TR 75150

JANUARY 1976

ROYAL AIRCRAFT ESTABLISHMENT

TECHNICAL REPORT 75150

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UDC 551.557 : 521.6

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Received for printing 3 December 1975

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SUMMARY

The orbits of Earth satellites with perigee heights less than 600km are liable to be appreciably perturbed by the aerodynamic forces resulting from winds in the upper atmosphere, and analysis of the changes in the orbits provides a method of determining zonal (west-to-east) and meridional (north-to-south) winds. The theory hitherto used has been developed for orbits of eccentricity $e < 0.2$. Here we develop the theory for the effect of zonal and meridional winds on the inclination i and right ascension of the node Ω for satellites in orbits with $e > 0.2$ moving in an oblate atmosphere. The results are expressed in terms of the change in orbital period, which is accurately known for actual satellites, so that the equations are independent of variations in air density and satellite cross-sectional area.

The results, summarized in equations (58) and (59), show that the changes depend on e through the function $(1 - e)^{\frac{5}{2}}(1 + e)^{-\frac{3}{2}}$. For zonal winds, the change in i is nearly proportional to $\sin i \cos^2\omega$ and the change in Ω to $\sin 2\omega$, where ω is the argument of perigee. For meridional winds, the change in i is nearly proportional to $(\sec^2\omega + \tan^2 i)^{-\frac{1}{2}}$, and the change in Ω to $\cot i \sin\omega(1 - \sin^2 i \sin^2\omega)^{-\frac{1}{2}}$.

Departmental Reference: Space 500

CONTENTS

1 INTRODUCTION

The orientation of the plane of an Earth satellite orbit may be significantly altered by persistent zonal (west-to-east) winds or meridional (north-to-south) winds in the upper atmosphere near perigee, and the theoretical variation of the relevant orbital elements needs to be established so that the observed changes in specific orbits can be analysed to evaluate these winds. The orbital parameters primarily affected are the inclination to the equator, i, and the right ascension of the ascending node Ω (see Fig.1 for definitions).

The theory for the effect of zonal winds on i and Ω for orbits of small eccentricity was first developed by Cook and Plimmer[1] for a spherically symmetrical atmosphere, and by Cook[2] for an oblate atmosphere. These results are recorded in a revised form by King-Hele[3]. This theory is for an atmosphere in which the density scale height H is constant, with the air density at height y proportional to $\exp(-y/H)$. In the real atmosphere, H varies with y, and the theory was extended by King-Hele and Scott[4] to take account of this variation. The real atmosphere also has a strong day-to-night variation above 200km, and theory appropriate in these conditions was developed by King-Hele and Walker[5]. The effect of meridional winds on orbits of small eccentricity in a spherical symmetrical atmosphere of constant H was studied by King-Hele[6].

In all this previous work the orbital eccentricity e is assumed to be small, and expansions in powers of e are used. Recently Brierley[7] has obtained accurate values of inclination for rapidly decaying orbits of Molniya satellites of high eccentricity $(e \simeq 0.5)$: a theory for high-eccentricity orbits is needed if upper-atmosphere winds are to be determined from the observed changes in the orbital parameters of such satellites.

In the present paper we develop the theory for the effect of zonal and meridional winds on both i and Ω for high-eccentricity orbits in an oblate atmosphere. Our results have already been successfully applied[8] to determine upper-atmosphere winds at heights near 110km from the orbit of 1970-114F.

2 SUMMARY OF PREVIOUS BASIC RESULTS

A spherical satellite moving through the upper atmosphere experiences an aerodynamic drag force in the direction opposite to its motion relative to the air. If the atmosphere did not rotate, this force would be opposite to the velocity relative to the Earth's centre, and would have no effect on i and Ω. Both these orbital elements are slightly altered, however, if the upper atmosphere is in motion; the most important effect is that caused by the steady west-to-east rotation of the upper atmosphere, generally at an angular velocity not very different from that of the Earth. This axial rotation produces a transverse force which has the effect of reducing the inclination in the course of a satellite's life, often by as much as 0.1°, which is readily measurable. The effect of axial atmospheric rotation (and hence zonal winds) on inclination is proportional to $\cos^2\omega$, where ω is the argument of perigee, and the effect therefore builds up over many revolutions of ω. In contrast, the effects of meridional winds on inclination are proportional to $\cos\omega$, and are therefore generally less important, because they tend to cancel out over each cycle of ω.

Both the zonal and meridional winds produce a change in inclination per revolution, Δi, say, which is proportional to the drag experienced by the satellite; and this drag also produces a decrease ΔT_d, say, in the orbital period T_d of the satellite, expressed as a fraction of a day. Since ΔT_d can be accurately measured, it is convenient to express Δi in terms of ΔT_d, thereby eliminating terms involving air density, satellite mass and cross-section, etc., which have the same effect on both Δi and ΔT_d.

For an orbit of eccentricity < 0.2 in an oblate atmosphere (of constant scale height H) rotating at Λ rev/day about the Earth's axis,

$$\frac{\Delta i}{\Delta T_d} = \frac{\Lambda \sin i}{6\sqrt{F}} \left\{1 + \frac{I_2}{I_0}\cos 2\omega + O(e,c)\right\} \qquad (1).$$

Here $\sqrt{F}$ is a factor which is nearly always between 0.95 and 1.05, and is given by $(1 - r_p w \cos i/v_p)$, where r_p and v_p are the radial distance and velocity at perigee, and w is the angular velocity of the atmosphere about the Earth's axis. The quantities I_n in equation (1) are Bessel functions of the first kind and imaginary argument, of order n and argument $z = ae/H$, where a is the semi major axis; the quantity c in (1) expresses the effect of atmospheric oblateness, being equal to $\varepsilon r_p \sin^2 i/2H$, where ε is the ellipticity of the atmosphere, which is usually taken equal to that of the Earth, 0.00335. The terms in e and c in (1) are given explicitly in Ref.3, and further terms, in ce and e^2, were evaluated by King-Hele and Scott[9].

Since the aerodynamic forces are important only near perigee, it is the wind in this region which is effective in altering i. If y_p and ϕ_p are the perigee height and latitude, the perigee is at a distance $(R + y_p) \cos \phi_p$ from the Earth's axis. Hence, if y_p is in km and R is taken as 6370km, the rotation rate Λ corresponds to a west-to-east wind near perigee of $(463 + 0.073y_p)(\Lambda - 1) \cos \phi_p$ m/s.

The change in Ω per revolution for an atmosphere with an axial rotation rate of Λ rev/day is given by

$$\frac{\Delta\Omega}{\Delta T_d} = \frac{\Lambda \sin 2\omega}{6\sqrt{F}} \left[\frac{I_2}{I_0}\left\{1 + O(e,c)\right\}\right] \tag{2}$$

Since the effects of atmospheric rotation on Ω tend to cancel over half a cycle of ω, it is usually better to determine upper-atmosphere winds by analysing the cumulative changes in i rather than the smaller changes in Ω.

When z is large enough, the Bessel functions in equations (1) and (2) may be replaced by their asymptotic expansions, with $I_2/I_0 = 1 - 2/z + O(1/z^2)$ etc. Equations (1) and (2), with the e and c terms restored, then reduce to

$$\frac{\Delta i}{\Delta T_d} = \frac{\Lambda \sin i}{3\sqrt{F}} \left\{(1 - 4e) \cos^2\omega - \frac{1}{z} \cos 2\omega + \frac{\varepsilon}{e} \sin^2 i \sin^2 2\omega + O(e^2, c^2, 1/z^2)\right\} \qquad \ldots\ldots(3)$$

$$\frac{\Delta\Omega}{\Delta T_d} = \frac{\Lambda \sin 2\omega}{6\sqrt{F}} \left\{1 - \frac{2}{z} - 4e - \frac{2\varepsilon}{e} \sin^2 i \cos 2\omega + O(e^2, c^2, 1/z^2)\right\} \qquad (4)^*$$

These simplified forms apply if $z > 10$, corresponding to $e > 0.05$ for a typical value of H/a, 0.005: so equations (3) and (4) may be regarded as applicable for $0.05 < e < 0.2$. These equations are of particular interest here, because the expressions we find for Δi and $\Delta\Omega$ due to atmospheric rotation should reduce to (3) and (4) if expanded in powers of e.

