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CONICAL STREAMLINES AND PRESSURE DISTRIBUTION
IN THE VICINITY OF CONICAL STAGNATION POINTS
IN ISENTROPIC FLOW

by

P.G. Bakker

DELT - THE NETHERLANDS
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SUMMARY.

The properties of the supersonic flow in the vicinity of conical stagnation points, occurring in isentropic conical flow, are studied in a more general way.

Especially the relation between conical streamline pattern and pressure distribution is discussed in detail. From this viewpoint some unsatisfactory results, occurring in the literature, could be clarified. To obtain a unique description of local flow pattern a parameter is introduced which takes into account the particular properties of the pressure distribution.
CONTENTS.

List of symbols 1

1. Introduction 3

2. The equations of inviscid conical flow 5

3. Types of conical stagnation points 7
   3.1. General considerations 7
   3.2. Particular stagnation points 10
   3.3. Stagnation points in the flow field 12
   3.4. Stagnation points on a conical body 19

4. Pressure distribution in the vicinity of conical stagnation points 23

5. Unique description of conical flow pattern 29
   5.1. Introductory remarks 29
   5.2. Outline of Melnik's analysis 29
   5.3. Properties of the pressure distribution 31

6. Conclusion 35

References 36
LIST OF SYMBOLS.

\( D_p \) parameter depending on the pressure distribution

\( D_s \) parameter depending on the streamline pattern

\( f(s) \) contour of conical body in the \( t,s \)-plane

\( I_p \) \( \frac{\partial (\frac{\partial p}{\partial s} \frac{\partial p}{\partial t})}{\partial (s,t)} \)

\( I_s \) \( \frac{\partial (v,w)}{\partial (s,t)} \)

\( k=m \) parameters for a unique description of flow pattern

\( n,\sigma \) polar coordinates in the \( t,s \)-plane

\( P \) pressure function (Ref. 2)

\( p \) pressure

\( p^\star \) non-dimensional pressure

\( q^' \) magnitude of conical velocity

\( q^\star,\omega \) polar coordinates in the \( v^\star, w^\star \)-plane

\( r,\theta,\phi \) spherical coordinate system

\( s \) entropy per unit mass

\( t,s \) rectangular coordinate system, having its center in a conical stagnation point

\( t^',s^' \) rotated coordinates in the \( t,s \)-plane

\( u,v,w \) spherical velocity components

\( V \) conical velocity vector

\( V_2,W_2 \) higher order terms in the Taylor expansion of the conical velocity

\( v^\star \) \( \frac{1}{u_0} \left( \frac{\partial v}{\partial t^'} \right)_0 \)

\( w^\star \) \( \frac{1}{u_0} \left( \frac{\partial w}{\partial t^'} \right)_0 \)
\( \alpha \)  angle between \( t' \)- and \( s \)-axis
\( \beta \)  slope of conical body in \( t,s \)-plane
\( \gamma_1, \gamma_2 \)  directions of asymptotes in the pressure field
\( \mu_{1,2} \)  exponents determining the flow pattern
\( \rho \)  density
\( \sigma \)  coordinate, constant along a conical streamline

**Indices.**

\( o \)  conical stagnation point
\( \text{irr} \)  irrotational
1. **INTRODUCTION.**

In general the treatment of a three dimensional inviscid gas flow problem is a very difficult task. Therefore an analysis delivering information concerning the nature of the problem is a useful step to attain a more tractable model of the problem. In three dimensional gas dynamics the concept of conical flow has been frequently applied for special conical shaped bodies. For example, the problem of the supersonic flow around a circular cone at incidence is considerably simplified by the assumption that the flow is conical. The main idea in conical flow is the constancy of the flow properties (velocity, density, pressure) along a ray emanating from the conical centre. At different rays the flow quantities have different values. The so called self similarity property of conical flow allows the reduction of the problem from three to two spatial dimensions.

The conical nature of the flow asks for a description in a spherical coordinate system \((r, \theta, \phi)\) with the origin in the conical centre (Fig. 1). In this coordinate system the flow quantities are independent of the radial distance and it is sufficient to analyse the conical flow problem on the unit sphere only. The polar angle \(\theta\) and azimuthal angle \(\phi\) determine the location of a point on the unit sphere. In any point on the unit sphere the velocity is determined by its components along the radius and perpendicular to the radius. The latter velocity component lies in a plane tangent to the unit sphere and defines a direction on the surface of the sphere. Integrating along this direction with respect to time, we get a streamline on the sphere, usually called a conical streamline, which may be defined as the intersection of a particular streamsurface with the unit sphere. Such a particular streamsurface is formed by all spatial streamlines going through the same ray. Points on the unit sphere where the tangential velocity component disappears are defined as conical stagnation points and consequently the direction of a conical streamline is undetermined in a conical stagnation point.
If the flow is not irrotational, some flow quantities become singular in stagnation points (Ref. 1,2). Due to this behaviour, different numerical solutions break down in the vicinity of singular points. In 1968 Melnik (Ref. 2) employed a coordinate expansion of the flow equations and gave a description of the possible streamline pattern in the vicinity of conical stagnation points. Melnik argued that the qualitative behaviour of the streamlines has not been affected by entropy gradients in the flow field and therefore, without loss of generality, the treatment of the streamline pattern in the vicinity of conical stagnation points may be restricted to isentropic conical flow.

In the present report the coherence between streamline pattern and pressure distribution in isentropic flow is studied and from this viewpoint some unsatisfactory results, occurring in Melnik's description could be clarified.

The terms streamline and stagnation point mentioned in this report will be frequently used for the indication of conical streamline and conical stagnation point unless stated otherwise.
2. THE EQUATIONS OF INVISCID CONICAL FLOW.

Because of the conical nature of the flowfield, it is convenient to formulate the problem in spherical coordinates \((r, \theta, \phi)\), Fig. 1, where \(r\) is the radial distance from the conical centre, \(\theta\) is the polar angle \((0 < \theta < \pi/2)\) and \(\phi\) is the azimuthal angle \((-\pi < \phi < \pi)\). The velocity components are \(u, v, w\) positive in the directions of increasing \(r, \theta, \phi\) respectively; the pressure, density and entropy are denoted by \(p, \rho\) and \(s\) respectively. The effects of viscosity and heat conduction are neglected.

