Underwater Drag Reduction: Air Layer Stability over SuperHydrophobic Surface under Turbulent Conditions

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Author:
Roberto Martinez de la Cruz (4631072)

TU Delft Supervisor:
Dr.ir. B.W. van Oudheusden

U.C. Berkeley Supervisor:
Prof. Simo Mäkiharju

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Abstract

Framed in the current trend of global energetic efficiency, active drag reduction techniques in water vehicles have regained popularity in the last decades thanks to the intense research in the air lubrication approach. Within the last decade, bubbly drag reduction has yielded its place to air layer drag reduction, in which a thin, continuous air layer is produced under the ship’s hull. Reductions in friction drag have shown to be between the 80 and 99% and net energy savings of around 8-12 % have been predicted, which is however probably not enough to compensate the complexity of the system. Nevertheless, by coating the lower part of the hull with a superhydrophobic coating, C. Peifer et al. (2019) showed that air flow requirements to obtain a stable air layer diminished by a factor of three. The report presented here is a first study of the mechanisms that enhance the air layer stability when combined with a superhydrophobic surface. Water impact dynamics and free surface - turbulence interaction are studied experimentally and numerically respectively.

Water behaviour upon impact has been investigated through an ascending jet impacting on the underside of four different surfaces with contact angles ranging from 45 to 150 degrees, obtaining the plate forces and top and side views of the water spread on the surface. Interesting results regarding the collapse of the spread area when a modified Weber number is used has been obtained. Two characteristic dewetting mechanisms for surfaces with low and high contact angle have been identified and explained by a simple theoretical model. Furthermore, possible independence of the friction coefficient on the surface wettability characteristic has been found. When plotted against the streamwise Reynolds number all surfaces collapse in the theoretical Blasius laminar friction coefficient, which suggests that once the surface is fully wetted (Wenzel state), the lower dimensional drag seen in (super)hydrophobic surfaces is due solely to their smaller contact area. To the best of our knowledge, these comprise the first experiments in which area and force were measured in a scenario in which viscosity, inertia, surface tension and gravity shape the water flow characteristics.

On the other hand, a numerical model based on the classic elastic membrane concept has been employed to represent the deformations due to turbulence pounding in an air-water interface. Although further adjustments are needed, the model has been able to predict the tow tank experiments in C. Peifer et al. (2019) by using the surface-dependent water spread obtained in the water impact experiment. The dependence of the critical air flux to form a stable air layer on the surface wettability characteristics can therefore be predicted.
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“Consistent Hard Work Over Time”
Cal Triathlon
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**List of Symbols**

- $X^A$: Referred to attached
- $A$: Water spread area
- $A_{jet}$: Jet cross-section area
- $\alpha$: Jet/pipe angle with respect to the horizontal
- $b$: Model Span
- $b_{max}$: Maximum water patch width
- $Bo$: Bond number $\left(\frac{\rho g L_t}{\sigma}\right)$
- $C_F$: Friction coefficient $\left(\frac{F_x}{1/2 \rho v^2 A}\right)$
- $X^D$: Referred to detached
- $d$: Jet/pipe diameter
- $d_{patch}$: Intersected water patch equivalent diameter
- $\delta$: Boundary layer thickness
- $\epsilon$: Sand grain dimensional roughness
- $Fr$: Froude number $\left(\frac{v}{\sqrt{gd}}\right)$
- $f$: Wet fraction
- $F_x$: Horizontal force
- $g$: Gravity acceleration
- $X_{GS}$: Referred to gas-solid interface
- $X_{GL}$: Referred to gas-liquid interface
- $X^J$: Referred to the jet
- $k^+$: Sand grain roughness $\left(\frac{\rho u \tau}{\mu}\right)$
- $L$: Water patch length
- $X_{LS}$: Referred to liquid-solid interface
- $\mu$: Dynamic viscosity
- $Oh$: Ohnesorge number $\left(\frac{\sqrt{We}}{Re}\right)$
- $p'_a$: Fluctuating air pressure
- $p'_w$: Fluctuating water pressure
- $Q$: Water flow rate
- $q, Q_a$: Air flow rate
- $q_{crit}$: Critical air flow rate for void fraction over 80%
- $R_a$: Arithmetic roughness average
- $Re$: Reynolds number $\left(\frac{\rho v d}{\mu}\right)$
- $Re_{L}$: Water patch length Reynolds number $\left(\frac{\rho v L}{\mu}\right)$
- $Re_x$: Downstream Reynolds number $\left(\frac{\rho v x}{\mu}\right)$
- $Re_\tau$: Friction Reynolds number $\left(\frac{\rho v \tau}{\mu}\right)$
List of Symbols

\( \rho \) Density
\( \rho_w \) Water density
\( \rho_a \) Air density
\( X^S \) Referred to spread
\( \sigma \) Surface tension
\( t \) Time / Water patch thickness
\( t_{AL} \) Nominal/real air layer thickness \( \left( \frac{Q_a}{U_\infty} \right) \)
\( t_{AL}|_{crit} \) Critical air layer thickness for void fraction over 80 % \( \left( \frac{Q_{crit}}{U_\infty} \right) \)
\( \theta \) Contact angle
\( u \) Velocity
\( U_\infty, U \) Upstream velocity
\( u' \) Fluctuating velocity
\( v \) Jet velocity
\( v_z \) Vertical jet velocity
\( v_x \) Parallel jet velocity
\( We \) Weber number \( \left( \frac{\rho v^2 d}{\sigma} \right) \)
\( We_\theta \) Contact-angle-modified Weber number \( \left( \frac{\rho v^2 d}{\sigma(1-\cos(\theta))} \right) \)
\( We_{\theta z} \) z velocity component Contact-angle-modified Weber number \( \left( \frac{\rho v^2 d}{\sigma(1-\cos(\theta))} \right) \)
List of Abbreviations

**List of Abbreviations**

ALDR  Air Layer Drag Reduction  
BDR   Bubble Drag Reduction 
CA    Contact Angle   
DNS   Direct Numerical Simulation 
HLC   Horizontal Load Cell   
JHTD  John Hopkings Turbulence Database 
KH    Kevin Helmholtz (instability) 
RA    Roll off Angle 
SHS   SuperHydrophobic Surface(s) 
TBL   Turbulent Boundary Layer 
TCF   Turbulent Channel Flow 
VLC   Vertical Load Cell
1. Introduction

Drag reduction techniques for marine vehicles have been long considered by naval engineers as an efficient manner of reducing the ship’s propulsive requirements. Multiple benefits are obtained along with it: lower fuel consumption with the economical impact attained, lower negative atmospheric emissions, increase of speed, lower bulk of fuel on board etc.

It is estimated that friction drag accounts for 60% of resistance in the typical marine vehicles operating conditions ($Fr < 0.2$) (Ceccio and Mäkiharju, 2010), and therefore, important efforts have been directed towards frictional drag reduction in particular.

![Figure 1: World freight sector energy consumption by freight modes, 2012 and projected 2040 (British Thermal Units - BTU). Retrieved from U.S. Energy Information Administration (2012) - Modified.](image)

Such efforts are understood from the perspective that transportation uses currently 25% of the global energy (U.S. Energy Information Administration, 2012), with rising trends due to globalization. Within these figures, marine vessels have a considerable impact (see fig 1). Moreover, although it is common to address their effects by quantitative means, and, indeed, the number and size of ships employed are very large, we often forget they are in direct contact with one of the most damaged and important resources in our planet: water.

This report presents the research carried out towards understanding one of the most innovative drag reduction techniques, namely, Air Layer Drag Reduction over SuperHydrophobic Surface. Developed at Prof. Mäkiharju’s Lab (FLOW Lab) at U.C. Berkeley, it has shown improvements above 60 % in energetic efficiency with respect to previous air injection techniques (Mäkiharju et al., 2012; C. Peifer et al., 2019).

After this introduction, a review of the most relevant drag reduction approaches is given in section 2. The particular problem statement is formulated in chapter 3, while its approach and results are contained in sections 4 and 5. Finally, the conclusions and future work are summarized in section 6 and a series of appendices regarding calibrations and measurements have been added at the end of the report.
2. State of the Art: Drag Reduction Techniques

2

STATE OF THE ART: DRAG REDUCTION
TECHNIQUES

Water vehicle friction drag reduction is not a novel idea. Latorre (1997), for instance, reports patents on the topic as early as 1880. A wide variety of techniques has been developed throughout the years. Some have been successful and lead to new routes of investigation, or even have been commercially applied (Mizokami et al., 2010). Others, on the contrary, have been discarded.

A first division can be established depending on whether external power is required for their functioning. Those which do not need any external energy are termed passive techniques. Polymer solutions (White and Mungal, 2008), SuperHydrophobic Surfaces (SHS) (Watanabe et al., 1999; Davis and Lauga, 2010), compliant walls (Hahn et al., 2002), natural cavitation (Gokcay et al., 2004) or riblets and other forms of surface patterns to reduce turbulence momentum exchange (Luchini et al., 1991) are the most common approaches.

On the other hand, active techniques require an external power input, either through direct pumping of gas, an electrical current to produce electrolysis, etc. The most relevant approaches are illustrated in figure 2: Bubble Drag Reduction (BDR) (Sanders et al., 2006), transitional air layer drag reduction (Elbing et al., 2008), Air Layer Drag Reduction (ALDR) (Ceccio et al., 2010; Mäkiharju et al., 2012; Elbing et al., 2013) and Partial Cavity Drag Reduction (PCDR) (Lay et al., 2010). ALDR is the most promising approach, as it has shown drops in drag of over 80%, and, combined with a SHS will comprise the main topic of the MSc Thesis. Thus, a brief overview of the recent development of SHS and ALDR is exposed below.
2. State of the Art: Drag Reduction Techniques

2.1 Passive Techniques: Superhydrophobicity

Hydrophobicity refers to the physical property of a molecule that is seemingly repelled from a mass of water. It is commonly characterized by the droplet-boundary Contact Angle (CA). Hydrophobic surfaces are those with CA larger than 90 degrees, while super(ultra)hydrophobic coatings have a CA larger than 150 degrees, as shown in fig. 3.

The structure of SHS is usually comprised of micro and nano roughness (posts) coated with a low energy (hydrophobic) painting (figure 4b). The key to obtain such large CA lies in the air that is trapped in between these posts or irregularities, as can be seen on the left panel of figure 4a. This state in which the droplet is suspended as a 'fakir' is termed Cassie-Baxter or just Cassie state. On the contrary, the Wenzel state occurs whenever the air pockets migrate and the drop wets all the solid surface (right in figure 4a). Their manufacturing processes and a more detailed explanation of these two and more intermediate states can be found in Ma and Hill (2006).

The use of hydrophobic surfaces for drag reduction purposes has popularized in the last decades, specially since Onda et al. (1996) first manufactured an artificial SHS. Micro- and nanotechnology have opened a new era in the field of surface properties. Multiple theoretical (Fukagata et al., 2006), numerical
(Min and Kim, 2004) and experimental (Daniello et al., 2009) studies have been carried out, resulting in, for instance, shark skin-like fabrics for competition swimsuits. Although its exact functioning is still unclear, it seems that by trapping a thin layer (or pockets) of gas, they reduce the effective contact surface between the solid and liquid, enabling an apparent slip (Tretheway and Meinhart, 2002).

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Several groups have reported drag reductions around 20% both in DNS and experimental studies (Daniello et al., 2009; Min and Kim, 2004; Fukagata et al., 2006), but, when applied to larger Reynolds numbers, some of its properties are lost. Turbulence in general, and specially at \( Re \approx 10^8 \), more characteristic of real ships, is nearly unaffected by the current surface patterns due to their small effect on large scale eddies (the SHS slip length\(^1\) is too small). Current work is therefore directed towards understanding its behaviour in turbulent flows at higher Reynolds numbers. Kim and Kim (2002) have shown that it is possible to fabricate a hydrophobic surface with much larger slip length, aiming for dramatic drag reductions in larger vehicles. Daniello et al. (2009) and Gose et al. (2018) have experimentally shown drag reductions up to 50% in high speed turbulent flows at Reynolds numbers comparable to those of marine vehicles. A further analysis of rough superhydrophobic surfaces textural statistics on drag reduction is given by Rajappan et al. (2019).

Moreover, the self-cleaning properties of SHS could serve, for instance, as a ecological barrier against seaweed and barnacles deposition on the hull. This is a major problem in commercial ships in which roughening of the surface due to marine creatures negatively affects the drag and fuel consumption.

Nevertheless, durability arises as the basic problem, not only of the coating itself, but of the gas pockets that enable drag reduction. Within some time (typically in the scale of hours maximum), the air trapped in the SHS is lost, the coating is wetted and the drag recovers values of regular paintings or even higher due to the rough surface. Accumulation of dirt or damage in the coating due to crashes are big drawbacks of this approach as well. This idea of gas trapped and lost in SHS is, nevertheless, of great value to understand the current research carried out at U.C. Berkeley described in section 2.2.3. Apparently, the air pockets can be reestablished by injecting air near the SHS (Du et al., 2017).

### 2.2 Active Techniques

The state-of-the-art active approach in academia is the so-called gas injection. By injecting air in the near-wall region, the flow structures change, to the point that the entire water Boundary Layer (BL) might be shifted further from the solid surface, potentially reducing overall resistance. Depending on the flux of air injected, a bubble region or a ‘stable’ thin Air Layer may be created.

\(^1\)Extrapolated distance relative to the wall where the boundary layer parallel velocity component vanishes
2. State of the Art: Drag Reduction Techniques

2.2.1 Bubble Drag Reduction (BDR)

Air injection in bubbly form is achieved by injecting relatively low amounts of air or a similar gas with low viscosity through a slot or hole. BDR attracted interest during the last decades of the 20th and first years of the 21st century, as laboratory experiments showed that friction drag reduction exceeding 80% was possible locally (McCormick and Bhattacharyya, 1973; Bogdevich and Malyuga, 1976). Several theories that account for the observed drag reduction have been posted e.g. purely viscosity and density modification (for instance, reducing the mix mean density would lead to a decrease in the Reynolds shear stresses \( -\rho \langle u'v' \rangle \)), turbulence structures alteration (Ferrante and Elghobashi, 2004), turbulent-induced bubble splitting (Meng and Uhlman, 1989), etc. but none has convince the academic community. Probably a combination of the aforementioned mechanisms is responsible for the experimental results.

Despite the encouraging drag reduction figures, Sanders et al. (2006) showed that the scaling of BDR to higher Reynolds number proved unstable, resulting in the bubbles detaching from the near-wall region after a few meters, losing its effect in drag reduction. Nagamatsu et al. (2002) and Kodama et al. (2006) carried out sea trials in 50% scale ship models, encountering similar results of bubble layer deterioration downstream. An extensive review of the work carried out in last two decades regarding BDR can be found in Hashim et al. (2015).

2.2.2 Air Layer Drag Reduction (ALDR)

Given the endurance problems BDR showed, part of the research efforts recently shifted towards Air Layer Drag Reduction (ALDR). Although air lubrication in marine vessels is a long-established idea, it was not until the last decade that the first consistent air films were obtained. Sanders et al. (2006) reported, while testing BDR at large Reynolds numbers, how, in some cases, a continuous air layer was formed beneath the test model. Indeed, at the lowest test speed and highest air injection rate, buoyancy pushed the air bubbles to the plate surface where they coalesced to form a nearly continuous gas film which persisted to the end of the plate with near-100% skin-friction drag reduction. Figure 5 clearly depicts this trend, showing constant near-100% drag reduction for the entire plate length (lower, two dashed lines). The \( y \) axis shows the ratio between friction coefficient with air injection (\( C_F \)) and without air injection (\( C_{F0} \)).

![Figure 5: Skin friction ratio as a function of the downstream distance. Solid lines represent cases with BDR and dashed lines transitional (upper) and continuous air layer (lower two lines). The two columns in the legend reference the water upstream speed and air flow rate. Retrieved from Sanders et al. (2006).](image-url)
2. State of the Art: Drag Reduction Techniques

The same result can be observed from the near-wall void fraction perspective (Figure 6). Starting from no drag reduction when no air is supplied, there is an asymptotic trend towards a 100% drag reduction \((\frac{C_{F}}{C_{F0}} \to 0)\) when the air flow rate is much larger than the water flow within the boundary layer, i.e. when the parameter below approaches one:

\[
\frac{Q_a}{Q_a + Ub(\delta_0 - \delta_0^*)} \to 1
\]

\(Q_a\) is the air flow rate, \(U\) is the water upstream velocity, \(b\) is the model span and \(\delta_0 - \delta_0^*\) represents the difference between the boundary layer 99% thickness and displacement thickness for the unmodified zero-pressure-gradient boundary layer. Albeit different void fraction definitions result in different data collapse, all cases exhibit a clear relationship between the obtained \(\frac{C_{F}}{C_{F0}}\) and the air fraction close to the plate.

Provided such remarkable results, a more detailed study of the transition from BDR to ALDR was studied by the same group in Elbing et al. (2008). A similar test model as in Sanders et al. (2006) was employed. A wide spectrum of parameters was scanned, including surface roughness, air injector geometry, freestream velocity, water-air surface tension and volumetric air injection. Following a series of sweeps in air flow \(q\) at a constant free-stream speed \(U\), a flow rate threshold \(q_{crit}\) was found such that, drag reduction was already 80% and further increases in gas flow rate did not decrease friction considerably. Three regimes, plotted in figure 7 were differentiated. At low air injections (I), BDR, discussed in section 2.2.1, is observed. If the gas flow is increased, there is a transition region (II) with a much steeper slope that leads to the ALDR regime once the drag reduction has surpassed 80% (III).

Elbing et al. (2008) found that \(q_{crit}\) scaled clearly with the square of the upstream velocity.

Given the great performance seen in ALDR, efforts were directed to reduce the gas flux required to form an air layer. Elbing et al. (2013) introduced Vortex Generators (VG) some centimeters upstream of the air injection slot. The idea behind was to introduce a vertical water velocity in order to reduce

Figure 6: Summary of different studies showing skin friction ratio as a function of void fraction. Retrieved from Sanders et al. (2006).
2. State of the Art: Drag Reduction Techniques

Figure 7: % Drag reduction versus volumetric gas injection per unit spanwise at $X = X_{inj} = 6.67m$. Test at $U = 11.1ms^{-1}$. Retrieved from Elbing et al. (2008).

the thickness of the air layer. Photographic evidence showed a clear set of confined streaks behind each VG, but air rate to maintain the air layer slightly increased.

