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(54) Title: METHOD AND DEVICE FOR FILTERING A DIGITAL SIGNAL WITH FRACTIONAL DELAY

(57) Abstract

     Method and device for filtering a digital signal with the aid of an Mth order finite impulse response filtering with a fractional delay d which is a fraction of the sampling period associated with the digital signal, in order to obtain an interpolated value. This involves predetermining (M+1)/2 window weights w_k of a window function which is applied to the infinite impulse response of the ideal interpolator, the window function being a symmetric stepped window function. Then simplified filter coefficients $(h_j)_{w0}$ are determined as a function of the fractional delay d, and the finite impulse response filtering is applied to the digital signal. Finally, the weighted result of the finite impulse response filtering is multiplied by a common normalisation factor $(F(d))_{w0}$. Each filter coefficient determination requires just three operations.

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Method and device for filtering a digital signal with fractional delay

The present invention relates to a method and device for filtering a digital signal in order to reconstruct a value of a signal sample at any time between two samples of the digital signal. More specifically, the invention relates to the filtering of a digital signal with the aid of an Mth order (M preferably odd) finite impulse response filtering (FIR) with a fractional delay d which is a fraction of the sampling period associated with the digital signal, in order to obtain an interpolated value, comprising the step of predetermining $(M+1)/2$ window weights w_k of a window function which is applied to the infinite impulse response of the ideal interpolator.

Such filters are used in various fields where it is necessary for signals which are limited in bandwidth and are sampled at twice the Nyquist frequency to be reconstructed at any time in respect to the sampling times. These interpolation filters are also referred to as fractional delay filters or variable phase delay filters.

Fields of application include, but are not limited to, synchronisation in digital modems, phase-controlled array antennae (for example "null-steering" antennae), asynchronous sample frequency conversion of previously sampled signals ("upsampling" and "downsampling"), speech coding, signal pattern recognition and temporal localisation of patterns (for example speech recognition), periodical or non-periodical signal pattern generation in any phase desired, physical modelling of musical instruments (for example vibrating strings), but also the tracking of signals in digital navigation receivers (GPS, LORAN-C, EUROFIX), the time-continuous tuning of numerically controlled oscillators (NCO) and adaptive averaging devices (in LORAN-C), the direct determination of signal characteristics such as zero crossings and maxima/minima in, for example, seismic measurement data, the change in the aspect ratio (3/4 <-> 9/16) in the case of video processing, simulating multipath phenomena in the case of antenna signals in mobile receivers, the rapid determination of (local) maxima and minima in sampled data (for example in GPS data processing), time-discrete Hilbert transformers for software radio receivers, signal pattern recognition for temporal and/or positional localisation in, for example, the medical sector (ECG (heart rhythm analysis), EEG, etc) and the synchronisation of many asynchronous digital data streams into a single data stream (for example the synchronisation of compass data, gyroscope data, level data, roll data, pitch data, (GPS) position data, speed data, acceleration data, and data

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regarding the change in acceleration into one navigational solution for aeroplanes), or the synchronisation of various asynchronous process data streams into one data stream for digital control systems having a plurality of inputs in all forms of industry (i.e. a simplification of Kalman filtering).

5　　　　Such filters are used as an alternative for known mathematical interpolation methods for determining the best curve through a particular set of equidistant points. If the original signal, after filtering by an ideal low pass filter has been sampled at twice the Nyquist frequency, the original signal can be reconstructed perfectly by convoluting the sample values with the impulse responses associated with the ideal low pass filter.

10　　The impulse response of the ideal low pass filter is a time-continuous sinc function where the period of the side lobes is equal to the inverse of twice the Nyquist frequency. For the ideal case, even one convolution operation will already take an infinitely long time for such a filter of infinite order M. In practice, however, the only possibility is to work with causal impulse responses and a limited order of the inter-

15　　polation filter, the so-called finite impulse response filters (FIR).

