THE COASTAL DYNAMICS OF SAND WAVES AND THE INFLUENCE OF BREAKWATERS AND GROYNES

by

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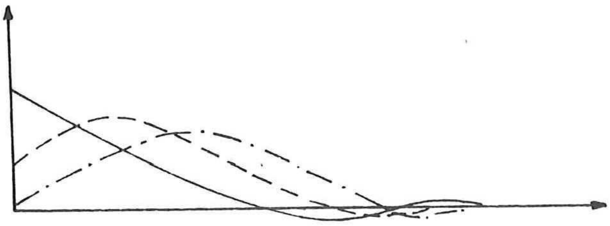

fig. 1

A mathematical theory will be given about "sand waves", an alternating accretion and erosion of the coast, which phenomenon moves along the coast (fig. 1).
The influence of breakwaters (preventing all transport) and groynes (preventing a part of the transport) is considered. Only the influence of waves is taken into account.

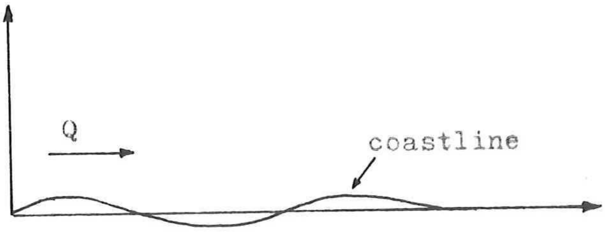

fig. 2

At a coastline, of which the direction does not vary very much, one can linearise the connection between littoral drift and coastal direction according to the following equation:

$$Q = Q_o - q \frac{\partial y}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1),$$

in which: Q is the littoral drift, x is the mean coastal direction and Q_o is the littoral drift at places where the coastal direction is parallel to the x-axis, and q is a proportionality constant.
Combining equation (1) with the continuity equation:

$$\frac{\partial Q}{\partial x} + D \frac{\partial y}{\partial t} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2),$$

one obtains the coastal equation:

$$\frac{\partial y}{\partial t} = \frac{q}{D} \frac{\partial^2 y}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3),$$

in which D means the depth. One finds $\frac{q}{D}$ as a "coastal constant".
This equation has been found earlier by Pelnard-Considère.

A periodical solution, not mentioned by Pelnard-Considère, forms:

$$y = Ae^{-kx} \cos(wt - kx + \varphi_1) + Be^{kx} \cos(wt + kx + \varphi_2) \quad \ldots \quad (4^a),$$

in which $k = \sqrt{\frac{wD}{2q}}$ . . . . (4^b)

The amount of A, B, φ_1 and φ_2 depends of the boundary conditions.
The first term of (4^a) is a "sandwave" moving and attenuating in positive x-direction, the second one a sandwave, moving and attenuating in negative x-direction.
Restricting us to the first term (B = 0), which occurs at a semi-infinite coastline, which is at rest at $x = \infty$, one obtains for the wave-length:

$$\lambda = \frac{2\pi}{k} = \sqrt{\frac{q}{D} \cdot 4\pi T} \quad \ldots\ldots\ldots\ldots \quad (5^a)$$

and for the velocity of propagation:

$$c = \sqrt{\frac{q}{D} \cdot \frac{4\pi}{T}} \quad \ldots\ldots\ldots\ldots \quad (5^b)$$

Of course, this will not be the velocity of the sediment, which can be in the opposite direction. According to (1), the littoral drift amounts:

$$Q = Q_o + q \cdot Ak\sqrt{2}\, e^{kx} \cos(wt - kx + \varphi_1 + \frac{\pi}{4}) \quad \ldots \quad (6)$$

From (6) it can be seen that there are many circumstances (Q_o negative, for instance), that the littoral drift is negative, although the sandwave is moving in positive direction.
The first annex and the dotted line in the 6th one give the shape of the sandwave. At a distance of half a wave-length, the amplitude is decreased to 4% of the original one.

Until now, we did not have to restrict ourselves to unvarying wave characteristics: the coastal constant $\frac{q}{D}$ can be found as a mean over a long period, say a year.
If we take breakwaters into account, we have to simplify more seriously. From fig. $3^{a,b,c}$ it will be seen, that with a varying wave direction a breakwater generates a sandwave. The coastline remains at rest at infinity and the coastal direction near the breakwater is allways the direction of the wavecrest (Q = 0).

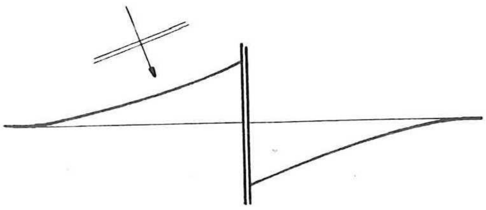

fig. 3^a

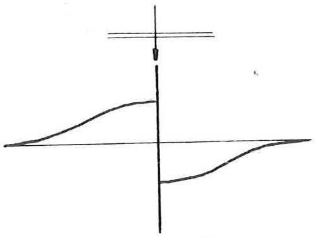

fig. 3^b

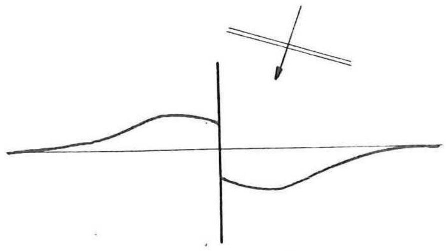

fig. 3^c

Intricate cases like this one, however, we will not take into account. We restrict ourselves for the next part to the cases in which the wave characteristics don't vary and in which the equation (1) can be applied near the breakwater as well, where the transport Q is zero. This means a small angle of incidence of the waves.