The changes in i and Ω caused by a meridional wind for orbits of small eccentricity, again with the Bessel functions expanded in powers of $1/z$, may be written[6]

* There is a serious misprint in this equation as given in Ref.3, equation (8.39): the cos 2ω is omitted.

$$\frac{\Delta i}{\Delta T_d} = -\frac{\mu \cos i}{3}\left\{\frac{2}{F(1+\cos^2 i)}\right\}^{\frac{1}{2}}\left\{\left(1+\frac{K}{4}\right)\left(1-\frac{1}{2z}\right)\cos\omega - \frac{K}{4}\cos 3\omega + O(0.1, 1/z^2)\right\} \quad \ldots\ldots(5)$$

$$\frac{\Delta\Omega}{\Delta T_d} = -\frac{\mu \cos i}{3}\left\{\frac{2}{F(1+\cos^2 i)}\right\}^{\frac{1}{2}}\left\{\left(1+\frac{K}{4}\right)\left(1-\frac{1}{2z}\right)\sin\omega - \frac{K}{4}\sin 3\omega + O(0.1, 1/z^2)\right\} \quad \ldots\ldots(6)$$

where $K = \sin^2 i/(1 + \cos^2 i)$

and μ rev/day is the equivalent south-to-north atmospheric rotation rate: in other words, the south-to-north wind near perigee is $(463 + 0.073y_p)\mu$ m/s.

Equations (3) to (6) are derived on the assumption that the satellite is spherical and experiences an aerodynamic force in the direction opposite to its motion relative to the ambient air. A non-spherical satellite may suffer aerodynamic lift forces perpendicular to the drag, but Cook[10] and Fiddes[11] have shown that lift generally has only a very small effect, except for satellites of peculiar shapes (such as flat plates) with specific variations of incidence (e.g. flip-over at perigee). The theory developed here for spherical satellites should apply equally well to satellites of irregular shape which rotate many times during the course of each revolution round the Earth, and should therefore be valid for nearly all unstabilized satellites.

It is worth emphasizing again that the results are independent of the irregular variations in upper-atmosphere density, and independent of any variations in satellite cross-sectional area or drag coefficient. Such variations affect Δi and ΔT_d equally and leave their quotient unchanged.

3 EFFECT OF AN OBLATE ROTATING ATMOSPHERE ON i AND Ω, FOR ORBITS OF ECCENTRICITY GREATER THAN 0.2

3.1 Effect on inclination

3.1.1 Δi in terms of ρ and δ

An atmosphere rotating at an angular velocity w about the Earth's axis produces a rate of change of i given by equation (8.11) of Ref.3 as

$$\frac{di}{dE} = -\tfrac{1}{2}r^2\rho w\delta\left\{\frac{r(1 + e\cos E)}{GMF(1 - e^2)}\right\}^{\frac{1}{2}}\sin i\cos^2(\omega + \theta) \qquad (7),$$

where ρ is the air density at distance r from the Earth's centre,

E is the eccentric anomaly,

θ is the true anomaly,

GM is the gravitational constant for the Earth ($398601 km^3/s^2$),

and $\delta = FSC_D/m$, where C_D is the drag coefficient of the satellite based on the effective cross-sectional area S, and m is its mass.

In equation (7) we first express w in the form $w = \Lambda w_E$, where w_E is the Earth's angular velocity and Λ is the axial atmospheric rotation rate in rev/day. Also the anomalistic orbital period expressed as a fraction of a day, T_d, is equal to w_E/n, where n is the satellite's mean motion, which is linked to a by Kepler's equation, $n^2a^3 = GM$. Hence

$$w = \Lambda T_d (GM)^{\frac{1}{2}} a^{-\frac{3}{2}} \qquad (8),$$

and equation (7) becomes

$$\frac{di}{dE} = -\tfrac{1}{2} r^{\frac{5}{2}} a^{-\frac{3}{2}} \Lambda T_d \rho \delta \left\{\frac{1 + e\cos E}{F(1 - e^2)}\right\}^{\frac{1}{2}} \sin i \cos^2(\omega + \theta) \quad . \qquad (9).$$

For the orbits we are studying, with $e > 0.2$, it proves fruitful to change the independent variable from E to λ, where

$$\cos E = 1 - \lambda^2/z \qquad (10)$$

with $z = ae/H$ as before, and then to use expansions in powers of λ^2/z. From equation (10),

$$\left.\begin{aligned} \sin E &= 2^{\frac{1}{2}} z^{-\frac{1}{2}} \lambda (1 - \lambda^2/2z)^{\frac{1}{2}} \\ \text{and} \qquad dE &= 2^{\frac{1}{2}} \{z(1 - \lambda^2/2z)\}^{-\frac{1}{2}} d\lambda \end{aligned}\right\} \qquad (11).$$

If we use the standard equations for an unperturbed ellipse,

$$r = a(1 - e\cos E); \quad r\cos\theta = a(\cos E - e); \quad \text{and} \quad r\sin\theta = a(1 - e^2)^{\frac{1}{2}} \sin E \qquad \ldots\ldots(12),$$

and write

$$x^2 = \frac{z(1 - e)}{2(1 + e)} \tag{13},$$

we find, on expanding in powers of λ/x ,

$$\left.\begin{aligned}\cos^2(\omega + \theta) &= \cos^2\omega - \frac{\lambda}{x}\sin 2\omega - \frac{\lambda^2}{x^2}\cos 2\omega + 0\left(\frac{\lambda^3}{x^3}\right)\\ \sin^2(\omega + \theta) &= \sin^2\omega + \frac{\lambda}{x}\sin 2\omega + \frac{\lambda^2}{x^2}\cos 2\omega + 0\left(\frac{\lambda^3}{x^3}\right)\end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned}\cos(\omega + \theta) &= \cos\omega\left\{1 - \frac{\lambda}{x}\tan\omega - \frac{\lambda^2}{2x^2} + 0\left(\frac{\lambda^3}{x^3}\right)\right\}\\ \sin(\omega + \theta) &= \sin\omega\left\{1 + \frac{\lambda}{x}\cot\omega - \frac{\lambda^2}{2x^2} + 0\left(\frac{\lambda^3}{x^3}\right)\right\}\end{aligned}\right\} \tag{15}$$

$$r = a(1 - e)\left\{1 + \frac{e\lambda^2}{z(1 - e)}\right\} \tag{16}.$$

If equations (10), (11), (14) and (16) are used to eliminate E, $(\omega + \theta)$ and r , equation (9) gives

$$\begin{aligned}\frac{di}{d\lambda} = &- a\Lambda T_d\rho\delta\left(\frac{1}{2F}\right)^{\frac{1}{2}}(1 - e^2)^{-\frac{1}{2}}(1 - e)^{\frac{5}{2}}\left\{1 + \frac{e\lambda^2}{z(1 - e)}\right\}^{\frac{5}{2}}\left\{1 + e - \frac{\lambda^2 e}{z}\right\}^{\frac{1}{2}}\\ &\times\left\{z\left(1 - \frac{\lambda^2}{2z}\right)\right\}^{-\frac{1}{2}}\sin i\left[\cos^2\omega - \frac{\lambda}{x}\sin 2\omega - \frac{\lambda^2}{x^2}\cos 2\omega + 0\left(\frac{\lambda^3}{x^3}\right)\right]\end{aligned} \tag{17}.$$

To obtain the change Δi in i during one revolution, we integrate equation (17) between $E = -\pi$ and $E = \pi$, that is between $\lambda = -\sqrt{2z}$ and $\lambda = \sqrt{2z}$. Equation (17) then becomes, after expanding the curly brackets in powers of λ^2/z ,

$$\Delta i = -\left(\frac{1}{2Fz}\right)^{\frac{1}{2}} a(1-e)^2 \Lambda T_d \delta \sin i \int_{-\sqrt{2z}}^{\sqrt{2z}} \left\{1 + \frac{(1 + 8e + 11e^2)\lambda^2}{4(1-e^2)z} + O\left(\frac{\lambda^4}{z^2}\right)\right\}$$

$$\times \left[\cos^2\omega - \frac{\lambda}{x}\sin 2\omega - \frac{\lambda^2}{x^2}\cos 2\omega + O\left(\frac{\lambda^3}{x^3}\right)\right]\rho d\lambda \quad (18).$$

3.1.2 Density ρ in terms of λ

In an atmosphere of constant scale height H and ellipticity ε (taken equal to the Earth's ellipticity, 0.00335), the density ρ may be expressed as

$$\rho = \rho_p \exp\{-(r-\sigma)/H\} \quad (19),$$

where σ is the radius vector of the surface of constant density that passes through the perigee point and ρ_p is the density at perigee. From equations (5.2) and (5.4) of Ref.3,

$$\sigma = r_p\left[1 - \varepsilon \sin^2 i \left\{\sin^2(\omega+\theta) - \sin^2\omega\right\} + O(\varepsilon^2)\right] \quad (20),$$

where r_p is the perigee distance, $a(1-e)$. On substituting this expression for σ into equation (19), we have

$$\rho = \rho_p \exp\left[\!\left[-\frac{1}{H}\left[r - r_p + \varepsilon r_p \sin^2 i\left\{\sin^2(\omega+\theta) - \sin^2\omega\right\} + O(\varepsilon^2)\right]\right]\!\right] \quad (21).$$