A different approach is that taken by C. Peifer et al. (2019). It combines the techniques of ALDR and SHS and will be the base for the MSc Thesis

2.2.3 Air Layer over Superhydrophobic Surface

The first encouraging results combining these two techniques were reported as early as in 2000. Fukuda et al. (2000) conducted experiments at moderate Reynolds numbers (up to $5 \times 10^7$ based on the model length) on flat plates and ships mock-ups. Drag reductions up to 80% (similar to those found in current research, see Sanders et al. (2006); Elbing et al. (2008)) were found at the lowest speeds. However, the hydrophobic surface characteristics where not measured, and the underlying mechanisms for such drag reduction were not studied neither explained. Furthermore, as the Reynolds number (free-stream velocity) increased, drag recovered gradually its nominal value. Probable explanation for this result is the constant air flow used ($t_{AL1} = 0.5$ and $t_{AL2} = 1mm$) with $t_{AL}$ being the so-called nominal air thickness defined as $t = Q / U_{\infty}b$, where $Q$ is the air flow rate, $U_{\infty}$ is the free-stream speed and $b$ the model span. Had they increased $Q$ as per $U_{\infty}^2$, as suggested by Elbing et al. (2008), constant drag reductions over 80% may had been found.

PIV-based velocity measurements and video recordings on air layer over smooth and SHS were conducted by Du et al. (2017). The videos clearly showed the ability of the SHS to retain and spread a small flux of air over the surface (fig. 8a). If the same flow conditions were applied on a hydrophilic smooth surface, the jet was dominated by buoyancy, divided into bubbles and rose in the water tunnel (fig. 8b). PIV measurements confirmed as well the rise in drag when the SHS lost all its trapped air. Indeed, the SHS became a simple rough surfaces when the air pockets were lost, rising its drag compared to a smooth surface. This trend worsened with increasing Reynolds number, as the boundary layer thickness decreases and the surface roughness protrudes beyond the viscous sublayer. However, the injection of air replenished the air pockets on the SHS and reduced drag once again. This same phenomena is observed by Wang et al. (2014). Besides these two efforts, no more attention was given to the topic.

Intuitively it may look obvious that SHS aids to the stability of the air layer, but not dedicated
2. State of the Art: Drag Reduction Techniques

Figure 8: (a) Air injected staying and spreading over SHS. (b) Air injected rising due to buoyancy on smooth surface. The Reynolds number is based on the upstream velocity and boundary layer momentum thickness ($\theta$). Retrieved from Du et al. (2017)

research into quantifying and understanding its underlying mechanisms was made until C. Peifer et al. (2019). In a similar fashion as the experiments of Elbing et al. (2008), two identical flat plates with a step to aid the air layer formation (see figure 9a) were coated with normal hydraulically smooth painting ($k_{max}^+ = 0.07$) and with a 158 ± 2 deg CA SHS. Electrical impedance measurement probes and high speed video recording was employed in an attempt to determine the void fraction close to the surface, i.e. if an air layer was formed.

Results, shown in figure 9b, demonstrate that the required air flux to obtain characteristic ALDR void fractions (over 80%) halved at least in the case of SHS coating when compared to smooth painting. Furthermore, for the Reynolds number tested of $5 \times 10^6$, the air layer was stable until the end of the model at lower gas injection levels. As mentioned above and discussed by Wang et al. (2014), with time, gas migrates from the surface that is eventually wetted. By injecting a certain amount of air, the entrapped air pockets mentioned in section 2.1 never migrate, allowing for a constant drag reduction over time thanks to the SHS. Moreover, as the potential of the water-SHS is now higher (higher contact angle), the stability of the air layer is increased, even at lower air fluxes. It resembles a perfect synergy.
Figure 9: (a) Test plate setup as per C. Peifer et al. (2019). 'E' marks the end of the plate and 'I' the air injection point. Note the air layer boundary layer starts developing further upstream. (b) Bow (top) and stern (bottom) void fraction measurements. Locations are 56.4 cm and 156.2 cm aft of injection slot for bow and stern respectively. The • corresponds to the visual observations of the void fraction on the painted plate while the ● corresponds to the SH coated plate. The □ marker indicates the data collected with the electrical impedance probe on the painted plate and the ○ indicates data collected on the SHS coated plate. The third configuration impedance probe data is included in both figures marked by *.

Only data in the SHS test model was taken with this probe configuration. Due to difficulties in impedance measurements at the bow location on the painted plate, the data is omitted. The uncertainty of visual coverage void fraction is estimated to be ± 5%. Retrieved from C. Peifer et al. (2019)
3. Problem Statement

As mentioned previously, C. Peifer et al. (2019) showed that coating the hull with a SHS reduced the air flux requirements to obtain a stable air layer. Although it may seem somehow intuitive, there is no clear explanation as what are the mechanisms through which the stability of a water-air interface is enhanced by a SHS.

After carefully examining the test high speed videos obtained as a result of the experiment described in C. Peifer et al. (2019), two main types of air layer breakdown were identified: Kelvin-Helmholtz (KH) instability, in which the air layer breaks along the entire width of the plate simultaneously (figure 10a); and liquid entrainment in the air layer (as if the air layer was punctured by the liquid) due to turbulence close to the interface, where different nucleation spots are created and grow until breakage (figure 10b).

![Figure 10: Test plate from C. Peifer et al. (2019). Bottom view. Plate moving upwards (flow moving downwards). (a) Supposed Kelvin-Helmholtz instability breaking mechanism, characterized by a horizontal simultaneous break location. (b) Turbulence pounding breaking mechanism, characterized by sparse random nucleations](image)

Regarding the KH breaking mechanism, Kim and Moin (2010), through a series of Direct Numerical Simulations (DNS) studied the ALDR phenomenon over a backward-facing step. They showed that, at air fluxes lower than the critical ($q_{crit}$), a KH instability developed, expanding downstream until the water touched the surface. The wet patch in the surface then grew and eventually provoked the air layer collapse. Nevertheless, when the air flux was above a critical value, the KH instability was suppressed downstream, resulting in a stable air layer. No hydrophobicity effects were taking into account in this case.

On the other hand, breakage due to pounding of the Turbulent Boundary Layer (TBL) is very similar to the KH instability described above, except for the driving force. Instead of an interface instability, it is the pressure fluctuation of the turbulent flow under the air layer which protrudes the gas film leading to liquid contacting the solid surface and further air layer collapse.

Interestingly, the breaking mechanism was found to be dependent on the gas flow rate, i.e. the nominal air layer thickness. As figure 11 suggests, higher flow rates tend to produce KH instabilities, probably due to a higher velocity difference in between phases, resulting in higher shear stress at the interface. Instabilities at lower air flow rates are in contrast associated with turbulence pounding. These ideas are only hypothesis, and work in understanding both mechanisms is ongoing.
Since the interest is to maintain the air layer with the minimum flow rate necessary, the research presented in this MSc Thesis is associated with the turbulent pressure break up. Other colleagues at FLOW Lab will as well work towards understanding KH instability break mechanism.

![Figure 11: Maximum and minimum air layer nominal thickness observed for each breaking mechanism. No AL indicates not enough air was pumped to form an air layer, KH refers to Kelvin-Helmholtz Instability, and Mixed - KH and Mixed - Turbulent are cases in which the breakage presents both topologies, although one of them is predominant. Data obtained from visual analysis of the experiments described in C. Peifer et al. (2019)]](image)

The question arises then as how does the SHS affect the turbulent pressure pounding and specially the water contact dynamics once the instability has grown enough. It is argued by S. Mäkiharju that the presence of a lower energy surface probably aids in the control of the water spot growth once the water has touched the solid surface. Furthermore, from a timing perspective, the water jet or drop residence time in the air layer might be lower in the cases of SHS, due to bouncing or faster retraction (Bartolo et al., 2006; Kibar et al., 2010; Kaps et al., 2014). In that case, SHS may aid stability by rising the water expulsion frequency from the air layer compared to the water entrainment frequency, so that water does not accumulate in the gas film.

It is the aim of this Thesis to investigate the difference in water spots nucleation, growth, timescales etc. between a normal coated surface and a SHS under the action of turbulent pressure fluctuations.

This allows us to formulate the following research question:

**What mechanisms enhance stability in a water-air layer interface over a superhydrophobic surface under turbulent pounding?**

Several sub-questions arise from it:

1. Related with the surface properties
   (a) How is the air layer affected by the roughness of the SHS
   (b) How is the water contact dynamics affected by the SHS

2. Related to the water-air interface
   (a) What is the water TBL pounding frequency and strength.
   (b) How is the water TBL affected by the presence of the interface.

Associated with each sub-question there is an entire fluid dynamics problem. The approach to study each of them is described in 3.1.

The ultimate goal would be to obtain the critical air flux ($q_{crit}$) as a function of the upstream velocity or a similar parameter, the SHS equilibrium contact angle ($\theta$), and other possible surface-related parameters.
3. Problem Statement

3.1 Approach

Figure 12: Detailed air layer problem setup. Lubricating air film in between the SHS surface and the water domain. Given the Reynolds number (> $5 \times 10^5$ based on downstream distance) there is a turbulent boundary layer developed in the water (Anderson Jr, 2010). Droplets and jets, or even the entire interface, might break into the air layer, impacting the SHS. Gravity acts down.

Typically in ALDR experiments, the ship’s hull is approximated by a long flat test plate, in which there is a first region for boundary layer development, followed by an air injection section and a last area where the measurements are performed (similarly to figure 9a).

Focusing on the C. Peifer et al. (2019) Tow Tank Experiment, a detail of the expected flow features for air layer over SHS is depicted in figure 12. It is a multiphase flow (water and air), not-fully separated as there are drops and jets breaking into the air layer. The interface is wavy, and non-stationary. The Reynolds number in C. Peifer et al. (2019), based on the downstream distance and water velocity far from the plate ($U_\infty$) and summarized in table 1, indicates both the water and probably air flows are turbulent. Both fluids are modelled incompressible given the maximum velocities throughout the experiment (below 10 m/s).

![Figure 12: Detailed air layer problem setup.](image)

<table>
<thead>
<tr>
<th>Injection Point</th>
<th>Plate End</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>$2.5 \times 10^6$</td>
</tr>
<tr>
<td>Air</td>
<td>$5.1 \times 10^4$</td>
</tr>
</tbody>
</table>

Table 1: Characteristic Reynolds number in the experiments from C. Peifer et al. (2019), based on downstream distance and water velocity far from the plate ($U_\infty = 1.7$ m/s)

In line with the research questions shown above, two main problems are distinguished.

3.1.1 Problem 1: Water Impact Dynamics

A large enough interface deformation results in a volume of water impacting against the ship’s hull, represented here by a flat surface. The wetting and roughness characteristics of the surface will determine the behaviour of the water film in contact with it: its spreading, contact time, dewetting etc. Inertia, surface tension, viscosity and gravity play potentially important roles in the water impact and spread.

In order to gain insight into the water contact dynamics dependency on the surface characteristics, an experiment is designed in which the impact of a volume of water against the underside of a flat plate is studied in a controlled environment. From the point of view of the hull, a water impact on the surface...
is produced by a jet of water with a horizontal velocity of the order of the upstream water velocity and a vertical velocity due to the turbulent fluctuations.

Even though the water jet arising to impact on the hull does not have a determined shape, for simplicity, it is simulated with a circular jet, which is directed at an angle against the underside of a coated surface. Figure 13 depicts the main idea behind this configuration. A sweep in different jet diameters, angles, velocities and surface characteristics within the order of magnitude of C. Peifer et al. (2019) Tow Tank experiment is carried out to obtain the influence of surface properties in the water impact. Section 4 contains the experimental setup, post processing algorithm and results for the water impact dynamics on a flat plate underside

![Figure 13: Reasoning behind the design of problem 1 experimental setup, in which the uprising water volume in the ALDR problem is simulated by an uprising jet on the underside of a flat plate](image)

### 3.1.2 Problem 2: Interface deformations due to turbulence

A second problem concerns the interface between the water and air presumably turbulent flows. It is of interest for the study to identify how different flow conditions might affect the shape of the air-water interface, and in which cases such deformation is large enough to produce a water impact on the ships hull.

As designing an experiment for this purpose requires of complex tools and a long development time, a numerical model based on the elastic membrane equation is developed. By inputting the pressure fluctuations obtained from external sources, the interface shape can be computed and the water impact on the underside of the plate simulated. The theoretical derivation, numerical model and results can be found in section 5.

### 3.1.3 Interaction

As a first approximation, both problems will be treated as independent.

For problem 1, the water impact on a flat surface will be treated as a separate study. The range of parameters to study will be determined by the expected interface deformation, but no simulation or measurement is done in order to obtain an exact value to input into the setup. In other words, we are assuming the change in the flow conditions does not affect the behaviour of the surface coatings when impacted by water.

Only the case in which the water jet impact velocity is below a certain threshold which marks the switch from Cassie to Wenzel state could make this assumption invalid. In such case, for low velocities the behaviour of the surface (Cassie state) would completely differ from the that at higher speeds (Wenzel state). However, looking at Zheng et al. (2005), typical pressure to wet a SHS and force a transition from Cassie to Wenzel state is on the order between 500 and 1000 Pascals, equivalent to water jet velocities below 1 m/s. Given the jet velocities studied are around 1 m/s or higher, as a first approximation, it is assumed the water jet wets the surface provoking a Wenzel behaviour. Deeper reasoning for this assumption can be seen in the results in section 4.5.4. Further work will look at the possibility of wetting transitions.
On the other hand, for problem 2, the solution of the interface shape does not depend on the surface characteristics. In this, we are assuming that the change in surface characteristics (mainly roughness) does not affect the interface shape. The most direct effect of surface roughness would be a change in the air layer TBL. However, air pressure fluctuations are considered to be much smaller than those form the water TBL, given the large difference in fluid densities. Further reasoning is provided in section 5.2.3.

Nevertheless, there will be a point of contact between the two problems. That is, once the interface shape is computed in problem 2, those instabilities that are large enough to contact the surface will have to use the data obtained in problem 1 in order to characterize their behaviour upon impact. This topic is treated in section 5.2.4.
4. Water Impact Dynamics

4.1 Approach and Methods

Multiple studies regarding water impact in superhydrophobic surfaces have been conducted in the last decade, mostly focused on vertical impingement of droplets (Bartolo et al., 2006; Antonini et al., 2014; Lee et al., 2015; Li et al., 2017) and jets (Celestini et al., 2010; Kaps et al., 2014; Kibar et al., 2010) on the upper side of a horizontal plate. Most results obtained concern smaller water spread and force measured in SHS in comparison with normal surfaces. Interestingly, for small enough jets, water rebounds have been observed on SHS (see figure 14).

As far as we know, research has been solely focused on the interaction between inertia and surface tension, including in some cases viscosity. No study has included gravity as one of the dewetting forces, since all experiments where performed on the upper side of an horizontal (or inclined) plate, or on a vertical flat surface. In marine air lubrication applications, however, water impingement is produced by an uprising volume of water impacting on the underside of an horizontal plate, being the dewetting aided by gravity. Indeed, a non dimensional analysis for a water patch observed in C. Peifer et al. (2019) confirms a Bond number of the order of $O(1)$ to $O(10)$, with the Bond number defined as:

$$Bo = \frac{\rho_w g L t}{\sigma} \quad (2)$$

where $t$ and $L$ represent water patch thickness -$O(0.001)$- and water patch length -$O(0.05)$- respectively and $\sigma$, $g$ and $\rho_w$ are the water surface tension, gravity acceleration and water density. In any case, the Bond number remains larger than one, implying that gravity plays a role at least as important as surface tension.

Consequently, trying to resemble as closely as possible water impact dynamics in the hull of the boat, the experimental setup will consist of an uprising jet impacting on the underside of a flat plate coated with different surface wetting properties (contact angle) and roughness.

4.2 Experimental Setup

The water spread on the underside of the surface and the dewetting process are the main variables of interest. The uprising volume of water impacting on the underside of the ship’s hull as hypothesized in section 3.1 is simulated in a controlled environment by an uprising jet impinging on a flat surface, coated with different water contact characteristics and roughness. Video recordings are taken from the top and side view in order to obtain the water spread and dewetting behaviour, while the horizontal force on the plate will be measured to gain an insight into the energy dissipation due to viscosity. Jet velocity, surface properties and pipe angle and diameter will be varied throughout the experiment according to table 4.

4.2.1 Optical Setup

Two cameras will be placed to obtain a top and side view of the water impact. Since placing the top view camera below the plate is not an option as, not only water falling from the plate could damage the
4. Water Impact Dynamics

Figure 14: (a) Take off of a liquid jet when experiencing transition from hydrophobic to SHS. (b) Multiple specular-like reflections of a water jet over two parallel SHS separated by a distance of 1.2 cm. (c) Double rebound against gravity over SHS. After each rebound, the jet displays stable and decreasing oscillations. Retrieved from Celestini et al. (2010).

camera electronics but also interfere in the view, it will be placed looking down above the plate. This adds to the test plates the requirement to be transparent, which will be solved as shown in section 4.2.6.

Given the setup structural restrictions, the top camera needs to be lightweight and small. A Basler ace acA2040-55um, which counts with a Sony IMX265 CMOS sensor and is able to deliver 55 frames per second at 3.2 MP resolution is used.

On the other hand, given restrictions for the side view are minimal, a high speed, high resolution Phantom v1210 will be used. Its high speed image acquisition (up to 660000 fps) will be key to determine the (de)wetting processes. Its high resolution (1280x800 pixels) may allow to obtain detailed pictures of the air pockets, if any, in between the SHS coating posts. A Godox SL-200W light source is placed opposite to the side view camera in order to illuminate the region and increase the water-background contrast.
4. Water Impact Dynamics

4.2.2 Force Measurement Setup

The inclusion of two load cells will make it possible to measure the horizontal ($x$) and vertical ($z$) forces on the plate due to the impinging jet. The Vertical Load Cell (VLC) needs to be able to withstand the 3.5 kg plate and frame weight, while still being able to sense force variations in the order of the jet dynamic pressure ($1/2 \rho v^2 A_{jet} \approx 8 mN$ with the variables values being the smallest from table 4). Thus, a 4 kgf load cell, with 0.25% accuracy connected to a 0-10VDC Load Cell sensor Amplifier, with an accuracy better than 0.2% Full Scale is used. To increase the resolution, 200 data points are averaged to form one final value. The load cell calibration shown in appendix C.1 shows repeatability errors below 0.01 % using the average technique. Further explanation of the data acquisition is shown in section 4.2.5.

