Topology in Two-Dimensional Systems
TOPOLOGY IN TWO-DIMENSIONAL SYSTEMS

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Topological states of matter have an insulating bulk and gapless states at their boundary. Besides being fascinating states on their own, certain topological states have attracted much attention for their potential application in quantum computing. This potential arises from Majorana fermions, quasiparticles that are their own anti-particle, that emerge in these topological states. These Majorana fermions fulfill non-Abelian exchange statistics, a property that could be exploited to obtain fault-tolerant quantum bits.

Two topological states that are expected to host Majorana fermions are the topological superconducting state and the fractional quantum Hall state at $\nu = 5/2$. In this thesis we investigate two semiconductor heterostructures in which these respective states could potentially be formed.

The first material investigated in this thesis is a high-quality GaAs/AlGaAs heterostructure in which a two-dimensional electron gas (2DEG) resides. Majorana fermions are expected when the magnetic field perpendicular to the 2DEG is such that the fractional quantum Hall state at $\nu = 5/2$ is reached. However, this fragile $\nu = 5/2$ state is only observed in heterostructures of sufficiently high quality. In this respect, it is known that nanofabrication processing degrades heterostructure quality.

We have therefore designed, developed and built a setup with the aim of preserving the pristine quality of the heterostructure. In this ‘flip-chip’ setup metallic gates hover ~100 nm above the surface of the GaAs/AlGaAs heterostructure. These remote gates allow electrostatic gating of the material without exposing it to invasive nanoprocessing. The stiff design of the setup results in a gate-to-heterostructure separation that fluctuates less than 1.5 pm. Using this set-up, we demonstrate electrostatically defined quantum point contacts, quantum dots and Fabry-Pérot interferometers in the 2DEG. Quantum interference is achieved at integer quantum Hall states in a Fabry–Pérot interferometer and it is shown that the studied interferometer is in the Coulomb dominated regime. It is recommended that future experiments towards study of fragile fractional quantum Hall states using the flip-chip approach use a gate-containing tip structure and that any direct contact between heterostructure and gate chip is avoided.

The second heterostructure studied in this thesis is a InAs/GaSb double quantum well (DQW). Electrons in the InAs can hybridize with holes in the GaSb and such hybridization opens an energy gap. This energy gap is predicted to be topologically distinct from vacuum and therefore the material would be a two-dimensional topological insulator.
(2D TI). A defining characteristic of a 2D TI is an insulating bulk and gapless helical channels at its perimeter. Majorana fermions are expected when a 2D TI is proximitized with a superconductor. The experiments in this thesis are aimed at establishing experimentally the 2D TI character of InAs/GaSb DQWs.

We first study an InAs/GaSb DQW contacted by two superconducting leads (Ti/Al or NbTiN). The induced superconductivity is shown to be tunable from electron mediated to hole mediated by means of top and bottom gate voltages. The spatial supercurrent distribution is extracted using superconducting quantum interference measurements. When the normal state resistance of the device is small, a supercurrent flows uniformly through the device. When the sample is gated to a high resistance regime, the supercurrent predominantly flows along the sample edge. The width of this edge conduction channel is 260 nm or less.

In a second experiment we study a dual-gated InAs/GaSb Hall bar device with normal metal leads. We demonstrate in-situ tuning between a trivial band alignment (trivial regime) and an inverted band alignment (inverted regime) as a function of top and bottom gate voltages. In the case of trivial band alignment an energy gap is observed between electron band and hole band. If instead the band alignment is inverted a hybridization gap is observed when the electron density equals the hole density.

A subsequent experiment focuses on the trivial (non-topological) regime of an InAs/GaSb DQW Hall bar device. Surprisingly, in this trivial gapped regime an insulating bulk and conducting edges are found. We expect that band-bending at the edge of the material is responsible for this conduction.

Furthermore, spin-orbit interaction is investigated by studying magnetoresistance in a dual-gated InAs/GaSb Hall bar device. In the trivial regime at high electron density the linear and cubic Dresselhaus contributions are found to be of similar strength. In this regime the Rashba spin-orbit strength can be tuned from $\alpha = (8 \pm 0.6) \times 10^{-12} \text{ eVm}$ to $\alpha = (6.5 \pm 0.5) \times 10^{-12} \text{ eVm}$. The inverted regime is also studied. Here the spin-orbit strength increases when the Fermi level is moved towards the hybridization gap, consistent with theoretical calculations.

While some of these experiments might be interpreted as signatures of a 2D TI, they can also be explained by more mundane, topologically trivial, phenomena such as edge conduction. Furthermore, a bulk conduction through the hybridization gap is observed. These two phenomena render the experiments in this thesis inconclusive with respect to the 2D TI characteristics of InAs/GaSb DQWs.

It is expected that removal of the trivial edge conduction represents a considerable challenge. It is however essential for further experiments with InAs/GaSb heterostructures as 2D TIs that this challenge is met. This thesis concludes with proposals for methods and devices to respectively reduce the bulk conductivity and circumvent the trivial edge conduction.
SAMENVATTING

Topologische toestanden van materie hebben een isolerende bulk en randtoestanden zonder energiekloof. Naast het fascinerende toestanden zijn, staan bepaalde topologische toestanden in de belangstelling voor hun potentiële toepassing in een topologische kwantumcomputer. Deze potentie volgt uit de aanwezigheid van Majorana fermionen, quasi-deeltjes die hun eigen antideeltje zijn. Deze Majorana fermionen vertonen niet-Abelse statistiek onder verwisseling van deeltjes, een eigenschap die gebruikt kan worden voor het maken van een fouttolerante quantum bit.

Twee topologische toestanden die Majorana fermionen kunnen bevatten zijn de topologische supergeleidende toestand en de fractionele quantum Hall toestand op $\nu = 5/2$. In dit proefschrift onderzoeken we twee halfgeleider heterostructuren waarin deze toestanden gevormd kunnen worden.

Eerst wordt een hoge-kwaliteit GaAs/AlGaAs heterostructuur onderzocht waarin zich een twee-dimensionaal electronen gas (2DEG) bevindt. Majorana fermionen worden verwacht waneer een magneetveld loodrecht op het 2DEG wordt aangelegd zodat de quantum Hall toestand $\nu = 5/2$ wordt bereikt. Echter, dezefragile $\nu = 5/2$ toestand is slechts waargenomen in heterostructuren van voldoende hoge kwaliteit. Het is bekend dat nanofabricatie de kwaliteit van een heterostructuur degradeert.

We hebben daarom een opstelling ontworpen, ontwikkeld en gebouwd met als doel de ongerepte kwaliteit van een heterostructuur te behouden. In deze ‘flip-chip’ opstelling zweven metallische gates $\sim 100$ nm boven de oppervlakte van een GaAs/AlGaAs heterostructuur. Deze verafgelegen gates maken het elektrostatisch gaten van een materiaal mogelijk zonder dat het aan invasieve nanofabricatie wordt blootgesteld. Het rigide ontwerp van deze setup resulteert in een gate-heterostructuur afstand die minder dan 1.5 pm fluctueert. Gebruikmakend van deze opstelling demonstreren we elektrostatisch gedefinieerde kwantumpuntcontacten, kwantumdoosjes en Fabry-Pérot-interferometers in het 2DEG. Kwantuminterferentie van integrale kwantum Hall toestanden is aange- toond met een Fabry-Pérot-interferometer en de bestudeerde interferometer is in het Coulomb gedomineerde regime. Voor toekomstige experimenten met het doel fractionele kwantum-Hall-toestanden te bestuderen met de flip-chip opstelling wordt aanbevolen om een spitse structuur te gebruiken waarop de gates liggen en elk contact tussen gate chip en heterostructuur te vermijden.

De tweede heterostructuur die in dit proefschrift wordt bestudeerd is een InAs/GaSb dubbele kwantum well (DQW). Electronen in InAs kunnen hybridizeren met gaten in
GaSb waardoor een energiekloof opent. Er is voorspeld dat deze energiekloof topologisch verschilt van vacuüm, waardoor dit materiaal een twee-dimensionale topologische isolator (2D TI) zou zijn. Kenmerkend van een 2D TI is een isolerende bulk en helische randkanalen. Majorana fermionen ontstaan wanneer supergeleidende paring geïnduceerd wordt in een 2D TI. De experimenten in dit proefschrift hebben als doel het 2D TI karakter van InAs/GaSb DQWs experimenteel aan te tonen.

Als eerste bestuderen we een InAs/GaSb DQW met twee supergeleidende contacten (Ti/Al of NbTiN). De geïnduceerde superstroom kan door electron of gaten gemedieerd worden, afhankelijk van de voltages op de gates boven en onder de heterostructuur. De spatiale distributie van superstroom wordt afgeleid uit metingen van supergeleidende kwantuminterferentie. Wanneer de weerstand in de normale toestand klein is vloeit de superstroom uniform door het device. Wanneer het device naar een hoge weerstand wordt afgestemd vloeit de superstroom langs de randen van het device. De breedte van deze randgeleiding is 260 nm of kleiner.

In een tweede experiment bestuderen we een InAs/GaSb DQW Hall bar device met normale metallicke contacten. We demonstreren in-situ tuning tussen een triviale bandoplijning en een geïnverteerde bandoplijning als functie van de voltages op de gates boven en onder de heterostructuur. In de triviale bandoplijning wordt een energiekloof waargenomen tussen de elektronenband en gatenband. In de geïnverteerde bandoplijning wordt een hybridizatiekloof waargenomen wanneer de elektronendichtheid gelijk is aan de gatendichtheid.

Het vervolgexperiment richt zich op de triviale (niet topologische) regime van een InAs/GaSb DQW Hall bar device. Verassend is dat hier een isolerende bulk en geleidende randen wordt waargenomen. We verwachten dat bandbuiging aan de rand van het materiaal verantwoordelijk is voor deze geleiding.

Verder is de spin-baaninteractie onderzocht door middel van magnetoweerstandmetingen in een InAs/GaSb Hall bar device. In het triviale regime bij hoge elektronendichtheid zijn de bijdragen van lineare en kubische Dresselhaus van gelijke sterkte. In dit regime kan de Rashba spin-baaninteractie sterkte van $\alpha = (8 \pm 0.6) \times 10^{-12}$ eVm naar $\alpha = (6.5 \pm 0.5) \times 10^{-12}$ eVm worden gevarieerd. Het geïnverteerde regime is ook bestudeerd. Hier neemt de spin-baaninteractie toe wanneer het Fermi-niveau naar de hybridizatiekloof wordt gebracht, consistent met theoretische berekeningen.

Sommige van deze experimenten kunnen worden geïnterpreteerd als aanwijzingen voor een 2D TI toestand, echter ze kunnen ook verklaard worden door triviale fenome- nen, zoals randgeleiding. Verder is een bulkgeleiding door de hybridizatiekloof waargenomen. De experimenten in dit proefschrift zijn dus niet afdoende om 2D TI karakteristieken van InAs/GaSb DQWs aan te tonen.

Verwacht wordt dat de eliminatie van deze triviale randgeleiding een aanzienlijke uitdaging is, maar wel essentieel is voor verdere experimenten met InAs/GaSb hetero-structuren als 2D TI. Dit proefschrift sluit af met voorstellen om de geleiding door de bulk en door triviale randkanalen te reduceren.
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Early 20th century a radically new theory was developed that explains nature to an unprecedented accuracy: quantum mechanics. Humanity became acquainted with new mechanisms in nature, such as quantum measurement, superposition states and entanglement; mechanisms that have no classical analog, and reflect properties inherent to nature. In a superposition state a particle can be in multiple states at the same time whereas a quantum measurement forces the particle to choose one of these states. Two entangled particles have their states coupled to each other. Instead of observing and describing these phenomena further, this quantum toolbox can be exploited to build a machine with unprecedented computational power: the quantum computer.

Since Richard Feynman suggested the idea of a quantum computer in 1982, mathematicians and physicist have proven that such a machine could solve certain problems that are intractable on the fastest conventional computers [1]. A typical example is the quantum Fourier transform that lies at the basis of Shor's algorithm (devised by Peter Shor). Shor's algorithm can be used to swiftly factor large numbers which is important for data encryption. Another algorithm, proposed by Lov Grover, speeds up searches in unstructured databases [1]. Furthermore, it is natural for quantum computers to simulate processes in nature, which are quantum systems itself, such as biological processes for predicting medicine effects or in the development of new materials. The strength of a quantum computer is that it explores a huge number of possible computational pathways in parallel, a property called parallelism.

Not all problems are solved faster on a quantum computer, it will therefore not replace its classical variant. Rather it will be used for specific problems that enjoy the 'quantum speed-up'. The quantum computer is envisioned as a tool to develop other technologies, especially in the field of material science and biology.
1. Introduction

Currently, the controllable logic units—quantum bits—are being developed and implemented in numerous systems [2]. The next section discusses why these quantum bits are capable of such computational speed.

1.1. Quantum Bit

Classical computers are built out of transistors that act like switches, they can be ‘on’ or ‘off’, 0 or 1. Grouping a number of transistors in a network allows the formation of logical gates that compute. A quantum bit, or qubit, is a two-level system with states $|0\rangle$ or $|1\rangle$. Quantum mechanics allows the qubit to be in a superposition state, e.g. $\Psi = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$, thus ‘0’ and ‘1’ at the same time. For quantum computing such a state needs to be initialized, manipulated and read out in order to function as computational unit [3]. Manipulation often proceeds using a control knob in the form of a light pulse or gate voltage pulse.

Computational speed-up is possible by virtue of superposition states. By comparison, eight bits in a normal computer can be assigned a value between 0 and 255. A register of eight qubits however can hold all values 0 – 255 simultaneously. In a sense, a quantum computer explores all possible solutions—including incorrect solutions—at once. The trick is to devise an algorithm that assigns the largest probability to the correct state. A measurement of the outcome singles out the most probable one, the solution.

Two qubits, say $a$ and $b$, can attain an entangled state, such as described by $\Psi = \frac{1}{\sqrt{2}} (|0_a\rangle|0_b\rangle + |1_a\rangle|1_b\rangle)$. Such a state cannot be separated into individual contributions from qubit $a$ and $b$, and operations on qubit $b$ affect the state of $a$. Entanglement, therefore, can be detrimental to the qubit state when it is entangled to the environment. The turbulence of the environment changes the qubit state such that after some time no correlation is found to the original state. This process is called decoherence [4] and remains one of the main obstacles to build a large scale quantum computer.

Decoherence can be reduced by dynamical decoupling techniques preserving the qubit state for a longer time [5]. Furthermore, quantum error correction techniques have been developed that spread quantum information over many qubits in such a way that errors can be detected and corrected [6, 7]. An alternative approach toward quantum computing uses qubits that are naturally deaf to decoherence. Such qubits can be defined using topological states of matter [8].

1.2. Topological Quantum Bit

In order to make a topological qubit resistant to decoherence, its quantum state is stored and manipulated in a non-local manner. Non-Abelian states of matter support such non-local storage of information. These states contain composite particles—particles emerging from the collective behavior of many electrons—that are neither fermions nor bosons. In order to understand what makes non-Abelian states different from an ordinary (Abelian) states, we consider the effects of interchanging these quasiparticles, i.e.
1.2. Topological quantum bit

Consider a wavefunction of $N$ identical particles whose coordinates are $r_1, \ldots, r_N$, $\psi(r_1, \ldots, r_N)$. Interchange of two particles is described by the operator $P$ acting on the wavefunction. Interchanging the same particles again recovers the original state, hence $P^2 = 1$. Therefore, the exchange operator $P$ has eigenvalues $\pm 1$. Particles corresponding to the exchange eigenvalue $-1$ are fermions and particles with eigenvalue $+1$ are bosons.

Figure 1.1: Interchanging particles twice (left situation) is topologically equivalent to braiding one particle around the other (right situation). Furthermore, the depicted braid operations in the right situation are topologically identical. Figure reproduced from [9].

The fermion-boson dichotomy breaks down for particles roaming in two dimensions. Exchanging particles twice does not necessarily lead to the initial state. Two successive particle interchanges is identical to encircling one particle around the other, see Fig 1.1. In three dimensions such a loop can always be continuously deformed to the starting point, without cutting through the other particle, hence we recover $P^2 = 1$. In two dimensions, however, such a braid path cannot be deformed to a point without cutting through the other particle and $P^2$ is not necessarily 1 (see Fig. 1.1). Exchange eigenvalues of arbitrary phase $e^{i\phi}$ are possible. Particles that possess such unconventional, but still Abelian, exchange statistics are called anyons.

The even more exotic non-Abelian exchange statistics can occur in the presence of a ground state degeneracy. The interchanging of two particles does not merely multiply the wave function by a phase factor, but can shift the system to a different ground state. In order to see how this can be used for quantum computing, suppose there are $M$ degenerate ground states $\psi_m(r_1, \ldots, r_N)$ that span a subspace. The qubit state can be in a superposition of all ground states, $\Psi = \sum_{m=1}^{M} c_m \psi_m$. Interchanging particles 1 and 2, might rotate the state within the subspace, $\Psi' \rightarrow R_{12} \Psi$, described by a unitary matrix $R_{12}$. Exchanging particles 2 and 3 constitutes a different operation $\Psi'' \rightarrow R_{23} \Psi$. When the matrices $R_{12}$ and $R_{23}$ do not commute, $[R_{12}, R_{23}] \neq 0$, the particles are said to obey non-Abelian braiding statistics. The final state depends not only on which particles braid, but also the order in which these braids are performed.

The state of a topological qubit is thus encoded in the ground state manifold of a non-Abelian system. Qubit operations are performed in a non-local manner by braiding the
non-Abelian quasiparticles. The operations depend only on the topology of the braid, that make them robust to local perturbations [10].

Several systems have been proposed to host non-Abelian anyons [9], such as the $v=5/2$ fractional quantum Hall effect and $p$-wave superconductivity, but so far there is no experimental proof of such particles. In these systems the non-Abelian quasiparticles take the form of Majorana zero-modes [10].

1.3. REALIZATIONS
Proposals to create Majorana zero-modes in solid state systems nearly all follow the same recipe [11]. Electrons are confined in reduced dimensional structures (2D or 1D) and are stripped from their spin degree of freedom. These ‘spinless’ fermions are then forced to pair, resulting in a $p$-wave superconducting state. Such a state can be mapped to the celebrated Kitaev chain [10] which is at the hart of many Majorana proposals [12–14]. Defects in this state, end-points or vortices, hold localized Majorana operators.

For this thesis two proposals are relevant. The first involves the fractional quantum Hall effect and the second a 2D topological insulator. These proposals will be discussed in subsequent sections.

1.3.1. THE FRACTIONAL QUANTUM HALL EFFECT
The first system which non-Abelian quasiparticles were predicted is the fractional quantum Hall effect at filling factor $v=5/2$. Moore and Read [15] proposed a trial wavefunction for this peculiar even denominator$^1$ state which possesses non-Abelian excitations. The $v=5/2$ state occurs in a 2D electron system subject to a strong magnetic field. The Moore-Read trial wavefunction can best be understood in terms of the composite fermion theory. In this theoretical description the problem of interacting electrons in a magnetic field is mapped to a problem of composite fermions that do not feel any external field. A composite fermion is an electron to which two flux quanta of magnetic field are attached. The trial wavefunction lets the composite fermions pair and condense into a Bose-Einstein condensate. The strong external magnetic field ensures that all electron spins point in the same direction, along the external field. Pairing is then only possible if the relative angular momentum per pair is odd, being $p$-wave pairing.

Shifting the magnetic field slightly away from the $v=5/2$ state introduces vortices. These vortices are the non-Abelian excitations that carry Majorana operators and have a fractional charge of $e/4$. It should be noted that the Moore-Read wavefunction is not the only candidate put forward. Some other candidates [17] have Abelian (non-topological) excitations, therefore experimental effort is ongoing to determine the true state [18–21]. If the $v=5/2$ is indeed a non-Abelian state, a topological qubit can be made by means of an electronic analogue to the Fabry-Perot interferometer [22]. The $v=5/2$ state is a very

$^1$A state at an even denominator filling factor is strange in that it could not be explained by the well established composite fermion picture [16].
1.3.2. 2D TOPOLOGICAL INSULATORS
Two dimensional insulators (2D TIs) proposed by Kane and Mele in 2005 [23] are materials that have a bulk energy gap but conductive modes along the perimeter of the material. These edge modes persist at energies inside the bulk energy gap where states in the interior of the material are forbidden. The edge modes have another interesting property, the spin is coupled to the momentum, i.e., left moving electrons have spin-up and right moving spin-down, a property dubbed helical [24, 25]. Two dimensional topological insulators were first predicted in graphene [23], however the bulk gap was too small to observe the effect. In 2006 Bernevig, Huges and Zhang proposed quantum wells of HgTe/CdTe to be a 2D TI [26] and in 2008 Liu et al. did the same prediction for InAs/GaSb/AlSb double quantum wells [27]. By interfacing a 2D TI with a superconductor a \( p \)-wave superconducting state in the edge modes is created [28, 29].

1.4. AIM AND CONTENTS OF THIS THESIS
This thesis includes two research directions both aim to discover a building block for topological quantum computing. First, in Chapter 3, a novel setup is designed, built, and tested, that can electrostatically gate a material without endangering the materials pristine quality. The setup was designed with the goal to reveal non-Abelian quasiparticles in the \( \nu=5/2 \) fractional quantum Hall effect. To stabilize these particles, material quality should be as high as possible.

Second, and the larger part of this thesis, is devoted to InAs/GaSb double quantum wells. They are investigated for their possible use in topological quantum bits. Unprecedented material growth allowed us to investigate this material beyond existing (gate) bounds. By interfacing this material with superconducting leads we set the first step towards topological quantum computing in this material. However, we also found topological-like signatures arising from trivial mechanisms.

The remainder of this thesis is structured as follows:

- In Chapter 2 a brief theoretical overview is provided to understand successive chapters. Section 2.1 starts with describing two-dimensional systems of electrons and/or holes subjected to electric and magnetic fields. The framework to describe quantum transport, the Landuaer-Büttiker approach, is explained in section 2.1.3. Spin-orbit interaction is reviewed in section 2.1.4, which is of importance to Chapter 6. Next, in section 2.2 the InAs/GaSb material system is introduced together with the arguments why, and under which circumstances, this system is believed to be a two-dimensional topological insulator. This background is important for understanding Chapters 4 and 5. Finally, in section 2.3 the theory behind interferometry in quantum Hall edge channels is reviewed, relevant to Chapter 3.
• Chapter 3 describes a setup designed to gate materials without harming their pristine properties. Preserving the materials high-quality is advantageous to stabilize topological states in the fractional quantum Hall effect.

• Chapter 4 presents measurements on a InAs/GaSb quantum well system using superconducting leads.

• Chapter 5 presents measurements that establish the key feature of the InAs/GaSb material system: the system can be tuned from a trivial insulator to a topological insulator by means of electrostatic gating.

• Chapter 6 presents measurements of spin-orbit interaction in InAs/GaSb quantum wells.

• Chapter 7 summarizes the experimental findings, presents an improved flip-chip setup and presents routes towards signatures of helical edge conduction in InAs/GaSb double quantum wells studied in this thesis.
2

Theory
This chapter starts in section 2.1 with an introduction to the realm of physics in two dimensions, including the classical and quantum mechanical behavior of particles in electric and magnetic fields. Then, in section 2.2, the InAs/GaSb material system is reviewed including the conditions under which this material becomes a 2D topological insulator. Lastly, section 2.3 covers electron interference in a Fabry-Perot interferometer in the quantum Hall regime.

2.1. TWO DIMENSIONAL SYSTEMS
In a two-dimensional system carriers are bound to move in just two directions, labeled \((x, y)\), and are confined in the perpendicular \(z\)-direction. Experimental realizations of a 2D system quench the movement in the \(z\)-direction by using a thin layer of material, such that only the lowest subband in \(z\)-direction is energetically accessible.

Electrical transport in large samples can be captured using Drude’s theory; electrons or holes are accelerated in an electric field until they undergo a scattering event that randomizes their direction of momentum \([30]\). The momentum relaxation time \(\tau_D\) is the average time between two of such scattering events. The particle movement is captured by the following equation of motion

\[
m^* \left(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau_D}\right) = q\vec{E},
\]

(2.1)

where \(\vec{v}\), \(q\) and \(m^*\) are the velocity, charge and effective mass of the carrier, respectively and \(\vec{E}\) is the driving electric field. At steady state, \(d\vec{v}/dt = 0\), carriers experience a drift velocity

\[
\vec{v}_D = \frac{q\tau_D}{m^*} \vec{E}.
\]

(2.2)

The proportionality constant between drift velocity \(\vec{v}_D\) and electric field \(\vec{E}\) is called mobility \(\mu = |q|\tau_D/m^*\).

The conductivity of such a sheet is the proportionality constant between current density \(\vec{j}\) and electric field \(\vec{E}\). Using \(\vec{j} = nq\vec{v}_D\) the conductivity \(\sigma_0\) reads

\[
\sigma_0 = \frac{nq^2\tau_D}{m^*} = n|q|\mu.
\]

(2.3)

The resistivity is obtained by inverting the conductivity \(\rho = \sigma^{-1}\).

A standard device geometry used to characterize a 2D system is a Hall bar device depicted in Fig. 2.1. The device has two contacts at top and bottom end (1-2) between which a current is forced to flow. Between contacts 3-5 the longitudinal voltage is measured and between contacts 4-6 the transverse, or Hall, voltage is measured. The longitudinal resistance of a sample is given by

\[
R_{xx} = \frac{V_{3-5}}{I_{1-2}} = \rho \frac{W}{L},
\]

(2.4)
where $W$ is the width of the sample and $L$ the distance between the voltage probes 3-5. Peculiar for 2D systems is that the resistance of a square ($W = L$) is constant, no matter its size (a direct consequence of eq. 2.4). The resistance of a sample is thus proportional to the number of squares between the voltage probes.

In a quantum mechanical treatment the free 2D electron states are described by plane waves $\psi_{k,n}(\vec{r}) = e^{-i(k_x x + k_y y)} \psi_n(z)$ with energy given by

$$E_n(k) = -\frac{\hbar^2}{2m^*} \left( k_x^2 + k_y^2 \right) + E_z(n), \quad (2.5)$$

where $E_z(n)$ is the energy of $n^{th}$ subband in the $z$-direction. Every $k$-value with an energy below the Fermi-energy ($E_F$) is occupied by an electron. The Fermi energy $E_F$ and wavevector $|k_F|$ are related to the electron density $n_s$ according to

$$k_F = \sqrt{2\pi n_s}, \quad E_F = \pi \frac{\hbar^2}{m^*} n_s. \quad (2.6)$$

Thus, for 2D systems $k_F$ and $n_s$ are directly related to each other without reference to the underlying bandstructure. The 2D density of states (DOS) is

$$DOS(E) = \frac{dN}{dE} = \frac{m^*}{\pi \hbar^2}, \quad (2.7)$$

and does not depend on energy, which is special for electrons in two dimensions.

**Figure 2.1:** Hall bar with standard four terminal measurement configuration. The current is applied between contacts 1 and 2 while the longitudinal voltage is measured between contacts 3 and 5 and the Hall voltage is measured between contacts 4 and 6. A magnetic field is applied perpendicular to the sample. Arrows in the sample indicate the current distribution at zero magnetic field. Figure adapted from [31].
2.1.1. Classical Magnetoresistance

In the presence of a magnetic field $\vec{B}$ a Lorentz force acts on the particles, in addition to the driving electric field $\vec{E}$. The equation of motion becomes

$$m^*(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau_D}) = q(\vec{E} + \vec{v} \times \vec{B}). \quad (2.8)$$

For very clean samples, which have a long $\tau_D$, equation 2.8 describes a circular motion for electrons with the cyclotron frequency

$$\omega_c = \frac{eB}{m^*}. \quad (2.9)$$

This clean regime is the subject of the next section. Here we will focus on the classical regime where $\tau_D$ is short, such that the particle is scattered before it can complete one full circle, i.e., $\omega_c \tau_D = \mu B \ll 1$.

Again we seek for the steady state situation $d\vec{v}/dt = 0$, but now with a perpendicular magnetic field $B = (0, 0, B_z)$. Rearranging eq. 2.8 gives

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \frac{m^*}{e\tau_D} \begin{bmatrix} v_x \\ v_y \end{bmatrix} - \begin{bmatrix} v_y B_z \\ -v_x B_z \end{bmatrix}. \quad (2.10)$$

Using $\vec{j} = nq\vec{v}$ the above set of equations can be written as

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} m/q^2\tau_D n \\ B_z/nq \end{bmatrix} \begin{bmatrix} -B_z/nq \\ m/q^2\tau_D n \end{bmatrix} \begin{bmatrix} j_x \\ j_y \end{bmatrix} = \rho \vec{j}. \quad (2.11)$$

Thus, in the presence of $B_z$, $\vec{j}$ and $\vec{E}$ need not be parallel and their relation is described by the resistivity tensor $\rho$, which is given in shorthand notation by

$$\rho = \rho_0 \begin{bmatrix} 1 & -\omega_c\tau_D \\ \omega_c\tau_D & 1 \end{bmatrix}, \quad (2.12)$$

with $\rho_0 = 1/\sigma_0 = m^*/(nq^2\tau_D)$. The diagonal elements of the resistivity tensor are equal and even functions of $B$. The off-diagonal elements are equal to each other in magnitude but opposite in sign, being odd functions of $B[32]$. The conductivity is the reciprocal of resistivity, $\sigma = \rho^{-1}$, and using

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \quad \rho_{xy} = \frac{-\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2},$$

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \quad \sigma_{xy} = \frac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}, \quad (2.13)$$

the conductivity tensor can be written as

$$\sigma = \frac{\sigma_0}{1 + (\omega_c\tau_D)^2} \begin{bmatrix} 1 & -\omega_c\tau_D \\ \omega_c\tau_D & 1 \end{bmatrix}. \quad (2.14)$$
Returning to the Hall bar device depicted in Fig. 2.1 we are now in the position to derive the Hall voltage between terminals 4-6. At steady state the current in \( y \)-direction should vanish, \( j_y = 0 \), while in the \( x \)-direction a constant current is enforced by the current source. From eq. 2.11 we see that with these constraints the electric field in \( y \)-direction must be

\[
E_y = -\frac{B_z}{nq} \cdot j_x,
\]

which is called the Hall effect. The physical picture of this steady state situation is that the Lorentz force is canceled by the force due to the Hall electric field of eq. 2.15. Multiplying both sides by the width \( W \) of the sample, the Hall voltage \( V_{\text{Hall}} = WE_y \) and current \( I = W j_x \) are related by the Hall resistance

\[
R_{xy} = \frac{V_{\text{Hall}}}{I} = \frac{B_z}{nq}.
\]

From measuring the Hall resistance as a function of magnetic field \( B \) the carrier type and density can be extracted.

### Magnetoresistance for Two Carriers

In previous sections the conductivity and resistivity were calculated assuming one carrier type. However, there are situations where several carriers contribute to the current. More than one subband may be occupied, or, as we will show in chapter 5 in some materials electrons and holes may coexist. The most striking consequence of two carrier transport is that the Hall resistance is no longer a linear function of \( B_z \). The following model extends Drude’s model to multiple independent carriers, i.e. carriers without any coupling between them.

Independent carriers respond in the same way to external electric fields, so that the total current density is found by adding the individual conductivity tensors \( \vec{j} = \sum_i \sigma_i \vec{E} \). Therefore the components of the conductivity tensor become [33]

\[
\sigma_{xx} = \sum_i \frac{n_i |q_i| \mu_i}{1 + (\mu_i B)^2},
\]

\[
\sigma_{xy} = \sum_i \frac{n_i q_i \mu_i^2 B}{1 + (\mu_i B)^2}.
\]

Note that in \( \sigma_{xx} \) the absolute value of the charge is used because holes and electrons constitute a current in the same direction upon an applied bias voltage. For \( \sigma_{xy} \) however the sign of \( q_i \) is important because it results from the Lorentz force \( \vec{F}_L = q(\vec{v} \times \vec{B}) \). As in the single carrier case, the net transverse current should vanish in the steady state, \( j_y = \sum_i n_i |q_i| v_{i,y} = 0 \), when driving current in \( x \)-direction. Although the net current vanishes in \( y \)-direction, currents carried by a particular carrier \( j_{i,y} \) may be non-zero.

Consider the specific case where electrons and holes coexist. The conductivity tensor can be built using eq. 2.17 where electrons are characterized by the set \( \{ q = -e, \mu_e, m_e^*, n_e \} \)
and holes by \( \{ q = +e, \mu_h, m_h^*, n_h \} \). The resistivity tensor components are obtained by inversion of the conductivity tensor (using eq. 2.13) and read
\[
\rho_{xx} = \frac{1}{e} \frac{n_e \mu_e (1 + \mu_h^2 B^2) + n_h \mu_h (1 + \mu_e^2 B^2)}{(n_e \mu_e + n_h \mu_h)^2 + \mu_e^2 \mu_h^2 B^2 (n_e - n_h)^2},
\]
\[
\rho_{xy} = \frac{1}{e} \frac{n_e \mu_e^2 B (1 + \mu_h^2 B^2) - n_h \mu_h^2 B (1 + \mu_e^2 B^2)}{(n_e \mu_e + n_h \mu_h)^2 + \mu_e^2 \mu_h^2 B^2 (n_e - n_h)^2}.
\]
(2.18)

As before, forcing a current in \( x \)-direction, \( j = (j_x, 0) \), the longitudinal resistance becomes \( R_{xx} = \rho_{xx} W/L \) and the Hall resistance is \( R_{xy} = \rho_{xy} \). The net transverse current is zero because the individual currents for electrons and holes are equal and opposite, \( j_y = e n_e v_{e,y} + e n_h v_{h,y} = 0 \).

2.1.2. THE QUANTUM HALL REGIME

The above section describes the diffusive case where electrons are scattered before they can complete a full orbit. In the regime where carriers complete at least one full orbit before being scattered, i.e. \( \omega_c \tau_D = B \mu > 1 \), interesting quantum effects arise. Assuming the magnetic field is applied along the \( z \)-direction the Hamiltonian for electrons in the 2D plane can be written as
\[
H = - \left( \hat{p} + e \hat{A} \right)^2 + \frac{1}{2} g \mu_B B z S_z.
\]
(2.19)

The first term is the generalized momentum of a particle subjected to a magnetic field. The second term describes the Zeeman energy for a particle with a spin. Using the Landau gauge \( \hat{A} = (0, B_x, 0) \) diagonalization of the Hamiltonian of eq. 2.19 gives an energy spectrum \[32, 34\]
\[
E_{n\pm} = \hbar \omega_c \left( n + \frac{1}{2} \right) \pm \frac{1}{2} g \mu_B B. \quad n = 0, 1, 2, \ldots
\]
(2.20)

The presence of a magnetic field splits the energy spectrum in discrete Landau levels (LL) labeled by \( n \), which are separated \( \hbar \omega_c \) in energy. Zeeman energy splits the the LLs further into a spin-up (\( + \)) polarized and spin-down (\( - \)) polarized state. The LL energies disperse linearly in \( B \) (see eq. 2.9). The wavefunctions corresponding to the eigenstates of eq. 2.20 are
\[
\phi_{nk\pm} \propto H_n \left( \frac{x-x_k}{l_B} \right) \exp \left[ -\frac{(x-x_k)^2}{2l_B^2} \right] \exp(iky)|\pm\rangle, \quad n = 0, 1, 2, \ldots
\]
(2.21)

where \( H_n \) are Hermite polynomials, \( x_k = -\hbar k / eB \) and \( l_B = \sqrt{\hbar / |eB|} \) is the magnetic length (which is 26 nm at \( B = 1 \) T).
Interestingly, the energy levels (eq. 2.20) do not depend on \( k \). They are therefore highly degenerate. Their degeneracy is given by the number of allowed \( k \) values, which depends on sample size \((L_x, L_y)\). The wavefunction in eq. 2.21 has plane waves in y-direction, and the assumption of periodic boundary conditions gives quantization in \( k \) of \( \Delta k_y = \frac{2\pi}{L_y} \).

