DISCUSSION


In this paper, Fokkema and Ziolkowski consider precritical and postcritical incidence and resulting implications for the reflection response of the earth. The ideas are interesting and thought provoking, but their inferences with regard to the whiteness assumption in standard deconvolution are misleading. We will endeavor to show why.

Practical deconvolution and the whiteness assumption

In standard deconvolution, it is assumed that the normal incidence seismic trace values \( x_i \) in a gate \( T_1 \leq t \leq T_2 \) (typically of 1.5 - 2 s duration) may be represented by

\[
x_i = w_i * r_i,
\]

where \( w_i \) is the seismic wavelet, \( r_i \) is normally taken to be the sequence of primary or primary plus very short period reinforcing multiples, and * denotes convolution. Such a gate length ensures an adequate ratio of data length to wavelet length for estimating the autocorrelation function, but should not be too prone to time-varying effects such as absorption. Let us write the random process generating the time series \( x_i \) from which our sample is drawn as \( X_i \), and similarly the process generating \( r_i \), as \( R_i \). The autocorrelation of \( X_i \), denoted \( c_X \), is the convolution of the autocorrelation of \( w_i \), denoted \( c_w \), with the autocorrelation of \( R_i \), denoted \( c_r \):

\[
c_X = c_w * c_r .
\]

Another way of interpreting this equation is to regard \( X_i \) and \( R_i \) as infinitely long sequences. If \( R_i \) is uncorrelated (spectrally white), its autocorrelation is a unit spike at lag zero, and then the autocorrelation of \( X_i \) is the same as that of the wavelet \( w_i \) (e.g., Robinson 1980). This is the standard assumption in the theoretical view of predictive deconvolution. However, the gate \( (T_1, T_2) \) used in practical processing is not infinitely long, and therefore we must rethink the meaning of equation (1). Since \( w_i \) is a transient wavelet, its autocorrelation is clear-cut. When we calculate the autocorrelation estimate \( \hat{c}_X \) from the gate \( (T_1, T_2) \) we are bringing in only a limited length of \( r_i \), and we can rewrite equation (1) as

\[
\hat{c}_X = c_w * c_r .
\]

We would like the estimated autocorrelation \( \hat{c}_X \), computed from the given length of \( r_i \), values to be as near to a unit spike at lag zero as possible for \( \hat{c}_X \) to be close to \( c_X \). What this means is that the limited length segment of \( r_i \) should be statistically indistinguishable from a segment of the same length extracted from an infinite length stationary uncorrelated sequence. This is the practical whiteness assumption of deconvolution. Fokkema and Ziolkowski say "We believe there is no theoretical justification for this [whiteness] assumption". Whether the whiteness assumption is true or not does not depend on a theoretical justification, but rather on the actual properties of seismic reflectivities (at a particular location and time interval) and ultimately on the geology of the subsurface.

Walden and Hosken (1985) tested well log primary reflectivities for whiteness using, among other things, the statistical technique of cumulative periodograms. They found the spectra of primary reflectivities from a set of wells scattered round the world to be deficient in low frequencies; i.e., the spectra were colored blue. It is certainly conceivable that at a certain location and time interval, reflectivities could be tested and found to satisfy the whiteness assumption; indeed, in some parts of the North Sea we have found sequences which only just fail the whiteness test. In other areas, such as the Middle East, the failure of the whiteness test is usually drastic.

Even if a limited length sequence is thus declared white, it definitely will not have an autocorrelation function which is simply a unit spike at lag zero, since it represents only a segment from an infinite length sequence with such an autocorrelation. Obviously, this causes distortions in wavelet estimation from seismic data (White and O'Brien, 1974). Nevertheless, on average, such white sequences will have autocorrelation functions closer to a unit spike than any other type of stationary sequence of the same length.

In the event that the sequence is declared colored, all is not lost, since the correction discussed in Walden and Nunn (1988) can be implemented. In the absence of well data, reflectivity characteristics obtained from wells in the same region can often show whether a nonwhiteness correction is needed (Walden and Nunn, 1988, p. 296).

