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DOI
10.1109/CDC45484.2021.9682967

Publication date
2021

Document Version
Accepted author manuscript

Published in
Proceedings of the 60th IEEE Conference on Decision and Control (CDC 2021)

Adaptive Synchronization of Uncertain Complex Networks under State-dependent a priori Interconnections

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***Abstract*— We address a distributed adaptive synchronization problem for complex networks composed of nonlinear nodes under state-dependent *a priori* interconnections, i.e. interconnection terms acting before control design. The interconnection terms are uncertain and the heterogeneous dynamics of the network nodes further contain state-dependent uncertainty and uncertain input matrix gain. Adaptive distributed control laws are proposed to tackle such an unsolved design. The proposed controller is verified in simulation via a multi-area load frequency control for power systems.**

I. Introduction

Complex networks are used to describe multi-agent systems that can interact with each other in order to achieve a common desiblack goal, such as synchronization. Synchronization of complex networks has wide application, including cooperative vehicles [1], robotic systems [2], social networks [3], smart grids [4]. Synchronization can be achieved without [5]–[7] or with a leader [8], [9], whose dynamics can be known or unknown [10]–[12].

With the increasing number of nodes in many modern networks, it is often impractical to have every node communicate with the leader. Pinning control was thus proposed in synchronization for complex networks, where only a small fraction of network nodes is directly controlled by the leader (which is often referblack to as pinner) [13] [14]–[16]. Network nodes might have different (heterogeneous) dynamics in most situations [17], and it is known that heterogeneity and uncertainty may destabilize synchronicity [18]. Heterogeneity and uncertainty can affect both the drift terms but also the input matrix gain [11], [19]. Typical uncertainty structures in the literature include linear-in-the-parameter (LIP) structure [10], [19] or Lipschitz-like condition [14]–[16].

While uncertainty is often consideblack in node dynamics, interconnection terms acting *a priori* before control design are often overlooked [11], [19], [20]. In fact, standard literature considers *a posteriori* linear or nonlinear couplings, which are the result of the control design but are nonexistent before design. However, in many practical applications, e.g. Kuramoto dynamics in power systems [21]–[24], state-dependent interconnection terms exist a priori. In some literature, a priori interconnection is taken to be known either in linear form [14], [18], [25] or in nonlinear (typically sinusoidal, i.e. a priori bounded) form [6], [17], [26].

The main contribution of this paper is proposing a novel adaptive distributed protocol targeting the synchronization for complex networks under heterogeneity, uncertainty and state-dependent interconnections. We focus on second-order nonlinear heterogeneous dynamics with uncertainty in the drift terms and the input matrix gain; most importantly, we consider state-dependent interconnection which is also uncertain. Synchronization is shown in the uniform ultimately bounded (UUB) sense, which is in line with the few existing literature considering a priori interconnection [5], [6], [14], [17], [18], [25], [26].

The rest of this paper is organized as follows: Section II presents some basic notation and the synchronization problem we want to address. In Section III, uncertainty is analyzed and the adaptive synchronization controller is designed. A numerical simulation is given in Section IV.

II. Problem formulation

The following notation is used: I represents the identity matrix of appropriate dimension; $\|\cdot\|$ and $(\cdot)^g$ denotes the 2-norm and generalized inverse of $(\cdot)$; $\underline{\lambda}(\cdot)$ and $\overline{\lambda}(\cdot)$ are the minimum and maximum eigenvalues of a symmetric matrix.

A complex network can be described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} \triangleq (v_1, \ldots, v_N)$ is the set of N nodes (or agents) in the network and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges interconnecting the nodes. A pair of nodes (v_j, v_i) represents that agent i has access to the information from agent j, i.e. agent j is a neighbour of agent i. Accordingly, $\mathcal{N}_i$ denotes the set of the neighbors of agent i.

The topology of a weighted graph is represented by the adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ with $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. We assume there are no self-loops, that is, $a_{ii} = 0$ for $i = 1, \ldots, N$. When the graph $\mathcal{G}$ is undirected, $a_{ij} = a_{ji}$. The Laplacian matrix $\mathcal{L} = [l_{ij}]$ of $\mathcal{G}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. The augmented graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ contains the aforementioned N agents and a leader node v_0, where $\bar{\mathcal{V}} = \{v_0, v_1, \ldots, v_N\}$ and $\bar{\mathcal{E}} = \mathcal{E} \cup \{(v_i, v_0) : b_i > 0\}$, with b_i being the pinning weight of the edge from the leader node

This work was partially supported by Natural Science Foundation of China grant 62073074, the Double Innovation Plan grant 4207012004, and the Special Funding for Overseas talents grant 6207011901 (corresponding author: S. Baldi).

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to node i; if agent i is pinned, then $b_i > 0$ and agent i can obtain information from the leader node. Otherwise, $b_i = 0$.

