The impact of vertical wing placement on the wave drag and sonic-boom performance at supersonic speeds
Master Thesis Report
H.W. Kinderman
The impact of vertical wing placement on the wave drag and sonic-boom performance at supersonic speeds

Thesis Report

by

H.W. Kinderman

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This research project aims at obtaining a better understanding of vertically translating the wing and the related wing-body interference effects on the drag and sonic boom. Computational Fluid Dynamics (CFD) analysis using the Euler equations has been used to evaluate an airplane with different vertical wing placements at a lift coefficient of 0.15 at a Mach-number of 1.6 and also in zero-lift conditions. Pressure distributions, drag forces and pressure signatures have been calculated in order to assess the performance in terms of wave drag and sonic booms. These results have been analysed to find out why certain effects are happening for these configurations. The low wing configuration has the highest lift-to-drag ratio due to interference on the upper wing surface close to the fuselage. The lift-to-drag ratio for $C_L = 0.15$ is found to be 4.79% higher compared to the worst performing configuration, the high wing configuration. Due to the local geometry of the low wing configuration it is possible to create additional suction on the upper wing surface, which positively affects the performance. Pressure signatures are extracted at 1 body-length distance ($70 \text{m}$) from the aircraft for several azimuth angles. These distributions show that the low wing configuration also has the lowest impulse and maximum overpressure. The higher wing configurations show an extra peak in overpressure emanating from the trailing end of the wing, which is created due to interference effects. Below the wing surface there is a large volume of the fuselage, while it is absent for the low wing configuration. Therefore the higher wing configurations show an extra peak in the pressure signature.

Next to this discovery, an analysis is presented to relate the geometry of the configurations to the wave drag by assessing the cross-sectional area distribution using different intersection methods. These methods are compared with other methods found in literature. Two methods which uses a single Mach-cone have been analysed, as well as a method incorporating a forward and a backward pointed Mach-cone. One method translates a Mach-cone vertically to align the vertex of the Mach-cone with the centroid of the intersection with the aircraft. This gives a $x,z$-position which can be used to adjust the area distribution. The drag for the methods using a single Mach-cone were overestimated by a factor of 2, but after multiplying these results by a factor of $\frac{1}{2}$ the results for the heigh-weighted Mach-cone method approached the wave drag results from CFD within 5%. The double Mach-cone method showed an even better agreement with less variation, while no multiplication factor was applied.

A further analysis has taken place to find out why some methods that incorporate a single Mach-cone to evaluate the cross-sectional area to calculate the wave drag, overestimate the drag by a factor of 2. It is found that these methods do not overestimate the drag for a simple shape, such as a Sears-Haack body. The methods simply overestimate the cross-sectional area, which needs smoothing to obtain an area distribution that is suitable for drag calculation. The double Mach-cone method is a method that smoothens the area distribution, since the forward cone lowers the cross-sectional area in the front of an aircraft and adds cross-sectional area in the tail part of the aircraft.

Further research is required to find out if these methods using a single Mach-cone can be applied to any geometry or not. Next to that it is recommended to research the sonic boom of other aircraft configurations with wings shifted further to backward to see if the sonic-boom from the tail can be reduced. Further research is necessary to see if the interference effects on the upper side of the wing of the low wing configuration are present in viscous flows. If the positive interference effects from this thesis are present in a viscous flow, this might see a reduction in fuel burn and maybe contribute to commercial supersonic flight becoming a reality again.
Acknowledgements

It would have been impossible to complete this MSc thesis without the help and support I received from some people. I would like to thank them for their guidance during my thesis work.

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I would also like to thank Egbert Torenbeek and Willem Bannink for their input in defining the problem statement and their critical view on some aspects. I would like to thank Reinier van Dijk, CEO and founder of ParaPy as well, since their software gave the basis to efficiently construct and mesh multiple airplane configurations. Nico van Beek helped me by supplying a server node to produce the final meshes, without his help it would not be possible to complete this research.

Last but not least I want to thank my family and friends for their support and keeping me positive. Especially I want to thank my girlfriend, Stefani Peters, for cheering me up when problems occured and for proof-reading my thesis.

Hendrik Wisse Kinderman
Delft, December 2017
# Contents

<table>
<thead>
<tr>
<th>List of Figures</th>
<th>ix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nomencature</td>
<td>1</td>
</tr>
<tr>
<td>1 Introduction</td>
<td>3</td>
</tr>
<tr>
<td>1.1 Research objective and goal</td>
<td>4</td>
</tr>
<tr>
<td>1.2 Research approach</td>
<td>5</td>
</tr>
<tr>
<td>1.3 Report structure</td>
<td>5</td>
</tr>
<tr>
<td>2 Literature review</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Supersonic flow</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Characteristics, shockwaves and expansion fans</td>
<td>7</td>
</tr>
<tr>
<td>2.3 Wave drag</td>
<td>9</td>
</tr>
<tr>
<td>2.4 Sonic boom and the boom carpet</td>
<td>9</td>
</tr>
<tr>
<td>2.5 Area rule</td>
<td>11</td>
</tr>
<tr>
<td>2.6 Supersonic area ruling</td>
<td>13</td>
</tr>
<tr>
<td>2.7 Differential area rule</td>
<td>13</td>
</tr>
<tr>
<td>2.8 Factors influencing the sonic boom</td>
<td>15</td>
</tr>
<tr>
<td>2.8.1 Ray tubes and the propagation of sound to the ground</td>
<td>16</td>
</tr>
<tr>
<td>2.8.2 Pressure signature</td>
<td>16</td>
</tr>
<tr>
<td>2.9 Relating the aircraft geometry to the sonic boom and wave drag</td>
<td>17</td>
</tr>
<tr>
<td>2.10 Low sonic boom design</td>
<td>19</td>
</tr>
<tr>
<td>2.11 Current low-boom designs</td>
<td>21</td>
</tr>
<tr>
<td>2.11.1 NASA Shaped Sonic Boom Demonstration program</td>
<td>21</td>
</tr>
<tr>
<td>2.11.2 NASA N+2 Supersonic program</td>
<td>22</td>
</tr>
<tr>
<td>2.12 Effect of wing placement</td>
<td>23</td>
</tr>
<tr>
<td>2.12.1 Non-axisymmetric fuselage shaping</td>
<td>23</td>
</tr>
<tr>
<td>2.13 3D-flowfield</td>
<td>24</td>
</tr>
<tr>
<td>2.13.1 Using a Mach-cone for area ruling and drag estimation</td>
<td>25</td>
</tr>
<tr>
<td>2.13.2 New method of area ruling and drag estimation by using a Mach-cone</td>
<td>26</td>
</tr>
<tr>
<td>3 Methodology</td>
<td>29</td>
</tr>
<tr>
<td>3.1 Numerical setup</td>
<td>29</td>
</tr>
<tr>
<td>3.2 Cross-sectional area distribution data</td>
<td>30</td>
</tr>
<tr>
<td>3.3 Flow definition</td>
<td>32</td>
</tr>
<tr>
<td>4 Geometry</td>
<td>33</td>
</tr>
<tr>
<td>4.1 Requirements</td>
<td>33</td>
</tr>
<tr>
<td>4.2 Sizing</td>
<td>33</td>
</tr>
<tr>
<td>4.3 Wing geometry</td>
<td>34</td>
</tr>
<tr>
<td>4.3.1 Wing planform</td>
<td>34</td>
</tr>
<tr>
<td>4.3.2 Airfoil definition</td>
<td>35</td>
</tr>
<tr>
<td>4.4 Fuselage geometry</td>
<td>35</td>
</tr>
<tr>
<td>4.4.1 Area ruling</td>
<td>35</td>
</tr>
<tr>
<td>4.5 Final geometry</td>
<td>36</td>
</tr>
<tr>
<td>4.5.1 Final base specifications</td>
<td>36</td>
</tr>
</tbody>
</table>
## CFD: meshing and validation

5.1 SU2 .............................................................. 39
5.2 Solver settings .................................................. 39
5.3 Meshing .......................................................... 40
5.4 Mesh refinement ................................................ 41
  5.4.1 Wing edges ................................................ 41
  5.4.2 Nose and tail .............................................. 41
5.5 Mesh validation ................................................ 42
  5.5.1 Difference in lift and drag ............................... 42
5.6 Effect of mesh parameters on pressure signature ......... 43
  5.6.1 Effect of domain size .................................... 43
  5.6.2 Effect of mesh element sizes in the domain .......... 45
5.7 Final mesh ....................................................... 47

## Results

6.1 Aerodynamic performance .................................. 49
  6.1.1 Flow description ........................................ 49
  6.1.2 Lifting conditions ...................................... 50
  6.1.3 Non-lifting conditions ................................. 51
  6.1.4 Analysis of airplane performance ..................... 52
6.2 Sonic boom performance in zero-lift conditions ....... 55
  6.2.1 Development of the shock pattern .................... 55
  6.2.2 Pressure signatures in zero-lifting conditions .... 57
6.3 Sonic boom performance in lifting conditions .......... 58
  6.3.1 Development of the shock pattern .................... 58
  6.3.2 Pressure signatures in lifting conditions .......... 59
6.4 Additional sonic-boom performance parameters ........ 60
  6.4.1 Differences in overpressures during lift and zero-lift 60
  6.4.2 Maximum overpressures at different azimuth angles 60
6.5 Impulses at different azimuth angles .................... 61
  6.5.1 Analysis of wing-body interaction on the sonic-boom 62
6.6 Drag estimation by using different shapes for intersection 65
  6.6.1 Mach-planes ............................................ 65
  6.6.2 Mach cone ............................................... 66
  6.6.3 Height-weighted Mach-cone ............................ 67
  6.6.4 Double Mach-cone ..................................... 67
  6.6.5 Method of Jumper and Nikolic for drag estimation 68
  6.6.6 Results of drag estimation and further analysis .... 68
  6.6.7 Comparing methods with a Sears-Haack body ........ 70
  6.6.8 Relating the pressure signature to the cross-sectional area distribution 72

## Conclusion & Discussion

7.1 Answers to the research questions ....................... 73
7.2 Discussion & recommendations ............................ 74

### Bibliography

A Configuration file for SU2 ................................ 81
B Overview of validation results ............................ 87
C Overview of results ........................................ 89
List of Figures

1.1 A comparison between drag at sub- and supersonic speeds [1].......................... 3
2.1 The formation of a Mach-cone, emanating from a point source flying at speed \( V \) [2] . 8
2.2 Left- and right-running characteristics along a streamline through point \( A \) [2] ....... 9
2.3 Acoustic and aerodynamic reference frames. [3] ................................................. 10
2.4 An illustration of the boom carpet in an atmosphere. ......................................... 11
2.5 Area rule of Whitcomb for an arbitrary wing-body combination [4] ..................... 11
2.6 The zero-lift drag-rise at transonic speeds reduces when a body is waisted. [4] ...... 12
2.7 A Sears-Haack body with a slenderness ratio of 10 ............................................ 13
2.8 Supersonic area rule of Whitcomb for an arbitrary wing-body combination. Note the oblique planes and difference in area distribution for different azimuth angles [5] .... 13
2.9 Differential area ruling example on the Northrop YF-17 [7] ............................... 15
2.10 Sketch of how a N-wave is produced (left) and some characteristics of an N-wave (right) [8] ........................................................................................................... 16
2.11 A sketch of how a N-wave is produced (left) and some characteristics of an N-wave (right) [8] ........................................................................................................... 16
2.12 A sketch of the true characteristics from a body in a supersonic flow, whereas for linearized theory they are described by \( x + \beta r = \) constant and thus are straight lines running diagonal [9] .......................................................... 17
2.13 Nomenclature for Whitham’s theory [8] ............................................................ 18
2.14 The definition of the azimuthal angle \( \theta \). In this figure the airplane is seen from behind. 18
2.15 Fore Mach-cone for the linearized solution. This fore cone shows which region of the aircraft directly influences the sonic boom on point \( x, r, \theta \) [3] ......................... 18
2.16 Pressure signature in black and the impulse (gray). Note that the area just before the leading shock and below the x-axis is also taken into account. ......................... 20
2.17 Sketch of typical N-waves of which the lower 3 are for low sonic boom design. The negative part is more or less a copy of the positive part for illustration. ................. 21
2.18 An overview of the modifications (highlighted in green) to the Northrop-Grumman F-5E to form the SSBD. It is clear that there is an increase in volume on the lower side of the frontal fuselage [10] ........................................... 22
2.19 A comparison in pressure signature for the SSBD and the Norghrop-Grumman F-5E. Visible is the decrease in frontal shock strength, a flat top and a lower shock strength at the aft of the SSBD [11] ...................................... 22
2.20 An impression of the NASA/Lockheed Martin N+2 concept. The airplane features some dihedral and a high-wing configuration [12] ............................................ 22
2.21 Results of wing-placement and wave drag [13] ............................................... 23
2.22 The effect of the different cuts of the Mach-plane with the geometry and the lift[13] .... 23
2.23 A sheared nose section of a fuselage, where the bottom centerline runs parallel to the axial axis [14] .................................................................................................... 23
2.24 Pressure contour of the axisymmetric nose section at Mach 1.8. Because this nose section is axisymmetric, only the upper half is pictured. [14] .............................. 24
2.25 Pressure contour of the sheared nose section at Mach 1.8 [14] ............................. 24
2.26 Intersected airplane by Mach-cones[15] ............................................................ 25
2.27 Area rule by making an axisymmetric body and using the projected area of Mach-cones along the fuselage [16] ................................................................. 25
2.28 The intersection of a Mach-cone with an airplane and the vertical shift of the Mach-cone to align the vertex with the centroid of the cut-through section. ................. 27
3.1 The different steps in the research that have to be taken to answer the research questions. 29
3.2 Different intersection methods for the evaluation of the zero-lift drag. The striped areas are the frontal projections of the intersections that will be used in the calculations for the wave drag. ................................................................. 31
3.3 Difference for different methods for determining the cross-sectional area distribution. 32
4.1 The final wing planform projected on the horizontal plane for the different configurations 34
4.2 Different configurations with unfavourable geometries. ............................................. 36
4.3 Final low-wing configuration as is used in the simulations. ........................................... 37
5.1 Convergence history of $C_L$ and the residual in SU2 for CFL=2 ................................. 39
5.2 Overview of the different blocks within the domain. ....................................................... 40
5.3 The black lines represent the lines where local mesh hypothesis are placed. .................. 41
5.4 The mesh around the wing tips. ....................................................................................... 41
5.5 Difference in pressure coefficient before (left) and after (right) refinement of the nose section. ........................................................................................................ 42
5.6 The pressure distributions over the airfoil at different spanwise locations for different minimum element sizes. The maximum element size is 0.2m. ............................... 43
5.7 Shock interaction at the position of the slice at 95% span. ............................................ 44
5.8 Convergence of minimum element size at the wing for the lift-to-drag ratio. .................. 44
5.9 Visible reflections in the lower regions of the domain when the pressure coefficient is plotted. ........................................................................................................ 44
5.10 Pressure signatures directly below the aircraft ($R = 70m$) for a varying radius of the domain. ........................................................................................................ 45
5.11 Pressure signatures directly below the aircraft for different maximum element sizes in the mid-field block. ........................................................................... 46
5.12 Pressure signatures directly below the aircraft for different maximum element sizes in the far-field block. ........................................................................... 47
6.1 Overpressures for different configurations seen from the side at $C_L = 0.15$. ............... 49
6.2 Mach contours and streamlines on the upper wing surface at $C_L = 0.15$ for the high wing configuration. ................................................................. 50
6.3 Comparison of some gathered results for $C_L = 0.15$ ..................................................... 51
6.4 The pressure distributions over the wing at different spanwise locations in lifting conditions. ........................................................................... 52
6.5 Comparison of the $C_p$-distribution on the upper surface of the wing for 2 configurations. The pressure regions are marked with a distinctive colormap. ........... 53
6.6 Comparison of the projections of the wetted upper wing surface. The projection is on the $XY$-plane. ....................................................................................... 54
6.7 Comparison of the visible ‘channels’ near the junction of the wing and fuselage as seen from behind, where streamlines are drawn for illustration. ........................ 54
6.8 Streamlines visualized on the upper wing surface of the low-wing and high-wing configuration. ....................................................................................... 54
6.9 Comparison between the low and mid wing configuration at zero lift. Note the shock coming from the wing trailing edge on the lower side. ................................. 55
6.10 Isometric view of the overpressure around the non-lifting case for the low-wing configuration. Several cuts normal to the $x$-plane are visible. ......................... 56
6.11 Side view at the plane $y = 5m$ of the overpressure generated by the low-wing configuration. ........................................................................... 56
6.12 Mach contours and streamlines on the upper wing surface at $C_L = 0.0$ for the low wing configuration. ........................................................................... 57
6.13 Pressure signatures for different configurations at zero lift. The signatures are normalized to the lenght of the aircraft. ................................................................. 57
6.14 Effect vertical wing position on locations of peaks in the pressure signatures. ............... 58
6.15 Side view on the symmetry plane of the overpressures generated by the low-wing configuration in lifting conditions. ................................................................. 58
6.16 Pressure signatures at $R = 70m$ for different azimuth angles. The signatures are normalized to the length of the aircraft. ................................................................. 59
6.17 Differences in pressure signatures in lift and zero-lift. For comparison the original pressure signatures are made transparent. ................................................................. 60
6.18 Maximum overpressure and absolute overpressures at lift and zero-lift at 70m distance. 61
6.19 Impulses for all configurations at various azimuth angles in lifting conditions. 61
6.20 A comparison between the low wing configuration and the mid wing configuration in zero-lift as seen from below in different colormaps. 62
6.21 Side view at the plane y = 5m of the overpressure generated by the mid wing configuration. 63
6.22 Comparison of shock development from the wing leading edge in lifting conditions. The slice for the high wing configuration (left) has been made at x = 22.26m to match the shocks for the low wing configuration (right) at x = 20m. 63
6.23 Comparison of overpressures below the wing between the mid wing configuration (left) and low wing configuration (right) during lift. 64
6.24 The striped area and the white area below it in the fuselage illustrates that for the low wing configuration (left) there is more relative decrease in fuselage volume below the wing than for a higher wing configuration (right). The aircraft is seen from the back. 65
6.25 Overview of an airplane which is cut by some Mach-cones. The yellow cone represents a Mach-cone which vertex lies on the longitudinal axis, the red Mach-cones their vertices are aligned with the z-position of their centroid. 66
6.26 Variation of the z-location of the Mach-cone which is used to cut the different geometries. 67
6.27 For a new method for the estimation of the drag, the z-locations of the vertices of the Mach-cone intersections are translated by the Mach-angle to a single axis as illustrated. 67
6.28 The area distribution of the double-cone method in comparison with a fore and backward facing cone distribution. 67
6.29 Difference in cross-sectional area distribution for different methods. 68
6.30 Relative error for the different methods compared to the calculations in CFD. 69
6.31 Convergence by varying several parameters for the calculation for the drag by using the area rule. 69
6.32 Comparison of S(x), S'(x) and S''(x) for the low-wing configuration for multiple methods for determining S(x)). 71
6.33 Calculated pressure signatures using the Whitham F-function. Different cross-sectional areas have been used. The calculated signatures are stretched to match the length of the zero-lift pressure signature. 72
# Nomenclature

## Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSM</td>
<td>Advection Upstream Splitting Method</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>KBE</td>
<td>Knowledge-Based Engineering</td>
</tr>
<tr>
<td>MDO</td>
<td>Multidisciplinary Design Optimization</td>
</tr>
<tr>
<td>SU2</td>
<td>Stanford University Unstructured, flow solver</td>
</tr>
</tbody>
</table>

## List of Symbols

### Latin symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AR$</td>
<td>Wing aspect ratio</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$a$</td>
<td>Speed of sound</td>
<td>$[\text{m/s}]$</td>
</tr>
<tr>
<td>$b$</td>
<td>Wing span</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag coefficient</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Friction coefficient</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift coefficient</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$D_{wave}$</td>
<td>Wave drag</td>
<td>$[\text{N}]$</td>
</tr>
<tr>
<td>$e$</td>
<td>Oswald efficiency factor</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$h$</td>
<td>Height</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$h_w$</td>
<td>Wing installation height</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$I$</td>
<td>Impulse</td>
<td>$[\text{Pa} \cdot \text{s}]$</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$L$</td>
<td>Lift</td>
<td>$[\text{N}]$</td>
</tr>
<tr>
<td>$L_{eff}$</td>
<td>Effective length</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$l$</td>
<td>Mesh element length</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$MAC$</td>
<td>Mean aerodynamic chord length</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of azimuthal angles</td>
<td>$[-]$</td>
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<tr>
<td>$P$</td>
<td>Pressure</td>
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<tr>
<td>$S$</td>
<td>Area</td>
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</tr>
<tr>
<td>$S_{ref}$</td>
<td>Wing reference area</td>
<td>$[\text{m}^2]$</td>
</tr>
<tr>
<td>$S_{w,wet,proj}$</td>
<td>Wetted wing projected area on the $XY$-plane</td>
<td>$[\text{m}^2]$</td>
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<tr>
<td>$t/c$</td>
<td>Thickness to chord ratio</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume</td>
<td>$[\text{m}^3]$</td>
</tr>
<tr>
<td>$W$</td>
<td>Weight</td>
<td>$[\text{kg}]$</td>
</tr>
<tr>
<td>$x_{w,start}$</td>
<td>$x$-coordinate of wing leading edge vertex</td>
<td>$[\text{m}]$</td>
</tr>
<tr>
<td>$z_{vertex}$</td>
<td>$z$-location of the vertex of the Mach-cone</td>
<td>$[\text{m}]$</td>
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</tbody>
</table>
Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Angle of attack</td>
<td>[$^\circ$]</td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>Wing incidence angle</td>
<td>[$^\circ$]</td>
</tr>
<tr>
<td>$\alpha_{eff}$</td>
<td>Effective wing incidence angle ($\alpha + \alpha_t$)</td>
<td>[$^\circ$]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Prandtl-Glauert parameter</td>
<td>[-]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Ratio of specific heats</td>
<td>[-]</td>
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<tr>
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<tr>
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<td>Azimuthal angle</td>
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<tr>
<td>$\theta$</td>
<td>Direction of the local streamline</td>
<td>[$^\circ$]</td>
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<tr>
<td>$\Lambda$</td>
<td>Sweep angle</td>
<td>[$^\circ$]</td>
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<tr>
<td>$\lambda$</td>
<td>Taper ratio</td>
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<tr>
<td>$\mu$</td>
<td>Mach-angle</td>
<td>[$^\circ$]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Prandtl-Meyer function</td>
<td>[$^\circ$]</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
<td>[kg/m$^3$]</td>
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Subscripts

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<tr>
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</tr>
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<td>Calculated by CFD</td>
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<td>Far-field parameter</td>
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<td>Fuselage</td>
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<td>Height-weighted Mach-cone method</td>
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<td>Inviscid</td>
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<tr>
<td>JN</td>
<td>Jumper &amp; Nikolic method</td>
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<tr>
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<td>Zero-lift value</td>
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<td>Leading edge</td>
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<td>Mach-cone method</td>
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<td>Trailing edge</td>
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<td>Total property</td>
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<tr>
<td>varied</td>
<td>A parameter which will be varied</td>
</tr>
<tr>
<td>vertex</td>
<td>Coordinate of the cone vertex</td>
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Introduction

Since the 1950s mankind has entered the supersonic regime of flight. Several attempts are made to make supersonic flight more economically feasible, but have failed due to high costs and high noise levels due to sonic booms, which prevented flight over land. The drop in oil prices in the recent years opens new opportunities to make supersonic flight economically feasible again. Also, there are new opportunities nowadays because of the higher fidelity computing options for designing airplanes compared to the era when, for example, the Concorde was developed. By saving fuel, the costs can be reduced for supersonic flight and a larger group of customers will be able to afford a supersonic flight. Even flying over land might be possible if the strength of the sonic boom is reduced enough, reducing the costs of travelling even more since less fuel has to be burned.