The following boundary conditions for this case will be taken:

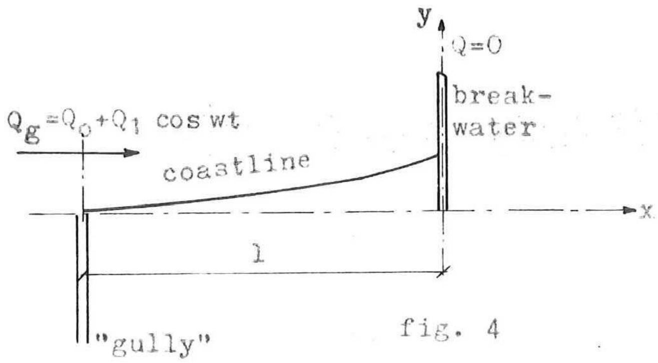

fig. 4

$Q = 0$ at the breakwater;

$Q_g = Q_o + Q_1 \cos wt$ at the other boundary, at a distance l. This boundary will be called "gully" for shortness.

As the coastal equation is linear, one finds the solution by superposition of two cases:

1^o $Q = 0$ at the breakwater and $Q_g = Q_o$ at the gully "stationary transport"	2^o $Q = 0$ at the breakwater and $Q_g = Q_1 \cos wt$ at the gully "alternating transport"

The first case can be found easily with the theory, given by Pelnard-Considère, delivering the coastal formation of annex 2.

There will be assumed, that the coastline does not reach the end of the breakwater.

The second case can be solved from equation (4^a) by fitting the correct values for A and B. In this case also a sandwave moving from the breakwater to the gully occurs: "a reflection of the sandwave".

At every point and time the y-coordinate can be given as $y = \hat{y}_x \cos(wt - \Theta_x)$; after some computations one finds the expressions mentioned on annex 3 and 4 for $\hat{y}$ and Θ respectively.

Annex 3 gives the envelope $\hat{y}$ of the sandwave for various values of $\frac{1}{\lambda}$;

Annex 4 gives the phase Θ of the wave;

Annex 5 gives the effect of the breakwater: the ratio $K = \frac{\hat{y}_{md}}{\hat{y}_{zd}}$,

in which:

$\hat{y}_{md}$ the amplitude of the sandwave in the case of a breakwater

and $\hat{y}_{zd}$ the amplitude of the sandwave in the case of a semi-infinite coastline with the same transport at the gully: $Q_g = Q_1 \cos wt$.

If $\frac{1}{\lambda}$ and $\frac{x}{\lambda}$ are large, then the phaseshift Θ of Q_g to y is about $- kx + kl + \frac{\pi}{4}$.
This agrees with (6), as $\varphi_1 = kl$ in this case.
If $\frac{1}{\lambda}$ is small, then the phaseshift is about $\frac{\pi}{2}$.
From annex 5 it can be seen, that the influence of the mole is about a doubling of the amplitude near the mole, if $\frac{1}{\lambda}$ is sufficiently large.
The envelope does not decrease exponentially from the gully to the mole; it is much more equalised. At some places the amplitude even decreases.
If $\frac{1}{\lambda}$ is very small, then the amplitude becomes much larger then without a mole.
Instead of a mole, preventing <u>all</u> transport, we now consider a groyne preventing a <u>part</u> of the transport.
For the mechanism of transport along the head of the groyne, we assume, that the transport is relative to the distance $y_l - y_r$,

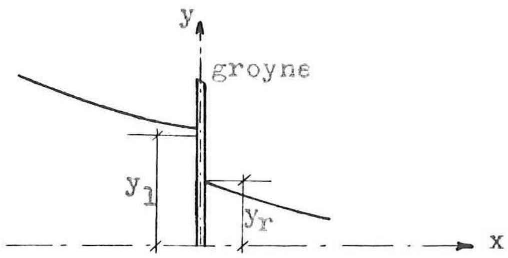

fig. 5

in which y_l and y_r are the y-coordinates of the coastline just on the left- and righthand side of the groyne.

$$Q = \mu (y_l - y_r)$$

The continuity equations near the groyne are:

$$\left.\begin{array}{l} \underline{a}\ \ Q \text{ on the lefthand side} = Q \text{ on the righthand side, so } \left(\frac{\partial y}{\partial x}\right)_l = \left(\frac{\partial y}{\partial x}\right)_r \\ \\ \underline{b}\ \ \text{The transport along the head of the groyne = transport just left (right)} \\ \quad\ \text{of the groyne} \quad Q = \mu (y_l - y_r) = Q_o - q \frac{\partial y}{\partial x} \end{array}\right\} \text{.. (}$$

The influence of the stationary transport Q_o is obviously the formation of a "step" in the coast-line of the amount $a = \frac{Q_o}{\mu}$.

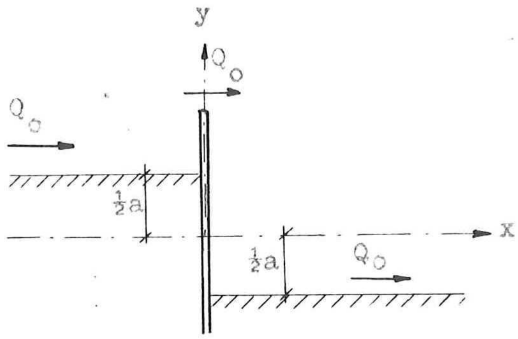

fig. 6

Therefore it is sufficient to solve the case in which $Q_o = 0$; afterwards we can add the above mentioned solution by shifting the left branch with relation to the right branch over a distance a.

Taking the y-axis at the groyne, we will solve the problem for the boundary conditions:

$$y = A\, e^{-kx} \cos(wt - kx) \qquad \text{for } x \rightarrow -\infty;$$

$$y = 0 \qquad \text{for } x = +\infty.$$

The lefthand branch ($x < 0$) will consist of the original sandwave plus a reflected sandwave:

$$y = A\, e^{-kx} \cos(wt - kx) + B\, e^{kx} \cos(wt + kx + \varphi).$$

The righthand branch only consists of one sandwave, because $y = 0$ at $x = \infty$:

$$y = C\, e^{-kx} \cos(wt - kx + \psi).$$

After substituting the conditions (7) at the groyne, one finds:

$$B = \frac{A}{\sqrt{\frac{1}{2} + \left(\frac{1}{\sqrt{2}} + \frac{\mu\sqrt{2}}{qk}\right)^2}} \qquad \varphi = \frac{\pi}{4} - \operatorname{arctg} \frac{qk}{2\mu + qk}$$

$$C = \frac{A}{\sqrt{\left(1 + \frac{qk}{2\mu}\right)^2 + \left(\frac{qk}{2\mu}\right)^2}} \qquad \psi = -\operatorname{arctg} \frac{qk}{2\mu + qk}$$

