Homogenous Orthotropic Masonry Material Model
Research, development and implementation for explicit analysis

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zonneveld ingenieurs®
Homogeneous Orthotropic Masonry Material Model
Research, development and implementation for explicit analysis

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Abstract

The Groningen region, in the Netherlands, experiences earthquakes since 1986. These earthquakes are new to the region and the structures built there are not designed for it. Therefore, they pose a great risk for the historical structures and people living in the area. Most of the buildings there are built using masonry and to analyze such structures on how they will be affected by earthquakes using currently available tools and methods is rather complicated.

In this report, the author investigates the use of available continuum damage mechanics models for the purpose of simulating orthotropic masonry behavior during an explicit integration analysis. The material behavior is described in Fortran programming language and is used as a custom user subroutine (VUMAT) in Simulia Abaqus finite element analysis software. The developed material model exhibits 3D elasticity and 2D plane plasticity. Furthermore, it is assumed that two general failure mechanisms are present. One associated with tensile and shear brittle fracture represented by Rankine type yield surface and other with distributed crushing of a material represented by Hill type yield surface. The model exhibits uncoupled damage evolution in the tension regime and coupled in compression. Additionally, the model supports tensile crack closure, while in compression it accumulates the plastic deformations and if an element is crushed it can be flagged for deletion from the mesh. The model is formulated in such a way that most of the properties in material directions are independent of one another.

The developed model was tested by examining its behavior in analyses where numerical models were composed out of one or few elements. Additionally, for experimental comparison, four shear walls were modeled, three subjected to monotonic loading and one to cyclic. The analyses closely agree to experimental results even when using raw test data. The material model is stable due to the explicit approach and provides qualitative results as it is flexible enough to be used for various types of analyses, either static or cyclic.

In the final part of the report, further developments are considered, including improvements to the code base, additional testing, and development of a custom element.
The thesis was carried out as a part of the Civil Engineering program at Delft University of Technology, for the master track of Structural Engineering with a specialization in Concrete Structures. The research and development were carried out in cooperation with Zonneveld ingenieurs B.V. from March 2016 to December 2016.

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## 6.2 Recommendations for analyses and future development

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In Europe masonry is one of the most used construction materials. Most of the historic heritage is built using such means of construction. In many places in Europe, these buildings are built in earthquake-prone locations. This causes a threat to the historic heritage. It is most prevalent in the locations that just start to experience earth shakes as all of the buildings there were built without taking into account seismic activities. One of such locations is Groningen region in the Netherlands.

In Groningen, seismic activities started quite recently, in 1986. At that time first heavier earthquakes were experienced. The second big quake happened in 1997 and had a magnitude of 3.4 in Richter scale. In 2003 The Royal Netherlands Meteorological Institute (KNMI) admitted the link between the natural gas extraction and the earthquakes in the region. Since then the earthquakes only increase in magnitude. The third quake happened in 2006 of a magnitude of 3.5 and the fourth in 2012, and had a magnitude of 3.6 in Richter scale. Not only that the big earthquakes get stronger but also smaller rumbles increase in magnitude and frequency.¹

Gas extraction produces earthquakes that are relatively near the surface. From such seismic activity, more damages are observed compared to natural earthquakes that have a deeper point of origin and the same magnitude.

![Figure 1.1: Earthquake depth due to gas extraction (~3 km) and natural causes (~15 km) (Source: [44])](image)

¹Data used in this paragraph is obtained from The Royal Netherlands Meteorological Institute [44]
1.1. Effects and solutions

The earthquakes produced by natural gas extraction do not pose an immediate risk to the structures as their magnitude is low. To the contrary of earthquake-prone regions that exhibit the seismic activity of moderate strengths (5.0 or more in Richter scale) [10]. But rather they damage buildings slowly. Cracks that appeared during one earthquake, can contribute to the collapse of the building during the other one, even if the next quake is several magnitudes weaker. Particularly considering that the buildings in the region were not designed to resist such type of load. This poses a great risk to historical heritage and safety of the people living in the area.

![Figure 1.2: Damage on buildings in Groningen. (a) mortar failure, already filled cracks under the windows and (b) mortar failure, loose bricks are taken out. Photos by Harm Hoorn.](image)

It can be prevented by taking measures. One of the possible solutions is to reinforce the buildings in the area. It can be accomplished by renovating load bearing elements or installing dampers at the foundations of such buildings. So that the seismic loading is taken into account. The solutions to the problem are yet to be implemented in the area. One of the currently potential wall reinforcement techniques is milling deep and shallow grooves into the masonry wall, then embedding carbon strips in the grooves using a special visco-elastic adhesive. The curing is done on only one side of the wall. After the strips are placed, a carbon net is placed on the surface. Finally, a cement based or a polymer based layer is applied. [90]

However, the problem arises when it has to be analyzed which buildings will need reinforcing or if the designed solutions are sufficient.

![Figure 1.3: Reinforcing masonry building from earthquakes](image)
1.2. Material properties of masonry

Masonry is a highly orthotropic composite material with strengths and properties dependent on workmanship, materials, and layouts used. It consists of large units and joints between them. The arrangement of these units and mortar joints determine anisotropy of the composite material. While the type of units and composition of mortar determines overall strength and elasticity. Mortar joints behave as the weakest plane, thus their orientation has the governing effect on the load bearing capacity of masonry.

(a) Common bond
(b) English Bond
(c) English Cross Bond
(d) Single Flemish Bond
(e) Double Flemish Bond

Figure 1.4: Examples of bond types in brick masonry

The failure behavior is determined by 5 basic failure mechanisms (see Figure 1.5). Two of these mechanisms are governed by the properties of the bond (a and b) and three by unit strengths. However, mechanism (c) is only likely [5] to form if tensile strength of masonry unit is weak, it would allow a crack passing along head mortar joints and through the center of the units (see Figure 1.6a) to form. But if masonry unit strengths are far greater than the strengths of mortar, a zigzag like pattern (see Figure 1.6b) would form instead.

The difference in elastic properties between a joint and a unit can cause failure [38] as well. This difference, particularly if the unit is stiffer than the joint, leads to a state of triaxial compression in the joint and compression or biaxial tension in the unit (see Figure 1.7). The cause is that the joint lateral extension is confined by the unit, this, in turn, causes a crack in the unit itself. Increased deformation produces additional vertical cracks until the unit fails.

(a) joint tensile cracking; (b) joint slipping; (c) unit direct tensile cracking; (d) unit diagonal tensile cracking; (e) masonry crushing.

Figure 1.5: Masonry failure mechanisms (from [58]): (a) joint tensile cracking; (b) joint slipping; (c) unit direct tensile cracking; (d) unit diagonal tensile cracking; (e) masonry crushing.
Figure 1.6: Modes of tension failure of masonry walls under direct tension (from [5]): (a) through type, (b) zigzag type

The strengths and failure mode change when different orientations of bed joints and loadings are used. In the case of uniaxial tension (see Figure 1.8) the mode of failure always remains the same, cracking of a joint. Therefore, for the tensile strength of masonry as the whole, it is safe to assume the strengths of the bond between masonry and the mortar. However, this assumption is only valid for loading direction perpendicular to bed joint as for other directions friction between units and joints cause additional strengths.

In the case of masonry with high strengths mortar and low tensile strengths in units e.g. bricks with a high number of perforations, failure might occur when stresses exceed tensile strengths of a unit. Then tensile strengths of the masonry can be considered to be the tensile strengths of a unit.
1.2. Material properties of masonry

During uniaxial compression (see Figure 1.9) the failure modes alternate between crushing of masonry when loading is applied perpendicular to the bed joints and tensile splitting of joints when loading is applied parallel. While at loading angles in between, combination of failure modes are obtained.

The behavior of masonry under biaxial stress cannot be described only by means of principal stresses. Masonry is an orthotropic material, thus, the strengths are fixed to the material axes and cannot rotate together with principle stresses. Therefore, the strength envelope has to be described in terms of full stress vector in a fixed set of material axes or in terms of principle stresses and the rotation angle $\theta$ between principal stresses and material axes.

Considering the test results obtained by Page [72, 73] (see Figure 1.10), that were carried out with half scale solid clay units. It can be seen that the failure strengths are heavily influenced by the orientation of the principle stresses in respect to the material directions.
A lateral compressive stress decreases the tensile strengths. The lowest strength is reached when the tension is perpendicular to the bed joints. In biaxial tension-compression tests failure generally occurs by cracking/sliding of the joints or a combined mechanism involving both – joints and units (see Figure 1.11). However, there is no available experimental results that determine the influence of lateral tensile stress to the tensile strengths of the masonry. Consequently, assumptions have to be made and no exact behavior can be derived.

![Figure 1.11: Modes of failure of solid clay unit masonry under biaxial tension–compression (from [73]).](image1)

In biaxial compression typically failure occurs by splitting of a specimen in mid-thickness (see Figure 1.12). The orientation of principal stresses does not influence the failure mechanism. However, if the ratio between principal stresses is not equal to one, the orientation has a significant influence to the formation of the failure mechanism in the specimen. In that case, failure occurs as a combined mechanism of both – joint failure and lateral splitting.

![Figure 1.12: Mode of failure of solid clay unit masonry under biaxial compression (from [72]).](image2)

The strengths envelope obtained by Page [72, 73] is of limited applicability in masonry. Different types of masonry composed of different materials, unit shapes and bonds will likely produce different failure modes and envelopes. There are further studies on characterization of biaxial strengths of masonry [27, 34, 60] that are advised to refer when designing masonry.

The determination of the shear response [4, 39, 92] depends on the ability of the test set-ups to generate a uniform state of stress in joints. The confinement stress increases the shear strengths due to frictional behavior of masonry in shear. Furthermore, the behavior of joint is non-associative, i.e. \( \delta_n \neq \delta_t \tan \phi \), where \( \delta_n \) and \( \delta_t \) are respectively the normal and tangential relative displacements between sliding surfaces at a masonry joint and \( \phi \) is the angle of friction.

Although some dilatation is likely to occur when two rough units pass over each other, experiments indicate that the real joint behavior is quite complex. The dilation of the masonry subjected to shear depends on the micro-scale geometrical and mechanical features of the masonry joint [93]. Furthermore its is observed that the angle of dilation tends to reduce by both means of increasing isotropic pressure and increasing tangential relative displacement (see Figure 1.13 and 1.14).

Another feature of masonry is softening behavior. This kind of behavior is prominent for quasi-brittle materials e.g. clay bricks, ceramics, rock or concrete. Softening is a gradual decrease of mechanical resistance under a continuous increase of deformation forced upon a material specimen or a structure. Materials exhibiting softening behavior fail due to a process of progressive internal crack growth. Such mechanical behavior is commonly attributed to the heterogeneity of the material, due to the presence of different phases and material imperfections, like defects and voids.
Initially, mortar and units contain micro-cracks and inclusions. The initial stresses and cracks, as well as variations of internal stiffness and strength, cause progressive crack growth when the material is subjected to progressive deformation. At the start of the loading, the micro-cracks are stable which means that they grow only when the load is increased. Around peak load an acceleration of crack formation takes place and the formation of macro-cracks starts. In a deformation controlled test, the macro-crack growth results in softening and localization of cracking in a small zone while the rest of the specimen unloads.

Characteristic stress-displacement diagrams for quasi-brittle materials in uniaxial tension, uniaxial compression and pure shear can be seen in Figure 1.17. The fracture energy, denoted by $G_f$ and $G_c$, is defined as the integral of the $\sigma - \delta$ diagram for tension and compression, respectively. In case of mode II failure mechanism, i.e. slip of the unit-mortar interface under shear loading, the inelastic behavior in shear can be described by the mode II fracture energy $G_{II,f}$, defined by the integral of the $\tau - \delta$ diagram. Figure 1.17c shows brittle behaviors in shear. The value of the fracture energy depends on the level of the confining stress. Shear failure is a salient feature of masonry behavior which must be incorporated in a micro-modeling strategy. However, for continuum macro-models, this failure cannot be directly included because the unit and mortar geometries are not discretized. Shear failure is then associated with tension and compression modes in a principal stress space.

The consideration of the cyclic behavior of masonry adds another layer of complexity. Furthermore, the exact behavior is not sufficiently documented. However, there are tests done regarding cyclic shear loading on masonry walls [4, 28, 84], but cyclic uniaxial and biaxial tests are very rare. In addition, most
of the masonry material models available in the market do not simulate cyclic behavior and are designed only for static pushover tests.

Tensile unloading of the material when it is already in the plastic phase causes reduction of initial stiffness. Such behavior occurs due to separation of the units and cracking of the joints while specimen is loaded, and units coming back to initial positions while unloading.

On the contrary, when material is loaded in compression, during softening or hardening of masonry specimens no damage to the stiffness is produced (see Figure 1.17). In such loading situation when hardening or crushing of the material occurs the specimen deforms and plastic strain will be generated as a result. When the load is removed, the specimen does not regain its initial form and remains deformed.
1.2. Material properties of masonry

Figure 1.17: Compression tests on masonry specimens (from [21]): (a) Calcium silicate units, vertical compression (perpendicular to the bed joint); (b) Calcium silicate units, horizontal compression (parallel to the bed joint); (c) Clay units, vertical compression (perpendicular to the bed joint); (d) Clay units, horizontal compression (parallel to the bed joint).

The cyclic shear behavior in the bed joints is quite simple (see Figure 1.18a). After the joint is fully softened only stress due to friction is remaining and no damage to shear stiffness is present. However, when a shear wall is subjected to a lateral load (see Figure 1.18b), various second order effects take place in the definition of cyclic shear response.

Figure 1.18: Typical cyclic shear response: (a) of the joints for solid clay units (from [4]); (b) of the masonry walls (from [84])
1.3. Current methods
As described in the previous section, it can be clearly seen that the masonry is a very complex composite material. This means that analyzing big masonry structures using empirical methods are highly ineffective and cost inefficient. Consequently, other methods should be used i.e. Finite Element Method (FEM). To model masonry successfully with FEM, simplifications are needed. Whereas, how much to simplify the model is up to the engineer. There are mainly two distinctively different modeling approaches commonly used in FEM, namely, micro- and macro-modeling (see Figure 1.19). However, a combination of the two approaches is also possible.

Figure 1.19: Modelling strategies for masonry structures (from [56]): masonry sample (a); detailed (b) and simplified (c) micro-modelling; macro-modelling (d).

1.3.1. Micro-modeling
Micro-modeling is so far the most accurate method available to simulate the behavior of masonry. Since the material is separated into units of continuous elements and interfaces (discontinuous elements) between them (see Figure 1.19b). Accordingly, only well established and accurate isotropic material models can be used to simulate the anisotropic behavior of masonry. Therefore, the results of such analyses are the most reliable (see Figure 1.21). On the contrary, the models require an extensive amount of time to be prepared and the analyses can be computationally taxing when analyzing big and complex structures.

Nevertheless, this shortcoming could be minimized by removing continuous elements that represent mortar and replacing them by interfaces with mortar-like properties (see Figure 1.19c). This way simplified micro-model is obtained [2, 25, 26, 52, 54, 58, 83, 87]. However, the high level of detail required to represent masonry accurately still makes this approach only suitable for analyses of small elements and small structural details.

Figure 1.20: Micro-modelling of masonry shear walls (from [58])
1.3. Current methods

Figure 1.21: Force – displacement graph. Comparison experimental shear wall and numerical micro modeling analysis (from [56]).

Even though micro-modeling is computationally taxing it is not limited to simple masonry walls (Figure 1.21), but also bigger structures or parts of buildings containing walls, columns and/or arches can be modeled. The Figure 1.22 shows a finite element representation with interface elements that was used in [52] to study a pillar-arch stone structure, analyzed under pseudo-dynamic loading.

Figure 1.22: Monastery S. Vicente de For a, in Lisbon. Deformation pattern using interface elements (from [52]).

As the computation power of computers increase, more and more complex models can be analyzed. A very good example of such analysis was presented by Alexandris et al. [2]. They investigated the collapse mechanisms of traditional one- and two-story houses under earthquake loading in 2D and 3D (see Figure 1.23). The models were used to evaluate alternative intervention options.

Figure 1.23: Collapse of a two-story house under seismic loading (from [2]).
1.3.2. Macro-modeling

As demonstrated in the previous subsection the power of modern numerical tools to represent the complex interaction is sufficient in specific cases. However, when the structure becomes larger the interactions between masonry components (units and joints) start to merge in a homogeneous behavior (see Figure 1.19c). Therefore for large-scale analyses, a faster material model can be assembled. However, making further simplifications reduces the accuracy of the simulation. Consequently, this type of analysis is only relevant when robustness and ease of use (no need to model complex structures consisting of lots of different elements and interactions) are more important than the slight reduction in accuracy of the behavior of the structure.

One of the good examples of such approach was presented in [80]. Finite element model for a block compound in Lisbon (see figure 1.24) was modeled in order to perform a seismic analysis. The model of this scale currently would not be possible to assemble using micro-modeling. Another complex model used for masonry analysis was assembled by Macchi [61]. There a St. Peter’s Basilica was modeled and analyzed (see Figure 1.25) in order to aid the studies of its restoration.

![Figure 1.24: Finite element model for a block compound in Lisbon (from [80]): (a) Finite element mesh (b) results for seismic (shading indicates damage levels).](image)

![Figure 1.25: Studies for the St Peter’s Basilica restoration (from [61]): (a) Model of the entire Facade; (b) tensile stresses on the central section under the effect of the settlement.](image)
Masonry macro models started from the usage of isotropic material description, because of its simplicity. Isotropic materials require few material parameters, therefore, it is easier to perform analyses on historic structures as not all needed material properties can be obtained. Papa [74] derived unilateral damage model for an orthotropic case from a material model originally developed for isotropic materials. The outcome included a homogenized technique to take into account the texture of brick and mortar. A year later Lourenco et al. [59], developed an orthotropic material model for masonry and demonstrated that the homogeneous material model is sufficiently accurate when modeling shear walls under monotonic loading. It was able to predict cracking patterns, peak and ultimate strengths of the structures (see Figure 1.26).

Figure 1.26: Results for an analysis of a masonry shear wall (from [59]): (a) load displacement diagram; (b,c) predicted cracking pattern at peak and ultimate load.

In all of the above examples, smeared damage models were adopted, even if they only provide general information about the level of damage expected on the structure. To point out, the damage simulated this way is unrealistic. It propagates through significant volumes and spreads over large regions of the structure. However, Clemente et al. [12] proposed an enhancement to traditional damage approaches. Their model was based on smeared-crack scalar damaged model and it was modified to reproduce localized (discrete) cracks. To achieve it a local crack-tracking algorithm was used. This model enables the simulation of more realistic damage distributions than the original smeared-crack model.

The localized cracks predicted by the crack tracking model reproduce consistently a set of expectable plastic hinges developing gradually in the structure and leading to the full collapsing mechanism. The model has been used to analyze the response of the structure of Mallorca Cathedral under gravity and seismic forces (see Figure 1.27).

Figure 1.27: Seismic analysis of Mallorca Cathedral (from [12]): (a) smeared damage approach versus (b) localized damage approach.

An interesting approach was used by Pela et al. [76], where they composed a material model from isotropic failure envelopes by mapping them from fictitious isotropic stress and strain space to a real orthotropic space. This allowed them to use well defined isotropic failure criteria for orthotropic material behavior simulation. Additionally, they used a local crack-tracking algorithm derived by Clemente et al. [12] in order to localize cracks produced by damage due to tension (see Figure 1.30).
1.3.3. Limit-modeling

Previously mentioned modeling techniques can be combined by modeling the structure subdivided into homogeneous elastic or rigid blocks that are connected with interfaces simulating most common failure mechanisms (see Figure 1.29). These modeled collapse mechanisms are then analyzed by applying kinematic limit analysis. The approach was first proposed by Giuffré [32, 33] where he observed a pattern in failure modes of historical and traditional buildings in Italy. This approach is particularly interesting as a tool for seismic analysis of buildings which do not conform to box behavior because of lack of stiff floor slabs or because of weaker partial collapses affecting the façade or inner walls. For more upper and lower bound limit analyses and techniques reader is referred to [30–33, 53, 62–66, 87].

Figure 1.28: Analysis of shear walls with opening (from [76]): (a) smeared tensile damage; (b) localized tensile damage; (c) smeared compression damage; (d) deformed mesh (x50).

Figure 1.29: Failure mechanisms for buildings embedded within urban texture (from [83] according to [16]).
1.4. Current numerical methods used in the Netherlands

Currently, there are several finite element software packages that are being used in the Netherlands to analyze masonry structures. Mainly Simulia Abaqus, TNO Diana and LS-DYNA. Simulia Abaqus and LS-DYNA both support implicit and explicit integration while TNO Diana has only an implicit solver. The implicit solvers are very difficult to work with when analyzing highly nonlinear problems that involve large deformations and large amounts of plasticity. Therefore, for masonry analyses under earthquake loading they are not an ideal solution and explicit solvers are preferred.

Even though LS-DYNA has a masonry material model it is developed and used only by ARUP and the model is not available for third parties. Because of the fact the thesis is sponsored by Zonneveld ingenieurs b.v. the use of LS-DYNA masonry model is not possible as Zonneveld ingenieurs b.v. uses Simulia Abaqus as their finite element analysis software.

For masonry analyses with Simulia Abaqus an adapted model of Concrete Damaged Plasticity is being used by Zonneveld ingenieurs b.v.. This model is one of the default materials that comes integrated in Simulia Abaqus and it is developed to be used as a tool to analyze reinforced concrete. Concrete Damaged Plasticity is an asymmetric isotropic continuum damage model that supports different behaviors in tension and compression, strengths hardening/softening, and damage to stiffness.

Although, by adapting the model to be used with an orthotropic material such as masonry causes several issues. E.g. due to the isotropic nature of the material model, engineer using it has to predict a critical failure mechanism in order to adapt real orthotropic properties of a material to the isotropic model. Therefore, to simulate masonry behavior more precisely a better material model has to be developed. Such model should exhibit asymmetric orthotropic plastic behavior.

1.5. Scope of the thesis

The main aim of the thesis is to develop a nonlinear material model based on the Continuum Damage Mechanics, that can be used in Simulia Abaqus Finite Element Analysis software. The material model should exhibit asymmetric orthotropic plastic behavior and it should be robust and accurate in small scale tests as well as large-scale complex analyses.
The development is focused on three-dimensional structures that are loaded by static as well as dynamic loading. The model is a macro type and it simplifies masonry as continuous homogeneous material, on the contrary to the micro modeling where units and mortar are modeled separately.

The objectives of the study are:

- To assemble information on the existing knowledge about Continuum Damage Mechanics models, through a comprehensive literature review;
- To develop an orthotropic masonry model that is able to accurately predict elastic and plastic behavior of masonry structures and incorporates the knowledge of nonlinear fracture mechanics used in crack propagation problems;
- To test the model by comparing the predicted behavior with the behavior observed in experiments on masonry. The developed model should be able to predict the failure modes and the ultimate load with a reasonable agreement with the experimental evidence.
- To demonstrate the applicability of the verified model in engineering practice case-studies i.e. in analysis of shear walls.