Now, from equations (10) and (12),

$$r - r_p = ae(1-\cos E) = ae\lambda^2/z = H\lambda^2 \quad (22),$$

since $z = ae/H$. With the aid of equations (14) and (22) we may rewrite equation (21) as

$$\rho = \rho_p \exp(-\lambda^2)\exp\left[-\frac{2c\lambda}{x}\left\{\sin 2\omega + \frac{\lambda}{x}\cos 2\omega + O\left(\frac{\lambda^2}{x^2}\right)\right\}\right] \quad (23),$$

where
$$c = (\varepsilon r_p \sin^2 i)/2H \quad (24).$$

Expanding the second exponential in equation (23) in powers of λ/x , and assuming that c is of order 1, we have

$$\rho = \rho_p \exp(-\lambda^2)\left[1 - \frac{2c\lambda}{x}\sin 2\omega + \frac{2c\lambda^2}{x^2}(c \sin^2 2\omega - \cos 2\omega) + 0\left(\frac{\lambda^3}{x^3}\right)\right] \quad (25).$$

3.1.3 <u>Δi in terms of $\rho_p\delta$</u>

We now substitute the expression (25) for ρ into the equation (18) for Δi , multiply out the expressions in square brackets, and ignore all terms in $\lambda, \lambda^3, \lambda^5 \ldots,$ which vanish when integrated between $-\sqrt{2z}$ and $\sqrt{2z}$. We find:

$$\Delta i = -\left(\frac{1}{2Fz}\right)^{\frac{1}{2}} a(1-e)^2 \Lambda T_d \rho_p \delta \sin i \int_{-\sqrt{2z}}^{\sqrt{2z}} \left\{1 + \frac{(1 + 8e + 11e^2)\lambda^2}{4(1-e^2)z} + 0\left(\frac{\lambda^4}{z^2}\right)\right\}$$

$$\times \left\{\cos^2\omega - \frac{\lambda^2}{x^2}\cos 2\omega + \frac{2c\lambda^2}{x^2}\cos^2\omega(6\sin^2\omega - 1 + c\sin^2 2\omega)\right\} \exp(-\lambda^2)d\lambda \quad (26).$$

We are assuming that $e > 0.2$ and $a/H > 150$, so that $z = ae/H > 30$ and $\sqrt{2z} > 7$: since $\exp(-49) = 5 \times 10^{-22}$, the value of the integrand is negligible for $\lambda > \sqrt{2z}$, and the limit $\sqrt{2z}$ may be replaced by ∞ . Hence, multiplying out the curly brackets, we may rewrite (26) as

$$\Delta i = -\left(\frac{1}{2Fz}\right)^{\frac{1}{2}} a(1-e)^2 \Lambda T_d \rho_p \delta \sin i \cos^2\omega$$

$$\times \int_{-\infty}^{\infty} \left[1 + \frac{\{8(1+e)^2\sec^2\omega - (15 + 24e + 5e^2)\}\lambda^2}{4z(1-e^2)}\right.$$

$$\left. + \frac{2c\lambda^2}{x^2}(6\sin^2\omega - 1 + c\sin^2 2\omega) + 0\left(\frac{\lambda^4}{z^2}\right)\right] \exp(-\lambda^2)d\lambda \quad (27).$$

We now eliminate x^2 using (13), and utilize the standard integrals

$$\int_{-\infty}^{\infty} \exp(-\lambda^2)d\lambda = \sqrt{\pi} \; ; \quad \int_{-\infty}^{\infty} \lambda^2 \exp(-\lambda^2)d\lambda = \tfrac{1}{2}\sqrt{\pi} \; ; \quad \int_{-\infty}^{\infty} \lambda^4 \exp(-\lambda^2)d\lambda = \tfrac{3}{4}\sqrt{\pi} \qquad (28).$$

Equation (27) then becomes

$$\Delta i = -\left(\frac{\pi}{2Fz}\right)^{\frac{1}{2}} a(1-e)^2 \Delta T_d \rho_p \delta \sin i \cos^2\omega$$

$$\times \left[1 + \frac{8(1+e)^2\sec^2\omega - (15 + 24e + 5e^2)}{8z(1-e^2)} + \frac{2c(1+e)}{z(1-e)}(6\sin^2\omega - 1 + c\sin^2 2\omega) + 0\left(\frac{1}{z^2}\right)\right] \qquad (29).$$

3.1.4 ΔT_d in terms of $\rho_p\delta$

From equation (4.14) of Ref.3 the change Δa in a during one revolution is

$$\Delta a = -a^2\delta \int_{-\pi}^{\pi} (1 + e\cos E)^{\frac{3}{2}}(1 - e\cos E)^{-\frac{1}{2}}\rho dE \qquad (30).$$

Changing the variable from E to λ by means of equations (10) and (11):

$$\Delta a = -\left(\frac{2}{z}\right)^{\frac{1}{2}} a^2\delta \int_{-\sqrt{2z}}^{\sqrt{2z}} \left(1 + e - \frac{\lambda^2 e}{z}\right)^{\frac{3}{2}} \left(1 - e + \frac{\lambda^2 e}{z}\right)^{-\frac{1}{2}} \left(1 - \frac{\lambda^2}{2z}\right)^{-\frac{1}{2}} \rho d\lambda \qquad (31).$$

Expanding in powers of λ^2/z, and using equation (25) for ρ, we have

$$\Delta a = -a^2\rho_p\delta\left(\frac{2}{z}\right)^{\frac{1}{2}} \frac{(1+e)^{\frac{3}{2}}}{(1-e)^{\frac{1}{2}}} \int_{-\sqrt{2z}}^{\sqrt{2z}} \left\{1 - \frac{(8e - 3e^2 - 1)\lambda^2}{4z(1-e^2)} + 0\left(\frac{\lambda^4}{z^2}\right)\right\}$$

$$\times \left\{1 + \frac{2c\lambda^2}{x^2}(c\sin^2 2\omega - \cos 2\omega) + 0\left(\frac{\lambda^4}{x^4}\right)\right\} \exp(-\lambda^2)d\lambda \qquad (32).$$

In equation (32) the terms in λ and λ^3 from equation (25) have been dropped because they lead only to terms of odd degree in λ, which vanish when integrated between $-\sqrt{2z}$ and $\sqrt{2z}$.

Since $T_d \propto a^{\frac{3}{2}}$, we have

$$\Delta T_d = \frac{3T_d}{2a} \Delta a \qquad (33).$$

Substituting for Δa from equation (32), altering the limits of integration to $-\infty$ and ∞, using the integrals (28), and expressing x^2 in terms of z by (13), we have

$$\Delta T_d = -3\left(\frac{\pi}{2z}\right)^{\frac{1}{2}} aT_d\rho_p\delta \frac{(1+e)^{\frac{3}{2}}}{(1-e)^{\frac{1}{2}}}$$
$$\times \left[1 - \frac{8e - 3e^2 - 1}{8z(1-e^2)} + \frac{2c(1+e)}{z(1-e)} (c \sin^2 2\omega - \cos 2\omega) + 0\left(\frac{1}{z^2}\right)\right] \qquad (34).$$

3.1.5 Expression for $\Delta i/\Delta T_d$

We now divide equation (29) by equation (34). From equation (24),

$$\frac{2c}{z(1-e)} = \frac{\varepsilon \sin^2 i}{e} \qquad (35).$$

On using (35) and expanding in powers of $1/z$, we obtain

$$\frac{\Delta i}{\Delta T_d} = \frac{\Lambda \sin i\ (1-e)^{\frac{5}{2}}}{3\sqrt{F}(1+e)^{\frac{3}{2}}} \left[\cos^2\omega + \frac{(1+e)^2 - (2 + 2e + e^2)\cos^2\omega}{z(1-e^2)}\right.$$
$$\left. + \frac{\varepsilon}{e}(1+e)\sin^2 i \sin^2 2\omega + 0\left(\frac{1}{z^2}\right)\right] \qquad (36).$$

This is the required expression for $\Delta i/\Delta T_d$; it reduces to the same form as equation (3) when expanded in powers of e.