Furthermore, requiring that \( x_k \) lies within \( L_x \) sets the number of degenerate states to

\[
N_L = \frac{eB}{h} \text{A}/h,
\]

or

\[
\nu = \frac{n_s}{n_L} = \frac{n_s\phi_0}{B}.
\]

The density of states (DOS) in perfect materials at zero temperature consists of \( \delta \)-functions at energies given by eq. 2.20 with an area given by eq. 2.22. In reality however the \( \delta \) functions are broadened due to scattering events. An electron spends a finite time, the quantum lifetime \( \tau_q \), in a quantum state\(^1\). The energy \( E \) can therefore only be de-

\(^1\)The quantum lifetime \( \tau_q \) is usually smaller than the transport lifetime \( \tau_D \) defined in section 2.1. Whereas \( \tau_D \)
fined with a precision of \( \Gamma = \hbar / \tau_q \), which broadens the \( \delta \) functions by \( \Gamma \). Assuming a Gaussian broadening\(^2\) the DOS can be written as [32]

\[
D(E, B) = \sqrt{\frac{2}{\pi \Gamma^2}} \sum_{n \pm} \left( \frac{eB}{\hbar} \right) \exp \left( -2 \frac{(E - E_{n \pm})^2}{\Gamma^2} \right).
\]

Figure 2.2 depicts the DOS as a function of \( E \) and \( B \). The energy scales corresponding to the LL splitting \( \hbar \omega_c \), the Zeeman energy \( g \mu_B B \), and the broadening \( \Gamma \) are indicated.

In experiments the perpendicular magnetic field is often swept while the electron density \( n_s \) is held constant. The Fermi energy \( (E_F) \) is then found by integrating the DOS until the sample density is found,

\[
n_s = \int_{-\infty}^{E_F} D(E) \, dE.
\]

Figure 2.2 indicates the \( E_F \) for \( n_s = 5 \times 10^{15} \) m\(^{-2}\) with a black line. It is seen that \( E_F \) fluctuates in energy as a function of magnetic field. Moreover, the Fermi-energy passes regions of high DOS in the midst of the LLs \((\nu = n + 1/2)\) and regions of low DOS in-between the LLs \( \nu = n \). This results in oscillatory behavior of \( \text{DOS}(E_F) \) in \( 1/B \) (see eq. 2.23). Material properties like conductance and magnetization are proportional to \( \text{DOS}(E_F) \) and therefore also display the same periodic behavior as a function of magnetic field. The next section describes the oscillatory conductivity, also known as Shubnikov-de Haas oscillations, in more detail.

**The Shubnikov-de-Haas Effect**

The oscillatory longitudinal resistivity of a sample as a function of magnetic field shown in Fig. 2.3 is called the Shubnikov-de Haas (SdH) effect. The low field \( (\hbar \omega_c \ll E_F) \) magnetoresistivity can be described in an analytical expression [35, 36]. Starting point is the Drude conductivity of a 2D system rewritten as a Fermi-level property \( \sigma = e^2 \mathcal{D} D(E_F) \) [34], where \( \mathcal{D} \) is the diffusion coefficient and \( D(E_F) \) is the density of states at the Fermi level. To calculate the magnetoconductivity one needs to obtain the \( D(E) \) from Landau levels convolved with some specific broadening function, depending on the type of scatterers. The resulting expression is expanded in terms of \( \cos(2\pi E/\hbar \omega_c) \) and integrated over the Fermi-Dirac distribution in the leads [32]. The result as derived in [35] are given by

\[
\rho_{xx}(B, T) = \frac{1}{\sigma_0} \left( 1 - 4 \cos \left( \frac{\pi h}{2eB} n_s B \right) \frac{2\pi^2 k_B T}{\hbar \omega_c} \sinh \left( \frac{2\pi^2 k_B T}{\hbar \omega_c} \right) e^{-\pi \tau_q \omega_c} \right).
\]

Here \( \sigma_0 \) is the conductivity at zero magnetic field, which contains the transport scattering time \( \tau_D \).

\(^2:\) weights scattering events with large momentum difference more heavily [32, 34], \( \tau_q \) is equally sensitive to all scattering events. Even small angle scattering destroys the coherence and therefore contributes to \( \tau_q \).

\(^2:\) The particular shape of the broadened Landau levels, however, is still under debate [32].
Three useful quantities can be extracted from the SdH oscillations [36, 37]. First, the density \( n_s \) is determined from the period of \( \rho_{xx}(1/B) \) provided by the cosine term. Second, the effective mass \( m^* \) can be extracted from the temperature dependence of the amplitude of the oscillations (term next to the cosine). Third, when the effective mass is known, the quantum lifetime \( \tau_q \) can be extracted from the exponential term.

Figure 2.3: Simulated of the longitudinal resistivity \( \rho_{xx} \) as a function of perpendicular magnetic field \( B_\perp \) of a two dimensional electron gas. At low fields \( B < 0.5 \) the oscillatory part is described by eq. 2.26. At higher magnetic field the integer quantum Hall sets in. Around \( B \sim 1 \) T the resistance peaks split, indicating that the Zeeman energy exceeds the disorder broadening. The calculation uses an effective mass of \( m^* = 0.04m_0 \), a g-factor of \( g = -10 \), an electron density of \( n_s = 5 \times 10^{15} \text{ m}^{-2} \), and a broadening of \( \Gamma = 0.3 \) meV.

2.1.3. EDGE TRANSPORT: THE LANDAUER-BÜTTIKER FORMALISM

The Landauer-Büttiker formalism describes a nanostructure using its Fermi level properties. The formalism divides a mesoscopic sample into three parts: contacts, leads and scattering region (see Fig. 2.4). The contacts are reservoirs for electrons that are in thermal equilibrium and perfectly absorb all impinging electrons. The leads are perfect waveguides between the contact and scattering region and contain 1D modes. Finally the device is characterized by a scattering matrix that relates outgoing waves from the scattering region to impeding waves from the contacts. The wave function in lead \( \alpha \) (oriented along the x-axis) is given by

\[
\psi(x, y, z) = 1/\sqrt{L} \sum_n \Phi_n(y, z) \left( a_n e^{i k_n x} + b_n e^{-i k_n x} \right),
\]

where the waves impinging the scattering region have amplitude \( a \) and the outgoing waves have amplitude \( b \). The outgoing waves are the result of reflection or transmission through the scattering region and relate to the incoming waves via the scattering matrix \( \hat{s} \),

\[
b_{\alpha n} = \sum_{\beta m} s_{\alpha n, \beta m} a_{\beta m},
\]

where the sum is over all leads \( \beta \) and the modes in such lead. The scattering matrix is unitary \( (\hat{s} \hat{s}^\dagger = 1) \), as charge is conserved. At zero magnetic field the scattering matrix is
symmetric ($s = s^T$) due to time reversal symmetry.

A wave leaving contact $\alpha$ (towards the scattering region) constitutes a current in the lead $\alpha$ of

$$I^\text{out}_\alpha = \sum_n \int_0^\infty -e\nu_n\rho_n f(E - \mu_\alpha) dE = -\frac{e}{\hbar} N\mu_\alpha,$$

where $\mu_\alpha$ is the chemical potential of lead $\alpha$ and $N$ the total number of modes in lead $\alpha$. The outgoing current is only determined by the chemical potential of the contact and the number of modes in the lead. The current flow towards contact $\alpha$ is

$$I^\text{in}_\alpha = -\frac{e}{\hbar} \mu_\alpha R_\alpha - \frac{e}{\hbar} \sum_{\beta \neq \alpha} T_{\beta \rightarrow \alpha} \mu_\beta,$$  

where the first term describes the current from lead $\alpha$ being reflected back from the scattering region. The second term describes current being transmitted via the scattering region from the other leads $\beta$ with chemical potential $\mu_\beta$. The reflection coefficient is

$$R_\alpha = \sum_n \sum_m |s_{\alpha n, \alpha m}|^2,$$

and the probability for an electron in lead $\beta$ to be transmitted to lead $\alpha$ is

$$T_{\beta \rightarrow \alpha} = \sum_n \sum_m |s_{\alpha n, \beta m}|^2.$$

The net current for an arbitrary lead $\alpha$ is the sum of incoming (eq. 2.30) and outgoing (eq. 2.29) currents,

$$\frac{\hbar}{e} I_\alpha = (N_\alpha - R_\alpha) \mu_\alpha - \sum_\beta T_{\beta \rightarrow \alpha} \mu_\beta.$$

In the next section we apply the Landauer-Büttiker formalism to the transport in the quantum Hall regime with chiral edge states. This is followed by a section on transport in the quantum spin Hall regime with helical edge states.

Figure 2.4: a) Hall bar with chiral edge states. b) Hall bar with helical edge states. c) Network equivalent of the QSH edge states of panel b. For quantum spin Hall edge channels $R_0 = \hbar/e^2$, while for trivial edge channels $R_0 = \hbar/(2e^2)$. 
2.1. TWO DIMENSIONAL SYSTEMS

QUANTUM HALL EDGE STATES

In this section we use the Landauer-Büttiker approach to calculate the four-terminal resistance for a standard six terminal Hall bar presented in Fig. 2.4a in the quantum Hall regime.

Assume the magnetic field is tuned such that \( \nu \) LLs are filled, resulting in \( N = \nu \) chiral edge channels around the perimeter of the sample, as depicted in Fig. 2.4. A current \( I \) is forced between the source (\( \mu_s \)) and drain (\( \mu_d = 0 \)) contacts. The other contacts are perfect voltage probes with zero net current. The system of equations eq. 2.33 now simplifies substantially, because the quantum Hall edge channels have unit transmission \( (T = 1) \) and only nearest neighbors directly contact each other:

\[
\begin{bmatrix}
  I \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  N & 0 & 0 & 0 & 0 & -N \\
  -N & N & 0 & 0 & 0 & 0 \\
  0 & -N & N & 0 & 0 & 0 \\
  0 & 0 & -N & N & 0 & 0 \\
  0 & 0 & 0 & -N & N & 0
\end{bmatrix}
\begin{bmatrix}
  \mu_s \\
  \mu_1 \\
  \mu_2 \\
  \mu_3 \\
  \mu_4
\end{bmatrix} = \begin{bmatrix}
  \frac{h}{e} \\
-1
\end{bmatrix}.
\]

Solving eq. 2.34 gives the two-terminal resistance

\[
R_{2t} = \frac{\mu_s - \mu_d}{I} = \frac{1}{N} \left( \frac{h}{e^2} \right),
\]

where \( h/e^2 = 25,812.807 \) \( \Omega \) is the resistance quantum (or von Klitzing constant) [38]. The four-terminal longitudinal resistance and Hall resistance become

\[
R_{4t} = (\mu_2 - \mu_1)/I = 0,
\]

\[
R_{\text{Hall}} = (\mu_1 - \mu_4)/I = \frac{h}{N \left( e^2 \right)}.
\]

An important consequence of the chirality of the edge channels is that the longitudinal resistance vanishes, no matter how many voltage probes are provided along the perimeter.

QUANTUM SPIN HALL EDGE STATES

The quantum spin Hall state can be viewed as a sum of two chiral edge states, of which one propagates clockwise and the other counter clockwise (see Fig. 2.4b). The spin orientation does not enter the transport equations. The system of equations 2.33 is

\[
\begin{bmatrix}
  I \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  2 & -1 & 0 & 0 & 0 & -1 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
  0 & -1 & 2 & -1 & 0 & 0 \\
  0 & 0 & -1 & 2 & -1 & 0 \\
  0 & 0 & 0 & -1 & 2 & -1
\end{bmatrix}
\begin{bmatrix}
  \mu_s \\
  \mu_1 \\
  \mu_2 \\
  \mu_3 \\
  \mu_4
\end{bmatrix} = \begin{bmatrix}
  \frac{h}{e} \\
-1
\end{bmatrix}.
\]
The two-terminal resistance obtained by solving eq. 2.38 is

\[ R_{2t} = \frac{(\mu_s - \mu_d) e I}{\frac{3}{2} \frac{h}{e^2}}. \] (2.39)

The four terminal and Hall resistances become

\[ R_{4t} = \frac{(\mu_2 - \mu_1) e I}{\frac{1}{2} \frac{h}{e^2}}, \] (2.40)

\[ R_{\text{Hall}} = \frac{(\mu_1 - \mu_3) e I}{0}. \] (2.41)

The electric equivalent of the QSH state in Fig. 2.4b is the resistor network presented in Fig. 2.4. Each edge, no matter how long, can be replaced by a resistance of \( R_0 = h/2e^2 \). In contrast to the QH edge states, the resistance that is measured depends on the number of contacts between source and drain.

**TRIVIAL EDGE CHANNELS**

Trivial edge channels with transmission \( T = 1 \) can be seen as the sum of two quantum spin Hall states with opposite spin directions. The matrix elements in eq. 2.38 are doubled and the system can be casted into the resistor network of Fig. 2.4 with individual resistances of \( R_0 = h/(2e^2) \), instead of \( h/e^2 \) for helical edge channels. The Hall resistance is therefore zero while the four terminal resistance equals \( R_{4t} = h/(4e^2) \).

However, it is expected that the lack of protection against backscattering results in transmission values smaller than unity, \( T < 1 \), for trivial edge channels (see section 2.2.4). Furthermore, when severe disorder is present they can easily be gapped out.

**2.1.4. SPIN-ORBIT COUPLING**

Spin orbit (SO) interaction originates from the non-relativistic approximation to the relativistic Dirac equation together with the Darwin and Zeeman terms [39]. The intuitive picture of SO interaction can be found in atomic physics where an electron feels the electric field of the nucleus while orbiting around it. In the rest frame of the electron this electric field Lorentz transforms into a magnetic field, which is felt by the spin of the particle. This leads to a Pauli SO term in the Hamiltonian [39]

\[ H_{SO} = -\frac{\hbar}{4m_0^2c^2} \mathbf{\sigma} \cdot \mathbf{p} \times \nabla V, \] (2.42)

where \( m_0 \) is the mass of a free electron, \( c \) is the speed of light, \( \mathbf{p} \) the momentum operator, \( V(\mathbf{r}) \) the Coulomb potential of the atomic cores, and \( \mathbf{\sigma} \) is the vector of Pauli matrices. Cores with a higher atomic number have stronger electric fields (\( \mathbf{E} = \nabla V \)) and therefore provide stronger SO coupling.

**SPIN-ORBIT IN A SEMICONDUCTOR**

The Pauli equation (eq. 2.42) describes an electron moving in vacuum in the presence of a potential \( V \). However, the same equation also holds for an electron traveling in a
2.1. TWO DIMENSIONAL SYSTEMS

crystal, where the electron feels a (periodic) crystal potential $V_0(r)$. The Hamiltonian then becomes [40]

$$\left( \frac{p^2}{2m_0} + V_0(r) - \frac{\hbar}{m_0 c^2} \mathbf{\sigma} \cdot \mathbf{p} \times \nabla V_0 + V(r) \right) \psi = E \psi. \quad (2.43)$$

The term $V(r)$ captures external potentials, e.g. from a gate or donors in a quantum well. This external potential is neglected in the SO term because the largest gradient comes from the crystal potential. The external potential however can cause an inversion asymmetry leading to a Rashba type spin orbit interaction.

The first two terms in equation 2.43 describe electrons in a periodic crystal lattice. Because of the periodicity of the crystal, the natural basis functions to solve this equation are the Bloch functions, $\psi_{n,k} = u_{n,k}(r)e^{ikr}$, where the index $n$ is the band index. The solution to eq. 2.43 can be approximated by inserting the Bloch wave functions and treating the resulting $k.p$ term as a perturbation. The result already gives some features of the band structure, where the conduction band is separated from the valence band by an energy gap $E_0$. The conduction band states (basis state $|S, \pm\rangle$) originate from the $s$ type atomic orbitals and are spin degenerate. The valence band states originate from $p$-type orbitals constitute (including spin) a six fold degenerate basis $\{|X, \pm\rangle, |Y, \pm\rangle, |Z, \pm\rangle\}$.

The SO term mixes these bands. They are then characterized by their total angular momentum numbers ($j$ and $m_j$) and the orbital momentum $l = 0$ ($l = 1$) for conduction (valence) bands. This has a profound effect even at $k = 0$, namely that the states characterized by $j = 1/2, m_j = \pm 1/2$ split off by an energy $\Delta_0$ (see Fig. 2.5), with

$$\Delta_0 = -\frac{3i\hbar}{4m^2 c^2} \langle X|\nabla V_0 \times \mathbf{p}|Z \rangle. \quad (2.44)$$

The SO split off band is thus obtained by coupling between the valence band states.

The idea of the $k.p$ approach is that the band edge eigenstates form a complete basis. To obtain eigenstates at finite $k$ we expand the wave function in the band edge states. Kane found that inclusion of just a few bands around the band gap is enough to capture the main physics [41]. In the original Kane model only the bands $\{S\pm, X\pm, Y\pm, Z\pm\}$, see Fig. 2.5, were taken into account, leading to the $8 \times 8$ Hamiltonian

$$H = \begin{bmatrix} H_{cc} & H_{cv} \\ H_{vc} & H_{vv} \end{bmatrix}. \quad (2.45)$$

Here, $H_{cc}(H_{vv})$ is the block of the conduction (valence) band states ($H_{vv}$ contains the $\Delta_0$ terms). The elements in $H_{cv}$ couple valence band (p-type) with conduction band (s-type) states and are therefore necessarily linear in $k$ with proportionality constant

$$P_0 = \frac{\hbar}{m} \langle S|p_x|X \rangle. \quad (2.46)$$
2. Theory

Figure 2.5: a) Heterostructure consisting of a bottom barrier \( l \), a quantum well \( c \) of thickness \( d \) and the top barrier \( r \). An external field tilts the band structure thereby providing structural inversion asymmetry. b) The bands that participate in the \( 8 \times 8 \) Kane model are the \( \Gamma_{6c} \) conduction band, the \( \Gamma_{8v} \) heavy hole (HH) and light hole (LH) bands and the \( \Gamma_{7v} \) spin-orbit split-off band.

We are interested in describing the electrons in the conduction band without having to refer to the valence band states. Such effective conduction band Hamiltonian can be obtained using a unitary transformation \( U \) such that

\[
U H U^\dagger = \begin{bmatrix}
\tilde{H}_{cc} & 0 \\
0 & \tilde{H}_{\nu v}
\end{bmatrix},
\]

where \( \tilde{H}_{cc} \) is the effective Hamiltonian describing electrons in the conduction band [42]. In this description the mass of the conduction band states and the g-factor are renormalized to an effective mass \( m^* \) and effective g-factor \( g^* \) [32].

In general, the matrix elements \( P_0 \) and \( \Delta_0 \) are not calculated but chosen such that they give the best agreement with experimental data [39]. The energy gaps \( E_g \) and \( \Delta_0 \) can often be directly determined from experiments. The values for the \( P_0 \) matrix elements can be derived from relations involving \( m^* \) and \( g^* \).

When a crystal lacks spatial inversion symmetry new terms due to SO arise in the effective Hamiltonian \( H_{cc} \). Inversion asymmetry can be due to a non-centrosymmetrical lattice, like in zincblende crystals, or due to an electric field. The latter type is called structural inversion asymmetry (SIA) while the former type is bulk inversion asymmetry (BIA). In the following we assume that the electrons are confined in the z-direction by a quantum well structure grown in the (001) direction and review the effect of BIA and SIA on the electrons in the conduction band.
Dresselhaus spin–orbit in heterostructures

Bulk inversion asymmetry is characterized by the Dresselhaus term in the Hamiltonian, which in 2D systems takes the form [43]

\[ \tilde{H}^D_{cc} = \gamma \left[ \langle p_z^2 \rangle (-k_x \sigma_x + k_y \sigma_y) + (k_x k_y^2 \sigma_x - k_y k_x^2 \sigma_y) \right] . \] (2.48)

Here \( \gamma \) is the Dresselhaus SOI material parameter. The first term is linear in \( k \) and the second term cubic in \( k \). These terms are referred to as the linear and cubic Dresselhaus contributions, respectively. The average \( \langle p_z^2 \rangle = \langle \psi^*(z) | p_z^2 | \psi(z) \rangle \) describes the average over the wave function in z-direction. Usually \( \beta = \gamma \langle p_z^2 \rangle \) is given as parameter for the linear Dresselhaus term.

In quantum wells where only the lowest subband in z-direction is populated \( \langle k_z^2 \rangle \approx \left( \frac{\pi}{d} \right)^2 \), where \( d \) is the quantum well thickness [44]. For small \( k_{x,y} \) (and thus small density) the linear Dresselhaus term dominates. However, for \( \left( \frac{\pi}{d} \right)^2 \sim \frac{1}{4} |k_{x,y}|^2 \) the cubic term cannot be neglected.

Rashba spin–orbit in heterostructures

Structural inversion asymmetry arises from the confinement potential, created by internal or external electric fields and from band edge variations (like at heterointerfaces). The external fields are modeled by \( V(r) \) in eq. 2.43 and result in the Rashba term in the effective conduction band Hamiltonian (eq. 2.47) [45, 46]

\[ H^R_{cc} = \alpha_R (\sigma_x k_y - \sigma_y k_x) , \] (2.49)

where \( \alpha_R \) is the Rashba SOI parameter. The Rashba parameter has contributions from electric fields in the valence band \( E_v(z) \) and contributions from the interfaces of the quantum well, and can be written as

\[ \alpha_R = \langle \alpha_0(z) \rangle + \langle \alpha_{\text{int}} \rangle , \] (2.50)

where the \( \langle \ldots \rangle \) again denotes the averaging over the wavefunction in z-direction [40]. Consider the quantum well depicted in Fig. 2.5. Three regions of different materials can be distinguished, \( l, c, \) and \( r \). The Rashba parameter \( \alpha_0 \) as a function of \( z \) can be written as

\[ \alpha_0(z) = \alpha^l(z) \Theta(-z - d/2) + \alpha^c(z) \Theta(d/2 - |z|) + \alpha^r(z) \Theta(z - d/2) , \] (2.51)

where \( \Theta(z) \) is the Heaviside step function [40]. The Rashba parameter for region \( i \) is

\[ \alpha^i = \frac{P_0^2}{3} \left( \frac{1}{[E_{i0}^i + \Delta_{i0}^i]^2} - \frac{1}{[E_{i0}^i]^2} \right) \frac{dV_{\text{ext}}}{dz} , \] (2.52)

with \( P_0 \) the \( k.p \) interaction parameter, \( E_0 \) the band gap and \( \Delta_0 \) the split-off energy for region \( i \). It is apparent that the Rashba parameter is proportional to the external field in the region. The interface contribution to the Rashba term is

\[ \langle \alpha_{\text{int}} \rangle = \left( \beta^c - \beta^l \right) |\Psi^L|^2 - \left( \beta^c - \beta^r \right) |\Psi^R|^2 , \] (2.53)
where $\Psi^i$ is the amplitude of the wavefunction at the interface and

$$\beta^i \approx \frac{P_0^2}{3} \left( \frac{1}{E_0^i + \Delta_0^i} - \frac{1}{E_0^i} \right). \quad (2.54)$$

The Rashba SO interaction strength $\alpha_0$ of eq. 2.50 can thus be enhanced by a strong external electric field ($dV_{\text{ext}}/dz$) in the quantum well. Furthermore, eq. 2.52 reveals that increasing the ratio $\Delta_0/E_0$ (e.g. by reducing the band gap) also enhances the Rashba SO interaction.

However, previous studies show that the Rashba effect of the applied external electric field $\alpha_0$ is insufficient to account for the large SO splitting observed in experiments [47–50]. It is therefore speculated that the SO interaction due to the heterointerface $\alpha_{\text{int}}$ dominates in $\alpha_R$ and that tuning can be accomplished by pushing the wave function into the interfaces, as described by 2.53.

**LANDAU LEVELS IN THE PRESENCE OF SPIN-ORBIT COUPLING**

Spin-orbit interaction has a profound signature in the low field magnetoresistance: it introduces beating in the Shubnikov-de Haas (SdH) oscillations. Effectively the spin-orbit interaction causes a difference in density of spin-up electrons and spin-down electrons at the Fermi energy. This in turn gives rise to two SdH frequencies ($f_{\text{SdH}} \propto n$) which interfere with each other. From the node positions the SO strength can be estimated. Such a beat node occurs when the Fermi-energy cuts a LL of one spin species and at the same time is precisely in between two LL of the other spin species, mathematically written [51]:

$$E_F = E^+(n)$$

$$E_F = E^-(m) + \frac{1}{2} [E^-(m+1) - E^-(m)]. \quad (2.55)$$

Where $n,m$ are the LL index for the higher energy spin band ($n$) and the lower energy spin band ($m$). The above equation does not tell much about the zero-field splitting without knowing the precise dispersion relations $\{E^+_n(B), E^-_n(B)\}$. This requires knowledge of the underlying band structure. We consider here two explicit band structures for which the LL dispersion is calculated. The first is a spin splitting due to an in-plane Zeeman energy (used in tilted magnetic field experiments). Second, the LLs are calculated for a bandstructure with a Rashba type spin splitting.

The conduction band Hamiltonian in the presence of an in-plane magnetic field $B_{||}$ has the form

$$\tilde{H}_{cc} = \frac{\hbar^2 k^2}{2m^*} \pm \Delta_z/2, \quad (2.56)$$

where $\Delta_z = g\mu_B B_{||}$ is the Zeeman spliting between spin-up and spin-down electrons. The LL dispersion derived from the Hamiltonian reads

$$E^\pm(n) = (n+1/2)\hbar\omega_c \pm \Delta_z/2, \quad n = 0, 1, 2, \ldots \quad (2.57)$$
and is plotted in figure 2.6. Insertion of eq. 2.57 into the node criteria of eq. 2.55 gives a relation between Zeeman energy and cyclotron energy (∼1/B) at the node,

\[ m - n + 1/2 = \Delta_z / \hbar \omega_c. \]  

Here \( m \) and \( n \) are the LL indices of the split bands. The first beat node occurs when \((n - m) = 0\), i.e., the LL of the higher energy spin band crosses halfway in between two LLs of the lower energy spin band. Note that only the difference in LL index matters, not the absolute LL index. This means that the magnetic field value of the node \( (B_n) \) is independent of density. Changing the density only affects the number of oscillations between nodes (see Fig. 2.6a). By knowing \( B_n \) (and therefore \( \hbar \omega_c \)) and \((n - m)\) one can easily extract the Zeeman spin-splitting \( \Delta_z \) from eq. 2.58.

The beating has an envelope function with a period and frequency. The beat frequency is obtained from the period \( \delta (1/B) \) between two nodes:

\[ f_{\text{beat}} = \frac{m \Delta_z}{\hbar e}. \]  

This frequency will not give any spectral weight in a Fourier transform, because the envelope is not a real signal. The beat is created by two closely spaced frequency components.

In case of Rashba type spin orbit interaction, the LL dispersion is more complicated and reads [51]:

\[ E(0) = \frac{1}{2} \hbar \omega_c, \]

\[ E^\pm(n) = \hbar \omega_c \left( n \pm \frac{1}{2} \sqrt{1 + n \frac{\Delta_R^2}{E_F \hbar \omega_c}} \right), \quad n = 1, 2, \ldots \]
where $\Delta_R = 2\alpha_R k_F$ and this dispersion is depicted in Fig. 2.6b. Inserting eq. 2.60 into eq. 2.55 does not give a closed form expression like in the case of in-plane Zeeman spin splitting. Besides the difference $n - m$ now also the absolute values of $n$ and $m$ matter. However, for large $n$ the Fermi energy can be approximated to $E_f \approx n\hbar\omega_c$ and for low magnetic fields ($\Delta_R/\hbar\omega \gg 1$) eq. 2.60 simplifies to

$$E^\pm = n\hbar\omega_c \pm \frac{\Delta_R}{2}, \quad n \gg 1$$

which is similar to the in-plane Zeeman type (see eq. 2.58) for large $n$. The LL dispersion mimics the dispersion of parabolic bands shifted by $\Delta_R$ in energy. Therefore the two SdH frequencies are a good approximation to the Fermi-surface area at zero field, and hence a good approximation to the zero-field spin split densities. Analysis of this kind to extract Rashba spin splitting is used in [51–55].

Rashba SO interaction together with Dresselhaus SO interaction leads to a more complicated LL dispersions [56]. It is expected that at low magnetic fields the frequencies in the SdH oscillations still indicate the zero-field spin split densities in a manner similar to the pure Rashba SO interaction case.

### 2.2. The InAs/GaSb Material System

The three semiconductors InAs, GaSb and AlSb, provide a lattice matched material system, dubbed the 6.1 family. All family members have unit cell dimensions around 6.1 Å (to within 1.3%) [57]. The three materials are easily grown on top of each other with sharp interfaces allowing for high carrier mobilities [33, 58]. The band structure and physical properties of the constituents of the 6.1 family are summarized in table 2.1. Note that some of these bulk properties (especially effective mass) may change significantly in thin layers, i.e., in confinement.

Combination of materials in the 6.1 family allow for radically new band lineups compared to the established (Al, Ga)As platform [61]. An overview of all possible lineups is schematically depicted in Fig. 2.8 [33]. The GaSb to AlSb interface is of type-I, AlSb-InAs type-II and a type-III broken-gap lineup is created at the interface between InAs and GaSb (type-III is sometimes grouped under type-II lineup). The extraordinary type-III InAs-GaSb lineup, first noticed by Sasaki et al. [62], is special in that the conduction band of InAs lies below the top of the valence band of GaSb with a break in the gap of 150 meV (see Fig. 2.8) [63]. The energy spectrum of such a heterostructure therefore does not exhibit a gap and the material was first thought to be a semimetal. As we will see later, actually a small energy gap is opened when electrons in the InAs hybridize with holes in the GaSb [64, 65].

The InAs in this material system plays a key role for device applications. First, due to its small electron effective mass InAs has a high electron mobility [61]. Second, InAs is easily contacted by metals or superconductors as a result of Fermi level pinning at the
2.2. The InAs/GaSb material system

Figure 2.7: Bandstructure calculations from [33]. All materials in the 6.1 Å family have a direct bandgap at the Γ point.

<table>
<thead>
<tr>
<th>Crystal type</th>
<th>InAs</th>
<th>GaSb</th>
<th>AlSb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice constant at 300K (Å)</td>
<td>6.0584</td>
<td>6.0959</td>
<td>6.1355</td>
</tr>
<tr>
<td>Band gap at Γ (meV)</td>
<td>415</td>
<td>813</td>
<td>1696</td>
</tr>
<tr>
<td>ΔSO (meV)</td>
<td>380</td>
<td>720</td>
<td>650</td>
</tr>
<tr>
<td>Electron mass (m₀)</td>
<td>0.023</td>
<td>0.041</td>
<td>0.12</td>
</tr>
<tr>
<td>HH mass (m₀)</td>
<td>0.41</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>LH mass (m₀)</td>
<td>0.026</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>Dielectric constant εᵣ</td>
<td>15.15</td>
<td>15.7</td>
<td>12.04</td>
</tr>
</tbody>
</table>

Table 2.1: Material parameters for the 6.1Å family, obtained from [59, 60]. m₀ is the free electron mass, HH (LH) stands for heavy (light) holes. The effective masses are averages over crystal directions.
Figure 2.8: Possible band alignments in the 6.1Å family. The InAs-GaSb band alignment has type II broken gap alignment where the bottom of the InAs conduction band is 150 meV below the top of the GaSb valence band. The GaSb-AlSb is type-I and InAs-AlSb type-II. Figure taken from [33].

surface at an energy 130 meV above the bottom of the conduction band [66]. Therefore a Schottky barrier is absent and clean metal-semiconductor interfaces and transparent superconducting contacts can be attained [67]. However, this advantage also means that uncovered InAs parts can accumulate a high electron density at the surface, which is difficult to remove by electrostatic gating [68, 69]. Furthermore, InAs is known for its strong non-parabolicity of the conduction band which sets in at relatively modest electron densities; $m^*$ increases 40% compared to the bare InAs effective mass for an electron density of $1 \times 10^{15}$ m$^{-2}$ in a 15 nm quantum well [70].

The interface between InAs and GaSb can have two bond configurations, In-Sb or Ga-As bonds. Interfaces with predominantly In-Sb bonds are expected to give higher carrier mobility [33] and a $\sim 44$ meV lower band offset [71] than Ga-As bonds. Which of the two possible interface bonds are grown can be controlled by the MBE operator by adjusting the shutter sequence during crossover between materials [72]. However, usually the preference for one bond type will not be perfect.

The InAs/GaSb system gained popularity in the optics community as a long wave infrared detector [73, 74]. Superlattices of InAs and GaSb are used to achieve effective band gaps that are narrower than those of InAs and GaSb itself, thus providing a promising alternative to HgCdTe based detectors [75].

2.2.1. InAs/GaSb Quantum Wells
Growing an InAs/GaSb heterojunction between two AlSb layers creates a double quantum well, as shown in Fig. 2.9a. The AlSb, having a larger band gap, provides a barrier for electrons in InAs and holes in GaSb, see Fig.2.8. Due to the resulting confinement the momentum of the carriers in growth direction ($z$-direction) is quantized and the carriers are bound to move in-plane ($x, y$-direction) of the quantum well. Assuming infinite
2.2. The InAs/GaSb Material System

Figure 2.9: a) Typical structure of a double quantum well of InAs and GaSb. The order of the InAs and GaSb layers may be reversed for some wafers. b) Schematic inverted band structure diagram, hybridization between electrons and holes opens up a gap at the crossing of the bands (charge neutrality point). The energy difference between the top of the hole band and bottom of the conduction band is $2M$, where $M$ is negative in the inverted case depicted here. Linear dispersing helical modes, depicted in green and purple, appear at the boundary of the sample. The schematic in a is taken from [27].

AlSb barrier height, the energy dispersion for electrons in InAs and holes in GaSb can be approximated by

$$E_n(k_{||}) = E_c + \frac{\hbar^2 k_{||}^2}{2m_e^*} + \frac{\hbar^2}{2m_e^*} \frac{n\pi^2}{L_{\text{InAs}}^2},$$  \hspace{1cm} (2.62)

$$H_n(k_{||}) = E_v - \frac{\hbar^2 k_{||}^2}{2m_h^*} - \frac{\hbar^2}{2m_h^*} \frac{n\pi^2}{L_{\text{GaSb}}^2}.$$  \hspace{1cm} (2.63)

Here $E_c$ is the bulk conduction band energy for InAs and $E_v = (E_c + 150 \text{ meV})$ the valence band energy for GaSb, $L_{\text{InAs}}$ and $L_{\text{GaSb}}$ are the thicknesses of the InAs and GaSb layers, respectively, and $n$ is the subband index. In the remainder we focus on $n = 1$ and assume that the confinement is so strong that energies for $n \geq 2$ are not accessible for carriers. Therefore a 2D electron system (2DES) is formed in the InAs layer and a 2D hole system (2DHS) emerges in the GaSb layer. By varying the layer thicknesses we can tune onset energies of the hole and electron gases, and a variety of phases can be accessed. Figure 2.10a shows the possible phases as a function of the layer thickness of InAs and GaSb. For a fixed GaSb thickness of 5 nm three different regimes can be accessed by varying the InAs width: the trivial phase ($L_{\text{InAs}} < 9.5 \text{ nm}$), the inverted phase ($9.5 < L_{\text{InAs}} < 15 \text{ nm}$) and the inverted semimetallic phase ($L_{\text{InAs}} > 15 \text{ nm}$).

