1. Het gebruik van een grid afhankelijke waarde voor de warmte-overdrachtscoëfficiënt reduceert de predictieve waarde van CFD berekeningen voor de warmtehuishouding in een stroming tot nul.

2. De aanschafkosten van CFD software en hardware zijn klein ten opzichte van de kosten van een gespecialiseerde gebruiker. Coöperatieve financiering van zo’n specialist door meerdere gieterijen kan leiden tot kostenbesparing en kwaliteitsverbetering voor iedere deelnemer.

3. De moeilijkst te bepalen parameter bij het gebruik van CFD software is de mate van onwetendheid.

4. De capaciteitstoename van computers met een factor 8 in 4 jaar bevoordeelt die promovendi, die pas laat de voordelen van CFD ontdekken.

5. Aan ouvrde over leven onder een communistische staatsvorm ligt dezelfde behoefte van de mens ten grondslag als aan de hardnekkige voorkeur voor autogebouw boven het openbaar vervoer: de mens wil graag inhalen.

6. Het door Marx geïntroduceerde begrip “Verelendung” kan direct vanuit het industriële tijdperk naar het informatietijdperk worden overgebracht. (K. Marx, Das Kapital)

7. Directe humanitaire hulp aan notoire rampgebieden leidt tot vergroting van het aantal slachtoffers in de toekomst.


9. Gezien de stemmingsgevoeligheid van de beurskoersen, is het vanuit psychologisch standpunt verstandig om ten tijde van sterke koersdalingen bij de grafische weergave van de koersontwikkeling te kiezen voor het omdraaien van een der coëdinaat-assen.

10. Mijn geloof gelooft niet in het gelijk van gelovigen.
MOULD FILLING OF HORIZONTAL THIN-WALL CASTINGS
MOULD FILLING OF HORIZONTAL
THIN-WALL CASTINGS

PROEFSCHRIFT

ter verkrijging van de graad van doctor

aan de Technische Universiteit Delft,

op gezag van de Rector Magnificus prof. ir. K.F. Wakker

in het openbaar te verdedigen ten overstaan van een commissie,

door het College voor Promoties aangewezen,

op maandag 30 november te 10:30 uur

door

Robert VAN TOL

doctorandus in de natuurkunde

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Chapter 1

Introduction

1.1 Castings

Using shape castings for structural applications is attractive, because it enables the design of multi-functional products, which are sometimes impossible to construct otherwise. In the automotive and aerospace industry in particular, there is an increasing demand for lighter products. In addition, foundries continuously aim at improved product performance and reductions in material consumption. Many of these demands may be realised by designing and manufacturing shape castings as thin as possible. For foundries, this creates a market for thin-wall castings with a guaranteed high quality.

A casting can be produced by a number of techniques, named after the driving force that is used to fill the mould cavity with liquid metal: high- and low-pressure casting, vacuum casting, and gravity casting. Since the focus of this thesis is on mould filling, the casting method is seen only as a way of delivering the metal to the gating system. The castings for this study are produced by use of gravity as a driving force.

Since the quest for ideal castings started, the geometrical design of gating systems has been studied extensively (Campbell 1981; Chandley and Flemings 1988; Wukovich and Metevelis 1989).

These studies have led to the following rules (Campbell 1991):

- fill the mould before the metal solidifies in the mould cavity
- prevent turbulence in the liquid metal
- prevent spray effects, i.e. do not introduce the metal in the mould cavity violently.
- reduce the momentum of the falling metal in the down sprue
- prevent the first, relatively cold, material from entering the mould cavity

By choosing the dimensions of the gating system in figure 1.1 properly, many requirements can be satisfied. The liquid metal from the pouring cup falls into a pouring well, where the liquid decelerates. The choke limits the volumetric flow rate to keep a certain pressure head in the down sprue. The liquid metal is transported from the choke through the runner to the ingate. To prevent spraying effects at the ingate, a depressurised gating system is used. In such a gating system, the cross-sectional area of the ingate is larger than the area of the runner, which again is larger than the choke area.
Figure 1.1: A typical gating system.

In practice, the flow rate from the gating system can be calculated from an integrated version of Bernoulli's law (Bird et al. 1960; Poirier and Geiger 1994) (see also figure 1.1)

\[ \rho g (h_2 - h_1) + (P_2 - P_1) + 1/2 \rho (U_2^2 - U_1^2) = 0 \]  

(1.1)

By setting the pressure \( P_1 = P_2 \), the velocity \( U_1 = 0 \), and the height \( h_2 = 0 \), the velocity \( U_2 \) at the choke as a function of \( h_1 \) becomes

\[ U_2 = \sqrt{2gh_1} \]  

(1.2)

To calculate the flow rate, the choke area is used. From the flow rate and the volume of the casting, the filling time can be calculated. This equation is valid for frictionless laminar flow and a completely filled gating system. Frictional losses can be incorporated by multiplying the right hand side of equation 1.2 with a discharge coefficient \( \alpha \), with a value between zero and one. Because \( \alpha \) is not known a priori, the calculated velocity may be inaccurate.

To compute the metal temperature, models were developed to approximate the heat loss to the gating system (Hlinka et al. 1961; Jones et al. 1963). None of these models give a reliable approximation of the temperature at the ingate on the basis of a given pouring temperature, because they assume a stationary state for flow as well as heat transfer. This is only a good approximation for very large castings, where metal-flow through the gating system sustains for a long time. In addition the gating system is too complex to
approximate it by a simple tube. Hence, the dimensions of the gating system are designed on basis of the foundry’s experience.

The gating system needs to produce a flow rate that enables filling of the entire mould cavity before the casting has solidified. In practice, the freezing time of a casting as a function of the volume to surface ratio can be approximated by Chvorinov’s rule (Poirier and Geiger (1994), page 332):

$$t_f = k \left( \frac{V}{A} \right)^2$$

(1.3)

with:

- \(t_f\) = freezing time
- \(k\) = geometrical constant \([\text{s/m}^2]\)
- \(\frac{V}{A}\) = volume to surface ratio

For aluminium sand castings, \(\frac{V}{A}\) can be approximated by half the casting thickness. A derivation of the constant \(k\) is given in Poirier and Geiger (1994). The freezing time obtained from equation 1.3 gives the minimum volumetric flow rate that is needed to fill the mould without a could run, i.e. the velocity \(U\) of equation 1.2. The above approximation is only valid for uniform cooling of the casting, thus not suitable to apply in thin-wall castings.

Thin-wall castings need relatively short filling times and typically have a high surface to volume ratio. This results in temperature gradients within the metal and in the mould, and thus to a cooling rate that depends on the position in the casting and which therefore may affect the micro-structure of the metal: the dendritic arm spacing and to some extent the grain size. These temperature gradients may also result in local obstruction of the flowing metal, while at another location the metal continues to fill the mould cavity. A typical example of mould filling is shown in figure 1.2. A top view of the mould cavity (black) for a horizontal plate is shown. The metal (grey) enters from the right in sub-figure (a). In sub-figure (e) the liquid tongue stops, and is by-passed by a separate stream, observable at the top of picture (f). In picture (g) this stream welds to the metal that was already present. At the welded contact area the surface oxides will be entrapped and a weak spot in the casting is formed.

This filling shows two trends that are typical for horizontal thin-wall castings:

- a permanent contact between the liquid metal and the top of the mould cavity (in figure 1.2 the glass plate)
- solidification during mould filling

The uncontrolled flow behaviour (where does a liquid tongue stop, and where does the mould filling continues ?) and the sensitivity to variations in heat transfer and pouring rate make the production of thin-wall castings a difficult task. Often a trial-and-error process is needed to obtain sound and reproducible castings. This is expensive, especially for small series. To make the process more consistent and more cost effective, computer modelling can make a major contribution.
CHAPTER 1. INTRODUCTION

1.2 Computational Fluid Dynamics

Computer simulations, using Computational Fluid Dynamics (CFD), have turned out to be a powerful tool to optimise the characteristics of the casting product and to study the sensitivity of the manufacturing process (Adams et al. 1992; Piwonka et al. 1993; Cross and Campbell 1995; Thomas and Beckerman 1998).

Mould filling simulations take a special place in CFD, because a free surface tracking algorithm is required. The MAC (Marker And Cell) method (Welch et al. 1966)) was the first algorithm for solving free surface flows, followed by an optimised version, the SMAC (Simplified MAC) method (Amsden and Harlow 1970). The SOLAVOF (Solution Algorithm with Volume Of Fluid) (Hirt and Nichols 1981) method introduced in 1981, has a much higher memory efficiency than the MAC method. Today, both methods have been extended to 3-D. The computational efficiency of both methods is compared by Chen and co-workers (1993).

The VOF algorithm allows application of surface tension at the free surface. The incorporation of surface tension is essential in simulation of horizontal thin-wall castings, because it may keep the metal in permanent contact with the top of the mould cavity. The VOF algorithm is one of the key features of the FLOW-3D software (1990) used to obtain the computational results in this thesis.

Another important issue that needs special attention is heat transfer during mould filling. Solidification during mould filling, as shown in figure 1.2, depends highly on the heat transfer from metal to mould.

The non-linearity of the flow equations and all types of small-scale phenomena make it difficult to produce reliable predictive computational results. Therefore, results of computations have to be compared with experimental results.

1.3 This thesis

The robustness of calculational methods for mould filling simulations is generally tested in simple shaped geometries (Van der Graaf 1995; Van der Graaf et al. 1993; Xu and Mampaey 1993).

Amongst mould filling studies this thesis takes a special place, because it aims at validating the mould filling calculation of a horizontal thin-wall casting, while other studies used vertical plates. This type of castings lays a heavy claim on the free surface algorithm. The horizontal casting studied in this thesis exhibits a strong interaction between fluid flow, heat transfer, solidification, surface tension and gravitational force. The gravitational force is normal to the plane of flow for the casting. Therefore, the upstream velocity has a major effect on the shape of the free surface in the plane of flow. Surface tension forces cause a permanent contact with the top of the mould cavity, increasing heat transfer. The separate streams as shown in figure 1.2 show local solidification of the metal, while mould filling is taken over by another stream (Nieswaag and Deen 1990; Van Tol et al. 1991).

These phenomena should all be taken into account by a numerical model for calculating the mould filling of horizontal thin-wall castings, and validated against experiments.

The main objectives of this thesis can therefore be summarised as follows:

- Modelling of horizontal mould filling and the effect of various casting parameters on
the freezing length.

- Experimental validation in a (cold) water model and in an actual casting.

To reach these goals, models including surface tension, heat transfer and solidification have to be evaluated.

Three methods are used to visualise mould filling: contact measurements and video registration to observe the propagation of the free surface, and Particle Image Velocimetry to measure flow velocities in a water model. In chapter 2, these techniques are discussed and experimental results will be given. The next two chapters discuss the mathematical and numerical model to describe horizontal thin-wall mould filling. In chapter 5 the numerical results are given, followed by concluding remarks in chapter 6.

The research has been carried out at two laboratories of the Delft University of Technology: the "Laboratory of Materials Science" and the "Kramers Laboratorium voor Fysische Technologie". It was initiated and partly financed by the "IOP-MetaLEN" program.
Figure 1.2: Mould filling of a horizontal plate of 285x300x5 mm in 6 seconds. Images are captured from video by a frame grabber. Hot metal enters from the right side (grey).
Chapter 2

Experimental methods and results

2.1 Introduction

The major objective of this thesis is the computation of mould filling of a thin-wall horizontal casting. This type of casting exhibits two characteristics: permanent contact between the liquid metal and the top mould of the mould cavity, and solidification effects during mould filling.

The filling pattern in a mould, such as in figure 1.2, is hard to reproduce, because small inaccuracies in levelling of the mould lead to a completely different filling pattern. To increase the reproducibility, a long bended strip (horseshoe) is chosen as a test-casting, see section 2.2. The filling of this geometry remains the two characteristics mentioned above, while the displacement of the metal-air surface in the horizontal plane takes place in a more consistent way.

Mould filling simulation requires four quantities to be computed: position, or motion, of the free surface, flow velocity, temperature, and the phase-transition from liquid to solid. To experimentally validate the computed values, different measurements are used:

- Section 2.3 describes how the free surface is tracked by the use of video recordings and contact measurements. The phase-transition from liquid to solid cannot be observed directly, but information about this process is deducted from the length at which the flow is arrested.

- Thermocouple measurements, to detect the local temperature of metal or mould, are described in this section as well. Results obtained with these three methods are presented in section 2.4.

- Particle Image Velocimetry, a technique to measure the integral velocity field in a water model of the casting, is described in section 2.5. Velocities measured with this technique are presented in section 2.6.

2.2 Geometry and materials

The present study of horizontal casting focusses on the mould filling of a thin-wall horse-shoe. A geometry as in figure 2.1 is chosen, because it combines a considerable strip length with a restricted width and length of the mould. This casting geometry is long enough
to observe solidification during mould filling, while it has overall mould dimensions that can be handled in the foundry.

The geometry is shaped in moulding sand for the metal castings. The water model has been manufactured out of perspex, with a black background, to prevent reflections.

The geometry of casting and water model are the same, except for the horseshoe’s thickness. In the water model the thickness of the horseshoe is 3 mm, whereas in the casting a thickness of 5 mm is used. The height of a water drop on perspex (3.75 mm) is much smaller than that of a liquid aluminium drop on moulding sand (11 mm)\(^1\). Since permanent contact between liquid and the top mould is one of the key features of horizontal mould filling, this has to be assured by a decrease of the thickness of the horseshoe in the water model. The width of the horseshoe is 40 mm, as in the casting.

The top of the down sprue has an area of 15 mm x 15 mm and a bottom area of 10 x 10 mm. The choke area is in both cases 10 mm x 7.5 mm. The area ratios of choke and ingate for the water model and the casting are 75:120 and 75:200, respectively; so the gating system is in both cases depressurised, which prevents spraying of the metal from the ingate (Campbell 1991).

For the mould, chemically bonded silica sand is used. The mould parts are dried overnight in a stove at 120 °C to evaporate water from the mould material. The mould parts are cooled down again and assembled with heat resistant adhesive. The mould is

\(^1\)These values are calculated with equation 3.2
2.3. OBSERVATION METHODS FOR MOULD FILLING WITH LIQUID METAL

<table>
<thead>
<tr>
<th>Element</th>
<th>wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pb</td>
<td>0.005</td>
</tr>
<tr>
<td>Zn</td>
<td>0.02</td>
</tr>
<tr>
<td>Cu</td>
<td>0.01</td>
</tr>
<tr>
<td>Ni</td>
<td>0.01</td>
</tr>
<tr>
<td>Fe</td>
<td>0.3</td>
</tr>
<tr>
<td>Mn</td>
<td>0.06</td>
</tr>
<tr>
<td>Cr</td>
<td>0.03</td>
</tr>
<tr>
<td>Ti</td>
<td>0.01</td>
</tr>
<tr>
<td>Si</td>
<td>12.1</td>
</tr>
<tr>
<td>Sn</td>
<td>0.005</td>
</tr>
<tr>
<td>Mg</td>
<td>0.03</td>
</tr>
<tr>
<td>Al</td>
<td>remainder</td>
</tr>
</tbody>
</table>

Table 2.1: Composition of the pouring material

placed in a horizontal position and weights are put onto the top mould, to prevent it from being pushed upwards by the liquid metal.

As pouring material, aluminium with 12.1 wt% Silicon is used. A more extensive list of alloying elements is shown in table 2.1. The aluminium is melted in a mid-frequency range induction furnace. It is superheated up to some 20 – 30 °C above the melting temperature in order to allow for some cooling during transfer from furnace to mould. The temperature is measured, while the liquid metal is gently stirred. When the required temperature is reached, the metal is poured manually.

Since the water model is used for velocity measurements, reflecting particles are added to the water, to visualise the velocity at the position of the particles. Polystyrene grains of 500μm are used, which have approximately the same density as water. They are stirred through the water overnight, to filter the heavy (sunk) and light (floating) particles.

### 2.3 Observation methods for mould filling with liquid metal.

The mould material is normally opaque, but can be made transparent by using thermoshock resistant glass. A thermal shock is introduced by sudden contact between the cold glass and the hot metal. High-resolution images of mould filling are obtained at a rate of 50 Hz, with a video recorder. These images are used to study the displacement of the metal air-surface, see figure 2.3.