3.2 Effect on right ascension of the node

For Ω, the equation corresponding to (7) is equation (8.10) of Ref.3:

$$\frac{d\Omega}{dE} = -\frac{1}{2}\left\{\frac{r(1 + e\cos E)}{GMF(1 - e^2)}\right\}^{\frac{1}{2}} r^2\rho w\delta \sin(\omega + \theta)\cos(\omega + \theta) \qquad (37).$$

Thus the analysis is the same as for i, except that $\sin i$ does not appear and $\cos^2(\omega + \theta)$ in (7) is replaced by $\sin(\omega + \theta)\cos(\omega + \theta)$. From equations (15),

$$\sin(\omega + \theta)\cos(\omega + \theta) = \tfrac{1}{2}\sin 2\omega\left\{1 + \frac{2\lambda}{x}\cot 2\omega - \frac{2\lambda^2}{x^2} + 0\left(\frac{\lambda^3}{x^3}\right)\right\} \qquad (38).$$

So the Ω-equation corresponding to (18) is

$$\Delta\Omega = -\frac{1}{2}\left(\frac{1}{2Fz}\right)^{\frac{1}{2}} a(1 - e)^2\Lambda T_d\delta\sin 2\omega \int_{-\sqrt{2z}}^{\sqrt{2z}}\left\{1 + \frac{(1 + 8e + 11e^2)\lambda^2}{4(1 - e^2)z} + 0\left(\frac{\lambda^4}{z^2}\right)\right\}$$

$$\times\left[1 + \frac{2\lambda}{x}\cot 2\omega - \frac{2\lambda^2}{x^2} + 0\left(\frac{\lambda^3}{x^3}\right)\right]\rho\, d\lambda \qquad (39).$$

In equation (39) we insert the expression (25) for ρ, multiply out, ignore odd powers of λ and evaluate the integrals, as with i. We find:

$$\Delta\Omega = -\frac{1}{2}\left(\frac{\pi}{2Fz}\right)^{\frac{1}{2}} a(1 - e)^2\Lambda T_d\rho_p\delta\sin 2\omega$$

$$\times\left[1 - \frac{15 + 24e + 5e^2}{8z(1 - e^2)} + \frac{2c(1 + e)}{z(1 - e)}(c\sin^2 2\omega - 3\cos 2\omega) + 0\left(\frac{1}{z^2}\right)\right] \qquad (40).$$

Finally, dividing equation (40) by equation (34) and eliminating c by means of (35), we find:

$$\frac{\Delta\Omega}{\Delta T_d} = \frac{\Lambda\sin 2\omega\,(1 - e)^{\frac{5}{2}}}{6\sqrt{F}(1 + e)^{\frac{3}{2}}}\left[1 - \frac{2 + 2e + e^2}{z(1 - e^2)} - \frac{2\varepsilon}{e}(1 + e)\sin^2 i\cos 2\omega + 0\left(\frac{1}{z^2}\right)\right]$$

$$\ldots\ldots(41).$$

This is the required expression for $\Delta\Omega/\Delta T_d$; it reduces to the same form as equation (4) when expanded in powers of e.

3.3 Critique of the assumptions

We assume in equations (14) that an expansion in powers of λ/x is legitimate and we go on to neglect terms of order λ^4/x^4 or λ^4/z^2, leading to the neglect of terms of $O(1/z^2)$ in the final solutions. Since $z > 30$ and is normally of order 10^2, this assumption has *a posteriori* justification.

The assumption that c is of order 1, made in equation (25), is nearly always valid, since c cannot appreciably exceed 1 unless $H < 10\text{km}$, i.e. the perigee height is less than 115km - low enough to make most satellites decay within a few hours. A dense satellite may survive with a slightly lower perigee and an extreme upper limit for c of 2 should be considered. A possible extra term of $O(c^4\lambda^4/x^4)$ would then appear in equation (26), giving a possible extra term in (36) of $O(c^4/z^2) = O(c^2\varepsilon^2/e^2) < 10^{-3}$ if $c < 2$ and $e > 0.2$. This is less than the $O(1/z^2)$ at its maximum, and so does not deserve to be included.

4 EFFECT OF MERIDIONAL WINDS ON i AND Ω FOR ORBITS WITH $e > 0.2$ IN AN OBLATE ATMOSPHERE

4.1 Effect on i

If the south-to-north atmospheric rotation rate near perigee is Φ, i.e. the south-to-north wind component is $r\Phi$, the change in i due to Φ is obtained from equation (35) of Ref.6 as

$$\frac{di}{dE} = \tfrac{1}{2}r^2\rho\Phi\delta\left\{\frac{r(1 + e\cos E)}{GMF(1 - e^2)}\right\}^{\frac{1}{2}} \frac{\cos i \cos(\omega + \theta)}{\left\{1 - \sin^2 i \sin^2(\omega + \theta)\right\}^{\frac{1}{2}}} \qquad (42).$$

As before, we express r and E in terms of λ by (10), (11) and (16), write $\cos(\omega + \theta)$ in powers of λ/x using (15), and write

$$\Phi = \mu w_E = \mu T_d (GM)^{-\frac{1}{2}} a^{-\frac{3}{2}} \quad \text{by (8)} \qquad (43).$$

Equation (42) then becomes

$$\frac{di}{d\lambda} = \left(\frac{1}{2Fz}\right)^{\frac{1}{2}} \frac{a\mu T_d\rho\delta}{(1 - e^2)^{\frac{1}{2}}} \left(1 - e + \frac{e\lambda^2}{z}\right)^{\frac{5}{2}} \left(1 + e - \frac{e\lambda^2}{z}\right)^{\frac{1}{2}} \left(1 - \frac{\lambda^2}{2z}\right)^{-\frac{1}{2}} \cos i \cos\omega$$

$$\times \left\{1 - \frac{\lambda}{x}\tan\omega - \frac{\lambda^2}{2x^2} + O\left(\frac{\lambda^3}{x^3}\right)\right\}\left\{1 - \sin^2 i \sin^2(\omega + \theta)\right\}^{-\frac{1}{2}} \qquad (44).$$

Now, from equations (14),

$$1 - \sin^2 i \sin^2(\omega + \theta) = \alpha \left\{1 - \frac{\lambda \sin^2 i}{\alpha x}\left(\sin 2\omega + \frac{\lambda}{x}\cos 2\omega\right) + 0\left(\frac{\lambda^3}{x^3}\right)\right\} \quad (45)$$

where $\alpha = 1 - \sin^2 i \sin^2\omega = \cos^2 i + \sin^2 i \cos^2\omega$ (46).

From equation (45),

$$\{1 - \sin^2 i \sin^2(\omega+\theta)\}^{-\frac{1}{2}} = \alpha^{-\frac{1}{2}}\left\{1 + \frac{\lambda \sin^2 i \sin 2\omega}{2\alpha x} + \frac{\lambda^2 \sin^2 i}{2\alpha x^2}\left(\cos 2\omega + \frac{3\sin^2 i \sin^2 2\omega}{4\alpha}\right) + 0\left(\frac{\lambda^3}{x^3}\right)\right\} \quad (47).$$

In equation (44) we may now expand the first three main factors in powers of λ^2/z, insert ρ from (25), use (47) and integrate. This gives:

$$\Delta i = \left(\frac{1}{2Fz\alpha}\right)^{\frac{1}{2}} a\mu T_d \rho_p \delta(1-e)^2 \cos i \cos\omega \int_{-\sqrt{2z}}^{\sqrt{2z}} \left\{1 + \frac{(1 + 8e + 11e^2)\lambda^2}{4z(1-e^2)} + 0\left(\frac{\lambda^4}{z^2}\right)\right\}$$

$$\times \left\{1 - \frac{\lambda}{x}\tan\omega - \frac{\lambda^2}{2x^2}\right\}\left\{1 + \frac{\lambda \sin^2 i \sin 2\omega}{2\alpha x} + \frac{\lambda^2 \sin^2 i}{2\alpha x^2}\left(\cos 2\omega + \frac{3\sin^2 i \sin^2 2\omega}{4\alpha}\right) + 0\left(\frac{\lambda^3}{x^3}\right)\right\}$$

$$\times \left\{1 - \frac{2c\lambda}{x}\sin 2\omega + \frac{2c\lambda^2}{x^2}(c\sin^2 2\omega - \cos 2\omega) + 0\left(\frac{\lambda^3}{x^3}\right)\right\}\exp(-\lambda^2)d\lambda \quad (48).$$

On multiplying out the brackets, ignoring all odd powers of λ, and replacing the limits by $-\infty$ and $+\infty$ as before, we find that equation (47) reduces to

$$\Delta i = \left(\frac{1}{2Fz\alpha}\right)^{\frac{1}{2}} a\mu T_d \rho_p \delta(1-e)^2 \cos i \cos\omega \int_{-\infty}^{\infty}\left[1 + \frac{(7e^2 - 3)\lambda^2}{4z(1-e^2)} + \frac{\lambda^2}{x^2} f(i,\omega) + \frac{2c\lambda^2}{x^2}\left\{1 - 2\cos 2\omega + \left(c - \frac{\sin^2 i}{2\alpha}\right)\sin^2 2\omega\right\} + 0\left(\frac{\lambda^4}{z^2}\right)\right]\exp(-\lambda^2)d\lambda \quad (49)$$

where $$f(i,\omega) = \frac{\sin^2 i}{2\alpha}\left(\cos^2\omega - \frac{3\cos^2 i \sin^2\omega}{\alpha}\right) \tag{50}$$

When we evaluate the integrals in (49), and eliminate x^2 using (13), we obtain

$$\Delta i = \left(\frac{\pi}{2Fz\alpha}\right)^{\frac{1}{2}} a\mu T_d \rho_p \delta (1-e)^2 \cos i \cos\omega$$
$$\times \left[1 + \frac{7e^2-3}{8z(1-e^2)} + \frac{1+e}{z(1-e)} f(i,\omega) + \frac{2c(1+e)}{z(1-e)}\left\{1 - 2\cos 2\omega + \left(c - \frac{\sin^2 i}{2\alpha}\right)\sin^2 2\omega\right\} + 0\left(\frac{1}{z^2}\right)\right] \tag{51}$$

