathcal{C}'$. Hence, S' is consistent.

Since S' is consistent, we can assume some arbitrary schedule σ' for S. We show that $\{[\sigma'(X_i^-), \sigma'(X_i^+)] : X_i \in \mathcal{X}\}$ is an interval schedule for S. If we take some arbitrary constraint $X_j - X_i \leq c_{ij} \in \mathcal{C}$, by Proposition 3.4, it is sufficient to show that $\sigma'(X_j^+) - \sigma'(X_i^-) \leq c_{ij}$. But this is immediate, since by construction of $\mathcal{C}'$, we know that $X_j^+ - X_i^- \leq c_{ij}$ is a constraint in $\mathcal{C}$, and σ' is a schedule for S'. □

In Figure 3.1, a commutative diagram is shown, depicting the suggested route to compute an interval schedule for an STN S, using the construction of the double STN.

Example 3.3 Consider the STN S_2 presented in Example 3.1. Let STN $S_2' = (\mathcal{X}', \mathcal{C}')$ be the double STN derived from S_2 where

$$\mathcal{X}' = \{X_0', X_0^-, X_1^-, X_2^-, X_3^-, X_0^+, X_1^+, X_2^+, X_3^+\} \tag{3.27}$$

$$\begin{aligned}\mathcal{C}' = {} & \{X_i^+ - X_j^- \leq 0 : 1 \leq i < j \leq 3\} \cup \\ & \{X_j^+ - X_i^- \leq 5 : 1 \leq i < j \leq 3\} \cup \\ & \{X_0^+ - X_i^- \leq 0 : i = 1, 2, 3\} \cup \\ & \{X_i^+ - X_0^- \leq 0 : i = 1, 2, 3\} \cup \\ & \{X_i^- - X_i^+ \leq 0 : X_i \in \mathcal{X}\} \cup \\ & \{X_0^+ - X_0' \leq 0, X_0' - X_0^- \leq 0\}\end{aligned} \tag{3.28}$$

One possible solution is

$$\begin{aligned}\sigma'(X_0^-) &= 0 & \sigma'(X_0^+) &= 0 \\ \sigma'(X_1^-) &= 0 & \sigma'(X_1^+) &= 1 \\ \sigma'(X_2^-) &= 1 & \sigma'(X_2^+) &= 3 \\ \sigma'(X_3^-) &= 3 & \sigma'(X_3^+) &= 4\end{aligned}$$

This schedule can be used to construct the following interval schedule for S:

$$I_S = \{[0,0],[0,1],[1,3],[3,4]\}. \tag{3.29}$$

Computing a maximal interval schedule for an STN

In the previous section we introduced the concept of a double STN S', with which it is possible to efficiently find interval schedules for the original STN S. Finding an arbitrary interval schedule is however not sufficient to determine the flexibility of an STN: the sum of the sizes of the intervals found is not necessarily maximal, as required by **F3**. We have thus to devise a method to find those solutions σ' to the double STN S' that maximizes the sum of the interval sizes.

It is easy to see that the following LP-formulation will solve this maximization problem, and thus also the problem of computing flex(S):

Theorem 3.1 Let $S = (\mathcal{X}, \mathcal{C})$ be an STN. Then flex(S) can be computed by solving the following linear program:

$$\max \quad \sum_{X_i \in \mathcal{X}} (X_i^+ - X_i^-) \tag{3.30}$$

$$\begin{aligned} \text{subject to} \quad & X_i^- \le X_i^+ && \forall X_i \in \mathcal{X} \\ & X_j^+ - X_i^- \le c_{ij} && \forall (X_j - X_i \le c_{ij}) \in \mathcal{C} \\ & X_0^+ = 0 \\ & X_0^- = 0 \end{aligned} \tag{3.31}$$

Proof. Let $S = (\mathcal{X}, \mathcal{C})$ be an STN. The constraints of the LP correspond to the constraints of the double STN S'. Hence, any solution σ' of this set of constraints can be used to create an interval set for S. So, according to Proposition 3.3, the function flex defined as

$$\text{flex}(X_i) = [\sigma'(X_i^-), \sigma'(X_i^+)] \quad \forall X_i \in \mathcal{X} \tag{3.32}$$

satisfies **F1**–**F2′**. Maximizing the sum of the sizes of the intervals $\sum_{X_i \in \mathcal{X}} (X_i^+ - X_i^-)$ thus obtained guarantees that this function also satisfies the last postulate, **F3**. □

Example 3.4 Applying the flexibility metric flex to the STNs S_1 and S_2 from our earlier example results in the following flex functions:

1. For S_1, we have flex$(X_i) = [0,5]$ for $i = 1,2,3$ and flex$(S_1) = 3 \times 5 = 15$.

2. For S_2, we have $\text{flex}(S_2) = 5$; one possible flex function is $\text{flex}(t_1) = [0, 5]$, $\text{flex}(t_2) = \text{flex}(t_3) = [5, 5]$.

So as intended, this flexibility function will assign 3×5 flexibility units to S_1, while assigning 5 units to S_2.

3.3 Applications

In this section we will discuss two applications of the flexibility metric introduced in this chapter. They are both focused on some of the problems encountered in dynamic schedule execution environments, such as found at NedTrain.

First, we apply the results of our flexibility metric to the *temporal decoupling problem*, in distributed scheduling, and we show that a globally optimal decoupling of an STN can be obtained. Next to being optimal, we also show that the flexibility of the subsystems is not affected, refuting a widespread belief in this research area.

A second application is in the field of the Simple Temporal Network with Uncertainty (STNU). Here, we show how the double STN construction can be applied to yield an elegant solution to the problem of *strong controllability* and its generalizations, in STNUs.

3.3.1 *Decoupling without loss of flexibility*

When events have to be scheduled in a distributed environment, STNs are often used as well (see Hunsberger, 2002; Boerkoel and Durfee, 2011, 2012; Brambilla, 2010). An STN $S = (\mathcal{X}, \mathcal{C})$ is called *distributed* if the events in $\mathcal{X}$ are distributed over k agents $A_1, A_2, \ldots, A_k$, such that each individual agent A_i is responsible for scheduling a (non-empty and disjoint) subset $\mathcal{X}_i \subseteq \mathcal{X} - \{X_0\}$ of events.[4] Together, we assume that the set $A = \{A_1, A_2, \ldots, A_k\}$ of agents induces a partition $\{\mathcal{X}_i\}_{i=1}^{k}$ of $\mathcal{X}$, i.e., the union of all sets $\mathcal{X}_i$ equals $\mathcal{X} - \{\mathcal{X}\}$, meaning that all time point variables are controlled by an agent.

The main reason for modeling the scheduling of the events $\mathcal{X}$ as a distributed environment is the assumption that the agents A_i act as *autonomous* schedulers—each agent A_i wants to determine its own schedule $\sigma_i : \mathcal{X}_i \to \mathbb{R}$ for the set $\mathcal{X}_i$ of tasks assigned to it, independent of the other agents. The problem with the requirement of autonomously scheduling agents however is that we still have to ensure, no matter what choices the agents make in constructing their individual schedule σ_i, that the composition of all these schedules satisfies *all* constraints $\mathcal{C}$ in the global STN $S = (\mathcal{X}, \mathcal{C})$. This means that, for every agent A_i, we have to find an STN $S_i - (\mathcal{X}_i \cup \{X_0\}, \mathcal{C}_i')$ such that the merge $\sigma = \bigcup_{i=1}^{k} \sigma_i$ of these individual schedules always constitutes a valid schedule for the global STN S.

[4] Note that the reference time point X_0 is excluded: it is fixed, and hence no agent controls it.

This problem, of ensuring that the merge σ of individual schedules σ_i for the STNs S_i controlled by the agents, is known as the *Temporal Decoupling Problem* Hunsberger (2002).

Definition 3.2 Let $S = (\mathcal{X}, \mathcal{C})$ be an STN. Suppose that $\mathcal{X} - \{z\} = \{\mathcal{X}_i\}_{i=1}^k$ is partitioned in k subsets $\mathcal{X}_i$. Then the *temporal decoupling problem* is to find k STNs $S_i = (\mathcal{X}_i \cup \{X_0\}, \mathcal{C}_i')$ such that, whenever $\sigma_1, \ldots, \sigma_k$ are independently constructed schedules for the individual STNs $S_1, \ldots, S_k$, respectively, their merge $\sigma = \bigcup_{i=1}^k \sigma_i$ is a schedule for the original STN S.

The heart of the problem are constraints involving events X_i and X_j that belong to different agents A_f and A_h. These agents may choose schedules σ_f and σ_h, respectively, that satisfy their own private constraints $\mathcal{C}_f$ and $\mathcal{C}_h$, respectively, but violate an *inter-agent constraint* $X_j - X_i \leq c_{ij}$ by setting $\sigma_f(X_j) - \sigma_h(X_i) > c_{ij}$.

Example 3.5 Suppose that in the STNs S_1 and S_2 of our earlier examples, we have three agents A_1, A_2 and A_3, where A_i is responsible for scheduling event X_i, with $i = 1, 2, 3$. Suppose that in $S_i = (\mathcal{X}, \mathcal{C}_i)$ we simply assign the constraints relating to X_i to A_i. This means that each agent A_i only needs to take care of the subset $\mathcal{C}_i = \{0 \leq X_i - X_0 \leq 5, 0 \leq X_0 - X_i \leq 5\}$. Hence, agent A_1 might choose $\sigma_1(X_1) = 4$, A_2 might choose $\sigma_2(X_2) = 2$, and A_3 might choose $\sigma_3(X_3) = 1$. Each of these assignment satisfies the local constraints, but together they violate the inter-agent constraints: for example, while $X_1 - X_2 \leq 0 \in \mathcal{C}$, we have $\sigma_1(X_1) - \sigma_2(X_2) > 0$.

Hunsberger (2002) gave an elegant solution to this problem. Assuming a minimal STN has been computed for the STN to be decomposed (i.e., the distance matrix D_S is known), he proposed an iterative procedure that basically makes each *inter-agent* constraint $X_i - X_j \leq D_S[j, i]$ obsolete by tightening the associated *intra-agent* constraints $X_i - X_0 \leq D_S[0, i]$ and $X_0 - X_j \leq D_S[j, 0]$.

These tightenings consist in selecting values $\delta_{i,1}$ and $\delta_{i,2}$ such that

1. $X_i - X_0 \leq \delta_{i,1} \leq D_S[0, i]$,
2. $X_0 - X_j \leq \delta_{i,2} \leq D_S[j, 0]$, and
3. $\delta_{i,1} + \delta_{i,2} \leq D_S[i, j]$.

Now, if we add the new constraints

$$X_i - X_0 \leq \delta_{i,1} \leq D_S[0, i] \tag{3.33}$$

and

$$X_0 - X_j \leq \delta_{i,2} \leq D_S[j, 0] \tag{3.34}$$

to S, while making sure that the resulting system remains consistent, the effect of adding these constraints is that an inter-agent constraint $X_i - X_j \leq D_S[i,j]$ is implied by the additional, tighter, inter-agent constraints, since we have

$$(X_i - X_j) = (X_i - X_0) + (X_0 - X_j) \leq \delta_{i,1} + \delta_{i,2} \leq D_S[i,j]. \tag{3.35}$$

The result is that such an inter-agent constraint can be removed from the updated STN. Applying this procedure for every non-implied inter-agent constraint ensures that all inter-agent constraints are implied by intra-agent constraints and the resulting system is a temporal decoupling. All the inter-agent constraints can be safely removed from the STN, and the resulting system is a temporal decoupling of S.

Example 3.6 Again referring to the two STN instances S_1 and S_2, note that in S_1, every inter-agent constraint $-5 \leq X_i - X_j \leq 5, 1 \leq i \neq j \leq 3$, is already implied by the intra-agent constraints $0 \leq X_i - X_0 \leq 5$ and $0 \leq X_j - X_0 \leq 5$, since

$$X_i - X_j = (X_i - X_0) + (X_0 - X_j) \leq 5 + 0 = 5.$$

Therefore, S_1 is in fact already a decoupled STN.

In S_2, however, we can achieve a decoupling by tightening the intra-agent constraints to (for example):

$$0 \leq X_1 - X_0 \leq 2 \quad 3 \leq X_2 - X_0 \leq 4 \quad 4 \leq X_3 - X_0 \leq 5.$$

The reader might verify that by these tightenings indeed every inter-agent constraint is now implied.

An arbitrary decoupling, however, is not always what we want, since the added constraints may limit the resulting flexibilities of the subsystems S_i. As a result, the sum $\sum_{i=1}^{k} \text{flex}(S_i)$ of the flexibilities of the subsystems S_i could be considerably less than the flexibility $\text{flex}(S)$ of the original system.

Example 3.7 To illustrate this flexibility loss due to decoupling, consider the decoupling of the STN S_2 as discussed in Example 3.6. We know that for the original STN, we have $\text{flex}_N(S_2) = 15$, $\text{flex}_H(S_2) = 30$ and $\text{flex}(S_2) = 5$. If we compute the sum of the flexibilities of all the subsystems $S_{2,i}$ it is evident that the given decoupling affects the flexibility of the system, regardless of the metric considered:

$$\sum_{i=1}^{3} \text{flex}_N(S_{2,i}) = 2 + 1 + 1 = 4 < 15$$

$$\sum_{i=1}^{3} \text{flex}_H(S_{2,i}) = 2 + 1 + 1 + 3 + 2 + 3 = 12 < 30$$

$$\sum_{i=1}^{3} \text{flex}(S_{2,i}) = 2 + 1 + 1 = 4 < 5$$

It is a popular belief in the research community that an optimal decoupling i.e., a decoupling that would maximize the flexibilities of the subsystems, would still suffer from some flexibility loss. In the literature we are aware of (e.g., Hunsberger, 2002; Boerkoel and Durfee, 2011, 2012) several experiments have been performed to measure this loss of flexibility. However, the metrics used all have been based on the flex_N and flex_H metrics discussed above.

Instead of using this metric we try to find a decoupling that maximizes the sum $\sum_{i=1}^{k} \text{flex}(S_i)$ of the flexibilities of the induced subsystems S_i, and we consider it an *optimal decoupling* if the ratio $(\sum_{i=1}^{k} \text{flex}(S_i))/\text{flex}(S)$ is maximal.

Contrary to the popular beliefs in the research community we will show now that

1. an optimal decoupling can be obtained in $O(k)$-time using the flexibility metric we discussed before, and

2. this optimal decoupling need not affect the flexibility of the system, i.e., we have $\sum_{i=1}^{k} \text{flex}(S_i) = \text{flex}(S)$.

Proposition 3.6 Let $\{S_i\}_{i=1}^{k}$ be an optimal decoupling of an STN S. Then $\sum_{i=1}^{k} \text{flex}(S_i) = \text{flex}(S)$.

Proof. Consider the set of intervals $\{[l_i, u_i] : X_i \in \mathcal{X}\}$ occurring as solutions of the LP from Theorem 3.1. Given the set $A = \{A_i\}_{i=1}^{k}$ of k agents, let $\mathcal{C}_{\text{inter}} \subseteq \mathcal{C}$ be the set of all inter-agent constraints, and for $i = 1, \ldots, k$, let $\mathcal{C}_i \subseteq \mathcal{C}$ be the set of constraints restricted to $\mathcal{X}_i$. For every inter-agent constraint $X_j - X_i \leq c_{ij}$, where X_i occurs in S_i and X_j occurs in S_j, with $i \neq j$, add the constraint $X_j - X_0 \leq u_j$ to $\mathcal{C}_i$ and add $X_0 - X_i \leq -l_j$ to $\mathcal{C}_j$. We show the following.

Claim 1 The resulting systems $\{S_i\}_{i=1}^{k}$ constitute a decoupling of S, and

Claim 2 The sum of the flexibilities $\text{flex}(S_i)$ of the systems S_i in the decoupling $\{S_i\}_{i=1}^{k}$ equals $\text{flex}(S)$.

These claims, together, obviously imply the proposition.

Proof of Claim 1 Suppose, on the contrary, that the merge σ of individual schedules σ_i violates a constraint in $\mathcal{C}$. This must be an inter-agent constraint $X_j - X_i \leq c_{ij}$ where X_i occurs in S_i and X_j occurs in S_j $(i \neq j)$. So we must have

$$\sigma_j(X_j) - \sigma_i(X_i) > c_{ij}, \tag{3.36}$$

but since σ_i satisfies all constraints in $\mathcal{C}_i$, it follows by construction that $\sigma_i(X_i) \geq l_i$, and similarly, $\sigma_j(X_j) \leq u_j$. Therefore,

$$\sigma_j(X_j) - \sigma_i(X_i) \leq u_j - l_i. \tag{3.37}$$

Observe that, since $X_j - X_i \leq c_{ij}$, the constraint $X_j^+ - X_i^- \leq c_{ij}$ occurs in the LP formulation. Since $X_j^+ = u_j$ and $X_i^- = l_i$ occur in the solution of this LP, we also know that $u_j - l_i \leq c_{ij}$. Hence, by (3.37),

$$\sigma_j(X_j) - \sigma_i(X_i) \leq c_{ij}, \tag{3.38}$$

contradicting (3.36).

Proof of Claim 2 Let $S' = (\mathcal{X}', \mathcal{C}')$, where $\mathcal{X}' = \bigcup_{i=1}^k \mathcal{X}_i$ and $\mathcal{C}' = \bigcup_{i=1}^k \mathcal{C}_i \cup \mathcal{C}$. Note that $\text{flex}(S') = \sum_{i=1}^k \text{flex}(S_i)$. Since $\mathcal{X}_i = \mathcal{X}$ and $\mathcal{C} \subseteq \mathcal{C}'$, it is easy to see that

$$\text{flex}(S') = \sum_{i=1}^{k} \text{flex}(S_i) \leq \text{flex}(S). \tag{3.39}$$

Let $I_S = \{[l_i, u_i] : X_i \in \mathcal{X}\}$ be an interval schedule for $S = (\mathcal{X}, \mathcal{C})$ realizing maximum flexibility. For every subsystem $S_i = (\mathcal{X}_i, \mathcal{C}_i)$, consider the set

$$I_{S_i} = \{[l_i, u_i] \in I_S : X_i \in \mathcal{X}_i\}, \tag{3.40}$$

being the restriction of I_S to $\mathcal{X}_i$. We show that for each $i = 1, \ldots, k$, I_{S_i} is an interval schedule for S_i. First, let σ_i be such that $l_j \leq \sigma_i(X_j) \leq u_j$ for every $X_j \in \mathcal{X}_i$. We show that σ_i is a valid schedule for S_i. So consider an arbitrary constraint $X_j - X_i \leq c_{ij} \in \mathcal{C}_i$. We consider the following cases:

$X_j = X_0$ In this case, $X_0 - X_i \leq c_{i0} \in \mathcal{C}_i$. If this constraint occurs in $\mathcal{C}$, too, we know that $0 - l_i \leq c_{i0}$ since $X_0 - X_i^- \leq c_{i0}$ occurs in the LP. Since $\sigma_l(X_i) \geq l_i$, it follows that $0 - \sigma_i(X_i) \leq c_{i0}$ and σ_i satisfies the constraint

If this constraint does not occur in $\mathcal{C}$, then it is added in the decomposition. Therefore, $c_{i0} = -l_i$. Since $\sigma_i(X_i) \geq l_i$, it follows that $0 - \sigma_i(X_i) \leq -l_i = c_{i0}$ and σ_i satisfies the constraint in this case as well.

$X_i = X_0$ In this case, $X_j - X_0 \leq c_{0j}$. Again, we have two sub-cases. If $X_j - X_0 \leq c_{0j}$ occurs in $\mathcal{C}$, we know that $u_j - 0 \leq c_{0j}$ since $X_j^+ - X_0 \leq c$ occurs in the LP. Since $\sigma_i(X_j) \leq u_j$, it follows that $\sigma_i(X_j) \leq c_{0j}$ and the constraint is satisfied.

If this constraint does not occur in $\mathcal{C}$, again, it is added to achieve the decomposition. Then, $c = u_j$, and since $\sigma_i(X_j) \leq u_j$ we have $\sigma(X_j) \leq c$, and the constraint is satisfied.

$X_i \neq X_0, X_j \neq X_0$ In this case $X_j - X_i \leq c_{ij}$ must occur in $\mathcal{C}$. Hence $X_j^+ - X_i^- \leq c_{ij}$ occurs in one of the conditions in the LP and $u_j - l_i \leq c_{ij}$. Since $\sigma_i(X_j) \leq u_j$ and $\sigma_j(X_i) \geq l_i$, we have $\sigma_i(X_j) - \sigma_i(X_i) \leq c_{ij}$ and σ_i satisfies the constraint.

This shows that σ_i is a valid schedule for S_i, so, for each $i = 1, \ldots, k$, I_S is an interval schedule for S_i. But, then it follows that for each $i = 1, \ldots, k$, we have

$$\sum_{I_j \in I_{S_j}} |I_j| \leq \sum_{X_j \in \mathcal{X}_i} \text{flex}(X_j) = \text{flex}(S_i) \tag{3.41}$$

Summing over $i = 1, \ldots, k$, we derive that

$$\text{flex}(S) = \sum_{i=1}^{k} \sum_{I_j \in I_{S_j}} |I_j| \leq \sum_{i=1}^{k} \sum_{X_j \in \mathcal{X}_i} \text{flex}(X_j) = \sum_{i=1}^{k} \text{flex}(S_i) \tag{3.42}$$

Now Claim 2 follows from Equations 3.39 and 3.42.

□

Example 3.8 Consider Example 3.6 again. A decoupling for S_2 realizing maximum flexibility can be created by adding the following decoupling constraints:

$$X_2 - X_0 \leq 5 \quad X_0 - X_2 \leq -5 \quad X_3 - X_0 \leq 5 \quad X_0 - X_3 \leq -5.$$

The sum of flexibilities of the subsystems equals the flexibility of the original system.

3.3.2 *Strong controllability in the* STNU

In the Simple Temporal Problem with Uncertainty (STNU) the STN is extended with an explicit distinction between *executable* and *contingent* events. This is an explicit way of modeling uncertainty: The set $\mathcal{X}^E$ contains the executable events, which are controlled by an agent executing the schedule, and the set $\mathcal{X}^C$ contains the contingent events, which are controlled by an external entity beyond our control, such as nature itself.

Given this distinction between $\mathcal{X}^C$ and $\mathcal{X}^E$, we can define the *strong controllability problem*:

Given: An STNU $S = (\mathcal{X}^C \cup \mathcal{X}^E, \mathcal{C})$, where $|\mathcal{X}^C| = m$ and $|\mathcal{X}^E| = n$.
Question: Is there a choice of (scheduled) time point values $v_0, \ldots, v_n$ for the events $X_i \in \mathcal{X}^E$ such that for every choice of time points $w_i, \ldots, w_m$ for events $X_j \in \mathcal{X}^C$, the function σ defined by $\sigma(X_i) = v_i$ for $X_i \in \mathcal{X}^E$ and $\sigma(X_j) = w_j$ for $X_j \in \mathcal{X}^C$ is a schedule for S?

In other words, is it possible to construct an assignment *a priori* for the executable events in $\mathcal{X}^E$, which will result in a valid schedule for S for *any* possible execution of the events in $\mathcal{X}^C$ that are beyond our control?

While it has been shown (see Vidal and Fargier, 1999) that this problem is tractable, we would like to show how our approach to flexibility can handle this problem in a very natural way, even extending the concept of strong controllability to a more general one. While the actual distinction in an STNU is between contingent and requirement links *between* events, for our purposes here we will focus on the events themselves instead, distinguishing between *required* flexibility of an event, and *computed* flexibility. The former refers to the contingent events, controlled by nature, and the latter to the events under control of the executing agent. For each contingent event X_i we only know a lower bound l_i and an upper bound u_i on the time of its occurrence. It is easy to see that we can re-cast this in terms of an interval schedule, such as introduced above, by stating that for each such event $X_i \in \mathcal{X}^C$, its required flexibility equals $I_i^C = [l_i, u_i]$. The next step then follows quite naturally, as instead of asking for the existence of a fixed schedule for the executable events in $\mathcal{X}^E$, we ask for a maximal interval schedule for these events:

Given: An STNU $S = (\mathcal{X}^C \cup \mathcal{X}^E, \mathcal{C})$, where $|\mathcal{X}^C| = m$ and $|\mathcal{X}^E| = n$.
Question: Is there a set I^E of intervals I_i^E for the events $X_i \in \mathcal{X}^E$ such that for the set I^C of required flexibility intervals I_j^C, $X_j \in \mathcal{X}^C$, the set $I^E \cup I^C$ is a maximal interval schedule for S extending I^C?

Note that in the case that $\mathcal{X}^C = \varnothing$, this problem reduces to that of finding a maximal interval schedule for an STN to obtain flex(S), as discussed in the previous section.

A simple solution to this problem can be given by a slight modification to our original method, which obtains a maximal interval schedule for S by using the double STN method, and solves an LP with maximization of the flexibility as the objective function. In order to represent the given (partial) interval schedule I^C in our method,

1. we should add required values for the lower bound variable X_i^- and the upper bound variable X_i^+ for every $X_i \in \mathcal{X}^C$ in the construction of the double STN, to ensure that the given flexibility interval I_i^C has its required width, and

2. we should restrict the objective function to the sum of the differences $(X_i^+ - X_i^-)$ where $X_i \in \mathcal{X}^E$, as the differences $(X_i^+ - X_i^-)$ for $X_i \in \mathcal{X}^C$ are fixed.

Adapting the formulation from Theorem 3.1, it is easy to see that the following LP formulation will solve this problem:

Proposition 3.7 Let $S = (\mathcal{X}^E \cup \mathcal{X}^C, \mathcal{C})$ be an STNU, where for each $X_i \in \mathcal{X}^C$ the uncertainty of X_i is given by the interval $I_i^C = [l_i, u_i]$. Then there exists a solution to the strong controllability problem for S if the following linear

program has a solution:

$$
\begin{aligned}
\text{max} \quad & \sum_{X_i \in \mathcal{X}^E} (X_i^+ - X_i^-) \\
\text{subject to} \quad & X_i^- \leq X_i^+ && \forall X_i \in \mathcal{X} \\
& X_j^+ - X_i^- \leq c_{ij} && \forall (X_j - X_i \leq c_{ij}) \in \mathcal{C} \\
& X_i^- = l_i && \forall X_i \in \mathcal{X}^C \\
& X_i^+ = u_I && \forall X_i \in \mathcal{X}^C \\
& X_0^+ = 0 \\
& X_0^- = 0
\end{aligned}
$$

Moreover, if there is a solution, we are guaranteed to have a maximum flexible schedule for S extending I^C.

