Hydrodynamic Erosion Process of Undisturbed Clay

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Abstract: This paper describes the hydrodynamic erosion process of undisturbed clay due to the turbulent flow, based on theoretical analysis and experimental results. The undisturbed clay has the unique and complicated characteristics of cohesive force among clay particles, which are highly different from disturbed clay and non-cohesive sand. Based on momentum equilibrium, the critical incipient velocity is derived from the forces of particle weight under water, cohesive force among particles around the clay particle, uplift force and drag force. Via the turbulent boundary layer flow theory, the critical stress can be connected with these forces. The formulae for the incipient stress and critical velocity have been calibrated and validated with the results of undisturbed clay tests. A new formula for the erosion rate is proposed. The study gives a new insight into the erosion process of undisturbed clay and the resulting sediment transport.

Keywords: erosion process; undisturbed clay; turbulent boundary-layer flow; incipient stress

1. Introduction

The erosion of undisturbed clay by flowing water is an important issue in many hydraulic engineering problems, such as the stability of clay coasts, the stability of clay layers in river beds and flood plains, scour around bridge piers, the stability of embankments and the breach development in dikes and dams (see Zhu et al., 2008; Knapen et al., 2007; Merritt et al., 2003). Clay offers resistance due to its strength, in particular its cohesive strength. Resistance of clay to erosion depends on a large number of parameters, i.e. size and shape of the grains, grain size distribution, mineral composition, chemical properties, water temperature, etc. and the characteristics of the flow over the sediments. Undisturbed clay has a large range of densities, i.e. from about 1500 kg/m³ to about 2000 kg/m³, while the density of non-cohesive sediment in streams is usually about 2000 kg/m³.

Due to the large number of parameters involved, is the identification and prediction of undisturbed clay erosion a complicated problem in geomorphology and hydraulic engineering. While the pick up of non-cohesive sediment through discrete particle entrainment may be quantified (if the flow velocities are not very large) by the magnitude of shear stress and particle size, undisturbed clay, however, is eroded through entrainment of aggregates. The cohesive strength between and within aggregates causes the erosion to be complex.

Only a few studies on undisturbed clay have been performed in flumes. These experiments were done in a modified rotating cylinder apparatus, by a drill-hole method, and in a wave tank (Howing et al., 1998; Julian, 2006). These tests confirm that clay is very resistant to erosion when undisturbed.

This paper discusses the characteristics of undisturbed clay and analyzes the strengths between and within undisturbed clay particles. Moreover, a primary dynamic erosion model is given for undisturbed clay in different flow conditions. Then an experimental study on the erosion of undisturbed clay is described. Finally the erosion model is analyzed with experimental data.

2. Characteristics of undisturbed clay affecting erosion

Undisturbed clay is usually collected from embankments (dikes, dams, etc.) or other fields without any disturbance. So, it is a type of complex sediment compared to non-cohesive sediment for its unique properties. The properties of undisturbed clay can be considered as a combination of clay properties and un-disturbance factors.
Ideally speaking, for a single clay particle, there are two forces on it, i.e. weight under water and cohesive forces.

I. Weight

The weight of a single clay particle under water is

\[ W_s = a_i \frac{\pi}{6} (\rho_s - \rho) g D^3 \]  

where \( W_s \) is the weight of a clay particle under water, \( a_i \) the shape coefficient, \( \rho_s \) the density of clay, \( \rho \) is the density of water and \( D \) the diameter of a clay particle.

II. Shear strength among soil particles

The clay’s resistance originates from the bonding forces that hold clay particles together and other material in the soil matrix, such as organic matter, plant roots and rock fragments. The various mechanical, adhesive, cohesive and electrostatic bonding forces acting on the undisturbed clay increase the resistance to erosion, and these can generally be considered as cohesive forces, i.e. can be written as

\[ \tau_f = \sigma \tan \varphi + c \]  

where \( \tau_f \) is the shear stress, \( \sigma \) the normal stress on the clay particle, \( \varphi \) the angle of friction, \( c \) the cohesive force of clay particle, and

\[ N = \frac{\pi D^2}{4} \tau_f \]  

where \( N \) is the shear strength force among clay particles.

3. Erosion model

Particle movement will occur when the instantaneous fluid force on a particle is just larger than the instantaneous resisting force related to the submerged particle weight and the friction coefficient. The driving forces are strongly related to the local near-bed velocities. In turbulent flow conditions the velocities are fluctuating in space and time. This makes together with the randomness of particle size, shape and position that initiation of motion is not merely a deterministic phenomenon but a stochastic process as well.