Our second measuring method involves placing sensors in the mould to detect the presence of liquid metal. A number of conducting wires are inserted into the mould and connected to an I/O card in a PC. The lose ends represent a logical "1" for the computer. The metal, which is grounded, induces a transition to a logical "0" when contact with the wires is established. By sweeping along all the contacts every 0.01 seconds, the location
of the interface between metal and air can be tracked with a sufficient time resolution. Sweep time is around 1 ms. A software program has been written to produce animations of mould filling, using the location of the wires and the time they make contact with the metal as input, see figure 2.4.

An alternative method (Sirrel et al. 1995) would be to make use of X-rays. Sand moulds have a high permeability for this kind of radiation, while metals are opaque for the radiation. An image of the location of the metal can be made on a photo-negative, and does not influence the mould filling. The method makes use of dedicated equipment and is not used in this thesis.

Temperatures are detected with Cr/Cr-Al-thermocouples. The thermocouple wires are placed into thin ceramic tubes to facilitate positioning of the weld, which is the actual measuring position, see figure 2.2. The thickness of the wires is preferably chosen as small as possible, to limit the heat extraction by the conducting wires. Wire with a diameter of 0.1 mm is used in a tube of 1.2 mm.

The methods described are used to directly obtain information about displacement of the metal-air surface and temperatures during mould filling and during cooling. Information about the phase transition from liquid to solid is obtained indirectly by measuring the freezing length. The freezing length is the length of the strip at the moment the flow
is arrested by solidification, e.g. the length of the strip when a cold shut has occurred.

2.4 Mould filling measurements

The results shown in this chapter are all obtained using the experimental techniques and procedures described in section 2.3, and using the materials described in section 2.2. The general trends of the filling of the horseshoe with liquid metal are investigated first, using video and contact measurements. The flow rate is measured, as well as the freezing length. The cooling after mould filling is recorded as well.

2.4.1 Free surface and flow rate

The mould filling of the horseshoe can be viewed clearly from video, using a glass top plate. Figure 2.3 shows such a controlled filling as compared to the mould filling of the horizontal plate shown in the introduction (figure 1.2). In the horseshoe, the liquid metal is guided through a channel, preventing the occurrence of separate streams.

The pouring cup as well as a complete top view of the mould are visible on these images. Four major trends can be observed, which are typical of the mould filling of the horseshoe.

- The liquid metal is in permanent contact with the top of the mould.

- Near to the ingate, the liquid metal is driven to the outside of the horseshoe. This is a result of the 90° angle between the runner and the first leg of the horseshoe. During filling of the first leg, the metal is spread across the width of the first leg. The air gap at the ingate remains until the flow front has passed the bend.

- During the filling of the bend, the liquid interface on the outside is ahead of the interface on the inside. In the second leg, the metal spreads again.

- Effects of solidification can be observed as an interrupted propagation of the free surface in the second leg.

Figure 2.4 shows the mould filling as measured with 84 electrical contacts. The white coloured cells are empty, while the cells that contain metal are black. The filling trends on the video recording are confirmed by contact measurements, with the exception for freezing, because of a higher pouring temperature. The air gap near to the ingate is only shown in figure 2.4 (a). It might persist till later stages of the mould filling, as observed in the video registration, but it is not seen by the limited number of contacts. The distance between the inner side of the horseshoe and the closest contact is 5 mm and the air gap is about this size.

A permanent contact is observed with the top of the mould, which holds the contacts. Displacement of the metal-air interface in the ingate and in the bend is similar to that in the video visualisations. Complete freezing of the channel is not found in this casting, due to a higher pouring temperature. The mould is completely filled, although partial freezing of the channel is observed as an interrupted propagation of the surface at the inner side of the second leg.
Figure 2.3: Mould filling trends (top view). The ingate velocity is 0.3 m/s at a pouring temperature of 750°C. In the left upper corner the pouring cup is visible.
Figure 2.4: Mould filling with aluminium (top view), detected with electrical contacts. The ingate velocity is 0.3 m/s at a pouring temperature of 780° C.
Figure 2.5: The filled volume as a function of time for three different castings poured at 700°C and three different castings poured at 750°C, measured with electrical contacts.

The liquid velocity as a function of time is obtained by measuring the volume of the horseshoe that has been filled in a certain time interval and dividing it by the cross-sectional area of the horseshoe. Typical measured volume to time relations for two different pouring temperatures are shown in figure 2.5. The velocity, proportional to the derivative of the curve shown, is constant in the beginning of the mould filling period, and reduces to zero when a cold run occurs.

2.4.2 Freezing length

The freezing length is defined as the distance the liquid-air interface has travelled from the ingate until it is arrested due to solidification. The dependence of the freezing length on ingate temperature, is shown in figure 2.6. The figure shows a freezing length that increases with temperature at approximately 3 mm/°C.

2.4.3 Solidification after mould filling

The high surface to volume ratio of a thin-wall casting results in fast cooling of the metal during mould filling. This will show up as a difference in solidification time at different locations in the casting. There are two contributions to this effect: (1) the longer the metal has been in the mould, the more the metal has cooled; (2) the longer the mould has been in contact with the metal, the more it will have been warmed. Near the ingate, the metal has lost the least heat with the warmest sand present. Hence, this is where the longest solidification time is expected.

In figure 2.7, the temperatures in three locations are compared during the first minute after the mould is full: near to the ingate (couple 1), just before the bend (couple 3) and just after the bend (couple 5). The temperature at couple 1 is still 25°C above the solidification temperature of 577°C at the moment the mould is full. This is not observed
for couple 5. The slopes of the curves show that cooling of the casting in the vicinity of the ingate is slower than in the bend. This is caused by the two contributions mentioned above.

2.5 Particle Image Velocimetry

2.5.1 Introduction

Particle Image Velocimetry (PIV) is a measurement technique that reveals flow velocities in transparent liquids or gases. PIV shows the integral velocity field over an extended area, where techniques as LDA (Durst et al. 1981) and the hot wire method show the velocity at one position only. PIV, that relies on image capturing and computational power, has come under the attention of researchers in the area of mould filling recently (Van der Graaf et al. 1992; Van der Graaf et al. 1993; Van der Graaf and Van den Akker 1994).

First, some attention will be drawn to the principles and to the optical system, next to the so-called interrogation of the image, and finally to post-processing of data.

2.5.2 Principles and optics

PIV relies on the multiply exposure of particles in a liquid flow. This can be realised by either taking separate photographs of the particles in the liquid, or by creating a multiply exposed picture with a stroboscope, see figure 2.8. All particles are found twice, at their original location (black) and at their displaced position (grey). Multiply exposed pictures are used here because in that case no exact reference point is needed to correlate separate pictures. Using multiply exposed pictures introduces a directional ambiguity, because it can not be seen which of the images of a particle in the picture was taken first.

An integral velocity field is created from a picture by dividing it into a number of interrogation areas and by assigning a velocity to each of them. The key issue of PIV is
Figure 2.7: Cooling of the casting after mould filling.

to obtain an average displacement of the particles in every interrogation area, see figure 2.9. The measured velocity depends on the displacement in the light plane:

\[
\vec{v}(\vec{x}, t) = \frac{\Delta \vec{x}(\vec{x}, t)}{\Delta t}
\]  

(2.1)

with:

\(\Delta t = \) time between exposures

\(\Delta \vec{x}(\vec{x}, t) = \) the average actual displacement of the particles.

A system of lenses is used to project the images onto the camera. For an ideal system of lenses as shown in figure 2.10, the object distance \(d_o\), the image distance \(d_i\), the focal length \(f\), and the magnification \(M\) obey the following relations:

\[
\frac{1}{d_i} + \frac{1}{d_o} - \frac{1}{f} = 0
\]  

(2.2)

and

\[
M = \frac{d_i}{d_o}
\]  

(2.3)

For a small displacement in the image plane \(\Delta \vec{X}(\vec{X}, t)\) of a particle, the actual velocity (in the light plane) is obtained from:

\[
\vec{v}(\vec{x}, t) = \frac{1}{M} \frac{\Delta \vec{X}(\vec{x}, t)}{\Delta t}
\]  

(2.4)

Hence, the key issue in PIV is obtaining the displacement in the image plane from multiply exposed pictures. The procedure to obtain the displacement is called interrogation and is described in more detail in section 2.5.4.
Figure 2.8: Exposure showing two images, both containing 5 particles, used to compute the displacement during the time interval between the images.

Figure 2.9: The marked area shows an interrogation area, used to compute one velocity vector. Doing this for all interrogation areas gives an integral velocity field.
Figure 2.10: An ideal system of lenses transforms a displacement $\Delta x$ in the light plane into a displacement $\Delta X$ in the image plane.

### 2.5.3 Parameters of PIV measurements

Interrogation, determining the average displacement, depends on the number of particles per interrogation area. If there are one or two particles in an interrogation area, one can identify the particles and their trajectory. This is called particle tracking. If the number of particles is so large that they overlap in the picture, it is called Laser Speckle Velocimetry. In the intermediate regime, the technique is called PIV. PIV can be distinguished from Particle Tracking Velocimetry (PTV) and Laser Speckle Velocimetry (LSV) by two dimensionless parameters. The particle source density ($N_S$) and the particle image density ($N_I$) can be used to express the information contents of an image (Adrian 1991). The source density is related to the diameter of the particles in the image plane ($d_s$).

$$N_S = C \Delta Z_0 \frac{\pi d_s^2}{4 M^2}$$  \hspace{1cm} (2.5)

The image density is related to the diameter of the interrogation area ($d_I$).

$$N_I = C \Delta Z_0 \frac{\pi d_I^2}{4 M^2}$$  \hspace{1cm} (2.6)

with
2.5. PARTICLE IMAGE VELOCIMETRY

\[ M = \text{Magnification} \]
\[ C = \text{Particle concentration} \]
\[ \Delta Z_0 = \text{Thickness of the light sheet} \]

For a PIV system, using autocorrelation, the distance between the particles has to be larger than the particle diameter, or \( N_S << 1 \). The probability of multiple images of a particle within the interrogation area has to be large, or \( N_I >> 1 \). Both the parameters that set the operational mode, are a function of experimental parameters. The other operational modes are Particle Tracking Velocimetry (PTV), with \( N_S << 1 \) and \( N_I << 1 \), and Laser Speckle Velocimetry, with \( N_S >> 1 \) and \( N_I >> 1 \) (Keane and Adrian 1990).

Losses of paired images occur if during the exposure time a particle is shifted into or out of the interrogation area. The in-plane loss pair effect can be described with a parameter \( F_I \) that is defined by:

\[ F_I = \frac{\Delta X}{d_I} = \frac{M v_I \Delta t}{d_I} \quad (2.7) \]

The out-of-plane loss pair effect is discussed by Adrian (1991) and described with a parameter \( F_O \):

\[ F_O = \frac{v_0 \Delta t}{\Delta Z_0} \quad (2.8) \]

On the basis of Monte Carlo simulations, Keane and Adrian (1990) proposed to choose \( (F_I \leq 0.25)(F_O \leq 0.25) \). In the present experiments, however, no out-of-plane losses will be observed, since the thickness of the casting will be used as the thickness of the light plane \( (Z_0) \), and the value of \( F_O \) is not critical.

The paired image density of a multiply pulsed system, e.g. more than 2 stroboscope flashes during the exposure time, is determined by

\[ N_P = N_I(n - 1) \quad (2.9) \]

where \( n \) is the number of exposures per image.

Keane and Adrian (1990) define the validity \( D_0 \), as the ratio of the displacement peak, \( R_D \), and the highest noise peak \( R_F \), in the auto correlation spectrum (see section 2.5.4):

\[ D_0 = \frac{R_D}{\max \{R_F\}} \quad (2.10) \]

Keane and Adrian (1991) reject measurements, that have a validity \( D_0 < 1.2 \). An other criterion, based on the \((S/N)\) ratio for determining the validity, is investigated by Coupeland and Pickering (1988).

With respect to \( D_0 \), the valid detection probability, \( \Gamma \), is introduced as the probability that a measurement is correct, i.e. has a \( D_0 > 1.2 \). \( \Gamma \) depends on the number of paired images and on the displacement relative to the size of the interrogation area \( (\Delta X/d_I) \), as shown in figure 2.11. From this figure, it can be deduced that the valid detection probability is better than 95 % if the paired image density \( (N_P) \) is larger than 15.
The last aspect that should be studied, with respect to the PIV system, is the effect on the signal of velocity gradients within the interrogation area. They cause a gradient bias, due to the in-plane loss of pairs. The larger the velocity, the more particles leave the interrogation area, during the exposure time, leading to an average velocity that is underestimated. An other relevant bias is the detection bias, that is caused by the broadening of the detection peaks (Huang and Fiedler 1993). These effects are acceptable, if the following two criteria are fulfilled, according to Keane and Adrian (1990) and (1991):

\[
\frac{M|\Delta v|\Delta t}{d_I} < 0.05 \tag{2.11}
\]

\[
\frac{M|\Delta v|\Delta t}{d_e} < 0.5 \tag{2.12}
\]

An overview of the parameter requirements for a PIV system is summarised in table 2.2.

2.5.4 Interrogation

Pictures from the area of interest are subdivided into a number of interrogation areas. In every interrogation area, a velocity is computed from the multiply imaged particles in the interrogation area. All particles are correlated with all of the remaining particles, this procedure being denoted as interrogation.
2.5. PARTICLE IMAGE VELOCIMETRY

Table 2.2: Criteria for the parameters in a PIV system.

<table>
<thead>
<tr>
<th>Validity criterion</th>
<th>$D_0$</th>
<th>$&gt; 1.2 - 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paired image density</td>
<td>$N_P$</td>
<td>$\geq 15$</td>
</tr>
<tr>
<td>Relative in-plane displacement</td>
<td>$\Delta X/d_l$</td>
<td>$\leq 0.25$</td>
</tr>
<tr>
<td>Relative out-of-plane displacement</td>
<td>$v_0 \Delta t/\Delta Z_0$</td>
<td>$\leq 0.25$</td>
</tr>
<tr>
<td>Gradient bias</td>
<td>$M \Delta v \Delta t/d_l$</td>
<td>$\leq 0.05$</td>
</tr>
<tr>
<td>Detection bias</td>
<td>$M \Delta v \Delta t/d_e$</td>
<td>$\leq 0.5$</td>
</tr>
</tbody>
</table>

Figure 2.12: Autocorrelation function of an interrogation area.

First, the average intensity of the $N \times N$ pixels in the interrogation area is computed according to:

$$
\bar{I} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} I_{i,j}
$$ (2.13)

Then the auto-covariance $\hat{R}_{II}$ is computed for the displacement over vector $[r,s]$:

$$
\hat{R}_{II}[r,s] = \frac{1}{N^2} \sum_{i}^{N-|r|} \sum_{j}^{N-|s|} (I_{i,j} - \bar{I})(I_{i+r,j+s} - \bar{I})
$$ (2.14)

with:

$$
\sum_{i}^{N-|r|} = \begin{cases} 
\sum_{i=1}^{N-r} & \text{for } r \geq 0 \\
\sum_{i=r}^{N} & \text{for } r < 0
\end{cases}
$$ (2.15)
The auto correlation spectrum looks as shown in figure 2.12. In general, the auto-
correlation spectrum can be written as the sum of 5 terms (Adrian 1988):
\[
\hat{R}_{II}[u,v] = \hat{R}_F[u,v] + \hat{R}_{D+}[u,v] + \hat{R}_{D-}[u,v] + \hat{R}_P[u,v] + \hat{R}_C[u,v]
\] (2.16)

The term \( \hat{R}_P \) represents the self-correlation peak. This peak is the highest of all and
represents the displacement over a distance zero by correlating every particle to itself. The terms \( \hat{R}_{D+} \) and \( \hat{R}_{D-} \) on both sides of the self-correlation peak represent the displacement of the particles during time-interval \( \Delta t \). Peaks appear on both sides of the self-correlation peak, because of the directional ambiguity. The term \( R_F \) contains all of the noise peaks, while \( \hat{R}_C \) is the average back ground correlation.