Dividing (51) by (34), eliminating c by (35), and restoring the explicit form (46) for α, we find

$$\frac{\Delta i}{\Delta T_d} = -\frac{\mu(1-e)^{\frac{5}{2}} \cos i \cos\omega}{3\sqrt{F}(1+e)^{\frac{3}{2}}(\cos^2 i + \sin^2 i \cos^2\omega)^{\frac{1}{2}}}$$
$$\times \left[1 + \frac{e^2+2e-1}{2z(1-e^2)} + \frac{(1+e)f(i,\omega)}{z(1-e)} + \frac{2\varepsilon(1+e)\sin^2 i \cos^2 i \sin^2\omega}{e(\cos^2 i + \sin^2 i \cos^2\omega)} + 0\left(\frac{1}{z^2}\right)\right] \tag{52}$$

4.2 Effect on Ω

Equation (36) of Ref.6 gives the change in Ω as

$$\frac{d\Omega}{dE} = \tfrac{1}{2}\Phi r^2 \rho\delta \left\{\frac{r(1+e\cos E)}{GMF(1-e^2)}\right\}^{\frac{1}{2}} \frac{\cot i \sin(\omega+\theta)}{\{1 - \sin^2 i \sin^2(\omega+\theta)\}^{\frac{1}{2}}} \tag{53}$$

Equation (53) is similar to equation (42) for i, except that (53) has $\cot i \sin(\omega+\theta)$ where (42) has $\sin i \cos(\omega+\theta)$. So the analysis is similar, and the Ω-equation corresponding to (48) is

$$\Delta\Omega = \left(\frac{1}{2Fz\alpha}\right)^{\frac{1}{2}} a\mu T_d \rho_p \delta (1 - e)^2 \cot i \sin \omega \int_{-\sqrt{2z}}^{\sqrt{2z}} \left[1 + \frac{(1 + 8e + 11e^2)\lambda^2}{4z(1 - e^2)} + O\left(\frac{\lambda^4}{z^2}\right)\right]$$

$$\times \left\{1 + \frac{\lambda}{x} \cot \omega - \frac{\lambda^2}{2x^2}\right\}\left\{\ldots\right\}\left\{\ldots\right\} \exp(-\lambda^2) d\lambda \qquad (54)$$

where the curly brackets filled with dots are identical to the last two curly brackets in (48).

On multiplying out the brackets, ignoring all odd powers of λ, and evaluating the integrals, as for i, we find, as the Ω-equation corresponding to (51):

$$\Delta\Omega = \left(\frac{\pi}{2Fz\alpha}\right)^{\frac{1}{2}} a\mu T_d \rho_p \delta (1 - e)^2 \cot i \sin \omega$$

$$\times \left[1 + \frac{7e^2 - 3}{8z(1 - e^2)} - \frac{1 + e}{2z(1 - e)} g(i,\omega)\right.$$

$$\left. - \frac{2c(1 + e)}{z(1 - e)} \left\{2 \cos 2\omega + 1 - \left(c - \frac{\sin^2 i}{2\alpha}\right) \sin^2 2\omega\right\} + O\left(\frac{1}{z^2}\right)\right] \qquad (55)$$

where $$g(i,\omega) = \frac{\sin^2 i}{\alpha}\left(1 - 4 \cos^2 \omega - \frac{3 \sin^2 i \sin^2 2\omega}{4\alpha}\right) \qquad (56).$$

Dividing (55) by (34), we obtain

$$\frac{\Delta\Omega}{\Delta T_d} = - \frac{\mu(1 - e)^{\frac{5}{2}} \cot i \sin \omega}{3\sqrt{F}(1 + e)^{\frac{3}{2}} (\cos^2 i + \sin^2 i \cos^2 \omega)^{\frac{1}{2}}}$$

$$\times \left[1 + \frac{e^2 + 2e - 1}{2z(1 - e^2)} - \frac{(1 + e)g(i,\omega)}{2z(1 - e)} - \frac{2\varepsilon(1 + e) \sin^2 i \cos^2 \omega}{e(\cos^2 i + \sin^2 i \cos^2 \omega)} + O\left(\frac{1}{z^2}\right)\right]$$

$$\ldots\ldots(57).$$

5 DISCUSSION OF RESULTS

5.1 Combined effects of zonal and meridional winds

From equations (36) and (52) the combined effect on i of an axial rotation rate Λ rev/day and a south-to-north meridional wind equivalent to a rotation rate of μ rev/day is given by

$$\frac{\Delta i}{\Delta T_d} = \frac{(1-e)^{\frac{5}{2}}}{3\sqrt{F}(1+e)^{\frac{3}{2}}}$$

$$\times \left[\Lambda \sin i \left\{ \cos^2\omega + \frac{(1+e)^2 - (2+2e+e^2)\cos^2\omega}{z(1-e^2)} + \frac{\varepsilon}{e}(1+e)\sin^2 i \sin^2 2\omega + 0\left(\frac{1}{z^2}\right) \right\} \right.$$

$$- \frac{\mu \cos i \cos\omega}{(\cos^2 i + \sin^2 i \cos^2\omega)^{\frac{1}{2}}} \left\{ 1 + \frac{e^2 + 2e - 1}{2z(1-e^2)} + \frac{(1+e)f(i,\omega)}{z(1-e)} \right.$$

$$\left.\left. + \frac{2\varepsilon(1+e)\sin^2 i \cos^2 i \sin^2\omega}{e(\cos^2 i + \sin^2 i \cos^2\omega)} + 0\left(\frac{1}{z^2}\right) \right\} \right] \qquad \text{......(58)}$$

where $f(i,\omega)$ is given by (50).

Similarly, from equations (41) and (57), the effect on Ω is given by

$$\frac{\Delta\Omega}{\Delta T_d} = \frac{(1-e)^{\frac{5}{2}}\sin\omega}{3\sqrt{F}(1+e)^{\frac{3}{2}}}$$

$$\times \left[\Lambda \cos\omega \left\{ 1 - \frac{2+2e+e^2}{z(1-e^2)} - \frac{2\varepsilon}{e}(1+e)\sin^2 i \cos 2\omega + 0\left(\frac{1}{z^2}\right) \right\} \right.$$

$$- \frac{\mu \cot i}{(\cos^2 i + \sin^2 i \cos^2\omega)^{\frac{1}{2}}} \left\{ 1 + \frac{e^2 + 2e - 1}{2z(1-e^2)} - \frac{(1+e)g(i,\omega)}{2z(1-e)} \right.$$

$$\left.\left. - \frac{2\varepsilon(1+e)\sin^2 i \cos^2\omega}{e(\cos^2 i + \sin^2 i \cos^2\omega)} + 0\left(\frac{1}{z^2}\right) \right\} \right] \qquad (59)$$

where $g(i,\omega)$ is given by (56).

5.2 The effect of zonal winds

Taking $\mu = 0$ in equations (58) and (59) gives the effect of zonal winds alone. The second and third terms within the curly brackets in (58) and (59) become negligible as e (and hence z) become large, so we may assess their importance by taking the lowest values of e and z, namely 0.2 and 30. The terms are then $(0.05 - 0.08\cos^2\omega)$ and $0.02\sin^2 i\sin^2 2\omega$, in equation (58). Thus both terms are small, although it should be noted that the main term is also small if ω is near 90° or 270°. So, for a given change in T and given Λ, Δi is normally proportional to $(1-e)^{\frac{5}{2}}(1+e)^{-\frac{3}{2}}\sin i\cos^2\omega$. Fig.2 shows the variation of $(1-e)^{\frac{5}{2}}(1+e)^{-\frac{3}{2}}$ with e, for $0.2 < e < 0.9$.

For $\Delta\Omega$, the variation with e is the same, $\Delta\Omega$ being proportional to $(1-e)^{\frac{5}{2}}(1+e)^{-\frac{3}{2}}\sin 2\omega$; but $\Delta\Omega$ is to a first approximation independent of i, and has its maxima at $\omega = 45^\circ, 135^\circ, \ldots,$ being zero for $\omega = 0, 90^\circ, 180^\circ, \ldots$

5.3 The effect of meridional winds

Taking $\Lambda = 0$ in equations (58) and (59) gives the effect of meridional winds alone. Again the second, third and fourth terms in curly brackets are largest when $e = 0.2$ and $z = 30$, and in the i-equation (58) these three terms have the values

$$-0.01,\ 0.05f(i,\omega) \quad \text{and} \quad \frac{0.04\sin^2 i\cos^2 i\sin^2\omega}{\cos^2 i + \sin^2 i\cos^2\omega}$$

respectively. Generally, therefore, these three terms are small compared with the main term, and the major variation is that due to e, given by Fig.2. The same conclusion applies to equation (59) for Ω.