In order to save fuel at speeds higher than the speed of sound there can be searched in several areas for a reduction of fuel burn in order to increase efficiency, namely:

- Engine efficiency or propulsive efficiency
- Aerodynamic efficiency
- Noise reduction in terms of sonic boom

When comparing subsonic to supersonic flight, there are differences in the drag breakdown of an aircraft. In the supersonic flight regime a substantial relative increase in wave drag is noted, as can be seen in figure 1.1, while the other contributions in their absolute form do not change that abruptly. When wave drag can be reduced, it will increase the aerodynamic efficiency of the aircraft.

Noise is a contribution to nuisance which can be reduced as well. Noise from supersonic aircraft is a problem in two regimes, subsonic and supersonic flight. In subsonic regime the noise level is higher because of the huge amount of thrust that is required for take-off since the design is suited for high-speed flight. In the supersonic flight regime there is another factor which is important, the sonic boom.

![Figure 1.1: A comparison between drag at sub- and supersonic speeds](image)
which is just like the aerodynamic efficiency dependent on the design and shape of the aircraft. The sonic boom can cause nuisance to people and animals on the ground [17]. Therefore more fuel efficient and quieter aircraft are required to enable commercial viable supersonic flight.

Research in the 1960s has shown differences in high- and low-wing configurations [13] and more recent research sees an opportunity in asymmetrical configuration shaping for better sonic boom performance [14]. When these research directions are combined with modern tools, as for example CFD, these effects can be further examined for other configurations to give more insights in the effect of the wing configuration on sonic boom performance and drag.

1.1. Research objective and goal
In the recent years researchers have shown that it is possible to shape an aircraft in such a way that the noise levels are suppressed [18]. However, these design studies rely heavily on multidisciplinary design optimization (MDO) methods in combination with CFD. This can give good results, however, the true understanding of wing-body interference at supersonic speeds and their relation to the sonic boom is still sketchy. Therefore it is useful to find out more about this relationship between the vertical wing position and the drag and sonic boom of a supersonic flying airplane. Although the research is practice-oriented, because experiments will be conducted, the research will be of a theory-testing type. The hypothesis that will be tested is:

“The wave drag and sonic boom performance of a supersonic airplane are dependent on the vertical position of the wing due to wing-body interference effects.”

The goal of the research is to provide a better understanding about the wing-body interaction and the effect on the drag and sonic boom at supersonic speeds. This goal can be achieved in multiple ways, for example by testing different geometries in a windtunnel or by using CFD to test these geometries. The latter method is preferred since windtunnel tests and especially making windtunnel models is expensive and these activities are time-consuming. By using CFD multiple configurations can be tested in less time compared to windtunnel tests and modifications to the geometry can be made easily. Hence, finding a relationship between a new method of area ruling and the drag and sonic boom performance will be helped by the fact that the geometry is digital and calculations and alterations on that model can be done by using specialized software. Therefore the research objective is:

“To test the impact of the vertical position of the wing on the wave drag and sonic-boom performance by performing CFD-analysis and by using a new method of drag estimation by using Mach-cones to calculate the wave drag, the sonic-boom performance and to find the interference effects for a low-wing and high-wing supersonic aircraft in lifting and zero-lift conditions.”

The research questions are defined, based on the framework and objective stated earlier. The first research question is:

“What is the impact on the wave drag and sonic-boom performance of a low-wing and a high-wing configuration supersonic airplane at supersonic speeds?”

The sub-questions to answer this research question are as follows:

• What is the effect of zero-lift for a low-wing and a high-wing configuration on the wave drag and pressure-signatures at different distances at supersonic speeds?
• What is the effect of lift for a low-wing and a high-wing configuration on the wave drag and pressure-signatures at different distances at supersonic speeds?
• Why and where are pressure disturbances and interference-effects emerging for a low-wing and high-wing configuration at supersonic speeds in lifting and zero-lift conditions?
1.2. Research approach

A second research question will be used in case the hypothesis is true. This second question is:

"Is it possible to predict the wave drag and sonic boom characteristics by using a new method of cross-sectional area evaluation by using a Mach-cone?"

For this question, the method described in section 2.13.2 should be used. The subsequent sub-questions are:

- What are the results when using a frontal projection of the intersection of the oblique area of a Mach-cone with the aircraft model?
- How do the results compare to the intersection methods of Jumper & Nikolic [16] and Rallabhandi [15]?

These research questions will be answered by using ParaPy, which is a computer program written in Python. ParaPy can be used to parametrically describe geometries. It uses a wrapped version of the open-source SALOME software for meshing, which can export these meshes to the SU2 format. This is also a reason why the CFD solver to be used will be SU2. This flow solver is often used in sonic boom research [18–20].

1.2. Research approach

This research will use CFD as a tool to evaluate the drag and sonic boom at a certain distance. CFD is a good alternative for tests in wind-tunnels. Especially for supersonic measurements which are technically advanced. Because the viscous effects are of secondary importance and pressure distributions and pressure signatures are of higher importance, viscosity can be neglected. The difference between viscous and inviscid analysis shows negligible differences in pressure signatures [19, 21]. Also, boundary layer effects are not particularly of interest when calculating the wave drag of a configuration. These reasons open up opportunities for faster calculations by using the Euler equations instead of Navier-Stokes equations.

By using Computational Fluid Dynamics, it is easier to adjust a model for a certain situation. An equal comparison between different models is required, so the different models need to generate the same lift forces since this is a basic criterion for an aircraft with a certain mission profile. This lift coefficient needs to be verified by using CFD. Multiple iterations are required to mesh the model, run a CFD-simulation, evaluate the results in terms of validity and lift and adjust this model, mesh it again, performing CFD analysis, and so on.

When the models have reached an equal lift coefficient by adjusting the wing incidence angle, results can be gathered to compare their performance in both lift and drag. The performance will also be compared with different theories on sonic-boom and drag estimation, such as the supersonic area rule and the Whitham F-function [5, 9]. By optimizing the angle of attack of the different configurations in SU2 for zero-lift, it is possible to deduce the zero-lift wave drag and the contribution of the lift to the inviscid drag terms such as wave drag due to lift and induced drag and also the effect of lift on the sonic booms at a certain distance.

1.3. Report structure

In the next chapter, chapter 2, a literature study is presented which covers all relevant items for the thesis. In chapter 3, the workflow and methods that are used will be discussed. The geometry of the model to be tested is discussed in chapter 4, where several choices in the design of the airplane will be elaborated such as the sizing and the wing planform. In chapter 5, the mesh, CFD solver and validation of the results are covered. The results of the CFD simulations regarding the effect of the vertical wing placement on the drag and the sonic boom performance are covered in chapter 6. In the same chapter an analysis will be made to relate the performance on sonic boom and drag back to the cross-sectional area distribution by using Mach-cones. The interference effects between the wing and fuselage will be analysed by using slices of overpressure in the domain and pressure distributions on the surface of the aircraft. Finally, in chapter 7, conclusions will be drawn and a critical discussion of this thesis is presented.
2

Literature review

2.1. Supersonic flow
This research is about supersonic flow and phenomena that occur at these high speeds. The name supersonic implies that the flow is going faster than the speed of sound, which is equal to formula (2.1). In a flow, small disturbances travel around at this speed of sound. When a flow is flowing faster than the speed of sound, these disturbances, which travel at the speed of sound, do not travel upstream, but downstream. The speed of sound is defined by:

\[ a = \sqrt{\gamma RT} \] (2.1)

The nondimensional Mach number, is defined as the ratio between flow speed and the speed of sound:

\[ M_\infty = \frac{V}{a} \] (2.2)

An object flying through an atmosphere will create continuously disturbances. These disturbances are emitted at different locations and times, but travel at roughly the speed of sound. Consider a point source flying through a gas with speed \( V \) as depicted in figure 2.1. When the object is at location \( A \) it disturbed the gas. The disturbance travels with the speed of sound \( a \) to arrive at point \( C \) at distance \( at \). However, when the sound has arrived point \( C \) at time \( t \), the object has already arrived at point \( B \). The source is moving at a supersonic speed and has created different wavefronts, which are visualized as circles. When lines are drawn at the tangents of these disturbances, these lines will describe the Mach-cone.

2.2. Characteristics, shockwaves and expansion fans
Consider the disturbance from the section above. In the flow the disturbances travel along Mach-waves which are straight Mach-lines which align with Mach-angle \( \mu \).

\[ \mu = \sin^{-1} \frac{1}{M} \] (2.3)

This is due to the fact that the disturbances are very small and thus the disturbances do not change the flow properties. This is an approximation of what is really happening in a supersonic flow and is called the linearized theory [22]. Linearized theory is often used in research in the past to calculate flows and has been really usefull. However, in a real supersonic flow there will be Mach-waves of finite strength which will change the properties rearward of the disturbance, which is discussed next.

When the properties after a disturbance in an adiabatic supersonic flow are changed, the local Mach-lines will be changed as well downstream. Therefore there need to be adjustments to the linearized theory to make it applicable for streams with larger disturbances. Consider the streamline in figure 2.2. This streamline is curved and so are the Mach-lines, or characteristics of this streamline. These characteristics are divided into left- and right-running characteristics, \( C_+ \) and \( C_- \) respectively. The
characteristics curve as the local Mach angle $\mu$ depends on the local Mach number and the direction of the local streamline $\theta$ varies. Along these characteristics, the so-called compatibility relations hold, which eventually relate the local Mach number and the local direction of the streamline by:

$$\theta + \nu(M) = \text{constant} = K_- \quad \text{(Along the } C_- \text{ characteristic)}$$

$$\theta - \nu(M) = \text{constant} = K_+ \quad \text{(Along the } C_+ \text{ characteristic)}$$

Where $\nu(M)$ is given by the Prandtl-Meyer function:

$$\nu(M) = \sqrt{\frac{y+1}{y-1}} tan^{-1} \left( \frac{y-1}{y+1} \right) (M^2 - 1) \tan^{-1} \sqrt{M^2 - 1}$$

These equations form the basis of the method of characteristics, which is a method to calculate the flow properties for a homentropic supersonic flow.

When a streamline comes across a disturbance, a ramp for example, there will be a compression of the flow. Mach waves are emitted before the disturbance and also at the disturbance. These Mach waves can intersect each other and should get information from 2 points at the same time. Physically this is impossible and therefore shockwaves emerge at these locations. These oblique shockwaves are stronger than simple sound waves and physically stronger than the Mach waves (which are in fact infinitely weak oblique shocks). The opposite can take place as well, for example when an expansion takes place. Then a continuous series of Mach waves form an expansion fan in which the flow expands isentropically. These expansion waves are often called Prandtl-Meyer expansion waves.
2.3. Wave drag

Drag, or air resistance, is the resultant aerodynamic force component parallel to the free-stream that has to be conquered to fly faster or more efficiently. There are different kinds of drag, as is depicted in figure 1.1. There are parasitic drag terms, such as skin friction drag due to roughness, pressure drag and interference drag. Inviscid drag consists of wave drag. Next to that there is induced drag, or drag-due-to-lift, which is an inviscid drag created because the lift that is being generated induces vorticity in the wake.

Wave drag is an additional kind of inviscid drag which starts to appear around the Mach drag-divergence number. It is a sudden increase in drag when the free-stream Mach number is getting close to 1 due to compressibility effects. This increase of wave drag can be attributed to the formation of shockwaves around the object because the flow is locally going faster than the speed of sound. These shockwaves can trigger boundary layer separation at the shock foot, which causes additional drag. Over this shockwave, the total pressure decreases because entropy is being generated and the static pressure increases.

Wave drag is often called a kind of pressure drag. Airplanes were first limited to the subsonic flight region because the sudden increase of wave drag near Mach 1 was limiting the flight speed. This led to the term "sound barrier". Wave drag can be generated because of the shape and therefore the resulting shocks emanating from the geometry, but also due to local pressures on the surface when more lift is produced. This kind of wave drag gives rise to stronger shocks at some locations and is called wave drag due to lift.

Improvements in aerodynamic design, which are discussed later on in section 2.5, made it possible to lower this wave drag to enter the supersonic regime of flight more easily. In figure 1.1 a comparison is made about which fraction of drag there is at sub- and supersonic speed. The interference drag and wave drag increase significantly at supersonic speeds.

2.4. Sonic boom and the boom carpet

At supersonic speeds, shockwaves are generated which travel through the atmosphere and can reach the ground. These shockwaves can cause loud bangs, which are called the sonic boom. It is created by a front and an tail shock, so two booms are created. First, the overpressure of the aircraft flying by and after that a second boom at which the pressure goes back to the atmospheric pressure. These generated shockwaves can be observed from two different perspectives, Seebass [3] makes a dis-
tinction between the aerodynamic and the acoustic approach of the sonic boom. In the aerodynamic approach the airplane is assumed to be fixed in a supersonic flow, whereas for the acoustic perspective the aircraft is flying through an atmosphere at rest. Consider the acoustic perspective and an aircraft flying by. The aircraft is flying supersonic, so it produces a shockwave at an angle of nearly Mach angle \( \mu \). The wavefront of this shockwave will travel at the same speed as the airplane flies. This wavefront, or acoustic wavefront propagates away from the aircraft, as can be seen in figure 2.3. This acoustic wave will travel through several distinctive regions which are of importance, they are discussed later on in section 2.8.1. When the distance the acoustic wavefront travels increases, the energy of the perturbation is spread out over a larger area. This area is the area of a ray tube at distance \( s \), which is shown in figure 2.3. A ray tube is a mathematical description of the normals to the acoustic wavefront.

![Figure 2.3: Acoustic and aerodynamic reference frames. [3]](image)

First, the 2-dimensional perspective is covered in the propagation of waves, however, sound waves are emitted in three dimensions. When figure 2.1 is mapped to a 3-dimensional perspective, a better understanding of how the soundwaves travel through the air can be made. When observing figure 2.4, different regions are of interest. The green lines represent ground level. The red arrow represents an airplane flying at supersonic speed at an altitude of 18 kilometers. The blue circles are similar to the circles in figure 2.1, but now they represent soundwaves transmitted in planes which have the flight path as the normal vector. As can be seen, these circles span a cone which intersects with the ground level. At this line where the intersection between the faces of the ground and the cone takes place, the sonic boom is heard at the same time. This region is the boom carpet. This figure also shows that the distance directly below the aircraft to the ground is shorter than at lateral points. The effect of this larger distance is discussed later on in section 2.8.1.
2.5. Area rule

In an attempt to reduce the wave drag at Mach 1, Richard T. Whitcomb introduced the area rule in the 1950s [4]. The rule of thumb stated that for an airplane to pass the sound barrier, the cross-sectional area distribution should be smooth (the second derivative should be continuous). To minimize the wave drag, the cross-sectional area contributions of the wing, tail, nacelles and so forth should be included (see figure 2.5) and the sum of these cross-sectional area contributions should be smooth over the longitudinal axis of the aircraft. Often, the cross-sectional area is smoothened by reducing the fuselage in diameter, but it can also be smoothened by adding displacement bodies to the airplane. As an effect, the sudden drag-rise in the transonic regime \( M \approx 0.9 \) up to \( M \approx 1.2 \) reduces when wings are added and the fuselage is indented, as can be seen in some results from Whitcomb in figure 2.6.
Figure 2.6: The zero-lift drag-rise at transonic speeds reduces when a body is waisted. [4]
One ideal cross-sectional area distribution is that of the Sears-Haack body. The Sears-Haack body (see figure 2.7) is a shape for the minimum wave for a certain volume of a body. The shape is independently derived and published by Wolfgang Haack in 1941 and William Sears in 1947 [23]. The derivation is based on the formula for the calculation of wave drag by evaluating the wave drag for a distribution of singularities along the axis of an axisymmetric body. By solving the wave resistance by using an approximate formula of Von Kármán [24] and by restricting the geometry of the body by requiring that the body is closed and pointed on both ends, the minimum theoretical wave drag for a body of revolution of a certain volume and length can be obtained. The formula for the radius as a function of the maximum radius $r_{max}$ and length $l$ is given by:

$$r(x) = r_{max} \cdot \left( \frac{x}{L} \left( 1 - \frac{x}{L} \right) \right)^{3/7}$$  \hspace{1cm} (2.7)$$

The wave drag for a Sears-Haack body is given by:

$$D_{wave} = \frac{9\pi^3 r_{max}^2}{4L^2} \rho_{\infty} U_{\infty}^2$$  \hspace{1cm} (2.8)$$

### 2.6. Supersonic area ruling

After the transonic area rule Whitcomb derived the supersonic area rule, which is valid for speeds in the supersonic regime [5]. This rule is used for conceptual designs, where quick iterations are necessary. The supersonic area rule uses oblique planes which are translated along the aircraft for different azimuth angles. The cross-sectional area, projected onto the frontal plane in combination with the integration of the lift distribution in each plane in the direction of the azimuth give the equivalent cross-sectional area. At each longitudinal location the mean of all equivalent cross-sectional areas for all azimuthal angles is then calculated to find the mean cross-sectional area distribution. This longitudinal cross-sectional area distribution is then used for area ruling. Harvard Lomax derived the mathematical basis of this supersonic area rule, which includes the lift that is being generated [25]. Harris used this subsequently in his code to compute the wave drag in lifting conditions [13]. The longitudinal lift distribution is being used at various azimuthal angles to calculate the equivalent area distribution, which influences the wave drag at lifting conditions. These calculations are discussed in more detail later on in section 2.9. The mathematical basis of this all lies in the linearized theory, derived by Hayes [22].

### 2.7. Differential area rule

In 1963 Lock and Rogers gathered results on asymmetric fuselage shaping [6]. By reducing the fuselage on the upper side of the wing and by adding volume to the fuselage below the wing, a better pressure distribution is obtained. This better pressure distribution results in isobars which are aligned with the local wing sweep angle. However, for aircraft with swept-back wings the flow on the upper side of the wing will curve inwards because the direction of the steepest pressure gradient is more or
less normal to the leading edge of the wing [26]. This will cause the flow to go towards the fuselage and there the flow cannot expand enough. By reducing the radius of the upper side of the fuselage, the flow can curve more inwards and the flow speed will be less reduced and therefore the isobars will be aligned better with the local sweep angle along the chord which reduces drag. An asymmetric fuselage for an optimum pressure distribution can be observed in figure 2.9. Note that when no tailoring is used, the wing incidence angle near the root has to increase to prevent the flow from decelerating near the wing-fuselage junction.
The method described on the last page is used for the design of the Northrop YF-17, which is developed into the F-18. This method is described as differential area ruling \cite{7}. The fuselage is tailored such that on the upper side the fuselage is waisted and on the lower side volume is added. This is done from forward of the wing onto the mid-chord of the wing. As an effect more negative pressures and therefore more lift is generated on the upper side, while on the lower side the addition of area creates more positive lift due to higher positive pressures. These increased and decreased pressure stretch out over a large part of the wing along Mach-lines. As an effect, the wing root incidence angle can be smaller and therefore a better lift-to-drag ratio is obtained.