Proof. Let $S = (\mathcal{X}^E \cup \mathcal{X}^C, \mathcal{C})$ be an STNU. The constraints of the LP are as least as tight as the constraints of the double STP S', such as in Theorem 3.1, so any solution σ' of this set of constraints can be used to create an interval set $I = I^E \cup I^C$ satisfying the constraints of S', which guarantees that at least one solution exists.

We know that this interval set will have $I_i^C = [l_i, u_i]$ for every $X_i \in \mathcal{X}^C$, guaranteeing us that any assignment within these bounds satisfies the constraints in S', therefore, we can conclude that the solution that has been proven to exist is a solution to the strong controllability problem.

Lastly, since the intervals given by I^C are fixed, and the size of the intervals for I^E is maximized, we are guaranteed that the solution we have found has maximal flexibility. □

This approach to solving the strong controllability problem can be generalized further. Suppose we have an instance of the strong controllability problem for which no schedule σ exists, solving it. Instead of stopping here, we might ask if it is possible to find a set of *maximal subintervals* $I_i'^C \sqsubseteq I_i^C$ for the contingent events $X_i \in \mathcal{X}^C$ such that a schedule *does* exist. The reason for this generalization is obvious: if we manage to find a subset of these intervals that covers a large portion of the original intervals, there is a high probability that a chosen schedule for the executable events does not violate the constraints.

This problem can be simply solved by another modification to the LP formulation. In this case, we want to find a solution minimizing the total sum of differences between the lower and upper bounds obtained, and the lower and upper bounds required by the contingent events. The following linear program

achieves this:

$$\begin{aligned}
\min \quad & \sum_{X_i \in \mathcal{X}^C} ((u_i - X_i^+) - (X_i^- - l_i)) \\
\text{subject to} \quad & X_i^- \leq X_i^+ && \forall X_i \in \mathcal{X} \\
& X_j^+ - X_i^- \leq c_{ij} && \forall (X_j - X_i \leq c_{ij}) \in \mathcal{C} \\
& X_i^- \geq l_i && \forall X_i \in \mathcal{X}^C \\
& X_i^+ \leq u_i && \forall X_i \in \mathcal{X}^C \\
& X_0^+ = 0 \\
& X_0^- = 0
\end{aligned}$$

3.4 Conclusion and discussion

In this chapter, the first research question was investigated, asking which criteria encapsulate our intuitive notion of flexibility, and how to devise a method measuring that flexibility. For the first part of this question, the rationality postulates **F1** through **F3** introduced in this chapter provide a rigorously defined answer to this question, based on the concept of an interval schedule, in which each time point variable has an associated interval from which a value can be picked. The flexibility of the schedule is then defined as the sum of all the interval sizes. The most significant contribution here is postulate **F2′**, describing the independence requirement. As Proposition 3.2 shows, satisfying this postulate indicates that all the choices made in assigning time point variable values from such intervals are *independent* from each other. Assigning a value to a time point variable has no impact on the range of choices for any of the other intervals. Hence, we can be assured that the sum of the sizes of these intervals is representative of the flexibility we can make use of while executing the schedule.

The second major contribution is Theorem 3.1, which shows a method to actually calculate the sum of these interval sizes, hereby answering the second part of the research question. A side result of the method is a valid interval schedule for the original STN, which can be used to efficiently execute the schedule under dynamic circumstances.

Besides schedule execution, two other applications are discussed as well. The first application is in the optimal decoupling of multi-agent schedules, partially answering the fourth research question, asking how to decouple schedules in such a way that disturbances in the schedule in one agent have low impact on the other agents. We disprove the wide-spread belief in the research community that the temporal decoupling of an STN leads to a reduction of the available flexibility, based on the flexibility definition using our rationality postulates: we show that our earlier result on the creation of maximally flexible interval schedules can be applied to create a temporal decoupling for an STN.

The second application is in solving the strong controllability problem for the STNU, the Simple Temporal Network with Uncertainty. Here, uncertain events are modeled explicitly, as contingent events over which the executing agent has no control. Our method to find interval schedules can be adapted to this problem, by explicitly assigning the required flexibility to these contingent events, and then finding a maximally flexible interval schedule for the controllable events. Additionally, we generalize the notion of finding a solution to the strong controllability problem. If a solution cannot be found, our method can, with another adaptation, find a solution that maximizes the probability of a consistent schedule, by finding new interval bounds for the contingent events, encapsulating the largest possible portion of the original intervals still resulting in a solution.

3.4.1 Discussion

As can be seen from Example 3.4, presenting an interval schedule with maximal flexibility, and Example 3.8, presenting a decoupling with maximal flexibility, interval schedules and decouplings can be obtained in many, sometimes unfair, ways. In both these examples, one event, and one agent, respectively, gets all the available flexibility, leaving none for the other events or agents. This indicates that there are multiple ways to make use of the available flexibility in a schedule, which shows the need for the second research question, asking how flexibility can be used best to achieve a schedule with high robustness. In the next chapter we will therefore investigate modifications to the LP formulation presented in Theorem 3.1, with the aim of influencing the flexibility distribution to improve fairness and robustness.

Another extension of the work in this chapter, of which the careful reader might have already realized the possibility, lies in the computation of the flexibility for general linear systems of the form $\mathbf{Ax} \leq \mathbf{b}$, as has already been done (see Witteveen et al., 2014). This offers the possibility to compute sets of solutions for this system, guaranteeing that any choice from within this set preserves the maximum value of the objective function. Moreover, the extension from schedule flexibility to schedule decoupling can also be made in this case, giving efficient solutions for decoupling problems, such as analyzed by Brodsky et al. (2005).

Distributing flexibility to improve robustness

4

In the previous chapter, a method for constructing a maximally flexible interval schedule for an STN was introduced, answering the first research question. But as already mentioned in the discussion section, there is more to the flexibility of a schedule than simply maximizing it: often, the distribution of available flexibility is very lopsided. We therefore continue exploring the concept of the flexibility of a schedule in this chapter, turning to the second research question: we analyze the effect of different ways of distributing the available flexibility over the schedule, with the goal of achieving high *robustness*. Flexibility and robustness are two interrelated concepts. *Flexibility*, as discussed in the previous chapter, is the ability of a schedule to adapt to unexpected events without changes, for example by utilizing the intervals assigned to each task to adapt to the new situation.

It is clear that some events have such a disruptive nature that this is no longer possible. In this case the schedule has to be changed to adapt, usually by delaying tasks and starting them at a later time than intended. We define the robustness of a schedule as the ability of a schedule to withstand such disruptions—the fewer the number of tasks that have to be delayed outside their planned execution time to accommodate disruptions the more robust the schedule is. Often, a schedule is not only used to plan the execution of the tasks itself, but also for the reservation of resources to be used by the tasks, or to make commitments with external parties involved in the execution of tasks. If tasks have to be delayed to accommodate unexpected events, such commitments are put at risk, which is undesirable. If the distribution of flexibility is lopsided, parts of the schedule have very low flexibility, and disturbances will be much more disruptive. In this chapter we will examine ways to arrive at a distribution of flexibility in which this problem is reduced, in an attempt to increase the robustness.

This chapter starts with a motivation, which shows using examples why there is a need to pay attention to how the available flexibility in a schedule is distributed over the individual tasks. In the next section, methods are proposed to actually achieve certain distributions of flexibility, with the aim of improving the robustness of the schedule during execution. These methods are then tested using a simulated execution of the schedule, and the results are discussed and explained.

4.1 Motivation

In the previous chapter, we introduced a new method for measuring the flexibility of an *interval schedule*, which represents a set of solutions to a resource-constrained project scheduling problem instance: the original RCPSP is transformed into an STN, by using constraint posting techniques that add temporal constraints to eliminate resource constraints; for this STN we construct a solution in the form of an interval schedule, which then also serves as a solution for the original RCPSP instance. Many different interval schedules can be found for a single STN, however. To calculate the flexibility of the STN, an interval schedule is found that maximizes the sum of the sizes of the intervals, and hence the flexibility. But even using this objective, many different interval schedules are still possible. Looking back to the original problems as discussed in Chapter 1, one of the motivations for our work is the need for *robust* schedules, i.e., schedules that are resistant, to a certain extent, to external disturbances. Next to their use in dispatching tasks for execution, schedules are often also used to reserve resources needed for the execution of a task at some time point in the future, or to make commitments to external parties needed to execute a task. This is were robustness comes into play: when disturbances are present, it is important to still be able to rely on the predictions of a schedule, such that commitments that have been made based on the schedule can be honored. Robustness is therefore operationalized as the ability of a schedule to dispatch the tasks as planned, even when disturbances are present. It is in this light that this chapter will evaluate the different interval schedules that can be constructed for a given STN transformation of the original planning problem. To motivate this more clearly, let us look at the following example.

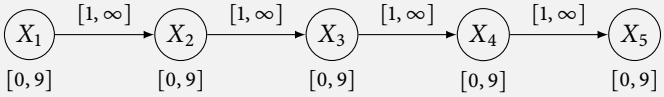
Example 4.1 Consider an RCPSP instance containing five tasks $t_1, \ldots, t_5$, where for all i, $s_i = 0$, $d_i = 10$ and $l_i = 1$, i.e., all tasks have length 1, and they have to execute within the time interval $[0, 10]$. Resource constraints dictate that none of the tasks can execute in parallel, such that one STN transformation for this problem would be $S_{\text{seq}} = (\mathcal{X}, \mathcal{C})$, with

$$\mathcal{X} = \{X_0, X_1, \ldots, X_5\}$$

representing the reference time point X_0 and the start times for the tasks, and

$$\begin{aligned}\mathcal{C} = &\{X_0 - X_i \leq 0 : i = 1, \ldots, 5\} \cup \\ &\{X_i - X_0 \leq 10 - l_i = 9 : i = 1, \ldots, 5\} \cup \\ &\{X_i - X_{i+1} \leq -l_i = -1 : i = 1, \ldots, 4\}\end{aligned}$$

representing, respectively, the release time constraints, due time constraints, and precedence constraints enforcing one particular sequential execution of the tasks. Below, the temporal network for this problem is shown (to reduce clutter, the constraints between a point and X_0 are shown as a label on that time point).



As the reader might verify, we have $\text{flex}(S_{\text{seq}}) = 5$ for this simple example. Two possible interval schedules achieving this flexibility score are

$$I_r = \{[0,0],[1,1],[2,2],[3,3],[4,9]\},$$

a schedule placing all available flexibility at the "rightmost" time point X_5, and

$$I_l = \{[0,5],[6,6],[7,7],[8,8],[9,9]\},$$

which places all flexibility at the "leftmost" time point X_1 (for both, $X_0 \in [0,0]$). Both schedules have the same amount of total flexibility, but I_l is much more vulnerable if task processing times are increased due to unexpected delays: for every task except X_1 such a delay has a direct effect on the makespan of the schedule. In contrast, I_r is much less vulnerable to delay with regard to the makespan of the schedule, since all flexibility is placed at the end of the schedule.

As the example shows clearly, simply maximizing the flexibility is not enough if we are interested in robustness of the schedule: Both I_r and I_l satisfy the objective of maximal flexibility, but one of them is much better at guaranteeing we can satisfy the imposed deadline than the other. Besides satisfying a due date, or guaranteeing a certain makespan, however, there might be other aspects in which we could want to ensure robustness, as motivated by the following example.

Example 4.2 Suppose that in the case of the scheduling problem outlined in the example above, task t_4 requires some sort of special tool, or an outside contractor, which has to be reserved at some time point well before executing the task itself. For both the interval schedules I_l and I_r, as presented in the previous example, there is a risk that tasks preceding t_4 have an impact on the starting time of t_4, causing it to be started outside of the interval I_4 assigned to it. While this might have no impact on the deadline of the entire schedule, any previous commitment made to reserve the aforementioned special resource will be violated.

A schedule tailored to this specific situation is

$$I_4 = \{[0,0],[1,1],[2,2],[3,8],[9,9]\},$$

where all available flexibility is given to t_4, which gives a high certainty that, whatever the delays occurring, it will be possible to execute it in the planned interval.

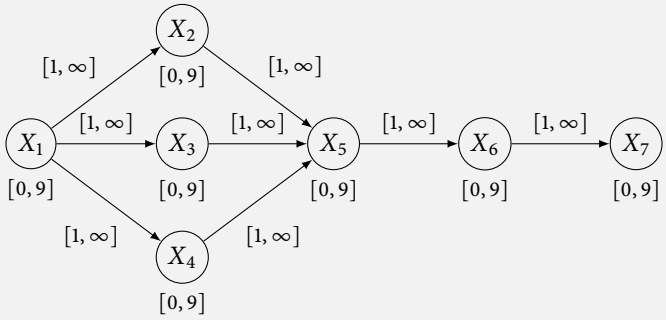
The above assumes a priori knowledge of which task(s) have external commitments that are to be honored. We can generalize this idea for situations where this is not known in advance, and try to improve the likelihood that the actual execution of a task t_i takes place in its assigned interval I_i, such that any (prior) commitments related to the execution of t_i will be honored. If, in our example, the delays to be expected are small relative to the task length (i.e., the delay is rarely larger than the length of the actual task), a more equalized distribution of flexibility over the schedule, such as in a schedule like

$$I_e = \{[0,1],[2,3],[4,5],[6,7],[8,9]\},$$

has a much lower risk of tasks execution outside their assigned interval due to delays. Note that we still have $\text{flex}(I_e) = 5$, meaning that the flexibility is still maximal.

We can even go a step further, and state that sacrificing some of the available flexibility, as seen over the total schedule, can be beneficial, if it improves the *distribution* of flexibility over the rest of the schedule. The following example provides a concrete motivation for this claim.

Example 4.3 Consider a variation on the RCPSP instance presented in Example 4.1, in which t_2 is split into three sub-tasks that can be executed in parallel, given the available resources. The temporal network for one possible STN transformation $S_{\text{par}} = (\mathcal{X}, \mathcal{C})$ is shown below.



Maximizing $\text{flex}(S_{\text{par}})$ yields, in this case, the unique interval schedule

$$I_{\text{par}} = \{[0,0],[1,6],[1,6],[1,6],[7,7],[8,8],[9,9]\}.$$

For this schedule, we have $\text{flex}(I_{\text{par}}) = 15$, but note that all flexibility is concentrated in the parallel part of the schedule, i.e., in tasks t_2, t_3 and t_4. The sequential part of the schedule receives no flexibility at all. Recalling the argument made in Example 4.2, there might be valid reasons for wanting to have a more equalized distribution of flexibility over the tasks. A schedule

that achieves this for this example is

$$I'_{\text{par}} = \{[0,1],[2,3],[2,3],[2,3],[4,5],[6,7],[8,9]\},$$

in which every task has an equal amount of flexibility. Note, however, that $\text{flex}(I'_{\text{par}}) = 7$: in global terms, this schedule offers lower flexibility. However, if honoring overall commitments made prior to execution is a concern, it is clear that I'_{par} is to be preferred over I_{par}: flexibility is distributed more evenly, such that the sequential parts of the schedule have higher robustness than when using I_{par}.

From these examples, it is clear that there is more to it than maximizing the flexibility of a schedule if our goal is to achieve some form of robustness. Moreover, we observe that, even if we keep our aim of having maximal global flexibility, there may be multiple schedules satisfying that goal, with differing properties regarding robustness during execution. Finally, sometimes there is a clear case to be made for sacrificing a certain amount of (global) flexibility, to be able to improve the distribution of flexibility over all individual tasks in a schedule. In the following section, we will formalize this notion of flexibility distribution.

4.2 Methods for distributing flexibility

As a first step in formalizing the distribution of flexibility over interval methods, we note that, while our aim is to distribute flexibility more equally, it is undesirable to *enforce* a strictly uniform distribution of flexibility—this could easily lead to schedules with unreasonably low or even zero flexibility: If there is a single task with low or zero flexibility, this would enforce this level of flexibility to all other tasks, needlessly reducing flexibility.

Equalizing flexibility To make this idea more explicit, we start by examining the intervals I_a and I_b assigned to two time point variables X_a and X_b in some STN $S = (\mathcal{X}, \mathcal{C})$ over which we need to distribute flexibility. Our goal is to discriminate between two situations, in which the total flexibility is equal, but in which the distribution differs. In this case, we would like to differentiate between $|I_a| = |I_b|$, and $|I'_a| - \varepsilon < |I'_b| + \varepsilon$. While $|I_a| + |I_b| = |I'_a| + |I'_b|$, the first assignment is to be preferred, since the distribution of the flexibility is more equal. Minimizing this deviation ε bears resemblance to the idea of least squares minimization, in which the squared residual error of a curve as compared to a set of data points is minimized. Therefore, we propose to use the objective function

$$\min \sum_{X_i \in \mathcal{X}} ((\text{lst}(X_i) - \text{est}(X_i)) - (X_i^+ - X_i^-))^2, \tag{4.1}$$

which is used to minimize the squared difference between the maximally available flexibility for each time point variable X_i, and the actual flexibility received

in the interval schedule. This will result in equalization of flexibility, since distribution of a certain amount of flexibility over multiple time point variables will yield a lower score than if all flexibility were concentrated on a single variable. This equation replaces Equation 3.30 in Theorem 3.1[1].

The above assumes that all tasks in a scheduling problem are equal, in terms of flexibility requirements. In practice, this is not always the case. If it is known in advance that some tasks are more susceptible to disturbances, this should be reflected in the flexibility distribution of the schedule, by favoring these tasks. Similarly, the structure of the dependency network connecting the tasks in the schedule can also have influence on the consequences of a disturbance. As an example, if a certain task t_i has a large number of predecessors, the chances that at least of them will suffer a delay is higher. Assigning more flexibility to the associated time point X_i as a prevention can increase the robustness of the schedule.

Influencing the flexibility distribution Reflecting this desire to more directly influence the flexibility distribution, we introduce the objective function

$$\min \sum_{X_i \in \mathcal{X}} w(X_i) \cdot ((\mathrm{lst}(X_i) - \mathrm{est}(X_i)) - (X_i^+ - X_i^-))^2, \tag{4.2}$$

a weighted objective function, where $w : \mathcal{X} \to \mathbb{R}$ is a weight function influencing the amount of flexibility X_i should have. The higher $w(X_i)$, the more flexibility is given to X_i, at the expense of tasks with a weight lower than $w(X_i)$.

An obvious choice for w would be some function representing a priori knowledge of delay probabilities for the tasks in the schedule. If it is known in advance that a certain task is highly probable to be delayed, it makes sense to give it a higher weight, to ensure that additional flexibility is available for it. However, we will assume that no such information is available beforehand, and that we will have to use other properties to influence the weight per time point variable. One evident property that we could use is of course the structure of the network of tasks surrounding a time point variable X_i, in particular the number of predecessors and successors.

To start, we consider the number of *predecessors* X_i has. Our rationale is that a time point variable with more predecessors has a higher risk of being impacted by a delay, since any of those predecessors could incur a delay that might propagate to X_i. A basic weight function $w(t_i)$ based on the number of predecessors of X_i can be defined as

$$w(X_i) = |\{X_j : X_j \prec X_i\}|. \tag{4.3}$$

This function simply counts the number of predecessors X_i has, and assigns that value to $w(X_i)$.

[1] Instead of a linear programming problem, we are now confronted with a quadratic programming problem. But as it turns out, the functions to be optimized are convex, making the use of the ellipsoid method possible, which solves this problem in polynomial time.

A disadvantage might be that this function heavily biases the "tail end" of the schedule: time point variables at the end of the schedule invariantly have more predecessors than those at the beginning. Furthermore, we can pose that a delay at the very beginning has a low chance of reaching the very end of a schedule. Time points in between will usually also have some flexibility, which absorbs the delay, especially if the delay is small. In the most extreme case, we could propose to use the function

$$w(X_i) = |\{X_j : X_j \ll X_i\}|, \tag{4.4}$$

where $\ll$ is the immediate predecessor relation, i.e., $X_j \ll X_i$ if, and only if, $X_j \prec X_i$ and there is no X_k such that $X_j \prec X_k \prec X_i$, with $i \neq j \neq k$. This function hence only counts the direct predecessors of a time point variable X_i and uses that value to assign a value to $w(X_i)$.

It might be desirable to have a weight distribution between these two extremes, i.e., we want to account for some of the non-direct predecessors, but not for *all* of them, in how we assign weight to time point variables. To this end, we can generalize our weight function to

$$w(X_i) = \sum_{X_j \in \mathrm{pre}(X_i)} \max(1 - \frac{\mathrm{distance}(\mathrm{X_j}, \mathrm{X_i}) - 1}{\varphi}, 0), \tag{4.5}$$

where $\mathrm{pre}(X_i) = \{X_j : X_j \prec X_i\}$ contains the set of predecessors of X_i, $\mathrm{distance}(X_j, X_i)$ is the length of the shortest path between X_j and X_i (i.e., the number of precedence constraints connecting them), and φ is a discounting factor. For $\varphi = 1$, this function reduces to that of Equation 4.4, counting only the direct predecessors. In general, for $\varphi = j$, predecessors up to j steps removed from X_i are considered, where their value is discounted linearly, i.e., a predecessor removed j steps contributes a weight of $1/j$ to the total value of $w(X_i)$.

Next to arguing that more predecessors cause more delay, and we should have more flexibility for time point variables that have more predecessors, we could as well reason more pro-actively: time point variables that have a high number of successors are at risk of propagating a delay to many other points in a network, hence we should give more weight to such variables.

Therefore, we can define

$$w(X_i) = |\{X_j : X_i \prec X_j\}|, \tag{4.6}$$

a weight function that assigns to $w(X_i)$ a value equal to the total number of successors of X_i. Similar objections can be raised here as for Equation 4.3, which counts *all* predecessors, so here we can also define

$$w(X_i) = |\{X_j : X_i \ll X_j\}|, \tag{4.7}$$

a function counting only the direct successors, in similar vein to Equation 4.4, and of course

$$w(X_i) = \sum_{X_j \in \mathrm{succ}(X_i)} \max(1 - \frac{\mathrm{distance}(\mathrm{X_i}, \mathrm{X_j}) - 1}{\varphi}, 0), \tag{4.8}$$

where $\text{succ}(X_i) = \{X_j : X_i \prec X_j\}$ is the set of successors of X_i.

4.3 Experiments

In Examples 4.1 and 4.3 the main motivation behind the ideas in this chapter was sketched: maximizing the flexibility of a schedule need not be the same as maximizing the robustness of that schedule. One reason, demonstrated in the first example, is that multiple such schedules can be constructed, which will have different properties during execution. An even more compelling reason is outlined in Example 4.3: maximizing can lead to very skewed distributions of flexibility, which in turn will have a negative impact on the robustness. While these examples prove the existence of such effects, more general conclusions cannot be drawn from them. For this reason, this section presents hypotheses and results from simulation experiments designed to see if these hypotheses indeed hold.

A first hypothesis is that the problem outlined by the examples in the motivation is not a contrived example, but that sacrificing flexibility to improve the flexibility distribution has a positive effect on the robustness of a schedule in general. Several new distributions have been proposed in this chapter, the next question is therefore which of these distributions works best. The hypothesis is that network structure plays a role in the propagation of disturbances, and that adapting the distribution to that structure has a positive effect on the performance. It is unclear from the theory *how* to adapt the distribution: one hypothesis is that tasks with many predecessors benefit from receiving more flexibility, another hypothesis is that tasks with many successors benefit instead.

To test these hypotheses, we will be performing simulation experiments, in which an interval schedule constructed using one of the proposed flexibility distribution methods will be executed repeatedly. For each execution, a set of tasks will be picked that will be artificially delayed by increasing their execution time. To evaluate the robustness of an interval schedule, we will be focussing on how well the simulated execution adheres to the original intervals. If a task has to start outside its assigned interval, this is seen as a *violation*. The more violations an interval schedule has, the lower its robustness. The rationale behind this idea is that it is our aim to construct a robust schedule, which can be used as a basis to make commitments. If a task starts outside its assigned interval, due to an unforeseen delay, this violates such a commitment.

We start by examining the overall performance, averaged over all experiments performed, then we will zoom in and analyze the effect of different types of delay. In particular, we will vary both the number of tasks affected by delay, and the severity of the delay itself. It is expected that there will be a clear correlation between increasing both of these parameters and the performance of the interval schedules. It is unclear however which of these will have the biggest impact.

Lastly, we will perform a more extended analysis of the relationships between sacrificing flexibility and increasing robustness, and between the trade-off of constraint violations and schedule tardiness. For the first relationship, two

opposing forces are at work: On one side, we expect that sacrificing flexibility will improve performance, as this is the idea behind the equalized flexibility distribution. On the other side, if too much flexibility is lost, schedule execution performance will eventually drop. For the second relationship, we refer back to Example 4.1: From these examples it is clear that if flexibility is placed at the end, we can expect lower tardiness. But, this also means that the other tasks have lower flexibility, which leads us to expect that more constraints will be violated.

Note that we are not able to compare the results optimizing our flexibility metric to the metrics discussed in Section 3.2, since those metrics do not provide us with an interval schedule.

4.3.1 Setup of Experiments

Task scheduling problem instances are obtained from the PSPLIB instances (Kolisch and Sprecher, 1996; Kolisch et al., 1998), whereby only the network part of the instance is used—all resource constraints are ignored. Four sizes are available (30, 60, 90 and 120 tasks), we will concentrate on the set with 120 tasks. Since the instances contain no due date, a due date is imposed by taking the earliest start time schedule of the instance, and increasing the end time of this schedule by 10%.