It must be mentioned, however, that results of masonry tests that are large and complex or small and simple, typically shows a wide scatter. Thus, the main concern is not too sharply reproduce the experimental results in the form of load-displacement curve but to demonstrate the ability of models to capture the behavior observed in the experiments.

It is to be noted that the model developed in this study can be used in a broader field than just masonry. It is applicable to any other anisotropic material like plastics, wood, and fiber-reinforced composites.

1.6. Outline of the Thesis

This thesis consists of six Chapters.

**Chapter 1** contains a brief introduction of the seismic situation in the Netherlands and effects of it on the masonry structures. This chapter also describes masonry and its properties as well as the overview of current approaches to masonry analysis. The scope of the thesis and its outlines are also included.

**Chapter 2** presents the review of several Continuum Damage Mechanics models and aspects corresponding to their numerical implementation.

**Chapter 3** describes the formulation of a damage model for masonry. Such a model accounts for different orthotropic behaviors in tension and compression. Individual damage criteria are considered for tension and compression, according to different failure mechanisms. The former is associated with cracking phenomenon, while the latter is associated with the crushing of the material. Entirely different elastic and inelastic behaviors can be predicted along the material axes, both in tension and compression. For compression, the model can predict residual plastic strain and it can simulate the cyclic behavior. The resulting formulation is implemented in a nonlinear finite element code for Simulia Abaqus Explicit software package.

**Chapter 4** validates the damage model developed in Chapter 3 by means of the FE analysis of cube tests in order to portray the pure theoretical characteristics of the model.

**Chapter 5** furthermore validates the damage model by means of the FE analysis of engineering practice case studies, e.g. shear walls with monotonic or cyclic loading. A smeared crack approach is considered.

**Chapter 6** presents an extended summary and the final conclusions together with suggestions for future work which can be derived from this study.
Basic concepts and theoretical formulations

This Chapter presents various approaches to material modeling using Continuum Mechanics, the possible descriptions of failure envelope and their comparability with available tests. Basic concepts are defined, together with the theoretical formulation. Then, a comparative discussion concerning different failure envelopes and damage models is carried out, in order to emphasize the implications arising from the different backgrounds. Furthermore, the chapter will also describe the relation between different analysis techniques and explanations where does the custom material models belong in Simulia Abaqus analysis procedure.

2.1. Implicit and Explicit algorithms

The Finite element method (FEM) is one of the most popular methods in both research and industrial numerical simulations. In the FEM codes, several different algorithms are implemented, these algorithms can have varying computational costs, accuracy or ease of use. Understanding the nature, advantages and disadvantages of these algorithms are very helpful for choosing the right algorithm for the particular problem.

Finite element algorithms can be classified into two categories: implicit and explicit algorithms. In implicit algorithms, a matrix system has to be solved one or more times per step through an iterative procedure in order to obtain a force equilibrium and advance the analysis. This type of technique generally has an advantage regarding numerical stability. In many cases, an “Unconditional Stability” may be obtained, resulting in no time step restrictions caused by stability considerations [40].

However, in explicit algorithms, the solution can be advanced without solving a system of equations. It generally requires that small time steps need to be taken to ensure numerical stability. Although, there are cases when step-size restrictions are more stringent than accuracy considerations might require. Though, on the other hand, due to the lack of equation solving the computational cost per step is generally much less for explicit algorithms than for implicit. None of the algorithms are perfect for all analysis situations, therefore an optimal one has to be chosen for a specific case.

Implicit algorithms are mostly used for linear and non-linear static or quasi-static analyses where slow loading conditions are applied. For non-linear analyses where a model is subjected to large amounts of plasticity or a complicated model with a high number of interactions is used, the unconditionally stable implicit method will encounter some difficulties [86]. In such cases, in order for the system to converge time increment has to be reduced and as the reduction of the time increment continues, the computational cost in the tangent stiffness matrix is dramatically increased and can even cause divergence. Furthermore, local instabilities cause force equilibrium difficult to be achieve. Therefore, the implicit algorithm loses its advantages and as the time increment for convergence approaches the stable time of an explicit analysis, the explicit method becomes more viable to use.

Explicit algorithms are mostly used for large, fine mesh, high phased and short analyses. For an explicit solution, the CPU cost per increment is approximately proportional to the size of the model. There is no drastic increase in memory or processing time as the problem size or complexity increases.
such as that associated with an implicit method [78]. However, the time increment in the explicit analysis depends only on the dimension of the elements and the properties of the materials used (see eq. 2.2). Therefore long static or quasi-static analyses even if not complex, become less feasible as it would take an extensive amount of computational time to complete them.

The stable time increment for explicit algorithm is defined as:

\[ \Delta t \leq \frac{2}{\omega_{max}} \]  

(2.1)

where \( \omega_{max} \) is the element maximum eigenvalue. A conservative estimate of the stable time increment is given by the minimum taken over all the elements. The above stability limit can be rewritten as

\[ \Delta t = \min \left( \frac{L_e}{c_d} \right) \]  

(2.2)

where \( L_e \) is the characteristic element dimension and \( c_d \) is the current effective, dilatational wave speed of the material.

Regarding modeling and analyzing masonry, the most widely used method is implicit, mostly due to a large number of FEM software that supports such method. Despite this, masonry FEM analysis would greatly benefit from an explicit algorithm as big models, earthquake loading, extensive amounts of damage and cracking heavily slow down implicit analyses as well as produces extensive amounts of convergence problems. However, in quasi-static analyses due to infeasibility to simulate long periods of time, the simulated time period has to be shortened, this induces undesirable kinematic effects onto the structure. On the other hand, these effects can be neglected if kinetic energy is less than 5% of total strain energy [40].

For further reading about implicit and explicit algorithms reader is referred to [7, 40, 41, 78, 82, 86].

2.2. Theory of plasticity

A fundamental difference between elastic and inelastic behavior is that in the elastic analysis the total stress can be evaluated from total strain alone, however, in inelastic solutions the total stress at time \( t \) also depends on stress and strain history (see Figure 2.1). There are a very large number of developed material models that simulate the distinctive phenomena of plasticity i.e. elastoplasticity, creep, and viscoplasticity [6].

![Figure 2.1: Material nonlinearity (from[45]): (a) non-linear elasticity (b) elasto-plasticity.](image)

Generally, for metals the theory required to describe plastic flow is simplistic since metals are insensitive to isotropic pressure, are generally incompressible and with approximation follow associated flow rules. For other materials such as concrete, rocks, masonry, fiber reinforced composites etc. the conditions are more complicated, but regardless, falling within the frame work of plasticity theory.

The theory of plasticity employs some of the fundamental concepts such as the yield criterion, the flow rule, and the consistency conditions. The yield criterion describes the limit at which material becomes plastic and the consistency conditions prevent stresses from exceeding that prescribed limit, while the flow rule describes the relationship between strains and stresses once material enters plasticity.
2.2. Theory of plasticity

2.2.1. Yield criteria
One of the first proposals for criteria of yielding of plastic solids, mainly soils, was made by Coulomb [13] in 1776, and had been applied by Poncelet [77] in 1840 and Rankine [81] in 1853 to problems such as the calculation of earth-pressure on retaining walls. At the end of 19th century, Mohr generalized the criterion and it became widely used, and known as Mohr-Coulomb yield criterion [95].

The criterion is pressure sensitive and is best used in describing the materials whose behavior is strongly depending on yield limit and hydrostatic pressure i.e. materials like soils, rocks or concrete. The Mohr–Coulomb criterion is based on the assumption that the phenomenon of macroscopic plastic yielding is, essentially, the result of frictional sliding between material particles [17]. The general formulation of Coulomb’s friction law states that critical combination (see eq. 2.3) of normal stress $\sigma_n$ and shear stress $\tau$ triggers a plastic yielding.

$$\tau = c - \sigma_n \tan \varphi$$ (2.3)

where $\sigma_n$ is tensile positive normal stress, $c$ is the cohesion and $\varphi$ is the angle of internal friction (see Figure 2.2).

The Mohr–Coulomb multi-surface representation (see Figure 2.9) can be derived from a yield function expressed in terms of the principle stresses (see eq. 2.4). Such representation consists of 6 yield surfaces described by every combination of $\sigma_{max}$ and $\sigma_{min}$ in the yield function, whose roots are $\Phi_i(\sigma) = 0$.

$$\Phi(\sigma) = (\sigma_{max} - \sigma_{min}) + (\sigma_{max} + \sigma_{min}) \sin \varphi - 2c \cos \varphi$$ (2.4)

Figure 2.2: The Mohr–Coulomb criterion. Mohr plane representation.

Figure 2.3: The Mohr–Coulomb criterion. Multi-surface representation in principal stress space (from [17]).
The first scientific study of the plasticity started from a study of plastic behavior in metals in 1884. At that time Tresca [89] published a preliminary account of experiments on punching and extrusion, which led him to state that a metal yielded plastically when the maximum shear stress attained a critical value. Unlike in the Mohr-Coulomb model in Tresca’s findings metals were not hydrostatic pressure sensitive, therefore a new yield criterion in principal stress space was assembled (see eq. 2.5).

\[ \Phi(\sigma) = \sigma_{max} - \sigma_{min} - \sigma_y \]  

(2.5)

Nevertheless, like Mohr-Coulomb, Tresca criterion can be represented as 6 surfaces in principle stress space (see Figure 2.4) composed from all possible combinations of \(\sigma_{max}\) and \(\sigma_{min}\) in the yield function (eq. 2.5) whose roots are \(\Phi(\sigma) = 0\).

![Figure 2.4: The Tresca criterion. Multi-surface representation in principal stress space (from [17]).](image)

After the findings of Tresca, there were numerous suggestions of the yield criteria for metals, however, more accurate works were yet to be presented. In 1913 von Mises [67] made an advancement in the definition of yield criterion for metals purely from a mathematical point of view. He described the failure in terms of second stress invariant \(J_2\) reaching a critical value.

\[ J_2 = k(\alpha) \]  

(2.6)

where \(k\) is critical value assumed to be a function of internal hardening variable \(\alpha\) and mostly used with a relation \(\tau_y = \sqrt{k}\), where \(\tau_y\) is the shear yield stress. Therefore, a yield function (see Figure 2.5) for the von Mises criterion can be defined as

\[ \Phi(\sigma) = \sqrt{J_2(s(\sigma))} - \tau_y \]  

(2.7)

In 1952 a smooth approximation to Mohr-Coulomb law (see Figure 2.6) was proposed by Drucker and Prager [20]. The new criterion was derived from von Mises criterion by adding an extra term to introduce pressure sensitivity. The Drucker–Prager criterion states that the critical combination of hydrostatic pressure \(p\) and second deviatoric stress invariant \(J_2\) triggers the yielding of a material. Therefore, the yielding is initiated when the following equation is satisfied

\[ \sqrt{J_2(s)} + \eta p = \hat{c} \]  

(2.8)

where \(\eta\) and \(\hat{c}\) are material parameters. However, in order to approximate the Mohr-Coulomb yield surface, Drucker-Prager’s yield function can be defined as follows

\[ \Phi(\sigma) = \sqrt{J_2(s(\sigma))} + \eta p(\sigma) - \xi c, \]  

(2.9)
Figure 2.5: The Tresca and von Mises yield surfaces in principal stress space (adapted from [17]): (a) 3d view and (b) view along hydrostatic axis.

Figure 2.6: The Drucker–Prager yield surface in principal stress space (from [17]).

Figure 2.7: Drucker–Prager approximations (adapted from [17]): (a) approximations in principle stress plane (b) approximations matching the Mohr–Coulomb surface in uniaxial tension and uniaxial compression.
where $c$ is the cohesion and the parameters $\eta$ and $\xi$ define the required approximation to the Mohr-Coulomb criterion. The stated yield criteria form the basis of yield descriptions in material mechanics. From these, other criteria are derived and described. It must be pointed out that all of the criteria previously mentioned are of isotropic nature. Hill [37] in 1948 described an anisotropic version of von Mises criterion, a modification of it will be used in this thesis as a part of the description of the failure envelope. For further information on various other yield criteria reader is referenced to [8, 9, 11, 18, 23, 35]

### 2.2.2. The plastic flow and stress return

The plastic flow is classified according to the relation of plastic strain increment and deviatoric stress directions. If direction of plastic strain increment is equivalent to the direction of the deviatoric stress it is called associated flow rule. However, if they are not equivalent it is called non-associated flow rule. The former is usually observed in metals, while the latter is valid for other materials such as rock, concrete and masonry. From here the yield function $f(\sigma)$, and the flow function $g(\sigma)$ can be defined.

For associated flow rule $f(\sigma) = g(\sigma)$ and for non-associated flow rule $f(\sigma) \neq g(\sigma)$.

The direction of the plastic strain is perpendicular to the flow surface and is equal to the direction of the flow vector for the isosurface $\Psi(\mathbf{A})$ (see Figure 2.8), where $\Psi(\mathbf{A})$ is $f(\sigma)$ or $g(\sigma)$ depending on the type of the flow:

$$ N = \frac{\partial \Psi(\sigma)}{\partial \sigma} \quad (2.10) $$

![Figure 2.8: Smooth potential (from [17]). (a) The flow vector and (b) stress return represented in a plane perpendicular to the hydrostatic pressure line in principal stress space](image)

For finite element calculation, not only the direction of the flow is important but also the magnitude $d\lambda$ of the stress return. Let $\sigma_n$ be a current stress, $\sigma_{trial}^{n+1}$ be trial stress for the current calculation increment and $\sigma_{n+1}$ be the final stress. The trial stress increment can then be defined as follows:

$$ d\sigma_{trial}^{n+1} = \sigma_{n+1}^{trial} - \sigma_n = D d\varepsilon \quad (2.11) $$

The “real” stress increment is equal to

$$ d\sigma_n = d\sigma_{trial}^{n+1} - D d\varepsilon_{pl} = D (d\varepsilon - d\varepsilon_{pl}) \quad (2.12) $$

In order to return back to the yield surface, the plastic strain increment ($d\varepsilon_{pl}$) has to be found. Which can be expressed as:

$$ d\varepsilon_{pl} = d\lambda \frac{\partial g}{\partial \sigma} \quad (2.13) $$
2.2. Theory of plasticity

The plastic strain is determined by two quantities: the scalar $d\lambda$ and the gradient of the loading surface $(\partial g/\partial \sigma)$ giving the direction. To determine the magnitude $d\lambda$, the following condition must be satisfied:

$$f(\sigma_B - d\sigma_p) = 0$$

Where plastic corrector $d\sigma_p$ can be expressed as:

$$d\sigma_p = d\lambda D \frac{\partial g}{\partial \sigma}$$

(2.15)

Considering Taylor expansion of the yield surface around the point B we have:

$$f(\sigma_B - d\sigma_p) \approx f(\sigma_B) - \left( \frac{\partial f}{\partial \sigma} \right)_B d\sigma_p = 0$$

(2.16)

Inserting eq. 2.15 to eq. 2.16 we obtain the following:

$$d\lambda = \frac{f(\sigma_B)}{\left( \frac{\partial f}{\partial \sigma} \right)_B D \left( \frac{\partial g}{\partial \sigma} \right)_C}$$

(2.17)

One should note that this solution is only valid if flow direction at point B and the flow direction at point C is the same. Otherwise, iterations are necessary. However, if the flow function has discontinuities or flow is described by several functions, the trial stress that falls in the sub-differential set of $\Psi$ (see Figure 2.9) should always return to the point of discontinuity $\sigma$. The sub-differential set of $\Psi$ is bounded by a set of normals as follows:

$$N_i = \frac{\partial \Psi_i}{\partial \sigma}$$

(2.18)

Figure 2.9: The flow vector (from [17]). Non-smooth potential.

2.2.3. Stress and strain hardening

Most materials exhibit some degree of hardening as an addition to plastic straining. The hardening of the material manifests as the change of the size and the shape of the yield surface during plastic loading. This change is complex and an accurate description of it is rather difficult to obtain. Therefore, hardening is often described as a combination of two different types of hardening, namely isotropic hardening and kinematic hardening (see Figure 2.10).

A material is said to experience isotropic hardening when the evolution of the yield surface is such that, at any state of hardening, it can be described as an isotropic (uniform) expansion, without translation, of the initial yield surface. In the case of von Mises yield criterion the elastic domain expands...
equally in tension and compression during plastic flow, i.e. isotropic hardening corresponds to the in-
crease in the radius of the von Mises cylinder in principal stress space. Therefore, the hardening can
be described by the following:

\[ f(\sigma) - \sigma_0(\alpha) = 0 \]  

(2.19)

where \( \sigma_0(\alpha) \) is the material strengths depending on dynamic hardening parameter \( \alpha \) when the following
is always true \( \dot{\alpha} \geq 0 \). On the other hand, for orthotropic materials, isotropic hardening is controlled by
changing each strength parameters in yield criteria \( \sigma_0(\alpha) \).

Kinematic hardening takes place when the shape and size of a yield surface is preserved but it
can translate in the stress space. Lemaitre and Chaboche [50] showed that many materials when
being loaded and hardened in one direction, exhibit a decreased resistance to plastic yielding in the
opposite direction. The phenomenon is called Bauschinger-effect and it can be simulated by introducing
kinematic hardening.

In the case of von Misses yield criterion kinematic hardening is introduced by replacing stress vector
in the criterion by the relative stress tensor (see eq. 2.21) that depends on stress deviator \( s(\sigma) \) and the
symmetric deviatoric stress-like tensor, \( \beta \), also known as backstress tensor.

\[ f(\sigma, \beta) = \sqrt{3J_2(\eta(\sigma, \beta))} - \sigma_0 \]  

(2.20)

where

\[ \eta(\sigma, \beta) = s(\sigma) - \beta \]  

(2.21)

There are a number of studies that propose a constitutive model for the definition of the elasto-
plastic behavior under cyclic loading conditions [50, 69, 85]. However, only the isotropic hardening
behavior will be implemented in this thesis as kinematic hardening behavior for composite material
such as masonry is poorly documented and any effort to implement it would be purely phenomenologi-

cal. Furthermore, considering explicit integration analysis, it is likely that kinematic hardening behavior
in the simulations of structures would appear naturally to some extent, without extra considerations in
the material model.

### 2.3. Continuum Damage Mechanics

In 1958 Kachanov [43] introduced a theory of continuum damage mechanics that provided a powerful
and general framework for the derivation of consistent material models suitable for many engineering
fields and the term itself was defined by Janson and Hult [42]. The advantage of continuum damage
mechanics is that it can be used to simulate a wide range of materials such as ceramics, plastics, met-
als, rock, concrete, and masonry. It is due to the simplistic approach in modeling damaged materials.
Such materials are assumed to remain continuous and the effect of cracks is modeled by changing the
mechanical properties, such as strength and stiffness.

Kachanov [43] defined the collective effect of deterioration by means of a field variable named
continuity and denoted \( \psi \). Completely deterioration free material was given the condition \( \psi = 1 \) and
completely damaged continuous material with no remaining load carrying capacity was defined as \( \psi = 0 \).
As continuity \( \psi \) defines absence of defects, the variable \( D = 1 - \psi \) defines state of deterioration or
damage [70, 79].
2.3. Continuum Damage Mechanics

Continuum damage mechanics is a counterpart of fracture mechanics, where the fracture is simulated by embedding a fixed crack in a usually non-degradable material. However, both of these approaches can be combined to simulate crack growth and degradation of load carrying capacity [42, 47].

The damage itself in solids is considered as discontinuities in the medium that on a macro scale is continuous. These discontinuities are generally caused by micro-cracks and micro-voids. The biggest volume over which such material can be treated as homogeneous is called representative elementary volume (REV) or unit cell (see 2.11). The size of this unit cell can vary from about 0.1 mm$^2$ for metals and ceramics to about 100 mm$^2$ for concrete (see Figure 2.12).

![Figure 2.11: Representative elementary volume in a composite body (from [55])](image1)

![Figure 2.12: Examples of damage (from [51]) in a metal (micro-cavities in copper), in a composite (micro-cracks in carbon-fiber/epoxy resin laminate), and in concrete (crack pattern).](image2)

The damage of the material is always related to the plastic strain (permanent strain) and more generally to a strain dissipation in the scale of REV (mesoscale) or the scale of discontinuities (microscale). In mesoscale three types of damage can occur: the nucleation and growth of voids in mesoscale of plastic strains under static loading causes ductile damage; low cycle fatigue damage occurs under repeated high-level loadings and elevated temperatures causes creep damage by intergranular decohesion in metals.

Regarding microscale, two different kinds of damage can be classified: quasi-brittle and high cycle fatigue damage. The former is when brittle failure is caused during monotonic loading. The latter is when a material is subjected to a loading of a large number of repeated cycles. Materials like ceramics, concrete, and metals exhibit this type of damage.
Considering an isolated unit cell (REV) in a damaged solid and letting $dS$ be the area of the section of the unit cell identified by its normal $\vec{n}$. On the section, cracks and voids leave traces and reduce the effective area of resistance $d\tilde{S}$ where ($d\tilde{S} < dS$). Let $dS_D$ be the difference between the area and the effective area:

$$dS = dS - d\tilde{S}$$  \hfill (2.22)

The Damage variable $D$ can be defined as the surface density of micro-cracks [51] therefore it can be written as a ratio between the area of the voids and cracks, and the area of the section:

$$D(\vec{n}) = \frac{dS_D}{dS}$$  \hfill (2.23)

The above expression provides the measure of local damage relative to the direction $\vec{n}$. $0 \leq D(\vec{n}) \leq 1$ characterizes the damaged state, where $D(\vec{n}) = 0$ is equivalent to initial undamaged state and $D(\vec{n}) = 1$ – to fully damaged material.

However, if the damage is isotropic, the crack and cavities are distributed evenly in all directions. Therefore, the damage variable $D(\vec{n})$ can be completely described by the scalar intrinsic variable $d$:

$$D(\vec{n}) = d \quad \forall \vec{n}$$  \hfill (2.24)

On the other hand, in the general case of anisotropic damage the variable $D(\vec{n})$ depends on the orientation of the normal. It was shown that during loading history, the micro cracks undergo irreversible growth in the direction perpendicular to the maximal tensile strain [24, 46]. The concept of the damage variable can further be advanced to the formulation of effective stress.