2.8. Factors influencing the sonic boom

Several factors influence the strength of the sonic boom, some of these variables are:

- Aircraft weight
- Aircraft lift-to-drag ratio
- Aircraft shape (slenderness, volume, smoothness)
- Flight altitude
- The location of the observer along the boom carpet
- Aircraft maneuvers

The weight of an aircraft influences the boom because more lift needs to be generated to keep the aircraft at level flight. This causes a larger disturbance in the atmosphere. By reducing the weight or by increasing the lift-to-drag ratio, the boom can be reduced. The length of the aircraft can also compensate for an increase in weight. When an aircraft is longer and the lift is spread out over a longer length, the intensity of the sonic boom can be reduced. Therefore the boom is also a function of the shape of the aircraft. This will be explained in section 2.9. An aircraft with a high slenderness with the same volume ‘spreads’ the disturbance over a longer distance, which causes less strong shocks. The altitude at which an aircraft flies influences the boom because the increased distance to the ground diffuses the shock more. This is also valid for the location of the observer. When the observer has a larger lateral distance from the flight path on the ground, the effective distance to the aircraft is larger and therefore the shock intensity is less as well. This can be seen in figure 2.4.

These relationships are wrapped into an equation for the intensity of the overpressure shock which is given by Seebass. In formula \eqref{2.9} \( \Delta p \) is the overpressure of the front shock, \( p_\theta \) is the air pressure on the ground, \( W \) is the aircraft weight, \( \theta \) the azimuthal angle, \( h \) the altitude, \( l \) the length of the aircraft and \( \beta \) the Prandtl-Glauert factor, which is defined as \( \sqrt{M^2 - 1} \).

\[
\frac{\Delta p}{p_\theta} \propto \frac{\beta^{\gamma/2} W^{\gamma/2} \cos \theta}{M h^{\gamma/2} l^{\gamma/2}}
\]  
\eqref{2.9}

Aircraft maneuvers influence the sonic boom as well, however, this is out of the scope of this research since an aircraft in steady flight is observed.
2.8.1. Ray tubes and the propagation of sound to the ground
The basics of the sonic boom are already explained in section 2.4. But more knowledge is required about sonic booms and their propagation through the atmosphere. When the aircraft produces a shockwave, the wavefront of this shockwave will travel at the same speed as the airplane flies. A ray tube is emanating from the aircraft, which is defined by the normals to the acoustic wavefront, as can be seen in figure 2.3. As the ray propagates through the atmosphere, it passes through several distinctive regions which are of importance.

First of all, there is the local region around the aircraft, often called the near-field region. This region extends to approximately 3 to 5 body-lengths away from the aircraft. Here, the flow is fully three-dimensional. Next comes the mid-field region, which can extend up to several hundred body-lengths. In the end, there is the far-field region, which is usually taken as the region where the pressure signature has taken its final shape. Sound rays are directed in all directions normal to the wavefront. This causes an increase in ray tube area as the ray tube length $s$ increases. The total pressure perturbation from the aircraft will therefore be spread out over a larger area as the length increases. This is the reason why the pressure perturbation varies with the square root of the altitude. This relationship is found as well in formula (2.9). Note that during the propagation through the atmosphere some rays will focus to become a larger pressure disturbance or never reach the ground (refraction). This last effect is happening because the temperature in the atmosphere and therefore speed of sound varies with the altitude. Focusing of rays is a result of the varying temperature in the atmosphere and the disturbance the aircraft is creating. Because the aircraft is disturbing the flowfield, the characteristics will change, which in turn influence the shockwave and the sound rays.

2.8.2. Pressure signature
The pressure perturbation of an airplane breaking the sound barrier has a specific shape. Because of atmospheric effects the pressure disturbances coalesce during propagation towards the ground. This results in a shock, originating from the front of the aircraft, after that shock there usually is an expansion which is linear with time and then a shock after which the pressure is going back to the normal atmospheric pressure on the ground. When this overpressure is measured and plotted against time, it looks like an N-shape, which is why the signature is often called the N-wave. In figure 2.11, an overview is shown of how a N-wave is produced.

![Figure 2.11](image.png)

Figure 2.11: A sketch of how a N-wave is produced (left) and some characteristics of an N-wave (right) [8]

Some performance characteristics of the sonic boom can be measured. These items are depicted in figure 2.11 on the right side. These performance parameters are the maximum overpressure $\Delta p$, the rise time $t_{rise}$ and the duration of the N-wave. Note that other shapes for N-waves are possible.
2.9. Relating the aircraft geometry to the sonic boom and wave drag

The pressure signature of a supersonic aircraft is an important measure for the sonic boom, and is related to the geometry of the aircraft. For designers it is useful to be able to calculate the effect of the geometry on the sonic boom. Whitham’s F-function describes this relationship between the cross-sectional area distribution and the pressure signature [9].

First of all, linearized theory describes the characteristics from a supersonic flow as an approximation \((x + \beta r = \text{constant})\). However, the true characteristics in a supersonic flow depend on the flowfield as can be seen in figure 2.12. These characteristics are described by the exact form, replacing \(x + \beta r = \text{constant}\) by \(y(x, r) = \text{constant}\). In a uniform atmosphere, equation 2.10 describes the relationship between characteristics and the F-function, assuming that the ratio \(\frac{\beta r}{x}\) is sufficiently large, which indicates that the function is only valid at large distances from the body. These variables are explained in figure 2.13, where \(y = 0\) is equal to \(x + \beta \cdot 0\). Note that this F-function is still an approximation, since higher order terms in the perturbation velocity are neglected [8].

\[
x = y + \beta r - \frac{(y + 1)M^4}{\sqrt{2}\beta^3} \sqrt{F(y)}
\]  
(2.10)

Where \(\gamma\) is the ratio of specific heats, \(M\) represents the freestream Mach number, \(r\) the vertical distance away from the body, \(\beta = \sqrt{M^2 - 1}\) and \(F(y)\) is the Whitham F-function, as described by:

\[
F(y) = \frac{1}{2\pi} \int_0^y \frac{S''(x)}{\sqrt{y - x}} \, dx
\]  
(2.11)

Where \(S''(x)\) is the second derivative of the cross-sectional area of the body as is measured by the normal projection of cuts aligned with the Mach angle \(\mu\). The value of \(y\) is measured from the nose along the body axis, as is shown in figure 2.13. The value \(p_0\) is the local static pressure at the altitude where the pressure signature is measured.

For the F-function, there can be seen that from the viewpoint of the observer (see figure 2.15) the integration makes sense, since the volume of the body that generates the disturbance is captured in the fore Mach-cone.

As is shown by Hayes et al [27], the pressure signature is related to the F-function by:

\[
\frac{\Delta p}{p_0} = \frac{\gamma M^2}{\sqrt{2}\beta r} F(y)
\]  
(2.12)

The Whitham F-function is only valid for axisymmetric bodies. To extend this theory to lifting non-axisymmetric bodies, Walkden used Lomax [25] his idea about equivalent areas which was then used for wave drag prediction. This idea is applied to the sonic boom theory [28] to predict the strength of
the sonic boom. The idea behind this approach is that an aircraft in lifting conditions is replaced by an equivalent axisymmetric body in non-lifting conditions which produces the same sonic boom. This theory was later on verified in wind tunnel tests \[29\].

The equivalent area \(A_E\) of an aircraft is a function of both azimuthal angle \(\theta\) and the longitudinal distance \(x\). It consists of two components, one from the body shape \(A_B\) and one for the longitudinal lift distribution of the aircraft \(A_L\).

\[
A_E(x, \theta) = A_B(x, \theta) + A_L(x, \theta)
\]  \quad (2.13)

The azimuthal angle \(\theta\) is defined according to the nomenclature of figure 2.14, where the airplane is seen from behind. At each azimuthal angle, the cross sectional area is determined by the frontal projection of the cut-through area aligned with the Mach-plane for the fuselage and wing (see figure 2.8), whilst the lift component is calculated according to formula 2.14, where \(U_\infty\) is the freestream velocity, \(\rho\) the density and \(L(x, \theta)\) the lift at axial location \(x\) in azimuthal plane \(\theta\). The lift is calculated from integrating the pressure distribution along the cut of the plane. The lift does make a contribution to the cross-sectional area because in the evaluated cross-section, a lift force is created by the pressure distribution. This pressure distribution can also be seen as a potential distribution. When this lift force is divided by the dynamic pressure, the area due to lift is left, which can be used to determine the wave drag due to lift of the lifting wing-body combination. The formula for the area due to lift is:

\[
A_L(x, \theta) = \frac{\beta}{\rho U_\infty} \int_0^x L(x, \theta) \, dx
\]  \quad (2.14)

To determine the wave drag of a lifting configuration, formula (2.15) can be used. In case when
an axisymmetric body is used without any lift that is being generated, \( A_e \) will reduce to \( A_B \).

\[
D_w = -\frac{\rho_{\infty}U_{\infty}^2}{4\pi} \int_0^L \int_0^L A''_e(x_1)A''_e(x_2) ln|x_1 - x_2| dx_1 dx_2
\]  

(2.15)

Where \( x_1 \) is the location of the evaluated cross-sectional plane, and \( x_2 \) is the variable which runs along the \( x \)-axis.

2.10. Low sonic boom design

The shockwaves emanating from a supersonic aircraft are heard as a loud boom on the ground. This boom can be annoying for people. The overpressures created can be converted to a sound pressure level (SPL) which is given in decibel (dB). In table 2.1, some aircraft and the strength of their sonic booms are shown. People get startled because of a sonic boom and experiments show interruptions in people’s responses when a sonic boom is heard [30]. These are some reasons why sonic boom is usually not allowed over land.

Table 2.1: A comparison of the strength of the sonic boom for some aeroplanes (converted to SI from [17]).

<table>
<thead>
<tr>
<th>Airplane</th>
<th>Speed (Mach)</th>
<th>Altitude (m)</th>
<th>Overpressure (Pa)</th>
<th>SPL (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lockheed SR-71</td>
<td>3.0</td>
<td>24384</td>
<td>43.09</td>
<td>126.67</td>
</tr>
<tr>
<td>Concorde</td>
<td>2.0</td>
<td>15850</td>
<td>92.89</td>
<td>133.34</td>
</tr>
<tr>
<td>F-104</td>
<td>1.93</td>
<td>14630</td>
<td>92.41</td>
<td>133.29</td>
</tr>
<tr>
<td>Space Shuttle</td>
<td>1.5</td>
<td>18288</td>
<td>59.85</td>
<td>129.52</td>
</tr>
</tbody>
</table>
To make supersonic flight over land practical, it is required to minimize the sonic boom as much as possible. There are several ways of doing this, most notably by shaping the aircraft in such a shape that a different pressure signature is obtained. Multiple characteristic parameters can be influenced, such as the maximum overpressure and the impulse. The impulse is the integral of the positive part of the overpressure of the signature over time, as is illustrated by the gray part in figure 2.16. The negative part, just before the first shock and the main positive part, is also included in the calculation of the impulse. Shaping the aircraft to influence the pressure signature was already described by R. Seebass and A.R. George in 1972 where multiple pressure signatures are discussed [31]. These signatures are depicted in figure 2.17 on page 21. Note that there are no values along the axis in these signatures since it only illustrates what a signature can look like. In figure 2.17 the waves are more or less antisymmetric, this does not have to be the case. A combination of signatures is possible, as is shown later in in figure 2.19 on page 22.

- **N-wave** The N-wave is the normal signature, with a boom, a linear decay and another boom from the aft of the aircraft, bringing the pressure back to the normal pressure.

- **Minimum impulse N-wave** This signature is characterized by a minimized total impulse. The impulse is calculated by calculating the integral over the positive part of the overpressure.

- **Minimum overpressure N-wave** This signature has a strong but minimized shock after which a section of nearly constant overpressure is present. After that, there is a linear decay.

- **Minimum shock N-wave** The pressure rise is minimized, so a small shock is to be expected, but the bigger part of the shock is spread over a longer timespan. After this the linear decay of the overpressure takes place.

The impact of sonic booms can be reduced by several measures, such as [3]:

- **Engine streamtube area reduction.** In theory, when a lifting body creates a compression wave below it, the full streamtube should be intercepted and processed by an engine, after which the streamtube leaves at the correct ambient pressure to prevent “pluming” and to prevent the occurrence of a sonic boom. [32]

- **Designing aircraft with less annoying signatures.** By shaping the aircraft in such a way that the pressure signature is changed. This is called “signature shaping”.

- **Increase the $c/D$-ratio.** When the lift-to-drag ratio is increased, less drag is produced for the same amount of lift.

- **Decrease specific fuel consumption and structural weight.** By reducing fuel consumption, less fuel is required for the same distance which decreases the fuel weight and can have its effects on structural weight.

- **Increase altitude capability and length.** By flying higher and making the aircraft longer, the disturbance generated by the aircraft will be spread out over a larger area on the ground. It is even possible to avoid shockwaves altogether by increasing the length of the aircraft, however, this is not always physically possible.
2.11. Current low-boom designs

In the past, there have been some attempts to decrease the sonic boom. Some of these designs will be discussed in this section.

2.11.1. NASA Shaped Sonic Boom Demonstration program

In the early 2000s, the Northrop-Grumman Corporation (NGC), Defense Advanced Research Projects Agency (DARPA) and NASA collaborated and conducted the Shaped Sonic Boom Demonstation program \[11\]. This program showed that the sonic boom of a supersonic aircraft can be shaped by modifying the geometry of the aircraft. Several linear analysis and CFD tools were used to design the shape of the nose of a supersonic aircraft, a Northrop-Grumman F-5E. The nose is shaped in such a way that the shockwaves and expansions that are arising from the nose do not coalesce into a single large sonic boom. The design is mostly based on cross-sectional area development and the correlation with the

Figure 2.17: Sketch of typical N-waves of which the lower 3 are for low sonic boom design. The negative part is more or less a copy of the positive part for illustration.
Whitham F-function. The goal was to develop a nose glove for the aircraft that made it possible to produce a flat-top pressure signature on the ground. The front part of the signature is therefore a minimum overpressure N-wave while the aft part is a normal N-wave (see figure 2.19). This makes sense, since only the frontal part of the airplane is changed and therefore the frontal part shock will be radically changed. Multiple flight tests have taken place and the goal was achieved. Full specifications on the nose-glove however, such as the precise shock-pattern, are not given.

![Figure 2.18: An overview of the modifications (highlighted in green) to the Northrop-Grumman F-5E to form the SSBD. It is clear that there is an increase in volume on the lower side of the frontal fuselage [10].](image)

![Figure 2.19: A comparison in pressure signature for the SSBD and the Norghrop-Grumman F-5E. Visible is the decrease in frontal shock strenght, a flat top and a lower shock strength at the aft of the SSBD [11].](image)

2.11.2. NASA N+2 Supersonic program

NASA Langley Research Center is funding a program where Lockheed Martin is a prime contractor for a supersonic aircraft, which can enter service in the next decade. The first design of this N+2 Supersonic [18] is based upon a report of the N+3 Supersonic program [33]. Several high-wing concepts are presented, which are developed into the NASA N+2 Supersonic program. In figure 2.20 an impression of the design is shown. Opted is for a high wing aircraft with some dihedral. Although no reference is presented, mentioned is that this dihedral is present to elevate the lift for a stretched boom. The different concepts all show a high wing configuration, even with an overwing engine placement. This could indicate that the engine placement is not a requirement for the choice of a high-wing aircraft.

![Figure 2.20: An impression of the NASA/Lockheed Martin N+2 concept. The airplane features some dihedral and a high-wing configuration [12].](image)
2.12. Effect of wing placement

In the report of Harris [13], attention is given to a body with alternative wing placements. A high-wing, mid-wing and low-wing configuration is tested and evaluated with the code, which incorporates the calculations of Lomax [25]. Figure 2.21 shows 7.5% less wave drag due to interference between volume and wave drag for the high-wing configuration compared to the low-wing configuration. No elaboration on these results is given except for the interference between the area distribution and the lift distribution in some azimuthal angles, directly below and to the top of the aircraft as shown in figure 2.22. The tips of the high-wing configuration produce lift for a longer time when the geometry is cut by a Mach-plane at the azimuthal angle of +90° (in this report by Harris defined from another starting point compared to the convention used in this thesis). This stretched equivalent area distribution is an interesting phenomenon which can be further explored.

![Image](image1.png)

Figure 2.21: Results of wing-placement and wave drag [13]

![Image](image2.png)

Figure 2.22: The effect of the different cuts of the Mach-plane with the geometry and the lift [13]

In a technical paper of NASA by Carlson and Mann a high and a low-wing configuration have been simulated and compared in terms of lift and drag [34]. This research uses an unconventional case of a wing with a wedge attached to it which functions as a fuselage. The mean camber surface is used to calculate drag and lift. Although this is an approximation of the real flow, the high wing configuration produces less drag compared to the low wing configuration. The term interference lift is mentioned, but no exact explanation is given on the results.

2.12.1. Non-axisymmetric fuselage shaping

In a report of Donald C. Howe from 2003, non-axisymmetric configuration shaping is used to decrease the sonic boom [14]. The main focus is on the nose of an aircraft. Instead of using a symmetrical nose, the nosecone is 'flattened' by aligning the lower centerline of the nose with the axial axis, as can be seen in figure 2.23.

The effect is that the disturbances in the supersonic flowfield are deflected upwards instead of downwards. A comparison is shown in figures 2.24 and 2.25, where it is obvious that at the lower part of the sheared section, less strong shocks are visible. By doing so for a supersonic configuration, the initial overpressure below the aircraft (at 5 bodylengths distance) is reduced by 14% and the peak overpressure by 22% relative to the axisymmetric case. When the propagation towards the ground is taken into account from an altitude of 55000 feet, or 16764 m altitude,

![Image](image3.png)

Figure 2.23: A sheared nose section of a fuselage, where the bottom centerline runs parallel to the axial axis [14]
this yields a reduction of the initial overpressure of 5% and the peak overpressure is reduced by 23% compared to the axisymmetric configuration. Next to that, the total impulse (the area of the N-wave described by the positive pressure portion of the signature) is reduced by 10%. These CFD results are validated by wind tunnel tests and they show excellent agreement, except for the aft shock. This disagreement is due to the viscous effects that can not be modeled in an Euler analysis. As part of a next phase of the effort, the author suggests to extend the research towards wing-body combinations. These combinations will need careful shaping to minimize disturbances below the airplane. Furthermore, the suggestion is made to use a grid which extends up to five body lengths for the near field distance, since that is about the distance up to which the sensitivity extends for non-axisymmetric distributions of the fuselage.

This concept is used also in a different way on a F-15B testbed [35]. The concept is the Quiet Spike, developed by Gulfstream. It features an extendable spike, housed in the nose of the aircraft. During flight, this boom extends. There are several conical transitions of which the goal is to divide the single shock emanating from the nose cone into several smaller shocks. The concept is tested, however, the ground signature was not reduced. This should only happen when the Quiet Spike is combined with a properly designed aircraft.

2.13. 3D-flowfield
Several topics within supersonic flight and the associated sonic boom and wave drag are discussed within this study. From different sections it is possible to draw some conclusions to give a direction on this research. First of all, to make supersonic flight more feasible it is beneficial to reduce the wave drag. This can be done by using better tools for area-ruling because nowadays better high-fidelity computers are available. Next, it is required to predict the sonic boom better and to focus on low-boom designs for aircraft. Several manufacturers are exploring commercial supersonic flight and it is necessary to have better tools available for prediction and minimization.

The relative simple methods to estimate wave drag of supersonic airplanes are focusing on the design of an aircraft in a 2-dimensional way. They evaluate the cross-sectional area by using planes instead of using a full 3-dimensional solution, for example an approximation of the shape of the flowfield at supersonic speeds: a Mach-cone. Next to that, the focus is on the longitudinal area distribution, but the vertical distribution is not taken into account. When looking at the researches of NASA and Lockheed with the SSBD and the N+2 concept, no configurations which are symmetrical about the wing plane are evaluated. Instead, more complex 3-dimensional shaped airplanes are evaluated. In the N+2 concept, the wing is mounted high on the fuselage. When a cross-sectional area distribution is made of this aircraft, and afterwards of each cut-through section the centroid is evaluated, the centroid will describe a line above the longitudinal axis. When observing the nose-glove in figure 2.18 that is put on to the Northrop Grumman SSBD, it shows that the glove adds additional volume to the underside of the nose of the aircraft. So when the position of this centroid of these cut-through areas is evaluated along the longitudinal axis of the aircraft, it will show that at longitudinal point where the wing starts,
the vertical position of this centroid will make a jump upwards, which could be a similar effect as raising the vertical position of the wing in a fuselage.

These relationships in combination with the results gathered by Harris and Carlson & Mann are interesting and lead to the thought that it could be possible that there is another relationship in area ruling. It may be that the centroid of the cross-sectional area influences the wave drag and the sonic boom performance.

### 2.13. Using a Mach-cone for area ruling and drag estimation

The usage of a Mach-cone in area ruling is already demonstrated by S. Rallabhandi and D. Mavris [15]. They make use of a streamwise Mach-cone to intersect the airplane. This technique is used to make a plot of the axial cross-sectional area distribution for different Mach-numbers. It shows that in the front of an aircraft the influence of the wing tips is taken into account, if the Mach-plane intersects these sections. A remark must be made to this method because the stream tube of inlet air for the engine is included in the geometry in the model, while this streamtube does not add to the cross-sectional area of the airplane. Another point of interest is where exactly the vertex of the Mach cone starts. In the research performed by S. Rallabhandi & D. Mavris, this is along the longitudinal axis, as can be seen in figure 2.26.