Moreover, it is assumed that the gas is perfect and that the flow is steady. With these assumptions the equations for conical flow, written in spherical coordinates, become:

Continuity equation

\[
\frac{\partial}{\partial \theta} (\rho v) + \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} (\rho w) + \rho v \cot \theta + 2 \rho u = 0
\]  \hspace{1cm} (2.1)

Momentum equations

\(r\)-momentum

\[
v \frac{\partial u}{\partial \theta} + \frac{w}{\sin \theta} \frac{\partial u}{\partial \phi} = v^2 + w^2
\]  \hspace{1cm} (2.2)

\(\theta\)-momentum

\[
v \frac{\partial v}{\partial \theta} + \frac{w}{\sin \theta} \frac{\partial v}{\partial \phi} + \frac{1}{\rho} \frac{\partial p}{\partial \theta} = w^2 \cot \theta - uv
\]  \hspace{1cm} (2.3)

\(\phi\)-momentum

\[
v \frac{\partial w}{\partial \theta} + \frac{w}{\sin \theta} \frac{\partial w}{\partial \phi} + \frac{1}{\rho \sin \theta} \frac{\partial p}{\partial \phi} = -vw \cot \theta - uw
\]  \hspace{1cm} (2.4)
Energy equation

\[ v \frac{\partial s}{\partial \theta} + \frac{w}{\sin \theta} \frac{\partial s}{\partial \phi} = 0 \]  

(2.5)

In conical flow any spatial streamline crossing the same ray possesses the same value of entropy and it may be concluded that a conical streamline is a line on the unit sphere on which the entropy is constant. Therefore conical streamlines may be deduced from Eq. (2.5) from which there is obtained:

\[ \frac{d\theta}{d\phi} _{\text{conical streamline}} = - \frac{\partial s}{\partial \phi / \partial \theta} \frac{\partial s}{\partial \phi} = \frac{v \sin \theta}{w} \]  

(2.6)
3. **TYPES OF CONICAL STAGNATION POINTS.**

3.1. **GENERAL CONSIDERATIONS.**

According to the definition of a conical stagnation point such a point is characterized by the condition \( v=w=0 \) and consequently a conical stagnation point coincides with a singular point of the streamline equation (Eq. 2.6). The direction of the conical streamlines in a stagnation point cannot be determined at first sight. To eliminate this difficulty it is assumed that the velocity components \( v \) and \( w \) may be expanded in a Taylor series with respect to the location of a conical stagnation point \((\theta_o, \phi_o)\).

This assumption seems to be reasonable in isentropic flow because the derivatives of the velocity component \( v \) and \( w \) are continuous due to the absence of a vortical singularity in the stagnation point. Introducing new variables \( t = \theta - \theta_o \) and \( s = (\phi - \phi_o) \sin \theta_o \), Eq. (2.6) may be rewritten in the vicinity of the stagnation point:

\[
\frac{dt}{ds} = \left( \frac{\partial v}{\partial s} \right)_o \cdot s + \left( \frac{\partial v}{\partial t} \right)_o \cdot t + V_2(s,t) \]

\[
\left( \frac{\partial w}{\partial s} \right)_o \cdot s + \left( \frac{\partial w}{\partial t} \right)_o \cdot t + W_2(s,t)
\]

Now the conical stagnation point \( v=w=0 \) corresponds to the singularity of the first order differential equation (Eq. 3.1). This singularity occurs in the origin of the \( s,t \)-plane. It has been shown by Poincaré (Ref. 3) that the differential equation (3.1), in which the constant coefficients of the first order terms are such that the Jacobian

\[ I_s = \left( \frac{\partial v}{\partial s} \right)_o \left( \frac{\partial w}{\partial t} \right)_o - \left( \frac{\partial v}{\partial t} \right)_o \left( \frac{\partial w}{\partial s} \right)_o \neq 0 \]

and in which \( V_2(s,t) \) and \( W_2(s,t) \) vanish like \( s^2+t^2 \) as \( s \to 0 \) and \( t \to 0 \), has a singularity equal to those of the much simpler equation:
\[
\frac{dt}{ds} = \left(\frac{\partial v}{\partial s}\right)_o \cdot s + \left(\frac{\partial v}{\partial t}\right)_o \cdot t \\
\left(\frac{\partial w}{\partial s}\right)_o \cdot s + \left(\frac{\partial w}{\partial t}\right)_o \cdot t
\]

(3.2)

If the quantity \(I_s\) is equal to zero, singularities of higher order may occur, which may be of a quite different type as compared to those of Eq. (3.2). However, in general the condition \(I_s \neq 0\) is fulfilled so that we may confine our attention to Eq. (3.2).

Equation (3.2) is a linear differential equation with constant coefficients and the type of the singularity depends on the value of the derivatives of the velocity-components in the singular point. The classification of possible singular solutions may be found in many textbooks about linear differential equations. The results may be summarized as follows.

It appears that the behaviour of integral curves is completely determined by the relation:

\[
D_s = \left(\frac{\partial v}{\partial t} - \frac{\partial w}{\partial s}\right)^2 + 4 \frac{\partial v}{\partial s} \frac{\partial w}{\partial t}
\]

(3.3)

and the Jacobian:

\[
I_s = \frac{\partial v}{\partial t} \frac{\partial w}{\partial s} - \frac{\partial v}{\partial s} \frac{\partial w}{\partial t}
\]

(3.4)

For \(D_s > 0\) the singularity is a nodal point if \(I_s < 0\), (Fig. 2 a,b) and a saddle point if \(I_s > 0\) (Fig. 2c).

\[2a. D_s > 0, I_s < 0 \]

\[2b. D_s = 0, I_s < 0 \]

\[2c. D_s > 0, I_s > 0 \]

Fig. 2 Integral curves if \(D_s \geq 0\)
For \( D_s < 0 \) a spiral point is formed if \( \frac{\partial v}{\partial t} + \frac{\partial w}{\partial s} \neq 0 \), Fig. 3a and a centre point occurs if \( \frac{\partial v}{\partial t} + \frac{\partial w}{\partial s} = 0 \), Fig. 3b, provided that second order terms are neglected.