The trivial phase is characterized by a semiconducting gap where the onset energy of the electron gas lies above the onset energy for the hole gas, as in trivial semiconductors. Figure 2.10b shows the typical band structure in this regime. Increasing the InAs layer thickness reduces the band gap (see Fig. 2.10a) in accordance with eq. 2.62. The onset energy for electrons is thus reduced for larger $L_{\text{InAs}}$ while the onset for holes does not
change. At $L_{\text{InAs}} = 9.5$ nm the electron and hole bands touch, resulting in the closing of the semiconducting gap, indicated by the white contour line in Fig. 2.10a. This closure of the semiconducting gap marks the boundary of the trivial phase.

The inverted phase is obtained when increasing $L_{\text{InAs}}$ further. The hole band rises above the electron band (eq. 2.62) and naively one would expect no band gap. Strangely, at the $\Gamma$-point (at $k = 0$) in the band diagram, the p-type hole orbitals are higher in energy than the s-type electron orbitals. Such a band structure is called an inverted band structure. For even wider InAs layers, the band structure remains inverted.

So far we have treated the system as two independent gases of electrons and holes located in their respective quantum wells. This is a good approximation in the trivial phase where at a particular energy the available state lives in either one of the two layers. However, in the inverted gap phase, electron states and hole states coexist at the same energy. The wave functions of the electron and hole gases leak into each others well, thereby creating a quantum mechanical coupling between the two gases. This coupling is described in the Hamiltonian by an off-diagonal element proportional to $|k|$ [76, 77]:

$$H = \begin{bmatrix} E_c(k) & A(k_x + i k_y) \\ A(k_x + i k_y) & E_h(k) \end{bmatrix}. \quad (2.64)$$

When the in-plane momenta of electrons and holes are similar, i.e., the electron density
equals the hole density (charge neutral), electrons hybridize with holes. This leads to
the opening of a hybridization gap of a few meV at finite momentum [27] as seen in Fig
2.10c. The occurrence of such anticrossing behavior was conjectured by [65] and first
calculated by [64]. Wavefunctions with energy close to the hybridization gap delocalize
over the two layers, and thereby probe properties of both InAs and GaSb. At energies
far away from the hybridization gap the gases are effectively uncoupled due to their mo-
menta mismatch. Here they can again be treated as independent gases.

The semimetallic closed gap phase occurs for even wider InAs layers. Figure 2.8d depicts
a typical band structure. Hybridization is still present, although no band gap is formed.
The structure is thus expected to behave as a semimetal with electron states present at
every energy.

2.2.2. TOPOLOGICAL TRANSITION

2D Topological insulators (2D TIs) are materials that are characterized by an insulating
bulk and gapless helical edge states, i.e., the quantum spin Hall state. The edge states
are dissipationless at zero magnetic field, as time reversal symmetry prohibits backscat-
tering. The quantum spin Hall (QSH) state was first discovered in HgTe/CdTe quantum
wells in 2007 by König et al. [78]. The research was motivated by the proposal of Bernevig, Hughes and Zhang (BHZ) in 2006 that predicted a QSH state in the ‘inverted’
phase of the HgTe/CdTe quantum wells [79]. A quantum well of HgTe was predicted to
be a trivial insulator for a thickness \( d < 6.1 \text{ nm} \) and a QSH insulator for \( d > 6.1 \text{ nm} \). Soon
thereafter, in 2008, Liu et al. [27] proposed InAs/GaSb DQWs as 2D TI candidate based
on the BHZ formalism. As seen in Fig. 2.10, the InAs/GaSb quantum well also possesses
an inverted phase for sufficiently thick layers of InAs and GaSb. In this phase a small
hybridization gap is opened as a result of the coupling between electrons and holes. The
Hamiltonian for the InAs/GaSb system in the inverted regime is adiabatically connected
to the BHZ Hamiltonian and therefore topological nontrivial [27].

2.2.3. BHZ MODEL AND EDGE STATES

The assumptions that go into the BHZ model are that of inverted bands and time re-
versal symmetry (TRS). The model focuses on a spin degenerate s-type electron band
\( |E, \pm 1/2 \rangle \) and a spin degenerate heavy hole band \( |H, \pm 3/2 \rangle \) which form the basis states of
the model. All other bands are remote in energy and are therefore neglected. The oppo-
site parity of the electron and hole states require the coupling matrix elements between
them to be linear in \( |k| \), in the lowest order [76, 77]. Furthermore, rotation symmetry
along the \( z \)-axis requires the matrix elements to take the form

\[
Ak_x = A(k_x \pm ik_y), \quad (2.65)
\]
where \( A \) is related to the wavefunction overlap between electron states and hole states. In the basis \( \{E_+, H_+, E_-, H_-\} \) the BHZ Hamiltonian reads

\[
\begin{pmatrix}
\frac{\hbar^2 k^2}{2m_e} + M & A(k_x + i k_y) & 0 & 0 \\
A(k_x - i k_y) & -\frac{\hbar^2 k^2}{2m_h} - M & 0 & 0 \\
0 & 0 & \frac{\hbar^2 k^2}{2m_e} + M & A(-k_x + i k_y) \\
0 & 0 & A(-k_x - i k_y) & -\frac{\hbar^2 k^2}{2m_h} - M
\end{pmatrix}

\begin{pmatrix}
E_+ \\
H_+ \\
E_- \\
H_-
\end{pmatrix}

= E

\begin{pmatrix}
E_+ \\
H_+ \\
E_- \\
H_-
\end{pmatrix}.
\] (2.66)

The blocks of spin-up states and spin-down states are decoupled, because inversion symmetry and axial symmetry around the growth axis is assumed. \( 2M \) is the energy difference between the s-type band and p-type band at the \( \Gamma \)-point (see Fig. 2.9b). If \( M < 0 \) the non-trivial inverted regime is attained, whereas \( M > 0 \) describes a normal insulator.

The coupling terms \( A|k| \) only have significant effect where the electron and hole state are of similar energy.

The underlying assumption that the system preserves bulk and structural inversion symmetry is generally not true for InAs/GaSb, as it has a zincblende crystal structure and a strong spin orbit coupling\(^3\). Therefore the Hamiltonian will acquire off-diagonal components coupling the spin-up to the spin-down block. However, numerical calculations show that for the InAs/GaSb quantum well widths of interest the Hamiltonian can be transformed into the BHZ Hamiltonian without closing of the hybridization gap. Hence the InAs/GaSb system and BHZ model belong to the same topological class [27].

In the topological non-trivial regime, \( M < 0 \), solving the BHZ model of eq. 2.66 for a half-space, \( x > 0 \) on the \((x, y)\)-plane, produces states within the bulk (topological) gap located at the boundary of the material, i.e., edge states. The lowest order effective Hamiltonian that describes the edge states propagating in the y-direction is [25]

\[
H_{\text{edge}} = A k_y \sigma_z.
\] (2.67)

Evident is that their dispersion is linear in momentum (for small energies) with states of opposite spin counter propagating, a property dubbed ‘helical’ motion. Due to time reversal symmetry the dispersions necessarily cross at \( k = 0 \) (see Fig. 2.9b). These very simple symmetry considerations indicate that the QSH phase has a built-in protection against elastic backscattering, will be discussed in more detail in the next section. Although the edge states are localized at the edge, they have a finite decay length in the bulk of

\[
L_{\text{decay}} = \max \left( |\Re \lambda_{1,2}|^{-1} \right),
\] (2.68)

with

\[
\lambda_{1,2} = \frac{1}{2B} \left( A \pm \sqrt{A^2 - 4MB} \right),
\] (2.69)

\(^3\)Strictly speaking these assumptions also do not hold for HgTe/CdTe quantum wells. However, the bulk inversion and structural inversion asymmetry are expected to be small compared to the topological gap in HgTe. They and are therefore in first approximation neglected.
2.2. THE InAs/GaSb MATERIAL SYSTEM

Figure 2.11: Top view of a 2D topological insulator sample with helical edge modes and a disordered region at the edge. Red (blue) arrows represent the spin-up (spin-down) edge channels. Illustration adapted from [81].

where \( B = \hbar^2/4(m_e - m_h)/m_em_h \) is the Newtonian mass. For an InAs/GaSb quantum well of 10 nm GaSb and 9 nm InAs, \( B = -78.3 \text{ eVÅ} \), \( M = -1.8 \times 10^{-4} \text{ eVÅ}^2 \) and therefore \( L_{\text{decay}} = 13 \text{ nm} \). This decay length determines the amplitude for inter-edge tunneling when the opposite edge is close by. To observe the quantum spin Hall state the sample width should exceed this decay length, as otherwise the inter-edge tunneling opens an energy gap.

2.2.4. TOPOLOGICAL PROTECTION OF HELICAL EDGE-MODES

In the previous paragraph we have seen that a 2D TI has pairs of edge channels related to each other by time reversal symmetry (TRS). This section considers transport through such edge channels in the presence of non-magnetic scattering. We will find that even for strong elastic disorder backscattering is forbidden, resulting in perfect transmission of the helical edge channels. The derivation below is along the lines of refs. [80, 81].

SCATTERING MATRIX WITH TRS

Consider the situation depicted in Fig. 2.11 where the red (blue) line represents an edge channel with spin-up (spin-down) at the boundary of a 2D TI. A disordered region (hatched in the figure) opens the possibility for elastic scattering. Incoming states on the left and right side are labeled \( |L, \uparrow\rangle \) and \( |R, \downarrow\rangle \), respectively. Outgoing states relate to incoming states by TRS, thus the left outgoing state is \( T|L, \uparrow\rangle \) and right outgoing state \( T|R, \downarrow\rangle \), where \( T \) is the time-reversal operator. The wavefunction on the left and right side is a superposition of the incoming and outgoing states:

\[
\Psi_L = \alpha_L |L, \uparrow\rangle + \beta_L T|L, \uparrow\rangle \\
\Psi_R = \alpha_R |R, \downarrow\rangle + \beta_R T|R, \downarrow\rangle. \tag{2.70}
\]

The coefficients \( \alpha_i, \beta_i \) are related by a scattering matrix \( \hat{s} \) (like in section 2.1.3),

\[
\begin{bmatrix} \beta_L \\ \beta_R \end{bmatrix} = \hat{s} \begin{bmatrix} \alpha_L \\ \alpha_R \end{bmatrix}. \tag{2.71}
\]
Charge conservation requires the scattering matrix to be unitary $\hat{s}\hat{s}^\dagger = 1$ and can be written as [82]

$$\hat{s} = \begin{bmatrix} r & t \\ t' & r' \end{bmatrix}.$$  \hspace{1cm} (2.72)

Before we continue we need a short intermezzo about TRS and its operator $T$. The time-reversal operator $T$ is antiunitary and commutes with the Hamiltonian $[T, H] = 0$ in the case of TRS. The square of the antiunitary operator $T$ is an unitary operator $T^2 = \pm 1$. Systems with no or integer spins have $T^2 = 1$. For half-integer spin systems $T^2 = -1$ and the operator can be written as a product of an unitary operation and the complex conjugation operator $K$, $T = i\sigma_y K$.

Now we can apply the time-reversal operator to our scattering states of eq. 2.70,

$$T\Psi_L = \alpha^*_L T|L, \uparrow\rangle - \beta^*_L |L, \uparrow\rangle$$

$$T\Psi_R = \alpha^*_R T|R, \downarrow\rangle - \beta^*_R |R, \downarrow\rangle,$$  \hspace{1cm} (2.73)

which is also a scattering state with the same energy (as $T$ commutes with $H$). This means that

$$\begin{bmatrix} \alpha^*_L \\ \alpha^*_R \end{bmatrix} = \hat{s} \begin{bmatrix} -\beta^*_L \\ -\beta^*_R \end{bmatrix}.$$  \hspace{1cm} (2.74)

Multiplying both sides with $-\hat{s}^\dagger$, taking the complex conjugate and comparing with eq. 2.71, we conclude that $\hat{s} = -\hat{s}^T$. Applying this relation to the scattering matrix of eq. 2.72 we obtain $r = -r'$, which can only be satisfied if $r = 0$. Due to charge conservation this directly means $t = 1$, i.e., unit transmission for the channel. This proves the protection against backscattering by TRS of the helical edge-channels.

The above argument can be expanded to $N$ pairs of counter propagating spin-polarized edge channels [83]. If $N$ is odd, the reflection eigenvalues come in pairs plus one single eigenvalue that equals to zero (with corresponding transmission eigenvalue of unity). By increasing the disorder the $N - 1$ states with paired eigenvalues eventually localize due to Anderson localization, leaving one perfectly transmitting channel.

**LIFTING TOPOLOGICAL PROTECTION**

There are a variety of processes that escape the protection by TRS. Most important, the presence of a magnetic field destroys the perfect transmission of the helical edge channels. The TRS is broken, such that $[T, H] \neq 0$, and the time reversed paths of eq. 2.73 are no longer degenerate. The same argument holds for magnetic impurities in the disordered region. Moreover, inelastic scattering of particles is not captured by the scattering matrix approach as such scattering does not conserve energy. In an inelastic scattering process particles have a chance to flip spin [23, 84–87]. The characteristic length scale for such inelastic scattering is the phase coherence length $l_\phi$. Therefore, only perfect transmission is expected within a phase coherence length $l_\phi$. For sample dimensions $L > l_\phi$, the scattering events can be modeled as phase breaking probes (i.e. contacts) in
the Landauer-Büttiker formalism of section 2.1.3 [88, 89]. Besides the presence of a magnetic field and inelastic scattering also higher-order processes such as two simultaneous spin flips cause backscattering (via the Umklapp process) [85, 90]. Also in the presence of an elastic impurity and Rashba spin orbit interaction a two-particle process can cause backscattering [91, 92]. Such processes are less likely when the topological gap is larger.

2.3. QUANTUM HALL INTERFEROMETER

Interference of light is a well established phenomenon and easily demonstrated in an optical interferometer. The Fabry-Pérot and Mach-Zehnder optical interferometers are the most widely known variants. Their electronic counterparts, aimed to achieve interference of electrons, are defined in two dimensional electron gases (2DEG). The Fabry-Pérot all electric interferometer emerged in the ‘80s [93], while the first electronic Mach-Zehnder interferometer (MZI) was made in 2003 [94].

Recently, these electronic interferometers have gained much interest as a probe of the non-Abelian exchange statistics of quasiparticles. Such non-Abelian particles are believed to exist in certain (even) denominator fractional quantum Hall (FQH) states, in particular at \( \nu = \frac{5}{2} \) and \( \nu = \frac{12}{5} \) [80, 95, 96].

The MZI is seen as the most pure interferometer as particles interfere only once. It is thus a ‘true two-path’ interferometer [94]. However, the MZI is difficult to fabricate as the drain for the electrons is at the center of the device. The Fabry-Pérot interferometer (FPI) on the other hand is much easier to make, namely by placing two split-gate quantum point contacts (QPCs) in series. In a FPI particles generally can interfere multiple times, which complicates the analysis. Furthermore, the interference characteristics of a FPI depend on its electrostatic environment. Two limiting cases can be studied, namely the Aharonov-Bohm regime and the Coulomb dominated regime.

Following a series of observations in [93, 97] and theoretical insights [94, 98, 99] the theory of the FPIs recently converged. In this section I will describe properties of a Fabry-Pérot interferometer. This section requires some basic knowledge about the integer quantum Hall effect (QHE), for which introductory literature can be found in many text books. I enjoyed reading [100–102] for the QHE basics and [103] for an introduction to non-Abelian statistics.

2.3.1. FABRY-PÉROT INTERFEROMETER

An electronic Fabry-Pérot interferometer is realized by fabricating two quantum point contacts (QPC) in series on a 2DEG, as shown in Fig. 2.12a. An additional plunger gate controls the area of the FPI. With all gates grounded the application of a strong perpendicular magnetic field \( B \) tunes the 2DEG into the integer quantum Hall (IQH) regime. Due to Landau quantization of the cyclotron orbits, electrons form energetically discrete Landau levels (LL). The bulk becomes insulating (conducting) if the Fermi-level lies out-
Figure 2.12: Fabry-Pérot interferometer at the IQH state $\nu = 4$. a) Standard 6-terminal Hall bar layout with two QPC constrictions, left and right. Plunger gate alters the area of encircling edge channels. b) Capacitor network of the central part of the interferometer. Inner disc is formed by the 4th LL which is totally reflected at the two QPCs and the isolated island has self-capacitance $C_L$. The outer ring is the interfering edge channel which couples to source and drain via capacitance $C_L$. A mutual capacitance $C_M$ couples the disc to the ring.

side (within) a LL. A quantum Hall state is characterized by its filling factor

$$\nu = \frac{n \Phi_0}{B},$$

(2.75)

where $\Phi_0 = h/e$ is the quantum of flux and $n$ the electron density. The filling factor reflects the number of LLs below the Fermi-level. For integer $\nu$ the bulk is insulating. For $\nu = m - 1/2$, with $m$ an integer, the Fermi-level lies within a LL and the bulk behaves as a Fermi-liquid [100, 101] (the $\nu = 5/2$ state is an exception to this statement). Consider $\nu = 4$, where the bulk is insulating. From the bulk towards the edge of the sample the electron density $n(r)$ gradually decreases (where $r$ is the position), hence the filling factor $\nu$ decreases as well (see eq. 2.75). Near the edge the lowest 4 LLs are pushed upward in energy and subsequently pass through the Fermi-level at positions $r_m$ ($m = 1, 2, 3, 4$) where the density is $n(r_m) = (m - 1/2)B/\Phi_0$. Four metallic regions (edge-channels) are thus created at the edge, while the bulk remains insulating (see Fig. 2.12a). These edge channels are chiral (solid blue lines), as electrons flow only in one direction, and carry the current dissipationlessly along the perimeter of the sample. The perfect transmission of edge-channels in the IQH regime originates from the fact that the probability of tunneling to the opposite edge is exponentially suppressed as a function of distance between those edges[104]. Also the probability of tunneling to neighboring forward propagating edge channels is small [105, 106]. Because the bulk is insulating, electric transport is only possible via the chiral edge channels.

We first investigate the influence of the left QPC (QPC L) gates on the conductance through
the sample. A negative voltage on the gates locally reduces the electron density $n(r)$ and forces the edge channels to flow around the QPC gates. The constriction pushes the forward and backward propagating edge channels towards each other. The innermost edge channel, corresponding to the highest LL, is first eligible for tunneling events, because it is closest to its backward moving counterpart. Tunneling is characterized by complex reflection ($r_L$) and transmission ($t_L$) probability amplitudes [107, 108] and determines whether the QPC is in the weak ($|r_L|^2 \sim 0$) or the strong ($|r_L|^2 \sim 1$) backscattering regime [97]. In Fig. 2.12a the QPC gate voltage is such that the 4th channel is fully reflected, the 3rd is in the weak or strong scattering regime and channels 1-2 pass the constriction unimpeded, as the distances to their backward moving counterparts are still rather large.

The reflection amplitude can be adjusted using QPC gate voltage $V_L$ and magnetic field $B$. Decreasing $V_L$ raises the potential in the constriction, thereby reducing $n(r)$ and pushing edge-channels on opposite edges towards each other. The increase of $B$ reduces the filling factor in the constriction (see eq. 2.75), causing the edge-channels to move inwards to regions of larger density. Further decreasing $V_L$ or increasing $B$ therefore increases the backscattering probability and eventually results in full reflection of the channel [32].

Electronic transport across a constriction (left or right) is characterized by the transmission probability $T = |t|^2$ via [93, 99, 104]

$$G_{L(R)} = (e^2/h) (N + T_{L(R)}),$$

(2.76)

where $G_{L(R)}$ is the diagonal conductance ($I_{1-4}/V_{2-5}$), $N$ is the number of fully transmitted edge channels and $T_{L,R} = 1 - R_{L(R)}$ is the transmission probability of the $(N+1)^{th}$ edge channel.

The Fabry-Pérot interferometer is formed by energizing the right QPC (QPC R) and left QPC with the same negative voltage $V_R = V_L$, leading to two constrictions in series [109] with identical reflection probabilities $R_R = R_L = R$. The resulting edge channel paths are depicted in Fig. 2.12a for $v = 4$. Two edge channels pass with unity transmission, $f_T = 2$. The third is partially back scattered, the interfering edge channel. The fourth channel is fully reflected, $f_R = 1$, which creates a small metallic island inside the interferometer. The total number of channels is $v = f_T + 1 + f_R$.

The fully transmitted $f_T$ and fully reflected $f_R$ channels are insensitive to small deviations in $B, V_{pl}$. the conductance of the interfering channel, however, shows a complicated oscillatory behavior. An electron in the interfering edge channel can undergo multiple reflections between the two QPCs before it exits at the right (transmitted) or left (reflected) side. Each time the electron completes a loop in the interferometer, it acquires an Aharonov-Bohm phase $\theta = 2\pi(AB)/\Phi_0$ as the interfering edge channel encloses a flux $\Phi = AB$. The diagonal conductance through the interferometer is obtained
by summing over all possible paths [93, 110, 111]

\[ G_d = \frac{e^2}{h} \left( \frac{T^2}{1 + R^2 - 2R\cos\theta} \right), \tag{2.77} \]

which is a periodic function of \( \theta \). The flux \( \Phi \) enclosed by the interfering edge channel is a function of gate voltage and magnetic field \( \Phi = \Phi(V_G, B) \).

The relation between the on the one hand enclosed flux \( \Phi \) and on the other hand \( B, V_g \) depends on the electrostatics of the FPI, which can be captured by an effective capacitor model depicted in Fig.2.12b [97–99]. Edge channels are treated as metallic regions with an effective capacitive coupling to each other \( (C_M) \) and to ground \( (C_I, C_L) \). Subscript ‘I’ stands for the interfering edge channel and ‘L’ for the localized island. The ratio

\[ \Delta = \frac{C_M}{C_L + C_M} \tag{2.78} \]
determines whether the interference is of Aharonov-Bohm (AB) type \( (\Delta \approx 0) \) or Coulomb dominated \( (\Delta \approx 1) \). The effective capacitances result from a combination of classical electrostatics and quantum mechanical energies. Prediction of these capacitances from solely device geometry remains inadequate; a detailed microscopic model and numerical calculations are needed to also capture electron interactions [98]. In the next section we first focus on weak backscattering \( (R \sim 0) \) and review the AB and CD regimes in more detail. At the end we review the strong backscattering regime \( (R \sim 1) \) and its implications for the AB and CD regimes.

### 2.3.2. AHARONOV-BOHM REGIME

The Aharonov-Bohm regime is the simplest to describe but difficult to attain in experiment [97, 112, 113]. In this regime the mutual capacitance between island and edge state is negligible compared to the self-capacitance of the island \( (C_M \ll C_L) \). Any excess charge on the island has no effect on the interfering edge channel. The plunger gate voltage, \( V_{pl} \), alters the area of the interferometer through a change of \( n(r) \). Modeling the coupling between plunger gate and edge state as a capacitor, the area changes linearly with gate voltage, \( A = \alpha V_{pl} \).

The differential of the phase acquired by an interfering electron is given by

\[ d\theta = 2\pi/\Phi_0(AdB + BdA). \tag{2.79} \]

The magnetic field period of the conductance oscillation \( (\Delta \theta = 2\pi) \) is \( \Delta B = \Phi_0/A \). The period in plunger gate voltage is \( \Delta V_{pl} = \Phi_0/\alpha B \) and depends also on magnetic field. As will be evident in the next section, the slope of the constant phase lines provides a useful tool to discriminate between the AB and CD regimes. By setting \( \theta = 0 \) in Eq. 2.79 (constant phase) we obtain the slope

\[ \frac{dV}{dB} = -\frac{A}{B}, \tag{2.80} \]
This negative slope of the constant phase has a simple physical meaning. If the magnetic field is increased, the plunger gate voltage should decrease (reducing area) in order to keep the flux constant. Such a negative slope is a characteristic of the AB regime and provides a way to discriminate the AB regime from the CD regime.

The experimental difficulty in obtaining the AB regime is to selectively increase $C_L$. Researchers achieved this by building a large interferometer with a grounded global top gate. [97, 112].

### 2.3.3. COULOMB DOMINATED REGIME

If the mutual capacitance dominates the total capacitance of the island ($C_M \gg C_L$), any excess charge on the island induces an equal but opposite charge on the interfering edge [98, 114]. Such excess charge can be induced by the plunger gate or by the magnetic field. First we reveal the role of the magnetic field and after that the effect of the plunger gate.

Changing the magnetic field $B$ such that one flux quantum $\Phi_0$ is added through the island causes the LL degeneracy to increase by one. There are thus $f_T + 1$ extra electron states created below the Fermi-level. These states will be filled by $f_T + 1$ electrons from the highest LL, i.e., the island. Thus, increasing $B$ by an infinitesimal amount $dB$ would reduce the electron charge on the island by

$$dQ = e(f_T + 1) \frac{A}{\Phi_0} dB. \tag{2.81}$$

However, a change $dQ$ smaller than an electron charge $e$ is not allowed due to Coulomb energy considerations. Therefore, in the case $|dQ| < e/2$, the island is effectively charged by $-dQ$ compared to the situation without Coulomb energy. This negative charge induces an equal but positive charge on the interfering edge through $C_M$ [114]. In the QH regime, each electron charge occupies an area related to the flux quantum $A_e = \Phi_0/B$. The negative charge therefore reduces the area of the interfering LL by

$$dA = \frac{\Phi_0}{eB} (-dQ). \tag{2.82}$$

The interfering area $A$ depends thus on the magnetic field. As a result, the enclosed flux $\Phi = AB$ depends not only on magnetic field directly, but also through the area $A(B)$. The change in phase $d\theta$ acquired by an interfering particle upon increasing $B$ by $dB$ is

$$d\theta = \frac{2\pi}{\Phi_0} (AdB + BdA)$$

$$= -2\pi f_T A dB. \tag{2.83}$$

The interference period ($\Delta \theta = 2\pi$) corresponds to $\Delta B = -\Phi_0/(f_T A)$. Quite unexpectedly, the magnetic field period $\Delta B$ of the conductance oscillations depends on the number of fully transmitted edge channels. Only if $f_T = 1$ the magnetic field period is equal to that of the AB case. Another interesting implication of Eq. 2.83 is that for $f_T = 0$ no conductance oscillations should appear.
The $1/f_T$ scaling of the magnetic field period is clearly observed in experiments. Ofek et al. [97] and Zhang et al. [112] measured magnetic field periods for different $f_T$ and found that the period indeed follows Eq. 2.83.

However, one might wonder why the charge deficit on the island is not directly canceled by surrounding mobile charge. For interferometers in the CD regime, the island has a small total capacitance resulting in a finite Coulomb charging energy that favors an integer number of electrons on the island. Only when the induced charge exceeds $\Delta Q = e/2$ it is energetically favorable for an electron from the interfering edge state to tunnel onto the island. This tunneling event has no influence on the phase of the interfering electrons as the phase jumps by $2\pi$ (Eq. 2.82 and 2.83).

Also in the Aharonov-Bohm regime the island is charged upon piercing one flux quantum through the interferometer. However, because $C_L \gg C_M$ the extra charge on the island is screened via $C_L$ such that effectively no charge is induced on the interfering edge-channel which therefore does not change in area.

Next we focus on the effect of the plunger gate voltage. Also $V_{pl}$ affects the area in two ways. First, a voltage on the gate varies the area directly, as in the AB regime, and is characterized by $\alpha = dA/dV$. Second, it induces a charge $Q$ on the island through $C_L = dQ/dV$. Assuming $C_L \gg C_I$, almost no charge is induced directly on the interfering edge mode. Equation 2.82 can be extended with the plunger gate voltage dependence

$$dA = -\frac{\Phi_0}{eB}dQ + \left(\frac{\Phi_0}{eB}C_L + \alpha\right)dV.$$ (2.84)

Using Eq. 2.81 and Eq. 2.83 we find a new expression for $\theta$

$$d\theta = -2\pi f_TA dB + B\left(\frac{\Phi_0}{eB}C_L + \alpha\right)dV.$$ (2.85)

Lines of constant phase $\theta = 0$ thus have positive slope

$$\frac{dV}{dB} = \frac{f_T A}{\Phi_0/eC_L + \alpha B}.$$ (2.86)

The plunger gate is expected to be much more efficient in charging the island than in altering the area. Therefore $\alpha$ can be neglected. Thus it follows from eq. 2.85 that the period in gate voltage is independent of $B$ and $f_T$, in contrast to the AB regime.

2.3.4. STRONG BACKSCATTERING

The strong backscattering regime is characterized by a reflection amplitude of the interfering edge that is close to unity. Conductance $G_{L(R)} \ll e^2/h$ and therefore Coulomb charging effects dominate the transport. The interfering edge channel becomes effectively a quantum dot with tunnel junctions to the leads. The charging energy prohibits charge to flow to the ring in the case of a charge deficit.
In the AB limit \( C_M \ll C_L \) the interfering edge channel is decoupled from the island. Adding one flux quantum to the interfering area increases the degeneracy of the interfering LL by one. In the strong coupling regime, electrons can only be added in discrete numbers when there is a vanishing energy cost for adding one electron to the interfering edge. Therefore there will be resonant transmission through the interferometer at a magnetic field period \( \Delta B = \Phi_0 / A \), the same as in the weak backscattering limit.

In the CD regime the large mutual capacitance makes one large quantum dot from the island and interfering edge, with \( f_T \) LLs below the Fermi level. Adding one flux quantum will move \( f_T \) electrons from the quantum dot to lower energy LLs. The magnetic field period of the resonant transmission is given by \( \Delta B = \Phi_0 / f_T A \), again the same as in the weak backscattering limit.

2.3.5. Conclusion

In conclusion we have reviewed the interference properties of a FPI in the CD and AB regimes in the presence of both weak and strong backscattering. The interference periods can qualitatively be understood from an effective capacitor model. Besides the extreme cases of pure AB or pure CD, an FPI can be in an intermediate regime, where \( C_M \sim C_L \). Such regime gives rise to interesting interference patterns that were experimentally measured [97] and theoretically analyzed in [98].

Proposals on interfering non-Abelian quasiparticles generally assume a FPI in the AB regime [80, 95, 96]. Pursuing experimental realizations of these proposals is difficult as the regime the interferometer will be in cannot be predicted beforehand [98]. However, non-Abelian statistics could in principle also be investigated using a FPI in the CD regime [95].
3

THE FLIP-CHIP SETUP: A NONINVASIVE METHOD FOR NANOSCALE ELECTROSTATIC GATING OF PRISTINE MATERIALS
Electrostatic gating is essential for defining and control of semiconducting devices. However, nanofabrication processes required for depositing gates inevitably degrade the pristine quality of the material of interest. Examples of materials that suffer from such degradation include ultrahigh mobility GaAs/AlGaAs two-dimensional electron gases (2DEGs), graphene, topological insulators, and nanowires. To preserve the pristine material properties, we have developed a flip-chip setup where gates are separated from the material by a vacuum, which allows nanoscale electrostatic gating of the material without exposing it to invasive nanoprocessing. An additional benefit is the vacuum between gates and material, which, unlike gate dielectrics, is free from charge traps. We demonstrate the operation and feasibility of the flip-chip setup by achieving quantum interference at integer quantum Hall states in a Fabry–Pérot interferometer based on a GaAs/AlGaAs 2DEG. Our results pave the way for the study of exotic phenomena including fragile fractional quantum Hall states by preserving the high quality of the material.

3.1. INTRODUCTION
Electrostatic gating is widely used to define functional electronic devices on semiconductors [115]. Various growth techniques persistently produce increasingly high-quality semiconducting materials that form the basis of a wide variety of devices. But degradation of the pristine material quality by nanofabrication processes hinders exploration of the devices’ full potential [116, 117]. One particular example is ultrahigh mobility two-dimensional electron gases (2DEGs) in GaAs/AlGaAs quantum wells with a carrier mobility exceeding $3.5 \times 10^7 \text{ cm}^2/(\text{V s})$ [118, 119]. Such high-quality material enables the study of novel interaction phenomena including the fractional quantum Hall (FQH) effect, where strong electron–electron interactions provide a wealth of new phases [18, 120–124]. In particular, the 5/2 and 12/5 FQH states are believed to obey non-Abelian quantum statistics [9, 15, 96, 125]. Attempts to discover the non-Abelian nature of the 5/2 state have been guided by promising proposals for a Fabry–Pérot interferometer (FPI) defined by electrostatic gating [22, 95]. Quantum interference has been achieved at integer quantum Hall (IQH) states [93, 97, 112, 126] as well as at some FQH states [127, 128] but remains a challenge when applied to the fragile 5/2 state [129–131]. Depending on the electrostatics, the mechanism for the interference can be of Aharonov-Bohm (AB) type or Coulomb-dominated (CD) [97–99, 112, 132]. So far, all devices for use in such experiments have been conventionally fabricated by depositing metal on the surface of the ultrahigh-mobility GaAs/AlGaAs using electron-beam lithography. However, it is widely accepted that nanofabrication processes degrade the quality of the pristine 2DEG and thus weaken the fragile states, [18, 116, 124, 133, 134] preventing exploration of the underlying physics. Therefore, it is crucial to maintain the as-grown material quality while inducing the electrostatic confining potential required for interferometry.

The present research takes a new approach that preserves the pristine quality of the heterostructure material while enabling nanoscale gating. The gate structure is fabri-
cated on a separate chip that is flipped over and brought close (~100 nm) to the heterostructure of interest, resembling a flip-chip assembly [135, 136]. FPIs are fabricated on the gate-chip and assembled onto a high mobility GaAs/AlGaAs chip. At low temperatures and in high magnetic fields, stable quantum interference patterns at IQH states are measured, an essential step toward interferometry at the FQH states.

The flip-chip technique has several unique advantages compared to the conventional gating method [136]. First, all the invasive nanofabrication processes are performed on a separate chip. Therefore, the GaAs chip is not contaminated with resist residues; no damage is inflicted on the surface; the device region under study is not exposed to any electron-beam radiation, [116, 133] etc. Second, the vacuum gap between the gate and the surface of the GaAs chip is an ideal dielectric layer, solving the problem of gate-leakage, interface charges, defects in the dielectric, and hysteresis in gating that are often found in traditional gate-dielectrics [18, 134, 137]. Third, the GaAs chip and the gate-chip are easily assembled and disassembled, and both can be reused. Fourth, the flip-chip setup enables complex and flexible device designs as the ‘dirty’ nanofabrication is performed on a separate chip. One can think of multi layered gate/dielectric structures for various functions, for example, single electron transistor sensors and superconducting quantum interference devices built into the gate-chip. Moreover, the setup can also be applied to other materials sensitive to nanofabrication processes, [117, 138] for instance, graphene, carbon nanotubes, HgTe/HgCdTe quantum wells, and three-dimensional topological insulators.

![Figure 3.1: Flip-chip setup. (a) Schematic of the flip-chip setup (not to scale). The Ti/Au (5 nm/40 nm) metallic gates (yellow) and Ti posts (100 nm, black) are deposited on a quartz chip (blue). The quartz surrounding the gates and posts is etched inward by ~4 µm. Afterward, the quartz chip is flipped over and put onto the GaAs target chip (gray), which has not undergone invasive fabrication. The posts, which are by design taller than the gates, ensure a minimum gate-heterostructure separation of ~60 nm. The 4 µm deep etch of the quartz chip reduces the probability that dust particles prohibits close separation. (b) Scanning electron micrograph of the quartz chip (from assembly A) showing the gate-pattern (inset) and posts. (c) Optical image of the coldfinger onto which the flip-chip assembly is mounted. Copper springs with adjustable forces press on the quartz chip to ensure a mechanically stable and small separation upon cooling to ~100 mK. The diameter of the PCB board is 3.5 cm.](image-url)
3. The Flip-Chip Setup

Several constraints have to be considered in the design of such a flip-chip setup: First, to electrostatically confine the potential landscape in the 2DEG on a length scale below 100 nm, the gates need to hover 100 nm or less above the surface of the target chip. Second, to prevent significant charge noise, the relative vibration amplitude between the two chips should be subangstrom. Third, the setup must sustain the cooling from room temperature to millikelvin temperatures as well as operation in high magnetic fields. Fourth, due to the close separation over a large area of $\sim 3 \text{ mm}^2$ the two chips need to be absolutely parallel (angle $\ll 1 \text{ mrad}$). Fifth, a precise positioning of the gate pattern with respect to the target chip is required, for example, positioning gates over a graphene flake. Finally, the setup should allow for illumination of the 2DEG.