![Figure 2.26: Intersected airplane by Mach-cones](image1)

Another variation of using a Mach-cone in drag estimation is posed by Jumper and Nikolic [16]. They make a simplification of the existing supersonic area rule by using an axisymmetrical model for the area distribution of the aircraft. By slicing this model with a plane at the Mach-angle $\mu$ of the freestream and by using this area distribution, they achieve accurate results from Mach 1.1 to 1.4. Nikolic suggests 4 new methods which use a cone for the intersection of an airplane. Of these methods, there is one which is using the cut-through projection of the lateral surface of a Mach-cone on a plane normal to the freestream. Compared to the cross-sectional area distribution with normal planes, the cross-sectional area distribution is now shifted to the front of the aircraft. The improvement over the supersonic area rule is that only a single set of Mach-cones are used. This decreases the computational effort compared to the supersonic area rule, where the cut-through areas for multiple azimuthal angles for a full aircraft configuration are used. Nikolic and Jumper found that the method with the lateral surface of the Mach-cone gave the good trend for Mach number and the value for the wave drag, however the wavedrag was often overpredicted by a factor of 2. By dividing the results by this factor, better agreement with test data was achieved. Data from the method of Nikolic and Jumper showed for a Northrop F-5E for
Mach numbers between 1.4 and 1.8 an accuracy of less than 10% compared to wave drag computer programs (124J Wave Drag Program of Northrop and the Langley Wave Drag Program).

Especially for wave drag results from a computer program of NASA Langley and Northrop data of a Northrop F-5E for Mach numbers between 1.4 and 1.8. From this experiment, results are achieved for the wave drag within 10% accuracy from Mach 1.4 to 1.8 relative to the supersonic area rule.

2.13.2. New method of area ruling and drag estimation by using a Mach-cone
By using a Mach-cone for area ruling it is required to find where the Mach-cone should be placed to evaluate the cross-sectional area of an airplane. A Mach-cone has 4 basic parameters, first of all the \( x, y, z \)-coordinates and also the Mach angle \( \mu \), which describes the half-angle of the Mach-cone. The cross-sectional area should be evaluated at several longitudinal coordinates, just as the normal area rule. Because (usually) every airplane is symmetrical about the \( x, z \)-plane, the \( y \)-coordinate for the tip of the cone should be placed in this plane, therefore \( y = 0 \). The last coordinate for a Mach-cone that needs to be found, is the \( z \)-coordinate. Earlier on there was mentioned in the beginning of section 2.13 that there might be a relationship between the centroid of the cross-sectional area and the wave drag. By placing the vertex of the cone on the same vertical position of the centroid of the cross-sectional area of the Mach-cone and the airplane geometry, a formulation for the \( z \)-location can be found. Note that this will be an iterative process since each time the Mach-cone is shifted upwards or downwards, the cross-sectional area and the position of the centroid will change as well. This procedure can also be seen in figure 2.28 on page 27. The cross-sectional area distribution that is obtained can be used for drag estimation by using the formula for the supersonic area rule and replacing the cross-sectional area terms with the one obtained. When this gives accurate results, this new method could be used for area ruling.
Figure 2.28: The intersection of a Mach-cone with an airplane and the vertical shift of the Mach-cone to align the vertex with the centroid of the cut-through section.
In this chapter, the methodology will be covered. First the experimental setup will be given where the main steps for the research are laid out. After that the different methods for intersecting the geometry for the zero-lift drag estimation will be shown. At the end of this chapter the flow parameters are given. 

In figure 3.1 a broad overview of the work that has to be undertaken is shown. This is a guideline for the main steps that have to be carried out.

3.1. Numerical setup
The goal of this research is to find out what the influence of the vertical wing position is on the drag and sonic boom performance at supersonic speeds. Therefore, different airplane models need to be defined which are as similar as possible in terms of total volume, lift and wing shape to be able to make a fair comparison. Below, the steps necessary for the full experiment are listed:

- Design a supersonic airplane model
  - ParaPy will be used for the design. This enables quick generation of a modified geometry.
  - The wing will be positioned on 5 vertical positions, varying from the highest to the lowest achievable point where the full wing lower and upper surface lines in the symmetry plane of the wing fall inside the fuselage.
• Apply area ruling on the airplane model by using an iterative algorithm.

• Mesh the airplane model
  - SALOME, wrapped in the ParaPy software will be used to quickly generate unstructured meshes for different configurations.

• Inviscid CFD analysis to find the amount of lift that is generated and change the wing incidence angle to obtain the same lift coefficient for each configuration.
  - A first guess for $\alpha_i$ will be made and the geometry will be created. The wing incidence angle will be adjusted after a CFD simulation so that the final configuration generates a specific amount of lift.
  - SU2 will be used for the CFD simulation, since it is often used in sonic boom research and can deal with unstructured grids.

• Perform inviscid CFD simulations for a large domain to evaluate the lift and drag and the sonic boom performance in lifting and non-lifting conditions.
  - The lift and drag characteristics will be measured for lifting and zero-lift conditions. The zero-lift conditions will be found by optimizing the angle of attack for zero-lift for each configuration in SU2.
  - The pressure signatures in the near-field will be evaluated at various azimuthal angles (0, 10, 20, 30, 40 & 50°) at 1/4 body-length and 1 body-length distance below the aircraft.

• Detailed analysis of the results by comparing the results from the CFD analysis to the results from the cross-sectional area distribution analysis for zero-lift wave drag.

3.2. Cross-sectional area distribution data
To give an answer to the second research question, it will be necessary to generate the different kinds of cross-sectional area distributions. The data extraction will be done using ParaPy, since the airplane models will be built using this software and since it is relatively easy to use different geometric shapes to intersect the geometry. For all intersection methods it is required to get a cross-sectional area distribution $S$ on the longitudinal axis $x$. One of these distributions also uses a $z$-coordinate to locate the vertical location of the centroid of the intersection at $x$. To convert a conical surface on for example a cone or on plane inclined at the Mach-angle $\mu$, one can easily divide this surface area by the Mach-number to calculate the projection of this area onto the plane normal to the $x$-axis [36]. The tool will generate the following area distributions, which are also graphically displayed in figure 3.2:

• Mach-planes
  This distribution is the one proposed by Whitcomb and used by Jones and Lomax in the calculation of the drag at supersonic speeds. The distribution uses planes which are aligned with the Mach-angle $\mu$. See figure 3.2a for illustration. For $N$ azimuthal angles $\theta$, varying from 0 to $2\pi$ radians a cross-sectional area distribution will be generated. The subscript $MP$, derived from Mach-planes will be used to indicate values associated with this distribution.

• Jumper & Nikolic
  This distribution uses a planar surface normal to the $x$-axis to create a cross-sectional area distribution $S(x)$. See figure 3.2b for illustration. This cross-sectional area distribution will be used to create an axisymmetrical body by using $r(x) = \sqrt{\frac{2\theta}{\pi}}$. This axisymmetrical body is then cut by a conical surface with a half-angle equal to the Mach-angle, of which the projected area onto a plane normal to the $x$-axis is used to get a final area distribution $S_{JM}(x)$. In fact, cutting an axisymmetrical body with conical planes only shifts the $x$-location of $S$ forward. This transformation is: $x_{JM} = x - Br(x)$.

• Mach-cone
  This distribution is made by using the projection on the plane normal to $x$-axis of a conical surface with a half-angle $\mu$. See figure 3.2c for illustration. Variables related to this area distribution will use the subscript $MC$, since a single Mach-cone will be used for this intersection.
3.2. Cross-sectional area distribution data

- **Height weighted Mach-cone**
  
  This distribution uses the same method as proposed by Rallabhandi, however the Mach-cones are translated over the $z$-axis as well to align the centroid of the cross-sectional area with the vertex of the conical surface. This gives the additional $z$-coordinate as has been pointed out earlier on. Values associated with this area distribution will be marked with the subscript $HWMC$, from height-weighted Mach-cone.

- **Double Mach-cone**

  The supersonic area rule uses planes aligned with the Mach-angle at $N$ azimuthal angles. When $N$ goes to infinity, all these planes span 2 Mach-cones. The intersections with two Mach-cones are illustrated in figure 3.2d. One cone is the fore Mach-cone and the second Mach-cone is the cone as is used by the method of Rallabhandi. When both intersections are projected on a plane normal to the $x$-axis and the average area of these 2 projections is taken, a new area distribution is generated. This distribution could be equal to the average of the intersections at each location $x$ as given by the supersonic area rule. The subscript $DC$ will be used to mark values associated with this method that uses double Mach-cones.

![Diagram](image)

(a) Airplane cut by a Mach-plane rotated to azimuthal angle $\theta$.

(b) Airplane cut by a plane normal to the $x$-axis.

(c) Airplane cut by a Mach-cone.

(d) Airplane cut by double Mach-cones. The average of the cross-sectional areas is used.

Figure 3.2: Different intersection methods for the evaluation of the zero-lift drag. The striped areas are the frontal projections of the intersections that will be used in the calculations for the wave drag.

These distributions will then be used to evaluate the zero-lift drag by using formula 2.15 and they will be used to calculate the pressure signature at $R = 70m$ from the aircraft by using the F-function from Whitham his theory (see equations 2.11 and 2.12) [9]. Figure 3.3 illustrates the difference in area distribution method for a Sears-Haack body of length $L = 70m$ and maximum radius $R_{max} = 2.5m$. When a very close look is taken on the different distributions, some differences can be observed. The area distribution according to Jumper & Nikolic follows the same distribution of that of the method of Rallabhandi (Mach-cone) and the height-weighted Mach-cone distribution. The peak is shifted forward by a certain amount for an axisymmetrical body which is also symmetrical in the plane normal ot the
The method which uses the double Mach cones nearly follows the same line as that of the supersonic area rule, although the method for double Mach-cones has a slightly lower peak, since it involves taking an average of 2 areas.

![Projection of cross-sectional area on planes normal to the x-axis](image)

Figure 3.3: Difference for different methods for determining the cross-sectional area distribution.

### 3.3. Flow definition

For the inviscid CFD simulations, air conditions at an altitude of 18000m will be used, since that is a typical altitude for supersonic aircraft to fly at cruising conditions [18, 37, 38]. A flow speed of Mach 1.6 will be used, since this is well within the supersonic regime and transonic effects will not be of influence. The farfield is given by the following properties of the flow, which are listed in table 3.1.

Table 3.1: Freestream properties of the flow [39].

<table>
<thead>
<tr>
<th>Flow parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td></td>
<td>1.6</td>
</tr>
<tr>
<td>$v$</td>
<td>m/s</td>
<td>472.115</td>
</tr>
<tr>
<td>$T_t$</td>
<td>K</td>
<td>327.58</td>
</tr>
<tr>
<td>$T_s$</td>
<td>K</td>
<td>216.65</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\text{kg/m}^3$</td>
<td>0.120674</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Pa</td>
<td>7504.84</td>
</tr>
<tr>
<td>$q_\infty$</td>
<td>Pa</td>
<td>13448.7</td>
</tr>
</tbody>
</table>
In this chapter the geometry of the model to be tested will be explained. First, the requirements for the model is laid out, after which an explanation on the sizing is given. Then the wing planform and airfoil definition are covered after which the steps for fuselage shaping are explained. In the end the final design is shown which will be used for the simulations.

4.1. Requirements

A geometry is required to test the hypothesis. The geometry to be tested is subjected to some requirements, which are:

- The fuselage must be a slender body, preferably comparable to that of a supersonic aircraft
- The fuselage will be aligned with the freestream direction
- The lift coefficient should be that of a typical value for supersonic aircraft
- The wing planform will be equal for all configurations
- The cross-sectional area distribution must be as smooth as possible

To make the study relevant to future aircraft or aircraft in development, the fuselage must be a slender body, preferably of a slenderness ratio comparable to that of a supersonic aircraft. However, when the slenderness ratio is high, this can cause some implications for the research. A high slenderness ratio for a fixed length implies that the diameter of the body will be low. This restricts the possibility of researching the effect of the vertical wing position. A trade-off must be made in this design. The fuselage will be aligned with the freestream to rule out the effect of lift created by the fuselage. Furthermore, the wing planform will be equal for all configurations of the airplane. The only parameters of the wing that are varied, are the vertical position and the incidence angle of the wing. This vertical position and incidence angle of the wing will have an effect on the cross-sectional area distribution of the configuration. Therefore the configurations will be area ruled for the same Mach number. The cross-sectional area distribution is optimized for Mach 1.0, which involves cutting the geometry using a plane normal the freestream. This choice is made to keep the area ruling algorithm quick, simple and to reduce the chance of errors.

4.2. Sizing

In table 4.1 the geometry of several supersonic aircraft is listed. In the column on the right, the mean value of some non-dimensional parameters are stated. These values can be used to size the airplane that will be evaluated with CFD.

The airplanes stated in table 4.1 all carry passengers, however, the market is different for each airplane. The Concorde and the Lockheed/NASA N+2 aircraft focus on carrier flights, while the Aerion AS2 focuses on the market for business jets. For supersonic flight to be cost-effective, the focus should
Table 4.1: Comparison of supersonic airplanes [18, 37, 38]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Concorde</th>
<th>Lockheed/ NASA N+2</th>
<th>Aerion AS2</th>
<th>Average of non-dimensional parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuselage length</td>
<td>m</td>
<td>61.66</td>
<td>70.10</td>
<td>51.82</td>
<td></td>
</tr>
<tr>
<td>Fuselage Diameter</td>
<td>m</td>
<td>2.87</td>
<td>2.9</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>Fuselage L/D</td>
<td>–</td>
<td>21.5</td>
<td>24.2</td>
<td>23.6</td>
<td>23.1</td>
</tr>
<tr>
<td>Wing span</td>
<td>m</td>
<td>25.6</td>
<td>25.6</td>
<td>18.59</td>
<td></td>
</tr>
<tr>
<td>Wing surface</td>
<td>m²</td>
<td>358.25</td>
<td>306.39</td>
<td>125.4</td>
<td></td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>–</td>
<td>1.83</td>
<td>2.14</td>
<td>2.76</td>
<td>2.24</td>
</tr>
<tr>
<td>Weight empty</td>
<td>kg</td>
<td>78700</td>
<td>76082</td>
<td>26219</td>
<td></td>
</tr>
<tr>
<td>Weight full</td>
<td>kg</td>
<td>187000</td>
<td>160572</td>
<td>54885</td>
<td></td>
</tr>
<tr>
<td>Average weight</td>
<td>kg</td>
<td>132850</td>
<td>118327</td>
<td>40552</td>
<td></td>
</tr>
<tr>
<td>Cruise speed</td>
<td>m/s</td>
<td>2.02</td>
<td>1.6</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Cruise Altitude</td>
<td>m</td>
<td>18000</td>
<td>15240</td>
<td>15545</td>
<td></td>
</tr>
<tr>
<td>Speed of sound</td>
<td>m/s</td>
<td>295.07</td>
<td>295.07</td>
<td>295.07</td>
<td></td>
</tr>
<tr>
<td>Airspeed</td>
<td>m/s</td>
<td>596.04</td>
<td>472.11</td>
<td>413.10</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>0.1207</td>
<td>0.1865</td>
<td>0.1777</td>
<td></td>
</tr>
<tr>
<td>Dynamic pressure</td>
<td>Pa</td>
<td>21436</td>
<td>20782</td>
<td>15164</td>
<td></td>
</tr>
<tr>
<td>$C_L$</td>
<td>–</td>
<td>0.170</td>
<td>0.182</td>
<td>0.209</td>
<td>0.187</td>
</tr>
<tr>
<td>Number of passengers</td>
<td></td>
<td>128</td>
<td>100</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

be on carrier flights. Therefore the focus has to be on the larger aircraft carrying about 100 passengers. A length of the airplane of 70 m is a reasonable choice when looking at the data from table 4.1. A cruise altitude of 18000 m can be taken, combined with a cruise speed Mach number of 1.6 to rule out any transonic effects in the airflow. When the average weight is about 120 tonnes and the lift coefficient is 0.15, this will result in a wing planform area of 583.66 m². In the next section, the wing geometry will be further discussed.

4.3. Wing geometry
The wing layout and geometry has a direct influence on the performance of the airplane. This is the section where lift is generated and this will directly influence the drag and the sonic boom. The wing planform will be discussed first and the airfoil definition next.

4.3.1. Wing planform
The wing planform should be suitable for supersonic flight. To keep the geometry as simple as possible, no twist and camber will be used. For supersonic applications, a high sweep angle is often used. A high sweep angle also helps to smoothen the cross-sectional area distribution, since there will be a smaller rise in cross-sectional area at the longitudinal point where the wing starts. This helps in keeping the airplane cross-sectional area distribution smooth, which will most likely help in reducing wave drag. Tapering the wings not only helps in reducing bending moments because the chordlength at the tip is smaller and the chordlength at the root is longer, it also helps in smoothing the area distribution, which is more important for this research. A relative long wing compared to the width has a low aspect ratio, typical for supersonic aircraft. First, some parameters will be varied to make a planform that is typical for a supersonic aircraft.
4.4. Fuselage geometry

The fuselage will be based on a Sears-Haack body. There are multiple reasons for this. First of all, a Sears-Haack body is a low-drag body in a supersonic stream. Secondly, the body needs to be indented at the position of the wing. This makes the body less indented at that section. First, a Sears-Haack body is constructed using 20 circles at linear spaced distances which span a ruled solid. Subsequently, the wing is fused with the fuselage. Then this body will be area-ruled to keep the wing planform the same for all configurations. The fuselage should be a slender body comparable to other supersonic airplanes, this means that the diameter must be lower than the length of the body. When a slenderness ratio of 20 is used and the incidence angle of the wing is increased, there is not much room to vertically translate the wing. Therefore a lower slenderness-ratio for the equivalent Sears-Haack body 14.0 is used, which led to a final slenderness ratio of $\approx 15.9$ for the final, area-ruled design.

4.4.1. Area ruling

The airplane configuration will be area-ruled for a speed of Mach 1. The choice for this speed is made to keep the algorithm simple and to reduce the possibility of any errors arising. When an aircraft is area-ruled for Mach 1, a plane can be used which is translated along the longitudinal axis of the aircraft. The cross-section of the airplane, consisting of a wing and a fuselage, should be the same as the cross-section of the Sears-Haack body of which the fuselage is based on. There is a surplus in cross-sectional area for the airplane at the point where the wing is added. Therefore the cross-sectional area of the airplane configuration must decrease. By reducing the local diameter of the fuselage at this point, the total cross-sectional area will reduce as well. An iterative loop is used in ParaPy, which makes the following steps in calculating the radii at several longitudinal stations of the fuselage:

1. Calculate the cross-sectional area of the wing $S_{\text{cross,}w}$
2. Calculate the total cross-sectional area of the airplane $S_{\text{cross,}\text{total}}$
3. Calculate the current cross-sectional area of the fuselage $S_{\text{cross,}\text{fus}}$
4. Calculate the difference between the cross-sectional area of the airplane and the one of the base Sears-Haack body $\Delta S = S_{\text{cross,}\text{total}} - S_{\text{cross,}SH}$
5. Calculate the ratio between the area of the fuselage that is not in the area covered by the wing cross-sectional area and the fuselage cross-sectional area. This ratio can also be seen as the ratio of area which can be changed by changing the radius of the fuselage, since the other part of the fuselage is included in the wing cross-sectional area. $\Pi_S = \frac{S_{\text{cross,}\text{total}}-S_{\text{cross,}w}}{S_{\text{cross,}\text{fus}}}$
6. Calculate the new aim for a cross-sectional area based on a new goal of the fuselage diameter. The aimed area is calculated by adding or subtracting the multiplication of the difference in area \( \Delta S \) with the ratio of fuselage that is not part of the wing. This must be done to prevent the new aim for the fuselage radius of oversooting which could prevent convergence. 

\[
R_{\text{new}} = \sqrt{\frac{S_{\text{cross fus}} - \Pi \Delta S}{\Pi}}
\]

7. The error is calculated by: 

\[ \text{error} = \frac{\Delta S}{S_{\text{Sh}}} \]

By repeating these steps until the relative error is below 0.1%, the cross-sectional area distribution of a Sears-Haack body can be approached.

### 4.5. Final geometry

Several attempts have been made to make sure an airplane model came together which satisfies the requirements in section 4.1. In figure 4.2 some geometries are visible which had some unfavourable properties. The main problem was that for a typical slenderness ratio of a supersonic aircraft, the fuselage became too thin at the longitudinal station of maximum thickness after area ruling the geometry for Mach 1.

- (a) A short wing with a small amount of leading edge sweep.
- (b) An airplane with a high slenderness. An extremely waisted body is the result, which also restricts the possibility to vary the wing installation height.
- (c) A configuration with a wing of \( t/c = 4\% \).

![Figure 4.2](image)

Figure 4.2: Different configurations with unfavourable geometries.

#### 4.5.1. Final base specifications

The final design for the airplane model has the following base specifications for each configurations, see table 4.2. Per configuration the incidence angle of the wing will be varied to approach a \( C_L \) of 0.15. After setting the wing incidence angle, the body has been area-ruled for Mach 1, where the equivalent Sears-Haack body used for area ruling, has a maximum radius of 2.5\( \text{m} \). A final low-wing configuration
Table 4.2: Base parameters for the construction of each configuration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airfoil</td>
<td></td>
<td>NACA 0002</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>$c_{root}$</td>
<td>m</td>
<td>28.47</td>
</tr>
<tr>
<td>MAC</td>
<td>m</td>
<td>19.61</td>
</tr>
<tr>
<td>AR</td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>$S_{ref}$</td>
<td>m$^2$</td>
<td>583.66</td>
</tr>
<tr>
<td>$\lambda$</td>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>$b_w$</td>
<td>m</td>
<td>34.17</td>
</tr>
<tr>
<td>$x_{w,start}$</td>
<td>m</td>
<td>15.00</td>
</tr>
<tr>
<td>$l_{fus}$</td>
<td>m</td>
<td>70.0</td>
</tr>
<tr>
<td>$R_{fus,max}$</td>
<td>m</td>
<td>$\approx$ 2.20</td>
</tr>
<tr>
<td>$D_{fus,max}$</td>
<td>m</td>
<td>$\approx$ 15.9</td>
</tr>
</tbody>
</table>

is depicted in figure 4.3a and the cross-sectional area distribution measured by planes normal to the $x$-axis is shown in figure 4.3b.