![Spiral and Centre Diagrams](image)

### 3a. \( \frac{\partial v}{\partial t} + \frac{\partial w}{\partial s} \neq 0 \)  
### 3b. \( \frac{\partial v}{\partial t} + \frac{\partial w}{\partial s} = 0 \)

**Fig. 3 Integral curves at \( D_s < 0 \)**

The previous analysis completes our discussion of the classification of singularities of first order differential equations, except for one additional remark. We have already stated that the singularities obtained in the present section are the same as those corresponding to the differential equation:

\[
\frac{dt}{ds} = \frac{(\frac{\partial v}{\partial s})_0 s + (\frac{\partial v}{\partial t})_0 t + V_2(s, t)}{(\frac{\partial w}{\partial s})_0 s + (\frac{\partial w}{\partial t})_0 t + W_2(s, t)}
\]

in case \( I_s \neq 0 \) and \( V_2 \) and \( W_2 \) vanish to at least second order at the origin. Poincaré also showed that the criteria for the classification of the types of singularities also remain unchanged in this more general case, with one exception: the condition \( (\frac{\partial v}{\partial t})_0 + (\frac{\partial w}{\partial s})_0 = 0 \) no longer suffices to distinguish between a centre- and a spiral point in the case \( D_s < 0 \) and the higher order terms \( V_2 \) and \( W_2 \) have to be retained in the analysis.
3.2. PARTICULAR STAGNATION POINTS.

It is evident that the integral curves of the differential equation (3.2) may be identified as conical streamlines in the vicinity of a stagnation point. Before discussing the general solutions of Eq. (3.2) in connection with the inviscid flow equations it is illustrative to consider for a moment those particular conical stagnation points which possess the property: $\frac{\partial v}{\partial s} = \frac{\partial w}{\partial t} = 0$. These stagnation points occur for example in the conical flow field generated by a circular cone at incidence in a supersonic flow. Eq. (3.2) now reduces to:

$$\frac{dt}{ds} = \frac{\frac{\partial v}{\partial t} o \cdot t}{\frac{\partial w}{\partial s} o \cdot s}$$

(3.5)

Expanding the continuity equation with respect to the location of a conical stagnation point and retaining only the leading term there is obtained:

$$\frac{\partial v}{\partial t} o + \frac{\partial w}{\partial s} o + 2u_o = 0$$

(3.6)

Eliminating the quantity $\frac{\partial w}{\partial s} o$, Eq. (3.5) may be written as:

$$\frac{dt}{ds} = \frac{\frac{\partial v}{\partial t} o \cdot t}{(-\frac{\partial v}{\partial t} o - 2u_o) \cdot s}$$

This equation may easily be integrated and the equation describing the conical streamlines becomes:

$$t = \text{const.} \cdot s^m$$

(3.7)

where

$$m = \frac{\frac{\partial v}{\partial t} o}{-\frac{\partial v}{\partial t} o - 2u_o}$$
The flow pattern depends on the value of \(\frac{1}{u_0} \left( \frac{\partial v}{\partial t} \right)_0\) (or \(m\), Fig. 4) and the constant varies from streamline to streamline. For positive values of \(m\) all streamlines have the origin in common and a nodal point is formed. If \(m\) is negative the streamlines have a hyperbolic character and a saddle point occurs. The different flow patterns that may occur in the vicinity of a stagnation point are sketched in Fig. 5.
In the case of a nodal point the streamlines are tangent to the t-axis if \( 0 < m < 1 \) and tangent to the s-axis if \( m > 1 \). It should be noted that the appearing singularities occur only as nodal- and as saddle-points, this in contrast to stagnation points occurring in two-dimensional plane flow where also centre points may appear.

3.3. STAGNATION POINTS WITHOUT ADDITIONAL PROPERTIES.

In the previous paragraph we have discussed conical stagnation points which possess the symmetry conditions \( \frac{\partial v}{\partial s} = \frac{\partial w}{\partial t} = 0 \). In that case the differential equation describing the streamline pattern Eq. (3.2), could easily be solved by the method of separation of variables. It appeared that the solution gives stagnation points which are only nodal- and saddle-points.

We will now show that in general this conclusion remains valid for stagnation points in an isentropic conical flow. For that purpose Eq. (3.2) is again taken as starting point, which is a first order differential equation with constant coefficients. These coefficients are the first order partial derivatives of the velocity components \( v \) and \( w \) in the stagnation point. The two partial derivatives \( \left( \frac{\partial v}{\partial s} \right)_0 \) and \( \left( \frac{\partial w}{\partial s} \right)_0 \) are interrelated according to Eq. (3.6).

It appears that also a relation exists between the remaining derivatives \( \left( \frac{\partial v}{\partial s} \right)_0 \) and \( \left( \frac{\partial w}{\partial t} \right)_0 \). This relation may be derived from the equations for \( \theta - \) and \( \phi - \) momentum.

If these equations are differentiated once with respect to \( \phi \) and \( \theta \) respectively, the quantity \( \frac{\partial^2 P}{\partial \theta \partial \phi} \) may be eliminated. Applying the result in a stagnation point there is obtained:

\[
\left( \frac{\partial v}{\partial s} \right)_0 - \left( \frac{\partial w}{\partial t} \right)_0 (\frac{\partial v}{\partial s} + \frac{\partial w}{\partial s} + u)_0 = 0
\]

From Eq. (3.6) it follows immediately that the latter expression may only be satisfied if:

\[
\left( \frac{\partial v}{\partial s} \right)_0 - \left( \frac{\partial w}{\partial t} \right)_0 = 0
\]

(3.8)
provided \( u_0 \) is a nonzero quantity.

It may be noted that the particular stagnation points treated in paragraph 3.2. have the property \( \left( \frac{\partial v}{\partial s} \right)_o = \left( \frac{\partial w}{\partial t} \right)_o = 0 \), which is a special solution of Eq. (3.8).

In the analysis we have used the expansion of the velocity components in a Taylor series and the reliability of this method is guaranteed only for irrotational flow. In a rotational flowfield the entropy, which remains constant along a conical streamline, causes a nonuniformity of the radial velocity and density in a nodal stagnation point. This nonuniformity of the radial velocity gives rise to the nonanalytic behaviour of the conical velocity in the stagnation point and the Taylor expansion of the velocity in terms of the distance fails.