If $F$ is the applied force onto the cross section of the unit cell, the uniaxial case is considered due to simplicity. The stress in the cross section is $\sigma = F/S$ satisfying the equilibrium conditions. Therefore, if the isotropic damage $d$ is present, the effective area of resistance is

$$\tilde{S} = S - S_D = S(1 - d)$$  \hfill (2.25)

hence effective stress is defined by

$$\tilde{\sigma} = \frac{\sigma}{\tilde{S}} = \frac{\sigma}{1 - d}$$  \hfill (2.26)

In case of multi-axial isotropic damage, the damage does not depend on the orientation of the normal and the operator $(1 - d)$ can be applied to all of the components. Therefore, we can consider the tensorial form

$$\tilde{\sigma} = \frac{\sigma}{1 - d}$$  \hfill (2.27)

or the inverse expression

$$\tilde{\sigma} = \sigma(1 - d)$$  \hfill (2.28)
The hypothesis of the strain equivalence states that the strain associated with a damaged state under applied stress $\sigma$ is equivalent to the strain associated with its undamaged state under effective stress $\bar{\sigma}$ [49] (see Figure 2.14). Therefore, by using simple relation $\bar{\sigma} = E\varepsilon$ we obtain:

$$\sigma = (1 - d)\bar{\sigma} = (1 - d)E\varepsilon$$ \hspace{1cm} (2.29)

Where $E$ is Young's modulus. From which can be derived that actual tension stress $\sigma$ is related to strain by means of a damaged stiffness:

$$E_d = (1 - d)E$$ \hspace{1cm} (2.30)

The damage is irreversible, therefore:

$$\dot{d} \geq 0, \quad \dot{d} \geq 0 \quad \rightarrow \quad \dot{E}_d \leq 0$$ \hspace{1cm} (2.31)

The damage is only initiated if when the stress or strain exceeds initial failure threshold $\sigma_0$ (or $\varepsilon_0$):

$$d = 0 \quad \text{if} \quad \begin{cases} \sigma < \sigma_0 \\ \varepsilon < \varepsilon_0 \end{cases}$$ \hspace{1cm} (2.32)

In case of unloading we have:

$$\dot{\varepsilon} < 0 \quad \rightarrow \quad \dot{\varepsilon}_d = 0 \quad \text{and} \quad \dot{d} = 0$$ \hspace{1cm} (2.33)

and, therefore,

$$\dot{\sigma} = (1 - d)E \dot{\varepsilon} - dE \ddot{\varepsilon} = E_d \dot{\varepsilon} \hspace{1cm} (2.34)$$
Unloading does not increase the damage, it will, however, follow the unloading branch according to the damaged stiffness until the point of origin. A further reloading follows the same loading paths until a failure criterion is reached once more. The damage constitutive law does not allow any occurrence of irreversible plastic deformation as after unloading all the deformations are recovered. However, the actual behavior of materials, especially masonry, do not fully comply with this law. As described in section 1.2 plasticity occurs in actual physical tests and the behavior of such mediums as masonry is not isotropic. Therefore, some additional considerations are necessary.

2.4. Damage models and failure envelopes for masonry

There were only a few attempts to describe the general failure criteria for masonry, due to the difficulties involved in developing a representative biaxial test and the fact that a large number of such tests is required. This problem was discussed by Yokel and Fattal [94] with reference to the failure of shear walls. Further efforts were made by Dhanasekar et al. [19] where they interpolated the test data of Page [72, 73] by means of three elliptic cones (see Figure 2.16). However, as it is described in the paper, the cones do not correspond with the observed distinct modes of failure. The elliptic cones have been expressed by a second-order tensor polynomial. A significant review of the subject can be found in [3, 36, 68].

![Figure 2.16: Failure surface idealized by Dhanasekar et al. [19].](image)

There were several proposals to use existing yield criteria of available composite materials for the expression of analytical failure models of masonry. For example, Syrmakesis and Asteris [88], used a Tsai and Wu [91] cubic tensor polynomial. Nevertheless, the adaptation did not return a satisfactory approximation of Page’s [72] experimental data (see Figure 2.17).

The further advancements in masonry behavior modeling were made by Lourenco [56]. He proposed a multi-surface plane-stress softening plasticity model for a masonry failure simulation. The model consists of two yield criteria: one Rankine-like for failure prediction of joints and one Hill-like for failure prediction of units (see Figure 2.18). He described different stress-strain behavior laws in tension and compression. In tension exponential softening law was adopted along material axes and in compression, he applied an isotropic parabolic hardening law followed by a parabolic/exponential softening law with different compressive fracture energies along each material axes. Therefore, the principal directions of damage are fixed and aligned with the initial orthotropy axes. However, although the model adopted two different fracture energies, a single internal scalar variable controlled the plasticity algorithm in order to determine the softening in two material axes. Furthermore, the damage was only described in terms of softening of the material strengths and the degradation of stiffness was not present.
2.4. Damage models and failure envelopes for masonry

Figure 2.17: Comparison between experimental results from Page [72, 73] and a Hoffman type yield surface (from [56]).

Figure 2.18: Multi-surface failure envelope in Cauchy stress space (from [56]).

Figure 2.19: Behavior of the model (from [57]) for (a) tension and (b) compression, along two orthogonal directions.
Since the works of Lourenco there was no further significant development of masonry constitutive material models until Pela [75] proposed the use of isotropic yield surfaces mapped on to orthotropic space as viable analysis approach for masonry structures. His model consisted of isotropic Faria [23] and isotropic Rankine-type failure envelopes (see Figure 2.21a). Furthermore, he used a modified method proposed by Oller et al.[71] where authors multiplied the transformation tensor by a "shape adjustment tensor", whose purpose is to adjust the isotropic criterion to the desired orthotropic one. But, the shape adjustment tensor must be derived by means of an iterative procedure. Thus, the non-linear solution of a quadratic system by the Newton-Rapson method is required which is not an easy task and is quite costly, since the shape adjustment tensor depends on the stress state at the point at each instant of the mechanical process. To circumvent this, Pela [75] simplified Oller’s et al. method to a standard form of a stress transformation tensor and obtained similar results to Lourenco [56] (see Figure 2.22).

Figure 2.20: Comparison between Lourenco model and experimental results from Page [72, 73] (from [56]).

Figure 2.21: Yield surfaces used by Pela [75]: (a) Composite yield surfaces in Cauchy stress space and (b) mapping each surface from fictitious isotropic stress space to real stress space.
2.4. Damage models and failure envelopes for masonry

Like in Lourenço’s [56] model in Pela’s [75] work, the damage was modeled by softening of the material strengths, but additionally, Pela used stiffness damage model proposed by Papa [74]. The model allows stiffness recovery when loading changes from tension to compression which is in agreement with experimental results. On the other hand, it does not allow any permanent plastic strains to occur, while it can be true for tensile behavior, it is not in agreement with the cyclic compression test results [21].

Figure 2.22: Comparison between Pela, Lourenco, Symakezis models and experimental results from Page [72, 73] (from [75]).

Figure 2.23: Behavior of the model (from [75]) for (a) tension and (b) compression, along two orthogonal directions. (c) represents cyclic behavior.

Figure 2.24: Damage model proposed by Papa [74] (from [75]).
2.5. Simulia Abaqus user material interface

Before writing a material model it is important to understand how custom user material (VUMAT) interfaces with Simulia Abaqus. This section will discuss the capabilities of user subroutines for Abaqus / Explicit package.

Figure 2.25 portrays the location of execution of VUMAT. The scheme is composed out of the experience gathered when working with various Abaqus subroutines and can differ from the actual execution procedure to some extent. The Abaqus manual does not discuss the actual global flow of explicit analysis.

When the whole model is prepared for the analysis it’s integration points are subdivided into blocks containing the maximum number of 136 points. For each of the blocks, the user subroutine is called. Each block only contains the integration points of the same material, and inside the subroutine, these points have to be iterated to provide the output for all of them. Furthermore it is important to note, that there is only one integration point per element in the explicit analysis. The exact procedure of user subroutine used for the thesis is described in Figure 3.1.

The User subroutine must output new stress vector and can output optional vectors such as state variables, stretch tensors, deformation gradients, temperature values, total internal energies, inelastic energies and field variables. The material properties for the calculation are passed to the subroutine through the props array, this array should be defined in the input file of the analysis. The strain is passed to the subroutine as strain increment and together with stresses have a direction corresponding to the local material orientation. The orientation can be defined in the input file and by default, it corresponds to the global axes of the analysis.

The state variables internally denoted as SDVs, act as information storage for a specific element and at the same time they can be used as output for the end user. Element deletion can be controlled by one of the variables and the SDV number for deletion control should be specified in the input file.

The energy output is beneficial as large sudden energy gain or loss in the system can signify a problem with the analysis. Most often such problem is caused by mesh distortion, interface elements, damping or drift of the solution and in order to let Abaqus calculate energies correctly, total internal energy and inelastic internal energy values have to be outputted from the VUMAT.

The other output possibilities such as temperature values, stretch tensors, deformation gradients and field variables fall outside the scope of this thesis and will not be used in the definition of the material model. For a full description of the input and output of the VUMAT subroutines the reader is advised to refer Abaqus Manual [15] Chapter 1.2.22.
As for the control of the time incrementation, there is no possibility to do it directly in VUMAT, however, Abaqus uses the subroutine indirectly to determine the stable time. It is done by dividing the analysis into two phases: phase one when the step time is zero and phase two when \( t > 0 \). During the time zero dummy values for strain \( \Delta \varepsilon_{1,2,3} = -0.001 \) and \( \Delta \varepsilon_{12,23,31} = 0.001 \) are passed into the subroutine. The subroutine is expected to output stresses as if the material would be perfectly elastic. Abaqus uses these stresses to assemble internal stiffness matrix for calculation of stable time, internal wave speed, eigenmodes etc. During the phase two user subroutine is expected to simulate the material as it was programmed for.

On the other hand, when using VUMAT for a material, some of the analysis functions stop working, one of these is the \( \beta \) component of Rayleigh damping. It is not applied and if such damping is wanted during analysis, it has to be calculated manually inside the subroutine. But this poses a problem, \( \beta \) damping heavily effects stable time increment, therefore special measures have to be taken to counteract and correct the stable time. These measures will be described in subsection 3.5.2 Stable Time.

Although Abaqus provides extensive flexibility and freedom to model custom material as the user wants, there are still shortcomings and behaviors that are not possible to model within VUMAT interface. E.g. in user material subroutine environment it is impossible to know adjacent elements or integration points, thus such algorithms as the one described by Clemente et al. [12], the local crack-tracking algorithm, are impossible to implement without developing a custom element.

2.6. Summary

In this Chapter, basic theoretical considerations were discussed. It was determined that the explicit algorithm is superior to implicit when the complex, highly plastic and rapidly loaded models have to be analyzed, yet, the opposite is true when static or quasi-static analysis are required. Furthermore, a brief overview of Plasticity was presented including the most common yield criteria such as Tresca, Mohr-Coulomb, von Mises and Drucker-Prager. The relation between the deviatoric stress tensor and plastic strain increment were defined together with the types of material hardening.

Followed by an overview of Continuum damage mechanics. Where, the basic concepts of damage variable, effective stress and strain-equivalence have been discussed. Further, available material models for masonry were reviewed. Two main distinctive works were identified, namely the work Lourenco [56] and the work of Pela [75]. Even though Pela used an interesting approach in modeling orthotropic behavior the results were not far from what was obtained by Lourenco. However, the damage model used by Pela is beneficial for the accuracy of cyclic analysis, although some improvements to the damage model could be made. These additional considerations will be further discussed in section 3.4.

And last but not least, the Abaqus user subroutine interface was analyzed and the structure of the analysis procedure was presented. It was determined that the user subroutine must output only new stress vectors each increment in order for the analysis to run. Of course, for better integration and clearer output more considerations are required.
Description of the material model

This model uses a simplified approach for 3d space. It means that material strengths in out of plane direction are ignored. The model exhibits plastic behavior in plane and elastic out of the plane. Such way of describing the material allows a fast and easy preparation of structural model for Finite Element Analysis, while in turn providing an easier calibration of the material properties and a faster calculation comparing it to full 3d orthotropic plasticity.

This chapter will present formulations used in describing the material model. The formulation includes orthotropic stiffness matrix, yield and flow surfaces, damage in elasticity and strengths and additional considerations for better integration with Abaqus software. For the simplified scheme of the material subroutine procedure see Figure 3.1.

Figure 3.1: Scheme of material subroutine procedure.
3.1. Stiffness matrix

The stiffness matrix (\(K\)) is an inverse of compliance tensor (\(D\)) which is derived for an element from an elastic stress–strain relationship (\(\varepsilon = D\sigma\)). For a purely anisotropic material, such compliance matrix consists of 36 elastic constants. Because of the symmetry of constitutive tensor, only 21 of 36 constants are independent.

\[
D = \begin{bmatrix}
D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\
D_{22} & D_{23} & 0 & 0 & 0 & 0 \\
D_{33} & 0 & 0 & 0 & 0 & 0 \\
D_{44} & 0 & 0 & 0 & 0 & 0 \\
D_{55} & 0 & 0 & 0 & 0 & 0 \\
D_{66} & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\] (3.1)

However, the material would require 21 independent tests for all of its parameters to be determined, which is impractical. Nevertheless, the amount of independent constant can be reduced if the material in question has planes of symmetry. The plane of symmetry is defined as a plane through a material that divides the material into two parts in which the elastic constants are a mirror image of each other. A material is called orthotropic if at every point it has 3 planes of symmetry. The intersections of these planes are called principle axes of orthotropy.

In case a material is orthotropic the amount of independent elasticity constants is reduced to 9 (see eq. 3.2). These constants can be expressed in 12 engineering terms, i.e. 3 Young moduli \(E_1, E_2, E_3\), 6 Poisson’s ratios \(\nu_{12}, \nu_{13}, \nu_{21}, \nu_{23}, \nu_{31}, \nu_{32}\) and 3 shear moduli \(G_{12}, G_{13}, G_{23}\) (see eq. 3.3)

\[
D = \begin{bmatrix}
1 & -\nu_{12} & -\nu_{13} & 0 & 0 & 0 \\
\frac{1}{E_1} & \frac{\nu_{12}}{E_2} & \frac{\nu_{13}}{E_3} & 0 & 0 & 0 \\
-\nu_{21} & 1 & -\nu_{23} & 0 & 0 & 0 \\
\frac{1}{E_1} & \frac{\nu_{21}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\
-\nu_{31} & -\nu_{32} & 1 & 0 & 0 & 0 \\
\frac{1}{G_{12}} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (3.2)

\[
D = \begin{bmatrix}
1 & -\nu_{12} & 0 \\
\frac{1}{E_1} & \frac{\nu_{12}}{E_2} & 0 \\
-\nu_{21} & 1 & 0 \\
\frac{1}{E_1} & \frac{\nu_{21}}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}} \\
\end{bmatrix}
\] (3.3)

where due to the symmetry of the matrix, the following holds true \(-\nu_{21}/E_2 = -\nu_{12}/E_1, -\nu_{31}/E_3 = -\nu_{13}/E_1\) and \(-\nu_{23}/E_2 = -\nu_{32}/E_3\). This way number of engineering constants is reduced to the number of elasticity constants.

Due to the fact the three-dimensional behavior is poorly documented and additionally that the masonry in practice is usually appears as walls, out of plane characteristic of such material can be ignored. Therefore, a 2d case of compliance matrix can be assembled by setting \(\sigma_3 = \sigma_5 = \sigma_6 = 0\):

\[
D = \begin{bmatrix}
1 & -\nu_{12} & 0 \\
\frac{1}{E_1} & \frac{\nu_{12}}{E_2} & 0 \\
-\nu_{21} & 1 & 0 \\
\frac{1}{E_1} & \frac{\nu_{21}}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}} \\
\end{bmatrix}
\] (3.4)
3.2. Failure envelope

A failure envelope describes a surface in a stress space. When stresses in the material reach this surface plastic yielding occurs. For masonry deriving such yield surface is a complex task. However, as described in section 2.4 there already are failure surfaces that were composed with masonry in mind. For this model the yield surface that was proposed by Lorenço et al. [59] was chosen. The chosen surface is comparatively well documented and has good agreement with experimental results. Furthermore, the author of the surface, proposed various methods to calibrate the surface with the experimental test results.

The surface is described by two yield criteria. These criteria simulate failure of a brick and mortar separately. For the description of a mortar failure, Rankine-like surface is used:

\[ f(\sigma) = \frac{\sigma_x - f_{tx}}{2} + \sqrt{\left(\frac{\sigma_y - f_{ty}}{2}\right)^2 + a\tau_{xy}^2} \]  

(3.9)
and for a brick – Hill-like:

\[ f(\sigma) = \sqrt{\sigma_x \cdot \left( \frac{\beta \cdot \sigma_y}{2} + \frac{f_{cy}}{f_{cx}} \cdot \sigma_y \right) + \sigma_y \cdot \left( \frac{\beta \cdot \sigma_x}{2} + \frac{f_{cx}}{f_{cy}} \cdot \sigma_x \right) + \gamma \cdot \tau_{xy}^2} - \sqrt{f_{cx} \cdot f_{cy}} \]  

(3.10)

Where parameters \( \alpha, \beta, \gamma \) describe the shape of the composite yield surface. These parameters could be obtained by performing least squares fit method for the yield surfaces on to experimental data or calculating them from following tests:

Figure 3.2: Natural tests to calibrate the composite model (from [59]): uniaxial tension (a) parallel to the bed joints and (b) normal to the bed joints; uniaxial compression (c) parallel to the bed joints and (d) normal to the bed joints

Figure 3.3: Possible non-standard tests to calibrate the composite model and calculate (a) parameter \( \alpha \), (b) parameter \( \beta \) and (c) parameter \( \gamma \) (from [59]).

Figure 3.4: Typical position of the natural tests and proposed non-standard tests with respect to composite model (from [59]).
With these tests, the model parameter $\alpha$, $\beta$ and $\gamma$, read:

$$\alpha = \frac{1}{9} \left( 1 + 4 \cdot \frac{f_{tx}}{f_{u}} \right) \left( 1 + 4 \cdot \frac{f_{ty}}{f_{a}} \right)$$  \hspace{1cm} (3.11)$$

$$\beta = \left( \frac{1}{f_{xx}^2} - \frac{1}{f_{mx}} - \frac{1}{f_{my}} \right) f_{mx} f_{my}$$  \hspace{1cm} (3.12)$$

$$\gamma = \left( \frac{16}{f_{yx}^2} - 9 \left( \frac{1}{f_{mx}} - \frac{\beta}{f_{mx} f_{my}} - \frac{1}{f_{my}^2} \right) \right) f_{mx} f_{my}.$$  \hspace{1cm} (3.13)$$

Furthermore, if the specific tests are unavailable an approximation could be made by defining them from the constituents of the masonry. Parameter $\alpha$ is defined by the shear strengths and tensile strengths of a joint:

$$\alpha = \frac{f_{tx} f_{ty}}{f_{u}};$$  \hspace{1cm} (3.14)$$

parameter $\gamma$ can be defined by the shear and compression strengths of a unit, the same way as $\alpha$ was defined:

$$\gamma = \frac{f_{tx} f_{cy}}{f_{u}};$$  \hspace{1cm} (3.15)$$

while for parameter $\beta$ a biaxial compression test must be performed in order to determine the strengths of the units of masonry (where $\sigma_x = \sigma_y = -f_{dy}$):

$$\beta = \left( \frac{1}{f_{xs}^2} - \frac{1}{f_{xx}^2} - \frac{1}{f_{yx}^2} \right) f_{cx} f_{cy}. $$  \hspace{1cm} (3.16)$$

Alternatively, the factor $\alpha$ is also related to the friction angle ($\varphi$) (see eq. 3.17). The friction angle is linear and true to its formulation in an isotropic pressure situation, however, the behavior is different in case the stresses are not isotropic, e.g. vertical load is applied onto the specimen while no horizontal pressure is present. In such cases, the friction angle starts at infinity when stress is zero and approaches zero when stress approaches infinity.

$$\alpha = \frac{1}{\mu^2};$$  \hspace{1cm} (3.17)$$

where, $\mu$ is the tangent of the friction angle ($\mu = \tan \varphi$).

By fitting the surfaces to the experimental data from Lurati et al. [60], the following shape can be obtained:

![Figure 3.5: Composite surface render in Wolfram Mathematica. It is presented in Cauchy stress space. Where the horizontal plane represents $\sigma_x$ and $\sigma_y$, while vertical $-$ $\tau_{xy}$. The fit was made according to test results from Lurati et al. [60]](image-url)
3.3. Stress-Strain relation

In FEM analyses the yielding is initiated when the trial stress \( \sigma_{\text{trial}} \) overpasses the yield surface \( f(\sigma) = 0 \). Theoretically, the stresses cannot exceed the yield surfaces therefore a corrector stress \( \sigma_{\text{corr}} \) has to be determined in order to return the trial stress back to the surface. The plastic corrector \( \sigma_{\text{corr}} = d\lambda \frac{\partial f}{\partial \sigma} \) was derived in section 2.2.2. The magnitude \( (d\lambda) \) of the plastic corrector is determined by the yield function \( f(\sigma) \), the direction – by flow function \( g(\sigma) \). When the flow function is equal to the yield function \( f(\sigma) = g(\sigma) \), the stress–strain relation is called associated, otherwise – non-associated.

The Hill-type part of the yield surface will be treated as associated flow and Rankine-type – as non-associated.

The flow function for Rankine-like criterion can be written as follows:

\[
g(\sigma) = \frac{\sigma_x - f_{1x} + \sigma_y - f_{1y}}{2} + \sqrt{\left(\frac{(\sigma_x - f_{1x}) - (\sigma_y - f_{1y})}{2}\right)^2 + \xi \tau_{2y}^2};
\]

where \( \xi \) represents parameter inversely related to the square of the tangent of the dilation angle \( (\psi) \), therefore:

\[
\xi = \frac{1}{\tan^2 \psi}.
\]

The dilation angle is defined as the angle between isotropic pressure line and the flow function. This angle determines the volume change of an element during yielding. If dilation angle is positive, at yielding element will expand, if negative – it will contract, but if the dilation angle is zero the element will retain its volume. The change of volume can be described as follows:

\[
\frac{\Delta V}{V} = (1 + \varepsilon_x)(1 + \varepsilon_y)(1 + \varepsilon_z) - 1 \approx \varepsilon_x + \varepsilon_y + \varepsilon_z;
\]

and the dilation angle relation to the change of volume can be expressed as:

\[
\frac{dV}{V} = d(\varepsilon_x + \varepsilon_y + \varepsilon_z) = d\lambda \tan \psi;
\]

therefore,

\[
\frac{dV}{V d\lambda} = \tan \psi.
\]

However, as in the case with the angle of friction, the angle of dilation in uniaxial loading situation changes from infinity to zero. As it can be observed in the figure 3.6 that in the case of yielding from point B, the dilation angle is much greater than when yielding occurs from point A.

Figure 3.6: Yield and flow functions together with dilation angles at \( \sigma_y = 0 \).

This poses a problem in cyclic analyses where only shear load and uniaxial compression is present. In such case, the analyzed structure would extensively dilate introducing unrealistic lift and in case of...
3.3. Stress-Strain relation

confinement – access sive internal pressure in the structure. To circumvent this it is possible to set the dil ation angle to zero by making the flow surface flat in $\sigma_x-\sigma_y$ plane at the start of the analysis or at a later point when some volumetric increase is accumulated. There is also a possibility to decrease the dilation angle gradually with the accumulated equivalent plastic strain. In the later case, it would be best to relate the dilation of an element to the maximal possible volumetric change in terms of equivalent plastic strain. Such relation can be expressed as:

$$\frac{\Delta V}{V}(\varepsilon_{pl}) = \frac{2 \arctan \left( \frac{\pi \varepsilon_{pl}^e}{2 d_{max}} \right)}{\pi d_{max}}$$

(3.23)

where $d_{max}$ represents maximal possible volume increase and $\varepsilon_{pl}^e$ equivalent plastic strain in the element.