For the meridional winds, the effect of variations in i and ω on Δi is best seen by writing

$$\frac{\cos i\cos\omega}{(\cos^2 i + \sin^2 i\cos^2\omega)^{\frac{1}{2}}} = \frac{1}{(\sec^2\omega + \tan^2 i)^{\frac{1}{2}}} .$$

This factor is greatest when $\sec^2\omega$ is smallest, i.e. when $\omega = 0$ or 180° - when perigee is on the equator: the factor then reduces to $\cos i$. Even this maximum effect of meridional winds, therefore, is not especially large.

The corresponding factor for $\Delta\Omega$ is $\cot i\sin\omega\,(1 - \sin^2 i\sin^2\omega)^{-\frac{1}{2}}$, and this has its greatest numerical value when $\sin\omega$ is greatest, *i.e.* at apex when $\omega = 90^\circ$ or 270°: the factor then becomes $\pm\operatorname{cosec} i$. So the rate of change of Ω is large for near-equatorial orbits, as always happens unless the force producing the perturbation in Ω vanishes when the orbit becomes equatorial: for zonal winds the force does vanish, but not for meridional winds.

The effect of variations in e is the same for meridional as for zonal winds, and is given by Fig.2.

Equation (58) shows that, to the first order, there is a symmetry between the effects of zonal and meridional winds on inclination if perigee is on the equator, for then

$$\frac{\Delta i}{\Delta T_d} = \frac{(1-e)^{\frac{5}{2}}}{3\sqrt{F}(1+e)^{\frac{3}{2}}}\left[\Lambda \sin i \pm \mu \cos i + O\left(\frac{1}{z}\right)\right] .$$

(The $+$ is for $\omega = 180^{\circ}$, the minus for $\omega = 0$.) No such symmetry exists for $\Delta\Omega$.

So far we have assumed that the functions $f(i,\omega)$ and $g(i,\omega)$ in equations (58) and (59) are not unduly large, but this assumption is not always valid. Figs.3 and 4 show the variations of $f(i,\omega)$ and $g(i,\omega)$ with i and ω . (Only the squares of sines and cosines occur in f and g , which are therefore only plotted for $0 \leqslant \{i,\omega\} \leqslant 90^{\circ}$.) When i and ω are both less than 60°, $|f| < 1$ and $|g| < 2.3$: the f and g terms are then likely to be negligible. However, when i and ω are both near 90°, large values of f and g may arise and the terms need to be computed. At $i = \omega = 90^{\circ}$ there is a singularity which ought to be noted and avoided in programming equations (58) and (59) for numerical computation. The singularity is of no significance physically, because Lagrange's planetary equations, from which (42) and (53) are derived, show that both Δi and $\Delta\Omega$ are proportional to the force normal to the orbit, which is zero when a polar orbit experiences a purely meridional wind*.

5.4 Day-to-night variations in density

In developing the theory, we have taken the surfaces of constant atmospheric density to be oblate spheriods. However, the actual constant-density

* The singularity arises through a substitution made in equation (19) of Ref.6, where the angle λ between the orbit and the easterly direction is eliminated by the equation $\cos\lambda = \cos i/\cos\phi$, where ϕ is the latitude $(\leqslant i)$. When $i = \phi = 90^{\circ}$ the value of λ from this equation is indeterminate, though a geographical approach shows that $\lambda = 90^{\circ}$ when $i = 90^{\circ}$, whatever the value of ϕ .

surfaces also have a slight diurnal bulge towards the Sun[12], the density having a maximum at local times near 14h and a minimum near 04h. We have previously[5] assessed the effect of this asymmetry for orbits of eccentricity less than 0.2, and found that it had a negligible influence except for certain special orbits with eccentricities of order 0.01 and perigee heights near 500km. When the asymtotic expansions of the Bessel functions are used in equation (27) of Ref.5, it is seen that the contribution to Δi of the day-to-night variation in density has $1/z$ as a multiplying factor, and hence becomes even less important as the eccentricity increases above 0.2. So the day-to-night variation in density can be ignored.

5.5 Variation of scale height H with height

We have developed the theory on the assumption that the density is given by equation (19), with constant density scale height H. In reality H varies with height, and the variation of density with r may more realistically be represented by the equation[13]

$$\rho = \rho_p \left\{1 + \frac{\kappa}{2H_p^2}(r - r_p)^2\right\} \exp\left(-\frac{r - r_p}{H_p}\right) \tag{61}$$

where H_p is the value of H at perigee and κ is, very nearly, the gradient of H, so that $\kappa \simeq dH/dr$: we assume that κ is constant and less than 0.2, and ignore terms of order $\frac{1}{2}\kappa^2$. Eliminating $(r - r_p)$ by use of equation (22), we may write equation (61) as

$$\rho = \rho_p\left\{1 + \tfrac{1}{2}\kappa Z^2(1 - \cos E)^2\right\} \exp\{- Z(1 - \cos E)\} \tag{62}$$

where $$Z = ae/H_p \tag{63}.$$

If we redefine λ by writing $z = Z$ in equation (10), equation (62) reduces to

$$\rho = \rho_p(1 + \tfrac{1}{2}\kappa\lambda^4) \exp(- \lambda^2) \tag{64}.$$

Substituting this expression for ρ into equation (18), we obtain equation (27) with $c = 0$, $z = Z$ and an extra factor $\{1 + \frac{1}{2}\kappa\lambda^4\}$ within the integrand. On integration this leads to an extra term, and equation (29) becomes

$$\Delta i = -\left(\frac{\pi}{2FZ}\right)^{\frac{1}{2}} a(1-e)^2 \Lambda T_d \rho_p \delta \sin i \cos^2\omega$$

$$\times \left[1 + \frac{3\kappa}{8} + \frac{8(1+e)^2 \sec^2\omega - (15 + 24e + 5e^2)}{8Z(1-e^2)} + 0\left(\frac{\kappa}{Z}, \frac{1}{Z^2}\right)\right] \qquad (65).$$

To reduce equation (65) to the same form as (29), to order κ, we have to write

$$Z = z'(1 + \tfrac{3}{4}\kappa) \qquad (66).$$

Substituting (66) into (65) gives

$$\Delta i = -\left(\frac{\pi}{2Fz'}\right)^{\frac{1}{2}} a(1-e)^2 \Lambda T_d \rho_p \delta \sin i \cos^2\omega$$

$$\times \left[1 + \frac{8(1+e)^2 \sec^2\omega - (15 + 24e + 5e^2)}{8z'(1-e^2)} + 0\left(\kappa^2, \frac{\kappa}{z'}, \frac{1}{z'^2}\right)\right] \qquad (67).$$

The neglected $0(\kappa^2)$ term in (67) is $9\kappa^2/128$, which is less than 0.003 if $\kappa < 0.2$. Apart from the 0 terms, equation (67) is the same as (29), with z replaced by z'. Hence, to the first order in κ, the effect of variation of H with height may be allowed for by using

$$z' = \frac{ae}{H_p(1 + \frac{3}{4}\kappa)} \qquad (68)$$

instead of $z = ae/H$. In other words, the equations previously derived may be used unchanged, provided the scale height H is evaluated at a height $\frac{3}{4}$ of a scale height above perigee: for, since κ is the gradient of H,

$$H = H_p(1 + \tfrac{3}{4}\kappa)$$

at a height $\frac{3}{4}H_p$ above perigee. The same conclusions apply for Ω and for meridional winds.

6 CONCLUSIONS

We have considered a satellite moving in an orbit of eccentricity > 0.2 through an oblate atmosphere, and have derived expressions for the changes in inclination and right ascension of the node caused by zonal and meridional winds, equations (58) and (59). The theory is developed for an atmosphere in which the scale height H is constant, but the effect of variations in H can be allowed for by evaluating H at $\frac{3}{4}$ of a scale height above perigee. Day-to-night variations in density have negligible effects. The equations, being expressed in terms of the change in orbital period, are independent of variations in air density or satellite cross-sectional area. They are also unaffected by lunisolar perturbations unless these are large during one revolution of the satellite.

The changes Δi and $\Delta\Omega$ depend strongly on the values of e, i and ω. All depend on e through the function $(1 - e)^{\frac{5}{2}}(1 + e)^{-\frac{3}{2}}$, plotted in Fig.2. The dependence on i and ω is more complicated. For zonal winds, Δi is proportional to $\sin i \cos^2\omega$, and $\Delta\Omega$ is proportional to $\sin 2\omega$, if only the main terms are considered. For meridional winds, Δi is proportional to $(\sec^2\omega + \tan^2 i)^{-\frac{1}{2}}$, which ranges between zero, when $i = 90^\circ$, and $\cos\omega$, when $i = 0$; $\Delta\Omega$ is proportional to $\cot i \sin\omega\,(1 - \sin^2 i \sin^2\omega)^{-\frac{1}{2}}$, which ranges between zero when $\omega = 0$ and $\operatorname{cosec} i$ when $\omega = 90^\circ$.