For the Horizontal Load Cell (HLC), taking as reference the values of plate-parallel ($x$) force shown in Kibar et al. (2010), in the order of milliNewtons, an extraordinary accuracy and precision is required. For that reason, a 100 gf mini load cell with a 600 $\mu$V/V rated output and a supply voltage of 3 - 10 VDC is employed. Its maximum repeatability error of 50 mg (0.5mN) ensures high enough resolution. To increase its sensitivity, it is hooked to a DMD4059 Omega Strain Gauge to DC Isolated Transmitter, which acts both as a load cell amplifier and noise filter. Calibration of both load cells can be found in the Appendix C.1

Furthermore, by hanging the plate from four 1.2-meter length wires (Stren SHIQS10-HG High Impact Monofilament), we suppress any possible friction between the plate and its holding frame, assuring quality of the results. The wires length is the maximum that fits inside the FLOW Lab room, in order to minimize the plate inclination due to a horizontal movement caused by the jet impingement (as if it were a swing). Indeed, a horizontal movement of the plate of 1 millimeter, will result in a plate inclination of 0.05 degrees, which is within the error of the angle measurement device the FLOW Lab counts with. Moreover, the plate is physically connected to the HLC, restricting most of its movement.

4.2.3 Hydraulic Circuit

The water circuit consists of a water tank, pump, flowmeter, valve and pipe. The water tank recovers the liquid falling from the plate, which is then drained by the 0.5 horsepower Goulds Water Technology...
MCS 1MS1C5E4 pump, with a maximum flow rate of 72 litres per minute. Water is passed through a universal whole house filter before entering the pump.

The flowmeter used is a coriolis Micromotion CMF025M319N0AMEZZZ equiped with a 2400S transmitter, which allows not only instantaneous readings of temperature and flowrate, but also outputs them through a 4-20 mA circuit if desired, as will be explained in section 4.2.5. Its maximum allowed flowrate for water is 18 litres per second with an accuracy better than 0.05% at nominal conditions. These specifications were confirmed by measuring the mass of water obtained after periods of 30, 60, 90 and 120 seconds and comparing the mean flowrate to the readings obtained.

Finally, a pipe responsible of obtaining the correct impinging jet characteristics is connected to the flowmeter. The different pipe diameters needed required of easy interchangeability and thus a quick release mechanism serves as link between these two elements. The pipe length (0.9 m) is considered long enough to obtain a turbulent fully developed jet at the exit point. Indeed, according to Cencel and Cimbala (2006), the hydrodynamic entrance length until a turbulent flow in a pipe is fully developed is \( L_{h,turb} = \frac{4DRe^{1/6}}{6} \) where \( D \) is the pipe diameter and \( Re \) is the Reynolds number. For the worst case scenario in our experiment, the hydrodynamic entrance is 0.2 m.

The temperature of the water was monitored by the flowmeter and was found to be within 20.4 - 21.8 Celsius for the hydrophobicity tests and 23.5 - 23.9 Celsius for the roughness experiments. Accordingly, the average dynamic viscosity and density at that temperature for tap water were found to be \( 9.7 \times 10^{-4} \) Pa.s and 998 kg/m\(^3\). Surface tension was measured as shown in appendix B and found to be 0.058 N/m. All values are summarized in Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>20 - 24</td>
<td>°C</td>
</tr>
<tr>
<td>Dynamic Viscosity</td>
<td>( 9.7 \times 10^{-4} )</td>
<td>Pa.s</td>
</tr>
<tr>
<td>Density</td>
<td>998</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>Surface Tension</td>
<td>0.058</td>
<td>N/m</td>
</tr>
</tbody>
</table>

Table 2: Water variable values

4.2.4 Electrical Wiring & Noise Suppression

Figure 16 depicts the wiring diagram built for the experiment. Cameras, loadcells and flowmeter were connected through a National Instruments Data Aquisition NI USB-6351, which is able to receive and output analogue and digital signals.

The side view camera power supply is directly connected to the wall plug, whereas the loadcells amplifiers, DAQ and flowmeter are ultimately connected to a Smart-UPS C1500 power supply in an attempt to reduce noise and grounding issues.

Nevertheless, despite all the considerations, the pump electrical noise was still noticeable in the measurements when turned on, making, for instance, illegible the HLC data. Seeing the noise remained under different grounding schemes as well as with the use of external batteries not connected to the electrical network, electromagnetic induced noise in the wires was deemed responsible. Therefore, the entire electrical pump circuit was modified. The wiring was replaced by copper shielded cable with three phases and three ground connections, a NAC-20-472-D AC power line filter from Mouser was included between the pump and its controller and a series of ferrite cores with impedances ranging from 100 to 300 Ohms were placed in most of the electrical circuits. All these measures were directed towards isolating the electromagnetic coupling the pump power cables had on the measurement wires. Their effect was noticeable, reducing, for instance, the shift in voltage measured in the VLC, due to the pump being turned on from 6% to under 0.1 %.

In summary, the connections for each devices are:
4. Water Impact Dynamics

- **Data Acquisition (DAQ):** Collects voltage readings from the HLC amplifier, VLC amplifier and Flowmeter\(^2\). Sends analogue and digital trigger signals to the top view and side view cameras respectively. Controlled from the main computer, is connected to the power supply.

- **Horizontal Load Cell (HLC):** Four data cables, positive and negative input, positive and negative output, connected to the HLC Omega amplifier.

- **HLC Amplifier:** Connected to the power supply and four data cables to the Data Acquisition: positive and negative input, positive and negative output.

- **Vertical Load Cell (VLC):** Four cables, two input, two outputs, connected to the VLC Omega amplifier.

- **VLC Amplifier:** Connected to the power supply and four data cables to the Data Acquisition: positive and negative input, positive and negative output.

- **Top View Camera:** Data sent directly to the computer through USB 3.0, while power is obtained from the power supply. Lastly, it is connected to the DAQ for triggering and synchronization

- **Side View Camera:** Data sent directly to the computer through an ethernet cable, while power is obtained directly from the wall plug and connected as well to the DAQ for triggering and synchronization.

- **Flowmeter:** 4-20 mA circuit to send data to the DAQ, and connected to the power supply.

- **Power Supply:** Plug to the wall plug, provides the required current to the horizontal and vertical amplifiers and loadcells, as well as to the flowmeter.

- **Computer:** Through a Labview code reads the data from the DAQ and stores the images from the top and side view cameras using PylonViewer and Phantom Camera Control (PCC) software respectively.

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Figure 16: Simplified wiring diagram for the water impact experiment. Green lines indicate data transmission, red is used for trigger signals and black for power cables

\(^2\)Flowmeter output is in mA but is converted to voltage using an external resistor of 120 ± 0.1 % Ohm nominal value.
4. Water Impact Dynamics

4.2.5 Data Acquisition

As exposed above, the DAQ is the central element for the data acquisition. A Labview code, which flow chart diagram is showed in figure 17, was developed to start, monitor, control and organize the data. Once the Run button is pushed, data packets of 2500 data points are read at 250 kHz (maximum rate allowed). Those 2500 points are averaged into one single value, which corresponds to one final datapoint that is stored in an excel file automatically. One entire dataset was comprised of 600 averaged final datapoints, with the following distribution: The first 100 points were taken with the pump off to be used as a zero reference. The pump was then turned on at a constant flowrate and 400 more averaged datapoints were taken. Finally, the pump was again turned off for a second zero reference and to observe the dewetting process. The total time elapsed was approximately 45 seconds per experiment.

At the same time the acquisition was started, two trigger signals were sent to the top and side view cameras to start recording, so that the images could be corresponded with the forces and flowrate graphs. The top view camera acquires 2 images per second (10 for the roughness test), while the side view has a higher rate of 100 fps, stored for both cases in the computer.

![Figure 17: Acquisition algorithm flow chart.](image)

4.2.6 Test Plates

As described in section 4.2.1, the top view camera being placed above the plate requires them to be transparent or translucent at least. Finding coatings with superhydrophobic properties while maintaining a certain level of transparency is however complex, and a world wide research had to be done to find the correct products. To the best of our knowledge this was the first study with an uprising jet, in which gravity was included in the dewetting process, and therefore no previous studies could be used to determine the appropriate coating.

Five different surfaces where prepared for the first batch of experiments, focused on understanding contact angle dependency (‘A’ plates). They are summarized in table 3. An extra two were used for the second set of tests to study the roughness effect on the water spread (‘B’ plates). Both rough study plates were coated with exactly the same commercial primer (‘RUST-OLEUM Specialty Lacquer Transparent’) to maintain the contact angle, but one contained 64.5 ml/m² of hand-sprinkled ceramic microspheres (‘Miaoxy 64’) with a distribution from 10 to 540 microns and a mean of 110 microns.

The surface characteristics studied were two: i) (super)hydrophobicity, determined by the Contact Angle (CA - water contact angle at the triple point of a droplet) and Roll off Angle (RA - plate angle at which a droplet deposited on top of the plate starts rolling down) and ii) roughness measurements, determined by the arithmetic average of the roughness profile \( R_a \) and equivalent sand grain (Adams et al., 2012a) based \( k^+ \). The procedure for each of the measurements is described in appendix A.
Water Impact Dynamics

<table>
<thead>
<tr>
<th>ID</th>
<th>Coating</th>
<th>CA (deg)</th>
<th>RA(^3) (deg)</th>
<th>(R_a) (nm)</th>
<th>(k^+)</th>
<th>Transparency</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Glass</td>
<td>45 ± 8</td>
<td>-</td>
<td>12 ± 0.5</td>
<td>0.0065</td>
<td>Transparent</td>
</tr>
<tr>
<td>A2</td>
<td>NeverWet</td>
<td>101 ± 3</td>
<td>17 ± 3</td>
<td>11 ± 0</td>
<td>0.007</td>
<td>Transparent</td>
</tr>
<tr>
<td>A3</td>
<td>Naisol C</td>
<td>105 ± 5</td>
<td>40 ± 5</td>
<td>14 ± 0.9</td>
<td>0.0067</td>
<td>Transparent</td>
</tr>
<tr>
<td>A4</td>
<td>Naisol SHBC</td>
<td>142 ± 4</td>
<td>12 ± 2</td>
<td>1932 ± 40</td>
<td>1.2</td>
<td>Translucent</td>
</tr>
<tr>
<td>A5</td>
<td>Cytonix 800M</td>
<td>150 ± 2</td>
<td>0 ± 1 (^4)</td>
<td>120 ± 6</td>
<td>0.06</td>
<td>Translucent</td>
</tr>
<tr>
<td>B1</td>
<td>Smooth</td>
<td>78 ± 4</td>
<td>-</td>
<td>515 ± 263</td>
<td>0.95</td>
<td>Transparent</td>
</tr>
<tr>
<td>B2</td>
<td>Rough</td>
<td>88 ± 5</td>
<td>-</td>
<td>2.1 \times 10^4 ± 8 \times 10^3</td>
<td>40</td>
<td>Transparent</td>
</tr>
</tbody>
</table>

Table 3: Set of test plates surface characteristics. CA and RA indicate Contact and Roll-off Angle respectively. The roughness parameter \(k^+ = \frac{\mu U}{\rho u_{\tau}}\) is defined as the hydraulic roughness, where \(\epsilon\) is the equivalent sand grain according to Adams et al. (2012a). Values below 5 typically indicate the roughness peaks are contained within the viscous sublayer of the boundary layer, and therefore the surfaces can be considered hydraulically smooth. Between 5 and 70 roughness effects start to become important, and over 70 the surface can be considered fully rough. The value of the friction velocity \(u_{\tau}\) is taken as the highest computed for each plate to be in the roughness worst case scenario. Therefore, no plate had a roughness higher than the one showed in the table.

The fact that the roughness parameter \(k^+\) is below 5 for all of the ‘A’ surfaces indicates that they can be considered hydraulically smooth, i.e. roughness does not play any role (13.021 Marine Hydrodynamics Lecture 18, 2004). This allows to separate the roughness results shown in section 4.6 from the contact angle studies (section 4.5). Furthermore, given the similarities among test plates A2 and A3, only A2 was tested during the experiment campaign. Depending on the coating, two different materials were used: for surfaces A1, A2 and A3, a 451 × 610 × 3.2 mm glass plate was used, while the rest required of 451 × 610 × 6.4 mm acrylic. Plate thickness was enough to ensure no deformation due to the jet or its own weight when hanged. Indeed, using Bernoulli beam theory and a plate with two simple supports on its extremes and a weight of around 2kg, maximum deflection was 500 microns.

4.3 Test Matrix

Each of the 600 data points described in section 4.2.5 is considered one test, which was run with a particular surface (characterized by a contact angle \(\theta\) and roughness \(R_a\)), flowrate \((Q)\), pipe diameter \((d)\) and jet angle \((\alpha)\). Those four are the main input test parameters, and their range was determined to match, to the extent it was possible, the experiments previously performed at the FLOW Lab Tow Tank (C. Peifer et al., 2019): upstream velocity around 1.7 m/s, which implies horizontal jet velocities of 1.7 m/s and vertical jet velocities of around 10% of that value according to Dai et al. (1998) \(^5\); jet angles of \(\tan^{-1}(0.1) \approx 6\) degrees from the horizontal; and uprising jet diameters between 5 and 20 mm, resulting in the water patches seen in C. Peifer et al. (2019) in between 5 and 30 cm in equivalent diameter. Although the contact angles used in C. Peifer et al. (2019) were 74 and 158 degrees, a wider, more populated range was studied in order to gain a deeper understanding on the contact angle effect. Such values, which would allow a full comparison to the C. Peifer et al. (2019) Tow Tank experiment data are denominated ‘desired’, and are summarized in table 4 against the actual parameter values that were used during the experiment.

As can be seen in table 4, the amount of different diameter pipes and jet angles is three while four

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3High region variability was found in the roll of angle measurement. The value shown in the table is that of the largest region, which usually covered at least 75% of the total surface.
4Droplet would move just after being deposited in the surface.
5Dai et al. (1998) shows that the vertical fluctuating velocities due to a liquid boundary layer at the free surface are of the order of 10% of the upstream horizontal velocity \((U_\infty)\).
6Computed considering the jet velocity profile radially uniform.
4. Water Impact Dynamics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Desired Range</th>
<th>Experiment Actual Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flowrate ($Q$) b</td>
<td>$4.8 - 222$</td>
<td>$1.2 - 9$</td>
<td>1/min</td>
</tr>
<tr>
<td>Pipe Diameter ($d$)</td>
<td>$5 - 20$</td>
<td>$4.72; 6.13; 9.2$</td>
<td>mm</td>
</tr>
<tr>
<td>Jet velocity ($v$)</td>
<td>$1 - 3$</td>
<td>$0.7 - 3$</td>
<td>m/s</td>
</tr>
<tr>
<td>Surface Contact Angle ($\theta$)</td>
<td>$74 - 160$</td>
<td>$45; 101; 142; 150$</td>
<td>deg</td>
</tr>
<tr>
<td>Jet Angle ($\alpha$)</td>
<td>$5 - 15$</td>
<td>$25; 35; 45$</td>
<td>deg</td>
</tr>
<tr>
<td>Weber no. ($We = \frac{\rho dv^2}{\sigma}$)</td>
<td>$71 - 2566$</td>
<td>$40 - 1440$</td>
<td>-</td>
</tr>
<tr>
<td>Reynolds no. ($Re = \frac{\rho dv}{\mu}$)</td>
<td>$5.6 \times 10^3 - 6.7 \times 10^4$</td>
<td>$3.7 \times 10^3 - 3.1 \times 10^4$</td>
<td>-</td>
</tr>
<tr>
<td>Ohnesorge no. ($Oh = \sqrt{\frac{We}{Re}}$)</td>
<td>$1.2 \times 10^{-4} - 9.1 \times 10^{-3}$</td>
<td>$2.04 \times 10^{-4} - 1.02 \times 10^{-2}$</td>
<td>-</td>
</tr>
<tr>
<td>Bond no. ($Bo = \frac{\rho g d^2}{\sigma}$)</td>
<td>$3.5 - 56$</td>
<td>$3.8 - 14.6$</td>
<td>-</td>
</tr>
<tr>
<td>Froude no. ($Fr = \frac{v}{\sqrt{g d}}$)</td>
<td>$2 - 13.5$</td>
<td>$2.3 - 13.9$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4: Range of desired and actual test parameters in the ascending jet impact experiment. $\rho$, $\sigma$, $\mu$ and $g$ denote density, surface tension, dynamic viscosity and gravity acceleration respectively. $v$ is the jet velocity, $d$ symbolizes the water jet diameter. The jet angle ($\alpha$) is measured from the horizontal.

plus two surfaces (hydrophobic and roughness) are tested. The flowrate not being exactly the same for each case results from the pump limitations, as for larger pipe sizes it did not have enough power for velocities up to 3 m/s. In any case, to swipe a larger range of Reynolds number, four different velocities (flowrates) within the range shown were studied.

For the hydrophobicity study, the number of predicted tests were $3^2 \times 4^2 = 144$. However, due to water reaching the plate borders in the A1 - Glass plate, the highest velocities for such surface could not be studied and the total test went down to 135.

On the other hand, the roughness tests comprised two surfaces and three flowrates, pipe angles and pipe diameters, adding to $2 \times 3^3 = 52$ more tests.

Thus, in total, 187 tests were conducted.

4.4 Data PostProcessing

The acquisition produces three main outputs: an excel spreadsheet containing the acquisition total time, horizontal and vertical load cells voltages and flowrate; a set of images taken from the top view camera; and a high resolution video from the side camera. In standard conditions, one single test contains around 600 datapoints per measurement, while the side and top view cameras produce 4400 and 100 images respectively. Given the high number of tests (187), it was considered important to spend some time developing algorithms for the automated analysis and organization of images and data. The code main flow chart, written in MATLAB, is shown in figure 18

It is basically formed by four functions which extract different information from each experiment output. They are ran in a loop so that once one test is finished post processing, the next one will automatically be loaded. The four main functions are:

4.4.1 ExtractParameters.m

Reads the name of the experiment output folder and identifies the flowrate, surface (assigning to it a contact and roll-off angle, roughness etc.), jet diameter and jet angle. From those values, it computes other test parameter such as the jet velocity, or different non-dimensional values, such as the Reynolds or Weber number.
4. Water Impact Dynamics

4.4.2 ExtractExcelData.m

Data output for both loadcells and the flowrate typically looks like figure 19, where there is a first region of 100 data points with the pump off, followed by roughly 400 with the pump on and a final 100 pump off.

