Void fraction and pressure waves in bubble and slug flow
design of experimental setup

P. J. W. Clarijs

Delft University of Technology
Faculty of Applied Physics
Kramers Laboratorium voor Fysische Technologie
Prins Bernardlaan 6, 2628 BW Delft

Professor: prof.dr.ir. H.E.A. van den Akker
Supervisor: ir. H.J.W.M. Legius
Summary

In vertical two phase flow, different flow patterns exist, varying from bubble flow at low gas fractions to annular flow at very high gas fractions. Changes in gas fraction or pressure propagate through the system like waves and can lead to unstable flow. The aim of the project is to investigate these waves in the bubble and slug flow regime.

For this purpose a column has been designed which makes it possible to investigate both bubble and slug flow. The column consists of two pipes with different diameters, connected via a restriction. In the column both upward water and air flow are possible. In the lower part of the column (15 cm i.d.) bubble flow and in the upper part (5 cm i.d.) slug flow will appear. Preliminary experiments have been performed in a shorter setup to investigate the use of several measurement techniques.

Measurements on "steady-state" properties of flow and on waves have been performed with pressure transducers, placed along the column. Also making use of a laser and a high speed camera has been investigated.

The results of the stationary slug flow experiments show little agreement with theory, because the length of the upper pipe of the preliminary setup is insufficient for developed slug flow and the pressure signals are disturbed by the knock out vessel on top of the column. Measuring with the laser does not suffer from these disturbances and is therefore recommended for further research in the large column.

The propagation of void fraction waves has been obtained indirectly by measuring the propagation velocity of a bubble front, which agrees with the Richardson & Zaki model for air/water systems. Continuous void fraction waves have been realized by making use of a compressor. As expected from theory, the deformation of the produced approximately sinusoidal profile to a damped sinusoid can be seen. However, due to the high frequency of void fraction fluctuations, the transition via a saw tooth profile to this damped sinus could not been shown experimentally.

Pressure waves are generated by applying a temporary overpressure in the gas inlet. In the bubble regime the propagation velocities agree with theoretical predictions. In the slug flow regime it was hard to obtain propagation velocities, and the results show little agreement with theory. The slug flow in the preliminary setup is not fully developed and therefore more experiments with a higher scan frequency in the larger setup are recommended.
Contents

Summary i

Contents iii

Symbols vii

1. Introduction 1
   1.1 Instabilities in vertical two phase flow 1
   1.2 Aim and approach of this project 1
   1.3 Arrangement of the report 3

2. Instable two phase flow 5
   2.1 Introduction 5
   2.2 Stationary flow 7
      2.2.1 Bubble flow 7
      2.2.2 Slug flow 17
      2.2.3 The bubble-slug flow transition 25
   2.3 Void fraction waves 31
   2.4 Pressure waves 35
      2.4.1 Introduction 35
      2.4.2 Bubble flow 35
      2.4.3 Slug flow 41

3. Experimental setup 45
   3.1 Introduction 45
   3.2 The test column 45
      3.2.1 The column-size 45
      3.2.2 The restriction 47
      3.2.3 The bubble generator 47
      3.2.4 Generation of pressure and void fraction waves 47
      3.2.5 The pump 49
   3.3 The preliminary setup 51
      3.3.1 Introduction 51
      3.3.2 Dimensions of the preliminary setup 51
      3.3.3 Air and water supply 51
      3.3.4 Generation of void fraction and pressure waves 51
4. Measurement methods

4.1 Introduction 53
4.2 Measurement techniques 53
   4.2.1 Pressure transducers 53
   4.2.2 Laser 53
   4.2.3 High speed camera 53
4.3 Evaluation of the signals 55
   4.3.1 Determination of the gas fraction 55
   4.3.2 Slug flow 55
   4.3.3 Void fraction waves 59
   4.3.4 Pressure waves 59

5. Results 61

5.1 Introduction 61
5.2 Stationary slug flow 61
   5.2.1 Results obtained with pressure transducers 61
   5.2.2 Results of laser and high speed camera 65
5.3 Void fraction waves 67
   5.3.1 Void fraction wave propagation determined
       by the bubble front velocity 67
   5.3.2 Producing void fraction waves with the compressor 67
5.4 Pressure waves 69
   5.4.1 Introduction 69
   5.4.2 Bubble flow 69
   5.4.3 Slug flow 69

6. Conclusions and recommendations 73

References 77

Appendices
   Appendix A 81
   Appendix B 83
   Appendix C 87
## Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>cross sectional area of the pipe</td>
<td>m²</td>
</tr>
<tr>
<td>a</td>
<td>pressure wave propagation velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>a_B</td>
<td>pressure wave propagation velocity in bubble flow</td>
<td>m/s</td>
</tr>
<tr>
<td>a_L</td>
<td>pressure wave velocity in gas part of bubble flow</td>
<td>m/s</td>
</tr>
<tr>
<td>a_L,L</td>
<td>pressure wave velocity in liquid part</td>
<td>m/s</td>
</tr>
<tr>
<td>a_G</td>
<td>acoustic speed in air</td>
<td>m/s</td>
</tr>
<tr>
<td>a_L</td>
<td>acoustic speed in liquid</td>
<td>m/s</td>
</tr>
<tr>
<td>a_m</td>
<td>acoustic speed in a mixture</td>
<td>m/s</td>
</tr>
<tr>
<td>a_d</td>
<td>acoustic speed in slug flow regime</td>
<td>m/s</td>
</tr>
<tr>
<td>C_d</td>
<td>drag coefficient</td>
<td>-</td>
</tr>
<tr>
<td>C_m</td>
<td>added-mass coefficient</td>
<td>-</td>
</tr>
<tr>
<td>C_o</td>
<td>distribution coefficient</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>diffusion coefficient</td>
<td>m²/s</td>
</tr>
<tr>
<td>D</td>
<td>pipe diameter</td>
<td>m</td>
</tr>
<tr>
<td>f</td>
<td>pipe roughness</td>
<td>-</td>
</tr>
<tr>
<td>F</td>
<td>lateral area of the pipe's section</td>
<td>m²</td>
</tr>
<tr>
<td>f_Ls</td>
<td>friction factor associated with the liquid slug</td>
<td>-</td>
</tr>
<tr>
<td>F_b</td>
<td>resultant force acting on a rigid spherical bubble</td>
<td>N</td>
</tr>
<tr>
<td>F_d</td>
<td>actual drag force on the bubble</td>
<td>N</td>
</tr>
<tr>
<td>F_s</td>
<td>gravitational force</td>
<td>N</td>
</tr>
<tr>
<td>F_v</td>
<td>virtual-mass force</td>
<td>N</td>
</tr>
<tr>
<td>F_p</td>
<td>force due to the pressure gradient in the liquid far from the bubble</td>
<td>N</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>I</td>
<td>impulse of the motion of the bubble</td>
<td>kgm/s</td>
</tr>
<tr>
<td>L</td>
<td>pipe's length</td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>length of a slug unit</td>
<td>m</td>
</tr>
<tr>
<td>L_G</td>
<td>length of the gas part in a slug unit</td>
<td>m</td>
</tr>
<tr>
<td>L_L</td>
<td>length of the liquid part in a slab unit</td>
<td>m</td>
</tr>
<tr>
<td>l_e</td>
<td>entrance length, required to form stable slugs</td>
<td>m</td>
</tr>
<tr>
<td>L_LS</td>
<td>length of the liquid slug</td>
<td>m</td>
</tr>
<tr>
<td>L_S</td>
<td>length of the slug unit</td>
<td>m</td>
</tr>
<tr>
<td>L_TB</td>
<td>Length of the Taylor bubble</td>
<td>m</td>
</tr>
<tr>
<td>L_LS</td>
<td>Length of the liquid slug</td>
<td>m</td>
</tr>
<tr>
<td>m_i</td>
<td>liquid slug mass</td>
<td>kg</td>
</tr>
<tr>
<td>n</td>
<td>the number of slug units</td>
<td>-</td>
</tr>
<tr>
<td>n</td>
<td>number of bubbles per unit volume</td>
<td>-</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
<td>N/m²</td>
</tr>
<tr>
<td>p_i</td>
<td>undisturbed liquid pressure</td>
<td>N/m²</td>
</tr>
<tr>
<td>p_0</td>
<td>pressure in front of and to the rear of a shock wave</td>
<td>N/m²</td>
</tr>
<tr>
<td>ΔP_A</td>
<td>acceleration pressure drop</td>
<td>N/m²</td>
</tr>
<tr>
<td>ΔP_H</td>
<td>hydrostatic pressure drop</td>
<td>N/m²</td>
</tr>
<tr>
<td>Symbol</td>
<td>description</td>
<td>unit</td>
</tr>
<tr>
<td>----------</td>
<td>-------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>$\Delta P_f$</td>
<td>frictional pressure drop</td>
<td>N/m²</td>
</tr>
<tr>
<td>$\Delta P_T$</td>
<td>total pressure drop</td>
<td>N/m²</td>
</tr>
<tr>
<td>$Q_g$</td>
<td>volume flowrate of the dispersed phase</td>
<td>m³/s</td>
</tr>
<tr>
<td>$Q_L$</td>
<td>liquid volume flow rate</td>
<td>m³/s</td>
</tr>
<tr>
<td>$Re_{LS}$</td>
<td>Reynolds number of the liquid slug</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>radius of the bubble</td>
<td>m</td>
</tr>
<tr>
<td>$S$</td>
<td>signal</td>
<td>-</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>$t_g$</td>
<td>time the pressure wave needs to pass the gas part of the slug unit</td>
<td>s</td>
</tr>
<tr>
<td>$t_l$</td>
<td>time the pressure wave needs to pass the liquid part</td>
<td>s</td>
</tr>
<tr>
<td>$U_{shock}$</td>
<td>shock speed</td>
<td>m/s</td>
</tr>
<tr>
<td>$u_G$</td>
<td>bubble velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$u_{Ga}$</td>
<td>bubble-phase superficial velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$u_{La}$</td>
<td>liquid superficial velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$u_l$</td>
<td>the interstitial liquid velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$u_L$</td>
<td>liquid velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$&lt;u_l&gt;$</td>
<td>the liquid average velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$&lt;u_g&gt;$</td>
<td>the gas average velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{GO}$</td>
<td>mean gas velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_t$</td>
<td>the terminal rise velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_0$</td>
<td>mixture velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{TB}$</td>
<td>Velocity of the Taylor bubble</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{LTB}$</td>
<td>Velocity of the liquid film around the Taylor bubble</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{GTB}$</td>
<td>Velocity of the gas in the Taylor bubble</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{GLS}$</td>
<td>Velocity of the gas in the liquid slug</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{LLS}$</td>
<td>Velocity of the liquid in the liquid slug</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{SG}$</td>
<td>superficial gas velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{SL}$</td>
<td>superficial liquid velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{TB}$</td>
<td>Taylor bubble velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_m$</td>
<td>mixture velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$V$</td>
<td>volume of the body</td>
<td>m³</td>
</tr>
<tr>
<td>$v$</td>
<td>void fraction propagation speed</td>
<td>m/s</td>
</tr>
<tr>
<td>$V_G$</td>
<td>bubble volume</td>
<td>m³</td>
</tr>
<tr>
<td>$V_G$</td>
<td>gas volume of the slug unit</td>
<td>m³</td>
</tr>
<tr>
<td>$V_{SU}$</td>
<td>total volume of the slug unit</td>
<td>m³</td>
</tr>
<tr>
<td>$x$</td>
<td>direction</td>
<td>m</td>
</tr>
<tr>
<td>$z$</td>
<td>direction</td>
<td>m</td>
</tr>
<tr>
<td>$a'$</td>
<td>amplitude of a void fraction wave</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>gas void fraction</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_{TB}$</td>
<td>Void fraction of the Taylor bubble</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_{LS}$</td>
<td>Void fraction of the liquid slug</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_{SU}$</td>
<td>Void fraction of the slug unit</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$L_{TB}/L_{SU}$</td>
<td>-</td>
</tr>
<tr>
<td>Symbol</td>
<td>description</td>
<td>unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>γ</td>
<td>ratio of specific heats, namely $c_p/c_v$</td>
<td>-</td>
</tr>
<tr>
<td>κ</td>
<td>rigidity</td>
<td>N/m</td>
</tr>
<tr>
<td>$\mu_{LS}$</td>
<td>viscosity of the liquid slug</td>
<td>Pa.s</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>liquid slug frequency</td>
<td>1/s</td>
</tr>
<tr>
<td>$\Delta \rho$</td>
<td>density difference</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>liquid density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>gas density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>surface tension</td>
<td>N/m</td>
</tr>
<tr>
<td>$\omega$</td>
<td>frequency of the signal</td>
<td>Hz</td>
</tr>
</tbody>
</table>
1. Introduction

1.1 Instabilities in vertical two phase flow

Gas-liquid two-phase flow, unlike single-phase flow, cannot be classified simply into laminar, transitional or turbulent flow, since the manner in which the phases are interdispersed affects the overall fluid properties. Basically there are four patterns in vertical upward gas-liquid flow, termed from bubbly flow for the low gas fractions, via slug and churn flow, leading to annular flow for high gas fractions. A peculiar regime is slug flow, because this is a kind of transition regime where liquid and gas are both separated and dispersed, which leads to periodic pressure fluctuations.