Figure 4.3: Final low-wing configuration as is used in the simulations.
In this chapter the solver for the CFD simulations is described and an analysis is presented of several meshing parameters and their influence on the results regarding lift, drag and pressure signatures.

5.1. SU2

SU2 is an aerodynamics solver for CFD. SU2 stands for Stanford University Unstructured and is a code which is aimed at unstructured meshes. The source-code is open which means it can be modified for specific applications. SU2 is used in researches which incorporate sonic boom optimization. It is also possible for SU2 to optimize the angle of attack of a configuration, which makes it very suitable for this research.

5.2. Solver settings

For SU2 many options can be chosen. The flow around the model is based upon the total freestream conditions, which are listed before in table 3.1 on page 32. For the convective numerical method, the AUSM (Advection Upstream Splitting Method) is used. This method is based on the upwind concept. The main features are, according to Younis et al [40]:

- Accurate in capturing shocks and in contact discontinuities
- Entropy satisfying solution
- Positivity-preserving solution
- Algorithmic simplicity
- Free of the so-called ‘carbuncle phenomena’
- A uniform accuracy and convergence rate for all Mach numbers

This method recognizes that the inviscid flux consists of two fluxes, the convective and pressure fluxes, which are physically different. This method is also used in aeroacoustic research [41]. Other methods have been tried as well for the convective numerical method, but they were not always suitable for supersonic simulations. Therefore AUSM is used. The numerical method for spatial gradients is the weighted least squares method. The CFL-number is kept at 2.0 and for the runs to optimize the angle of attack for zero lift, it needs to be at 1.0. The CFL-number has been varied up to 2.0 and no noticable difference
was observed in the output except that the simulations were quicker. BCGSTAB is used as the linear solver. The limiter coefficient for the Venkatakrishnan slope limiter has been varied, but had no effect on the results, just as the artificial dissipation coefficients. For the time discretization the Euler implicit formulation is used, since the other methods diverged. The termination of the simulation occurs when the residual value for the density drops below $10^{-12}$, which is reached in about 4200 iterations with a CFL-number of 2.0, but the lift-and drag numbers did not change after 1700 iterations.

5.3. Meshing

Since the airplane model is symmetrical, the domain around the airplane and the airplane itself can be split in the symmetry plane. This reduces the required computations that are necessary. A domain with with a radius of 120m around the longitudinal axis is made, which is split up into several blocks:

- Near-field
- Mid-field
- Far-field low
- Far-field high
- Far-field extended

Note that these blocks are given the names near-field, mid-field etcetera for convenience, but their distances to the aircraft do not specifically agree with the usual distances of a true mid-field and far-field. In these blocks there are different maximum element lengths $l_{\text{max}}$. Near the airplane the mesh must be more refined compared to the farfield. The far-field high block has large elements because the flow information in this region is not of interest. The far-field low block however has a small element size comparable to the near-field block. The near-field block is the block with the smallest maximum element length. In this near-field block, the airplane is present of which the mesh is more refined at some places. The layout of the domain can be seen in figure 5.2. Note that in this domain an extra block, the far-field extended block is visible. This block has been added after the validation, since the radius of the domain needed to be larger. An explanation is given in section 5.6.1.

The mesh is built by using triangular elements. The volume of the domain is then meshed by using tetrahedral elements.
5.4. Mesh refinement

In several areas of the mesh, refinement is required. Early results showed some sections with a rapid rise or decrease in pressure. To make sure the results are valid in these sections, the mesh needs to be refined at some places, such as the wing edges, the nose and the tail and the frontal shockwave. It is beneficial to refine the regions where the shockwaves from the nose and the leading and trailing edge of the wing are, however due to problems with the meshing algorithms of SALOME this was not possible.

5.4.1. Wing edges

Around the leading edge of the upper side of the wing there was a low pressure field visible. When analyzing the Mach numbers at this section, it showed a rapid increase in velocity. These sections are refined with elements of a length of $5\, \text{cm}$ and a growth rate of 5%. A smaller mesh size is required to make a better approximation of the geometry, since elements of $5\, \text{cm}$ cannot describe the leading edge radius of the airfoil. The used element length at the leading edge of the airfoil will return a sharp leading edge for the simulations. The growth rate of the mesh in SALOME is defined as how much the linear dimensions of two adjacent cells can differ [42]. The wing is partitioned into smaller spanwise chunks. On the leading- and trailing-edges of these chunks the elements have a length of $5\, \text{cm}$. On the chordwise edges, the elements have a distribution with a growth rate of 5% towards the mid-chord location to help the SALOME algorithm to mesh. On the surfaces toward the mid-chord, the mesh can be coarser, since smaller deviation of the flow is present and the properties of the flow do not change that much. This is not valid for the wing tips, which is why the element length at that section is kept small. The effect of using extra mesh guide lines on the wing surfaces was negligible in terms of lift and drag.

5.4.2. Nose and tail

Around the nose and the tail of the aircraft, the triangular elements remain very large. This has an effect on the drag of the aircraft. By using a local hypothesis in the meshing algorithm, the element size near the nose and tail are drastically decreased from $20\, \text{cm}$ to $1\, \text{cm}$ which results in a smoother approximation of the nose and tail sections. Because of this modification, the drag coefficient of the aircraft decreased from $C_D = 0.011499$ to $C_D = 0.011301$, which is a decrease of 1.75%. Close-ups of the nose section with the standard and the refined mesh are shown in figure 5.5 and an overview of the results of the modification can be found in table B.3 in appendix B.
5.5. Mesh validation

A sensitivity analysis is performed to see the effect of the mesh coarseness on several performance parameters of the aircraft. This is necessary to ensure that the effects that will be measured are truly due to the variation of several parameters of the aircraft and that they are not the effect of variations in the mesh.

5.5.1. Difference in lift and drag

For the same case, the chordwise pressure distribution is measured for different meshes. This is done at different spanwise locations to ensure the mesh is fine enough for the different locations. These locations are at \( \chi/\alpha = 0.25, 0.5, 0.75, 0.95 \). The results can be found in figure 5.6. When the minimum element size at the leading- and trailing edge decreases. At the leading edge along the wingspan, more suction is present. On the upper surface a shock is visible, which is captured well. However, at 95\% span it looks like the shock is captured better when the element size decreases. When the minimum element size is smaller, the dip after the shock is lower, but the shock starts at the same chordwise position. Also, the rate of change of the \( C_p \) is nearly equal. Looking further at what happens at the surface in figure 5.7, it seems that there is some shock interaction from the outboard leading edge and the shock on the surface of the wing. The point where these shocks meet on the upper surface happens to be at about 95\% span. At this point the mesh size does matter for the position of the shock, however, it will not influence the lift coefficient as much, as is depicted in figure 5.8.

In table 5.1 the effect of the wing minimum element size on the lift and drag of the configuration is stated. As can be seen in both this table and figure 5.8, the effect of the element size on the lift and drag is not so big. When the minimum element size is below 0.10m, the lift and drag converges and does not vary extremely. Since some problems occurred every now and then with wing minimum element sizes equal or smaller than 0.02m, a value of 0.05m will be used for the final simulations. Note that this makes the wing leading edge sharp, which is unfavourable. However, the differences in lift and drag for smaller elements on the leading and trailing edge are fairly small and for all configurations the same parameters will be used, which makes that this is not a big problem. It might be better to switch to a dedicated computer program for meshing in the future since this gives more and better control over the total mesh.

Table 5.1: Effect of surface mesh parameters on the lift and drag.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>0.02m</th>
<th>0.05m</th>
<th>0.10m</th>
<th>0.15m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements in airplane surface mesh</td>
<td>#</td>
<td>129508</td>
<td>73568</td>
<td>56700</td>
<td>52578</td>
</tr>
<tr>
<td>( C_L )</td>
<td>–</td>
<td>0.149242</td>
<td>0.149442</td>
<td>0.149261</td>
<td>0.149033</td>
</tr>
<tr>
<td>( C_D )</td>
<td>–</td>
<td>0.0110407</td>
<td>0.0109995</td>
<td>0.0109241</td>
<td>0.0115949</td>
</tr>
<tr>
<td>( l/l_0 )</td>
<td>–</td>
<td>13.517</td>
<td>13.586</td>
<td>13.663</td>
<td>12.853</td>
</tr>
<tr>
<td>Difference w.r.t. average ( l/l_0 )</td>
<td>%</td>
<td>0.84</td>
<td>1.35</td>
<td>1.93</td>
<td>-4.12</td>
</tr>
</tbody>
</table>
5.6. Effect of mesh parameters on pressure signature

The pressure signature away from the aircraft must be measured to assess the sonic boom performance. The pressure at several azimuthal angles will be extracted from the CFD results at different radial distances from the longitudinal axis of the aircraft. Early pressure signatures show a lower pressure at the beginning of the pressure signature, where a higher pressure than the ambient pressure was expected. This is probably due to the lack of mesh fineness in the frontal shockwaves since experimentation with different convective numerical methods show the same behaviour. Recent research shows the same phenomena in the near field pressure signatures \[43, 44\]. The same research also shows that the maximum overpressure varies with the mesh element sizes throughout the domain. An investigation on the effect of several mesh parameters, such as the element length in the near-field, the mid-field and far-field is conducted to see the effect on the pressure signatures at 40 and 70 meters below the aircraft.

5.6.1. Effect of domain size

The radius of the domain is varied, since some distortions were visible in plots of pressure coefficients on the edges of the domain. These distortions seem to be reflections of the shockwaves on the boundary of the domain. A close-up is shown in figure 5.9. Therefore the domain needs to be larger to ensure these reflections do not distort the pressure signatures. In this extended far-field, the mesh size can be larger since no results are required from this region.

Figure 5.6: The pressure distributions over the airfoil at different spanwise locations for different minimum element sizes. The maximum element size is 0.2 m.
Figure 5.7: Shock interaction at the position of the slice at 95% span.

Figure 5.8: Convergence of minimum element size at the wing for the lift-to-drag ratio.

Figure 5.9: Visible reflections in the lower regions of the domain when the pressure coefficient is plotted.

Figure 5.10 shows 3 pressure signatures for a varying domain, where the outer radius is the only parameter for the mesh that has been varied. Up to 110m axial distance at 70m below the aircraft, the signatures are nearly equal, but then the reflections of the shockwaves distort the pressure signature. The outer radius of the mesh must be at least 35m further away than the farthest pressure signature that will be measured because the characteristics of the shockwaves that hit the outer radius of the domain will not be able to distort the pressure signature. To be sure that there is no distortion of the pressure signature for the final meshes, a radius of 120m will be used for the final mesh. To decrease the memory usage of the computer required for the mesh generation, an extra block will be added, the far-field extended block, which will have a coarse mesh with a maximum element length of 4.0m. An overview of the CFD results regarding the validation can be found in table B.2 in appendix B.
5.6. Effect of mesh parameters on pressure signature

5.6.2. Effect of mesh element sizes in the domain

An investigation has been made to find out the effect of the element length of the triangular and tetrahedral elements on the pressure signatures. In table 5.2 the properties of the meshes are stated, where $l_{\text{varied}}$ is the maximum element size that has been varied for the different blocks. The growth rate throughout the domain is $0.05$, since SALOME gave irregularities in some meshes for smaller growth rates. The algorithm not always meshes all surfaces according to the input parameters and sometimes surfaces were triangulated partially. Another problem was that the volume mesh is not generated because of a random error in SALOME. Overall, using a growth rate of $0.05$ gave the most consistent performance while generating a mesh with small enough cells to answer the research questions.

Table 5.2: Mesh parameters for the validation of the domain.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Near-field block</th>
<th>Mid-field block</th>
<th>Far-field block</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{min}}$</td>
<td>m</td>
<td>0</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td>$R_{\text{max}}$</td>
<td>m</td>
<td>20</td>
<td>35</td>
<td>80</td>
</tr>
<tr>
<td>$l_{\text{max}}$</td>
<td>m</td>
<td>0.3</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>$l_{\text{varied}}$</td>
<td>m</td>
<td>0.3, 0.4, 0.5, 0.6</td>
<td>0.4, 0.6, 0.8</td>
<td>0.6, 0.8, 1.2</td>
</tr>
</tbody>
</table>

The impulses have been calculated and validated, but only for the lifting configurations since the zero-lift signatures have multiple parts of positive overpressure. The lifting configurations however first have a section of positive overpressure, after which a negative overpressure is measured. This positive portion of the overpressure, integrated over time is the pressure impulse. The average of the pressure impulses for the variation of the mesh for all element sizes stated in table 5.2 was $16.2430 \text{Pa} \cdot \text{s}$ with a higher outlier of $0.292\%$ and a lowest variation of $-0.745\%$. These results can be viewed in appendix B in table B.1.

Element sizes in the near-field block The element sizes in the near-field block (up to $R = 35m$ from the longitudinal axis of the airplane) only have a minor influence on the near-by pressure signature. The maximum element length is varied from $0.3, 0.4, 0.5$ to $0.6 \text{ m}$ length. When the mesh size further away from the aircraft is increased, the minor effects dissipate even further. At $1$ body length distance below the aircraft, the pressure signatures remain the same for different maximum element sizes. The maximum difference in impulse for the different element lengths is $0.0033 \text{ Pa} \cdot \text{s}$ from the average of $16.257 \text{ Pa} \cdot \text{s}$, which is very small.

Element sizes in the mid-field block The maximum element sizes in the mid-field block are tested for $0.4, 0.6$ and $0.8 \text{ m}$. The effect on the pressure distribution at $40m$ below the aircraft is clearly visible in figure 5.11. Smaller element sizes steepen the pressure signatures at the places where shocks are
present. Some local minima and maxima are larger for the smallest element size. However, the absolute maximum is the same for all element sizes. These effects are almost gone when the signatures are propagated to a body-length distance below the aircraft. When a close look is taken on the signatures, it is still visible that for the smaller element sizes in the mid-field the pressure rises are steeper. The mean impulse of these measurements 70\,m below the aircraft is $16.254 \pm 0.028\,Pa \cdot s$, which is again a small difference.

**Figure 5.11**: Pressure signatures directly below the aircraft for different maximum element sizes in the mid-field block.

**Element sizes in the far-field block** In the far-field block, the maximum element size ranged from a length of 0.6\,m up to 1.2\,m. The pressure signature did not vary that much 40\,m below the aircraft, but at 70\,m below the aircraft more differences can be seen. The pressure rises are less steep when the element length increases and the local peaks are higher for smaller element sizes, as is shown in figure 5.12. Note that again the absolute maximum pressure peak remains the same for all element sizes. The mean positive impulse of these 3 pressure signatures is $16.223\,Pa \cdot s$ and the signature with the largest size is the biggest outlier with $0.102\,Pa \cdot s$ less impulse, which is for a very coarse far-field mesh with element sizes of 1.2\,m.
5.7. Final mesh

Concluding from the work above the mesh size definitely influences the results. A smaller element length in the volume of the domain shows steeper pressure rises in the areas where shocks are present. Next to that, at these local areas the pressure signatures also show an overshoot of the pressure. On the other side the absolute maximum overpressure remains the same. Also the positive part impulse of the pressure signatures does not vary a lot for different maximum element sizes. However, the results of these maximum overpressures might not be reliable. Other researches which have implemented mesh adaptation, show that the maximum overpressures increase with a finer mesh [43]. The impulse however should remain constant for different mesh sizes, since the energy is contained in the system. For the final mesh, the parameters of tables 5.3 and 5.4 are used. This gives a mesh with approximately 6.5 – 7.5 million nodes and ≈ 42 million tetrahedral elements.

Table 5.3: Parameters for the volume mesh

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$R_{\text{min}}$</th>
<th>$R_{\text{max}}$</th>
<th>$l_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near-field block</td>
<td>0</td>
<td>25</td>
<td>0.3</td>
</tr>
<tr>
<td>Mid-field block</td>
<td>25</td>
<td>41</td>
<td>0.4</td>
</tr>
<tr>
<td>Far-field low</td>
<td>41</td>
<td>80</td>
<td>0.5</td>
</tr>
<tr>
<td>Far-field high</td>
<td>41</td>
<td>80</td>
<td>4.0</td>
</tr>
<tr>
<td>Far-field extended</td>
<td>80</td>
<td>120</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 5.4: Parameters for the surface mesh of the airplane

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$l_{\text{min}}$</th>
<th>$l_{\text{max}}$</th>
<th>Growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuselage</td>
<td>0.01</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Wing</td>
<td>0.02</td>
<td>0.2</td>
<td>0.05</td>
</tr>
</tbody>
</table>
In this chapter the results that are gathered by the CFD simulations will be presented. They are split up into a section on the aerodynamic performance of the different configurations, a section on the sonic boom performance and a section on the relationship between different drag estimation methods based upon the supersonic area rule. In this last section a method proposed in chapter 2 will be analyzed.

6.1. Aerodynamic performance

In this section the flow around the aircraft will be discussed as well as some results on the performance for $C_L = 0.15$ and $C_L = 0.0$. Using the results, an explanation will be given about why a certain configuration has a better performance compared to another configuration.

6.1.1. Flow description

For the lifting cases, the flow in proximity of the nose is similar. A first attached shockwave is generated by the nose-cone. Since the nose-cone is an axisymmetrical shape aligned with the flow

Figure 6.1: Overpressures for different configurations seen from the side at $C_L = 0.15$.
direction, the flow has an equal pressure distribution around the nose. From figures 6.1a and 6.1b can be seen that just in front of the location where the wing starts, the overpressure drops to about 0Pa because the radius of the fuselage reduces, so an expansion takes place. The leading edge of the wing is visible, from which shockwaves to the upper and lower side of the wing emanate. This shockwave will continue as the distance progresses away from the body.

Since the leading edge sweep angle is larger than the Mach-angle, there will be a bow shock in front of the wing, which is generated at the junction of the wing leading edge with the fuselage as can be seen in figure 6.2. In this figure can be observed that the flow will accelerate over the wing which causes a shockwave. Since the streamlines on the upper surface are curved inwards and the local Mach number is about 2.0, this will cause a shockwave which extends towards the tip of the wing. At the tip region there is some shock interaction visible from the shock created by leading edge of the tip. From the trailing edge of the wing another shock is visible which hits the fuselage. The wing-fuselage junction for the low wing configuration (see figure 6.1a) is closer to the $y=0$ symmetry plane than the trailing edge of the higher wing configurations, whose trailing edges are placed higher on the fuselage (see figure 6.1b). Therefore the shock from the wing trailing edge hits the fuselage more heavily. Note the red region just aft and above of the trailing edge of the wing in figure 6.1a, while this is absent in figure 6.1b. Directly behind this shock, the fuselage radius decreases, which will cause the flow to expand and so the pressure drops in these expansion waves. At the tail there is a recompression shock visible.

### 6.1.2. Lifting conditions

The different configurations have been constructed. An effort is made to adjust the wing incidence angle to obtain a lift coefficient of 0.15 for all configurations. In table 6.1 the final additional parameters of the geometry and the lift and drag results are listed.

<table>
<thead>
<tr>
<th>Parameter $h_w$</th>
<th>Units</th>
<th>Low wing</th>
<th>Mid-Low wing</th>
<th>Mid wing</th>
<th>Mid-High wing</th>
<th>High wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>$^\circ$</td>
<td>-0.3</td>
<td>0.2</td>
<td>0.8</td>
<td>1.3</td>
<td>1.8</td>
</tr>
<tr>
<td>$C_L$</td>
<td></td>
<td>3.55</td>
<td>3.74</td>
<td>3.87</td>
<td>3.92</td>
<td>3.91</td>
</tr>
<tr>
<td>$C_D$</td>
<td></td>
<td>0.150089</td>
<td>0.150435</td>
<td>0.150246</td>
<td>0.150133</td>
<td>0.15024</td>
</tr>
<tr>
<td>$t/\delta$</td>
<td></td>
<td>0.0107841</td>
<td>0.011043</td>
<td>0.011194</td>
<td>0.011272</td>
<td>0.0113115</td>
</tr>
<tr>
<td>$S_w,proj$</td>
<td>$m^2$</td>
<td>501.9</td>
<td>483.5</td>
<td>473.7</td>
<td>473.1</td>
<td>481.0</td>
</tr>
<tr>
<td>$S_w,wet$</td>
<td>$m^2$</td>
<td>1007.8</td>
<td>971.2</td>
<td>951.6</td>
<td>950.5</td>
<td>966.9</td>
</tr>
<tr>
<td>$S_{tot,wet}$</td>
<td>$m^2$</td>
<td>1721.7</td>
<td>1691.6</td>
<td>1674.8</td>
<td>1672.9</td>
<td>1683.9</td>
</tr>
<tr>
<td>$V_{tot}$</td>
<td>$m^3$</td>
<td>813.2</td>
<td>812.1</td>
<td>812.5</td>
<td>812.7</td>
<td>813.0</td>
</tr>
</tbody>
</table>

The data in table 6.1 shows that the low wing configuration has the lowest drag of all configurations and the airplane with the high wing configuration produces the largest amount of drag for the same lift as the other configurations. The mid-high wing configuration has the largest wing incidence angle to generate the same amount of lift as the other configurations. Note that the projection of the wing wetted surface area on the $XY$-plane is the largest for the low-wing configuration. Some of the results are visualized in figures 6.3a to 6.3d as their values are plotted against the wing installation height $h_w$. There is no linear trend observable in these values, but more a quadratic trend. It looks like the drag scales more or less proportionally with the wing planform wetted area, however this is not true when figures 6.3b and 6.3c are compared to eachother. The planform area...
6.1. Aerodynamic performance

definitely influences the performance parameters, but it looks like there is another effect since for the high-wing configuration the lift-to-drag ratio decreases while the projected area of the wetted wing part increases. The low wing configuration has a 4.79% better lift-to-drag ratio compared to the high-wing configuration, while the wing wetted projected area is 4.35% higher. For the mid-high wing configuration this is a 2.56% better ratio while the wetted wing area projection is only 0.52% higher when the high wing configuration is taken as a baseline. This proves that there is another effect in play, which will be further researched in section 6.1.4.

