Feedback Signal of a Seismic Vibrator

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Summary

In this paper, we discuss the wavefield emitted by a seismic vibrator. The shape of the far field wavelet is of practical interest, since removal of this wavelet from the recorded seismogram results in the desired (bandlimited) impulse response of the earth. The determination of the far field wavelet from measurements on the vibrator involves two steps. First, from the earth model it follows that the far field particle velocity essentially equals the time derivative of the ground force. What remains, then, is to determine the ground force from measurements on the vibrator. A vibrator model is developed which takes into account all forces acting on the baseplate, including the bending forces. The assumptions underlying two feedback signals that are currently being used (reaction mass acceleration, and a weighted sum of baseplate acceleration and reaction mass acceleration) are examined with the aid of this vibrator model. Also the vibrator model including the baseplate bending is used to derive an estimate for the ground force.

The theoretical results are compared with measurements made on a seismic vibrator. It is shown that the theory that includes the baseplate bending yields an accurate amplitude prediction of the vibrator behaviour, but that, as yet, unexplained phase shift of approximately 90° is introduced.

1. The earth model

It is well known (Miller and Pursey 1954; Baeten 1989) that the particle velocity in the far field of an elastic halfspace is equal to the time derivative of the ground force. This result is true for arbitrary loading conditions at the surface, provided that the source dimensions are small compared with a wavelength. This condition will in general be fulfilled for a single vibrator, but a frequency-dependent directivity pattern is introduced when an array of vibrators is used.

2. The vibrator model

To describe the vibrator behaviour, first an expression is needed for the dynamic input force exerted by the drive system of the seismic vibrator on the top of the baseplate. For a hydraulic P-wave vibrator, Lerwill (1981) showed that this input force \( p_{applied} \) is equal to

\[
p_{applied} = -M_A \frac{dA}{dr},
\]

in which \( A \) denotes the acceleration of the reaction mass, and \( M \) denotes its mass. Second, an expression is needed for the baseplate deflection under the action of a certain pressure distribution \( P_{in} \). Using classical plate theory, it can be shown that the baseplate deflection \( U_3 \) satisfies the equation (Timoshenko and Woinowsky-Krieger, 1959)

\[
D \left( \frac{\partial^4 U_3}{\partial x_1^4} + 2 \frac{\partial^4 U_3}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 U_3}{\partial x_2^4} \right) - \sigma \omega^2 U_3 = P_{in},
\]

in which \( \sigma \) denotes the surface density of mass of the baseplate, \( \omega \) denotes the angular frequency and \( D \) denotes the flexural rigidity of the baseplate. The flexural rigidity of the baseplate depends on the elastic properties and the mechanical structure of the baseplate.

Although structural components like stiffeners present in the baseplate are thus included in the analysis, anisotropic baseplate behaviour due to non-uniform stiffness of the plate is neglected.

For the vibroseis configuration, the applied load \( P_{in} \) consists of the dynamic input force per unit area, \( p_{applied} \), and the reaction force of the earth per unit area, \( T_3 \). Thus, equation (2) becomes

\[
D \left( \frac{\partial^4 U_3}{\partial x_1^4} + 2 \frac{\partial^4 U_3}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 U_3}{\partial x_2^4} \right) - \sigma \omega^2 U_3 = p_{applied} + T_3.
\]

The solution to differential equation (3), subject to the boundary conditions of vanishing bending moments and shearing forces at the plate edges, cannot be obtained analytically. However, if the plate dimensions are neglected, the baseplate deflection due to the action of a point force can be calculated. From this point force response, the response of the baseplate due to an arbitrary pressure distribution can be obtained by spatially convolving the point force response \( G^{flex} \) with the pressure distribution:

\[
U_3(x_1, x_2) = \int_S \left( p_{applied}^{H_0}(x_1', x_2') + T_3(x_1', x_2') \right) \times G^{flex}(x_1 - x_1', x_2 - x_2') \, dx_1' \, dx_2',
\]

where \( S \) denotes the baseplate area. The point force response (or Green's function) \( G^{flex} \) is given by

\[
G^{flex}(r, \omega) = \frac{1}{8 \pi \omega D} \left[ H_0^{(1)}(ar) - H_0^{(1)}(a\rho) \right],
\]

in which \( r \) denotes the distance between observation point and integration point, \( H_0^{(1)} \) denotes the Hankel function of the first kind and \( a \) is given by

\[
a = \left( \frac{\omega \rho}{D} \right)^{1/4}.
\]

The theory just described that includes the baseplate bending will be referred to as the flexural rigidity method.

3. Comparison with measurements

An experiment was carried out in which the accelerations of the baseplate and the reaction mass, as well as the pressure distribution directly underneath the baseplate were measured. The pressure distribution directly underneath the baseplate was measured using a crystal bed containing 40 pressure transducers. In the experiment, a Prakla Seismos VVDA vibrator was used.