Assumption 1. *The augmented graph $\bar{\mathcal{G}}$ is connected, and contains a spanning tree with the root being the leader node.*

A. Synchronization Problem

We consider a complex network with N second-order agents. The nonlinear dynamics of each agent i for $i = 1, \dots, N$ are given as:

$$\ddot{x}_i(t) = f_i(x_i(t), \dot{x}_i(t)) + h_i(e_i(t), \dot{e}_i(t)) + L_i u_i(t) \quad (1)$$

where $x_i, \dot{x}_i \in \mathbb{R}^n$ are the states, $u_i \in \mathbb{R}^m$ with $m \geq n$ is the control input, $f_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is the unknown drift term, $L_i \in \mathbb{R}^{n\times m}$ is an uncertain full rank input gain matrix, and $h_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ denotes the nonlinear interconnection term dependent on the local synchronization errors $e_i \in \mathbb{R}^n$ and on its derivative $\dot{e}_i \in \mathbb{R}^n$ defined as [9]

$$e_i(t) = \sum_{j=1}^{N} a_{ij}\big(x_i(t) - x_j(t)\big) + b_i\big(x_i(t) - x_0(t)\big) \quad (2a)$$

$$\dot{e}_i(t) = \sum_{j=1}^{N} a_{ij}\big(\dot{x}_i(t) - \dot{x}_j(t)\big) + b_i\big(\dot{x}_i(t) - \dot{x}_0(t)\big). \quad (2b)$$

Finally, x_0 is the desiblack trajectory of the leader node, which, as standard in complex network literature [11], is generated by autonomous bounded dynamics, i.e. $x_0,\ \dot{x}_0,\ \ddot{x}_0 \in \mathcal{L}_\infty$.

To describe the uncertainty in L_i, let us decompose $L_i = \hat{L}_i + \Delta L_i$ into a known (nominal) $\hat{L}_i$ and an unknown ΔL_i. The following assumption on a priori knowledge is made:

Assumption 2. *There exists a known scalar $\bar{J}$ such that for $J_i \triangleq (L_i \hat{L}_i^g - I)$ the following holds*

$$\|J_i\| \leq \bar{J} < 1. \quad (3)$$

Assumption 2 is common for practically relevant fully-actuated ($m = n$) [27], [28] and over-actuated ($m > n$) systems [29], [30].

The assumptions on the uncertain terms $f_i(x_i, \dot{x}_i)$ and $h_i(e_i, \dot{e}_i)$ are:

Assumption 3. *There exist unknown scalars $\bar{f}_{0i}, \bar{f}_{1i}, \bar{f}_{2i}, \bar{h}_{0i}, \bar{h}_{1i}, \bar{h}_{2i} \in \mathbb{R}^+$ such that $\|f_i(x_i, \dot{x}_i)\| \leq \bar{f}_{0i} + \bar{f}_{1i}\|x_i\| + \bar{f}_{2i}\|\dot{x}_i\|$, $\|h(e_i, \dot{e}_i)\| \leq \bar{h}_{0i} + \bar{h}_{1i}\|e_i\| + \bar{h}_{2i}\|\dot{e}_i\|$.*

A few remarks on the importance of Assumption 3, as compablack to the state-of-the-art, are given.

Remark 1. *The upper bound structure in Assumption 3 appears in mechanical dynamics, power flows, biochemical reactions (e.g. with Monod dynamics) [28], [31], [32]. However, we are not aware of works studying how to address such upper bounds when the agents are interconnected in a network.*

Remark 2. *Heterogeneous nodes are sometimes consideblack in the literature [10], [11], [19], [20], but without interconnection terms before the control design. It is an open problem to consider heterogeneous agents interconnected by heterogeneous terms without a priori constant bound.*

From (2), we can obtain that

$$e = -(\mathcal{L} + B) \otimes (x - \underline{x}_0) = -(\mathcal{L} + B) \otimes \delta \quad (4)$$

where $B = \text{diag}(b_1, \dots, b_N) \in \mathbb{R}^{N\times N}$, $e = [e_1, \dots, e_N]^T \in \mathbb{R}^{nN}$, $x = [x_1, \dots, x_N]^T \in \mathbb{R}^{nN}$, $\underline{x}_0 = (1_N \otimes x_0) \in \mathbb{R}^{nN}$, and $\delta = (x - \underline{x}_0) \in \mathbb{R}^{nN}$ represents the global synchronization error with the leader state.

Lemma 1. *[33] Owing to Assumption 1,*

$$\|\delta\| \leq \frac{\|e\|}{\underline{\lambda}(\mathcal{L} + B)} \quad (5)$$

with $\underline{\lambda}(\mathcal{L} + B)$ being the minimum eigenvalue of $(\mathcal{L} + B)$, which is positive.

