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Design and evaluation of a new biaxial testing method for sheet material (SUPERBAT)

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1. Introduction

Many engineering components operate in stress environments significantly more complicated than uniaxial tension, which is usually adopted for research studies. During the past two decades, a wide variety of techniques have been used to fatigue test laboratory specimens under multi-axial stress states. However, non of those techniques seams to be suitable for testing sheet materials (ref.1).

A new biaxial testing technique has been developed for testing sheet materials. The design and evaluation of this new test method will be outlined in this report. Some preliminary test results will be discussed in detail.

2. Design of the specimen: SUPERBAT

The Poisson ratio effect is a wellknown phenomenon in material science. For a biaxial state of stress, according to Hooke's law:

$$\varepsilon_x = \sigma_x / E_x - \nu_{xy} \sigma_y / E_y$$

If the Poisson contraction in the x-direction is prevented: $\varepsilon_x = 0$ and it follows that:

$$\sigma_x / \sigma_y = \lambda = \nu_{xy} E_x / E_y$$

where λ is the biaxial stress ratio.

For isotropic material where $\nu_{xy} = \nu$ and $E_x = E_y$ it reduces to:

$$\lambda = \nu$$

Clearly, a complete restraining of the Poisson contraction in a sheet material will create a biaxial stress field with a biaxial stress ratio, $\lambda = \nu$, for an isotropic material.

After careful considerations, a method for local restraining of the Poisson ratio effect has been chosen, which is obtained by fastening steel strips onto a sheet specimen, see fig.1. Because of the high stiffness of the steel strips and the low stiffness of the aluminium sheet material, a local biaxial stress field can be created. This type of test specimen is further called:SUPERBAT(Specimen Using Poisson's ratio Effect Restraining for Bi-Axial Testing).

3. Stress analyses

The steel strips are fastened symmetrically with respect to the test area (fig.1). As required by equilibrium, the bolt forces act in opposite directions on the sheet and the strips. All forces acting on the sheet and strips were assumed to act at the mid-thickness plane of the sheet.
The equations necessary to determine the unknown bolt forces P were obtained by equating the displacements at the bolt locations in the sheet and the strips. The displacements were written in terms of the following influence coefficients: A_{ij} and B_i, which represent the displacements in the sheet at bolt no.i because of unit values of P_i and S, respectively; and C_i, which represents the displacements in the strips at bolt no.i because of unit value of P_i.
Thus the displacement at bolt no.i in the sheet and strips can be written as:

$$u_i = \sum_j A_{ij} P_i + B_i$$

$$u_i^s = C_i \quad P_i$$

For symmetry reasons i and j can be restricted to the bolts in one quadrant of the sheet. Equating these displacements and collecting terms gives:

$$\sum_j A_{ij} P_i - C_i P_i = -B_i S$$

or

$$(\sum_j A_{ij} - C_i) P_i + B_i S = 0 \qquad (i=1,2,3\ldots)$$

These simultaneous equations can be solved when the influence coefficients are known.
The analytical solutions for the influence coefficients are given in appendix A (based on solutions by Love, Ref.2).

4. Calculations and static tests

Calculations have been made for a prototype SUPERBAT specimen (fig.1). In this specimen, steel strips of 1.5 mm in thickness and 15 mm width were used. The other dimensions of the specimen are given in fig.1. The Al 7075-T6 bare sheet material used in the specimens has a thickness of 1.55 mm. Strain-gauge measurements have been conducted on 4 locations on the specimen.

The results of the strain-gauge measurements in the middle of the test area along the X-axis before and after introducing the strips, are given in fig.2. The specimen appears to behave slightly more stiff in the loading direction after the strips are fastened onto the sheet (lower ε_y).

An excellent linearity was found for the deformations in both directions. The biaxial stress ratio can then be calculated according to the following formula:

$$\lambda = \sigma_x^b / \sigma_y^b = (\varepsilon_x^b + \nu\varepsilon_y^b)/(\varepsilon_y^b + \nu\varepsilon_x^b)$$

where b stands for biaxial stress state.
The stress efficiency-factor in the loading direction (α) is defined as:

$$\alpha = \sigma_y^b / \sigma_{appl.} = (\varepsilon_y^b + \nu\varepsilon_x^b)/(1 - \nu^2) / \varepsilon_y$$

The biaxial stress ratio's and the stress efficiency-factors have been calculated for all strain-gauge locations. The results of these calculations and the results of the theoretical calculations are given in fig.3 and fig.4.. A good agreement between the measurements and the theoretical calculations has been found.
Strain-gauge measurements on the strips were also carried out. Two strain gauges were applied to one strip of each pair, one at each side to eliminate possible bending effects. The results are presented in fig.5 as load factor defined as: $f_i = K_i/(dt\sigma_{appl.})$ where K_i is the sum of the reaction forces of the strip-pair transferred by the corresponding bolt; d is the diameter of the bolt; t is the thickness of the sheet and $\sigma_{appl.}$ is the remote loading stress. The results show differences with the calculated values, but the load factors of the two strips remain constant throughout the whole loading range of the specimen(see fig.5)

5. Fatigue tests

Two uniaxial tests and two biaxial tests have been carried out on 1.55 mm thick Al 7075-T6 bare sheet using the SUPERBAT technique. The results are given in fig.6. Scatter is low and a difference between the crack growth behaviour of the Al 7075-T6 sheets under uniaxial and biaxial loading conditions can not be observed.

Strain-gauge measurements on the steel strips were also carried out during the fatigue tests. The loading-factors of the strips are plotted in fig.7 as a function of the crack lengths. Clearly, the loading-factors remain constant throughout the full range of crack lengths measured during the test.

6. Conclusion and remarks

A new biaxial fatigue testing method called: SUPERBAT (Specimen Using Poisson's ratio Effect Restraining for Bi-Axial Testing) has been developed.
Static tests were carried out with an excellent agreement between the test results and theoretical calculations. A test area having a biaxial stress ratio of about λ=0.24 was obtained with the SUPERBAT technique.
Biaxial fatigue tests on 1.55 mm thick AL.7075-T6 sheets showed no difference between the fatigue behaviour of the material under uniaxial and biaxial load conditions.
It should also be pointed out, that further theoretical calculations have shown (see fig.8) that a higher biaxial stress ratio and a better stress distribution in the test area can be achieved by increasing the number of the steel strip-pairs from 4 to 6. As a result of these calculations an improved SUPERBAT specimen will be studied.

