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# A liquid of optical vortices in a photonic sea of vector waves

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**Abstract:**    Phase singularities arise in scalar random waves, with spatial distribution reminiscent of particles in liquids. Supporting near-field experiment with analytical theory we show how such spatial distribution changes when considering vector waves.

**OCIS codes:**  (260.6042) Singular optics; (140.1540) Chaos; (180.4243) Near-field microscopy.

Chaotic systems are everywhere around us, and unearthing correlations in their structure is a powerful route towards understanding them. With respect to optics, the investigation of random wave fields has already led to outstanding phenomena like the Anderson localization of light [1], or to more recent fascinating observations as rogue waves in photonic seas [2]. However, when considering the structure of a random wave field is useful to recognize the presence of deep-subwavelength dislocations known as phase singularities [3]. These points of undetermined phase abundantly arise in scalar random waves, where their spatial distribution exhibits a clear correlation structure, reminiscent of that of particles in a liquid [4,5]. Light is a wave field, but it is in general vectorial in nature. We investigate the distribution of phase singularities in a photonic sea of random optical waves, of which the vectorial nature cannot be ignored.

We generate optical random waves by injecting monochromatic light ($\lambda = 1550$ nm) in a silicon-on-insulator photonic crystal cavity. The shape of this chaotic cavity is a quarter of a stadium, so that the resulting field pattern consists of a superposition of plane waves interfering with the same momentum $k$ and random phases $\delta_{\mathbf{k}}$ [2], i.e.,

$$\mathbf{E}(\mathbf{r}) = \sum_{\mathbf{k}} \mathbf{A}_k \exp(i\mathbf{k} \cdot \mathbf{r} + i\delta_{\mathbf{k}}). \tag{1}$$

With phase- and polarization-resolved near-field measurement we map the in-plane optical field $(E_x, E_y)$ above the cavity [6]. Figure 1 presents as an example a subsection of the measured amplitude $A_x$ (a) and phase $\varphi_x$ (b) of the $E_x$ field. Additionally, figure 1(b) shows the phase singularities that arise in such field component. The position of phase singularities is determined together with their topological charge $s$, defined by the circulation of the phase around the singular points [Fig. 1(b)].
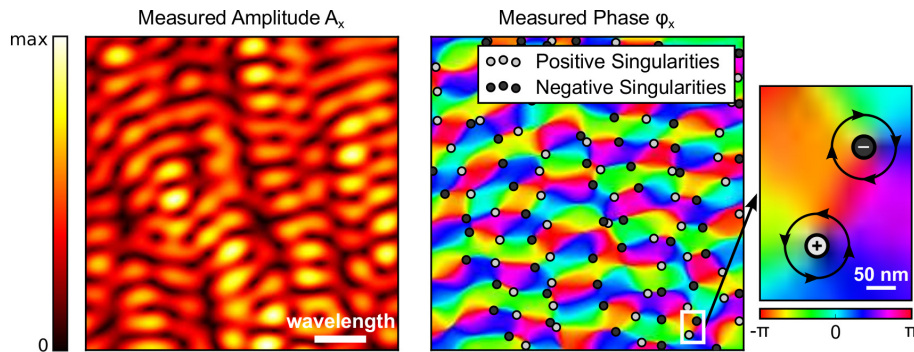


Fig. 1. Measured amplitude (a) and phase (b) of the $x$ component of the in-plane optical near-field measured above a chaotic cavity. Phase singularities are represented in the phase map with their topological charge: +1 (positive) or -1 (negative). The zoom highlights how the direction of the circulation of the phase around the singular point determines its topological charge.

A natural way of describing the distribution of phase singularities is by calculating their pair ($g$) and charge ($g_Q$) correlation function [4, 5]:

$$g(r) = \frac{1}{N\rho} \langle \sum_{i \neq j} \delta(r - |\mathbf{r}_j - \mathbf{r}_i|) \rangle \quad \text{and} \quad g_Q(r) = \frac{1}{N\rho} \langle \sum_{i \neq j} \delta(r - |\mathbf{r}_j - \mathbf{r}_i|) s_i s_j \rangle, \tag{2}$$

where $N$ is the total number of singularities, $\rho$ the surrounding density, and the Dirac function $\delta$ selects the pairs $(i, j)$ of singularities displaced by a distance $r$. Figure 2(a) presents $g(r)$ and $g_Q(r)$ calculated from our experimental data (circles) of the $E_x$ field (similar for $E_y$). We compare these distribution functions with the analytical model for scalar random waves [4] (solid gray lines), observing some significant quantitative and qualitative deviation.

The model for scalar random waves is built on the assumption of an isotropic distribution of waves. Contrarily, in vector waves there is a direct relation between field and propagation direction, that causes an anisotropy in the distribution of the waves in the single field components. This anisotropy violates the assumption of ref. [4].
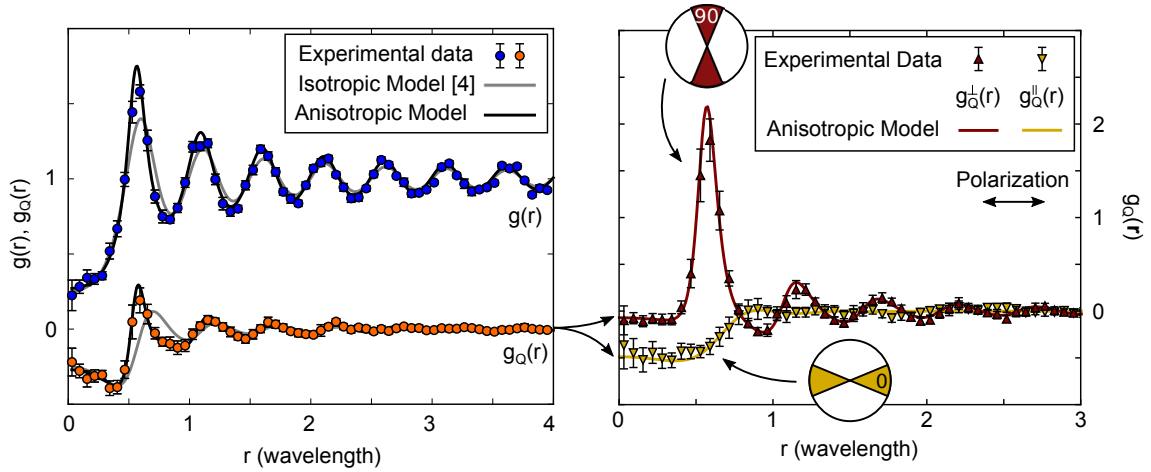


Fig. 2. (a) Pair (blue) and charge (orange) correlation function of phase singularities in the $E_x$ field. The experimental results (circles) are compared with the analytical model for isotropic scalar random waves (gray lines) and with our new model for anisotropic random waves of light (black lines). (b) Direction-dependent charge correlation function. The $g_Q(\mathbf{r})$ is averaged within angles $\pi/4$ around the directions parallel and perpendicular to the field projection axis.

Starting from the theory for scalar random waves, we develop a new theory that accounts for the vectorial nature of light. The outcome of this model (Anisotropic Model) is reported with solid black lines in Fig. 2(a). The agreement between experiment and new theory is now excellent. Moreover, the anisotropic wave distribution also results in an anisotropic distribution of phase singularities. A clear evidence of this is reported in Fig. 2(b), in which we show that the charge correlation function $g_Q(\mathbf{r})$ presents a strong direction-dependence. Agreement between experiment and the new theory is now excellent.

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