The earth's reflection response

As pointed out by Fokkema and Ziolkowski, the "all-pass theorem states that the normal-incidence reflection response of a plane-layered earth to an impulsive plane wave is white, provided there is a perfect reflector at the bottom of the stack of layers . . . therefore the reflection is all-pass, or white." This is perfectly true, but unlike Fokkema and Ziolkowski claim, this has nothing to do with the whiteness assumption in deconvolution (discussed above). For one thing, where is the concept of the presence or absence of a perfect reflector in statistical deconvolution? There isn't one; the only thing that matters is the autocorrelation or spectrum of the finite-length sequence of \( r \), in the gate \( (T_1, T_2) \). Secondly, the reflection response being white in the all-pass theorem refers to the spectrum of reflections including all orders of multiple, over all time. Hence, whether the spectrum of the reflection response (over all time) of a stack of layers is white because there is a perfect reflector, or nonwhite because there is not, does not affect the whiteness or otherwise of the finite segment of \( r \) used in deconvolution, since the latter's
whiteness depends only on whether its statistical properties are those of a finite length sample from an infinite uncorrelated sequence.

The critical reflection theorem may well prove useful in its own right; however, the all-pass theorem is an irrelevance with respect to the whiteness assumption of standard deconvolution, as can be readily discerned from our comments above and the huge body of theory and practice to be found in the geophysical and statistical literature on stationary random processes and their application to deconvolution (e.g., Webster, 1978), but not referenced in the paper.

Summary

The concept of whiteness in predictive deconvolution is concerned only with the statistical properties of a finite segment of primary reflectivity. As explained above, it is a concept quite different from the deterministic white reflection response in the all-pass theorem. Suppose a processor assumes white reflectivity for predictive deconvolution. The correctness or falseness of the decision depends upon the geology of the subsurface. It can be tested by quantitative assessment, for example, by using the methods of Walden and Hosken (1985) based on well logs.

The paper by Fokkema and Zdolikowski does not say anything about the statistical whiteness assumption used in predictive deconvolution, even though it claims (e.g., in the abstract) to show that the whiteness assumption is invalid at precritical incidence. Their work is concerned with the deterministic reflection response (over all time) of a stack of layers with or without a perfect reflector at the bottom, using precritical and postcritical incidence, and not with the temporal distribution of reflectors.

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References


Reply by the authors to A. Walden and R. White

We thank Andrew Walden and Roy White for their interest in our paper and their explanation of the practical whiteness assumption in deconvolution. As we understand it, what they are saying is this: True whiteness is not at issue when we are dealing with finite chunks of data. The only thing that matters is whether the statistical properties of a finite segment of the impulse response of the earth (what Walden and White call the reflection response \( r \)) are those of a finite length sample from an uncorrelated sequence. Quite. And how are we going to find that out unless we first do the signature deconvolution with a known signature? In other words, we can only test this assumption in circumstances where we have no need of it.

We agree with Walden and White that there is no theoretical justification for the whiteness assumption in conventional predictive deconvolution. This was one of the main points of our paper. We mentioned the all-pass theorem of Teitel and Robinson (1966) only partly because we were under the misapprehension that this lent credence to the whiteness assumption. Our main reason for mentioning it is because the notion of the perfect reflector in the all-pass theorem gave us the idea for the critical reflection theorem: if you go to critical incidence, the lower layer behaves like a perfect reflector.

On the practical whiteness assumption, we avoided quoting from papers in Webster (1978), not because we were unaware of them, but because we could not find a totally satisfactory practical argument to justify the assumption. However, now that we have been challenged to quote from these famous papers, we feel we must quote one argument to show how the whiteness assumption is justified to the practicing geophysicist.

Using Webster (1978), p. 109, we quote from Robinson’s (1954) thesis:

“Our assumption that the impulses \( i \) are mutually uncorrelated with each other is an orthogonality assumption, and is a weaker assumption than the assumption that the \( i \) are statistically independent, which we need not make.

Returning again, for the moment, to our discussion about the “sure” nature of the impulses \( i \), we see that the assumption that they are mutually uncorrelated in time and in strength does not hold in a completely deterministic system. Nevertheless, such an assumption is a reasonable one again for the working geophysicist whose knowledge of the entire deterministic setting is far from complete and who is faced with essentially a statistical problem.

“In other words, we assume that knowledge of the arrival time and strength of one wavelet does not allow us to predict the arrival time and strength of any other wavelet. In particular, we assume that the arrival time and magnitude of a reflection from a certain reflecting horizon does not allow us to predict the arrival time and magnitude of a reflection from a deeper reflecting horizon.”

These words are also to be found in Robinson and Teitel (1980), p. 242.

It is hard to find a better practical justification for the whiteness assumption than this. However, if we read the sentence