The unexpected delays that we simulate are controlled by two parameters. The first parameter controls the *number of tasks* that are delayed. For each set of simulations, a fixed number of tasks are delayed. We delay between 10% and 50% of the tasks in the schedule, in steps of 10%, for a total of 5 different settings. The second parameter controls the *amount of delay*, relative to the length of the selected task. The delay is implemented by increasing the processing time of the selected task, *without* updating the schedule to take this increased time into account. This way, an accurate simulation can be performed of an unexpected delay, of which no a priori information is known. The processing time is increased by a factor between 10% and 100% in steps of 10%, and between 100% and 200% in steps of 25%, for a total of 14 different settings.

Having created an interval schedule for a problem instance, and having selected the tasks to be delayed, simulation is performed by dispatching the tasks of the instance in their assigned intervals. We choose to dispatch a task as soon as possible, within the constraints of its assigned interval. This accurately simulates, for instance, most production environments, in which personnel executing tasks is expected not to sit around idling when tasks are available to be executed. Since the tasks to be delayed are selected randomly, the interval schedule resulting from each combination of parameters is simulated 150 times for each setting of the delay parameters.

The performance of a simulated schedule execution is measured by determining the number of schedule violations occurring during execution. A violation occurs as soon as a task has to be started outside its execution interval. The size of the violation is equal to the distance of the upper bound of the execution interval, and the realized start time of the task.

Table 4.1
Flexibility loss of the different flexibility distributions

Distribution	Flexibility		Flexibility loss	
	average	std.dev	average	std.dev
Maximal (3.30)	1347.8	334.2	0	0
Equalized (4.1)	1281.2	305.9	66.1 (4.0%)	37.2
Weighted, direct predecessors (4.4)	1192.1	277.3	155.2 (11.0%)	72.1
Weighted, discounted ($\varphi = 5$, 4.5)	1091.2	262.5	256.1 (18.0%)	96.4
Weighted, all predecessors (4.3)	1045.1	251.2	302.1 (22.0%)	112.3
Weighted, direct successors (4.7)	1221.8	279.7	125.5 (9.0%)	72.9
Weighted, all successors (4.6)	1134.0	255.7	213.1 (15.0%)	105.3

Example 4.4 Consider for example two tasks X_1 and X_2, with $l_1 = 2$, $l_2 = 2$ and $X_1 \ll X_2$. If $\sigma_i(X_1) = [0, 1]$ and $\sigma_i(X_2) = [3, 4]$, we will start X_1 at time point 0. If X_1 is now delayed, such that $l_1' = 5$, X_2 cannot be started in $\sigma_i(X_2)$. We start X_2 at the earliest possible time, which is directly after X_1 ends at time point 5. This is counted as a violation, of size $5 - 4 = 1$, since the upper bound of $\sigma(X_2)$ is 4.

4.3.2 *Comparing Flexibility Distributions*

We start with an overall comparison of the performance of the different flexibility distributions proposed, averaged over the full range of delay parameters on all instances, as discussed in the previous section. The different flexibility distributions proposed in Section 4.2 are compared with the one resulting from the flexibility metric introduced in Theorem 3.1, having maximal flexibility. Since there may be many different distributions maximizing the flexibility for a certain STN, we pick one which also maximizes Equation 4.1. This means that the maximally flexible distribution we use is as equalized as possible, such that we do not give unfair disadvantage to this distribution.

In Table 4.1, the resulting average flexibility is shown for each of the flexibility distributions, as well as the loss, when comparing the flexibility of each distribution to the maximal flexibility. We see that the price we have to pay in terms of flexibility loss to equalize flexibility is not very high: 95% of the available flexibility is retained. If we modify the equal distribution by applying a weight function w however, we see that the more w deviates from an equal distribution, the more flexibility we lose.

In Table 4.2, the average performance over all delay parameters is summarized for the different flexibility distributions, in terms of the number of violations, and the tardiness relative to the (undelayed) makespan. Here, we see that equalizing the flexibility gives an advantage in terms of the number of violations, even at the price of some flexibility loss. From this we can thus

Table 4.2
Average performance for the different flexibility distributions

Distribution	Violations			Tardiness, %		
	average		std.dev	average		std.dev
Maximal (3.30)	42.9		16.9	22.7		21.8
Equalized (4.1)	37.0	(−13.8%)	19.3	22.2	(−2.2%)	22.0
Weighted, direct predecessors (4.4)	42.9	(0.0%)	21.8	21.4	(−5.7%)	22.2
Weighted, discounted (φ = 5, 4.5)	50.1	(+16.8%)	23.5	20.7	(−8.8%)	22.5
Weighted, all predecessors (4.3)	53.6	(+24.9%)	23.5	20.5	(−9.7%)	22.7
Weighted, direct successors (4.7)	35.3	(−17.7%)	18.0	22.7	(0.0%)	21.8
Weighted, all successors (4.6)	33.9	(−21.0%)	16.1	23.7	(+4.4%)	21.7

conclude that sacrificing some flexibility can indeed improve the performance of schedule execution.

For the weighted flexibility distributions, this is not the case everywhere, contrary to our expectations. Weight functions distributing more flexibility to tasks with more predecessors (Equations 4.3, 4.4 and 4.5) lead to a higher number of violations, on average, but to a slight decrease in the average tardiness (i.e., the finish time of the latest task, as compared to the undelayed schedule). It is easy to understand why this happens: all these distributions, to a greater or lesser extent, concentrate flexibility in tasks with many predecessors. Tasks at the end of the schedule of course have more predecessors than tasks at the beginning, so this leads to a concentration of flexibility at the end of the schedule. A delay occurring anywhere in the schedule can thus be more easily absorbed by the tasks at the end of the schedule, but this comes at the price of more violations, for the tasks between the delayed task and the end of the schedule.

For the weight functions distributing more flexibility to tasks with more successors (Equations 4.6 and 4.7) we see the reverse, for the exact same reason. Tasks near or at the end of the schedule will have lower flexibility, explaining the slightly higher average tardiness. The number of violations is lower however, and from this we can conclude that it is better to distribute flexibility in such a way that delays can be compensated for immediately, before they spread to other tasks.

These numbers are all averages however, taken over *all* delay parameter combinations (i.e., number of tasks delayed, amount of delay per task). In the next section we will more closely analyze the influence these parameters have for the various distributions.

4.3.3 Influence of Delay Parameters on Performance

So far, we have discussed the performance of interval schedules averaged over all delay parameters used. We will now examine the dependency of the performance of the various flexibility distributions, in terms of the number of violations and

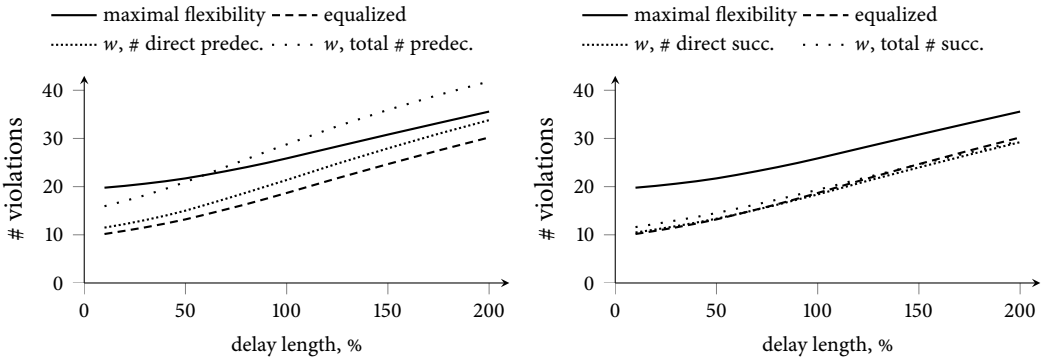


Figure 4.1
Influence of delay length on violations, for a delay in 10% of the tasks.

the tardiness, on both the number of tasks delayed and the amount of delay inserted.

In Figure 4.1 the effect of the *length* of the delay (relative to the task affected) is shown, for a constant number (10%, relative to the total number of tasks) of delayed tasks. The same picture emerges as from the summarized data: equally distributed flexibility consistently outperforms interval schedules having maximal flexibility, and interval schedules where flexibility is concentrated on tasks with many predecessors (right part of the figure). It can also be seen that maximally flexible schedules are influenced more for smaller delay lengths, relative to the other distributions. A possible explanation is that these schedules have more tasks with no flexibility at all: a delay has to propagate through all such tasks, until a task with enough flexibility is reached to absorb it. For the distributions concentrating the flexibility on tasks with many successors (left part of the figure), we see that they perform almost identical to the equalized distribution. This suggests that the slight performance increase we observed in Table 4.2 occurs for a higher number of delayed tasks only.

In Figure 4.2 the effect of the *number* of delayed tasks (relative to the instance size) is shown, for a fixed amount of delay (10%, relative to the task affected). The effects are largely similar to the effect of the length of the delay. Again, maximally flexible schedules are less affected by larger percentages of tasks delayed; the possible explanation is the same, as a large percentage of small delays give similar effects as a small percentage of large delays. The number of violations increases more slowly for larger percentages of delayed tasks; a possible explanation is that, due to re-convergence in the task graph, the effects of additional delays have less impact on the number of violations: if one delay already causes a series of violations, an additional delay cannot cause more violations in that path. Interestingly enough, here, we also see (right figure) that distributions concentrating flexibility on tasks with many successors perform virtually identical to distributions with maximal flexibility. We must therefore

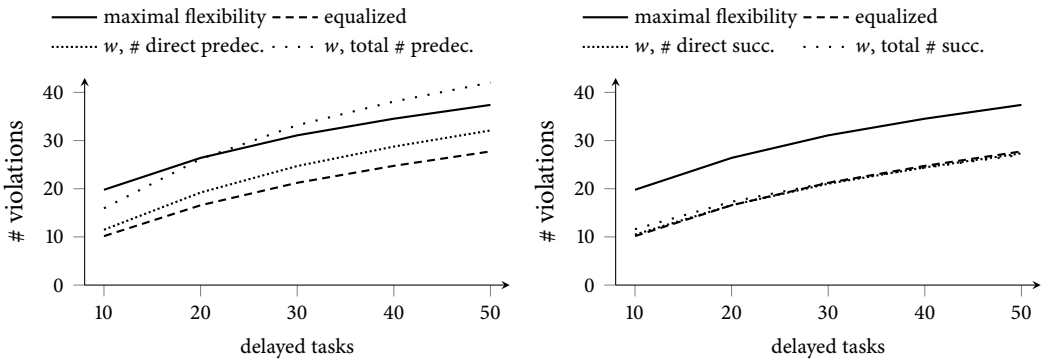


Figure 4.2
Influence of the number of delays on violations, for a delay length of 10%.

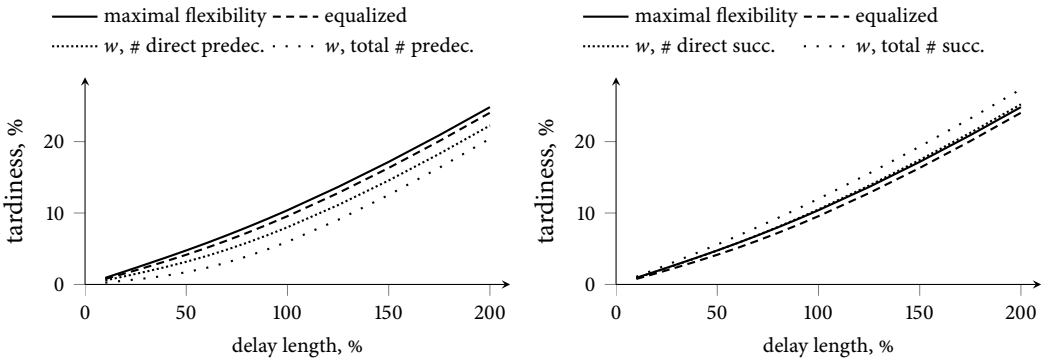


Figure 4.3
Influence of delay length on tardiness, for a delay in 10% of the tasks.

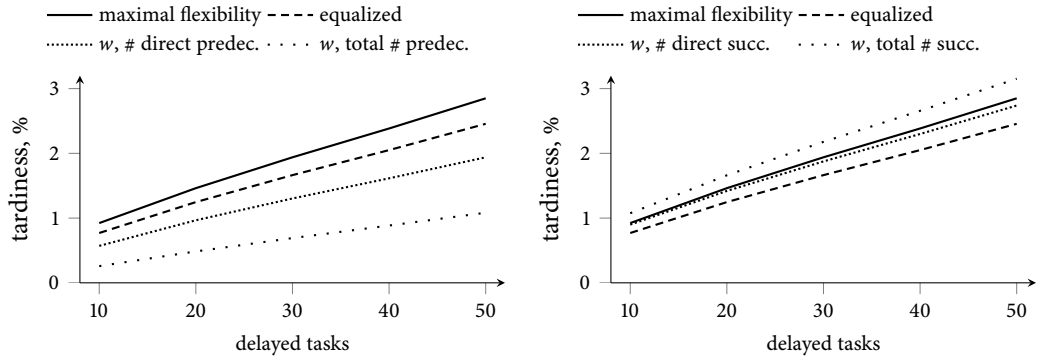


Figure 4.4
Influence of the number of delays on tardiness, for a delay length of 10%.

conclude that the advantage for these distributions we see in Table 4.2 occurs when both the number of tasks delayed *and* the size of the delay is large.

Next to the number of violations during execution, the tardiness is also of importance. In Figures 4.3 and 4.4, the dependence of the tardiness on the different delay parameters is shown. The results closely follow the summarized data in Table 4.2, and the relative performance of the distributions is not dependent on the delay parameters.

4.3.4 Influence of Flexibility on Performance

The discussion in the previous section centers on the relationship between the chosen distribution and the performance, in terms of violations and tardiness. It already shows that, sometimes, sacrificing some flexibility compared to the maximum achievable can give performance improvements for these criteria. Now, we will analyze this in some more detail by comparing the relative flexibility of certain instances to the relative performance on that instance, in terms of the number of violations. To be exact, 60 instances are selected at random, and the flexibility loss using the various distribution methods is computed, compared to the maximum achievable using Equation 3.30. The flexibility loss is plotted on the horizontal axis, and on the vertical axis the performance is shown, also relative to the performance, for a representative setting of the delay parameters (30% of the tasks are delayed by 30%, other settings yield comparable results). The data is separated across multiple graphs, to improve legibility.

In Figure 4.5 the data for equalized flexibility is shown, as well as the data for distributions based on the number of predecessors. For both the equalized distribution as well as the distribution based on the number of direct predecessors, the graph shows a correlation between the loss of flexibility and the performance. For the distribution based on the total number of predecessors, this correlation is almost indiscernible.

At first glance, this correlation appears to be in the wrong direction: the less flexibility remains, the higher the performance improvement, as compared to the maximally flexible schedule for the same instance. If we examine the basic idea behind the distribution of flexibility, this result does make sense after all: if an instance has a lot of parallel sections, the maximally flexible schedule will have more concentrations of flexibility, as demonstrated in Example 4.3. These concentrations are distributed when using the equalized objective function, and this will result in a relatively large difference in flexibility, but also in improved performance. In other words: the more a schedule can be changed using our equalized flexibility distribution, compared to a maximally flexible schedule, the higher the performance increase will be.

We do see a decrease in performance for the distributions based on the number of predecessors, compared to equalized flexibility distribution; this was also reflected in the previous section. The distributions based on the number of successors are shown in Figure 4.6, and here we also see the slight increase in performance, compared to the equalized distribution. For both distributions, the correlation between flexibility loss and performance holds. Here, we also see

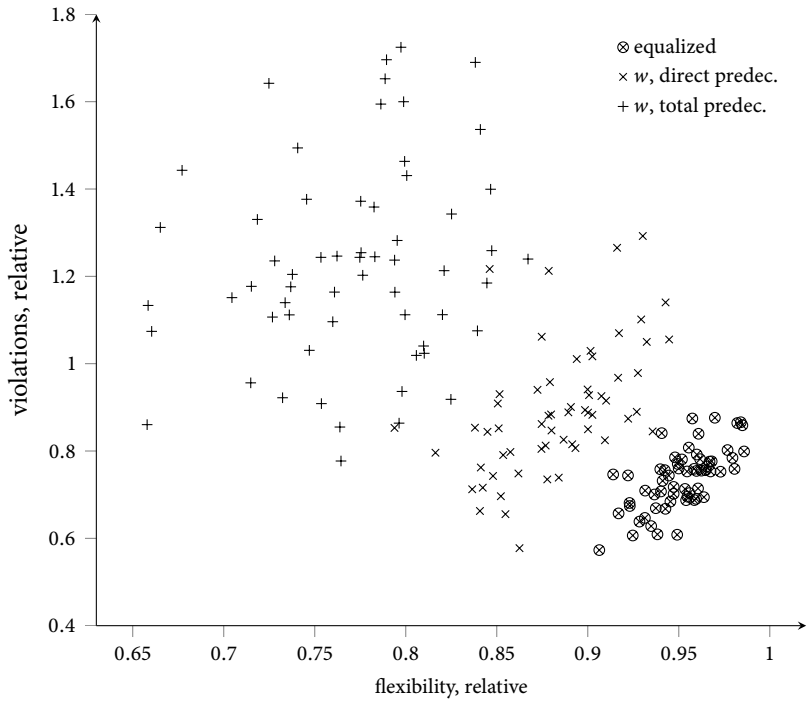


Figure 4.5
Relative number of violations versus loss of flexibility, for 30% delay in 30% of the tasks, for equalized flexibility and distributions based on predecessors.

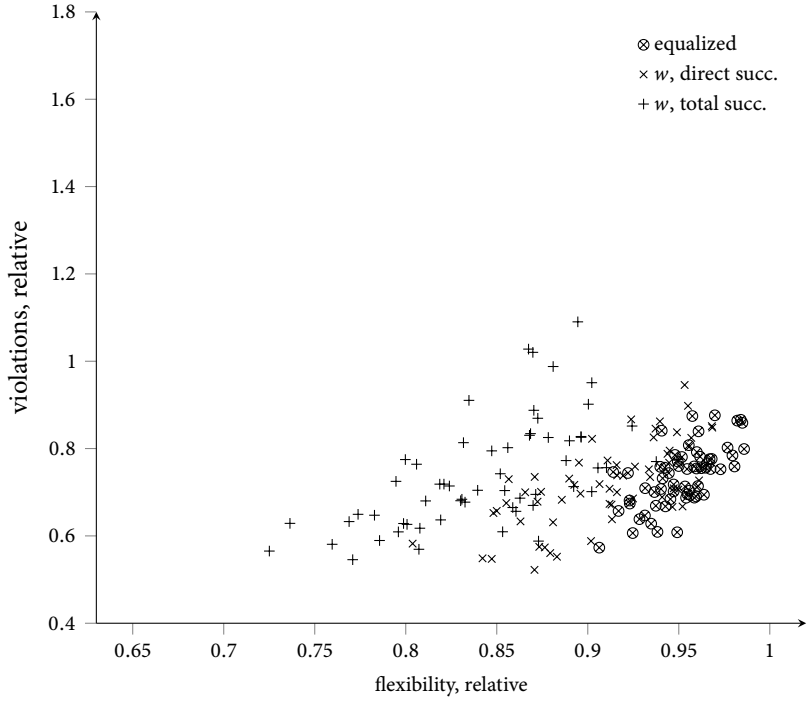


Figure 4.6
Relative number of violations versus loss of flexibility, for 30% delay in 30% of the tasks, for equalized flexibility and distributions based on successors.

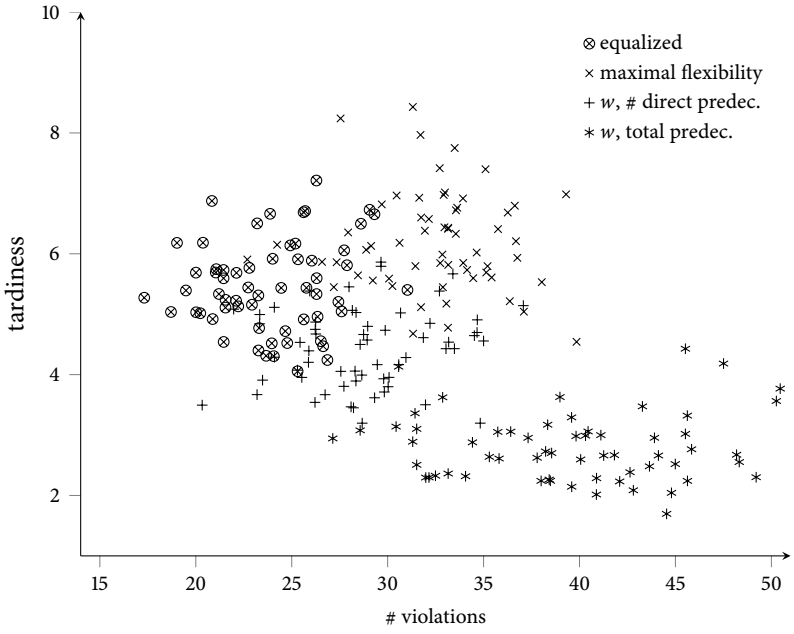


Figure 4.7 Violations versus tardiness, for 30% delay in 30% of the tasks, for equalized and maximal flexibility, and for distributions based on predecessors.

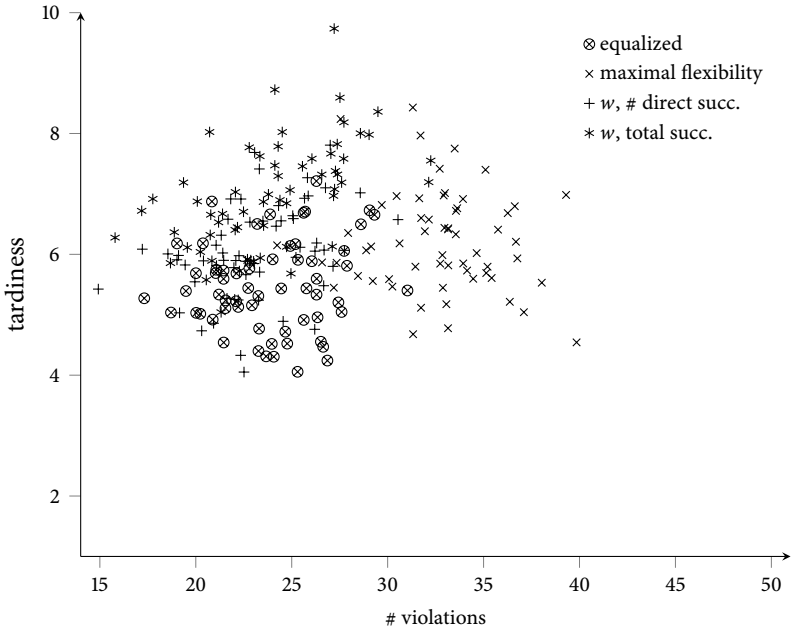


Figure 4.8 Violations versus tardiness, for 30% delay in 30% of the tasks, for equalized and maximal flexibility, and for distributions based on successors.

that deviating too far from the equalized distribution results in larger flexibility losses, which eventually begin to impact the performance, as seen in the results for the distribution based on the total number of successors.

4.3.5 *Violations Versus Tardiness*

As can already be seen from the data in Table 4.2, there is a relationship between the number of violations and the tardiness of a schedule, which can be noted especially in the distributions based on the number of predecessors. If flexibility is concentrated at the tasks at the end of the schedule, there is more slack available in this portion to compensate delays that would otherwise lead to tardiness. But, this also leads to more violations in the first part of the schedule. Conversely, a more equalized distribution avoids these violations, at the cost of tardiness.

In Figure 4.7 this trade-off is visualised for 60 instances simulated using 30% delay in 30% of the tasks. The difference between the schedules with equalized flexibility and the two weighted distributions based on the number of predecessors is very clear: an equalized schedule optimizes towards a low number of violations, a schedule weighted using the total number of predecessors optimizes towards low tardiness, and a schedule weighted using the number of direct predecessors compromises between the two. This graph shows again that maximizing the flexibility offers the worst performance of the differing flexibility distribution: it neither scores good on tardiness nor on the number of violations.

Next, we look at the two distributions based on the number of successors, these are also compared with the equalized flexibility distribution and the distribution with maximal flexibility, in Figure 4.8. Irregardless of the fact that, using a distribution based on the number of successors, less flexibility is concentrated at the end of the schedule, the tardiness does not increase a lot. The distribution based on the number of direct successors offers a slight increase in terms of the number of violations, with (almost) equal tardiness. Only when we deviate even more, by basing the distribution on the total number of successors, we indeed see a slight increase in the tardiness. This effect is visible even more in Table 4.2, this is caused by the fact that delay parameters causing more extreme delays are represented here as well.

The likely cause for the difference between Figures 4.7 and 4.8 lies in the different focus of the two distribution types. If we concentrate flexibility at the end of the schedule, we manage to lower tardiness, but the consequence is that more violations occur before delays are absorbed by the available flexibility. The other distributions, focussing on the number of successors, manage to decrease the number of violations slightly. While there might be less flexibility near the end of the schedule, the higher risk of tardiness is compensated by the lower propagation of delay.