![Figure 19: Typical HLC data output from the data acquisition setup described in 4.2.5. The pump is turned on around data point 100, rising the value of the load cell voltage some iterations after due to liquid inertia. The pump is turned off around iteration 500, and therefore the drop in load cell voltage. Although there is some sparsity in the results, the noise is lower than the change in voltage, which ensures good quality data.](image)

The three regions are separated using the MATLAB function `findchangepts`, which divides the data in as many levels as desired (3 in our case), computing for each region the mean and standard deviation. In case the first and third regions do not have similar values (below a 5% tolerance), the code reports an error and jumps to the next test. By using the calibrations shown in appendix C.1, voltages from the loadcells and flowmeter are transformed into force and flowrate. In order to reduce system variabilities, the final value of force or flowrate is computed from the difference in voltage between pump-off and pump-on regions.

4.4.3 SideView.m

The interest of the side view lays in obtaining an estimation of the water patch length as well as the detachment point and mechanism. For that, a series of image processing subroutines are used to isolate the water from the background. After loading the 4400 frames into MATLAB, the following operations are performed:
4. Water Impact Dynamics

1. **Reference Image:** The first image of the video, in which the pump is still off, is saved as the reference background image.

2. **Average:** Using the iteration at which the pump was turned on and then off again, the code selects and averages all the images within the pump-on region. The reason to perform this task at the very beginning was to reduce computational time and smooth possible noise. Figure 20b shows an example of an averaged image, in contrast with 20a which is shows a single frame.

3. **Subtraction:** Since the difference between the background image and the averaged image is the water flow, those pixels with different intensity between them correspond to the water patch. Thus, subtracting both images most of the background disappears, leaving only a bright region corresponding to the water patch. An example is shown in figure 20c.

4. **Contrast Enhancement:** In order to further separate the bright pixels corresponding to the water and the possible dimmer background noise pixels, a contrast enhancement is performed.

5. **Intensity Threshold:** The dimmer pixels are then erased using an intensity threshold.

6. **Region Masking:** As the plate height is almost stationary (changes are in the order of cm between plates), the higher and lower areas of the image do not contain any interesting information and are, therefore, deleted.

7. **Hole Filling:** Possible holes in the main water patch region are filled to obtain a smooth, continuous surface. At this point, all other noise has been deleted, so only the actual water patch is corrected.

8. **Dilation and Erosion:** In order to obtain a smoother contour, a dilation and subsequent erosion with identical core is performed to maintain the same thickness and length while smoothing out possible border irregularities.

9. **Border Computation:** Finally, the water patch is reduced to its border pixels, as shown in figure 20d.

![Figure 20: Three intermediate steps towards obtaining the final water patch shape. (a) Frame extracted from the raw side view video. (b) Average image as describe in item no. 2. from section 4.4.3 (c) Image resulting from subtracting the reference background image from the mean. (d) Final shape obtained.](image-url)
4. Water Impact Dynamics

4.4.4 TopView.m

The top view shape algorithm follows a very similar image processing as the one described in the side view section (4.4.3): a reference image is taken when the pump is off and subtracted from the average pump-on frames (figures 21a, 21b and 21c). Afterwards, identically to the side view code, a contrast enhancement, intensity threshold, region masking, dilation and erosion is performed.

From there, the process is slightly different since the shape of interest is as well unique. Instead of obtaining the border pixels, using the MATLAB function \texttt{bwskel} we obtain the figure skeleton, that is, all objects are reduced to the middle line without loosing connectivity among them. Finally, since it was seen that the top view water patch shape resembled that of an ellipse in the vast majority of the cases, it is fitted to the obtained skeleton through two methods: one, by examining all possible major axes (all pairs of points) and getting the minor axis using a Hough transform (function from MATLAB File Exchange) and two, obtaining the maximum width and length of the skeleton in the horizontal and vertical direction respectively. Both methods are then compared for sanity check, storing the ellipse major and minor axis as the water patch length and width if their value is consistent. Figures 21d and 21e show an example of this. In any case, a separate tool was developed to check manually the ellipse was a good fit.
4. Water Impact Dynamics

Figure 21: Three intermediate steps towards obtaining the final water patch shape, and a last comparison with the actual averaged image. (a) Series of frames obtained from the top view camera. (b) Average image (c) Image resulting from subtracting the reference background image from the mean. (d) Final skeleton obtained with the ellipse fitted. (e) Obtained ellipse superimposed to the original averaged image.

All results mentioned above (forces, flowrate, water patch sizes etc.) are saved into a single spreadsheet which contains all the tests parameters and results. Furthermore, all the variables in the MATLAB workspace are saved into a MATLAB file in case further information or revision is needed.
4.5 Results: Hydrophobicity Study

In this section, the results from the five 'A' plates described in table 3 are shown. First, a comment on the typical water patch shape is exposed, to continue with the quantitative measurements such as water patch size and measured forces.

4.5.1 Water Patch Topology

Two different water spreading patterns were found within the 135 hydrophobic tests, closely related to the hydrophobic or hydrophilic characteristic of the coating.

In the hydrophobic cases (surfaces A2, A4 and A5), shown in figure 22a, the impinging jet spreads in the surface following an ellipse / falling droplet shape. Most of the water is concentrated within two rims created slightly behind the jet impact point. They constitute a clear border of the wet region, and enclose a very thin film of liquid (probably laminar as shown in section 4.5.4) between them. The rims, due to the inertia of the impacting jet, are pushed outwards, increasing the wet region until approximately half of the patch length. At the point of maximum width, as will be argued in section 4.5.2, all the perpendicular-to-the-plate (z direction) momentum is transformed into surface energy. During the second half of the patch, the surface tension energy is transformed back into kinetic energy, pushing the inner region fluid together until the rims touch again. A similar trend and explanation for vertical and horizontal SHS (with jet impinging on top of the surface) is detailed in Kibar (2018) and Kaps et al. (2014) respectively. The width variation can be easily understood with a mass-spring analogy. When the mass passes through the center of the oscillation, all its energy is kinetic. However, once one of the extremes is reached, the entire kinetic energy budget is transformed into potential energy. Similarly, at beginning of the patch, the flow has mainly outward (y direction) kinetic energy, which is later transformed into surface tension/potential energy at the maximum width. On the second half of the water patch, surface tension is transformed back to inward kinetic energy, forcing the rims to close. This behaviour in hydrophobic surfaces has already been seen in other studies in which they were placed vertically against a down-facing jet (Kibar et al., 2010).

Nevertheless, unlike the spring scenario, the rims do not oscillate more than once. Instead, once they touch, the water detaches from the plate in a jet shape. This is substantially different from some cases shown in Kibar et al. (2010), where the oscillation between rims open and closed was denominated braiding.

Hydrophilic surfaces (A1 surface), such as depicted in figure 22b, show a different dewetting pattern. No rims are observed, but the water spreads equally in all directions, forming a nearly constant film thickness. For the flowrates used, it looks as if water does not detach from the plate but accumulates on the edges of the patch due to friction drag. Once enough water is accumulated in a certain location, a drop falls almost vertically (see figure 24a). Indeed, looking at the side view video, droplets with a certain radius fall periodically from the edges, as was previously described for jet impacts on the underside of a flat plate in Jameson et al. (2010) and Brunet et al. (2004).

Figure 22 shows a top view and section of the two cases described above. The bright curved lines in figure 22a are the rims that enclose a laminar region (transparent). In the section view, it is hypothesized that the two rims form a partial cylinder with a contact angle in the neighbourhood of the surface contact angle (see table 3). On the right panel, figure 22b, the film is thin and laminar (transparent again) in most of the wet patch. At the border or the wet region, water concentrates as described above and falls into droplets of a given sized, discussed below.
4. Water Impact Dynamics

Figure 22: (a) Top view of the Neverwet (A2) surface. Flowrate, pipe angle and pipe diameter were 4.8 l/min, 45 deg and 9.3 mm respectively ($Re = 1.25 \times 10^4$, $We = 231$). Notice the two turbulent brighter rims delimiting a thinner laminar (clear) region in the middle. (b) Top view from the Glass (A1) surface. Flowrate, pipe angle and pipe diameter were 4.8 l/min, 45 deg and 9.3 mm respectively ($Re = 1.25 \times 10^4$, $We = 231$). Note in this case how the two rims are absent and the water accumulates in certain regions (time dependent) from where drops detach. Note as well the much larger area wetted in the A1 case. (c) Section view at the lines indicated in figures (22a) and (b). Note the contact angle ($\theta$) at the edge of the water patch.

To understand the two distinct dewetting scenarios and detachment mechanisms, and in an attempt to model the water patch rims, the energy balance of a water cylinder attached on the underside of a plate is studied. Two possible scenarios are investigated: i) the cylinder is attached to the surface, with a contact angle $\theta$ and a constant volume $V$ per unit length that determines the cylinder radius when attached $r_A$ and detached $r_D$. And ii) the cylinder is tangent to the surface but detached. It has perfect circular cross-section and there is no contact with the surface (figure 23a).

The volume $V$, contact area between the liquid and solid $A_{SL}$, area between liquid and gas when detached $A_{DG}$ and attached $A_{LG}$ and the centroid $c$ per unit length can be computed as a function of the contact angle $\theta$ and attached $r_A$ and detached radius $r_D$. Table 5 summarizes those values:

Establishing an energy balance between the detached and attached cylinder, taking into account surface tension and gravitational potential energies, we obtain:

$$A_{SL}^A \sigma_{SL} + A_{LG}^A \sigma_{LG} - \rho_w V^A g c^A = A_{DG}^D \sigma_{LG} + A_{SG}^D \sigma_{SG} - \rho_w V^D g c^D$$

(3)

where the superscripts $A$ and $D$ indicate the attached and detached value from table 5 respectively; $g$ is the gravity acceleration; $\rho_w$, the water density and $\sigma$ the surface tension between the two different media, as indicated with the subscript. Notice the negative sign in front of the potential energy terms since the zero potential energy line is taken at the plate surface height.

Using the Young’s Equation that relates the surface tension forces at the triple point and the contact angle ($\sigma_{SG} - \sigma_{SL} = \sigma_{LGC} \cos(\theta)$), and realizing that the detached solid-gas area is equal to the attached

\[^{7}\]New area of solid-gas contact created when the drop is detached
4. Water Impact Dynamics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Attached</th>
<th>Detached</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area Liquid - Gas ($A_{LG}$)</td>
<td>$2\theta r_A \Delta L$</td>
<td>$2\pi r_D \Delta L$</td>
</tr>
<tr>
<td>Area Solid - Liquid ($A_{SL}$)</td>
<td>$2\sin(\theta) r_A \Delta L$</td>
<td>-</td>
</tr>
<tr>
<td>Area Solid - Gas ($A_{SG}$)</td>
<td>-</td>
<td>$2\sin(\theta) r_A \Delta L$</td>
</tr>
<tr>
<td>Volume ($V$)</td>
<td>$\frac{\Delta L r^2}{3}(2\theta - \sin(2\theta))$</td>
<td></td>
</tr>
<tr>
<td>Centroid ($c$)</td>
<td>$r_A \left(\frac{4\sin^2(\theta)}{3(2\theta - \sin(2\theta))} - \cos(\theta)\right)$</td>
<td>$r_D$</td>
</tr>
</tbody>
</table>

Table 5: Values of different areas, volumes and centroids of the attached/detached cylinder as a function of their radius ($r_A$ and $r_D$), unit length ($\Delta L$) and the contact angle ($\theta$) in radians.

...where $\sigma_{LG}$ is the typical water surface tension that can be found in literature, and that was measured to be 0.058 N/m (see appendix B).

Finally, the detached and attached cylinder radius can be related by the cylinder volume. Consequently, we can write equation 4 as:

$$r_D = f(\theta)$$

which gives a curve that separates the cases in which the attached total energy is higher than the detached one and therefore the rim will detach and vice versa. Such function corresponds to the continuous line in figure 23b. To the left of the line, the energy of the attached rim is lower than that of the detached, and therefore it will preferably stay attached, and vice versa for the right region. Superimposed with the continuous line, the discrete points represent the actual rim diameter and surface contact angles used in the test as shown in table 4. The actual rim diameter is obtained assuming the flowrate through the laminar middle film is negligible, and therefore, the jet flowrate is divided into two rims. Mass conservation dictates $d_{rim} = d_{jet}/\sqrt{2}$.

Interestingly, for the glass (surface A1), the jet diameters used are too small to detach from the surface. That is, energetically, it is more stable to remain attached in the surface, even more if we consider that the jet immediately spreads into a thinner film after the impingement. However, as water is slowed down by the plate friction, it accumulates in the edges, increasing the film thickness, i.e. moving to the right on the graph. At some point, drops are too heavy for the surface tension to keep them attached, and fall almost vertically. A totally analogous analysis for attached and detached drops, not shown in this report, shows detaching drops from the glass surface (A1) have a diameter between 10 and 15 mm, which is the size seen in figure 24a.

On the other hand, the Neverwet Surface (A2), lies around the borderline. To explain this case, it is convenient to remember that upon impact, the jet is divided into two rims preventing their detachment. However, as soon as the rims merge at the end of the water patch, the size surpasses the borderline and therefore detachment occurs.

For the superhydrophobic surfaces (Nasil SHBC - A4 and Cytonix 800M - A5), the most energetically efficient state is completely detached. And, even though in most cases the detachment is produced when the two rims merge as well, for the same reasons stated above, in some particular situations water detached in a film continuously around the borderline (see figure 24b), or even two or three jets were ejected before the rims joined (figure 24c).
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Figure 23: (a) Cylinder cross-section used to develop the energy model to distinguish attached and detached rim states. $\theta$, $r_A$ and $c$ denote the contact angle, attached cylinder radius and centroid distance to the surface. On the detached side, only the parameter $r_D$, detached radius is necessary. The cylinder is considered to be tangent to the surface. (b) Map of the most energetically stable position for a rim on the underside of a plate. The solid line represents equation 5. To the left of the line the most energetically favourable position of the rim is attached to the surface, while on the right side, the rim will tend to detach from the surface. The $x$ axis contains the rim detached diameter from the model for the solid line, and the 'actual' rim diameter seen in the tests computed as $d_{rim} = d_{jet}/\sqrt{2}$.

Figure 24: Different detaching mechanisms (a) Detaching in glass surface. Note the almost vertical drop trajectories thanks to the high exposure time. Drop size is of the order of 10-15 mm, as predicted by figure 23b. Flowrate, pipe angle and pipe diameter were 2.46 l/min, 35 deg and 6.13 mm respectively ($Re = 9.75 \times 10^3$, $We = 212$). (b) Continuous film detachment on the border of the Cytonix800M (A5) plate. Flowrate, pipe angle and pipe diameter were 4.62 l/min, 45 deg and 9.3 mm respectively ($Re = 1.21 \times 10^4$, $We = 214$). (c) Three jets emerging from the NaisolSHBC (A4) plate. Flowrate, pipe angle and pipe diameter were 4.26 l/min, 35 deg and 9.3 mm respectively ($Re = 1.11 \times 10^4$, $We = 182$).
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4.5.2 Water Patch Width

One of the key arguments to understand the water patch topology described in section 4.5.1 was the interplay between the vertical velocity component (perpendicular to the plate, \( z \) direction in figure 15a) and the ‘elastic’ surface energy. Indeed, as argued by Kaps et al. (2014), the \( x \) velocity component plays a minor role in the expansion of the water in \( y \) direction. The behaviour of the water after impacting the surface is analogous to that of a spring. The jet inertia is transformed into potential elastic energy at the maximum spread, in which the interface acts as an elastic membrane. Neglecting the change in gravitational potential energy (that is, assuming the shift in center of mass throughout the wet area length has a small impact), the energy per unit length before the water touching the surface is formed by the jet kinetic and surface tension component:

\[
E_J = \frac{1}{2} \rho_w v_z^2 \frac{d^2}{4} \pi + \sigma_{GS} A^J_{GS} + \sigma_{GL} \pi d
\]

where \( \sigma \) is the surface tension, with \( GS, GL \) meaning gas-solid, gas-liquid interface respectively, \( v_z \) is the perpendicular jet velocity component, \( d \) indicates the jet diameter and \( \rho_w \) is the water density. The superscript \( J \) refers to jet properties before the impact.

At the point of maximum spread, we can assume all the perpendicular velocity is null, obtaining:

\[
E^S \frac{\Delta l}{\Delta y} = \sigma_{GL} A^S_{GL} + \sigma_{LS} A^S_{LS} + \sigma_{GS} A^S_{GS} - E_d
\]

where \( LS \) indicates the liquid-solid interface, and \( E_d \) is the energy dissipated due to viscosity per unit length. The superscript \( S \) indicates the patch properties at maximum spread.

As a first approximation, and acknowledging it is only valid for a handful of cases, we assume dissipation is small (\( E_d \approx 0 \)). The different areas per unit length (\( \Delta l \)) are \( A^S_{GS} = b_{max}, A^S_{GL} = (2-f)b_{max}, A^S_{LS} = f b_{max} \) and \( A^S_{GS} = (1-f)b_{max} \), where \( f \) is fraction of plate that is actually in contact with the water and \( b_{max} \) is the maximum water patch width. Recall that water in contact with superhydrophobic surfaces can be in either Wenzel \((f = 1, \text{all surface wetted})\) or Cassie state \((f < 1, \text{"fakir" position})\), as shown in image 4a. Note as well that the liquid-air interface area \( (A^S_{GL}) \) is computed neglecting the extra interface surface from the rims curvature and film thickness, which incurs in a 2% error approximately. Using once again Young’s equation to incorporate the contact angle instead of the solid-gas and solid-liquid surface tension, we obtain, for the maximum width:

\[
\frac{b_{max}}{d} = \frac{\pi}{8} \sigma_{GL} \left( 2f + (1 - f \cos(\theta)) \right) + \frac{\pi}{2 - f(1 + \cos(\theta))}
\]

where \( \theta \) is the contact angle.