Therefore, to continue the analysis we have to make the additional assumption of irrotational flow and there may be written:

\[
\text{rot} \, \mathbf{v} = 0
\]

In spherical coordinates the three components of \( \text{rot} \, \mathbf{v} \) are:

\[
\frac{\partial}{\partial \theta} (w \sin \theta) - \frac{\partial v}{\partial \phi} = 0
\]

\[
\frac{\partial u}{\partial \phi} - w \sin \theta = 0
\]

\[
v - \frac{\partial u}{\partial \theta} = 0
\]

In a conical stagnation point where \( v \) and \( w \) vanish, the components of the rotation vector reduce, in the \( t, s \) coordinate system to:

\[
\left( \frac{\partial w}{\partial t} \right)_0 - \left( \frac{\partial v}{\partial s} \right)_0 = 0
\]

\[
\frac{\partial u}{\partial s}_0 = 0 \quad , \quad \left( \frac{\partial u}{\partial t} \right)_0 = 0
\]

The first of this set of equations is identically the same as Eq. (3.8) and that implies that the radial component of the rotation vector vanishes in a stagnation point.
Substitution of Eqs. (3.6) and (3.8) into the expressions for \( D_s \) and \( I_s \) we obtain for a conical stagnation point:

\[
D_s = 4\left(\omega^2 + (\nu^2 + 1)^2\right) \quad (3.9)
\]

\[
I_s = \omega^2 + (\nu^2 + 1)^2 - 1 \quad (3.10)
\]

where \( \nu = \frac{1}{u_0} \left( \frac{\partial T}{\partial t} \right)_0 \) and \( \omega^2 = \frac{1}{u_0} \left( \frac{\partial w}{\partial t} \right)_0 \).

From Eq. (3.9) it is obvious that \( D_s \) is always positive or zero and it may be concluded that only nodal and saddle points can exist in isentropic conical flow (Fig. 5).

With the aid of Eqs. (3.6) and (3.8) equation (3.2) may be rewritten as:

\[
\frac{dt}{ds} = \frac{\omega.s + \nu.t}{(-\nu^2 - 2).s + \omega^2.t} \quad (3.11)
\]

By means of a coordinate transformation \( t' = t'(t,s) \) and \( s' = s'(t,s) \).

Eq. (3.11) may be rewritten as:

\[
\frac{dt'}{ds'} = \frac{\lambda_1 t'}{\lambda_2 s'} \quad (3.12)
\]

Integrating of this equation yields the solution:

\[
\frac{\lambda_1}{\lambda_2} t' = \text{const.} s' \quad (3.13)
\]
Eq. (3.13) is of a similar type as Eq. (3.7) and the singular flow pattern now depends on the coefficients \( \lambda_1 \) and \( \lambda_2 \).

When the calculations, describing the coordinate transformation, are carried out the transformation appears to be a rotation in the \( t-s \) plane and the corresponding coefficients are:

\[
\lambda_{1,2} = -1 \pm \sqrt{(1+\nu^*)^2 + (\omega^*)^2} = -1 \pm \sqrt{I_s + 1}
\]  

(3.14)

From the definition of \( I_s \) (Eq. 3.10) it appears that the smallest possible value of \( I_s \) is \(-1\) so that a positive argument of the square root is assured. The behaviour of conical streamlines in the vicinity of the conical stagnation point depends on the ratio \( \lambda_1/\lambda_2 \) which is:

\[
\frac{\lambda_1}{\lambda_2} = \frac{-1 + \sqrt{I_s + 1}}{-1 - \sqrt{I_s + 1}}
\]  

(3.15)

The dependence of \( \lambda_1/\lambda_2 \) is shown in Fig. 6.

For \(-1 < I_s < 0\) we have \( 0 < \lambda_1/\lambda_2 < 1 \) and it may be concluded that the singular streamline pattern is that of a nodal point with streamlines tangent to the \( t' \)-axis.
If $I_s > 0$, $\lambda_1 / \lambda_2 < 0$, the streamlines have a hyperbolic character and a saddle point is formed. The case $I_s = 0$ must be ruled out because then the expression given in Eq. (3.13) is not the solution of the original differential equation (Eq. 3.12). As has been mentioned before for $I_s = 0$ higher order singularities may be expected and second order terms have to be retained in the analysis. An example of a second order singularity appears in the flow field of a circular cone if the angle of incidence is just equal to that particular angle of incidence at which lift-off starts (Ref. 4, 5).

We just concluded that if nodal points are present the conical streamlines are directed along the $t'$-axis. The direction of the $t'$-axis may be determined from the coordinate transformation, which gives:

$$
\frac{dt}{ds}_{t'-axis} = \frac{w^*}{-(v^*+1) + \sqrt{I_s+1}}
$$

(3.16)

The direction of the $s'$-axis may be calculated in a similar way as:

$$
\frac{dt}{ds}_{s'-axis} = \frac{w^*}{-(v^*+1) - \sqrt{I_s+1}}
$$

(3.17)

It appears that the $t'$-$s'$ system is an orthogonal coordinate system because $\left(\frac{dt}{ds}_{t'-axis}\right) \cdot \left(\frac{dt}{ds}_{s'-axis}\right) = -1$.

The position of the $t'$- and $s'$-axis in the physical plane ($t$-$s$ plane) may be elucidated by summarizing the foregoing results in the $v^* - w^*$ plane. In this plane we may distinguish between the field points which represent either nodal or saddle points. The separation line between the region where nodal points occur and that of saddle points is given
by Eq. (3.10) since on that line \( I_S = 0 \). Hence it follows that the transition from a nodal-to a saddlepoint or vice-versa takes place through a second-order singularity.

The \( I_S = 0 \) curve is a circle of radius 1 in the \( v^* - w^* \) plane (Fig. 7).

The minimum value of \( I_S \) (\( I_S = -1 \)) occurs at \( v^* = -1 \) and \( w^* = 0 \).

The interior of the circle represents all nodal points and necessarily the saddle points are found in the outer region. All particular conical stagnation points, appearing in the problem of a supersonic flow around a circular cone at angle of incidence, have their image in the \( v^* - w^* \) plane on the \( v^* \)-axis.

The direction of the \( t' \)-axis, given in Eq. (3.16), may easily be interpreted by introducing the variables:

\[
\begin{align*}
v^* + 1 &= q^* \cos \omega \\
q^* &= q^* \sin \omega
\end{align*}
\]  

(3.18)

with \( q^* > 0 \) and \( 0 < \omega < 2\pi \).

If the angle between the \( t' \)-axis and \( s \)-axis (Fig. 8) is denoted by \( \alpha \) then Eq. (3.16) yields:

\[
\tan \alpha = \frac{q^* \sin \omega}{-q^* \cos \omega + q^*}
\]  

(3.19)

Because \( q^* \) may be interpreted as \( \sqrt{I_S + 1} \) we may conclude that the position of the \( t' \)-axis in the physical plane is independent of \( I_S \).
Because the angle $\alpha$ is independent of $I_s$, the axes $t'$ and $s'$ conserve their positions if at the same value of $\omega$ a nodal point changes into a saddle point and vice versa.