By differentiating eq. 3.23 in respect to $\varepsilon_{pl}$, the relation between tangent of the dilation angle and equivalent plastic strain can be derived:

$$\tan \psi = \frac{4d_{max}^2}{\pi^2 \varepsilon_{pl, eq}^2 + 4d_{max}^2}$$

(3.24)

by substituting the obtained relationship into eq. 3.19 and then into eq. 3.18 the plastic strain dependent flow function is obtained. However, it has to be noted that this relation is purely phenomenological as exact dilatory behavior of masonry is poorly documented and furthermore due to the shape of the flow function the dilation angle in orthotropic stress situation will always be different than the set value, and that the imposed maximal volume change constraint will only be valid in isotropic loading conditions.

As discussed in the Section 2.2.2, if the flow surfaces have discontinuities a subdifferential zone of flow directions have to be defined. The most simplistic approach would be is to iterate the return stress until the wanted solution is obtained. However, such iteration would heavily slow down the explicit analysis which is not a wanted side effect. Therefore a faster, but potentially less accurate method has to be derived.

First, yield condition is checked for both surfaces (Figure 3.8). If both of these surfaces are yielding it means that the predictor stress is in a stress space where advanced return algorithm has to be initiated. The algorithm determines if the predictor stress is in the subdifferential zone by checking the yield conditions once more, but this time using the returned stresses of both yielding surfaces determined in first conditional check. In such case, only the other yield surface has to be checked for the obtained values of the returned stresses (e.g. if Rankine-like surface returned to stress point A then only Hill-like surface has to be checked at that point A and vice versa). If after this check both surfaces are yielding,
then the initial predictor stress lies in the subdifferential stress zone. Otherwise, a returned stress is assumed to the one returned by a yield surface which the second check resulted in a not yielding state (see Figure 3.9).

In case the initial predictor stress falls into the subdifferential stress zone, then the stresses have to be returned to the intersection curve of both yield surfaces so that the return direction is perpendicular to the curve in strain space. However, finding a return to the intersection of two implicit surfaces is an extremely computationally taxing procedure, therefore an approximation of the intersection point is used. Such approximation involves defining a plane that is tangent to the Hill-like surface at a point of first stress return (point B) and a line passing through points A and B*. The intersection of the plane and the line would estimate the intersection of the yield surfaces. This approach is only accurate if the strain and stress increments are infinitesimal. In that case, both Hill-like and Rankine-like surfaces act as flat planes and the error from the approach is minimal.

In figures 3.8 and 3.9 the stresses are presented in $\zeta - \chi$ plane. It is a fictitious plane that goes through the points AOB. The rest of the vectors and the points presented in the schemes are only the projections to the plane. The scale of the plane is adjusted so that the stress return would appear to be perpendicular to the yield or flow surfaces. This is done in order to simplify the description and presentation of the return algorithm.

Figure 3.8: The stress return if the predictor stress originates in a subdifferential stress zone. Plotted in $\zeta - \chi$ stress plane, vectors $N_1$ and $N_2$ represents the projections of the subdifferential zone boundaries on to the plane.

Figure 3.9: The stress return if the predictor stress does not originate in a subdifferential stress zone. Plotted in $\zeta - \chi$ stress plane, vectors $N_1$ and $N_2$ represents the projections of the subdifferential zone boundaries on to the plane.
Another region when flow is not defined by the general algorithm is at the apex of Rankine-like surface cone when the shear stress is zero. There, two issues have to be taken into account: stress at the subdifferential zone and stresses when the numerator of the partial derivative of the flow function with respect to stress tensor is zero (see Figure 3.10). Let’s call the latter case “Line of undefined flow”. If the trial stress point does not fall on to the line but when returned has at least one of its stress components higher than the corresponding yield limit the returned stress has to be corrected to be on the apex of the cone. Alternatively, if the trial stress point does fall on the line of undefined flow it has to be immediately returned to the apex of the cone as otherwise there are no solutions. Therefore, it can be concluded that in this situation the subdifferential zone can be defined by the outcome of the first return and the line of undefined flow by the numerator of the flow function.

![Figure 3.10](image-url): The stress return if the predictor stress does (Point O) and does not (point O’) originate in a subdifferential stress zone. Plotted in stress yield/flow plane, when $\tau_{12} = 0$. Vectors $N_1$ and $N_2$ represents the projections of the subdifferential zone boundaries on to the plane and point B is the apex of Rankine-like yield/flow surface cone.

### 3.4. Damage

The damage in the model is described by means of reduction in stiffness and strengths of the material. The strengths reduction (softening) is subdivided into two regions: tension and compression. For former fracture energy based linear softening (Figure 3.11a) is adopted. Because the behavior is fracture energy based it is mesh-insensitive. The evolution of softening curve can be expressed as:

$$f_{yt1}(\varepsilon) = f_{yt0} - \frac{f_{yt0} h (\varepsilon_{max} - \varepsilon_y)}{2G_f}$$

(3.25)

where $f_{yt0}$ is initial tensile strengths of the material, $\varepsilon_{max}$ is the maximal total strain in the analysis, $G_f$ is the fracture energy, $h$ is the element height and $\varepsilon_y$ is strain at initial yield expressed as $\varepsilon_y = f_{yt0} / E$ with $E$ the stiffness of the material.

Material tensile strengths together with softening behavior are uncoupled in both material directions. This means that when the material is subjected to tension along one of its axes the strengths do not get reduced along the other one. Furthermore, it is important to note that the lowest possible strengths are set to 1% of the initial material strengths in that direction. This is done to ensure numerical stability during the analysis.

For compression (see Figure 3.11b) a mesh sensitive softening/hardening curve is used, the curve does not have a predefined shape and it is specified in terms of equivalent plastic strain and compressive strengths as a tabular input by the end user. Such approach was chosen in order to not restrict the definition of different compressive behaviors as strain and compressive strengths relationship vary extensively from one masonry type to another. Unlike tension softening, compression hardening is coupled in material axes through equivalent plastic strain ($f_c(\varepsilon_{eq,c})$), where compressive equivalent plastic...
strain is equal to the sum of scalar plastic multipliers \(d \lambda_h\) of corrector stress increments calculated due to the yielding in Hill-like surface:

\[
e_{eq,c} = \sum d e_{eq,c} = \sum d \lambda_h
\]  

(3.26)

Figure 3.11: Tension (a) and compression (b) softening/hardening curves.

Due to homogenized approach, the shear softening behavior is directly related to the tensile strengths of the material. Therefore, yielding due to shear on Rankine-like yield surface will reduce the tensile strengths of the material. However, the tensile softening is controlled by total strain, therefore, it becomes difficult to introduce shear effects into the softening behavior. Two regions need to be declared in the Cauchy stress space, namely when the material is in compression and a region when at least one of the material axes are in tension. If yielding occurs in the first region the tensile strengths are controlled by equivalent plastic strain (see eq. 3.27), if in the second – by the combination of plastic shear and total axial strain.

\[
f_{yt2}(e_{eq,r}) = f_{yt0} \frac{f_{yt0}^2 h + 2EG_f - E f_{yt0} h e_{eq,r}}{f_{yt0}^2 h + 2EG_f}
\]  

(3.27)

Furthermore, the softening accumulated in compression causes additional softening in tension. E.g. during compression loading, compressive strengths got softened by 10%, the tensile strengths will get reduced by the same amount of percent as well. Additionally, because the softening behavior in compression is coupled in material axes, the reduction in tensile strengths is also coupled. The full tensile strengths relationship for a given material direction can be expressed as:

\[
f_{yt}(e_{eq,r}, e_s, r, e) = r \cdot (f_{yt1}(e) - (2f_{yt0} - f_{yt2}(e_{eq,r}) - f_{yt2}(e_{eq,r})))
\]  

(3.28)

where \(r\) is the ratio between current compressive strengths in the softening regime and initial ultimate compressive strengths (\(r = f_c/f_{uc0}\)); \(e_{s,r}\) is plastic shear strain accumulated when stresses at least in one of the material axes are in tension.

As for the stiffness damage, the model exhibits a reduction of stiffness only when it is subjected to tension. Variable \(d\) controls the reduction in elasticity (\(E_d = (1 - d)E_0\)). The damage serves purely a crack closure function, therefore it is based on total strain, it is uncoupled in material directions and is calculated with a rather simple expression:

\[
d = 1 - \frac{f_y}{E_0 \varepsilon_{t,max}}
\]  

(3.29)
where $\varepsilon_{t,max}$ is maximal total tensile strain during the analysis and $f_y$ is current yield strengths in the element.

The damage parameters are applied to the diagonal and off-diagonal terms of the stiffness matrix. The only exceptions are the stiffness parameters representing out of plane elasticity. The off-diagonal terms get full damage as soon at least one of the material directions get damaged. The damage recovers fully if the material becomes compressed. The damaged stiffness matrix can be represented as follows:

$$
K_d = \begin{bmatrix}
(1 - d_x)E_1 \Delta & \kappa E_2 \Delta & 0 & 0 & 0 & 0 \\
\kappa E_2 \Delta & (1 - d_y)E_2 \Delta & 0 & 0 & 0 & 0 \\
0 & 0 & E_3 & 0 & 0 & 0 \\
0 & 0 & 0 & \sqrt{\kappa} 2G_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 2G_{23} & 0 \\
0 & 0 & 0 & 0 & 0 & 2G_{31}
\end{bmatrix}
$$

where $\kappa = (1 - d_x)(1 - d_y)$. The brackets $[\cdot]$ represent the floor function.

During the cyclic loading, the damaged stiffness does not transfer from compression to tension (Full recovery). During the compression, there is no damage to the elasticity of the material. Though the softening that occurs during compression get proportionally transferred to all tensile components.

This model uses incremental return approach when the stresses are returned to the yield surface of yield initiation and only then the surface is softened (contracted) or hardened (expanded). This type of approach creates stress delay in terms of strain increment. The resultant stress is delayed behind actual stress by the magnitude of one strain increment. If time step is infinitesimal, the strain increment is infinitesimal as well, thus the deviation becomes negligible.

Figure 3.12: Stress – Strain curve. Damage to stiffness.
3.5. Additional considerations

3.5.1. Damping

If the damping of the material is desired it has to be explicitly implemented into the user subroutine. In Abaqus, generally, mass and stiffness proportional damping is used. Such damping is also known as Rayleigh damping and it consists of a mass proportional term \( \alpha \) and a tangent stiffness proportional term \( \beta \) (see eq. 3.31). The mass proportional term can be specified in Abaqus and it will be applied to user subroutine. However, even though stiffness proportional term can be specified it is not applied for user defined materials. Therefore, \( \beta \) damping factor has to be implemented in user subroutine.

\[
\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K}
\] (3.31)

Numerically, \( \beta \) part of Rayleigh damping is implemented by multiplying stiffness proportional damping factor \( \beta \) by a stress rate and adding the result to the stress increment. Furthermore, this additional stress has to be removed at the beginning of next increment.

\[
\sigma_{\text{new}} = \sigma + \beta \frac{d\sigma}{dt}
\] (3.32)
3.5.2. Stable time estimation

During the analysis at \( t = 0 \) Abaqus determines the stable time increment. The calculations of it are based on the stress output from the subroutine. However, the stress output and calculated stable time increment relationship is quadratic, this means that added Beta damping to an element does not produce sufficient increment reduction. Therefore, the stresses have to be altered by a factor to represent the true influence of beta damping to the stable time.

The stable time increment condition is as follows:

\[
\Delta t \leq \frac{2}{\omega_{\text{max}}} \left( \sqrt{1 + \xi^2} - \xi \right) \tag{3.33}
\]

Where,
- \( \omega_{\text{max}} \) – Highest eigen-frequency of the element
- \( \xi \) – Fraction of critical damping, can be also expressed as \( \xi = \beta \omega / 2 \)

Highest eigenfrequency of the element can be calculated by determining eigenmodes of an element, or it can be determined by the following relationship:

\[
\omega_{\text{max}} = \frac{2c_d}{l_{\text{e,min}}} \tag{3.34}
\]

where \( l_{\text{e,min}} \) is the smallest characteristic length in the element and \( c_d \) is dilation wave speed in the material. The \( c_d \) can be expressed through the relation of Lamé's constants (\( \lambda \) and \( \mu \)) and material density \( \rho \) as follows:

\[
c_d = \sqrt{\frac{\lambda + 2\mu}{\rho}} \tag{3.35}
\]

The Lamé's constants can be found:

\[
\mu = \frac{1}{2} \frac{\Delta S : \Delta e}{\Delta e : \Delta e} \tag{3.36}
\]

\[
\lambda = \frac{2}{3} \mu \tag{3.37}
\]

where \( \Delta S \) is the deviatoric stress increment, \( \Delta e \) is the deviatoric strain increment and \( \tilde{K} \) is the effective bulk modulus expressed as:

\[
\tilde{K} = \frac{-\Delta p}{\Delta e_{\text{vol}}} \tag{3.38}
\]

where \( \Delta p \) is increment of equivalent pressure stress defined as \( p = -\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) \), \( \Delta e_{\text{vol}} \) is the increment of the volumetric strain.

If we assume that the strain increments \( \Delta \varepsilon \) is:

\[
\Delta \varepsilon = \begin{bmatrix}
-0.001 \\
0.001 \\
0.001 \\
-0.001 \\
0.001 \\
0.001 \\
0.001
\end{bmatrix} \tag{3.39}
\]

Then the stable time increment can be obtained:

\[
t_{\text{min}} = \frac{1.5 L_e \left( \sqrt{1 + \xi^2} - \xi \right)}{\sqrt{\frac{0.25 \Delta (E_1 + 2E_2) + 0.25 E_5 + G_{12} + G_{23} + G_{31}}{\rho}}} \tag{3.40}
\]

where \( \Delta = 1/(1 - \nu_{12} \nu_{21}) \).
3.5.3. Element deletion
Element deletion is implemented by flagging the element for deletion when the tabular input of compression hardening/softening curve runs out of values. Abaqus deletes the element by setting the stress and strain increments in the element equal to zero. By default when an element is deleted mesh does not contact with itself, however, it is possible to reinitialize a contact after the deletion using specific modeling strategies. For further reading on contact re-initiation after the deletion, the reader is referenced to Abaqus Analysis User’s Guide, Section 36.4.1 [14].

3.6. Summary
The model consists of three-dimensional stiffness matrix that is compatible with two-dimensional plasticity model. The failure is initiated through one of the two yield surfaces, namely Rankine like and Hill like. The plastic flow is non-associated for the former and associated for the latter. For the Rankine like non-associated flow, three types of dilatary behavior were considered: always constant; constant until prescribed volume increase is reached and then dilation is disabled; dilation angle decreases gradually with plastic strain. The intersections of the yield surfaces and the apex of the Rankine-like surface cone cause a problematic stress return in the sub-differential zone. In this chapter, the underlying algorithms used to mitigate this issue were discussed.

Furthermore, the model exhibits tensile strengths softening based on total strain when the material is subjected to tension and on plastic strain when it is subjected to compression and shear. Compressive behavior supports both hardening and softening of the material strengths and the softening percentage is transferred to the tensile strengths. Additionally, in tension, crack closure is present while in compression plastic deformations are accumulated. The softening and hardening is applied in increments where the material strengths in current increment depend on the plastic strains and total strains generated in the previous one.

Finally, additional measures were taken to ensure stable and smooth analysis, stiffness proportional therm of Rayleigh damping was incorporated together with stable time estimation based on the effects of the added damping. To even further increase the stability of an explicit analysis an option to remove crushed element was considered.
Material model verification

For the model, verification tests several different kinds of cube setups were used. The tests were subdivided into two categories tension-compression and shear. Each category contains tests subjected to monotonic loading and cyclic loading. It is so to show that the model functions as expected in all loading situations.

4.1. Cube Setups

For tension–compression tests the size of the cube is $1 \times 1 \times 1 \text{m}$ and it is composed of 64 single integration elements is used. It results in the $0.25 \times 0.25 \times 0.25 \text{m}$ size of each element. The second order accuracy is enabled in the settings of Abaqus. The cube is fixed at the bottom plane in the vertical direction. Above the cube, there is a reference point tied with an equation to the top surface of the cube. On the reference point, depending on the nature of the test, either upwards or downwards displacement is applied. The analyses themselves were performed with the double precision setting.

![Cube set-up for tension-compression tests.](image)

The cube for the shear test has the same dimensions as the cube for compression/tension tests, but it is only composed of one element. It was done like this due to the difficulty to obtain pure shear without over-constraining the cube.

There is a reference point tied with an equation to the top right edge of the cube. On the reference point, horizontal displacement is applied. The cube is restrained in the vertical direction at the bottom and in horizontal – on bottom right edge. Out of plane displacements were restricted in order to obtain a more stable output. The whole cube was exposed to isotropic pressure. The analyses themselves were performed with the double precision setting.
Note that this way of modeling induces extensive hourglass effect on the element. However, because the element has only one integration point, hourglassing does not have a significant effect on the results of analyses.

![Cube set-up for shear tests.](image)

Figure 4.2: Cube set-up for shear tests.

In the tests material properties described in the table 4.1 are used. In some tests, some of these properties will be alternated in order to better verify the model, but in that case changing values will be explicitly stated. Note that the properties used are not usually found in real masonry specimens, such properties were chosen in order to easier check and display the results.

| Table 4.1: Material properties used through out the tests. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Elastic properties                      |                   |                   |                   |                   |                  |
| $10000 \cdot 10^6$ | $5000 \cdot 10^6$ | $10000 \cdot 10^6$ | 0.2 | $3000 \cdot 10^6$ |                  |

| Inelastic properties                      |                   |                   |                   |                   |
| $f_{tx}$ [MPa]  | $f_{ty}$ [MPa]  | $G_{fx}$ [Nm/m]  | $G_{fy}$ [Nm/m]  | $\alpha$ |                   |
| $1 \cdot 10^6$ | $0.5 \cdot 10^6$ | $15$  | $15$  | 1.56 |                   |

| $f_{cx}$ [MPa]  | $f_{cy}$ [MPa]  | $\beta$ | $\gamma$ | Soft. Type. |                  |
| $10 \cdot 10^6$ | $5 \cdot 10^6$ | $-1$  | $3$ | hyp. $f_{cc} = 30$ |                  |

### 4.2. Uniaxial tension

In this test, the tensile behavior of the model will be examined. Initiation and softening of yield surface will be tested with various different fracture energies. The results will be verified by comparing them to hand calculations. Furthermore, plasticity localization will be examined as well as high strain situations and problems that can occur.

The analysis was performed with four different fracture energies (in the direction of loading), namely: $G_f = 1.5Nm^{-1}$, $G_f = 5.0Nm^{-1}$, $G_f = 10.0Nm^{-1}$ and $G_f = 15.0Nm^{-1}$.

The observed behavior was as expected and it can be verified by calculating the ultimate tensile strain $\varepsilon_u$ and comparing it to obtained results:

$$\varepsilon_u = \frac{F_y}{E} + 2 \frac{G_f}{hF_y}.$$  \hspace{1cm} (4.1)

This yields strains $\varepsilon_u = 0.124\%$, $\varepsilon_u = 0.18\%$, $\varepsilon_u = 0.26\%$ and $\varepsilon_u = 0.34\%$, respectively. It can be observed from figure 4.3 that the values match the results.
Even though the smeared crack approach is used, the cracks localize (Figure 4.4). This behavior is present because unlike in static implicit analysis in dynamic explicit analysis displacements and forces propagate through the material at a specific speed. Because of this propagation in each increment, the stresses do not increase uniformly throughout the mesh, therefore there will be places in the model where stresses reach the yield values first. As soon as that happens, at that location, tensile softening of the material starts. Because of the softening, the affected elements will act as the weakest link in the model and all of the stresses will redistribute to them, therefore a localized crack will appear.

Sometimes in analyses, a big crack openings can be observed. It is important for a material model to allow the separation of these cracks without additional excessive forces. When analyzing such situation with default Abaqus settings large ever increasing resistance forces can be observed with high strains (see Figure 4.5). It is due to the Linear viscosity setting that tries to suppress such deformations. It works by applying additional forces to the element when the volumetric strain rate is high. It is most apparent when the element is damaged and it’s volume increases quite rapidly. Nevertheless, it can be avoided by setting linear viscosity to 0 (see [14], Section 27.1.4) but in that case stiffness damping should be applied as after appearance of each crack the structure will have high-frequency vibrations (Figure 4.6).
Figure 4.5: Force – Displacement curve. Tensile softening behavior in high strains.

Figure 4.6: Oscillations of elastic element.
4.3. Uniaxial compression

In this test, the compressive behavior of the model will be examined. Initiation and softening of yield surface will be tested. The results will be verified by comparing them to hand calculations. Furthermore, damage localization will be examined as well as high strain situations.

As an input for the compression hardening/softening behavior, Thornfeldt softening curve for concrete was used.

\[
f(\varepsilon) = f_u \frac{\varepsilon}{\varepsilon_u} \left( \frac{n}{n - \left(1 - \frac{n}{\varepsilon_u} \right)^{nk}} \right)
\]  
\[
\varepsilon_u = f_u \frac{n}{E \cdot (n - 1)};
\]
\[
k = \begin{cases} 
1, & \text{if } \varepsilon_u < \varepsilon < 0 \\
0.67 + \frac{f_{cc}}{\varepsilon_u}, & \text{if } \varepsilon \leq \varepsilon_u
\end{cases}
\]
\[
n = 0.8 + \frac{f_{cc}}{17}.
\]

Where,
- \(f_u\) – Ultimate compressive strength of the material
- \(\varepsilon_u\) – Strain at ultimate strengths
- \(E\) – Elasticity modulus of the material
- \(n, k\) – Shape parameters
- \(f_{cc}\) – Factor defining softening shape

This curve was generated for yield strength of \(F_u = 5 \cdot 10^6 Pa\) and a varied \(f_{cc}\) factor, namely – 15, 20, 30 (See Table 4.2).

Table 4.2: Properties used in different analyzes

<table>
<thead>
<tr>
<th>No.</th>
<th>(E_x)</th>
<th>(E_y)</th>
<th>(f_{ux})</th>
<th>(f_{uy})</th>
<th>(f_{cc})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 \cdot 10^6 Pa</td>
<td>5 \cdot 10^6 Pa</td>
<td>10 \cdot 10^6 Pa</td>
<td>5 \cdot 10^6 Pa</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>10 \cdot 10^6 Pa</td>
<td>5 \cdot 10^6 Pa</td>
<td>10 \cdot 10^6 Pa</td>
<td>5 \cdot 10^6 Pa</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>10 \cdot 10^6 Pa</td>
<td>5 \cdot 10^6 Pa</td>
<td>10 \cdot 10^6 Pa</td>
<td>5 \cdot 10^6 Pa</td>
<td>30</td>
</tr>
</tbody>
</table>

During the tests, downwards displacement of 17.5mm was applied to the cube. The hardening/softening behavior was as expected, but it must be noted that the plastic strain localization only happens when the transition from hardening to softening is sudden (\(f_{cc} = 30\)).