3.1 Test of the flexural rigidity method

First, the flexural rigidity method (equation (4)) is used to predict the acceleration in the centre of the baseplate from measurements of the pressure distribution underneath the plate and the acceleration of the reaction mass. The result is shown in Figures 1 and 2, where a comparison is made between amplitude and phase of the exact (measured) and the predicted baseplate acceleration. It is observed
that the flexural rigidity method accurately describes the amplitude of the baseplate acceleration, but that a phase difference of approximately 90° exists between measured and predicted acceleration. At this moment, the origin of this phase error is still unknown.

3.2 Comparison between true and predicted ground force

In this section, three methods to obtain the ground force $F^G$ from measurements on the vibrator (reaction mass acceleration, a weighted sum of the baseplate acceleration and the reaction mass acceleration, and a new method based upon the flexural rigidity method) are compared with the exact, directly measured ground force. The exact ground force is obtained by summing the output of the 40 pressure transducers underneath the baseplate. A discussion of the three feedback signals is presented using the integrated equation of motion of the baseplate (see equation (5)):

$$
\int_{S} \left( \frac{\partial^4 U_3}{\partial x_1^4} + 2 \frac{\partial^2 U_3}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 U_3}{\partial x_2^4} \right) dA - \int_{S} \omega^2 U_3 dA = \int_{S} F_{\text{applied}} dA + \int_{S} T_3 dA.
$$

(7)

The equation of motion consists of four terms: the bending force, the inertial force, the applied force and (minus) the ground force.

Using reaction mass acceleration as a feedback signal neglects the inertial and bending forces in the baseplate, and therefore does not give a reliable estimate of the ground force.

The weighted sum approximation of the ground force is based on the rigid body assumption of the baseplate, yielding the following equation of motion of the baseplate:

$$
F^G = - \int_{S} T_3 dA = - M \rho_0 \frac{\partial^2}{\partial t^2} U_3 + F_{\text{applied}}.
$$

(8)

In this equation, the applied force can be obtained from measurements of the reaction mass acceleration (see equation (1)). The weighted sum method neglects the bending forces acting in the baseplate, and the effect a non-uniform displacement distribution has on the magnitude of the inertial force. The magnitude of these errors depends on the stiffness of the baseplate, the elastic parameters of the earth and the frequency of operation. The amplitude and phase of the exact ground force, and the ground force predicted using the weighted sum method, are shown in Figures 3 and 4. It is observed that the weighted sum method gives a reasonable estimate for the phase of the ground force, but that the amplitude prediction of the weighted sum method is less accurate.

The flexural rigidity method can be rewritten by assuming that the baseplate is stiff, and consequently that $G_{\text{flex}}$ is approximately uniform over the baseplate. This yields the following estimate for the ground force:

$$
F^G = - \int_{S} T_3 dA = - \left[ - \frac{8 \sqrt{\rho D}}{\omega A_3 (x_1^e, x_2^e)} \right] - F_{\text{applied}}.
$$

(9)

where $x_1^e$ and $x_2^e$ denote the coordinates of the centre of the baseplate, and $A_3$ denotes the acceleration of the plate. It should be noted that the assumption of a stiff plate does not result in the rigid-body equation of motion of the baseplate (equation (8)). The reason for this is that an infinitely stiff plate is not physically realizable; for a large, finite positive value of $D$, even small variations in the displacement over the baseplate area affect the baseplate behaviour.

The flexural rigidity method only yields an accurate estimate for the ground force if the previously mentioned 90° phase shift is accounted for. Amplitude and phase of the predicted ground force using the phase-corrected Green’s function, and the exact ground force are shown in Figures 5 and 6. The amplitude match is excellent, and the phase difference is similar to the phase difference observed when using the weighted sum method.

4. Conclusions

A vibrator model is developed which takes into account the bending of the baseplate. It is shown that even for a very stiff plate the baseplate behaviour cannot be described by the rigid body equation of motion of the baseplate, but that the bending of the plate should be included in the description. Comparison of true and predicted ground force shows that the flexural rigidity method gives a reliable estimate of both amplitude and phase of the ground force, although the method still suffers from an, as yet, unexplained phase shift of approximately 90°.

References


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**Fig. 2.** Phase difference in degrees between predicted baseplate acceleration using flexural rigidity method and measured acceleration.

**Fig. 3.** Amplitude of predicted ground force using weighted sum method (solid line) and amplitude of measured ground force (dotted line).

**Fig. 4.** Phase difference in degrees between predicted ground force using weighted sum method and measured ground force.

**Fig. 5.** Amplitude of predicted ground force using phase-corrected flexural rigidity method (solid line) and amplitude of measured ground force (dotted line).

**Fig. 6.** Phase difference in degrees between predicted ground force using phase-corrected flexural rigidity method and measured ground force.