References:

1. D.Chen New techniques for aircraft fuselage skin tests. LR-Report, Faculty of aerospace engineering,Delft University of Technology, the Netherlands.

2. A.Love A treatise on the mathematical theory of elasticity. Fourth ed.(First Amer. Printing),Dover Publ.,1944,p.209.

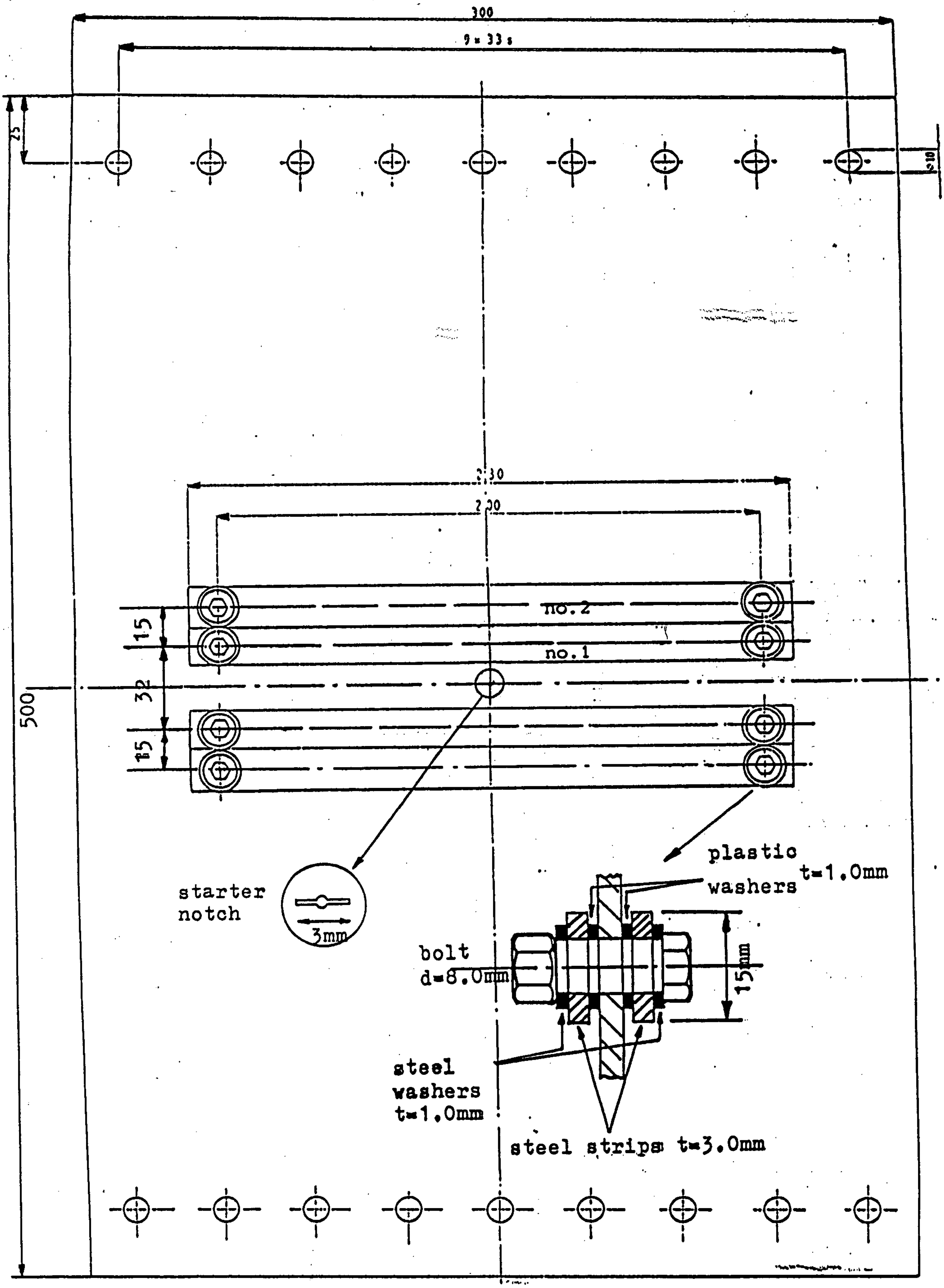

Fig.1 SUPERBAT specimen (prototype) with four pairs of steel strips

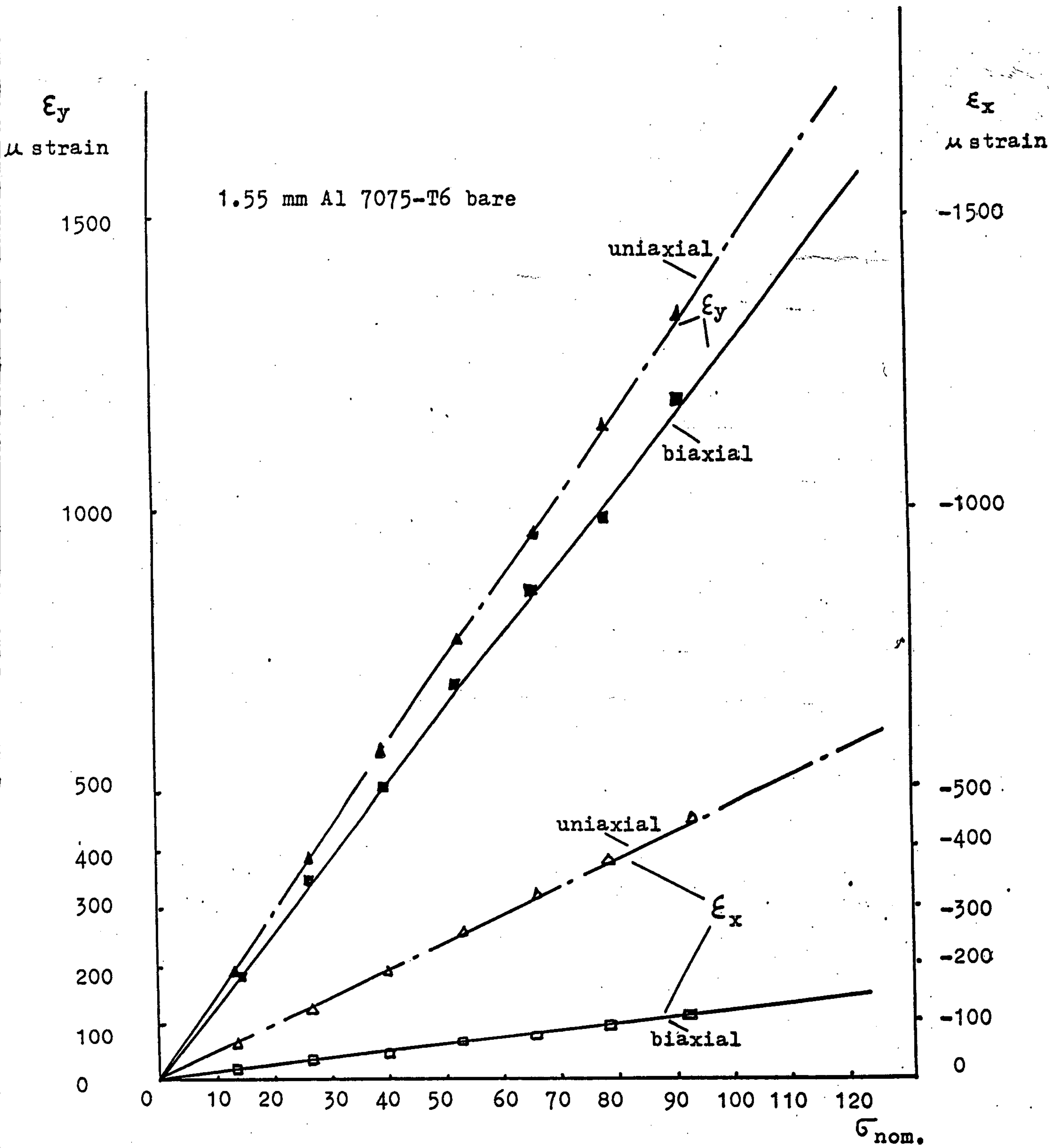

Fig.2 Strain-gauge measurements in the center of the specimen