By comparing the output to the input provided, it can be seen that curves do not match (see Figure 4.9). There is ever increasing drift of stresses from the expected result. This is because during analysis, the hardening/softening behavior for compression is being tracked using equivalent plastic strain and that the yield value of Rankine-like yield surface is not equal to the material strengths along each of the material directions. This means that a perfect equivalence in the stress–strain diagram is not achieved. It can be solved by formulating \(\dot{\varepsilon}_{eq} \neq \dot{\lambda}_q\), but due to the usual lack of the experimental results current used simplification is sufficiently accurate.

By plotting the difference ratio between equivalent plastic strain and the strain obtained from the analysis (see Figure 4.10) it can be seen that the equivalent plastic strain is always \(\sqrt{\frac{1}{2}}\) of the actual plastic strain. Therefore, one could adjust stress-strain curve depending on the type of analysis expected in order to minimize the deviation.
4. Material model verification

Figure 4.7: Stress – Strain curve. Compression hardening/softening behavior.

Figure 4.8: Force – Displacement curve. Compression hardening/softening behavior.
Figure 4.9: Stress – Strain curve. Comparison of compression softening to expected results.

Figure 4.10: Equivalent plastic strain versus plastic strain in loading direction.
4.4. Shear

In this test, the shear behavior of the material model will be tested. Shear and tension relation together with dilation angle will be examined. For material properties, the Tensile fracture energy, and the dilation angle will be alternated between the analyses. The fracture energies are set to be the same both for $x$ and $y$ directions. The properties used are presented in table 4.4.

<table>
<thead>
<tr>
<th>No.</th>
<th>$p$ [Pa]</th>
<th>$G_{fr}$</th>
<th>$\tan \psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 000</td>
<td>20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>100 000</td>
<td>20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>200 000</td>
<td>20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>400 000</td>
<td>20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>1 000</td>
<td>15.0</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>1 000</td>
<td>10.0</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>1 000</td>
<td>20.0</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>1 000</td>
<td>20.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

From the Figure 4.11 it can be seen that increased pressure results in increased shear. The difference from shear at low pressure can be described by:

$$\tau = \mu p \quad (4.6)$$

Friction coefficient $\mu$ in this case is 0.8 (from Eq. 3.17)

![Figure 4.11: Stress – Strain curve. Influence of different isotropic pressures](image)

This is only valid in isotropic loading situation. If the loading is orthotropic the shear strain – shear stress relation is parabolic meaning that with the lower uniaxial compressive force the friction would approach infinity and with a higher – zero.

From above figure it can be seen that the softening of tension influences the softening of shear, the retaliation comes through plastic strain. The fracture energy in tension defines ultimate plastic strain and that, in turn, defines the Mode-II fracture energy.

Next, the dilation angle will be examined. The results of the analysis can be checked by:

$$\frac{dV}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = d\lambda \tan \Phi \quad (4.7)$$
4.5. Cyclic: Tension – Compression

In this test, cyclic behavior of the material will be analyzed. The effects of the change of the state (Tension/Compression) will be tested. The default material properties were used for the text, except for tensile fracture energy which is \( G_{ff} = 30 \, N/m \).

Cyclic vertical displacement (Figure 4.15) was applied to the test cube. The displacement was prescribed in such a way that specimen does not fully soften in the first cycle, but only in the second.

During tension, all of the plastic strain is converted to the damage of the stiffness. Thus, after unloading no plastic deformations are present. To the contrary, in compression, the plastic strain is accumulated and stiffness does not experience any damage (see Figure 4.16). When a specimen is compressed it’s stiffness fully recovers and when put back to tension the damage is resumed.

4.6. Cyclic: Shear

In this test, a cyclic shear behavior of the material model will be examined. For this analysis, the same element as in the previous shear analysis is used. However, the boundary conditions were changed. The horizontal displacement is applied on the top face of the element instead of the rightmost edge, as well as the bottom support was expanded from the edge of the element to the whole face. It was done so to prevent hourglassing in the element. Even though the hourglassing did not have a

\[
\tan \Phi = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{d\lambda}
\]  

(4.8)
4. Material model verification

Figure 4.13: Volume change in respect to plastic strain

Figure 4.14: Dilation angle in respect to plastic strain
4.6. Cyclic: Shear

**Figure 4.15:** Dilation angle in respect to plastic strain

**Figure 4.16:** Stress – Strain curve. Tension – Compression cycles.
negative effect on analysis results during the monotonic loading, during cyclic loading it had significant influence. These new boundary conditions over-constrain the element, therefore pure shear behavior is not obtained.

Harmonic cyclic shear loading was applied to the test elements (see Figure 4.18). The duration of the analyses was 10s in order to minimize the dynamic effects. During this period 10 full shear cycles were applied.

First, constant dilation was examined. From Figure 4.19 it can be seen that the dilation is much higher than the set dilatory angle of $\tan \psi = 1.0$. This is due to several reasons. First of all the element is over-constrained, therefore, stresses in the constrained material axes get accumulated over the time of the analysis (see Figure 4.20), furthermore, because of the shape of the flow function, the lower the uniaxial stress, the greater is the dilation angle. Such high dilation is unrealistic and it is never seen in experimental results.

From Figure 4.20 it can be also observed that even tough the dilation is ever increasing, the stress after reaching certain point starts softening. It is due to the fact that the confining stress grows so high that the material starts yielding and softening due to the stresses reaching the compression surface.

When examining the dynamic dilation behavior it can be seen that the volumetric strain change is significantly lower than in the previous analysis. This type of behavior would be expected from experimental results. However, in the material properties it was that the maximal possible volumetric change is 0.001 of total volume, but in the analysis 3 times the higher value was achieved (see Figure 4.21). This is, again, due to the shape of the flow function, does not matter how low the dilation angle will be set, at low values of uniaxial pressure, the dilation angle will still be high.

As in the previous analysis, the stress appears to be growing (see Figure 4.22) due to the confinement of the element. However, this time softening in compressive strengths was not present.

The final shear test is a control for the confining stresses. The dilation in this test was set to zero. Not only that but the flow surface was made to be completely flat. In this case, no dilation is experienced by the element (see Figure 4.23), though a sudden volume decrease can be observed at the start of
Figure 4.18: Input signal for the analyses.

Figure 4.19: Volume change per shear strain, for an element with constant dilation angle.
Figure 4.20: Force – Displacement curve. Shear force resistance in an element with a constant dilation.

Figure 4.21: Volume change per shear strain, for an element with decreasing dilation angle.
the analysis. This decrease is due to elastic volume change due to the instantaneous application of vertical pressure.

Because there is no dilation, there are no confining stresses. From Figure 4.24 it can be observed a continuous cycle of the shear force after first tensile strength softening, during the analysis. From the cyclic shear test, it was determined that the model behaves as expected.

4.7. Element deletion

In this test, the element deletion is analyzed. For the test, a masonry wall is placed between two platforms. The platforms are disconnected from the wall and a general contact is defined in the whole model with “Hard” normal behavior and a tangential behavior with the friction of 0.7. The top platform is loaded with imposed downward displacement. The bottom – clamped.

The input used for compressive behavior is depicted in Figure 4.26. The failure of an element is initiated by the last equivalent plastic strain entry in the input table. In such case output variable SDV29
During the analysis localization of crushing can be observed (See Figure 4.27b). Such localization introduces stress redistribution and propagation of crushed elements (See Figure 4.27c). After the whole line of elements is removed the top part of the wall drops onto the bottom part and closing the gap.

It must be noted that after element deletion Abaqus recalculates the contact surfaces and wall does not fall through. But for this to happen special measures has to be taken (Consult Abaqus User Guide 36.4.1 Modeling surface erosion).

By further examining the stress-strain relationship of a failed element, it can be seen that indeed the stress is set to zero as soon as the input is exceeded (Figure 4.28)
Figure 4.26: Stress – Strain relationship of input.

Figure 4.27: Equivalent compressive plastic strain throughout the analysis
4.8. Summary

In this chapter, tensile, compressive and shear behaviors subjected to both monotonic and cyclic loadings were tested. All of the obtained results were true to formulations described in Chapter 3. It has to be noted, that although the dynamic dilation angle worked as expected, it introduced some instabilities into the system. These instabilities can cause substantial problems in larger scale analyses, therefore it is recommended to only use this feature if confining pressure is always present in the affected elements or to avoid it at all. Alternatively, dilation can be set to zero immediately or after wanted volumetric change is reached as these settings provided much more stable results.

In the end, the capabilities of element deletion were presented. The contact between the elements can be re-initiated after elements are deleted and the appeared openings can be closed.

Figure 4.28: Stress – Strain relationship of a failed element.
Comparison to experiments

In this chapter, developed material model will be compared to the results of experimental tests and its ability to simulate the behavior of different masonry tests will be assessed. It is important to note that the masonry macro-models will always include some degree of approximation as all of the failure mechanisms of masonry cannot be simulated with the smeared out approach.

The goal of this chapter is to verify the material model in different loading situations. Namely, shear behavior with low pressure so the failure mechanisms prescribed by the Rankine-like yield surface can be tested; shear behavior with high pressure so the failure mechanisms prescribed by Hill-like surface can be tested and shear behavior under cyclic loading so the cyclic capabilities of the model can be examined. Therefore, walls from experiments carried out by Ganz and Thürlimann \[27, 28\] will be analyzed. The tests will be referenced as ETH Zurich tests throughout the chapter.

ETH Zurich wall tests were chosen mainly because of two reasons. First of all the researchers that carried out the tests had provided sufficient amount of material data with which an accurate failure envelope can be mapped in Cauchy stress space. Second, the underlying failure surface was compared by Lourenço \[56\] to specifically these test results, therefore, the tests act as a good control for the developed material model.

5.1. Determination of the material properties

To determine the material properties 12 wall panels reported by Ganz and Thürlimann \[27\] of dimensions $1200 \times 1200 \times 150 \text{[mm]}^3$ and denoted $K$ were considered. The panels were loaded proportionally in principle stress directions $\sigma_1$ and $\sigma_2$ along different rotations $\theta$ of the bed joint. To determine the inelastic properties of the masonry the test results of these wall panels were mapped in Cauchy stress space. For the mapping panels, $K5$ and $K9$ were disregarded. The reason is that the boundary conditions affected the failure of the panel $K5$ and panel $K9$ included reinforcement.

The mapping of the results was done using a least square fit method. To determine the ratio between experimental and predicted failure was calculated by determining the stress vector $\sqrt{\sigma_1^2 + \sigma_2^2 + \tau_{xy}^2}$ and predicted failure envelope intersection point. The comparisons between the fit and experimental results can be seen in table 5.1.

The fit resulted in the following material properties: $f_{tx} = 0.246 \text{[MPa]}$; $f_{ty} = 0.0 \text{[MPa]}$; $\alpha = 1.716$; $f_{cx} = 1.730 \text{[MPa]}$; $f_{cy} = 7.505 \text{[MPa]}$; $\beta = -1.171$; $\gamma = 1.0$, however for numeric analysis it is more stable to have some amount of strengths in the material. Therefore, for further calculations tensile strength in y direction will be used as follows: $f_{ty} = 0.05 \text{[MPa]}$.

The elastic properties were determined directly from test data and they are as follows: $E_x = 2460 \text{[MPa]}$; $E_y = 5460 \text{[MPa]}$; $\nu_{xy} = 0.18$; $G_{xy} = 1130 \text{[MPa]}$. However, data to determine some of the inelastic parameters was missing, therefore, these parameters: tensile fracture energy, compression softening curve and ultimate equivalent plastic strain for compression had to be assumed.

For the tensile softening behavior, the fracture energies of $20 \text{Nm/m}$ are used. For the compression regime, parabolic hardening/softening law are used (see Figure 5.1) in order to obtain a softening behavior that is not sensitive to the element size. However, as examined in the previous chapter the
Table 5.1: Comparison between plasticity model and the experimental results.

<table>
<thead>
<tr>
<th>Panel</th>
<th>Regime</th>
<th>$\sigma_1 / \sigma_2$</th>
<th>$\theta$</th>
<th>Plasticity model</th>
<th>Experimental results</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_x$ [MPa]</td>
<td>$\sigma_y$ [MPa]</td>
<td>$\tau_{xy}$ [MPa]</td>
</tr>
<tr>
<td>K1</td>
<td>Tens.</td>
<td>$-0.092$</td>
<td>$22.5$</td>
<td>$-0.069$</td>
<td>$-0.825$</td>
<td>$0.389$</td>
</tr>
<tr>
<td>K2</td>
<td>Tens.</td>
<td>$-0.050$</td>
<td>$22.5$</td>
<td>$-0.142$</td>
<td>$-1.145$</td>
<td>$0.509$</td>
</tr>
<tr>
<td>K3</td>
<td>Comp.</td>
<td>$0.000$</td>
<td>$9.0$</td>
<td>$-0.000$</td>
<td>$-7.505$</td>
<td>$0.000$</td>
</tr>
<tr>
<td>K4</td>
<td>Comp.</td>
<td>$0.000$</td>
<td>$9.0$</td>
<td>$-1.730$</td>
<td>$0.000$</td>
<td>$0.000$</td>
</tr>
<tr>
<td>K5</td>
<td>Tens.</td>
<td>$0.000$</td>
<td>$45.0$</td>
<td>$0.000$</td>
<td>$-3.444$</td>
<td>$0.344$</td>
</tr>
<tr>
<td>K6</td>
<td>Tens.</td>
<td>$0.000$</td>
<td>$22.5$</td>
<td>$-0.344$</td>
<td>$-2.005$</td>
<td>$0.830$</td>
</tr>
<tr>
<td>K7</td>
<td>Tens.</td>
<td>$0.000$</td>
<td>$67.5$</td>
<td>$0.337$</td>
<td>$-0.057$</td>
<td>$0.140$</td>
</tr>
<tr>
<td>K8</td>
<td>Tens.</td>
<td>$0.000$</td>
<td>$67.5$</td>
<td>$-0.344$</td>
<td>$-2.005$</td>
<td>$0.830$</td>
</tr>
<tr>
<td>K9</td>
<td>Comp.</td>
<td>$0.000$</td>
<td>$0.0$</td>
<td>$2.019$</td>
<td>$-4.675$</td>
<td>$1.146$</td>
</tr>
<tr>
<td>K10</td>
<td>Comp.</td>
<td>$0.000$</td>
<td>$90.0$</td>
<td>$2.019$</td>
<td>$-2.072$</td>
<td>$1.146$</td>
</tr>
<tr>
<td>K11</td>
<td>Comp.</td>
<td>$0.000$</td>
<td>$45.0$</td>
<td>$2.019$</td>
<td>$-2.072$</td>
<td>$1.146$</td>
</tr>
<tr>
<td>K12</td>
<td>Comp.</td>
<td>$0.000$</td>
<td>$45.0$</td>
<td>$2.019$</td>
<td>$-2.072$</td>
<td>$1.146$</td>
</tr>
</tbody>
</table>

Compression localization heavily depends on the softening rate of the material and the speed of stress propagation in the analyzed structure. Therefore the compressive fracture energy: $G_{fcy} = 3280 \text{ Nm/m}$ and $G_{fcy} = 10000 \text{ Nm/m}$, and the ultimate plastic strain $\kappa_p = 8 \cdot 10^{-4}$ is chosen. For the effective compressive region $h = 375 \text{ [mm]}$ was selected to cover the height of 5 elements.

5.2. Description of the tests and analysis setup

The shear walls analyzed consist of hollow clay brick masonry. The tests were reported by Ganz and Thürlimann [28] and denoted $W$. There were in total seven walls tested. However, only four of them will be used for the comparison, namely walls $W1$, $W2$, $W4$ and $W6$. The walls $W3$ and $W5$ had reinforcement in them, therefore they fall outside of the scope of this thesis. The wall $W7$ was removed due to the indecisive crack patterns, while simulation could be performed it would be difficult to compare the results.

The geometry of the walls consist of a masonry panel of $3600 \times 2000 \times 150 \text{ [mm]}$ and flanges of $150 \times 2000 \times 600 \text{ [mm]}$ (see Figure 5.2). The wall is subjected to the boundary conditions by two reinforced concrete slabs at top and bottom of the wall. Additionally, the wall is given prescribed uniformly distributed load $p$ over the length of the wall with a resultant $P$. Furthermore, a horizontal displacement $d$ is applied at the top concrete slab. The magnitudes of $P$ and $d$ alternate between the walls tested. The walls $W1$, $W2$ and $W4$ are subjected to monotonic displacement increment, while wall $W6$ is loaded with a dynamic signal.

All of the numerical analyses are performed using dynamic explicit integration method. The continuum elements used are 3D brick elements consisting of 8 nodes and 1 integration point. The size of these elements is $75 \times 75 \times 75 \text{ [mm]}$. The mesh is regular throughout the whole model (see Figure 5.3). Aside from the loads mentioned before, self-weight of the wall and concrete slabs are also included in

Figure 5.1: Parabolic hardening/softening law for compression (from [56])
the analysis. The monotonic analyses are performed in a period of 1 second while quasi-static in a 12s period.

Figure 5.2: Geometry and loads for ETH Zurich shear walls (from [56])

Figure 5.3: Mesh and boundary conditions of walls used in the numerical analyses (different colors denote different material properties applied to the elements)
Figure 5.4: Wall W1. Experiment failure patterns: (a) at peak load; (b) at end stage
5.3. Results of the analyses

First, the wall $W1$ is analyzed. The wall has an applied distributed load $P = 416 \text{ kN}$. The numerical results show good agreement to the experimental tests both in horizontal force – displacement curve and in crack patterns of the wall. Furthermore, the developed material model combined with an explicit integration analysis provides exceptional stability. The test was completed successfully without an extra effort or special modeling techniques, by contrast, where numerical analysis (implicit integration) reported by Lourenço diverged without reaching 15mm horizontal displacement point.

By analyzing the force – displacement diagrams (see Figure 5.5) it can be observed that the cracking of the wall starts at around 1mm of displacement. There are some fluctuations in the magnitude of the horizontal force, due to a sudden tensile failure of the elements. Furthermore, because the analysis is executed with 1s of simulation time, the dynamic effects further increase the fluctuations in the force-displacement curve.

It further has to be noted that up until 8mm of displacement the dilation throughout the whole wall is constant. After that, first elements surpass the set limit of 0.03 volumetric strain and the dilation angle for those elements is set to zero. It forces shear localization and strength degradation.

![Figure 5.5: Wall W1. Horizontal force – displacement diagrams.](image)

The behavior is the wall is shown in figures 5.6 and 5.7 in terms of deformed meshes and principal strain directions. For the plots of principle strain, the lowest 5% of the values are discarded in order to obtain more legible results. A reasonable agreement can be seen between numerical and experimental results. At the beginning, (see Figure 5.6b) of the analysis the right part of the wall starts to crack in a diagonal direction. This cracking is accompanied by the flexural cracks at the left toe of the wall. As seen later the later crack will close and will be replaced by flexural cracks at the upper part of the left flange.

Upon increasing of the loading (see Figure 5.7b), cracking starts to concentrate around the initially cracked diagonal strip. The cracking band angle and position coincides with the experimental results. Furthermore, after a point of 8mm of displacement shear localization starts appearing at top left and bottom right parts of the wall. This is due to the dilation angle being set to zero at the elements in those locations. However, the localization appears to propagate horizontally instead of diagonally due to it being sensitive to the element orientation. In this case, it does not alter the accuracy of the results significantly as in the experiment, flange flexural failure was observed at the approximate position. But it means that in cyclic shear analysis dilation angle cannot be set to zero as that would induce localized shear in a horizontal strip that might significantly alter the analysis results. On the other hand, non-zero dilation angle produces shear failure mode that seems to be determined by the loading conditions and the geometry of the wall rather than the orientation of the elements.
5. Comparison to experiments

Figure 5.6: Wall W1. Results of the analysis at the displacement of 2mm: (a) deformed mesh (scale 100); (b) max. principle strain directions

Figure 5.7: Wall W1. Results of the analysis at the displacement of 20mm: (a) deformed mesh (scale 5); (b) max. principle strain directions

Figure 5.8 shows the contour of minimum principal stresses at the same stages of analysis. Upon the increasing of the loading, it can be observed that the stresses concentrate in the narrower band with peak stresses at the bottom right corner especially the flange. However even at the displacement of 20mm, it can be seen that the stresses do not exceed maximal compressive strengths, therefore it can be concluded that the failure is completely governed by the tension regime.

Figure 5.8: Wall W1. Minimal principal stresses at the displacement of (a) 2 mm and (b) 20 mm.

Next wall W2 is analyzed. The wall is subjected to the vertical load $P$ of 1287 [kN]. At the beginning of the analysis (see Figure 5.10) the behavior of the wall is ductile followed by a later brittle failure due to compression in the bottom right toe of the wall. As in analysis executed by Lourenço, the force displacement curve is around 10-20% higher than obtained in the experiments. The slight difference in peak strengths comes from slightly different fit obtained for the material properties. However, in general,
5.3. Results of the analyses

Figure 5.9: Wall W2. Experiment failure patterns: (a) at peak load; (b) at end stage
the good agreement is found between the analysis and the experiment. Even though the strengths of the wall are higher, the force-displacement curve follows parallel path after 3mm of displacement. At around 8mm of displacement, a sudden failure of the wall is observed.

![Force vs Displacement Diagram](image)

Figure 5.10: Wall W2. Horizontal force – displacement diagrams.

By examining the principal strains (see Figure 5.11) it can be seen that reduction in tangential stiffness around 3mm of displacement is caused by the formation of two crack bands. One at the left side of the wall propagating from the top part of the left flange and another on the right side of the wall propagating from the bottom of the right flange. This behavior can also be observed in the experimental test (see Figure 5.9a) as well.