Figure 3.1a schematically depicts the design of our flip-chip assembly. A double-side polished quartz chip (blue) supports the Ti/Au (5 nm/40 nm) gates (yellow). Titanium posts (black) with a thickness of 100 nm are evaporated onto the quartz chip. When the quartz chip is flipped over and pressed against the target chip, these taller posts ensure a fixed height of the gates with respect to the surface. Furthermore, the direct contact of the Ti posts with the target substrate results in a good mechanical stability.

Although all the aforementioned fabrication processes are performed in a clean-room, irregularities such as dust particles that prevent the two chips from approaching each other can still be present on the $\sim 3 \text{ mm}^2$ surfaces. The quartz surrounding the gates and posts is etched $\sim 4 \mu\text{m}$ inward by dry reactive ion etching to reduce the area that is in close proximity to the target chip. All the structures on the gate chip are patterned by standard electron-beam lithography. The transparent material of the gate-chip allows for precise alignment of the gates relative to the target chip simply by optical means. Moreover, it enables illumination of the 2DEG through the gate-chip. The insulating nature of the quartz substrate requires several precautionary measures. First, to prevent charge accumulation during the electron-beam exposure, a 15 nm thick Cr conductive layer is sputtered onto the quartz chip prior to spin coating of PMMA resist and is subsequently etched away after development. Second, as the insulating substrate is prone to static electricity, the fine gates are easily damaged and dust particles can be attracted onto the surface. Therefore, throughout the fabrication process, additional metal bridges are used to interconnect all the gates to impose an equal potential on all gates. Once the setup assembly is complete, the bridges are scratched away.

In the present study, we used a high mobility GaAs/AlGaAs heterostructure as the target material, where the 2DEG is 100 nm deep with single-side Si doping. To minimize the amount of fabrication on the heterostructure, six ohmic contacts are formed by soldering indium droplets on the edges of the heterostructure followed by a rapid thermal annealing step. Occasionally the surface roughness ($\sim 100 \text{ nm}$ on a millimeter scale) of the target wafer prevents a uniform spacing between the two chips. Regions with a larger separation require more negative gate voltage to deplete the electrons un-
derneath the gate, resulting in gate leakage at regions with small separation. To avoid such gate leakage, we performed one electron-beam lithography and wet etching step on the GaAs heterostructure to define a narrow (60 µm) transport channel, which is then isolated from the regions with possible gate leakage. For future applications this step needs to be avoided by ensuring the surface roughness of the wafers is much smaller than 100 nm over 3 mm².

Figure 3.1b shows a scanning electron microscope picture of a typical gate-chip. The inset shows the interferometer gate pattern for assembly A. The gate-chip and the GaAs chip are assembled and mounted on the coldfinger of the dilution refrigerator, as shown in Figure 3.1c (see Methods section).

Figure 3.2: Characterization of the 2DEG and gates for assembly A. (a) Schematic of the flip-chip assembly with contact and gate numbers. (b) Longitudinal (black) and Hall (red) traces as a function of perpendicular magnetic field B with all gates grounded. (c) Conductance as a function of gate voltage $V_{7,10}$ on the left QPC (gates 7 and 10) in both sweeping directions, indicating depletion of the 2DEG underneath the gates around $V_{7,10} = -2.2$ V and pinch-off of the QPC around $V_{7,10} = -3.4$ V. (d) Zoom-in on (c) near pinch-off showing quantized conductance steps.

3.3. CHARACTERIZATION OF THE SETUP

Flip-chip assembly A uses a gate-chip with an interferometer gate pattern depicted in the inset of Figure 3.1b. Figure 3.2a shows a schematic of the assembly numbering contacts and gates. The interferometer consists of two quantum point contacts (QPCs) formed by the left two (7 and 10) and right two (9 and 12) gates, and a pair of plunger gates (8 and 11). The QPCs bring the edge channels on the opposite edges in the quantum Hall

regime together so that interedge scattering becomes possible. The plunger gates control the area of the interferometer and thus the number of electrons. The assembly is cooled down in a wet dilution refrigerator hanging on a passive vibration isolation frame. The still pumping line is modified to isolate the sample from vibrations of the environment (e.g., pumps) but without extra damping stages inside the fridge.

At a base temperature of ~100 mK, the 2DEG is first examined without energizing the gates (grounded). Standard low frequency lock-in techniques are used to measure the longitudinal \( R_{xx} = R_{3,4} = (V_3 - V_4)/I_{2-5} \) and Hall resistances \( R_{xy} = R_{3,1} = (V_3 - V_1)/I_{2-5} \) employing a current bias of 2 nA between contacts 2 and 5 \((I_{2-5})\) in perpendicular magnetic field \(B\), as shown in Figure 3.2b. A sheet density of \(1.35 \times 10^{11} \text{ cm}^{-2}\) and a mobility of \(3.3 \times 10^6 \text{ cm}^2/(\text{V s})\) are extracted.

Next the QPCs are characterized. As Figure 3.2c shows, the conductance \(G = I_{2-5}/(V_3 - V_4)\) drops when the left QPC is energized with negative voltages \((V_{7,10})\) because the electron density underneath the gates is reduced. At a voltage of \(V_{7,10} = -2.2 \text{ V}\) the 2DEG underneath the gates is depleted (see Supporting Information Figure 3.6). Using the depletion voltage as an input to a capacitor model we estimate the distance between the gate and the surface of the GaAs chip to be ~84 nm. In an independent test of a flip-chip with two quartz chips containing capacitor plates, a separation of ~120 nm is confirmed by measuring the capacitance at high frequencies (see Supporting Information Figure 3.7). As more negative voltages are applied to the gates, the QPC is pinched off gradually. Note that almost no hysteresis is observed in opposite sweeping directions of the gate voltages, illustrating the advantage of using a vacuum as a gate dielectric. A close-up of the QPC trace in the depleted regime, as Figure 3.2d illustrates, shows quantized conductance plateaux due to depopulation of 1D subbands. The right QPC presents an identical behavior (see Supporting Information Figure 3.8). To investigate its reproducibility, the assembly is cooled down five times in two different cryostats. Depletion and pinch-off voltages do not change (within \(\pm 0.1 \text{ V}\)) after each thermal cycle, which demonstrates the robustness of the mechanical structure.

To study the vibrational stability of the setup, we define a quantum dot in the 2DEG. The voltages of the plunger gates are set past the depletion point \((V_{8,11} = -2.5 \text{ V})\) and both QPCs are close to pinch-off \((V_{9,12} = -3.27 \text{ V} \text{ and } V_{7,10} = -3.32 \text{ V})\). A voltage bias \(V_b\) is applied to contact 2, and current is measured between contact 5 and ground. Figure 3.3a shows the current as a function of \(V_b\) and gate voltage \(V_8 = -2.5 \text{ V} + \Delta V_8\). Regular and stable Coulomb diamonds are observed. A charging energy of ~55 µeV is extracted from the bias voltage at the top of the diamonds. Because a quantum dot is very sensitive to induced charges and hence to changes in capacitance, it is a perfect tool for investigating the stability of the gate-heterostructure separation. Vibrations are expected to shift the Coulomb diamonds up and down in gate-voltage and also change their size. Large vibrations induce extra electrons in the quantum dot and would distort the Coulomb diamonds beyond recognition. Small vibrations result in blurring of
3.3. Characterization of the Setup

The edges of the Coulomb diamonds and broadening of the zero-bias crossing. Therefore, an upper bound on the vibration amplitude can be estimated from the blur at the zero-bias crossing of the diamonds. Using 0.15 mV as the uncertainty of the Coulomb resonance peak position and taking into account the length ratio (~1/3) between gate 8 and all six gates, the upper bound on the vibration amplitude is estimated to be 1.5 pm (see Supporting Information Figure 3.6). Note that as the Ti posts are more than 50 µm away from the quantum dot, the influence of the floating potential is negligible. The duration of the measurement of the Coulomb diamonds of Figure 3.3a is ~15 min. On longer time scales, (~hour), though, a drift of the Coulomb diamonds is observed (see Supporting Information Figure 3.9). However, such slow drift is easily corrected using a feedback loop on the gate voltages (not yet implemented). Nonetheless, as discussed later, the current setup is stable enough to perform quantum interference measurement in a time scale of ~30 min.

Figure 3.3: Quantum dot and interferometer formed by the gates in assembly A. (a) Coulomb diamonds measured on a quantum dot defined by energizing all gates. (b) Diagonal conductance Gd as a function of perpendicular magnetic field B. The gate voltages are tuned to \( V_{8,11} = -3.5 \) V, \( V_{7,10} = -1.6 \) V, and \( V_{9,12} = -1.9 \) V. The quantum interference oscillations are plotted in (c) for the region between plateaux 2 and 3, and (d) between plateaux 1 and 2.
3.4. Quantum Hall Interference

Having confirmed the stability of the setup, we continue to investigate the quantum interference at IQH states using the flip-chip setup. We first explore the response of the QPCs in a perpendicular magnetic field. $V_{9,11}$ are set past depletion to -2.2 V. The magnetic field is set to 1.3 T, where the bulk of the 2DEG is in the incompressible state with filling factor $\nu = 4$. A current of $I_{2-5} = 300$ pA is applied, which is small enough to avoid self-heating. Diagonal conductance $G_d = I_{2-5}/(V_3-V_6)$ is probed as a direct measure of the filling factor at the QPC, that is, the number of channels transmitted by the QPC. As each of the two QPCs is energized with negative voltages, quantized conductance plateaux of $f_0 e^2/h$ ($f_0 = 1, 2, 3, 4$) are observed (see Supporting Information Figure 3.10), verifying the functioning of the QPCs and identifying the regions where backscattering occurs. Note that $f_0$ here represents the number of fully transmitted chiral quantum Hall edge-modes, unlike 1D subbands for a QPC without magnetic field, as Figure 3.2d depicts.

For both QPCs, $G_d = 2e^2/h$ at the depletion voltage of -2.2 V, indicating that the density in the QPCs is around half of that in the bulk (where $\nu = 4$). This is due to the smooth potential landscape induced by the gates. In pursuance of an interferometer, gate voltages for the QPCs are set to the low voltage side of plateau 3 ($V_{7,10} = -1.9$ V and $V_{9,12} = -1.6$ V) such that the two outer edge channels are fully transmitted and the third (inner) edge channel is partially transmitted, while the fourth channel is fully reflected. Note that although the QPC voltages are not yet beyond the depletion voltage of -2.2 V, the backscattering observed in the third edge channel indicates that the edge channels are brought close to each other. At plunger gate voltages $V_{8,11} = -3.5$ V, the amplitude of the quantum interference oscillations is maximized.

Figure 3.3b shows the dependence of $G_d$ on $B$ at the above gate voltage settings. Conductance plateaux of $f_T e^2/h$ ($f_T = 1, 2, 3, ...$) are observed, where $f_T$ equals the number of edge-modes that are fully transmitted through the interferometer. At the low magnetic field side of the fully developed plateaux, the innermost edge modes on opposite sides of the QPCs couple, leading to finite interedge tunneling. An electron in the innermost edge-channel entering the interferometer is partially reflected at the first QPC. The transmitted electron wave is again partially reflected at the second QPC. The two backscattered electron waves interfere constructively or destructively depending on the flux through the area between the QPCs. As Figure 3.3c,d shows, quantum interference oscillations of Gd are observed between plateaux 2 and 3 ($f_T = 2$), and 1 and 2 ($f_T = 1$) with magnetic field periods of 0.275 and 0.547 mT, respectively. From the latter magnetic field period, the estimated enclosed area by the edge-modes is 7.6 $\mu$m$^2$, close to the design area of 10 $\mu$m$^2$ (as discussed below).
Figure 3.4: Quantum oscillations at IQH states as a function of magnetic field and gate voltage for assembly B. (a) $G_d$ as a function of $B$ showing well-quantized integer plateaux. Inset depicts the gate-pattern for assembly B, with a QPC separation of 1.7 µm and an enclosed area of 13.6 µm$^2$. (b,d,f) and (c,e,g) Quantum oscillations as a function of $B$ and plunger gate voltage $V_g = -4.5 V + \Delta V_g$ between plateaux 3 and 4, 2 and 3, and 1 and 2, respectively.
Depending on sample geometry and size, quantum oscillations in a FPI fall into one of two regimes: the AB regime, or the CD regime [97–99, 112, 132]. To investigate the effect of the interferometer size, we switch to flip-chip assembly B containing a larger interferometer. The GaAs chip used for assembly B is from the same wafer as in assembly A. The QPCs have an opening of 1.7 µm and the lithographic area enclosed by the interferometer is 13.6 µm² (see inset of Figure 3.4a). The electrons underneath the gates are depleted at a gate voltage of -3 V, implying a gate-heterostructure separation of 115 nm (see Supporting Information Figure 3.11). Despite their large opening, the QPCs can still reach full pinch-off at a voltage of -9 V without hysteresis (see Supporting Information Figure 3.11).

The buildup of the interferometer follows the same procedure as for assembly A. To observe oscillations with significant amplitude in $G_d$, all gate voltages are set to -4.5 V. As shown in Figure 3.4a, fully developed quantized plateaux in $G_d$ are visible over a large magnetic field range. Between the plateaux, oscillations are again observed due to interedge tunneling events. Figure 3.4b,d,f zooms in on the regions between plateaux 3–4 ($f_T = 3$), 2–3 ($f_T = 2$), and 1–2 ($f_T = 1$) and shows interference oscillations as a function of magnetic field with periods of 0.22, 0.29, and 0.56 mT, respectively. In addition to oscillations in the magnetic field, we observe oscillations as a function of plunger gate voltage between plateaux 3-4, 2-3, and 1-2 as depicted in Figure 3.4c,e,g, respectively. The periods are 2.4, 2.02, and 2.11 mV, respectively, and exhibit no obvious dependence on $1/B$ (as discussed below and in Supporting Information Figure 3.12).

Furthermore, the interference pattern can be mapped out as a function of both perpendicular magnetic field and plunger gate voltage, as shown in Figure 3.5a-c for the regions between plateaux 3-4, 2-3, and 1-2, respectively. In all the measured 2D maps, the parallel lines of constant phase disperse with a positive slope (as discussed below). Each plot is obtained in ~35 min, indicating a very stable operation of the flip-chip setup with negligible charge noise, consistent with the quantum dot characterization.

In a FPI, an electron wave in the inner edge state partially scatters at each QPC to the opposite backward-flowing edge channel. These backscattered electron waves interfere with each other with a relative phase set by the flux enclosed by the trajectories, $\Phi = BA$ (where $B$ is the magnetic field, and $A$ the area of the interferometer), that is, the Aharonov-Bohm phase. The period of the interference in $B$ is $\Delta B = \phi_0/A$ ($\phi_0 = h/e$) regardless of the filling factors at the QPCs. On the other hand, the period in gate voltage is $\Delta V_g = a\phi_0/B$, where $a$ represents how fast the area changes upon changing the gate voltage. However, this single-particle picture breaks down when Coulomb interactions are strong. The interior of the FPI forms an island similar to a quantum dot with a finite charging energy [99, 139]. Depending on the electron-electron interaction strength and the capacitances from the island to the gates and the edge channels, the FPI can enter
the CD regime. The charging energy of the island is given by [99, 112]

\[
E = \frac{1}{2C} \left( eN + e f_T \frac{BA}{\phi_0} - CV_g \right)^2
\] (3.1)

where \(N\) is the number of electrons on the island, \(C\) and \(A\) are the total capacitance and the area of the island, respectively, and \(f_T\) is the number of fully occupied (transmitted) Landau levels. \(C_g\) and \(V_g\) are gate-to-island capacitance and gate voltage, respectively. Magnetic field couples to the charge on the island through the \(f_T\) underlying Landau levels. The island can be charged with one electron by changing the flux through the island by \(1/ f_T \cdot \phi_0\) or the gate voltage by \(e/C_g\). The induced charge on the island alters the area of the interference loop because of Coulomb interactions. Therefore, the magnetic field affects the enclosed flux \(\Phi = BA\) through \(B\) and \(A\), in contrast to the AB regime. The period of the oscillations in \(B\) and \(V_g\) are \(\Delta B = 1/ f_T \cdot \phi_0 / A\) and \(\Delta V_g = e/C_g\) (as both generate a phase difference of \(2\pi\) between the two interfering waves), respectively.

In addition, \(G_d\) can be plotted as a function of \(V_g\) and \(B\), as shown in Figure 3.5. For the AB case, increasing \(B\) is equivalent to increasing \(V_g\) as they both increase the flux. A negative slope is therefore expected for lines with constant phase. However, for the CD case increasing \(B\) is equivalent to reducing \(V_g\) (see eq. 1), thus leading to a positive slope of the constant phase lines.

In assembly A, the oscillations of \(G_d\) follow \(\Delta B \propto 1/ f_T\), indicating operation in the CD regime. In assembly B, \(\Delta B \propto 1/ f_T\), and oscillations in \(V_g\) show no dependence on \(f_T\). In addition, the constant phase lines in the 2D maps (Figure 3.5) have a positive slope at all measured plateaux. Thus, assembly B also falls into the CD regime, despite the larger dimension and wider QPCs of the interferometer. The CD type of interference is attributed to the small capacitance of the island, which is dominated by the capacitance to the edges of the gates. A top gate can increase the island’s total capacitance and turn the FPI into the AB regime [97, 112]. Moreover, a top gate would generate a sharper potential landscape (see Supporting Information Figure 3.13) and thus a better control over the interferometer.

We observe a small drift of the conductance peaks in the quantum dot, which we attribute to slow relaxation of the material and/or the setup. This slow drift can be actively corrected by a feedback loop on the gate voltages. Nevertheless, the current setup is stable enough to study the physics of the IQH and FQH states. In this flip-chip design, the gate-chip and material chip are mechanically connected by the posts. The influence of this mechanical contact on the material properties needs further study. The yield of flip-chip assemblies that meets all requirements is mostly determined by the flatness of both chips. Substrates without mechanical deformations would increase the yield and eliminate the need to etch channels in the target chip.
3.5. CONCLUSION

We have developed a flip-chip setup that allows for nanoscale electrostatic gating without the requirement of extensive nanofabrication processing on the target material. By measuring quantized conductance in a QPC and Coulomb blockade features in a quantum dot, we have shown that the flip-chip setup operates as required with a gate - heterostructure separation of $\sim 100$ nm and vibration amplitudes of several picometers. After characterization of the setup, we performed measurements at high magnetic fields. Using an FPI gate-pattern on the gate-chip, we probed the interference of IQH edge states. In both small and large interference loops (assembly A and B, respectively), the quantum interference is dominated by electron–electron interactions, that is, the Coulomb-dominated regime. To obtain pure Aharonov-Bohm interference the capacitance of the interferometer needs to be increased, achievable by implementing an extra top gate on the gate-chip.

In addition to its application in the high-mobility heterostructures investigated in this work, the flip-chip setup can be used on a wide range of materials, such as flakes of graphene or of 3D topological insulator. The platform developed for noninvasive nanoscale gating presented in this research opens a promising route toward the development of devices of unprecedented quality.

3.6. METHODS

The flip-chip assembly consists of a gate-chip and a material chip, an $8 \times 8$ mm quartz piece used as base and additional quartz supports to hold the gate-chip in place. The material chip is first glued onto the quartz base using S1805 photoresist. Then two quartz supports with the same thickness as the material chip are glued on either side of the material chip on the $8 \times 8$ mm piece. Afterwards, droplets of photoresist are placed on these quartz supports and the gate-chip is positioned over the material chip and quartz.
supports in a mask aligner. After alignment, a small force is applied onto the gate-chip to fix the setup while the glue dries (for \(\sim 12\) h). The flip-chip assembly can then be safely transferred out of the cleanroom without dust entering between the chips. The assembly is glued directly on the cold finger of the cryostat and all contacts are connected to a PCB board through a combination of wire bonding, indium soldering and use of silver paint. Next, copper clamps are placed onto the gate-chip. Springs attached to the cold finger can adjust the force exerted by the copper clamps. The forces and positions of the clamps are fine-tuned such that a uniform dark blue color is observed suggesting close and uniform spacing between the chips. This color originates from the interference between the reflected light from the surface of the material chip and the reflected light from the bottom surface of the gate-chip. A dark blue color suggests a separation of \(\sim 100\) nm between the gate and the surface of the material chip, and a uniform color signals uniform spacing. Figure 3.1c shows the flip-chip assembly mounted on the cold finger including the copper springs. The whole setup is free from magnetic parts and fits in a standard 2 inch magnet bore.

**3.7. Supporting Figures**
3. THE FLIP-CHIP SETUP: A NONINVASIVE METHOD FOR NANOSCALE ELECTROSTATIC GATING OF PRISTINE MATERIALS

Figure 3.6: Depletion voltage for the continuous gate that spans the width of the 2DEG in assembly A in the main text. (a) Four-terminal conductance as a function of the voltage on the continuous gate (indicated by red in the false-colored SEM image in a). The gate has a width of 8.3 µm, much wider than the average gate-heterostructure separation of 100 nm. Therefore, the parallel plate capacitor model (b) is justified for estimating the vacuum gap between the two chips. At a voltage of $V_{\text{continuous}} = -2.2$ V the conductance drops to zero, indicating that all electrons underneath the gate are depleted. The gate-2DEG coupling can be described by the capacitor network depicted in b. $C_{\text{GaAs}}$ is the capacitance per unit area between the 2DEG and the surface of GaAs. The capacitance between the gate and the surface of GaAs is $C_{\text{vac}} = \epsilon_0 \epsilon_r / d$ per unit area. The total capacitance between the 2DEG and the gate is $C_t = 1/(1/C_{\text{vac}} + 1/C_{\text{GaAs}})$.

Using $e \cdot n = C_t V_{\text{continuous}}$ and $V_{\text{continuous}} = -2.2$ V, a density of $n = 1.35 \times 10^{11}$ cm$^{-2}$ (obtained from Hall measurements), a depth of the 2DEG of 100 nm, we estimate that $d = 84$ nm.

The vibration amplitude between the two chips is estimated using the blur at the zero-bias crossings of the Coulomb diamonds. From Fig. 3.3a in the main text we estimate a 20% blur (0.15 mV in $V_8$). A change in $d$ affects the island potential through all six gates ($V_{8,11} = -2.5$ V, $V_{9,12} = -3.27$ V and $V_{7,10} = -3.32$ V). The capacitance of the island is $C_I = C_0 + C_g$, where the $C_0$ is the self-capacitance and $C_g$ the capacitance to the gates, which is predominantly the capacitance over the vacuum gap (because the dielectric constant of GaAs is 12x larger than vacuum). The plunger gate capacitance is $\sim C_g / 3$ (4 µm over 13 µm) by comparing the lengths of the gates. Using the capacitor model in b) we relate the change in capacitance to a change in separation $\Delta C_g / C_g = \Delta d / d$. The charge induced by change of the plunger gate $\Delta V_8$, $\Delta Q = C_g / 3 \cdot \Delta V_8$, should equal the charge induced by a change in chip separation, $\Delta Q = \Delta C V_g = C_g (\Delta d / d) V_g$. Thus an upper bound on the vibration amplitude is given by $\Delta d = d \Delta V_8 / 3 V_g = 1.5$ pm, using $V_g = -2.8$ V, $d = 84$ nm and $\Delta V_8 = 0.15$ mV.
3.7. SUPPORTING FIGURES

**Figure 3.7: Parallel plate capacitor.** (a) Tilted scanning electron micrograph of a quartz chip with a metallic square of 240 µm x 240 µm. The posts are ~100 nm higher than the metallic square and the surrounding quartz is etched 4 µm deep. The scale bar represents 100 µm. This quartz chip is flipped over and put onto the other quartz chip with the same metallic square but with neither posts nor etching. (b, c) Optical images of the two chips during and after alignment. (d) The full flip-chip assembly consisting of a parallel plate capacitor.

The two metallic squares act as a parallel plate capacitor from which the average distance is extracted by measuring the capacitance. The assembly is placed inside a nano-manipulator that is used to apply a variable force on the top of the assembly. Capacitance is measured using a lock-in amplifier. A sinusoidal voltage of $1 \text{ V}_{\text{rms}}$ is applied between the two plates at a frequency of 7.9 kHz. A measured current of 200 nA (Y-component) translates into 4 pF between the two capacitor plates resulting in an average separation of 120 nm. Precautions are taken to minimize the shunt capacitance of the wires to <0.2 pF.

**Figure 3.8: Characterization of the right QPC for assembly A in the main text.** (a) Conductance as a function of the voltage on the right QPC (gates 9 and 12) in both sweeping directions. The electrons in the 2DEG underneath the gates are depleted at -2.2 V and pinch-off of the QPC occurs at -3.3 V. (b) Quantized conductance steps near pinch-off, similar to the left QPC shown in Fig. 2c and d in the main text.
Figure 3.9: Coulomb peaks as a function of time for assembly A in the main text. By scanning the plunger gate voltage \( V_8 = -2.5 \text{ V} + \Delta V_8 \) over a 3 mV range, five Coulomb peaks of a quantum dot are probed at a voltage bias of \( V_b = 10 \mu\text{V} \). This trace is repeated 50 times during \( \sim 90 \) minutes. A slow drift in the Coulomb peak positions is observed. After 90 minutes, one extra electron has entered the quantum dot. Such drift may result from relaxation in the 2DEG and/or the setup. Note that except for the slow drift, no charge noise is observed, indicating no significant vibrations between the two chips. In the case that the drift is solely due to the mechanics of the setup, the gate-heterostructure separation needs to change by \( \Delta d = d \Delta V_8 / 3 V_g = 7.5 \) pm, using \( \Delta V_8 = 0.75 \text{ mV} \), \( V_g = -2.8 \text{ V} \) and \( d = 84 \text{ nm} \).
3.7. SUPPORTING FIGURES

Figure 3.10: QPC traces at a magnetic field of \( B=1.3 \) T for assembly A in the main text. The figure shows the diagonal conductance through the left (black) and right (red) QPC as a function of the gate voltages \( V_{7,10} \) and \( V_{9,12} \), respectively, with the plunger gates set at the depletion point of -2.2 V. When \( V_{7,10}=0 \) or \( V_{9,12}=0 \), \( G_{d}=4e^{2}/h \), consistent with a filling factor of \( \nu=4 \) in the bulk at \( B=1.3 \) T. Interference is observed if both QPC voltages are set on the low voltage side of plateau 3, \( V_{7,10}=-1.6 \) V and \( V_{9,12}=-1.9 \) V, as shown in Fig. 3.3c in the main text. At these gate settings the innermost edge channel (belonging to the fourth Landau level) is fully reflected at the QPC, while the third edge channel has a small backscattering amplitude. The outer two channels belonging to \( \nu=1 \) and \( \nu=2 \) are fully transmitted.
Figure 3.11: QPC traces for assembly B in the main text. (a, b, c, d) Diagonal conductance $G_d$ as a function of gate voltages $V_{9,12}$ for the right QPC of the interferometer at $B = 0, 0.1, 0.3, 0.5$ T, respectively. At $V_{9,12} = -3$ V the electrons underneath the gates are depleted. Using a parallel plate capacitor model and the depletion voltage, a gate-heterostructure separation of $\sim 115$ nm is derived. At $V_{9,12} = -9$ V the QPC closes. However, due to the large QPC opening of $1.7 \mu$m, quantized conductance is absent at zero magnetic field. In small perpendicular magnetic fields, quantized plateaux resulting from the backscattering of the quantum Hall edge channels are observed. Note that in all traces (a-b) no hysteresis is observed between down (black) and up (red) sweeps of the gate voltages.
3.7. SUPPORTING FIGURES

Figure 3.12: Magnetic field and gate voltage period of quantum oscillations as a function of $1/f_T$ for Assembly B, where $f_T$ is the number of fully transmitted edge-channels. (a) The magnetic field periods for different $f_T$ fall on a straight line through zero with a slope of $488.6 \, \mu T/(1/f_T)$. (b) The period in the plunger gate voltage has no obvious dependence on $f_T$. The periods in (a) and (b) are determined from the interference patterns in Fig. 3.4(b-g) in the main text after removing the background conductance, which is obtained by a 6th order polynomial fit to the data. The change in the background conductance as a function of gate likely originates from details of the tunnel rates in the QPCs.
3. THE FLIP-CHIP SETUP: A NONINVASIVE METHOD FOR NANOSCALE ELECTROSTATIC GATING OF PRISTINE MATERIALS

Figure 3.13: Electrostatic potential simulation. A double-layered gate structure can be implemented in the quartz gate-chip, as shown in the cross-section in the top panel. The electron density in the 2DEG can be simulated electrostatically for certain gate voltages. The red curve in the bottom figure shows the density profile in the 2DEG by applying a gate voltage of $V_G = -2.7$ V to the split gates (QPC gates in the interferometer) without implementing the global top gate. The black curve represents the density profile for a geometry with a grounded global top gate. The induced potential landscape is sharper in the latter case, and thus a better control can be achieved. In addition, as discussed in the main text, the top gate also increases the capacitance between gates and island (region enclosed by the interferometer) and can drive the interferometer from the Coulomb-dominated regime into the Aharonov-Bohm regime.

Figure 3.14: Layout of the GaAs/AlGaAs heterostructure used in this experiment.
3.8. EPILOGUE: CHALLENGES

Previous sections presented data of a flip chip assembly that has a gate-heterostructure spacing below 100 nm without influencing the heterostructure's properties. However, the yield of such assemblies is low. In this section we describe two reasons for this low yield.

The first reason is the wavy surface of the AlGaAs/GaAs heterostructure and gate chip. Figure 3.15a depicts the height profile of 4.3 mm$^2$ heterostructure surface. The heterostructure surface shows height fluctuations of $\sim$100 nm. Figure 3.15b and c presents the horizontal and vertical line cuts corresponding to the white dashed lines in Fig. 3.15a. This waviness of the surface complicates uniform <100 nm separation between gate chip and target heterostructure material. If you are 'lucky' to position the gates over a hill of the heterostructure surface, a local separation of 100 nm is possible. However, when gates are positioned over a valley, the ‘unlucky’ case, the separation exceeds 100 nm.

Figure 3.15: a) Height profile of the heterostructure surface. The white arrow indicates one of the etched trenches. b) Horizontal line cut of the height profile at $y = 1.4$ mm and in c) the vertical line cut at $x = 0.8$ mm. Note that the dip in b) around 0.9 mm is due to the etched trench.

The second reason for the poor yield is the sensitivity of the heterostructure to external forces. The gate chip is pressed onto the heterostructure chip by clamps, see Fig. 3.1c, thereby exerting a force on the heterostructure material. Figure 3.16 depicts magneto resistance traces for different clamp forces. In cool down 1 the sample resistance is larger than in the cool down without gate chip. From cool down 1 to 4 the clamp force is reduced by loosening the nut depicted in Fig. 3.1c. Reduction of the clamp force recovers the unperturbed device resistance (blue and black line). However, loosening the clamp results in an increase of the gate-material separation beyond 100 nm.
Figure 3.16: Magnetoresistance traces for different clamp forces. The force is decreased by loosening the nuts (depicted in Fig. 3.1c) that press the clamp onto the gate chip.
4

**Edge-mode superconductivity in a two-dimensional topological insulator**

Topological superconductivity is an exotic state of matter that supports Majorana zero-modes, which have been predicted to occur in the surface states of three-dimensional systems, in the edge states of two-dimensional systems, and in one-dimensional wires [24, 25]. Localized Majorana zero-modes obey non-Abelian exchange statistics, making them interesting building blocks for topological quantum computing [96, 140]. Here, we report superconductivity induced in the edge modes of semiconducting InAs/GaSb quantum wells, a two-dimensional topological insulator [26, 27, 78, 79, 141, 142]. Using superconducting quantum interference we demonstrate gate-tuning between edge-dominated and bulk-dominated regimes of superconducting transport. The edge-dominated regime arises only under conditions of high-bulk resistivity, which we associate with the two-dimensional topological phase. These experiments establish InAs/GaSb as a promising platform for the confinement of Majoranas into localized states, enabling future investigations of non-Abelian statistics.

4.1. Introduction

Several studies have reported on topological superconductivity in three-dimensional (3D) [143] and 1D [144–147] materials. In 2D semiconductor quantum wells a topological insulator (TI) is identified by the observation of a quantum spin Hall effect [26, 141]. In this phase the 2D bulk is a gapped insulator and transport only occurs in gapless edge states. These edge modes are spin-polarized and counter-propagating channels, known as helical modes, which are protected against elastic backscattering in the presence of time-reversal symmetry. To date, only two 2D TI systems have been identified experimentally — HgTe/HgCdTe quantum wells [78] and InAs/GaSb double quantum wells [142, 148]. In each of these, the origin of the TI phase is different: relativistic band-bending for HgTe/HgCdTe [79] and type-II broken band alignment for InAs/GaSb [27]. Recent scanning microscopy experiments have confirmed the presence of edge currents in both 2D TIs [149, 150]. The two different material classes are considered to be interesting complementary alternatives for topological studies.

4.2. Methodology

Effects arising from proximitizing TIs with superconductors have been investigated, including excess currents due to Andreev reflection [151] and Josephson effects in superconductor–normal–superconductor (SNS) junctions [152]. To demonstrate topological superconductivity (TS), however, it needs to be shown explicitly that superconducting transport takes place along the helical edges. Here, we demonstrate edge-mode superconductivity in InAs/GaSb. A similar experiment was reported recently by Hart et al. in the HgTe material [153].

A straightforward consequence of the conventional SNS junction configuration (Fig. 4.1a), in contrast to an edge-mode superconducting junction (Fig. 4.1b), can be observed in a superconducting quantum interference (SQI) measurement, where a perpendicular
Figure 4.1: a,b, Top panels: Schematic band diagrams for InAs/GaSb quantum wells. Owing to the type-II broken band alignment within the heterostructure, the electron (red) and hole (blue) 2D-bulk bands cross. Coupling between these bands opens up a topological gap, which is crossed by gapless, linearly dispersive helical edge states. These states are shown by the pink and green lines. Arrows indicate the spins of the states. When the Fermi level is in one of the bulk bands (a, orange rectangles) the critical current density profile is spatially uniform (middle panel) and the corresponding SQI has a Fraunhofer-like shape with a central lobe of width $2\phi_0$ and side lobes of width $\phi_0$ (bottom panel). When the Fermi level is in the topological gap and crosses the helical edge modes (b, orange rectangle), the current density profile is localized at the edges (middle panel) and the corresponding SQI has a SQUID-like shape (bottom panel). For a conventional SQUID, the SQI has $\phi_0$ periodicity (bottom panel, solid line). A $2\phi_0$-periodic SQI is expected for the helical edge modes in the absence of quasiparticle poisoning (two phases are possible, as shown by the dashed lines in the bottom panel of Fig. 4.1b, depending on whether or not the two edges have the same fermion parity) [154]. Quasiparticle poisoning can induce fermion parity switches that restore the $\phi_0$ periodicity, even for helical magnetic field induces oscillations in the amplitude of the superconducting current. A wide conventional SNS junction yields the Fraunhofer pattern, as shown in the bottom panel of Fig. 4.1a. In the case of edge-mode superconductivity the junction effectively acts as a superconducting quantum interference device (SQUID) with a well-known $\phi_0$-periodic interference pattern (bottom panel of Fig. 4.1b). A $2\phi_0$-periodic SQI is expected for the helical edge modes in the absence of quasiparticle poisoning (two phases are possible, as shown by the dashed lines in the bottom panel of Fig. 4.1b, depending on whether or not the two edges have the same fermion parity) [154].
modes.