To obtain the average displacement, the spectrum is inspected for the second highest
maximum. Because of symmetry only half the spectrum is investigated. This gives the
displacement at pixel level. By assuming a peak shape the displacement can be determined
at a sub-pixel level. The peak is fitted through the pixel values and gives a location for
the maximum that is more accurate than a pixel. A Gaussian estimate is used as a peak
shape (Westerweel 1993).

2.5.5 Post-processing

Wrong, or spurious, vectors can be computed, mostly in interrogation areas where the
number of paired images is too low. As detecting the spurious vectors on a basis of the
S/N-ratio is not robust, every vector is compared to its neighbours. Whether or not a vector can be considered to be a spurious vector, is decided with a median test, see
Westerweel (1993)): a correct vector \( \hat{V}_{i,j} \) is detected with a certain error \( \varepsilon_{i,j} \). A spurious vector is at the centre of a noise peak \( \hat{U}_{i,j} \) in a search window around the expected
displacement. The measured velocity \( \hat{V}_{i,j} \) is considered as a stochastic signal:
\[
\hat{V}_{i,j} = \gamma_{i,j} \cdot (\hat{V}_{i,j} + \varepsilon_{i,j}) + (1 - \gamma_{i,j}) \cdot \hat{U}_{i,j}
\] (2.17)

The parameter \( \gamma_{i,j} \) is either 1, for a good vector, or 0 for a spurious vector. The deviation
of \( \hat{V}_{i,j} \) relative to \( \hat{V}_{i,j} \) is described with a residual value \( r_{i,j}^2 \):
\[
r_{i,j}^2 = ||\hat{V}_{i,j} - \hat{V}_{i,j}||^2
\] (2.18)

Since in practice, the correct value \( \hat{V}_{i,j} \) is not known, an estimation \( \hat{V}_{i,j} \) is used. Three
estimators have been tested by Westerweel (1993)), who found the median test to be the
most effective.

The median is defined as the centre value of \( 2n+1 \) elements of a continuously inclining
row. The median of a row of \( 2n \) elements is the average of element \( n \) and element \( n + 1 \). The median of a vector \( \text{med}[\hat{V}_{i,j}] \), is defined by Westerweel (1993) as the median of
surrounding vectors: Inside the flow domain, this number amounts to 8, while in the
corners 3 of such vectors are available and at the edges 5. This is done separately for the
x- and y-direction. In this way a residue \( r^2 \) is computed for all vectors.

A critical value can be found above which the residual of a vector can no longer
be caused by white noise (Westerweel 1993). If, according to this criterion, a vector is
recognized as a spurious vector, it is replaced by a new vector obtained via interpolation
of its neighbours.
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Figure 2.13: Experimental setup for PIV measurements in a horizontal thin-wall horseshoe.

2.5.6 Experimental set-up and parameters.

The experimental setup is shown in figure 2.13. The important parameters for this PIV system are shown in table 2.3, as compared to the criteria in table 2.2.

The volume fraction of particles ($\alpha$) is 3% in the PIV experiments. This leads to an increase in dynamic viscosity ($\mu$) of 5%, according to Barnea and Mizrahi (1973).

$$\mu_{\text{apparent}} = \mu_{\text{liquid}} e^{\frac{5\alpha}{3(1-\alpha)}}$$  \hspace{1cm} (2.19)

For optical registration of the particles, 6 pictures per second are obtained, dictated by the shutter speed of the camera. The flash rate of the stroboscope is set to give 3 light

<table>
<thead>
<tr>
<th>Validity criterion</th>
<th>$D_0$</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paired image density</td>
<td>$N_P$</td>
<td>47</td>
</tr>
<tr>
<td>Source density</td>
<td>$N_S$</td>
<td>$8.1 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>Relative in-plane displacement</td>
<td>$\Delta X/d_f$</td>
<td>0.1</td>
</tr>
<tr>
<td>Relative out-of-plane displacement</td>
<td>$V_0\Delta t/\Delta Z_0$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.3: Values of critical PIV parameters as used in this study.
pulses per exposure. The negatives are piecewisely digitised, using a CCD camera, with a resolution of 256x256 pixels. The photos are used to obtain an integral velocity field of the complete horseshoe with sufficient resolution.

In the area near to the ingate, the CCD camera is used for direct digital registration of the particle distribution. Only a limited area can be scanned with the CCD camera because of its resolution. A particle has to be covered by at least 3x3 pixels for an accurate intensity distribution.

2.6 PIV results

2.6.1 Displacement of the water-air surface and the integral velocity field.

From photographic registrations of the water experiments, the displacement of the water-air surface can be deduced. From measured locations of the water-air surface, the filled volume as a function of time can be calculated. This function can be used to check the average velocity, through a cross-section of the horseshoe, as observed with PIV. The displacement of the water-air surface is shown in figure 2.14. An average velocity of 0.2 m/s is measured throughout the mould filling. In the bend, the surface on the outer side is ahead with respect to the free surface at the inner side of the bend. This is the same as observed in the mould filling with liquid metal.

The integral velocity field, as observed in PIV measurements, with spurious vectors, is shown in figure 2.15. This integral velocity field does not show the velocity differences in the flow very clearly. Hence, more detailed pictures will be shown in the following sections.

It is observed that, compared to the velocity of the water-air surface, the average velocity measured with PIV is larger by some 30% -70%. Since the visually observed velocity of the water-air surface is in agreement with the imposed flow rate, the difference must be caused by the PIV measurements. This difference can not be explained by the fact that the channel thickness \((d)\) is only 6 particle sizes \((d_p)\) in thickness. If the average laminar velocity is computed over the channel, only taking into account the area between \(1/12\ d_p\) and \(11/12\ d_p\), it will be 8 % larger than the average velocity computed for the complete channel thickness. A possible explanation may be that the particles are mainly located at half the thickness of the horseshoe. This might be due to a lift force, which, for Poiseuille flow in a tube, causes small particles to move away from the wall and concentrate at \(r = 0.6\ \text{R}\) (Segré and Silberberg 1962; Clift et al. 1978). The precise effect of this force for plane Poiseuille flow and large particles is unknown, but may result in a concentration of particles halfway the height of the channel. The maximum flow velocity is found in this position, and would be 1.5 times the average velocity for laminar flow. Hence the velocity measured with PIV is 1.5 times the velocity of the water-air surface.

2.6.2 Velocity in the bend

The shape of the water air-surface in the bend suggests a larger velocity at the outer side of the bend. Figure 2.16 shows a PIV-image of the bend and the computed velocity field. The velocity vectors found at the middle (plane B in figure 2.15) of the bend suggest that
Figure 2.14: The free surface for mould filling at a velocity of 0.2 m/s.
the velocity at the inner side of the bend is larger than at the outer side of the bend. This can be seen more clearly in figure 2.17, where the velocities during the transition from first to second leg are shown. This can not be in agreement with the water-air surface moving faster at the outer side of the bend, unless a reversal of the velocity gradient should take place just behind this surface.

To check this, PIV measurements are performed when the water-air surface passes the bend, observing velocities as closely as possible to the water-air surface (see figure 2.18). The velocities in the bend are presented in figures 2.19. At $t = 1.8$ s, the velocity in plane B (see figure 2.15) is larger at the outer side of the bend than at the inner side of the bend, while at $t = 2.2$ s, the opposite is found. This means that the velocity gradient is inverted in the bend, just behind the water-air surface.

In the PIV measurements the average velocity appears to decrease along the bend from $\sim 27.5$ cm/s in plane A to $\sim 24.0$ cm/s in plane C, see figure 2.17. A possible explanation might be that the particles concentrated at half the channel height before the bend, are distributed more evenly over the channel height, due to flow disturbances in the bend.

### 2.6.3 Velocities near to the ingate

In casting the horizontal horseshoe, a gap arises near to the ingate, at the inner side of the first leg, see figure 2.3. At a velocity of 20 cm/s, this gap is not found in the water model. The velocities, in an area of 40 mm x 40 mm near to the ingate, are shown in figure 2.20. The time averaged u-components of the velocity, along lines D and E of this area (see figure 2.15), are shown, in figure 2.22. This figure shows a larger value of the velocity at the inside of the first leg. This observation is in contradiction with the larger velocity at the outside of the first leg, observed in aluminium castings. This might be due to a difference in inertia between the water ($Re = 1200$) and the liquid aluminium ($Re = 1875$). Therefore, the same experiment is performed with a higher flow rate ($Re = 2280$). The results are given in figure 2.21 and figure 2.22. At the entrance line D the velocity at the
2.6. PIV RESULTS

(a) PIV-image

(b) Computed velocity vectors

Figure 2.16: Velocity field in the bend.
Figure 2.17: Velocity during transition from first to second leg.
Figure 2.18: Velocity field during filling of the bend.
inner side of the leg is still the largest, whereas at the exit line E the velocity at the outer side is the largest.

2.7 Summary

The filling of a thin-wall horizontal horseshoe is investigated. The experimental techniques used for analysing mould filling are: video recording, contact measurements, and thermocouples (section 2.3).

Video recordings and contact measurements show similar mould filling trends, that are described in section 2.4.1. The volumetric flow rate (figure 2.5) as well as the freezing length (figure 2.6) are determined. A difference in solidification time is observed between a location near to the ingate and after the bend (figure 2.7).

In the water model, PIV is used to obtain the integral velocity field (section 2.5). The flow rate on a basis of the measured velocities is larger than the flow rate that is expected from the displacement of the free surface, possibly due to a concentration of seeding particles halfway the channel height (section 2.6.1). The velocities measured in the water show gradients in the bend and near the ingate (section 2.6.2). In the bend, the velocity just behind the liquid-air surface is largest at the outer side. When the interface has passed, the velocity at the inner side is larger (figure 2.18). Near to the ingate of the water model, PIV shows the largest velocity at the inner side (figure 2.22), whereas it appears to be largest on the outer side in video registrations of a real casting (figure 2.3).
Figure 2.20: Velocity field near to the ingate (see figure 2.15) of mould filling with with water, and an ingate velocity of 0.2 m/s.
Figure 2.21: Velocity field near the ingate (see figure 2.15) during of mould filling with water, and an ingate velocity of 0.38 m/s.
Figure 2.22: The time averaged $u$-component of the velocity at the horseshoe inlet area, along lines D and E (see figure 2.15).
Chapter 3

Mathematical model

3.1 Introduction

The previous chapter showed typical filling conditions of the horizontal thin-wall casting under consideration: permanent contact with the top of the mould cavity, solidification and cold run during mould filling, and the flow behaviour at the ingate and in the bend. The first two phenomena are common features of horizontal thin-wall castings, while the third is typical for the casting under consideration.

Permanent contact with the top of the mould cavity is controlled by the surface tension, \( \sigma \), of the liquid aluminium and the contact angle with the moulding sand. A stream of aluminium in a mould cavity behaves like a liquid drop lying on a flat horizontal support. It maintains a certain height owing to adhesive and cohesive forces, see figure 3.1. The interfacial tension forces define the contact angle (\( \theta \)):

\[
\cos \theta = \frac{\sigma_{sa} - \sigma_{sm}}{\sigma}
\]  

(3.1)

where \( \sigma_{sa} \) and \( \sigma_{sm} \) represent the adhesive forces between metal and mould, and the cohesive forces in the metal.

Hence, \( \theta \) is a material property, depending on the three materials at the triple point of liquid, mould and air. By assuming a circular shape of the sides of the drop and setting the pressure due to gravity equal to the pressure due to surface tension, one obtains the following relation for the height, \( H \), of a liquid drop:

\[
H = \sqrt{\frac{2\sigma}{\rho g}} (1 - \cos \theta)
\]  

(3.2)

The exact shape of a liquid drop can be obtained by minimising the energy of a liquid drop, see figure 3.1, according to Pitts (1974):

\[
E = \int_0^H \left[ 2\pi \sigma x \frac{ds}{dy} + \pi \rho g (H - y) x^2 \right] dy - \sigma \pi x_0^2 \cos \theta
\]  

(3.3)
in which a slice $dy$ is considered at a radius $x(y)$. If $ds$ is the length of the profile of the drop, the energy due to surface tension is $2\pi \sigma x \frac{ds}{dy}$. The second term in the integral represents the potential energy of the slice. If $x_0$ is the radius of the liquid-support interface, the interfacial energy can be written as $\sigma \pi x_0^2 \cos \theta$.

Pitts (1974) applied variational analysis to this energy equation, and gives a numerical recipe to solve the obtained equations.

The height of a liquid drop on a flat horizontal support, called the natural height, makes it possible to distinguish between thick-wall and thin-wall horizontal castings on a basis of material parameters. In the following, a casting will be called thick-wall if its thickness is larger than the natural height, and thin-wall if the thickness is smaller than the natural height of a liquid drop.

Cold runs may be predicted by a mathematical model developed by Schröder (1982, 1983). In this model, the metal enters a 2-D channel, with a constant average velocity. The temperature of the metal is determined by conduction, convection, the heat transfer rate to the mould and the heat release per unit of time. The heat release rate and the heat transfer rate to the mould are constants.

This model is based on the following assumptions:

- Constant average flow velocity $u_{avg}$
- A given release rate of the latent heat per unit volume ($L$)
- A constant heat transfer coefficient ($h$) at the metal-mould interface
- A given wall temperature ($T_w$)
- A constant temperature ($T_p$) at the inlet
- The flow stops when the flowing metal has reached the solidification temperature ($T_{sol}$).

Conduction, convection of liquid and the heat transfer to the wall are equal to the time rate change of temperature ($T$) minus the temperature rise due to release of latent heat. This leads to the following differential equation:
\[
\frac{\partial^2 T'}{\partial x^2} - \frac{u_{avg}}{a} \frac{\partial T'}{\partial x} - \frac{2hT'}{\kappa d} = \frac{1}{a} \frac{\partial T'}{\partial t} - \frac{\dot{L}}{\kappa}
\] (3.4)

with \( T' = T - T_{wall} \). The thermal diffusivity (\( a \)) and the thermal conductivity (\( \kappa \)) are metal constants. Solving the differential equation under the conditions mentioned above, using a given temperature (\( T_{wall} \)) for the surrounding, and pouring temperature \( T_{pour} \) the following solution for the freezing length \( (L_f) \) is obtained (see (Schröder 1982)):

\[
L_f = \frac{-ln\left( \frac{T_{pol} - \frac{L_d}{2h}}{T_{pour} - \frac{L_d}{2h}} \right)}{-\frac{u_{avg}}{2a} + \sqrt{\frac{u_{avg}^2}{4a^2} + \frac{2h}{\kappa d}}} \] (3.5)

The presented models for the drop height and the freezing length, can be applied to simple geometries. Because the easy production of complex geometries is one of the most important advantages of casting over other manufacturing techniques, a model that also applies to complex geometries is preferable. In this chapter a general mathematical model is described. In horizontal thin-wall mould filling the quantities to be calculated are: Flow velocity, location of the free surface, surface tension, temperature and the fraction of solidified metal. The model consists of a set of conservation equations for momentum, mass and thermal energy, combined with an appropriate set of boundary and inlet conditions. The conservation laws of these three quantities will be described in a differential form. First, equations and boundary conditions will be evaluated for isothermal laminar flow with a free surface. Then the heat transfer equation with the boundary conditions at the metal-mould interface will described. The model will be completed by imposing a relation between the amount of solidified material and the friction felt by the liquid. The last section will consider the inlet condition to to be imposed on the set of differential equations.

### 3.2 Flow in a horizontal thin-wall cavity

Applying the conservation law of momentum to an infinitely small volume leads to the Navier-Stokes equation:

\[
\frac{\partial \rho \bar{u}}{\partial t} = -\nabla \cdot \rho \bar{u} - \nabla P - \nabla \cdot \tau + \rho \bar{g} \] (3.6)

where:

- \( \bar{u} \) : velocity
- \( t \) : time
- \( \rho \) : density
- \( P \) : pressure
- \( \tau \) : stress tensor (\( \tau_{ij} = -\mu \partial u_j / \partial x_i \))
- \( \bar{g} \) : gravitational acceleration
- \( \mu \) : dynamic viscosity
Mass conservation leads to the continuity equation:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{u} = 0
\]  

(3.7)

These equations are sufficient to describe isothermal laminar flow. The Reynolds number in tube flow is defined as:

\[
Re = \frac{\rho D_h u}{\mu}
\]  

(3.8)

where \(D_h\) is the hydraulic diameter of the cavity (or any other channel).