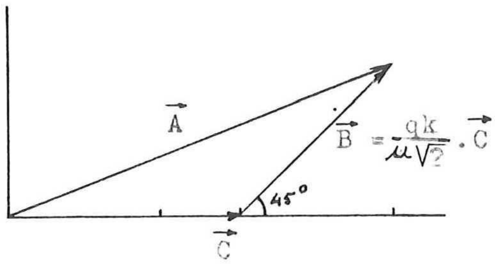

fig. 7

In fig. 7 the incident wave $\vec{A}$, the reflected wave $\vec{B}$ and the right-branch wave $\vec{C}$ are represented in a phase diagram for $x = 0$.
$\vec{B}$ makes an angle of 135° with $\vec{C}$, and the vectorial sum of $\vec{B}$ and $\vec{C}$ is $\vec{A}$.
The amount $\frac{B}{A}$ is a "reflection coefficient", varying between 0 and 1.

$$\frac{B}{A} = \frac{1}{\sqrt{1 + \frac{2\mu}{qk} + \frac{2\mu^2}{q^2k^2}}} \qquad \text{in which} \qquad \frac{\mu^2}{q^2k^2} = \frac{\mu^2 T}{\pi q D}$$

Annex 6 gives the shape of the sandwave (the time $t = 0$ is taken at the moment of maximum y_r).

The theory about the <u>breakwater</u> holds only for a small angle of wave incidence. This condition is not necessary for the theory about the <u>groyne</u>, in case that the reflection coefficient is not too large (otherwise the linearisation of equation (1) may not be applied).

CONCLUSIONS

1. At a coastline where the littoral drift by waves prevails, progressive sandwaves can exist, if there is a periodical varying disturbance in one point at the coast and the coastline is at rest at more than half a wave-length distance.

2. These sandwaves always move from the disturbance to the part of the coastline that is at rest, independant of the direction of the littoral drift.

3. The wave-length increases with the period, but the velocity of propagation decreases.

4. A groyne reflects a part of the sandwave. Therefore the amplitude of the sandwave near the groyne on the side of the disturbance increases. On the other side of the groyne the amplitude is less than without a groyne.

Literature

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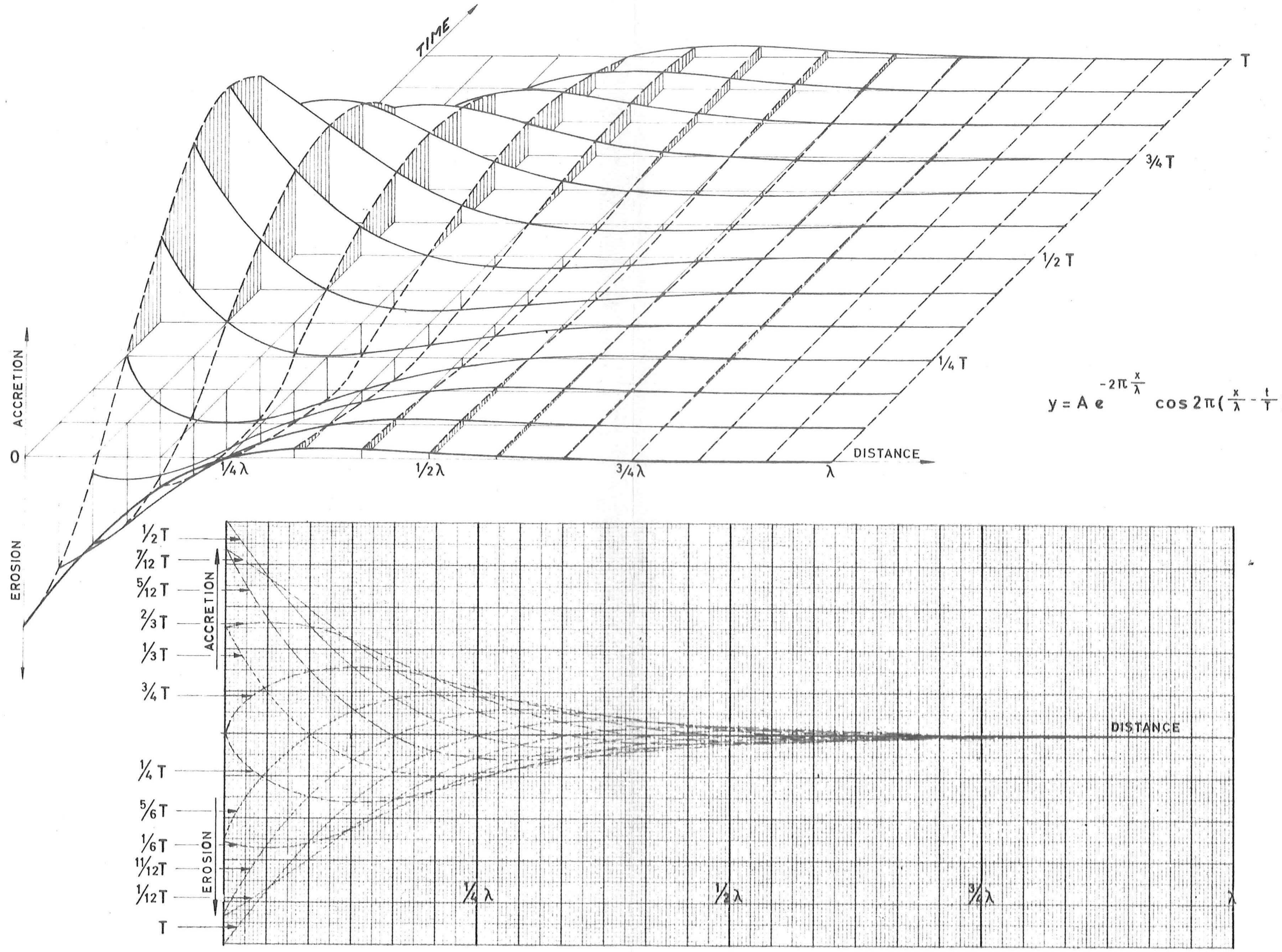