Fig. 9 Singular flow pattern at $D_s > 0$ and $u_o > 0$

For different values of $\omega$ the results are summarized in Fig. 9. The particular case $D_s = 0$ occurs in the center of the circle $I_s = 0$. In the center $I_s = -1$ and the ratio $\lambda_1/\lambda_2$ is equal to 1. According to Eq. (3.13) the streamlines are straight lines through the origin and a starlike node is formed. This starlike node was already sketched in Fig. 5 (case $m=1$).
The question in which direction the flow moves along a conical streamline may be answered by considering the sign of the velocity on the $t'$-axis. For a certain value of $\omega$ we have found:

$$v^H = \sqrt{1 + I_s} \cos \omega - 1$$

$$w^H = \sqrt{1 + I_s} \sin \omega$$

Because the $t'$-axis is given by $\cot \frac{1}{2} \omega = t/s$ there is obtained on the $t'$-axis:

$$\frac{v}{u_o} = \left( \sqrt{1 + I_s} \cos \omega - 1 \right) \cot \frac{1}{2} \omega + \sqrt{1 + I_s} \sin \omega \right) \cdot s$$

$$\frac{w}{u_o} = \left[ \sqrt{1 + I_s} \sin \omega \cot \frac{1}{2} \omega - 1 - \sqrt{1 + I_s} \cos \omega \right] \cdot s$$

If the velocity in the direction of the $t'$-axis is denoted by $q'$ there may be found:

$$\frac{q'}{u_o} = \left( \sqrt{1 + I_s} - 1 \right) t'$$

(3.20)

If $u_o$ is a positive quantity then Eq. (3.20) yields that in case of a nodal point ($I_s < 0$) $q' < 0$ and for a saddle point ($I_s > 0$) $q' > 0$ $t'$ is positive. The flow directions are indicated in Fig. 9 and must be reversed if $u_o$ is negative.

3.4. STAGNATION POINTS ON A CONICAL BODY.

In the previous paragraph the singular flow pattern in the vicinity of conical stagnation points was analysed in general. If the conical stagnation point is located on the boundary of a conical body it is interesting to investigate which additional constraints must be taken into account and how they affect the possible streamline pattern. Assume again that a conical stagnation point exists in the origin of a $t$-$s$ coordinate system, but that it is now located at the surface of a conical body. In the neighbourhood of the origin, the surface of the
body is defined in the $t,s$ system by $t=f(s)$ (Fig. 10).

On the body surface the boundary condition of tangent flow must be satisfied. If $V$ is the conical velocity vector and the function $T = t - f(s) = 0$ describes the body surface then the boundary condition reads:

$$V \cdot \nabla T = 0 \text{ on } T = 0$$

or

$$v - w \frac{df}{ds} = 0 \quad (3.21)$$

Fig. 10

Expanding the velocity components $v$ and $w$ in a Taylor series we obtain on $t = f(s)$:

$$\frac{v}{u_0} = \frac{v^*}{u_0} f + \frac{v}{u_0} s + \frac{v}{u_0} t \frac{t^2}{2} + \frac{v}{u_0} t s + \frac{v}{u_0} s^2 + \ldots$$

$$\frac{w}{u_0} = \frac{w^*}{u_0} f - (v^* + 1) s + \frac{w}{u_0} t \frac{t^2}{2} + \frac{w}{u_0} t s + \frac{w}{u_0} s^2 + \ldots$$

The contour of the conical body may be expanded as:

$$f = (f')_0 s + (f'')_0 \cdot \frac{s^2}{2} + \ldots$$

Substituting these expressions in the boundary condition and equating terms of the same order we find:

$$w^* (1 - (f')_0)^2 + (v^* + 1) 2 (f')_0 = 0 \quad (3.22)$$

From the previous equation we deduce that the slope of the body surface at a conical stagnation point depends only on the angle $\omega$ defined in Eq. (3.18). Introduction of the variable $\omega$ in Eq. (3.22) yields:

$$\tan \omega = \tan(\omega + \pi) = \frac{-2 f'}{1 - (f')^2}$$
If we denote by $f' = \tan \beta$ the local slope of the cone surface, the next two solutions of $\omega$ may be written:

$$\omega_1 = -2\beta$$
$$\omega_2 = -2\beta - \pi$$

Because one value of $\omega$ corresponds to one direction of the $t'$-axis we may conclude that a given slope of the body surface corresponds to two possible directions of the $t'$-axis. Denoting the angle of the $t'$-axis with the $s$-axis by $\alpha$, we may write:

$$\tan \alpha_1 \cdot \tan \beta = -1$$
and
$$\tan \alpha_2 \cdot \tan \beta = +1$$

where the subscripts 1 and 2 indicate the two solutions. These two equations point out that in cases where the conical stagnation point is a nodal point the conical streamlines can only reach the stagnation point normal or tangent to the body surface.

A similar conclusion may be drawn for a saddle point, because then one asymptotic streamline is tangent to the body surface while the other is perpendicular to this surface.

From experiments it is known that the flow around a circular cone at high incidences separates at the leeward side of the cone surface. A streamline may be observed, dividing an outer "inviscid" flow field and a flow regime where a "conical" vortex may be distinguished. (Fig. 11) It is evident that the last flow regime is strongly influenced by viscous and possibly entropy effects.

Ref. 6.

Fig. 11
A way of modelling the vortex flow would be not to consider the generation of the vortex which occurs in the layer close to the body surface, but to analyse the resulting vortex behaviour by means of the inviscid flow equations.

The foregoing analysis suggests that the dividing streamline is perpendicular to the cone surface, provided that the separated streamline starts on the body in a stagnation point.
4. PRESSURE DISTRIBUTION IN THE VICINITY OF CONICAL STAGNATION POINTS.

Considering the equations for \( \theta \)- and \( \phi \)-momentum (Eqs. (2.3) and (2.4)) in a stagnation point it may be seen that the pressure derivatives \( \frac{\partial p}{\partial \theta} \) and \( \frac{\partial p}{\partial \phi} \) both vanish. The lines of constant pressure (isobars) on the unit sphere are determined by the differential equation:

\[
\frac{d\theta}{d\phi} \text{ p=const.} = -\frac{\frac{\partial p}{\partial \phi}}{\frac{\partial p}{\partial \theta}}
\]  
(4.1)

and consequently, in a conical stagnation point, the pressure distribution shows a singular behaviour. On the other hand not every singular point in the pressure distribution corresponds necessarily to a conical stagnation point.

