An Analytical Model to Extract Wind Turbine Blade Structural Properties for Optimization and Up-scaling Studies

Turaj Ashuri
PhD Researcher
Department of Aerodynamic and Wind Energy
Delft University of Technology
t.ashuri@tudelft.nl

Michiel Zaaijer
Assistant Professor
Department of Aerodynamic and Wind Energy
Delft University of Technology
m.b.zaayer@tudelft.nl

Gerard van Bussel
Professor
Department of Aerodynamic and Wind Energy
Delft University of Technology
G.J.W.vanBussel@tudelft.nl

Gijs van Kuik
Professor
Department of Aerodynamic and Wind Energy
Delft University of Technology
G.A.M.vanKuik@tudelft.nl

Abstract

A wind turbine blade has a complex shape and consists of different elements with dissimilar material properties. To do any aeroelastic simulation, the structural properties of the blade such as stiffnesses and mass per unit length should be known in advance, and extracting these properties is a difficult task. This paper presents an analytical model to extract these structural properties in a simple way. It starts with calculating an equivalent material property of the cross section using weighting method. Then the centroid of each section is obtained. Next the second moment of inertia of each element relevant to its local coordinates system is calculated and transferred to the centroid of the section using parallel axis theorem. A coordinate transformation is employed to rotate these second moment of inertias around any arbitrary axis. Finally, flapwise and edgewise stiffnesses are found by multiplying the equivalent modulus of elasticity to the second area moment of inertia in each section. Mass per unit length is calculated by multiplying the equivalent density to the real area of each section. The method is verified with the structural properties of a commercial 660 kW wind turbine blade. Despite the simplicity of the method the results show a good agreement.

Keywords: Wind turbine blade, structural properties, analytical model

1. Introduction

The blades of a wind turbine are complex structural components to design. Orthotropic material properties, nonuniform distribution of different materials and the complex shape of the blade are the most important reasons for this complexity.

To model this complexity, usually FE models are employed. These models are of value in investigating the distribution of strains and stresses within the blade, thus suitable for the final design stage [1]. They are very detailed for an aeroelastic design optimization, which in general only requires mass and stiffness distribution along the blade.

To avoid FE models, many design optimization methods use either a linear elastic I-beam model for the blade [2], or a prescribed mathematical distribution of design variables along the blade [3]. These models can easily represent the stiffness and mass distribution of the blade, but to ensure their dependability they should be tuned with measurements. Despite their simplicity, they need model verification which is the main drawback, particularly for upscaling studies where there is no data available to do any proper tuning.

This paper provides a new methodology that enables the extraction of structural properties of a composite wind turbine blade, while avoiding model verification or tuning and yet simple to implement. This new methodology is based on an analytical model of the composite blade.

It starts with calculating the second area moment of inertia of different structural elements (shear webs, spar caps and shell) of a blade section to cope with the geometrical complexity. Generally, each structural element has a directional property. That is, the elastic constants depend on the orientation. Since each element consists of different materials with different thicknesses, the weighting method is used to find the equivalent directional properties of each section.

Clearly, the complicated nonuniform distribution of materials can be represented by the bulk representation that has equivalent properties as the original. The actual thickness of each element is maintained, since it is used to extract weights. Combining the contribution of different elements, the bending centroid of each section is calculated.

Parallel-axis theorem is used to move each second moment of inertia to the section centroid. An intermediate step, transform each sectional property to any rotated coordinates system, using a rotation matrix. This enables the extraction of properties in any arbitrary plane.

Stiffnesses are found by multiplying each element equivalent modulus to its second moment of inertia and integrated over all elements. Mass per unit length of each section is found by the same strategy. That is, the weighting method is applied on the density of each material, and using equivalent density of each element multiplied by its area, the mass per unit length is calculated. Other properties are also calculated in the same manner.