The fluid forces acting on a clay particle consist of skin friction forces and pressure forces (see Figure 1). The skin friction forces acts on the surface of the particle by viscous shear. The pressure forces force consisting of a drag and lift force is generated by pressure differences along the surface of the particle, theses forces per unit surface area can be reformulated in a time-averaged bed-shear stress.

I. Uplift force

The uplift force can be described with

\[ F_y = a_2 c_y \frac{\pi D^2}{4} \frac{\rho U_2^2}{2} \]
where \( F_y \) is the uplift force of clay particle, \( a_2 \) the shape coefficient, \( c_y \) the friction coefficient in the direction of \( y \), \( U_d \) the velocity through the clay particle.

II. Drag force

Similarly, the drag force can be described as

\[
F_x = a_3 c_x \frac{\pi D^2 \rho U_d^2}{2}
\]

where \( F_x \) is the drag force on a clay particle, \( a_3 \) the shape coefficient, \( c_x \) the friction coefficient in the direction of \( x \).

Considering \( O \) as fulcrum, according to the moment equilibrium, an equation can be written as

\[
F_x d_1 + F_y d_2 - W_s d_3 - N d_4 \geq 0
\]

So the incipient velocity of clay particle can be written as

\[
U_d = \sqrt{\frac{2}{(a_3 c_x d_1 + a_2 c_y d_2) \rho}} \left[ a_1 \frac{2\pi}{3} (\rho_s - \rho) g D d_3 + \tau_f d_4 \right]
\]

Supposing \( C' = \frac{4\pi a_2 d_3}{3(a_3 c_x d_1 + a_2 c_y d_2)} \), \( C = \frac{2d_4}{(a_3 c_x d_1 + a_2 c_y d_2)} \), then

\[
U_d = \sqrt{\frac{C' (\rho_s - \rho) g D + C \tau_f}{\rho}}
\]

According to Prandtl Law, the shear stress at any point in a turbulent flow over a solid surface can be given as

\[
\tau = \rho l^2 \left( \frac{dv}{dy} \right)^2
\]

where \( v \) is the velocity over the clay particle.
For the region near the solid surface, Prandtl proposed two assumptions: (I) that the mixing length is proportional to \( y \), and (II) that the shear stress is constant. Since the shear stress at the surface is equal to the unit tractive force, the second assumption gives \( \tau = \tau_0 \). With the two assumptions, Equation (9) can be written as

\[
dv = \frac{1}{\kappa} \left( \frac{\tau_0}{\rho} \right) dy
\]

where \( \kappa \) is 0.40 according to many experiments. Consequently

\[
v = \kappa \sqrt{\frac{\tau_0}{\rho}} \ln \left( \frac{y}{y_0} \right)
\]

(11)

\[
\rho v^2 = \kappa^2 \tau_0 \left( \ln \left( \frac{y}{y_0} \right) \right)^2
\]

(12)

Substituting Equation (12) into Equation (8) yields

\[
k^2 \tau_c \left( \ln \left( \frac{y}{y_0} \right) \right)^2 = C' (\rho_s - \rho) gD + C \tau_f
\]

(13)

\[
\frac{\tau_c}{(\rho_s - \rho) gD} = \frac{1}{k^2 \left( \ln \left( \frac{y}{y_0} \right) \right)^2} \left[ C' + C_2 \frac{\tau_f}{(\rho_s - \rho) gD} \right]
\]

(14)

where \( y \) is equal to 0.5\( D \), \( y_0 \) depends on the characteristics of the clay particle surface, such as roughness, and boundary layer transition. In Equation (14), the term \( \frac{1}{k^2 \left( \ln \left( \frac{y}{y_0} \right) \right)^2} C' \) is associated with particle weight and \( \frac{\tau_f}{(\rho_s - \rho) gD} \) is associated with cohesive stress. In the clay particle, the cohesive part plays a far more important role than the weight part. If the weight part is ignored, then Equation (14) can be written as

\[
\frac{\tau_c}{(\rho_s - \rho) gD} = \frac{1}{k^2 \left( \ln \left( \frac{y}{y_0} \right) \right)^2} C \frac{\tau_f}{(\rho_s - \rho) gD}
\]

(15)

In the non-cohesive sediment incipient motion, shields number can be expressed as

\[
\theta_c = \frac{\tau_0}{(\rho_s - \rho) gD}
\]

(16)
where $\theta_c$ is the critical mobility parameter or the Shields parameter, $\tau_0$ is critical shear stress of non-cohesive sediment particle’s incipient with the same size.