Two phase flow is frequently encountered in industry. One may think of two phase flow in large pipelines, two phase reactors, reboilers and vaporisers in distillation columns.

Sudden changes in gas fraction or pressure can cause instabilities which propagate through the system like waves. One of the problems which can arise with these different types of waves is that pressure fluctuations in two phase systems cause vibrations which can lead to noise production or even breakage of pipes.

The aim of this project is to investigate these instabilities in order to be able to predict when these phenomena take place. This research may make it possible in the future to design industrial equipment properly so that for example two phase flow pulsations are taken into account.

1.2 Aim and approach of this project

At the Kramers Laboratorium voor Fysische Technologie a new project has been started recently, which is concentrated on the understanding of two phase flow instabilities. The object of the research project is to develop physical-mathematical models for some two phase flow systems occurring in practical problems.

In order to validate these models experiments are done where two phase flow instabilities are measured. For this purpose measurements have been done in a preliminary setup. These experiments are meant to gain a better understanding of the possibilities of a larger setup which has been built recently.

The preliminary setup consists of three sections, namely two vertical pipes, with a large diameter in the lower part compared to a small diameter in the upper part, and a contraction inbetween. This may lead to bubbly flow in the lower part of the setup, a transition to slug flow in the contraction and slug flow in the upper part.
In order to measure two phase flow instabilities different types of measurement techniques have been used. The basic technique is the usage of pressure transducers along the pipe in order to measure pressure differences during experiments. The pressure transducers are used for obtaining slug flow parameters and propagation speed of different instability waves.

In order to compare pressure measurements with the actual view a high speed camera has been used.

A new technique is making use of a laser which will probably give better results for slug flow parameters than the pressure transducers. In the column a laser beam goes at a certain point through the pipe, the light signal is received and send to a computer. Because slug flow consists of alternating liquid slugs and large bubbles the value of the signal when liquid passes the beam differs when gas passes the beam. Only preliminary measurements have been performed for this technique.

This research has been carried out for a master thesis of graduation project at the department of Chemical Engineering in Delft. The research has been done at the Kramers Laboratorium voor Fysische Technologie, as a part of the Ph. D. project of ir. H.J.W.M. Legius. The research has been supervised by ir. Legius and by prof. dr. ir. H.E.A. van den Akker.

1.3 Arrangement of the report

In chapter 2 some existing theories about two phase flow and especially instabilities in the bubbly and slug flow regime will be explained. The experimental setups and measurement methods are given in chapter 3 and 4.

Results of the measurements are explained and discussed in chapter 5. In chapter 6 the conclusions of this research are summarized and recommendations for further research are done.
Figure 1: Flow patterns in vertical flow (Taitel et al.)
2. Instable two phase flow

2.1 Introduction

In gas-liquid two phase flow one can distinguish basically four different flow patterns, termed "bubble", "slug", "churn", and "annular" flow, as illustrated in figure 1.

Bubble flow is characterized by a dispersion of gas bubbles in a liquid. At high void fractions the bubbles tend to coalesce into large bubbles alternated by liquid slugs, where the so-called Taylor bubbles have a diameter almost equal to the pipe diameter. This is the slug regime. At even higher gas fractions the transition to churn flow will take place, where the flow is violent and the phases are interdispersed in packets of irregular shape. Annular flow, found for the highest gas fractions consists of a film of liquid flowing along the pipe wall, surrounding a core of flowing gas. Additional liquid may be dispersed as small droplets within the gas core.

In these different flow patterns instabilities can occur which may propagate through the two phase system in the form of waves. One type of waves are void fraction waves. They are characterized by a sudden change in gas fraction which passes as a wave through the pipe system. The propagation of these waves is dependent on the gas fraction.

Another type of wave is a pressure wave. The difference with void fraction waves which are described before is that the source for pressure wave propagation is momentum. The propagation of pressure waves is equal to the velocity of sound in a two-phase system. This velocity depends on the gas fraction and is much larger than the propagation of void fraction waves.

Both void fraction and pressure waves can lead to shock waves. A shock wave is characterized by a discontinuity in for example pressure.

In order to describe unsteady two phase flow first some of the theory of stationary bubble and slug flow will be explained in sections 2.2. Void fraction and pressure waves are described in sections 2.3 and 2.4.
2.2 Stationary flow

2.2.1 Bubble flow

In order to be able to examine two phase flow in a vertical pipeline it is necessary to have a theoretical model that describes important flow parameters like void fraction, density and velocities of the two phase system at every point in the pipe. In the experiments both bubble and slug flow are examined in a setup which consists of a contraction inbetween two pipes with different diameters. An excellent model for describing bubble flow in a contraction is given by Couët et al. [1991]. This model describes the flow in a vertical pipeline through a contraction.

Definitions

The flow is considered to be one-dimensional which implies that all bubbles at a given cross-section move with the same velocity. The bubbles are assumed to be rigid spheres with the same density and volume. Some of the nomenclature will now be described.

Bubbles are assumed to move with a velocity \( u_g \). The bubble-phase superficial velocity \( u_{Q,s} \) is given by

\[
u_{Q,s} = \frac{Q_g}{A}
\]  

where \( Q_g \) is the volume flowrate of the dispersed phase and \( A \) the cross sectional area of the pipe. The bubble velocity (which is the same as the average gas velocity) is then:

\[
u_g = \frac{u_{Q,s}}{\alpha}
\]  

For the liquid, the superficial velocity \( u_{L,s} \) is given by

\[
u_{L,s} = \frac{Q_L}{A}
\]  

where \( Q_L \) is the liquid volume flow rate. The average liquid velocity, for an illustration see figure 2, is then

\[
\langle u_L \rangle = \frac{u_{L,s}}{1 - \alpha}
\]  

The interstitial velocity \( u_L \) serves to represent the liquid velocity in the space between the bubbles,
Figure 2: Illustration showing the bubble velocity $u_b$, the interstitial velocity $u_l$, and the liquid average velocity according to Couët et al.
Unsteady two phase flow in pipes

, see figure 2. \( u_i \) can be thought of as the background velocity field determining the motion of any bubble. \( u_i \) and \( \langle u_i \rangle \) can be expressed in terms of the interstitial velocity by making use of the concept of virtual mass in the displacement of the bubbles through the liquid. When a body moves through an infinite volume of incompressible inviscid fluid at rest, it induces a drift in the fluid such that the drift-volume of fluid is equal to \( C_m V \), where \( V \) is the volume of the body and \( C_m \) is known as the added-mass coefficient. For a rigid sphere \( C_m \) has the value 0.5. The liquid flowrate across a surface area \( A \) is

\[
Q_L = u_{Ls} A = [(1 - \alpha)u_L + \alpha C_m (u_G - u_L)] A
\]  

(5)

The liquid average velocity can now be written as

\[
\langle u_i \rangle = u_L + \frac{\alpha}{1 - \alpha} C_m (u_G - u_L)
\]  

(6)

The interstitial velocity can also be rewritten in terms of the volume flowrates and the bubble velocity:

\[
u_L = \frac{(Q_L - C_m Q_G) u_G}{A u_G - (1 + C_m) Q_G}
\]  

(7)

Continuity and momentum equations for the gas phase

The continuity equation for the gas phase for constant densities and varying cross-sectional area is:

\[
\frac{\partial}{\partial t} (A \alpha) + \frac{\partial}{\partial x} (A \alpha u_g) = 0
\]  

(8)

If the x-axis is streamwise and upwards, the momentum equation for the gas phase in a non-steady state can be written as follows:

\[
\rho_g \frac{\partial}{\partial t} (u_g) + \rho_g \frac{\partial}{\partial x} (u_g^2) = n F_b
\]  

(9)
Unsteady two phase flow in pipes

where \( F_b \) is the resultant force acting on a bubble of volume \( V_b \) moving at a velocity \( u_O \). \( F_b \) can be decomposed into four uncoupled contributions,

\[
F_b = F_p + F_g + F_v + F_d
\]  

which are described below.

The force \( F_p \) due to the pressure gradient in the liquid far from the bubble is given by

\[
F_p = -p \frac{\partial p}{\partial x} = \rho V g \left( g + \frac{\partial u_L}{\partial t} + \frac{\partial^2 u_L}{\partial x^2} \right)
\]  

where \( p \) is the undisturbed liquid pressure and \( g \) is the gravitational acceleration. The gravitational force \( F_g \) exerted on the bubble is

\[
F_g = -\rho g V g
\]

Couët et al. say that the virtual-mass force \( F_v \) can be thought of as a drag force in its inviscid limit, i.e. the inertia force due to the local acceleration of the added mass of liquid travelling with the bubble, and is written as

\[
F_v = -\left( \frac{D I}{D t} + I \frac{\partial u_L}{\partial x} \right)
\]

with:

\[
\frac{D g}{D t} = \frac{\partial}{\partial t} + u_O \frac{\partial}{\partial x}
\]  

where \( I \) is the impulse of the motion of the bubble. \( I \) is proportional to the slip velocity.

\[
I = \rho V g c_m (u_O - u_L)
\]
Substitution gives:

\[ F_v = -p_1 V g C_m \left[ \frac{\partial}{\partial t} (u g - u l) + u g \frac{\partial}{\partial x} (u g - u l) + (u g - u l) \frac{\partial u l}{\partial x} \right] \]  

(16)

The actual drag force \( F_d \) on the bubble is due to the viscous stresses changing the pressure distribution around the bubble. Neglecting the non-uniformity and unsteadiness of the surrounding flow, \( F_d \) can be expressed as

\[ F_d = \frac{1}{2} p_1 (u g - u l) |u g - u l| C_D \pi r^2 \]  

(17)

where \( r \) is the bubble radius and \( C_D \) is the drag coefficient. At high Reynolds number, \( C_D \) is approximately constant. Therefore, as an alternative, particularly when the bubble radius and the drag coefficient are unknown, the drag force can be rewritten in terms of \( U_t \), the terminal rise velocity of a bubble in an infinite stationary liquid, corresponding to a balance of the gravitational and drag forces (Wallis):

\[ U_t^2 = \frac{8 \Delta p g g}{3 p_1 C_d} \]  

(18)

where \( \Delta p = \rho_r - \rho_g \). This formula leads to

\[ F_d = -|\Delta p| V g \frac{(u g - u l) |u g - u l|}{U_t^2} \]  

(19)

Continuity and momentum equations for the liquid phase

The continuity equation for the liquid phase is:

\[ \frac{\partial}{\partial t} (A(1-\alpha)) + \frac{\partial}{\partial x} (A(1-\alpha) <u_l>) = 0 \]  

(20)

The momentum-balance equation for the liquid component by integrating the Navier-Stokes equations without viscosity terms over a fixed control volume is:
Unsteady two phase flow in pipes

\( \frac{\partial}{\partial x} \left[ \rho_l A u_l^2 + \frac{2 \alpha}{5(1-\alpha)} A \rho_l C_M (u_G - u_L)^2 \right] = -A (1-\alpha) \rho_g \frac{\partial A <p>}{\partial x} - A \alpha \left( \frac{F_b}{V_g} + \rho_g u_g - \rho_l u_L \frac{du_L}{dx} \right) \) (21)

Results and assumptions

Couët and Brown combine equations 8, 9, 20 and 21, in order to be able to describe at every point in the pipe and contraction;

- liquid and gas velocities
- void fraction
- density,

with the following basic assumptions:

- The terminal rise velocity for a small bubble in a stagnant liquid is given by the Harmathy relationship (Harmathy):
  \[ U_t = 1.53 \left( \frac{\sigma (\rho_l - \rho_g)}{\rho_l^2} \right)^{1/4} \] (22)

where \( \sigma \) is the surface tension.

- The bubble velocity is set to be
  \[ u_G = C_o (u_{L,t} + u_{G,r}) - \frac{\rho_l}{\Delta \rho} U_t \] (23)

where \( C_o \) is an empirical distribution coefficient related to the liquid velocity profile and the bubble concentration profile.

Shortcomings in this model are:

- The fact that the equations of motion are only correct for low void fractions, because otherwise interaction forces play a dominant role and coefficients \( C_o \) and \( C_M \) are difficult to describe.

- Furthermore, the use of a terminal rise velocity of a single bubble in a stationary liquid in the formulation of the drag force on the bubbles is also an obvious source of discrepancies since it is well-known that as the void fraction increases, the rise velocity may change due to mutual interaction. Another phenomenon is that gas bubbles coalesce as the void fraction increases, giving larger faster-moving bubbles.
Figure 3: A slug unit containing a Taylor bubble and a liquid slug (according to Sylvester).
Another approximation of less importance is the fact that the model is one-dimensional and thus assumes that at a given cross-section the bubbles all travel at the same speed.