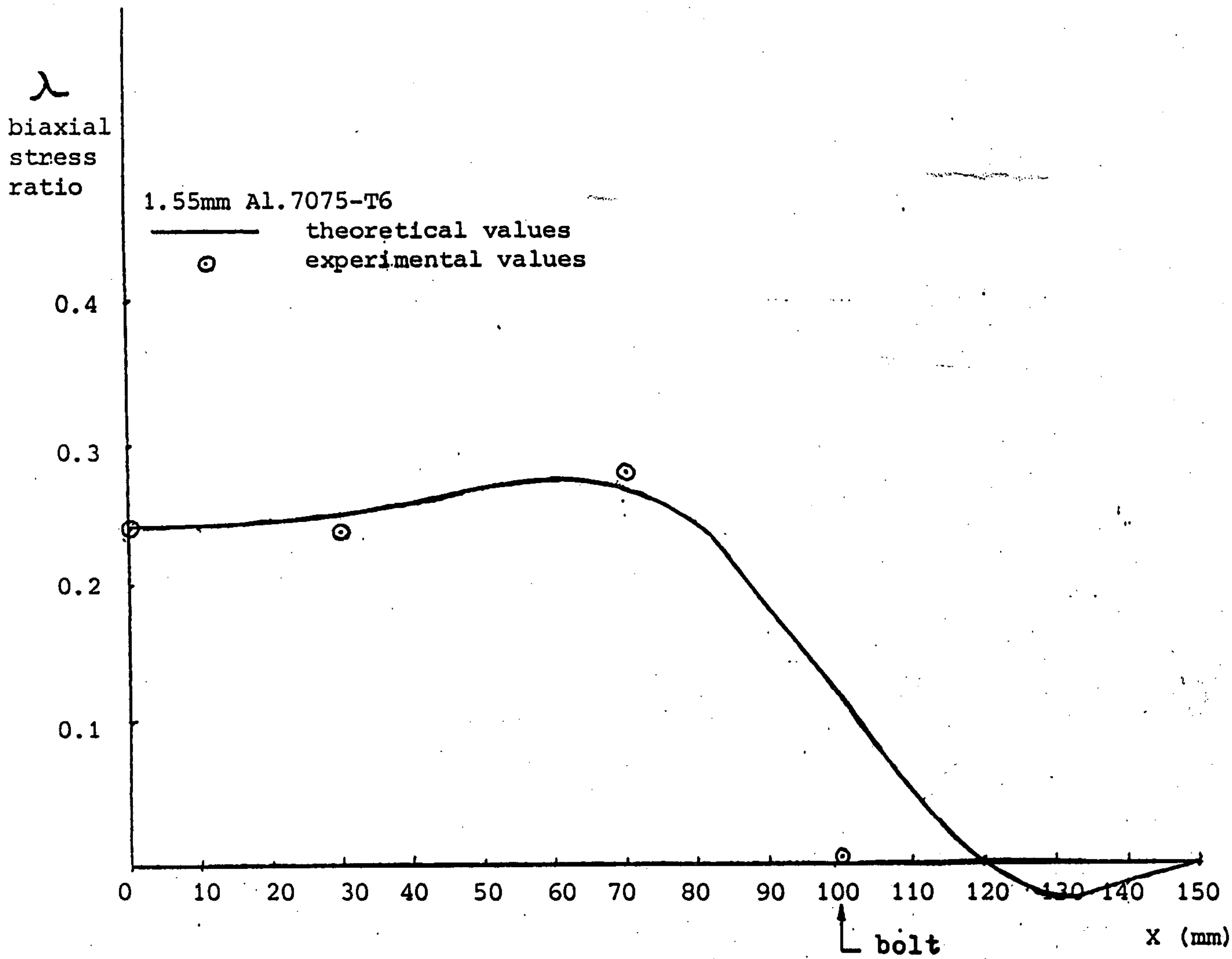

Fig.3 Biaxial stress ratio distribution along the X-axis

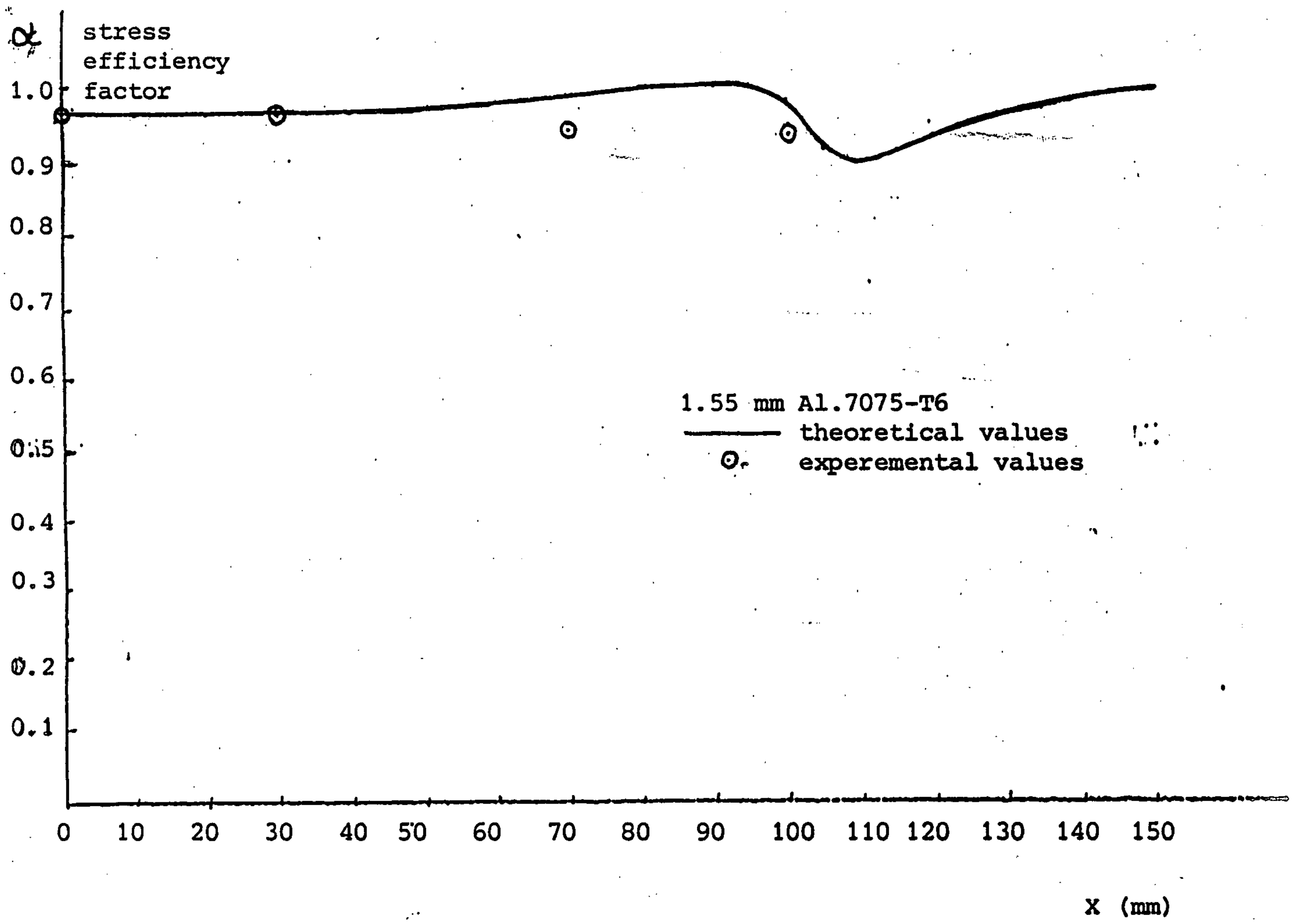

Fig.4 Stress efficiency-factor distribution along the X-axis in the test area