In a pipe, transition from turbulent to laminar flow takes place at \(Re \approx 2000\). In the horseshoe, \(u = 0.3 \text{ m/s}\) results in a Reynolds number below 2000.

There is always a moving surface involved in mould filling, and the above equations apply to the liquid. Therefore it is necessary to track the free surface, e.g. the metal-air interface, to know where the liquid is. The presence of liquid can mathematically be represented by a function \(F(x, y, z, t)\). This function \(F\) satisfies the conservation equation:

\[
\frac{\partial F}{\partial t} + \nabla \cdot F \vec{u} = 0
\]  

(3.9)

The function \(F\) has the value 1 at locations where fluid is present and a value 0 at locations with no fluid present. So \(F\) is a distribution, which is discontinuous and not differentiable at the interface. To solve this equation properly at the interface, special precautions have to be taken, as will be discussed in section 4.3.3. Equations 3.6 and 3.7 have to be solved only where \(F = 1\). The location of the free surface is at the transition from \(F=1\) to \(F=0\).

Equations 3.6 and 3.7 have to satisfy three different types of boundary conditions: an inlet condition (see section 3.5), a metal-mould interface boundary condition, and a boundary condition at the free surface.

At the metal-mould interface the liquid is at rest, i.e. the normal as well as the tangential velocity components are zero at this interface.

The free surface is subjected to adhesive and cohesive forces. The resulting capillary force resists the filling of a thin-wall cavity (Campbell 1988), if the contact angle (\(\theta\)) exceeds 90°. The natural height introduced in section 3.1 is also caused by these forces, which therefore can not be omitted.

Surface tension is introduced into the model as an excess pressure boundary condition \(\Delta P\) at the metal-air surface. The magnitude of this pressure is given by:

\[
\Delta P = \pm \frac{\sigma}{R}
\]  

(3.10)

where \(R\) is the radius of a circle approximating the local curvature of the surface. The sign of the excess pressure due to surface tension is positive for convex surfaces and negative for concave surfaces. At locations where the free surface is in contact with the mould, the contact-angle serves as a boundary condition for the curvature.
3.3 Heat transfer

3.3.1 Heat transfer in the liquid

Heat is transferred in the liquid by advection and diffusion. Free convection processes are of minor importance due to the small thickness of the casting under consideration. The internal energy is a conserved quantity that satisfies the following continuity equation:

$$\frac{\partial (\rho H)}{\partial t} + \nabla \cdot (\rho H \vec{u}) = \kappa \nabla^2 T$$  \hspace{1cm} (3.11)

The enthalpy $H$ is defined as:

$$H = \int_{T_0}^{T} C_p dT + H_L$$  \hspace{1cm} (3.12)

with:

$$H_L = (1 - f_s) L$$  \hspace{1cm} (3.13)

where $f_s$ is the fraction of solidified material and $L$ the latent heat. The thermal conductivity ($\kappa$) of the liquid is assumed to be a constant.

3.3.2 Heat transfer to the mould

During mould filling, heat is transferred from the metal to the sand. In the mould, conduction is the only heat transfer mechanism and no phase changes take place. So equation 3.11 can be reduced to

$$\frac{\partial T}{\partial t} = a \nabla^2 T$$  \hspace{1cm} (3.14)

with the thermal diffusivity $a$:

$$a = \frac{\kappa}{\rho C_p}$$  \hspace{1cm} (3.15)

Under the assumptions of ideal heat transfer at the interfaces and a constant fluid temperature $T_f$, the sand can be modelled as a finite slab with thickness $d$, initially at $T_\infty$, heated from one side at a constant temperature ($T_f$). This leads to a temperature distribution as in figure 3.2 (ideal contact). Two heat transfer regimes can be observed if equation 3.14 is solved for this case. First, heat will penetrate into the slab, causing a strongly varying internal temperature gradient. Then, when heat has penetrated throughout the slab, temperature is linear in the slab. If the Fourier number ($Fo$),

$$Fo = \frac{at}{d^2}$$  \hspace{1cm} (3.16)
Figure 3.2: Dimensionless temperature, with $T_\infty$ at $x = \infty$ in the sand at constant fluid temperature $T_f$.

is smaller than 0.1, penetration theory is valid. On a time scale of the mould filling period this is almost always the case for sand castings, because of the low thermal diffusivity of the moulding sand.

The heat transfer rate ($\phi''$) resulting from this penetration temperature profile is:

$$\phi'' = \frac{\kappa}{\sqrt{\pi at}} (T_f - T_\infty) = h_{pen}(T_f - T_\infty) \quad (3.17)$$

where $T_\infty$ is the temperature of the surrounding. The above only applies to the case with an ideal contact between the sand slab and the given boundary temperature. However, the contact between metal and sand is far from ideal. A schematic picture of the contact between sand and metal at the scale of the sand grains is shown in figure 3.3. Heat is transferred through such an interface by conduction through metal-mould contact area, conduction through the air between the sand grains and by radiation.

Such an interface can be incorporated in the mathematical model, by assuming two different temperatures at the interface. Although the temperature becomes a discontinuous quantity, the heat flux remains continuous, so that energy is conserved. The temperatures will be referred to as the temperature on the metal side of the interface and the temperature on the mould side of the interface, but they are mathematically at the same location. The heat flux through the interface is correlated to the temperature difference across the interface by an interfacial heat transfer coefficient $h_i$. The relative importance of the interfacial heat transfer and the heat transfer in the sand is given by the Biot-number ($Bi$):

$$Bi = \frac{h_i d}{\kappa} \quad (3.18)$$

where, $d$ is the thickness of the sand layer, and $\kappa$ the thermal conductivity of the
Figure 3.3: Schematic picture of the contact at the metal mould interface.

sand. Substituting the heat transfer coefficient associated with penetration theorie (equation 3.17), $h_{pen}$, into equation 3.18, gives for the Biot number:

$$Bi = \frac{h_i}{h_{pen}} = \frac{h_i \sqrt{\pi at}}{\kappa} \quad (3.19)$$

If $Bi \gg 1$, heat transfer is mainly limited by the temperature gradient in the sand. If $Bi \ll 1$, heat flow is controlled by the interfacial heat transfer coefficient and therefore the temperature gradient in the sand can be omitted. The Biot-number shows a transition from $Bi \ll 1$ to $Bi \gg 1$ during the mould filling period. Therefore, both heat transfer modes should be taken into account. To translate both heat transfer coefficients to a single heat transfer coefficient ($h$), the harmonical average of both values should be taken, i.e.

$$\frac{1}{h} = \frac{1}{h_i} + \frac{1}{h_{pen}} \quad (3.20)$$

The interfacial heat transfer coefficient is assumed to be constant in this study, although it might depend on shrinkage resulting in an air gap between metal and mould (Ho and Pehlke 1984).

3.4 Solidification during mould filling

Due to the large surface to volume ratio of a thin-wall casting, the metal may easily solidify during mould filling. Pure metals and eutectic alloys start to solidify on the mould wall, see figure 3.4 (case I), while for alloys in general solidification will take place in the melt itself, see figure 3.4 (case II).

The solidification mode has implications for the interactions between solidified fractions and the flowing liquid metal. For case I the metal will stick to the wall, narrowing
Figure 3.4: Solidifying material during mould filling.

the flow channel, and the solidified part will remain stationary with respect to the mould wall, see (Campbell 1988). In case II, the metal moves along with the fluid and has a velocity with respect to the mould wall. These two cases are extremes of the way of solidification, and for the presently studied casting it is most likely to have a mixture of both.

In the simulations, the influence of type I solidification on the liquid flow is automatically accounted for by a modification of the liquid flow domain. To account for the influence of type II solidification, deceleration of the liquid by the solid fraction is assumed to be linear with the velocity of the liquid. This can be expressed mathematically by adding an acceleration term to the right side of equation 3.6, leading to

\[
\frac{\partial \rho \bar{u}}{\partial t} = -\nabla \cdot \rho \bar{u} \bar{u} - \nabla P - \nabla \cdot \tau + \rho g - K \rho \bar{u}
\] (3.21)

where the constant K depends on the amount of solid fraction and the way the solid fraction is distributed, e.g. the way the material solidifies on a microscopic scale. For packed beds the function K is of the form:

\[
K = C_{dr g} \frac{1}{1 - \frac{f_s}{f_s}} (\frac{f_s}{1 - f_s})^2
\] (3.22)

with $C_{dr g}$ [1/s] a constant depending only on the shape of the particles in the packed bed and not on the degree of packing. This equation is similar to the Ergun equation, see (Poirier and Geiger 1994) (page 95).

The value of $C_{dr g}$ depends on the unknown shape of the solid particles and therefore has to be measured. The viscosity as a function of the solid fraction has been determined (figure 3.4) for an aluminium alloy, by Quaak (1996). To derive a value for $C_{dr g}$ the function K is scaled to an equivalent viscosity difference, using the density, $\rho$, of the liquid and the channel height, $d$:

\[
\mu - \mu_0 = \frac{1}{12} \rho d^2 K
\] (3.23)

For $C_{dr g}$ a value of 1000 s$^{-1}$ is chosen, which approximates the measured viscosity.
3.5 Flow inlet conditions

Now that flow and heat transfer equations have been described together with their boundary conditions, only an inlet condition is left to be defined. Either pressure or velocity has to be set somewhere, because the other quantity may then be computed from the Navier-Stokes and continuity equations. To make a choice, a closer look at gating systems is necessary.

The design of gating systems for manual pouring is based on controlling the pressure or the velocity, in order to be able to control the flow-rate from the gating system. An example for both cases is given in figure 3.6. In system I the pressure head is held constant and in system II the flow rate from the pouring cup.

To predict the flow rate, generally a Bernoulli approach is chosen. For gating system
I the Bernoulli equation can be reduced to

\[ v_3 = \sqrt{2g(h_1 - h_3) + \frac{2}{\rho}(P_1 - P_3)} \]  \hspace{1cm} (3.24)

and for gating system II to

\[ v_3 \sim v_2 = \sqrt{2g(h_1 - h_3)} \]  \hspace{1cm} (3.25)

This means that for gating system I the inflow rate depends on the pressure in the outlet, while it does not for gating system II. As a result, if the pressure in the outlet increases during mould filling, system I will react by decreasing the inflow-rate and II by increasing the pressure height in the down sprue, to maintain the flow-rate constant until the down sprue flows over.

For the experiments a gating system as in system II is used. This means that a constant flow rate is expected until the flow is obstructed, either because the mould is full or because a cold run occurs. The experimentally found flow rate is shown in figure 2.5.

The only function of the gating system is to deliver the flow to the inlet of the mould cavity. So it may be lumped into a given velocity or pressure at the inlet of the mould cavity. Since a constant flow rate is observed at the inlet, it is obvious to use the velocity as an inlet condition. The pressure at the ingate is then a computed value.

The pressure computed at the inlet can be used as a cold run criterion for a casting. It is clear that the pressure at the inlet can never be larger than the hydrostatic pressure of the liquid in the down sprue. However if a cold run appears, while a constant inlet velocity is used the pressure at the ingate will go to infinity. The moment the computed pressure at the inlet is equal to the maximum hydrostatic pressure in the down sprue, a cold run occurs.

### 3.6 Summary

In the above sections, differential equations have been selected for describing heat and momentum transfer on a local scale. The differential equations for momentum conservation (3.6), continuity (3.7) and heat transfer (3.11) have to be solved in the metal, and equation 3.14 in the sand. The location of the metal is given by the function \( F \) in equation 3.9, and the influence of solid fractions on the flow velocity is given by equation 3.21 and 3.22. These equations are to be solved simultaneously with the following boundary conditions: at the free surface the pressure depends on the local curvature of the surface, see equation 3.10, while at the metal-mould interface the normal and tangential velocities are zero. An interfacial heat transfer coefficient is used to approximate the non-ideal contact between metal and sand. The way these equations are solved, is described in chapter 4.
Chapter 4

Numerical model

4.1 Discretisation

Computations of the mould filling with liquid metal require solution of the set of differential equations of chapter 3. Flow with a free surface is described by equations 3.6 to 3.9 and heat transfer with solidification is described by equations 3.11 to 3.14. The solid fraction is coupled to the flow equation 3.6 by equations 3.21 and 3.22. To solve these equations computationally, they are written in a discrete form.

To discretise the equations a control volume (CV) method is used. This method requires the computational domain to be divided into a number of non overlapping control volumes. The differential equations are piecewisely integrated over the control volumes, using a constant value over the control volume for the variable to be solved. This results in difference equations in conserved form. Although boundary fitted coordinate systems have become available in many CFD codes in the last years (Minkowycz et al. 1988), brick shaped mesh cells are used in the present study.

A typical CV computational mesh cell with the stored parameters is shown in figure 4.1. According to the staggered grid definition (Patankar 1980), the velocities are located at mesh cell boundaries.

The use of a structured mesh makes it possible to address a computational mesh cell directly by its coordinates i,j,k. Its neighbours are then (i ± 1, j, k), (i, j ± 1, k) and (i, j, k ± 1). These indices will be used as subscripts to identify the cells in the finite difference equations that are to be solved. The cell size in x,y or z-direction is denoted by \( \delta x_i, \delta y_j \) and \( \delta z_k \). The distance between the cell centre and the centre of a neighbouring cell is denoted by \( \delta x_{i±1/2}, \delta y_{j±1/2} \) and \( \delta z_{k±1/2} \). For example:

\[
\delta x_{i±1/2} = \frac{(\delta x_i + \delta x_{i±1})}{2} \tag{4.1}
\]

Time has to be discretised as well. The most common schemes are implicit, explicit and Crank-Nicholson. In the implicit scheme the solution for a variable is computed from the newly computed values in neighbouring cells. Thus, an iteration procedure is needed. This method is unconditionally stable, i.e. the solution converges independently of the time-step size. In the explicit scheme, the new value of a variable is computed from the
values of the previous time step. No iteration is needed, but the maximum time step that is allowed is limited to prevent divergence. In the Crank-Nicolson scheme, the new value of a variable is computed from the values halfway the time step (Patankar 1980).

An explicit solution scheme is used here, because of the dynamical nature of mould filling. During mould filling, the shape of the free surface changes continuously. This shape is wanted at small time intervals to observe its dynamical behaviour. Since output with a high time resolution is wanted, limitations of the time step introduced by the explicit scheme are not disadvantageous. The limiting conditions for the time-step are described in section 4.6.

4.2 Solving the flow equations

To solve the discretised flow equations, a solution algorithm is needed. The method used in this thesis is basically the SIMPLE method (see Patankar (1980)). This algorithm advances the solution a time step $\delta t$ in three stages:

- Explicit approximation for the first guess new time level velocities, using previous time level values for all advective, pressure and other accelerations.

- The pressures are adjusted iteratively, so the velocities satisfy the continuity equation. Iteration steps are needed, because changing the pressure will upset the balance
in the adjacent cells.

- Update the scalar quantities, i.e. fluid fraction and temperature.