6.1.3. Non-lifting conditions

The lifting configurations are also simulated for zero lift. Therefore an optimization in $SU2$ has been made to find $a_{L=0}$ for all configurations. This value is of interest, since the zero-lift drag can be calculated and the effect of lift on the sonic-boom can be investigated. Also, the wing planform efficiency $e$ can be calculated by using this value. In table 6.2 the zero-lift values for each configuration are found.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Low wing</th>
<th>Mid-Low wing</th>
<th>Mid wing</th>
<th>Mid-High wing</th>
<th>High wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_w$</td>
<td>m</td>
<td>-0.3</td>
<td>0.2</td>
<td>0.8</td>
<td>1.3</td>
<td>1.8</td>
</tr>
<tr>
<td>$a_{L=0}$</td>
<td>$^\circ$</td>
<td>-3.25600</td>
<td>-3.25670</td>
<td>-3.24811</td>
<td>-3.23750</td>
<td>-3.22860</td>
</tr>
<tr>
<td>$a_{\text{eff}}$</td>
<td>$^\circ$</td>
<td>0.294</td>
<td>0.483</td>
<td>0.622</td>
<td>0.683</td>
<td>0.681</td>
</tr>
<tr>
<td>$C_L$</td>
<td></td>
<td>0.000136</td>
<td>0.000274</td>
<td>0.000110</td>
<td>0.000095</td>
<td>0.000072</td>
</tr>
<tr>
<td>$C_{DL=0}$</td>
<td></td>
<td>0.002864</td>
<td>0.002840</td>
<td>0.002832</td>
<td>0.002830</td>
<td>0.002827</td>
</tr>
<tr>
<td>$e$</td>
<td></td>
<td>0.453</td>
<td>0.439</td>
<td>0.430</td>
<td>0.425</td>
<td>0.423</td>
</tr>
<tr>
<td>$C_{L_{w}}$</td>
<td>$rad^{-1}$</td>
<td>2.639</td>
<td>2.642</td>
<td>2.648</td>
<td>2.655</td>
<td>2.665</td>
</tr>
</tbody>
</table>

Figure 6.3: Comparison of some gathered results for $C_L = 0.15$
Contrary to the aerodynamic drag in the lifting case, the low-wing configuration now shows the largest amount of drag, albeit the difference is only 291N. Still, this difference is about 1.31% compared to the high-wing configuration. As can be seen, for the angles of attack tested there is still some lift visible.

6.1.4. Analysis of airplane performance

With the data from tables 6.1 and 6.2 the span efficiency and true zero-lift drag coefficient can be calculated. For this analysis the assumption is made that the lift slope behaves linear between the measured angles of attack. By solving the following formula for the Oswald efficiency factor $e$, the zero-lift drag coefficient can be obtained.

$$C_D = C_{D_{\text{Leo}}} + \frac{C_L^2}{\epsilon \pi AR} \quad (6.1)$$

This coefficient however is very close to the already obtained value from the CFD simulations, so the value in table 6.2 does not change. Further analysis shows that the lift slope for the high-wing configuration is a little higher compared to the low-wing configuration, nearly 1%.

To explain the differences in the performance in the lifting conditions it is useful to have a look at the pressure distributions over the wing. The pressure distributions at 14.6, 25, 50 and 75% span are depicted in figure 6.4.

![Pressure distributions over the wing at different spanwise locations in lifting conditions.](image)

(a) $C_p$-distribution at 14.6% span  
(b) $C_p$-distribution at 25% span  
(c) $C_p$-distribution at 50% span  
(d) $C_p$-distribution at 75% span

Figure 6.4: The pressure distributions over the wing at different spanwise locations in lifting conditions.

At 50% and 75% span the pressure distributions are almost the same. Closer to the fuselage, however, differences begin to appear. The high-wing configuration has a pressure peak near the leading edge. This peak drops when the wing installation height is lower. On the upper side there is more
suction visible when the wing is placed lower on the fuselage. This is accompanied by a higher pressure on the lower wing surface between $0.25 < \frac{y}{c} < 0.8$, which is clearly visible for the low wing in this region. This larger area between the upper and lower curve also means that more lift is generated at this section. This extra suction on the upper surface and extra pressure at the lower surface is reduced at 25% span, but it still exists. On the outboard areas of the wing, there is less suction on the leading part of the upper wing surface. It is necessary to have a clear view on the upper and lower surfaces of the wing to see how the pressure distributions look like and if there is a cause for the difference in pressure coefficients. In figure 6.5 the low-wing and the mid-wing configurations are shown for the lifting cases. Note that the colormap is a distinctive one and that the boundaries between the different colors do not represent shocks. The distinctive colors clearly show that there is a large region of negative pressure on the upper wing surface near the fuselage for the low wing configuration. Apparently there is some interference between the fuselage and the wing in this region. Note that the region where $-0.09 < C_{\mu} < -0.08$ extends close to the fuselage further aft towards the trailing edge of the wing compared to the mid-wing configuration. Such a large region with a lower pressure will definitely influence the lift and drag performance of the configuration.

![Image](image.png)

(a) Low-wing configuration  
(b) Mid-wing configuration

Figure 6.5: Comparison of the $C_P$-distribution on the upper surface of the wing for 2 configurations. The pressure regions are marked with a distinctive colormap.

In order to find an answer to why this region of lower pressure exists, the different shapes of the projection of the upper wing surfaces on the $XY$-plane will be looked at. This is shown in figure 6.6. The leading edge of the upper surface of the high-wing configuration starts close to the symmetry plane, since the wing is mounted high on the fuselage with an incidence angle. When the junction of the wing to the fuselage is followed towards the trailing edge of the wing, the junction extends further away from the center. When the mid-wing configuration is taken, the line stays nearly straight, since the wing is mounted near the symmetry plane of the fuselage, where for a wing incidence angle of $3.87^\circ$ the wing-fuselage junction does not differ that much. But when the low-wing configuration is taken, where the trailing edge nearly touches the lower side of the fuselage, the wing-fuselage junction describes a line which extends smoothly further towards the symmetry plane.
This junction for the lower wing configuration, which extends far inboard on the lower side of the fuselage could be the answer to why there is a large region of lower pressure near the fuselage for this configuration. There is probably a gradual expansion of the supersonic flow, which lowers the pressure on the upper surface. When seen from behind, there is a ‘channel’ visible where streamlines curve outboard for the high-wing configuration and curve inwards and therefore expand for the low-wing configuration. This is illustrated in figure 6.7.

When some streamlines are visualized, as shown in figure 6.8, one can see that for the low wing configuration the streamlines diverge near the trailing edge, as where they converge for the high wing configuration. This indicates that the flow is accelerating towards the trailing edge and the fuselage more for the low wing configuration, which indicates that more suction and therefore more lift will be present. The flow on the lower side of the wing and fuselage interaction will be covered in section 6.5.1, since the lower side of the configurations have a direct effect on the sonic-boom.

Figure 6.6: Comparison of the projections of the wetted upper wing surface. The projection is on the $XY$-plane.

Figure 6.7: Comparison of the visible ‘channels’ near the junction of the wing and fuselage as seen from behind, where streamlines are drawn for illustration.

Figure 6.8: Streamlines visualized on the upper wing surface of the low-wing and high-wing configuration.
6.2. Sonic boom performance in zero-lift conditions

In this section the results will be presented about the pressure signatures at half a body-length and at a full body-length distance from the fuselage in both lifting and zero-lift conditions. First a description of the flow and shock interactions around the airplane in zero-lift is given.

6.2.1. Development of the shock pattern

![Diagram of pressure distribution](image)

(a) Side view at the symmetry plane of the overpressure generated by the low-wing configuration.

(b) Side view at the symmetry plane of the overpressure generated by the mid-wing configuration.

Figure 6.9: Comparison between the low and mid wing configuration at zero lift. Note the shock coming from the wing trailing edge on the lower side.

In figure 6.9a an overview of the overpressure generated in the symmetry plane \((y = 0.0m)\) is shown for the low wing configuration and in figure 6.9b for the mid wing configuration. From the nose, a shockwave is emanating which is propagated in all radial directions along the local characteristics. After this first shock, the overpressure decreases slightly and just before the section where the wing starts, an expansion fan is visible. On the lower side of the fuselage, there is a 'pocket' with a lower pressure which exists because the airplane has a negative angle of attack. Just after this region with a lower pressure, again a shockwave is visible. This one is emanating from the wing-fuselage junction, as can be deduced from figure 6.10. Further away from the symmetry plane this shock will be a bow shock, as can be seen in figure 6.11. On the wing at the leading edge, the flow is curving slightly
inwards. Another shock is visible, which is aligned almost perfectly with the leading edge of the wing, see figure 6.12 for the flow pattern and Mach-contours on the upper wing surface in zero-lift.

Figure 6.10: Isometric view of the overpressure around the non-lifting case for the low-wing configuration. Several cuts normal to the $x$-plane are visible.

Figure 6.11: Side view at the plane $y = 5m$ of the overpressure generated by the low-wing configuration.
At the trailing edge of the wing, there is a compression shock, since the upper and lower side flows over the wing meet again. At the trailing edge junction with the fuselage, the fuselage also compresses the flow. Next to that, the shock from the wing trailing edge junction hits the fuselage, which is reflected. This reflected shock is visible at both upper and just at the lower side of the fuselage at \( x = 45m \) in figure 6.9a. A little further to the back of the aircraft, there is an expansion visible, which extends towards the tail of the fuselage since the cross-sectional area decreases in that region. After the tail there is recompression which causes the trailing shockwave.

In figure 6.9a and 6.11, the downward propagation of shocks is clearly visible. After the lines where shocks exists, there are areas visible which converge to a thinner area with a higher pressure than the surrounding pressure. This is the so-called coalescence of shockwaves. In contrary, when there is an expansion, the characteristics diverge and the lower pressure dissipates into a wider region until another shock or recompression occurs. In the next subsection, the pressure signatures at zero-lift will be explained.

### 6.2.2. Pressure signatures in zero-lifting conditions

In figure 6.13a the pressure signatures for the 5 configurations are visible. They are measured in the symmetry plane at 70m below the aircraft. Note that the signatures have been normalized to the length of the aircraft. The first pressure rise starts at about \( \frac{x}{L} = 0 \) and the pressure signatures are nearly equal to about \( \frac{x}{L} = 0.2 \). After that, the low-wing configuration first keeps a higher overpressure compared to the other configurations. Then, the high wing configuration then shows a sudden overpressure increase at \( \frac{x}{L} = 0.3 \), higher than all the other configurations. When roughly calculated at which longitudinal station this sudden increase is coming from, it seems that it is emanating from the leading edge of the wing, according to equation 6.2.

\[
x_{R=0} = x_{R=0} - \beta R
= 107m - 1.249 \cdot 70m
= 19.57m
\]
Figure 6.14 shows an illustration of why the shock from the leading edge of the low wing configuration is shifted forward in the pressure signature. The shocks from the leading edge (red) of the low wing arrive earlier at the location where the signature is measured. The high wing configuration keeps a higher pressure, which drops further compared to the other configurations at \( \gamma/L = 0.45 \) and then climbs a bit higher, followed by a less negative overpressure of the low wing configuration. Note that all signatures cross the same point. Next the pressure drops further, followed by a recompression shock, which brings the pressure to just a higher pressure than the ambient pressure.

6.3. Sonic boom performance in lifting conditions

In figure 6.15 an overview of the overpressures generated by the low-wing configuration in lifting conditions is given. Note that the overpressures that are being generated are larger compared to the non-lifting conditions, especially below the wing, since these configurations generate lift.

![Figure 6.15: Side view on the symmetry plane of the overpressures generated by the low-wing configuration in lifting conditions.](image)

6.3.1. Development of the shock pattern

For the development of the shock pattern it is good to have a look again at figures 6.1a and 6.1b. Up to just before the wing leading edge, the overpressures are nearly equal. But at the start of the wing leading edge, differences appear. Directly below the fuselage, a region of negative overpressure is visible, just before the shock of the leading edge of the wing appears. This region of negative overpressure is lower for the high-wing configuration since the shock arrives further backwards at the bottom of the fuselage. Then, below the wing, there is a region of higher overpressure. For the low-wing configuration this higher pressure is more spread out, where the high-wing configuration in figure 6.1b is more concentrated backwards of the wing and has a short region at nearly the trailing edge with a high overpressure. After the trailing edge of the wing, on the upper side a recompression shock appears, where for the lower side an expansion fan is visible. At the tail a recompression shock is visible for the full airplane.
6.3.2. Pressure signatures in lifting conditions

In figure 6.16a pressure signatures are shown for an azimuthal angle of $0^\circ$. The low-wing configuration has a lower maximum positive overpressure. This overpressure is also spread out more compared to the other configurations, since the characteristics from the leading edge of the wing hit the measurement line earlier due to the lower wing placement. When the wing is placed higher, the second shock from the leading edge of the wing also looks a little steeper and there is a nearly flat section visible. This flat section in the signature is coming from the region just behind the leading edge when equation 6.2 is applied. The characteristics from the wing trailing edge let the overpressure drop to negative values. Note that the high-wing configuration has a less negative overpressure compared to the other signatures. The recompression at the trailing edge is nearly equal for all signatures. At $30^\circ$ azimuth angle, maximum overpressures are nearly equal and the highest negative overpressures are generated by the lower wing configurations.

![Pressure signatures at $0^\circ$ azimuth angle at $R = 70m$ for different configurations at $C_L = 0.15$](image)

(a) Azimuth angle of $0^\circ$.

![Pressure signatures at $30^\circ$ azimuth angle at $R = 70m$ for different configurations at $C_L = 0.15$](image)

(b) Azimuth angle of $30^\circ$.

Figure 6.16: Pressure signatures at $R = 70m$ for different azimuth angles. The signatures are normalized to the length of the aircraft.
6.4. Additional sonic-boom performance parameters

This section will list some additional results on the sonic-boom in both lifting and zero-lift conditions. The differences will be discussed in signature, overpressure and an analysis of the wing-fuselage interaction of these parameters will be given.

6.4.1. Differences in overpressures during lift and zero-lift

In figure 6.17 the differences in overpressures in lifting and zero-lift condition are plotted. The arrows indicate which type of overpressure each set of lines represents. The non-lifting signatures are normalized to their effective body length to match all distinctive locations of the signatures. By doing this, the immediate effect of the geometry on the signature can be compared. It shows that the effect of lift on the pressure signature is large, especially where the wing starts. The difference is the largest for the high wing configuration, since during zero-lift the wing generates a negligible amount of overpressure, so high overpressure of the high wing configuration during lift still persists. The differences in overpressure show an equal trend when compared to the overpressures during lift, they look very similar except for the nose region of the aircraft.

6.4.2. Maximum overpressures at different azimuth angles

The maximum overpressures when lift is generated are plotted in figure 6.18a and the maximum absolute overpressure for zero-lift is plotted in figure 6.18b. The difference is easily observable between both situations. For the lifting case the overpressures directly below the aircraft are over 2½ times higher compared to the non-lifting cases. Off-track however there are different trends visible. When lift is generated the overpressures decay for increasing azimuth angles, whereas they become larger with increasing azimuth angles when no lift is generated. Note that for the zero-lift overpressures the absolute value is taken, since the negative overpressures are larger than the positive overpressures.

According to Seebass & George the rear shock strength can be stronger than the front shock [31]. Later analysis in section 6.5.1 will try to give an answer to why this negative overpressure is so large.

When looking at figure 6.18a, where maximum overpressures for the lifting cases are plotted, it shows that the best performer with the lowest maximum positive overpressure is the low wing configuration. When the wing is placed higher relative to the fuselage, the maximum overpressures become higher. The maximum is achieved by the mid-high wing configuration with 341.5 Pa, which
makes the difference at 0° azimuth angle 48.0\textit{Pa} compared to the low wing configuration. Note that towards the side the high wing configuration its overpressure will be the lowest from 30° onwards.

(a) Maximum overpressures in lifting condition. (b) Maximum absolute overpressures in zero-lift.

Figure 6.18: Maximum overpressure and absolute overpressures at lift and zero-lift at 70m distance.

6.5. Impulses at different azimuth angles

The impulses have been measured for different azimuth angles, varying from 0° up to 50°. These are plotted in figure 6.19. The impulses for the zero-lift cases have not been calculated, since the pressure signature cross the $\Delta P = 0$ line multiple times, which makes a comparison fairly hard. As expected, the impulse below the aircrafts is the highest, since also the maximum overpressure is the highest for the azimuthal angle of 0°. The impulses rapidly decay when the azimuthal angle increases. The values for the impulses are close to eachother, so for convenience the results are listed in table 6.3 as well. It is interesting to see that the impulses for the low wing configuration and the high wing configuration at 0° are the lowest. When the azimuthal angle increases, the high-wing configuration will have the highest impulse from 30° onwards, while the impulses for the low wing configuration remain the lowest for all azimuthal angles. The impulse of the high wing configuration however will remain the lowest for all azimuthal angles.

Figure 6.19: Impulses for all configurations at various azimuth angles in lifting conditions.
Table 6.3: Impulses in lifting configuration for different azimuth angles.

<table>
<thead>
<tr>
<th>Wing configuration</th>
<th>( \theta = 0^\circ )</th>
<th>( \theta = 10^\circ )</th>
<th>( \theta = 20^\circ )</th>
<th>( \theta = 30^\circ )</th>
<th>( \theta = 40^\circ )</th>
<th>( \theta = 50^\circ )</th>
</tr>
</thead>
</table>

6.5.1. Analysis of wing-body interaction on the sonic-boom

It has been found that at the trailing edge section some overpressures are being generated which are responsible for very low overpressures and a sudden increase in overpressure moments later. A comparison is made between the low wing configuration and the mid wing configuration in figure 6.20, since for this difference in wing height of 1.1 m there is already a big difference in overpressure. For the mid wing configuration (each image on the left side of the symmetry line) the minimum overpressure extends further back compared to the low wing configuration (right of the symmetry line). The shock, coming from the wing leading-edge, after which an increase of overpressure is visible arrives further back for the mid wing configuration since that wing is placed higher. The effect is that for the low wing configuration the pressure will rise earlier on over the full length of the wing compared to the mid wing configuration. This explains the larger negative overpressure for the higher wing configurations directly below the aircraft.

![Overpressure comparison](image1.png)

(a) Overpressure comparison. Mid wing configuration (left) and low wing configuration (right)

![Overpressure comparison](image2.png)

(b) Overpressure comparison in a different colormap. Mid wing configuration (left) and low wing configuration (right)

Figure 6.20: A comparison between the low wing configuration and the mid wing configuration in zero-lift as seen from below in different colormaps.
6.5. Impulses at different azimuth angles

Reflections from shocks are visible as was already mentioned in subsection 6.2.1. This might be the reason that all pressure signatures cross each other in the region where the shocks from the wing trailing edge hit the pressure signature measurement line. In figure 6.20 for the low wing configurations just behind the trailing edge section there is a higher overpressure visible when the vertical wing placement is lower on the fuselage. This shock impinges on the fuselage and might be reflected further into the domain. When taking a close look at the same section in figure 6.9 around \( x = 45\, \text{m} \) on the lower side of the fuselage, there can be seen that for the low wing configuration a sharp overpressure develops just after the wing, whereas this is a wider, more spread out and less intense region of overpressure for the mid wing configuration. In figure 6.21 a side view is visible, 5m from the symmetry, which can be compared with figure 6.11 on page 56. It shows that the high overpressure region extends further back downward. This is because the shockwave from the wing trailing edge is reflected away from the fuselage, towards the ground.

The flat section in the overpressure in lifting conditions, mentioned in subsection 6.3.2, is traced back to the section just after the leading edge junction of the wing and fuselage. This can be seen in figure 6.22. The slices have been adjusted according to the illustration in figure 6.14 to match the shocks from the leading edge that propagate to the ground. Note that the overpressures from the leading edge of the high wing configuration are shielded by the fuselage, whereas they can travel more directly to the ground for the low wing configuration. This also causes higher overpressures for the high wing configuration off-track, since they are focused because of reflection to the fuselage.

![Figure 6.21: Side view at the plane \( y = 5\, \text{m} \) of the overpressure generated by the mid wing configuration.](image1)

![Figure 6.22: Comparison of shock development from the wing leading edge in lifting conditions. The slice for the high wing configuration (left) has been made at \( x = 22.26\, \text{m} \) to match the shocks for the low wing configuration (right) at \( x = 20\, \text{m} \).](image2)
The lower overpressure of the low wing configuration just before the trailing edge of the wing can be best explained by figures 6.23a for a slice at $x = 40m$ and 6.23b for a slice at $x = 45m$. The slices on the left side of the figures, the mid wing configuration, are shifted $1.37m$ further back to $x = 41.17m$ and $x = 46.17m$ respectively to match the shocks that are emanating from the wing leading edge for both configurations. The overpressures coming from the mid wing configuration are higher, since the relative decrease in volume below the wing is higher for the low wing configuration compared to the mid wing configuration.

Figure 6.23: Comparison of overpressures below the wing between the mid wing configuration (left) and low wing configuration (right) during lift.

(a) The slice on the left for the mid wing configuration is normalized to $x = 40m$ for the low wing configuration.