The methodology is verified with the sectional structural properties of a commercial 660 kW wind turbine blade over 20 stations along the blade. Good agreement is found between our model and the available data of the 660 kW wind turbine blade.

This new methodology can also be used in up-scaling studies, in which there is a need to have a parametric model for design optimization at different scales.

2. Methodology

In any wind turbine blade design study, structural properties (area and mass moment of inertia, stiffness, strength, radius of gyration, mass per unit length and …) are needed. These properties are used to calculate the mass of the blade, natural frequencies, deflections, buckling, stresses, fatigue and so on.

This section explains how to extract these properties for any arbitrary cross section of the blade. It starts with explaining different sectional definitions and assumptions made for the rest of the work. Then, based on these assumptions different sectional properties relative to the principal axis are calculated. Finally, using a coordinate rotation matrix, these properties can be transformed to any arbitrary plane.

2.1 Sectional description and assumptions

Usually a wind turbine blade cross section consists of two parallel shear webs located on any point along the chord, two spar cups atop and under the shear webs and a shell that surrounds the section form leading-edge to the trailing-edge, passing over the spars.

To extract the structural properties for such a section, certain assumptions based on Euler-Bernoulli theory are made [4]. These assumptions are as follow:

- The strain is proportional to the distance from the neutral surface
- Deflections remain small
- Plane sections of the beam remain plane an perpendicular to the neutral axis

2.2 Sectional properties calculation

Since the cross section of a wind turbine blade consists of several elements like shell and shear webs that have different material properties, the weighting method is used to extract the equivalent properties of each section.

This helps to represent the complicated nonuniform distribution of materials with a gross representation that has equivalent properties as the original [5]. The actual thickness of each element is maintained, since it is used to extract weights, see figure 1.

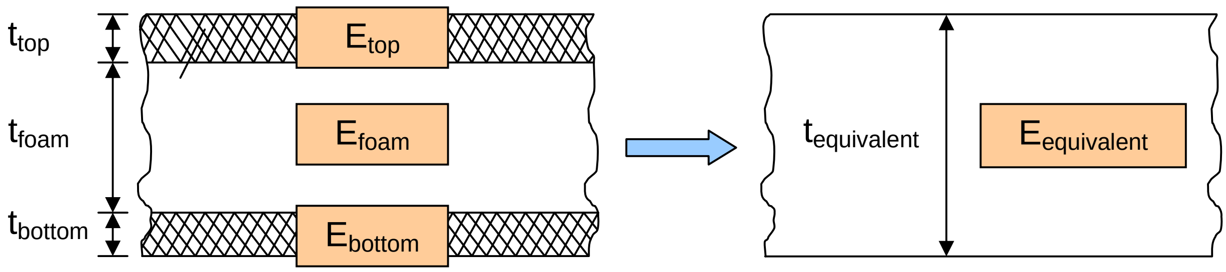

Figure 1: Equivalent representation of properties

For instance the equivalent module of elasticity of the shell, $E^{shell}_{equivalent}$ consisting of some ±45° layers of composite in top, t^{shell}_{top} and bottom, t^{shell}_{bottom} that is filled with foam having a modulus of elasticity of E^{shell}_{foam} and thickness of t^{shell}_{bottom} in between (figure 1) can be calculated as:

$$E^{shell}_{equivalent} = \frac{\sum_{m=1}^{n} E_m t_m}{\sum_{m=1}^{n} t_m} = \frac{\left(E^{shell}_{top} t^{shell}_{top} + E^{shell}_{bottom} t^{shell}_{bottom} + E^{shell}_{foam} t^{shell}_{foam}\right)}{\left(t^{shell}_{top} + t^{shell}_{bottom} + t^{shell}_{foam}\right)} \tag{1}$$

$$t^{shell}_{equivalent} = t^{shell}_{top} + t^{shell}_{foam} + t^{shell}_{bottom} \tag{2}$$

However, the same approach can be used to extract equivalent densities for each element of the cross section, by simply replacing the modulus of elasticity with the density.