### 4.2.1 The momentum equations

The finite difference approximation to the momentum equations can be written in a generic form:

\[
\begin{align*}
    u_{i,j,k}^{n+1} &= u_{i,j,k}^n + \delta t^{n+1} \left[ -\left(p_{i,j,k}^{n+1} - p_{i,j,k}^n \right) / (\rho \delta x)_{i+1/2,j,k} \right] \\
                    & \quad - F_{UX}^n - F_{UY}^n - F_{UZ}^n + V_{UX}^n + V_{UY}^n + V_{UZ}^n \\
    v_{i,j,k}^{n+1} &= v_{i,j,k}^n + \delta t^{n+1} \left[ -\left(p_{i+1,j,k}^{n+1} - p_{i,j,k}^n \right) / (\rho \delta y)_{i,j+1/2,k} \right] \\
                    & \quad - F_{VX}^n - F_{VY}^n - F_{VZ}^n + V_{VX}^n + V_{VY}^n + V_{VZ}^n \\
    w_{i,j,k}^{n+1} &= w_{i,j,k}^n + \delta t^{n+1} \left[ -\left(p_{i,j+1,k}^{n+1} - p_{i,j,k}^n \right) / (\rho \delta z)_{i,j+1/2,k} \right] \\
                    & \quad + g - F_{WX}^n - F_{WY}^n - F_{WZ}^n + V_{WX}^n + V_{WY}^n + V_{WZ}^n
\end{align*}
\]  

(4.2) (4.3) (4.4)

In these equations the terms \( F_{UX} \) to \( F_{WZ} \) express the velocity change, per unit of time, due to the local convective fluxes of momentum, \( V_{UX} \) to \( V_{WZ} \) the viscous decelerations, and \( g \) the gravitational acceleration. The treatment of the flux terms will be described in section 4.2.3. The superscripts denote the current time level, \( n \), or the next time level, \( n+1 \). The first step in time advancing the solution is using a pressure of the current time level, \( n \).

### 4.2.2 Pressure solution algorithm

The velocities computed from the momentum equations in section 4.2.1, also have to satisfy the discrete approximation to the continuity equation 3.7:

\[
\begin{align*}
    \left( u_{i,j,k}^{n+1} A_{x(i,j,k)} - u_{i-1,j,k}^{n+1} A_{x(i-1,j,k)} \right) / \delta x_i + \\
    \left( v_{i,j,k}^{n+1} A_{y(i,j,k)} - v_{i,j-1,k}^{n+1} A_{y(i,j-1,k)} \right) / \delta y_j + \\
    \left( w_{i,j,k}^{n+1} A_{z(i,j,k)} - w_{i,j,k-1}^{n+1} A_{z(i,j,k-1)} \right) / \delta z_k &= 0
\end{align*}
\]  

(4.5)

During the iterative procedure, the calculated velocities do not necessarily satisfy equation 4.5, and the left hand side of this equation may, in general, have a certain value, \( S_{i,j,k} \). With a view to forcing \( S_{i,j,k} \) to zero, the pressure field should be adjusted and a new velocity field calculated from the momentum equations. Locally, pressure is adjusted by an amount estimated from:

\[
\delta p = - \frac{S_{i,j,k}}{(\partial S_{i,j,k} / \partial p)}
\]  

(4.6)
4.2.3 The flux of momentum

The computation of the terms $F_{UX}, ..., F_{WZ}$ in equations 4.2 to 4.4 has to be done with considerable care in a mesh with, in general, a variable cell spacing. For stability reasons, a so-called upwind differencing scheme is used (Patankar 1980). If the flux is computed from the divergence form, conservation of momentum is guaranteed by the Gauss theorem. This however, will not lead to accurate flux computation if donor cell differencing is used.

If a donor-cell approximation is made for the divergence form $\nabla \cdot \bar{u} \bar{u}$, the flux term $F_{UX}$ will be:

$$\frac{F_{UX}}{\delta x_{i+1/2}} = \frac{(u_R < u_R > - u_L < u_L >)}{\delta x_{i+1/2}}$$  \hspace{1cm} (4.8)$$

with:
\[ u_R = \frac{(u_{i+1} + u_i)}{2} \]
\[ u_L = \frac{(u_i + u_{i-1})}{2} \]
\[ < u_L > = u_{i-1}, \text{ if } u_i \geq 0 \text{ else } < u_L >= u_i \]
\[ < u_R > = u_i, \text{ if } u_i \geq 0 \text{ else } < u_R >= u_{i+1} \]

Expanding equation 4.8 in a Taylor series results in:
\[
F_{UX} = \frac{1}{2} \left( \frac{3\delta x_i + \delta x_{i+1}}{\delta x_i + \delta x_{i+1}} \right) \frac{\partial u^2}{\partial x} + O(\delta x) \tag{4.9}
\]

This approximation makes clear that the zeroth-order term may only be correct, if \( \delta x_i = \delta x_{i+1} \), i.e., if the mesh is equidistant. To maintain first-order accuracy for variable meshes in the donor-cell differencing approximation, the flux is approximated by \( \vec{u} \cdot \nabla \vec{u} \).

The flux in the x-direction can then be written as:
\[
F_{UX} = \frac{1}{2} ((u_R - |u_R|)D_{adr} + (u_L + |u_L|)D_{add}) \tag{4.10}
\]

with:
\[ u_R = \frac{(u_{i+1} + u_i)}{2} \]
\[ u_L = \frac{(u_i + u_{i-1})}{2} \]
\[ D_{add} = \frac{(u_i - u_{i-1})}{dx_i} \]
\[ D_{adr} = \frac{(u_{i+1} - u_i)}{dx_{i+1}} \]

For this formulation the zeroth-order term is independent of cell spacing and the solution is first-order accurate for varying cell sizes, as it should be. This scheme is used for all advective flux terms in equations 4.2 to 4.4. For the other acceleration terms, central differencing is used.

### 4.3 Free surface tracking and surface tension

#### 4.3.1 Numerical diffusion

Numerical diffusion of scalar quantities is caused by convection of the quantity through a Eulerian mesh. As for each variable only one value is stored in the centre of a mesh cell, no gradient can exist within a cell. Convection then leads to averaging the quantity that is entering a mesh cell and the amount that is remaining in a mesh cell during a computational time step. This results in unphysical (numerical) diffusion and smoothing of gradients.

The free surface is a discrete step of material. In the next section, it will be shown that an efficient way of keeping track of the interface is by representing the amount of fluid in a mesh cell by a scalar quantity, \( F \). To maintain the discrete character of the interface, numerical diffusion has to be prevented. Therefore, different approaches have been developed over the past two decades. Some of them will be discussed in the next sections.
4.3.2 Free surface tracking

The most simple way of tracking the free surface is by defining a height function \( h = f(x, y, t) \). This method is very efficient with respect to computer storage space, but does only work if \( f(x, y, t) \) is a function, i.e. the surface is not multi valued, and if \( \frac{dh}{dx} \) does not exceed the cell aspect ratio \( dy/dx \). The variation of the height function \( h \) in time obeys the conservation equation:

\[
\frac{\partial h}{\partial t} = -u \frac{\partial h}{\partial x} - v \frac{\partial h}{\partial y}
\]  

(4.11)

The line segment method uses points that are connected by line segments. The points have to be closer together than the cell spacing. The method needs more computer storage, but is not limited to single valued surfaces. Describing the time evolution of the line segments is accomplished by moving the points with the local velocity of the liquid, that is determined by interpolation of the surrounding mesh cell values. The problem with this method is in the folding of surfaces. At the locations where surfaces intersect, the chains have to be reordered, which is in general not a trivial task.

The marker and cell (MAC) method (Harlow and Welch 1965; Nichols and Hirt 1971) makes use of lots of randomly distributed particles, that represent the position of the free surface. A cell containing no particles is an empty cell, a cell containing particles with at least one empty neighbour is a surface cell, and a cell containing particles with no empty neighbours is a full cell. The particles are moved according to the interpolated velocities again. No drawback from folding surfaces occurs in this methods. The main problem in this method is the computer storage (three reals per particle) for the many particles that are needed to define the location of the liquid.

To overcome this problem, the Volume Of Fluid method (Hirt and Nichols 1981) was designed. This method has become commonly used in the simulation of mould filling (Hwang and Stoehr 1983; Hwang and Stoehr 1987; Hwang and Stoehr 1988; Lin and Hwang 1989; Jong and Hwang 1992; Mampaey and Xu 1992; Xu and Mampaey 1993).

4.3.3 The VOF method

The VOF method uses only one scalar variable (F) per computational mesh cell. The definition of empty cells, surface cells, and full cells is the same as in the MAC method. The normal direction to the fluid interface is in the direction in which F changes most. When both the normal direction and the value of F are known, the exact orientation and location of the surface within a computational cell are known. Because the VOF method tracks a region rather than surfaces, no problems arise in the intersection of surfaces.

The time dependence of F is described by equation 3.9. This equation implies the convection of the scalar quantity F through a Eulerian mesh; this approach may lead to smoothening and the loss of the discrete character of F. Fortunately, the discrete nature of F can be preserved by a donor-acceptor flux approximation.

The basic idea for the donor-acceptor flux approximation is to use the upstream as well as the downstream value of F, and to use a crude interface shape in the flux computation. Consider two computational cells as in figure 4.3, with a cell face of unit area between them. The volume transported, per unit of area, from donor cell to acceptor cell then
becomes $u\delta t$. The amount of fluid transported, per unit area, $(\delta F)$ is the amount of fluid in this volume, and can be calculated from

$$\delta F = \min\{F_{AD} u\delta t + C_f, F_D \delta x_D\}$$  \hspace{1cm} (4.12)

with:

$$C_f = \max\{ (1 - F_{AD}) u\delta t - (1 - F_D)\delta x_D, 0 \}$$  \hspace{1cm} (4.13)

In these equations, $F_D$ denotes the fluid fraction of the donor cell, and $\delta x_D$ the cell size of the donor cell. The functions "min" and "max", the minimum and the maximum, respectively, of the two values between the brackets. $F_{AD}$ denotes the fluid fraction of either the donor cell or the acceptor cell, depending on the orientation of the surface.

When the surface is advected parallel to itself, see figure 4.3 b, then $F_{AD} = F_D$, and $C_f = 0$, because, according to the Courant condition, $\delta t < \frac{\delta x_D}{u}$. Then equation 4.12 states that the amount of fluid transported is the product of the fluid fraction of the donor ($F_D$) and the volume $u\delta t$. This amount of fluid is cross-hatched in figure 4.3 b.

When the surface is directed normal to the surface, see figure 4.3 c and 4.3 d, $F_{AD} = F_A$. The amount of fluid transported, per unit area, is the product the acceptor cell fluid fraction $F_A$ and the volume transported, per unit area. No more than the total amount of fluid available in the donor cell can be fluxed, however. To prevent this, the minimum of both values for $\delta F$ is taken, see figure 4.3 c. The max function in equation 4.13 works in a similar way, but preventing the flux of more than the amount of void present in the donor cell $(1 - F_D)$. So if the amount of void in volume $u\delta t$ is calculated to be larger than the amount of void available in the donor cell, the difference is to be occupied by liquid and has to be added to the amount calculated from $F_A u\delta t$.

There are just two exceptions to the selection rule for the fluid fraction to be used. The acceptor cell value is used regardless the orientation of the surface, if the acceptor cell is empty or if the cell upstream of the donor cell is empty.

### 4.3.4 Determining the surface normal

To know whether the flow is normal or tangential to the surface, an approximate normal to the surface is set. This is explained here for the 2-D case (x-y). The surface is determined by a function $Y(i)$ and a function $X(j)$. The function $Y(i)$ for column $i$ with surface cell $i,j$ is approximated by

$$Y(i) = F_{i,j-1} \delta y_{j-1} + F_{i,j} \delta y_j + F_{i,j+1} \delta y_{j+1}$$  \hspace{1cm} (4.14)

Similarly, for the neighbouring columns $i-1$ and $i+1$, the surface heights are approximated by

$$Y(i \pm 1) = F_{i\pm1,j-1} \delta y_{j-1} + F_{i\pm1,j} \delta y_j + F_{i\pm1,j+1} \delta y_{j+1}$$  \hspace{1cm} (4.15)

The function $X(j)$ can be developed in the same way. The value of $X(j)$ and $Y(i)$, together with $F$ and the slope of the surface, make it possible to construct a line in the
Figure 4.3: The donor-acceptor method. The cross-hatched surfaces represent the amounts of F that are transported.

computational cell, that approximates the position and orientation of the surface within a computational cell. The direction of the surface is approximated by assigning a value 1, 2, 3 or 4 to an integer parameter NF, representing one of the four neighbouring cells that is closest to the direction of the inward surface normal. If a cell has one empty neighbour (figure 4.4), the inward surface normal is in direction away from this cell. If a cell has more than one empty neighbour (figure 4.4), the inward normal points in the direction of the neighbouring cell with the largest F value. The extension of this procedure to three dimensions is rather straightforward.

4.3.5 Free surface boundary conditions

The VOF algorithm allows to set boundary conditions at the surface, for variables such as surface tension and the contact angle at the fluid interface at obstacles. To illustrate the procedure, consider a mostly horizontal surface (figure 4.4(a)).

To compute the excess pressure due to surface tension, equation 3.10 is used. In this equation R represents the radius of curvature. The curvature \( K = \frac{1}{R} \) of a mainly horizontal surface is given by

\[
K = \frac{d}{dx} \left[ \frac{\frac{dY}{dx}}{\sqrt{1 + (\frac{dY}{dx})^2}} \right]
\]  

(4.16)
Figure 4.4: Illustration of the principle of determining the surface normal.

The function $Y$ is given by equation 4.14. The pressure $p_s$ due to surface tension is the product of curvature and surface tension. This pressure has to be applied at the surface. In the finite volume method, however, the pressure is a cell centred value. Hence, the pressure at the surface has to be extrapolated to the cell centre. In figure 4.5, the pressure $p_k$ in the centre of a surface cell is set such that the pressure at the surface is $p_s$, i.e. the pressure in the surface cell is computed from

$$p_k = (1 - \frac{\delta k_i - 1/2}{\delta s}) p_{k-1} + \frac{\delta k_i - 1/2}{\delta s} p_s$$

(4.17)

4.4 Heat transfer and solidification

4.4.1 Heat transfer

Heat transfer in the mould is simply a conduction problem, with constant diffusivity, and is described in an explicit way, because the time step size is mostly limited by flow and not by heat transfer. To compute heat transfer in the mould, a central difference version of equation 3.14 is used. In the metal, a discrete version of equation 3.11 is solved.

Heat transfer from metal to mould is calculated using the two cell-centred values of the temperature on either side of the interface. The heat flux from metal to sand is computed as:

$$\phi'' = h_{num}(T_m - T_s)$$

(4.18)

Where $h_{num}$ is the heat transfer coefficient, and $T_s$ and $T_m$ denote the temperature of the cells on the sand side and the metal side of the interface, respectively. If the interfacial
heat transfer coefficient \( h_i \) is used as a value for \( h_{num} \) (model 1), the temperature \( T_s \) should be the temperature of the interface on the sand side. Since cell-centred values are used, this is only true in the limiting case of small Computational cells in the sand, i.e. the heat transfer through the interface depends on the computational cell size, \( \delta_s \), at the interface.

In FLOW-3D, at least in Version 5, only the approximation \( h_{num} = h_i \) was available. Detailed calculations revealed that this approximation was too crude, and resulted in inaccurate heat balances.

Two modifications of \( h_{num} \) can be suggested to cure this problem: (i) the use of a heat transfer coefficient \( h_{num} \) that is corrected for the distance between the cell centre and the interface (model 2) or (ii) the use of a time dependent heat transfer coefficient based on penetration theory (model 3). Both modifications were introduced into the FLOW-3D source code.

In this chapter, the effect of using these three approximations of \( h_{num} \) will be illustrated:

Model 1: \( \psi'' = h_i(T_m - T_s) \)
Model 2: \( \psi'' = \left( \frac{1}{h_i} + \frac{\delta_s^2}{k} \right)^{-1}(T_m - T_s) \)
Model 3: \( \psi'' = \left( \frac{1}{h_i} + \frac{\sqrt{\rho c_p \alpha_i}}{k} \right)^{-1}(T_m - T_\infty) \)

Models 1 and 2 use the local, transient sand temperature, \( T_s \), resulting from the simulation. Model 3 computes the heat transfer from the initial mould temperature. While, on one hand, the mould temperature does not need to be calculated, on the other hand model 3 depends on the contact time between metal and mould, and thus requires administration of the local contact time between liquid and sand.
Application of model 3 is restricted, because some assumptions are made. The Fourier-number has to be smaller than 0.1 for penetration theory to be valid. The wetted areas of the mould should stay wetted. Errors are introduced at curved walls, because in the model the walls have been assumed to be straight.