An example of this fact may be found in the leeward symmetry plane of a cone at high angle of incidence in a supersonic flow. In this chapter special attention is given to the correspondence between the pressure distribution and the conical streamline pattern.

In order to analyse the pressure distribution we expand the pressure derivatives in the \( t,s \)-coordinate system to first order:

\[
p_\theta = (p_{tt})_o \cdot t + (p_{ts})_o \cdot s
\]

and

\[
\frac{1}{\sin \theta_o} p_\phi = (p_{st})_o \cdot t + (p_{ss})_o \cdot s
\]

from which there is obtained:

\[
\frac{dt}{ds} \text{ p=const.} = -\frac{(p_{ss})_o \cdot s - (p_{st})_o \cdot t}{(p_{ts})_o \cdot s + (p_{tt})_o \cdot t}
\]

(4.3)

where the subscripts denote derivatives with respect to \( t,s \).

The differential equation (4.3) may be treated in a way similar to that of Eq. (3.2); the behaviour of integral curves (isobars) is completely determined by the quantities:

\[
D_p = \Delta (p_{ss} p_{tt} - p_{st}^2)
\]

(4.4)

and
\[ I_p = - (p_{ss} p_{tt} - p_{st}^2) \]  

(4.4)

From Eq. (4.4) the simple but important relation follows:

\[ I_p = \frac{1}{4} D_p \]  

(4.5)

From Eq. (4.4) follows that \( I_p \) and \( D_p \) have always the same sign, which implies that when \( D_p < 0 \) the constant pressure lines can only form a saddle point. If \( D_p > 0 \) the possibility of a center, or spiral point exists in principle, however, in general, \( p_{ts} = p_{st} \) so that the spiral point is ruled out. Hence we may conclude that for \( D_p > 0 \) the pressure distribution exhibits a center point only.

Next we want to find a relation between the quantities \( I_p \) and \( I_s \) to obtain more information about the expected correspondence between the streamline pattern and the pressure distribution. For that purpose the higher pressure derivatives may be expressed as a function of the velocity derivatives in the stagnation point.

If the equations for \( \theta \)- and \( \phi \)-momentum are differentiated with respect to \( \theta \) and \( \phi \) we obtain in a conical stagnation point:

\[ \frac{\partial^2 p}{\partial \theta^2} = - \rho_o \left[ \left( \frac{\partial v}{\partial \theta} \right)^2 + \frac{\partial w}{\partial \theta} \frac{\partial v}{\partial \phi} \frac{1}{\sin \theta_o} + u_o \frac{\partial v}{\partial \phi} \right] \]

\[ \frac{\partial^2 p}{\partial \theta \partial \phi} = - \rho_o \frac{\partial v}{\partial \phi} \left[ \frac{\partial v}{\partial \theta} \frac{\partial w}{\partial \phi} \frac{1}{\sin \theta_o} + u_o \right] \]  

(4.6)

\[ \frac{\partial^2 p}{\partial \phi^2} = - \rho_o \sin \theta_o \left[ \frac{1}{\sin \theta_o} \left( \frac{\partial w}{\partial \phi} \right)^2 + \frac{\partial v}{\partial \phi} \frac{\partial w}{\partial \theta} + u_o \frac{\partial w}{\partial \phi} \right] \]

Then, expressing Eq. (4.6) in the \( t,s \)-system and using the Eqs. (3.6) and (3.8) we achieve:

\[ p_{tt} = - (\nu^2 + v^* + w^*) \]

\[ p_{ts} = w^* \]

\[ p_{ss} = - (v^* + 3 v^* + 2 + w^* ) \]  

(4.7)
where
\[ p^* = \frac{p}{\rho_o u_o^2} \]

It appears that the second order pressure derivatives depend only on the quantities \( v^* \) and \( w^* \).

Substitution of Eq. (4.7) in the relation for \( D_p \) yields us:
\[ D_p = -4 \left[ \frac{\omega^*}{\omega^* + (v^* + 1)^2} \right] \left[ \frac{\omega^*}{\omega^* + (v^* + 1)^2} \right] \]
\[ (4.8) \]

If both expressions between square brackets are eliminated by using Eq. (3.10) the following equation may be found:
\[ D_p = -4 I_s (I_s + 1) \]
\[ (4.9) \]

and also from Eq. (4.5):
\[ I_p = -I_s (I_s + 1) \]
\[ (4.10) \]

In the previous chapter it is shown that the behaviour of conical streamlines depends on the sign of the quantity \( I_s \) (nodal points if \( I_s < 0 \), saddle points if \( I_s > 0 \)).

Eq. (4.10) shows that \( I_p \) (and also \( D_p \)) only depends on \( I_s \), see Fig. 12. Because \(-1\) is the smallest possible value of \( I_s \), it appears that the signs of \( I_s \) and \( I_p \) are always opposite.

This means that, in general, the following conclusions may be drawn:
(i) A nodal point of streamlines corresponds to a saddle point of isobars, and
(ii) a saddle point of streamlines corresponds to a centre point of isobars.

The behaviour of the isobars in the vicinity of a stagnation point depends on the sign of the quantity \( I_p \). If \( I > 0 \) the isobars form a
saddlepoint and for \( I_p < 0 \) a centre point occurs. Because \( I_p \) only depends on \( I_s \) it may be noted that according to Eq. (3.15) the quantity \( I_p \) determines the value of the exponent in the streamline equation (Eq. 3.13) but it gives no information about the direction in which conical streamlines approach the conical stagnation point. To elucidate this point it is useful to consider the pressure distribution in some detail. In the case of a saddle point of isobars \((I_p > 0)\) the asymptotic directions of this saddle point are given by:

\[
\frac{dt}{ds \text{ asymp.}} = \frac{p^*_{ss}}{p^*_{st} + \sqrt{I_p}}
\]

Elimination of the quantities \( p^*_{ss} \), \( p^*_{st} \) and \( I_p \) from Eqs. (4.7) and (4.10) gives:

\[
\frac{dt}{ds \text{ asymp.}} = \frac{\sqrt{I_s + 1} + \cos \omega}{\sin \omega + \sqrt{-I_s}}
\]  \(4.11\)