III. Controller Design

A. Uncertainty Analysis

Define a local error state $\xi_i = [e_i\ \dot{e}_i\ q_i\ \dot{q}_i]^T$ and a variable

$$r_i = [P_i \quad I_n]\xi_i \quad (6)$$

where $P_i \in \mathbb{R}^{n\times n}$ is a user-defined positive definite matrix.

The controller for each agent is designed as

$$u_i = \hat{L}_i^g(-K_i r_i - \Delta u_i) \quad (7a)$$

$$\Delta u_i = \rho_i \text{sgn}(r_i) \quad (7b)$$

$$\rho_i = \frac{1}{(1 - \bar{J})}\Big(\hat{\theta}_{0i} + \hat{\theta}_{1i}\|\xi_i\| + \gamma_i\Big) \quad (7c)$$

where $K_i \in \mathbb{R}^{n\times n}$ is a user-defined positive definite matrix, $\text{sgn}(r_i) = \frac{r_i}{\|r_i\|}$. The variables $\hat{\theta}_{li}$ and γ_i for $l = 0, 1$ are updated by adaptive laws that will be designed in Section III.B.

The dynamics of $\dot{e}_i$ can be calculated from (2b) as

$$\ddot{e}_i = \breve{a}_i \ddot{x}_i - \sum_{j=1}^{N} a_{ij}\ddot{x}_j - b_i \ddot{x}_0 \quad (8)$$

where $\breve{a}_i = (b_i + \sum_{j=1}^{N} a_{ij}) > 0$. We multiply (8) by $\frac{1}{\breve{a}_i}$, and calculate the dynamics of the local synchronization error:

$$\begin{aligned}
&\frac{1}{\breve{a}_i}\ddot{e}_i = \ddot{x}_i - \sum_{j=1}^{N}\frac{a_{ij}}{\breve{a}_i}\ddot{x}_j - \frac{b_i}{\breve{a}_i}\ddot{x}_0 \\
&= f_i(x_i, \dot{x}_i) + h_i(e_i, \dot{e}_i) + (L_i \hat{L}_i^g - I)(-K_i r_i - \Delta u_i) \\
&\quad - \frac{b_i}{\breve{a}_i}\ddot{x}_0 - (K_i r_i + \Delta u_i) - \sum_{j=1}^{N}\bar{a}_{ij}\Big[f_j(x_j, \dot{x}_j) + h_j(e_j, \dot{e}_j) \\
&\quad + (L_j \hat{L}_j^g - I)(-K_j r_j - \Delta u_j) + (K_j r_j + \Delta u_j)\Big] \\
&= -K_i r_i - e_i - (I + J_i)\Delta u_i + \sum_{j=1}^{N}\bar{a}_{ij}(I + J_j)\Delta u_j + \psi_{ij} \quad (9)
\end{aligned}$$

where $\bar{a}_{ij} = \frac{a_{ij}}{\breve{a}_i}$ and ψ_{ij} acts as an aggregate uncertainty

$$\psi_{ij} \triangleq \left[f_i(x_i, \dot{x}_i) + h_i(e_i, \dot{e}_i) - J_i K_i r_i\right] - \frac{b_i}{\breve{a}_i}\ddot{x}_0 + e_i - \sum_{j=1}^{N} \bar{a}_{ij}\left[f_j(x_j, \dot{x}_j) + h_j(e_j, \dot{e}_j) - J_j K_i r_j\right]. \quad (10)$$

According to (6), we get

$$\frac{1}{\breve{a}_i}\ddot{e}_i = \frac{1}{\breve{a}_i}\dot{r}_i - \frac{1}{\breve{a}_i}P_i\dot{e}_i. \quad (11)$$

Substituting (11) into (9), the dynamic of r_i are

$$\frac{\dot{r}_i}{\breve{a}_i} = -K_i r_i - (I + J_i)\Delta u_i + \sum_{j=1}^{N}\bar{a}_{ij}(I + J_j)\Delta u_j + \bar{\psi}_{ij} \quad (12)$$

where $\bar{\psi}_{ij} = \psi_{ij} + \frac{1}{\breve{a}_i}P_i\dot{e}_i$.

According to the definition of $\|\xi_i\|$, it holds that $\|e_i\| \le \|\xi_i\|,\ \|\dot{e}_i\| \le \|\xi_i\|,\ \|x_i\| \le \|\xi_i\|,\ \|\dot{x}_i\| \le \|\xi_i\|$. Combined with $\|r_i\| \le (1 + \|P_i\|)\|\xi_i\|$, we obtain