SHAPE OF SANDWAVE AS FUNCTION OF TIME					COASTAL DYNAMICS SANDWAVES	ANNEX 1
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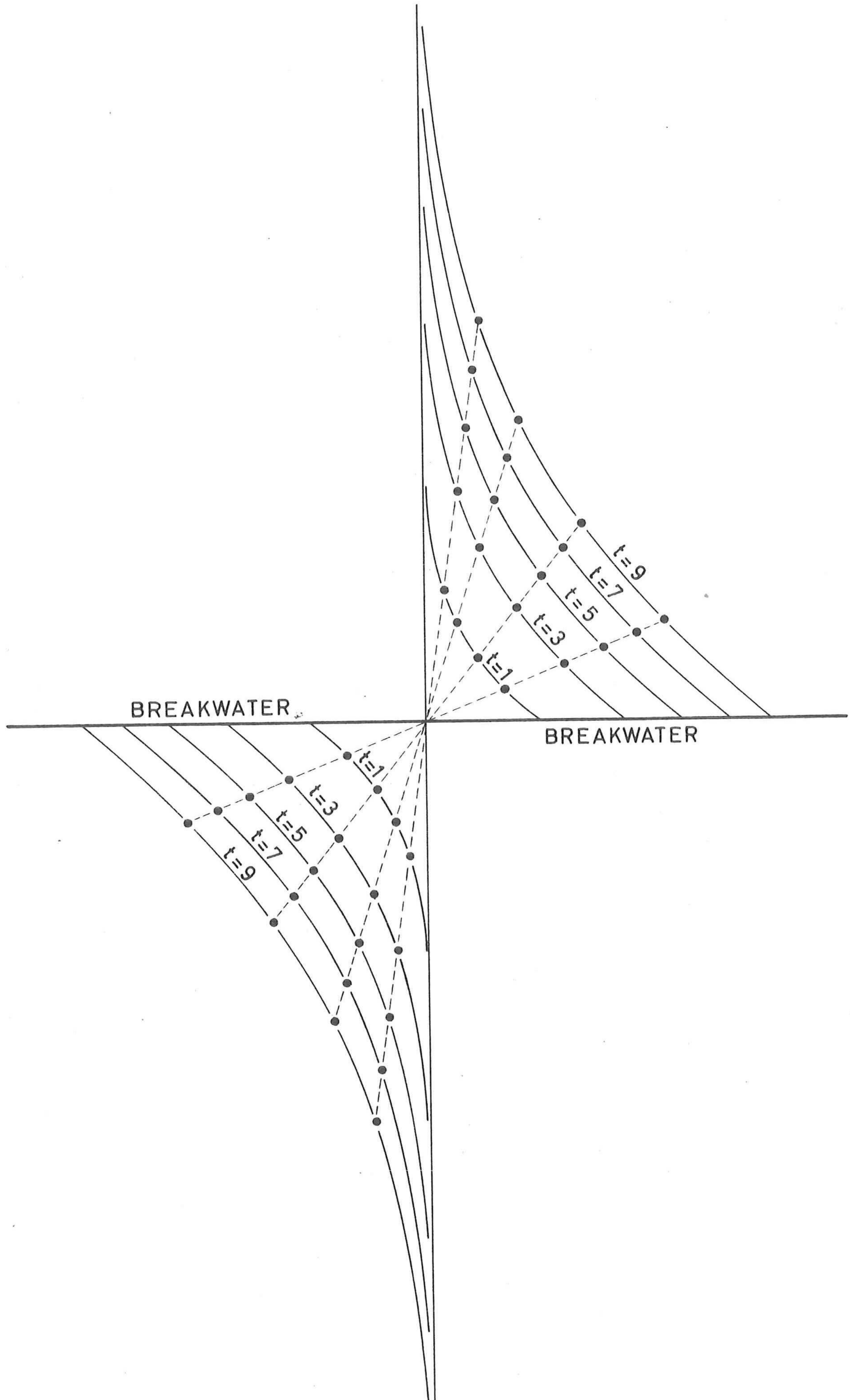
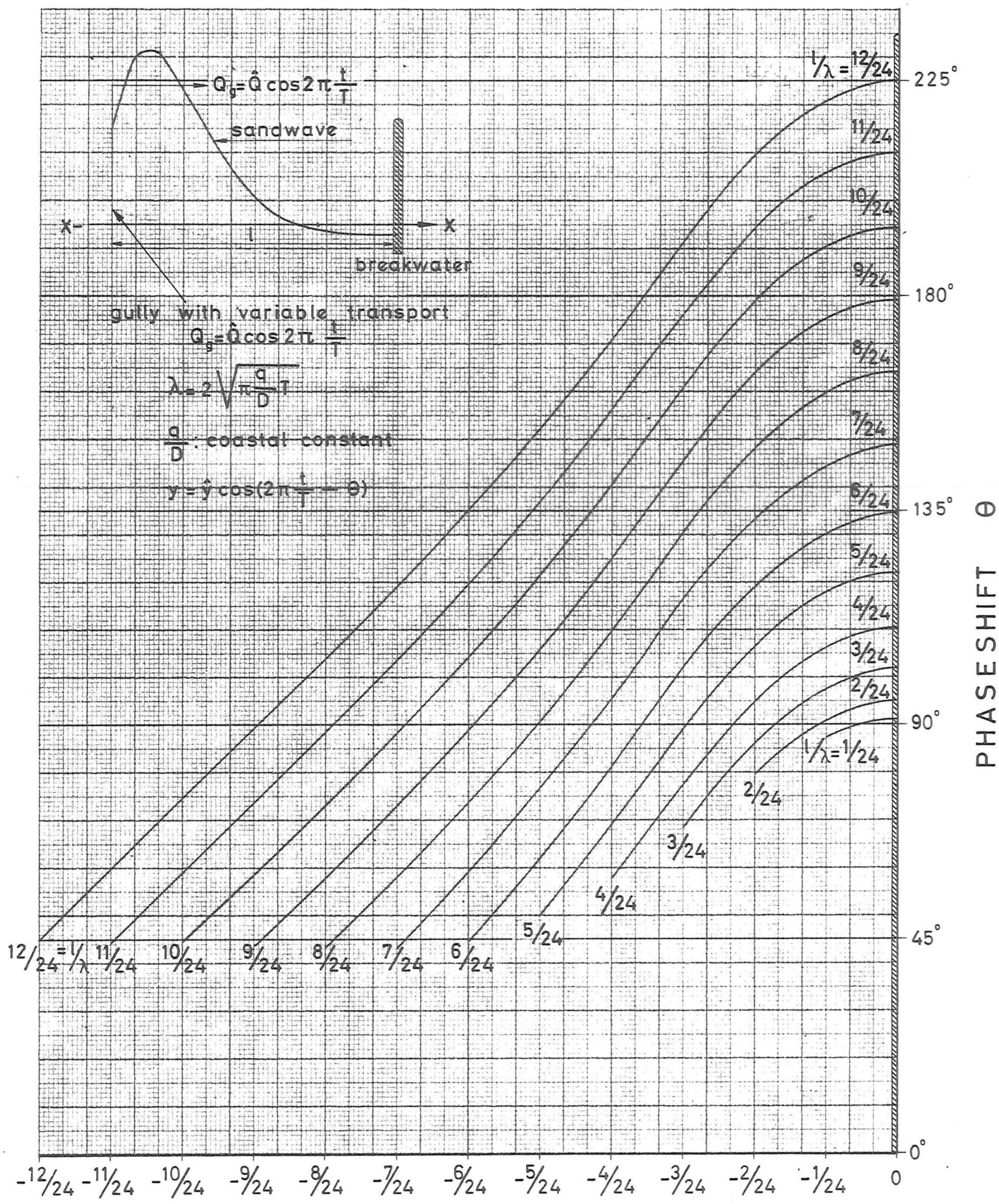
EFFECT OF A BREAKWATER ON STATIONARY TRANSPORT