If the asymptotic directions are denoted by the angles \( \gamma_1 \) and \( \gamma_2 \) it may be derived from Eq. (4.11) that

\[
\tan \gamma_1 = \frac{\sqrt{I_s + 1} + \cos \omega}{\sin \omega + \sqrt{-I_s}}
\]

and

\[
\tan \gamma_2 = \frac{\sqrt{I_s + 1} + \cos \omega}{\sin \omega + \sqrt{-I_s}}
\]

Both directions have physical significance when the quantity \( I_s < 0 \) \((I_p > 0)\) hence, for a nodal point of conical streamlines. Because the angle \( \omega \) determines the direction in which streamlines approach the stagnation point, it is useful to eliminate \( I_s \) from the above equation to obtain \( \omega \) as a function of \( \gamma_1 \) and \( \gamma_2 \). When we do so we obtain:

\[
\tan \left(\frac{\gamma_1 + \gamma_2}{2}\right) = \frac{1 + \cos \omega}{\sin \omega} = \tan \alpha
\]  \(4.12\)

From this equation we conclude that the direction of the streamlines approaching a stagnation point in the physical plane, coincides with the bisectrix of the two asymptotes of the pressure distribution. The angle between these asymptotes varies from zero at \( I_s = 0 \) to its
maximum value ($\pi/2$) at $I_s = -1$. According to Eq. (4.12) the angle $\frac{\gamma_1 + \gamma_2}{2}$ is independent of $I_s$ (and therefore independent of $I_p$) thus the direction of the conical streamlines in a nodal point coincides with the direction of both asymptotes of the pressure distribution at $I_p = 0$.

For a saddle point of conical streamlines in a stagnation point, $I_p < 0$ and necessarily in the pressure-distribution no asymptotic directions are found. Therefore the previous identification of these directions with the direction of conical streamlines fails.

However, if the pressure distribution exhibits a centre point ($I_p < 0$) in the conical stagnation point, the second derivative of the pressure in some direction may have an extreme value in that direction. In the $t, s$-plane we introduce the polar system $n, \sigma$ (Fig. 13). The second derivative of the pressure in the $n$-direction may now be written as:

$$p^{n^*}_{nn} = -(I_s + 1) - \sqrt{I_s + 1} \cos (\omega + 2\sigma)$$

(4.13)

The derivative $p^{n^*}_{nn}$ has an extreme value if:

$$\sin (\omega + 2\sigma) = 0 \text{ or } \sigma_1 = -\frac{\omega}{2}, \quad \sigma_2 = \frac{\pi - \omega}{2}$$

Substitution of these values in the expression for $p^{n^*}_{nn}$ results in:

$$p^{n^*}_{nn_1} = -(I_s + 1) - \sqrt{I_s + 1}$$

(4.14)

$$p^{n^*}_{nn_2} = -(I_s + 1) + \sqrt{I_s + 1}$$

(4.15)

Because the smallest possible value of $I_s$ is $-1$, so it is clear that $p^{n^*}_{nn_2} \geq p^{n^*}_{nn_1}$ and the pressure has its smallest variation in the $\sigma_2$ direction. From Eq. (3.19) it may be verified that at a certain value of $\omega$ the $\sigma_2$-direction corresponds to the direction of the $t'$-axis. Therefore in general the conical streamlines will approach or leave the conical
stagnation point in a direction causing the smallest pressure decrease (Fig. 14).

\[ I_s < 0 \left( p_{n_1 n_2} > 0 \right) \]

\[ I_s > 0 \left( p_{n_1 n_2} < 0 \right) \]

Fig. 14 Conical streamlines and related pressure distribution.
5. UNIQUE DESCRIPTION OF A CONICAL FLOW PATTERN.

5.1. INTRODUCTORY REMARKS.

In the conical flow field generated by a circular cone in a supersonic stream where different conical stagnation points may be present, the type and/or location of these stagnation points may change depending on Mach number, cone half angle and incidence. In several references (Ref. 1 and 2) different parameters have been introduced describing this behaviour in a more or less appropriate way. Especially Melnik (Ref. 2) gives a detailed description of a conical flow pattern in the vicinity of conical stagnation points. In his analysis a parameter $P$, proportional to the second derivative of the pressure in $\phi$-direction occurs. The description based on this parameter $P$ gives much detailed information concerning the behaviour of the conical flow pattern. Nevertheless the introduction of $P$ gives rise to some questions about breakdown and non-uniqueness of the local solutions. With the aim of the results obtained in the previous chapters these problems will be clarified in the present chapter and a suggestion for a unique description will be given.

First an outline of Melnik's analysis will be presented in the next section.

5.2. OUTLINE OF MELNIK'S ANALYSIS.

In Ref. 2 Melnik has given a local analysis of the streamline pattern in the vicinity of conical stagnation points occurring in the conical flow field around a circular cone in a supersonic stream. In this section we want to restrict the attention to the conical stagnation point $A$ located in the leeward symmetry plane on the body surface (Fig. 15).

Melnik's analytic approach is based on the equations of rotational flow and a solution of the flow problem in the vicinity of $A$ is constructed by a coordinate expansion of these equations.
When the calculations are carried out for the inviscid flow field, as a main result, an expression describing the streamline pattern is found of the following form:

\[ t = \int_{\sigma_0}^{\sigma} N(\lambda) \lambda^\mu_1 \lambda^\mu_2 \, d\lambda \quad (5.1) \]

where

\[ t = \theta - \theta_0 \quad (5.2) \]

\[ \mu_1,2 = \frac{1}{3} \sqrt{1-P} \quad (5.3) \]

and

\[ P = \frac{4}{\rho_0 u_0^2 \sin^2 \theta_0} \left( \frac{\partial P}{\partial \phi} \right)^2 \quad (5.4) \]

Eq. (5.1) gives the dependence of \( t \) on \( \phi \) along a conical streamline (\( \sigma \text{=constant} \)). In this streamline equation two exponents \( \mu_1 \) and \( \mu_2 \) appear which depend on a parameter \( P \) and \( P \) is proportional to the curvature \( \partial^2 \phi \) of the pressure distribution along the cone.

Assuming irrotational flow, it may be shown that the parameter \( P \), and therefore the exponents \( \mu_1 \) and \( \mu_2 \), are independent on the streamline coordinate \( \sigma \).