The one-dimensional model has the disadvantage that lift- and related forces are not taken into account, so that axi-symmetrical flow is not described.

2.2.2 Slug flow

Four major types of modelling approaches have been used in literature to describe two-phase flow. These models are the homogeneous flow models, the drift flux models, the two-fluid models and the flow pattern-based models.

For slug flow, the flow pattern-based model is the most suited, since slug flow is a well defined pattern. In the flow pattern based models, the two phases are considered to be arranged in a prescribed geometrical configuration or flow pattern. One of the most important models has been presented by Taitel et al. [1980]. It is a fully deterministic method for predicting flow patterns for the steady upward flow of gas-liquid mixtures in vertical pipes. Another model, presented by Sylvester [1987], is a mechanistic model of two-phase, steady vertical upflow in pipes for the slug flow regime. The advantage of the Sylvester model compared to the model presented by Taitel is that the Sylvester model is more extensive so that less estimations have to be made. It permits the calculation of important properties like lengths and velocities of liquid slug and Taylor bubble, slug and Taylor bubble frequency and the pressure drop over a slug unit. The model is based on the assumption that the flow is fully developed and stable and will now be described.

Nomenclature

The flow is assumed to consist of so-called slug units. A slug unit contains one Taylor bubble with its surrounding liquid and one liquid slug. Liquid flows downwards between the Taylor bubble and the pipe wall as a thin falling film. The idealized slug unit and its model variables are shown in figure 3.

With in this figure:

- \( L_{SU} \) Length of the slug unit
- \( L_{TB} \) Length of the Taylor bubble
- \( L_{LS} \) Length of the liquid slug
- \( \alpha_{TB} \) Void fraction of the Taylor bubble
- \( \alpha_{LS} \) Void fraction of the liquid slug
- \( \alpha_{SU} \) Void fraction of the slug unit
- \( U_{TB} \) Velocity of the Taylor bubble
- \( U_{LTB} \) Velocity of the liquid film around the Taylor bubble
- \( U_{OTB} \) Velocity of the gas in the Taylor bubble
- \( U_{GLS} \) Velocity of the gas in the liquid slug
- \( U_{LLS} \) Velocity of the liquid in the liquid slug
Average void fraction in a slug unit

The average void fraction $\alpha_{SU}$ of the slug unit is defined by:

$$\alpha_{SU} = \frac{V_G}{V_{SU}}$$  \hspace{1cm} (24)

where $V_G$ is the gas volume of the slug unit and $V_{SU}$ is the total volume of the slug unit. It can be written in the form

$$\alpha_{SU} = \beta \alpha_{TB} + (1-\beta) \alpha_{LS}$$  \hspace{1cm} (25)

where $\beta = L_{TB}/L_{SU}$.

Gas and liquid phase mass balance

The mass balance for the gas phase of the slug unit is expressed in terms of the superficial gas velocity $U_{SG}$

$$U_{SG} = \beta \alpha_{TB} U_{GTB} + (1-\beta) \alpha_{LS} U_{GLS}$$  \hspace{1cm} (26)

Similarly the mass balance for the liquid phase of the slug unit can be written in terms of the superficial liquid velocity $U_{SL}$.

$$U_{SL} = (1-\beta)(1-\alpha_{LS})U_{LLS}-\beta(1-\alpha_{TB})U_{LTB}$$  \hspace{1cm} (27)

The Taylor bubble travels through the two-phase mixture with the velocity $U_{TB}$ which is greater than either $U_{GLS}$ or $U_{LLS}$. Relative to the Taylor bubble, the gas flow from the preceding liquid slug is equal to that flowing in the Taylor bubble if we assume there are no small gas bubbles in the liquid film surrounding the bubble. This can be expressed by the relation

$$(U_{GLS}-U_{TB})\alpha_{LS} = (U_{GTB}-U_{TB})\alpha_{TB}$$  \hspace{1cm} (28)

Similarly, the liquid flow approaching a Taylor bubble is equal to that being drained from the liquid film, which can be expressed by

$$(U_{TB}-U_{LLS})(1-\alpha_{LS}) = (U_{TB}+U_{LTB})(1-\alpha_{TB})$$  \hspace{1cm} (29)
Taylor bubble velocity

The Taylor bubble velocity can be written in the form

\[ U_{TB} = C_{0}(U_{SG}+U_{SL})+C_{1}\left[\frac{gD(\rho_{L}-\rho_{G})}{\rho_{L}}\right]^{1/2} \]  (30)

where \( D \) is the pipe diameter and \( g \) the gravity constant. Various values of \( C_{0} \) and \( C_{1} \) have been reported in literature. For air-water systems \( C_{0} \) is 1.2 and \( C_{1} \) equal to 0.35. The first contribution to the Taylor bubble velocity is due to the mixture velocity, where the second term takes into account the density difference between gas and liquid which gives an extra slip velocity.

Bubble velocity in liquid slugs

Assuming that we have fully developed bubble flow in the liquid slugs, the average rise velocity of the bubbles in the liquid slug is the sum of the average velocity of the liquid and the bubble buoyancy velocity

\[ U_{GLS} = U_{LLS}+U_{0} \]  (31)

Another expression than the Harmathy expression (formula 22) is the one by Zuber and Hench:

\[ U_{0} = 1.53 \left[ \frac{\alpha g(\rho_{L}-\rho_{G})}{\rho_{L}^{2}} \right]^{1/4} (1-\alpha_{LS})^{1/2} \]  (32)

The difference between this relation and the Harmathy equation is that this relation is especially for the turbulent bubble regime where the bubbles loose their separate identity.

Falling film velocity

A relation between the liquid film velocity and the liquid film thickness \( \delta_{L} \) has been obtained by Brotz, and can be written in the form

\[ U_{LFB} = 9.916 \ gD_{L}^{1-\sqrt{\alpha_{TB}}}^{1/2} \]  (33)
Now equations 25 to 33 can be solved numerically, if \( \alpha_{LS} \) is fixed.

Sylvester showed that an equation which gives \( \alpha_{LS} \) can be written in the form

\[
\alpha_{LS} = \frac{U_{SG}}{C_2 + C_3(U_{SG} + U_{SL})}
\]

(34)

with \( C_2 = 0.425 \) and \( C_3 = 2.65 \) found.

**Pressure drop in the slug unit**

To obtain the total pressure drop in the slug unit one has to know its three components, namely the acceleration pressure drop \( \Delta P_A \), the hydrostatic pressure drop \( \Delta P_H \) and the frictional pressure drop \( \Delta P_F \).

The acceleration pressure drop is taken to be that required to reverse the direction and accelerate the liquid film falling around the Taylor bubble to the velocity \( U_{LLS} \). This pressure drop can be written as

\[
\Delta P_A = \rho_L(U_{LTB} + U_{TB})(1 - \alpha_{TB})(U_{LTB} + U_{TB} + U_{LLS})
\]

(35)

The hydrostatic pressure drop of the slug unit can be written in the form

\[
\Delta P_H = \rho_L(1 - \alpha_{LS})g L_{LS}
\]

(36)

The largest contribution to the frictional pressure drop is the pressure drop for the liquid slug

\[
\Delta P_F = \frac{L_{LS}}{2D} \left[ \frac{\rho_G \beta U_{TB}^2}{(1 - \beta) [1 - (1 - \alpha_{TB}^2)]} + \frac{U_{LLS} \rho_L (1 - \alpha_{LS}) f_{LS} (1 - \beta)}{f_{LS}} \right]
\]

(37)

with (Colebrook)

\[
f_{LS} = \frac{1}{[-2.0 \log \left( \frac{f/D}{3.7} \right) - \left( \frac{5.02}{Re_{LS}} \log \left( \frac{f/D}{3.7} + \frac{13}{Re_{LS}} \right) \right)]^2}
\]

(38)

and

\[
Re_{LS} = \frac{\rho_L (1 - \alpha_{LS}) U_{LLS} D}{\mu_{LS}}
\]

(39)

where \( f_{LS} \) is the friction factor associated with the liquid slug, \( Re_{LS} \) the Reynolds number of the liquid slug and \( f \) the pipe roughness.
Adding up all the contributions for the pressure drop in the slug unit gives:

$$\Delta P_T = \rho_L(U_{LTB} + U_{TB})(1 - \alpha_{TB})(U_{LTB} + U_{TB} + U_{LLS}) + \rho_L(1 - \alpha_{LS})g L_{LS} + \frac{L_{LS}}{2D} \left[ \frac{\rho_G f_{TB} U_{GTB}^2}{(1 - \beta)(1 - \alpha_{TB})^{1/2}} + (U_{SG} + U_{SL}) \right] \rho_L(1 - \alpha_{LS} f_{LS}(1 - \beta))$$

A new variable is the length of the liquid slug $L_{LS}$, which can be scaled by multiplying the pipe diameter by a constant $\gamma$ which varies in literature from 6 to 48. Sylvester uses $\gamma = 40$.

**Liquid slug frequency**

The liquid slug frequency may then be written as

$$\nu_s = \frac{U_{TB}(1 - \beta)}{L_{LS}}$$

### 2.2.3 The bubble-slug flow transition

For the experimental setup it is important to know when the transition from bubble to slug flow takes place. The experimental setup consists of two pipes with different diameters and a contraction. In this paragraph the Taitel et al. [1980] flow map will be explained which gives bubble and slug flow area's as a function of superficial liquid and gas flow.

Now let us first consider what happens when bubble flow changes to slug flow. The transition from the condition of dispersed bubbles at low gas rates to slug flow requires a process of agglomeration and coalescence. In this way the discrete bubbles combine into larger gas pockets, having a diameter nearly that of the tube, alternated by liquid slugs. As the gas flow rate increases (in the contraction), the bubble density is increased. This closer bubble spacing results in an increase in the coalescence rate. However, because the liquid rate increases too, the turbulent fluctuations associated with the flow can cause breakup of larger bubbles formed as a result of agglomeration. If this breakup is sufficiently intense to prevent recoalescence, then a dispersed bubble pattern exists. So a transition from bubble to slug flow takes place, or a transition from bubble to dispersed bubble flow, according to Taitel. A model which describes these transitions has been given by Taitel et al. [1980].

At low gas flow rates in a large diameter vertical column of liquid gassed via a sieve plate, the gas phase is distributed into discrete bubbles. If the bubbles are very small, they behave as rigid spheres rising vertically in rectilinear motion. However, above a critical size (about 0.15 cm for air-water at low pressure) the bubbles begin to deform, and the upward motion is a zig-zag path with considerable randomness.
Figure 4: Flow pattern map for vertical tubes 5.0 cm diameter (Taitel et al. [10]).
The bubbles randomly collide and coalesce, forming a number of somewhat larger individual bubbles with a spherical cap form, more or less similar to the Taylor bubbles of slug flow, but with diameters smaller than the pipe. Thus, even at low gas and liquid flow rates, bubble flow is characterized by an array of smaller bubbles moving in zig-zag motion and the occasional appearance of larger, Taylor-type bubbles. The spherical cap shaped bubbles are not large enough to occupy the cross section of the pipe so as to cause slug flow. Instead, they behave as free rising spherically capped voids.

When, however, the gas flow rate increases sufficiently to cause so closely packed bubbles that many collisions occur, the rate of agglomeration to larger bubbles increases sharply. This results in slug flow.

Experiments suggest that the bubble void fraction at which this happens is up to 0.25 to 0.30. An approach to this problem was given by Radovicich et al. They postulated that the maximum void fraction is reached when the frequency of collision is very high, and it was shown that this happens around a void fraction of 0.30.

An alternative approach is to consider this problem from the point of view of maximum allowable packing of bubbles. If we consider the bubbles to have spherical shape and arranged in a cubic lattice, the void fraction of the gas can be, at most 0.52. However, as a result of their random path, the rate of collision and coalescence will increase sharply at void fractions well below this lattice spacing at which they touch. Therefore, the closest distance between the bubbles before transition must be the one which permits some freedom of motion for each individual bubble. If the spacing between the bubbles at which coalescence begins to increase sharply is assumed to be approximately half their radius, this corresponds to a void fraction of 25%.

Considering the transition to slug flow to occur when $\alpha=0.25$ results by using equations 2, 4 and 22 in an equation, which relates the superficial gas and liquid velocity (Taitel et al.):

$$U_{L,s} = 3.0U_{G,s}^{1.15} \left[ \frac{g(\rho_1 - \rho_g)\alpha}{\rho_1^2} \right]^{3/4}$$

The theoretical transition curve can be plotted on $U_{L,s}$ vs $U_{G,s}$ coordinates and should remain invariant with tube size. Such a curve is shown in figure 4 (curve A) for a water-air system of 25 °C and 1 bar.

At higher liquid flow rates, turbulent forces act to break up and disperse the gas phase into small bubbles even for void fractions higher than 0.25. This transition is shown in curve B. Since in the experimental setup only bubble and slug flow are examined, only curve A and D are of interest.
Figure 5: Flow pattern map for vertical tubes 2.5 cm diameter (Taitel et al. [10]).
Unsteady two phase flow in pipes

To the right of A and below B in zone III, one expects to see the slug pattern.