To specify this further, we consider a short Josephson junction (defined as \( L \ll \zeta \)), where \( L \) is the contact separation and \( \zeta = \hbar v / \Delta_{\text{ind}} \) is the superconducting coherence length in the junction material with Fermi velocity \( v \) and induced gap \( \Delta_{\text{ind}} \), which has a sinusoidal current-phase relation. In this case, the Josephson supercurrent \( I_s(B_z) \) is given by the Fourier transform of the density profile of the critical current \( J_c(x) \) taken at a perpendicular magnetic field \( B_z = 0 \), \( I_s(B_z) = \Im \left[ \int_{-\infty}^{\infty} J_c(x) e^{i k x + \phi_0} d x \right] \), with the effect of the magnetic field included in \( k = 2 \pi L_{\text{eff}} B_z / \phi_0 \) (ref. [155]), where \( L_{\text{eff}} \) is the effective junction length taking into account magnetic flux focusing due to the Meissner effect. The critical current becomes \( I_c(B_z) = \max |I_s(B_z)| = \left| \int_{-\infty}^{\infty} J_c(x) e^{i k x} d x \right| \). For a wide junction with spatially uniform \( J_c(x) = \text{constant} \), the SQI pattern has the typical Fraunhofer form, \( \left| \sin \left( \pi L_{\text{eff}} W B_z / \phi_0 \right) / \left( \pi L_{\text{eff}} W B_z / \phi_0 \right) \right| \), with a central lobe of width \( 2 \phi_0 \) and side lobes of width \( \phi_0 \) (\( \phi_0 = \hbar / 2 e \) is the superconducting flux quantum) (Fig. 4.1a). In contrast, for edge-mode superconductivity, the SQI is simply \( \phi_0 \)-periodic (Fig. 4.1b).

Note that this analysis does not include effects with a topological origin, such as when the edge modes have helical character. In that case the SQI can become \( 2 \phi_0 \)-periodic [24, 25], as illustrated in Fig. 4.1b and discussed later in this Letter.

4.3. CHARACTERIZATION OF THE JUNCTION

Before investigating the superconducting regime we first describe the normal state transport in our Ti/Al–InAs/GaSb–Ti/Al junctions (for details of the device geometry see Fig. 4.2a,b). We focus on one device (device A) and map out the normal state resistance \( R_N \) when superconductivity is suppressed by \( B_z = 0.1 \, \text{T} \) (Fig. 4.2a). The junction has width \( W = 3.9 \, \mu m \) and contact separation \( L = 400 \, \text{nm} \), significantly shorter than the edge mode decoherence length of \( \sim 2 - 4 \, \mu m \) (refs [142, 148]). Transport is gate-tuned using the n+ GaAs substrate as a back gate, and a Ti/Au top gate. As the top-gate voltage \( V_{tg} \) is tuned from positive to negative, a resistance peak develops, indicating a charge neutrality point (CNP) [148, 156] when the Fermi energy is located in the topological gap (upper panel in Fig. 4.1b). For more positive \( V_{tg} \) the Fermi level is moved up into the conduction band and the dominant charge carriers are electrons, while for more negative \( V_{tg} \) the Fermi level is moved down into the valence band and charge transport is dominated by holes. This interpretation is confirmed by measurements in the quantum Hall regime performed on material from the same growth batch [156]. The position of the CNP shifts to more positive \( V_{tg} \) as the back-gate voltage \( V_{bg} \) is tuned more negative, as shown in the line cuts in Fig. 4.2b, in qualitative agreement with band structure calculations [27]. The maximum resistance at the CNP is \( \sim 7 \, \text{k}\Omega \). This value is smaller than the ideal quantized value of \( \hbar / 2 e^2 \) (\( \sim 13 \, \text{k}\Omega \)) expected for transport only via helical edge modes, indicating a residual bulk conductivity of \( \sim 15 \, \text{k}\Omega \).

For \( B_z < 11 \, \text{mT} \) we observe a supercurrent, a direct consequence of the d.c. Josephson effect. We define the switching current, \( I_{SW} \), as the value of the applied bias current when the developed voltage jumps from virtually zero to a finite value (Fig. 4.3b).
Figure 4.2: a, False-colour scanning electron microscope image of a typical S–InAs/GaSb–S junction, where S represents the superconducting material, which is composed of Ti(5 nm)/Al(150 nm) (see Supplementary Fig. 4.16 for devices with NbTiNx contacts). b, Cross-sectional view of device layout. c, Phase diagram measured on InAs/GaSb (device A, cooldown 1). $R_N$ is measured using d.c. excitation current $I_{sd} = 5$ nA. The Ti/Al contacts are driven into the normal state by an applied field $B_z = 100$ mT. The dashed rectangle refers to the data discussed in Fig. 4.5. d, Line cuts showing $R_N$ as a function of $V_{tg}$ for three different values of $V_{bg}$ (corresponding to the dashed lines in c).
Figure 4.3: a, $dV/dI$ versus $I_{sd}$ and $V_{tg}$ at $B = 0$, showing gate-tunable supercurrent through the junction ($V_{bg} = 0.1$ V). The three main transport regions are indicated by labels n (Fermi level in the conduction band), p (Fermi level in the valence band) and CNP (Fermi level at the charge neutrality point). b, $I–V$ traces without microwaves (black) and with microwaves (red), with $f_{RF} = 1.288$ GHz. Inset: Frequency dependence of the Shapiro step height, showing the expected linear dependence. Data are taken in the n-region ($V_{tg} = 5$ V and $V_{bg} = 0.2$ V). c, Dependence of the Shapiro plateaux on microwave field amplitude $P^{1/2}$. The white dashed line indicates the line cut corresponding to the red curve in b. Note that all Shapiro steps are observed, as expected deep in the electron regime.

$I_{SW}$ is tuned by means of gate voltages: as $V_{tg}$ becomes less positive $I_{SW}$ first decreases, then saturates at a minimum value for $V_{tg}$ near the CNP, and then increases again for more negative $V_{tg}$ due to hole-mediated transport through the bulk (Fig. 4.3a). To unambiguously establish the Josephson nature of our junctions, we irradiated the device with microwaves of frequency $f_{RF}$. The familiar Shapiro ladder [157, 158] is observed, with steps at $V = nh f_{RF}/2e$ ($n = 1, 2, \ldots$). Figure 4.3b shows a comparison of I-V curves measured without and with microwaves, the latter showing characteristic Shapiro steps, which are a consequence of the a.c. Josephson effect. The step heights exhibit the expected linear dependence when $f_{RF}$ is varied (inset of Fig. 4.3b). Figure 4.3c shows the characteristic modulation of the widths of the Shapiro steps by the magnitude of the applied microwave field. Similar data near the CNP are presented in Supplementary Fig. 4.12.

4.4. SUPERCONDUCTING QUANTUM INTERFERENCE

Having established the d.c. and a.c. Josephson effect in our InAs/GaSb junctions, we next analyse the spatial distribution of the supercurrent by performing SQI measurements at different gate values (Fig. 4.4). As shown by Dynes and Fulton [155], the current density profile $J_c(x)$ can be determined from the measured SQI provided the phase of the complex Fourier transform can be reliably estimated. We first comment on the validity of the Dynes and Fulton approach for our devices. The superconducting coherence length for an edge mode velocity $v \approx 4.6 \times 10^4$ m s$^{-1}$ in InAs/GaSb [159] is $\xi \geq 240$
4.4. SUPERCONDUCTING QUANTUM INTERFERENCE

Figure 4.4: a-f, Patterns and current density profiles in the n-region (a,b, \( V_{tg} = 4.8 \) V and \( V_{bg} = 0.2 \) V), at the CNP (c,d, \( V_{tg} = -0.3 \) V and \( V_{bg} = -0.4 \) V) and in the p-region (e,f, \( V_{tg} = -4.8 \) V and \( V_{bg} = 0.15 \) V). Gate voltages are indicated (I–III) in Fig. 4.2c. The effective device area used to extract \( J_c(x) \) was determined by requiring that the nodes of the SQI pattern be at multiples of \( \phi_0 \). Given the lithographic width \( W = 3.9 \mu m \), we compute an effective junction length \( L_{eff} \approx 640 \) nm. This is longer than the contact separation \( L = 400 \) nm, due to flux focusing by the superconducting contacts.

nm (using \( \Delta_{ind} \leq \Delta \approx 125 \mu eV \), where \( \Delta \) is the superconducting gap of the electrodes, Supplementary Fig. 4.13). We have verified that in our limit (where \( L \) is of order \( \zeta \) ) the SQI pattern is only weakly sensitive to deviations from a perfect sinusoidal \( I - \Phi \) relation (Supplementary Fig. 4.19), so the Dynes and Fulton short junction approach is indeed justified for obtaining qualitative supercurrent distributions. Recently, Hui et al. performed independent numerical calculations based on our data and support our results [160].

Figure 4.4 summarizes our main result: gate-tuning from bulk to edge-mode superconductivity. The figure shows SQI data at the representative points in gate space indicated in Fig. 4.2c, together with the current density profiles extracted using the Dynes and Fulton approach [153, 155] with \( L_{eff} = 640 \) nm. We observe three regimes: (I) a distinct Fraunhofer-like pattern when the Fermi energy is in the conduction band. The corresponding current density profile indicates that most of the current is carried by the bulk (Fig. 4.4a,b); (II) a SQUID-like interference when the Fermi energy is near the CNP. In this regime, the supercurrent density is clearly edge-mode dominated (Fig. 4.4c,d); (III) a return to a Fraunhofer-like pattern as the Fermi energy enters the valence band.
Here, the current distribution acquires a large bulk contribution, but edge modes also contribute over the range of accessible gate voltage values (Fig. 4.4e,f). Supplementary Fig. 4.7 presents additional SQI patterns measured at other points within gate space. Taken together, these data clearly demonstrate gate tuning between bulk and edge-mode superconductivity in InAs/GaSb and provide upper bounds on the edge mode widths (Supplementary Fig. 4.8). As a further check, we studied a non-topological InAs-only junction (device B), where, as expected, a SQUID-like SQI was not observed (Supplementary Fig. 4.15).

4.5. $2\phi_0$ EFFECT

The edge-mode SQI data typically show conventional $\phi_0$-periodicity (for example, as in Fig. 4.4c). However, over a certain gate range (dashed rectangle in Fig. 4.2c) we observe a striking even–odd pattern in the interference lobes. An example is shown in Fig. 4.5a. This $2\phi_0$-periodic effect is also seen in another device with different contact material (Supplementary Fig. 4.16). In the conventional Dynes and Fulton analysis this would require a current density profile containing three peaks, two at the edges and one in the middle (Fig. 4.5b). Simulations of such $2\phi_0$-SQI (Supplementary Fig. 4.17) indicate that this conventional analysis would require the middle channel to be within 10% of the device centre. It is improbable that such an effect would occur in two separate devices from different growth batches and different superconductors, although we cannot exclude this possibility.

The scenario above considers the possibility that interference paths enclose half the junction area. Alternatively, one could consider interference around the full junction area by particles of charge e instead of Cooper pairs with charge $2e$. The occurrence of e-interference is rare because supercurrent probes the coherence between superconductors by exchange of Cooper pairs. Nevertheless, several scenarios have been proposed involving processes with an electron travelling along one edge being Andreev-reflected as a hole into the other edge [161–165]. Such processes require phase coherence in excess of the sample circumference, $\sim 9 \mu m$, which seems large given previous transport data for InAs/GaSb quantum wells [148]. Another mechanism involves the fractional Josephson effect [29]. In this interpretation the edge modes need to have a helical structure and therefore contain Majorana zero-modes. Josephson-coupled Majoranas transport a charge e, indeed resulting in a doubling of the SQI periodicity [154, 166]. This interpretation, however, requires a quasiparticle poisoning time that is in excess of the measurement time (tens of seconds), which also seems improbably. Using existing techniques [163], future experiments should directly measure the rate of quasiparticle poisoning to further investigate the origin of this $2\phi_0$-periodic effect.
4.6. CONCLUSION

Using superconducting quantum interference, we demonstrate tuning between edge-dominated and bulk-dominated superconducting transport regimes as a function of electrostatic gating in InAs/GaSb quantum wells. This work establishes InAs/GaSb quantum wells as a platform for topological superconductivity and Majorana physics.

4.7. METHODS

The InAs/GaSb quantum wells were grown using molecular beam epitaxy on n+ (001) GaAs substrates. Two different material batches were used: a batch grown using high-mobility Ga (HM) and a batch using lower-mobility Ga (LM) [167]. The LM batch has lower residual bulk conductance near the CNP. Measurements were performed in a dilution refrigerator with a mixing chamber temperature of 16 mK equipped with a three-axis vector magnet. SQI patterns corresponding to an edge-mode current density profile were observed in three devices: device A from the main text (HM heterostructures and Al contacts) and devices C and D (based on LM heterostructures and with NbTiNx contacts, Supplementary Fig. 4.16). Device A was measured in two separate cooldowns. No significant changes in the device properties were observed between cooldowns. Offsets in $B_z$ of up to a few mT due to trapped flux in the superconducting magnets or leads were subtracted in the plotted SQI data. The spatial resolution of the current density profiles extracted from SQI patterns is $\sim W/\Delta\phi_0/\Delta\phi$, where $\Delta\phi$ is the magnetic flux range of the SQI measurement. In each of the plots, the full-width at half-maximum of the InAs/GaSb edge modes is near the Fourier resolution limit and represents an upper bound on the actual width of the edge mode. The maximum $\Delta\phi$ is limited by reduced visibility of the oscillations for $B_z \geq 11$ mT in the case of Al contacts and by switches along the $B_z$ axis in the case of NbTiNx contacts (presumably due to flux depinning in the leads, Supplementary Fig. 12). We attribute the non-zero values of $J_c(x)$ outside the device width to finite resolution and Fourier windowing effects (we used a standard rectangular window).
4.8. Supplementary Figures

Figure 4.6: Device images and measurement setup. Optical microscope image of a completed S-InAs/GaSb-S junction, similar to device A discussed in the main text. The inset shows a scanning electron microscope image for the InAs/GaSb mesa, defined using electron beam lithography and wet etching. The mesa was isolated by a selective wet etch that stops at the 50 nm-thick AlSb barrier (see heterostructure schematic in Fig. 4.2b in the main text). The stack was also selectively wet-etched down to the InAs layer, leaving behind an unetched ridge (600 nm-wide in the inset). Ridges were selected based on optical inspection and contacted by depositing Ti (5 nm)/Al (150 nm) onto the exposed InAs layer using e-beam evaporation. This was followed by sputtering a 100 nm-thick Si$_3$N$_4$ gate dielectric layer and evaporating the Ti/Au top gate. For the measurements, two wires were bonded to each Ti/Al contact, enabling quasi-four-terminal measurements with separate current and voltage wires from room temperature down to the device bonding pads.
Figure 4.7: Gate-dependence of SQI patterns. a-c, Differential resistance, \(dV/dI_{sd}\), as a function of perpendicular magnetic field, \(B_z\), and source-drain current bias, \(I_{sd}\), for several bottom and top gates settings (device A). a, For \((V_{bg}, V_{tg}) = (0, 4.8)\) V the device has a normal state resistance \(R_N \sim 900\) \(\Omega\), an intermediate value between the deep electron regime and the charge neutrality point. The corresponding current density profile obtained by reverse Fourier transform [153, 155] (as described in the main text) is shown in d. It indicates supercurrent contributions from both the bulk (c.f. Fig. 4.4a in the main text) and the edge modes (c.f. Fig. 4.4b in the main text). b, For \((V_{bg}, V_{tg}) = (0, -0.8)\) V the device is in the CNP regime, with \(R_N \sim 5000\) \(\Omega\). The data shows a SQUID-like SQI pattern. The corresponding current density profile in e is dominated by the edge modes. c, For \((V_{bg}, V_{tg}) = (-0.8, -4.8)\) V the device is in the hole regime, with \(R_N \sim 2600\) \(\Omega\). The corresponding current density profile is shown in f. In general, we observe that the SQI pattern becomes SQUID-like, corresponding to edge-mode-dominated superconducting transport, whenever \(R_N \gtrsim 900\) \(\Omega\).
**Figure 4.8**: Even-odd effect in the switching current. a, $dV/dI_{sd}$ vs. $B_z$ and $I_{sd}$ measured on device A at $(V_{bg}, V_{tg}) = (-0.8, 4.8)$ V. The data show an even-odd alternation of the switching current amplitude, as in Fig. 4.5, but here measured at different gate settings. We emphasize that this pattern, which effectively doubles the period of the SQI to $2\phi_0$, is robust. We observed it across a wide range of gate space (see Fig. 4.2c and Fig. 4.5a), as well as in one other device (see Fig. 4.16a). Based on the Dynes and Fulton approach [153, 155], for a $2\pi$-periodic current-phase relation the observed pattern translates into a current density profile with a third peak near the device center, as shown in b. As explained in the main text, this period doubling effect could also result from the fractional Josephson effect [168–171], which is expected to lead to an SQI periodicity of $2\phi_0$ for 2D TI helical edge modes. However, this topological interpretation requires a quasiparticle poisoning timescale longer than the measurement time, which is improbable.

**Figure 4.9**: SQI pattern over a large magnetic field range. a, $dV/dI_{sd}$ vs. $B_z$ and $I_{sd}$ measured on device A at $(V_{bg}, V_{tg}) = (-0.4, -0.15)$ V over a larger magnetic field range. The supercurrent oscillations disappear at $\pm 11$ mT, which we attribute to the suppression of superconductivity in the contacts. By resolving more SQI oscillations over a flux range $\Delta \phi$ corresponding to $\Delta B_z \sim 22$ mT, we can enhance the spatial resolution ($\sim W\phi/\Delta \phi$) of the current density profile in b. The extracted full width at half maximum of the current density peaks (marked by the two pairs of arrows) sets an upper bound of $\sim 260$ nm for the edge mode width.
Figure 4.10: Magnetic-field dependence of the normal state resistance. No magnetic-field dependence is observed for moderate magnetic fields (device A).  

a, $R_N$ vs. $V_{tg}$ and perpendicular field, $B_z$, for fixed $V_{bg} = -0.8$ V. No change in $R_N$ is observed when sweeping $B_z$ from 0 T to 0.8 T.  
b-d, $R_N$ vs. $V_{bg}$ and $V_{tg}$ at different magnetic fields, $(B_x, B_y, B_z) = (0.5, 0, 0.1)$ T (b), $(1.9, 0, 0)$ T (c) and $(2.9, 0, 0)$ T (d). The $R_N$ phase diagram is almost unchanged between the three fields with increasing in-plane component $B_x$. This is consistent with the lack of in-plane magnetic field dependence reported by Du et al. [142] and could be due to a very small effective g-factor of the edge modes. To the best of our knowledge, this observation is not well described by existing theoretical models.
Figure 4.11: Temperature dependence of the critical current. a-e, SQI patterns from device A measured for $T = 16$ mK, $40$ mK, $70$ mK, $100$ mK and $130$ mK, respectively. The data sets were measured for fixed $V_{bg} = -0.8$ V and $V_{tg} = 5.5$ V, the same values as for Fig. 4.55a. No difference is observed between 16 mK and 40 mK. The switching current begins to decrease at 70 mK, which likely indicates an effective junction temperature between 40 mK and 70 mK while the mixing chamber is at base temperature. At temperatures above $\sim 130$ mK, the switching currents become small, however the even-odd effect is still observed.
4.8. Supplementary Figures

Figure 4.12: Shapiro steps near the charge neutrality point (CNP). Shapiro steps [157] from device A, at gates settings $V_{bg} = -0.4 \, \text{V}$ and $V_{tg} = -0.15 \, \text{V}$ ($R_N \sim 3500 \, \Omega$). When microwaves of frequency $f_{RF} = 1.288 \, \text{GHz}$ are applied, several Shapiro steps develop (numbered -3 to +3 in the figure). This dataset was measured near the CNP, while the Shapiro steps discussed in the main text (Fig. 4.3c)) were measured in the electron regime. A signature of topological superconductivity is the suppression of the odd-number steps due to the fractional Josephson effect [168–171]. Here, we do not observe such suppression, which could be due to quasiparticle poisoning.

Figure 4.13: Line-cuts showing representative $I-V$ curves in the three transport regimes. a-c, $I-V$ curves from device A measured by sweeping $I_{sd}$ in the two opposite directions at three different gates settings. a. $(V_{bg}, V_{tg}) = (0.1, 5.5) \, \text{V}$, b. $(V_{bg}, V_{tg}) = (0.1, -1.2) \, \text{V}$ and c. $(V_{bg}, V_{tg}) = (0.1, -5) \, \text{V}$. The finite slope around $I_{sd} = 0$ in b and c is likely due to temperature broadening effects, which are more effective at low switching currents. d, $I-V$ curve over a large range of $I_{sd}$. The blue dotted line is a fit for small positive $I_{sd}$. The $I-V$ curve deviates from the low-$I_{sd}$ behaviour above $V \sim 250 \, \mu\text{V}$, (see arrow) from which we extract $\Delta \sim 125 \, \mu\text{eV}$ in the contacts, as expected for our Ti/Al material [172]. Since the $I-V$ slopes for low and high $I_{sd}$ differ by less than 10%, we use the slope at low current bias to extract the normal state resistance, $R_N$. 
Figure 4.14: Gate-dependence of the switching current. Differential resistance as a function of $I_{sd}$ for device A at $V_{bg} = 0$ V. As in Fig. 4.3a, when sweeping top gate from electron side through charge neutrality point to hole side, the switching currents decrease and then increase again. The green line shows the $I_{SW}R_N$ product, which closely follows the switching current, $I_{SW}$. We note that the $I_{SW}R_N$ product is considerably smaller than the superconducting gap of the Ti/Al contacts, $\Delta/e \sim 125 \mu$V. This may indicate a small induced superconducting gap, $\Delta_{\text{ind}}$, in the InAs/GaSb quantum well or a large suppression of the switching current, $I_{teSW}$, with respect to the critical current, $I_c$, as a result of the electromagnetic environment or thermal activation (note that $k_B T$ is of the order of the $I_{SW}R_N$ product).
Figure 4.15: Superconducting and normal transport for an S-InAs-S junction. Having induced superconductivity in the edge modes (see main text), here we investigated the SQI patterns on a similar junction fabricated on InAs only, where the helical edge modes should be absent. This device (device B) was fabricated by contacting the uncovered InAs layer with Ti/Al. a, Schematic layout. b, Optical microscope image of the device before topgate deposition. Device B underwent the same fabrication process as InAs/GaSb device A (both are on the same chip). c, Normal state resistance as a function of $V_{bg}$ measured at $B_z = 0$ using a DC excitation current $I_{sd} = 200$ nA. The top gate was kept floating due to a disconnected bonding pad. d, $dV/dI$ as a function of $B_z$ at three $V_{bg}$ values marked by 1, 2 and 3 in c. e, the corresponding current density profiles, showing a uniform current density through the bulk of the InAs layer, as expected for this non-topological junction. We see no evidence for edge modes in InAs, however, completely ruling out the existence of any non-topological edge modes requires gating the device to resistances above $\sim 900 \Omega$ (see Fig. 4.7), which could not be achieved due to the onset of backgate leakage for $V_{bg} < -1.5$ V.
Figure 4.16: SQI patterns for an S-InAs/GaSb-S junction based on InAs/GaSb grown using a lower mobility Ga source and contacted with NbTiNx. This device (device C), was fabricated using an InAs/GaSb quantum well structure grown with a lower mobility Ga source, which suppresses the residual bulk conductivity [167]. Differently from devices A and B, the superconducting contacts are made from 200 nm-thick sputtered NbTiNx, and have a width of 1 \( \mu \)m. a, Without any gating, we observe a SQUID-like SQI pattern as a function of \( B_z \) (for \( R_N \sim 1600 \Omega \)), which corresponds to edge-mode dominated superconducting transport. By analogy with device A, this suggests that the Fermi level resides in the bulk gap (near the CNP). Similar results were also observed in another similar device (device D, not shown). Further, we note the presence of an even-odd effect, similar to that observed in device A (see Fig. 4.5 and Fig. 4.8a). b, The corresponding current density profile extracted using a \( 2\pi \)-periodic current-phase relation. c, SQUID-like pattern for a larger range in \( B_z \). For \( B_z > -1.8 \) mT the pattern shows switches along the \( B_z \)-axis, presumably due to flux depinning in the NbTiNx contacts (the 200 nm contact thickness is below the London penetration depth of \(~ 250 \text{ nm} \) [173] of NbTiNx). The switching behaviour is hysteretic in the field-sweep direction (not shown), which is consistent with flux-depinning. Despite the switches, it is clear that the oscillations in c show very little attenuation at larger magnetic field amplitudes, consistent with a SQUID behaviour.
Figure 4.17: Simulated SQI patterns. Simulation results for a SQUID, assuming a third transport channel located at a distance $\Delta x$ from the center. We use a conventional, $2\pi$-periodic current-phase relation. The inset shows the current distribution for which the SQI patterns are calculated. The edge and center modes each have a width of 0.25 $\mu$m. The central mode has a critical current density 4 times smaller than that of the edges, in order to fit the measured data. An even-odd effect similar to experimental observations (see Fig. 4.5a in the main text, Fig. 4.8a, and Fig. 4.16a) is obtained for $\Delta x = 0$ (third mode exactly in the middle). The effect is not seen in simulations where $\Delta x > 0.2 \mu$m. This restricts the position of a possible third channel within $\sim 200$ nm from the center. Curves are offset from each other by 0.12 nA for clarity.
Figure 4.18: $I - V$ and $dV/dI$ traces for small switching currents. a, Color plot of $dV/dI$ vs. $I$ corresponding to Fig. 4.4c. b, $I - V$ traces corresponding to the line cuts in a. Traces are offset by 1 µV for clarity. c, $dV/dI$ vs. $I$ traces obtained by differentiating the $I - V$ traces in b.
Figure 4.19: Calculated SQI patterns for bulk and edge-mode supercurrent, with sawtooth and sinusoidal current-phase relations (CPR). The figure shows a matrix of four different combinations of CPR and supercurrent distribution. A sawtooth current-phase relation describes the extreme case of a junction with contact spacing much longer than the superconducting coherence length; a sinusoidal relation describes a short contact separation. We notice that the uniform supercurrent distribution always results in a ‘Fraunhofer’ type SQI (central lobe twice as wide as the outer lobes) irrespective of the CPR used. In contrast, for the edge-mode supercurrent distribution the SQI always shows an uniform SQUID-like pattern (all lobes equal width, no amplitude modulation). This demonstrates that the Dynes and Fulton approach can qualitatively distinguish between uniform and edge-mode supercurrent distribution.
### Table 4.1: Details of the devices discussed in the main text and Supplementary Information

<table>
<thead>
<tr>
<th>Device name</th>
<th>Width (W) (µm)</th>
<th>Contact spacing (L) (nm)</th>
<th>Contact material</th>
<th>Heterostructure type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3.9</td>
<td>400</td>
<td>Ti/Al</td>
<td>HM Ga</td>
</tr>
<tr>
<td>B (InAs)</td>
<td>3.9</td>
<td>400</td>
<td>Ti/Al</td>
<td>HM Ga</td>
</tr>
<tr>
<td>C</td>
<td>3.9</td>
<td>450</td>
<td>NbTiNₓ</td>
<td>LM Ga</td>
</tr>
<tr>
<td>D</td>
<td>3.9</td>
<td>550</td>
<td>NbTiNₓ</td>
<td>LM Ga</td>
</tr>
</tbody>
</table>
Electric and magnetic tuning between the trivial and topological phases in InAs/GaSb double quantum wells
Among the theoretically predicted two-dimensional topological insulators, InAs/GaSb double quantum wells (DQWs) have a unique double-layered structure with electron and hole gases separated in two layers, which enables tuning of the band alignment via electric and magnetic fields. However, the rich trivial-topological phase diagram has yet to be experimentally explored. We present an in situ and continuous tuning between the trivial and topological insulating phases in InAs/GaSb DQWs through electrical dual-gating. Furthermore, we show that an in-plane magnetic field shifts the electron and hole bands relatively to each other in momentum space, functioning as a powerful tool to discriminate between the topologically distinct states.

5.1. INTRODUCTION

Two-dimensional topological insulators (2DTIs), known also as quantum spin Hall insulators, are a novel class of materials characterized by an insulating bulk and gapless helical edges [23, 26, 78, 79]. Double quantum wells (DQWs) of indium arsenide and gallium antimonide (InAs/GaSb) have a unique type-II broken gap band alignment and are especially interesting since the electron and hole gases which form a topological band structure are spatially separated [27, 142, 148, 174]. For the appropriate layer thicknesses, the top of the hole band in GaSb lies above the bottom of the electron band in InAs, hence for small momentum (around $k = 0$) the band structure is inverted. At the crossing point ($k_{\text{cross}}$) of the two bands, coupling of the electrons and holes opens up a bulk hybridization gap [64, 156, 175–179] with gapless helical edge modes [27]. The size of the gap is determined by both $k_{\text{cross}}$ and the overlap of the electron and hole wave functions [180]. Due to the spatial separation of the two gases, electric and magnetic fields can induce relative shifts of the bands in energy and momentum [176, 181, 182], respectively. By controlling such shifts, it is possible to in situ tune between the trivial and topological insulating phases, which is the key advantage of InAs/GaSb compared to the other known 2DTIs [27, 151, 183, 184].

Here, for the first time, we map out the full phase diagram of the InAs/GaSb DQWs by independent control of the Fermi level and the band alignment through electric dual-gating. In particular, we observe the phase transition between the trivial insulator (normal gap) and topological insulator (hybridization gap). Moreover, the evolution of the resistance for in-plane magnetic fields is different in the two distinct phases, consistent with the fact that one is trivial, and the other topological.

5.2. THEORY

In InAs/GaSb DQWs, the band alignment can be controlled by top and back gate electrodes [27, 181] (see the structure shown in Fig. 5.1(a)). The two gates control the perpendicular electric field $E_z$, which shifts the electron and the hole band relatively to each other in energy by $\Delta E = eE_z\langle z \rangle$ ($\langle z \rangle$ is the average separation of the electron and hole gases), and the position of the Fermi level $E_F$. The resulting trivial-topological phase di-
5.2. Theory

Figure 5.1: (a) Schematic of the InAs/GaSb DQWs structure. E1 and H1 mark the bottom of the conduction band and the top of the valence band respectively, showing an inverted band alignment. (b) Phase diagram as a function of the applied electric field ($E_z$) and the Fermi level position ($E_F$), which can both be tuned by dual-gating using top gate and back gate. The insets show the band structures and the position of Fermi level at different points, A to I. The red and blue background colors represent the electron and hole densities, respectively. The vertical white line, along which the electron (red) and hole (blue) bands touch, separates the normal and inverted band alignment regimes. (c) Sketch of the phase diagram as a function of back gate voltage ($V_{BG}$) and top gate voltage ($V_{TG}$). The labeled points in (b) are indicated accordingly in (c). The red and blue lines mark constant electron and hole densities, respectively. The yellow dashed lines indicate constant band overlap for the inverted case or constant band separation for the normal case. The black dashed line represents charge neutrality. The white regions above and below point E correspond to the hybridization gap and the normal gap, respectively.
agram is shown in Fig. 5.1(b). On the left side of the vertical white line that marks the electric field for which the two bands touch, the DQWs are in the trivial regime. Independently from the electric field, Fermi level can be tuned either to cross the electron band (point A), be in the normal gap (point B) or cross the hole band (point C). The red and blue background colors denote the electron and hole densities, respectively. By increasing $E_z$ the two bands move towards each other and the size of the normal gap decreases until the two bands touch at a certain value of $E_z$ (marked by the white line). By increasing $E_z$ further, the two bands overlap (invert) and the system enters the topologically non-trivial regime. In such inverted regime, the electrons and holes have the same density at $k_{\text{cross}}$ and mixing between them opens up a hybridization gap (white region around point H at the right side of the white line). The green lines separate the region with a single type of carriers (electrons or holes) from the region where both types of carriers are present. Figure 5.1(c) shows the phase diagram as a function of the back gate ($V_{BG}$) and top gate ($V_{TG}$) voltages (assuming the same coupling strength from both gates) [185] (see supporting information). The yellow dashed lines indicate constant $E_z$, and thus a fixed band alignment. Along the black dashed line ($V_{TG} = -V_{BG}$), the total carrier density is zero (charge neutrality). The red and blue curves represent the constant density lines for electrons and holes, respectively. These curves bend when the Fermi level starts to cross both electron and hole bands (along the green lines) as the total density of states increases and screening sets in. The constant electron and hole density lines bend differently according to the effective masses of the two types of carriers and the asymmetric quantum well structures. Note that, the phase diagram shown in Fig. 5.1(c) follows the calculations by Liu et al. [27] qualitatively.

5.3. HETEROSTRUCTURE

Our heterostructure was grown by molecular-beam epitaxy [58]. A 100 nm buffer layer was first grown on a doped GaSb substrate, followed by a 50 nm AlSb bottom barrier. The DQWs consist of 5 nm GaSb and 12.5 nm InAs, followed by a 50 nm AlSb top barrier and a 3 nm GaSb cap layer (see supporting figure 5.10). Importantly, the GaSb substrate is lattice matched with the subsequent layers, which eliminates the requirement of a thick buffer layer compared to commonly used GaAs substrate and therefore enables a strong coupling between the back gate and quantum wells. Furthermore, for such choice of substrate, strain and the amount of dislocations are reduced resulting in record values of carrier mobility for this type of DQWs [58] (see supporting Fig.5.11). The Hall bars of 100 µm by 20 µm used in our measurements are chemically wet etched (inset of Fig. 5.2(a)). Ohmic contacts are fabricated by etching to the InAs layer prior to evaporation of Ti (50 nm)/Au (300 nm) layers. A sputtered 70 nm thick Si$_3$N$_4$ gate dielectric layer is used to isolate the Ti/Au top gate from the heterostructure. Longitudinal and Hall resistances are measured using standard lockin techniques at 300 mK unless otherwise stated. Two nominally identical devices are studied in detail.
5.4. ELECTRIC FIELD TUNING

First we map out the phase diagram of the InAs/GaSb DQWs by measuring the longitudinal resistance ($R_{xx}$) as a function of $V_{BG}$ and $V_{TG}$ for device #1 (Fig. 5.2(a)). The phase diagram reveals two regions of high resistance, labeled as I and II. Line L (R) crosses region I (II) and the corresponding resistance trace is shown in Fig. 5.2(b) (Fig. 5.2(c)). Region I has a maximum resistance of 8 kΩ, while the resistance in region II reaches 90 kΩ. The two regions touch around $V_{BG}=-0.2$ V and $V_{TG}=-1.2$ V, where the resistance shows a lower value of 4 kΩ, indicating a closing of a gap at this point in gate space. From this point, two less pronounced resistance peaks extend out (highlighted by the green lines, see supporting Fig. 5.12), indicating the onset of the coexistence of electrons and holes, as explained below. Note that, the finite conductance in the gapped regions I and II may result from disorder potential fluctuations in the bulk. In addition, for the inverted regime, level broadening will result in a finite residual bulk conductivity even at

**Figure 5.2:** (a) Four terminal longitudinal resistance ($R_{xx}$) as a function of $V_{BG}$ and $V_{TG}$ for device #1 measured at 300 mK, showing the phase diagram of the InAs/GaSb DQWs. Lines L and R cross the two different gapped regions (resistance peaks), labeled as I and II. Colored dots indicate the positions in gate-space where longitudinal resistance and Hall traces are taken, as shown in Figs. 5.3(a) and (b). The two green lines indicate the two less pronounced resistance peaks (see text). Inset shows the optical image of the Hall bar. The scale bar represents 20 µm. (b), (c) Resistance along lines L and R, respectively.
Figure 5.3: (a), (b) Hall resistance as a function of perpendicular magnetic field for five out of seventeen measured points along lines L and R in Fig. 5.2(a), respectively. The colors of the traces here correspond to the colored dots in Fig. 5.2(a). (c), (d) Carrier densities and longitudinal resistance for the seventeen uniformly distributed positions along lines L and R in Fig. 5.2(a), respectively. Black squares represent the electron densities ($n_{\text{SdH}}$) obtained from SdH. Red dots are the total densities ($n_{\text{Hall}}$) extracted from the Hall slope. Zero magnetic field resistances ($R_{xx}$) are shown by the green circles. In (c), the blue open triangles show the difference between the electron density and the total density ($n_{\text{SdH}} - n_{\text{Hall}}$), i.e., the hole density. At the left side of the resistance peak, the blue solid triangles and the red solid triangles represent the hole density ($p_{\text{fit}}$) and the electron density ($n_{\text{fit}}$) obtained from the fit of the bended Hall traces with a two-carrier model, respectively. In (d), the hole density near gapped region II deduced from fitting is shown by the blue solid triangles.