$$\begin{aligned}\|\bar{\psi}_{ij}\| \le& \Big(\bar{f}_{0i} + \bar{f}_{1i}\|x_i\| + \bar{f}_{2i}\|\dot{x}_i\| + \bar{h}_{0i} + \bar{h}_{1i}\|e_i\| + \bar{h}_{2i}\|\dot{e}_i\|\Big) \\ &+ \frac{b_i}{\breve{a}_i}\|\ddot{x}_0\| + \sum_{j=1}^{N}\bar{a}_{ij}\Big(\bar{f}_{0j} + \bar{f}_{1j}\|x_j\| + \bar{f}_{2j}\|\dot{q}_j\| + \\ &\bar{h}_{0j} + \bar{h}_{1j}\|e_j\| + \bar{h}_{2j}\|\dot{e}_j\|\Big) + \|J_i\|\|K_i\|\|P_i\|\|\xi_i\| \\ &+ \sum_{j=1}^{N}\bar{a}_{ij}\|J_j\|\|K_j\|\|P_j\|\|\xi_j\| + \frac{1}{\breve{a}_i}\|P_i\|\|\dot{e}_i\| + \|e_i\| \\ &\le \theta^*_{0i} + \theta^*_{1i}\|\xi_i\| + \sum_{j=1}^{N}\varphi^*_{1j}\|\xi_j\| \end{aligned} \quad (13)$$

where $\theta^*_{0i}, \theta^*_{1i}, \varphi^*_{1j} \in \mathbb{R}^+$ defined as

$$\theta^*_{0i} = \bar{f}_{0i} + \bar{h}_{0i} + \frac{b_i}{\breve{a}_i}\breve{x}_0 + \sum_{j=1}^{N}\bar{a}_{ij}\left(\bar{f}_{0j} + \bar{h}_{0j}\right) \quad (14a)$$

$$\theta^*_{1i} = \bar{f}_{1i} + \bar{f}_{2i} + \bar{h}_{1i} + \bar{h}_{2i} + \|P_i\|\Big(\frac{1}{\breve{a}_i} + \bar{J}\|K_j\|\Big) + 1 \quad (14b)$$

$$\varphi^*_{1j} = \bar{a}_{ij}\left[(\bar{f}_{1j} + \bar{f}_{2j}) + \bar{h}_{1j} + \bar{h}_{2j} + \bar{J}\|K_j\|\|P_j\|\right] \quad (14c)$$

are unknown constants with $\breve{x}_0 \in \mathbb{R}^+$ such that $\|\ddot{x}_0\| \le \breve{x}_0$.

B. Adaptive Synchronization Controller Design

The adaptive laws in (7c) are designed as:

$$\dot{\hat{\theta}}_{0i} = \|r_i\| - \alpha_0\hat{\theta}_{0i} \quad (15a)$$

$$\dot{\hat{\theta}}_{1i} = \|r_i\|\|\xi_i\| - \alpha_1\hat{\theta}_{1i} \quad (15b)$$

$$\dot{\gamma}_i = -(\epsilon_0 + \epsilon_1\|\xi_i\|^5 - \epsilon_2\|\xi_i\|^3)\gamma_i + \beta_i \quad (15c)$$

$$\text{initial condition } \hat{\theta}_{0i}(0) > 0, \hat{\theta}_{1i}(0) > 0, \gamma_i(0) > \nu_i \quad (15d)$$

$$\alpha_0, \alpha_1, \epsilon_0, \epsilon_1, \epsilon_2, \beta_i, \nu_i \in \mathbb{R}^+ \quad (15e)$$

with the following inequalities

$$\epsilon_0 \ge 1 + \epsilon_2, \epsilon_1 \ge \epsilon_2. \quad (15f)$$

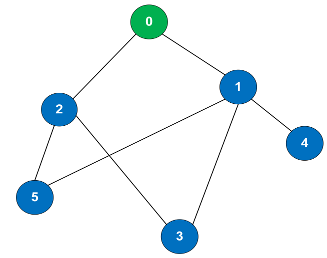


Fig. 1: Network topology for five-area load frequency control

The inequalities in (15f) are designed to guarantee that the term '$\epsilon_0 + \epsilon_1\|\xi_i\|^5 - \epsilon_2\|\xi_i\|^3$' in (15c) is positive for all $\|\xi_i\|$.

Theorem 1. *Under Assumptions 1-3, the trajectories of the closed-loop composed of the complex network dynamics* (1)*, the distributed control law* (7) *and distributed adaptive law* (15) *are Uniformly Ultimately Bounded (UUB) with the ultimate bound on the local synchronization error e as*

$$U = \sqrt{\frac{2\chi}{(\zeta - \kappa)}} \quad (16)$$

where $\chi = \sum_{i=1}^{N}\Big(\frac{\alpha_0{\theta^*_{0i}}^2}{2} + \frac{\alpha_1{\theta^*_{1i}}^2}{2}\Big) + \sum_{i=1}^{N}\frac{2\zeta\bar{\gamma}_i}{\underline{\gamma}_i}$*;* κ *is a scalar satisfying* $0 < \kappa < \zeta$ *with* $\zeta = \frac{\min\{\min_{i\in\Omega}\underline{\lambda}(K_i), \min_{i\in\Omega}\underline{\lambda}(P_i), \alpha_0/2, \alpha_1/2\}}{\max\{1/2\hat{a}, 1/2\}}$*, where* $\hat{a} = \min_{i\in\Omega}\{\breve{a}_i\}$.