Section centroids in chordwise, $\bar{X}^{section}_{c}$ and flapwise direction, $\bar{Y}^{section}_{c}$ are calculated as follow:

$$\bar{X}^{section}_{c} = \frac{\left(E^{shell}_{equivalent} A^{shell}_{equivalent} \bar{X}^{shell}_{c} + E^{spar}_{equivalent} A^{spar}_{equivalent} \bar{X}^{spar}_{c} + E^{web}_{equivalent} A^{web}_{equivalent} \bar{X}^{web}_{c}\right)}{\left(E^{shell}_{equivalent} A^{shell}_{equivalent} + E^{spar}_{equivalent} A^{spar}_{equivalent} + E^{web}_{equivalent} A^{web}_{equivalent}\right)} \tag{3}$$

And:

$$\bar{Y}^{section}_{c} = \frac{\left(E^{shell}_{equivalent} A^{shell}_{equivalent} \bar{Y}^{shell}_{c} + E^{spar}_{equivalent} A^{spar}_{equivalent} \bar{Y}^{spar}_{c} + E^{web}_{equivalent} A^{web}_{equivalent} \bar{Y}^{web}_{c}\right)}{\left(E^{shell}_{equivalent} A^{shell}_{equivalent} + E^{spar}_{equivalent} A^{spar}_{equivalent} + E^{web}_{equivalent} A^{web}_{equivalent}\right)} \tag{4}$$

These section centroids are used to transfer the area moments of inertia that are calculated relevant to each element centroids to the section centroids using parallel-axis theorem, which will be afterwards explained in this section.

Area moments of inertia can simply be calculated using an integration scheme. In this scheme, the cross section (X, Y) coordinates of the shell, spars and webs are separately used to find the area moment of inertia relevant to their local axis. However, with the same integration scheme, these integrals also need to be calculated for other elements like spar and web. These integrals for the shell are as follow:

$$I_{xx}^{shell} = \int y_{shell}^2 dxdy \tag{5}$$

$$I_{yy}^{shell} = \int x_{shell}^2 dxdy \tag{6}$$

$$I_{xy}^{shell} = \int x_{shell} y_{shell} dxdy \tag{7}$$

Now the calculated area moments of inertia that are calculated relative to their local centroids need to be transferred to the section centroids using the parallel-axis theory [5]. For the shell, it is as follow (note the difference between shell and SHELL in the equation):

$$I_{xx}^{SHELL} = I_{xx}^{shell} + A_{shell}\left(\bar{X}_c^{shell} - \bar{X}_c^{sec\,tion}\right)^2 \tag{9}$$

$$I_{yy}^{SHELL} = I_{yy}^{shell} + A_{shell}\left(\bar{Y}_c^{shell} - \bar{Y}_c^{sec\,tion}\right)^2 \tag{10}$$

$$I_{Xy}^{SHELL} = I_{Xy}^{shell} + A_{shell}\left(\bar{X}_c^{shell} - \bar{X}_c^{sec\,tion}\right)\left(\bar{Y}_c^{shell} - \bar{Y}_c^{sec\,tion}\right) \tag{11}$$

These area moments of inertia can be rotated around any arbitrary axis. This is a necessary step to take when one wants to use the in-plane and out-of-plane values instead of flapwise and edgewise and can be done using the following transformation [5]:

$$I_{x'x'}^{SHELL} = \frac{I_{xx}^{SHELL} + I_{yy}^{SHELL}}{2} + \frac{I_{xx}^{SHELL} - I_{yy}^{SHELL}}{2} cos\,2\theta - I_{xy}^{SHELL}\,sin\,2\theta \tag{12}$$

$$I_{y'y'}^{SHELL} = \frac{I_{xx}^{SHELL} + I_{yy}^{SHELL}}{2} - \frac{I_{xx}^{SHELL} - I_{yy}^{SHELL}}{2} cos\,2\theta + I_{xy}^{SHELL}\,sin\,2\theta \tag{13}$$

$$I_{x'y'}^{SHELL} = \frac{I_{xx}^{SHELL} - I_{yy}^{SHELL}}{2} sin\,2\theta + I_{xy}^{SHELL}\,cos\,2\theta \tag{14}$$

Based on the calculated flapwise area moment of inertia around the section centroid and the calculated section equivalent modulus of elasticity in flapwise direction for all different elements, section flapwise stiffness can be calculated using the following equation:

$$E_{total}^{sec\,tion} I_{xx}^{sec\,tion} = E_{equivalent}^{shell} I_{xx}^{SHELL} + E_{equivalent}^{spar} I_{xx}^{SPAR} + E_{equivalent}^{WEB} I_{xx}^{WEB} \tag{15}$$

Using the same strategy the stiffness in the edgewise direction can also be calculated.

For the case of calculating the torsional stiffness following replacement in the above equations needs to be done:

- Modulus of elasticity with modulus of rigidity
- Flapwise or edgewise second moment of inertia with polar moment of inertia

Mass distribution calculation along the blade can be simply done using the following equation:

$$\left(\frac{M}{L}\right)_{sec\,tion} = \left(\rho_{equvalent}^{shell} A_{equivallent}^{shell} + \rho_{equvalent}^{spar} A_{equivallent}^{spar} + \rho_{equvalent}^{web} A_{equivallent}^{web}\right) \tag{16}$$

Where ρ is the density and L the length of each section.

3. Analysis and results

A 660 kW wind turbine blade which all structural properties are known is selected for the verification. Three representative structural properties of the blade, flapwise and edgewise stiffness, and mass per unit length of 20 stations along the blade are used. However, the same level of accuracy has been found in comparing other properties (torsional stiffness, radius of gyration and …), but only three of them are presented in this study.

Figure 2 shows the nondimensional flapwise stiffness distribution of the blade. However, due to the confidentiality their nondimensionalized values are presented. As the figure shows, there is a good agreement between the calculated flapwise stiffness using this analytical model and the real data.

Results of the edgewise stiffness are presented in figure 3. Comparing the accuracy of the edgewise and flapwise stiffness, the edgewise stiffness shows more discrepancy between the analytical model and the 660 kW wind turbine blade data. This is due to the manufacturing modifications applied on the leading and trailing edges of the commercial blade, while in the analytical model the X-Y coordinates of the airfoil are used for calculating the stiffness. This manufacturing modification has more influence on the edgewise stiffness than the flapwise stiffness, since they are more away from the bending centroid in the edgewise direction.

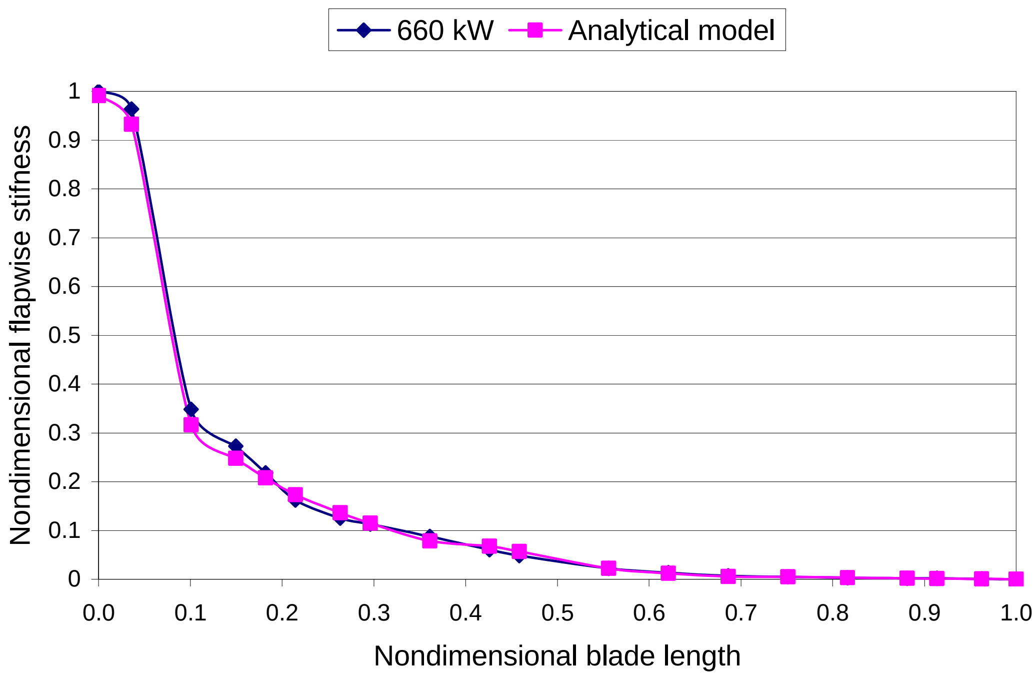

Figure 2: Comparison of flapwise stiffness of the analytical model with the 660 kW wind turbine blade

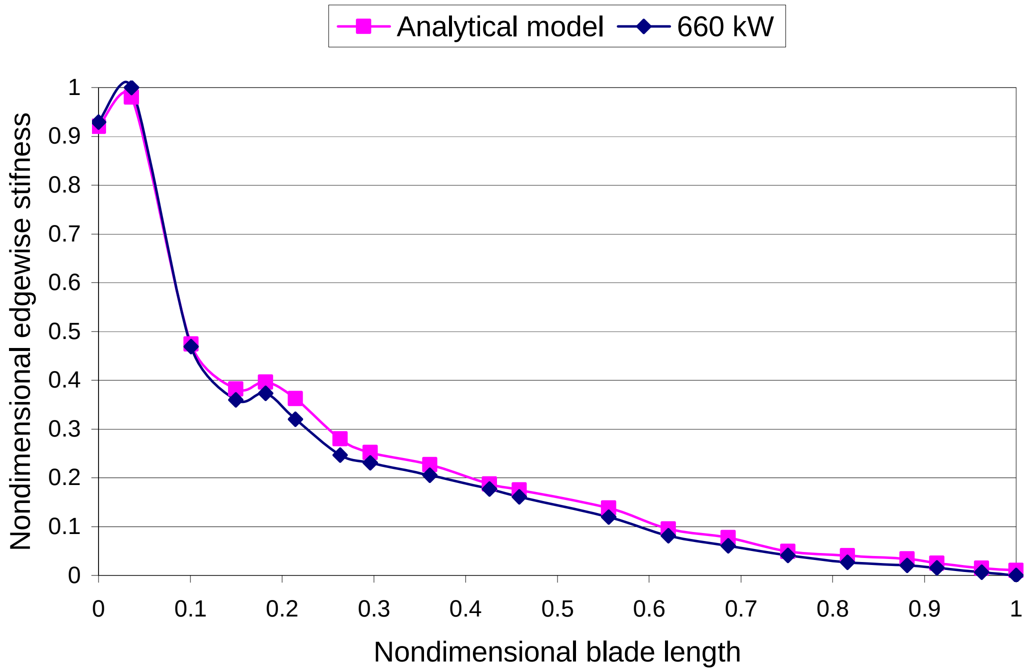

Figure 3: Comparison of edgewise stiffness of the analytical model with the 660 kW wind turbine blade