ANNEX 2

Θ = phaseshift between adding of sand at the gully ($x = -l$) and accretion of the coast y at a distance x of the breakwater.

$$\Theta = \varphi - \psi_x = \frac{\pi}{4} + \frac{2\pi(l+x)}{\lambda} + \arcsin \frac{e^{-\frac{2\pi l}{\lambda}} \sin 4\pi l/\lambda}{\sqrt{2(\cosh 4\pi l/\lambda - \cos 4\pi l/\lambda)}} - \arcsin \frac{e^{+\frac{2\pi x}{\lambda}} \sin 4\pi x/\lambda}{\sqrt{2(\cosh 4\pi x/\lambda + \cos 4\pi x/\lambda)}}$$



SANDWAVE AT A BREAKWATER
PHASESHIFT Θ

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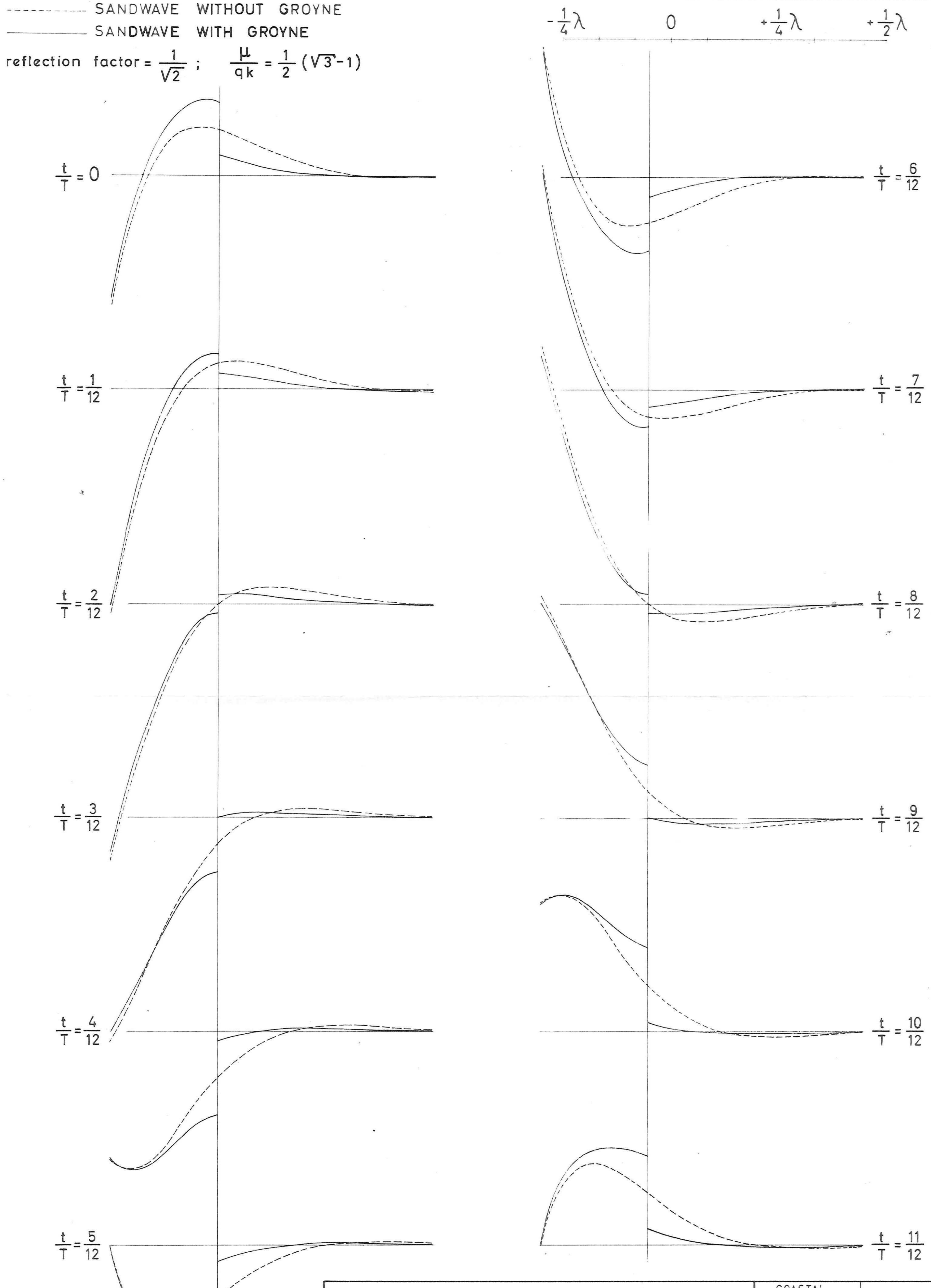
COASTAL DYNAMICS SANDWAVES