4.3.6 *Flexibility guarantees*

A downside to the approach discussed so far is that all methods based on the objective function from Equation 4.1 focus on minimizing flexibility *loss* differences,

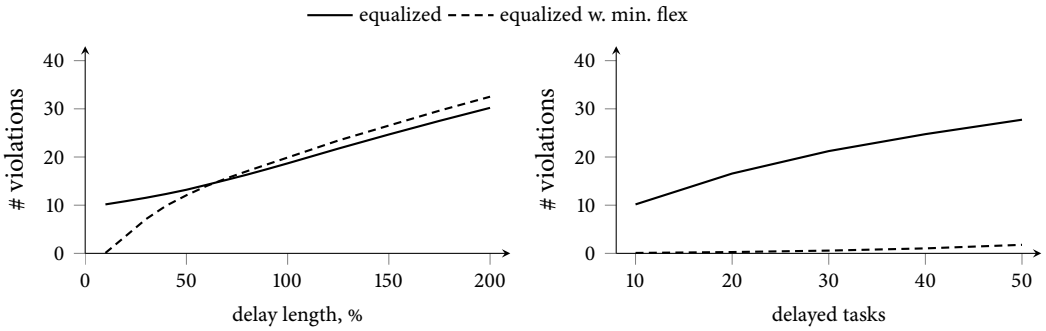


Figure 4.9
Influence of guaranteed flexibility on the number of violations, for a delay in 10% of the tasks (left) and for a delay of 10% (right)

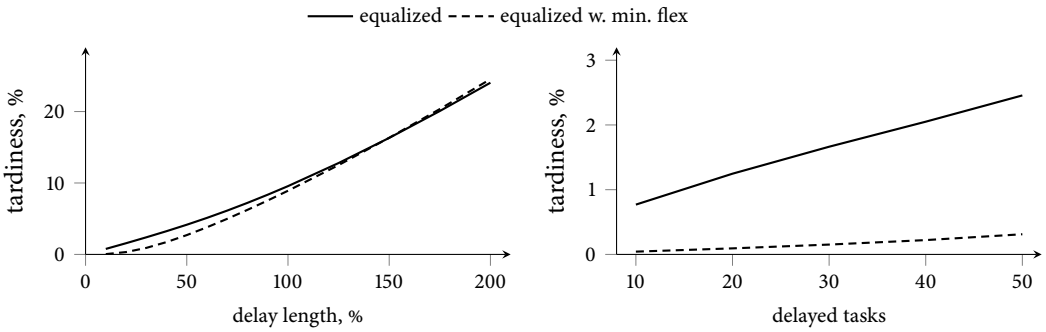


Figure 4.10
Influence of guaranteed flexibility on tardiness, for a delay in 10% of the tasks (left) and for a delay of 10% (right)

without taking into account the absolute value of the resulting flexibilities.

Example 4.5 Consider two time point variables X_a and X_b, with $\mathrm{lst}(X_a) - \mathrm{est}(X_a) = 4$ and $\mathrm{lst}(X_b) - \mathrm{est}(X_b) = 2$. Imagine that, due to the surrounding network structure, applying the linear programming method presented in the previous chapter combined with the objective function from Equation 4.1, both X_a and X_b lose two units of flexibility. Both time points are treated equally in terms of flexibility loss, but X_a has two units of flexibility remaining, whereas X_b has no flexibility at all.

As can be seen from this example, the method based on Equation 4.1 might result in tasks receiving no flexibility at all. A method to avoid this problem is to add constraints of the form

$$X_i^+ - X_i^- \geq f \tag{4.9}$$

to all time point variables, which forces the allocation of at least f units of flexibility to each of them. Note however that large values of f might make the problem infeasible. A simple solution is to adapt our linear program, again, using a new objective function that is able to find the largest value of f we can use:

$$\max \min_{X_i \in \mathcal{X}} |(X_i^+ - X_i^-)|. \tag{4.10}$$

This objective function finds an interval schedule in which the *smallest* interval is as large as possible. It is easy to see that the problem remains feasible if we use this value for f, forcing *all* intervals to be at least this size: the solution found when using the objective function in Equation 4.10 is one such solution for which these constraints all hold. There might however still be many different solutions for this new problem. To find the most suitable one, we can use the original objective function from Equation 4.2 combined with any of the weight functions, to distribute the remaining flexibility over the time point variables: first, we use a linear program to find the highest lower bound on the flexibility per task, and then this bound is enforced using additional constraints. The rest of the flexibility can then be distributed using any of the other methods. In this case, we choose to equalize the flexibility, and we compare the results with and without the lower bound enforcing a minimal amount of flexibility.

In Figure 4.9 we see that this approach works extremely well if the amount of delay is small. This is to be expected: small delays can be absorbed immediately, if the minimal amount of flexibility is high enough. The right part of the figure shows that, for a small amount of delay (10%, in this case), the number of delayed tasks has almost no influence on the performance at all. If delays are larger, there is a small performance penalty if a minimal amount of flexibility is enforced per task—the likely cause is a smaller total flexibility, over the entire instance.

The same trend is shown for the effect on the tardiness, in Figure 4.10. It is less apparent, since the tardiness is relatively low already, for small delay lengths (left figure), but on the right it is clear that there is a large influence on the tardiness if delays are small enough, independent of the amount of tasks delayed.

4.4 Conclusion and discussion

This chapter investigated the second research question, which asks how available flexibility can be best distributed over the tasks in a schedule, to achieve high robustness. The need for this is first illustrated using examples, and different distributions are proposed and analyzed in an experimental setting. These experiments show that some, but not all our intuitions behind these distributions are valid.

Our idea of equalizing the flexibility distribution over the tasks in the schedule is shown to have positive influence on both the number of violations, i.e., the number of tasks executing outside their assigned interval, as well as on the tardiness of the schedule, i.e., how much the deadline of the original schedule

was exceeded due to delay. Our idea to further improve the robustness by allocating more flexibility to certain tasks, based on either the number of predecessors or the number of successors, showed that concentrating flexibility based on predecessors improves the performance with regard to the tardiness, at the cost of a larger number of violations. Concentrating flexibility based on the number of successors however is shown to decrease the number of violations with little effect on the tardiness. The likely explanation for the first effect is that flexibility is concentrated at the end of the schedule, and as such any delays can be absorbed before the final task of the schedule is executed, at the cost of more propagation through the schedule. The likely explanation for the fact that tardiness does not suffer if flexibility is concentrated based on the number of successors is that it reduces violations, and thus the spread of delay over the schedule, before the final task is executed.

If the delays are known to be small, the best performance is obtained however by guaranteeing that a certain minimal amount of flexibility is allocated to every task in the schedule. We propose a method to maximize this enforced minimal amount of flexibility, and the experiments show that this distribution performs very good, with almost no violations at all, if the delays are reasonably small.

4.4.1 Discussion

The idea of robustness discussed in this chapter is important in the context of schedule execution at NedTrain, since here, we have highly uncertain task durations, combined with the use of shared resources by independently operating teams. If the schedules used are not robust in the face of the uncertain execution times, subsequent tasks in the schedule will be delayed. Resources can be the cause of such a delay propagation, as these might not be freed in time to start the next task. They can however also be a reason to want to avoid delays: if some type of (external) commitment has been made for a resource needed for a certain task, like the hiring of special machinery or extra personnel, delays can lead to penalties. The independently operating teams at NedTrain are another reason to strive to avoid the spread of delay: if unforeseen circumstances lead to extra work for one team, it is desirable if this has no effect on the schedule for the other teams, since this would lessen their independence.

This chapter, as well as the previous one, focused exclusively on the level of the STN and the properties of, and effects on execution performance of the different interval schedules that can be constructed for those STNs. The original problem that we are trying to solve, however, is the more complex RCPSP, which is reduced using precedence constraint posting to an STN as a convenient and flexible solution representation. Important to note is that a single RCPSP instance can be reduced to many different STNs—this is an important direction for further research: instead of maximizing the flexibility of a single STN solution to an RCPSP instance, it might be a much better strategy to modify the choices that are made in the precedence constraint posting process such that a different STN is obtained that admits an interval schedule with more flexibility.

Sequential flexibility

5

In solving the RCPSP in a flexible way, the previous two chapters explored the temporal flexibility of simple temporal networks, which are used to represent a set of solutions to an RCPSP instance. Chapter 3 explored the intuition behind the concept of temporal flexibility, in an effort to answer research question 1. We have discovered that existing methods do not conform to these expectations and we introduced a new method to measure the flexibility of a simple temporal network. The second research question represents our our aim to use the flexibility present in a solution to create robust solutions for the scheduling problem at NedTrain, and this question is explored in Chapter 4, where various ways to distribute flexibility over a simple temporal problem are tested for their performance in this regard.

A shared feature of the previous chapters (and research questions 1 and 2) is that they did not focus on the RCPSP *directly*: both are based on a flexibility measure for the STN. An RCPSP instance was converted to an STN, and using our flexibility metric, this in turn was converted into an interval schedule, yielding an efficient and flexible method of executing a portfolio of temporal solutions for the original RCPSP instance. This chapter will focus on the third research question, and explore how we can extend the concept of temporal flexibility to include a form of *sequential* flexibility. For this, we need to move a level up, to the conversion of an RCPSP instance to an STN.

We start by noting that the flexibility of the solution of an RCPSP instance, in the form of an STN, is determined by the temporal constraints in that STN, and if this STN is the result of a reduction from an RCPSP instance obtained using constraint posting techniques, we can make a distinction between two types of temporal constraints: one part of the constraints stems from temporal constraints in the original RCPSP instance, such as due times and precedence constraints between tasks, and the other part of the constraints is added by the reduction procedure to eliminate resource contention. The second type of constraint usually originates from heuristic solution procedures. Different types of heuristics can be employed, some even with different parameters. This means that there are many different choices, and hence many different sets of

constraints, which will all result in an STN that contains solutions[1] for the same associated RCPSP instance. Some of these choices will be more arbitrary than others, based on the information the heuristic is able to provide. To briefly recap, constraint posting is an iterative process, consisting of the following steps:

- Compute the resource usage profile of the current STN, and identify resource contention peaks, i.e., places in the current solution where the combined resource usage of the executing tasks exceeds the capacity of that resource.
- Select a pair of tasks contributing to said resource contention peak, select an ordering for these two tasks, and add (post) a precedence constraint to the STN enforcing this ordering.

These steps are repeated until the resource profile as computed in the first step no longer contains any resource contention peaks.

One obvious research avenue is to tune or develop heuristics that are able to make these decisions in such a fashion that the flexibility of the resulting STN is as high as possible. However, in this chapter a different approach is proposed: instead of making these choices and arriving at an STN where these constraints are *fixed*, we present an approach in which these choices are *postponed*—this results in a schedule in which these choices can be made during *execution time*, giving the agent executing the schedule the flexibility to re-order the tasks before dispatching them. To distinguish this from the flexibility discussed in the previous chapters, we propose to call the earlier form of flexibility *temporal* flexibility, as this form is concerned with the exact time at which the tasks can start, and to call this form of flexibility *sequential* flexibility, to indicate that the flexibility lies in the possibility to re-order the sequence of tasks

Being able to (partially) determine the order of tasks at the time of execution fits well within the goal of wanting to have a schedule that is flexible in the face of uncertainty. Imagine that a maintenance engineer at NedTrain has to start with a certain task, which he has to perform together with a colleague with certain special skills. If this colleague is unavailable, because he is still working on a prior, delayed, task, it is very natural for the maintenance engineer to decide to start with another task first, for which the second engineer is not needed. The effects of swapping the execution order of these tasks will however propagate through the schedule. In this chapter we will formalize this concept of flexibility in such a way that it can be encoded in a solution to the RCPSP, to be used during execution of the schedule to combat the spread of delays due to uncertainty. The method we use, in short, consists of creating *grouped* tasks at certain points in the constraint posting procedure, instead of adding precedence constraints. Such a grouped task is a placeholder, representing any ordering of the tasks contained therein. At execution time, the final order can be determined.

[1] Note that we are only interested in STNs in which *all* solutions are also solutions to the original RCPSP. There are also many STNs in which only part of the solutions are solutions to the original RCPSP, other solutions are resource-infeasible.

After this introduction, this chapter will continue with a slightly more detailed motivation of why and how we want to achieve sequential flexibility. Having motivated the why of sequential flexibility, we first describe the existing method of constraint posting, on which our idea is based. Then, task grouping itself is described, the actual method we propose to achieve sequential flexibility, and we follow this formal description by a section containing experiments to gain further insight into how task grouping achieves the desired flexibility. The chapter is ended with a conclusion and discussion.

5.1 Motivation

Just as in the previous chapters, we continue to aim at building schedules that are prepared for possible disruptions to occur. Instead of having a fixed start time assignment $\sigma : T \to \mathbb{R}$, we want to have a set S of schedules with which we are able to react to unforeseen circumstances during execution—if a delay in a task would cause the current schedule $\sigma \in S$ to violate some of the constraints, we are then able to find a different $\sigma' \in S$ that performs better under the new circumstances. In the previous chapters we introduced the concept of interval schedules, which can be seen as a restricted form of such a set of schedules S for a certain RCPSP problem: they only allow schedules σ in which the order of tasks is an extension of the partial ordering as defined by the STN reduction from the original RCPSP problem, i.e., only those schedules adhering to the extra precedence constraints added by the constraint posting procedure are admitted.

To lift this restriction, this chapter examines the method by which the RCPSP is converted into an STN. We will base our method on the class of precedence constraint posting methods, as described in Chapter 2. These methods are well-suited in the context of our problem: since they are based on heuristics, the are able to quickly provide a solution, and the solution is in the form of an STN, which is a flexible representation of multiple, fixed-time solutions.

A central component of constraint posting methods is the resolution of resource conflicts between two tasks. If a resource conflict is detected between two tasks t and t', constraint posting tries to remove this conflict by adding (posting) an additional precedence constraint between t and t', adding either $t \prec t'$ or $t' \prec t$. Selection of the tasks t and t' between which a constraint is posted, as well as the direction of this constraint, is based on heuristics. Sometimes, however, the heuristic to decide on the direction of the constraint does not indicate a very strong preference for one direction or the other, and it is not clear which choice is the best one to make. After suitable pairs of tasks t and t' are selected, our approach is therefore to *group* these tasks together without specifying the exact order (t before or after t'). In this way, the resource conflict is (partially) solved, since the tasks are no longer allowed to execute concurrently, but the order is not specified, since the group is constructed in such a way that either order is still possible. Note that this principle can be applied recursively: one, or both, of the tasks t and t' being grouped can be a grouped task itself. Note that the original grouping is not kept in such a case, i.e., groups with more than

two tasks do not contain sub-groups or any other form of hierarchy. This way, the final result can contain groups with an arbitrary number of tasks. At execution time we can pick any ordering we like for the tasks within these groups—in this way, the solution can be seen to represent a large set of possible schedules for the original RCPSP instance.

To summarize, an agent executing a classical non-grouped schedule only has two available options to handle delayed tasks: adhering to the schedule and delaying successive tasks as well, or generating a new, different schedule. Our approach construct a schedule with grouped tasks, which allows a third option: re-ordering the tasks in a task group such that execution of this group can commence with a different task without changing the global characteristics of the schedule. In this chapter we will explore the possibilities of this grouping approach to absorb disruptions that might happen during execution of the schedule.

5.2 Constraint posting

Before discussing task grouping, we will briefly recap the resource-constrained project scheduling problem (RCPSP), which is the problem we are solving, and how precedence constraint posting works.

The RCPSP consists of the following components:

- A set T with n tasks, where task $t_i \in T$ has a length $l_i \in \mathbb{R}$.
- A precedence relation $\prec$ inducing a partial order on T; the intuitive interpretation of $t_i \prec t_j$ is that t_i has to be finished before t_j can start.
- A set R of m renewable resources, where each resource $r_k \in R$ has integer capacity $\mathrm{cap}(r_k) \geq 1$.
- For each $r_k \in R$ and $t_i \in T$, $\mathrm{req}(t_i, r_k) \in \mathbb{N}$ specifies the number of units task t_i needs of resource r_k in order to be executed.

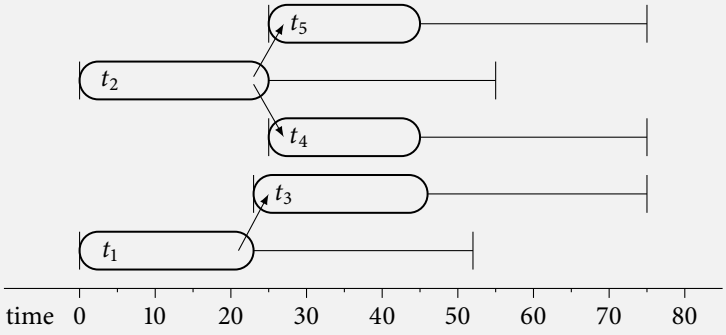
In addition to these constraints, we also have a release time $s_i \in \mathbb{R}$, and a due time $d_i \in \mathbb{R}$, which are in this case used to model when a train enters and leaves the workshop. The classical goal for the RCPSP is to find a start time assignment (schedule) $\sigma : T \to \mathbb{R}$ such that both the precedence relations and the resource constraints are satisfied. In our case this is also the final goal, but we would like there to be an intermediate step, in the form of a set of schedules S from which we can (efficiently) derive a multitude of final solutions σ.

Example 5.1 As a running example throughout this chapter, we will use a simple RCPSP instance containing five tasks using two different resources, r_a and r_b. The capacities of the resources are $\mathrm{cap}(r_a) = 2$ and $\mathrm{cap}(r_b) = 1$,

respectively. The length and resource usage of the tasks are shown in the following table:

task	t_1	t_2	t_3	t_4	t_5
length	23	25	23	20	20
use of r_a	0	2	1	1	1
use of r_b	1	0	0	1	0

The five tasks share a common release time of 0, and a due time of 75. In addition, there are three precedence constraints between the tasks: $t_1 \prec t_3$, $t_2 \prec t_4$ and $t_2 \prec t_5$. The temporal information from this example is combined in the following diagram:



In this diagram, the earliest starting times of the tasks are calculated based on the given precedence constraints. For example, since we have $t_1 \prec t_3$, and $l_1 = 23$, we can derive $\text{est}(t_3) = 23$, the earliest starting time of t_3. Instead of the latest starting time, the diagram displays the latest ending time of a task, since this makes the diagram easier to read.

Note that the location of the tasks along the lines indicating their earliest start time and latest end time is but one of many possible locations, and any combination of locations together represents a schedule σ. These schedules, however, do not necessarily present a resource-feasible solution. While some of the start times satisfying the temporal constraints will yield a resource-feasible solutions, many others do not. In fact, the shown assignment is resource-infeasible, since t_3, t_4 and t_5 execute concurrently, and $\text{req}(t_3, r_a) + \text{req}(t_4, r_a) + \text{req}(t_5, r_a) = 3 > 2 = \text{cap}(r_a)$.

To find schedules that satisfy both the temporal and the resource constraints, some techniques have already been discussed in Chapter 2. The technique most suited to our purposes is precedence constraint posting, such as described by Cesta et al. (1998): due to its heuristical nature, it is able to solve large problems very quickly, and it is able to represent solutions in a flexible way, because it is essentially a reduction from the original RCPSP to an STN. This reduction works by adding precedence constraints to the original problem specification

such that any start time assignment respecting the combined set of precedence constraints results in a resource-feasible schedule. This is a reduction to an STN in the sense that any solution to the STN is also a solution to the original RCPSP instance. However, the reduction is one-way: not all solutions to the RCPSP instance necessarily have a counterpart in the STN constructed by constraint posting.

In this section we will describe the existing constraint posting techniques upon which our task grouping method is based. The constraint posting technique starts with constructing a provisional schedule for the problem instance, taking into account only the *temporal constraints*. These constraints are represented using an STN. In such a representation, for each task t_i a time point variable X_i is introduced. Additionally, the time point variable X_0 is introduced as reference point (usually, $X_0 = 0$). To represent the release and due time constraints, we add the constraints $X_0 - X_i \leq -s_i$ and $X_i - X_0 \leq d_i - l_i$, respectively. These constraints ensure that the start time of t_i is at or after the release time, and that the end time of t_i (i.e., the start time represented by X_i, plus the length l_i of the task) is at or before the due time, respectively. To represent a precedence constraint $t_i \prec t_j$, we add the constraint $X_i - X_j \leq -l_i$, to ensure that the start time of t_j is at or after the end time of t_i.

Example 5.2 Applying this transformation to our running example, the release time and due time constraints are represented by

$$\begin{aligned} X_1 - X_0 &\leq 75 - 23 = 52 & X_0 - X_1 &\leq 0 \\ X_2 - X_0 &\leq 75 - 25 = 50 & X_0 - X_2 &\leq 0 \\ X_3 - X_0 &\leq 75 - 23 = 52 & X_0 - X_3 &\leq 0 \\ X_4 - X_0 &\leq 75 - 20 = 55 & X_0 - X_4 &\leq 0 \\ X_5 - X_0 &\leq 75 - 20 = 55 & X_0 - X_5 &\leq 0, \end{aligned}$$

and the precedence constraints in the problem are represented by

$$\begin{aligned} X_1 - X_3 &\leq -23 \\ X_2 - X_4 &\leq -25 \\ X_2 - X_5 &\leq -25. \end{aligned}$$

A solution to such an STN can be produced efficiently, as outlined in Chapter 2. In particular, like is done in Cesta et al. (1998), one can obtain the earliest start time solution that satisfies all the temporal and precedence constraints of the RCPSP instance in polynomial time. Using such an initial schedule σ, the constraint posting technique aims to satisfy the resource constraints as well, by

posting additional precedence constraints. To do so, for every resource r_k its *resource usage profile* with respect to the current schedule σ is computed. Such a profile indicates for every time point the total amount of resource r_k required, if each task t_i is started at time $\sigma(X_i)$.

More exactly, given a resource r_k and the set $T_k = \{t_j \in T \mid \text{req}(t_j, r_k) > 0\}$ of tasks using this resource, the demand of this resource r_k at time τ is defined as

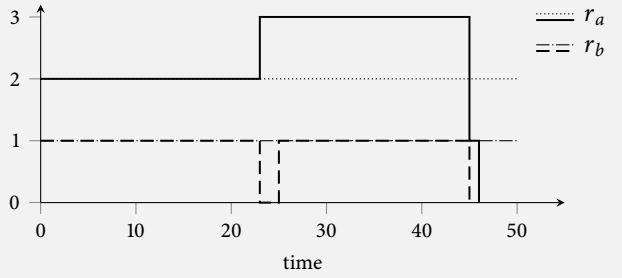
$$D_k(\tau) = \sum_{t_j \in T_k} \delta_j(\tau) \cdot \text{req}(t_j, r_k), \tag{5.1}$$

with

$$\delta_j(\tau) = \begin{cases} 1 & \text{when } \sigma(X_j) \leq \tau < \sigma(X_j) + l_j \\ 0 & \text{otherwise.} \end{cases} \tag{5.2}$$

This equation sums the resource usage for r_k for all tasks executing at time point τ. Usually, D_k is only computed for the time points τ corresponding to the start times of the tasks $t_i \in T$ of the current solution. This is more efficient, because we want to compute the resource profile to know at which time points a resource becomes oversubscribed—this can only happen at the time points at which a task is started: resource use is constant over the duration of a task, meaning that the start time points are the only time points at which resource oversubscription can occur.

Example 5.3 Applying this technique to the earliest start time solution of our example (this is the solution represented in the diagram in Example 5.1), we arrive at the following resource profiles, for r_a and r_b:



The capacity for each of the resources is indicated by the dashed lines, ····· for r_a, and —·— for r_b. Note that, for clarity, the resource profile is shown as a line, whereas in fact it is only computed at the start point of the tasks using the resources. It can be seen that the resource capacity of r_a is exceeded using this schedule.

Now the constraint posting method focuses on the *peaks* in this profile. Such a peak is a point in time where the use of a resource r_k increases, and is above its available capacity $\text{cap}(r_k)$. Formally, the set of tasks forming a *peak* on r_k at time point τ is defined as

$$C_{k,\tau} = \{t_j \in T_k \mid \delta_j(\tau) = 1\}, \tag{5.3}$$

where

$$D_k(\tau) > \text{cap}(r_k). \tag{5.4}$$

The peak $C_{k,\tau}$ represents a resource constraint violation, or a conflict, on resource r_k at time point τ, and contains all tasks contributing to this resource constraint violation.

Example 5.4 Continuing the previous example, there are *two* peaks using the current schedule: what looks like one violation of r_a are actually two violations, first by t_3 combined with t_2 starting at time 23, $C_{r_a,23} = \{t_2, t_3\}$, and then by t_3, t_4 and t_5 starting at time 25, $C_{r_a,25} = \{t_3, t_4, t_5\}$.

A resource constraint violation can be lowered by posting a precedence constraint between a pair of tasks $t_i, t_j \in C_{k,\tau}$. As a consequence of posting this constraint, we ensure that the resource peak causing the violation is lowered and spread. Note that the peak is not necessarily solved, only reduced. Such a set of tasks $\{t_i, t_j\}$ is called a (*pairwise*) *conflict*. Now, we define

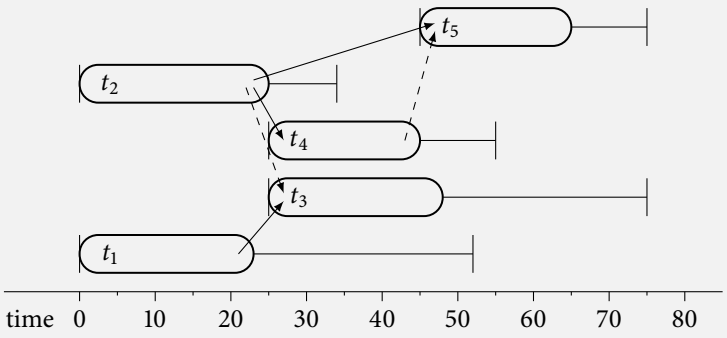
$$d(t_i, t_j) = \text{lst}(t_j) - (\text{est}(t_i) + l_i) \tag{5.5}$$

to represent the maximally allowed temporal distance between two tasks t_i and t_j (also known as the *slack*) in the current problem specification. In other words, $d(t_i, t_j)$ is the temporal distance between t_i and t_j if we start t_i as early as possible, and t_j as late as possible. Remark that, usually, $d(t_i, t_j) \neq d(t_j, t_i)$. For a given set $\{t_i, t_j\}$, we can define four possible conflict cases (Oddi and Smith, 1997):

1. $d(t_i, t_j) < 0 \wedge d(t_j, t_i) < 0$,
2. $d(t_i, t_j) < 0 \wedge d(t_j, t_i) \geq 0$,
3. $d(t_j, t_i) < 0 \wedge d(t_i, t_j) \geq 0$, and
4. $d(t_i, t_j) \geq 0 \wedge d(t_j, t_i) \geq 0$,

The first case is *pairwise unresolvable*: t_i cannot finish before t_j must start, and t_j cannot finish before t_i must start; this means that this conflict cannot be resolved. The second and third conditions are *pairwise uniquely resolvable*: there is only one feasible ordering, the decision is unconditional. The last condition is *pairwise resolvable*: both orderings are possible, and a choice needs to be made.