Whenever $U_{\text{gs}}$ increases slug flow will change into churn flow. This transition is depending on the entrance length $L_e$ and the pipe diameter $D$, because a certain entrance length to cause "stable" churn flow is necessary. A relation that describes the location of this transition has been given by Taitel et al. (curve D, equation in SI units):

$$L_e = 40.6 \left( \frac{U_{\text{gs}} + U_{\text{Ls}}}{\sqrt{gD}} + 0.22 \right)$$

(43)

For tubes with a diameter smaller than 5 cm bubble flow cannot exist in zone I (see figure 4), according to Taitel. This phenomenon will now be explained.

The rise velocity of the bubbles relative to the liquid, $U_o$, is given by the Harmathy equation (equation 22) and depends only on the properties of the fluids. The rise velocity of the Taylor bubbles relative to the mean velocity of the liquid on the other hand is given by (Taitel et al.):

$$U_o = 0.35 \sqrt{gD}$$

(44)

Whenever $U_g > U_o$ the rising bubbles approach the back of the Taylor bubble coalescing with it, increasing its size. Under these conditions bubbly flow cannot exist in zone I. On the other hand, when $U_g < U_o$ the Taylor bubble rises through the array of distributed bubbles and the relative motion of the liquid at the nose of the Taylor bubble sweeps the small bubbles around the larger one, and coalescence does not take place. The properties of air and water at normal pressure are such that $U_o = U_g$ at $D = 5.0$ cm (bubble size around 2.5 mm). Thus, for tubes smaller than 5 cm in diameter, no bubble flow can exist below the curve B and the entire zones I and III exist as the slug flow pattern. Only at high liquid rates, zone II, can bubbly flow exist for small tubes where dispersion occurs due to turbulence. The flow pattern map for 2.5 cm diameter tubes with the system air-water is shown in figure 5, where zones I and III are combined.

Thus, a system having a "small" diameter, is one which satisfies the criterion below (from $U_g \leq U_o$, equations 22 and 44):

$$\left[ \frac{\rho_g D^2}{(\rho_L - \rho_g)^2} \right]^{1/4} \leq 4.36$$

(45)

It is of interest that the range of diameters used in most laboratory air/water experiments, i.e., 2-6 cm, spans this critical diameter of 5.0 cm. Therefore in the upper part of the experimental setup usage of a pipe with a diameter of 5 cm or smaller is excellent for our purpose, because in that case slug flow will always appear.

Now the steady state flow is explained, and in the next two sections disturbances like void fraction and pressure waves will be discussed.


2.3 Void fraction waves

Void fraction waves or continuity waves are porosity fronts which move in one direction with a certain speed. The reason why void fraction waves are also called continuity waves is that they are primary based on the continuity equation. The continuity equations for the two phases are given below.

\[
\frac{\partial}{\partial t}(\rho_L(1-\alpha)) + \frac{\partial}{\partial x}(\rho_L(1-\alpha)u_L) = 0
\]

(46)

\[
\frac{\partial}{\partial t}(\rho_g\alpha) + \frac{\partial}{\partial x}(\rho_g\alpha u_g) = 0
\]

(47)

If one assumes that the gas phase is incompressible and its velocity is only a function of the gas fraction equation 46 can be written as:

\[
\frac{\partial}{\partial t}(u_g(\alpha)) + u_g(\alpha) \frac{\partial u_g}{\partial \alpha} = 0
\]

(48)

which gives the following void fraction propagation speed relation:

\[
v = u_g + \alpha \frac{\partial u_g}{\partial \alpha}
\]

(49)

Often in one-dimensional two-phase flow the velocities of liquid and gas are taken with respect to the mixture velocity \( U_0 \):

\[
U_0 = \alpha u_g + (1-\alpha)u_L
\]

(50)

The gas velocity can be defined as:

\[
u_g = U_0 + u_{go}
\]

(51)

with \( u_{go} \) as the mean gas velocity with respect to the mixture velocity \( U_0 \). This results in the following equation for the propagation speed:

\[
v = U_0 + u_{go} + \alpha \frac{du_{go}}{d\alpha}
\]

(52)
Figure 6: A continuity wave which develops into a shock wave (Rietema[1982]).

Figure 7: Periodical void fraction shock waves.
For the mean gas velocity $u_{go}$ the Richardson & Zaki correlation can be chosen:

$$u_{go} = C_1(1-\alpha)^{C_2}$$

(53)

with for air water systems $C_1$ is 0.214 m/s and $C_2$ 1.7 respectively (Lammers [1994]). Another formula is given by Barnea & Mizrahi (for high Reynolds numbers):

$$u_{go} = 1.53\frac{\sigma g(\rho_1 - \rho_L)}{\rho^2}1^{1/4} \left[1 - \frac{1-\alpha}{3(1-\alpha)^{1/3}}\exp\left(-\frac{\alpha}{\alpha^{1/3}}\right)\right]^{1/3}$$

Substitution of the Richardson & Zaki equation gives the propagation speed $v$ only depending on the gas fraction ($U_0$ in SI units):

$$v = U_0 + 0.214(1-\alpha)^{1.7} - 0.3638\alpha(1-\alpha)^{0.7}$$

(55)

**void fraction shock waves**

The velocity of a void fraction wave is depending on void fraction. This means that void fraction waves with different values of $\alpha$ propagate with different velocities. The continuity waves may overtake each other and then develops into a so-called continuity shock wave. This phenomenon is shown in figure 6.

Shock waves also arise when liquid and/or gas velocities suddenly change. When $U_{GS}$ is changed from a value $U_{GS1}$ to a value $U_{GS2}$ and $U_{LS}$ is varied from $U_{LS1}$ to $U_{LS2}$ relation 49 can be integrated to give:

$$v_{\text{shock}} = \alpha \frac{u_{GS1} - u_{GS2}}{\alpha_2 - \alpha_1} - (1 - \alpha) \frac{u_{LS1} - u_{LS2}}{\alpha_2 - \alpha_1}$$

(56)

**periodical void fraction waves**

Another type of void fraction waves which differ from the non-periodic waves which have been discussed above are periodic waves. An example is an air flow in a two phase system which is produced by a compressor. The compressor produces a sinusoidal void fraction profile. Again, different void fractions have different propagation velocities, which can result in a void fraction shock wave. See figure 7. For periodical signals this means that the sinusoidal profile (line 1 in figure 7) changes into a sawtooth profile (line 2). After some time due to diffusion the sawtooth profile will be damped (line 3) and finally result in a damped sinus (line 4).

The sawtooth signal can be given by (Biesheuvel [1993]):

$$\frac{\alpha' - \Sigma \sum \sin n\theta}{\alpha z}$$

(57)
with \( \alpha' \) as the amplitude of the sinus and \( \alpha \) the average void fraction, and:

\[
\theta = \omega (t - x/c_0), \quad \text{and} \quad z = (\omega \alpha c_1/c_0^2) x
\]

with \( \omega \) as the frequency of the signal, \( x \) the distance from the point where the signal was produced and the two constants \( c_0 \) and \( c_1 \) come from the following relation for the shock speed:

\[
v_{\text{shock}} = c_0 + 1/2(\alpha_1 + \alpha_2)c_1,
\]

where \( \alpha_1 \) and \( \alpha_2 \) denote the void fractions just before and after the shock.

An approximation of the damping sinusoid is given by (Biesheuvel):

\[
\frac{\alpha'}{\alpha} = 4 \nu e^{-\nu^2 \sin \theta} \tag{58}
\]

with:

\[
\nu = \omega D/(c_0 c_1 \alpha)
\]

where \( D \) is a diffusion coefficient. A typical value for \( D \) is \( 2.5 \times 10^{-3} \text{ m}^2/\text{s} \) (Biesheuvel).

### 2.4 Pressure waves

#### 2.4.1 Introduction

Pressure waves in two phase flow are characterized by a pressure difference which propagates through this system with a certain speed. This velocity is equal to the acoustic velocity. A medium containing gas and liquid reduces this acoustic velocity compared to the velocity in pure liquid or gas. The speed is depending on the gas fraction and on the flow regime. In the following two paragraphs different models describing the propagation speed in bubble and slug flow respectively will be discussed and compared with each other.

#### 2.4.2 Bubble flow

**pressure waves**

It is well known that gas bubbles reduce the propagation speed of pressure waves compared to that in the pure liquid itself.
Imaginary tubes consisting of gas and liquid zones according to the Nguyen et al. [1981] model.

Figure 8: Imaginary tubes consisting of gas and liquid zones according to the Nguyen et al. [1981] model.
The propagation speed of small-amplitude pressure waves is equal to the acoustic velocity \( a \), which can be approximated according to Henry [1969], following from the for the two phases combined continuity and momentum equations (homogeneous model):

\[
a = \frac{\gamma p}{\rho_l a (1 - a)}
\]

(59)

where \( p \) is the pressure, and for adiabatic conditions \( \gamma \) is the ratio of specific heats, namely \( c_p/c_v \). When assuming isothermal conditions between the phases \( \gamma \) goes to unity. When \( a \) goes to infinity or zero this equation is invalid. In those limiting cases the acoustic speeds for gas or liquid have to be used.

According to Eigner [1969] the pressure wave velocity takes a linear dependency of the voidage fraction, meaning

\[
a = a_G (-3a + 4)
\]

(60)

where \( a_G \) is the acoustic speed in air. Compared to experiments in literature this relation is not very useful.

Nguyen et al. [1981] also derived an expression for the propagation speed of pressure waves in bubble flow. In this model the system can be divided into imaginary parallel axial tubes. See figure 8. in each tube the wave front passes zones of the liquid phase and gas phase, just like in slug flow. Therefore, first the propagation speed in slug flow has to be derived. Assuming that the propagation speeds in liquid slug and Taylor bubble are equal to the acoustic velocity in liquid \( (a_L) \) and gas \( (a_G) \) respectively, the compound acoustic velocity becomes:

\[
a_s = \frac{a_L}{t_L} + \frac{a_G}{t_G} = \frac{L}{t_L} + \frac{L}{t_G} = \frac{L}{a_L} + \frac{L}{a_G}
\]

(61)

where \( t_i \) indicates the time the pressure wave needs to pass the liquid \( L \) or gas part \( G \) of a slug unit. Since the void fraction \( \alpha \) equals \( L_G \) over \( L \) relation 61 becomes:

\[
a_s = \frac{a_L a_G}{(1 - \alpha)a_G + \alpha a_L}
\]

(62)

In bubble flow \( a_G \) and \( a_L \) are replaced by \( a_{E,G} \) and \( a_{E,L} \) which are the propagation velocities in the liquid and gas parts of the imaginary tubes.
Figure 9: Comparison of the different models for predicting the pressure wave propagation speed in the bubble flow regime.
Nguyen derived the following relations:

\[
\alpha^2_{E,L} = \frac{1}{1 - \alpha + \frac{\alpha \rho_L}{\rho_G \alpha^2 G}} \tag{63}
\]

\[
\alpha^2_{E,G} = \frac{1}{\alpha + \frac{(1 - \alpha) \rho_G}{\rho_L \alpha^2 L}} \tag{64}
\]

Combining relations 62, 63, and 64 gives the propagation speed \(a_B\) in bubble flow:

\[
a_B = \frac{1}{(1 - \alpha) \sqrt{1 - \alpha + \frac{\alpha \rho_L}{\rho_G \alpha^2 G} + \alpha \sqrt{\frac{\alpha}{\alpha^2 G} + \frac{(1 - \alpha) \rho_G}{\rho_L \alpha^2 L}}} \tag{65}
\]

For the acoustic speed normalised with the acoustic velocity in air as a function of the void fraction for the models of Henry and Nguyen see figure 9. One may conclude that these two models have relatively the same curve. The advantage of the Nguyen model is that this model is valid for the whole gas fraction range, in the contrary to the other model which goes to infinity when approaching gas fractions of 0 and 1.

**Shock waves**

Just as with void fraction waves, also pressure waves velocities are dependent on the gas fraction, every part of the wave front propagates with different velocities. Steepening of the pressure wave front, illustrated in figure 10, can lead to the formation of a shock wave.

The propagation speed of shock waves is higher than the acoustic speed and is dependent on the strength of the shock wave \(p_t/p_0\), where \(p_0\) and \(p_t\) denote the pressure in front of and to the rear of the shock wave, respectively. For isothermal bubble behaviour, the Hugoniot relationship for a stationary shock wave in a gas-liquid mixture is

\[
\left(\frac{u_{\text{shock}}}{a_m}\right)^2 = \frac{p_t}{p_0} \tag{66}
\]

\(u_{\text{shock}}\) is the shock speed, and \(a_m\) is the acoustic velocity in the mixture at pressure \(p_0\).
Figure 10: Steepening of compression waves (Lighthill [1979]).