$T=0$ K [180]. The contribution from helical edge modes to the in-gap conductivity is expected to be small as the length of the Hall bar is significantly larger than the coherence length of the helical edge mode [156, 178].

To investigate the nature of the two gapped regions, we perform magnetoresistance measurements in perpendicular magnetic field ($B_z$) at the indicated points along lines L and R shown in Fig. 5.2(a). For clarity, five out of the seventeen measured Hall traces are shown in Figs. 5.3(a) and (b), for lines L and R, respectively. In Fig. 5.3(a), Hall traces at positions 1 (black) and 4 (red) taken along line L, start with a negative slope for small magnetic fields but bend up when magnetic field increases, indicating the coexistence of a majority of holes and a minority of electrons. Across the gap (e.g. point 12), the Hall trace also has a bend but a negative slope prevails, implying a majority of electrons. This interpretation can be clearly recognized in the extracted carrier types and densities.
5.4. Electric field tuning

along line L presented in Fig. 5.3(c).

In Fig. 5.3(c), the electron density (black squares) is obtained from the Shubnikov-de Haas (SdH) quantum oscillations which are present at all points along line L. These SdH oscillations are exclusively generated by the electrons in the InAs layer as a direct result of the much higher mobility for electrons [186] (see supporting information). To extract the hole densities, two different approaches are taken. First, at the left side, the hole concentration (blue solid triangles) is derived from a fit to the Hall traces using a two-carrier model [187]. Such a fit also provides the electron density, as indicated by the red solid triangles (see supporting information Fig. 5.6). Second, on the right side of the gap, the hole density (blue open triangles) equals to the difference between the total density and the electron density (obtained from SdH oscillations). The total density is calculated from the Hall slope at high magnetic fields, since for $B \gg 1/\mu_e, 1/\mu_h$, $R_{xy} \approx B/(p-n)e$, where $\mu_e (\mu_h)$ is the electron (hole) mobility and $p, n$ are the hole density and the electron density, respectively. This analysis maps out both the electron and hole densities across gapped region I. The extrapolated hole density (brown dashed line) crosses the electron density near the center of the resistance peak at $n = p \approx 4 \times 10^{15} \text{ m}^{-2}$. Accordingly, the wave vector at the crossing of the electron and hole bands is $k_{\text{cross}} = 1.59 \times 10^8 \text{ m}^{-1}$ [156, 178]. The above analysis for gapped region I is consistent with an inverted band alignment and a hybridization gap opening at the crossing of the two bands.

We now turn to region II which is crossed by line R (Fig. 5.2(a)). In contrast to region I, the Hall traces along line R are nearly linear, as shown in Fig. 5.3(b). First, at the right side of the resistance peak, density values are obtained from the Hall slope (red dots) and the SdH period in $1/B$ (black squares). The close agreement between the Hall and SdH densities implies that solely electrons are present. Secondly, close to the gap, as the resistance becomes large and the carrier density is low, the Hall traces are characterized by fluctuations (such as at position 9). Finally, at the left side of the resistance peak, the transport is dominated by holes as obvious from the positive slope of the Hall traces. Importantly, in contrast to gapped region I, here both electron and hole densities are vanishingly small at the resistance peak, typical for a trivial insulator with a normal band gap. We note however that in the hole regime the Hall traces do have a slight bend. A two-carrier model is used to extract the hole density (blue solid triangles in Fig. 5.3(d)) and a small residual electron density of $\sim 5 \times 10^{14} \text{ m}^{-2}$, which may indicate a parallel conducting path.

To substantiate the above identification of the two gaps, we investigate the band alignment and Fermi level position for the distinct regions in gate space. We apply a 2 T perpendicular magnetic field and measure the phase diagram, as shown in Fig. 5.4. At the right side of the 2D map, the parallel lines correspond to SdH oscillations. The uniform spacing indicates a linear change of electron density as a function of both $V_{BG}$ and $V_{TG}$. The red curve follows a fixed Landau level, along which the electron density is constant. However, following this line towards the left, at position G the curve bends, indicating a coexistence of electrons and holes [186]. Such a bend arises when the Fermi
5. Electric and magnetic tuning between the trivial and topological phases in InAs/GaSb double quantum wells

Figure 5.4: Longitudinal resistance as a function of $V_{BG}$ and $V_{TG}$ at a perpendicular magnetic field of 2 T for device #1. In a fixed perpendicular magnetic field, Landau levels will cross the Fermi level when the carrier density is tuned by gates. The parallel lines correspond to SdH oscillations and each line represents a contour of constant carrier density. From B to E, the electron and hole bands get closer and an insulator to semimetal transition occurs. From E to H, in the inverted regime, the overlap of the two bands increases and the Fermi level lies in the hybridization gap. The red curve denotes a constant density line for electrons, where the Fermi level is fixed relative to the bottom of the electron band and only the hole band moves. The two green lines separate the regions with a single type and two types of carriers.

level crosses the top of the hole band (green solid line). A similar effect happens when electrons come in at the hole side (green dashed line). The two green lines originating from point E follow the kinks on the constant density lines and separate the regions of single type and two types of carriers. The position of the green lines here is consistent with the two less pronounced resistance peaks in Fig. 5.2(a). Note that, at the gate regime just below gapped region I (between the green dashed line and the gap in Fig. 5.4), the observed SdH oscillations are primarily from electrons because of the lower mobility for holes than electrons, even though the holes have a higher density (see Fig. 5.6). Importantly the phase diagram taken in a finite magnetic field (Fig. 5.4) as well as the one taken at zero magnetic field (Fig. 5.2(a)) are fully consistent with the interpretation that: (i) Along the resistance peak from B to E, the electron and hole bands approach each other in energy with the Fermi level lying in the middle of the normal gap; (ii) At point E the two bands touch and the transition from normal to inverted band alignment takes place; and (iii) Towards position H, the overlap increases and the Fermi level lies in the hybridization gap.
5.5. MAGNETIC FIELD TUNING

A further confirmation for the origin of the high resistance regions is the dependence of the resistance peaks on in-plane magnetic field [176, 182]. An in-plane magnetic field shifts the electron and hole bands in momentum (in the direction perpendicular to the magnetic field) by a relative amount of \( \Delta k_y = \frac{e B_z}{\hbar} \), which is expected to reduce the hybridization gap but not the normal gap [176, 182]. To investigate this prediction, we focus on device #2 which was mounted in the plane of the two main axes (x and y) of a vector magnet. Device #2 is identical to device #1, in the sense of material, dimensions and all fabrication processes. The phase diagram for device #2 is nominally the same as device #1 (see Fig. 5.12). Figs. 5.5(a) and 5.5(b) show the in-plane magnetic field, \( B_y \), dependence of the resistance for device #2 along the same lines L and R as device #1, respectively, while Figs. 5.5(c) and 5.5(d) show the \( B_x \) dependence accordingly.

For the case of region I (line L), the height of the resistance peak decreases\(^2\) and the peak position shifts slightly in gate space for both \( B_y \) and \( B_x \) (Fig. 5.5). The decrease of the resistance peak is anisotropic as it decays faster in \( B_y \) (perpendicular to the transport direction). These observations are consistent with a relative shift of the two bands in momentum for the inverted regime. The anisotropy is expected since \( B_x \) (parallel to transport direction) shifts the bands in \( k_y \), and \( B_y \) shifts the bands in \( k_x \) [176, 182]. The inset of Fig. 5.5(a) ((c)) shows a sketch of the band structure along \( k_x \) for finite \( B_y \) (\( B_x \)) and the hybridization gap closes more quickly for \( B_y \) because of the ‘tilted’ gap. The relative shift at 4 T is estimated to be \( \Delta k = \frac{e B \langle z \rangle}{\hbar} = 5.3 \times 10^7 \text{ m}^{-1} \), which is smaller than \( k_{\text{cross}} \). In clear contrast, for region II (line R) the resistance remains unchanged up to 9 T, although the same amount of band shift as region I is expected. This insensitivity on \( B_x \) and \( B_y \) demonstrates the trivial nature of gapped region II. Hence, the in-plane magnetic field dependence further corroborates the different types of the two gapped regions.

5.6. HYBRIDIZATION GAP SIZE AND DISORDER POTENTIAL

Finally, to give a rough estimate on the size of the hybridization gap, we take gate positions 6 and 10 in Fig. 5.2(a) as the gap edges. Combining the electron density difference between the two selected points based on a linear extrapolation of \( n_{\text{SdH}} \) and the constant 2D density of states \( m_e/\pi \hbar^2 \), we estimate a gap size of \( \Delta = \pi \hbar^2 (n_{10} - n_6)/m_e = 9.3 \text{ meV} \), which is larger than values reported in the literature [148, 156, 176, 178, 179, 188]. The effective mass of \( m_e = 0.04 m_0 \) for electrons is deduced from the temperature dependence of SdH oscillations, i.e., the Dingle plot [189], deep in the electron regime \( (V_{\text{BG}}=1 \text{ V}, V_{\text{TG}}=2 \text{ V}) \) where \( n = 2.2 \times 10^{16} \text{ m}^{-2} \) and \( \mu_e = 70 \text{ m}^2/\text{Vs} \) (see supplementary Fig. 5.7). The relatively large deduced gap may be overestimated due to the inaccuracy of the selected gap edge positions, or it is indeed large because of strong

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\(^2\) By measuring the Hall resistance, a ~7 degree angle between \( B_y \) and the surface of the device is determined. As perpendicular magnetic field induces a large positive magnetoresistance, the peak resistance increases when \( B_y \) is small as a result of the competition between the positive magnetoresistance and the gap closing, as shown in Fig. 5(a).
5. ELECTRIC AND MAGNETIC TUNING BETWEEN THE TRIVIAL AND TOPOLOGICAL PHASES
IN INAS/GaSb DOUBLE QUANTUM WELLS

Figure 5.5: (a), (b) In-plane magnetic field ($B_y$) dependence of the longitudinal resistance for device #2, along the same lines L and R in Fig. 5.2(a). (c), (d) The same as (a), (b) but for $B_x$. The resistance for line L drops as $B_x$ or $B_y$ increases, while for line R, it stays constant. Insets in (a)-(d) show the band structure along $k_x$. As the relative shift of the bands in momentum is perpendicular to the in-plane magnetic field, the hybridization gap closes more quickly in $B_y$ (perpendicular to the transport direction) than in $B_x$ (parallel to the transport direction).

coupling between electrons and holes for these gate values. We also notice that at the upper-left side of gapped region I (larger $k_{cross}$) in Fig. 5.2(a) the peak resistance drops, indicating a decrease of the hybridization gap. Such decrease is presumably due to a reduced wave function overlap and hence a reduced coupling strength, in spite of the increased $k_{cross}$. From the Dingle plot deep in the electron regime we extract a quantum scattering time of $\tau_q = 0.27$ ps, corresponding to a quantum level broadening [156, 178] of $\Gamma_e = \hbar/2\tau_q = 1.22$ meV. However, close to the gap, the electron mobility drops by more than one order of magnitude ($\mu_e = 5.5$ m²/Vs at position 5), suggesting a much larger $\Gamma_e$. Hence, the total level broadening will be $\Gamma = \Gamma_e + \Gamma_h \gg 1.22$ meV (where $\Gamma_h$ represents the hole contribution), which could account for the relatively low resistance at the hybridization gap.
5.7. CONCLUSION

In conclusion, we explored the full phase diagram of InAs/GaSb DQWs structure by measuring dual-gated Hall bars. We observed two gapped regimes manifested by regions with a longitudinal resistance peak. For one gapped region, the extracted electron and hole densities both vanish at the resistance peak. While for the other gapped region finite equal electron and hole densities are present around the resistance peak. Our findings are fully consistent with a scenario that one gapped region corresponds to a trivial insulator with a normal gap while the other corresponds to a topological insulator with a hybridization gap. Moreover, the dependence of the two resistance peaks on in-plane magnetic fields further corroborates the different origins of the two gapped regions.

5.8. SUPPLEMENTARY INFORMATION

5.8.1. TWO-CARRIER MODEL

When electron and hole gases coexist in a two-dimensional system, the dependence of Hall resistance $R_{xy}$ on perpendicular magnetic field $B$ is [187]:

$$R_{xy} = \frac{-B\left[(n\mu_e^2 - p\mu_h^2) + B^2\mu_e^2\mu_h^2(n-p)\right]}{e\left[(n\mu_e + p\mu_h)^2 + B^2\mu_e^2\mu_h^2(n-p)^2\right]}$$

(5.1)

where $n$ and $p$ are the densities for electrons and holes, respectively; $\mu_e$ and $\mu_h$ are the corresponding mobilities. The measured Hall trace can be fitted using the formula above, and a constraint is given by the zero-field longitudinal resistance:

$$B = 0 : R_{xx} = \frac{L}{e(n\mu_e + p\mu_h)W}$$

(5.2)

where $L$ and $W$ are the length and width of the Hall bar, respectively.

In the limit of $B \gg 1/\mu_e, 1/\mu_h$, $R_{xy} \approx \frac{B}{(p-n)e}$.

5.8.2. DINGLE PLOT

For a two-dimensional electron gas in a perpendicular magnetic field $B$, the amplitude of the Shubnikov-de Haas (SdH) oscillations, $\Delta R$, is given by [189]:

$$\Delta R = 4R_0\exp\left(-\frac{\pi m_e}{eB\tau_q}\right)\frac{2\pi^2 m_e k_B T}{ehB} \sinh\left(\frac{2\pi^2 m_e k_B T}{ehB}\right)$$

(5.3)

where $R_0$ is the zero-field resistance, $m_e$ the effective electron mass, and $\tau_q$ the quantum scattering time. At a fixed magnetic field $B$, by measuring the temperature dependence of $\Delta R$, $m_e$ and $\tau_q$ can be extracted from the fit, the so called Dingle plot.

5.8.3. CALCULATION OF THE PHASE DIAGRAM

A simple capacitor model (Fig. 5.8) is used to simulate the phase diagram of the InAs/GaSb double quantum wells (DQWs). The result (Fig. 5.9) is in qualitative agreement with the
5. Electric and Magnetic Tuning Between the Trivial and Topological Phases

In InAs/GaSb Double Quantum Wells

Figure 5.6: An example of two-carrier model fitting (red) of the Hall trace (black) taken at position 1 on line L in Fig. 5.2(a) in the main text. The zero-field longitudinal resistance is used as the constraint. The extracted carrier densities and mobilities are shown in the figure.

calculations by Liu et al. [27].

The device operation can be explored and semi-quantitatively understood by considering electrostatics of its stack in the simplistic equivalent capacitance model [185] which is shown in Fig. 5.8. Here \( C_{TG} \), \( C_M \), and \( C_{BG} \) are geometric capacitances between the top gate and the two-dimensional electron gas (2DEG) (or, more accurately, 2DEG electron density center plane located inside the InAs layer), between 2DEG and two-dimensional hole gas (2DHG), and between 2DHG and the back gate, respectively. \( C_e \) and \( C_h \) are quantum capacitances reflecting an energy penalty associated with the gradual filling of electron/hole states. It is proportional to the density of states \( D \). For a 2D subband in a parabolic approximation, \( D = \frac{|m|}{\pi \hbar^2} \) is a constant defined by the subband in-plane effective mass \( m \). The penalty is zero when the corresponding subband is fully depleted, as \( D = 0 \) in this case. With the Fermi surfaces of electron and hole 2D gases grounded through the Ohmic contacts, electron and hole accumulation can be parameterized by the energies of the electron and hole subband extrema, \( -|e|V_e \) and \( -|e|V_h \). Thus, \( C_{e,h} \) are piecewise constant functions of \( V_{e,h} \): \( C_e = e^2 D_e \) when \( V_e > 0 \) (and is zero otherwise), while \( C_h = e^2 D_h \) only when \( V_h < 0 \). This model can be numerically solved for \( V_{e,h} \) at arbitrary back and top gate biases \( (V_{BG}, V_{TG}) \), to obtain the electron \( n = C_e V_e / |e| \) and hole \( p = -C_h V_h / |e| \) densities.

This simplistic model reproduces the anticipated physics well, resolving conditions in gate space when a normal gap forms (i.e., no conduction: \( n = p = 0 \)), single carrier situations (either \( n = 0 \) or \( p = 0 \)), and when both electrons and holes are present simultaneously. The resulting phase diagram is shown in Fig. 5.9. It marks electron and hole...
5.8. Supplementary Information

Figure 5.7: Dingle plot taken deep in the electron regime, where $V_{TG}=2$ V and $V_{BG}=1$ V for device #1 in the main text. The fit (red) of the temperature dependence of the SdH oscillation amplitude (black) gives the effective electron mass $m_e = 0.04m_0$ ($m_0$ is the electron mass), and the quantum scattering time $\tau_q = 0.27$ ps. (Deep in the hole regime, the Dingle plot gives the effective hole mass $m_h = 0.09m_0$.)

**Figure 5.8:** Capacitor model of the InAs/GaSb double quantum wells device.

depletion boundaries $V_{e,h} = 0$ (green lines), charge neutrality line $n = p$ (dashed), and a zero-gap condition $V_e = V_h$ (yellow line). Constant density contours for both electrons and holes are also shown. Geometric capacitances $C_{TG} = 3.6 \times 10^{11} \frac{|e|V^{-1} cm^{-2}}{cm^2} = 58$ nF cm$^{-2}$, $C_M = 9.8 \times 10^{12} \frac{|e|V^{-1} cm^{-2}}{cm^2} = 1.6$ $\mu$F cm$^{-2}$, and $C_{BG} = 4.9 \times 10^{11} \frac{|e|V^{-1} cm^{-2}}{cm^2} = 79$ nFcm$^{-2}$ have been estimated for the actual multilayer dielectric/semiconductor stack using $\epsilon_{SiN} = 7.0$, $\epsilon_{GaSb} = 15.7$, $\epsilon_{AlSb} = 10.9$, and $\epsilon_{InAs} = 15.5$ dielectric constants, assuming that electron and hole central planes coincide with centers of InAs and GaSb layers, and that $V_{BG}$ is applied to the edge of the n-doped GaSb substrate. Experimentally determined $m_e = 0.04m_0$ and $m_h = 0.09m_0$ have been used to set quantum capacitance values at $C_e = 1.7 \times 10^{13} \frac{|e|V^{-1} cm^{-2}}{cm^2} = 2.7$ $\mu$Fcm$^{-2}$ and $C_h = 3.8 \times 10^{13} \frac{|e|V^{-1} cm^{-2}}{cm^2} = 6.0$ $\mu$Fcm$^{-2}$.

The above simple model does not include $C_{e,h}$ modulation due to electron and (especially) hole non-parabolicities, the spatial shift of the wave functions inside the InAs and GaSb layers, as well as other details related to the band alignment in the device.
Figure 5.9: Simulation result from the capacitor model. Red and blue shaded areas indicate regions of purely electrons and holes, respectively. The purple shaded area indicates a region where electrons and holes coexist. Red and blue lines represent equal density of electrons and holes, respectively. The yellow lines indicate points where the two bands touch each other. The white region at the bottom right part of the figure indicates the normal insulating gap. The dotted white line at the top left part connect points where electrons and holes have equal density and the hybridization gap is expected.

5.8.4. Supplementary Figures
Figure 5.10: Cross-section of the InAs/GaSb DQWs used in the main text. The quantum well contains a 12.5 nm InAs layer on top of a 5 nm GaSb layer. The structure is grown on a doped GaSb substrate.

Figure 5.11: Densities and mobilities for electrons and holes for device #1 along line R in Fig. 5.2(a). The electron mobility is more than one order of magnitude higher than the hole mobility. Because of the application of the GaSb substrate, record values of the electron mobility are achieved.
5. ELECTRIC AND MAGNETIC TUNING BETWEEN THE TRIVIAL AND TOPOLOGICAL PHASES IN INAS/GaSb DOUBLE QUANTUM WELLS

Figure 5.12: (a) Longitudinal resistance as a function of back gate and top gate voltages for device #2 in the main text measured at 300 mK, which shows nominally the same phase diagram as Fig. 5.2(a) for device #1 in the main text. The green line indicates the onset of the coexistence of electrons and holes. (b) Line cuts taken from (a) between \( V_{TG} = 0.75 \) V and 2 V. The arrow shows the shift of the small resistance peak, consistent with the arrow in (a).

Figure 5.13: Hall resistance as a function of both top gate and back gate voltages at a fixed perpendicular magnetic field of 2 T for device #1 in the main text. This figure was measured simultaneously with Fig. 5.4. For regions with only holes or a majority of holes, the Hall resistance shows positive values, as shown by the red color. In contrast, the blue color indicates regions with only electrons or a majority of electrons.
5.9. Epilogue: Trivial Edge Conduction

Experiments that followed the measurements described in the previous section show conduction along the edge of the sample in the trivial gapped phase [190]. This epilogue presents three experiments that relate to this unexpected result. First, in section 5.9.1, the trivial gap size is measured. Second, in section 5.9.2, we show existence of edge conduction in the trivial gapped phase. Third, in section 5.9.3, using a device containing side gates we try to remove this trivial edge conduction.

The devices used in these experiments were fabricated on wafer B, which has the same structure as wafer A of the previous section, except for a thinner InAs layer of 10.5 nm (which was 12.5 nm for wafer A). First, wafer B is characterized and the trivial and topological regions are identified.

Characterization of Wafer B
Wafer B is characterized using a Hall bar device following the same procedure as described in section 5.4. Figure 5.14a shows the longitudinal resistance \( R_{xx} \) of the device as a function of top gate voltage \( V_{tg} \) and back gate voltage \( V_{bg} \), measured using a current bias of 5 nA. Two regions I and II can be identified. Figure 5.14b(c) shows the Hall resistance as a function of perpendicular magnetic field for indicated points along line L(R) crossing regions I(II). Around gapped region II the Hall traces are linear while around region I they bend. Following section 5.4 we therefore ascribe region II to a trivial gapped phase and region I to the inverted gapped phase.

By comparing the phase diagram of wafer B (Fig. 5.14a) to wafer A (Fig. 5.2), we see that at zero back gate voltage wafer B is deeper in the trivial regime (i.e., the phase diagram is shifted to lower \( V_{bg} \)). This is consistent with the thinner InAs layer thick-
Figure 5.15: Corbino device on the wafer with 10.5 nm InAs layer. (a) Schematic of the Corbino device and the electrical connection. The dark gray regions indicate contacts while the light gray region represents the gated (top gate not shown) bulk material. The source contact is voltage biased while current to ground is measured. (b) Conductance of the Corbino device as a function of $V_{bg}$ and $V_{tg}$ at a bias voltage of 100 $\mu$V and temperature of 20 mK. The data is corrected for the series resistance of 15.5 kΩ. Line G starts at $(V_{bg}, V_{tg}) = (0, -3.5)$ V and ends at (-2.2,-0.9) V. (c) Line cut of (b) at $V_{bg} = -1$ V. The conductance drops to zero inside the trivial gapped region. (d) Temperature dependence on the in-gap conductance for indicated points in (b).

ness of wafer B, resulting in a larger confinement energy for electrons, and therefore the structure has a smaller inversion. The resistance in the trivial gapped region is far from insulating and does not exceed 200 kΩ.

### 5.9.1. Trivial Gap Size

From the capacitor model (using geometric capacitances) a maximum trivial energy gap size around 40 meV ($\sim 500$ K) is expected for the gate voltages used. At a sample temperature of 300 mK and bias voltage $\sim 5$ mV such a large gap is expected to give an insulating behavior ($R_{xx} \sim G\Omega$). A surprising result of Fig. 5.2 and Fig. 5.14a is the low sample resistance of 90 kΩ and 200 kΩ inside the trivial gapped region for a Hall bar device. The data indicates a smaller energy gap than is expected.

This section presents data obtained from a Corbino device. The gap size in this device is extracted from the temperature and voltage dependent resistance of the trivial gap. The data corroborates to indeed a smaller gap size than expected from the capacitor model.
CORBINO DEVICE

Figure 5.15a shows a schematic of the Corbino device used in this experiment. It consists of two concentric metallic rings (with radii \( r_1 = 50 \mu m \) and \( r_2 = 80 \mu m \)) forming the drain and source electrode respectively. A top gate is fabricated over the bulk region using a 90 nm thick Si\(_3\)N\(_4\) gate dielectric. Current can only flow in the ring shaped bulk between the source and drain electrode. There are no sample edges connecting the source to the drain and therefore possible edge modes do not contribute to measured conductance.

The source is biased with a voltage \( V_{sd} = 20 \mu V \) while the current \( I \) in the drain is measured. Figure 5.15b shows the conductance of the Corbino disc as a function of top gate voltage \( V_{tg} \) and back gate voltage \( V_{bg} \) at a temperature of 20 mK. The data is corrected for a series resistance of 15.5 k\( \Omega \) from filters and contacts.

The conductance map looks qualitatively similar to the resistance map of the Hall bar presented in Fig. 5.14a (high resistance regions occur at similar gate positions). Therefore the region of low conductivity (high resistivity) at \( V_{bg} > -2 V \) is ascribed to the trivial gapped region. Remarkably, the conductance in the trivial gapped region, in-gap conductance, drops to zero (infinite resistance) whereas it was finite for the Hall bar device (see Fig. 5.14a). For clarity, Fig. 5.15c shows a line cut at \( V_{bg} = -1 V \).

TEMPERATURE DEPENDENCE

The trivial gap size is obtained from the temperature dependence of the in-gap conductivity. Figure 5.15d shows the in-gap conductivity for several points along the trivial gapped region as a function of temperature. Good agreement with the Arrhenius law \( \sigma_{xx} \propto \exp(-\Delta/k_B T) \) indicates activated transport. The gap sizes \( 2\Delta \) increases from 1.4 K to 8.2 K when moving to more positive back gate voltages \( V_{bg} \geq -1 V \) along line G. The temperature dependence for \( V_{bg} = -1.35 V \) does not follow an Arrhenius law, nor a variable range hopping model. This point is presumably close to the semi-metallic point without a well defined gap.

VOLTAGE DEPENDENCE

Next we investigate the bias voltage dependence of the mid-gap conductance in the trivial gapped regime, the mid-gap gate voltages are indicated by line G in Fig. 5.15b. Figure 5.16a shows the differential conductance as a function of bias voltage \( V_{sd} \) and mid-gap gate voltage. For \( V_{bg} > -1.25 V \) the plateau of zero conductance opens and extends over a larger bias range for larger \( V_{bg} \). Three bias voltages traces indicated in Fig. 5.16a are plotted in Fig. 5.16b. The I-V traces show an insulating behavior at low bias (\( \sim 1 mV \)), whereas for higher bias the device switches to a conductive state. Such a behavior resembles tunnel spectroscopy of an energy gap, where the bias voltage drops over the tunnel barrier.

We speculate that the voltage drops close to the contact. Figure 5.16c sketches the proposed band alignment in the contact region. In first approximation, the metallic con-

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\( ^3 \)This resistance is determined from the total resistance at \( V_{tg} = V_{bg} = 0 V \) (deep electron regime), where the sample resistance < 10 \( \Omega \) (see Fig. 5.14a).
**Figure 5.16**: Voltage bias dependence in the trivial gap. (a) Differential conductance as a function of bias voltage $V_b$ and gate position along line G in Fig. 5.15. The series resistance of 15.5 kΩ is included. (b) I-V traces at the gate positions indicated in (a). (c) Schematic of the band alignment close to the contact region. Left of (c) is the region underneath the source electrode. The Fermi-level ($E_F$) resides above the conduction band (CB). Right is the bulk material which electrostatics is dominated by the top gate and back gate. The Fermi-level is forced inside the trivial gap by $V_{tg}$ and $V_{bg}$.

Contact (left region) acts as a top gate with $V_{tg} \approx 0$, which drives the sample underneath in a $n$-doped conductive phase, according to Fig. 5.14b. In the bulk (right region) the top gate and back gate voltages fix the Fermi-level inside the band gap. For a region close to the contact, the voltages on the top gate, back gate, and source electrode determine the Fermi-level position through their capacitances. For the region far from the contact, the capacitance to top and back gate dominate and determine the Fermi level. Therefore, the source voltage is expected to drop fully over a lateral region comparable to the back gate set back, which is $\sim 50$ nm.

In this scenario, a bias voltage moves the Fermi-level in the lead independently from the mesa. Therefore, the bias voltage at which the current onsets is a direct measure for half the gap size. Thus, by inspecting the width of the zero-conductance plateaus in Fig. 5.16b the gap increases from 0 to 4 meV ($\sim 35$ K).

Estimates of the trivial gap size from the temperature dependence and bias voltage dependence are of the same order ($\sim 10$ K). However, they are one order lower than expected from a capacitor model ($\sim 500$ K). This discrepancy is yet not understood. Perhaps the assumptions introduced in the capacitor model need to be reexamined. The model assumes that the 2D gases are located in the center of the InAs and GaSb layers. However in the trivial gap, the negative (positive) top (back) gate voltages push the electrons (holes) more towards the interface, resulting in larger mutual capacitance $C_M$ between the electron gas and hole gas (see Fig. 1.8 for the capacitor model). The gap size is proportional to the voltage drop over $C_M$, which is $\Delta V_M = \frac{C_T C_B}{C_T + C_B + C_M} (V_{tg} - V_{bg})$. In the
usual case that \( C_M \gg \{C_T, C_B\} \) the trivial gap size is proportional to \( 1/C_M \). The mutual capacitance, hence separation between wave functions, should be reduced by a factor 3 to attain the expected 500 K gap.

Note that the ‘conductive’ high-bias state is actually still quite resistive. Using a dc current bias of 5 nA, the Corbino resistance in the trivial gap is >250 kΩ (data not shown). Converting to resistivity via \( \rho_{xx} = 2\pi R/\ln(r_2/r_1) \) gives \( \rho_{xx} > 2.6 \text{ MΩ}/\square \). For a 5 square Hall bar device this would result in 16.7 MΩ resistance, which is almost two orders larger than the in-gap resistance of 200 kΩ measured on the Hall bar of Fig. 5.14a. This discrepancy will be covered in the following section.

### 5.9.2. Trivial Edge Channels

Presence of edge conduction would manifest itself in a finite resistivity for a Hall bar device compared to insulating behavior for a Corbino device. This is indeed observed. The Corbino resistance in the trivial gap is infinite, as has been shown in Fig. 5.15b. The Hall bar resistivity at low bias (50 µV) is ~800 kΩ/□ (data shown in the next section). This makes us believe that edge conductance exists in the trivial gapped phase of InAs/GaSb quantum wells. This trivial edge conduction is investigated in detail in our report Nichele et al. [190], of which a part is summarized below.

Another way to determine edge conduction is by comparing devices of different width
5. ELECTRIC AND MAGNETIC TUNING BETWEEN THE TRIVIAL AND TOPOLOGICAL PHASES

while fixing the length. The bulk resistance scales with the number of squares and differs for samples of different width, while the resistance of edge modes scales with edge length and should not change as a function of device width.

Figure 5.17a shows the geometry of the two devices used. The devices are long mesas of width $W = 1 \mu m$ and $W = 2 \mu m$ across which multiple gates of length $L_i$ are patterned, ranging from $L_1 = 300$ nm to $L_9 = 20$ \mu m. Starting with all gates grounded ($V_{bg} = V_{tg,i} = 0$) the entire mesa was $n$-doped and highly conductive, see also Fig. 5.14a. The two-terminal mesa resistance is monitored while energizing one gate at a time. Below such an energized gate the sample is tuned into the trivial gapped phase. Therefore, the maximum resistance increase $\Delta R$ is the resistance of a trivial gapped region of length $L_i$.

Figure 5.17b shows for the $W = 2$ \mu m device the mesa resistance as a function of $V_{tg,i}$ for several top gate lengths $L_i$ together with $\Delta R$. Figure 5.17c shows $\Delta R$ as a function of gate length $L_i$ for the two devices $W_{1,2}$. The resistances for different device width $W_{1,2}$ fall onto each other, thus no width dependence, a direct proof of edge-conduction. From the length dependence we obtain from a linear fit to the data points a resistance of 10 k\Omega/\mu m for the edge conduction (see inset for fitting formula, the factor 1/2 takes into account that two edges conduct in parallel).

Furthermore, we notice that the edge resistance varies between devices from different fabrication batches using the same fabrication recipe. Possible mechanisms will be discussed in section 5.9.4.

5.9.3. SIDE-GATED DEVICE
The conductive edges discovered in previous section are likely to persist even in the inverted regime where helical edge channels are expected. The (co)existence of edge conduction hinders observation of possible quantum spin Hall edge channels. The question arises whether the edge channels can be tuned away by side gating, which we try to answer in this section.

The device used in this experiment is shown in Fig. 5.18a. It is a 8 terminal Hall bar device of 20 \mu m wide and 320 \mu m long fabricated on wafer B. Between contacts 3-27 and between contacts 6-36 an extra gate (hereafter named side gate) is patterned, which overlaps ~250 nm with the side of the mesa. The overlap of 250 nm is inspired by the measurement in chapter 4 which give an upper bound on the width of the edge conduction of 260 nm. A gate dielectric of 90 nm Si$_3$N$_4$ separates the side gates from the mesa. A top gate is defined on top of a 186 nm Si$_3$N$_4$ gate dielectric. Between contacts 36-33 and 27-30 no side gate is present. These sections therefore act as a reference.