The wet fraction \( f \) can be considered 1 for the A1 surface, glass, meaning that all the surface in between the water and plate is wetted. For the (super)hydrophobic cases (A3, A4, A5), the wet fraction value is unknown. However, as it is detailed in section 4.5.4, the collapse of all the surfaces when using the usual friction coefficient, and the dynamic pressure arguments showed in section 3.1, might indicate that, for the jet velocities used, the water in contact with the surface is in Wenzel state. Therefore, considering the wet fraction equal to 1 \((f = 1)\), equation 8 becomes:

\[
\frac{b_{max}}{d} = \frac{\pi}{8} \sigma_{GL} \left( (1 - \cos(\theta)) \right) + \frac{\pi}{(1 - \cos(\theta))}
\]

Realising the term \( \frac{\rho dv_z^2}{\sigma_{GL}(1-\cos(\theta))} \) is the contact-angle-modified (CA-modified) Weber number of the perpendicular jet velocity component, we can write:

\[
\frac{b_{max}}{d} = \frac{\pi}{8} W_{EGZ} + \frac{\pi}{(1-\cos(\theta))}
\]

Although not much attention has been given to such Weber number modification, some papers such as Son and Kim (2009) already used it to characterize the spreading diameter at the equilibrium of a inkjet droplet.
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Figure 25 shows the 135 test carried out in the hydrophobicity study classified by surface and pipe size in mm. The model prediction is represented with a continuous line for the Cytonix800M case, with a contact angle $\theta = 150$.

![Figure 25: Non dimensional water patch width against perpendicular, CA-modified Weber number for all the 135 test carried out. The legend only distinguishes surfaces, but different pipe diameters, angles and flowrates are included in the graph. Note the collapse of the (super)hydrophobic coated plates, different from the glass surface.](image)

The two different wetting and dewetting patterns explained in section 4.5.1 for hydrophilic and (super) hydrophobic surfaces can be clearly observed.

All the cases with contact angle above 90 degrees collapse well under the CA modified Weber number and fit to a certain extent the theoretical model. For larger Weber numbers, the patch width is bigger and therefore viscosity effects start to become important, killing the linear relationship between the water maximum width and CA-modified Weber number based on the $z$ velocity component.

In the case of hydrophilic surfaces (A1 - glass), given its dewetting procedure, we would expect that the maximum width is not given by the interplay between inertia and surface energy, but between inertia and viscosity. That is, the water will advance until viscous friction slows it down enough to form big enough droplets to fall as explained in 4.5.1.

4.5.3 Water Patch Area

The area is computed by the post processing algorithm as the area within an ellipse with major and minor axis equal to the length and width described above. Even though it is an approximation to the actual water patch shape, after examining around 10 particular cases, the error was lower than 4%, which is already on the order of the error due to the rim thickness.

As it is a variable directly dependent on the width and length, the scaling should be somehow related as well to the Weber number. Furthermore, for ALDR-SHS related work, finding a successful scaling for it is of utter importance, as it would be the link that would relate the incoming water mass on the test model with its spread on the surface after impact depending on the surface characteristics. Thus, instead of looking for a more theoretical approach, we have used some of the parameters that have been used in the previous sections, obtaining an interesting data collapse when the contact-angle-modified Weber number is used, defined as:
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\[ We_\theta = \frac{\rho dv^2}{\sigma_{GL}(1 - \cos(\theta))} \]  

(11)

in which the absolute value of the jet velocity itself and not any of its projections is used. Plotting the non-dimensional wet area with the incoming jet area against the Weber number defined above, we obtain figure 26b, in contrast with figure 26a, in which the non-modified Weber number is used:

![Figure 26](image)

Figure 26: (a) Non dimensional area with the incoming jet cross-section against Weber number. (b) Non dimensional area with the incoming jet cross-section against the contact-angle-modified Weber number. The fit curve is forced to pass through the origin.

The use of the CA-modified Weber number clearly enhances the collapse, especially among the (super)hydrophobic surfaces, but also the glass slope is brought closer to the other data, allowing for an empirical fit, which is used for the spread model detailed below in section 5.1. Although it is not
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It is extremely clear, it can be seen how the scatter flattens for larger Weber numbers, due, once again, to the appearance of viscosity.

4.5.4 Force

Although there have been multiple previous studies regarding SHS force measurements (Kibar et al., 2010; Daniello et al., 2009), to the best of our knowledge no experiment has obtained the area and net horizontal force due to a jet impact. Kibar et al. (2010), for instance, measured the force on a vertically placed surface. However, this setup may induce to errors in the measurements due to possible drops of water stationary on the plate, whose weight will have a considerable effect on the final results considering the small forces seen.

On the other hand, in the setup here described, the horizontal position of the plate and the one-meter length wires from which the plate is suspended ensure a minimal error when measuring the friction force of the water impinging on the surface.

The force measurement was found to be clearly dependent on the jet angle, jet velocity, surface and pipe diameter. A good collapse is however possible by using the friction factor constructed with the velocity $x$ component. Indeed, figure 27 shows the collapse of the 135 test by using the friction coefficient and Reynolds number defined as:

$$C_f = \frac{F_x}{\frac{1}{2} \rho_w v_x^2 A} \quad \text{Re}_L = \frac{\rho_w v_x L}{\mu}$$

where $F_x$ is the horizontal force measured by the load cell, $\rho_w$ is the water density, $v_x$ is the $x$ component of the velocity and $A$ and $L$ are the water patch area and length measured.

Figure 27: Parallel-velocity-based friction coefficient against Reynolds number based on the water patch length. The continuous lines represent the turbulent and laminar approximations to boundary layer friction

The continuous dashed lines represent the theoretical friction coefficient for turbulent and laminar boundary layer given by Prandtl and Blasius respectively for smooth surfaces. Note they correspond to the local Blasius and the Prandtl one-seventh-power law (Cencel and Cimbala, 2006) integrated over a length from 0 to the water patch length $L$:

$$C_{fLAM} = \frac{1}{L} \int_0^L \frac{0.664}{\sqrt{Re_x}} dx = \frac{1.33}{\sqrt{Re_L}}$$

$$C_{fTURB} = \frac{1}{L} \int_0^L \frac{0.027}{Re_x^{1/7}} dx = \frac{0.032}{Re_L^{1/7}}$$

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with \( \text{Re}_x = \frac{\rho_w v_x x_{trans}}{\mu} \). Errors regarding rectangular integration instead of ellipse have been numerically studied and bounded below 16 %. \( v_x \) is assumed constant.

Even though the net force is smaller for superhydrophobic surfaces, the collapse not taking into account any of the surface properties indicates that such force reduction is in this case is solely an effect of the smaller area wetted. Probably, due to the high impact velocity, no air pockets are retained in the superhydrophobic surfaces, entering a Wenzel state from the moment the water touches the surface \( (f = 1) \). Furthermore, the better agreement with the laminar theoretical prediction indicates that most of the water film is laminar, which could have been seen by observing that most of the surface wetted, located between the rims, is totally clear (turbulent film would not be clear). Another reason to believe most of the data is laminar is the critical Reynolds number:

\[
\text{Re}_{x,\text{crit}} = \frac{\rho_w v_x x_{trans}}{\mu}
\]

where \( \rho \), \( v \), \( x_{trans} \) and \( \mu \) represent the density, velocity, transition downstream location and dynamic viscosity. Although transition is currently an active topic of research, in engineering practices, and following Anderson Jr (2010), it is typical to consider flows around and below the region \( 5 \times 10^5 \) laminar in cases of zero pressure gradient, which is the region for most of the data in graph 27.

Therefore, we can conclude that when the wetting state is of Wenzel type (no air pockets), the reduction in force displayed by superhydrophobic surfaces corresponds solely to their ability to control the water spread in them due to the lower energy coating.

On a separate but related topic we can compare the jet incoming momentum with the force measured in the plate. Using the integral momentum Navier-Stokes law applied to a control volume delimited by the water patch boundary, we obtain

\[
\int \int \rho_w \mathbf{v} \mathbf{v} \, ds = \int \int \tau \, ds
\]

Assuming all the horizontal momentum inlet comes from the jet and that the fluid leaves the control volume vertically (which is only partially true for the glass A1 surface), we obtain, for the horizontal component:

\[
\rho_w v^2 \cos(\alpha) \sin(\alpha) \frac{A_{\text{jet}}}{\sin(\alpha)} = F_x
\]

where \( \alpha \) is the incident angle of the jet.

The actual measured force is plotted in the \( x \) axis against the jet incoming momentum in the \( y \) axis in figure 28. In the case all incoming momentum was lost, the points would lie in a 45 deg line. This is the case for most of the glass cases, which supports the idea proposed in section 4.5.1 of the water departing the surface once it stops (losses all its momentum) and accumulates. On the other hand, for the hydrophobic cases the points should lie in the lower-right triangle, which indicates not all momentum is transformed into force, i.e. the water departs with an angle from the surface due to surface tension dewetting. Figure 29 shows how the detaching angle is clearly close to vertical in the glass surface and almost the same as the incident angle in the Cytonix800M (little momentum lost).
Figure 28: Horizontal incoming jet momentum computed from the jet velocity and cross section area and pipe angle against measured horizontal force. The dashed line represents the case in which all horizontal momentum dissipated via viscous friction and therefore its equal to the measured force. Markers below the line indicate that not all its horizontal momentum is transformed into horizontal force.

Figure 29: (a) Water rebound on Cytonix800M. Flowrate 2.58 l/min, Pipe size and angle 6.13 mm and 25 deg respectively ($Re = 1.02 \times 10^4$, $We = 233$). (b) Water vertically detachment on Glass. Flowrate 1.98 l/min, Pipe size and angle 6.13 mm and 25 deg respectively ($Re = 7.84 \times 10^3$, $We = 137$). Higher flowrates on the glass surface could not be reached due to limitations on the plate size.
4. Water Impact Dynamics

4.5.5 Visual Comparison

As a final recap for the water patch topology, area, momentum loss, etc. figure 30 shows the effect surface and flowrate have in the water patch area, dewetting mechanism and angle. For a constant pipe angle of 35 degrees and pipe diameter of 9.3 mm, the pictures show four surfaces and flowrates, increasing contact angle left to right and flowrate top to bottom. The top view (top in each panel) is the result of averaging around 3000 images whereas the side view (bottom of each panel) is an instantaneous capture.

Figure 30: Top and side view for 14 different test cases. Pipe angle and size are constant and equal to 35 degrees and 9.3 mm. Flowrate is, from top to bottom 2.12, 4.25, 6.4 and 8.52 l/min. Reynolds no. is therefore $5.54 \times 10^3$, $1.11 \times 10^4$, $1.67 \times 10^4$, $2.22 \times 10^4$. Weber no. is 45, 181, 411, 728. Surface is, from left to right, glass (A1, 45 deg CA), Neverwet (A2, 101 deg CA), Naisol SHBC (A4, 142 deg CA) and Cytonix 800M (A5, 150 deg CA). No higher flowrates for glass were possible due to plate size limitations.

The difference in water spread on the plate is significant among the glass and the superhydrophobic Naisol SHBC and Cytonix 800M. In fact, no further flowrates could be explored in the glass due to plate size limitations.

Looking at the side view, the angle of departure from the plate increases with the hydrophobicity of the plate, as has been argued in section 4.5.4.
4.6 Results: Roughness

Surface roughness can as well have an impact on both the pressure fluctuations and wetted area. Elbing et al. (2013) showed a successful scaling for Air Layer Drag Reduction in rough and smooth surfaces with a different approach as the one described in this report. Therefore, it is of interest to investigate if the current hypothesized theory can account as well for the changes observed among different plate roughness, unifying the scaling of roughness and hydrophobicity in ALDR.

It is as well of interest to obtain the possible influence of roughness in the water patch nucleation and growth, as many superhydrophobic surfaces are, to a certain degree, rough. Surfaces B1 (smooth) and B2 (rough) are studied in terms of friction, and more importantly, water spread. Time limitations resulted in only two surfaces being produced in a manual labour. Therefore, inhomogeneities are perhaps larger than expected, which may have had a negative impact in the final results.

They are however shown in the two next sections as a first approach to understand roughness effects, a work that will be continued at FLOW Lab.

4.6.1 Force

There is extensive literature regarding force of an incoming jet on a flat plate. Therefore, the horizontal force measured is mostly considered a reliability check. Similarly to the hydrophobic studies, the horizontal force is non-dimensionalized with the \( x \) velocity component, the water density and patch area, obtaining the following friction coefficient:

\[
C_f = \frac{F_x}{\frac{1}{2} \rho_w v_x^2 L}
\]  

(17)

It is plotted in figure 31 against the \( x \) component Reynolds number based on the patch length, defined as:

\[
Re_L = \frac{\rho_w v_x L}{\mu}
\]

(18)

Figure 31: Parallel-velocity-defined friction coefficient against Reynolds number based on the water patch length. Solid lines represent the turbulent and laminar approximations to boundary layer friction. For reference, the Blasius and Prandtl theoretical laminar and turbulent coefficients integrated along the water patch length as shown in section 4.5.4 have been included.

\[
C_{fLAM} = \frac{1.33}{\sqrt{Re_L}}, \quad C_{fTURB} = \frac{0.032}{Re_L^{1/7}}
\]

(19)
The smooth plate friction coefficient tends, as the Reynolds number increases, to the laminar theoretical prediction as expected. For lower Reynolds number however, the measured friction is larger than the theoretical, which can be probably due to two main reasons: 1) the larger relative errors as the jet velocity becomes smaller (deceleration of the water jet before actually touching the surface begins to be important, not all water jet might touch the plate as the liquid coming out of the lower part of the pipe falls without reaching the surface etc.) and 2) possible roughness as a result of the manual coating application. Even though the coating was sprayed, some minor non-uniform thickness areas appeared.

On the other hand, the rough surface seems to fit better the turbulent Prandtl one seventh law, which again is within expectation. Indeed, looking at the $k^+$ value in table 3, the surface stands in the transition region from hydraulically smooth to fully rough. Therefore, turbulence might appear in the liquid film in between the rims, raising the friction coefficient closer to the still smooth surface but turbulent flow.

In general, values are higher than, for instance, for the glass and hydrophobic surfaces depicted in figure 27. Larger roughness values might be the clearest explanation for this behaviour.

### 4.6.2 Wet Area

Finally, the water patch area follows an interesting behaviour clearly dependent in the surface coating and somehow counter intuitive. Firstly, it is worth noting that the area computation algorithm performed slightly worse than in the hydrophobic studies since the water patch did not resemble in some cases, specially the rough surfaces, such a characteristic ellipse. Moreover, although inhomogeneities were mitigated as much as possible, some non-symmetric water patches, specially again in the rough surface, indicated different properties depending on location.

The area non-dimensionalized with the jet cross section is plotted against the CA-modified Weber number in figure 32.

![Figure 32: Non dimensional wet area with the jet diameter cross-section against jet CA-modified Weber number for rough (B2) and smooth (B1) surface](image)

Although the surfaces were manufactured with the objective of having identical contact angle, pinning of the water in the sand grains on the rough surface resulted in the droplets not spreading as much.

---

8Pinning, within the field of dynamics of contact line, is the mechanism by which the water lamella that surrounds a droplet on a surface gets stuck in one of the surface defects or roughness (Bonn et al., 2009).
4. Water Impact Dynamics

as in the smooth, obtaining larger contact angles. Therefore, in order to mitigate as much as possible such difference, the CA-modified Weber number is used since it showed to enhance data collapse in the hydrophobic study.

Nonetheless, even employing the CA-modified Weber number, different slopes can be observed. Smooth surfaces have lower spread at low Weber number, but increase rapidly to achieve almost twice the spread at Weber numbers around a thousand. The growth of the rough surface water patch is on the contrary much slower with the Weber number.

As a check, three different flowrates for the same pipe angle and diameter are shown in figure 33. The left images correspond to the smooth surface while the right column shows the rough behaviour. For the flowrates of 7.71, 5.78 and 3.86 l/min (top to bottom), the water patch clearly grows faster on the smooth surface, while on the rough, specially between the two top images, remains almost constant.

![Figure 33: Water patch top view. All six images correspond to a pipe size of 9.3 mm diameter and incidence angle of 25 degrees. From top to bottom for both left and right panel the flowrates are: 7.71, 5.78 and 3.86 l/min (Re = 2.01 × 10^4, We = 596; Re = 1.51 × 10^4, We = 335; Re = 1.01 × 10^4, We = 149 respectively). All pictures have been cropped to the same number of pixels to maintain the real differences in patch size. (a) Smooth surface - B1 (b) Rough Surface - B2.](image)

Further work is required to understand this trend, as, for large Weber numbers the difference in area wetted is more than double. It is however still far from the one caused by different hydrophobicities, where at medium Weber numbers the spread on the glass surface (A1) was already four times the spread in the NeverWet (A2).
Section 4 discussed how different surface properties affect the water spread, differentiating two behaviors for hydrophilic and hydrophobic coatings. That serves only as an answer to the first of the two problems posed in section 3, leaving the turbulence - interface interaction unsolved.

The behavior of the interface between water and air has to be characterized as well in order to have a complete prediction model for ALDR over SHS. The basic research question to answer in this case is related with the deformations that appear in the water-air interface due to the turbulent boundary layer present in the water and air. Given the complexity of the problem and the absence of the appropriate tools for an experimental approach, it was decided to construct a theoretical/numerical model. This section discusses the first iteration in the construction of the model and its results. Given the problem complexity, further iterations are needed to refine the model in order to obtain more accurate results. It is thought however, to have the potential to serve as a powerful simulation tool, and therefore it is included in this report.

The underlying concept behind the model is the idealization of the water-air interface as an elastic membrane, in which the surface tension acts as the classic elastic membrane tension. Pressure fluctuations, water weight, inertia and surface energy will shape the membrane differently at each point in space and time.

5.1 Theory Background

5.1.1 Assumptions

As stated above, the interface is treated as an elastic membrane from solid mechanics. The main assumptions are:

- The membrane is linear elastic, homogeneous, continuous and isotropic
- The vertical deformation of the membrane is small compared to the characteristic wavelength of such deformations
- The membrane has no mass, as it is just an idealization of the interface
- It is perfectly flexible and offers no resistance to bending.
- The tension per unit length (σ) is the same at all points and in all directions and does not change during the motion.
- Only transverse vibrations are allowed.