ANNEX 4

A2 Nr. 67.056b



EFFECT OF A GROYNE ON THE SHAPE OF A SANDWAVE

COASTAL DYNAMICS SANDWAVES

ANNEX 6

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A. Roos			

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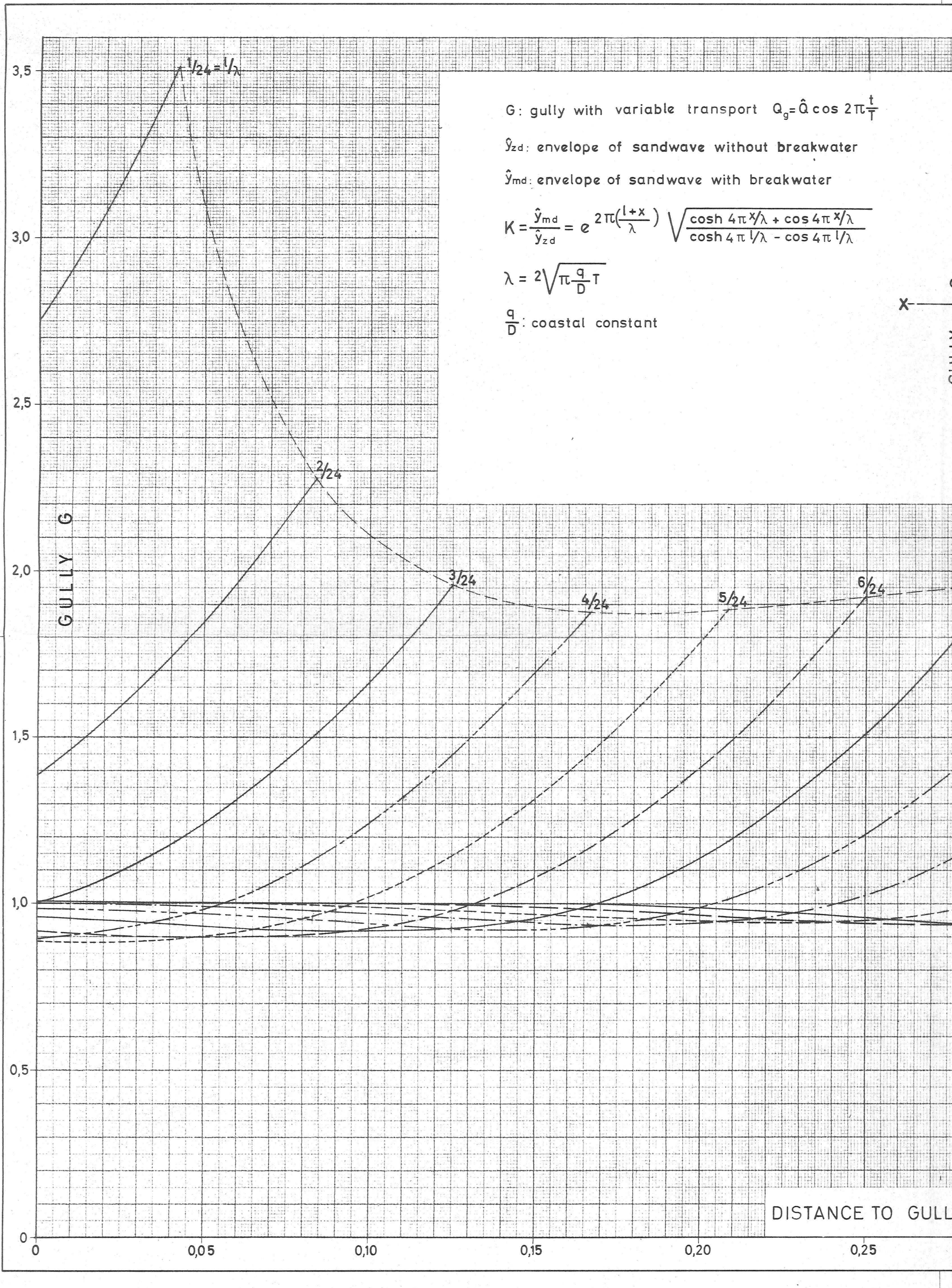
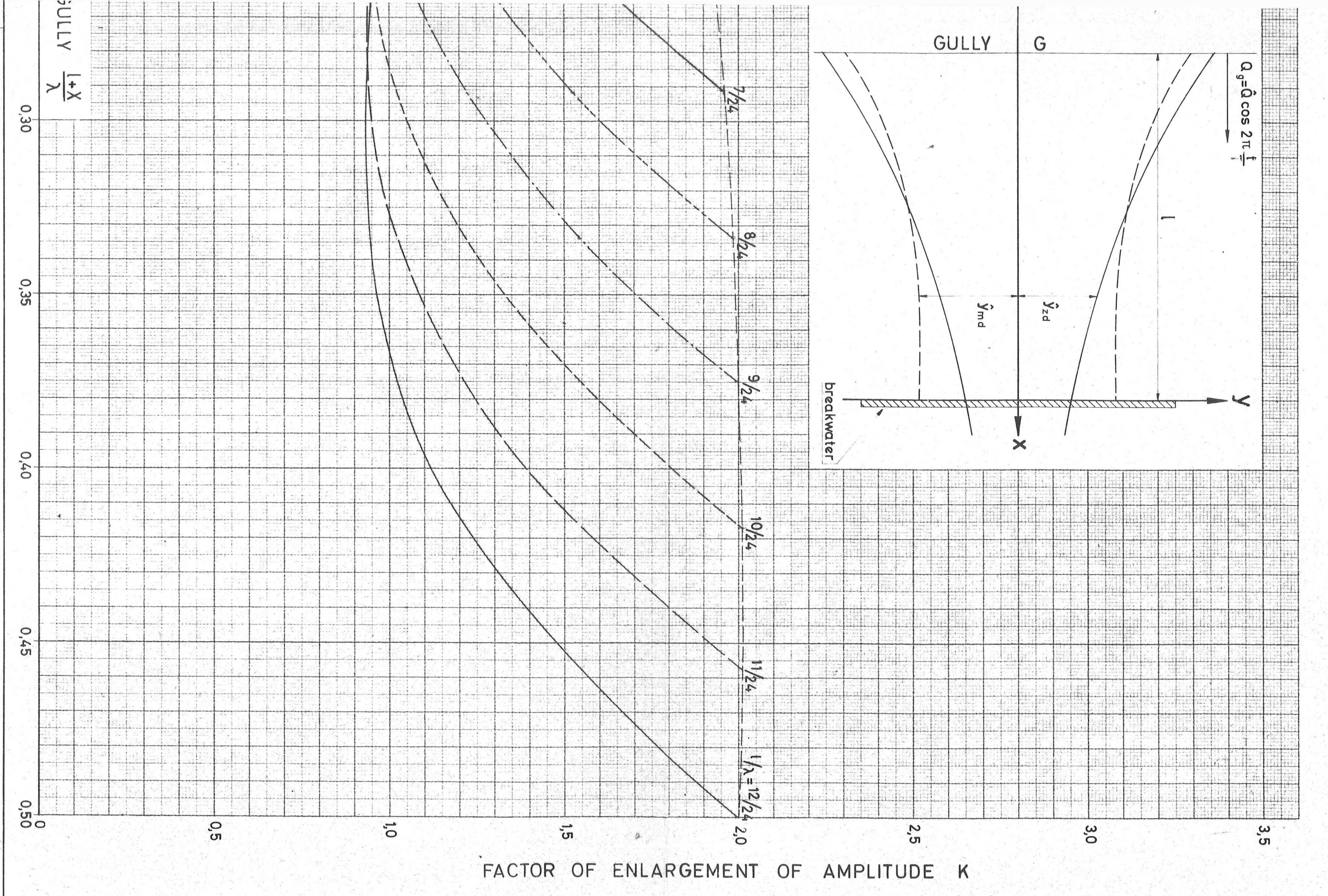