Proof. See Appendix. □

According to Lemma 1, the UUB on local synchronization error e implies the UUB on global synchronization error δ.

IV. Simulation Example

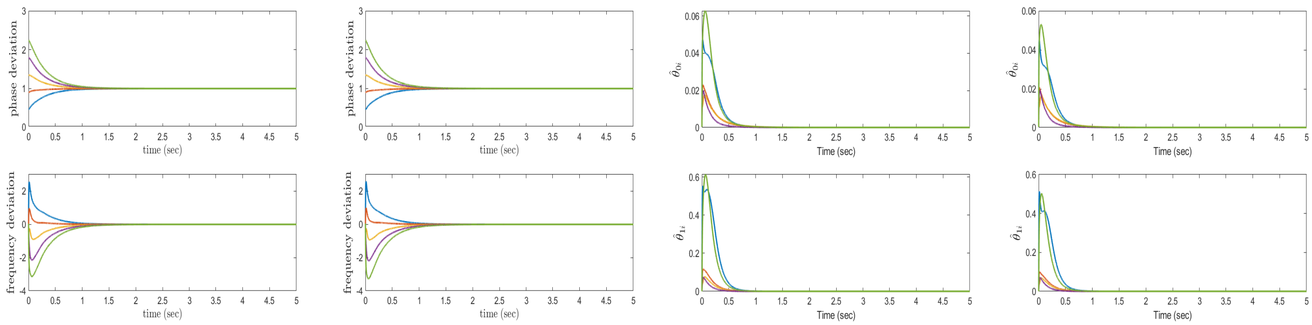
To validate the proposed design, we consider the power network dynamics of a five-area load frequency control (LFC). The five areas are connected as in Fig. 1.

The dynamics of LFC can be written as (1) where $f(x_i, \dot{x}_i) = (-\frac{1}{T_{pi}} - \frac{k_{pi}}{T_{pi}R_i})\dot{x}_i - \frac{k_{pi}}{T_{pi}}(\Delta P_{di} + \Delta E_i)$, $h(e_i, \dot{e}_i) = \Delta P_{ij}$, $L_i = \frac{k_{pi}}{T_{pi}}$. These parameters represent generator inertias for each area (T_{pi}) and local droop gains (k_{pi}, R_i): the interested reader is referblack to [22], [34] for more details in the model and its parameters.

Here $x_i,\ \dot{x}_i \in \mathbb{R}$ represent the deviation of phase and frequency of each area from the operating point; ΔP_{ij} is power flow coming from the interaction among neighbour areas; ΔP_{di} is an unmeasurable load disturbance and ΔE_i is the measurable area control error. We consider both

- linear interconnection $\Delta P_{ij} = 2\pi T_i \sum_{j\in\mathcal{N}_i}(x_i - x_j)$;
- nonlinear interconnection $\Delta P_{ij} = 2\pi T_i \sum_{j\in\mathcal{N}_i}\sin(x_i - x_j)$.

Power dynamics are a special case of (1) with the uncertainties $f(x_i, \dot{x}_i),\ h(e_i, \dot{e}_i)$ conforming to Assumption 3. These interconnection terms exist before the control design, and they are unknown for control purposes.



(a) Phase and frequency deviation of the five areas with linear interconnection

(b) Phase and frequency deviation of the five areas with nonlinear interconnection

(c) Adaptive gains $\theta_{li},\ l=0,1$ of the five areas with linear interconnection

(d) Adaptive gains $\theta_{li},\ l=0,1$ of the five areas with nonlinear interconnection

Fig. 2: Synchronization performance with both linear (1st and 3rd plot) and nonlinear (2nd and 4th plot) interconnection

Without loss of generality, Area 0 is taken as the leading area, with autonomous dynamics $x_0=1,\ \dot{x}_0=0,\ \ddot{x}_0=0$. For each area, the parameters are:

Area-1: $T_{p1}=10,\ \dfrac{k_{p1}}{T_{p1}}=0.1,\ R_1=0.05,\ T_1=2,\ \tilde{B}_1=41,\ k_1=0.5$

Area-2: $T_{p2}=8,\ \dfrac{k_{p2}}{T_{p2}}=0.083,\ R_2=0.05,\ T_2=5,\ \tilde{B}_2=81.5,\ k_2=0.5$

Area-3: $T_{p3}=8,\ \dfrac{k_{p3}}{T_{p3}}=0.063,\ R_3=0.05,\ T_3=8,\ \tilde{B}_3=62,\ k_3=0.6$

Area-4: $T_{p4}=10,\ \dfrac{k_{p4}}{T_{p4}}=0.09,\ R_4=0.03,\ T_4=2,\ \tilde{B}_4=50,\ k_4=0.4$

Area-5: $T_{p5}=7,\ \dfrac{k_{p5}}{T_{p5}}=0.075,\ R_5=0.06,\ T_5=3,\ \tilde{B}_5=55,\ k_5=0.7$

These parameters are used for simulation purposes, but unknown for control design.