Figure 11: Mechanical model of a two-phase flow in the slug flow regime. (Korolev et al. [1989])
For adiabatic behaviour, the corresponding relationship reported by Van Wijngaarden is

$$\left(\frac{u_{\text{shock}}}{a_m}\right)^2 = \frac{1}{\gamma} \frac{(p_{1}/p_{2})^{-1}}{1 - (p_{2}/p_{1})^{1/\gamma}}$$  \hspace{1cm} (67)

2.4.3 Slug flow

In the bubble regime several pressure wave propagation models have been published. This in contrast to the slug flow regime, because of the complexity of this regime. Korolev et al. [1989] use a mechanical model to describe the propagation of pressure waves in horizontal slug flow. The model consists of slug units, where the liquid slugs are represented by a concentrated mass $m$, while Taylor bubbles are represented by springs with "rigidity" $k$, connecting neighbouring masses. For an illustration see figure 11.

From this model it follows that the speed of wave propagation can be written as:

$$a = \left(\frac{k}{m}\right)^{1/2} \frac{L}{n+1}$$  \hspace{1cm} (68)

where $L$ is the pipe's length, $n$ is the number of slug units. From the equation for an adiabatic gas charge given below

$$PV^\gamma = \text{const.}$$  \hspace{1cm} (69)

, we obtain an expression for the expression for the "rigidity" of the gas charge

$$k_i = \frac{\gamma p_{1}A}{L_{TB}}$$  \hspace{1cm} (70)

where $A$ is the area of the pipe section; $\gamma$ is the ratio of specific heats $c_p/c_v$, and $L_{TB}$ is the length of the Taylor bubble.

Now considering the case that no phase slip occurs in the flow, the number of slug units can be determined from the pipe's length and the length of a unit:

$$n = \frac{L}{(k_{LS} + k_{TB})}$$  \hspace{1cm} (71)

where $L_{LS}$ is the length of the liquid slug.
Figure 12: Comparison of the pressure wave models in the slug flow regime.
For the mass of a unit one can write:

$$m_1 = AL_{LS} \rho = (1-\alpha)(L_{LS}+L_{TB}) \rho A$$

(72)

For the length of a Taylor bubble:

$$L_{TB} = L \frac{\alpha}{n}$$

(73)

Rewriting the expressions for the wave propagation speed gives

$$a = \sqrt{\frac{\gamma P}{\alpha(1-\alpha)\rho}} \cdot \frac{1}{\left(1+\frac{1}{n}\right)}$$

(74)

This formula is not applicable at the boundaries of the two-phase region $\alpha$ to 0 or $\alpha$ to 1. The difference between this relation and the Korolev relation for bubbly flow is that the predicted propagation speed is always lower than in bubbly flow. When $n$ goes to infinity the two relations are equal. The minimal value for $n$ is 1, this means that in that case according to Korolev the propagation speed in slug flow is half of the one in bubbly flow. A value of $n$ is 1 means that there is one unit which fills the total pipe.

Another model has been presented by Eigner. The model again takes slug units, which consist of a Taylor bubble and a liquid slug for horizontal flow. The wave propagation speed for horizontal slug flow is found to be:

$$\frac{a}{a_g} = \frac{4}{3\alpha+1}$$

(75)

where $a_g$ is the acoustic speed in air. Just as the Eigner model for bubble flow this relation for slug flow is not very useful because it doesn't fit the experimental data.

The Nguyen relation for slug flow has already been explained in paragraph 2.4.1 and equals:

$$a = \frac{a_L a_G}{(1-\alpha)a_G + \alpha a_L}$$

(76)

Comparison of the models of Korolev and Nguyen for the whole gas fraction range is shown in figure 12. The value for the number of slug units has been set to be 4, because in the experimental setup slug units of .75 m are quite normal and the length of the pipe is approximate 4 m. Experiments have shown that the Nguyen relation predicts a far too high propagation velocity. So the best available model is the one by Korolev.
3. Experimental setup

3.1 Introduction

In order to be able to investigate the propagation of void fraction and pressure waves in the cocurrent upward bubble and slug flow regime, an experimental setup has been built. This setup consists of a column with two pipe diameters and a restriction inbetween. Due to the smaller upper pipe diameter compared to the lower pipe, both the liquid and gas velocity increase, which leads from the existing bubble flow to slug flow.

The design of this test column will be discussed in the next paragraph. An important design property is the entrance length in lower and upper part of the column needed to form fully developed bubble and slug flow.

Experiments have been done on a preliminary setup which also consists of a column with two different pipe diameters. At first only gas flow in stagnant liquid was possible in this setup. Therefore the preliminary setup has been adjusted to one where water circulation is possible. The dimensions of this column are discussed in paragraph 3.3.

3.2 The test column

3.2.1 The column-size

In order to form developed stable bubble flow a minimum entrance length for the lower part of the column is needed. Kapteijn [1989] reports that this length is in the order of 40 pipe diameters, and Liu [1993] found that the length needed is even up to 100 pipe diameters. This means that if we use a pipe diameter of 0.08 m a minimum pipe length of 3.2 to 8.0 m is necessary. A length of 7 m has been chosen.

For the upper part of the column where the transition to slug flow takes place Dukler and Taitel [1986] presented a formula to evaluate the entrance length $l_e$, required to form stable slugs

$$\frac{l_e}{D} = 42.6\left(\frac{U_m}{\sqrt{gD}} + 0.29\right)$$

where $U_m$ is the superficial mixture velocity, i.e. the sum of the superficial velocities of the two phases, $g$ is the acceleration due to gravity and $D$ is the tube inner diameter. When using a pipe diameter of 0.05 m (the diameter at which slug flow always takes place, see paragraph 2.2.3) in the upper part this results in a pipe length of approximately 3.7 m. Fabre and Akagawa [1992] found fully developed slug flow after 50-200 pipe diameters, which in our case leads to pipe lengths from 2.5 to 10.0 m. Since there is enough space in the laboratory a length of 10.0 m has been chosen. The material of the column is transparent acrylate.
Figure 13: restriction
3.2.2 The restriction

A conical restriction (see figure 13) is in our case better than a sudden one because a sudden restriction would lead to more mixing in the contraction and then the transition to slug flow will take place in the pipe and not in the restriction. From a physical point of view, friction forces from the pipe wall acting on the passing fluid can possibly be neglected. For the restriction in the large test column an angle of 60 ° with the horizontal has been taken, which is the same angle the restriction of the preliminary setup has.

3.2.3 The bubble generator

The way in which the bubbles at the bottom of the column are generated is very important, because in most theories describing bubble flow identical spherical bubbles are assumed. Probably one of the best ways to produce identical bubbles is by using hypodermic needles as reported by Kapteijn. Kapteijn uses a special distribution plate with entrance channels, in which the hypodermic needles are placed.

The forming of stable bubbles of an approximately identical size can also be realized by using a bronze sieve plate and a longer column, which gives the bubbles more time to stabilize. This method has been chosen because the produced bubbles are good enough for our experiments. The method by Kapteijn is too expensive and overdone to apply.

3.2.4 Generation of pressure and void fraction waves

In order to perform experiments on dynamical flow systems, i.e. the propagation of void fraction and pressure waves, these waves have to be generated in the experimental setup.

By mounting a fast closing valve on the gas inlet the gas fraction can be instantaneously reduced to zero leading to a void fraction wave. Another method to produce void fraction waves is making use of a compressor which generates a sinusoidal void fraction profile.

Secondly, mounting a pipe in the column above the sieve plate, in addition with an electrically regulated valve on the air supply, gives the possibility to generate a temporary overpressure in the column by introducing a large gas bubble in the column. The air supply has an airpressure of 6 bar. This will lead to the formation of a pressure wave which propagates through the column.
Figure 14: Dimensions of the large column.
3.2.5 The pump

In order to calculate the capacity of the liquid pump, one has to know the maximum needed superficial liquid velocity in the column. For that purpose the flow map of figure 4 can be used, which gives the different flow patterns at different superficial liquid and gas velocities. The aim is obtaining slug flow in the upper part of the column and bubble flow in the lower part.

This results in the following relation between the superficial velocity in the upper and lower part of the column:

\[
\frac{U_{s,\text{upper}}}{U_{s,\text{lower}}} = \frac{g^2}{5^2} = 2.5
\] (78)

Two boundaries of the flow map in figure 4 which result in slug flow in the upper part and bubble flow in the lower part of the column are given below:

<table>
<thead>
<tr>
<th></th>
<th>upper part</th>
<th>lower part</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_{G,s} )</td>
<td>0.125 m/s</td>
<td>0.05 m/s</td>
</tr>
<tr>
<td>( U_{L,s} )</td>
<td>0.1 m/s</td>
<td>0.04 m/s</td>
</tr>
<tr>
<td>upper part</td>
<td>0.4 m/s</td>
<td>0.04 m/s</td>
</tr>
<tr>
<td>( U_{G,s} )</td>
<td>1.0 m/s</td>
<td>1.0 m/s</td>
</tr>
<tr>
<td>( U_{L,s} )</td>
<td>2.5 m/s</td>
<td>0.04 m/s</td>
</tr>
</tbody>
</table>

So the maximum flows for gas and liquid are approximately:

\[
\Phi_{V,\text{gas}} : \pi/4 \times 0.08^2 \times 0.4 = 2 \text{ l/s}
\]

\[
\Phi_{V,\text{liquid}} : \pi/4 \times 0.08^2 \times 1.0 = 5 \text{ l/s}
\]

This means that the capacity of the liquid pump should be approximately 5 l/s.

A setup of the large column with all the design criteria discussed before is given in figure 14.
Figure 15: Preliminary setup
3.3 Preliminary setup

3.3.1 Introduction

In order to study unsteady two phase flow a preliminary setup has been built. This setup consists of a 15 cm i.d. perspex pipe with a restriction at the end and a 5 cm i.d. pipe on top. Pressure transducers are mounted along both pipes. The column is filled with water. In order to make water flow possible pipes with small holes are placed at the bottom of the pipe. To separate water and air on top of the column a knock out vessel has been used. Important properties like dimensions, air and water supply and placement of the pressure transducers are discussed in the following paragraphs.

3.3.2 Dimensions of the preliminary setup

The dimensions of the preliminary setup are shown in figure 15. The distances for placement of the pressure transducers are expressed in meters.

3.3.3 Air and water supply

For the production of bubbles with a small bubble size distribution a bronze sieve plate is used. Technical specifications of the sieve plate are given in appendix A. The way in which the air supply is taking place is shown in figure 15. The air is reduced from 6 bar to 1 bar with a reducement valve. Water is supplied through 8 pipes with in each pipe 6 small holes of 3 mm. The way in which they are placed is shown in figure 16. For specifications of the pump see appendix A. In order to measure the gas and liquid flow two gas and one liquid flowmeter are used. The calibration curves are given in appendix A.

3.3.4 Generation of void fraction and pressure waves

The generation of void fraction waves is realized by placing a fast closing valve on the air supply. By closing the valve a void fraction wave is created. Also a compressor has been used to generate void fraction waves. The compressor gives an approximately sinusoidal profile in the air volume flow which will deform due to different propagation velocities of the void fractions. Specifications of the closing valve are shown in appendix A. The electrically regulated fast closing valve can also be used for producing pressure waves. The way to do this is by mounting a small pipe in the column and creating a temporary overpressure by means of introducing a large gas bubble, which will give a pressure wave.
Figure 16: Placement of the water supply pipes.

Figure 17: Placement of the different measuring devices.
4. Measurement methods

4.1 Introduction

One of the aims of this research project is to find out which type of measurement techniques is to be preferred for use in the large test column. The most important technique which has been investigated is pressure measurements with pressure transducers placed along the length of the pipe of the column. Other techniques which have only been preliminary investigated are using a laser and a high speed camera. These two techniques have been compared with the pressure transducers results. In the next paragraph pressure transducers, laser and the high speed camera are explained. Determination of for example gas fraction, Taylor bubble speed and void fraction wave propagation via pressure signals is explained in paragraph 4.3.

4.2 Measurement techniques

4.2.1 Pressure transducers

Pressure transducers were placed at different positions along the pipe wall, see figure 15. When for example a void fraction wave propagates through the pipe the pressure will change in time. By amplifying the pressure signals and then sending these signals to a computer via a multiplexer the signal can be processed. An illustration of the setup is given in figure 17. The technical specifications of the transducers and the data acquisition apparatus can be found in appendix B.

4.2.2 Laser

The laser equipment consists of an apparatus producing a laser beam and a photodiode (for technical information see appendix B). When these two are placed on opposite sides of the column the laser beam will pass the two phase medium. Since air and water give a different light diffraction and reflection, the passage of air, water or a mixture can be detected. In this manner the laser is ideal to investigate slug flow. For technical information see appendix B. An illustration of the placement of the laser is given in figure 17.

4.2.3 High speed camera

Using a camera is very useful to view what is exactly happening in the slug flow regime. A high speed camera differs from a normal camera in that manner that the high speed camera takes 200 pictures per second against 50 per second for a normal camera. Technical information of the camera is given in appendix B. The camera is shown in figure 17.
Figure 18: Pressure signal of one Taylor bubble passing a sensor.