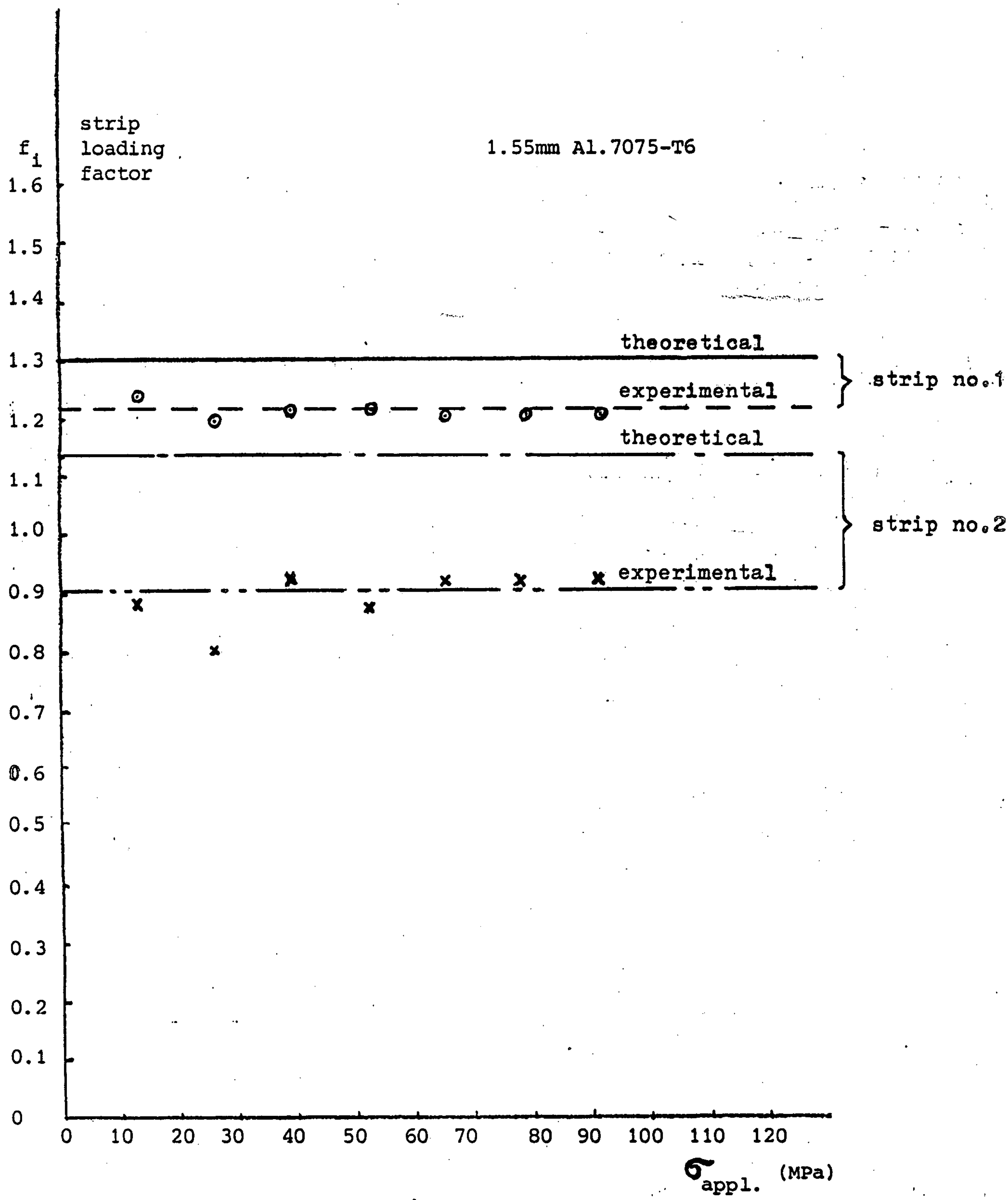

Fig.5 Loading factors of the steel strip-pairs vs. loading stress

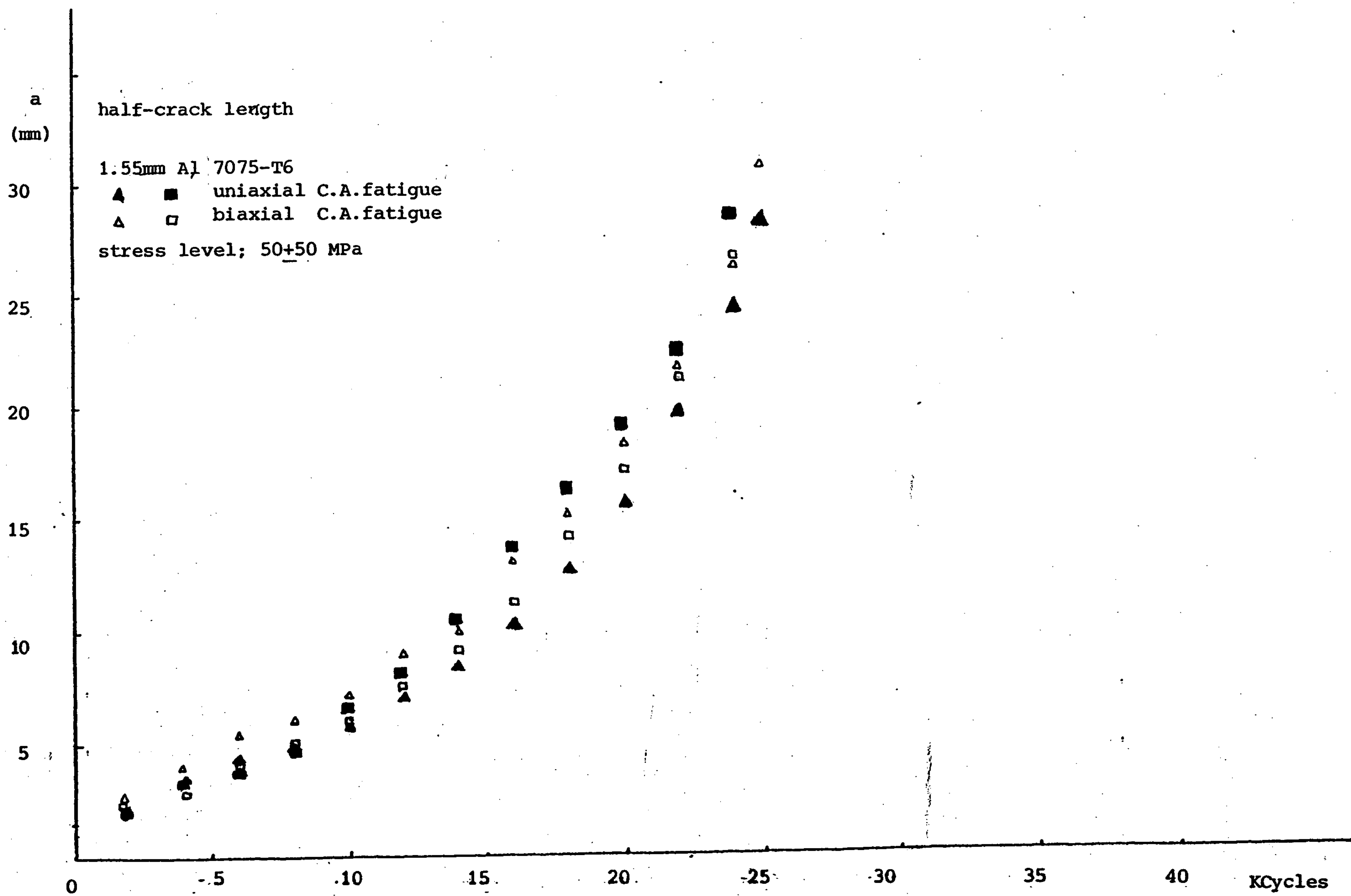

Fig.6 Constant- amplitude fatigue tests on 1,55mm Al 7075-T6 sheet, uniaxial and biaxial