### 4.4.2 Solidification

The fraction of solid, \( f_s \), that is formed within a computational cell as a result of solidification of a computational cell is computed from the internal energy of that cell. The assumption is made that the solidified fraction decreases linearly with enthalpy between the liquidus and the solidus enthalpy; hence,

\[
    f_s = \frac{H - H_S}{H_L - H_S} = \frac{H - H_S}{L} \quad (H_S \leq H \leq H_L)
\]  

(4.19)

Since partially solidified cells can only have a single temperature, the cells are always between solidus and liquidus temperature, when they contain some amount of solidified material.

The solidified fraction formed is assumed to be in rest with respect to the computational grid. The effect of the solidified parts on the flow is in terms of a drag force \( \vec{f}_{\text{drag}} \) per unit volume on the flow in the cell under consideration with magnitude

\[
    \vec{f}_{\text{drag}} = -K \rho \vec{u}
\]

(4.20)

The drag coefficient \( K \) is computed from the solid fraction \( (f_s) \) using equation 3.22. Because the drag coefficient \( K \) goes to infinity if all the liquid is solidified \( (f_s = 1) \), it is implemented in an implicit way to avoid instabilities.

The parameter \( C_{\text{drag}} \ [1/s] \), which determines the value of \( K \) and was set to \( C_{\text{drag}} = 1000 \text{s}^{-1} \) (see section 3.4), should be chosen in accordance with the packed bed model described in Poirier and Geiger (1994).

### 4.5 Obstacle representation

To describe an arbitrary geometry in the CV method with brick shaped cells, it is represented by FAVOR (Fraction Area/Volume Obstacle Representation), see Hirt and Sicilian (1985). This method requires four additional real variables to be stored for each computational mesh cell: three fractional areas on the mesh cell interfaces and the fractional volume of the computational cell that are open to flow, which are denoted by \( P_x, P_y, P_z \) and \( P_v \) corresponding to \( A_x, A_y, A_z \) and \( V \) in figure 4.1.

Equations 3.6, 3.7, 3.9 and 3.11 are all convection-diffusion type equations, of the form

\[
    \frac{\partial \rho \phi}{\partial t} + \nabla \cdot (\rho \vec{u} \phi - \Gamma_{\phi} \nabla \phi) = S
\]

(4.21)

with a diffusion constant \( \Gamma_{\phi} \) of the quantity \( \phi \). Integration over a time step \( \Delta t \) and considering only cell faces in the x-direction, equation 4.21 becomes

\[
    \Delta \left( \rho \phi \right) = -\frac{\Delta t}{V} \left( J_c - J_w \right) + S \Delta t
\]

(4.22)
with the flux \( J_e \) through the east cell face with an area \( A_e \),

\[
J_e = (\rho u_i \phi - \Gamma \phi \frac{\delta \phi}{\delta x_{i+1/2}}) A_e
\]  
(4.23)

For \( J_e \), a similar expression can be written down. To incorporate the open cell face areas and the open volume of a cell in the differential equations, the cell volume \( V \) is replaced by \( P_V V \), where \( P_V \) is the fraction of the cell volume open to flow. The area of the cell face \( A_e \) is replaced by \( P_e A_e \), where \( P_e \) is the fraction of the east cell face open to flow. The inter node distance \( \delta x_{i+1/2} \) is replaced by \( P_V \delta x_{i+1/2} \). Similar modifications are made for the other directions.

### 4.6 Stability considerations

Since an explicit method is used, the maximum time step to assure a converged solution is limited. Three limiting criteria for the time step are of importance for the computations under consideration. The most general limit is the Courant condition (see Patankar (1980)), that prevents liquid to travel a distance larger than the computational cell size. The time step is limited in the following way:

\[
\delta t < \min \left( \frac{\delta x_i}{u}, \frac{\delta y_j}{v}, \frac{\delta z_k}{w} \right) \tag{4.24}
\]

Viscous fluid computations lead to the limitation that:

\[
\delta t < \frac{\rho}{4 \mu} \left( \frac{1}{\delta x_i^2 + \frac{1}{\delta y_j^2} + \frac{1}{\delta z_k^2}} \right) \tag{4.25}
\]

For the surface tension the limit is:

\[
\delta t^2 < \frac{\rho}{\sigma \max \left( \left( \frac{1}{\delta x_i^2} \right), \left( \frac{1}{\delta y_j^2} \right), \left( \frac{1}{\delta z_k^2} \right) \right)} \tag{4.26}
\]

The most stringent condition is used to limit the next time step. Furthermore, the time step is changed in portions of 5% to keep the number of iterations at 25 for each time step \( \delta t \) if none of the convergence criteria is violated. The computation of values for the next time step is repeated with half the time step if one or more of the convergence criteria are violated.

### 4.7 Software and hardware

The mould filling and heat transfer computations have been carried out by the use of the commercial software code FLOW-3D (1990), Version 5, from Flow Science Inc.. This software tool was chosen because of its capabilities to describe flow with a free surface and solidification, its general modelling of transport equations and the availability of the source code. Other work, with the same code, in the field of mould filling has been reported by (Barkhudarov et al. 1992; Barkhudarov et al. 1993). The software is used on a HP735 Workstation running at 120 MHz, with an internal memory of 128 Mb. Reported computation times have been obtained on this type of computer.
4.8 Summary

A solvable set of difference equations has been selected, with an appropriate approximation of the boundary conditions and with suitable stability criteria. Special care is taken to make the convection of momentum first-order accurate with changing mesh sizes.

The VOF method is used to track the free surface in time. Surface tension is implemented as a boundary condition at the free surface.

Heat transfer at the metal mould interface is computed using an interfacial heat transfer coefficient at the wall. To translate the interfacial heat transfer coefficient to a value that can be used in computations, three approximations are suggested in section 4.4.1.

A commercial software code (FLOW-3D) is used to solve the equations. Modifications to the source code were made to implement model 3 for the heat transfer coefficient (section 4.4.1).
Chapter 5

Computational Results

5.1 Introduction

In this chapter, computational results are presented which were obtained with the mathematical model and the numerical procedures described in chapters 3 and 4. As a basis for these computations, the commercial software code FLOW-3D (Version 5) was used.

Keeping track of the free surface is essential in modelling mould filling. In horizontal mould filling with liquid metals, surface tension and poor wetting lead to a substantial fluid height. In horizontal thin-wall mould filling, surface tension causes metal to be in permanent contact with the top mould. Therefore, not only the surface has to be tracked by the VOF-algorithm, but also the surface tension has to be accounted for. As a first test of the software's capability of accurately modelling surface tension, the height of a liquid drop residing on a flat support is computed as a function of the contact angle, and compared with analytical models (section 5.2.1). The influence of surface tension on the pressure drop in a 2-D horizontal channel is studied in section 5.2.2.

An accurate description of the heat transfer is also important. The loss of heat to the mould may cause the metal to solidify during mould filling, and obstruct the liquid flow. Accuracy should preferably be obtained with a low computational overhead on the mould filling computations, i.e. the available mesh cells should be in the fluid and not in the mould (Van Tol et al. 1997a; Van Tol et al. 1997b; Van Tol et al. 1997c). The three suggested models for the heat transfer coefficient, described in chapter 4, will be evaluated in section 5.3.1.

The third subject of investigation concerns the cold-run criterion proposed in chapter 3. Eliminating part of the gating system from the computational domain reduces the computational overhead on the mould filling computations (section 5.3.2).

The findings of these tests will then be used to model the filling of the horizontal horseshoe (section 5.4). The computed freezing length will be compared with the freezing length measured in mould filling experiments with liquid metal, as presented in chapter 2. Cooling after mould filling is computed as well. Computed velocities will be compared with PIV measurements.
5.2 Numerical results concerning surface tension

5.2.1 The liquid drop

To investigate the capabilities of FLOW-3D in applying surface tension at a free surface, the height of a liquid drop on a flat horizontal support is computed in a Cartesian mesh. A cylindrical grid might be preferable for this situation, because of the cylinder symmetry of a drop. However, the algorithm is going to be used in a general geometry, where the surface may cut through several computational cells. Therefore a Cartesian mesh is used in this test.

In 2-D simulations of half the liquid drop, an equidistant grid of $50 \times 20$ cells is used in a domain of $0.05 \text{ m}$ horizontally times $0.02 \text{ m}$ vertically. In 3-D, a quarter of the drop is modelled, and an equidistant grid of $50 \times 50 \times 20$ cells in a domain of $0.05 \text{ m} \times 0.05 \text{ m} \times 0.02 \text{ m}$ is used. On all sides of the domain, symmetry boundary conditions are used, except for the bottom side, which is a rigid wall. The initial fluid configuration is built up out of a torus overlapping a cylinder, resulting in a quarter of a drop as shown in figure 5.1(a,c). Figure 5.1(a) shows a 3-D view of the iso-surface $F = 0.5$, and figure 5.1(c) shows a gray value plot of the fluid fraction. This initial shape is favourable over a square, because no sharp edges are present, that would introduce large numerical oscillations in the drop, leading to a long damping time and therefore to a large computational effort in these transient computations. The height of the initial drop is in all cases 10 mm. In figure 5.1(b,d), the shape of the same drop, with a contact angle of $140^\circ$ and a surface tension of $0.84 \text{ N/m}$, is shown after 3 seconds. The height and shape of the drop have changed during this period. In figure 5.5, the fluid fraction $F$ is plotted along the $x$-axis at $z=0$, for two time instants. These plots show a transition from 1 to 0 over a distance of 1 computational cell. Smearing of the liquid is efficiently controlled by the VOF algorithm.

The height, at the centre of the drop, during the computational period is shown in figure 5.3. The oscillations that appear on the surface of the liquid drop are much smaller than the height of the drop. To compute the average drop height, the average of the height of the drop is taken over the last 1.5 seconds, to eliminate the influence of the initial drop shape.

The time averaged drop heights in the simulations are shown in figure 5.4, together with the results of the analytical models of equations 3.2 and 3.3. The height of the liquid drop in 2-D simulations is smaller than would be expected from these analytical equations. In a 3-D simulation of the liquid drop in a Cartesian grid, the computed height resembles that of the 3-D analytical model, described by equation 3.3.

The drop shape computed with FLOW-3D is presented in figure 5.2, together with the shape of the drop calculated with equation 3.3. The shapes are virtually identical. The largest difference in the curves is found at the the triple point metal-air-sand. This is due to the resolution of the plotting program, where the surface is plotted as a straight line across a mesh cell. Near the triple point the curvature of the surface is large and will therefore be poorly estimated by a straight line.

These results show that the way surface tension is incorporated in the model, see section 4.3, leads to results that correspond well with the outcome of analytical models. Height and shape are computed correctly.

From these results it may be expected that surface tension will keep liquid metal in contact with the top mould in a horizontal thin-wall cavity.
Figure 5.1: Initial shape and shape after 3 seconds of the drop. The height of the drop increases due to surface tension forces. The initially circular side of the drop changes to a more realistic shape. Numerical values along the axes in (c) and (d) are in meters, while grey values relate to the fluid fraction F.
Figure 5.2: The shape of the liquid-air interface of a liquid metal drop lying on a flat support, according to FLOW-3D and due to formula 3.3.

Figure 5.3: Fluctuations observed in the liquid drop height during relaxation from an initial height of 10 mm. The height given in figure 5.4 is computed as the average height over a period from 1.5 seconds to 3.0 seconds.
Figure 5.4: Difference between analytical calculation and CFD approximation of the average liquid height. 2-D and 3-D computations are compared to models for the height of a liquid drop, described by equations 3.2 and 3.3.

Figure 5.5: The fluid function $F$, at half the initial drop height ($z=0$), shows a sharp transition from 1 to 0 for $t=0.0$ s and $t=3.0$ s. The cell size is 1 mm.
(a) One cell, no surface tension  (b) One cell, with surface tension

(c) Three cells, no surface tension  (d) Three cells, with surface tension

Figure 5.6: The interface shape, with and without surface tension, for different numbers of cells in vertical direction.

5.2.2 2-D study of isothermal horizontal thin-wall mould filling

The effect of surface tension on horizontal thin-wall mould filling can also be shown computationally in a two-dimensional strip as in figure 5.6. The boundary velocity at the left is set to \( u_0 = 0.3 \text{ m/s} \). If three computational cells are used in the vertical direction, surface tension keeps the liquid in permanent contact with the top mould, as can be seen from figure 5.6(d). If surface tension is set to zero, the liquid wets the bottom of the cavity first (fig. 5.6(c)). In cases where one cell is used across the thickness of the cavity, no differences are found in the shape of the free surface, because gradients are not resolved within one computational cell, figure 5.6 (a) and (b). Surface tension, however, has an effect on the pressure in the liquid.

The computed pressures for the cases of figure 5.6 are shown in figure 5.7. For three cells in vertical direction, the computed inlet pressure due to surface tension is \( 242 \text{ Pa} \). This value is obtained from the intersection with the y-axis of a straight line fitted through the pressure curve, as indicated in figure 5.7 with an arrow. Analytically, the pressure increase \( \Delta P \) due to surface tension in a horizontal channel with height \( d_{\text{channel}} \) can be calculated from (Campbell 1971)

\[
\Delta P = \frac{2\sigma \cos \theta}{d_{\text{channel}}} \quad (5.1)
\]

Using a contact angle (\( \theta \)) of \( 140^\circ \) and a surface tension of \( 0.84 \text{ N/m} \), an inlet pressure of \( 257 \text{ Pa} \) is computed. The computed pressure of \( 129 \text{ Pa} \) with one cell in the thickness is only around half this value. So, the pressure increase due to surface tension is not computed correctly for one computational cell with two adjacent wall cells. For multiple computational cells in the thickness of the channel the correct value is computed.

It is found that the free surface introduces pressure fluctuations in horizontal thin-wall simulations. These fluctuations do not depend on the number of cells in the thickness only, but also on the x to z aspect ratio of the computational cells. In figure 5.8, pressure curves are shown for aspect ratios 1 to 4 together with the computed surface. Figure 5.8 shows that an increasing aspect ratio leads to increasing pressure fluctuations. If the aspect
ratio is larger than 3, the surface breaks up and shows an unrealistic fluid distribution (figure 5.8d).

5.3 Solidification during mould filling

5.3.1 Heat transfer at the metal-mould interface

Sharp temperature gradients may be expected in the sand near the liquid metal, as indicated in chapter 3. To resolve these temperature gradients, which may vary in time, small computational cell sizes have to be chosen in this area, see chapter 4. This may lead to a substantial number of computational cells in the mould. Preferably, the cells are in the mould cavity, because this is the actual domain of interest. The three approximations of the numerical heat transfer coefficient described in chapter 4, viz. model 1 ($h_{\text{computational}} = h_{\text{interface}}$), model 2 ($1/h_{\text{computational}} = 1/h_{\text{interface}} + \delta/(2k)$), and model 3 ($1/h_{\text{computational}} = 1/h_{\text{interface}} + \sqrt{\pi} \alpha t/k$), are compared for different cell sizes. A constant liquid temperature is defined at one side of the interface. The total heat transferred through a metal-mould interface of unit area is computed for cell sizes of 5mm, 0.5mm and 0.05mm. An interfacial heat transfer coefficient of 1250 $W/m^2\cdot C$ is used. Note that model 3 approaches to the analytic solution of the problem for this one-dimensional case most closely, whereas the models 1 and 2 represent steady state approximations of different degree of simplicity.

The results from model 1 (dashed lines) and the analytical model (model 3, solid line) are shown in figure 5.9. It is clearly shown that the result in the case of model 1 depends on the cell sizes chosen. Near $t=0$, the result is correct, but deviations occur for longer times, because the heat transfer is overestimated. The model converges downwards to the analytical solution for decreasing cell sizes, as should be expected.

For model 2, the results are shown in figure 5.10 (dashed lines). The heat transfer is underestimated compared to model 3, but converges upwards to the analytical solution for decreasing cell sizes.
Figure 5.8: Computed inlet pressures in a strip of 0.3 m length and with 5 computational cells across the thickness. Surface break-up is shown for aspect ratios larger than 3.
Figure 5.9: Total heat transferred as a function of time, from metal at 552°C to a sand mould, computed with a constant heat transfer coefficient of \( h = 1250 \frac{W}{s\cdot\text{cm}^2} \), and cell sizes of 0.05 mm, 0.5 mm and 5 mm, respectively (model 1), and compared with analytical prediction (model 3).