Figure 19: The pressure signal of slug flow.
4.3 Evaluation of the signals

4.3.1 Determination of the gas fraction

The mean gas fraction of the two phase mixture can be obtained in two manners. The first is only applicable when the knock out vessel is not filled. The method is simply dividing the volume of gas $V_G$ by the total volume $V_L + V_G$. In that case one has to know the exact dimensions of the lower and upper pipe and the restriction.

Since this method is a little round-about and is only possible in a system without water circulation it is better to make use of pressure transducers. First the signal $S$ of a pressure transducer is measured when a certain height $h$ of water is above the transducer. Because this signal is proportional to the pressure,

$$S = C \rho_L gh$$  \hspace{1cm} (79)

the constant $C$ can be calculated. When now the average signal is measured in a two-phase mixture and the height between the pressure transducer and the water level is known one can calculate the density of the mixture and then the void fraction $\alpha$ because the density of the mixture is equal to:

$$\rho = \alpha \rho_G + (1-\alpha) \rho_L$$  \hspace{1cm} (80)

The void fraction is then:

$$\alpha = \frac{S}{C gh \rho_L} \frac{\rho_G - \rho_L}{\rho_G - \rho_L}$$  \hspace{1cm} (81)

This method does not take friction into account.

4.3.2 Slug flow

The theory of the slug flow regime has already been given in chapter 2. Some characteristic parameters for slug flow are Taylor bubble and liquid slug length, velocity and frequency. In order to evaluate signals the in house developed program AV_1.6 has been used. This program gives the possibility to scan a signal for a certain time and with various sample frequencies. For extra information see appendix B. The scans are further processed with the program VEETEST by Hewlett Packard. The program VEETEST can be used for a number of mathematical operations like Fourier transformations and cross correlations between signals. This program is given in appendix B.

First one has to determine what kind of signal the pressure transducer gives when a Taylor bubble and a liquid slug pass the sensor. This is schematically illustrated in figure 18.
Figure 20: A cross correlation of two slug flow pressure signals.

Figure 21: Fourier transform of a slug flow pressure signal.
The different points in the figure are explained below.

A : Top of the Taylor bubble passing the sensor.
B : End of the bubble, the pressure is descending because of the lowering of the density above the sensor.
B-C : Beginning of the liquid slug.
C-D : Taylor bubble leaving the knock out vessel, pressure is rising because the air is replaced by liquid.
D-E : Small bubbles leaving the knock out vessel, again replaced by liquid.

In real slug flow the signal looks rather the same like a repeatment of figure 18, an example is given in figure 19. The descending signal is coming from the Taylor bubble and the ascending signal from a liquid slug.

**Determination of the Taylor bubble and liquid slug velocity**

When at two places in the pipe a pressure signal is scanned the velocities of Taylor bubble and liquid slug can be determined. Because one knows the distance between the two pressure sensors, by obtaining the time the Taylor bubble or the liquid slug takes to travel from the lower to the upper sensor, the velocities can be calculated.

For this purpose the program VEETEST has been used. By making a cross correlation (for more explanation see appendix B) between the two signals the velocity can be determined. If two signals are exactly the same a cross correlation will have a time shift of zero. If there is a certain time delay between the two signals this peak will shift with that time delay. This is illustrated in figure 20. The velocity is then the distance between the two sensors divided by this time shift.

In this manner the combined velocity of Taylor bubble and liquid slug is calculated. If one assumes that these velocities are equal this method is correct. But in practice it is clear that the Taylor bubble velocity is higher than the liquid slug velocity. So the assumption that both velocities are equal is a disadvantage of the calculation method.

**Determination of the slug frequency**

Determination of the slug frequency is done with the aid of a Fourier transformation of the signal. This will give the slug frequency, see also figure 21. A more accurate result is obtained by using a Hanning filter in addition with the Fourier transformation. This is further explained in appendix B.

**Determination of the Taylor bubble and liquid slug length**

The Taylor bubble length is obtained by measuring the time the bubble takes to pass the sensor. In figure 18 this is distance A-B. When the Taylor bubble velocity is known multiplying the time with this velocity gives the Taylor bubble length. A disadvantage of obtaining results with this
Figure 22: Calculation of the void fraction propagation speed.

Figure 23: Passage of a pressure wave detected by two pressure transducers.
method is that sometimes overlap occurs between a Taylor bubble leaving the knock out vessel (rising signal), and another Taylor bubble passing the sensor (descending signal). This will distort the Taylor bubble signal and thus result in a smaller Taylor bubble length than in the real situation. The slug length can in theory be calculated in the same manner by measuring the distance of the ascending part of the signal of one slug unit. The problem is that this ascending part originates either from a liquid slug or a Taylor bubble leaving the knock out vessel. So one can never be sure whether the results obtained from this method are right. Therefore it is better to calculate the average liquid slug length via the other parameters which have already been measured. If the average slug velocity is divided by the slug frequency this results in the length of the slug unit. This unit consists of the Taylor bubble and the liquid slug length. The Taylor bubble length has been measured, so the liquid slug length is simply the unit length minus the Taylor bubble length.

4.3.3 Void fraction shock waves

Void fraction wave propagation velocities are obtained by measuring the propagation velocity of a void fraction front. This gives the bubble velocity and via a mathematical operation (see paragraph 2.3) the void fraction wave velocities are calculated. This is done by abruptly changing the void fraction from a certain value to zero by closing the air inlet. This will give a density difference in time which can be detected by pressure transducers. A schematic representation of this situation is given in figure 22. These measurements have been done when the knock out vessel was not filled. Signal 1 in the picture is from a sensor which was placed below the sensor which gave signal 2. Point A in the figure is the time when the air inlet is closed. The bubble front (which is below the sensor) will move upwards and due to the replacement of bubbles by water the water level above the sensor will move downward. This means that due to the lowering of the water level the pressure above the sensor will become less. This situation takes place from point A to B. Point B is reached when the bubble front passes the sensor. After that time the pressure will stay equal because the volume of water above the sensor does not change anymore. The bubble front will pass the lowest sensor first. The time difference between points B1 and B2 gives, with the distance between the sensors known, the average propagation velocity of the bubble front.

4.3.4 Pressure waves

Pressure wave propagation in the bubble and slug flow regime are measured with two pressure transducers. Since the propagation velocity of pressure waves is approximately 30 m/s or more the time the wave takes to travel between the sensors is of the order of one hundredths of seconds, so the scan frequency has to be in the order of 2500 scans per second. An example of the signal produced by the introduction of one large bubble in the column is shown in figure 23. The large bubble results in the diminishing sinusoidal signal due to damped fluctuations of the water level. At the beginning of this signal the pressure wave peak can be seen. By measuring the time between the pressure wave peaks of two signals the propagation speed can be calculated since the distance between two sensors is known. In this manner the propagation speed of pressure waves at the bubble and slug flow regime can be measured.
Figure 24: Different combinations of liquid and gas superficial velocities in the slug flow regime.

Table 1: Gas fractions at different superficial liquid and gas velocities.

<table>
<thead>
<tr>
<th></th>
<th>α (-)</th>
<th></th>
<th>α (-)</th>
<th></th>
<th>α (-)</th>
<th></th>
<th>α (-)</th>
<th></th>
<th>α (-)</th>
<th></th>
<th>α (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.39</td>
<td>E</td>
<td>0.30</td>
<td>I</td>
<td>0.39</td>
<td>M</td>
<td>0.46</td>
<td>Q</td>
<td>0.54</td>
<td>U</td>
<td>0.56</td>
</tr>
<tr>
<td>B</td>
<td>0.37</td>
<td>F</td>
<td>0.43</td>
<td>J</td>
<td>0.38</td>
<td>N</td>
<td>0.46</td>
<td>R</td>
<td>0.53</td>
<td>V</td>
<td>0.56</td>
</tr>
<tr>
<td>C</td>
<td>0.33</td>
<td>G</td>
<td>0.42</td>
<td>K</td>
<td>0.49</td>
<td>O</td>
<td>0.46</td>
<td>S</td>
<td>0.52</td>
<td>W</td>
<td>0.54</td>
</tr>
<tr>
<td>D</td>
<td>0.32</td>
<td>H</td>
<td>0.40</td>
<td>L</td>
<td>0.48</td>
<td>P</td>
<td>0.55</td>
<td>T</td>
<td>0.51</td>
<td>X</td>
<td>0.52</td>
</tr>
</tbody>
</table>
5. Results

5.1 Introduction

In this chapter results of experiments done with the preliminary setup will be given. Paragraph 5.2 deals with the stationary slug flow regime. Important parameters like velocities, frequencies and lengths of Taylor bubble and liquid slug are given and discussed.

Paragraph 5.3 gives the results of the void fraction wave propagation experiments, and in paragraph 5.4 experiments and theory of propagation velocities of pressure waves at different void fractions in the bubble and slug flow regime are compared.

5.2 Stationary slug flow

5.2.1 Results obtained with pressure transducers

Slug flow can be characterized by slug units consisting of a Taylor bubble and a liquid slug, as explained in chapter 2. The most important parameters are Taylor bubble and liquid slug lengths, velocities and frequencies. In the experiments these parameters are obtained at a number of different combinations of liquid and gas superficial velocities, see figure 24 and appendix C. Use has been made of two pressure transducers at places V and W (see figure 15 in chapter 3).

*The gas fraction*

The gas fraction is calculated with formula's 79 to 81 given in chapter 4. The results are shown in table 1. One can see that the gas fraction is high and decreases when the liquid flow is increased. The gas fraction is high due to the fact that the superficial gas velocities are large. This in order to cover an area of superficial gas and liquid velocities as large as possible in the flow map in chapter 2 (see figure 4). The void fraction decreases with increasing liquid flow, because with increasing liquid velocities the residence time of the gas decreases and thus the void fraction.

*Taylor bubble and liquid slug velocity*

The velocities of Taylor bubble and liquid slug are assumed to be equal, as explained in chapter 4. The calculation of the velocities has been done by a cross correlation of two pressure signals. The surprising result of this mathematical operation is that the obtained time difference between places V and W in the column is 0.02 s for all experiments. This equals one scan with the used scan frequency of 50 Hz. The velocity would then be 10 m/s for all different liquid and gas superficial velocities. This result is not in agreement with theory.

A probable reason is that the cross correlation method is right, but the influence of the knock out vessel is too large. When a Taylor bubble leaves the vessel, the mass above the sensors changes and the pressure at both places V and W changes immediately. So every change in mass above the sensor is directly changing the pressure signal.
Figure 25: Pressure signals of two sensors at places V and W in the slug flow regime.

Figure 26: Taylor bubble velocity as a function of the superficial liquid and gas velocity.
Results

This results in time differences of almost zero between the sensors. An illustration is given in figure 25. Because these small time differences are so numerous a cross correlation of these two signals will result in a time difference of almost zero or one scan.

An option to avoid this effect is lowering the scan frequency, which lowers the number of these disturbances in the signals. This is an unfavourable option, because the calculated velocities will then be too inaccurate. The only probable solution to get good results with cross correlations is doing experiments in the large column, because then the distance between sensors and knock out vessel is much larger, and the influence of these disturbances is less. When for example in the preliminary setup the signal due to the pressure above the sensor is 2 V, and the leaving of a Taylor bubble will increase the signal by 0.5 V this results in $2 \pm 0.5$ V. When using the large column the distance between sensor and knock out vessel is much larger, and thus the pressure will give a signal of for example 10 V. One Taylor bubble leaving the knock out vessel still changes the pressure by 0.5 V and thus will influence the total pressure signal relatively much less.

Therefore, the velocity of the Taylor bubble has been obtained by hand. In this way the results are better. The disadvantage of this method is that it is much less accurate. The time shift has been measured 30 times for every measurement. The calculated velocities are shown in figure 26 against the sum of superficial gas and liquid velocities, and are compared with the theoretical formula for the Taylor bubble velocity (formula 30 in chapter 2). For values see appendix C.

As can be seen in the figure the theoretical curve predicts a Taylor bubble velocity which is always lower than in the experiments. There are some possible reasons for this effect. One is that acceleration of the Taylor bubbles takes place, because of expanding of the bubbles when rising due to the lowering hydrostatic pressure. When using a closed system this effect will not occur. Secondly the flow is unstable due to the length of the column and thirdly results obtained by hand are too inaccurate.

Length of the Taylor bubble

The length of the Taylor bubble is obtained via the Taylor bubble velocity and the time the bubble needs to pass a pressure sensor. For every combination of velocities this time is measured 50 times and averaged.

The results are shown in figure 27, for values see appendix C. The Taylor bubble length decreases with increasing superficial liquid and gas velocities. This is logical because when the velocity increases the Taylor bubbles do not have the time to coalesce into larger Taylor bubbles. But even more important is the effect of breakup of bubbles at high velocities due to the more chaotically behaviour of the two phase system. At low gas and liquid velocities in the preliminary setup the Taylor bubbles accelerate and catch up with each other and form larger Taylor bubbles.
Figure 27: The Taylor bubble length for different values of liquid and gas superficial velocity.