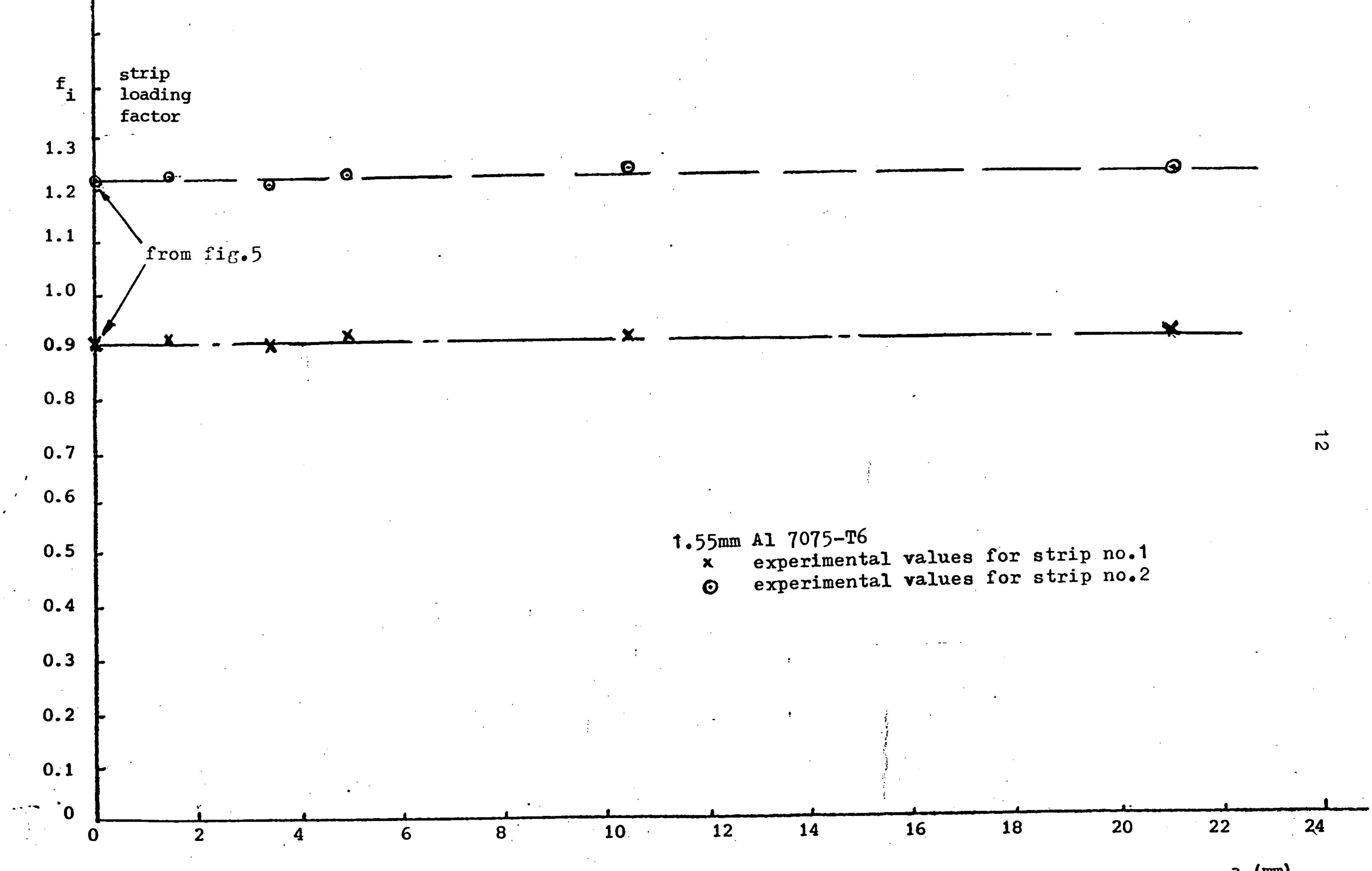

Fig.7 Strip loading factors vs. half crack length during fatigue tests

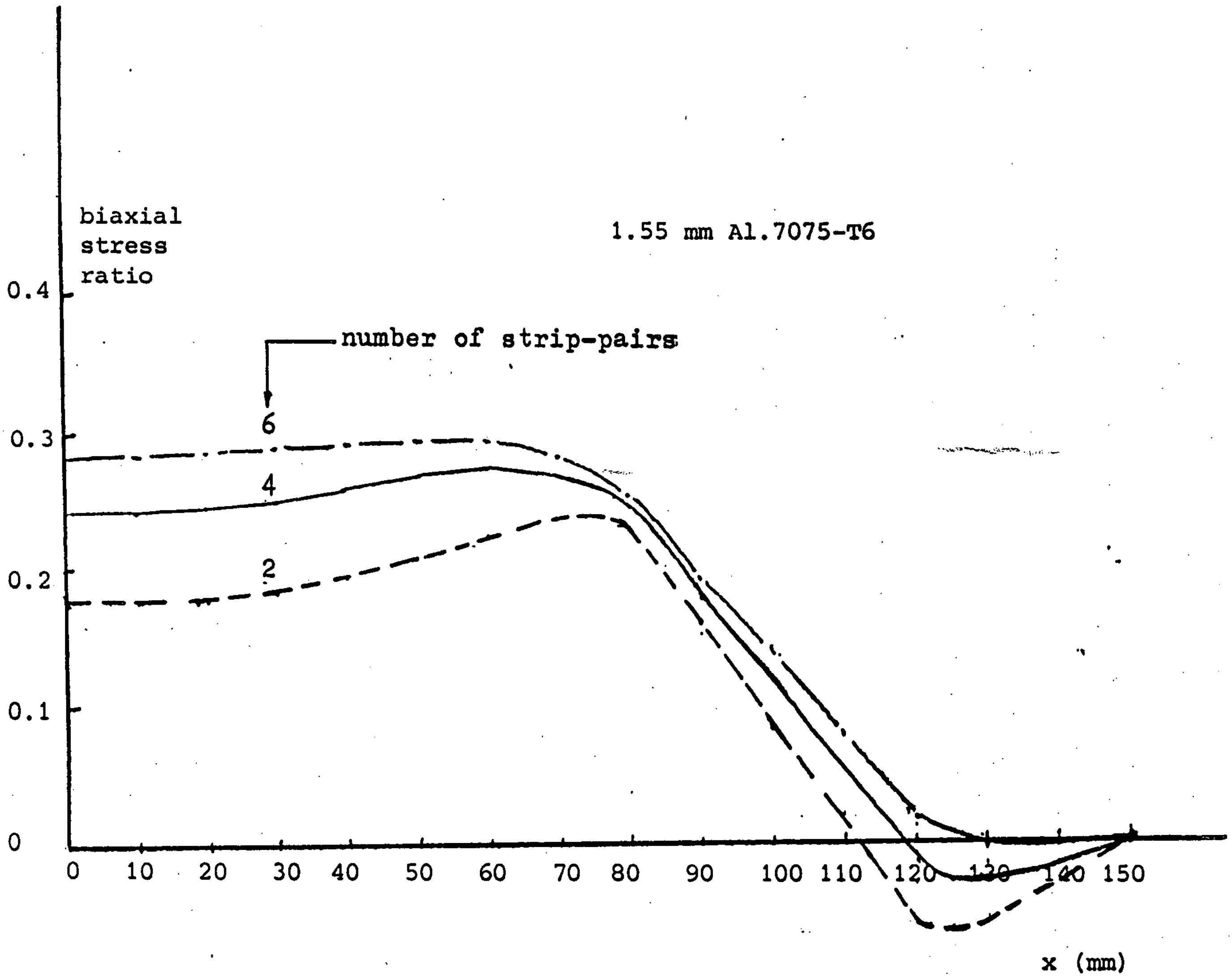

Fig. 8 Theoretical biaxial stress ratio distributions for three SUPERBAT configurations (2 strip-pairs, 4 strip-pairs and 6 strip-pairs resp.)

Appendix A

Equations for determining the influence coefficients

Applied uniaxial stress:

For the infinite sheet loaded in one direction (fig.A1) the displacements of any point in the X-direction can be calculated according to the following formula:

$$u_i = -\nu x S \;/\; E$$

The influence coefficients B_i are determined by setting the stress S equal to unity.

Bolt reaction forces:

According to Love (ref.2), the displacement in the X-direction of an infinite plane for the single point force shown in fig.A2 can be obtained for the plane stress state from the following equation:

$$u_{II} = -\frac{(1+\nu)P}{4\pi t E}\left(\frac{1}{2}(3-\nu)\ln(x^2+y^2)+(1+\nu)\frac{y^2}{x^2+y^2}\right)$$

This equation cannot be used in this form to calculate the displacement caused by the force P, because there is a singularity at the origin.
To circumvent this problem, the point force P is assumed to be distributed uniformly over the bolt diameter d. Replacing x by $x-\bar{x}$ in the above equation and integrating with respect to $\bar{x}$ from -d/2 to d/2, the displacement becomes:

$$u_{II} = -\frac{(1+\nu)(3-\nu)}{16\pi t E}P\left\{\left(\frac{2y}{d}+1\right)\ln\left[\left(\frac{2y}{d}+1\right)^2+\frac{4x^2}{d^2}\right]\right.$$
$$-\left(\frac{2y}{d}-1\right)\ln\left[\left(\frac{2y}{d}-1\right)^2+\frac{4x^2}{d^2}\right]$$
$$\left.-\frac{1-\nu}{3-\nu}\,\frac{8x}{d}\arctan\left[\frac{\frac{x}{d}}{\frac{x^2}{d^2}+\frac{y^2}{d^2}-\frac{1}{4}}\right]\right\}$$