We select $P_i=3.3,\ K_i=60,\ \varepsilon=0.1,\ \Delta P_{di}=-0.1\sin((0.5t)i)$. The parameters in adaptive distributed control law (15) are $\epsilon_0=55,\ \epsilon_1=3,\ \epsilon_2=0.003,\ \alpha_{0i}=\alpha_{0i}=9,\ \beta_i=3150$.

The simulation results shows that the synchronization behavior of network nodes with linear interconnection and nonlinear interconnection follows a similar pattern. The phase and frequency deviation of five areas converge to the desiblack values both for linear and nonlinear interconnection, cf. Figures 2a and 2b. The adaptive gains $\theta_{li},\ l=0,1$ for $i=1,\dots,5$ for linear and nonlinear interconnection are shown in Figures 2c and Figures 2d, respectively.

V. Conclusion

An adaptive synchronization problem for complex networks with state-dependent uncertainty and uncertain nonlinear interconnection has been consideblack under the challenging assumption that the interconnection terms are state dependent and exist before control design. This work is a preliminary study and further investigations are of interest: it is of interest to generalize the approach in the sense of handling more general dynamics and more general structures of the interconnections. In view of the bounded error result, it is also of interest to replace the sign function with a saturation function so as to avoid discontinuities in the ontrol action.

Appendix

Proof of Theorem 1. Construct a Lyapunov function as:

$$V(t)=\frac{1}{2}\sum_{i=1}^{N}\left\{\frac{1}{\breve{a}_i}r_i^T(t)r_i(t)+e_i^T(t)e_i(t)+\frac{2\gamma_i(t)}{\underline{\gamma}_i}+(\hat{\theta}_{0i}(t)-\theta_{0i}^*)^2+(\hat{\theta}_{1i}(t)-\theta_{1i}^*)^2\right\}. \quad (17)$$

Investigating the adaptive laws (15a)-(15c) and the initial gain conditions (15d)-(15e), it can be verified that there exists positive fixed scalars $\underline{\gamma}_i$ such that

$$\hat{\theta}_{li}(t)\geq 0,\ \gamma_i(t)\geq\underline{\gamma}_i>0\ \forall t\geq 0 \quad (18)$$

with $l=0,1$. In the following, we ignore the argument (t) of time-varying variables, i.e., $V=V(t)$ to simplify the notation.

From (12), the time derivative of (17) is obtained as

$$\begin{aligned}\dot{V}\leq&-\sum_{i=1}^{N}r_i^TK_ir_i+(I+\bar{J})\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\bar{a}_{ij}\rho_jr_i^T\operatorname{sgn}(r_j)\\&-\sum_{i=1}^{N}\left\{(1-\bar{J})\rho_ir_i^T\operatorname{sgn}(r_i)-\sum_{j\in\mathcal{N}_i}\|r_i\|\|\bar{\psi}_{ij}\|+e_i^TP_ie_i\right\}\\&+\sum_{i=1}^{N}\left\{\frac{\dot{\gamma}_i}{\underline{\gamma}_i}+\sum_{l=0}^{1}(\hat{\theta}_{li}-\theta_{li}^*)\dot{\hat{\theta}}_{li}\right\}.\end{aligned} \quad (19)$$

According to (13), and according to (6) we have $\|r_i\|\leq(1+\|P_i\|)\|\xi_i\|$, it follows that

$$\|r_i\|\|\bar{\psi}_{ij}\|\leq\|r_i\|\left\{\theta_{0i}^*+\theta_{1i}^*\|\xi_i\|+\sum_{j\in\mathcal{N}_i}\varphi_{1j}^*\|\xi_j\|\right\} \quad (20)$$

$$\varphi_{1j}^*\|r_i\|\|\xi_j\|\leq\varphi_{1j}^*(1+\|P_i\|)\|\xi_i\|\|\xi_j\|. \quad (21)$$

Since $\|\operatorname{sgn}(r_j)\|=1$ and $\hat{\theta}_{0j}\leq\bar{\theta}_{0j}+\check{\theta}_{0j}\|r_j\|,\ \hat{\theta}_{1j}\leq\bar{\theta}_{1j}+$