Figure 5.10: Total heat transferred as a function of time, from metal at 552°C to a sand mould, computed with a heat transfer coefficient of \( h = \frac{1}{1250 + \frac{5}{k} \cdot W} \frac{W}{s\cdot\text{cm}^2} \), and cell sizes of 0.05 mm, 0.5 mm, and 5 mm, respectively (model 2), and compared with analytical prediction (model 3).
For both models an acceptable approximation is reached at cell sizes of 0.05 mm only. This is just 1% of the thickness of the horseshoe and therefore inefficient in mould filling computations. Therefore, the time dependent heat transfer coefficient of model 3 was introduced. This model 3 is expected to give a more accurate computation of the heat transfer to the sand than the other two approximations. However, it suffers from the drawbacks mentioned in section 4.4.1: wetted areas must stay wetted, penetration in the sand must be smaller than the thickness of the sand, and an error is introduced in curved walls. Under these conditions the heat transfer to the sand becomes independent of the number of cells in the sand. Thus, although model 3 is far from ideal, it might serve as useful option for calculating the heat transfer.

5.3.2 2-D computations with heat transfer and solidification

To compute the freezing length, heat transfer as well as solidification are incorporated into the 2-D isothermal computations of section 5.2.2. A constant inlet temperature is specified at the left of the horizontal strip. The interfacial heat transfer coefficient is set to 1250 W/m²C and all 3 models suggested in section 4.4.1 are tested.

The moment of freezing of the flow channel is determined using a value of 1 kPa as the excess pressure criterion at the ingate, as described in section 3.5. The freezing length as a function of the inlet temperature, ranging from 600°C to 750°C, is shown in figures 5.11 to 5.13. A grid of 396 × 1 cells, 396 × 3 cells, and 396 × 5 cells, respectively, is used for a domain with dimensions 660 mm × 5 mm. Four additional cell-layers, two at the top and two at the bottom, are used to compute the heat transfer to the moulding sand; these cells are 5 mm in size.

In figures 5.11 to 5.13, the sub-figures (a) to (c) show the computed pressure at the ingate during the filling of the channel, for the three different heat transfer models. Figure 5.11 shows that for 1 computational cell in the thickness the pressure rise is sudden for all three models, and that the pressure before freezing resembles the pressure curves from figure 5.7. This means that the additional pressure rise is caused just by solidification of the liquid during filling. For three computational cells, the results are shown in figure 5.12. Solidification in these computations starts at an earlier time and pressure rises less steeply. This is shown more clearly in figure 5.13, where 5 computational cells are used in vertical direction. Plateaus are revealed at a pressure of 800 Pa, leading to an increase of the freezing length. These plateaus are caused by partial solidification and partial blocking of the channel, see section 3.4.

In figures 5.11 to 5.13, subfigures (d) summarise the computed freezing length for all three heat transfer models. For 1 cell (fig. 5.11) and 3 cells (fig. 5.12), the freezing lengths are almost identical. For 5 computational cells (fig. 5.13), the freezing length as a function of temperature differs from the computations with 1 cell and 3 cells. The increase in freezing length for model 3 is caused by the plateaus in the pressure curves.

In figure 5.14, for three computational cells in vertical direction, the computed freezing length is compared with the measured freezing length in the horseshoe, as presented in figure 2.6. The freezing length is underestimated by model 1, owing to the overestimation of the heat transfer shown in figure 5.9. Model 2 (not shown) shows freezing lengths that are far larger than found in the experiments, due to underestimation of the heat transfer, especially near the flow tip. Model 3 leads to overestimation of the freezing length as well. This may be due to the fact that the inlet temperature of the computational strip
Figure 5.11: Computed pressure rise and freezing length using one computational cell in vertical direction and three different approximations for the heat transfer.
Figure 5.12: Computed pressure rise and freezing length, using three computational cells in vertical direction and three different approximations for the heat transfer.
Figure 5.13: Computed pressure rise and freezing length using five computational cells in vertical direction and three different approximations for the heat transfer.
Figure 5.14: Freezing length as a function of temperature, experimental and for three computational cells.

is assumed to be equal to the pouring temperature of the metal. In a casting, the metal may have lost heat in the gating system at this moment and therefore is entering the mould cavity at a lower temperature. This results in a lower freezing length compared to the ideal situation where no heat is lost in the gating system. Another cause may be the simplification of the horseshoe to a two-dimensional case.

Since estimating the interfacial heat transfer coefficient accurately is difficult, the computational sensitivity to this parameter is tested. The results in figure 5.15 are obtained using an inlet temperature of 675°C, 1 and 3 cells in vertical direction, and a heat transfer coefficient ranging from 1000 W m⁻² K⁻¹ to 1500 W m⁻² K⁻¹. This figure shows that the results do not heavily depend on the heat transfer coefficient. Changing the heat transfer coefficient by 50% leads to a change of only 25% in the freezing length. This figure also shows that the choice of the heat transfer model has a larger impact on the absolute freezing length than the inaccuracy in the interfacial heat transfer coefficient.

5.4 The horseshoe

5.4.1 Computations with water

To compare CFD computations with the PIV experiments, the physical parameters are set to values of water, and the channel height of the horseshoe is reduced from 5 mm to 3 mm.

The calculated velocity profile is studied before the bend, in the middle of the bend, and after the bend, just like in the PIV experiments (see section 2.6). For an inlet velocity of 20 cm/s and 1 computational cell in vertical direction, the computed velocity profiles are shown in figure 5.16. Before the bend hardly any velocity gradients are found, see figure 5.16(a). When the liquid enters the bend (figure 5.16(b)), the velocity on the outer side is larger than on the inner side. The opposite, however, is found 0.4 seconds later.
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Figure 5.15: Freezing length as a function of the interfacial heat transfer coefficient for model 1 and model 3 and an ingate temperature of 675°C.

After the bend (figure 5.16(c)), the velocity on the outer side of the second leg is larger than on the inner side. Although the gradient decreases in time, it does not vanish. The computations with three computational cells in the thickness show the same behaviour (figure 5.17), except that a gradient is found before the bend as well. Wall shear is shown in this simulation by lower velocities near the extremes of the x-axis. Note that the overall velocities in this figure are larger than in figure 5.16, because the velocities in the central plane are plotted here, and not the averages over the channel height.

These figures may be compared with the PIV results shown in figures 2.17 and 2.18. Before the bend, the time averaged velocity, presented in figure 2.17, is a little larger at the inside of the first leg. The same is found in the CFD computations. PIV shows a velocity in the middle of the bend that is larger on the inner side, like figures 5.16(b) and 5.17(b) do for times larger than 3.2 seconds. After the bend, the largest velocity is found on the outside of the second leg, in both PIV measurements and CFD computations. The transition in velocity gradient in the middle of the bend is confirmed by figure 2.18.

The computed velocities in the ingate, presented in figure 5.18, may be compared with the PIV results in figure 2.22. Velocities along the entrance line show a velocity gradient, with the larger velocities on the inner side of the leg, in both computations and experiments. Along the exit line, the computed velocity is largest on the outside of the first leg. This is also found in the experiments, although less pronounced.

If the assumption is correct that the particles in the PIV experiments are located halfway the horseshoe's channel height (see section 2.6.1), the computed average velocities are roughly in agreement with the PIV results.

5.4.2 Computations of the casting.

Since FLOW-3D is based on a structural mesh, computing the mould filling in 3-D, especially with multiple cells in the thickness of the horseshoe, requires an unpractical number of mesh cells, if the whole gating system is taken into account in the computations.
Figure 5.16: Velocities during filling of the bend with water, using 1 computational cell across the channel height.
Figure 5.17: Velocities during filling of the bend with water, using 3 computational cells across the channel height.
Figure 5.18: $u$-component of the velocity at the horseshoe inlet area, along lines D and E (see figure 2.15).
Therefore, it was suggested to lump at least part the gating system into a single valued inlet velocity and to use a pressure criterion at the inlet, see section 3.5.

The choice of the location of the inlet conditions for the 2-D computations (section 5.3.2) was not a difficult one: The metal enters from the left side of the strip. If the location for the inlet condition for computations of the horseshoe geometry is chosen to be at the ingate, the entrance effects, that are caused by the geometry of the runner, are missed (Van Tol et al. 1994; Van Tol et al. 1996; Van Tol et al. 1998). Therefore, not only this case will be studied, but also the choke will be used as an alternative location for the inlet condition.

First, the complete gating system is omitted and a single-valued inlet velocity is defined at the ingate. The freezing length as a function of temperature is presented in figure 5.19, for 1 and 3 computational cells across the channel height, using model 3 for the heat transfer coefficient. The pressure curves show a steep rise, like in the 2-D strip of figures 5.11 and 5.12. For the computations with a temperature of 700°C, a gradual increase of pressure is shown before the pressure rise marking the moment of freezing. This is indicative of a partially solidified channel, while filling continues. Computed freezing lengths, plotted versus the temperature, exhibit larger values and a steeper slope than found in the experiments, as presented in figure 2.6 in section 5.3.2 (figure 5.19 c). This was expected, because the computational inlet temperature is taken equal to the experimental pouring temperature, while in the experiment the metal loses heat to the gating system. This heat loss increases with increase of the pouring temperature. Thus, the overestimation of the freezing length is larger at higher temperatures. Near 600°C the freezing length is underestimated. This might be due to the fact that solidification starts, where the flow is still influenced by the inlet, because the velocity profile is still developing.

Taking the runner into account makes it possible to learn more about the effect of the gating system on the mould filling (Van Tol et al. 1994; Van Tol et al. 1996; Van Tol et al. 1998). The observed pressure curves and freezing length are shown in figure 5.20. These computations show a smaller slope when freezing length is plotted versus inlet temperature than in the case without a runner (figure 5.19). With runner, the calculated slope is closer to the measured slope, because part of the heat loss to the gating system is taken into account in this case. The freezing length near 600°C is not underestimated here. Therefore, these simulations are in better agreement with the experiments.

### 5.4.3 Cooling of the casting.

When mould filling has been completed, the local cooling rate can be computed by solving the heat transfer equations without flow. This is a good approximation, because there is no free convection in a thin horizontal plate. Temperature curves have been plotted near the ingate, before the bend and after the bend, at the same locations as used to obtain figure 2.7. The computational data is plotted in figure 5.21, together with the experimental data from figure 2.7.

The computed results show a constant temperature for some period of time. During this stage, the material solidifies, i.e. the solid fraction changes from 0 to 1. The amount of latent heat that is released, balances the heat transfer to the mould. The solidifying computational cell is allowed to have only a single temperature, i.e. the solid material in the cell cannot be cooled below the solidification temperature as long as the cell is not completely solidified.
Figure 5.19: Pressure build-up and freezing length, without the runner.
Figure 5.20: Pressure build-up and freezing length, with the runner.
The computed temperatures are systematically higher than the measured temperatures. This is caused by the thermocouples, which give a systematic error due to cooling through the wires connecting the measuring weld to the surroundings, see figure 2.2. This is most clearly shown by the measured temperature when solidification starts: it is about 10°C below the solidification temperature of 577°C.

5.5 Discussion

Critical aspects in computational simulations of horizontal thin-wall castings relate to surface tension, heat transfer, and cold run criteria. In addition velocity gradients determine the shape of the metal-air interface near the ingate and in the bend.

Simulated surface shapes and heights of a liquid drop on a flat horizontal support are identical to theoretically expected shapes and heights. Provided that more than a single cell in vertical direction is used in simulations of a horizontal channel, surface tension gives expected excess pressure at the inlet. This pressure is found too low, if just one computational cell in vertical direction is used.

The locally time dependent model for the heat transfer coefficient, that has been added to the code, is suitable to reduce the size of the computational cells required. In 2-D simulations, as well as in simulations of the full horseshoe, computations with this model show freezing lengths between two approximate models for the heat transfer coefficient. This was expected, because these two approximations underestimate and overestimate the heat transfer to the mould, respectively. Although the validity of the time dependent model is restricted, it gives proper results in computations of the freezing length in the horseshoe.

The criterion for the excess pressure, used to detect cold runs at a constant inlet velocity, serves well, provided that the observed pressure rise due to solidification is steep. The computed freezing length is in the order of the measured freezing length. This means that the gating system does not need to be fully incorporated in simulations of mould filling, if the flow rate is known. However, care must be taken that flow effects due to the gating system, are not overlooked.

For 1 and 3 computational cells in vertical direction, respectively, the simulated freezing lengths are fairly identical. However, if 5 computational cells are used in the thickness, the freezing length is larger. This is due to plateaus in the pressure curves at 800 Pa, which is below the critical pressure that indicates a cold run. In such a case, the freezing length is sensitive to the value chosen for the critical pressure.

Simulations of the full horseshoe without the runner show a larger freezing length than measured in the casting, for temperatures above 600°C. The inlet temperature in the computations is taken equal to the pouring temperature in the casting, while actually it is lower. The difference at high temperatures is larger, because, depending on the temperature difference between metal and mould, the heat loss in the gating system is larger. The difference decreases, if the runner is taken into account.

Computed and measured velocities in the water model show similar gradients in the bend as well as near the ingate. The difference in measured average flow rate and the
Figure 5.21: Computed cooling of the casting near the ingate, before the bend and after the bend.
applied flow rate leads to the conclusion that particles may have been located halfway the horseshoe's height. Therefore, the measured velocities are compared to simulated velocities at half the height of the horseshoe, and not to the velocity averaged across the channel height.

The steady state velocity gradients in the bend as well as the change in the velocity gradient during filling of the bend, found in computations, are confirmed by the experiments. In the inlet area, similar trends in computations and measurements are found. Near the ingate, the computed velocities at the inner side of the bend are lower than the measured velocities. This is caused by air entrapment in the gating system, disturbing in the PIV procedures the determination of the displacement of the particles in the interrogation areas.
Chapter 6

Concluding remarks

In foundry practice Computational Fluid Dynamics (CFD) is being considered as a promising tool to arrive at a better understanding of casting defects introduced during the mould filling period of a casting. The use of CFD is believed to be advantageous in predicting possible defect locations, pouring times, and the lowest possible casting temperature without the risk of cold shuts. In addition, CFD may be used to optimise pouring system geometries. As opposed to experimental studies in e.g., cold water models, CFD allows for the flexible modification of the casting geometry and the pouring system, and for the inclusion of physical effects such as solidification and heat transfer. Thus, a proper use of CFD may lead to a reduction of the casting rejection rate without expensive trial-and-error procedures, and to a decrease of production costs in foundries.

The purpose of this thesis was to investigate to what extent CFD can be useful in optimising horizontal thin-wall casting. To this end, CFD simulations with the commercial FLOW-3D software (Version 5) were performed as to mould filling conditions and gating systems for manufacturing a horizontal horseshoe. These simulations were validated against data obtained with various experimental techniques.

The results presented in this thesis show that CFD can indeed be a useful tool in predicting and optimising:

- flow patterns, i.e. the transient velocity field during mould filling
- metal-air interface shapes, i.e. the position of the free surface and its shape due to surface tension
- freezing lengths, i.e. the prediction of cold shuts

in horizontal thin-wall casings.

Predicted flow patterns and interface shapes are in general agreement with experimental observations obtained by use of video registration and Particle Image Velocimetry. Freezing lengths, that are experimentally found to depend linear on the pouring temperature, can be predicted at an accuracy of about 15 %.

Such a use of CFD does not require more computer power than is offered by the present generation of PC's. Although the choice of various settings, the definition of the grid, and the treatment of (inlet) boundary conditions are non-trivial and require special
attention, the FLOW-3D software is reasonably accessible to dedicated users. This makes CFD a feasible tool to be used in foundry practice, provided that sufficient precautions are taken to assure proper handling of the code.

Some issues with respect to the application of CFD to mould filling are important:

- The code used for this study, FLOW-3D (Version 5), makes use of a structured rectangular grid. Grid sizes needed somewhere in the casting are to be extended throughout the whole computational domain; this leads to small cell sizes even where not really needed and, hence, to long computational times.

Although the applied porosity method allows the simulation of flow within curved geometries, body fitted coordinates might still be a better solution.