By translating the origin from $(0,0)$ to (x_0, y_0) and by adding the displacements for forces acting at $(\pm x_0, \pm y_0)$, the displacement field can be written as:

$$u_{II} = \frac{(1+\nu)(3-\nu)}{16\pi t E} P\,\Omega$$

where

$$\begin{aligned}\Omega = {} & (\alpha_3+1)\ln\left[\frac{(\alpha_3+1)^2+\alpha_2^2}{(\alpha_3+1)^2+\alpha_1^2}\right] - (\alpha\ -1)\ln\left[\frac{(\alpha_3-1)^2+\alpha_2^2}{(\alpha_3-1)^2+\alpha_1^2}\right] \\ & -(\alpha_4-1)\ln\left[\frac{(\alpha_4-1)^2+\alpha_2^2}{(\alpha_4-1)^2+\alpha_1^2}\right] + (\alpha\ +1)\ln\left[\frac{(\alpha_4+1)^2+\alpha_2^2}{(\alpha_4+1)^2+\alpha_1^2}\right] \\ & +4\left(\frac{1-\nu}{3-\nu}\right)\left\{\alpha_2\left[\arctan\left(\frac{2\alpha_2}{\alpha_2^2+\alpha_3^2-1}\right)+\arctan\left(\frac{2\alpha_2}{\alpha_2^2+\alpha_4^2-1}\right)\right]\right. \\ & \left. -\alpha_1\left[\arctan\left(\frac{2\alpha_1}{\alpha_1^2+\alpha_3^2-1}\right)+\arctan\left(\frac{2\alpha_1}{\alpha_1^2+\alpha_4^2-1}\right)\right]\right\}\end{aligned}$$

and

$$\alpha_1 = \frac{2(x-x_0)}{d};\quad \alpha_2 = \frac{2(x+x_0)}{d};\quad \alpha_3 = \frac{2(y-y_0)}{d};\quad \alpha_4 = \frac{2(y+y_0)}{d}.$$

The coefficients A_{ij} can then be determined by setting P's equal to unity.

Forces in the strips:

The deformations, in this case, the negative elongations of the strips can easily be calculated according to Hooke's law:

$$P_i/(2B_s t_s E_s) = -u_i^s / x_0$$

or

$$u_i^s = -\frac{x_0}{2B_s t_s E_s} P_i$$

where

B_s = width of a strip;

t_s = thickness of a strip;

E_s = Young's modulus of the strip.

Hence, the coefficients C_i's are a constant, when P_i is set equal to unity.