3,5
3,0
2,5
2,0
1,5
1,0
0,5
0
0
0,05
0,10
0,15
0,20
0,25
GULLY G
1/24 = 1/λ
2/24
3/24
4/24
5/24
6/24
G: gully with variable transport $Q_g = \hat{Q} \cos 2\pi \frac{t}{T}$
$\hat{y}_{zd}$: envelope of sandwave without breakwater
$\hat{y}_{md}$: envelope of sandwave with breakwater
$K = \frac{\hat{y}_{md}}{\hat{y}_{zd}} = e^{2\pi\left(\frac{l+x}{\lambda}\right)} \sqrt{\frac{\cosh 4\pi x/\lambda + \cos 4\pi x/\lambda}{\cosh 4\pi l/\lambda - \cos 4\pi l/\lambda}}$
$\lambda = 2\sqrt{\pi \frac{q}{D} T}$
$\frac{q}{D}$: coastal constant
X
DISTANCE TO GULL



SANDWAVE AT A BREAKWATER EFFECT OF BREAKWATER ON THE AMPLITUDE					COASTAL DYNAMICS SANDWAVES	ANNEX 5
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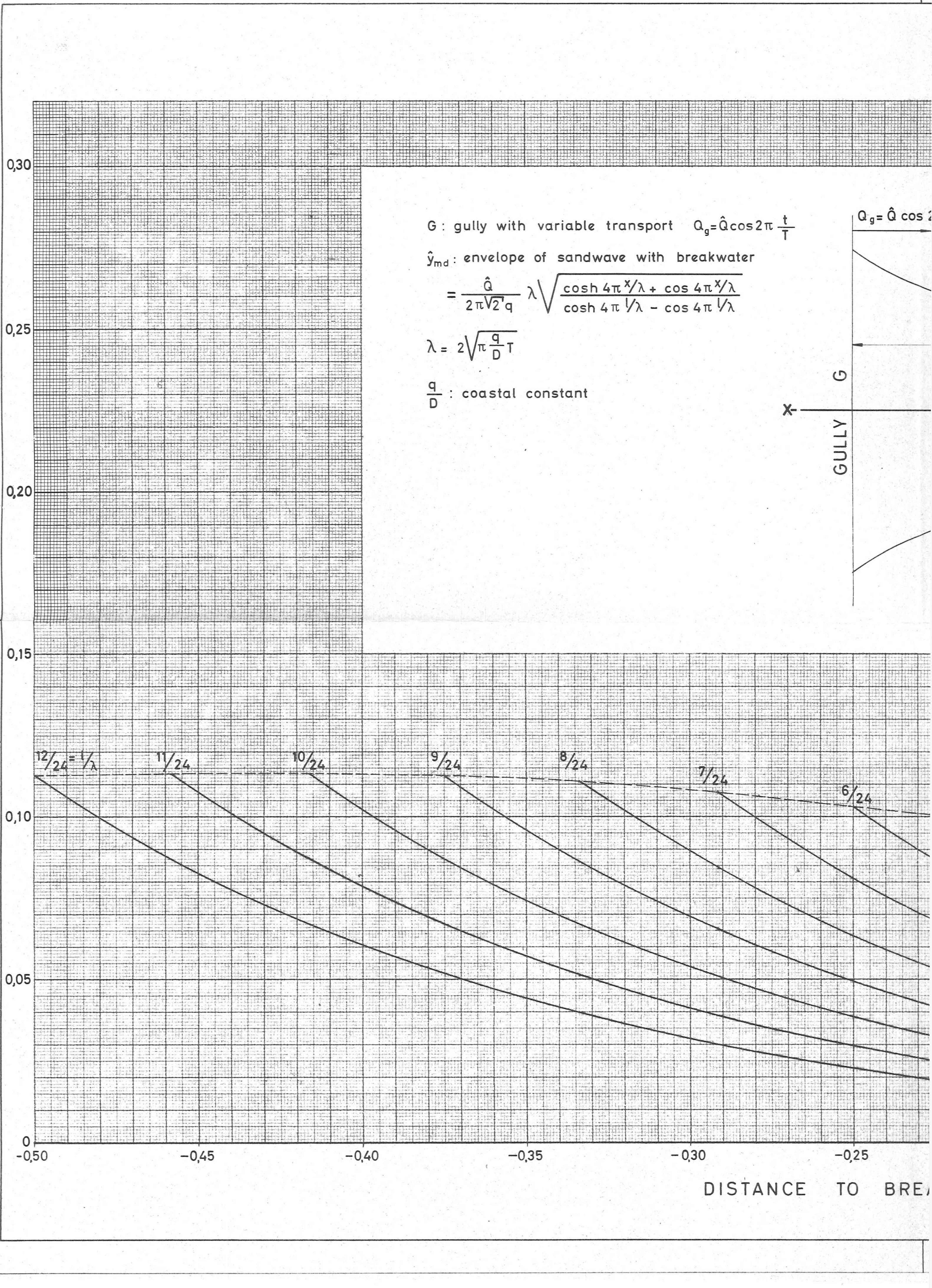
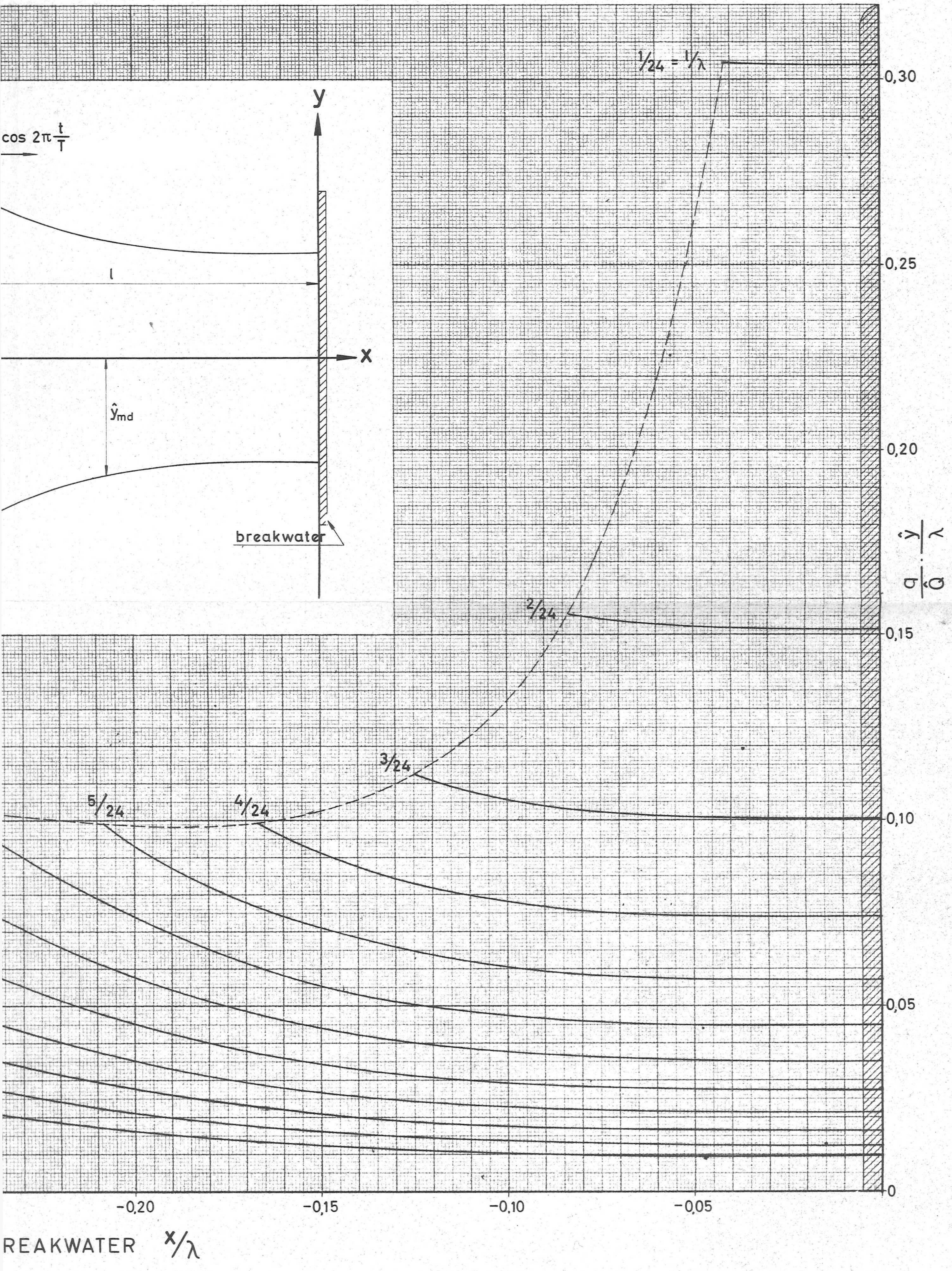


G : gully with variable transport Qg=Q̂cos2π t/T
ŷmd : envelope of sandwave with breakwater
= Q̂/(2π√2 q) λ √((cosh 4π x/λ + cos 4π x/λ)/(cosh 4π l/λ − cos 4π l/λ))
λ = 2√(π q/D T)
q/D : coastal constant
Qg= Q̂ cos 2
X-
G
GULLY
12/24=l/λ
11/24
10/24
9/24
8/24
7/24
6/24
0,30
0,25
0,20
0,15
0,10
0,05
0
-0,50
-0,45
-0,40
-0,35
-0,30
-0,25
DISTANCE TO BRE



COASTAL DYNAMICS SANDWAVES

ANNEX 3

B 2 Nr. 67.055[b]

SANDWAVE AT A BREAKWATER

ENVELOPE OF SANDWAVE

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