5.1.2 Model Derivation

Figure 34 shows a side view of a membrane element. The neutral line is defined as the rest position of the membrane in the case of no exterior forces applied. When a pressure difference across the membrane is applied, as in the case in the Youngs-Laplace equation, the membrane deforms to the side of lower pressure so that the tension, tangent at the membrane boundaries, compensates for such
5. Interface Model

Figure 34: ∆x-sized membrane model element diagram force. For ease of understanding, the view is restricted to the x axis, even though the model is 3D. $p'_a$ and $p'_w$ represent the boundary layer air and water pressure fluctuations respectively (dependent on time and x and y coordinates), $\sigma$ is the surface tension, $g$ refers to gravity and $z$ is the ∆x-element control point height.

pressure difference. In the case of this model, the typical elastic tension is replaced by the air-water surface tension. As mentioned above, the main driving force is the pressure fluctuations within the water and air boundary layer. Furthermore, the mass of water above or below the neutral line has a non-negligible mass, which will be taken into account as a finite volume integration; that is, the weight will be computed as the water density multiplied by the cube volume composed by the element area ($\Delta x \times \Delta y$) times the height at the element’s control point ($z_{ij}$). Finally, the inertia of the water mass will be considered as the same water mass multiplied by its acceleration. Air inertia on the contrary is neglected based on its density a thousand times smaller than water.

The final partial differential equation which gives the membrane vertical displacement ($z$) reads:

$$
\rho_w \frac{\partial^2 z}{\partial t^2} = \sigma \left( \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right) - \rho_w \frac{z}{g} - p'_a + p'_w
$$

(20)

where for ease of reading, the variable dependencies $z(x, y, t), p'_a(x, y, t), p'_w(x, y, t)$ have not been included. The z axis is placed perpendicular to the neutral membrane plane, positive upwards. From left to right we can observe the terms corresponding to water inertia, 'elastic' surface tension force, water mass, air and water pressure fluctuations. For further details in the elastic model derivation see appendix D

Given its complexity (non-linear second order partial differential equation), it is worth having a look at the order of magnitude of their terms. The typical values for the different variables, obtained from visual inspection of videos, database or hypothesis are summarized in table 6.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Order of Magnitude</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water density ($\rho_w$)</td>
<td>$10^3$</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Water-air surface tension ($\sigma$)</td>
<td>$5 \times 10^{-2}$</td>
<td>N/m</td>
</tr>
<tr>
<td>Gravity ($g$)</td>
<td>10</td>
<td>m/s²</td>
</tr>
<tr>
<td>Height variation ($z$)</td>
<td>$5 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>Temporal scale ($t$)</td>
<td>1</td>
<td>s</td>
</tr>
<tr>
<td>Length scale ($x, y$)</td>
<td>$10^{-2}$</td>
<td>m</td>
</tr>
<tr>
<td>Water Pressure Fluctuations ($p'_w$)</td>
<td>10</td>
<td>Pa</td>
</tr>
<tr>
<td>Air Pressure Fluctuations ($p'_a$)</td>
<td>0.01</td>
<td>Pa</td>
</tr>
</tbody>
</table>

Table 6: Typical order of magnitude for the variables and parameters in equation 20

The order of magnitude of each member of equation 20 is computed and shown in table 7.
5. Interface Model

<table>
<thead>
<tr>
<th>Term</th>
<th>Order of Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water inertia ( \left( \rho_w \frac{\partial^2 z}{\partial t^2} \right) )</td>
<td>0.1</td>
</tr>
<tr>
<td>'Elastic' surface tension ( \left( \sigma \frac{\partial^2 z}{\partial x^2} \right) )</td>
<td>1</td>
</tr>
<tr>
<td>Water Mass ( (\rho_w z g) )</td>
<td>10</td>
</tr>
<tr>
<td>Water Pressure Fluctuations ( (p'_w) )</td>
<td>10</td>
</tr>
<tr>
<td>Air Pressure Fluctuations ( (p'_a) )</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 7: Typical order of magnitude for each term in equation 20

It is clear therefore from table 7 that the water pressure fluctuations are the driving force, mainly opposed by the water mass and, when the deformation size is small enough, the surface tension. The fact that the air pressure fluctuations are several orders of magnitude below the water fluctuations supports the assumption that changes in surface roughness, which affects directly only the air layer, does not have an important effect on the interface shape. Furthermore, from now on, air pressure fluctuations are neglected.

5.2 Numerical Implementation

5.2.1 Equation Discretization

A finite difference scheme is used to discretize equation 20. Spatial second order derivatives are discretized using a centered finite difference scheme, resulting in:

\[
\rho_w z_{i,j} \frac{\partial^2 z_{i,j}}{\partial t^2} = \sigma \left( \frac{z_{i-1,j} - 2z_{i,j} + z_{i+1,j}}{\Delta x^2} + \frac{z_{i,j-1} - 2z_{i,j} + z_{i,j+1}}{\Delta y^2} \right) - \rho_w z_{i,j} g - p_{i,j}^a + p_{i,j}^w
\]

where \( i \) and \( j \) are the \( x \) and \( y \) direction indices respectively.

For time, an implicit Euler method for higher stability is employed. Such an implicit scheme is constructed by taking the time average values for each node in the right hand side. The non-linearity in the left hand side is 'solved' by employing the solution from the previous timestep, giving:

\[
\rho_w z_{i,j} \frac{z_{i,j}^{n+1} - z_{i,j}^n}{\Delta t} = \sigma \left( \frac{z_{i-1,j}^{n+1} - 2z_{i,j}^{n+1} + z_{i+1,j}^{n+1}}{\Delta x^2} + \frac{z_{i,j-1}^{n+1} - 2z_{i,j}^{n+1} + z_{i,j+1}^{n+1}}{\Delta y^2} \right) - \rho_w z_{i,j}^{n+1} g - p_{i,j}^{n+1,a} + p_{i,j}^{n+1,w}
\]

where the time step of interest is \( n+1 \) while \( n \) and \( n-1 \) are already known. In the case of a 1D wave equation (one spatial coordinate), the updating scheme would look as shown in figure 35

Reorganizing equation 22 a linear system of the type \( Az = b \) can be formed, in which \( A \) is a pentadiagonal matrix, \( z \) is the vector containing the unknown solutions at the time step \( n+1 \) and \( b \) encloses the previous time step information and pressure fluctuations.

5.2.2 Solver Algorithm Flow Chart

The computation algorithm, depicted in figure 36, is the following:

1. **Inputs:** The simulation time and space are input through the test plate dimensions and the total time of simulation. The number of elements both in stream and spanwise directions are input too.
5. Interface Model

![Diagram](image)

Figure 35: Updating scheme for the element $z_{i+1}^j$, which depends on the neighbouring points, and data from previous two time steps.

The number of elements per meter is kept over 100, as instabilities in the code were identified to occur otherwise. Regarding time discretization, the time step ($\Delta t$) used in the PDE discretization is modified when the pressure is extracted in order to match the speed at which the TBL structures pass by. That is:

$$\Delta t_{\text{pressure}} = \frac{U_{\text{real}}}{U_{\text{JHTD}}} \Delta t$$

Besides, density, viscosity, surface tension and upstream velocity of both air and water are given, as well as the surface characteristics through the contact angle.

2. **Extract Pressure:** Since the only parameter not known in equation 20 is the water pressure fluctuation (recall the air pressure is negligible), it is extracted from a DNS database in a 3D mesh ($x, y, t$) defined by the inputs above. More details in the pressure extracting process are given in section 5.2.3.

3. **Scale Pressure:** Given the different flow conditions in the DNS database and the C. Peifer et al. (2019) experiment, a scaling process is applied to the extracted pressure. More details are given as well in section 5.2.3.

4. **System Solution:** The membrane equation 20, discretized as shown in section 5.2.1 is solved in two steps. First, two time steps are solved without the inertia term, neglecting the possible variation in time that, nevertheless, has been shown to be small (Table 7). These two will serve as initial condition for the entire system solution. The boundary conditions, applied on the edges
of the rectangular membrane, are be fixed at $z = 0$ as if the air layer would extend to the infinite from the simulation domain.

5. **Water Impact and Spread**: Once the membrane shape is computed, it is intersected with an horizontal plane in order to simulate the position of the ship hull. Different plane heights model different air layer thicknesses (see figure 37b). The result of such intersection is a series of 2D shapes or 'patches' on the plane, resulting from the intersection, which correspond to the locations wetted by the water (see black contour in figure 37a). The patch layout changes with time and plane height (air layer thickness $- l_{AL}$).

Secondly, the effect of the surface properties is input through a water spread algorithm. Depending on the surface characteristics, the patches laying on the plane resulting from the previously mentioned intersection spread more or less. Since the impact of water on the underside of flat plate has already been studied in section 4, we can use equation 32 to determine the final extension of the water after impact. Numerically, the water spread is done with the dilate defined MATLAB function, which expands the wet patches a given radius. More details regarding the water impact and spread can be found in section 5.2.4.

6. **Average and obtain void fraction**: After the water spread algorithm, we obtain a two dimensional binary array with the domain dimensions $(x \times y)$ with ones where the horizontal plane is wet (water has contacted the plane, either by directly impacting on it or after the spreading) or zeroes where it is not. Recall that the wet patches change depending on time (the membrane shape is time dependant), air layer thickness (different heights result in larger or smaller patches) and contact angle (since the horizontal plane simulates the hull, a lower contact angle will result in a larger wet area after the spreading and viceversa). Thus, it is possible to construct a five dimensional array which encloses the plane points that have been wetted for each timestep, air layer thickness and contact angle. To reduce its dimension and be able to visualize the result, first, an algorithm computes the plane wet percentage for each timestep, air layer thickness and contact angle. Secondly, a time average similar to that of C. Peifer et al. (2019) is employed to obtain a void fraction averaged percentage per air layer thickness and contact angle.

5.2.3 **Pressure Fluctuations**

Solving equation 20 allows to obtain the membrane height $(z(x, y, t))$ at each spatial and temporal location. All inputs (surface tension, water density, spatial discretization, etc.) are known but for the water pressure fluctuations $(p'_w(x, y, t))$. Ideally, the pressure fluctuations on the free surface would be obtained from a DNS simulation or experimental measurements. However, time and resources limitations invite for a compromise solution.

Given multiple studies regarding pressure fluctuations in Turbulent Boundary Layers (TBL) have been conducted in facilities much more prepared than FLOW Lab for this regard, it seems logical to make use of that knowledge instead of trying to solve a problem more complex than the membrane model itself. Acknowledging it is a big but necessary assumption, we neglect the presence of an interface and use the pressure fluctuations extracted from the wall values in a turbulent channel flow database as an input for the model.

In that sense, we would like to thank the generosity of Prof. I. Marusic at Monash University Melbourne (Marusic et al., 2013, 2015) and the John Hopkins Turbulence Database (JHTD) (Li et al., 2008; Perlman et al., 2007; Graham et al., 2016) for allowing us to use their experimental TBL data and numerical Turbulent Channel Flow (TCF) data respectively.

Of special interest is the channel flow data from the third case in the JHTD corresponding to a fully developed channel flow at $Re_\tau = 1000$, with $Re_\tau = \frac{\rho u_\tau}{\mu}$, where $\delta$ is the boundary layer thickness, friction velocity ($u_\tau$) equal to 0.05 and kinematic viscosity ($\nu$), $5 \times 10^{-5}$. Citing from their website:

'... it is a Direct Numerical Simulation (DNS) of channel flow in a domain of size $8\pi \times 2 \times 3\pi$ , using 2048 x 512 x 1536 nodes. Incompressible Navier-Stokes equations are solved using the pseudo-spectral
(Fourier-Galerkin) method in wall-parallel (x, z) planes, and the 7th-order B-spline collocation method in the wall-normal (y) direction."

Even though there exist fundamental differences between the TBL (case of Prof. Marusic data and the flow under the model in C. Peifer et al. (2019)) and TCF (case of the JHTD), we are interested solely in the near wall characteristics, which are similar between the two cases. Furthermore, mean and fluctuating values of the velocity were compared between the experimental data from Prof. Marusic and a scaled version of the TCF DNS, showing good agreement. Therefore, the pressure fluctuations in equation 20 ($p'_{w}$) are obtained from the JHTD third case - TCF, and scaled before inputing them in the model.

<table>
<thead>
<tr>
<th>JHTD</th>
<th>Marusic et al.</th>
<th>Tow Tank (Water)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Type</td>
<td>TCF</td>
<td>TBL</td>
</tr>
<tr>
<td>Data Type</td>
<td>Experimental</td>
<td>DNS</td>
</tr>
<tr>
<td>Available Data</td>
<td>Velocity</td>
<td>Velocity and Pressure</td>
</tr>
<tr>
<td>$Re_{τ}$</td>
<td>1000</td>
<td>2500 - 13000</td>
</tr>
<tr>
<td>$Re_{x}$</td>
<td>$4.0 \times 10^{6} - 2.1 \times 10^{7}$</td>
<td>$2.5 \times 10^{6} - 5.7 \times 10^{6}$</td>
</tr>
</tbody>
</table>

Table 8: Differences and parameters among the three data sources available for the study: John Hopkins Turbulence Database (JHTD), wind tunnel data from Prof. I. Marusic et al. and tow tank experiments as per C. Peifer et al. (2019). $Re_x$ is based in the downstream distance range and the upstream velocity (channel height and bulk velocity for the JHTD).

The scaling for the pressure fluctuations attends to the different flow characteristics between the JHTD - TCF and the C. Peifer et al. (2019) Tow Tank experiments, enclosed mainly in different Reynolds numbers. It is based in an empirical correlation obtained from Tsuji et al. (2007), which is a review of Farabee and Casarella (1991). Accordingly, the wall pressure root mean square is scaled with:

$$\left( p_{rms}^{+} \right)^{2} |_{\text{wall}} = 6.5 + 1.86 \ln \left( Re_{τ} / 333 \right)$$

Given the definition of the root mean square:

$$p_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} p_{i}^{2}}$$

multiplying the $p_{rms}$ is equivalent to multiplying all $p_i$, and therefore, the scaling in equation 24 can be applied also to single values of pressure.

The scaling procedure, which uses equation 24, the turbulent boundary layer approximations given in Schlichting and Gersten (2016) and Prandtl one-seventh law for the friction coefficient, is detailed below:

1. The downstream distance-based Reynolds of the injection point ‘I’ in figure 9a is computed

2. The Reynolds number ($Re_I$) computed above is used to obtain the boundary layer thickness and the friction coefficient at the injection point as:

$$\frac{\delta_{I}}{x} = \frac{0.14}{\ln(Re_I)} G(Re_I) = \frac{0.21}{\ln(Re_I)} \quad C_{f,I} = \frac{0.027}{Re_{I}^{1/7}}$$

where the function $G(Re_I)$ is taken as constant and equal to 1.5 since the Reynolds numbers are among $10^{5}$ and $10^{6}$, as suggested by Schlichting and Gersten (2016).
3. Once the friction coefficient is known, the computation of the friction velocity is trivial:

\[
\tau_{\text{wall},I} = \frac{1}{2} C_f \rho U^2_\infty, \quad u_{\tau,I} = \sqrt{\frac{\tau_{\text{wall},I}}{\rho}}
\]  

(27)

4. The \( Re_{\tau} \) at the injection point is computed as per definition:

\[
Re_{\tau,I} = \frac{\rho u_{\tau,I} \delta_I}{\mu}
\]

(28)

5. Finally, dividing equation 24 particularized for the JHTD TCF (\( Re_{\tau} = 1000 \)) by the one particularized for injection point 'I' we obtain:

\[
\frac{(p^{+}_{TCF})^2|_{\text{wall}}}{(p_{I}^+)^2} = \frac{6.5 + 1.86 \ln(Re_{\tau,TCF}/333)}{6.5 + 1.86 \ln(Re_{\tau,I}/333)}
\]

(29)

Isolating the pressure of interest and taking into account that \( p^+ = \frac{P}{\rho u^2} \), where \( P \) is the dimensional pressure in Pascals we obtain:

\[
P_I = f_P P_{TCF}|_{\text{wall}}
\]

(30)

where \( f_P \) is the scaling factor given by:

\[
f_P = \frac{\rho_I u^2_{\tau,I}}{\rho_{TCF} u^2_{\tau,TCF}} \frac{\sqrt{6.5 + 1.86 \ln(Re_{\tau,I}/333)}}{\sqrt{6.5 + 1.86 \ln(Re_{\tau,TCF}/333)}}
\]

(31)

The fluctuating pressure value at the wall for each location \((x,y)\) at each time \(t\) can be extracted from the TCF DNS data, scaled with the procedure above and input into the interface model equation 20.

A relevant extension to the current work would consider the impact of the existence of a free surface instead of a solid wall in the evolution of the TBL, as has been done, qualitatively by Brochini and Peregrine (2001a,b).

5.2.4 Water Impact and Spread Simulation

Once the surface shape is computed, it is intersected with an horizontal plane in order to simulate the position of the ship’s hull. Different plane heights give different simulated air layer thicknesses. The result of such intersection is a series of series of 2D shapes or patches on the plane, which correspond to the locations wetted by the water. The intersection result changes with time and plane height (air layer thickness - \( t_{AL} \)). Figure 37a shows a detail of the membrane intersected with the plane. The patch is defined as the interior of the intersection between the plane and membrane, depicted in the figure with a black line.
5. Interface Model

Figure 37: (a) Intersection detail between the membrane model and the hull simulated by the grey translucent plane. The intersection is depicted with a black line, and the arrows make reference to the spread direction of the water. (b) Intersection detail side view. $h$ is the column height used to compute the proxy velocity in the CA-modified Weber number, while $d_{\text{patch}}$ represents the equivalent patch diameter used as well for the CA-modified Weber number computation. Note the plane height in this particular case is 0.75 mm, which would correspond to an air layer thickness $t_{AL} = 0.75 \text{mm}$

In the real world, when water impacts with some velocity in a surface, it spreads on it. In order to simulate this, the model computes the final extension of the water after impact. The problem of a water jet spreading on the underside of a horizontal surface has already been solved in section 4, and therefore, the fit obtained in figure 26b and detailed on equation 32 is employed. It is here when the surface characteristics play their role, and the water impact dynamics experiment interacts with the interface model. Lower contact angle will result in larger water spreading and therefore lower void fraction, and vice versa.