Example 5.5 To flatten the peaks listed in Example 5.4, we pick two tasks from both sets, and post a constraint between them. In this case we will choose to add the constraints $t_2 \prec t_3$ and $t_4 \prec t_5$. Note that both pairs are of the fourth type, both orderings are possible. This will result in a new temporal network, yielding the following solution diagram (in the format of Example 5.1):



In this solution, no peaks exceeding the resource capacity are present, so additional constraints do not have to be posted. Note that this is caused by the simplicity of this example, usually a peak needs more than a single constraint to be eliminated completely.

Summarizing, the structure of a constraint posting method generally has the following form:

1. Using a provisional schedule σ, compute the resource usage profile over time and select a resource peak violating the resource capacity.

2. Select two (partially) concurrent tasks t_i, t_j contributing to the selected peak.

3. Decide on adding a precedence constraint between these tasks: $t_i \prec t_j$ or $t_j \prec t_i$.

4. Add this precedence constraint, and solve the underlying STN to generate a new schedule.

5.2.1 *Heuristics in constraint posting*

At three different points in the outline above, choices must be made:

1. a peak must be chosen,

2. a pair of (partially) concurrent tasks within this peak must be selected, and

3. a direction for the precedence constraint between these tasks must be decided.

To make these choices, *heuristics* are employed.

To select a peak, we look at the overcommitment each peak represents. The peak that exceeds the capacity of a resource by the largest amount is chosen first. Intuitively, this points the algorithm towards the areas where resource contention

is largest. In many algorithms (e.g., Cesta et al. (1998)), this step is omitted, and the algorithm selects a task pair from *all* peaks in the current solution. We opted to limit task pair selection to a single peak for two reasons. First, this causes the algorithm to give priority to the largest overcommitment on a resource. The intuition behind this is that such a peak usually requires a lot of constraints to be entirely flattened—the more constraints are added to the problem the less flexibility remains, and the higher the chance that the problem cannot be solved anymore. A second reason is that the number of task pairs grows rapidly with the number of tasks from which the algorithm can choose, so limiting ourselves to the tasks in a single peak gives a performance increase.

Within a peak, a conflict $\{t_i, t_j\}$, which is a pair of (partially) concurrent activities, must be chosen. Recall the four different conflict types described before. If all conflicts in this peak are of the first type, we can immediately conclude that the instance is no longer solvable without backtracking. If there are conflicts of the second or third type (i.e., conflicts that can only be resolved in one way), we select the conflict for which

$$\omega_{\text{res}}(t_i, t_j) = \min(d(t_i, t_j), d(t_j, t_i))$$

is lowest (Cesta et al., 1998). The lower this value, the closer the two tasks are to being forced into a resource-consistent state. In other words, the temporal flexibility we lose by posting a constraint between these two tasks is the lowest.

For conflicts of the fourth type (i.e., conflicts that can be resolved both ways), we cannot completely rely on this measure to select a conflict to resolve.

Example 5.6 Consider two conflicts $\{t_a, t_b\}$ and $\{t_c, t_d\}$, for which the temporal distance $d(t_a, t_b) = d(t_b, t_a) = 4$, but $d(t_c, t_d) = 3$ and $d(t_d, t_c) = 100$. Here, $\omega_{\text{res}}(t_a, t_b) = 4$ and $\omega_{\text{res}}(t_c, t_d) = 3$. However, it is clear that $\{t_a, t_b\}$ has less temporal flexibility in which it can be resolved than $\{t_c, t_d\}$: $d(t_d, t_c) = 100$ indicates that posting $t_d \prec t_c$ leaves a very large amount of flexibility in the schedule.

Therefore, ω_{res} is adapted to be biased towards conflicts that have similar slack in both directions (Cheng and Smith, 1996):

$$\omega'_{\text{res}}(t_i, t_j) = \frac{\min(d(t_i, t_j), d(t_j, t_i))}{\sqrt{S}}, \tag{5.6}$$

with

$$S = \begin{cases} \frac{\min(d(t_i,t_j),d(t_j,t_i))}{\max(d(t_i,t_j),d(t_j,t_i))} & \text{for } \max(d(t_i, t_j), d(t_j, t_i)) \neq 0 \\ 1 & \text{otherwise} \end{cases} \tag{5.7}$$

Example 5.7 For $\{t_a, t_b\}$, we have $S = 1$, resulting in $\omega'_{\text{res}}(t_a, t_b) = 4$. But, for $\{t_c, t_d\}$, we have $S = 3/100$, resulting in $\omega'_{\text{res}}(t_c, t_d) = 3/\sqrt{3/100} \approx 17.3$. This results in $\{t_a, t_b\}$ being selected for resolution before $\{t_c, t_d\}$.

As shown by the example, constraints that are close to being unresolvable in *both* directions are now selected for resolution first. If other constraints are posted first instead, we risk reducing the flexibility of this constraint even further, turning it into an unresolvable conflict. If this happens, the only way to arrive at a solution is to backtrack, which is highly undesirable. Our algorithm, like many other constraint posting algorithms, does not perform backtracking due to reasons of efficiency, and will simply give up if an unresolvable conflict is encountered.

The last step is to determine a direction for the constraint to be posted between the selected tasks. For $\{t_i, t_j\}$, we post $t_i \prec t_j$ if $d(t_i, t_j) > d(t_j, t_i)$, and $t_j \prec t_i$ otherwise. The intuition for this decision is that we want to preserve the largest amount of temporal flexibility.

To summarize, the three decisions are made using the following criteria:

1. Select the peak with the *largest* amount of resource overcommitment.

2. Within this peak, select the conflict with the *least* amount of scheduling flexibility left.

3. Post a constraint preserving the *largest* amount of flexibility.

This way, the algorithm focuses on the most critical part of the problem, where it tries to make decisions keeping the amount of flexibility remaining as high as possible.

5.3 Task grouping

In the preceding section, the technique of constraint posting has been described, with which the RCPSP can be solved easily, up to the scale of the problems that are present at NedTrain. However, flexibility still remains low: while the technique of interval schedules can be used, as described in the previous chapter, to give the schedule some amount of temporal flexibility, the added dependency constraints impose a certain rigidity on the *ordering* of the tasks. Ideally, we would like to have sequential flexibility as well, such that we are not forced to use a certain ordering, decided in advance, at execution time—conditions might have changed, making another ordering more attractive.

Therefore, this section introduces an addition to the precedence constraint posting method, one by which we can include multiple execution orderings in a solution to an RCPSP problem instance. The method operates on the basis of the idea of *task grouping*: instead of posting a constraint between two tasks and enforcing an ordering, the two tasks are grouped together in an encapsulating group task, which reserves resources such that both execution orders remain feasible at execution time. The concept of grouping is recursive, meaning that such an encapsulating group task can take part in *another* grouping operation,

resulting in a group task containing three tasks, which can be executed in *any* order at execution time.

This section starts by describing the exact procedure by which tasks are grouped together, followed by a description of the procedure by which such a schedule can be executed, i.e., how this sequential flexibility can actually be used.

5.3.1 *The grouping operation*

We propose to extend the constraint posting technique by adding the operation of grouping two tasks together, forming an encapsulating group task, as an alternative to posting a precedence constraint to the schedule: instead of always adding such a constraint we offer the possibility to group two tasks t_i and t_j together in order to *postpone* the decision on their ordering to the time of execution of the schedule. The grouping $\textsc{Group}(t_i, t_j)$ of t_i and t_j results in a composite task $t_{i,j}$ where t_i and t_j are executed sequentially without specifying the exact order: either t_i is executed first, and t_j second, or execution starts with t_j, and t_i is second. $\textsc{Group}(t_i, t_j)$ consists of the following steps:

1. Both t_i and t_j are removed from T, and the composite task $t_{i,j}$ is added to T instead.

2. Release time, due time and duration of this group task $t_{i,j}$ are computed as follows:

$$s_{i,j} = \max(s_i, s_j), \tag{5.8}$$
$$d_{i,j} = \min(d_i, d_j), \tag{5.9}$$
$$l_{i,j} = l_i + l_j. \tag{5.10}$$

   For the release time, the maximum of the release times of t_i and t_j is taken. This ensures that $t_{i,j}$ can start with either t_i or t_j. If the minimum were taken, we would be forced to commence with the earliest task of the two, during execution. In similar vein, the minimum of the due times d_i and d_j is taken. For the task length of $t_{i,j}$, the sum of l_i and l_j is used, since the two tasks are to be executed sequentially.

3. A similar reasoning is used behind the resource usage of the grouped task. Since the ordering of the two tasks is not known until execution time, the grouped task is given a resource profile that ensures that either task can run at any time within the time limits of the grouped task. Hence, for each resource r_k, we compute the resource usage of the group task to be equal to the maximum of the resource usage of the individual tasks:

$$\text{req}(t_{i,j}, r_k) = \max\{\text{req}(t_i, r_k), \text{req}(t_j, r_k)\}. \tag{5.11}$$

   Note that this creates a worst-case resource profile, which ensures that enough resources are available to execute either t_i or t_j at any time during $t_{i,j}$.

4. Lastly, when forming a grouped task, existing precedence constraints must be updated. All constraints of the form $t_k \prec t_l$ with $l = i$ or $l = j$ are replaced by $t_k \prec t_{i,j}$, and all constraints of the form $t_l \prec t_k$ with $l = i$ or $l = j$ are replaced by $t_{i,j} \prec t_k$. This means that all tasks that were constrained to take place before (respectively, after) either t_i or t_j are now constrained to take place before (respectively, after) $t_{i,j}$ instead.

These steps are formalized in Algorithm 5.1.

Algorithm 5.1: Operation to group tasks together.

function GROUP(t_i, t_j)**:**
  $T \leftarrow T - \{t_i, t_j\}$
  $l_{i,j} \leftarrow l_i + l_j$
  $s_{i,j} \leftarrow \max(s_i, s_j)$
  $d_{i,j} \leftarrow \min(d_i, d_j)$
  forall the $r_k \in R$ **do**
    $\text{req}(t_{i,j}, r_k) \leftarrow \max\{\text{req}(t_i, r_k), \text{req}(t_j, r_k)\}$
  end
  forall the $t_k \prec t_l : l = i \vee l = j$ **do**
    $C \leftarrow C - \{t_k \prec t_l\}$
    $C \leftarrow C \cup \{t_k \prec t_{i,j}\}$
  end
  forall the $t_l \prec t_k : l = i \vee l = j$ **do**
    $C \leftarrow C - \{t_l \prec t_k\}$
    $C \leftarrow C \cup \{t_{i,j} \prec t_k\}$
  end
  $T \leftarrow T \cup \{t_{i,j}\}$
end

To show that the grouping operation has no effect on the validity of the solution space, we need the following proposition:

Proposition 5.1 Any temporally and resource-feasible start time assignment σ for the tasks in a group task is a temporally and resource-feasible start time assignment for the tasks in the original problem as well.

Proof. To start, we note that since $l_{i,j} = l_i + l_j$, the tasks contained in the group can be executed sequentially during the time the group is scheduled for execution. For any of the tasks t_l with $l \in \{i, j\}$ in the task group, we have

$$\sigma(t_l) \geq s_{i,j} = \max(s_i, s_j) \Rightarrow \sigma(t_l) \geq s_i \wedge \sigma(t_l) \geq s_j, \tag{5.12}$$

showing that the release time constraint is satisfied.

Similarly, we have

$$\sigma(t_l) + l_l \leq d_{i,j} = \min(d_i, d_j) \Rightarrow \sigma(t_l) + l_l \leq d_i \wedge \sigma(t_l) + l_l \leq d_j, \tag{5.13}$$

showing that the due time constraint is satisfied.

For the resource constraints, we know that $\text{req}(t_{i,j}, r_k)$ of capacity is available for resource r_k during the execution of $t_{i,j}$, since σ is a resource-feasible assignment. Since

$$\text{req}(t_{i,j}, r_k) = \max(\text{req}(t_i, r_k), \text{req}(t_j, r_k)) \geq \text{req}(t_l, r_k), \tag{5.14}$$

we know that there must also be enough capacity available to execute t_l ($l \in \{i, j\}$).

Lastly, for the precedence constraints, for any of the tasks t_l for which there is a constraint $t_k \prec t_l$ in the original problem, we have $t_k \prec t_{i,j}$, such that

$$\sigma(t_k) + l_k \leq \sigma(t_{i,j}) \leq \sigma(t_l), \tag{5.15}$$

showing that the precedence constraint is satisfied. Similarly, if we have $t_l \prec t_k$ in the original problem, we have $t_{i,j} \prec t_k$, such that

$$\sigma(t_l) + l_l \leq \sigma(t_{i,j}) + l_{i,j} \leq \sigma(t_k). \tag{5.16}$$

□

Now that we have specified how to group two tasks together, we need to be able to select two suitable tasks in an RCPSP on which to perform this grouping. For this, we must find two tasks that should be executed sequentially, due to resource constraints, and for which the exact execution order is not very important. This idea bears some resemblance to the stochastic procedure used by Oddi and Smith (1997): here, for tasks that cannot be executed in parallel but for which the heuristic used has low *discriminatory power*, a random ordering is chosen. Instead, in this case our grouping method will simply replace such tasks with a grouped task.

More precisely, while we normally would post $t_i \prec t_j$ if $d(t_i, t_j) > d(t_j, t_i)$, and $t_j \prec t_i$ otherwise, we now employ a threshold parameter γ, enforcing a minimal difference before the algorithm turns to posting a constraint. If $|d(t_i, t_j) - d(t_j, t_i)| \leq \gamma$, we decide to group t_i and t_j into the composite task $t_{i,j}$ using the procedure outlined above.

In Algorithm 5.2, the structure of the resulting algorithm is shown. Note that task grouping and constraint posting are interleaved: if the chosen conflict falls below the threshold γ, the tasks are grouped, otherwise, a constraint is posted. This is possible because both constraint posting and task grouping are essentially problem transformations:

- constraint posting takes one RCPSP instance, and returns another RCPSP instance, with an additional precedence constraint.

- task grouping takes one RCPSP instance and returns another RCPSP instance, with two tasks replaced by one other task representing a task group.

Algorithm 5.2: Using task grouping and constraint posting to solve an RCPSP instance

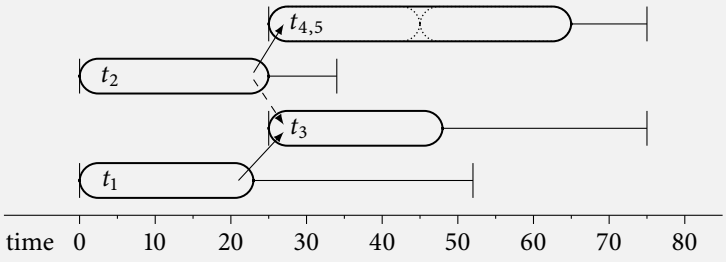
```
1  while the RCPSP instance contains a peak do
2  |  p ← largest peak
3  |  if p is unresolvable then terminate
4  |  {t_i, t_j} ← SelectConflict(p)
5  |  if |d(t_i, t_j) − d(t_j, t_i)| ≤ γ then
6  |  |  Group(t_i, t_j)
7  |  else if d(t_i, t_j) > d(t_j, t_i) then
8  |  |  add t_i ≺ t_j
9  |  else
10 |  |  add t_j ≺ t_i
11 |  end
12 end
```

Example 5.8 Consider again the conflicts from Example 5.4. An alternative way of resolving them would be to first post the constraint $t_2 \prec t_3$. For the next peak, we can, among other options, choose to post $t_4 \prec t_5$ (as done before) or $t_5 \prec t_4$. From a temporal perspective, there is little difference between the two options. Therefore, we choose to apply Group(t_4, t_5) to the problem, which results in the following transformation:

before Group(t_4, t_5)	task	t_1	t_2	t_3	t_4	t_5
	length	23	25	23	20	20
	use of r_a	0	2	1	1	1
	use of r_b	1	0	0	1	0
after Group(t_4, t_5)	task	t_1	t_2	t_3	$t_{4,5}$	
	length	23	25	23	40	
	use of r_a	0	2	1	1	
	use of r_b	1	0	0	1	

Note that the length of $t_{4,5}$ is the sum of the lengths of t_4 and t_5, and that $t_{4,5}$ uses r_b over its entire length—even though t_5 does not need this resource.

Graphically, this solution looks as follows:



Now, consider an execution where t_1 is delayed. Using the solution in Example 5.5, task t_3 will be delayed too, due to the precedence constraint. However, t_4 cannot start until t_1 is finished either, due to the use of r_b, so this task, and t_5, will be delayed as well. Contrast this with the solution in this example: here, the agent executing the schedule can still choose to start with t_5 instead. This task does not need r_b, and, since $\text{cap}(r_a) = 2$, t_5 can execute concurrently with t_1. Hence, if t_1 is not delayed past the end of t_5, the makespan of this instance is not influenced at all.

5.4 Execution of grouped schedules

Having defined how to *build* a schedule containing sequential flexibility, in the form of grouped tasks, this section describes how to *execute* such a schedule, and to make use of this flexibility. Execution of a schedule containing grouped tasks differs from that of a normal schedule, since at run time, an ordering needs to be determined for the tasks contained in a group. If a schedule is executed without any tasks being delayed, executing a group task is trivial: since the group task has ensured that resources needed by *any* task in that group are available for the duration of that group, any ordering picked is guaranteed to be feasible.

When tasks are delayed during their execution special precautions need to be taken, both for normal and for grouped tasks: the start of tasks succeeding the delayed task might need to be delayed, since the delayed task might now finish after the planned start time of these tasks. As concurrent execution is not possible, due to either resource constraint violations, or violations of precedence constraints present in the original problem, any task that can not start at its planned time is added to the end of a queue of *pending tasks*. Whenever a task finishes executing, this queue is examined first, in order, to see if any of these tasks are now eligible to be started. Notice that this causes delay propagation: resources freed by the finished task might be claimed by one of the pending tasks, resulting in tasks that would otherwise be able to start being added to the pending queue.

Grouped tasks can be executed using this mechanism as well: a grouped task is in essence a set of pending tasks, one of which can be executed at the start time of that grouped task. This means that, if a grouped task is encountered, all tasks in the group are added to the pending queue, and then the first eligible task in the queue is selected for execution. Recall that at any time a task finishes, the queue is examined again for eligible tasks. This ensures that all tasks from the group are executed eventually.

Note that the order in which the tasks from a group are added to the queue can be determined at execution time. This is the mechanism by which grouped tasks achieve sequential flexibility: the tasks can be added to the pending queue in any order, and if there are no delays, the construction of the schedule ensures that they can be executed in sequence. If there are delays, the mechanism of picking eligible tasks from the queue ensures that the group task is automatically reordered to adapt to the changed circumstances during execution.

Example 5.9 To show how delays are handled during the execution of a schedule, consider the (grouped) schedule shown in Example 5.8. Assume that t_1 is lengthened by 20%, such that its length will be $l_1' = 1.2 \cdot l_1 = 27.6$. Execution proceeds normally, until time point 25, when t_3 and $t_{4,5}$ are to be started. Task t_3 depends on t_1 to be finished, so we cannot execute this task now, and we add it to the queue of postponed tasks.

While $t_{4,5}$ does not directly depend on t_1, it does make use of r_b, which is still in use by t_1 at its start time. If $t_{4,5}$ was a normal task, we would have to delay its execution, but in this case, we can make use of the option to reorder the execution of this group task: if we choose to execute t_5 before t_4, no delay is necessary, since t_5 does not make use of r_b. For larger group tasks we check for all tasks in the group if all needed resources are available, and if all predecessors have finished. From the set of tasks for which this holds, we pick one (here, only t_5 is eligible), and the rest (in this case, t_4) is added to the queue of postponed tasks.

At time point 27.6, t_1 is finished, and the queue of postponed tasks is examined. Task t_4 can not start, since t_5 is still executing, but t_3 can now be started, such that it finishes at time point 50.6. The last task in the queue is t_4, which can be started after t_5 has finished. Since the start time of t_5 was not delayed, t_4 finishes at the scheduled time.

5.5 Experiments

As shown in the preceding section, task grouping offers increased flexibility in executing a schedule. This is demonstrated by the increased number of topological orderings allowed by such a schedule, as well as by Example 5.8 in which execution-time reordering is demonstrated. Recall, however, that our reason for constructing flexible schedules was born out of a desire to improve robustness during execution, if delays were to be expected. Both these observations offer a basis for the hypothesis that schedules using task groups are more robust when delays are present. To test this hypothesis, this section presents experiments using simulated executions, with delays, of such schedules.

We start with a description of how the experiments are setup. After this, we begin with a general analysis of the structure of the schedules resulting from our experiments, which is followed by a series of experiments to determine if our method indeed leads to higher robustness, and under which circumstances and by which mechanism it does so.

5.5.1 Setup of experiments

To be able to simulate schedule executions and measure the effect of task grouping on the robustness, we must be able to construct these schedules, with and without task groups. For this, the well-known benchmark instances from PSPLIB are used (see Kolisch and Sprecher (1996); Kolisch et al. (1998) for an in-depth

Table 5.1 Summary of the PSPLIB benchmark instances.

Name	Number of tasks	Number of instances
j30	30	480
j60	60	480
j90	90	480
j120	120	600

discussion of the design of these instances). From this set we use the single-mode instances only.[2] Note that the PSPLIB benchmark instances were originally designed to explore the various levels of difficulty of RCPSP instances, therefore the instances in the benchmark have a high variety, as different parameters potentially affecting the difficulty of the instances are varied over the instances. For each parameter combination, ten instances are included in the benchmark set. Van Nijs and Klos (2014) analyze these instance complexity issues further, and improvements are suggested to generate instances that are more constant over some of the complexity parameters. However, for our purposes, having a varied set of instances is beneficial, since this will show the performance improvement independent of instance difficulty parameters. Table 5.1 presents a short summary of the properties of the instances contained in this benchmark.

5.5.2 *Characteristics of grouped schedules*

To convert these RCPSP instances into schedules, Algorithm 5.1 is used. If $\gamma = 0$ is used as threshold, the grouping operation is never applied. When $\gamma > 0$, the algorithm creates group tasks in an opportunistic fashion: tasks are grouped whenever the heuristic that decides on their ordering falls below the threshold γ. The number of group tasks in the solution is thus dependent on the combination of the structure of the problem, and the value of γ—there is no method to directly control the number or size of group tasks in the solution. Since the idea behind task grouping was to create groups to add flexibility, it is natural to start with an analysis examining the structure of the schedules created by the algorithm, in terms of the number of grouped tasks, and the size of these groups. The hypothesis here is clear: for higher values of γ, more and bigger groups are expected in the resulting schedule.

In Table 5.2 these data are summarized for $\gamma \in \{2, 4, 6, 8\}$. The threshold parameter has a clear effect on both the number of group tasks as well as on the number of tasks in a group: the higher the threshold the more grouping takes place, up to a certain level, confirming our hypothesis. The difference between $\gamma = 2$ and $\gamma = 4$ is much higher than the difference between $\gamma = 6$ and $\gamma = 8$, and since values of $\gamma > 8$ give little additional benefit in terms of number of group tasks and size of task groups, further experiments have been limited to schedules constructed using $\gamma \leq 8$.

[2] Multi-mode instances have multiple versions of each task, with a different length and different resource usage pattern. These represent a different type of scheduling problem, outside of the scope of our research, and are thus not used in the experiments.

Table 5.2
Characteristics of grouped schedules for different parameters.

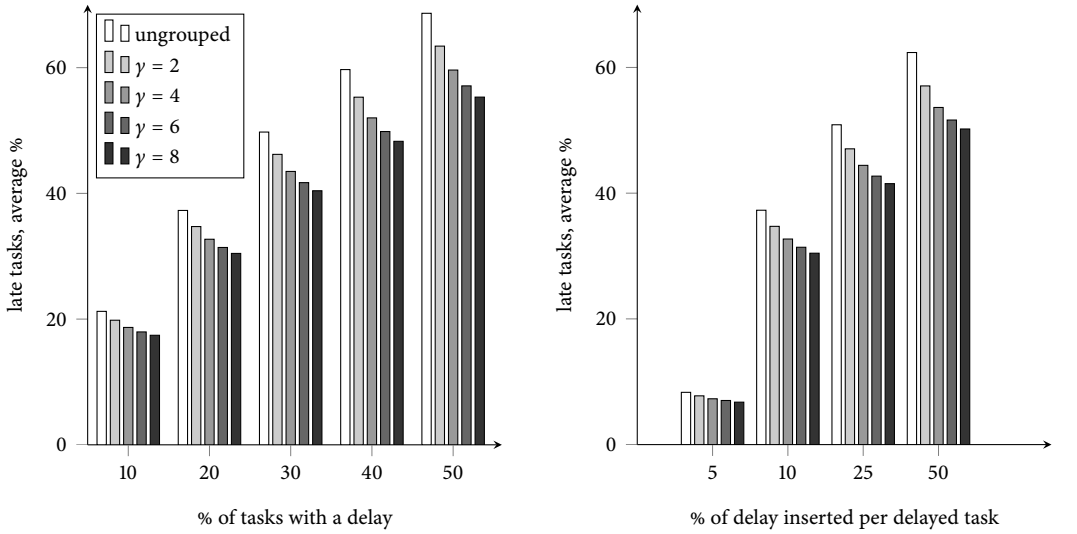
	Average number of group tasks				Average group size			
Set	$\gamma = 2$	$\gamma = 4$	$\gamma = 6$	$\gamma = 8$	$\gamma = 2$	$\gamma = 4$	$\gamma = 6$	$\gamma = 8$
j30	1.79 (6.0%)	2.43 (8.1%)	2.69 (9.0%)	2.82 (9.4%)	2.26	2.46	2.62	2.77
j60	2.58 (4.3%)	3.39 (5.6%)	3.83 (6.4%)	4.08 (6.8%)	2.35	2.61	2.84	3.02
j90	3.21 (3.6%)	4.20 (4.7%)	4.64 (5.2%)	4.94 (5.5%)	2.39	2.69	2.94	3.13
j120	8.13 (6.8%)	9.85 (8.2%)	10.42 (8.7%)	10.62 (8.8%)	2.47	2.86	3.17	3.44

An interesting observation is that in general, the (relative) number of group tasks gets lower as instances get bigger, except for the instances in the j120 set. A likely cause for this difference is the different set of *resource strength* parameters used in generating the instances. The resource strength parameter *RS* sets the capacities of the resources of an instance (Kolisch and Sprecher, 1996), as the ratio between the minimally needed capacity (i.e., the capacity needed to be able to sequentially execute each task) and the highest possible capacity (i.e., the capacity needed to be able to execute the schedule without any additional leveling constraints). The sets j30, j60 and j90 use $RS \in \{0.2, 0.5, 0.7, 1.0\}$, whereas j120 uses $RS \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ (Kolisch et al., 1998) — this is also the reason that j120 has more instances. The (on average) lower resource strength for the j120 instances lead to more resource peaks, which in turn lead to more constraints being posted and tasks being grouped.