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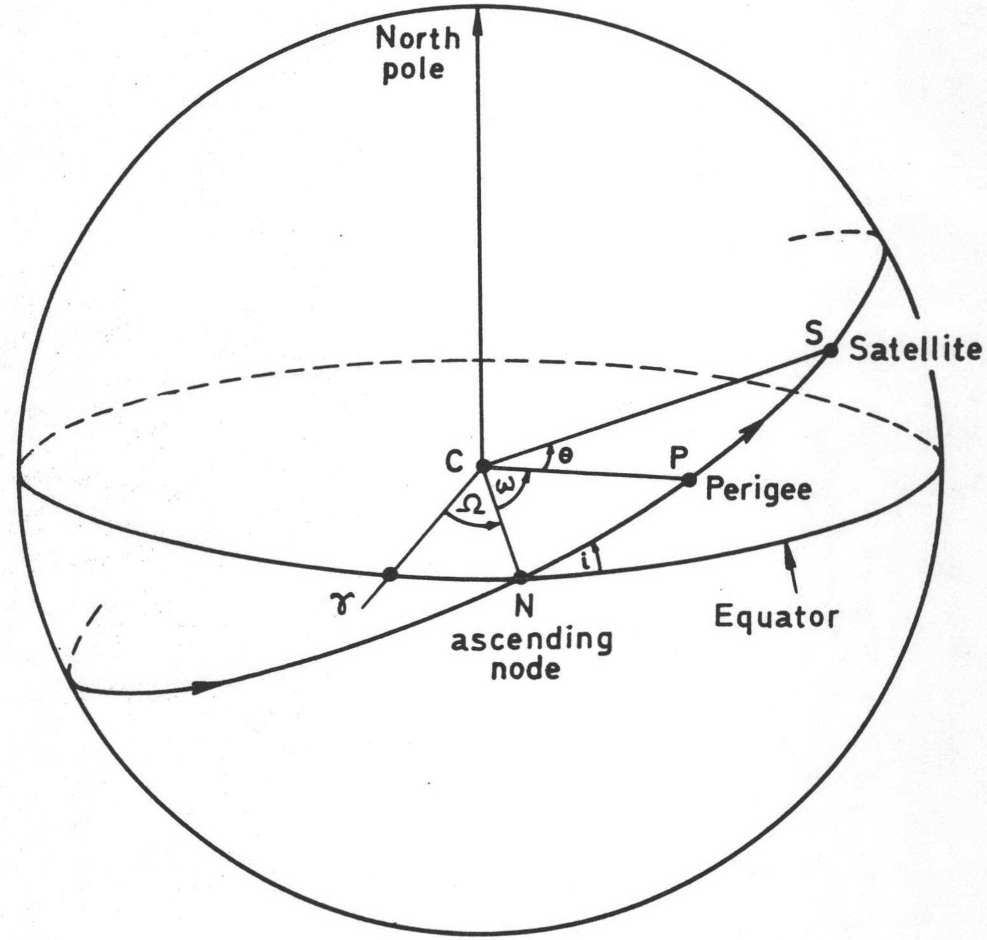


Fig.1 Projection of satellite orbit on unit sphere with centre at the Earth's centre C, showing inclination i, right ascension Ω, argument of perigee ω and true anomaly θ.
(γ is first point of Aries)

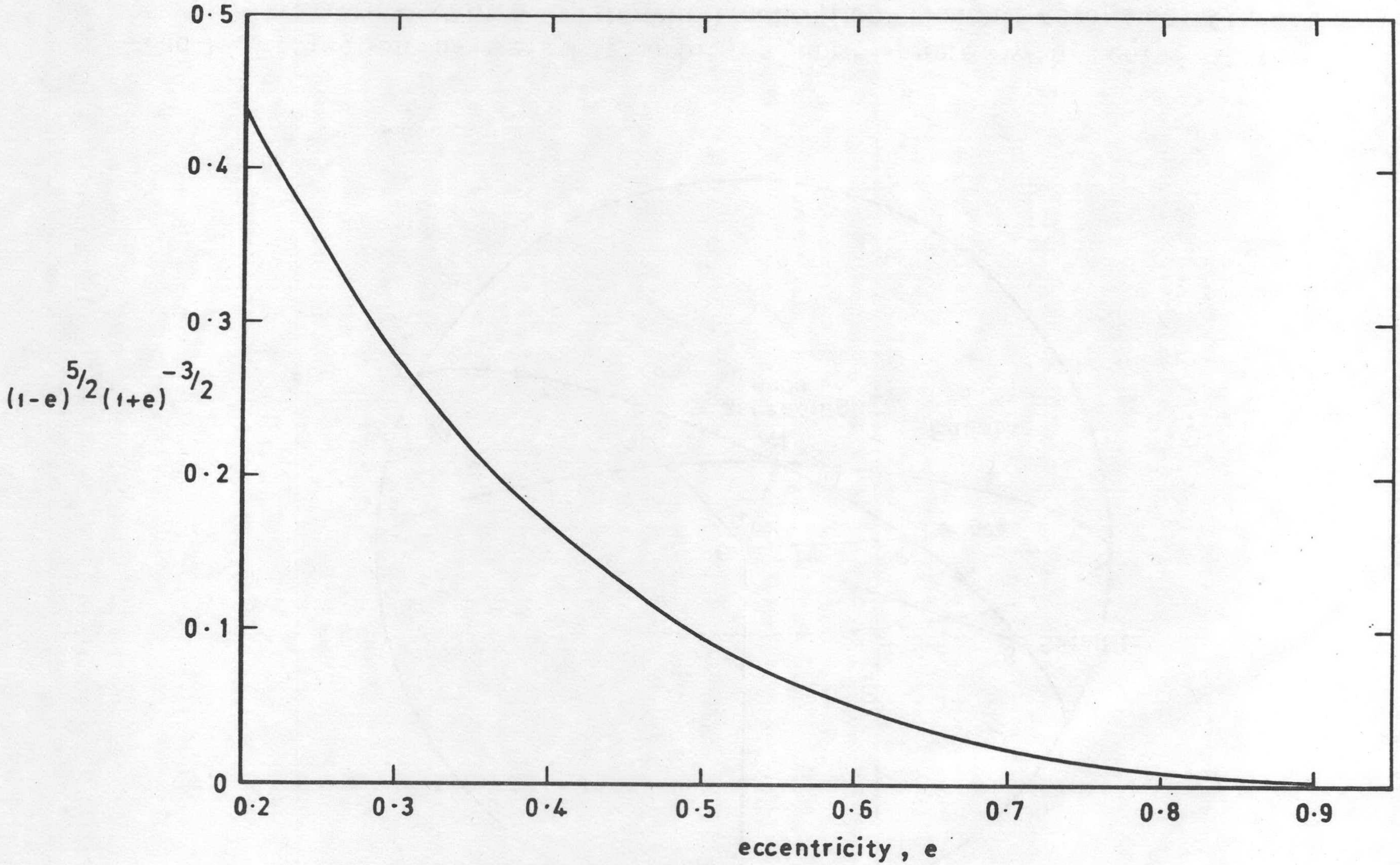
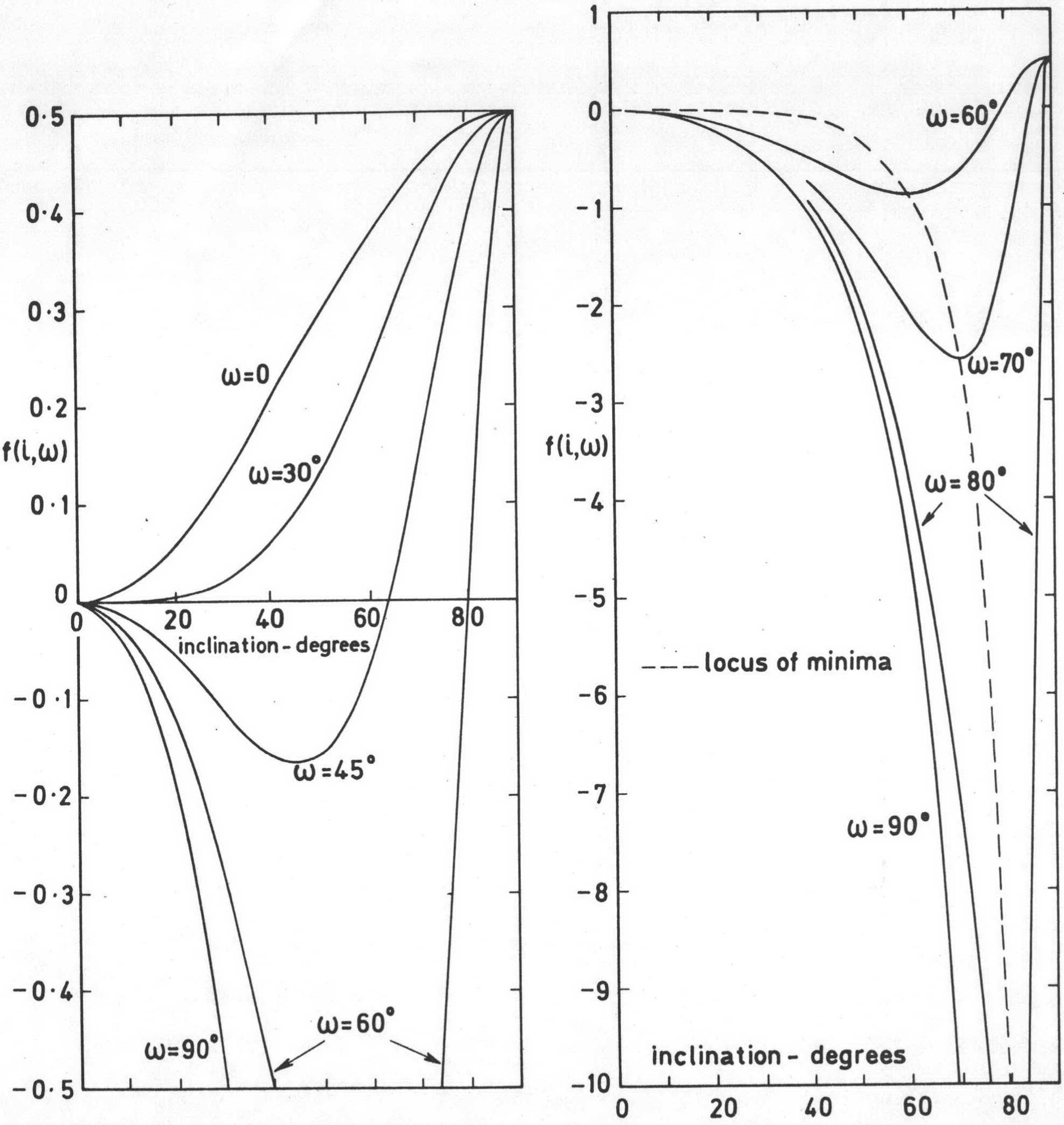