The modified Weber number in equation 32 is computed with the water density ($\rho_w$), the diameter of the patch before spreading ($d_{\text{patch}}$) and the surface contact angle ($\theta$).

$$\frac{A_{\text{spread}}}{A_{\text{patch}}} = 1.6W_{\theta}g, \quad W_{\theta} = \frac{\rho_w v_{\text{proz}}^2 d_{\text{patch}}}{\sigma(1 - \cos(\theta))} \quad (32)$$

The jet velocity, for simplicity, is computed from the column height of the water patch ($h$), as shown in figure 37b.

$$v_{\text{proz}} = \sqrt{2gh} \quad (33)$$

where $h$ is the water volume height above the intersecting plane. 37b. Note that $d_{\text{patch}}$ is computed as the radius of the circle with equivalent area as the patch.
5. Interface Model

5.3 Results

In this section, the main results from the numerical interface model are shown and compared to the experimental data obtained in C. Peifer et al. (2019). No further comparison with other ALDR experiments are shown as, to the best of our knowledge, C. Peifer et al. (2019) was the first measurement of the actual void fraction inside the air layer on surfaces with different wettability characteristics. Previous studies just looked at the effective drag reduction, which is not possible to quantify within this algorithm.

5.3.1 Cases of Study

Given the large number of physical and numerical parameters in the model (shown in table 9 in appendix E), approximately 55 simulations were run additionally to the ones show in table 9. Finally, a good balance was obtained with 100 elements per meter in the $x$ and $y$ coordinates. Time discretization, interestingly, became unstable when the number of elements was over 25 per second, probably due to the larger importance of the inertia term, which is destabilizing. The test model length and width was chosen to be similar or equal to that in C. Peifer et al. (2019) (1.81 and 0.91 m) as well as the time of simulation (6 seconds).

The boundary conditions for the membrane height $(z)$ are of Dirichlet type and equal to zero, as if the air layer would extend until infinity. Other values for the Dirichlet boundary conditions were tested but no important influence was observed. Finally, the air layer thickness $(t_{AL})$ and surface contact angle $(CA)$ are equally spaced vectors which typically range from 0 to 0.006 m and 30 - 160 degrees respectively. Water and air density and viscosity were taken from the standard values at 25 Celsius, which are summarized in table 2.

Once the numerical parameters (number of elements, time discretization, boundary conditions etc.) were set, four main studies were carried out:

1. **Baseline:** The baseline corresponds to the final dataset in terms of parameters adjusting, and matches as closely as possible the model data from C. Peifer et al. (2019).

2. **Contact Angle (CA) Study:** In order to obtain the dependence on the contact angle of the air flux required to form a stable air layer (void fraction above 80% ), a case with the same parameters as the baseline was run, including a wider and denser contact angle vector space.

3. **Pressure Scaling Test:** Even though pressure is already scaled as shown in section 5.2.3, the presence of an interface and the assumptions might affect the results obtained, and therefore a sensitivity analysis was done in order to obtain the effect of different pressure scaling by just multiplying the scaling shown in section 5.2.3 by an integer.

4. **Effect of pressure extracting location:** Since the TCF data from the JHTD has a larger domain both in space and time than the Interface Model, it was considered important to determine the influence of the location in time and space from where the data was obtained. Big variations in the results solely due to extracting the pressure in different locations are not desired.

5.3.2 Topology

Before diving into the results, it is interesting to look at the typical membrane shape. The result of the algorithm, as stated above, is a deformed membrane intersected with a horizontal plane. Figure 38 shows the surface shape at a particular timestep, intersected by the plane at a height $(t_{AL})$ of 1 mm. The black contours indicate the intersection between the interface shape and the plane. The interface maximum deformation is in the order of mm while the wavelength is measured in meters or decimeters, which validates our small deformation assumption.

The rising water volumes have a top view diameter on the order from a few millimeters to tens of centimeters. Maximum height of the uprising peaks is in the vast majority of cases below 5mm, and its
5. Interface Model

Figure 38: Deformed membrane intersected by the horizontal translucent plane. The intersections are highlighted with a black thick line. Those black lines are the contour of the wet patches, i.e., where the water would have impacted against the hull. The spread algorithm then takes those patches and spreads them following the fit obtained from the experimental results. Note the vertical axis is in mm and the horizontal are in m for ease of viewing.

It is interesting to observe however that the vertical deformation downwards is larger than upwards, being the fluctuating pressure values responsible for it.

The resemblance to the actual surface can be assessed by comparing the wet surface observed in the videos taken during the experiment in C. Peifer et al. (2019) to the predictions of the current code. Figure 39 shows results for the patches obtained at the end of the plate for a nominal air layer of 1.8 mm. The numerical solution is shown on the right panel whereas one frame from the tow tank experiment video can be seen on the left. The topology and shape of the void fractions change continuously, and therefore it is difficult to compare the exact shape. Furthermore, the smallest structures cannot be captured in the model since the spatial resolution is not fine enough (1 membrane element has a size of $1 \times 1$ cm). However, the size and characteristics of the main wet areas are similar between the two of them.

Figure 39: (a) Experimental bottom view (camera pointing upwards). Clearer regions indicate the presence of water touching the test model. Nominal air layer thickness is 1.8 mm (163 l/min) and upstream water velocity is 1.7 m/s. The model was painted with a superhydrophobic coating. Axes are approximate. (b) Numerical result for the interface model. Similarly, it can be understood as a bottom view with the camera looking up. Clear gray indicate wet surface while darker regions indicate presence of air layer. Air layer thickness is 1.77 mm (162 l/min).
5.3.3 Baseline Comparison with Experimental Results

Figure 40 shows the data obtained with the superhydrophobic coating (156 degrees contact angle) and ‘painted’ coating (74 degrees contact angle) in C. Peifer et al. (2019) Tow Tank experiment for the rear (stern) and front (bow) part of the test model. Superimposed, the results for the numerical model for surfaces with contact angles of 74 and 156 degrees are shown. The numerical parameters employed in the simulation can be observed in table 9 under the Case 1 - Baseline (Appendix E). This case is intended to replicate the conditions in the C. Peifer et al. (2019) Tow Tank experiment, with identical test plate dimensions (1.81×0.91 m) and upstream velocity (1.7 m/s).

The bow and stern differentiation in the figure corresponds to the two different void fraction impedance probes used in the test model, located 56.1 and 156.3 cm aft of the injection slot (see figure 9a). The numerical data, obtained from the average over the entire surface is overlaid as solid and dashed lines.

Figure 40: Comparison between experimental and baseline numerical data for void fraction percentage as a function of the air layer thickness. Experimental data was obtained from C. Peifer et al. (2019) at the front and aft of the test model. Numerical data corresponds to case 1 - Baseline from table 9.

For the lowest air layer thickness, where turbulence is the main breaking mechanism (see figure 11) the model agrees well with data obtained in the case of the SHS coated model. However, at larger air layer thickness the model overpredicts the void fraction. Perhaps the Kevin-Helmholtz instability could play a role here, as the waves formed due to the shear stress in the interface might increase the amount of water protruding the air layer.

On the other hand, the difference between the superhydrophobic and painted surfaces void fraction as predicted by the model is much smaller than that obtained in the experiment. From the model point of view, there are multiple reasons that could be deemed responsible for this difference between model and experiment.

- Including the residence time of the water in the model could be one of the possible solutions. Currently, the void fraction is computed independently for each timestep. Therefore, each timestep assumes all the water from the previous impacts is no longer present in the air layer, and takes into account only the current water impacts to compute the wet area. No accurate value of the time of residence of the water for each surface was easily obtained in the experiments discussed in section 4, but given the larger water patch length and viscous behaviour in the hydrophilic surfaces, the time of residence in hydrophilic surfaces is expected to be above those in (super)hydrophobic ones.

- On another line, an important source of error might be the neglected difference between the nominal and real air layer thickness: The nominal air layer thickness, used in multiple ALDR reports is defined as the air thickness that would result if the injected air spanned the entire model and moved at the same speed as the water upstream flow \( t_{AL} = \frac{Q}{U_{\infty}} \). The actual air layer thickness \( t_{AL\text{actual}} = \frac{Q}{U_{\infty} b} \) in the experiments (C. Peifer et al. (2019) for instance) probably differs.
from this value since the air speed in the air layer is different from the water upstream velocity. Furthermore, work as Kim and Moin (2010) suggests the air speed is actually higher than the water speed \(U_a > U_\infty\). If such is the case, the actual air layer thickness would be smaller than the nominal air film thickness.

The problem in figure 40 arises from using the actual air layer thickness as horizontal axis for the interface model data and the nominal air layer thickness for experimental data. Although both are being assumed to be the same for convenience, this is not fully correct. If we were able to measure in the experiments the actual air layer thickness, and used those values to plot the experimental data in figure 40, the scattered points would move to the left (recall actual air layer thickness is thought to be smaller than nominal air layer thickness). The model solid line would probably then sit closer to the experimental data.

- The pressure fluctuations that are inputted in the model are obtained with a strong series of assumptions, such as the null effect of the free surface, the empirical scaling, etc. Actually measuring the pressure fluctuations, or improving the extraction and scaling algorithm would change the model solution. A first insight into the influence of the pressure fluctuations in the final results is shown in 5.4.1

- There are as well a series of assumptions and models such as the empirical spreading fit, the Reynolds number computation in the pressure scaling, etc. that must be assessed and have an important influence in the results.

Moreover, taking a step back, it is important to recall that the model described in this section is just a first iteration of a simulation tool that will be further developed in the future.

5.3.4 Critical Air Flux \(q_{crit}\) dependence on \(\theta\)

Finally, as an answer to the main research question proposed at the beginning of the thesis, in this section the dependence of the critical air flux required on the surface contact angle is shown in figure 41. The critical nominal air layer thickness \(t_{AL\,crit}\) to form a consistent air film is defined as the value corresponding with a void fraction of 80%. Multiple contact angle surfaces were simulated, obtaining for each surface their critical nominal air layer thickness, which can be then converted to the required flowrate as \(q = bt_{AL\,crit}U_\infty\), where \(b\) is the model span, and \(U_\infty\) is the upstream velocity.

![Figure 41: Air layer thickness necessary to obtain a void fraction of 80% as a function of the surface contact angle.](image)

In agreement with results in figure 40, the difference among different contact angles is not as large as has been seen experimentally. C. Peifer et al. (2019) reported decreases on the critical air flux of
nearly 70%. The shape of the curve indicates a reduction of air layer thickness at lower contact angles but flattens for the hydrophobic range.

Noting that the spread depends on the contact angle as \( \frac{1}{\cos(\theta)} \) as seen in the definition of the CA-modified Weber number, it reasonable to expect larger slopes in low contact angles, which diminishes when moving into the hydrophobic range. However, given the small difference obtained theoretically compared to experiments, a steeper \( q_{\text{crit}} \) slope in all regions can be expected in experimental data (not available yet).

5.4 Parameters dependence

5.4.1 Pressure Scaling Dependence

As discussed in section 5.2.3, the pressure fluctuations are taken from the JHTD and scaled to match the flow conditions (Reynolds number) of the C. Peifer et al. (2019) Tow Tank experiment. Although as a first approximation the numerical results obtained are in line with the empirical data (see section 5.3.3), it is important to verify the sensitivity of the model predictions to changes in the pressure scaling. Not only for the correctness of the solution obtained, but also to quantify how roughness, upstream turbulence etc. might affect the air layer behaviour. Cases 3.1 and 3.2 from table 9 are compared with 4.1, which has identical parameters but for a unity pressure scaling multiplying factor (i.e. the pressure scaling is not modified with respect to section 5.2.3). In the other two cases the already scaled pressure is multiplied by 2 and 3.

Figure 42 shows the void fraction against the air layer thickness for the three different pressure scaling.

![Void Fraction vs Air Layer Thickness](image)

Figure 42: Change in void fraction depending on pressure scaling. Cases 3.1, 3.2 and 4.1 as shown in table 9 with factors of 2, 3 and 1 multiplying the already scaled pressure as per section 5.2.3. Scatter points represent part of the data obtained from C. Peifer et al. (2019) as a reference.

It is easy to understand that larger pressure fluctuations produce larger peaks and valleys in the membrane (figure 38), producing wet spots at higher air layer nominal thickness. Therefore, the shape of the model prediction curve is flatter, i.e. slower increase of void fraction with air layer thickness. This implies that for a given air layer thickness the wet spots are larger, reducing the void fraction. This affects as well the result of the water spread algorithm (see fifth point in section 5.2.2). Larger wet spots induce even larger differences in the final area wetted after the spread.

Interestingly, the average agreement with the experimental data is best when doubling the pressure fluctuations after they have already been scaled according to equation 24. Plotting the critical air layer thickness dependence in the contact angle for this larger pressure fluctuations figure 43 is obtained. With respect to the baseline curve, which is the same as plotted in figure 41, the curve is not only shifted upwards but also steeper, coming closer to that 70 % reduction of the critical air flow \( q_{\text{crit}} \) observed in
5. Interface Model

C. Peifer et al. (2019) when SHS are employed. Further corrections or a more complex water spread model need to be implemented in the code.

Figure 43: Air layer thickness necessary to obtain a void fraction of 80% for different contact angles and scaling pressure factors. Numerical parameters for cases 1 and 2.2 are summarized in table 9. Experimental data extracted from C. Peifer et al. (2019)

5.4.2 Pressure Extraction Location

The domain of the TCF data from the JHTD is considerably larger than the size of the test plate in the Interface Model. Therefore, a choice has to be made regarding the piece of channel and time that will be inputted in the Interface Model. A sensitivity analysis is thus considered in order to acknowledge the variability induced by the different options. Cases 4.1, 4.2, 4.3 and 4.4 from table 9 vary one parameter for each test, namely, x and y location of the sample as well as time section. Several locations were tested, and figure 44 depicts the differences among the most characteristic (larger differences).

Figure 44: Change in void fraction depending on pressure extracting location. Case 4.1 is considered as ‘baseline’, while cases 4.2, 4.3 and 4.4 change time, x location and y location in the TCF database respectively. More information about the cases can be found in table 9. Scatter points represent part of the data obtained from C. Peifer et al. (2019) as a reference.

It is interesting to note that differences are in the order of tens of percents, which, although might be large for results variability, are not that important when compared to the changes the results ex-
experience when other parameters are modified. The trend furthermore seems unaltered, and therefore fetching location cannot be accounted responsible for some of the mismatches between experimental and numerical data.

### 5.5 Turbulence Spectrum Decomposition

On a separate but related topic, and in order to understand the basics of the model described in section 5, a simple Fourier analysis was carried out. The interface, which now will be considered to be 2 dimensional and non-time dependent, is simulated with a series of sines (Fourier Series). The wavelength from the first mode is the length of the model, and the successive N-1 modes have decreasing wavelength. Mathematically we can write:

\[
z(x) = \sum_{n=1}^{N} A_n \sin \left( \frac{2\pi nx}{L} + \phi \right)
\]

where \(x\) is the length coordinate which goes from 0 to \(L\), \(n\) is the wave mode, \(L\) represents the model length, \(\phi\) is a phase generated randomly to obtain arbitrary interface shapes and \(A_n\) is the mode amplitude. The amplitudes follow a Gaussian curve with zero mean (largest amplitude at the first mode) and variable standard deviation. The modes lay on the horizontal axis and their amplitude value are on the vertical axis. Small standard deviations create a very sharp Gaussian, meaning that only the first mode is dominant. A larger standard deviation, on the contrary, flattens the Gaussian, generating more equally important harmonics and resulting in a more chaotic interface.

![Figure 45: Interface shape produced by 50 modes with amplitudes determined by a Gaussian with standard deviation of 1 and 20. z axis has been normalized by dividing by the maximum absolute value for each graph.](image)

Once the interface shape is computed, obtaining the wet area is trivial. An horizontal line is drawn at a certain height, corresponding with the air layer thickness of interest. The intersections with the interface shape are computed, assigning as wet those portions below the graph and dry (void) those above it. Plotting, as in the interface model, the void fraction against the normalized nominal air layer thickness we obtain figure 46

Firstly, notice how the minimum void fraction is approximately 50 %. This is due to the minimum air layer thickness being 0, and therefore around half of the interface is still below it. Secondly, and most importantly, the different interface shapes result in graphs with different curvature. While the lower standard deviation gives a concave curve, the higher one results in a convex shape. C. Peifer et al. (2019) Tow Tank experiment data can be interpret as having a more convex curvature, as can be seen in figure 9.

Acknowledging the model simplicity, this might indicate that the typical shape of the interface in the tow tank experiment is probably closer to a series of more random oscillations than a single dominant
Figure 46: Void Fraction against air layer thickness ($t_{AL}$) for Fourier interfaces with amplitudes with Gaussian distributions 1 and 20. Note the air layer thickness is normalized with the maximum absolute value for each plot.

It is possible too to compare both shapes with the two hypothesized breaking mechanisms in section 3: typically, Kevin Helmholtz instabilities are related with one dominant wavemode that grows until breaking the air layer, as suggested by Kim and Moin (2010). On the other hand, turbulence probably produces a more random, wide-scale interface, similar to the high standard deviations in this model. The fact that the void fraction - air layer graph shape in the experiment is convex, might indicate that the main mechanism producing the water protrude into the air layer is turbulence.
Active drag reduction techniques in water vehicles have regained popularity thanks to the intense research in the air lubrication approach. Bubble drag reduction has yielded its place to air layer drag reduction in the last decade, in which a thin, continuous air layer is produced under the ship hull. Reductions in friction drag shown to be between the 80 and 99%. Net energy savings of around 8-12% have been predicted, probably not enough to compensate the complexity of the system. Nevertheless, by coating the lower part of the hull with a superhydrophobic coating, C. Peifer et al. (2019) showed air flow requirements to obtain a stable air layer reduced by a factor of three.

The report presented here is a first study of the mechanisms that enhance the air layer stability when combined with a superhydrophobic surface. By examining the videos obtained in the tests from C. Peifer et al. (2019) two different breakage mechanisms were identified, namely Kevin-Helmholtz instability and turbulent boundary layer pounding. Since the latter was responsible for the air layer breakage with lower air fluxes, it was decided to specially focus on it.

An experiment was designed in order to obtain the influence of roughness and especially contact angle in the water spread. Two different detaching mechanisms were found: gravity-viscosity dominated for hydrophilic surfaces and surface tension-inertia dominated for (super)hydrophobic surfaces. It was observed that for lower contact angles the energy of the water in contact with the plate was higher when attached to the surface for the thicknesses studied, and therefore only when enough water accumulated the gravitational pull was stronger than the hydrophilic characteristics of the surface. On the other hand, high contact angle surfaces produced a rebound of the incoming water jet, decreasing considerably the area wetted, time of residence in contact with the plate and barely reducing the horizontal momentum of the jet. A successful scaling for the wetted area was obtained using the contact-angle-modified Weber number ($W_{\theta} = \frac{\rho v^2 d}{\sigma (1 - \cos(\theta))}$). No clear dependence was found on other surface characteristic parameters such as the roll off angle.