First, the Hall bar is characterized by measuring a phase map show in Fig. 5.18b using voltage bias between contacts 33 and 36 and current measurement in lead 36 using a bias voltage $V_{sd} = 50 \mu V$. The back gate range is limited to -0.6 V due to leakage. The phase diagram is qualitatively similar to that of Fig. 5.14a and Fig. 5.15b for the back gate range used. The top gate voltages are twice as large due to the two times thicker gate dielectric. The maximum in-gap resistance is 3.5 M\Omega at gate position $V_{tg} = -5.9$ V and
Figure 5.18: (a) Optical photograph of a 8 terminal Hall bar including top gate and side gates between contacts 3-27 and 33-36. The scale bar is 20 µm. (b) Current as a function of $V_{bg}$ and $V_{tg}$ at a bias voltage of $V_{sd} = 50$ µV between contacts 33-36 (without side-gate). (c) I-V trace between contacts 33-36 at $V_{tg} = -5.9$ V and $V_{bg} = -0.6$ V. For small bias the resistance is 3.5 MΩ from a linear fit (red line) and 160 kΩ from a fit to the large bias part $V_{sd} > 0.2$ mV. (d) Current as a function of bias voltage $V_{sd}$ and side gate voltage $V_{sd}$ between contacts 3-27. The bias voltage is corrected for a series resistance of 105.5 kΩ. (e) The resistance between contacts 3-27 as a function of $V_{side}$ for low bias ($dI/dV$ of (d) around $V_{sd} = 0$ mV) in blue and for large bias ($V_{sd} > 0.25$ mV) in red.
$V_{bg} = -0.6$ V. Figure 5.18c shows the I-V trace corresponding to this gate position; the low-bias differential resistance is 3.5 M$\Omega$ which we believe to be the resistance of trivial edge channels.

Next we turn to the side gated section between contacts 3-27 and see whether the edge conduction can be tuned by electrostatic gating. The top and back gate are again set to $V_{tg} = -5.9$ V and $V_{bg} = -0.6$ V. Figure 5.18d shows the current as a function of bias voltage and side gate voltage. The data is corrected for a series resistance of 105.5 k$\Omega$. The current reduces for more negative $V_{side}$ but never reaches zero (never insulating). Figure 5.18e depicts the low-bias ($|V_{sd}| < 50 \mu$V) differential resistance in blue and the high-bias resistance ($V_{sd} > 0.25$ mV) in red. The low-bias resistance fluctuates around 1 M$\Omega$ for $V_{side} > -5$ V and around 3 M$\Omega$ for $V_{side} < -5$ V. The high-bias resistance increases from 100 k$\Omega$ to 600 k$\Omega$.

In conclusion, the side gates that overlap $\sim 250$ nm with the mesa can reduce edge conductance, but for the side gate voltage range considered here the device could not be tuned to an insulating state similar to that of the Corbino device of Fig. 5.15b. However, as will be shown in chapter 7 the trivial edge conduction likely occurs on a smaller scale than 250 nm. In such a case, this side gate covers too much of the mesa and induces conduction in the bulk of the material.

Furthermore, the gapped feature in bias voltage is not yet understood. It could relate to the same gapped feature present in the Corbino device. However, the resistivity at high bias (160 k$\Omega$) is much smaller than expected from comparison to the Corbino device at high bias ($\sim 13$ M$\Omega$). A recent report explains this suppressed low bias conduction as result of Luttinger liquid behavior in InAs/GaSb edge modes [191].

5.9.4. DISCUSSION

Using temperature and bias voltage dependence in the trivial gap of a Corbino device we measured a trivial gap size of at most $\sim 0.7$ meV (8.2 K) for the accessible gate range. The bias voltage used for measurements on Hall bar devices of Figs. 5.2a and 5.14a exceed the $\sim 1$ mV in the trivial gap, which implies that the bulk conducts. However, this bulk conduction has a resistance in the order of M$\Omega$s and is insufficient to explain the low resistance (200 k$\Omega$) of the trivial gap of the Hall bar devices.

The low in-gap resistance is caused by conduction along the samples edge. An edge channel resistivity of 10 k$\Omega$/µm is determined by comparing samples of different widths and lengths. This linear resistance holds for edge lengths down to 300 nm with corresponding resistance $R_{300 \text{ nm}} \approx 3$ k$\Omega$. A single trivial edge mode with $T = 1$ would result in 6.4 k$\Omega$ resistance (see section 2.1.3). Therefore, we conclude that at least 2 edge channels are involved with less than unity transmission, hence a mean free path shorter than 300 nm.

The mechanisms that underpin trivial edge conduction should permit varying edge conductivities for differently processed samples. There are several mechanisms that qualify. The first is the band bending of the InAs conduction band at a vacuum interface,
which depends on the precise termination of the semiconductor crystal. This effect is typically in the order of the bulk InAs gap (350 meV) [61, 67–69]. Second, the fabrication process itself might turn the side wall of the mesa conductive. This could occur by redeposition of amorphous Sb during AlSb etching, or to dangling bonds resulting from the exposure of the etched semiconductor to air [192, 193]. Third, the electrostatics of the structure can enhance the electric field at the sample edges. The top and back gate form a parallel plate capacitor, at the edges the fringing field causes the electric field to increase and can alter the band structure. Furthermore, if the top gate overlaps with the side of the mesa the potential at the edge is enhanced.

It is expected that these trivial edge modes persist across the whole phase diagram. In the topological gap the trivial edge channels would conduct in parallel to the helical edge channels, quenching their quantization effect. However, signatures of quantization from the helical modes can still be observed if the trivial edge resistance is large compared to $h/e^2$. In that case the length dependence presented in Fig. 5.17c would saturate at $R_0 = h/2e^2$ for edge lengths smaller than the spin-flip length.

With a side gated sample an attempt was made to in-situ eliminate the trivial edge conduction. A large negative voltage on the side gate decreased the edge conduction by a factor 6.5 but was unable quench the edge conduction. Concluding from these observations we believe that electrons are responsible for the trivial edge conduction. However, the overlap of the side-gates with the mesa edge might be too large and therefore induce conductive paths in the bulk. Samples with different side-gate overlap should be investigated before conclusions can be drawn on the effect of side gating on the trivial edge conduction.
ZERO-FIELD SPIN SPLITTING IN INAS/GaSb HETEROSTRUCTURES
We investigate spin-orbit interaction (SOI) in double gated InAs/GaSb quantum wells in the single carrier and double carrier regime. We observe beating in the Shubnikov-de-Haas oscillations from which the zero-field spin-splitting is extracted.

In the single carrier regime at high electron density we observe anomalous magnetoresistance, resulting from the intermixing of Dresselhaus and Rashba spin-orbit interaction. In this regime the Rashba SOI strength is tuned from $\alpha = (8 \pm 0.6) \times 10^{-12}$ eVm to $\alpha = (6.5 \pm 0.5) \times 10^{-12}$ eVm by changing the electric field at constant electron density.

We further investigate the spin-splitting for in double carrier regime. The spin-splitting is largest close to the hybridization gap. Increasing the top gate voltage, the spin-splitting first reduces and subsequently increases. This behavior is consistent with the calculated band structure.

6.1. INTRODUCTION

InAs/GaSb quantum wells are an interesting system to study spin-orbit interaction (SOI) for several reasons. The constituents, InAs and GaSb, are both small band gap materials and therefore known to have a strong SOI [40]. Furthermore, the InAs/GaSb quantum well has a highly asymmetric confinement potential which is key for obtaining strong Rashba SOI [175, 194, 195]. Finally, dual gated InAs/GaSb systems can be tuned to a regime where electrons and holes coexist and interact with each other [27, 196]. It is interesting to study whether this interaction enhances the SOI for electrons.

In this chapter we investigate the SOI in the conduction band of a dual-gated InAs/GaSb DQW in both the pure electron regime and in the regime where electrons and holes coexist, the double carrier regime. In the pure electron regime we fix the electron density and tune the confinement potential by electric fields generated by the top and bottom gates. Our data in this regime shows a spin-splitting in the conduction band which can be tuned by $\sim$20%. The data furthermore suggest an anisotropic Fermi-surface having vanishing spin-splitting for certain momentum directions. Spin-splitting for these direction is restored by application of an electric field. Next we tune to the double carrier regime and find that the spin-splitting shows oscillatory behavior upon increasing the electron density. Finally, also in the double carrier regime, we fix the electron density and monitor the spin-splitting as a function of the hole density.

This chapter is structured as follows. Section I covers the experimental details and introduces the phase map on which gate points are selected to study SOI. Section II reviews SOI in the pure electron regime; section III investigates SOI in the regime where electrons and holes coexist as a function of top gate voltage. In section IV the electron density is fixed while SOI is measured as a function of hole density.
Figure 6.1: a) Schematic of the InAs/GaSb quantum well structure. b) Device layout, Hall bar is biased with a current of 50 nA while longitudinal and Hall voltage are measured simultaneously; the sample has 4 squares between the voltage probes. c) Calculated band structure of our InAs/GaSb quantum well. Positive (negative) $k$-vectors point along [100] ([110]). Blue and red bands indicate different spin-species of the conduction band. The valence bands are depicted in black. d) Phase diagram of the longitudinal resistance as a function of top gate voltage and back gate voltage taken at 2 T perpendicular magnetic field. Region A and B are the topological and trivial gapped regions, respectively. Insets A and B depict the band alignment of the gapped region A and B, respectively. At each white point in gate space magnetoresistance traces are measured. The white points are grouped into lines, line I contains 10 points along a constant electron density $n_e = 18 \times 10^{15}$ m$^{-2}$. Line II contains 16 points all situated in the two-carrier regime. Line III contains 7 points with constant electron density ($n_e = 8 \times 10^{15}$ m$^{-2}$) and starts (point 1) in the double-carrier regime and ends (point 7) in the single carrier regime.
Figure 6.2: Spin-splitting at constant electron density line I. a) Magnetoresistance traces for points 1 to 10 on line I indicated in Fig. 6.1d. A constant background is subtracted from the traces and afterwards are offset by 10 Ω from each other. b) Fourier power spectrum $|F[R_{xx}(1/B)]|^2$ of the traces in a). For further details on the Fourier extraction method, see Supplementary Information. The colored dots below the power spectrum indicate the peak positions. c) Electron density (extracted via Hall resistance and Shubnikov-de-Haas period) and derived spin-splitting at each data point along line I.
6.2. SECTION I: PREREQUISITES

6.2.1. MATERIAL

The InAs/GaSb heterostructure used in this experiment is characterized in an earlier report [58, 196] (see Chapter 5). The heterostructure is epitaxially grown on a doped GaSb substrate. The subsequent stack consist of a 100 nm GaSb buffer layer, 50 nm AlSb bottom barrier, 5 nm GaSb and 12.5 nm InAs forming the quantum well, followed by a 50 nm AlSb top barrier and a 5 nm GaSb capping layer. All layers, including the substrate, are lattice matched providing high carrier mobility due to small strain and a small number of dislocations [58].

An optical image of the Hall bar used in this measurement is depicted in Fig. 6.1b. The mesa is defined using a wet etch and Ohmic contacts are fabricated by etching to the InAs layer prior to evaporation of Ti (50 nm)/Au (300 nm). A sputtered 70 nm thick Si$_3$N$_4$ gate dielectric layer is used to isolate the Ti/Au top gate from the heterostructure. The distance between voltage contacts is 80 $\mu$m and the Hall probes are separate by 20 $\mu$m. Longitudinal and Hall resistances are measured using standard lock-in techniques at 300 mK, unless otherwise stated.

6.2.2. PHASE DIAGRAM

Figure 6.1a depicts schematically the quantum well structure where thin InAs (12.5 nm) and GaSb (5 nm) layers are surrounded by the large band gap material AlSb. Electrostatic gates at both side of the quantum well can tune the electric field and Fermi-level independently. Figure 6.1b shows the longitudinal resistance ($R_{xx}$) as a function of top gate voltage $V_{TG}$ and back gate voltage $V_{BG}$ at a perpendicular magnetic field of 2 T. Two regions of high resistance, indicated by A and B, were attributed in ref. [196] to the topological and trivial gapped phase, respectively. The band alignments in regime A and B are depicted in the insets A and B of Fig. 6.1d respectively.

The oscillatory resistance of Fig. 6.1d in gate space reflects the Shubnikov-de-Haas (SdH) effect upon tuning the carrier density. The SdH stripes change slope at the green and orange dashed lines. The green line marks the boundary between the double carrier regime (left) and single carrier regime (right).

The region in gate space above the trivial gap (B) has a trivial band alignment (as illustrated in the inset) and electrons are the only carriers. The region above the topological gap (A) contains both electrons and holes. Here the band structure is distorted compared to the bare InAs and GaSb band structures due to the hybridization of electrons with holes. Close to the topological gap A, it is expected that SOI strongly affects the band structure (as indicated in Fig 6.1c) [194, 197, 198].

In the trivial regime, SOI is investigated for points along line I in Fig. 6.1d, the electron density is held constant while the electric field is tuned. We investigate the inverted regime using points along line II in the phase diagram. Line II starts at ($V_{TG},V_{BG}$) = (0 V, -0.8 V) close to the hybridization gap and subsequent points are reached by increasing
6.3. SECTION II: TRIVIAL REGIME

The trivial regime is characterized by a band alignment where the electron band is higher in energy than the hole band (see right inset of Fig. 6.1d). The energy gap between these bands is called the trivial gap. In this section, we focus on the regime where the Fermi-energy is located in the electron band. In this single carrier regime, only electrons are present and their wave function is mainly located in the InAs. The GaSb layer is depleted from holes and is insulating. In this regime 10 points are selected with equal electron density (white points along line I in Fig. 6.1d); this is verified by extracting the density from the Hall voltage (Fig. 6.2c). Although the electron density is constant, each point experiences a different external electric field (see Fig. 6.1d) imposed by the top and back gate voltages $V_{TG}$ and $V_{BG}$.

Figure 6.2a shows magnetoresistance traces for the points 1-10 along the constant density line I, which are offset for clarity. Each trace shows Shubnikov-de-Haas (SdH) oscillations periodic in $1/B$ (for $B > 0.15$ T) modulated with a clear beat pattern. The magnetoresistance data is analysed up to 0.6 T, before Zeeman energy splits the Landau levels (Supplementary Fig. 6.8 shows a similar trace extending to larger magnetic fields). Fourier transforms of $R_{xx}(1/B)$ of the traces are plotted in Fig. 6.2b (see Supplementary Information section 6.7.1 for the details on the Fourier transform).

The magnetoresistance data for gate point 1 is depicted in the upper green trace 1 of Fig. 6.2a. The beat labeled A contains $N_A = 23(\pm 1)$ oscillations and beat B $N_B = 20(\pm 1)$. Such similar number of oscillations in neighboring beats $N_A \approx N_B$ point to two frequency components in the power spectrum; indeed the Fourier transform for trace 1 (Fig. 6.2b) shows two pronounced peaks. For magnetoresistance traces 2-10 the number of oscillations in beat A increase while that in B decrease and $N_A \neq N_B$, see supplementary Fig. 6.9. Furthermore, the amplitude of beat B is smaller than its neighbor A. Trace 10 has the largest beat-asymmetry, here beat A has 41 oscillations whereas beat B has 12 oscillations. Such even-odd pattern indicates a third oscillatory component with a frequency exactly the average of the outer two frequency components as also displayed by the Fourier transform in Fig. 6.2b.

The total number of oscillations in beat A and B ($N_A + N_B$) increases from gate point 1 to 10 (see Fig. 6.9). This trend indicates that the outer two oscillation frequencies move towards each other as is also reflected in the Fourier transforms of Fig. 6.2b.

The electron density at each gate point is extracted via two methods. First $n_{\text{Hall}}$ is obtained from the Hall resistance ($n_{\text{Hall}} = B/(R_{\text{Hall}}e)$). The second method adds the densities corresponding to the outer SdH frequency components, $n = (f_1 + f_2) \cdot h/e$. Figure
6.2c plots these two densities together and shows that they deviate by less than 3%.

At points 1 and 10 an effective mass of $m_1^* = 0.042 m_0$ and $m_{10}^* = 0.041 m_0$, respectively, is extracted by fitting the Dingle formula to the temperature dependence of the SdH oscillations (see Supporting Fig. 6.10). The electron mobility does not change along line I and equals 650,000 cm$^2$/Vs.

6.3.1. Analysis

Each spin non-degenerate band that crosses the Fermi energy gives rise to oscillations in the low field magnetoresistance $R_{xx}(1/B)$ with a frequency given by the area of the Fermi-surface, which in 2D is directly proportional to the carrier density via $n_i = f_i \cdot h/e$ [55, 199]. Multiple Fermi surfaces thus give rise to multiple frequencies and the total carrier density is given by $n = \sum_i f_i \cdot h/e$. Thus, two spin non-degenerate bands constitute in general to two frequencies $f_1$ and $f_2$ that sum up to the total carrier density $n = (f_1 + f_2)e/h$. Two spin degenerate bands are a special case where $f = f_1 = f_2$ and $n = 2f \cdot e/h$ [200].

Other known origins for multiple frequencies in the magnetoresistance are the population of a second subband [201], the population of low-lying electron bands at distant $k$-points [202] or a density inhomogeneity in the sample [203]. Besides frequencies that directly correspond to a Fermi-surface, also mechanisms that scatter between different existing Fermi-surfaces can produce frequency components. Such mechanisms are magnetic inter subband scattering (MIS) [52, 201] and magnetic breakdown (MB) [204–206].

The magnetoresistance trace 1 in Fig. 6.2a has two oscillatory components with frequencies $f_1$ and $f_2$. Because their corresponding densities sum up to the Hall density (see Fig. 6.2c) these frequencies are related to spin-split bands. In case of spin-degenerate bands (e.g. second subband) addition of the two frequencies would give half the Hall density, which is clearly not observed. The difference $\Delta n/n = (f_1 - f_2)/(f_1 + f_2)$ is a measure of the spin splitting energy and is plotted in Fig. 6.2c using the outer frequencies of each trace in Fig. 6.2b. Spin splitting energy decreases by 20% ($\Delta n/n$ changes from 0.026 to 0.021) when moving to larger $V_{BG}$ along line I.

In non-magnetic materials, spin-splitting can be of Rashba, Dresselhaus (linear and cubic) or Zeeman type [39] or a combination of these. Zeeman energy due to a perpendicular field cannot produce beating, whereas a stray in-plane field is too small to account for the large spin-splitting observed $^2$.

Linear Dresselhaus can be estimated as $\beta \sim \gamma (\pi/d)^2 = 2.5 \times 10^{-12}$ eVm [44] using an InAs layer thickness $d_{\text{InAs}} = 12.5$ nm and the material parameter $|\gamma| = 40$ eVÅ$^3$ [59].

$^2$Considering the B-field is misaligned by $10^\circ$ from the perpendicular direction, then $\Delta n/n = [g\mu_B B \sin(10^\circ)] \times (m^*/\pi h^2)/18 \times 10^{15} = 0.1 \times 10^{-2}$, using $g = 15$, $m^* = 0.04 m_0$ and $B = 0.6$ T. This is much smaller than the observed $\Delta n/n > 2 \times 10^{-2}$. 

6.3. SECTION II: TRIVIAL REGIME

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Such spin-orbit coupling would give rise to a spin splitting $\Delta n/n = 0.015$ which is insufficient but close to explaining the lowest observed spin splitting of $\Delta n/n = 0.021$ (gate point 10). Cubic Dresselhaus cannot be neglected at these high densities as the in-plane electron wavevector $\frac{1}{4}k_F^2 = 0.028 \, 1/\text{nm}^2$ is comparable to the confinement wavevector $\langle k_z^2 \rangle \sim (\pi/d_{\text{InAs}})^2 = 0.063 \, 1/\text{nm}^2$ [207]. Indeed, from band structure calculations, presented in Fig. 6.6b, we see an anisotropic Fermi surfaces at large $k$-vector that has strong resemblance to a Fermi surface subjected to a significant cubic Dresselhaus strength (see Fig. 6.3b). Therefore, we expect a complicated spin-texture due to the interplay between Rashba, linear and cubic Dresselhaus for all points along line I.

Traces 9-10, show an even-odd pattern resulting from an extra frequency component in the Fourier transforms at $f_{\text{ave}} = (f_1 + f_2)/2$. Because the electron density is constant for all traces (1-10), the peak at $f_{\text{ave}}$ cannot correspond to a separate Fermi area, but should combine the areas corresponding to the left $f_1$ and right $f_2$ frequency peak.

As mentioned earlier, two mechanisms could account for an ‘artificial’ peak in the Fourier spectrum. The first, magnetic inter subband scattering cannot explain our data. The MIS frequency is fixed by the subband spacing and should not depend on density. However, magnetoresistance measurements at a lower density (see Supplementary Fig. 6.11) have the center frequency as well, ruling out MIS.

The second mechanism is magnetic breakdown [204–206], a term originating from the semiclassical treatment, and explains the spurious peak as carriers tunneling from one spin polarized Fermi-surface to the other at spin-degeneracy points. Linear and cubic Dresselhaus of comparable strength create such highly anisotropic Fermi-surface with four spin-degenerate crossings of the spin-polarized Fermi surfaces, see Fig. 6.3b (for details on the model see Supplementary Information section 6.7.2). At these crossings the spin-splitting vanishes and carriers have a probability to tunnel to the other Fermi surface [204, 205]. For a tunnel probability close to unity, a carrier switches Fermi-surface at every crossing, and effectively encircles an area which is the average of both real surfaces, leading to a pronounced center frequency component $f_{\text{ave}}$. This is a viable explanation at point 10 where the center frequency has a larger amplitude than the outer two frequencies.

Addition of Rashba type SOI lifts the spin degeneracy at the crossing points (see Fig. 6.3c) and quenches the tunnel probability. Presence of strong Rashba SOI therefore reduces the Fourier spectrum to just two frequencies that correspond to the spin-split Fermi-surfaces.

Following the above reasoning, the Rashba SOI would be largest at point 1 in line I and weakens towards point 10 in gate space. Such behavior can qualitatively be explained from the location of the wave function in the quantum well. Gate point 1 has strong positive top gate voltage, pulling the electron wave function into the upper interface where the wave function experiences a strong electric field. This electric field increases
the Rashba SOI (see Supplementary Fig. 6.12 for schematic band diagram under bias) [47, 49, 208]. At gate point 10 on the other hand, $V_{TG} \sim V_{BG}$, and the electron wave function is expected to sit in the middle of the InAs quantum well, resulting in a small Rashba SOI.

In order to validate such scenario, knowledge is needed on the individual Rashba and Dresselhaus contributions, which is not accessible using the single axis magnetoresistance measurements presented here. One way is discriminate between the SOI contributions is by the angular dependence of a spin-galvanic current [209] or by optically monitoring the angular dependence of the electron spin [210]. Another approach is to measure weak-localization in etched nanowire arrays as a function of magnetic field direction [211]. Also by measuring magnetization (de Haas-van-Alphen effect) in tilted magnetic fields one can extract the individual absolute values of Rashba and Dresselhaus including their relative sign [212].

A rough estimate on the Rashba contribution can be given though. Figure 6.4 presents calculated spin splittings as a function of Rashba strength $\alpha$ and Dresselhaus strength $\gamma$ for an InAs quantum well of 12.5 nm at an electron density of $n = 18 \times 10^{15} \text{m}^{-2}$. The calculation includes Rashba, linear- and cubic Dresselhaus terms, as described in section 6.7.2. If the Dresselhaus parameter is $|\gamma| \approx 40 \text{eVÅ}^3$ [59], then from the spin splittings found in Fig. 6.2c the Rashba parameter for point 1 is $\alpha = (8 \pm 0.6) \times 10^{-12} \text{eVm}$ and for point 2 $\alpha = (6.5 \pm 0.5) \times 10^{-12} \text{eVm}$. Which is a tuning of the Rashba parameter of 20%.

![Figure 6.3: Calculated Fermi-surfaces for small (inner circles) and large (outer circles) energy (or $k$-vector) in the presence of specific types of SOI (see Supplementary Information section 6.7.2 for model details). a) Fermi-surfaces for only linear Dresselhaus contribution. Blue and red indicate the Fermi-surface for different spin-species. b) Cubic and linear Dresselhaus. The cubic term becomes relevant for large $k$-values where it drives the Fermi surface anisotropic and introduces four spin-degenerate points, it leaves small $k$-values unaffected. c) Addition of Rashba to the cubic and linear Dresselhaus lifts the degeneracy points in b), whereas for small $k$-values it introduces 2 spin-degeneracy points.](image)
Figure 6.4: Calculation of the spin splitting $\Delta n/n$ as a function of the Dresselhaus strength $\gamma$ and Rashba strength $\alpha$ for a carrier density $n = 18 \times 10^{15} \text{ m}^{-2}$. See supplementary section 6.7.2 for the model.
Figure 6.5: a) Magnetoresistance traces for points along line II in the phase diagram of Fig 6.1a. b) Fourier transform amplitude for the traces in a). c) Electron density and mobility along trace II as a function of $V_{TG}$. Electron density is extracted from the SdH oscillation frequency ($n_{SdH}$) and the Hall resistance ($n_{Hall}$) d) Relative spin splitting $\Delta n/n_{SdH}$ along trace II, using the frequencies indicated by the colored dots in b).
6.4. Section III: Inverted Regime

Next we investigate the regime where both electrons and holes are mobile at the Fermi energy, and necessarily the band structure is inverted such as depicted in inset A of Fig. 6.1d. We focus on points along the vertical line II in the phase diagram of Fig. 6.1a. Starting from point 1, \((V_{BG}, V_{TG}) = (-0.8 \text{ V}, 0 \text{ V})\), close to the hybridization gap \((n_e \approx n_h)\) the other 15 points are reached by increase of \(V_{TG}\) in steps of 0.2 V. The gate point 16 \((-0.8 \text{ V}, 3.0 \text{ V})\) is close to the green dashed line which indicates that the onset of the hole band touches \(E_F\). The hole concentration at point 16 is therefore negligible. By changing \(V_{TG}\) along line II, both the band structure (overlap of electron with hole band) and the Fermi level (with respect to the bottom of the conduction band) are altered.

Figure 6.5a presents the magnetoresistance traces up to 0.65 T for each point along line II. The SdH oscillations are solely the result of electrons as the mobility for holes is much lower \[196\]. The traces are normalized to 1 and offset by 1 for clarity. Some of the traces show a clear beat pattern in the magnetoresistance oscillations whereas for others beating is absent. Following line II from \(V_{TG} = 0\) towards larger \(V_{TG}\), the beating is absent for traces 1-4, then present for traces 5-8, absent for traces 9-12, and is reappears in traces 13-16. This ‘oscillatory behavior’ is also reflected in the Fourier transforms of 6.5b. Traces that lack beating display only a single frequency peak, traces that show beating have multiple frequency peaks. Next we will review the 16 traces in more detail.

Trace 1 at \(V_{TG} = 0 \text{ V}\) shows clear oscillations in the longitudinal resistance on top of a parabolic background. No beating is observed. The parabolic background is due to the presence of two types of carriers \[213\]. Traces 2-4 \((V_{TG} = 0.2 – 0.6 \text{ V})\) show similar behavior. Trace 5 \((V_{TG} = 0.8)\) shows a beat pattern and in the Fourier transform of Fig. 6.5b two frequency components \((f_1, f_2)\) can be identified. Note that the beat nodes do not close which indicates spin-dependent SdH amplitudes. The different amplitudes are reflected in the unequal height of the frequency peaks (as will be discussed later). Trace 6 \((V_{TG} = 1.0)\) shows similar behavior as trace 5, here an effective mass \(m^* = 0.039 m_0\) is measured using a Dingle plot (see Supplementary Fig. 6.13). Traces 7-8 \((V_{TG}=1.2-1.4 \text{ V})\) show asymmetric beating (one node closes fully while the second does not) indicating a third frequency component. Traces 9-12 \((V_{TG} = 1.6-2.2 \text{ V})\) do not show beating. For traces 13-16 \((V_{TG}=2.4-3.0)\) the beat pattern returns.

Using the method as described in section 6.3.1, the relative spin-splitting \(\Delta n/n\) can be extracted for each trace. This relative spin-splitting is plotted in Fig. 6.5d as a function of the top gate voltage \(V_{TG}\), and clearly reflecting the ‘oscillatory’ trend in spin-splitting already expected from the plain data.

Figure 6.5c presents the density and mobility of the carriers as a function of \(V_{TG}\). The Hall density is obtained from a linear fit to the Hall resistance at fields above 1 T. The SdH density, \(n_{SdH} = (f_1 + f_2)e/h\), is obtained from the frequencies \(f_1, f_2\) of the outer two peaks in the Fourier transform. If only one frequency peak is observed then \(f_1 = f_2\). The Hall density has contributions from both electrons and holes while the SdH frequency singles out the electron density. As expected, the density obtained from SdH frequency
deviates from Hall density close to the hybridization gap where \( n_h/n_e \approx 1 \). The electron mobility increases with electron density as shown in Fig. 6.5c.

### 6.4.1. Analysis

The measured appearance and disappearance of beating along line II indicates a spin-splitting that ‘oscillates’ as a function of \( V_{TG} \). This oscillating spin-splitting can be matched qualitatively to band structure calculations.

Figure 6.1c shows the band structure calculated for the InAs/GaSb quantum well neglecting built-in and external electric fields. Negative \( k \)-vectors point along the [110] direction whereas positive \( k \) is along the [100] direction. Different spin species in the conduction band are indicated by red and blue. The hybridization gap opens at \( k = 0.15 \) 1/nm with a strength (~5 meV) depending on the spin species [194, 195, 197, 214, 215].

Figure 6.6a zooms in on Fig. 6.1c close to the hybridization gap, as indicated by the dashed rectangle in Fig. 6.1c. The blue (red) traces are in the [100] ([110]) direction. The red traces are offset in \( k \) by 0.2 nm\(^{-1} \) for clarity. Focusing on the [100] direction, the energy splitting \( \Delta E = E_\uparrow(k) - E_\downarrow(k) \) between the two blue spin-split bands behaves indeed non-monotonically: a large splitting just above the hybridization gap (at \( k \sim 0.16 \) 1/nm), then at \( k \sim 0.195 \) 1/nm the splitting vanishes (spin-degeneracy point) and at larger \( k \)-values the splitting reopens. Similar trend is observed for the [110] direction. Such non-monotonic behavior has been attributed to the different hybridization gaps for the spin-up and spin-down band [197].

However, the magnetoresistance measurements from Fig. 6.6a provide density differences, \( \Delta n \), between the spin-split bands and not energy differences \( \Delta E \). In order to compare the measurement with the band structure calculations we need to extract the contours of constant energy, i.e. Fermi-surfaces, for each spin species. The area difference between these contours in \( k \)-space \( \Delta A_k \) is then proportional to the density difference \( \Delta n \) extracted from magnetoresistance measurements.

Figure 6.6b plots these Fermi-surfaces for \( E_F = 112 \) meV, 120 meV and 130 meV, these energies are indicated as horizontal lines in Fig. 6.6a. The Fermi-surfaces are highly anisotropic with a four-fold symmetry and therefore only 1/4th of the surface is depicted. At a Fermi energy of 112 meV, both spin-split electron (blue) bands and spin-split hole (red) bands are present. Focusing on the spin-split electron bands, the area difference between their Fermi-surfaces \( \Delta A_k \) is proportional to the density difference via \( \Delta n = \Delta A_k/(4\pi^2) \). Dividing by the average density \( n_{ave} = (A_{k,\uparrow} + A_{k,\downarrow})/4\pi^2 \) gives the relative spin-splitting \( \Delta n/n \). This procedure can be done for each Fermi-energy to obtain the plot in Fig. 6.6c\(^3 \).

It is clear that the calculated relative spin-splitting \( \Delta n/n \) show a similar trend as the measured spin-splitting depicted in Fig. 6.5d. The relative spin-splitting is large for small energies, minimal for energies close to the spin-degeneracy points and increases

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\( ^3 \) Note that the areas \( A_{k,(\uparrow,\downarrow)} \) of the Fermi-surfaces are estimated from the spin-splitting \( \Delta k \) in [110] and [100] direction of Fig. 6.6a instead of calculating the full Fermi-surface for each energy.
in strength for larger energies. The minimum could be matched to the measured traces 10 and 11 in Fig. 6.5a (at $V_{TG} \sim 2$ V) where no spin-splitting can be resolved. For larger and smaller $V_{TG}$ beating reappears.

However, in contrast to the measured data where the spin-splitting cannot be resolved close to the hybridization gap (traces 1-4 in Fig. 6.5a,b), the calculated spin-splitting keeps increases close to the hybridization gap. We speculate that for the measured traces 1-4 in Fig. 6.5b the spin splitting is indeed large, but that the amplitude of the left peak is unresolvably small. Indeed the left frequency component shrinks in size from trace 7 to trace 5, and this trend is expected to continue. The height of the peak in the frequency domain is a measure of the amplitude of the SdH oscillations corresponding to the spin-species. The SdH amplitude $A_{SdH}$ is proportional to scattering time ($\tau$) and effective mass ($m^*$) according to [44]

$$A_{SdH} \sim \left(\frac{eB}{m^*}\right)^3 \exp\left[-\frac{\pi}{\omega_c \tau}\right]. \quad (6.1)$$

Using the second derivative of the band structure we see that the effective mass can change by a factor 2 close to the hybridization gap (see Supplementary Fig. 6.14) explaining the large amplitude mismatch of the peaks in the Fourier spectrum. Besides spin-dependent effective mass also spin-dependent scattering, $\tau_\uparrow \neq \tau_\downarrow$, could occur and lead to different amplitudes in the Fourier spectrum. Such spin-dependent scattering has been seen before [44] and could be due to the different spatial location of the spin-polarized wave functions in the quantum well [194, 195, 214]. This would imply that the different spin-species have different mobilities close to the hybridization gap.

As for the trivial line I, we also see a pronounced middle frequency component for traces 7-9 and 10-14 close to the spin-degeneracy points. This middle peak could again be explained by the anisotropic Fermi-surface where carriers transit to the other spin-polarized Fermi surface via spin-degeneracy points. This anisotropy is indeed observed in the calculated Fermi-surfaces of Fig. 6.6b

Quantitative comparison of the spin-splitting $\Delta n/n(E_F)$ obtained from band structure calculations with the spin-splitting $\Delta n/n(V_{TG})$ obtained from measurement is not straightforward. The calculation presented in Fig. 6.6 assumes a fixed band structure on which the Fermi-level can be moved. However, as already mentioned before, along line II in phase diagram the band structure is not static due to the finite coupling of $V_{TG}$ to the hole band. Increase of $V_{TG}$ does not only shift the Fermi-energy but also raises the hole band in energy. Thus, only qualitative comparison is possible between the band structure calculations presented in Fig. 6.6 and the measurement of Fig. 6.5. The important qualitative result from these calculations is that the ‘oscillatory’ spin-splitting observed in the measurement of Fig. 6.5d is reproduced.
6.5. SECTION IV: FROM INVERTED TO TRIVIAL REGIME

Next, we study SOI along line III in Fig. 6.1b where the electron density is held constant and band alignment is tuned from inverted (points 1-4) to trivial (points 5-7) (the band gap from negative to positive). Using a capacitor model we estimate that at gate point 1 \((V_{BG}=-1 \, V, V_{TG}=1.0 \, V)\) approximately 30% of the carriers are holes. The hole concentration decreases linearly to \(n_h \sim 0\) at point 4. The electron density is constant at \(n_e = 8 \times 10^{15} \, m^{-2}\) as can be seen from Fig. 6.7c.

The magnetoresistance traces at points 1 and 2 show clear background resistance due to the presence of two-carriers [213]. A clear beating is observed for with nodes that do not close. Traces 3 and 4 show stronger asymmetry in the beat pattern, where one node closes while its neighbor does not. This can be attributed to three peaks of which two have similar amplitude. Traces 5-7 are taken in the pure electron regime for which only trace 7 a beating could be identified however too small to identify in the Fourier

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**Figure 6.6:** Band structure and Fermi-surfaces. a) Zoom-in on the band structure of Fig. 6.1c for the [100] (blue) and [110] (red) direction. The trace for the [110] direction (red) is shifted by \(\Delta k=0.2 \, 1/nm\) for clarity. Dashed lines indicate the spin-degenerate positions. b) Fermi-surfaces for energies indicated by the black lines in a). For 112 meV the Fermi-energy crosses both hole band (red) and electron band (blue). 120 meV is in between the spin-degeneracy points of both directions and only the electron band is present at this energy. At 130 meV the spin-splitting increases. The area of the spin-split Fermi-surfaces is a direct measure for the density, according to \(n(\uparrow, \downarrow) = A_k(\uparrow, \downarrow)/4\pi^2\). c) Calculated relative spin-splitting \(\Delta n/\bar{n}\) as a function of the Fermi-energy.
transform. Figure 6.7d. presents the spin-splitting which sharply drops when holes are depleted.