![Wall W2 Analysis Results](image)

Figure 5.11: Wall W2. Results of the analysis at the displacement of 3mm: (a) deformed mesh (scale 5; (b) max. principle strain directions

Just before the failure of the wall (see Figure 5.12) tensile crack concentration around the initial left crack band can be seen. Furthermore, at the top left flange shear failure between the wall and the flange forms. At the collapse stage (see Figure 5.13) the crushing of the bottom right row of the elements is evident. The crushing forced the stable time increment to reduce and the analysis stopped. One way to avoid such behavior is to enable element deletion. Therefore, the crushed elements would be deleted and the analysis could continue. In this specific test this measure was not required but in other cases e.g. in an analysis of full buildings, element deletion might be required as the analysis should continue even though some elements are crushed in the structure.
5.3. Results of the analyses

Figure 5.12: Wall W2. Results of the analysis at the displacement of 8mm just before the collapse: (a) deformed mesh (scale 5); (b) max. principle stress directions

Figure 5.13: Wall W2. Results of the analysis at the displacement of 8mm at the moment of collapse: (a) deformed mesh (scale 5); (b) min. principle stress directions

Figure 5.14: Wall W2. (a-b) Minimal principal stresses at the displacement of 3 mm and 8 mm (before failure); (c) equivalent plastic strain at 8 mm (after failure).
Figure 5.15: Wall W 4. Experiment failure patterns: (a) at peak load; (b) at end stage
Further, wall \( W^4 \) is analyzed, the wall has a concentrated force \( P = 423 \text{ kPa} \) applied at the distance of \( e = 840 \text{ mm} \) from the center of the wall and distributed through a width of 500 \( \text{mm} \). Moreover, this wall had wider flanges 900 \( \text{mm} \) instead of 600 \( \text{mm} \) like in previous walls. The results from the analysis are again closely related to the experiment (see Figure 5.16). The force-displacement curve follows the experimental results closely, with some minor fluctuations around 3 mm of displacement. The fluctuations are caused due to cracking at the top of the left flange.

![Force-displacement curve](image)

**Figure 5.16:** Wall \( W^4 \). Horizontal force – displacement diagrams.

Initially, three cracking bands can be identified (see Figure 5.17). The most major one is at the top left, flange and the concrete pad separation, then bottom left toe and bottom pad separation and almost vertical cracking originating from the bottom right corner of the wall and following upwards along the right flange. The crack band right of the wall and the crack and top left of the wall can be also identified in the experiment (see Figure 5.15a). However, none of the cracks at the bottom right toe of the wall are visible.

At later stages of the analysis (see Figure 5.18) the cracking at the top left part of the wall expands further and merges with the cracking on the right side of the wall, while the crack at the bottom left toe of the wall closes. These patterns can be also observed in the experiment (see Figure 5.15b). There it appears as the horizontal and vertical cracks between the bricks.

By analyzing minimum principle stresses in the wall at later stages of the analysis (see Figure 5.19b). It can be observed that the crushing starts to appear at the bottom right part of the wall, just nearby.

![Cracking bands](image)

**Figure 5.17:** Wall \( W^4 \). Results of the analysis at the displacement of 3mm: (a) deformed mesh (scale 10); (b) max. principal strain directions
the right flange. Additionally, some crushing appears at the bottom right part of the right flange. In the experiment, the crushing crack patterns are also observed at the right side of the wall and at the bottom of the right flange.

Finally, the cyclic analysis of wall W6 is executed. The wall has an applied distributed load resulting in \( P = 418 \, [kN] \). Using the previous analysis settings resulted in a horizontal shear localization at the top part of the wall. The localization has high effects on crack patterns and reaction forces, therefore, for this analysis, the dilation angle was not set to zero and was kept constant. Furthermore, the experiment was carried out through a long period of time and it had 9 phases (denoted \( L_{S1} \) to \( L_{S9} \)) where each phase consisted of 10 cycles. Such analysis is infeasible with explicit integration method as it would take indefinitely long to complete the simulation. Therefore, a simplified signal was constructed. The new signal consists of 6 phases (see Figure 5.21) starting from a phase \( L_{S4} \) as the lower phases act only in the elastic domain of the wall. Each of the phases contains 3 full cycles and lasts 2s.

Each of the phase used in numerical and experimental analysis had an imposed maximal horizontal displacement at the top of the wall of \( 0.4, 0.8, 1.5, 3, 6, 9 \, [mm] \) respectively starting from phase \( L_{S4} \). It is import to note that this analysis is heavily simplified, not only by the shorter duration of the test but also by fewer cycles. Furthermore, in the experiment each cycle in a phase had a longer period than in a previous phase, wherein the numerical analysis all of the periods are the same. Therefore, it is extremely difficult to compare the results of the experimental test and the numerical analysis.
5.3. Results of the analyses

Figure 5.20: Wall W6. Experiment failure patterns: (a) at peak load; (b) at end stage
Despite the carried out simplifications, by analyzing force-displacement graph (see Figure 5.22), good agreement with the strengths envelope is found. At the last phase, in the experiment, some strength degradation is observed, while in the numerical analysis, this strength degradation is not seen. The main reason might be that the phases in the analysis only had 3 cycles instead of 10. Although at the end of the last cycle, there is a peak of lower strengths, this suggests that if the analysis were to continue, strength degradation would be present.

However, the dissipated energy in the numerical simulation is much higher than in experimental analysis. This issue could be explained by incorrectly modeled boundary conditions of the wall, as it is seen in the experimental results, the wall experiences rocking behavior, however, in the analysis, some degree of rocking was only observed at later cycles. Further examining the pictures provided in the experiment it can be seen that the wall has cracked at the connection of the wall and concrete slab, both, at the bottom and at the top.
To compare the cracking patterns is a difficult task as both in the experiment and in the numerical analysis, the whole wall was cracked. However, in an experiment, a prominent “X” pattern on the wall is seen. By analyzing the results of the analysis it can be observed that the “X” crack pattern can be first seen to form at the analysis time of 7.25 [s] (see Figure 5.23a). Unfortunately, at the later stages of the analysis, the crack pattern is lost in the noise of cracks as the whole wall is cracked. Nevertheless, the crack concentrations can be examined by looking at the equivalent plastic strain generated by the yielding on the Rankine type surface (see Figure 5.23b). There, a higher concentration of equivalent plastic strain can be seen at the mid-wall, flanges, and diagonals spanning from the wall center to the top corners of the wall. Even though, in the experiment a clear Mode-II crack can be seen at the center of the wall, it had more prominent cracks spanning from the center to the bottom corners of the wall.

![Figure 5.23: Wall W6. Results of the numerical analysis: (a) Tensile strengths degradation at the time of 7.25[s] (blue – fully degraded, 1% remaining strength, red – not degraded, 100% remaining strengths); (b) equivalent plastic strain generated by Rankine like yield surface at time of 12[s] (red – more strain, green – less strain); in both figures, deformations are to scale.](image)

The extra noise added to the late stage crack patterns can be attributed to the fact that the plastic strain is recorded every time the Rankine type surface is yielding, however, the model also exhibits crack closure in tension. Therefore, in some regions, the generated and displayed plastic strain is fictitious and does not affect the behavior of the wall. As an example, such regions are the flanges of the wall. While on tension, if they are cracked, the cracks would open up and generate fictitious plastic strain, however later when tension is removed and compression is applied, the cracks would close due to the damage in stiffness but the generated plastic strain would remain recorded. Additionally, it should be noted that due to the fact the dilation angle was kept constant throughout the analysis, the wall dilated substantially, furthermore, because of the flanges providing the horizontal constraint, higher stresses was to be expected than in the experiment.

Despite that, a strain increment output could be generated, it would show regions where the strain rate is highest at any given increment. These regions would approximately (the main crack path can change throughout analysis) show the crack pattern in the structure. This kind of output is not available by default in Abaqus. Therefore, a Python script has to be written in order to obtain it (See Appendix B). By examining such output (see 5.24) a reasonable agreement with experimental results is found. Diagonal cracks can be observed starting from the top left and top right parts of the wall. Also, a wall and a concrete slab separation at the top can be observed in both peaks of the displacement. In the experiment, more prominent cracking can be seen in the lower left bottom area of the wall while in the analysis – upper right. But, then again, these differences can be influenced due to the imperfections in the experimental specimens or due to the numerical model simplification for the analysis (i.e. the way the displacement is applied).
5.4. Summary

Four shear walls were analyzed, three subjected to monotonic in-plane horizontal displacement and one to dynamic cyclic horizontal displacement. By examining the obtained results, good agreement with the experimental data was found. The crack patterns and F-U curves go hand in hand with the results of the experiments. As for cyclic analysis, similar behavior in terms of strength envelope was observed, however, the analysis yielded higher energy dissipation than it was present in the experiments. It was concluded that in the experiment rocking behavior was observed and in the analysis it was lacking due to strict boundary conditions that did not allow the separation of the wall base and the bottom concrete pad. Furthermore, it was difficult to define the compression softening curve as the experiment was lacking such data. A curve was assumed, but it did not yield the most favorable results ($W_2$ and $W_6$) as wanted strength degradation was not observed. Better fitting the compression curve would have yielded better results.
Conclusions and recommendations

6.1. Conclusions
In this thesis the following objectives were achieved:

- Through a discussion of basic theoretical considerations it was found that the explicit algorithm is more favorable than the implicit.
- Through a comprehensive literature review the information on the existing knowledge about Continuum Damage Mechanics models were assembled. Two main distinctive works were identified, namely the work Lourenco [56] and the work of Pela [75]. After investigation of strengths and drawbacks of both models, it was decided to use Lourenco [56] model as the basis of the thesis.
- A stable, accurate and robust orthotropic composite masonry model for explicit solver has been developed. The model exhibits 3D elasticity and 2D plane plasticity. Therefore, the modeling and application of boundary conditions are simplistic. Furthermore, it is assumed that two general failure mechanisms are present. One associated with tensile and shear brittle fracture represented by Rankine type yield surface and another with distributed crushing of a material represented by Hill type yield surface. The model exhibits uncoupled damage evolution in tension regime and coupled in compression. The shear brittle fracture and distributed crushing are controlled by $\varepsilon_{\text{pl},t}$ and $\varepsilon_{\text{pl},c}$ respectively. Additionally, the model supports tensile crack closure, while in compression it accumulates the plastic deformations. The model is formulated in such a way that most of the properties in material directions are independent of one another.
- The intersections of the yield surfaces and the apex of the Rankine-like surface cone cause a problematic stress return in the subdifferential zone. The underlying robust algorithms used to mitigate these issues were derived that yielded stable and accurate results.
- Additional measures were taken to ensure stable and smooth analysis, stiffness proportional term of Rayleigh damping was incorporated together with stable time estimation based on the effects of the added damping. To even further increase the stability of an explicit analysis an option to remove crushed element is considered.
- The developed model was tested by examining its behavior in one or few element tests and by comparing it with experimental results. For experimental comparison, four shear walls were modeled, three subjected to monotonic loading and one to cyclic. The analyses closely agree to experimental results even when using raw test data. The model is stable due to explicit approach and it is flexible enough to be used and provide qualitative results in various types of analyses, static or cyclic.

However, during the testing of the model, several flaws emerged:

- The developed nonlinearly decreasing dilation angle appeared to be unstable when elements are subjected to tension regime, by setting of the dilation to zero did not help much as it forced the
shear to localize in horizontal bands and just by having constant dilation influenced the deformations of the cyclic tests as too extensive lift of the wall was observed.

• The way that equivalent plastic strain is being tract appears to be flowed as it does not take in to account the closure of the cracks. So the plastic strain output is not what one would expect. However, this issue could be eliminated with the future iterations of the code. Or temporary mitigated by analyzing strain increment output data.

In addition to the main objectives, two other programs were developed to aid the analysis of masonry:

• A test data fitter in order to obtain the material properties from raw experimental results. It can also be used to check the quality of the fit (see Appendix A).

• An output database formatter, that can be used to rename and vectorize custom output variables and also create a strain increment output (see Appendix B).

6.2. Recommendations for analyses and future development

6.2.1. Further development on non-associated flow

Current model incorporates a smooth flow function for Rankine yield surface. However, this function has drawbacks. It is highly recommended to describe a more stable flow algorithm that does not loose stability when the dilation angle is reduced. Such algorithm would be especially useful in cyclic analyses in which the dilation of the structure could be controlled and excessive lift could possibly be avoided.

6.2.2. Localized damage

Shear damage appears to be localizing in horizontal bands (mesh orientation dependent) when the dilation angle is set to zero and partially localizing (wide bands of cracks) if the dilation angle is kept constant. To this, there could be several reasons. First, the single integration elements are subjected to hour-glassing. Elements affected by hour-glassing will have lower strengths than they are supposed to, therefore, when the dilation angle is set to zero the path of the least resistance for shear propagation are in the orthogonal directions. However applying some means of hour-glassing control exerts unrealistic forces on to the structure (see chapter 27.1.4 Section controls in [14]) and that may influence the results of the analysis in a negative way. Second, the brick elements may be unsuitable for analyses where diagonal localizations are expected as the localization tends to propagate in orthogonal directions. Third, the time step even in explicit analyzes is too big to obtain expected localization. The stress increment per one time increment encompasses several elements, therefore, damaging bigger bands of a mesh.

If the first issue is governing the solution would be to develop a custom element that would have at least four integration points. This way the hour-glassing would be avoided and elements would have appropriate stiffness and strengths when subjected to hour-glassing modes. If the second issue is governing, it is possible to use different type of element already provided by Abaqus such as solid tetrahedral elements. However, they should be used with caution as many types of them are either overly stiff and extremely fine mesh is required for accurate analysis or they exhibit “volumetric locking” at large plastic deformations (see Section 28.1.1 Solid (Continuum) elements in [14]). And finally, if the last case is governing, the solutions would be either to lower the stable time increment,which could lead to a drastic increase of the analysis time until the desired result is obtained or to develop a custom element that would only let the damage propagate in the directions perpendicular to the direction of principal strain. Such algorithm was proposed and discussed by Clemente [12].

6.2.3. Improve compression behavior

The compression behavior in this material model was developed using a rather simplistic approach. No stiffness damage was applied during the crushing of the material, while in experimental tests [22], such damage can be observed to some extent. Even though, the masonry analysis relevant for Groningen region the compressive crushing is not governing, it would increase the field of application and make the model more universal. Additionally, the localization for the compression crushing appears to be mesh and stress rate dependent, therefore often the failure due to compression may be unpredictable
and volatile. This issue could be minimized using higher values for distortion control or for quadratic bulk viscosity (see chapter 27.1.4 Section controls and chapter 6.3.3 Explicit dynamic analysis in [14]).

6.2.4. Further testing

Even though the material model is tested and the results are compared to 4 experimental tests. Further testing still has to be done. Such additional tests could include shear walls with openings, walls subjected to out of plane two-way bending, and full-scale buildings in order to fully verify the material model.


Bibliography


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Determining the material properties for wall tests

In order to obtain the material properties for wall tests one of the ways to do it is to apply a least square fit method on the experimental data. This can be easily achieved with mathematical packages like Maple or Wolfram Mathematica. However, these software packages are proprietary and expensive. Therefore, it is more convenient to develop a fitting solution whose usage would not require buying expensive software that would not be used for any other purpose.

To this, a program composed of MS Excel spreadsheet and python, a high-level general-purpose programming language, was developed. The program functions so that the user has to input principle stress data obtained from biaxial tension/compression tests made on square wall specimens. Then the program rotates the stresses to Cauchy stresses. These stresses then are sorted depending on the regime (specified by user) and the data is transfered to python code where the fit is executed. After obtaining the fit parameters, the data is transfered back to the excel spreadsheet where the the accuracy of the fit is estimated and the results are displayed for the end user.

A.1. Input

For the input of the program a material data points are requested. They are provided by entering the data point name, the principle stresses \( \sigma_x \) and \( \sigma_y \) specified in MPa, and the rotation of the bed-joint \( \theta \) specified in degrees. Each data point should be attributed either to be fit on Hill-like yield surface or Rankine-like. The Data points can be enabled or disabled on demand specifying either yes or no in the column titled “Included”. The example of the input is provided in figure A.1.

When the input is entered, the principle stresses are rotated to the material directions so that the new stresses are either perpendicular or parallel to the bed-joint. For the rotation the following formulas are used:

\[
\sigma_x = \sigma_1 \cdot \cos^2 \left( \frac{\theta}{180^\circ \pi} \right) + \sigma_2 \cdot \sin^2 \left( \frac{\theta}{180^\circ \pi} \right) \quad (A.1)
\]

\[
\sigma_y = \sigma_1 \cdot \sin^2 \left( \frac{\theta}{180^\circ \pi} \right) + \sigma_2 \cdot \cos^2 \left( \frac{\theta}{180^\circ \pi} \right) \quad (A.2)
\]

\[
\tau_{xy} = (\sigma_1 - \sigma_2) \cdot \cos^2 \left( \frac{\theta}{180^\circ \pi} \right) \cdot \sin^2 \left( \frac{\theta}{180^\circ \pi} \right) \quad (A.3)
\]

When the input is all entered, “Execute Fit Procedure” should be clicked. Then the excel will sort all of the input in two tables, for Rankine-like and Hill-like surfaces and pass the data to Python code for fitting.
A.2. Fitting

The fitting process is semi-automatic. Even though the fit itself is automatic, the outliers are not excluded automatically. After the fit is performed the results have to be inspected and if a larger error is present the outliers have to be disabled in the spreadsheet and the fit has to be run again.

The data fit itself is accomplished using `least_squares` function from `scipy.optimize` package.

A.3. Results

The display of the results is provided in two mediums. First the data points are plotted against the yield surfaces as depicted in figure A.2 using `mayavi` visualization package, then the fit data is exported back to excel where the material parameters and the quality of the fit is then displayed (see Figure A.3).
A.4. Code

A.4.1. Visual Basic for excel

```vbnet
Sub Fit()
    Dim rng As Range, cell As Range, i As Integer, j As Integer
    Dim ActSheet As Worksheet
    Dim SelRange As Range
    On Error GoTo errHandler
    Set ActSheet = ActiveSheet
    Set SelRange = Selection
    If Range(“A3”).Value = “#” Then
        Range(“J2”).Value = “Running...”
    Application.Wait (Now + TimeValue(“0:00:01”))
    Application.ScreenUpdating = False
    Call SortSurf(Range(“B4:B100”), i, j)
    Call ExportCsv(Range(“M4:S” & 3 + i), “temp_Hill”)
    Call ExportCsv(Range(“U4:AA” & 3 + j), “temp_Rankine”)
    Call ExportCsv(ThisWorkbook.Sheets(“Settings”).Range(“B3:D9”), “temp_Bounds”)
    Call ExportCsv(ThisWorkbook.Sheets(“Settings”).Range(“G3:H5”), “temp_Plot_Bounds”)
End Sub
```

Figure A.3: Output after running the fit procedure.

The results should be taken with caution as the fit can be easily be usable due to the wide scatter of the experimental results. In that case the parameters can be forced in to the logical boundaries using settings of the program (see Figure A.4).

Figure A.4: Settings for better fit and visualization control.
Call ExportCsv(ThisWorkbook.Sheets(“Settings”).Range(“H7:H9”), “temp_Plot_density”)
If ThisWorkbook.Sheets(“Settings”).Range(“B15”) = “Yes” Then
    python_loc = ThisWorkbook.Sheets(“Settings”).Range(“B11”)
Else
    python_loc = Replace(ThisWorkbook.Sheets(“Settings”).Range(“B11”), “python.exe”, “pythonw.exe”)
End If
script_loc = ThisWorkbook.Sheets(“Settings”).Range(“B12”)
Location = Application.ActiveWorkbook.Path
RetVal = Shell(python_loc & “ ” & script_loc & “ “ & Location & “ “, vbNormalFocus)
Range(“J20”) =RetVal
If IsItDone() Then
    Call ReadOutput
End If
Call DeleteFile
Columns(“M”).AutoFit
Columns(“U”).AutoFit
Application.CutCopyMode = False
Range(“J2”).Value = “Done!”
Else
    MsgBox “Code is being executed on the wrong sheet!”
Exit Sub
End If
Application.ScreenUpdating = True
ActSheet.Select
SelRange.Select
errHandler:
    Application.ScreenUpdating = True
End Sub
Sub ExportCsv(mycels As Range, name As String)
    filename = name & “.csv”
    Open Application.ActiveWorkbook.Path & “\” & filename For Output As #1
    Set myrng = mycels
    For i = 1 To myrng.Rows.Count
        For j = 1 To myrng.Columns.Count
            lineText = IIf(j = 1, “”, lineText & “ “)
            myrng.Cells(i, j) = lineText
        Next j
        Print #1, lineText
    Next i
    Close #1
End Sub
Sub SortSurf(rng As Range, i As Integer, j As Integer)
    i = 0
    j = 0
    Range(“M4:S999”).ClearContents
    Range(“U4:AB999”).ClearContents
For Each cell In rng
    If cell.Value = "" Then
        Exit For
    ElseIf cell.Value = "Hill" Then
        If Range("I" & cell.Row).Value = "Yes" Then
            Range("C" & cell.Row & ":H" & cell.Row).Copy
            Range("N" & i + 4).PasteSpecial xlPasteValues
            Range("M" & i + 4) = Range("A" & cell.Row)
            i = i + 1
        End If
    Else
        If Range("I" & cell.Row).Value = "Yes" Then
            Range("C" & cell.Row & ":H" & cell.Row).Copy
            Range("V" & j + 4).PasteSpecial xlPasteValues
            Range("U" & j + 4) = Range("A" & cell.Row)
            j = j + 1
        End If
    End If
Next cell
End Sub
Sub ReadOutput()
    PathF = Application.ActiveWorkbook.Path & "\temp_output.csv"
    Open PathF For Input As #1
    r_no = 0
    l_no = 0
    inc = 0
    Do Until EOF(1)
        Line Input #1, LineFromFile
        LineItems = Split(LineFromFile, ",")
        inc = inc + 1
        If LineFromFile = "" Then
            r_no = r_no + 1
            l_no = 0
        Else
            If r_no = 0 Then
                Range("K8") = LineItems(0)
                Range("K9") = LineItems(1)
                Range("K10") = LineItems(2)
                Range("K12") = LineItems(3)
                Range("K13") = LineItems(4)
                Range("K14") = LineItems(5)
                Range("K15") = LineItems(6)
            ElseIf r_no = 2 Then
                Range("N" & 4 + l_no) = LineItems(0)
                Range("O" & 4 + l_no) = LineItems(1)
                Range("P" & 4 + l_no) = LineItems(2)
                Range("T" & 4 + l_no) = 1 / LineItems(3)
                l_no = l_no + 1
            ElseIf r_no = 1 Then
                Range("V" & 4 + l_no) = LineItems(0)
                Range("W" & 4 + l_no) = LineItems(1)
                Range("X" & 4 + l_no) = LineItems(2)
                Range("AB" & 4 + l_no) = 1 / LineItems(3)
                l_no = l_no + 1
            ElseIf r_no = 1 Then
                Range("V" & 4 + l_no) = LineItems(0)
                Range("W" & 4 + l_no) = LineItems(1)
                Range("X" & 4 + l_no) = LineItems(2)
                Range("AB" & 4 + l_no) = 1 / LineItems(3)
                l_no = l_no + 1
            Else