Furthermore, it was found that in the case of laminar full wet (Wenzel state), the difference in drag between hydrophilic and hydrophobic surfaces is merely related to the water patch area size, contrary to what has been found in other SHS studies. All drag values collapsed when using the typical definition of friction factor.

Roughness effects were also studied from the wet area perspective. Surprisingly perhaps, roughness clearly affects the relation between wetted area and jet incoming inertia (Weber number). The water patch grows on the order of two times faster in smooth surfaces than in rough, having the crossover around a Weber number equal to 200.

On the other hand, a numerical model based on a membrane analogy was developed to study the deformation of the interface between air and water due to the turbulent pressure fluctuations in both fluids. The water nucleation topology observed was qualitatively similar to the ones obtained in C. Peifer et al. (2019) Tow Tank experiments. Combining the wet patches from the code with the water spread upon impact found from experimental data described in the paragraph above, a decent agreement with the tow tank data, specially at low flow rates (where turbulence is the dominant breaking mechanism), was found.

Therefore, as a first approximation, we can describe the influence of the surface characteristics on the air layer stability as following: a low energy surface aids with the control of the water spread and contact time, reducing the amount of liquid that stays within the air layer, and thus rising the void fraction. The scale to which this occurs is associated with the parameter $\frac{1}{1 - \cos(\theta)}$, where $\theta$ is the surface
6. Conclusions & Recommendations

contact angle.

The interface model code described in this report is just a first iteration of what is thought to have the potential of simulating the stability of the air layer depending on different surface characteristics. In that sense, the future works concerns are shown in the section below.

6.1 Recommendations for Further Work

Regarding the water impact experiments:

- **Side view:** Obtain a clearer, lower exposure water impact side view. With some of the lenses FLOW Lab owns it might be possible to zoom in in order to actually observe the state of the water below the SHS (Wenzel or Cassie). Studies with droplets (Lee et al., 2015) have shown it is possible to directly observe the air pockets trapped in the SHS, but no images have been obtained regarding jets to the best of our knowledge.

- **Wet Fraction:** The wet fraction has been mentioned and included in some of the models above (4.5.2 for instance). However, no analysis or measurement has been done in this regard, whereas it could affect the results obtained in this report.

- **Explore the attached-detached map:** The droplet attached-detached map showed in figure 23b has been key to understanding some of the water dewetting processes upon impact on the different contact angle surfaces. It is impressive how such a simple model can predict such different behaviours. Further data can be obtained to complete more regions from the map.

- **Water patch length model:** Attempts have been done in order to obtain a reliable model for the water patch length. Unfortunately, these efforts have been unsuccessful, mainly because of the water film thickness, which is key to the viscous effects in the film, and therefore length, and it is known only with a high uncertainty. Measurements of the film thickness in the water patch would greatly aid in the understanding of the water patch length.

- **Side view 'rain zone':** The rain zone has been defined as the region of the side view average image (figure 20c) where there is water falling. For some cases this zone is small and placed at the very aft of the water patch (when the two rims merge). For others, water detaches from the surface from the very beginning. It is thought that the size of this zone is directly related to the type of dewetting determined by the attached-detached map.

- **Roughness:** Expand and understand the experimental data obtained with rough surfaces, especially the two different water spread behaviour for smooth and rough surfaces.

Regarding the interface model numerical tool:

- **Interface influence on the TBL and vice versa:** For simplicity in the model, the interdependence between the water-air interface and the turbulent boundary layer has been neglected. That is, the TBL properties have not been altered due to the presence of the interface, but just extracted from the fluctuating pressure at the wall in a TCF, scaled to the correct Reynolds number. Therefore, quantitative studies regarding the behaviour of a boundary layer under a multiphase interface have to be done, either numerically or experimentally, to obtain a more accurate description of what might be happening under a ship hull when air lubrication is used. Better scaling for the pressure fluctuations might result from this process.

- **Flow loop studies:** Current work is studying the water patch topology for different surfaces by submerging a series of transparent plates with different contact angle in a flow loop (water wind tunnel). By introducing an air injection system at the beginning of the plate, for the first time, the wetted patch topology and evolution will be observed directly from above. Comparison of such information with the interface model results might provide information on the correctness of the interface model data.
6. Conclusions & Recommendations

- **Difference between nominal and actual air layer thickness**: The nominal air layer thickness, used in multiple ALDR reports is defined as the air thickness that would result if the air injected spanned the entire model and moved at the same speed as the upstream flow. The actual air layer thickness in the experiments probably differs from this value, as the air might move at a different speed within the lubricating film. The importance of this difference is relevant to the comparison between numerical and experimental data showed for instance in figure 40. The numerical data is computed and then plotted using the real air layer thickness, while the experimental data is plotted against the nominal air layer thickness. Both are being assumed to be the same, which is not correct. Furthermore, it is thought that air moves at a higher speed than the upstream flow. This would result in lower real air thickness compared to nominal $t_{AL}$, which would bring the experimental scattered data to the left in graph 40, closer to the numerical results.

- **Water spread implementation**: Water spread dependence on the surface characteristics in the interface model was implemented through the fit shown in equation 32. However, as has been argued, not only the spread is different between surfaces types, but also, for example, residence time in contact with the surface. For hydrophilic surfaces the residence time is higher, resulting in water being already in contact with the surface when a new volume of water hits the coating. This might explain the poor match between surface types in the baseline numerical results when compared to the experimental results.
References


References


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References


Appendix A. Test Plate Measurements

A

Test Plate Measurements

The surfaces described in table 3 were characterized using three different parameters: Contact angle, roll off angle and roughness.

A.1 Contact Angle

The contact angle is defined as the inside angle a droplet forms with the surface it is statically resting on. Its measurement was performed by depositing 20 µL of tap water on top of each surface positioned horizontally. The volume was chosen to be the smallest possible measurable so that gravity would have a minimal effect on the drop shape. The capillary length ($L_C$) for water in normal conditions is 2.74 mm, which corresponds to the radius of a 43 µL semi sphere. The smaller volume of the test drop ensures that gravity does not play a dominant role.

15 different drops were studied, distributed in 5 different points in a cross pattern, in which the cross ends were separate 127 mm from the center, located itself in the center of the plate. 3 measurements were carried out in each location. The interval shown in table 3 for the contact angle is the standard deviation from all the 15 measurements.

The equipment used included a Basler ace acA2040-55um camera, which counts with a Sony IMX265 CMOS sensor and is able to deliver 55 frames per second at 3.2 MP Resolution. One single picture per drop was taken. To enhance contrast, a light source was located opposite from the camera looking at it, so that light diffraction within the water will aid in the contact angle measurement. The images were analyzed with the free software image J plus the Contact Angle measuring plug-in. By selection three points in the water air interface, the software identifies the drop boundaries, the triphase line and uses the best fit between an ellipse and a circle to obtain the angle of the tangent line at the triple point (point in the 2D picture where the water, air and solid are in contact).

![Figure 47: Original pictures taken for the contact angle measurement. (a) A1 - Glass (45 ± 8). (b) A2 - NeverWet (101 ± 3). (c) A4 - NaisolSHBC (142 ± 4). (d) A5 - Cytonix800M (150 ± 2).](image)

A.2 Roll off Angle

The roll off angle indicates the angle at which a drop positioned statically on top of a surface starts moving down the slope. There has been some attempts of relating it to the contact angle and roughness, or advancing and receding contact angle, but their link is still unclear (Smyth, 2010).
Appendix A. Test Plate Measurements

The setup employed tilted the plate in the longitudinal direction (direction of the jet). The angle was measured with a digital inclinometer (GemRed Digital Level) with a 0.05 degree resolution. The plate was secured using three aluminum 50x50 mm profiles bolted to an optical table in three of the sides, while the forth, higher border was supported by a lab jack table in order to slowly change the plate angle.

Drops were positioned along the span of the plate at 5 different longitudinal positions. Three measurements were carried for each span line. There was a considerable non-homogeneity in the rolling angle for each test, depending especially in the plate region. For that reason, different regions with lower and higher roll off angle were identified. In most of the cases, the center region of the plate was homogeneous, and the (super) hydrophobic characteristics are deteriorated close to the borders of the plate. The nominal roll off angle that is shown in table 3 corresponds then to the roll of angle of the center region, which is where the jet impinged and water spread.

![Figure 48: Roll off measurement setup.](image)

A.3 Roughness

The surface roughness of each of the plates was measured in a very similar manner as the previous surface characteristics. Five different locations were used to take measurements, with five readings in one of them, and four in each other spot.

A Mitutoyo Surftest SJ-210 optical roughness meter with a 360 µm range and 0.006 µm resolution in the vertical direction was employed. A profile of 17.5 mm is obtained for each of the measurements, formed by 9600 discrete points. The device was calibrated with the calibration plate provided, giving the expected values within one standard deviation.

Several roughness parameters are given by the Surftest SJ-210, from which the arithmetic average \(R_a\) was chosen to be the most representative. The arithmetic average can be transformed to sand-grain roughness (typical roughness measurement used in wall friction studies) according to Adams et al. (2012b) as:

\[
\epsilon = 5.86R_a \tag{35}
\]

where \(\epsilon\) is the surface sand-grain roughness. Such can be later used to obtain the hydraulic roughness \(k^+\) as:

\[
k^+ = \frac{\rho u_r \tau \epsilon}{\mu} \tag{36}
\]
where the friction velocity \( u_\tau \) is computed from the friction coefficient obtained for each plate. The values are summarized in table 3, in which the ± indicates one standard deviation.
Appendix B. Water Surface Tension Measurement

B

WATER SURFACE TENSION MEASUREMENT

Water surface tension in the water impact dynamics setup (see section 4) was measured following the method shown in Lapham et al. (1999). A hollow polished glass cylinder of 29.5 and 27.5 mm exterior and interior radius respectively was employed. While hanging from a calibrated Omega 600 grams load cell, it was submerged a few mm into the water and allowed to rest for some seconds. The water-holding container was then lowered extremely slowly, resulting in the glass cylinder being pulled out of the water. The surface tension formed a round meniscus around the cylinder that was broken once the water container reached low enough. The force drop registered by the load cell when the water detached from the cylinder (see figure 49 indicates the surface tension from the liquid.

Figure 49: Voltage output given by the load cell during one of the surface tension measurements. Dashed lines indicate maximum voltage (force) before the water film breaking and minimum after breakage.

After 6 tests, the surface tension was found to be 58 mN/m ± 1.5, slightly lower than pure water, but similar to previous measurements done by other lab members with the same tap water. Furthermore, the addition of bleach to prevent algae from growing in the setup probably lowered even more the water surface tension.
Appendix C. Calibration

C

CALIBRATION

C.1 Load Cell Calibration

The two load cells (vertical and horizontal force measurement) were calibrated using a series of certified weights. After being fix in a cantilever position, different weights were hanged from the free extreme. Following the same sampling rate as in the experimental setup (250 kHz reading rate, with packets of 2500 points averaged into 1 that is written in an Excel file), 200 hundreds data points were obtained for each weight. The average and standard deviation were computed for each mass.

C.1.1 Horizontal Load Cell

The horizontal load cell is calibrated by increments of 1 gram in the range between 0-10 grams, and then in increments of 10 grams until the maximum allowed of 100 grams. Repeatability within the same mass was below 0.7 %, which agrees with the load cell specifications. Within on test (one weight), the standard deviation relative to the average value was below 0.001 %, which shows the low level of noise obtained thanks to the DMD4059 Omega Strain Gauge to DC Isolated Transmitter. The fit, which had a R$^2$ value of 1, can be seen in figure 50a.

C.1.2 Vertical Load Cell

Since the vertical load cell had to support the frame for the plate weighting approximately 3.5 kg its calibration was carried out with a preload of 3.5 kg as well. On top of them, decrements of 50 grams were registered with the same sampling rate as for the horizontal load cell. Decrements were chosen given that the jet impinging on the underside of the plate would provoke a decrease in the force measured by the load cell. Again, repeatability within the same test was below 0.01 % and relative standard deviation (noise) during one weight test was below 10^{-6} %.

C.2 Flowmeter Calibration

The Micromotion Coriolis flowmeter employed in the test consists of a CMF025M319N0AMEZZZ sensor and a 2400S transmitter, which converts the sensor voltages into flowrate, liquid density etc. Two calibrations where necessary in order to ensure the correct functioning of the device.

On a first step, the correct functioning of the sensor was checked by running a constant flow of water for 30, 60, 90 and 120 seconds. The final water mass was measured with a Sartorius Entris 6202-1S Lab Balance with a maximum weight ad precision of 6200 and 0.01 grams. Given the time, the flowrates were computed through the measured water mass and the readings from the flowmeter, giving, in the worst case, a mismatch below 1%.

Secondly, the 4-20 mA circuit that extracts continuous data from the sensor was calibrated in order to match each voltage (the 4-20 mA circuit consisted of a 120 ± 0.1% resistor in parallel with the DAQ measuring point to transform intensity into voltage) to a flowrate. A series of 200 data points, being each already an average of 1000 data points were acquired at a 250 kHz rate. Figure 51 shows the final calibration curve obtained.
Figure 50: (a) Horizontal Strain Gage Calibration. (b) Vertical Strain Gage Calibration.

Figure 51: Flowmeter Calibration. Read voltage is shown in the y axis while data shown in the flowmeter transmitter lays in the y axis.
The derivation for the interface model is based on the derivation for the elastic membrane seen in Kreyszig et al. (2013), adding as external forces the pressure from the air and water boundary layers, the water weight and the inertia term.

Let the interface be considered as a perfectly elastic membrane with the simplifications given in section 5. We can construct an orthogonal grid on the membrane so that now there is a number of square elements with four edges orthogonal to each other, four edge points and one central point per element. Additionally to the assumptions given in section 5, we will assume the following regarding the element behaviour:

- Pressure is considered constant within one element.
- The deformation angle of the membrane (θ in figure 52) is considered constant along the edges. In other words, as it is a 2D membrane that bends in a 3D space, there are two angles that determine the x and y deformation at any given point. Along the edges however, those angles are constant, although they might change from edge to edge within the same element. This means the surface is actually formed of a series of linear tiles.

Figure 52: Geometry of a membrane element. Element might be non-linear inside but the edges are linear so that the angle (θ) is constant along it. σ, p'_a and p'_w indicate surface tension, and air and water pressure fluctuations respectively. Left, oblique view, right side and top view.

The forces acting on each element are:

- Surface Tension σ acting alongside the four edges of each element, tangent to the element surface and perpendicular to the edge. Constant in time and space
Appendix D. Interface Model: Equation Derivation

- Pressure from the air and water TBL, \( p'_w \) and \( p'_a \) respectively, acting per unit surface and dependent both in time and location. Within an element however, they are constant.
- Gravity pulling from the water down, constant and vertical down.

Applying Newton’s second law in the vertical direction we obtain:

\[
\rho_w |V_w| \frac{\partial^2 z}{\partial t^2} = -\sigma \Delta y \sin(\theta(x, y)) + \sigma \Delta y \sin(\theta(x + \Delta x, y)) - \\
- \sigma \Delta x \sin(\theta(x, y)) + \sigma \Delta x \sin(\theta(x, y + \Delta y)) - \rho_w V_w g - p'_a \Delta x \Delta y + p'_w \Delta x \Delta y
\]  

where for ease of understanding the dependence on the position \((x, y)\) and time \(t\) of the vertical displacement \(z\), membrane angle \(\theta\), water volume \(V_w\), air pressure fluctuation \(p'_a\) and water pressure fluctuation \(p'_w\) have not been included. Taking a volume element approach, the water volume can be easily computed as \(V_w = \Delta x \Delta y z(x, y, t)\). Notice this has two effects: first, it is possible for a volume to become negative, which is the reason for the absolute value in the inertia term, but is useful in the case of the water weight, as, when the water is below the neutral line, buoyancy will push it upwards; second, we acknowledge that for small vertical displacements, the inertia term vanishes, which might not be true. However, as it is a stable equilibrium position, and the order of magnitude is small attending to table 7, the error is small.

Dividing equation 37 by \(\Delta x \Delta y\), and taking the limit when the elements tend to 0, we obtain:

\[
\rho_w |z| \frac{\partial^2 z}{\partial t^2} = \sigma \left( \frac{\partial \sin(\theta)}{\partial x} + \frac{\partial \sin(\theta)}{\partial y} \right) - \rho_w z g - p'_a + p'_w
\]  

Geometrically, the sine and tangent of the membrane deformations can be written as:

\[
\sin(\theta) = \frac{\partial z}{\partial x} \sqrt{1 + \left( \frac{\partial z}{\partial x} \right)^2} \quad \tan(\theta) = \frac{\partial z}{\partial x}
\]

where \(x\) is interchangeable for \(y\) when looking at the perpendicular direction. Considering small vibrations, as suggested in the assumptions, means that \(\theta \ll 1\) and therefore \(\tan(\theta) \ll 1, \frac{\partial z}{\partial x} \ll 1\) and finally, \(\sin(\theta) \simeq \frac{\partial z}{\partial x}\), recovering the elastic membrane equation given by:

\[
\rho_w |z| \frac{\partial^2 z}{\partial t^2} = \sigma \left( \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right) - \rho_w z g - p'_a + p'_w
\]

The derivation was supported by Kreyszig et al. (2013) and Feldman (2000).
### Interface Model: Cases of Study

Table 9: Cases studied for the interface model shown in this report. Nx, Ny and Nt correspond to the number of elements for the model length (x) and width (y) and the time (t) of simulation. Similarly, Nt\textsubscript{AL} shows the number of discrete air layer thicknesses investigated, from zero to Max t\textsubscript{AL}. CA is the surface contact angle and x\textsubscript{Pini}, y\textsubscript{Pini}, t\textsubscript{Pini} denote the position and time within the JHTD data from where the pressure is obtained. Finally, P scale indicate the factor by which the pressure fluctuations were multiplied besides the scaling shown in equation 24 in section 5.2.3. Different value Dirichlet boundary conditions were also studied, obtaining similar results independently of their value and therefore using z = 0 at the boundaries.

* From 20 to 170 degrees in steps of 10 degrees

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