Stress field and biaxial stress ratio:

Love has also given the stress components in a point force stress field(ref.2):

$$\sigma_{x_{II}} = -\frac{P(1+\nu)}{4\pi t}\,\frac{x}{x^2+y^2}\left(\frac{3+\nu}{1+\nu} - \frac{2y^2}{x^2+y^2}\right)$$

$$\sigma_{y_{II}} = \frac{P(1+\nu)}{4\pi t}\,\frac{x}{x^2+y^2}\left(\frac{1-\nu}{1+\nu} - \frac{2y^2}{x^2+y^2}\right)$$

By using the same procedure as for the displacements the stress field can be written as:

$$\sigma_{x_{II}} = \frac{P(1+\nu)}{\pi t d}\left\{\frac{1}{2}\left(\frac{1-\nu}{1+\nu}\right)\ \mathrm{Term1} + \mathrm{Term2}\right\}$$

$$\sigma_{y_{II}} = \frac{P(1+\nu)}{\pi t d}\left\{\frac{1}{2}\left(\frac{1+3\nu}{1-\nu}\right)\ \mathrm{Term1} - \mathrm{Term2}\right\}$$

where

$$\mathrm{Term1} = \frac{\alpha_2}{\alpha_3^2+\alpha_2^2} + \frac{\alpha_2}{\alpha_4^2+\alpha_2^2} - \frac{\alpha_1}{\alpha_3^2+\alpha_1^2} - \frac{\alpha_1}{\alpha_4^2+\alpha_1^2}\ ;$$

$$\mathrm{Term2} = \alpha_2\left[\left(\frac{\alpha_2}{\alpha_3^2+\alpha_2^2}\right) + \left(\frac{\alpha_2}{\alpha_4^2+\alpha_2^2}\right)^2\right] - \alpha_1\left[\left(\frac{\alpha_1}{\alpha_3^2+\alpha_1^2}\right) + \left(\frac{\alpha_1}{\alpha_4^2+\alpha_1^2}\right)^2\right]$$

After superposition with the uniaxial stress field, it follows:

$$\sigma_x = \sigma_{x_{II}}$$

$$\sigma_y = \sigma_{y_{II}} + S$$

The biaxial stress ratio λ then becomes:

$$\lambda = \sigma_x / \sigma_y = \sigma_{x_{II}} / (\sigma_{y_{II}} + S)$$

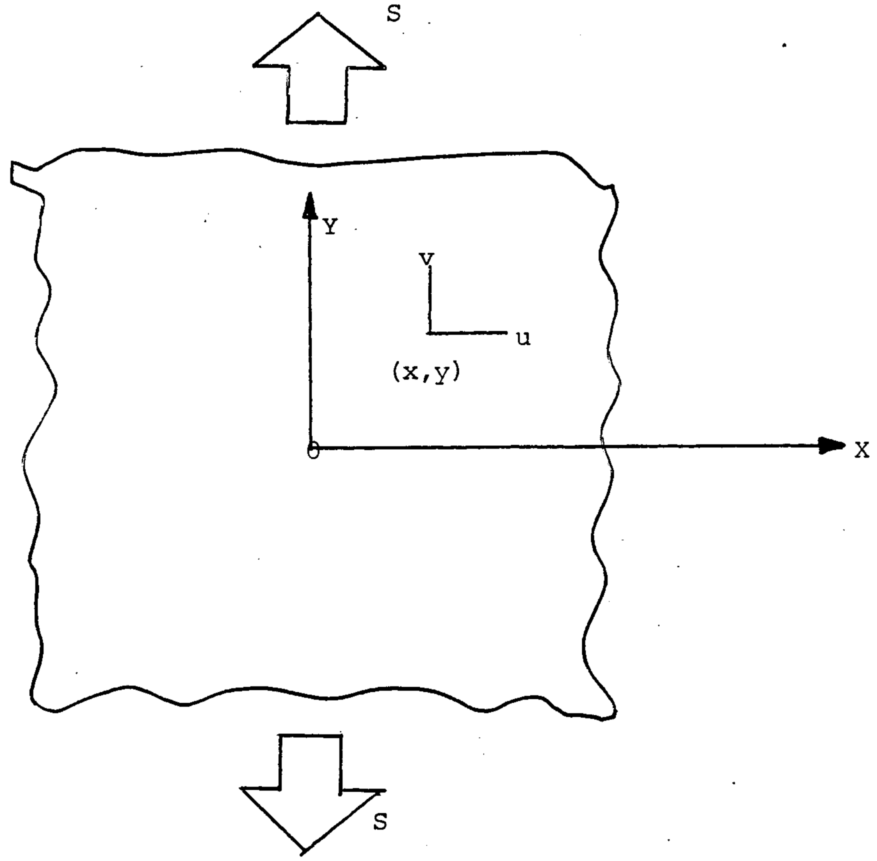

Fig.A1 Uniaxially loaded infinite sheet

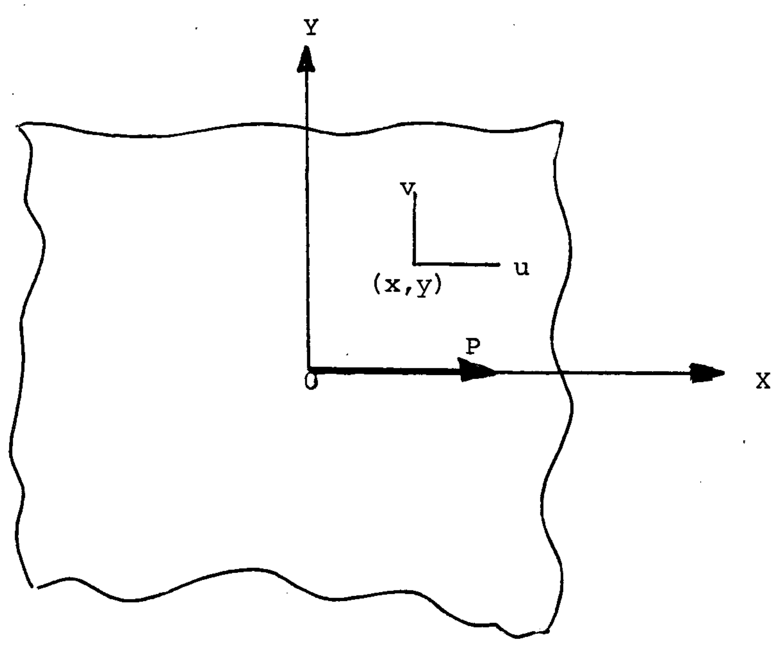

Fig.A2 An infite sheet loaded by a single point force

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