5.5.3 *Robustness*

The main purpose of task grouping, increasing the flexibility of the schedules to improve the robustness during execution, is investigated next. To analyze the robustness, a certain percentage of the tasks in the constructed schedule is selected, and their lengths are extended to simulate a delay. These schedules are then executed as described in Section 5.4. For every instance, a percentage of the tasks is delayed with a constant factor. Each instance is simulated 150 times, where the tasks to be delayed are selected at random. The robustness is measured by how well execution adheres to the schedule: tasks that complete after their projected finish time are counted as late, and the robustness of a schedule is measured as the average number of late tasks over all simulated executions. Note that this differs from the concept of violations as used in the previous chapter: in that chapter, we counted how often a task had to be executed outside of its assigned interval. As we do not use intervals here, this approach is not applicable, and we resort to counting the number of tasks that fail to finish at their scheduled time.

The delays inserted during execution are characterized by two parameters: the *number* of tasks that are delayed for a given execution, and the *amount* of delay given to each of these tasks. The first parameter, the number of delayed



(a) Increasing the number of delayed tasks. The delay per task is kept constant, at 10%.

(b) Increasing the amount of delay of tasks. The number of delayed tasks is kept constant, at 20%.

Figure 5.1
The effect of delay characteristics on the robustness of executed schedules, for various settings of the grouping threshold parameter γ. Simulations are performed on all PSPLIB instances, shown is the average percentage of late tasks.

tasks, is taken from the set $\{10\%, 20\%, 30\%, 40\%, 50\%\}$, relative to the total number of tasks in the instance. The second parameter, the amount of delay in a task, is taken from the set $\{5\%, 10\%, 25\%, 50\%\}$, relative to the length of the task receiving the delay.

The effect of these parameters on the performance, in terms of robustness, is investigated separately. We start by fixing the delay per task at 10%, and varying the number of tasks with a delay. The results are shown in Figure 5.1(a). The number of tasks that complete in time generally drops rapidly when increasing the number of delayed tasks, which is in line with expectations. The performance of grouping increases for larger values of γ; the cause is the larger number of groups present in the solution. For a low number of tasks with a delay, γ has less effect on the performance. A reason might be that all the delays that can be absorbed are already absorbed with a low number of groups—adding more groups does not give an increase in performance in this case.

Similarly, in Figure 5.1(b), the predictability for increasing the amount of delay in each task is shown, for a delay in 20% of the tasks. A similar, but more pronounced, effect is seen for simulations where only 5% delay is inserted in a delayed task: adding more groups, by increasing γ, has only very little effect.

We have so far analyzed the performance of the algorithm by looking at the delay parameters and the various values for γ. While the γ parameter can be used to indirectly influence the formation of groups by the algorithm, as evidenced

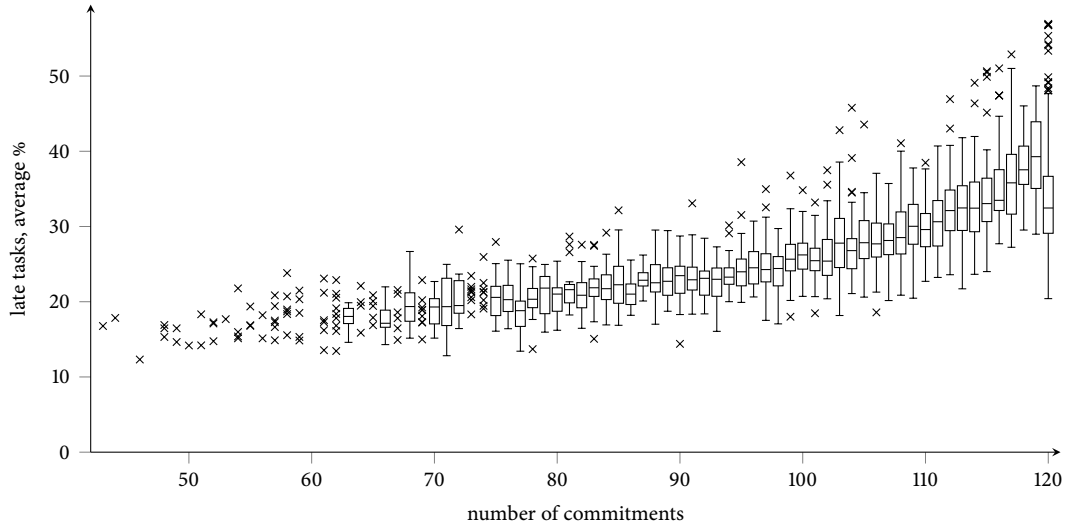


Figure 5.2
The relationship between the number of commitments and the robustness. Results are shown for the j120 set, with 20% of the tasks delayed by 10%. For each level of commitment, a box and whiskers plot is shown representing the distribution of the percentage of late tasks. If less than ten instances are available for a given level, the individual results are shown.

by Table 5.2, the effects we witness are not directly coupled to this parameter, but to its result: a higher participation of tasks in groups. This higher participation can be viewed as a reduction of the level of commitment present in the schedule: if we have a task group containing m tasks, this is a single commitment, whereas m separate tasks represent m separate commitments. Our hypothesis is thus that schedules with lower commitment levels will show an increase in performance in terms of robustness. Since the size of the problem also determines the level of commitment in the resulting schedule, we will restrict ourselves to the j120 set: as discussed before, these instances have, on average, lower resource strength, and contain more/bigger resource peaks, leading to more complex solutions. We also opt to fix the delay parameters at a delay of 10% per task, for 20% of the tasks.

In Figure 5.2 the lateness percentage is shown for the various levels of commitment present in all solutions for the j120 instances. Solutions with equal levels of commitment are grouped together, represented by a box plot. This means that all schedules constructed without task grouping are represented by the box plot on the far right. The schedules constructed using $\gamma > 0$ are represented by the other box plots, depending on the amount of grouping that took place.

There is indeed a clear correlation between the level of commitment and the lateness, with diminishing returns for lower levels of commitment. A more interesting observation is that the lateness distribution for the ungrouped solutions has a lower median than that of the first few boxplots of the solutions containing a low number of task groups. There is apparently a common factor

in the structure of these instances that makes them both less amenable to the creation of task groups and more susceptible to the effects of delay. A possible reason might be the existence of a larger number of precedence constraints in the original problem. These may lead to longer chains of tasks in the solution, which result in more tasks being affected by delay propagation. At the same time, instances with more precedence constraints have a lower number of tasks in parallel during the construction of the schedule, resulting in fewer opportunities to group tasks together.

5.5.4 *Solution structure*

An astute reader may have already observed that the preceding analysis based on the level of commitment neglects an important distinction in the structure of the solution. Schedules with a similar level of commitment can have a very different composition in terms of task groups: one solution might have only a few larger groups, whereas another solution might contain many smaller groups. From our earlier discussion of the effect of task groups on the flexibility, it would seem that increased group sizes would have more effect than an increased number of groups:

Example 5.10 Consider a task group with $m > 2$ tasks. At execution time, such a group permits $m!$ possible orderings. If we now divide these tasks over two smaller, sequentially executed groups of size $m/2$ the number of possible orderings is reduced, to $2 \cdot (m/2)!$.

As the number of groups is also dependent on the number of tasks in the problem, we again restrict our analysis to the j120 set, with delay parameters fixed to the same values as earlier, a delay of 10% per task, for 20% of the tasks.

In Figure 5.3 the lateness is plotted relative to both the number of groups and the average size of the groups. These two solution structure parameters form a discrete, two-dimensional space. At each point in this space where solutions are present, a circle is plotted, where its size is scaled according to the average lateness for solutions at this point. As a visual aid, the circles are also given a shade of gray according to the quartile in which they belong.

Looking at the figure, it is clear that both the number of groups as well as the size of the groups contributes to decreasing the lateness percentage. What is surprising, however, is that our hypothesis that the increased flexibility offered by a smaller number of large groups as compared to a larger number of smaller groups should lead to higher robustness and thus lower lateness does not seem to hold. To validate this visual impression, the Spearman rank-order correlation was calculated between both average group size and number of groups, and the lateness. For the group size, a correlation of $\rho = -0.567$ was found, and for the number of groups, a correlation of $\rho = -0.749$ was found. This confirms the visual impression that having more groups has a more consistent effect on lateness than having large groups.

An explanation for why our hypothesis does not seem to hold lies in the difference between flexibility and robustness. While a schedule with few large

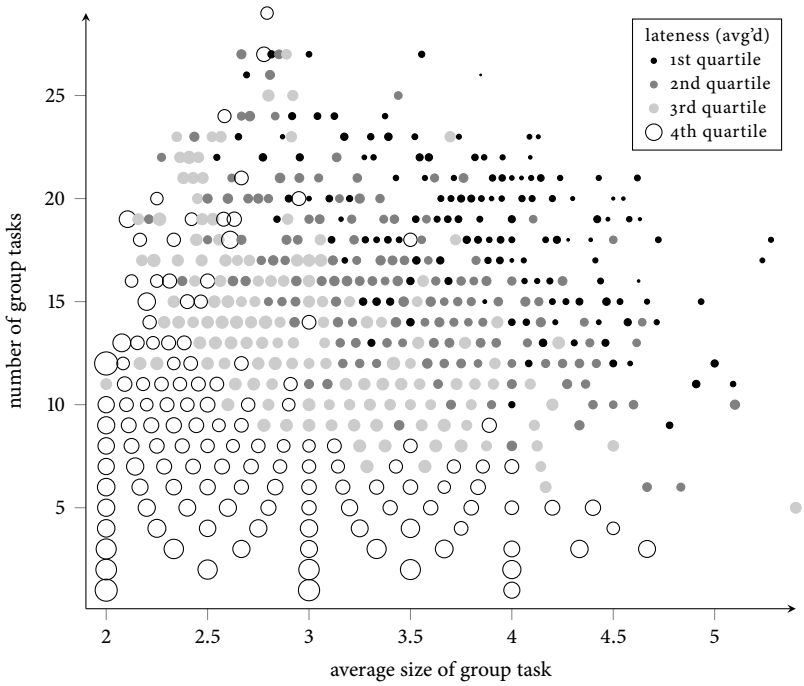
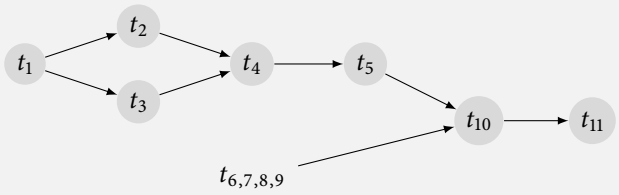


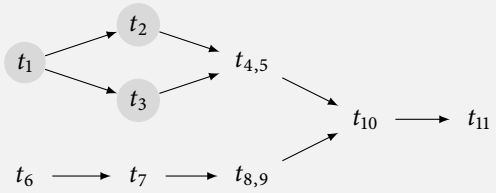
Figure 5.3
Lateness versus both the number and average size of group tasks. Results are shown for the j120 set, with 20% of the tasks delayed by 10%.

group tasks might have high flexibility, this flexibility is necessarily concentrated in the areas containing these large groups. Our method of assessing robustness is based on introducing delays in a schedule during execution, and measuring how many tasks are affected by these delays. Since these delays can be inserted at any location in the tasks, a delay might impact many other tasks before a task group, which can absorb it by reordering as demonstrated by Example 5.8, is reached. If the schedule contains more, smaller task groups, less tasks will be affected, explaining the higher correlation between the number of tasks and lateness.

Example 5.11 To illustrate this effect, consider the schedule depicted below (for clarity, the lengths of the tasks are not shown). Imagine that an unexpected event causes t_1 to be delayed. This delay propagates to both t_2 and t_3, and then further down the schedule. The background of affected tasks is colored. Tasks all the way up to, and including, t_{11} are affected. The single, large group task $t_{6,7,8,9}$ is not in the "path" of the delay, and therefore cannot absorb it.



Contrast this with the schedule below, for the same network structure, in which two smaller group tasks are present. Here, only t_2 and t_3 are affected, and then group task $t_{4,5}$ can absorb the delay.



5.6 Conclusion and discussion

This chapter presents a novel way to create flexible schedules, addressing the third research question raised by this thesis. The method works by creating solutions to scheduling problems containing *groups* with multiple tasks in them. The order of the tasks in a group is not fixed in the solution, and can be freely determined at execution time. This serves a dual purpose: it answers the needs of the third research question, by giving a method with which it is possible to efficiently represent solutions to the RCPSP containing sequential flexibility, and it gives another way of constructing schedules that are robust in the presence of uncertainty.

The algorithm we present forms task groups based on a threshold parameter γ. Experiments confirm our expectation that higher values of γ lead to both more and bigger groups. Independent control of these parameters is not possible with the current form of the algorithm. Simulating executions of schedules with delays demonstrates that schedules with group tasks are better able to combat the spread of those delays. Larger values of γ lead to better performance in that regard: fewer tasks are started late as a result of the simulated delays in the execution of the schedule.

Since γ offers no direct control of either the number or the size of groups, we next looked at the level of commitment versus the lateness. Each task, whether it is a grouped task or a normal one, is seen as one commitment, to represent the flexibility we still have in executing a grouped task. In this way, the more tasks take part in a grouping operation, either in the form of more groups, or bigger groups, the lower the total level of commitment in the schedule. There is indeed a clear correlation between commitment and lateness, but it also leads to the next question: what influences the performance more, the number of groups or the size of the groups?

This question leads to a rather surprising result. Our initial hypothesis was that bigger groups will give higher robustness, since they offer higher flexibility: if we split a big group in two smaller, sequential groups, the number of possible orderings is made smaller. The experiments show however that the reverse is

true: the number of groups has a bigger effect on the robustness than the size of the groups. An explanation for this effect is that the origin of a delay is random. In an instance with few large groups, such a delay might impact a large number of (ungrouped) tasks before a task group that can absorb it, by reordering, is reached.

5.6.1 Discussion

The concept of task grouping offers a lot of opportunities for future research. The experiments show that the number of groups in the schedule offers a good indication of the performance in terms of late tasks. Using the current method however, it is hard to directly control the number of groups. This could be very desirable in a situation in which we want to ensure that the resulting schedule has a certain amount of sequential flexibility, be it to combat uncertainty or to offer autonomy to an executing agent. The current method is opportunistic in performing the grouping operation: tasks are grouped only if the heuristic, using the threshold parameter γ, indicates so. For some situations it would be more practical to determine the number of required groups beforehand, and to have a method that would perform grouping operations until that level was reached.

A second important point that has so far not been addressed is the selection of the tasks to be grouped. In essence, two opposing objectives can be identified here. On the one hand, it is beneficial if tasks to be grouped have a similar resource usage. This way, the over-estimation of resource usage for the group task will be kept to a minimum, such that the makespan will be less affected. On the other hand, to be able to prevent delay propagation, tasks in a group should have as diverse a resource usage as possible. This way, if one resource is still in use due to a delay, the probability that a task can be found in a group that does not use this resource is higher. Different policies for selecting tasks to group should therefore be tested, to see if these effects are indeed present.

Discussion 6

The research presented in this thesis was undertaken in the context of the applied research and development program at NedTrain, which performs the maintenance operations for the rolling stock of the Dutch national railway services. It focuses on the planning challenges encountered at the operational level, dealing with the scheduling of maintenance jobs in a single workplace.

The changing environment of the railway industry introduces new challenges to maintenance operations. Two developments are discussed that present new challenges to maintenance operations at the operational level. In the past, new rolling stock was developed in very close cooperation with national industries, leading to a large build-up of knowledge among engineers long before the new stock was taken into service. In modern railway operations, there is a preference to buy existing designs, tailored to the local specifications. Combined with higher complexity of on-board systems, this leads to maintenance operations being more difficult and less predictable. A second development is the increased need for punctuality due to the desire to have multiple smaller maintenance visits such that trains are only in workshops during off-peak hours. The inherent uncertainties in the maintenance jobs tend to average out if a visit is fairly long, but this effect is lost if the visits are made much shorter. For such visits it is therefore important to have a flexible method of planning, with flexible results.

In this last chapter, we want to re-examine the research goals as introduced in the first chapter, and evaluate if these goals were met. To do this, we work back by first re-examining the research questions together with the answers as presented in this thesis. Then, we turn to the research goals, and also re-examine them, to see if they have been met. Along the way, some insights on future research avenues will be presented.

6.1 Research questions

We will start by re-examining the research questions presented at the end of the second chapter, and we will compare them with the research presented in this thesis.

The nature of the scheduling problem at NedTrain is such that flexibility, to be used during execution of a schedule, is of high importance. Existing research shows that the Simple Temporal Network (STN) can be used as a flexible solution representation for an instance of the Resource-Constrained Project Scheduling Problem (RCPSP), which is able to capture the most important aspects of our scheduling problem. But from this existing literature it is not clear how to quantify the flexibility of a solution exactly, or how to compare two solutions to the same problem based on their flexibility. Therefore, we posed our first research question:

RQ1 What criteria encapsulate our intuitive notion of the concept of *flexibility*, as applied to solutions of scheduling problems, and can we devise a method to measure flexibility, conforming to these criteria?

The rationality postulates **F1**, **F2**, **F2′** and **F3** introduced in Chapter 3 capture this notion of flexibility. Of special interest is our contribution of **F2′**, which states that the flexibility of one task has to be independent of the chosen start time of any of the other tasks: picking a certain start time for one task should not reduce the flexibility of another task.

Applying these postulates to existing metrics, we discovered that these severely over-estimate the available flexibility, and we introduced a new measure that better takes into account dependencies between tasks, satisfying **F2′**. This measure is constructive: as a result of calculating the flexibility of an STN we arrive at an assignment of intervals to tasks. Such an assignment has the property that the intervals are *independent*: a choice in one interval has no effect on the available choices in any of the other intervals.

Having determined how flexible a certain solution is, the next question is how to best make use of this flexibility. Our goal here is to arrive at a *robust* schedule—a schedule in which a disturbance in one task has low impact on the rest of the schedule.

RQ2 Given a schedule with a certain potential flexibility, in what ways can this flexibility be used best to achieve a schedule with high *robustness*?

The experiments performed in Chapter 4 show that the best way to distribute flexibility depends on the exact aim. A more even distribution of flexibility lowers the number of violations (i.e., tasks executing outside their assigned interval because of a delay), and lowers the tardiness (i.e., how severely the deadline of the schedule is violated because of delays). If this distribution is skewed such that tasks having more predecessors receive more flexibility, the effect on tardiness is more pronounced, while the number of violations increases.

One result we encountered is contradictory at first glance: using a solution with lower flexibility can improve the robustness when executing the schedule. This seems to contradict our desire that our measure of flexibility corresponds to *intuition*, such as expressed in the first research question. The effect occurs because sometimes, a higher global level of flexibility can be achieved if multiple

tasks or regions of a schedule are 'sacrificed' in terms of available flexibility such that one (or a few) tasks can be allocated more flexibility. The result is a solution with high level of flexibility, concentrated in only a small part of the schedule. The apparent contradiction is caused because our operationalization of the intuitive idea of flexibility is applied at a global scale, looking at the complete schedule. The effects influencing robustness operate at a much more local scale: for a certain disturbance, only tasks that are successors of the delayed tasks are influenced.

So far, we have solely considered the temporal aspect of flexibility in a schedule. The third research question extends the scope of our research to sequential flexibility:

RQ3 In what way can the concept of temporal flexibility of a schedule be extended to include some *sequential flexibility*?

In answering the previous research questions, we considered solutions at the level of the STN. For the third research question, we widen our scope to include the conversion of an RCPSP to an STN, and we propose a task grouping extension to the STN with which not only temporal flexibility can be encoded, but sequential flexibility as well. Constraint posting normally adds precedence constraints between tasks to convert resource constraints into temporal constraints. Our task grouping extension adds the option to *group* two tasks together, enforcing sequential execution (and thus avoiding a resource conflict), but allowing the order of the two tasks to be decided upon at execution time. Experiments show that this method can give improvements to robustness during execution as well.

The last research question focuses on the workplace structure of parallel maintenance by independent teams. Ensuring team independence can be seen as an instance of the *decoupling problem* in multi-agent scheduling. Here, again, our focus is on robustness: we want disturbances in the execution of the schedule of one team to have minimal effects on the other teams in the workplace.

RQ4 How can our earlier work on ensuring robustness of entire schedules be applied to the *decoupling* of multi-agent schedules, in which we strive to decouple in such a way that disturbances in the schedule of one agent minimally affect the schedules of the other agents?

Our work in Chapter 3 in answering the first research question yielded a method to construct interval schedules, and as it turns out, the fact that these intervals are constructed to be *independent* enables such a schedule to serve as a *decoupling*, precisely because of this independence, thereby answering the last research question as well. Additionally, we show that our method to construct an interval schedule by definition leads to an *optimal* decoupling: an optimal decoupling has no loss of flexibility compared to the original schedule. As our decoupling is derived directly from the interval schedule constructed to achieve maximal flexibility it is intuitively evident that the decoupling is optimal.

6.2 Research goals and further research

Having discussed how the research questions posed at the end of Chapter 2 have been answered, we will now turn to the research goals formulated at the end of Chapter 1, and we will compare the answers to the research questions with the original goals.

The first research goal is to determine a way in which the scheduling process at NedTrain workshops can be automated.

RG1 The scheduling process employed in the NedTrain workshops relies on a large amount of slack to keep unexpected events under control. In addition, it is labor-intensive and inflexible. In what way can this process be at least partially automated? Note that automation alone is not sufficient: the process of constructing and adapting a schedule has to be quick enough to make it part of an interactive process, and it has to be flexible, to be able to adapt to the incomplete and changing nature of the environment.

This goal is already partially met by existing research discussed in Chapter 2, since the scheduling problem at NedTrain can be represented as a Resource-Constrained Project Scheduling Problem, for which Precedence Constraint Posting is a flexible process resulting in a flexible solution.

The requirement of having a flexible process is solved by this method, but as stated in the second research goal, the requirement of a flexible and robust solution needs further research:

RG2 Schedules, by their nature, prescribe a certain way to execute a set of tasks, giving rise to a form of rigidity. To be able to adapt to uncertain events it is desirable to reduce this rigidity.

a) As a primary goal, we want to have a schedule that prescribes task execution in a *flexible* way, such that we can adapt to changing events without needing to revise the schedule, and without having to resort to using large amounts of slack.

b) As a secondary goal, we want the resulting schedule to be *robust*: if the available flexibility is not enough to be able to adapt to some events, we want the unavoidable revisions we have to make to continue executing the needed tasks to be as minor as possible.

These goals are investigated with the first three questions. The answer to RQ1 gives a method to prescribe task execution in a non-rigid way, by assigning intervals to each task. This method is further investigated in answering RQ2, when we determine how the flexibility in a schedule can be used most effectively. And finally, RQ3 investigates a different method of achieving non-rigid prescription of task execution, by postponing some ordering decisions to execution time. Experiments performed using the proposed methods show that clear improvements

in robustness can be achieved when delays are present during the execution of a schedule.

The answer to in particular the second research question raises an important question for further research, as we show that there is a discrepancy between flexibility and robustness: our answer to the first research question gives an efficient method to determine the flexibility of a schedule a priori—but the experiments performed to determine the robustness show that high flexibility does not translate directly to high robustness. It would be advantageous to have a method by which the (expected) robustness of a solution could be measured directly, especially if such a metric could then be used as optimization target.

Our investigation of the third research question yielded the concept of task grouping, to also have a form of sequential flexibility. An interesting approach for future research would be to combine this concept with the construction of interval schedules, such as introduced in this thesis as well. This combination would give even more robustness during execution. Another limitation of the task grouping approach is that while a group task can contain an arbitrary number of tasks, they have to be executed sequentially. One could think of more advanced group task constructions, where such tasks can also contain parallel components. This would create more opportunities to use the grouping operation, at the expense of a more complex algorithm.

The last research goal describes the need for isolation between teams in the workplace that make use of shared resources.

RG3 The need to use shared resources to complete tasks, combined with uncertainties in the schedule negatively affects the independence of the teams in the workshop. Our goal is to construct schedules such that there is more isolation between the teams, in terms of disturbances in the execution of the schedule in one team not affecting the other teams.

This is translated to the decoupling problem, which is a well-known problem in scheduling research. The answer to RQ1 shows that it is possible to decouple a schedule without loss of flexibility: the flexibility is calculated using an interval schedule that maximizes flexibility, and we prove that such an interval schedule is in itself a decoupling.,

However, as already described in the preceding section, our decoupling strategy focuses on individual tasks instead of teams. This method works well if each executing agent is in control of a single task, but in real-life situations agents are often in control of a group of tasks. An adaptation of our method to only consider the constraints between such groups of tasks is fairly straightforward. The expected benefit of such an adaptation is that more flexibility can be allotted to these inter-agent constraints, leading to better isolation between the agents: a delay in one agent has a higher chance of being absorbed by inter-agent constraints if the available flexibility on these constraints is higher.

6.2.1 *The greater context*

When placing the results of our research in the greater context of the maintenance scheduling problem at NedTrain, some additional opportunities for future research can be identified.

Real-life problem sets
All experiments performed in this thesis use synthetic benchmark instances from the PSPLIB set of scheduling problems. This set of instances is constructed with variation in mind: it is constructed as a full factorial design using several instance parameters. The advantage is that this leads to varied instances, showing that our methods work in very different settings. The disadvantage is that there are relatively few instances per parameter combination. An important validation step would be the use of problem instances more closely resembling the actual scheduling problems present in the NedTrain workplaces.

Stochastic extensions
As has been noted at many points in this thesis, the uncertainty in the tasks to be executed is one of the major problems. The approach taken has been to hedge against the uncertainty by ensuring that the schedules contain enough flexibility to adapt to the uncertainty. If the amount of uncertainty is known beforehand, one of the obvious opportunities for extension is to make use of this information to better control schedule execution: the start time and duration of tasks can be extended with probabilistic information, enabling us to make informed predictions about schedule deadline violation probabilities. Mountakis (2015) presents a proof-of-concept of this idea, where techniques for timing analysis of complex digital integrated circuits are adapted to analyse schedules with tasks having probabilistic information.

Interactive constraint posting
Another important direction for future research in constraint posting methods is their use in interactive environments. Precedence constraint posting methods take a temporal and resource constrained network of tasks, and add temporal constraints to eliminate the resource constraints step by step—the end result is a purely temporal problem. The form of this temporal problem is identical to the temporal part of the problem used as input, which shows that constraint posting is well-suited for iterative or interactive environments: adding resource constraints or removing unsuited temporal constraints can be followed by another application of the algorithm resulting in a new solution. This thesis has not investigated the effects of such a scheme on the flexibility and robustness. Such interactive usage is important if our methods are to be used in a workshop—often, not all the constraints of the scheduling problem can be encoded in an RCPSP instance, and manual corrections might be necessary. Intuition suggests that having high flexibility would be of use in this case, but more research is needed to validate this claim.

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Summary

This thesis presents research on scheduling in an uncertain environment, which forms a part of the rolling stock life cycle logistics applied research and development program funded by Dutch railway industry companies. The focus therefore lies on scheduling of maintenance operations on rolling stock in the railway industry.

The first chapter describes some of the history of the Dutch railways, focusing on the rolling stock used, and it introduces the context in which NedTrain, a major Dutch rolling stock maintenance company, operates. While maintenance of rolling stock is not a new problem, recent changes in the field are identified in this chapter which introduce new challenges: the first is the declining involvement of maintenance experts in the procurement of new rolling stock, the second is the on-going demand for increasing efficiency. Based on this discussion, the goal of this thesis is to devise a method with which schedules can be created based on a *flexible process*. The schedules resulting from this method are to be both *flexible*, meaning that they can be adapted easily, and *robust*, meaning that they are resilient to the effects of uncertain events. Lastly, our goal is to create the schedules in such a way that the effects of disturbances in one team have little or no effect on any other teams in the same workshop.

Chapter 2 examines existing research in the context of the formulated research goals. It is shown that the well-known Resource-Constrained Project Scheduling Problem (RCPSP) can be mapped to the problem of scheduling maintenance tasks on trains in the NedTrain workshops. The most important methods to solve the RCPSP are discussed, among which are many exact methods. Both the scale of the problem and the requirement of having a flexible schedule instead of a fixed time assignment for the tasks prohibit using these methods. Among the heuristic methods discussed, the Precedence Constraint Posting method turns out to be an especially good fit: it is able to solve large-sized problems very quickly, and it uses a Simple Temporal Network (STN) as a way of representing many different solutions to the original problem, offering a lot of flexibility. Based on the research discussed, research questions are formulated to answer the research goals.

- The first question asks how to define criteria encapsulating the notion of

flexibility as posed by our research goals, and how to measure the flexibility of a schedule.

- The second question asks how we can make the best use of available flexibility to attain robustness.

- The third question asks if there is also a way to extend the concept of temporal flexibility to include a form of sequential flexibility.

- The last question asks how we can use our work on robustness to decouple schedules, ensuring that disturbances in the schedule for one agent have no impact on any other agents.

The third chapter investigates criteria corresponding to our intuitive notion of flexibility in scheduling. The concept of *interval schedules* serves as a basis for this investigation: the idea is that each task in the schedule is assigned an interval from which its start time can be picked, and the length of all these intervals serves as a representation for the flexibility of the schedule. We show that flexibility measures in existing research lead to an over-estimation of the total available flexibility, because dependencies between tasks are not taken into account properly: simply adding the interval sizes for two dependent tasks causes over-estimation, because the picked time for one task can reduce the interval size for another task. Our first major contribution is an independence requirement for the creation of these intervals: we state that intervals should be constructed in such a way that any choice in one interval has no impact on the available choices in any of the other intervals. Our second major contribution is a method to actually construct such intervals, which results in a valid interval schedule for an STN, allowing efficient schedule execution under dynamic circumstances. Additionally, we note that an interval schedule can also serve as a mechanism to decouple a schedule, giving a partial answer to the last research question.

In the fourth chapter, we investigate how to make use of the available flexibility to ensure that schedules are robust. We show that having a maximal total amount of flexibility does not imply that the schedule is as robust as possible: maximization might lead to a very skewed distribution of flexibility over the schedule. Different ways of distributing flexibility over a schedule are proposed and analyzed in an experimental setting. From the experiments it can be clearly concluded that sacrificing some of the total flexibility to improve the distribution of flexibility can have a positive influence on the robustness of the schedule. We also propose a different maximization strategy: one which maximizes the minimum of the interval sizes for the schedule. This strategy is shown to work especially well for small delays.

So far we only discussed strategies concerning temporal flexibility, i.e., those in which start times remain flexible instead of fixed. Chapter 5 introduces a novel representation of solutions for planning problems in which not all ordering decisions needed to avoid resource contention are made in advance. This is achieved by grouping tasks which need to be executed sequentially together, in

such a way that either ordering of the tasks remains feasible during execution time. An analysis using simulation experiments shows that this technique results in higher robustness, at the cost of somewhat lower resource utilization. The proposed algorithm to construct such schedules offers limited control over the size of the grouped tasks. The experiments show a slightly counter-intuitive results: schedules with larger groups offer lower robustness than those with smaller groups. A plausible explanation is the fact that delays can occur at any place in a schedule. Having multiple smaller groups instead of a few very large ones increases the chance that such a delay can be compensated for using a task group.

Samenvatting

Dit proefschrift bevat onderzoek naar planning in een onzekere omgeving, en maakt onderdeel uit van het "rolling stock life cycle logistics applied research and development program", gefinancierd door bedrijven in de Nederlandse spoorsector. Het accent ligt derhalve op het plannen van de onderhoudswerkzaamheden aan rollend materieel in de spoorwegindustrie.

Het eerste hoofdstuk beschrijft een deel van de geschiedenis van de Nederlandse spoorwegen, gaat in op het gebruikte materieel en beschrijft de context waarbinnen NedTrain, één van de belangrijkste onderhoudsbedrijven van Nederlands spoorwegmaterieel, functioneert. Ondanks het feit dat het onderhoud van rollend materieel geen nieuw probleem is, worden in dit hoofdstuk recente veranderingen geïdentificeerd die leiden tot nieuwe uitdagingen: de eerste is de afnemende betrokkenheid van onderhoudsexperts bij de aankoop van nieuwe treinen, de tweede is de alsmaar toenemende wens naar het verhogen van de efficiëntie. Het doel van dit proefschrift is derhalve het verzinnen van een methode waarmee planningen kunnen worden gemaakt op basis van een *flexibel proces*. De planningen die deze methode oplevert moeten zelf zowel *flexibel* als *robuust* zijn, wat betekent dat ze makkelijk kunnen worden aangepast, maar ook dat ze bestand zijn tegen de effecten van onvoorziene gebeurtenissen. Bovendien moeten deze planningen een zodanige structuur hebben dat de effecten van verstoringen in het ene team weinig of geen effect hebben op andere teams in de werkplaats.

Hoofdstuk 2 behandelt bestaand onderzoek in samenhang met de geformuleerde onderzoeksdoelen. Er wordt aangetoond dat het welbekende "Resource-Constrained Project Scheduling Problem" (RCPSP, projectplannen met beperkt aanwezige hulpbronnen) goed correspondeert met het probleem van het plannen van onderhoudstaken op treinen binnen de werkplaatsen van NedTrain. De belangrijkste methoden voor het oplossen van het RCPSP worden besproken. Een aantal van deze methoden zijn exact; echter, dergelijke methoden zijn niet inzetbaar vanwege de schaal van het probleem en de eis voor het hebben van een flexibele planning in plaats van vaste starttijden voor de diverse taken. Onder de methoden op basis van heuristieken blijkt de "Precedence Constraint Posting" methode (een methode op basis van het incrementeel toevoegen van ordeningsrelaties) bijzonder goed te passen bij deze eisen: zij is in staat proble-

men van grote afmeting zeer snel op te lossen, en zij maakt gebruik van een Simple Temporal Network (STN) als methode om een scala aan verschillende oplossingen voor het originele probleem te bevatten, wat een grote mate van flexibiliteit oplevert. Op basis van het besproken onderzoek worden vervolgens onderzoeksvragen geformuleerd om te beantwoorden aan de onderzoeksdoelen.

- De eerste vraag is hoe criteria kunnen worden gedefiniëerd die de intuïtieve notie achter flexibiliteit omvatten, zoals gesteld door de onderzoeksdoelen, en hoe de flexibiliteit van een planning kan worden gemeten.

- De tweede vraag is hoe we het beste gebruik kunnen maken van bovengenoemde flexibiliteit om tot een robuuste planning te komen.

- De derde vraag is op welke wijze het idee van flexibiliteit in de tijd kan worden uitgebreid om ook een vorm van flexibiliteit qua ordening te omvatten.

- De laatste vraag is hoe het resultaat van robuustheid kan worden ingezet voor het "ontkoppelen" van planningen, waarbij het doel is dat verstoringen in de planning voor de ene partij geen effect hebben op andere deelnemende partijen.

In het derde hoofdstuk wordt ingegaan op de criteria die corresponderen met de intuïtieve notie van flexibiliteit in planning. Het concept van een *intervalplanning* fungeert als basis voor het onderzoek: het idee is dat elke taak in de planning een interval krijgt toegekend waaruit de starttijd kan worden gekozen. De lengte van al deze intervallen geeft een maatstaf voor de flexibiliteit van de planning. Er wordt aangetoond dat flexibiliteitsmaten uit bestaand onderzoek leiden tot een overschatting van de totaal beschikbare flexibiliteit. De oorzaak is dat afhankelijkheden tussen taken onvoldoende in beschouwing worden genomen: het simpelweg optellen van de intervallengtes van twee afhankelijke taken leidt tot een overschatting, aangezien de gekozen starttijd voor de ene taak het beschikbare interval van de andere taak kan beïnvloeden. De eerste bijdrage is een onafhankelijkheidseis voor het opstellen van deze intervallen: de intervallen moeten worden gekozen op zodanige wijze dat een keuze in één interval geen invloed heeft op de beschikbare keuzes in willekeurig welk ander interval. De tweede bijdrage is een methode om daadwerkelijk dergelijke intervallen op te stellen, hetgeen resulteert in een geldige intervalplanning voor een STN, waarmee efficiënte uitvoer van taken bewerkstelligd kan worden, ook in een dynamische omgeving. Ten laatste wordt opgemerkt dat een intervalplanning ook als mechanisme kan dienen om een planning te ontkoppelen, hetgeen ten dele de laatste onderzoeksvraag beantwoordt.

In het vierde hoofdstuk wordt onderzocht hoe gebruik te maken van de beschikbare flexibiliteit zodanig dat planningen robuust zijn. Er wordt getoond dat het hebben van een maximale hoeveelheid flexibiliteit niet impliceert dat een planning zo robuust als mogelijk is: maximaliseren kan leiden tot een zeer scheve verdeling van flexibiliteit over de planning. Verschillende manieren om

flexibiliteit te verdelen over een planning worden geïntroduceerd en geanalyseerd door middel van experimenten. Uit de experimenten kan worden geconcludeerd dat het opofferen van een deel van de totale flexibiliteit ter bevordering van de verdeling een positieve invloed kan hebben op de robuustheid van een planning. Ook wordt een nieuwe strategie van maximalisatie voorgesteld: één waarbij de minimaal voorkomende intervalgrootte wordt gemaximaliseerd. Deze strategie werkt met name goed bij het voorkomen van kleine vertragingen.

Tot dusver zijn slechts strategieën besproken die betrekking hebben op temporele flexibiliteit, dat wil zeggen, strategieën waarbij de starttijden van taken flexibel zijn in plaats van vast. Hoofdstuk 5 introduceert een nieuwe wijze om oplossingen van planningsproblemen vast te leggen waarbij niet alle ordeningsbeslissingen ter voorkoming van overgebruik van hulpbronnen volledig vast liggen. Dit wordt bereikt door het groeperen van taken die achtereenvolgend moeten worden uitgevoerd, op zodanige manier dat beide volgordes nog mogelijk zijn tijdens de uitvoer van de planning. Een analyse van deze techniek met behulp van simulatie toont aan dat op deze wijze een grotere robuustheid wordt verkregen, ten koste van een wat lagere benutting van de hulpbronnen. Het voorgestelde algoritme biedt beperkte controle op de grootte van de gegroepeerde taken. De experimenten geven een ietwat tegen-intuïtief resultaat: planningen met grotere groepen resulteren in een lagere robuustheid dan planningen met kleinere groepen. Een verklaring voor dit feit is dat verstoringen op elke willekeurige plaats in de planning op kunnen treden. Het hebben van meerdere kleine groepen in plaats van slechts enkele grotere vergroot de kans dat een dergelijke vertraging opgevangen kan worden met de flexibiliteit in een groep.

Curriculum vitæ

Michel Wilson was born in Rozenburg on Saturday, April 18, 1981. From 1994 to 1997 he received secondary education at OSG 'De Ring van Putten' in Spijkenisse, and from 1997 to 1999 at CSG 'Blaise Pascal', also in Spijkenisse, where he obtained his VWO diploma. Starting in the fall of 1999, he attended six months of community college at Green River Community College, in Auburn, WA, USA.

In the fall of 2000, Michel started his education at Delft University of Technology, beginning with a bachelor in Technical Informatics, followed by a master in Computer Science, specializing in Software Technology. As a master student, he researched the application of spectrum-based fault localization techniques to hardware systems, supervised by prof.dr.ir. A.J.C. van Gemund, resulting in an MSc degree in January 2010.

Michel began his work as PhD researcher in February 2010, at the Algorithmics group of the faculty of Electrical Engineering, Mathematics and Computer Science, also at Delft University of Technology, under the supervision of prof.dr. C. Witteveen and dr. T.B. Klos. In this period, he published five peer-reviewed papers as principal author, one of which received an IJCAI Distinguished Paper Award. During his time as PhD researcher, Michel was also involved in the teaching of courses offered by the Algorithmics group, and he supervised three MSc students. In May 2014 he started work at West IT Solutions B.V. in Delft.

In his spare time, Michel likes to tinker around with electronics and small embedded devices. Starting as a child, when his parents gave him an old manual SLR camera, he has been interested in photography. As a child he was also already interested in trains and railroads. This interest was revived during his time as PhD researcher, in which he enjoyed the collaboration with NedTrain, and the opportunities to visit maintenance workshops.

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13 Rutger Rienks (UT) *Meetings in Smart Environments; Implications of Progressing Technology*
14 Niek Bergboer (UM) *Context-Based Image Analysis*
15 Joyca Lacroix (UM) *NIM: a Situated Computational Memory Model*
16 Davide Grossi (UU) *Designing Invisible Handcuffs. Formal investigations in Institutions and Organizations for Multi-agent Systems*
17 Theodore Charitos (UU) *Reasoning with Dynamic Networks in Practice*
18 Bart Orriens (UvT) *On the development an management of adaptive business collaborations*
19 David Levy (UM) *Intimate relationships with artificial partners*
20 Slinger Jansen (UU) *Customer Configuration Updating in a Software Supply Network*
21 Karianne Vermaas (UU) *Fast diffusion and broadening use: A research on residential adoption and usage of broadband internet in the Netherlands between 2001 and 2005*
22 Zlatko Zlatev (UT) *Goal-oriented design of value and process models from patterns*
23 Peter Barna (TUE) *Specification of Application Logic in Web Information Systems*
24 Georgina Ramírez Camps (CWI) *Structural Features in XML Retrieval*
25 Joost Schalken (VU) *Empirical Investigations in Software Process Improvement*

2008

1 Katalin Boer-Sorbán (EUR) *Agent-Based Simulation of Financial Markets: A modular, continuous-time approach*
2 Alexei Sharpanskykh (VU) *On Computer-Aided Methods for Modeling and Analysis of Organizations*
3 Vera Hollink (UVA) *Optimizing hierarchical menus: a usage-based approach*
4 Ander de Keijzer (UT) *Management of Uncertain Data—towards unattended integration*
5 Bela Mutschler (UT) *Modeling and simulating causal dependencies on process-aware information systems from a cost perspective*
6 Arjen Hommersom (RUN) *On the Application of Formal Methods to Clinical Guidelines, an Artificial Intelligence Perspective*
7 Peter van Rosmalen (OU) *Supporting the tutor in the design and support of adaptive e-learning*
8 Janneke Bolt (UU) *Bayesian Networks: Aspects of Approximate Inference*
9 Christof van Nimwegen (UU) *The paradox of the guided user: assistance can be counter-effective*
10 Wauter Bosma (UT) *Discourse oriented summarization*
11 Vera Kartseva (VU) *Designing Controls for Network Organizations: A Value-Based Approach*
12 Jozsef Farkas (RUN) *A Semiotically Oriented Cognitive Model of Knowledge Representation*
13 Caterina Carraciolo (UVA) *Topic Driven Access to Scientific Handbooks*
14 Arthur van Bunningen (UT) *Context-Aware Querying; Better Answers with Less Effort*
15 Martijn van Otterlo (UT) *The Logic of Adaptive Behavior: Knowledge Representation and Algorithms for the Markov Decision Process Framework in First-Order Domains.*
16 Henriette van Vugt (VU) *Embodied agents from a user's perspective*
17 Martin Op 't Land (TUD) *Applying Architecture and Ontology to the Splitting and Allying of Enterprises*
18 Guido de Croon (UM) *Adaptive Active Vision*
19 Henning Rode (UT) *From Document to Entity Retrieval: Improving Precision and Performance of Focused Text Search*
20 Rex Arendsen (UVA) *Geen bericht, goed bericht. Een onderzoek naar de effecten van de introductie van elektronisch berichtenverkeer met de overheid op de administratieve lasten van bedrijven.*
21 Krisztian Balog (UVA) *People Search in the Enterprise*
22 Henk Koning (UU) *Communication of IT-Architecture*
23 Stefan Visscher (UU) *Bayesian network models for the management of ventilator-associated pneumonia*
24 Zharko Aleksovski (VU) *Using background knowledge in ontology matching*

25 Geert Jonker (UU) *Efficient and Equitable Exchange in Air Traffic Management Plan Repair using Spender-signed Currency*
26 Marijn Huijbregts (UT) *Segmentation, Diarization and Speech Transcription: Surprise Data Unraveled*
27 Hubert Vogten (OU) *Design and Implementation Strategies for IMS Learning Design*
28 Ildiko Flesch (RUN) *On the Use of Independence Relations in Bayesian Networks*
29 Dennis Reidsma (UT) *Annotations and Subjective Machines—Of Annotators, Embodied Agents, Users, and Other Humans*
30 Wouter van Atteveldt (VU) *Semantic Network Analysis: Techniques for Extracting, Representing and Querying Media Content*
31 Loes Braun (UM) *Pro-Active Medical Information Retrieval*
32 Trung H. Bui (UT) *Toward Affective Dialogue Management using Partially Observable Markov Decision Processes*
33 Frank Terpstra (UVA) *Scientific Workflow Design; theoretical and practical issues*
34 Jeroen de Knijf (UU) *Studies in Frequent Tree Mining*
35 Ben Torben Nielsen (UvT) *Dendritic morphologies: function shapes structure*

2009

1 Rasa Jurgelenaite (RUN) *Symmetric Causal Independence Models*
2 Willem Robert van Hage (VU) *Evaluating Ontology-Alignment Techniques*
3 Hans Stol (UvT) *A Framework for Evidence-based Policy Making Using IT*
4 Josephine Nabukenya (RUN) *Improving the Quality of Organisational Policy Making using Collaboration Engineering*
5 Sietse Overbeek (RUN) *Bridging Supply and Demand for Knowledge Intensive Tasks—Based on Knowledge, Cognition, and Quality*
6 Muhammad Subianto (UU) *Understanding Classification*
7 Ronald Poppe (UT) *Discriminative Vision-Based Recovery and Recognition of Human Motion*
8 Volker Nannen (VU) *Evolutionary Agent-Based Policy Analysis in Dynamic Environments*
9 Benjamin Kanagwa (RUN) *Design, Discovery and Construction of Service-oriented Systems*
10 Jan Wielemaker (UVA) *Logic programming for knowledge-intensive interactive applications*
11 Alexander Boer (UVA) *Legal Theory, Sources of Law & the Semantic Web*
12 Peter Massuthe (TUE, Humboldt-Universitaet zu Berlin) *Operating Guidelines for Services*
13 Steven de Jong (UM) *Fairness in Multi-Agent Systems*
14 Maksym Korotkiy (VU) *From ontology-enabled services to service-enabled ontologies (making ontologies work in e-science with ONTO-SOA)*
15 Rinke Hoekstra (UVA) *Ontology Representation—Design Patterns and Ontologies that Make Sense*
16 Fritz Reul (UvT) *New Architectures in Computer Chess*
17 Laurens van der Maaten (UvT) *Feature Extraction from Visual Data*
18 Fabian Groffen (CWI) *Armada, An Evolving Database System*
19 Valentin Robu (CWI) *Modeling Preferences, Strategic Reasoning and Collaboration in Agent-Mediated Electronic Markets*
20 Bob van der Vecht (UU) *Adjustable Autonomy: Controling Influences on Decision Making*
21 Stijn Vanderlooy (UM) *Ranking and Reliable Classification*
22 Pavel Serdyukov (UT) *Search For Expertise: Going beyond direct evidence*
23 Peter Hofgesang (VU) *Modelling Web Usage in a Changing Environment*
24 Annerieke Heuvelink (VUA) *Cognitive Models for Training Simulations*
25 Alex van Ballegooij (CWI) *"RAM: Array Database Management through Relational Mapping"*
26 Fernando Koch (UU) *An Agent-Based Model for the Development of Intelligent Mobile Services*
27 Christian Glahn (OU) *Contextual Support of social Engagement and Reflection on the Web*
28 Sander Evers (UT) *Sensor Data Management with Probabilistic Models*
29 Stanislav Pokraev (UT) *Model-Driven Semantic Integration of Service-Oriented Applications*
30 Marcin Zukowski (CWI) *Balancing vectorized query execution with bandwidth-optimized storage*
31 Sofiya Katrenko (UVA) *A Closer Look at Learning Relations from Text*
32 Rik Farenhorst (VU) and Remco de Boer (VU) *Architectural Knowledge Management: Supporting Architects and Auditors*
33 Khiet Truong (UT) *How Does Real Affect Affect Affect Recognition In Speech?*

34 Inge van de Weerd (UU) *Advancing in Software Product Management: An Incremental Method Engineering Approach*
35 Wouter Koelewijn (UL) *Privacy en Politiegegevens; Over geautomatiseerde normatieve informatie-uitwisseling*
36 Marco Kalz (OUN) *Placement Support for Learners in Learning Networks*
37 Hendrik Drachsler (OUN) *Navigation Support for Learners in Informal Learning Networks*
38 Riina Vuorikari (OU) *Tags and self-organisation: a metadata ecology for learning resources in a multilingual context*
39 Christian Stahl (TUE, Humboldt-Universitaet zu Berlin) *Service Substitution—A Behavioral Approach Based on Petri Nets*
40 Stephan Raaijmakers (UvT) *Multinomial Language Learning: Investigations into the Geometry of Language*
41 Igor Berezhnyy (UvT) *Digital Analysis of Paintings*
42 Toine Bogers (UvT) *Recommender Systems for Social Bookmarking*
43 Virginia Nunes Leal Franqueira (UT) *Finding Multi-step Attacks in Computer Networks using Heuristic Search and Mobile Ambients*
44 Roberto Santana Tapia (UT) *Assessing Business-IT Alignment in Networked Organizations*
45 Jilles Vreeken (UU) *Making Pattern Mining Useful*
46 Loredana Afanasiev (UvA) *Querying XML: Benchmarks and Recursion*