$\check{\theta}_{1j}\|r_j\|\|\xi_j\|$ with $\bar{\theta}_{lj}, \check{\theta}_{lj} \in \mathbb{R}^+,\ l = 0, 1$, we have

$$
\begin{aligned}
&(1+\bar{J})\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\bar{a}_{ij}\rho_j r_i^T \mathrm{sgn}(r_j)\\
\leq&(1+\bar{J})\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\bar{a}_{ij}\rho_j\|r_i\|\\
\leq&J^{'}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\Big\{\bar{a}_{ij}\bar{\theta}_{0j}(1+\|P_i\|)\|\xi_i\|\\
&+\bar{a}_{ij}\check{\theta}_{1j}(1+\|P_i\|)(1+\|P_j\|)\|\xi_i\|\|\xi_j\|^3+\bar{a}_{ij}\gamma_j\|r_i\|\Big\}\\
&+\bar{a}_{ij}\big[\check{\theta}_{0j}(1+\|P_j\|)+\bar{\theta}_{1j}\big](1+\|P_i\|)\|\xi_i\|\|\xi_j\| \qquad (22)
\end{aligned}
$$

where $J^{'} = \dfrac{1+\bar{J}}{1-\bar{J}}$ is a constant.

Similarly, define an overall term Δ_{ij} coming from agents j as

$$
\begin{aligned}
\Delta_{ij} =& \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\Bigg\{(1+\bar{J})\bar{a}_{ij}\rho_j r_i^T\mathrm{sgn}(r_j)+\varphi_{1j}^*\|r_i\|\|\xi_j\|\Bigg\}\\
\leq& J^{'}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\Bigg\{\bar{a}_{ij}(\bar{\theta}_{0j}+\bar{\gamma}_j)(1+\|P_i\|)\|\xi_i\|\\
&+\Big[\bar{a}_{ij}\big(\check{\theta}_{0j}(1+\|P_j\|)+\bar{\theta}_{1j}\big)+\varphi_{1j}^*\Big](1+\|P_i\|)\|\xi_i\|\|\xi_j\|\\
&+\bar{a}_{ij}\check{\theta}_{1j}(1+\|P_i\|)(1+\|P_j\|)\|\xi_i\|\|\xi_j\|^3\Bigg\}. \qquad (23)
\end{aligned}
$$

According to (7c), we have

$$
\begin{aligned}
&-(1-\bar{J})\sum_{i=1}^{N}\rho_i r_i^T\mathrm{sgn}(r_i) = -(1-\bar{J})\sum_{i=1}^{N}\rho_i\|r_i\|\\
&=-\sum_{i=1}^{N}\Bigg[\Big(\hat{\theta}_{0i}+\hat{\theta}_{1i}\|\xi_i\|+\gamma_i\Big)\Bigg]\|r_i\|. \qquad (24)
\end{aligned}
$$

According to (19), (23) and (24), we obtain

$$
\begin{aligned}
\dot{V} \leq& -\sum_{i=1}^{N}r_i^T K_i r_i - \sum_{i=1}^{N}\Bigg\{\sum_{l=0}^{1}(\hat{\theta}_{li}-\theta_{li}^*)\|\xi_i\|^l\|r_i\|\Bigg\}\\
&+\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\Delta_{ij}-\sum_{i=1}^{N}e_i^T P_i e_i\\
&+\sum_{i=1}^{N}\Big\{\frac{\dot{\gamma}_i}{\underline{\gamma}_i}+\sum_{l=0}^{1}(\hat{\theta}_{li}-\theta_{li}^*)\dot{\hat{\theta}}_{li}\Big\}. \qquad (25)
\end{aligned}
$$

Using (15a)-(15c), we have

$$
(\hat{\theta}_{li}-\theta_{li}^*)\dot{\hat{\theta}}_{li} = (\hat{\theta}_{li}-\theta_{li}^*)\|\xi_i\|^l\|r_i\| + (\alpha_{li}\hat{\theta}_{li}\theta_{li}^* - \alpha_{li}\hat{\theta}_{li}^2) \qquad (26)
$$

for $l = 0, 1$ and $i = 1, \ldots, N$. The last terms of (26) can be written as

$$
\alpha_{li}\hat{\theta}_{li}\theta_{li}^* - \alpha_{li}\hat{\theta}_{li}^2 \leq -\frac{\alpha_{li}(\hat{\theta}_{li}-\theta_{li}^*)^2}{2}+\frac{\alpha_{li}{\theta_{li}^*}^2}{2}. \qquad (27)
$$

Similarly, with $\gamma_i(t) \geq \underline{\gamma}_i > 0$, (15c) leads to

$$
\begin{aligned}
\sum_{i=1}^{N}\frac{\dot{\gamma}_i}{\underline{\gamma}_i} =& \sum_{i=1}^{N}\frac{1}{\underline{\gamma}_i}-(\epsilon_0+\epsilon_1\|\xi_i\|^5-\epsilon_2\|\xi_i\|^3)\gamma_i+\beta_i\\
\leq&\sum_{i=1}^{N}\Big[-\big(\epsilon_0+\epsilon_1\|\xi_i\|^5-\epsilon_2\|\xi_i\|^3\big)+\frac{\beta_i}{\underline{\gamma}_i}\Big]. \qquad (28)
\end{aligned}
$$