According to Equation (11) and Equation (16), Equation (15) can be written as

$$\frac{\tau_c}{(\rho_s - \rho)gD} = C\frac{\tau_f}{\rho v^2} \theta_c$$

or

$$\tau_c = C\frac{\tau_0}{\rho v^2} \tau_f$$

### 4. Experimental study

The undisturbed clay erosion experiments were conducted by GeoDelft (GeoDelft, 2003). The rotating cylinder erosion device (see Figure 2) consists of a vertical placed metal cylinder with 1cm wide blades attached on the inside. The internal diameter of the metal cylinder is 16 cm. The cylindrical soil sample, having a diameter of 6.6 cm and a height of 5 cm, is placed between two spindles with spikes which penetrate the sample for a few millimeters. The sample and the two spindles are placed on a vertical metal axis. The axis is pierced through the center of the sample and the spindles. The spindles are fixed onto the axis which is placed between two ferrules, ensuring an independent rotation of the sample with respect to the rest of the apparatus.

By rotating the metal cylinder, the water inside it flows around the sample, which applies torsion on the sample. Due to the fact that the sample is independently hung from the rest of the apparatus, this torsion can be measured constantly. The erosion rate is measured every 10 minutes by measuring the weight of the sample.

The rotating cylinder erosion device is controlled by the computer. The test procedure is generally such that the rotation speed is kept constant during a certain period and increases in steps. The working procedure is shown in Figure 3.

The clay samples EG and CG were collected from Elisabethgroden and Cäciliengroden without disturbance, respectively. Their relevant properties are given in Table 1.
Figure 3. Sketch of working procedure of rotating cylinder erosion device

<table>
<thead>
<tr>
<th>Clay type</th>
<th>EG</th>
<th>CG</th>
</tr>
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<tbody>
<tr>
<td>Soil type</td>
<td>Cs13</td>
<td>Csi2</td>
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<table>
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<tr>
<th>Particle size distribution</th>
<th>EG</th>
<th>CG</th>
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<tr>
<td>&lt; 2 μm</td>
<td>24.2</td>
<td>48.8</td>
</tr>
<tr>
<td>&lt; 16 μm</td>
<td>38.6</td>
<td>77.3</td>
</tr>
<tr>
<td>&gt; 2 μm and &lt; 63 μm</td>
<td>36.8</td>
<td>40.4</td>
</tr>
<tr>
<td>&gt; 63 μm</td>
<td>39.0</td>
<td>10.8</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Water content in-situ [%]</th>
<th>EG</th>
<th>CG</th>
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<tr>
<td></td>
<td>23</td>
<td>45</td>
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</table>

<table>
<thead>
<tr>
<th>Undrained shear strength [kN/m²]</th>
<th>EG</th>
<th>CG</th>
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<tbody>
<tr>
<td></td>
<td>29.32</td>
<td>35.55</td>
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</table>

<table>
<thead>
<tr>
<th>Specific density [g/cc]</th>
<th>EG</th>
<th>CG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.66</td>
<td>2.72</td>
</tr>
</tbody>
</table>

4. Analysis and discussion

In the experiment above, two clay samples (EG and CG) were tested with the rotating cylinder erosion device. The shear stress was measured every 10 minutes. The shear stress to velocity was plotted for EG and CG in Figure 5 and Figure 6, respectively.

In Equation (17) and Equation (18), the shear stress for the non-cohesive sediment should be used with the same diameter of undisturbed clay particle. Therefore, the Shields curve (Figure 4) should be used to confirm the Shields parameter \( \theta_c \) in the calculation of shear stress of clay particle. For simplicity, \( D_{50} \) is used to represent the particle diameter. Then the shear stress of clay particles can be calculated with Equation (18) for EG and CG, and be plotted in Figure 4 and Figure 5.
In the calculation of shear stress for sample clay, the calculated data can be fitted to the measured data with a suitable accuracy. So, Equation (18) and Equation (19) can be used to calculate the shear stress for undisturbed clay. There is a parameter $C$ in Equation (17) and Equation (18), however, which should be calibrated in future research. In this paper, $C$ is set from 0.75 to 1.0. How to select the parameter still needs to be considered in the future.

The relationships between shear stress and velocity of EG and CG are plotted in Figure 5 and Figure 6. These relationships can be used to predict for other conditions without doing tests.

Figure 4. Shields curve

Figure 5. Comparison of shear stress between measured data and calculated data for EG
5. Conclusions

The undisturbed clay has the unique and complicated characteristics which are highly different from disturbed clay and non-cohesive sand. In this paper, a new dynamic erosion model is given for the undisturbed clay in different flow conditions. The model has been calibrated with the data of an experiment conducted by GeoDelft (2003). It has been found that the agreement between model predictions and experimental data is good. So, the model can be used to simulate the shear stress for undisturbed clay. The relationships between shear stress and velocity can be calculated.

References

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