        End If
    Loop
End Sub
A. Determining the material properties for wall tests

```vba
Exit Do
End If
End If
Loop
Close #1
End Sub

Function IsItDone() As Boolean
Count = 0
Do While True
    If Application.Wait(Now + TimeValue("0:00:02")) Then
    If Not Dir(Application.ActiveWorkbook.Path & "\" & "done", vbDirectory) = vbNullString Then
        IsItDone = True
        Exit Do
    Else
        IsItDone = False
    End If
    End If
    Count = Count + 1
    If Count > 5 Then
        MsgBox "Did not receive an answer from python! Did the Python script get stuck?"
        Exit Do
    End If
Loop
End Function

Sub DeleteFile()
Dim LRandomNumber As String
Dim Loc As String
Loc = Application.ActiveWorkbook.Path & "\"
If ThisWorkbook.Sheets("Settings").Range("B14") = "Yes" Then
    LRandomNumber = Format(Now(), "yyyyMMdd_hhmmss")
    FileCopy Loc & "temp_rankine.csv", Loc & LRandomNumber & "_rankine.csv"
    FileCopy Loc & "temp_hill.csv", Loc & LRandomNumber & "_hill.csv"
    FileCopy Loc & "temp_output.csv", Loc & LRandomNumber & "_output.csv"
    FileCopy Loc & "temp_Bounds.csv", Loc & LRandomNumber & "_bounds.csv"
    FileCopy Loc & "temp_Plot_Bounds.csv", Loc & LRandomNumber & "_Plot_Bounds.csv"
    FileCopy Loc & "temp_Plot_density.csv", Loc & LRandomNumber & "_Plot_density.csv"
End If
Kill Loc & "temp_rankine.csv"
Kill Loc & "temp_hill.csv"
Kill Loc & "temp_output.csv"
Kill Loc & "temp_Bounds.csv"
Kill Loc & "temp_Plot_Bounds.csv"
Kill Loc & "temp_Plot_density.csv"
Kill Loc & "Done"
End Sub
```
A.4.2. Python code

```python
from scipy import optimize
import numpy as np
from mayavi import mlab
import sys
import csv
import time

def process_csv(file_loc):
    try:
        with open(file_loc + '\temp_rankine.csv', 'rb') as fl:
            reader = csv.reader(fl)
data1 = list(reader)
        with open(file_loc + '\temp_hill.csv', 'rb') as fl:
            reader = csv.reader(fl)
data2 = list(reader)
        with open(file_loc + '\temp_Bounds.csv', 'rb') as fl:
            reader = csv.reader(fl)
data3 = list(reader)
        with open(file_loc + '\temp_Plot_Bounds.csv', 'rb') as fl:
            reader = csv.reader(fl)
data4 = list(reader)
        with open(file_loc + '\temp_Plot_density.csv', 'rb') as fl:
            reader = csv.reader(fl)
data5 = list(reader)
    except:
        print "ERROR! Could not find the files in:"
        print file_loc

    names1 = []
    names2 = []

    for row in data1:
        names1.append(row[0])
        del row[0:4]

    for row in data2:
        names2.append(row[0])
        del row[0:4]

data1 = np.array(data1, dtype=float)
data2 = np.array(data2, dtype=float)
data3 = np.array(data3, dtype=float)
data4 = np.array(data4, dtype=float)
data5 = np.array(data5, dtype=float)

    return [names1, names2, data1, data2, data3, data4, data5]

def rankine_func(A, B, C, x, y, z):
    return (x - A + y - B) / 2 + np.sqrt(((x - A - y + B) / 2) ** 2 + C * z ** 2)

def hill_func(D, E, F, G, x, y, z):
    return np.sqrt(x**((F*y)/2 + (E*x)/D) + y**((F*x)/2 + (D*y)/E) + G*z**2) - np.sqrt(D*E)

def residuals_r(coeff, data):
```

# Function that returns the squared loss.
# We want the function to choose A, B, C such that all values are close
to zero
A, B, C = coeff
x, y, z = data.T
# The function we care about
objective = rankine_func(A, B, C, x, y, z)
losses = (objective - 0)
return losses

def residuals_h(coeff, data):
  # Function that returns the squared loss.
  # We want the function to choose A, B, C such that all values are close
to zero
  A, B, C, D = coeff
  x, y, z = data.T
  # The function we care about
  objective = hill_func(A, B, C, D, x, y, z)
  losses = (objective - 0)
  return losses

def Surf1(x, y, z):
  F1 = rankine_func(A, B, C, x, y, z)
  F2 = hill_func(D, E, F, G, x, y, z)
  t = F1 if F2 > 0 else None
  return F1

def Surf2(x, y, z):
  F1 = rankine_func(A, B, C, x, y, z)
  F2 = hill_func(D, E, F, G, x, y, z)
  t = F2 if F1 > 0 else None
  return F2

def write_csv(file_loc, data):
  with open(file_loc + '\temp_output.csv', 'ab') as fp:
    a = csv.writer(fp, delimiter=',')
    try:
      a.writerows(data)
    except:
      a.writerow(data)
    a.writerow('')

def intersection_r(A, B, C, x0, y0, z0):
  t = (-A * y0 - B * x0 + np.sqrt(A ** 2 * y0 ** 2 + (4 * C * z0 ** 2 - 2 * x0 * y0) * B * A + B ** 2 * x0 ** 2)) / (2 * C * z0 ** 2 - 2 * x0 * y0)
  return [x0 * t, y0 * t, z0 * t]

def intersection_h(D, E, F, G, x0, y0, z0):
  t = D * E / np.sqrt(D ** 2 + E ** 2 + F ** 2 + G ** 2)
  return [x0 * t, y0 * t, z0 * t]
location = r"C:\Users\aukselis\Desktop\Test worksheet".replace("\", "/")

else:
    location = sys.argv[-1]
data = process_csv(location)

names_r = data[0]
names_h = data[1]
print "------------INPUT------------"
print "Defined names"
print "Rankine:"
print names_r
print "Hill:"
print names_h
print 

data_r = data[2]
data_h = data[3]
print "Defined data:"
print "Rankine:"
print data_r
print "Hill:"
print data_h
print 

bounds = data[4]
print "Defined bounds:"
print bounds
print 

plot_i = [data[5], data[6]]

coeff_r0 = bounds.T[2][0:3]
coeff_r = coeff_r0
coeff_h0 = bounds.T[2][3:]
coeff_h = coeff_h0

print "\n\n--------------OUTPUT--------------"
rankine = optimize.least_squares(residuals_r, coeff_r0, args=(data_r,),
    bounds=((bounds[0][0],
          bounds[1][0],
          bounds[2][0]),
    (bounds[0][1],
          bounds[1][1],
          bounds[2][1])))

ftx, fty, alpha = rankine.x
print('Rankine Surface\n -- ftx = \{0\} \n -- fty = \{1\} \n -- alpha = \{2\}
    \n\n'.format(ftx, fty, alpha))

hill = optimize.least_squares(residuals_h, coeff_h0, args=(data_h,),
    bounds=((bounds[3][0], bounds[4][0],
          bounds[5][0], bounds[6][0]),
    (bounds[3][1], bounds[4][1],
          bounds[5][1], bounds[6][1])),
Determining the material properties for wall tests

```
fcx, fcy, beta, gamma = hill.x
print('Hill Surface
-- fcx = {0}
-- fcy = {1}
-- beta = {2}
-- gamma = {3}'.format(fcx, fcy, beta, gamma))

A, B, C = rankine.x
D, E, F, G = hill.x
write_csv(location, [A, B, C, D, E, F, G])

data_r = np.array(intersection_r(A, B, C, x0, y0, z0))
ratio = np.array([np.sqrt(xyz1[0]**2 + xyz1[1]**2 + xyz1[2]**2) / np.sqrt(x0**2 + y0**2 + z0**2)])
write_csv(location, np.append(xyz1, ratio, axis=0).T)

open(file_loc + 'done', 'a').close()

minx = -D * plot_i[0][0][0]
maxx = A * plot_i[0][0][1]

miny = -E * plot_i[0][1][0]
maxy = B * plot_i[0][1][1]

minz = max(D, E) * plot_i[0][2][0]
maxz = max(D, E) * plot_i[0][2][1]

minx = -D * plot_i[0][0][0]
maxx = A * plot_i[0][0][1]

miny = -E * plot_i[0][1][0]
maxy = B * plot_i[0][1][1]

minz = max(D, E) * plot_i[0][2][0]
maxz = max(D, E) * plot_i[0][2][1]


app = np.append(data_r, data_h, axis=0)

names = names_r + names_h

f = mlab.figure(fgcolor=(0., 0., 0.), bgcolor=(1, 1, 1))

mlab.contour3d(x, y, z, Surf1, contours = [0], transparent=True)
mlab.contour3d(x, y, z, Surf2, contours = [0], transparent=True)
mlab.axes(xlabel=r'$\sigma_x$', ylabel=r'$\sigma_y$', zlabel=r'$\tau_{xy}$')
mlab.points3d(x1, y1, z1, mode='axes', scale_factor=0.1)
```
text=[]
for i in range(len(x1)):
    text.append(mlab.text(x1[i],y1[i],names[i],z=z1[i]))
    text[i].actor.text_scale_mode = 'none'
mlab.show()
In order to obtain more readable output and compute variables that are not available as output request in Abaqus, the output database has to be formatted using Python scripting interface. For this specific case a Python script was written in order to replace the SDV to a more legible output to custom named variables and vectors that can be later be plotted as a vector plots. Furthermore, the strain increment variable was also added, as it was necessary for the dynamic cyclic test to see the crack patterns.

```python
from abaqusConstants import *
from odbAccess import *

# *******************************************************************
odbPath = "C:\Some\Path\to\an\Output_database.odb" # path to output database
# *******************************************************************

# Do you want to copy non-SDV variables to the new step?
copy = False

# List of sdv's (name, tuple(no),description,type,tuple(validInvariants)):
l_sdv = [["Yt",(1,2),"Tensile yield strengths in material directions",VECTOR,(MAGNITUDE,)],
        ["Yc",(4,5),"Compressive yield strengths in material directions",VECTOR,(MAGNITUDE,)],
        ["rYt",(10,11),"Ratio of remaining tensile yield strengths in material directions",VECTOR,(MAGNITUDE,)],
        ["rYc",(12,),"Ratio of remaining compressive yield strengths",SCALAR,()],
        ["dVol",(17,),"Volume change",SCALAR,()],
        ["rEq",(18,),"Equivalent Rankine plastic strain",SCALAR,()],
        ["hEq",(19,),"Equivalent Hill plastic strain",SCALAR,()],
        ["tE",(20,21,22),"Total strain in material directions",VECTOR,(MAGNITUDE,)],
        ["tEm",(23,24,25),"Total maximum strain in material directions",VECTOR,(MAGNITUDE,)],
        ["dam",(27,28),"Tension continuity parameters in material directions",VECTOR,(MAGNITUDE,)],
        ["ElDel",(29,),"Element deletion parameter",SCALAR,()],
        ["Sdamp",(30,31,32,33,34,35),"Damping stress increment",TENSOR_3D_FULL,(PRESS,MAX_PRINCIPAL,MID_PRINCIPAL,MIN_PRINCIPAL)]
# *******************************************************************

# Open Odb
odb = session.openOdb(name=odbPath,readOnly=False)
```
# Create an object with all steps
stepRepository = odb.steps

# Get the names of all steps
allSteps = stepRepository.keys()

# Iterate all steps
for i in range(len(allSteps)):
    pr.enable()
    # Assign a step
    step = stepRepository[allSteps[i]]
    # Create a list containing all frame numbers
    allFrames = range(len(step.frames))
    # Get the total time of the step
    StepTime = step.frames[-1].frameValue
    # Create a new step to store all of the custom field entries
    newStep = odb.Step(name=allSteps[i]+'_c_out', description='Step for new fields', domain=TIME, timePeriod=StepTime)
    # Create an object with all frames (This is recommended by abaqus manual to improve the speed of the script)
    frameRepository = step.frames
    # Get a list of all of the field variables available in the step
    fields = frameRepository[1].fieldOutputs.keys()
    # Iterate all frames in the step
    for j in allFrames:
        # Try assigning previous frame if it does not work. Previous frame is a new frame. This is used for the calculation of strain increment.
        try:
            pframe = frame
        except:
            pframe = frameRepository[j]
        # Assign a frame to a new frame
        frame = frameRepository[j]
        # Create a frame for the custom field variables in the new step created before this loop
        newFrame = newStep.Frame(frameId=j, frameValue=frame.frameValue,
                                  description='Increment: ' + str(j) + ', Time: ' + str(frame.frameValue) )
        # Define or clean sdv list
        SDV = []

        # Iterate through the available field variables
        for field in fields:
            # Assign field output to an object
            current = frame.fieldOutputs[field]
            # If field output is not an SDV
            if 'SDV' not in field:
                # If copying existing variables is allowed
                if copy:
                    # Copy existing variables to a new step
                    newField = newFrame.FieldOutput(name=current.name,
                                                     description=current.description, field=current)
                # If the field output variable is strain calculate strain increment
if 'LE' in field:
    previous = pframe.fieldOutputs[field]
    comp = current - previous
    newField = newFrame.FieldOutput(name='dLE',
         description='Logarithmic strain increment',
         field=comp)
    # Otherwise collect the SDV objects in to a list
else:
    SDV.append(frame.fieldOutputs[field])
# Define or clean field variable list
l_field = []
# Define the structure of the field variable list
for output in l_sdv:
    l_field.append([output[0],newFrame.FieldOutput(name=output[0],
         description=output[2],type=output[3],validInvariants=output[4]),output[1],[]])
# Iterate trough values of sdv to separate different instances and
# construct a list of elements
for k in range(len(SDV[0].values)):
    # Element numbers
    labl = SDV[0].values[k].elementLabel
    # Element belongs to an instance
    inst = SDV[0].values[k].instance
    # For each field output in list of fields construct proper tuples
    for field in l_field:
        if len(field[3][0]) == 1:
            field[3][0]=[inst,[],[]]
        elif inst != field[3][-1][0]:
            field[3].append([inst,[],[]])
        field[3][-1][1].append(labl)
        temp = []
        for tup in field[2]:
            temp.append(SDV[tup-1].values[k].data)
        if len(temp) == 2:
            temp.append(0)
        field[3][-1][2].append(tuple(temp))
# For each field output in list of fields
for field in l_field:
    # Iterate trough all instances in each SDV
    for inst in field[3]:
        # Add data of an instance to a field
        field[1].addData(position=INTEGRATION_POINT,
             instance=inst[0], labels=tuple(inst[1]),
             data=tuple(inst[2]))
    # Print progress to screen
    print >> sys.__stdout__, '', allSteps[i], '', allFrames[j]
    pr.disable()
# Save the odb
odb.save()
# Close the odb
odb.close()
### Definition of input

*Depvar
28,

*User Material, constants=YY

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>state, Ex, Ey, Ez, Nuxy, bdamp, tpsi, Gxy, Gyz, Gzx, Ftx, Fty, alpha, Fcx, Fcy, beta, gamma, tss, dmax, Gfx, Gfy, Fuc, Fcy, ecplu, XX, ncomp, {comp_epl}, {comp_stress_r}</td>
<td></td>
</tr>
</tbody>
</table>

**State** – Definition of analysis state:
- 1 – Orthotropic elastic
- 2 – Orthotropic damaged plasticity

**Ex** – Stiffness modulus in material direction X

**Ey** – Stiffness modulus in material direction Y

**Ez** – Stiffness modulus in material direction Z

**Nuxy** – Poisson’s ratio \( \nu_{xy} \)

**bdamp** – Stiffness proportional Rayleigh damping coefficient

**tpsi** – Tangent of the dilation angle

**Gxy** – Shear stiffness modulus in material plane xy

**Gyz** – Shear stiffness modulus in material plane yz

**Gzx** – Shear stiffness modulus in material plane zx

**Ftx** – Tensile yield strengths in direction X

**Fty** – Tensile yield strengths in direction Y

**Alpha** – Yield shape parameter [common from 1.0 to 2.0]

**Fcx** – Compressive yield strengths in direction X

**Fcy** – Compressive yield strengths in direction Y

**Beta** – Yield shape parameter [common from -1.1 to -0.9]

**Gamma** – Yield shape parameter [common from 1.0 to 10.0]

**tss** – Type of shear softening:
- 0 – weakest direction,
- 1 – isotropic

**dmax** – Maximal volume change for the element in the analysis

**Gfxt** – Fracture energy in X direction

**Gfyt** – Fracture energy in Y direction

**Fuc** – Ultimate yield strengths for compression, must be consistent with {comp_stress_r}

**ecplu** – Plastic strain at \( f_{uc} \), must be consistent with {comp_epl}

**Ncomp** – Size of input softening table describing compressive behavior

{comp_epl} – Plastic strain part of compression table
{comp_stress_r} – Yield strength part of compression table
XX – Unused material property position. Just set 0.0
YY – Number of material properties (26+ncomp*2)
Definition of output

SDV01 – Tensile yield strengths X direction
SDV02 – Tensile yield strengths Y direction
SDV03 – Compressive yield strengths X direction
SDV04 – Compressive yield strengths Y direction
SDV05 – Ratio of remaining tensile material strengths in x direction
SDV06 – Ratio of remaining tensile material strengths in y direction
SDV07 – Ratio of remaining compressive strengths in the material
SDV08 – Compressive plastic strain in X direction
SDV09 – Compressive plastic strain in X direction
SDV10 – Volumetric change in an element
SDV11 – Equivalent Rankine plastic strain
SDV12 – Equivalent Hills plastic strain
SDV13 – Total strain in X direction
SDV14 – Total strain in Y direction
SDV15 – Total strain in Z direction
SDV16 – Max strain in tension in X direction
SDV17 – Max strain in tension in Y direction
SDV18 – Max strain in tension in Z direction
SDV19 – Damage parameter $\sqrt{d_2^t + d_3^t}$
SDV20 – Damage parameter for X direction
SDV21 – Damage parameter for Y direction
SDV22 – Element deletion parameter
  0 – Not deleted
  1 – Deleted
SDV23 – Storage for damping stress – xx
SDV24 – Storage for damping stress – yy
SDV25 – Storage for damping stress – zz
SDV26 – Storage for damping stress – xy
SDV27 – Storage for damping stress – yz
SDV28 – Storage for damping stress – zx
The modules `def_par_math`, `def_par_state`, `def_par_dir` containing parameters that define memory positions of variables in arrays and mathematical constants were removed due to request from Zonneveld Ingenieurs B.V.

```fortran
module utility_routines
public :: ut_elastic, ut_int_energy, ut_asgn_fstate,
     n ut_asgn_props, ut_assem_K_mtx, ut_asgn_tstate,
     n ut_vuhard
contains

pure subroutine ut_elastic(Sn, So, de, D)
use def_par_dir
include 'vaba_param.inc'
Real(8), intent(out)::Sn(:)
Real(8), intent(in)::So(:,), de(:,), D(:)
Sn(xx)= So(xx) + D(K11)*de(xx) + D(K12)*de(yy) + D(K13)*de(zz)
Sn(yy)= So(yy) + D(K21)*de(xx) + D(K22)*de(yy) + D(K23)*de(zz)
Sn(zz)= So(zz) + D(K31)*de(xx) + D(K32)*de(yy) + D(K33)*de(zz)
Sn(xy)= So(xy) + D(K44)*de(xy)
Sn(yz)= So(yz) + D(K55)*de(yz)
Sn(zx)= So(zx) + D(K66)*de(zx)
return
end

pure subroutine ut_int_energy(eIntNew, eIntOld, Sn, So, de, rho)
use def_par_dir
use def_par_math
include 'vaba_param.inc'
Real(8), intent(out)::eIntNew
Real(8), intent(in):: rho, eIntOld
Real(8), intent(in):: Sn(:), So(:,), de(:)

STRESS_POWER = HALF * ( 
   n(So(xx) + Sn(xx)))*de(xx) + (So(yy) + Sn(yy))*de(yy)
   + (So(zz) + Sn(zz))*de(zz) + (So(xy) + Sn(xy))*de(xy)
   + (So(yz) + Sn(yz))*de(yz) + (So(zx) + Sn(zx))*de(zx)
```

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eIntNew = eIntOld + STRESS_POWER/rho

return
end

subroutine ut_ine_energy(eIneNew, eIneOld, Sn, So, depl, depl1, rho)
  use def_par_dir
  use def_par_math
  include 'vaba_param.inc'
  dimension Sn(:), So(:), depl(:), plastic_work(size(So))
  real*8 rho, eIntNew, eIntOld, depl(:)
  plastic_work = (Sn+So)*half*(depl+depl1)
  eIneNew = eIneOld + sum(plastic_work)/rho

return
end

pure subroutine ut_asgn_fstate(yield, epl, te, tem, props, state, n dmg, eeqpl, ryield, dV)
  use def_par_dir
  use def_par_props
  use def_par_state
  use def_par_math
  include 'vaba_param.inc'
  real(*), intent(out):: yield(:,), epl(:,), te(:,), tem(:,), dmg(:,), n eeqpl, ryield, dV
  real(*), intent(in):: props(:,), state(:,)

  if (state(sYxt).eq.zero) then
    yield(xxt:yyt) = props(pYxt:pYyt)
    yield(xxc:yyc) = props(pYxc:pYyc)
    dmg(xxt:yyt) = one
    dV = zero
  else
    yield(xxt:yyt) = State(sYxt:sYyt)
    yield(xxc:yyc) = State(sYxc:sYyc)
    ryield = State(sPc)
    epl(xxt:yyt) = State(sepxt:sepyt)
    epl(xxc:yyc) = State(sepxc:sepyc)
    te(xx:zz) = State(stex:stez)
    dV = State(dvol)
    tem(xxt:zzt) = State(semxt:semzt)
    dmg(xxt:yyt) = State(dmgt:xmgyt)
    eeqpl = State(seeqp)
  end if
return
end
subroutine ut_asgn_props(E, xNu, G, Gf, props, ctable)
use def_par_dir
use def_par_props
include 'vaba_param.inc'
dimension E(:,), xNu(xy:), G(xy:), Gf(:,), props(:,), ctable(:, :)

e(XX:ZZ) = props(Ex: Ez)
xNu(xy:zx) = props(Nuxy: Nuxz)
G(xy:zx) = props(Gxy: Gzx)
Gf(xxt:yyt) = Props(Gfxt: Gfyt)
ctable(1,:) = Props(ncomp+1: Props(ncomp) + ncomp)
ctable(2,:) = Props(Props(ncomp) + ncomp+1: Props(ncomp) + ncomp*2)
return
donend

subroutine ut_assem_K_mtx(D, E, xNu, G)
use def_par_dir
use def_par_math
include 'vaba_param.inc'
dimension D(:,), E(:,), xNu(xy:), G(xy:)
xNu(yx) = xNu(xy) * E(yy)/E(xx)
Delta = one / (1 - xNu(yx) * xNu(xy))
D(K11) = E(xx) * Delta
D(K22) = E(yy) * Delta
D(K33) = E(zz)
D(K12) = E(xx) * Delta * xNu(yx)
D(K13) = zero
D(K32) = zero
D(K44:K66) = two * G(xy:zx)
return
donend

subroutine ut_asgn_tstate(state, yield, epl, te, tem, dmg, eeqpl, n ryield, reeqpl, props, dV)
use def_par_dir
use def_par_props
use def_par_state
include 'vaba_param.inc'
dimension yield(:,), epl(:,), te(:,), tem(:,), state(:,), dmg(:,), n props(:)