(b) The slice on the left for the mid wing configuration is normalized to $x = 45m$ for the low wing configuration.
This relative decrease in volume below the wing can be explained by using figure 6.24. On the left side a low wing configuration is pictured and on the right a configuration with a higher placed wing. The tail of the wing is placed lower when walking backward over the wing surface, as is illustrated by the arrow. On the right side the same is visible. When the area below the striped area is compared with the area in the fuselage below the striped area, one can see that there is a lot less white area visible for the low wing configuration. The relative decrease in fuselage volume is higher for the low wing configuration than for the higher wing configurations. For the higher wing configurations there is no possibility for the flow to flow towards the fuselage where this possibility exists for the low wing configuration. Therefore higher pressures are visible at the tail for the higher wing configurations.

6.6. Drag estimation by using different shapes for intersection

In the past, several researchers have tried to come up with solutions to calculate the drag based solely on the geometry and the freestream flow conditions. A good example is the supersonic area rule, where the geometry is cut by local planes inclined at the freestream Mach angle \( \mu \), which are called Mach-planes. For the 5 configurations of this thesis, the zero-lift drag has been calculated by using the supersonic area rule. The resulting drag comes close to the zero-lift values that have been calculated by the CFD-simulations. Other methods which use Mach cones to evaluate the cross-sectional area distribution, have also been tested. These results are compared with methods proposed by Jumper and Nikolic [16]. An explanation of a new method for evaluating the drag at zero lift for supersonic airplane configurations is discussed as well in subsection 6.6.3.

6.6.1. Mach-planes

First, the supersonic area rule is used to calculate the wave drag of the various configurations. The drag can be estimated with formula 2.15, which is recalled here for convenience:

\[
D_{wave} = -\frac{\rho_\infty U_\infty^2}{4\pi} \int_0^L \int_0^L S^{\text{cross}}(x_1)S^{\text{cross}}(x_2) \ln|x_1 - x_2| dx_1 dx_2
\]

(6.3)

This formula can be rewritten to the following formula, which makes it suitable for further processing by using Riemann sums in a computational code [45]:

\[
D_{wave} = -\frac{\rho_\infty V_\infty^2}{2\pi} \int_0^L S^{\text{cross}}(x) dx \int_0^x S^{\text{cross}}(x_1) \ln(x - x_1) dx_1
\]

(6.4)

For the value of \( S^{\text{cross}}(x) \), the cross-sectional area distribution of choice can be used. First, the cross-sectional area as dictated by R.T. Jones is used. This uses planes tilted in the Mach-angle \( \mu \) which cut the geometry along the longitudinal axis of the aircraft. This is done for multiple azimuthal angles, which requires the Mach-plane to be rotated around the longitudinal axis, or to rotate the aircraft about the longitudinal axis. For all other methods stated next, this rotation is not required, since an axisymmetrical shape will be used, which already has the shape of the flowfield in a supersonic stream.

Following this procedure, each cross-sectional area distribution, which is a function of \( x \), is described using an interpolation function which fits a cubic spline in Python over a number of linear spaced points, 1000 in this case. Then, the second derivative is calculated, which is in turn integrated by calculating
the Riemann sum using trapezoidal elements. This can be plugged into equation 6.4, which gives the drag of the configuration. For the supersonic area rule, the drag for the cross-sectional area distribution for each azimuthal angle (88 angles in this case) will be calculated and then the mean value will be calculated. For all other methods stated next, the drag will only be calculated once, since there is only 1 cross-sectional area distribution. The results for this method using Mach-planes can be found in table 6.4 under $D_{\text{wave,MP}}$ on page 69.

6.6.2. Mach cone

The different configurations have been cut by using a Mach-cone, just as Rallabhandi proposed in his article [15]. The vertex of the Mach cone lies on the longitudinal axis of the fuselage. By implementing the method described above, the drag was steadily overpredicted by a factor larger than 2, just as Nikolic and Jumper described in their article [16]. When these drag forces are multiplied by $\frac{\omega}{\Omega}$, the results in table 6.4 at $D_{\text{wave,MC}}$ are obtained.

(a) Isometric view. (b) Side view.

Figure 6.25: Overview of an airplane which is cut by some Mach-cones. The yellow cone represents a Mach-cone which vertex lies on the longitudinal axis, the red Mach-cones their vertices are aligned with the $z$-position of their centroid.
6.6.3. Height-weighted Mach-cone

Another method to evaluate the cross-sectional area distribution of a supersonic airplane, mentioned in section 2.13.2 has been used. The geometry is cut by Mach-cones, which are aligned with the free-stream at several longitudinal locations. For each cone at each longitudinal station, the Mach-cone is shifted vertically so that the vertex of the Mach-cone is aligned with the centroid of the cut area. This gives an \( xz \)-location of this vertex according to figure 6.26. These values, the frontal area of the Mach-cone \( S_{cone} \) and the \([x, z]\)-location, are then used in the following way to place all cross-sectional areas on a single axis:

\[
x_{\text{aligned}} = x - z(x) \cdot \beta
\]  

This places all cones on a single line (see figure 6.27 for illustration), which makes it suitable for further use in formula 6.4 for the drag estimation. The drag was again overestimated by a factor of 2 for the aircraft; however, when the results are multiplied by one-half, the results came close to the real zero-lift drag as can be seen in table 6.4 under \( D_{\text{wave,HWMC}} \). To see the difference in position and lateral surface of the Mach-cones between this method and the one stated before, figure 6.25a and 6.25a can be observed.

6.6.4. Double Mach-cone

Double cones, using a forward faced Mach-cone and a backward facing Mach-cone, are used since the original supersonic area rule incorporates Mach-planes for the determination of the cross-sectional area distribution at multiple azimuth angles, varying from \( 0^\circ \) to \( 360^\circ \). When an infinite amount of planes is used at these azimuthal angles, they will form a forward and a backward facing Mach-cone. When the average area of the frontal projections of these Mach-cones is used, there will be a new area distribution. This area distribution is shown in figure 6.28. The higher
curvature and extreme peaks of the area distributions using a single Mach-cone are damped. In table 6.4 the results are listed under $D_{\text{waveDC}}$. As can be seen from these results, the drag came very close to the value calculated in CFD. It is even more interesting that there is not a factor of $\frac{1}{\varepsilon}$ applied to get the results closer to these values. Only for the high-wing configuration the drag is underestimated, but for the rest of the results this method generates values that are close to the measurements.

### 6.6.5. Method of Jumper and Nikolic for drag estimation

The method as described by Jumper and Nikolic is used for the estimation of the wave drag for comparison. First a cross-sectional area distribution is obtained by using the plane normal to the $x$-axis. Since the configurations are area-ruled for Mach 1, a distribution for an axisymmetrical body, which is in fact a nearly perfect Sears-Haack body with a maximum radius of 2.5m will be obtained. Instead of intersecting this axisymmetrical body with Mach-cone as Jumper and Nikolic have used, one may as well calculate the radius of this body at each $x$-location and then shift the $x$-location by a method similar to the one mentioned in equation 6.5:

$$x_{\text{JN}} = x - \beta \sqrt{\frac{S(x)}{\pi}}$$

$$= x - \beta r(x)$$

The values for the drag for the method of Jumper and Nikolic have not been multiplied by $\frac{1}{\varepsilon}$, although it was suggested to multiply by this factor by Jumper and Nikolic for the method that they developed. The results are solely based on the outcome of plugging the area distribution according to Jumper and Nikolic into formula 6.4. There is even an underestimation of all values for the drag, as can be seen in table 6.4. This could be due to the fact that the area distribution is very close to that of a Sears-Haack body. The locations of the cross-sectional area do shift more forward when the cross-sectional area is larger and thus the radius of the body is larger, but still the distribution remains close to that of a Sears-Haack body. The area distribution according to Jumper and Nikolic is compared with earlier methods in figure 6.29 and it is visible that the area distributions which use a cone have more curvature, which indicates that the integral of a second derivative can vary more as well.

### 6.6.6. Results of drag estimation and further analysis

The results of the various methods for the drag estimation are shown in table 6.4. Note that for $D_{\text{waveMC}}$ and $D_{\text{wave,NWMC}}$ the results from formula 6.4 have been multiplied by $\frac{1}{\varepsilon}$ to achieve the final results presented in this table.
From the results listed in table 6.4 the errors with respect to the CFD simulations have been calculated. An overview of these relative errors is visualized in figure 6.30. The method by Jumper and Nikolic underestimates the drag by nearly 30% compared to the CFD simulations. The new height-weighted method using Mach-cones comes very close to the values calculated in CFD, while the method which incorporates only Mach-cones overestimates the drag by more than 10% for the low and mid-low wing configurations and underestimates the drag for the high wing configurations. The method which uses Mach-planes according to the supersonic area rule underestimates the drag by more than 5%, but this underestimation remains stable whereas the other methods have more variation in the results. The method which incorporates the double Mach-cones shows a little less variation compared to the height-weighted Mach-cones, while the double Mach-cone method does not use a factor of $1/2$ to come close to the CFD values. Furthermore, the convergence for the number of cuts that is used to get an area distribution has been calculated, seen in figure 6.31a, where more than 100 cuts do not give a significant improvement of the drag calculation. It is noteworthy that using 1000 or 2000 interpolation points for the numerical integration give the best results for drag estimation, as is pictured in figure 6.31b. This is done for the method which incorporates 100 longitudinal Mach-planes over 45 azimuth angles from $0^\circ$ to $180^\circ$, where the other angles (from $180^\circ$ to $360^\circ$) are mirrored to achieve $N = 45 + 43 = 88$ wave drag values which have been averaged.

![Figure 6.30: Relative error for the different methods compared to the calculations in CFD.](image)

![Figure 6.31: Convergence by varying several parameters for the calculation of the drag by using the area rule.](image)

Table 6.4: Results from various implementations of drag estimations at supersonic speeds.

<table>
<thead>
<tr>
<th>Method</th>
<th>Units</th>
<th>factor $1/2$ applied</th>
<th>Low</th>
<th>Mid-Low</th>
<th>Mid</th>
<th>Mid-High</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_D$wave,CFD</td>
<td>$10^{-3}$ [-]</td>
<td>No</td>
<td>2.8640</td>
<td>2.8400</td>
<td>2.8320</td>
<td>2.8300</td>
<td>2.8270</td>
</tr>
<tr>
<td>$C_D$wave,MP</td>
<td>$10^{-3}$ [-]</td>
<td>No</td>
<td>2.6768</td>
<td>2.6598</td>
<td>2.6435</td>
<td>2.6341</td>
<td>2.6326</td>
</tr>
<tr>
<td>$C_D$wave,Mc</td>
<td>$10^{-3}$ [-]</td>
<td>Yes</td>
<td>3.1763</td>
<td>3.1322</td>
<td>3.0183</td>
<td>2.8924</td>
<td>2.7605</td>
</tr>
<tr>
<td>$C_D$wave,HVMC</td>
<td>$10^{-3}$ [-]</td>
<td>Yes</td>
<td>2.8967</td>
<td>2.9427</td>
<td>2.9375</td>
<td>2.8831</td>
<td>2.8017</td>
</tr>
<tr>
<td>$C_D$wave,DC</td>
<td>$10^{-3}$ [-]</td>
<td>No</td>
<td>2.8859</td>
<td>2.8833</td>
<td>2.8428</td>
<td>2.7840</td>
<td>2.7058</td>
</tr>
<tr>
<td>$C_D$wave,JN</td>
<td>$10^{-3}$ [-]</td>
<td>No</td>
<td>2.0249</td>
<td>2.0253</td>
<td>2.0244</td>
<td>2.0179</td>
<td>1.9899</td>
</tr>
</tbody>
</table>
Still, it is strange that the results for a single Mach-cone with or without vertical translation need to be multiplied by a factor of $\frac{\omega}{a}$ to approach drag forces that have been calculated by CFD, while other methods do not use this multiplication.

### 6.6.7. Comparing methods with a Sears-Haack body

A method to find out where this multiplication factor is coming from is to use the same methods for drag estimation on a shape of a body of which the wave drag is well known: the Sears-Haack body. For all methods the drag has been calculated, based on a Sears-Haack body constructed in ParaPy by the same method and size as the shape of the aircraft. This gives an axisymmetrical body with a maximum radius at $x = 35m$ of $2.5m$ and a length of $L = 70m$. The drag can be calculated with formula 2.8, where the maximum radius is used, measured by the cross-sectional area distribution from the intersection methods used in ParaPy. This is the theoretical drag in table 6.5. So for each of the methods 100 intersections have been made where for each intersection the frontal area is used to calculate the drag. This gives the following results:

<table>
<thead>
<tr>
<th>Method</th>
<th>$C_D$ (Theoretical)</th>
<th>Error</th>
<th>$C_D$ (Theoretical)</th>
<th>Error</th>
<th>$C_D$ (Theoretical)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach-planes</td>
<td>0.05531</td>
<td>-1.77%</td>
<td>0.05631</td>
<td>0.00%</td>
<td>0.05631</td>
<td>19.51600</td>
</tr>
<tr>
<td>Mach-cone</td>
<td>0.05846</td>
<td>3.19%</td>
<td>0.05974</td>
<td>5.46%</td>
<td>0.05665</td>
<td>19.63500</td>
</tr>
<tr>
<td>HW Mach-cone</td>
<td>0.05845</td>
<td>3.18%</td>
<td>0.05974</td>
<td>5.45%</td>
<td>0.05665</td>
<td>19.63500</td>
</tr>
<tr>
<td>Double Mach-cones</td>
<td>0.05528</td>
<td>-1.25%</td>
<td>0.05637</td>
<td>0.71%</td>
<td>0.05597</td>
<td>19.40080</td>
</tr>
<tr>
<td>Jumper</td>
<td>0.05867</td>
<td>3.58%</td>
<td>0.05999</td>
<td>5.91%</td>
<td>0.05664</td>
<td>19.63210</td>
</tr>
</tbody>
</table>

From these results, where no multiplication by a random factor of $\frac{1}{2}$ has taken place to approach the theoretical drag value, there can be concluded that the methods work since they approach the theoretical drag quite well. Still the question remains why the factor of $\frac{1}{2}$ is required when the drag is calculated for an aircraft configuration. Therefore a comparison is given in figure 6.32 for the area distribution according to different methods for the low wing configuration on page 71. In the graph in the middle, which displays the first derivative, it is clear that the method of Jumper & Nikolic has a completely different trend in the region where the effect of the wing in the distribution is visible from $\frac{r}{h} = 0.25$ to $\frac{r}{h} = 0.7$. The other 3 methods show an equal trend in derivative, and also in second derivative. The second derivative of the Jumper & Nikolic method remains nearly constant, where the other methods have sharp peaks of which the double Mach-cone method has lower peaks. These sharp peaks do influence the outcome of the integral, but another section has more influence. A negative value must come out of the integral for formula 6.4 to calculate the wave drag. When a look is taken on the second derivative, which is in fact the integral of the second derivative, it looks like the negative portion for the Mach-cone and height-weighted Mach-cone method have a large area below the $x$-axis from $\frac{r}{h} = 0.25$ to $\frac{r}{h} = 0.4$. This portion is mostly responsible for the overprediction of the drag. The double Mach-cone method however “flattens” these peaks of the integral of the second derivative, since the average area of 2 cones is used. Also note that the both methods which use a single Mach-cone for the area distribution have a large peak in the cross-sectional area, since they incorporate the cross-sectional area of the full wing around $\frac{r}{h} = 0.25$, which causes a large spike in the cross-sectional area distribution. Some smoothening is required to get an area distribution for the drag estimation which does not require a factor of $\frac{1}{2}$ to calculate the drag. By using 2 cones, one forward an 1 backward cone for the intersection of the geometry the drag can be calculated quite good. Further research on exotic aircraft configurations is required to find out if the drag can be simply estimated by using single cones for intersection and by multiplying the drag by $\frac{1}{2}$ or that the double Mach-cone method is better because it simply is the supersonic area rule for $N = \infty$ azimuthal angles for Mach-planes.
Figure 6.32: Comparison of $S(x)$, $S'(x)$ and $S''(x)$ for the low-wing configuration for multiple methods for determining $S(x)$. 
6.6.8. Relating the pressure signature to the cross-sectional area distribution

The different area distributions have been used to try and relate the pressure signature to the geometries of the configurations. The results can be observed in figures 6.33a to 6.33d. The zero-lift pressure signatures have been used to compare the different calculated signatures, and these calculated signatures are stretched to try to match some peaks and dips to the extracted pressure signature. For all signatures it is clear that the calculations and the measured signature do not match. However, some trends in the calculated signatures match the trends of the measured signatures, especially when the solution of the Mach-plane in the corresponding azimuthal angle of the signature is matched. This should not come as a surprise, since Whitham his theory uses these planes as well.

Figure 6.33: Calculated pressure signatures using the Whitham F-function. Different cross-sectional areas have been used. The calculated signatures are stretched to match the length of the zero-lift pressure signature.

It has not been researched if a forward Mach-cone from the perspective of the observers point of view gives better results. The radius of this cone which cuts the geometry of the aircraft is very large, the intersection will be nearly a plane, just as has been used by cutting the geometry by using Mach-planes. Therefore it is not worth researching.
7

Conclusion & Discussion

In this chapter an answer to the research questions stated in chapter 1 will be given after which conclusions will be drawn. In the end a discussion is given, after which some recommendations are presented for future work.

7.1. Answers to the research questions
The research questions of this research can be answered using the results of this thesis. First, the research questions will be recalled:

1. What is the impact on the wave drag and sonic-boom performance of a low wing and a high wing configuration supersonic airplane at supersonic speeds?

2. Is it possible to predict the wave drag and sonic boom characteristics by using a new method of cross-sectional area evaluation by using a Mach-cone

To answer the first question, it seems that a low-wing configuration in this case has the lowest wave drag and the high-wing configuration the highest amount of wave drag. In zero-lift condition there is nearly no difference between the configurations since the largest amount of drag is coming from the lift that is generated. The low wing configuration has the best lift-to-drag ratio since there is favourable interference between the fuselage and the upper surface of the wing near the fuselage. In this region extra suction is being generated since the supersonic flow expands because of the local geometry, which causes the flow to accelerate locally. This causes a lower pressure on the wing upper surface. The other wing configurations do not have this extra suction, because the local area is not shielded in the same way by the fuselage, so no expansion will take place in this region. The low wing configuration obtained a 4.79% better lift-to-drag ratio compared to the worst performing configuration, the high wing configuration.

When the pressure signatures at a body-length distance from the aircraft are observed, there has been found that the low-wing configuration generates the lowest maximum overpressures. The difference at 0° azimuth angle between the low wing configuration and the mid-high wing configuration, which is the worst performer in maximum overpressure, is 48Pa which is significant. The overpressures are mainly dependent on the lift that is generated, so the low-wing configuration shows the lowest overpressures here as well. Also, there is some interference on the lower side of the wing. Because the wing is placed lower for the low wing configuration, the shockwave from the leading edge wing-fuselage junction will reach the bottom of the fuselage earlier. This causes the pressure on the lower side of the fuselage to rise earlier on, which causes more pressure on the lower side of the fuselage. At the aft section around the symmetry plane on the lower surface of the wing, there are higher pressures visible for the higher wing configurations. This is due to the fact that there is less relative decrease in cross-sectional area of the fuselage below the wing surface, when walking along a streamline on the lower wing surface. The relative decrease in fuselage cross-sectional area below the wing is higher for a low-wing configuration. This makes that at the trailing end of the wing on the
lower surface near the fuselage, the flow cannot spread out easily and a higher pressure will develop.

The impulses have been measured for the lifting conditions. Below the aircraft the highest impulse is measured and the impulse strength decays rapidly with increasing azimuth angle. The impulses for the high wing configuration are becoming stronger off-track due to reflection of the shock from the leading edge of the wing to the fuselage. These shocks are a little more focused to the side. The signatures show some wing-fuselage interference at the trailing edge of the wing as well. For the low wing configuration, the recompression shock after the wing hits the fuselage and this shock is reflected directly to the ground. The same shock reflects to the fuselage for the higher wing configurations, but then the shock propagates further to the side. These effects can be used to cause stronger impulses to the side of the flying path or to make an effort to cancel out the overpressures below the aircraft by using this interference. Also shifting the wing further backward can be a possibility to restrict the strength of the trailing shockwave.

To answer the second research question, different methods for intersecting the geometry have been used. Intersecting the geometry by using a Mach-cone has originally been proposed by Rallabhandi. Making an axisymmetrical body of the geometry and then intersecting this body by using a Mach-cone is a method that has been proposed by Nikolic and Jumper. A method of intersecting the aircraft models by using a Mach-cone that translates in the $z$-direction to match the centroid of the cross-sectional area and the vertex of the Mach-cone is proposed for drag estimation in this thesis. By using the $z$-coordinate and the Prandtl-Glauert parameter $\beta$ all cross-sectional areas can be placed on a single axis, which makes the cross-sectional area distribution suitable for calculating the drag of the geometry. The methods which used a single cone for the cross-section overestimated the drag by over a factor of 2, except for the method of Jumper and Nikolic. By multiplying these overestimated results by a factor of $\frac{1}{2}$ the drag was estimated within a margin of 5%, even better than the traditional method by R.T. Jones and H.M. Lomax [25, 46]. Multiplying the results by a factor of $\frac{1}{2}$ is happening as well in the article by Nikolic and Jumper, however, when the method of Nikolic and Jumper is applied to the geometry of this research, the drag is underestimated, which is strange. To find out where this multiplication factor comes from, a study has been conducted by applying these methods to a Sears-Haack body. This gave results on the drag that came within 5% of the theoretical value of a Sears-Haack body, while no multiplication factor of $\frac{1}{2}$ was applied. This gives the idea that the multiplication factor appears out of the blue. When further analyzing the area distributions for the different configurations it is found that the cross-sectional area distributions of the methods that use a Mach-cone have high and sharp peaks in them. It might be required to smooth these distributions in a certain way to assess the zero-lift drag.