Fig.2 Variation of the function $(1-e)^{5/2}(1+e)^{-3/2}$



(a) When |f| < 0·5

(b) When |f| < 10

Fig. 3a & b Variation of f(i,ω) with i and ω

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Fig. 4

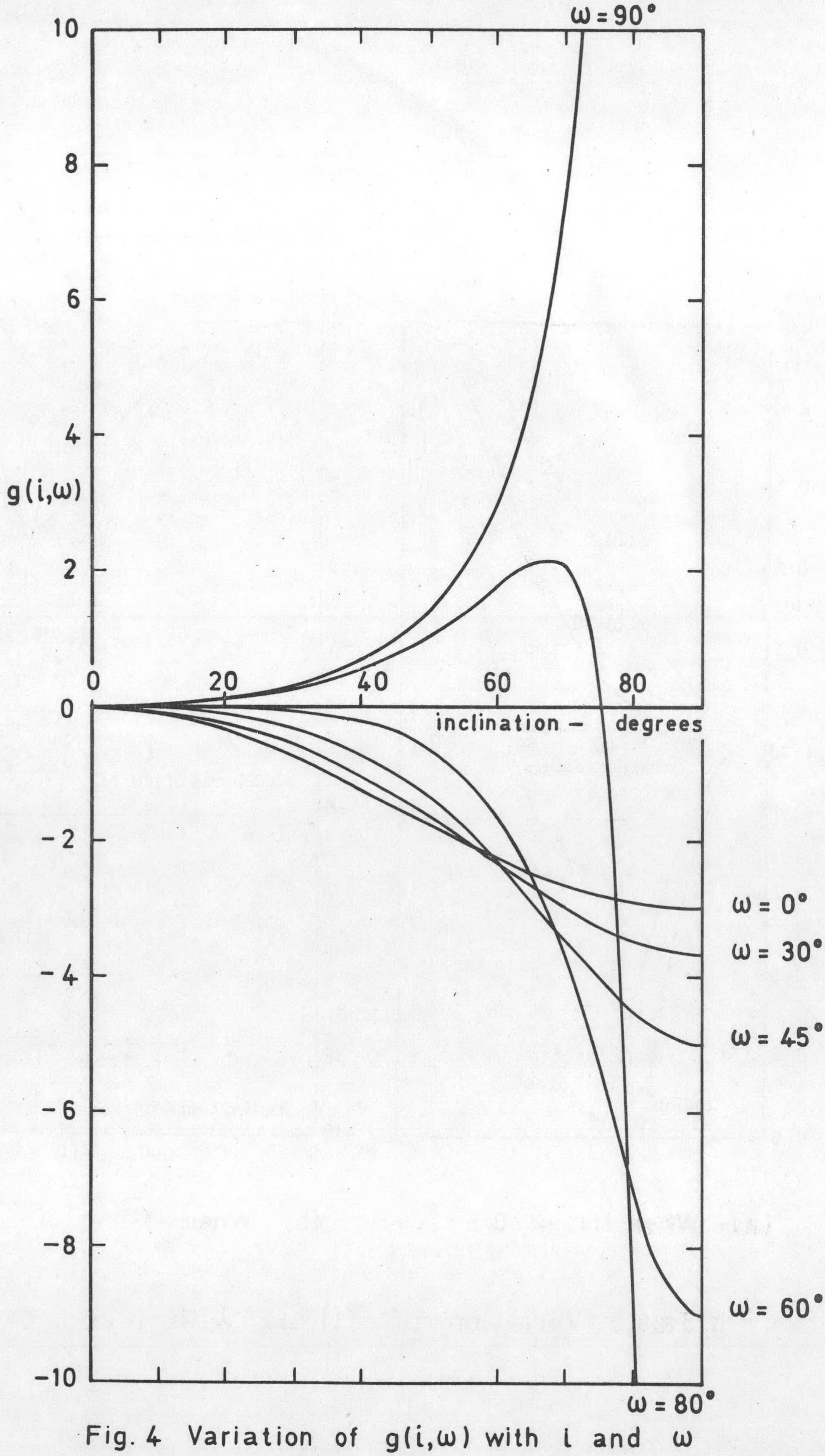

Fig. 4 Variation of g(i,ω) with i and ω

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1. DRIC Reference (to be added by DRIC)	2. Originator's Reference	3. Agency Reference	4. Report Security Classification/Marking
	RAE TR 75150	N/A	UNCLASSIFIED

5. DRIC Code for Originator	6. Originator (Corporate Author) Name and Location
850100	Royal Aircraft Establishment, Farnborough, Hants, UK
5a. Sponsoring Agency's Code	**6a. Sponsoring Agency (Contract Authority) Name and Location**
N/A	N/A

7. Title The effect of atmospheric winds on satellite orbits of high eccentricity

7a. (For Translations) Title in Foreign Language

7b. (For Conference Papers) Title, Place and Date of Conference

8. Author 1. Surname, Initials	9a. Author 2	9b. Authors 3, 4	10. Date	Pages	Refs.
King-Hele, D.G.	Walker, Doreen M.C.		January 1976	29	13
11. Contract Number	**12. Period**	**13. Project**	**14. Other Reference Nos.**		
N/A	N/A				

15. Distribution statement
(a) Controlled by –
(b) Special limitations (if any) –

16. Descriptors (Keywords) (Descriptors marked * are selected from TEST)

Upper atmosphere. Winds. Orbital theory.

17. Abstract

The orbits of Earth satellites with perigee heights less than 600km are liable to be appreciably perturbed by the aerodynamic forces resulting from winds in the upper atmosphere, and analysis of the changes in the orbits provides a method of determining zonal (west-to-east) and meridional (north-to-south) winds. The theory hitherto used has been developed for orbits of eccentricity $e < 0.2$. Here we develop the theory for the effect of zonal and meridional winds on the inclination i and right ascension of the node Ω for satellites in orbits with $e > 0.2$ moving in an oblate atmosphere. The results are expressed in terms of the change in orbital period, which is accurately known for actual satellites, so that the equations are independent of variations in air density and satellite cross-sectional area.

The results, summarized in equations (58) and (59), show that the changes depend on e through the function $(1 - e)^{\frac{5}{2}}(1 + e)^{-\frac{3}{2}}$. For zonal winds, the change in i is nearly proportional to $\sin i \cos^2\omega$ and the change in Ω to $\sin 2\omega$, where ω is the argument of perigee. For meridional winds, the change in i is nearly proportional to $(\sec^2\omega + \tan^2 i)^{-\frac{1}{2}}$, and the change in Ω to $\cot i \sin\omega(1 - \sin^2 i \sin^2\omega)^{-\frac{1}{2}}$.

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