State(sYxt:sYyt) = yield(xxt:yyt)
State(sYxc:sYyc) = yield(xxc:yyc)
State(sYxy) = yield(xy)

State(sep_xt:sepyt) = epl(xxt:yyt)
State(sep_xc:sepyc) = epl(xxc:yyc)
State(seepq) = eepl
State(stex:stey) = te(xxt:yyt)

State(seqr) = reeqpl

State(SF_x:SF_yt) = yield(xxt:yyt)/props(pY_xt:pY_yt)
State(sFc) = ryield
State(dvol) = dV
State(tem_xt:tem_zt) = tem(xxt:zzt)
State(sdmg) = (dmg(xx)*dmg(yy))**(1.0/2.0)
State(dmg_xt:dmg_yt) = dmg(xxt:yyt)
return
END

subroutine sumepl(So, yield, epl, depl, eeqpl, reeqpl, syield_r, n syield_h, props)
use def_par_dir
use def_par_math
use def_par_props
include ’vaba_param.inc’
dimension epl(:), depl(:), tdepl(3), So(:), yield(:), props(:)

if (syield_r.ge.zero) then
  if (So(xx).lt.zero. and .So(yy).lt.zero) then
    if (props(pSc).eq.one) then
      epl(xxt) = epl(xxt) + reeqpl
      epl(yyt) = epl(yyt) + reeqpl
    else
      if (yield(xxt).lt.yield(yyt)) then
        epl(xxt) = epl(xxt) + reeqpl
      else
        epl(yyt) = epl(yyt) + reeqpl
      end if
    end if
  else
    if (props(pSc).eq.one) then
      epl(xxt) = epl(xxt) + abs(depl(xy))
      epl(yyt) = epl(yyt) + abs(depl(xy))
    else
      if (yield(xxt).lt.yield(yyt)) then
        epl(xxt) = epl(xxt) + abs(depl(xy))
      else
        epl(yyt) = epl(yyt) + abs(depl(xy))
      end if
    end if
  end if
end if
return
end

subroutine generate_dump(array)
include 'vaba_param.inc'
dimension array(:)
Logical OK
INQUIRE( UNIT=1, OPENED=OK )
IF ( OK ) Then
  write(1,"(9999(G12.5,:,','))") array
else
  open(unit = 1, file = "D:\Temp\Dump1.txt")
end if
return
end
subroutine sune(te,tem,de,So,syield_h,depl)
  use def_par_dir
  use def_par_math
include 'vaba_param.inc'
dimension te(:), de(:), tem(:),So(:),depl(:)

where (So(xx:yy) > zero)
  te(xx:yy) = te(xx:yy) + de(xx:yy)
endwhere

if (syield_h .ge. zero) then
  where (So(xx:yy) > zero)
    te(xx:yy) = te(xx:yy) - de(xx:yy)
  endwhere
end if

tem = max(te,tem)
return
end

subroutine ut_dev_stress(So, Sp, dS)
  use def_par_dir
  use def_par_math
include 'vaba_param.inc'
dimension So(:), Sp(:), dS(:)
dS = Sp - So
return
end

subroutine ut_vuhard(syield, hard, eqplas, table, nvalue)
include 'vaba_param.inc'
dimension table(2, nvalue)
parameter(zero=0.d0)
syiel=table(2, nvalue)
hard=zero

if(nvalue.gt.1) then
  do k1=1, nvalue-1
    eqpl1=table(1, k1+1)
    if(eqplas.lt.eqpl1) then
      eqpl0=table(1, k1)
      deqpl=eqpl1-eqpl0
      syiel0=table(2, k1)
      syiel1=table(2, k1+1)
      dsyiel=syiel1-syiel0
hard=dsyiel/deqpl
syield=syiel0+(eqplas-eqpl0)*hard
go to 10
   end if
   goto 10
  end do
end if
continue
end if
return
end

pure subroutine stable_time(D, beta, factor, cL, rho)
use def_par_dir
use def_par_math
include 'vaba_param.inc'
Real(8), intent(out):: factor
Real(8), intent(in) :: beta, cL, rho
Real(8), intent(in) :: D(:)

dimension omega(3)

omega(1) = Sqrt((D(K11)+D(K22)+sqrt(D(K11)**2-2*D(K11)*D(K22)
  + D(K12)**2*4+D(K22)**2))/(rho*cL**2))
omega(2) = Sqrt((D(K11)+D(K22)-sqrt(D(K11)**2-2*D(K11)*D(K22)
  + D(K12)**2*4+D(K22)**2))/(rho*cL**2))
omega(3) = Sqrt(2*D(K33)/(rho*cL**2))

omega_max = max(omega(1),omega(2),omega(3))

factor = sqrt(one+((half)*beta*omega_max)**2)-(half)*beta
  * omega_max
factor = (one/factor)**2
return
end
endmodule

module sb_other
public damage,assem_dmg_K,damping
contains

subroutine assem_dmg_K(dmg, Dd, D, So, tem, E,yield,epl,props)
use def_par_dir
use def_par_math
include 'vaba_param.inc'

integer*2 status
dimension dmg(:,), Dd(:,), D(:,), So(:,), tem(:,), E(:,), yield(:)
dimension status(size(dmg)), epl(:,), props(:)
status(xx:zz) = zero
if (So(xx) .gt. zero) status(xx) = 1
if (So(yy) .gt. zero) status(yy) = 1

Dd(K11:K22) = dmg(xx:yy)**status(xx:yy)*D(K11:K22)
Dd(K33) = D(K33)

Dd(K12) =floor(dmg(yy)**status(yy)*dmg(xx)**status(xx))*D(K12)
Dd(K13) = D(K13)
Dd(K32) = D(K32)

Dd(K44) = sqrt(dmg(yy)**status(yy) * dmg(xx)**status(xx) * D(K44)
D(K55) = D(K55)
D(K66) = D(K66)

return
end

subroutine damping(So, Sn, dt, Sdamo, beta)
use def_par_dir
use def_par_props
use def_par_math
include 'vaba_param.inc'

dimension So(:), Sn(:), Sdamo(:), Sdamn(size(Sdamo))

b = beta/dt
Sdamn = b * (Sn - So)
Sn = Sn + Sdamn
Sdamo = Sdamn

return
end

subroutine damage(dmg, Dd, D, Gf, E, yield, depl, props, cL, nte, tem)
use def_par_dir
use def_par_math
use def_par_props
include 'vaba_param.inc'

parameter (ratio_t=1.d0/100.d0, ratio_c=1.d0/100.d0)
integer*2 status

dimension dmg(:,), Dd(:,), D(:,), Gf(:,), E(:,), yield(:,), te(:,), tem(:)
dimension status(size(dmg)), depl(:,), props(:)

status = 0
protect = zero

Dd(K11:K22)

return
end

endmodule

module sb_yield_surfaces
public Yield_Rankine, Yield_Hills, hardening
contains

subroutine Yield_Rankine(syield, depl, dSpl, Sp, yield, Dd, cL, props, Gf, E, de, dir3, D, reeqpl,
n
use def_par_math
use def_par_dir
use def_par_props
use utility_routines
include 'vaba_param.inc'

integer  dirc
logical stat

dimension depl(:), Sp(:), yield(:), Dd(:), props(:), Gf(:),
nE(:), dSPl(:), de(:), D(:),Stn(:)
dimension a(size(Sp)),hard(size(Gf)),b(size(Sp))

parameter (power = 10)

alpha = props(pYat)

phi  = props(pYpt)

zmax = props(mdil)

temp = Sp(xy)

if (phi.le.zero) then
  factor = zero
  alphal = one
else
  if (zmax.eq.0) then
    factor = one
    alphal = one/(phi**2)
  elseif (dV.ge.zmax) then
    factor = zero
    alphal = one
  else
    factor = one
    alphal = one/(phi**2)
  end if
end if

Stn(3) = alphal
Stn(6) = factor

if ((Sp(xx).ge.yield(xxt).OR.Sp(yy).ge.yield(yyt))
  .AND.factor.eq.zero) then
  Sp(xy) = zero
  stat = .true.
end if

syield1=sqrt((half*(Sp(xx)-yield(xxt)-Sp(yy)+yield(yyt)))**2
n+alpha*Sp(xy)**2 )
syield = half*(Sp(xx)-yield(xxt)+Sp(yy)-yield(yyt))+syield1

if (syield.gt.zero) then
  if (syield1.ne.zero) then
    a(xx) = half+half**2*(Sp(xx)-yield(xxt)-Sp(yy)+yield(yyt))
    n / syield1
    a(yy) = half+half**2*(-Sp(xx)+yield(xxt)+Sp(yy)-yield(yyt))
    n / syield1
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\[ a(xy) = \text{two} \times \alpha \times \frac{Sp(xy)}{(\text{two} \times \text{syield1})} \]

\[
\text{Syield2} = \sqrt{\left(\frac{\text{half} \times (\text{factor} \times (\text{Sp(xx)} - \text{Sp(yy)}) - \text{yield(xxt)})}{\text{Syield2}} \right)^2 + \alpha \times \text{Sp(xy)}^2} 
\]

\[
\text{if (stat) then} \\
\quad b = a
\]

\[
\text{else} \\
\quad b(xx) = \text{half} \times \text{factor} \times \frac{(\text{factor} \times (\text{Sp(xx)} - \text{Sp(yy)}) - \text{yield(xxt)})}{\text{syield2}} \\
\quad b(yy) = \text{half} \times \text{factor} \times \frac{(\text{factor} \times (-\text{Sp(xx)} + \text{Sp(yy)}) + \text{yield(xxt)})}{\text{syield2}} \\
\quad \text{end if} \\
\]

\[
dlambdal = \frac{(\text{Dd(K11)} \times a(xx) + \text{Dd(K21)} \times a(yy)) \times b(xx)}{\text{two}} \\
dlambd = \text{syield} / \text{dlambdal} \\
depl = \text{zero} \\
depl(xx:yy) = \text{dlambd} \times b(xx:yy) \\
depl(xy) = \text{dlambd} \times b(xy) \\
\]

\[ \text{reeqpl} = \text{dlambd} \]

\[
\text{call ut_elastic(dSpl, \((/0.0d0,I=1,6)/\), depl, Dd)} \\
dSpl(xy) = \frac{dSpl(xy)}{\text{two}} \\
\]

\[
\text{else} \\
\quad dSpl(xx:yy) = \text{Sp(xx:yy)} - \text{yield(xxt:yyt)} \\
\quad \text{depl(xx)} = \frac{-(\text{Dd(K12)} \times dSpl(yy) - \text{Dd(K22)} \times dSpl(xx))}{\text{(Dd(K11)} \times \text{Dd(K22)} - \text{Dd(K12)} \times 2)} \\
\quad \text{depl(yy)} = \frac{(\text{Dd(K11)} \times dSpl(yy) - \text{Dd(K12)} \times dSpl(xx))}{\text{(Dd(K11)} \times \text{Dd(K22)} - \text{Dd(K12)} \times 2)} \\
\quad \text{reeqpl} = \sqrt{(\text{depl(xx)}^2 + \text{depl(yy)}^2)} \\
\]

\[
\text{if (stat) then} \\
\quad dSpl(xy) = \text{temp} \\
\quad \text{depl(xy)} = \frac{\text{two} \times \text{temp} / \text{Dd(K44)}}{\text{two}} \\
\quad \text{reeqpl} = \text{reeqpl} + \text{abs(depl(xy))} \\
\]

\[
\text{else} \\
\quad dSpl(xy) = \frac{dSpl(xy)}{\text{two}} \\
\]

\[
\text{end if} \\
\text{end if} \\
\]

\[
\text{return} \\
\text{end} \\
\]

\[
\text{subroutine return_to_inter(Sint,dSpl,a,Sr1,Sr2,Sh1,Sp)} \\
\text{use def_par_math} \]
if (dlambdal .ge. 0) then
    dlambdal = max(tiny(dlambdal), dlambdal)
end if

dlambda = syield/dlambdal
depl = zero
depl(xx:yy) = dlambda * a(xx:yy)
depl(xy) = dlambda * a(xy)
eeqpl = dlambda
call ut_elastic(dSpl, ($(\text{0.0d0, 1, 6})$), depl, Dd)
if(present(u)) then
    u = a
end if

return
end if

return
end

subroutine hardening(yield, bhard, epl, props, cL, Gf, E, D, tem, n nstate, dmg, eeqpl, ctable, ryield)
use def_par_math
use def_par_dir
use def_par_props
use utility_routines

include 'vaba_param.inc'
dimension yield(:), Gf(:), epl(:), props(:), bhard(:), tem(:), E(:)
dimension ein(size(E)), ctable(:,:), tyield(size(yield))
dimension dmg(:,), D(:)
parameter (ratio_t = 1.0d0/100.0d0, ratio_c = 1.0d0/100.0d0)
call ut_vuhard(ryield, hard, eeqpl, ctable,
    int(props(ncomp)))
yield(xxc:yyc) = max(props(pYxc:pYyc) * ratio_c, props(Fucx:Fucy) * ryield)
if (eeqpl.gt.props(ecpl)) then
    ryield = yield(yyc) / (props(Fucy))
else
    ryield = one
end if

tyield(xxt) = min(props(pYxt), max(props(pYxt) * ratio_t,
    -props(pYxt)
    * (D(K11) * props(pYxt) * cL * epl(xxt)
    - props(pYxt)**two * cL - two * D(K11)
    * Gf(xxt)) / (props(pYxt)**two * cL + two
    * D(K11) * Gf(xxt))))
yield(xxt) = min(props(pYxt), max(props(pYxt) * ratio_t,
    props(pYxt) - props(pYxt)**2 * cL


yield(xxt) = min(props(pYxt), max(props(pYxt) * ratio_t, ryield
n * (yield(xxt) - (props(pYxt) - tyield(xxt)))))

yield(yyt) = min(props(pYyt), max(props(pYyt) * ratio_t, n
(D(K22) * props(pYyt) * cL * epl(yyt)
- props(pYyt)**two * cL - two * D(K22)
* Gf(yyt)) / (props(pYyt)**two * cL + two
* D(K22) * Gf(yyt))))

yield(yyt) = min(props(pYyt), max(props(pYyt) * ratio_t, ryield
n * (yield(yyt) - (props(pYyt) - tyield(yyt)))))

return
end
endmodule

 subroutine vumat(
1 nblokc, ndir, nshr, nstatev, nfieldv, nprops, lanneal, 2 stepTime, totalTime, dt, cmname, coordMp, charLength, 3 props, density, strainInc, relSpinInc, 4 tempOld, stretchOld, defgradOld, fieldOld, 5 stressOld, stateOld, enerInternOld, enerInelasOld, 6 tempNew, stretchNew, defgradNew, fieldNew, 7 stressNew, stateNew, enerInternNew, enerInelasNew )

 use def_par_dir
 use def_par_props
 use def_par_math
 use def_par_state

 use utility_routines
 use sb_yield_surfaces
 use sb_other

 include 'vaba_param.inc'

dimension props(nprops), density(nblock),
1 coordMp(nblock, *), charLength(nblock),
2 strainInc(nblock, ndir+nshr), relSpinInc(nblock, nshr),
3 tempOld(nblock), stretchOld(nblock, ndir+nshr),
4 defgradOld(nblock, ndir+nshr+nshr),
5 fieldOld(nblock, nfieldv), stressOld(nblock, ndir+nshr),
6 stateOld(nblock, nstatev), enerInternOld(nblock),
7 enerInelasOld(nblock), tempNew(nblock),
8 stretchNew(nblock, ndir+nshr),
9 defgradNew(nblock, ndir+nshr+nshr),
10 fieldNew(nblock, nfieldv),
2 stressNew(nblock, ndir+nshr), stateNew(nblock, nstatev),
3 enerInternNew(nblock), enerInelasNew(nblock)

character*80 cmname
logical idam
parameter (idam = .TRUE.)
integer dirc
LOGICAL OK

dimension Sp(ndir+nshr), yield(2*ndir+nshr),
nepl(ndir*2+nshr), Gf(ndir*2+nshr), dmg(ndir), tote(ndir),
totemax(max(dir)), E(dir), xNu(xy:xy+dir*2-1), G(xy:xy+nshr-1),
ndepl(dir+nshr), t_depl(4,ndir+nshr), t_dSpl(4,ndir+nshr),
nD(dir*2+nshr), Dd(dir*2+nshr), dummy(2), ds(dir+nshr),
ndSpl(dir+nshr),ctable(2,props(ncomp)), syield(4),
na(size(Sp)),eeq(4), depl1(dir+nshr),Sr1(dir+nshr),
nSr2(dir+nshr),Sh1(dir+nshr),Sint(dir+nshr)

Sp = zero
yield = zero
epl = zero
Gf = zero
dmg = zero
tote = zero
totemax = zero
E = zero
xNu = zero
G = zero
depl = zero
t_depl = zero
t_dSpl = zero
D = zero
Dd = zero
dummy = zero
dS = zero
dSpl = zero
ctable = zero
yield = zero
a = zero
eeq = zero
depl1 = zero
Sr1 = zero
Sr2 = zero
Sh1 = zero
Sint = zero
t_dS = zero

eeqpl = zero
factor = zero
dV = zero
ryield = zero
syield = zero

call ut_asgn_props(E, xNu, G, Gf, props, ctable)
call ut_assem_K_mtx(D, E, xNu, G)

if(steptime.eq.zero) then
  do CONCURRENT (n = 1:nblock)
    call ut_elastic(StressNew(n,:),StressOld(n,:),
    n StrainInc(n,:),D)
    call stable_time(D, props(bdamp), factor,
    n charLength(n), density(n))
    StressNew(n,:) = StressNew(n,:) * factor
    call ut_int_energy(enerInternNew(n),enerInternOld(n),
    n StressNew(n,:),StressOld(n,:),
    n StrainInc(n,:),density(n))
  end do
else
  do CONCURRENT (n = 1:nblock)
    syield_h= -one
    syield_r= -one
    STRESSOLD(n,:)=STRESSOLD(n,:) - stateOld(n,dampx:dampzx)
    call ut_asgn_fstate(yield, epl, tote, totemax, props,
    n stateOld(n,:),dmg,eeqpl,ryield,dV)
    dV = dV + Sum(StrainINC(n,1:2))
    if (eeqpl.ge.props(ncomp+props(ncomp))) then
      if (SUM(StrainInc(n,:)).ne.zero) then
        stateNew(n,delp) = zero
      else
        StressNew(n,:) = StressOld(n,:)
        goto 1212
      end if
    else
      stateNew(n,delp) = one
    end if
    if (idam) then
      call assem_dmg_K(dmg, Dd, D, STRESSOLD(n,:),
      n totemax, E, yield, epl, props)
    else
      Dd = D
    end if
    call ut_elastic (Sp, STRESSOLD(n,:), StrainINC(n,:), Dd)
    call ut_dev_stress (StressOld(n,:), Sp, dS)
  call Yield_Rankine (syield(1),t_depl(1,:),t_dSpl(1,:),
  n Sp, yield,Dd, charLength(n), props, Gf, E,
  n StrainInc(n,:), dirc,D,eeq(1), Stateold(n,seqr),
  n stateNew(n,:),dV)
call Yield Hill (syield(2), t_depl(2,:), t_dSpl(2,:),
    Sp, yield, Dd, charLength(n), props, Gf,
    n E, StrainInc(n,:), D, eeq(2), a)

if (syield(1).ge.0. AND syield(2).ge.0.) then
    call Yield_Rankine (syield(3), t_depl(3,:),
        t_dSpl(3,:), Sp-t_dSpl(2,:), yield, Dd, charLength(n),
        props, Gf, E, StrainInc(n,:), dirc, D, eeq(3),
    n Stateold(n,seqr), stateNew(n,:),dV)
    call Yield_Hill (syield(4), t_depl(4,:),
        t_dSpl(4,:), Sp-t_dSpl(1,:), yield, Dd, charLength(n),
        props, Gf, E, StrainInc(n,:), D, eeq(4))

if (syield(3).ge.0. AND syield(4).ge.0.) then
    Sr1 = Sp - t_dSpl(1,:)
    Sh1 = Sp - t_dSpl(2,:)
    Sr2 = Sh1 - t_dSpl(3,:)
    call return_to_inter(Sint,dSpl,a,Sr1,Sr2,Sh1,Sp)
    syield_h = syield(2)
    syield_r = syield(1)
    reeqpl = eeq(1)
    eeqpl = eeqpl + eeq(2)
    depl1 = t_depl(2,:)
    depl = t_depl(1,:)
elseif (syield(3).lt.0. AND syield(4).ge.0.) then
    dSpl = t_dSpl(2,:)
    depl = t_depl(2,:)
    syield_h = syield(2)
    syield_r = -1.0d0
    reeqpl = zero
    eeqpl = eeqpl+eeq(2)
    depl1 = zero
else
    dSpl = t_dSpl(1,:)
    depl = t_depl(1,:)
    syield_h = -1.0d0
    syield_r = syield(1)
    reeqpl = eeq(1)
    eeqpl = eeqpl
    depl1 = zero
end if
elseif (syield(1).lt.0. AND syield(2).ge.0.) then
    dSpl = t_dSpl(2,:)
    depl = t_depl(2,:)
    syield_h = syield(2)
    syield_r = -1.d0
    reeqpl = zero
    eeqpl = eeqpl+eeq(2)
    depl1 = zero
else
    dSpl = t_dSpl(1,:)
    depl = t_depl(1,:)
    syield_h = -1.d0
    syield_r = syield(1)
reeqpl = eeq(1)
eeqpl = eeqpl
depl1 = zero

end if

continue

call sume(tote,totemax,STRAININC(n,:),StressOld(n,:),
syield_h,depl)

if (syield_r .ge. zero .or. yield_h .ge. zero) then

call sumep(stressOld(n,:), yield, epl, depl,
eeqpl, reeqpl, syield_r, yield_h,
props)

call hardening(yield, dummy, epl, props, charLength(n),
Gf, E, D, totemax, 1, dmg, eeqpl,
ctable, ryield)

call damage(dmg, Dd, D, Gf, E, yield, depl, props, charLength(n), tote, totemax)

STRESSNEW(n,:) = Sp - dSp
else

STRESSNEW(n,:) = Sp

end if

call damping(STRESSOLD(n,:), STRESSNEW(n,:), dt,
statenew(n,dampx:dampzx), props(bdamp))

call ut_asgn_tstate(stateNew(n,:),yield,epl,tote,
totemax,dmg,eqeqpl,ryield,
Stateold(n,seqr)+reeqpl,props,dV)

E = props(Ex:Exz)

call ut_int_energy(enerInternNew(n), enerInternOld(n),
StressNew(n,:), StressOld(n,:),
StrainInc(n,:), density(n))

call ut_ine_energy(enerInelasNew(n), enerInelasOld(n),
StressNew(n,:), StressOld(n,:),
depl, depl1, density(n))

continue
end if
return
end