Hence, the optimum CFD code for mould filling simulations would have to combine many aspects of the FLOW-3D version used in this study with a more advanced capability of handling generalised grids.

- The VOF-algorithm, used to describe the position of the metal-air surface, was found to be stable, as long as the cell aspect ratio does not exceed the value of 3. Larger values lead to unrealistic surface shapes. In thin-wall casting simulations, resolving the velocity profile across the small height of the flow to some degree of accuracy requires several computational cells across this height. As a result of the requirement as to aspect ratio, and in view of the large aspect ratio of thin-wall castings, the number of computational grid cells may easily become very large. In combination with the use of small time steps in an explicit time integration, as persued by FLOW-3D, this may lead to relatively long CPU times of the order of hours.

In foundry practice, CFD will be used to quickly evaluate the feasibility and complexity of casting new mould shapes. To some extent the application of CFD may therefore depend on the possibility to reduce CPU times. One way to do this might be to use implicit time integration schemes. This requires further study.

- A second possibility to reduce CPU efforts is to limit the number of grid cells:
  - As shown in this study, the number of grid cells may be somewhat reduced by not including the complete gating system into the simulations. In doing so, however, one should take care not to disturb the inlet flow pattern.
  - Another possibility to reduce the number of grid cells is the use of a penetration-theory based, locally transient, heat transfer coefficient for the heat transfer to the mould, as implemented in the present study. Thus, the need of many small cells in the mould is eliminated and heat transfer becomes grid size independent, as shown in this thesis. However, this solution somewhat complicates the bookkeeping in the computer code.

- Finally, since an equilibrium condition is used in the solidification model, i.e. the kinetics of solidification is omitted, it is not possible to describe undercooling during solidification. A micro-solidification model needs to be implemented into the code to predict metal structures.
In CFD simulations of mould filling, the value of the heat transfer coefficient is critical. This value, that is reasonably universal for aluminium-sand castings, should be measured with greater accuracy to improve the reliability of the computations. The same is true for the drag coefficient, that takes the solidification morphology into account, and which should be determined for every alloy applied.

In spite of the remaining problems, the present study has shown that a properly chosen CFD model can describe important aspects of mould fillings, even in the case of a horizontal thin-wall casting, that turns complex owing to the combination of free surface liquid flow, gravity, and surface tension, along with a phase change from liquid to solid. Although cold hydrodynamic experiments, using Particle Image Velocimetry and water as a model fluid, have been shown to be very useful, such experimental techniques cannot be a full alternative for CFD simulations. Compared to experimental studies, CFD is faster, less expensive, and more flexible, allows for a more realistic representation of the actual mould filling process, and gives more detailed information.
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List of Symbols

Abbreviations:

- CFD: Computational Fluid Dynamics
- PIV: Particle Image Velocimetry
- MAC: Marker and Cell
- VOF: Volume of Fluid
- FAVOR: Fractional Area and Volume Obstacle Representation
- SIMPLE: Semi Implicit Method for Pressure Linked Equations

Dimensionless numbers:

- Fo: Fourier number
- Bi: Biot number
- Re: Reynolds number

Symbols:

- $A_x, A_y, A_z$: Area of a cell face in x,y,z direction [m$^2$]
- C: PIV particle concentration [m$^{-3}$]
- $C_f$: Constant (equation 4.13) [m]
- $C_{drag}$: Constant in drag function $K$ [s$^{-1}$]
- $C_p$: Heat capacity [J s$^{-1}$ °C$^{-1}$]
- $D_0$: Validity criterion
- $E$: Energy of a liquid drop [J]
- $F_A, F_D$: Fluid fraction in donor, acceptor cell
- $F$: In plane loss of pairs
- $F_0$: Out of plane loss of pairs
- $J_x, J_w$: Transported mass per unit of time [kg s$^{-1}$]
- $H$: Enthalpy [J kg$^{-1}$]
- $H_L$: Drop height in equation 3.2 [m]
- $H_S$: Enthalpy at liquidus temperature [J kg$^{-1}$]
- $H_{SL}$: Enthalpy at solidus temperature [J kg$^{-1}$]
- $K$: Drag constant [s$^{-1}$]
- $K$: Curvature (equation 4.16) [m$^{-1}$]
- $L$: Latent heat [J kg$^{-1}$]
- $L_f$: Freezing length [m]
- $M$: Magnification
- $N_S$: Source density
- $N_I$: Image density
- $N_P$: Image density for multiply pulsed systems
- $R$: Radius of curvature [m]
- $S_{i,j,k}$: Left hand side of equation 4.5 [m$^3$ s$^{-1}$]
- $T$: Temperature [°C]
- $T_{sol}$ ($T_{eq}$): Solidus (Liquidus) temperature [°C]
LIST OF SYMBOLS

\( T_p \) Pouring temperature [°C]
\( T_\infty \) Temperature of the surrounding [°C]
\( T_f \) Fluid temperature [°C]
\( T_m \) Metal temperature [°C]
\( T_s \) Mould temperature [°C]
\( P_f, P_w \) Fractional area of a cell face open for flow
\( P_V \) Fractional volume open for flow
\( \Delta t \) Time interval [s]
\( \Delta X \) Displacement in image plane [m]
\( V \) Cell volume in x,y,z direction [m³]
\( \Delta Z_0 \) Thickness of the light plane [m]
\( a \) Thermal diffusivity [m² s⁻¹]
\( d \) Channel height [m]
\( d_e \) Particle diameter in the image plane [m]
\( d_l \) Equivalent diameter of the interrogation area [m]
\( d_e \) Particle diameter [m]
\( f \) Focal length [m]
\( f_{drag} \) Drag force per unit volume [N m⁻³]
\( f_s \) Solid fraction
\( g \) Gravitation acceleration [m s²]
\( h \) Heat transfer coefficient [J m⁻² s⁻¹ °C⁻¹]
\( h \) Height in Bernoulli equation [m]
\( h_i \) Interfacial heat transfer coefficient [J m⁻² s⁻¹ °C⁻¹]
\( h_{num} \) Numerical heat transfer coefficient [J m⁻² s⁻¹ °C⁻¹]
\( h_{pen} \) Heat transfer coefficient associated with penetration theory
[ J m⁻² s⁻¹ °C⁻¹]
\( s \) Path length [m]
\( t \) Time [s]
\( \delta t \) Time step size [s]
\( v_i \) Velocity in the light plane [m s⁻¹]
\( v_0 \) Velocity orthogonal to the light plane [m s⁻¹]
\( u, v, w \) Velocity in x,y,z direction [m s⁻¹]
\( v_{avg} \) Average velocity [m s⁻¹]
\( x, y, z \) Cartesian coordinate system [m]
\( x_0 \) Position of the triple point air-liquid-support (figure 3.1) [m]
\( \Delta x, \Delta y, \Delta z \) Cell size in x,y,z direction [m]
\( \delta x_k, \delta y_j, \delta z_k \) Spacing between cell faces x,y,z direction [m]
\( \delta x_{i+1}, \delta y_{j+1}, \delta z_{k+1} \) Spacing between cell faces x,y,z direction [m]
\( \Delta z \) Displacement in light plane [m]
\( \Delta Z_0 \) Thickness of the light plane [m]
\( \Gamma_\phi \) Diffusion coefficient of \( \phi \) [kg m⁻¹ s⁻¹]
\( \kappa \) Thermal conductivity [J m⁻² s⁻¹ °C⁻¹]
\( \mu \) Viscosity [N s m⁻²]
\( \rho \) Density [kg⁻¹]
\( \sigma \) Surface tension [N m⁻¹]
\( \sigma_{sa} \) Adhesive force per length [N m⁻¹]
\( \sigma_{sm} \) Cohesive force per length [N m⁻¹]
\( \sigma \) Surface tension [N m⁻¹]
\( \theta \) Contact angle [degrees]
References


Summary

In this thesis the usefulness of Computational Fluid Dynamics (CFD) in the optimization of mould filling of horizontally casted thin wall castings is investigated. A bended horizontal strip, horseshoe shaped, with overall dimensions of $340 \times 120 \times 5$ mm, has been selected for this research.

Several experimental techniques have been used to obtain information about the mould filling: video registration, contact and thermo-couple measurement, and Particle Image Velocimetry (PIV) in a water model. These measurements yield the displacement of the free surface and its shape, temperatures during and after mould filling, and a transient integral velocity field.

A number of features of the mould filling of this casting have been explicated by these experiments: permanent contact with the top mould, a gap near the ingate during mould filling, a time dependent velocity gradient in the bend, the freezing length as a function of temperature and local cooling of the casting after the mould is full. These experimental results have been used to validate CFD computations.

Transient CFD computations have been performed on this casting, using the commercial flow solver Flow-3D, and the VOF algorithm to compute the location of the free surface. Heat transfer is computed, and solidification during mould filling is accounted for.

For the casting simulations, a constant inlet velocity is issued, with a cold run criterion, that states that the pressure at the inlet may not rise above the maximum hydrostatical in the down sprue. This way of approaching freezing allows control of the inlet velocity and allows computation of the time at which a cold run occurs.

The total heat transfer coefficient between liquid metal and mould is implemented in the code in three approximations: direct use of the interfacial heat transfer coefficient, a heat transfer coefficient that has been corrected for the amount of mould between the computational cell centre and the interface, and a time dependent heat transfer coefficient based on penetration theory. The latter does not suffer from spatial dependency on the cell sizes.

Computation of surface tension is done carefully within FLOW-3D, resulting in the correct computation of height and shape of a liquid drop lying on a horizontal support. In horizontal thin wall casting this surface tension force results in permanent contact with the top mould, as found in the experiments. However, the correct pressure drop due to surface tension, is only found in computations with more than one computational cell in vertical direction. If the cell aspect ratio exceeds 1:3, instabilities arise at the surface.

For 1,3 and 5 computational cells in vertical direction the freezing length as a function
of temperature has been investigated for all three approximations of the heat transfer coefficient. For 1 and 3 computational cells the computed freezing lengths are fairly identical, however, at 5 computational cells a longer freezing length is found. This is due to channel forming at half the casting thickness, while the channel near the wall is frozen.

Computations show an inversion of the velocity gradient in the bend, just behind the free surface. When liquid enters the bend, the velocity on the outer side of the bend is the largest, while a little later the opposite is found. Experimentally as well as computationally velocity gradients are found near the ingate.

Differences in cooling rates of the cast are found at different locations. This is caused, by local (near the flow tip) cooling of the metal and local (near the ingate) heating of the sand.

Comparison of experimental and computational results leads to the conclusion that important phenomena of horizontal thin wall mould filling, like flow patterns, metal-air interface shapes and freezing lengths, can be well described with FLOW-3D. However, the large number of computational cells needed for accuracy in heat transfer through the metal-mould interface and for stability of the metal-air surface leads to a large computational effort. This problem has partly been solved by using a time dependent heat transfer coefficient at the metal-mould interface, such that a modern PC is sufficient to compute the mould filling and solidification of a horizontal thin wall casting.

Thus, the main conclusion of this thesis is that CFD can be a practical and useful tool in foundry practise for optimising thin wall metal casting processes.
Samenvatting

Binnen het kader van dit proefschrift is onderzocht in hoeverre CFD-software voor stromings-simulaties een bruikbaar instrument kan zijn bij de optimalisatie van de vormvulling van horizontale dunwandige gietstukken. Voor dit onderzoek is uitgegaan van een gietstuk in de vorm van een gebogen strip van 5 mm dikte, in een vormoppervlak van 340 mm × 120 mm.

Er is gebruik gemaakt van verscheidene technieken om informatie te verkrijgen over vormvulling: video-opnamen, contactmetingen, thermokoppelmetingen en Particle Image Velocimetry (PIV) in een watermodel. Met behulp van deze technieken kon een beeld worden gevormd van de plaats en vorm van het vrije oppervlak, de temperaturen gedurende en na vormvulling en het integrale transiënte snelheidsveld.

Uit deze experimenten zijn een aantal belangrijke karakteristieken bij vormvulling van het onderhavige gietstuk naar voren gekomen: een permanent contact met de bovenkant van de gietvorm, een onvolledige bevochtiging bij de aansnijding, een tijdsafhankelijke snelheidsgradient in de bocht, de koudlooplengte als functie van de giettemperatuur en locale afkoeling van het gietstuk als de vorm vol is. Deze experimentele resultaten zijn gebruikt om CFD berekeningen te valideren.

De transiënte CFD berekeningen zijn uitgevoerd met het commerciële softwarepakket FLOW-3D, dat gebruik maakt van het VOF-algoritme voor de bepaling van de plaats van het vrije oppervlak. In de berekeningen is rekening gehouden met warmtetransport en stolling gedurende en na de vormvulling.

Bij de berekeningen werd uitgegaan van een constante instroom snelheid. Als koudloopcriterium gold het stijgen van de druk in de inlaatopening tot boven de maximale hydrostatische druk in de gietstap. Dit heeft het voordeel, dat een constante instroom snelheid als inlaatconditie kan worden gebruikt, terwijl het tijdstip van koudloop kan worden bepaald.

Voor de numerieke berekening van de warmte-overdrachtscoëfficiënt door het grensvlak tussen metaal en gietvorm zijn drie benaderingen bestudeerd: direct gebruik van de empirische warmte-overdrachtscoëfficiënt, een warmte-overdrachtscoëfficiënt die gecorrigeerd is voor de hoeveelheid zand tussen het grensvlak en het eerste gridpunt in de vorm, en een tijdsafhankelijke warmte-overdrachtscoëfficiënt gebaseerd op penetratietheorie. Deze laatste benadering is onafhankelijk van de celafmetingen.

Een accurate implementatie van de oppervlaktespanning in FLOW-3D maakt een nauwkeurige berekening mogelijk van hoogte en vorm van een druppel op een horizontaal oppervlak. Bij de berekenen vulling van horizontale dunwandige gietstukken zorgt de oppervlaktespanning voor een permanent contact met de bovenkant van de gietvorm, zoals die wordt gevonden in
experimenten. De druk in de vloeistof wordt echter uitsluitend goed berekend als meer dan één reokcel in de hoogte van het kanaal wordt gebruikt. Wordt dit niet gedaan, dan wordt slechts de helft van de werkelijke druk gevonden.

Bij lengte-breedte verhoudingen van reokcellen groter dan 1:3 treden instabiliteiten op aan het oppervlak.

De koudlooplengte als functie van de temperatuur is onderzocht met gebruikmaking van 1,3 en 5 roostercellen over de hoogte van het kanaal en van de drie genoemde benaderingen voor de warmte-overdrachtscoëfficiënt. De koudlooplengte is bij het gebruik van 1 en 3 cellen ongeveer gelijk, maar korter dan bij gebruik van 5 cellen in vertikale richting. Dit is het gevolg van kanaalvorming in het midden van het gietstuk, waar aan de wand stolling optreedt.

Berekeningen en experimenten geven een inversie van de snelheidsgradiënt in de bocht te zien, een paar centimeter achter het vrije oppervlak. Bij het ingaan van de bocht is de snelheid van de vloeistof in de buitenbocht het grootst, terwijl even later het tegenovergestelde wordt gevonden. Experimenten en berekende resultaten geven verder snelheidsgradiënten te zien nabij de aansnijding.

Afkoeling van het gietstuk is onderzocht nabij de aansnijding, vóór de bocht en na de bocht. Verschillen in stoltijden worden gevonden ten gevolge van lokale afkoeling bij het stromingsfront en lokale opwarming van het zand nabij de aansnijding.

Vergelijking van experimenten en berekeningen leiden tot de conclusie dat de karakteristieken van de vulling van horizontale dunwandige gietstukken goed beschreven kunnen worden met FLOW-3D. Hierbij moet worden opgemerkt, dat vereiste nauwkeurigheid in de berekening van het warmtetransport en numerieke eisen t.a.v. de stabiliteit van het metaal-lucht oppervlak leiden tot een significante overhead in het aantal reokcellen.

De belangrijkste conclusie van dit proefschrift is dat CFD een voor gieterijen bruikbaar en praktisch instrument is voor optimalisatie van de vormvulling van dunwandig gietwerk.
List of publications

English:


Dutch:


Hungarian:

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Notes:
Notities: