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Limit State Imprecise Probabilistic Analysis in Geotechnical Engineering

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Abstract. The treatment of imprecision attracted continuous interest since the origin of probability as the science of uncertainty. As a matter of fact, probability has been developed to deal with uncertainty and the first argument supporting precise rather than imprecise probability counts on the nature of ordinary measurements. Starting from only a few papers wherein imprecise probability has been explored as a marginal alternative to precise probability, a number of devoted works appeared with intensified frequency in the second half of the 20th century. From first developments, imprecise probability emerged in the field of engineering by different approaches. The key feature consists in the identification of probability bounds for scenarios of interest with extended application in model validation, provided the ingredients for a systematic investigation of sensitivities. In this way, regardless of the increased interest in probabilistic methods within the geotechnical engineering community, unresolved issues may comprehend a discrepancy in reliability studies regarding the frequency of failure whereas enough tests are very seldom available for a definite choice of the best statistical model. A mixed approach that admits imprecise information as well as probabilistic information is therefore desirable. In this paper the conventional probabilistic approach to uncertainty is extended to include imprecise information in the form of intervals. For demonstration, results are provided for a strip spread foundation designed by the Eurocode 7 methodology, wherein the shear strength parameters of the foundation soil are implemented as intervals and then combined with other uncertain parameters in the form of random variables under dependence.

Keywords. uncertainty; imprecise probabilistic analysis; geotechnical engineering

1. Introduction

Imprecise probability does not exist! is the epigraph of a recent review of Vicig and Seidenfeld (2012) about the mathematician Bruno de Finetti and imprecision. The review is focused on influential ideas for the theoretical development of imprecise probabilities in the form of fundamental contributions, while notices some critical remarks about future prospects, given the limited development of imprecise probability theory over the first thirty years of Bruno de Finetti's writings. Actually, the idea of replacing one exact probability value by introducing an indecision interval with two different exact one-sided values as endpoints is discussed by arguments in the sphere of existence and consistency of the indecision interval, and an evidence-based precise two-sided probability is then proposed in the form of a coherent approach to practical examples. From this point of view, imprecision does not open a space to a new uncertainty measure, but only translates incomplete knowledge.

Beer and Kreinovich (2013) further analysed the appreciation of a representative interval or moments in situations characterised by scarce information due to limited observation. In these cases, engineering systems are primarily related to a context dependent truth and the reflective engineering practice is based on subjective judgement. From a comparative view, the interval representation is considered more informative whenever normal distributions are approachable by typical confidence levels, but for heavy tailed distributions or higher levels of confidence, the moments representation prevails. Under these circumstances, a mixed approach gathering imprecise information in the form of intervals may be jointly explored with standard probabilistic information, noted that sometimes no information apart from bounds is available.

In recent years, a considerable number of papers devoted to imprecise probabilistic analysis have been published, namely the study of Beer et al. (2013) for a reliability analysis with scarce information in a geotechnical engineering context, or the geotechnical applications of

Oberguggenberger and Fellin (2008) for determination of reliability bounds and the slope analysis of Rubio et al. (2004) by using probabilistic and imprecise information. For demonstration, results are provided for a strip spread foundation designed by the Eurocode 7 methodology, wherein the shear strength parameters of the foundation soil are implemented as intervals and then combined with other uncertain parameters in the form of random variables under dependence.

2. Probability Bounds from Probability Boxes

The underlying idea in the imprecise probability theory consists in modelling an imprecise probability distribution by a set of candidate probability distributions which are derived from the available data. The family is represented through a probability bounding approach applied to specify the lower and upper bounds of the imprecise probability distribution. Thereafter, the indecision interval reflects the imprecision of the model draft derived from a set of competing intervals considered separately.

A number of set-based uncertainty models derived from the probability bounding approach have been considered, namely the probability box structure, see for instance Zhang et al. (2013) in a description of techniques for construction of a probability box. Designed from different approaches, probability boxes may differ meaningly from each other. Nonparametric approaches as the Kolmogorov Smirnov confidence limits and the Chebyshev inequality do not require a distributional assumption. A parametric approach may involve distributions with interval parameters or an envelope of competing probabilistic models, noted that the latter may be either nonparametric. From search amid the number n of candidate c cumulative distribution functions $F_c(x)$ the envelope of competing probabilistic models is expressed by the probability box function in Eq. (1):

$$\left[\underline{F}_X(x), \overline{F}_X(x)\right] = \underset{c \in [1,n]}{\min,\max} F_c(x) \; \forall x \qquad (1)$$

if $\underline{F}_X(x)$ and $\overline{F}_X(x)$ are respectively the lower and upper bounding distributions of the probability box function for $\underline{F}_X(x) \leq \overline{F}_X(x)$.

It is noted that discretisation techniques may be applied to build a probability box structure, see for instance the study of Ly and Hyman (2004) involving probabilistic discretisation in computer arithmetic under dependency, wherein the uncertainty in system response is detailed for the challenge problems in Oberkampf et al. (2004). Sliced the domain to get slices of probability, the assessment of reliability intervals under input distributions with uncertain parameters and dependencies may be pursued, see for instance the study of Hurtado (2013).

3. Design Example

The design example is referred to the strip spread foundation on a relatively homogeneous c-φ soil shown in Figure 1, wherein groundwater level is away. Considered the vertical noneccentric loading problem and the calculation model for bearing capacity, the performance function may be described by the simplified Eq. (2):

$$M = f(B, D, \gamma_s, c_f, \varphi_f, \gamma_f, P, Q) \qquad (2)$$

if B is the foundation width; D is the soil height above the foundation base; γ_s is the unit weight of the soil above the foundation base; c_f is the cohesion of the foundation soil; φ_f is the friction angle of the foundation soil; γ_f is the unit weight of the foundation soil; P is the dead load; and Q is the live load.

Table 1 summarises the description of basic input variables, with different distributions. The considered coefficients of correlation between the basic input variables are either presented in the correlation matrix shown in Table 2.

The strip spread foundation is designed by the Eurocode 7 methodology, Design Approach DA.2*, noted that the imprecise probabilistic analysis is performed in a set of scenarios wherein the shear strength parameters of the foundation soil are implemented as intervals, but separately in the format of a conditional analysis.

4. Results and Discussion

A limit state function histogram in probabilistic scenario is presented in Figure 2, supplemented by the summary description of reliability estimates in probabilistic scenario detailed in Table 3. Considered that the cohesion and friction angle parameters of the foundation soil are implemented as intervals but separately in the format of a conditional analysis, the summary description of reliability estimates in interval scenario is detailed in Table 4, wherein results are obtained from Monte Carlo simulation and by distribution fitting. The probability box structure for the cases cohesion and friction angle interval scenario, wherein results are obtained from Monte Carlo simulation, is lastly sketched in Figure 3 and Figure 4, respectively. Comments on this basic information are added hereafter.

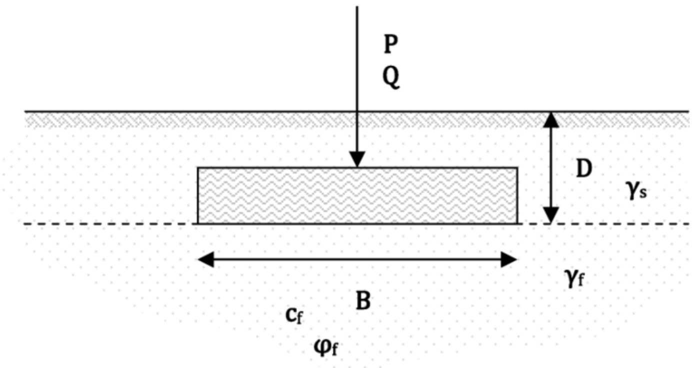

Figure 1. Strip spread foundation.

Table 1. Summary description of basic input variables.

Basic input variables	Distributions	Mean value	Coefficient of variation	Statistics
B (m)	Deterministic	-	-	1.30
D (m)	Deterministic	-	-	1.00
γ_s (kN/m^3)	Normal	16.80	0.05	-
c_f (kN/m^2)	Lognormal	14.00	0.40	μl=2.5648 σl=0.3853
	Interval	-	-	[0.00,35.00][a]
φ_f (°)	Lognormal	32.00	0.10	μl=3.4608 σl=0.0998
	Interval	-	-	[25.00,35.00][b]
γ_f (kN/m^3)	Normal	17.80	0.05	-
P (kN/m)	Normal	370.00	0.10	-
Q (kN/m)	Normal	70.00	0.25	-

μl-log mean; σl-log standard deviation.
[a]case cohesion interval scenario.
[b]case friction angle interval scenario.

Table 2. Coefficients of correlation between the basic input variables.

Correlation matrix

$$\begin{bmatrix} \rho_{x1x1} & \rho_{x1x2} & \rho_{x1x3} & \rho_{x1x4} & \rho_{x1x5} & \rho_{x1x6} \\ \rho_{x2x1} & \rho_{x2x2} & \rho_{x2x3} & \rho_{x2x4} & \rho_{x2x5} & \rho_{x2x6} \\ \rho_{x3x1} & \rho_{x3x2} & \rho_{x3x3} & \rho_{x3x4} & \rho_{x3x5} & \rho_{x3x6} \\ \rho_{x4x1} & \rho_{x4x2} & \rho_{x4x3} & \rho_{x4x4} & \rho_{x4x5} & \rho_{x4x6} \\ \rho_{x5x1} & \rho_{x5x2} & \rho_{x5x3} & \rho_{x5x4} & \rho_{x5x5} & \rho_{x5x6} \\ \rho_{x6x1} & \rho_{x6x2} & \rho_{x6x3} & \rho_{x6x4} & \rho_{x6x5} & \rho_{x6x6} \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 & 0.5 & 0.9 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.0 & 1.0 & 0.5 & 0.0 & 0.0 \\ 0.9 & 0.0 & 0.5 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

ρ-coefficient of correlation; x_1-γ_s; x_2-c_f; x_3-φ_f; x_4-γ_f; x_5-P; x_6-Q.

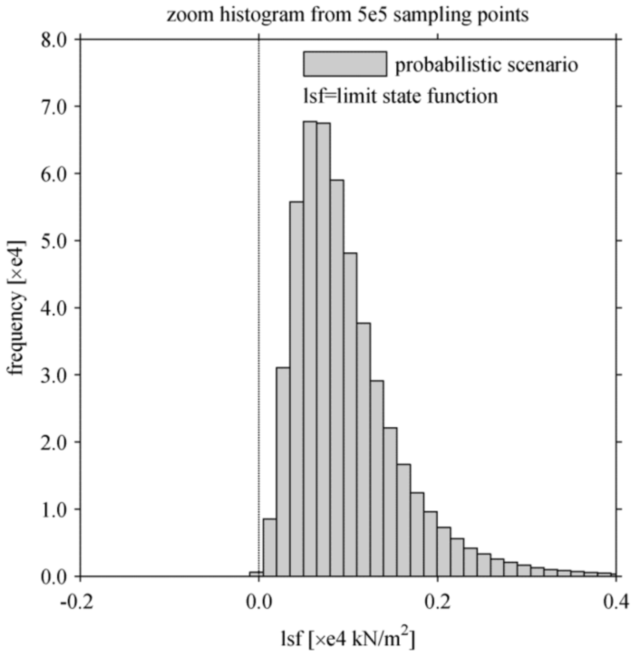

Figure 2. Limit state function histogram in probabilistic scenario.

Table 3. Summary description of reliability estimates in probabilistic scenario.

Reliability index β_{FORM}	Reliability index β_{SORM}	Reliability index β_{MCS}	β_{FORM} and β_{MCS} relative error (%)	β_{SORM} and β_{MCS} relative error (%)	MCS failure probability	MCS failure probability error (%)
3.3716	3.4392	3.4388	-1.9542	0.0102	0.0292e-2	5.2317

MCS results from 5e6 simulation steps in probabilistic scenario.

Table 4. Summary description of reliability estimates in interval scenario.

Case	Value	MCS failure probability	Reliability index β_{MCS}	FIT failure probability	Reliability index β_{FIT}
Cohesion interval scenario	0.00	4.4845e-2	1.6970	4.4800e-2	1.6974
	5.00	4.5420e-3	2.6089	5.0000e-3	2.5727
	10.00	2.0200e-4	3.5375	2.8887e-4	3.4419
	15.00	3.2000e-6	4.5127	9.1553e-6	4.2846
	20.00	≈0	≈∞	1.8173e-7	5.0872
	25.00	≈0	≈∞	2.5786e-9	5.8420
	30.00	≈0	≈∞	2.9561e-11	6.5459
	35.00	≈0	≈∞	3.0374e-13	7.1988
Friction angle interval scenario	25.00	2.2054e-3	2.8472	1.8000e-3	2.9189
	30.00	≈0	≈∞	≈0	≈∞
	35.00	≈0	≈∞	≈0	≈∞

MCS results from 5e6 simulation steps in interval scenario.
FIT results from 5e6 sampling points for the generalised extreme value distribution.

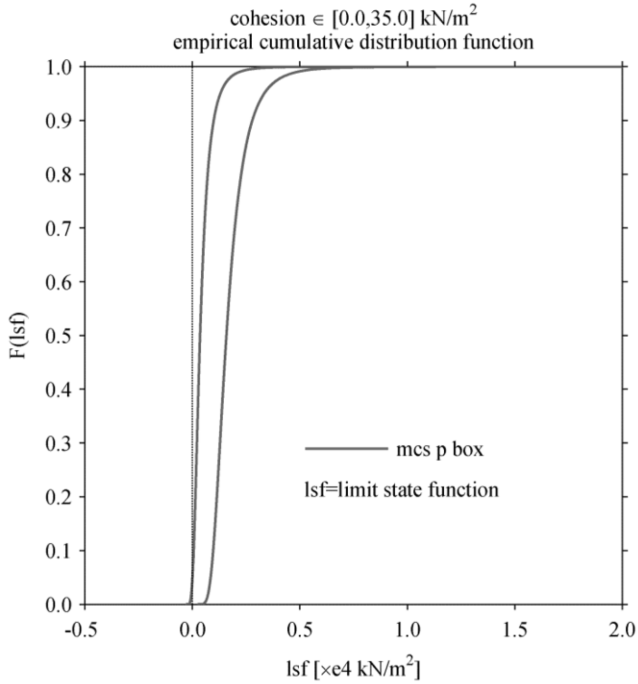

Figure 3. Monte Carlo simulation probability box for the case cohesion interval scenario.

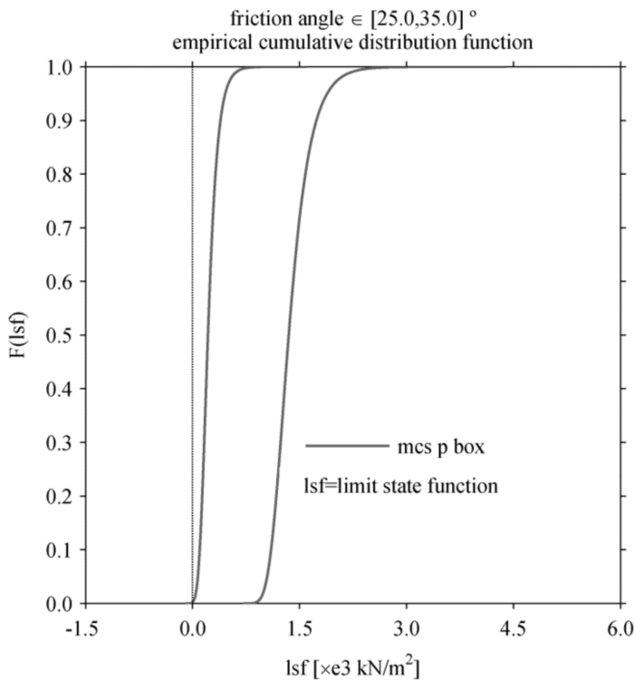

Figure 4. Monte Carlo simulation probability box for the case friction angle interval scenario.

Firstly, the behaviour of the nonlinear bearing capacity model is illustrated by the histogram in Figure 2, wherein the limit state function shows nonnormal distribution under the probabilistic scenario characterised by uncertain parameters and dependencies formerly described. Thorough observation reveals the presence of a small amount of data on the left of the zero limit state boundary, unsafe side. Thereby, failure may be considered a quite rare event among the considered 5e5 trials on nonsymmetric right skewed arrangement and with no single center.

The reliability estimates in probabilistic scenario are provided in Table 3, wherein three methods are compared: FORM, SORM and MCS, the latter from counting among 5e6 trials. Regardless of the type of function and within acceptable margin of error, the global behaviour is approachable in every case. It is further noted that neglecting the effects of dependence results in a Monte Carlo simulation higher reliability index of 3.6041, even so nonsatisfactory when compared with the Eurocodes 3.8 target ultimate limit state reliability index. In the Eurocode 7 Design Approach DA.2* partial factors are coupled with characteristic values, remarked that a cautious estimate of the 95% reliable mean value for a known coefficient of variation is considered for every geotechnical parameter. Recent experience suggests a review of the partial factors calibration to comply with the choice of characteristic values.

The reliability estimates in interval scenario are provided in Table 4 in the format of a conditional analysis for the cases cohesion and friction angle interval scenario, wherein results are obtained from 5e6 points by Monte Carlo simulation and distribution fitting. In particular, the generalised extreme value distribution is selected after a study with Kolmogorov Smirnov, Anderson Darling and Chi Squared goodness of fit tests. A comparative analysis of results from both methodologies shows minor differences with exception to the cohesion value of 15 kN/m^2, noted that for the correspondent level of reliability the Monte Carlo simulation failure probability error is estimated in about 50%. Moreover, it is clearly shown that small variations in the friction angle input are very influential in that the median and the imprecise lower bound of probability correspond to a boundless immeasurable reliability, noted that according to the Eurocodes, satisfactory levels of reliability estimates are separately attained for a cohesion of 12 kN/m^2 and for a friction angle of 26º-27º. Following the vagueness of the indecision interval in uniform discretisation, this imprecise probabilistic approach may be crossed now with a more informative limit state imprecise interval analysis for sensitivity, validation and decision. At last, the probability box structure for each case is sketched respectively in Figure 3 and Figure 4, wherein results are obtained from Monte Carlo simulation, noted the monotonicity of the limit state function.

5. Conclusion

A geotechnical engineering design example is detailed and precise versus imprecise probabilistic approaches are discussed.

References

Beer, M., Kreinovich, V. (2013). Interval or moments: which carry more information?, *Soft Computing* 17(8), 1319-1327.

Beer, M., Zhang, Y., Quek, S.T., Phoon, K.K. (2013). Reliability analysis with scarce information: comparing alternative approaches in a geotechnical engineering context, *Structural Safety* 41, 1-10.

Hurtado, J.E. (2013). Assessment of reliability intervals under input distributions with uncertain parameters, *Probabilistic Engineering Mechanics* 32, 80-92.

Li, W., Hyman, J.M. (2004). Computer arithmetic for probability distribution variables, *Reliability Engineering & System Safety* 85(1-3), 191-209.

Oberguggenberger, M., Fellin, W. (2008). Reliability bounds through random sets: non-parametric methods and geotechnical applications, *Computers & Structures* 86(10), 1093-1101.

Oberkampf, W.L., Helton, J.C., Joslyn, C.A., Wojtkiewicz, S.F., Ferson, S. (2004). Challenge problems: uncertainty in system response given uncertain parameters, *Reliability Engineering & System Safety* 85(1-3), 11-19.

Rubio, E., Hall, J.W., Anderson, M.G. (2004). Uncertainty analysis in a slope hydrology and stability model using probabilistic and imprecise information, *Computers and Geotechnics* 31(7), 529-536.

Vicig, P., Seidenfeld, T. (2012). Bruno de Finetti and imprecision: imprecise probability does not exist!, *International Journal of Approximate Reasoning* 53(8), 1115-1123.

Zhang, H., Dai, H., Beer, M., Wang, W (2013). Structural reliability analysis on the basis of small samples: an interval quasi-Monte Carlo method, *Mechanical Systems and Signal Processing* 37(1-2), 137-151.