2010

1 Matthijs van Leeuwen (UU) *Patterns that Matter*
2 Ingo Wassink (UT) *Work flows in Life Science*
3 Joost Geurts (CWI) *A Document Engineering Model and Processing Framework for Multimedia documents*
4 Olga Kulyk (UT) *Do You Know What I Know? Situational Awareness of Co-located Teams in Multidisplay Environments*
5 Claudia Hauff (UT) *Predicting the Effectiveness of Queries and Retrieval Systems*
6 Sander Bakkes (UvT) *Rapid Adaptation of Video Game AI*
7 Wim Fikkert (UT) *Gesture interaction at a Distance*
8 Krzysztof Siewicz (UL) *Towards an Improved Regulatory Framework of Free Software. Protecting user freedoms in a world of software communities and eGovernments*
9 Hugo Kielman (UL) *A Politiele gegevensverwerking en Privacy, Naar een effectieve waarborging*
10 Rebecca Ong (UL) *Mobile Communication and Protection of Children*
11 Adriaan Ter Mors (TUD) *The world according to MARP: Multi-Agent Route Planning*
12 Susan van den Braak (UU) *Sensemaking software for crime analysis*
13 Gianluigi Folino (RUN) *High Performance Data Mining using Bio-inspired techniques*
14 Sander van Splunter (VU) *Automated Web Service Reconfiguration*
15 Lianne Bodenstaff (UT) *Managing Dependency Relations in Inter-Organizational Models*
16 Sicco Verwer (TUD) *Efficient Identification of Timed Automata, theory and practice*
17 Spyros Kotoulas (VU) *Scalable Discovery of Networked Resources: Algorithms, Infrastructure, Applications*
18 Charlotte Gerritsen (VU) *Caught in the Act: Investigating Crime by Agent-Based Simulation*
19 Henriette Cramer (UvA) *People's Responses to Autonomous and Adaptive Systems*
20 Ivo Swartjes (UT) *Whose Story Is It Anyway? How Improv Informs Agency and Authorship of Emergent Narrative*
21 Harold van Heerde (UT) *Privacy-aware data management by means of data degradation*
22 Michiel Hildebrand (CWI) *End-user Support for Access to Heterogeneous Linked Data*
23 Bas Steunebrink (UU) *The Logical Structure of Emotions*
24 Dmytro Tykhonov *Designing Generic and Efficient Negotiation Strategies*
25 Zulfiqar Ali Memon (VU) *Modelling Human-Awareness for Ambient Agents: A Human Mindreading Perspective*
26 Ying Zhang (CWI) *XRPC: Efficient Distributed Query Processing on Heterogeneous XQuery Engines*
27 Marten Voulon (UL) *Automatisch contracteren*
28 Arne Koopman (UU) *Characteristic Relational Patterns*

29 Stratos Idreos (CWI) *Database Cracking: Towards Auto-tuning Database Kernels*
30 Marieke van Erp (UvT) *Accessing Natural History—Discoveries in data cleaning, structuring, and retrieval*
31 Victor de Boer (UVA) *Ontology Enrichment from Heterogeneous Sources on the Web*
32 Marcel Hiel (UvT) *An Adaptive Service Oriented Architecture: Automatically solving Interoperability Problems*
33 Robin Aly (UT) *Modeling Representation Uncertainty in Concept-Based Multimedia Retrieval*
34 Teduh Dirgahayu (UT) *Interaction Design in Service Compositions*
35 Dolf Trieschnigg (UT) *Proof of Concept: Concept-based Biomedical Information Retrieval*
36 Jose Janssen (OU) *Paving the Way for Lifelong Learning; Facilitating competence development through a learning path specification*
37 Niels Lohmann (TUE) *Correctness of services and their composition*
38 Dirk Fahland (TUE) *From Scenarios to components*
39 Ghazanfar Farooq Siddiqui (VU) *Integrative modeling of emotions in virtual agents*
40 Mark van Assem (VU) *Converting and Integrating Vocabularies for the Semantic Web*
41 Guillaume Chaslot (UM) *Monte-Carlo Tree Search*
42 Sybren de Kinderen (VU) *Needs-driven service bundling in a multi-supplier setting—the computational e3-service approach*
43 Peter van Kranenburg (UU) *A Computational Approach to Content-Based Retrieval of Folk Song Melodies*
44 Pieter Bellekens (TUE) *An Approach towards Context-sensitive and User-adapted Access to Heterogeneous Data Sources, Illustrated in the Television Domain*
45 Vasilios Andrikopoulos (UvT) *A theory and model for the evolution of software services*
46 Vincent Pijpers (VU) *e3alignment: Exploring Inter-Organizational Business-ICT Alignment*
47 Chen Li (UT) *Mining Process Model Variants: Challenges, Techniques, Examples*
48 *Withdrawn*
49 Jahn-Takeshi Saito (UM) *Solving difficult game positions*
50 Bouke Huurnink (UVA) *Search in Audiovisual Broadcast Archives*
51 Alia Khairia Amin (CWI) *Understanding and supporting information seeking tasks in multiple sources*
52 Peter-Paul van Maanen (VU) *Adaptive Support for Human-Computer Teams: Exploring the Use of Cognitive Models of Trust and Attention*
53 Edgar Meij (UVA) *Combining Concepts and Language Models for Information Access*

2011

1 Botond Cseke (RUN) *Variational Algorithms for Bayesian Inference in Latent Gaussian Models*
2 Nick Tinnemeier (UU) *Organizing Agent Organizations. Syntax and Operational Semantics of an Organization-Oriented Programming Language*
3 Jan Martijn van der Werf (TUE) *Compositional Design and Verification of Component-Based Information Systems*
4 Hado van Hasselt (UU) *Insights in Reinforcement Learning; Formal analysis and empirical evaluation of temporal-difference learning algorithms*
5 Base van der Raadt (VU) *Enterprise Architecture Coming of Age—Increasing the Performance of an Emerging Discipline.*
6 Yiwen Wang (TUE) *Semantically-Enhanced Recommendations in Cultural Heritage*
7 Yujia Cao (UT) *Multimodal Information Presentation for High Load Human Computer Interaction*
8 Nieske Vergunst (UU) *BDI-based Generation of Robust Task-Oriented Dialogues*
9 Tim de Jong (OU) *Contextualised Mobile Media for Learning*
10 Bart Bogaert (UvT) *Cloud Content Contention*
11 Dhaval Vyas (UT) *Designing for Awareness: An Experience-focused HCI Perspective*
12 Carmen Bratosin (TUE) *Grid Architecture for Distributed Process Mining*
13 Xiaoyu Mao (UvT) *Airport under Control. Multiagent Scheduling for Airport Ground Handling*
14 Milan Lovric (EUR) *Behavioral Finance and Agent-Based Artificial Markets*
15 Marijn Koolen (UvA) *The Meaning of Structure: the Value of Link Evidence for Information Retrieval*
16 Maarten Schadd (UM) *Selective Search in Games of Different Complexity*
17 Jiyin He (UVA) *Exploring Topic Structure: Coherence, Diversity and Relatedness*

18 Mark Ponsen (UM) *Strategic Decision-Making in complex games*
19 Ellen Rusman (OU) *The Mind ' s Eye on Personal Profiles*
20 Qing Gu (VU) *Guiding service-oriented software engineering—A view-based approach*
21 Linda Terlouw (TUD) *Modularization and Specification of Service-Oriented Systems*
22 Junte Zhang (UVA) *System Evaluation of Archival Description and Access*
23 Wouter Weerkamp (UVA) *Finding People and their Utterances in Social Media*
24 Herwin van Welbergen (UT) *Behavior Generation for Interpersonal Coordination with Virtual Humans On Specifying, Scheduling and Realizing Multimodal Virtual Human Behavior*
25 Syed Waqar ul Qounain Jaffry (VU)) *Analysis and Validation of Models for Trust Dynamics*
26 Matthijs Aart Pontier (VU) *Virtual Agents for Human Communication—Emotion Regulation and Involvement-Distance Trade-Offs in Embodied Conversational Agents and Robots*
27 Aniel Bhulai (VU) *Dynamic website optimization through autonomous management of design patterns*
28 Rianne Kaptein (UVA) *Effective Focused Retrieval by Exploiting Query Context and Document Structure*
29 Faisal Kamiran (TUE) *Discrimination-aware Classification*
30 Egon van den Broek (UT) *Affective Signal Processing (ASP): Unraveling the mystery of emotions*
31 Ludo Waltman (EUR) *Computational and Game-Theoretic Approaches for Modeling Bounded Rationality*
32 Nees-Jan van Eck (EUR) *Methodological Advances in Bibliometric Mapping of Science*
33 Tom van der Weide (UU) *Arguing to Motivate Decisions*
34 Paolo Turrini (UU) *Strategic Reasoning in Interdependence: Logical and Game-theoretical Investigations*
35 Maaike Harbers (UU) *Explaining Agent Behavior in Virtual Training*
36 Erik van der Spek (UU) *Experiments in serious game design: a cognitive approach*
37 Adriana Burlutiu (RUN) *Machine Learning for Pairwise Data, Applications for Preference Learning and Supervised Network Inference*
38 Nyree Lemmens (UM) *Bee-inspired Distributed Optimization*
39 Joost Westra (UU) *Organizing Adaptation using Agents in Serious Games*
40 Viktor Clerc (VU) *Architectural Knowledge Management in Global Software Development*
41 Luan Ibraimi (UT) *Cryptographically Enforced Distributed Data Access Control*
42 Michal Sindlar (UU) *Explaining Behavior through Mental State Attribution*
43 Henk van der Schuur (UU) *Process Improvement through Software Operation Knowledge*
44 Boris Reuderink (UT) *Robust Brain-Computer Interfaces*
45 Herman Stehouwer (UvT) *Statistical Language Models for Alternative Sequence Selection*
46 Beibei Hu (TUD) *Towards Contextualized Information Delivery: A Rule-based Architecture for the Domain of Mobile Police Work*
47 Azizi Bin Ab Aziz (VU) *Exploring Computational Models for Intelligent Support of Persons with Depression*
48 Mark Ter Maat (UT) *Response Selection and Turn-taking for a Sensitive Artificial Listening Agent*
49 Andreea Niculescu (UT) *Conversational interfaces for task-oriented spoken dialogues: design aspects influencing interaction quality*

2012

1 Terry Kakeeto (UvT) *Relationship Marketing for SMEs in Uganda*
2 Muhammad Umair (VU) *Adaptivity, emotion, and Rationality in Human and Ambient Agent Models*
3 Adam Vanya (VU) *Supporting Architecture Evolution by Mining Software Repositories*
4 Jurriaan Souer (UU) *Development of Content Management System-based Web Applications*
5 Marijn Plomp (UU) *Maturing Interorganisational Information Systems*
6 Wolfgang Reinhardt (OU) *Awareness Support for Knowledge Workers in Research Networks*
7 Rianne van Lambalgen (VU) *When the Going Gets Tough: Exploring Agent-based Models of Human Performance under Demanding Conditions*
8 Gerben de Vries (UVA) *Kernel Methods for Vessel Trajectories*
9 Ricardo Neisse (UT) *Trust and Privacy Management Support for Context-Aware Service Platforms*

10 David Smits (TUE) *Towards a Generic Distributed Adaptive Hypermedia Environment*
11 J.C.B. Rantham Prabhakara (TUE) *Process Mining in the Large: Preprocessing, Discovery, and Diagnostics*
12 Kees van der Sluijs (TUE) *Model Driven Design and Data Integration in Semantic Web Information Systems*
13 Suleman Shahid (UvT) *Fun and Face: Exploring non-verbal expressions of emotion during playful interactions*
14 Evgeny Knutov (TUE) *Generic Adaptation Framework for Unifying Adaptive Web-based Systems*
15 Natalie van der Wal (VU) *Social Agents. Agent-Based Modelling of Integrated Internal and Social Dynamics of Cognitive and Affective Processes.*
16 Fiemke Both (VU) *Helping people by understanding them—Ambient Agents supporting task execution and depression treatment*
17 Amal Elgammal (UvT) *Towards a Comprehensive Framework for Business Process Compliance*
18 Eltjo Poort (VU) *Improving Solution Architecting Practices*
19 Helen Schonenberg (TUE) *What's Next? Operational Support for Business Process Execution*
20 Ali Bahramisharif (RUN) *Covert Visual Spatial Attention, a Robust Paradigm for Brain-Computer Interfacing*
21 Roberto Cornacchia (TUD) *Querying Sparse Matrices for Information Retrieval*
22 Thijs Vis (UvT) *Intelligence, politie en veiligheidsdienst: verenigbare grootheden?*
23 Christian Muehl (UT) *Toward Affective Brain-Computer Interfaces: Exploring the Neurophysiology of Affect during Human Media Interaction*
24 Laurens van der Werff (UT) *Evaluation of Noisy Transcripts for Spoken Document Retrieval*
25 Silja Eckartz (UT) *Managing the Business Case Development in Inter-Organizational IT Projects: A Methodology and its Application*
26 Emile de Maat (UVA) *Making Sense of Legal Text*
27 Hayrettin Gurkok (UT) *Mind the Sheep! User Experience Evaluation & Brain-Computer Interface Games*
28 Nancy Pascall (UvT) *Engendering Technology Empowering Women*
29 Almer Tigelaar (UT) *Peer-to-Peer Information Retrieval*
30 Alina Pommeranz (TUD) *Designing Human-Centered Systems for Reflective Decision Making*
31 Emily Bagarukayo (RUN) *A Learning by Construction Approach for Higher Order Cognitive Skills Improvement, Building Capacity and Infrastructure*
32 Wietske Visser (TUD) *Qualitative multi-criteria preference representation and reasoning*
33 Rory Sie (OUN) *Coalitions in Cooperation Networks (COCOON)*
34 Pavol Jancura (RUN) *Evolutionary analysis in PPI networks and applications*
35 Evert Haasdijk (VU) *Never Too Old To Learn—On-line Evolution of Controllers in Swarm- and Modular Robotics*
36 Denis Ssebugwawo (RUN) *Analysis and Evaluation of Collaborative Modeling Processes*
37 Agnes Nakakawa (RUN) *A Collaboration Process for Enterprise Architecture Creation*
38 Selmar Smit (VU) *Parameter Tuning and Scientific Testing in Evolutionary Algorithms*
39 Hassan Fatemi (UT) *Risk-aware design of value and coordination networks*
40 Agus Gunawan (UvT) *Information Access for SMEs in Indonesia*
41 Sebastian Kelle (OU) *Game Design Patterns for Learning*
42 Dominique Verpoorten (OU) *Reflection Amplifiers in self-regulated Learning*
43 *Withdrawn*
44 Anna Tordai (VU) *On Combining Alignment Techniques*
45 Benedikt Kratz (UvT) *A Model and Language for Business-aware Transactions*
46 Simon Carter (UVA) *Exploration and Exploitation of Multilingual Data for Statistical Machine Translation*
47 Manos Tsagkias (UVA) *Mining Social Media: Tracking Content and Predicting Behavior*
48 Jorn Bakker (TUE) *Handling Abrupt Changes in Evolving Time-series Data*
49 Michael Kaisers (UM) *Learning against Learning—Evolutionary dynamics of reinforcement learning algorithms in strategic interactions*
50 Steven van Kervel (TUD) *Ontologogy driven Enterprise Information Systems Engineering*
51 Jeroen de Jong (TUD) *Heuristics in Dynamic Sceduling; a practical framework with a case study in elevator dispatching*

2013

1 Viorel Milea (EUR) *News Analytics for Financial Decision Support*
2 Erietta Liarou (CWI) *MonetDB/DataCell: Leveraging the Column-store Database Technology for Efficient and Scalable Stream Processing*
3 Szymon Klarman (VU) *Reasoning with Contexts in Description Logics*
4 Chetan Yadati (TUD) *Coordinating autonomous planning and scheduling*
5 Dulce Pumareja (UT) *Groupware Requirements Evolutions Patterns*
6 Romulo Goncalves (CWI) *The Data Cyclotron: Juggling Data and Queries for a Data Warehouse Audience*
7 Giel van Lankveld (UvT) *Quantifying Individual Player Differences*
8 Robbert-Jan Merk (VU) *Making enemies: cognitive modeling for opponent agents in fighter pilot simulators*
9 Fabio Gori (RUN) *Metagenomic Data Analysis: Computational Methods and Applications*
10 Jeewanie Jayasinghe Arachchige (UvT) *A Unified Modeling Framework for Service Design.*
11 Evangelos Pournaras (TUD) *Multi-level Reconfigurable Self-organization in Overlay Services*
12 Marian Razavian (VU) *Knowledge-driven Migration to Services*
13 Mohammad Safiri (UT) *Service Tailoring: User-centric creation of integrated IT-based homecare services to support independent living of elderly*
14 Jafar Tanha (UVA) *Ensemble Approaches to Semi-Supervised Learning Learning*
15 Daniel Hennes (UM) *Multiagent Learning—Dynamic Games and Applications*
16 Eric Kok (UU) *Exploring the practical benefits of argumentation in multi-agent deliberation*
17 Koen Kok (VU) *The PowerMatcher: Smart Coordination for the Smart Electricity Grid*
18 Jeroen Janssens (UvT) *Outlier Selection and One-Class Classification*
19 Renze Steenhuizen (TUD) *Coordinated Multi-Agent Planning and Scheduling*
20 Katja Hofmann (UvA) *Fast and Reliable Online Learning to Rank for Information Retrieval*
21 Sander Wubben (UvT) *Text-to-text generation by monolingual machine translation*
22 Tom Claassen (RUN) *Causal Discovery and Logic*
23 Patricio de Alencar Silva (UvT) *Value Activity Monitoring*
24 Haitham Bou Ammar (UM) *Automated Transfer in Reinforcement Learning*
25 Agnieszka Anna Latoszek-Berendsen (UM) *Intention-based Decision Support. A new way of representing and implementing clinical guidelines in a Decision Support System*
26 Alireza Zarghami (UT) *Architectural Support for Dynamic Homecare Service Provisioning*
27 Mohammad Huq (UT) *Inference-based Framework Managing Data Provenance*
28 Frans van der Sluis (UT) *When Complexity becomes Interesting: An Inquiry into the Information eXperience*
29 Iwan de Kok (UT) *Listening Heads*
30 Joyce Nakatumba (TUE) *Resource-Aware Business Process Management: Analysis and Support*
31 Dinh Khoa Nguyen (UvT) *Blueprint Model and Language for Engineering Cloud Applications*
32 Kamakshi Rajagopal (OUN) *Networking For Learning; The role of Networking in a Lifelong Learner's Professional Development*
33 Qi Gao (TUD) *User Modeling and Personalization in the Microblogging Sphere*
34 Kien Tjin-Kam-Jet (UT) *Distributed Deep Web Search*
35 Abdallah El Ali (UvA) *Minimal Mobile Human Computer Interaction*
36 Than Lam Hoang (TUe) *Pattern Mining in Data Streams*
37 Dirk Börner (OUN) *Ambient Learning Displays*
38 Eelco den Heijer (VU) *Autonomous Evolutionary Art*
39 Joop de Jong (TUD) *A Method for Enterprise Ontology based Design of Enterprise Information Systems*
40 Pim Nijssen (UM) *Monte-Carlo Tree Search for Multi-Player Games*
41 Jochem Liem (UVA) *Supporting the Conceptual Modelling of Dynamic Systems: A Knowledge Engineering Perspective on Qualitative Reasoning*
42 Léon Planken (TUD) *Algorithms for Simple Temporal Reasoning*
43 Marc Bron (UVA) *Exploration and Contextualization through Interaction and Concepts*

2014

1 Nicola Barile (UU) *Studies in Learning Monotone Models from Data*
2 Fiona Tuliyano (RUN) *Combining System Dynamics with a Domain Modeling Method*

3 Sergio Raul Duarte Torres (UT) *Information Retrieval for Children: Search Behavior and Solutions*
4 Hanna Jochmann-Mannak (UT) *Websites for children: search strategies and interface design—Three studies on children's search performance and evaluation*
5 Jurriaan van Reijsen (UU) *Knowledge Perspectives on Advancing Dynamic Capability*
6 Damian Tamburri (VU) *Supporting Networked Software Development*
7 Arya Adriansyah (TUE) *Aligning Observed and Modeled Behavior*
8 Samur Araujo (TUD) *Data Integration over Distributed and Heterogeneous Data Endpoints*
9 Philip Jackson (UvT) *Toward Human-Level Artificial Intelligence: Representation and Computation of Meaning in Natural Language*
10 Ivan Salvador Razo Zapata (VU) *Service Value Networks*
11 Janneke van der Zwaan (TUD) *An Empathic Virtual Buddy for Social Support*
12 Willem van Willigen (VU) *Look Ma, No Hands: Aspects of Autonomous Vehicle Control*
13 Arlette van Wissen (VU) *Agent-Based Support for Behavior Change: Models and Applications in Health and Safety Domains*
14 Yangyang Shi (TUD) *Language Models With Meta-information*
15 Natalya Mogles (VU) *Agent-Based Analysis and Support of Human Functioning in Complex Socio-Technical Systems: Applications in Safety and Healthcare*
16 Krystyna Milian (VU) *Supporting trial recruitment and design by automatically interpreting eligibility criteria*
17 Kathrin Dentler (VU) *Computing healthcare quality indicators automatically: Secondary Use of Patient Data and Semantic Interoperability*
18 Mattijs Ghijsen (VU) *Methods and Models for the Design and Study of Dynamic Agent Organizations*
19 Vinicius Ramos (TUE) *Adaptive Hypermedia Courses: Qualitative and Quantitative Evaluation and Tool Support*
20 Mena Habib (UT) *Named Entity Extraction and Disambiguation for Informal Text: The Missing Link*
21 Kassidy Clark (TUD) *Negotiation and Monitoring in Open Environments*
22 Marieke Peeters (UU) *Personalized Educational Games—Developing agent-supported scenario-based training*
23 Eleftherios Sidirourgos (UvA/CWI) *Space Efficient Indexes for the Big Data Era*
24 Davide Ceolin (VU) *Trusting Semi-structured Web Data*
25 Martijn Lappenschaar (RUN) *New network models for the analysis of disease interaction*
26 Tim Baarslag (TUD) *What to Bid and When to Stop*
27 Rui Jorge Almeida (EUR) *Conditional Density Models Integrating Fuzzy and Probabilistic Representations of Uncertainty*
28 Anna Chmielowiec (VU) *Decentralized k-Clique Matching*
29 Jaap Kabbedijk (UU) *Variability in Multi-Tenant Enterprise Software*
30 Peter de Cock (UvT) *Anticipating Criminal Behaviour*
31 Leo van Moergestel (UU) *Agent Technology in Agile Multiparallel Manufacturing and Product Support*
32 Naser Ayat (UvA) *On Entity Resolution in Probabilistic Data*
33 Tesfa Tegegne (RUN) *Service Discovery in eHealth*
34 Christina Manteli (VU) *The Effect of Governance in Global Software Development: Analyzing Transactive Memory Systems.*
35 Joost van Ooijen (UU) *Cognitive Agents in Virtual Worlds: A Middleware Design Approach*
36 Joos Buijs (TUE) *Flexible Evolutionary Algorithms for Mining Structured Process Models*
37 Maral Dadvar (UT) *Experts and Machines United Against Cyberbullying*
38 Danny Plass-Oude Bos (UT) *Making brain-computer interfaces better: improving usability through post-processing.*
39 Jasmina Maric (UvT) *Web Communities, Immigration, and Social Capital*
40 Walter Omona (RUN) *A Framework for Knowledge Management Using ICT in Higher Education*
41 Frederic Hogenboom (EUR) *Automated Detection of Financial Events in News Text*
42 Carsten Eijckhof (CWI/TUD) *Contextual Multidimensional Relevance Models*
43 Kevin Vlaanderen (UU) *Supporting Process Improvement using Method Increments*

44 Paulien Meesters (UvT) *Intelligent Blauw. Met als ondertitel: Intelligence-gestuurde politiezorg in gebiedsgebonden eenheden.*
45 Birgit Schmitz (OUN) *Mobile Games for Learning: A Pattern-Based Approach*
46 Ke Tao (TUD) *Social Web Data Analytics: Relevance, Redundancy, Diversity*
47 Shangsong Liang (UVA) *Fusion and Diversification in Information Retrieval*

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2 Faiza Bukhsh (UvT) *Smart auditing: Innovative Compliance Checking in Customs Controls*
3 Twan van Laarhoven (RUN) *Machine learning for network data*
4 Howard Spoelstra (OUN) *Collaborations in Open Learning Environments*
5 Christoph Bösch (UT) *Cryptographically Enforced Search Pattern Hiding*
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7 Maria-Hendrike Peetz (UvA) *Time-Aware Online Reputation Analysis*
8 Jie Jiang (TUD) *Organizational Compliance: An agent-based model for designing and evaluating organizational interactions*
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10 Henry Hermans (OUN) *OpenU: design of an integrated system to support lifelong learning*
11 Yongming Luo (TUE) *Designing algorithms for big graph datasets: A study of computing bisimulation and joins*
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13 Giuseppe Procaccianti (VU) *Energy-Efficient Software*
14 Bart van Straalen (UT) *A cognitive approach to modeling bad news conversations*
15 Klaas Andries de Graaf (VU) *Ontology-based Software Architecture Documentation*
16 Changyun Wei (UT) *Cognitive Coordination for Cooperative Multi-Robot Teamwork*
17 André van Cleeff (UT) *Physical and Digital Security Mechanisms: Properties, Combinations and Trade-offs*
18 Holger Pirk (CWI) *Waste Not, Want Not!—Managing Relational Data in Asymmetric Memories*
19 Bernardo Tabuenca (OUN) *Ubiquitous Technology for Lifelong Learners*
20 Loïs Vanhée (UU) *Using Culture and Values to Support Flexible Coordination*
21 Sibren Fetter (OUN) *Using Peer-Support to Expand and Stabilize Online Learning*
22 Zhemin Zhu (UT) *Co-occurrence Rate Networks*
23 Luit Gazendam (VU) *Cataloguer Support in Cultural Heritage*
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27 Sándor Héman (CWI) *Updating compressed column-stores*
28 Janet Bagorogoza (TiU) *Knowledge Management and High Performance; The Uganda Financial Institutions Model for HPO*
29 Hendrik Baier (UM) *Monte-Carlo Tree Search Enhancements for One-Player and Two-Player Domains*
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31 Yakup Koç (TUD) *On Robustness of Power Grids*
32 Jerome Gard (UL) *Corporate Venture Management in SMEs*
33 Frederik Schadd (UM) *Ontology Mapping with Auxiliary Resources*
34 Victor de Graaff (UT) *Geosocial Recommender Systems*
35 Junchao Xu (TUD) *Affective Body Language of Humanoid Robots: Perception and Effects in Human Robot Interaction*

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1 Syed Saiden Abbas (RUN) *Recognition of Shapes by Humans and Machines*
2 Michiel Christiaan Meulendijk (UU) *Optimizing medication reviews through decision support: prescribing a better pill to swallow*
3 Maya Sappelli (RUN) *Knowledge Work in Context: User Centered Knowledge Worker Support*

4 Laurens Rietveld (VU) *Publishing and Consuming Linked Data*
5 *Evgeny Sherkhonov (UVA) Expanded Acyclic Queries: Containment and an Application in Explaining Missing Answers*
6 Michel Wilson (TUD) *Robust scheduling in an uncertain environment*