Figure 6.7c shows that the mobility increases steeply when holes are depleted.

6.5.1. **Analysis**
The spin splitting reduces when hole band is removed and becomes eventually unresolvable small when pure electrons are present. This observation is consistent with section III where the spin-splitting is strong in the regime close to the hybridization gap where a significant number of holes are present.

![Image of Figure 6.7](image)

**Figure 6.7:** a,b) Magnetoresistance and Fourier amplitude of points along line III. c) Electron density and electron mobility as a function of $V_{BG}$. d) Spin-splitting as a function of $V_{BG}$.

6.6. **Conclusion**
In conclusion we extracted the density difference (between spin-up and spin-down electrons) resulting from the zero-field spin-splitting in double gated InAs/GaSb quantum wells using beating in high-quality magnetoresistance data. In the trivial regime at high electron density (line I) the observed magnetoresistance traces can be explained by the interplay between Rashba, cubic and linear Dresselhaus types of spin-orbit interaction. The estimated Rashba SOI value changes from $\alpha = (8 \pm 0.6) \times 10^{-12}$ eVm to $\alpha = (6.5 \pm$
0.5) \times 10^{-12} \text{eVm along a constant density line. At small Rashba SOI value the anisotropic Fermi-surface due to linear and cubic Dresselhaus produces a manifold of frequencies in the low field magnetoresistance data.}

In the inverted regime (line II) the data can be matched qualitatively to the calculated band structure. The presence of the hole band alters the band structure resulting in an oscillating spin splitting when tuning the Fermi energy across the band structure. We speculate that at top gate values close to the hybridization gap the mobility of one spin species is much smaller than the other, which could find possible applications in spin-accumulation devices [216].

Recent experiments on InAs/GaSb quantum wells show diffusive trivial edge channels [190] throughout the phase diagram. We note that such edge-conduction, estimated to be 10 k\Omega/\mu m, does not alter our results as the SdH oscillations probe only the bulk conduction. On top of that, the edge conduction for 80 \mu m length edges would be 800 k\Omega, more than three orders larger than the bulk conduction $\sim 100 \Omega$. 

6.7. Supplementary Figures

Figure 6.8: Magnetoresistance trace for $V_{TG} = 3.0$ V and $V_{BG} = -0.8$ V. Clear beating is visible for $B < 1$ T. Zeeman energy splits the SdH peaks around $B = 1.4$ T.

6.7.1. Fourier Transforms

The Fourier transforms in Figs. 6.2b, 6.5b and 6.6b are obtained using the method described here. Starting from a magnetoresistance curve, first a magnetic field range is chosen. The lower bound is fixed at 0.15 T. The upper bound is chosen such that the interval ends at 40% of a beat maximum. Truncating the signal in this way causes minimal deviation from the true frequency components. Next, the background resistance is estimated using a 6th order polynomial fit, which subsequently is subtracted from the signal. The remaining signal is interpolated on a uniform grid in $1/B$ and padded with zeros on both sides. No extra window function is applied. A Fast Fourier transform converts the signal to the frequency domain $R(\omega)$ and the power spectrum is obtained using $P(\omega) = R(\omega) \times R^*(\omega)$.

6.7.2. Theoretical model used to calculate the Fermi-surfaces

The calculated Fermi-surfaces in Fig. 6.3 are obtained by diagonalizing

$$H_{SO} = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2) \hat{I} + \alpha (k_y \sigma_x - k_x \sigma_y) + \beta (k_x \sigma_x - k_y \sigma_y) - \gamma (k_x k_y^2 \sigma_x - k_y k_x^2 \sigma_y),$$

where $\sigma_i$ are the Pauli matrices, $\hat{I}$ is the identity matrix. $\alpha, \beta, \gamma$ are the spin-orbit strength parameters for Rashba, linear Dresselhaus and cubic Dresselhaus, respectively. Linear Dresselhaus is related to $\gamma$ via $\beta = \gamma (\pi/d_{InAs})^2$. 

**Figure 6.9:** Number of oscillations in beat A (blue) and B (red) and the total number of oscillations in A and B (yellow) for points along line I in the phase diagram of Fig. 6.1d.

**Figure 6.10:** Dingle plots trivial regime. a) SdH oscillation amplitude, black squares, as a function of temperature for point 1 ($V_{BG} = 0.1 \text{ V}, V_{TG} = 3.0 \text{ V}$) in gate space. The fit (blue) of the Dingle formula to the temperature dependence gives an effective mass $m_1^* = 0.042 m_0$ ($m_0$ is the electron mass) and quantum scattering time $\tau = 0.64 \text{ ps}$. b) Similar analysis for point 10 ($V_{BG} = 1.0 \text{ V}, V_{TG} = 1.3 \text{ V}$) gives $m_2^* = 0.041 m_0$ and $\tau = 0.96 \text{ ps}$. From these extreme points we conclude that the effective mass does not change along the constant density line.
Figure 6.11: Magnetoresistance and corresponding Fourier transform for constant back gate $V_{BG} = 1.0$ V. a) Phase diagram at 2 T indicating the points where magnetoresistance traces are obtained, $V_{TG} =$ [0-3] V and $V_{BG} = 1.0$ V for all traces. b) The magnetoresistance traces for points 1 (bottom red) to 9 (top yellow). Traces are shifted to zero and offset by 20 Ω for clarity. c) Normalized Fourier power spectra of $R_{xx}(1/B)$ offset by 1. The number at the left indicates the point in gate-space in a). The SdH oscillations show even-odd, where the number of oscillations per beat is not constant. This is reflected in the Fourier transforms in c). Note that the central Fourier peak moves with density indicating that it cannot originate from magnetic inter-subband scattering (MIS).
**Figure 6.12:** Schematic band alignment for the InAs/GaSb quantum well under external applied gates. a) Band alignment for point 1 in Fig. 6.1d ($V_{TG} = 3.0\, V$, $V_{BG} = 0.1\, V$). The potential kinks down at the left interface of the InAs layer; the wave function is therefore pushed into the top AlSb barrier. b) Band alignment for point 10 ($V_{TG} = 1.2\, V$, $V_{BG} = 1.0\, V$). The potential is more symmetric and the wave function locates in the center of the InAs layer. The schematic is based on the capacitor model which provides the approximate potentials at the center of the InAs and GaSb quantum wells. Note that at both gate points (1-10) the GaSb layer is fully depleted from holes (and electrons) and therefore acts as an insulator.

**Figure 6.13:** Dingle plot inverted regime. SdH oscillation amplitude, black squares, as a function of temperature for point 1 ($V_{BG} = -0.8\, V$, $V_{TG} = 1.0\, V$) in gate space. The fit (blue) of the Dingle formula to the temperature dependence gives an effective mass $m^* = 0.039\, m_0$ ($m_0$ is the electron mass) and quantum scattering time $\tau = 0.51\, \text{ps}$. Note that the least square fit does give a 10x larger error than the Dingle plots taken in the trivial regime. We speculate that the second carrier affects the temperature dependence of the electrons in this regime.
Figure 6.14: a) Zoom on the band structure calculations in the [100] direction. b) Effective mass extracted using $m^* = \hbar^2 \left( \frac{d^2 E}{dk^2} \right)^{-1}$.
This chapter summarizes in section 7.1 the main findings of the preceding experimental chapters 3-6. Section 7.2 presents an improved version of the flip-chip setup of chapter 3 that is expected to provide a higher device yield. Section 7.3 presents routes towards signatures of helical edge conduction in InAs/GaSb double quantum wells using transport measurements.

7.1. CONCLUSION
The main conclusions from Chapters 3-6 are as follows:

- Chapter 3: A setup is designed in which electrostatic gates hover ∼100 nm above the surface of a GaAs/AlGaAs heterostructure, separated by vacuum. Nano-scale gating is demonstrated by electrostatically defined quantum point contacts, quantum dots and Fabry-Pérot interferometers based on the 2D electron gas of the target heterostructure. The mechanical vibration amplitude of the gates with respect to the heterostructure is less than 1.5 pm. Quantum interference is achieved at integer quantum Hall states in a Fabry-Pérot interferometer and the interference is shown to be dominated by charging effects.

- Chapter 4: Using superconducting quantum interference measurements on an InAs/GaSb double quantum well SNS device, we demonstrate the tuning between edge dominated and bulk dominated superconducting transport regimes. An upper bound on the spatial extend of the edge conduction of 260 nm is derived from the measurements.

- Chapter 5: The phase diagram of InAs/GaSb quantum wells is mapped out as a function of top and back gate voltages. The topological insulating and trivial in-
7. CONCLUSIONS AND OUTLOOK

Concluding phases are identified. The trivial insulating regime is found to have an insulating bulk together with diffusive edge conduction.

- Chapter 6: Spin-orbit interaction in InAs/GaSb heterostructures is investigated by studying the magnetoresistance. In the trivial phase at high electron density the linear and cubic Dresselhaus contributions have similar strength. In this regime the Rashba SOI strength can be tuned from $\alpha = (8 \pm 0.6) \times 10^{-12} \text{eVm}$ to $\alpha = (6.5 \pm 0.5) \times 10^{-12} \text{eVm}$. In the inverted regime the spin-orbit strength increases when the Fermi level is moved towards the topological gap.

**Figure 7.1:** a) Proposed flip chip setup. Gate chip (blue) is mounted on piezo stages that allow the chip to move in $x$, $y$ and $z$ direction together with a tilt $\Theta, \phi$ around the $x$ and $y$ axis, respectively. The heterostructure chip (red) is mounted onto the cage which provides a good thermal link to the mixing chamber of the refrigerator. b) Tip-like structure with an interferometer gate pattern defined at the flat head. A flattened pyramid of 25 $\mu$m tall is etched in a silicon wafer. Electrodes contacting the interferometer run down the sides of the tip. Each side forms a $45^\circ$ angle to the flat head.

7.2. FLIP-CHIP SETUP IMPROVEMENTS

As discussed in section 3.8, two obstacles prevent the flip-chip setup from reliably producing devices that have a gate-heterostructure separation below 100 nm and that preserve the pristine heterostructure properties. First, the setup relies on physical contact between gate chip and heterostructure chip. The small force related to this contact is sufficient to affect the heterostructure’s resistivity. Second, the setup does not deal well with surfaces that deviate from perfectly flat. The surface of the GaAs/AlGaAs heterostructure has height differences of 100 nm over a length scale of millimeters. This prevents a uniform and close separation of gate chip to the target heterostructure chip. However, over a length scale of $\sim 50 \mu$m the surface height changes by less than 5 nm.
In order to tackle these obstacles we propose the setup depicted in Fig. 7.1a. Here, the gate chip (blue) and heterostructure chip (red) are mounted on brass metallic parts inside a stiff brass cage. This configuration eliminates any physical contact between gate chip and heterostructure chip. The heterostructure chip is cleaved into 6 × 2 mm and indium ohmic contacts are soldered at the long edges. The gate chip also has dimensions of 6 × 2 mm and contains a ‘tip’-like structure in the middle on which the fine gates are patterned. This tip structure is ~2 µm tall and has a flat head with dimensions 50 × 50 µm and sides that make an angle of 45° to the surface. Fig. 7.1b shows an example of such a tip structure but with larger dimensions (25 µm high). As a result of this tip structure, the gates can approach the wavy heterostructure surface to within 100 nm at any lateral position. Fine gates are defined onto the flat part of the tip with metallic leads running down the tip side towards the bond pads at the long ends of the gate chip. The gate chip is furthermore equipped with capacitance sensors that probe the distance and angle of the gate chip to the heterostructure. This angle and distance is controlled by piezo stages below the gate chip, as indicated in Fig. 7.1a.

The tip structure on the gate chip can be fabricated using an anisotropic TMAH etch of a silicon wafer. Figure 7.1b depicts such an etched silicon gate chip with a tip structure of 25 µm high and a flat head of 60 × 60 µm. The side walls are at an angle of 45° and allow continuous metallic leads to connect to the gate structure on top of the tip. The tip head is the original (flat) surface of the silicon wafer on which small features can be defined. The inset of Fig. 7.1b shows such small gates for a Fabry-Pérot interferometer on this tip.

Operation of this setup is envisioned as follows. First, the gate chip and target chip are mounted on the brass parts and aligned parallel with an accuracy of <1° at room temperature. Next, the setup is cooled to a temperature of 4 K. Using the piezo stages the distance between gate chip and heterostructure chip is tuned to be <100 nm. At the same time the angle is adjusted such that the chips do not touch. The heat released by the movement of the coarse piezo stages is easily drained by the large cooling power at 4 K. When the gate chip is in the desired position, the setup is further cooled to millikelvin temperatures. The position of the gate chip is monitored by the capacitance sensors and corrected for drift and vibration using the fine X,Y,Z direction piezo stages. The heat caused by the slight movement of the fine piezo stages can be drained by the smaller cooling power at millikelvin temperatures.

With an interference measurements in the $\nu = 5/2$ state in mind, the setup needs to be designed such to meet following requirements:

- Electron temperature of the 2DEG should be much smaller than 100 mK to observe a fully developed $\nu = 5/2$ state (see [217] and references therein).

- Vibrations between gate chip and heterostructure chip should be smaller than 10 pm. Larger amplitude vibrations can induce extra electrons in an interferometer device with an area of 1 µm².
• The angle of the gate chip needs to be set with an accuracy of <0.01° in order not to touch the heterostructure chip.

• The capacitance sensors on the gate chip need to be sensitive enough to measure 10 pm changes in distance between the gate chip and the heterostructure. Considering a 1 µm² sensor, this translates to capacitance changes of $1 \times 10^{-20}$ F.

• A gate voltage of 200 V is required for the coarse gates in order to deplete an electron gas with a density of $n = 3 \times 10^{15}$ m$^{-2}$ that is at a 2 µm distance from the gates. An electron temperature below 100 mK is easily achieved in standard fractional quantum Hall measurement setups using low-pass filtering combined with thermalization on all measurement wires. However, in the proposed setup of Fig. 7.1, the coarse piezo stages (at millikelvin temperatures) need to connect to their control unit (at room temperature) by low Ohmic $\sim 10$ Ω leads in order to work. Standard filtering and thermalization techniques easily increase the resistance of wires by $\sim 2.5$ kΩ. Furthermore, the wires to the fine piezo stages and coarse gates need to be capable of holding voltages in the order of 200 V. Regular components in the low-pass filters break down at such high voltages and new filter designs are required. Techniques used in millikelvin STM setups may help to resolve these problems [218].

The requirement on the vibrational stability of <10 pm provides a more stringent constraint. Chapter 3 presents a setup that has a vibrational stability smaller than 1.5 pm. This setup is very stiff because the two chips directly touch each other. In the setup of Fig. 7.1, the chips are mechanically connected via coarse and fine piezo stages and the brass cage. Such a long loop is more susceptible to mechanical vibrations from the cryostat. Therefore, it is crucial to measure the vibrations of the cryostat as input for the design of the cage and choice of the piezo stages. However, with good isolation techniques this vibration level can be attained. However, such vibration isolation is expected to be within technological reach.

A limitation of the flip-chip setup is the sharpness in which a potential can be induced in the target material, i.e., the minimum feature size. This minimum size is determined by the distance $d$ between the gates and the heterostructure. With $d = 100$ nm the minimum feature size is $\sim 100$ nm. For smaller feature sizes, $d$ needs to be smaller. In the case the gates define a quantum dot in the heterostructure, a reduction of $d$ by a factor $a$ also reduces the maximum allowed vibration level by $a$. 
7.3. INAS/GaSb DQWs AS 2D TOPOLOGICAL INSULATOR

The boundary of a 2D topological state of matter contains gapless edge channels [23–25]. Chiral edge channels occur at the boundary of a quantum Hall state, whereas helical edge channels occur at the boundary of a 2D topological insulating state, see section 2.2.3. To establish InAs/GaSb double quantum wells (DQWs) as topological insulators it is sufficient to show gapless edge channels that have a quantized conductance value consistent with that of helical channels [24, 25, 219]. However, besides these gapless edge channels, other states may exist at the edge. These states may affect the conductance of the edge.

Consider for example the edge of a quantum Hall state. Figure 7.2a depicts the electrostatic potential and Landau level dispersion as a function of distance from the edge. The bulk of the material is described by a (topological) insulating state with filling factor $\nu = 2$. Near the sample edge, the electrostatic potential increases and consequently two chiral edge channels arise at the edge. However, the situation sketched in Fig. 7.2b also has $\nu = 2$ in the bulk, but four channels are found at the edge [220, 221]. The extra two counter propagating channels appear because the electrostatic potential bends down at the edge of the material. Scattering between these two counter propagating modes raises the longitudinal resistance from zero as reported experimentally in ref. [220], and raises the Hall resistance from its quantized value.

The above example shows that the signature of a topological state, i.e. gapless edge channels, may be obscured by trivial conductance due to band bending.

7.3.1. TOWARDS QUANTIZED CONDUCTION THROUGH HELICAL STATES

Existence of helical channels in InAs/GaSb DQWs can be determined by transport measurements when the Fermi-level in the bulk is within the topological gap. A quantization
of the conductance is then expected at zero magnetic field. In a standard six terminal Hall bar device the longitudinal conductance is quantized to \( G_0 = \frac{2e^2}{h} \) \( (R_0 = \frac{1}{G_0} \approx 12.9 \text{ k}\Omega) \). The six terminal Hall bar device and its conductance has been described in section 2.1.3.

Future research that is to be performed on the InAs/GaSb double quantum wells of Chapters 4-6 faces two challenges that should be overcome before experiments can probe this conductance quantization. The first challenge is to reduce the bulk conductance that is present in the topological gap. The second challenge is to remove or significantly reduce the trivial diffusive edge conduction. In the next sections we propose routes to overcome these challenges.

**REDUCING BULK CONDUCTIVITY**

The bulk resistivity in the topological gap is 1.6 k\(	ext{\Omega} / \square \) for the InAs/GaSb quantum well of chapter 5. Such a large bulk conduction (small resistivity) shunts possible helical edge conduction and thus prohibits the observation of a conductance quantization at \( G_0 \). The bulk conduction should be reduced at least two orders in magnitude to establish quantized conductance. We propose two approaches to reduce the bulk conductivity.

The first approach is to increase the topological gap size. The thickness of the InAs and GaSb layers that form the quantum well should be optimized for the largest topological gap. In this respect it should be noted that the band structure calculations of Fig. 2.10, do not include the electric field contributions from gate voltages and should only be used for qualitative understanding. Optimization of the gap size is expected to require growth and subsequent characterization of dual-gated InAs/GaSb quantum wells with different layer thicknesses. Further increase in gap size can be obtained through application of strain, as band structure calculations show that application of 2% compressive strain can increase the gap size by a factor 3.5 [223]. Strain can be incorporated by replacing GaSb with (Ga, In)Sb with up to 35% indium [73].

The second approach is to introduce elastic scatterers in the material. Incorporation of such impurities reduces the bulk conductivity, but does not affect the conduction through helical edge channels, as the electrons in helical channels cannot backscatter from elastic scatterers (see section 2.2.4). This approach is pursued by several groups. A study by one of these groups relies on the use of a ‘dirty’ gallium source during MBE growth of InAs/GaSb quantum wells [167]. They show an increase in bulk resistivity of three orders of magnitude compared to a heterostructure grown with a high-purity gallium source. In another study silicon dopants are incorporated between the InAs and

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1Quantized conductance of helical edge channels could be observed when the edge length is smaller than the inelastic scattering length. The inelastic scattering time \( \tau_i \) can be approximated from weak-localization measurements and universal conductance fluctuations [222] in the conductive phase outside of the topological gap. The edge state velocity for helical edge states is in the order of \( v_\text{e} \approx 2.4 \times 10^4 \text{ m s}^{-1} \), when considering a linear dispersion. We expect that the inelastic scattering length \( l_i = \tau_i v_\text{e} \) is in the order of 5 \( \mu \text{m} \). If we would make a Hall bar device from our InAs/GaSb heterostructure with a length \(<5 \mu\text{m} \) and width of 1 \( \mu\text{m} \), a bulk resistance of at most 8 k\(\Omega \) is expected. A bulk resistance of at least 800 k\(\Omega \) is required to measure an quantized edge resistance of 12.9 k\(\Omega \) within 2% accuracy.
GaSb layers of the quantum well [142]. The subsequent measurements on a Corbino device show an insulating bulk, and conductance measurements on 2 µm long Hall bar devices give quantized values consistent with helical edge channels.

This quantized conductance from [142], however, persists in in-plane magnetic field up to 12 T, which is not understood in the current theoretical framework of InAs/GaSb quantum wells. It should be noted that this experiment can also be explained by \( N \) trivial edge channels with on the average a transmission value of \( 1/2N \) per channel. Such a situation is observed in [190] where all the expected quantization values for helical edge states are produced with trivial edge channels, by an ‘unlucky’ choice of device dimensions.

However, also shown in [190], the transmission values depend strongly on sample size. For a robust signature of helical edge states it is therefore crucial to show quantized conductance values consistent with helical edge channels for at least two different edge lengths.

**REMOVING TRIVIAL EDGE CONDUCTION**

It is known [68, 224, 225] that conduction and valence bands of III-V semiconductor materials bend at the surface of the material. This band bending is a likely cause for the observed trivial edge conduction. The remainder of this section first reviews the origin of this band bending. Next, band bending approximations at the surface of InAs and GaSb and at the surface of InAs/GaSb double quantum wells are given. From these estimates the trivial edge conduction is evident. Finally, three routes are presented to remove or circumvent this trivial edge conduction.

The band bending is caused by a large density of surface states that appear at the end of a truncated crystal. These surface states ‘pin’ the Fermi-level. It is believed that these surface states are caused by either so called native defects’ which are missing anions (e.g., As) or missing cations (e.g. In) or by an anion(cation) occurring at a cation(anion) site [68, 225, 226]. These defect states are localized at the surface and have a donor-like or acceptor-like character. Donor states are neutral when they are occupied, whereas acceptor states are neutral when empty. Thus, charge neutrality is obtained with the Fermi level situated between donor and acceptor surface states. Shifting the surface Fermi-level away from the neutrality condition leaves the surface charged. An equal amount of charge with opposite sign arises in the bulk material to maintain overall neutrality. The resulting electric field between the charge at the surface and the charge in the bulk causes the bands to bend.

In the case of a large density of surface states (density > \( 10^{16} \text{ m}^{-2} \)), a small deviation from the charge neutrality point introduces a strong band bending that leaves the Fermi-level practically unchanged at, at first approximation an energy in between those of the donor and acceptors states [68]. In other works, the Fermi-level at the surface is ‘pinned’. For InAs this pinning occurs \( \sim 150 \text{ meV} \) above the conduction band edge [61, 68, 69, 227]. For GaSb the Fermi-level at the surface is pinned \( \sim 250 \text{ meV} \) above the valence band and therefore resides inside the band gap [228, 229]. The pinning energies are similar for all
Figure 7.3: a) Sketch of the band bending at the surface ($x = 0$) for a crystal (3D) of InAs (red) and GaSb (blue) when the Fermi-level (solid black line) resides in the conduction band of InAs and in the valence band of GaSb. The neutrality point of the surface states for InAs is 150 meV above the conduction band of InAs. The neutrality point for GaSb is 250 meV above the GaSb valence band. b) Position of the bands of bulk InAs and bulk GaSb with respect to each other. The InAs conduction band is positioned 150 meV below the GaSb valence band.

crystal facets when the crystal is exposed to air [224].

Figure 7.3a sketches the band bending for bulk InAs and GaSb as a function of distance $x$ from the surface. The red (blue) lines represent the band edges for InAs(GaSb). The black solid line represents the Fermi-level, which is situated between the conduction band of InAs and the valence band of GaSb. In InAs at low bulk electron density electrons accumulate near the surface due to the band bending [225]. For GaSb both electrons and holes are depleted near the surface, as the Fermi-level is pinned inside the band gap. The spatial region over which the InAs(GaSb) bands bend is indicated by $d_I (d_G)$ and depends on the ability of the material to screen the surface charge [230]. The accumulation layer in InAs provides a strong screening and band bending occurs on a scale of $d_I \sim 5$ nm [67]. The depletion layer in GaSb is expected to be thicker, since no mobile carriers are present near the surface.

Band bending is also expected at the mesa edges of a 12.5 nm/5 nm InAs/GaSb double quantum well. However, there are two differences compared to the band bending in a 3D crystal presented in Fig. 7.3a. First, the confinement provided in the quantum
well raises (lowers) the onset energy for electrons (holes) in InAs(GaSb). Second, the InAs accumulation layer is expected to screen the surface states of GaSb and thereby reduce the depletion layer of GaSb to a magnitude similar to that of InAs, that is \( d_I \approx d_G \).

The hypothesized band bending for this double quantum well is depicted in Fig. 7.4a. The thin solid red and blue lines denote the bulk bands shown in Fig. 7.3a for \( d_I \approx d_G \). The thick dashed red and blue lines represent the onset energy for electron and hole gases, respectively. The black double line indicates the position of the topological gap. The solid black line indicates the Fermi-level, which is positioned inside the topological gap in the bulk of the material. Figure 7.4b depicts the band alignment in growth direction in the bulk.

![Figure 7.4: a) Sketch of the band bending near the edge of the mesa (x = 0) for InAs (red) and GaSb (blue). Solid lines denote the bulk conduction and valence bands. Red(Blue) dashed lines represent onset of electron(hole) gas in InAs(GaSb). Back solid line denotes the position of the hybridization gap. Energy bands, band position and expected band bending are to scale. b) Band line up along the growth direction. c) Schematic representation of the band structure, where a hybridization gap opens at the crossing of the bands.](image)

As in the bulk material (Fig. 7.3a) electrons in the InAs/GaSb quantum well accumulate near the edge, while holes are depleted. The effect of this accumulation layer on the trivial edge conductance will be discussed after the positions of the topological gap and trivial gap are identified.

Consider points 1-4 in Fig. 7.4a, that are positioned on the Fermi-level near the mesa edge. Figure 7.4c shows the schematic band structure at these points. Far in the bulk, point 1, the Fermi-level resides in the hybridization gap. Moving towards the mesa edge, point 2, the bands start bending and the hybridization gap falls below the Fermi-level. At point 3 the topological gap closes. Even closer to the edge, at point 4, a trivial band
alignment is reached, with the Fermi-level in the conduction band of InAs. The helical edge channels propagating in the \( y \)-direction are predicted to occur at \( x = d_H \), where the topological gap closes and the trivial gap opens (see green dot in Fig. 7.4a). Whether these helical edge channels are the only channels present at the Fermi-level depends on the confinement potential of the accumulation layer.

Electrons in the accumulation layer experience approximately a triangular well potential, bounded by the trivial gap and the topological gap. Because the accumulation layer is very thin (\( \sim 5 \) nm) quantization effects are important. The first state in such a perfect infinite triangular well would lie 170 meV above the bottom of the conduction band and therefore also above the Fermi-level. However, the barrier formed by the topological gap is very thin as shown by the zoom-in in the inset of Fig. 7.4a. The electric field at the surface tilts the band such that the barrier for electrons is approximately 5 meV high and \( \sim 2.5 \) Å wide \(^2\). Tunneling probability across such a gap is close to unity\(^3\) and therefore no quantized state are expected within this gap.

The helical edge channels therefore coexist with trivial electron channels, which prohibits the observation of quantized conductance.

In the remainder of this section we propose three routes to observe the quantized conductance of the helical edge channels in InAs/GaSb double quantum wells.

The first route is to fully ‘unbend’ the bands of both materials at the edge to reach a flat band condition. The Fermi-level can then be positioned inside the topological gap which extends all the way to the edge. In this scheme, the bands need to be flat within \( \sim 5 \) meV throughout the material for both InAs and GaSb. There are, however, no reports in literature that show unbending of the bands of two types of material with such a precision.

A more promising route is to bend the InAs conduction band up such that it crosses the Fermi level at the edge, i.e., so that it depletes all the electrons in InAs near the surface. The Fermi-level at the edge is then in the trivial gap. The Fermi-level in the bulk remains in the topological gap and at the boundary between these gaps helical edge channels are expected. The band structure at point 4 would then be similar to Fig. 7.4c, but with the Fermi level inside the trivial gap. In such a way transport is only due to the helical edge channels. The density at the InAs edge is believed to be in the order of \( 10^{16} \) m\(^{-2}\) \(^{67, 68}\) which allows for depletion with a gate positioned against the side \(^{231, 232}\). The electric field from the side gate would also affect the GaSb bending, but since the band bending for GaSb is larger there must be a situation wherein both materials are depleted. However, as we will show in the next paragraph, such a situation still allows for possible trivial edge channels. The occurrence of these trivial channels depends crucially on the length scale over which the trivial gap closes and evolves into the topological gap. Furthermore, the energy at which the gap closes must occur within the energy gap

\(^2\) Using an electric field of \( E = \Delta V/\Delta x = 100 \) mV/5 nm = \( 2 \times 10^6 \) V/m.

\(^3\) Using a WKB approximation for tunneling across a parabolic barrier, \( T \approx \exp \left( -\sqrt{\frac{2mV}{\hbar^2}} \Delta d \right) \) [34].
of the trivial and topological regime. Likely, the freedom of one side gate potential is not sufficient to tune into this delicate crossing.

Finally, all edge details can be completely circumvented when the topological phase transition is forced to occur in the interior of the material. This is in principle possible in InAs/GaSb quantum wells, as the phase of the material depends on electric fields that can be applied by gate voltages (see Chapter 3). A region in the bulk can therefore be tuned to the topological insulating regime, while the adjacent region can be tuned to the trivial insulating regime.

Figure 7.5a depicts a cross section of the proposed setup. A split top gate \( V_{tg,A} \) and \( V_{tg,C} \) and split back gate \( V_{bg,A} \) and \( V_{bg,C} \) are fabricated at the top and bottom of the quantum well, respectively. The region between \( V_{tg,A} \) and \( V_{bg,A} \) \( V_{tg,C} \) and \( V_{bg,C} \) is denoted by A(C) in Fig. 7.4a. Region B is situated in between regions A and C, and has a length \( \Delta \). The gates are a distance \( d = 50 \) nm from the InAs/GaSb quantum well. A third gate pair, \( V_{tg,B} \) and \( V_{bg,B} \), is fabricated above the split top gate and below the split back gate, respectively. This third gate pair is first neglected and will be introduced later. Region A is tuned to the trivial insulating regime by setting gate voltages \( V_{tg,A} \) and \( V_{bg,A} \) to point I in the phase diagram of Fig. 7.5c. Region C is tuned to the topological insulating regime, point II in Fig. 7.5c, using gate voltages \( V_{tg,C} \) and \( V_{bg,C} \). The energy spectrum for this configuration is presented in figure 7.4b. Region A has a trivial band gap of \( \sim 8 \) meV, while the topological gap in region C has a size of \( 5 \) meV. Therefore, the regions A and C combined have an effective energy gap of \( 5 \) meV, denoted by \( \Delta \) in Fig. 7.4b. Helical edge channels are expected in region B, at the interface between the topological (C) and trivial insulating phases (A). However, as is noted in the beginning of section 7.3 besides the helical states also trivial channels could be present at the Fermi-level in region B.

In order to reach a regime without trivial edge channels, the energy where the topological gap closes (and crosses over into the trivial gap) must occur within the effective energy gap \( \Delta \). Such a situation is depicted in Fig. 7.4b where the solid black lines denote the band edge. Trivial electron and hole states within \( \Delta \) can occur in the red and blue shaded region of region B, respectively. These electrons and holes are bound by the trivial gap on the left and the topological gap on the right, and effectively experience a finite triangular potential well. When region B is large, a bound state of the triangular well can be found at every energy within \( \Delta \). These states result in trivial conduction in the \( y \)-direction, i.e., along the edge. By reducing \( \Delta \), bound states (1D channels) can only occur at discrete energies, as depicted by the solid red and blue lines in region B of Fig. 7.5b. The minimal width of the triangular well potential (when \( \Delta = 0 \)) is in the order of the barrier thickness \( d = 50 \) nm. Approximating region B by a square well potential of 5 meV high and 50 nm wide, the first bound state for electrons is expected at 0.8 meV above the crossing of the gaps and for holes 0.3 meV [233] below this crossing. When the Fermi-energy is in between these levels, no trivial channels are present and quantized conduction due to helical states could be observed. The Fermi-level needs to be placed
within an energy window of 1.1 meV, or 12 K.

We will now discuss the need for the third gate pair. Suppose region B is large, \( L \gg d \), and that the third gate pair is absent. The potential in the middle of region B (at \( x = 0 \)) is not influenced by the gate voltages at region A and C. The phase at \( x = 0 \) is therefore described by the phase at \((V_{tg}, V_{bg}) = (0, 0)\), point IV, in Fig. 7.4c. Although for smaller \( L \), the potential in region B is affected by gate voltages at region A and C, it is not clear that it would converge to point III in the phase diagram of Fig. 7.4c. Detailed electrostatic simulations are needed to obtain the band alignment in region B for each \( L \). However, it is unlikely that it naturally resides at point III in the phase diagram of Fig. 7.4c. Therefore an additional gate pair \( V_{tg,3}, V_{bg,3} \) is needed to tune region B into this regime. It should be noted that these gate voltages are not simply the voltages of point III in Fig. 7.5c, because nearby gates heavily screen their effect. With regions A and C tuned to the trivial and topological gaps, respectively, the gate voltages \( V_{tg,3} \) and \( V_{bg,3} \) should be scanned over a large gate range in order to find regions of a quantization of conductance.

The most difficult fabrication step of the proposed device is the split gate at the back of the double quantum well. The opening between the split gates needs to be in the order of \( \sim 20 \) nm. Fabrication of a split back gate has been reported in [234–238]. Using the approach of [237], the distance between the split gates can be made as narrow as 20 nm. However the alignment between top and bottom gates within an accuracy of 20 nm will be challenging. Furthermore, this technique requires highly involved fabrication steps which limit the yield of working devices. Recently a more promising technique was developed [239] for a structured back gate in GaAs/AlGaAs that allows for high-mobility of the carriers in the 2DEG (\( \sim 18 \times 10^6 \) cm\(^2\)/Vs at an electron density \( n = 3.7 \times 10^{15} \) m\(^{-2}\)). The back gate consists of a silicon doped GaAs layer which is selectively made insulating by oxygen ion implantation. Using this technique a split back gate can be made, however it is not known whether this technique is capable of defining a split gate with an opening \( \sim 20 \) nm without having leakage current between the parts of the split back gate. Furthermore, it is unclear if the extra back gate below the split gate can be made with this technique.

Further research is needed on split back gates before the proposed device with six gates can be fabricated.
Figure 7.5: Proposed split-gate setup. a) A cross section of the heterostructure. The split top gate and split back gate are separated by 50 nm from the InAs/GaSb quantum well. A third gate pair is fabricated over the other gates to control the potential in region B. b) The band alignment for regions A, B and C of panel a). The red (blue) shaded regions in region B indicate the triangular well in which electrons (holes) are bounded. The solid red (blue) lines in region B indicate quantized electron (hole) levels. c) The phase diagram of a InAs/GaSb double quantum well. The Fermi-level at point I is situated in the trivial gap, at point II in the topological gap, at point III at the semimetallic point, and at point IV in the topological regime.


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