<table>
<thead>
<tr>
<th>#</th>
<th>$L_{ls}$ (m)</th>
<th>#</th>
<th>$L_{ls}$ (m)</th>
<th>#</th>
<th>$L_{ls}$ (m)</th>
<th>#</th>
<th>$L_{ls}$ (m)</th>
<th>#</th>
<th>$L_{ls}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.884</td>
<td>E</td>
<td>6.084</td>
<td>I</td>
<td>1.905</td>
<td>M</td>
<td>1.648</td>
<td>Q</td>
<td>1.728</td>
</tr>
<tr>
<td>C</td>
<td>2.88</td>
<td>G</td>
<td>1.515</td>
<td>K</td>
<td>2.661</td>
<td>O</td>
<td>3.232</td>
<td>S</td>
<td>3.232</td>
</tr>
<tr>
<td>D</td>
<td>2.693</td>
<td>H</td>
<td>1.254</td>
<td>L</td>
<td>1.411</td>
<td>P</td>
<td>1.595</td>
<td>T</td>
<td>3.237</td>
</tr>
</tbody>
</table>

Table 2: Calculated liquid slug lengths.
Slug frequency and length of the liquid slug

The slug frequency is obtained with the Fast Fourier transform of a pressure signal, see also chapter 4.

The slug frequency against the liquid and gas superficial velocity is shown in figure 28. One can see a slightly increasing slug frequency with increasing superficial velocities. This effect is easily explained, because with increasing velocities the Taylor bubble length will decrease and thus result in more slug units per unit length in the column.

In order to compare the experimentally obtained slug frequency with theory one can use equation 41 in chapter 2. The problem with this equation is that the liquid slug length is needed in this formula.

In order to calculate the length of a liquid slug the velocity of the Taylor bubble, which is assumed equal to the velocity of the slug unit, is divided by this slug frequency. This results in the length of the total slug unit. The length of the liquid slug is the slug unit length minus the Taylor bubble length. For the calculated length of the liquid slug at all measurement points see table 2. The only theoretical prediction for the liquid slug length is that the length is equal to 40 times the pipe diameter (chapter 2), thus 2 m. The average of the experimental data is 2.35 m.

A comparison between theory and practice for the slug frequency will lead to exactly equal results, because the liquid slug length is obtained via the slug frequency. So directly measured liquid slug lengths are needed for an evaluation of the results.

5.2.2 Results of laser and high speed camera

The laser

The laser has been used to investigate the possibilities of this measurement method. It is very useful to investigate the stationary slug flow regime. The passage of air and water or a mixture and its transition are sharp distinguished in the signal. An illustration is given in figure 29. The signal from A to B is a Taylor bubble and the signal from B to C from a liquid slug. With this method it is very well possible to obtain slug flow parameters like velocities, lengths and frequencies. For that purpose however of course two lasers are necessary. Since there was only one available, it was not possible to further investigate the measurement method. The advantage of this method for studying stationary slug flow compared to using pressure transducers is that the signals are not influenced by the knock out vessel, which is disturbing the pressure signal.

The high speed camera

The high speed camera is very useful to investigate slug flow. With this apparatus it is possible to make 200 pictures a second, so velocities and lengths can easily be calculated. The disadvantage of this method is that it is very labour-intensive, because the whole processing has to be done by hand. In order to process by a computer a program has to be written.
Figure 28: Slug frequency against liquid and gas superficial velocities.

Figure 29: Signal of the slug flow obtained with the laser.
The primary purpose of using the camera was to recalculate the velocities of the Taylor bubble at different liquid and gas flows and compare them with the pressures signals results. The velocities were measured at some different gas and liquid flows. The result was that the velocity varied from 0.5 to approximately 2 m/s, which is about the same order like was calculated with the pressure signal method.

5.3 Void fraction waves

5.3.1 Void fraction wave propagation determined by the bubble front velocity.

The void fraction wave propagation velocity is determined via the bubble front velocity at different void fractions. The measurements are done with pressure transducers. Using laser was not possible because only one was available. The third option of using the camera has not been done because the results with the transducers are sufficient.

In figure 30 measurement points and the two models are shown. See also appendix C. For the bubble front velocity two different models in chapter 2 are applicable, namely the Richardson & Zaki equation and the Barnea & Mizrahi equation (equations 53 and 54).

One may conclude that the simple Richardson & Zaki model fits the data quite well and much better than the Barnea & Mizrahi model. The reason is that the two constants in the Richardson & Zaki model are fitted by Lammers for air water systems. In the Barnea & Mizrahi equation the Harmathy model parameters for air/water are used. This will always give worse results than simply fitting data.

This means that equation 55, where the Richardson & Zaki equation has been used, gives the propagation of void fraction waves depending on void fraction.

5.3.2 Producing void fraction waves with the compressor.

For the production of an approximately sinusoidal void fraction signal a compressor has been used. The frequency of the produced sinusoid is around 4 Hz. The pressure has been measured at three points in the lower part of the column, namely 3.8, 8.5, 22 and 62 cm above the sieve plate. The waterlevel has been at three levels, 40, 70 and 100 cm above the sieve plate. The air overpressure was set at 1, 2 and 2.5 bar.

The results of the experiments show that the sinusoidal profile damps out as expected from theory, see figure 31. What cannot be detected, is the changing of the sinusoid from a saw tooth profile to a damped sinusoid. The best results are obtained with only 40 cm water above the sieve plate and a 2.5 bar overpressure. This is due to the fact that at these conditions as little water as possible will enter the air pipe. Entering of water in the air feed pipe will lead to deformation of the sinusoidal profile, and thus give worse results.
Figure 30: Propagation of the bubble front at different void fractions.

Figure 31: An example of the compressor experiments.
5.4 Pressure waves

5.4.1 Introduction

Pressure waves are formed in the experimental setup due to the introduction of a large gas bubble with 6 bar overpressure. This leads to a pressure wave which moves upward in the column. In the lower part bubble flow exists and in the upper part slug flow. Detection of the pressure wave is done with pressure transducers at four places in the column, two in the lower part and two in the upper part. The method of using lasers or the high speed camera is unfavourable, because with these methods the pressure waves cannot be visualized.

5.4.2 Bubble flow

In the bubble flow regime it was quite easy to detect the pressure waves. An example of a measurement is given in figure 32. The scan frequency is 2500 Hz. The small first peak in the picture is coming from the pressure wave. The damping sinusoidal profile is caused by the fluctuations of the water level due to the introduction of the large bubble.

A number of experiments at different void fractions have been performed. The results of these are shown in figure 33, see also appendix C. The bubble flow models by Henry (formula 59) and Nguyen (formula 65) are also shown in the figure. The experiments show good agreement with both models.

5.4.3 Slug flow

Contrary to experiments in bubble flow, the determination of the propagation velocity in the slug flow regime gave a lot of problems. What occurred most of the time was that the pressure wave peak couldn't be detected anymore, because the pressure peak was not large enough. The reason for this disappearing of the pressure wave peak is that in the slug flow regime the pressure fluctuations are large compared to the fluctuations in the bubble regime. Secondly, the pressure wave is generated in the foot of the column, what means that the strength of the wave will be weakened on the upper part of the column. Another possible reason is that the scan frequency is still too low, so that the maximum of the peak is not been detected. A much higher scan frequency means a smaller measurement time with the program AV_1.6 due to the maximum of 32768 possible scans, which makes it very difficult to measure the pressure peak.
Figure 32: An example of a pressure wave in the bubble regime.

Figure 33: Pressure wave experiments in the bubble flow regime.
The experiments (figure 34, values in appendix C) show that the measured velocities are not very dependent on the gas fraction. The speed is around 40 m/s. The Korolev model for different values of n (see figure) fits the data with reasonable accuracy. An important result is that especially for lower gas fractions the pressure wave speed is lower than in bubble flow as the Korolev model predicted (see chapter 2). The model by Nguyen doesn't fit the data at all, the predicted pressure wave velocities are far too high. The Nguyen model predicts a velocity always between the velocity of sound in pure water or air. Experimentally this is not found, because of the higher compressibility that leads to a much lower velocity than in air. In order to be sure whether the Korolev model is correct much more experimental data should be available.
Figure 34: Pressure wave velocities in the slug flow regime at different void fractions.
6. Conclusions and recommendations

The experiments with the preliminary setup are especially aimed at testing the possibilities of various measurement methods for usage in the test column. This chapter is divided into parts where problems with experiments are discussed and possible recommendations for further research are given. Stationary slug flow, void fraction waves and pressure waves will be reviewed.

*Stationary slug flow*

The measurements with pressure transducers show that in our case there is not so much agreement with the predictions of the parameters by theory. An interesting result is that the Taylor bubble velocity is always higher than the theoretical prediction. A reason for this effect is that acceleration takes place due to expanding of the bubbles because of decreasing hydrostatic pressure. The experiments showed qualitative tendencies like a decreasing Taylor bubble length and an increasing slug frequency with increasing void fraction. The slug frequency could not been compared with theoretical predictions because for that purpose the liquid slug length has to be measured directly. This length has been obtained via the slug frequency.

The problem with obtaining the slug flow parameters by using pressure transducers is that in the preliminary setup disturbances due to the knock out vessel influence the pressure signals too much. Therefore it is uncertain whether the obtained results are correct. Using cross correlations for calculation of Taylor bubble and liquid slug velocities is impossible.

A method which is not influenced by the knock out vessel is making use of a laser beam. Using this technique is recommended for the large column. Of course using pressure transducers still has to be examined, because probably the influence of the knock out vessel is much less than in the preliminary setup.

The disadvantage of the preliminary setup is that the upper part of the column is not long enough to create stable slug flow. For stable slug flow a length of at least 7 meters is necessary. The large test column has an upper part of 10 meters, so stable slug flow is more easily achieved.

*Void fraction waves*

The method to calculate void fraction shock wave propagation velocities has been realized by measuring the propagation velocity of a bubble front. The results of these measurements are good compared with the Richardson & Zaki model. A disadvantage of this method is that void fraction wave propagation velocities are measured indirectly.

The study of void fraction waves has been done by making use of a compressor which generates an approximate sinusoidal signal. The experiments have shown that the damping of the profile to a damped sinusoid due to diffusion and dispersion exists. The sawtooth profile which arises from the sinusoid because the velocities depend on gas fraction could not be seen.
Conclusions and recommendations

In future experiments generation of sinusoidal profiles with a larger amplitude or a smaller frequency has to be realized, because in that case the transition to the damped sinusoid takes more time. Maybe then the transition is more easily examined. Making use of a compressor with these options is recommended.

Pressure waves

Pressure wave propagation velocities have been examined in the bubble and slug flow regime. The detection takes place with pressure transducers, where a high scan frequency is needed due to the high propagation velocities.

In the bubble flow regime the pressure wave results are consistent with the theoretical models by Nguyen and Korolev. The introduction of an overpressure with a large gas bubble generates pressure waves which are strong enough to detect with the pressure transducers.

Detection of the pressure waves in the slug flow regime is much more difficult. The pressure waves damp out quite fast and it is hard to detect the pressure waves. This is a reason that the results of the pressure wave propagation velocity measurements are so inaccurate. Secondly since the slug flow is not fully developed it is not sure whether the results are correct. The experiments show little agreement with the theoretical models. The propagation velocities are around 40 m/s and do not vary that much with void fraction. The theoretical models that describe propagation velocities in the slug flow regime are very simple and are only applicable to horizontal flow. Modelling improvement could be realized by taking into account that the flow is vertical, that the Taylor bubbles deform and that the length of the bubbles varies.

In the large test column additional experiments for obtaining the propagation velocities should be done. If the generation of a pressure wave by a large gas bubble does not work, making use of a loudspeaker is an alternative.
References


Harmathy, T. Z., Velocity of large drops and bubbles in media of infinite or restricted extent, AIChE 6, 1960, pp. 281-288.


Lighthill, J., Waves in fluids, 1979, Cambridge University Press, p.151


References


Figure i: calibration line of the small air flowmeter.

Figure ii: Calibration line of the large air flowmeter.

Figure iii: Calibration line of the liquid flowmeter.
Appendix A

Calibration lines

The calibration lines of the two air flowmeters and liquid flowmeter are shown in fig i, ii and iii.

Specifications of the liquid pump

The capacity of the liquid pump in liters per hour depending on sucking or overcoming height is given in the next table. The shaded area in the table is the usable capacity of the pump in the experimental setup.