When going back to the original supersonic area rule and the intersection method that is used in that theory, one can find that when the number of azimuthal angles goes to infinity, the shape of a fore Mach-cone and a streamwise aligned Mach-cone appear. This gave the idea of using two cones, which both have their vertex at the same point, but one cone is pointed forward while the other is pointed backwards. By intersecting the geometry with this method and by using the average of both frontal projection of the cross-sectional areas of these cones, results on drag show that this method even better approaches the zero-lift drag for some geometries, while the results did not require a multiplication factor of $\frac{1}{2}$. The area distributions are smoothed by itself, since the forward cone adds less area in the fore part of the aircraft and adds area to the aft part of the aircraft, giving a smoother area distribution.

In the end, using a single or double cone to assess the cross-sectional area distribution greatly simplifies the number of times a cross-sectional area distribution needs to be made to assess the inviscid zero-lift drag. Instead of performing the calculations for 50, 100, 1000 or even more azimuthal angles, only 1 distribution needs to be calculated and processed. Further research is required to check if this method is valid only for the 5 configurations of this thesis or also for more exotic configurations with a tail, engines and a different wing that is shifted further back or forth.

### 7.2. Discussion & recommendations

The research presented here has some points open for discussion, which in turn, which are listed below.

- The mesh that has been used in this research could be better, however, this was not possible
with the software that has been used. ParaPy is a great software package in which aircraft configurations can be easily modelled. A configuration can be changed very simply by varying a single parameter since the geometry is described parametrically. So for configuration creation while also incorporating iterative processes, such as area-ruling a configuration is simple. The drawback is that the open-source software used for meshing, SALOME, is not optimized for complex geometry and when large meshes have to be made, the algorithm does not always work properly. The meshing algorithm does not always work properly when elements become small and when surfaces become too large, the algorithm does not mesh the full surface. Therefore it is sometimes a trial-and-error process to find out whether a mesh can be made, especially when after a couple of hours meshing there is no more memory available in the operating system and the meshing fails. In the end a switch is made to a computing node with 64 GB RAM to mesh the configurations. This amount of memory was required since the domain not only needs to be large for sonic boom modelling, but the mesh also needs to be fine. Further research can definitely use ParaPy for geometry creation and meshing to assess the lifting and drag of a configuration, but when the sonic-boom needs to be modeled as well, one might better switch to specific software for meshing, such as Pointwise, which also includes the possibility to easily refine meshes at some places.

- An effort is made to apply an unstructured mesh in the near- and mid-field and a structured mesh in the far-field, however, the SALOME algorithm was not able to connect some points in the mesh. It is recommended to use other software packages, such as Pointwise or GMSH to mesh the domain to overcome this. This might also enable better possibilities in refining the mesh locally.

- The mesh for sonic-boom modelling can be optimized by enabling mesh adaptation in SU2. The mesh used in this research could not be manually refined on the locations where shocks exist. Researches in the past have used mesh adaptation and the shocks happen to be very sharp and overpressures increase as well for the signatures [43]. Better and sharper signatures also open possibilities to extract these signatures and to plug them into a code which calculates the ground signature. Then the direct effect of shifting a wing vertically can be measured and adjustments could be made to the design.

- A very thin wing has been used (\(t/c = 0.02\)) which is not a wing that could be used on a real aircraft. A thicker wing near the root chord or a different wing shape with a longer root chord will reduce bending moments and will give a better case for a viable supersonic aircraft.

- A further research could also investigate the effect of shaping the fuselage by placing the wing more aft to make an effort to cancel the negative overpressure of the signature. By doing this, the trailing shockwave can be reduced, which also helps in reducing the noise created by a sonic boom, especially the tail boom emanating from the aircraft.

- An effort was made to couple the airplane surface pressure information from Tecplot to ParaPy to be able to calculate the wave drag due to lift. Unfortunately it is a tedious process and more coding is necessary to enable this option. Tecplot also has a library which works with Python, so it is possible to couple ParaPy to this process. CFD simulations are still required to calculate the pressure information, which makes the coupling of all these computer applications obsolete, unless a relationship between the geometry and wave drag due to lift can be found.

- The relationship between aircraft geometry and drag mentioned in the last point and throughout this thesis is interesting and could be extended. Assume an aircraft, for example one that has been analysed in this thesis, and let it fly in a supersonic flow. In the frontal part there is compression in the region where the cross-sectional area distribution is increasing. This cross-sectional area distribution forms the basis of the Mach-contour on the surface of the airplane by an equation that might be there or needs to be invented. Because of this compression in the nose region, the flow decelerates and the Mach-numbers on the surface will be lowered. It might be possible to make a ray-tracing code that uses local characteristics as rays and propagates these characteristics. These characteristics coalesce and refract and might hit the surface of the aircraft again. If it is possible in some way to generate a flow solution on the surface or below
the aircraft, then it might be possible to predict sonic booms or the drag of an airplane. If this works, then it might even be possible to trace back some of these characteristics to shape an aircraft according to the desired pressure signature. Note that this is a highly conceptual idea, but it might be an idea worth researching.

During this thesis an aircraft design variable has been researched which has not been researched in the past for supersonic configurations. It is found that a low wing configuration can have favourable interference effects for lift and also for sonic boom, while the opposite was expected during the literature research. These interference effects can be used for future aircraft to increase lift or to decrease the sonic boom by shaping the aircraft in a specific way. However, aircraft shaping is a very tedious art and a large amount of parameters influence the design and the performance. More research is required to find out what the eventual effects are for the configurations of this thesis when a viscous flow is used or if the reduction in sonic boom is also measurable when the signatures are propagated to the ground. In the recent years there is more interest in supersonic flight and research has enabled new configurations and design tools for less annoying sonic-booms. These developments might open a new era for commercial supersonic flight in the coming years, but more research is required in this field.

Hendrik Wisse Kinderman
Bibliography


% SU2 configuration file
% Case description: Supersonic flow over a wedge in a channel (regression)
% Author: Thomas D. Economon
% Institution: Stanford University
% Date: 2012.10.07
% File Version 5.0.0 “Raven”

% DIRECT, ADJOINT, AND LINEARIZED PROBLEM DEFINITION

% Physical governing equations (EULER, NAVIER_STOKES, WAVE_EQUATION, HEAT_EQUATION, FEM_ELASTICITY, POISSON_EQUATION)
PHYSICAL_PROBLEM= EULER

% Mathematical problem (DIRECT, CONTINUOUS_ADJOINT)
MATH_PROBLEM= DIRECT

% Restart solution (NO, YES)
RESTART_SOL= NO

% COMPRESSIBLE AND INCOMPRESSIBLE FREE–STREAM DEFINITION

% Mach number (non–dimensional, based on the free–stream values)
MACH_NUMBER= 1.6

% Angle of attack (degrees)
AOA= 0.0
% Side-slip angle (degrees)
SIDESLIP_ANGLE = 0.0
%
% Free-stream pressure (101325.0 N/m^2 by default, only Euler flows)
FREESTREAM_PRESSURE = 7504.84
%
% Free-stream temperature (288.15 K by default)
FREESTREAM_TEMPERATURE = 216.650

%REFERENCE VALUE DEFINITION

% Reference origin for moment computation (Not used)
REF_ORIGIN_MOMENT_X = 0.25
REF_ORIGIN_MOMENT_Y = 0.00
REF_ORIGIN_MOMENT_Z = 0.00
%
% Reference length for pitching, rolling, and yawing non-dimensional moment
REF_LENGTH_MOMENT = 1.0
%
% Reference area for force coefficients (Half the total wing reference area is used)
REF_AREA = 291.83
%
% Reference element length for computing the slope limiter epsilon
REF_ELEM_LENGTH = 1.0
%
% Flow non-dimensionalization (DIMENSIONAL, FREESTREAM_PRESS_EQ_ONE, FREESTREAM_VEL_EQ_MACH, FREESTREAM_VEL_EQ_ONE)
REF_DIMENSIONALIZATION = DIMENSIONAL

%BOUNDARY CONDITION DEFINITION

% Inlet boundary type (TOTAL_CONDITIONS, MASS_FLOW)
INLET_TYPE = TOTAL_CONDITIONS
%
% Euler wall boundary marker(s) (NONE = no marker)
MARKER_EULER = (AIRPLANE)
%
% Far-field boundary marker(s) (NONE = no marker)
MARKER_FAR = (FARFIELD)
%
% Symmetry boundary marker(s) (NONE = no marker)
MARKER_SYM = (SYMMETRY)
%
% Outlet boundary marker(s) (NONE = no marker)
% Format: (outlet marker, back pressure (static), ...)
MARKER_OUTLET = (NONE)
%
% Marker(s) of the surface to be plotted or designed
MARKER_PLOTTING = (AIRPLANE)
%
% Marker(s) of the surface where the functional (Cd, Cl, etc.) will be evaluated
MARKER_MONITORING = (AIRPLANE)
%
%COMMON PARAMETERS DEFINING THE NUMERICAL METHOD

% Numerical method for spatial gradients (GREEN_GAUSS, LEAST_SQUARES, WEIGHTED_LEAST_SQUARES)
NUM_METHOD_GRAD = WEIGHTED_LEAST_SQUARES
%
% Courant–Friedrichs–Lewy condition of the finest grid
CFL_NUMBER = 2.0
%
% Adaptive CFL number (NO, YES)
CFL_ADAPT = NO
%
% Parameters of the adaptive CFL number (factor down, factor up, CFL min value, CFL max value)
CFL_ADAPT_PARAM = (1.5, 0.5, 1.0, 100.0)
%
% Runge–Kutta alpha coefficients
RK_ALPHA_COEFF = (0.66667, 0.66667, 1.000000)
%
% Number of total iterations
EXT_ITER = 18000
%
% Linear solver for the implicit formulation (BCGSTAB, FGMRES)
LINEAR_SOLVER = BCGSTAB
%
% Min error of the linear solver for the implicit formulation
LINEAR_SOLVER_ERROR = 1E−6
%
% Max number of iterations of the linear solver for the implicit formulation
LINEAR_SOLVER_ITER = 5
%
% --- FLOW NUMERICAL METHOD DEFINITION ---
%
% Convective numerical method (JST, LAX–FRIEDRICH, CUSP, ROE, AUSM, HLLC, TURKEL_PREC, M5W)
CONV_NUM_METHOD_FLOW = AUSM
%
% Spatial numerical order integration (1ST_ORDER, 2ND_ORDER, 2ND_ORDER_LIMITER)
SPATIAL_ORDER_FLOW = 2ND_ORDER
%
% Slope limiter (VENKATAKRISHNAN, MINMOD)
SLOPE_LIMITER_FLOW = VENKATAKRISHNAN
%
% Coefficient for the limiter (smooth regions)
LIMITER_COEFF = 0.3
%
% 1st, 2nd and 4th order artificial dissipation coefficients
AD_COEFF_FLOW = (0.15, 0.5, 0.02)
%
% Time discretization (RUNGE–KUTTA_EXPLICIT, EULER_IMPLICIT, EULER_EXPLICIT)
TIME_DISCRE_FLOW = EULER_IMPLICIT
%
% --- CONVERGENCE PARAMETERS ---
%
% Convergence criteria (CAUCHY, RESIDUAL)
CONV_CRITERIA = RESIDUAL
%
% Residual reduction (order of magnitude with respect to the initial value)
RESIDUAL_REDUCTION= 11
% % Min value of the residual (log10 of the residual)
RESIDUAL_MINVAL= −12
% % Start convergence criteria at iteration number
STARTCONV_ITER= 10
% % ——————————————————— INPUT/OUTPUT INFORMATION ———————————————————-%
%
% Mesh input file
MESH_FILENAME= mesh.su2
%
% Mesh input file format (SU2, CGNS, NETCDF_ASCII)
MESH_FORMAT= SU2
%
% Mesh output file
MESH_OUT_FILENAME= mesh.su2
%
% Restart flow input file
SOLUTION_FLOW_FILENAME= solution_flow.dat
%
% Restart adjoint input file
SOLUTION_ADJ_FILENAME= solution_adj.dat
%
% Output file format (PARAVIEW, TECPLOT)
OUTPUT_FORMAT= TECPLOT
%
% Output file convergence history (w/o extension)
CONV_FILENAME= history
%
% Output file restart flow
RESTART_FLOW_FILENAME= solution_flow.dat
%
% Output file restart adjoint
RESTART_ADJ_FILENAME= restart_adj.dat
%
% Output file flow (w/o extension) variables
VOLUME_FLOW_FILENAME= flow
%
% Output file adjoint (w/o extension) variables
VOLUME_ADJ_FILENAME= adjoint
%
% Output objective function gradient (using continuous adjoint)
GRAD_OBJFUNC_FILENAME= of_grad.dat
%
% Output file surface flow coefficient (w/o extension)
SURFACE_FLOW_FILENAME= surface_flow
%
% Output file surface adjoint coefficient (w/o extension)
SURFACE_ADJ_FILENAME= surface_adjoint
%
% Writing solution file frequency
WRT_SOL_FREQ= 100
%
% Writing convergence history frequency
WRT_CON_FREQ = 1
Overview of validation results

Table B.1: Validation results for varying mesh sizes in the volume domain.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$l_{NF}$</th>
<th>$l_{MF}$</th>
<th>$l_{FF}$</th>
<th>$C_L$</th>
<th>$C_D$</th>
<th>$l/F$</th>
<th>$\Delta_{1/0-1/\text{avg}}$</th>
<th>$I$</th>
<th>$\Delta I-I_{\text{avg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
<td>$%$</td>
<td>$P_a\cdot s$</td>
<td>$%$</td>
</tr>
<tr>
<td>Run049_1</td>
<td>0.3</td>
<td>0.6</td>
<td>0.8</td>
<td>0.149858</td>
<td>0.011227</td>
<td>13.297</td>
<td>0.008</td>
<td>16.2593</td>
<td>0.100</td>
</tr>
<tr>
<td>Run049_2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>0.149849</td>
<td>0.011228</td>
<td>13.284</td>
<td>-0.087</td>
<td>16.2567</td>
<td>0.084</td>
</tr>
<tr>
<td>Run049_3</td>
<td>0.5</td>
<td>0.6</td>
<td>0.8</td>
<td>0.149853</td>
<td>0.011271</td>
<td>13.295</td>
<td>-0.004</td>
<td>16.2598</td>
<td>0.103</td>
</tr>
<tr>
<td>Run049_7</td>
<td>0.6</td>
<td>0.6</td>
<td>0.8</td>
<td>0.149823</td>
<td>0.011265</td>
<td>13.300</td>
<td>0.029</td>
<td>16.2542</td>
<td>0.069</td>
</tr>
<tr>
<td>Run049_8</td>
<td>0.3</td>
<td>0.4</td>
<td>0.8</td>
<td>0.149836</td>
<td>0.011265</td>
<td>13.301</td>
<td>0.038</td>
<td>16.2761</td>
<td>0.203</td>
</tr>
<tr>
<td>Run049_9</td>
<td>0.3</td>
<td>0.8</td>
<td>0.8</td>
<td>0.149852</td>
<td>0.011227</td>
<td>13.297</td>
<td>0.004</td>
<td>16.2555</td>
<td>-0.108</td>
</tr>
<tr>
<td>Run049_10</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
<td>0.149854</td>
<td>0.01127</td>
<td>13.297</td>
<td>0.005</td>
<td>16.2905</td>
<td>0.291</td>
</tr>
<tr>
<td>Run049_11</td>
<td>0.3</td>
<td>0.6</td>
<td>1.2</td>
<td>0.149857</td>
<td>0.01127</td>
<td>13.297</td>
<td>0.007</td>
<td>16.1220</td>
<td>-0.751</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>13.296</td>
<td></td>
<td></td>
<td>16.2430</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table B.2: Effect of the domain size on the lift and drag coefficients.

<table>
<thead>
<tr>
<th>Simulation name</th>
<th>$R_{FF}$</th>
<th>$R_{MF}$</th>
<th>$l_{NF}$</th>
<th>$l_{MF}$</th>
<th>$l_{FF}$</th>
<th>$C_L$</th>
<th>$C_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
<td>$%$</td>
</tr>
<tr>
<td>Run049_4</td>
<td>90</td>
<td>41</td>
<td>0.4</td>
<td>0.5</td>
<td>0.7</td>
<td>0.149833</td>
<td>0.011269</td>
</tr>
<tr>
<td>Run049_5</td>
<td>100</td>
<td>41</td>
<td>0.4</td>
<td>0.5</td>
<td>0.7</td>
<td>0.149839</td>
<td>0.011266</td>
</tr>
<tr>
<td>Run049_6</td>
<td>110</td>
<td>41</td>
<td>0.4</td>
<td>0.5</td>
<td>0.7</td>
<td>0.149864</td>
<td>0.011272</td>
</tr>
</tbody>
</table>
Table B.3: Validation results including nose refinement and wing refinements.

| Simulation name | Parameters to be checked | $\alpha_1$ | $h_w$ | $\alpha$ | $l_{\text{airplane, max}}$ | $l_{\text{fus, min}}$ | $g_{\text{fus}}$ | $l_{w, \text{min}}$ | $g_{\text{w}}$ | $l_{\text{NF}}$ | $l_{\text{FF}}$ | $R_{\text{NF}}$ | $R_{\text{max}}$ | Number of nodes | $C_L$ | $C_D$ |
|-----------------|------------------------|---------|-------|--------|------------------|-------------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|--------|--------|
| Run018          | -                      | 3.90    | 1.80  | 0      | 0.3              | 0.05              | 0.10           | 0.05            | 0.05           | 0.5            | 0.7            | 20             | 40                | 1440302 | 0.149772 | 0.0114986 |
| Run019          | Refinement on nose/tail| 3.90    | 1.80  | 0      | 0.3              | 0.01              | 0.10           | 0.05            | 0.05           | 0.5            | 0.7            | 20             | 40                | 1679723 | 0.14972  | 0.0113007 |
| Run031          | Element size wing      | 3.90    | 1.80  | 0      | 0.2              | 0.01              | 0.10           | 0.02            | 0.1            | 0.4            | 0.6            | 10.5           | 75                | 1187436 | 0.149242 | 0.0110407 |
| Run035          | Element size wing      | 3.90    | 1.80  | 0      | 0.2              | 0.01              | 0.10           | 0.05            | 0.1            | 0.4            | 1              | 10.5           | 50                | 1777567 | 0.149442 | 0.0109995 |
| Run038          | Element size wing      | 3.90    | 1.80  | 0      | 0.2              | 0.01              | 0.10           | 0.10            | 0.1            | 0.4            | 1              | 10.5           | 50                | 922571  | 0.149261 | 0.0109241 |
| Run039          | Element size wing      | 3.90    | 1.80  | 0      | 0.2              | 0.01              | 0.10           | 0.15            | 0.1            | 0.4            | 1              | 10.5           | 50                | 790964  | 0.149033 | 0.0115949 |
On the next page an overview is given of the final CFD simulations of which the results are gathered. The first 2 simulations for each configuration have a smaller domain size, since only the wing incidence angle and the lift that is produced were of interest. Based on these mesh files also the optimization for the angle of attack $\alpha$ for zero-lift is based. The number of required iterations is so high because of 2 reasons.

1. The angle of attack needs to be optimized, this needs a couple of iterations. A maximum of 12 iterations was used.

2. A CFL-number of 1 was required for the optimization. If this value was changed to another value, the simulation would converge, but the optimization process would not take place.
Table C.1: Overview of the final simulations in CFD.

<table>
<thead>
<tr>
<th>Name</th>
<th>Details</th>
<th>$h_w$</th>
<th>$\alpha_i$</th>
<th>$\alpha$</th>
<th>Nodes</th>
<th>Tetrahedra</th>
<th>Iterations</th>
<th>$C_L$</th>
<th>$C_D$</th>
<th>$L/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td></td>
<td>$m$</td>
<td>°</td>
<td>°</td>
<td>#</td>
<td>#</td>
<td>#</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low wing config</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Run090</td>
<td>Check the wing incidence angle</td>
<td>-0.3</td>
<td>3.55</td>
<td>0</td>
<td>1980041</td>
<td>11619873</td>
<td>7816</td>
<td></td>
<td></td>
<td>13.9175</td>
</tr>
<tr>
<td>Run090a</td>
<td>Adjust $\alpha$ for $C_L = 0$</td>
<td>-0.3</td>
<td>3.55</td>
<td>-3.2560</td>
<td>1980041</td>
<td>11619873</td>
<td>39999</td>
<td>0.150032</td>
<td>0.002871</td>
<td>0.011146</td>
</tr>
<tr>
<td>Run091</td>
<td>Final run for results</td>
<td>-0.3</td>
<td>3.55</td>
<td>0</td>
<td>6513235</td>
<td>38073754</td>
<td>5239</td>
<td>0.150089</td>
<td>0.0107841</td>
<td>13.9176</td>
</tr>
<tr>
<td>Run092</td>
<td>Final run for zero-lift results</td>
<td>-0.3</td>
<td>3.55</td>
<td>-3.2560</td>
<td>6846571</td>
<td>40550045</td>
<td>5452</td>
<td>0.150017</td>
<td>0.010779</td>
<td>13.9176</td>
</tr>
<tr>
<td>Mid-low wing config</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Run072</td>
<td>Check the wing incidence angle</td>
<td>0.2</td>
<td>3.74</td>
<td>0</td>
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<td>14201462</td>
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