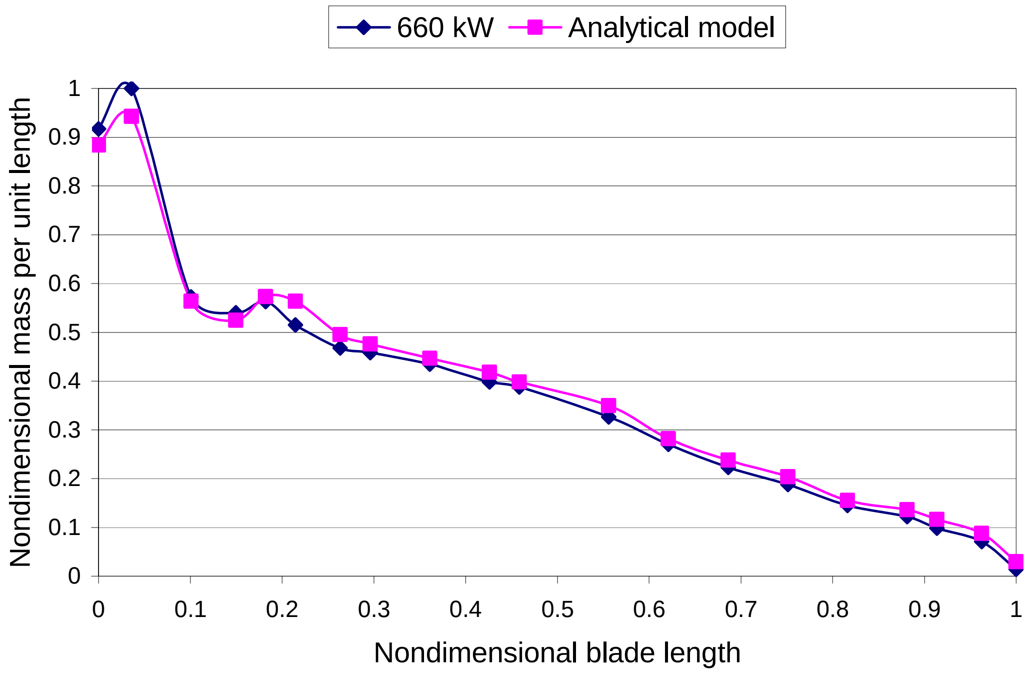

Figure 4: Comparison of mass per unit length of the analytical model with the 660 kW wind turbine blade

The comparison of the mass per unit length is presented in figure 4. As the figure shows, the predicted values from the analytical model are very close to the values of the 660 kW wind turbine blade.

For different stations along the blade, our analytical model needs the blade geometry, the thickness of composite laminates including related material properties as inputs. The geometry should be given in terms of chord, twist, aerodynamic thickness and airfoil X-Y coordinates along the blade. Since, it is a parametric code it allows any parameter such as thicknesses and the location of the shear webs to be introduced as design variable in a design optimization study. The code is very fast and runs in a fraction of a second, and requires only a little knowledge of composites materials used in wind turbine blades.

4. Conclusion

Despite the simplicity of the approach, it provides good results. However, there is no bend-twist coupling that makes the method less accurate for detailed design studies. Therefore the usage of this method should be limited to up-scaling and optimization studies.

Due to the analytical approach of the method, it is an excellent tool for blade's internal layout optimization as well. This can be done for example by playing with the location and the number of shear webs along the chord or adding ribs to see the influence of that on the stiffness and mass, thus the associated cost.

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References

[1] D. J. Malcolm, D L. Laird, "Extraction of Equivalent Beam Properties from Blade Models", Wind Energy Journal 2007, 10:135–157

[2] P. Fuglsang, H.A. Madsen, "Optimization method for wind turbine rotors", Journal of Wind Engineering and Industrial Aerodynamics 1999, 80:191-206

[3] E. Benini, A. Toffolo, "Optimal design of horizontal-axis wind turbines using blade-element theory and evolutionary computation", Journal of Solar Energy Engineering, 2002, 24(11): 357-363

[4] O. A. Bauchau , J. I. Craig, "Structural analysis", Springer publication, 2009

[5] F.P. Beer, E.R. Johnston, "Mechanics of Materials", McGraw-Hill publication, 2006