<table>
<thead>
<tr>
<th>height in meters</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>height to over­­come</td>
<td>15</td>
<td>4100</td>
<td>3000</td>
<td>2500</td>
<td>1800</td>
<td>1200</td>
<td>700</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3300</td>
<td>2500</td>
<td>1800</td>
<td>1200</td>
<td>700</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3700</td>
<td>3350</td>
<td>2600</td>
<td>1900</td>
<td>1300</td>
<td>750</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3000</td>
<td>3000</td>
<td>2560</td>
<td>1950</td>
<td>1400</td>
<td>900</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sucking height</td>
<td>2</td>
<td>2700</td>
<td>2700</td>
<td>2300</td>
<td>1700</td>
<td>1150</td>
<td>700</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2300</td>
<td>2300</td>
<td>2100</td>
<td>1500</td>
<td>1000</td>
<td>500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1300</td>
<td>800</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1250</td>
<td>1250</td>
<td>1250</td>
<td>1200</td>
<td>600</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Specifications of the sieve plate

material: sintered bronze
hole diameter: 30-70 µm
% holes: 40%
thickness: 5 mm
pressure drop: less then 0.1 bar

Closing valve

Closing or opening time: 1 ms
Figure iv: Picture and schematic view of a pressure transducer.

Figure v: Program made in Veetest.
Appendix B

Technical specifications of the pressure transducers

The measurement method of the transducers (for a schematic view of a transducer see figure iv) is based on the deformation of a silicon 'window' in the transducer due to the pressure difference inside and outside the equipment. The deformation results in a change of resistance of the window, which is processed to a voltage. The measurement inaccuracy is lower than 0.5 % of the maximum value of 1.350 bar.

High speed camera

Dalsa camera CA-D2, max. 200 pictures per second, 256 x 256 square pixels.

Laser

Nec Corporation Helium Neon Laser, max 5 mW, 632.8 nm.

Telefunken Photodiode, type BPW 24.

Data acquisition

The use of pressure transducers and laser results in a maximum of 4 signals which have to be measured simultaneously. This is accomplished by using a Hewlett Packard HP 3852A data acquisition and control unit. Simultaneous measurements are made possible by a HP 44711A high speed FET multiplexer.

Program AV_1.6

This program is used to control and process for example pressure signals. The number of scans and the scan time can be regulated. The maximum allowable scans are $2^{15}$ which is 32760.

Program Veetest

The program VEETEST by Hewlett Packard has been used to further process the pressure signals. VEETEST makes it possible to process mathematical operations with the data from AV_1.6. The program is shown in figure v. The different blocks will now be explained:

Open file: Block in order to open the AV_1.6 datafile.

Load AV-datafile: This block makes it possible to load in the AV-datafile and gives parameters like number of scans, all data points and scan frequency.

Integer: A simple integer slider to choose a channel to be processed.
Figure vi: Hanning filter weighing function.
**Get channel:** Builds a waveform from the datapoints and gives parameters like the average of the signal, measurement points which are averaged around zero (for the Fast Fourier Transformation (FFT)).

**Average:** Gives the average of the signal in order to calculate the void fraction.

**Xcorrelate:** Correlates two signals.

**Hanning filter:** The Hanning filter is used to give better Fourier transformation results. A further explanation is given below.

**FFT of a crosscorrelation:** Graphical representation of the Fourier transform of a cross correlation, used to obtain the slug frequency.

**Waveform:** Gives the signal against time in order to measure the Taylor bubble length.

**Crosscorrelation:** The crosscorrelation is used to calculate the combined Taylor bubble and liquid slug velocity.

**FFT:** Used for the same purpose as the FFT of a crosscorrelation.

**FFT with a Hanning filter:** Just like the FFT block in order to obtain the slug frequency.

**Further explanation of cross correlations and the Hanning filter in use with Fourier transformations:**

The cross correlation function of two continuous-time signals $x(t)$ and $y(t)$ is given by

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)y(\tau)d\tau$$  \hspace{1cm}(i)

The auto correlation of a signal is obtained by setting $y(t) = x(t)$ in eq $i$.

$$\phi_{xx}(t) = \int_{-\infty}^{\infty} x(t+\tau)x(\tau)d\tau$$  \hspace{1cm}(ii)

The Hanning filter is used to give the Fast Fourier Transform of a data input a more discrete and better signal. It is a mathematical weighting factor function, as can be seen in figure vi. The advantage of using a filter is that the Fourier spectrum with filter consists of more discrete peaks in comparison to a Fourier transform of the pure signal.
Appendix C

Stationary slug flow measurements

The different combinations of liquid and gas superficial velocities in the slug flow regime are given below.

<table>
<thead>
<tr>
<th>$U_{gs}$ (m/s)</th>
<th>$U_{ls}$ (m/s)</th>
<th>#</th>
<th>$U_{gs}$ (m/s)</th>
<th>$U_{ls}$ (m/s)</th>
<th>#</th>
<th>$U_{gs}$ (m/s)</th>
<th>$U_{ls}$ (m/s)</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.147</td>
<td>0</td>
<td>A</td>
<td>0.298</td>
<td>0</td>
<td>F</td>
<td>0.448</td>
<td>0</td>
<td>K</td>
</tr>
<tr>
<td>0.147</td>
<td>0.0222</td>
<td>B</td>
<td>0.298</td>
<td>0.0239</td>
<td>G</td>
<td>0.448</td>
<td>0.0239</td>
<td>L</td>
</tr>
<tr>
<td>0.147</td>
<td>0.0457</td>
<td>C</td>
<td>0.298</td>
<td>0.0439</td>
<td>H</td>
<td>0.448</td>
<td>0.0457</td>
<td>M</td>
</tr>
<tr>
<td>0.147</td>
<td>0.0639</td>
<td>D</td>
<td>0.298</td>
<td>0.0674</td>
<td>I</td>
<td>0.448</td>
<td>0.0639</td>
<td>N</td>
</tr>
<tr>
<td>0.147</td>
<td>0.0857</td>
<td>E</td>
<td>0.298</td>
<td>0.0874</td>
<td>J</td>
<td>0.448</td>
<td>0.0831</td>
<td>O</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$U_{gs}$ (m/s)</th>
<th>$U_{ls}$ (m/s)</th>
<th>#</th>
<th>$U_{gs}$ (m/s)</th>
<th>$U_{ls}$ (m/s)</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.598</td>
<td>0</td>
<td>P</td>
<td>0.748</td>
<td>0</td>
<td>U</td>
</tr>
<tr>
<td>0.598</td>
<td>0.0239</td>
<td>Q</td>
<td>0.748</td>
<td>0.0239</td>
<td>V</td>
</tr>
<tr>
<td>0.598</td>
<td>0.0457</td>
<td>R</td>
<td>0.748</td>
<td>0.0457</td>
<td>W</td>
</tr>
<tr>
<td>0.598</td>
<td>0.0674</td>
<td>S</td>
<td>0.748</td>
<td>0.0892</td>
<td>X</td>
</tr>
<tr>
<td>0.598</td>
<td>0.0892</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Taylor bubble velocity

The Taylor bubble velocities in meters per second are given below.

<table>
<thead>
<tr>
<th>#</th>
<th>$U_{tb}$</th>
<th>#</th>
<th>$U_{tb}$</th>
<th>#</th>
<th>$U_{tb}$</th>
<th>#</th>
<th>$U_{tb}$</th>
<th>#</th>
<th>$U_{tb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.92</td>
<td>E</td>
<td>1.01</td>
<td>I</td>
<td>1.24</td>
<td>M</td>
<td>1.22</td>
<td>Q</td>
<td>1.31</td>
</tr>
<tr>
<td>B</td>
<td>0.81</td>
<td>F</td>
<td>1.09</td>
<td>J</td>
<td>2.26</td>
<td>N</td>
<td>1.25</td>
<td>R</td>
<td>1.46</td>
</tr>
<tr>
<td>C</td>
<td>0.96</td>
<td>G</td>
<td>1.05</td>
<td>K</td>
<td>1.49</td>
<td>O</td>
<td>1.35</td>
<td>S</td>
<td>1.39</td>
</tr>
<tr>
<td>D</td>
<td>0.88</td>
<td>H</td>
<td>1.06</td>
<td>L</td>
<td>1.06</td>
<td>P</td>
<td>1.23</td>
<td>T</td>
<td>1.43</td>
</tr>
</tbody>
</table>
### Taylor bubble length

<table>
<thead>
<tr>
<th></th>
<th>L_{TB} (m)</th>
<th></th>
<th>L_{TB} (m)</th>
<th></th>
<th>L_{TB} (m)</th>
<th></th>
<th>L_{TB} (m)</th>
<th></th>
<th>L_{TB} (m)</th>
<th></th>
<th>L_{TB} (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.61</td>
<td>E</td>
<td>0.30</td>
<td>I</td>
<td>0.31</td>
<td>M</td>
<td>0.28</td>
<td>Q</td>
<td>0.24</td>
<td>U</td>
<td>0.21</td>
</tr>
<tr>
<td>B</td>
<td>0.77</td>
<td>F</td>
<td>0.35</td>
<td>J</td>
<td>0.29</td>
<td>N</td>
<td>0.25</td>
<td>R</td>
<td>0.23</td>
<td>V</td>
<td>0.25</td>
</tr>
<tr>
<td>C</td>
<td>0.52</td>
<td>G</td>
<td>0.27</td>
<td>K</td>
<td>0.19</td>
<td>O</td>
<td>0.20</td>
<td>S</td>
<td>0.23</td>
<td>W</td>
<td>0.14</td>
</tr>
<tr>
<td>D</td>
<td>0.52</td>
<td>H</td>
<td>0.34</td>
<td>L</td>
<td>0.29</td>
<td>P</td>
<td>0.29</td>
<td>T</td>
<td>0.22</td>
<td>X</td>
<td>0.12</td>
</tr>
</tbody>
</table>

### Bubble front velocity

<table>
<thead>
<tr>
<th>α (-) (10^2)</th>
<th>(u_{front} (m/s) (10^2)</th>
<th>α (-) (10^2)</th>
<th>(u_{front} (m/s) (10^2)</th>
<th>α (-) (10^2)</th>
<th>(u_{front} (m/s) (10^2)</th>
<th>α (-) (10^2)</th>
<th>(u_{front} (m/s) (10^2)</th>
<th>α (-) (10^2)</th>
<th>(u_{front} (m/s) (10^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.44</td>
<td>0.215</td>
<td>4.42</td>
<td>1.97</td>
<td>6.82</td>
<td>1.87</td>
<td>9.90</td>
<td>1.78</td>
<td>15.3</td>
<td>1.55</td>
</tr>
<tr>
<td>2.66</td>
<td>0.203</td>
<td>5.17</td>
<td>1.97</td>
<td>7.12</td>
<td>1.83</td>
<td>11.5</td>
<td>1.74</td>
<td>15.6</td>
<td>1.55</td>
</tr>
<tr>
<td>2.75</td>
<td>0.203</td>
<td>5.39</td>
<td>1.92</td>
<td>8.13</td>
<td>1.83</td>
<td>12.3</td>
<td>1.66</td>
<td>16.6</td>
<td>1.52</td>
</tr>
<tr>
<td>4.19</td>
<td>0.209</td>
<td>6.06</td>
<td>1.92</td>
<td>8.53</td>
<td>1.78</td>
<td>13.3</td>
<td>1.62</td>
<td>16.6</td>
<td>1.52</td>
</tr>
</tbody>
</table>

### Pressure wave velocity in the bubble flow regime

<table>
<thead>
<tr>
<th>α (%)</th>
<th>a (m/s)</th>
<th>α (%)</th>
<th>a (m/s)</th>
<th>α (%)</th>
<th>a (m/s)</th>
<th>α (%)</th>
<th>a (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.19</td>
<td>124</td>
<td>5.69</td>
<td>44.9</td>
<td>8.47</td>
<td>33.3</td>
<td>14.0</td>
<td>24.8</td>
</tr>
<tr>
<td>4.22</td>
<td>78.8</td>
<td>6.18</td>
<td>44.4</td>
<td>10.8</td>
<td>30.4</td>
<td>16.0</td>
<td>27.9</td>
</tr>
<tr>
<td>4.98</td>
<td>58.5</td>
<td>7.31</td>
<td>42.9</td>
<td>12.0</td>
<td>29.9</td>
<td>17.6</td>
<td>29.3</td>
</tr>
<tr>
<td>5.24</td>
<td>45.4</td>
<td>8.07</td>
<td>57.7</td>
<td>12.5</td>
<td>30.7</td>
<td>21.8</td>
<td>24.3</td>
</tr>
</tbody>
</table>
Pressure wave velocity in the slug flow regime

<table>
<thead>
<tr>
<th>$\alpha$ (%)</th>
<th>$a$ (m/s)</th>
<th>$\alpha$ (%)</th>
<th>$a$ (m/s)</th>
<th>$\alpha$ (%)</th>
<th>$a$ (m/s)</th>
<th>$\alpha$ (%)</th>
<th>$a$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.19</td>
<td>44.9</td>
<td>6.18</td>
<td>39.4</td>
<td>8.47</td>
<td>36.9</td>
<td>16.0</td>
<td>36.3</td>
</tr>
<tr>
<td>4.22</td>
<td>46.7</td>
<td>7.31</td>
<td>40.9</td>
<td>10.1</td>
<td>40.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.24</td>
<td>36.3</td>
<td>7.47</td>
<td>46.7</td>
<td>10.8</td>
<td>44.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.69</td>
<td>36.3</td>
<td>8.07</td>
<td>33.3</td>
<td>12.5</td>
<td>30.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>