Substituting (26)-(28) into (25) yields

$$
\begin{aligned}
\dot{V} \leq& -\min_{i\in\Omega}\underline{\lambda}(K_i)\sum_{i=1}^{N}\|r_i\|^2-\min_{i\in\Omega}\underline{\lambda}(P_i)\sum_{i=1}^{N}\|e_i\|^2\\
&-\sum_{i=1}^{N}\sum_{l=0}^{1}\Big[\frac{\alpha_{li}(\hat{\theta}_{li}-\theta_{li}^*)^2}{2}-\frac{\alpha_{li}{\theta_{li}^*}^2}{2}\Big]+Z(\|\xi\|) \qquad (29)
\end{aligned}
$$

where $\Omega = \{1, \ldots, N\}$ and $\xi = [\xi_0, \ldots, \xi_N]^T$ with

$$
\begin{aligned}
Z(\|\xi\|) \triangleq& -\epsilon_1\sum_{i=1}^{N}\|\xi_i\|^5+\epsilon_2\sum_{i=1}^{N}\|\xi_i\|^3+\sum_{i=1}^{N}\Big(-\epsilon_0+\frac{\beta_i}{\underline{\gamma}_i}\Big)\\
&+J^{'}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\Bigg\{\bar{a}_{ij}(\bar{\theta}_{0j}+\bar{\gamma}_j)(1+\|P_i\|)\|\xi_i\|\\
&+\Big[\bar{a}_{ij}\big(\check{\theta}_{0j}(1+\|P_j\|)+\bar{\theta}_{1j}\big)+\varphi_{1j}^*\Big](1+\|P_i\|)\|\xi_i\|\|\xi_j\|\\
&+\bar{a}_{ij}\check{\theta}_{1j}(1+\|P_i\|)(1+\|P_j\|)\|\xi_i\|\|\xi_j\|^3\Bigg\}. \qquad (30)
\end{aligned}
$$

Using Descartes' rules of sign change and Bolzano's Theorem [35], the polynomial Z has exactly one positive real root $\eta \in \mathbb{R}^+$. The coefficient of the highest degree of Z is negative, $-\epsilon_1$. Therefore, $Z(\|\xi\|) \leq 0$ when $\|\xi\| \geq \eta$, where $\xi = [\xi_1, \ldots, \xi_N]^T$.

Lyapunov function (17) satisfies

$$
\begin{aligned}
V \leq& \frac{1}{2\check{a}}\sum_{i=1}^{N}\|r_i\|^2+\frac{1}{2}\sum_{i=1}^{N}\|e_i\|^2\\
&+\frac{1}{2}\sum_{i=1}^{N}\Big[\frac{2\gamma_i}{\underline{\gamma}_i}+\sum_{l=0}^{1}\big(\hat{\theta}_{li}^2-\theta_{li}^*\big)^2\Big] \qquad (31)
\end{aligned}
$$

where $\check{a} = \min_{i\in\Omega}(\check{a}_i)$. Let us define a positive scalar κ such that $0 < \kappa < \zeta$. Then combined with (31), $\dot{V}$ in (29) is further simplified to

$$
\dot{V} \leq -\zeta V + \sum_{i=1}^{N}\Big[\frac{2\zeta\bar{\gamma}_i}{\underline{\gamma}_i}+\sum_{l=0}^{1}\frac{\alpha_{li}{\theta_{li}^*}^2}{2}\Big]+Z_1(\|\xi\|). \qquad (32)
$$

Using $0 < \kappa < \zeta$, (32) is further simplified to

$$
\dot{V} \leq -\kappa V - (\zeta-\kappa)V + \chi \qquad (33)
$$

where $\chi = \sum_{i=1}^{N}\Big[\frac{2\zeta\bar{\gamma}_i}{\underline{\gamma}_i}+\sum_{l=0}^{1}\frac{\alpha_{li}{\theta_{li}^*}^2}{2}\Big]$.

Combine (33) and define $Y = \chi/(\zeta-\kappa)$ when $\|\xi\| \geq \eta$. It can be concluded that $\dot{V} \leq -\kappa V$ when $V \geq Y$. These two cases leads to the bound

$$
V \leq \max\{V(0), Y\}. \qquad (34)
$$

The definition of the Lyapunov function (17) satisfies

$$V \geq \frac{1}{2}\|e\|^2 \tag{35}$$

where $e = [e_1, \ldots, e_N]^T$. Using (34) and (35), it can be obtained $\|e\|^2 \leq 2\max\{V(0), Y\}$. Finally, an ultimated bound U in (16) on the local synchronization error e is obtained, implying an UUB on the global synchronization error δ from Lemma 1.

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