In that case the streamline equation (Eq. (5.1)) may easily be integrated and it reduces to:

\[ t_{\text{irr.}} = G(\sigma) \phi^{\mu_1,2} \quad (5.5) \]

The functional relation \( G(\sigma) \) is left unspecified because \( G \text{=constant} \) along a conical streamline.

In Ref. 2 Melnik showed that the influence of rotationality is only quantitative and we may confine ourselves to the aforementioned
irrotational solution. It is worthwhile noting that Eq. (5.5) is very similar to Eq. (3.7). However, in Eq. (3.7) only the exponent \( m \), depending on \( \frac{1}{u_o} \left( \frac{\partial v}{\partial \theta} \right)_o \) appears, whereas in Eq. (5.5) two exponents \( \mu_1 \) or \( \mu_2 \), both depending on \( P \), are present.

The non-uniqueness of the description of the conical flow pattern is demonstrated clearly by this appearance of the exponents \( \mu_1 \) and \( \mu_2 \) in the solution, which both correspond to one value of \( P \).

If \( P > 1 \) then the exponents \( \mu_1 \) and \( \mu_2 \) take complex values and necessarily for \( P > 1 \) no real solution of the flow pattern exists.

The problems of non-uniqueness and breakdown of the local solutions may be clarified considering the properties of the pressure distribution in the vicinity of a stagnation point in more detail.

5.3. PROPERTIES OF THE PRESSURE DISTRIBUTION.

In chapter 4 of this present report we have found that the type of conical flow pattern in the vicinity of conical stagnation points depends completely on the pressure distribution.

Particularly the three second order derivatives of the pressure are necessary and sufficient in the complete description, since the first order derivatives vanish in a stagnation point.

Even the dependance of the flow field on the cross-derivative \( p_{ts}^x \) may be removed by a suitable rotation of the coordinate system. If this transformation is carried out we arrive at conical stagnation points with the additional property:

\[
\frac{1}{u_o} \frac{\partial v}{\partial s} = \frac{1}{u_o} \frac{\partial w}{\partial t} = w^x = p_{st}^x = 0
\]

In the conical flow generated by a circular cone in a supersonic flow only these conical stagnation points may be expected and therefore without loss of generality we may restrict ourselves to this type of stagnation points.

The second order derivatives of the pressure may now be written as:
\[ p_{ss}^* = -(v^* + 3v^* + 2) \]
\[ p_{st}^* = 0 \]
\[ p_{tt}^* = -(v^* + v^*) \]

The relations given in Eq. (5.6) point out that the quantities \( p_{ss}^* \) and \( p_{tt}^* \) are both negative in the region \( v^*<2, \ v^*>0 \), and they have the opposite sign for \( -2<v^*<0 \) (see fig. 16). This behaviour excludes the possibility of an absolute pressure minimum in a conical stagnation point. Also the second order derivatives of the pressure both change sign at \( v^*=-1 \) in such a way that both derivatives remain of opposite sign with respect to each other.

This means that due to a continuous variation of Mach number or angle of incidence causing a continuous change of \( v^* \), the conical flow pattern of type E (Fig. 5) always changes, through type D, into type C if \( v^* \) is decreasing (or vice versa if \( v^* \) is increasing).

From fig. 16 we also observe that one value of \( p_{ss}^* \) corresponds in general to two values of \( p_{tt}^* \) which have, depending on \( p_{ss}^* \), the same sign or the opposite sign. This phenomenon gives the possibility of two different pressure distributions at one value of \( p_{ss}^* \), which in turn give rise to two different conical flow patterns.

Because the parameter \( P \) used by Melnik is proportional to \( p_{ss}^* \) it is evident now that the parameter \( P \) cannot give a unique description of
the conical flow pattern, because the influence of the pressure
distribution in t-direction is not taken into account. The corresponding
values of \( p_{tt}^* \) determined by a given value of \( p_{ss}^* \) (and a given value of
\( P \)) may be identified with the \( \mu_1 \)- and \( \mu_2 \)-solutions occurring in Melnik's
analysis.

The question, concerning the breakdown of the local solutions at
values of \( P \) larger than 1 may easily be answered now by considering
again the second order derivatives of the pressure. The functional
relation of \( p_{ss}^* \) with \( v^* \) is a parabolic curve and \( p_{ss}^* \) reaches its
maximum value \( (\frac{1}{4}) \) at \( v^* = -3/2 \). Because \( P = 4 \cdot p_{ss}^* \), it is evident that
the parameter \( P \) never exceeds values larger than 1. Therefore
solutions for \( P > 1 \) are beyond physical reality. And the problem of the
breakdown of the local solutions of conical flow pattern for \( P > 1 \) is
an immaterial problem.

To obtain a unique description of the local flow pattern we have to
take into account the pressure distributions in both \( t \)- and \( s \)-
directions. This may be done by introducing the parameter:

\[
k = \frac{\frac{-p_{tt}^*}{p_{ss}^*}} \quad (5.7)
\]

which is suggested by the special behaviour of \( p_{ss}^* \) and \( p_{tt}^* \) (Eq. 5.6).

\[\text{Fig. 17}\]
When \( k \) is evaluated in terms of \( v^m \) and the resulting equation is identified with Eq. (3.7) it appears that \( k \) and \( m \) are identical and the sketch shown in Fig. 17 gives the desired unique description of the conical flow pattern and pressure distribution in the vicinity of a conical stagnation point.
6. CONCLUSION.

The main results obtained in the present report may be summarized as follows.

- In isentropic conical flow conical stagnation points may occur; the corresponding flow pattern in the vicinity of such points either appears as a node or as a saddle.

- If a conical stagnation point is located on the boundary of a conical body, the streamlines of a nodal point are normal or tangent to the body surface. The same is true for the asymptotic directions of a saddle point on the body surface.

- There exists a strong relation between pressure distribution and conical flow pattern; in general a nodal point of streamlines corresponds to a saddle point of isobars and a saddle point of streamlines corresponds to a centre point of isobars.

- The conical streamlines approach or leave the conical stagnation point in that direction where the pressure decrease is the smallest.

- Within the scope of the present theory it is found that in a conical stagnation point the pressure cannot have a minimum value.

- The second order partial derivatives of the pressure with respect to \( \theta \) or \( \phi \) can achieve a maximum value; this case corresponds to the case \( P = 1 \) in Melnik's analysis. Therefore solutions based on higher values of those derivatives are beyond physical reality.
REFERENCES.