The frequency response of a fractional delay filter, implemented as a finite impulse response filter, is generally given by

$$H(z) = h_0 + h_1 * z^{-1} + h_2 * z^{-2} + \ldots\ldots + h_M * z^{-M} = \sum_{j=0}^{M} h_j * z^{-j}$$

20　　where h_j represents the impulse response weights and M is the filter order.

Finite impulse response filters with fractional delay are disclosed, for example, by the article "Splitting the Unit Delay" by T.I. Laakso et al., in IEEE Signal Processing Magazine, Volume 13, No. 1, pp. 30-60, January 1996.

This publication discloses, for example, a design for a finite impulse response

25　　filter with fractional delay, where the infinite sinc impulse response is multiplied by a bell-shaped window function. The zero values outside the bell shape ensure that the ultimate impulse response will be constrained or finite. The computation of the filter coefficients for such a finite impulse response filter is complex (for each coefficient determination, a value of a computation-intensive bell-shaped window function must be

30　　multiplied by a value of a computation-intensive sinc function), and the error introduced by the filter is not minimised, moreover.

Another way, disclosed in this publication, of designing finite impulse response

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filters with fractional delay is the so-called Lagrange interpolation. Here, the filter coefficients $\mathbf{h_j}$ are given by

$$\left(\mathbf{h_j}\right)_{\text{Lagr}} = \prod_{\substack{n=0 \\ n \neq j}}^{M} \frac{n - \dfrac{M}{2} - \mathbf{d}}{n - j} \qquad j = \{0,1,...,\mathbf{M}\}$$

5   Here, too, many computation-intensive operations are required for each coefficient, thus increasing the total operation time per interpolation of the filter.

Since the known filter designs require many, often complex computations for each filter coefficient of the finite impulse response filter, it is often not possible to

10   achieve the accuracy, speed of adjustment to new values of the fractional delay $\mathbf{d}$, bandwidth and/or maximum allowable total filter delay which are required for a particular application.

It is an object of the present invention to provide a method for obtaining an interpolated value of a digital signal with a fractional delay $\mathbf{d}$ as a variable component of

15   a delay of M/2 sampling times, with the aid of Mth order finite impulse response filtering, which requires the smallest possible number of operations per filter coefficient, so that highly accurate, rapidly adjustable filtering with fractional delay can take place at the highest possible bandwidth even for small values of the total filter delay M/2+$\mathbf{d}$.

This object is achieved in a method of the type defined in the preamble, charac-

20   terised in that the window function is a symmetric stepped window function and in that the method further comprises the following steps:

- determining simplified filter coefficients $(\mathbf{h_j})_{w0}$ as a function of the fractional delay $\mathbf{d}$ according to the formula

$$\left.\begin{array}{l}
\left(\begin{array}{c} \mathbf{h(k,d)} \\ {}_{j=i} \end{array}\right)_{w0} = (-1)^{k-1} * \dfrac{\mathbf{w_k}}{\left(k - \frac{1}{2}\right) + \mathbf{d}} \\[4mm]
\left(\begin{array}{c} \mathbf{h\ (k,d)} \\ {}_{j=M-i} \end{array}\right)_{w0} = (-1)^{k-1} * \dfrac{\mathbf{w_k}}{\left(k - \frac{1}{2}\right) - \mathbf{d}}
\end{array}\right\}
\begin{array}{l}
\mathbf{i} = \left\{0,1,...,\frac{M-1}{2}\right\} \\[2mm]
\mathbf{k} = \left\{\frac{M+1}{2},...,2,1\right\} = \frac{M+1}{2} - \mathbf{i} \\[2mm]
|\mathbf{d}| < \frac{1}{2}
\end{array}$$

and subsequently applying the finite impulse response filtering to the digital signal;

- multiplying the weighted result of the finite impulse response filtering by a common normalisation factor $(F(d))_{w0}$, given by the formula:

$$\left(F(d)\right)_{w0} = \cfrac{1}{\sum\limits_{k=1}^{\frac{M+1}{2}}\left[\left(\prod\limits_{j=1}h(k,d)\right)_{w0} + \left(\prod\limits_{j=M-1}h\ (k,d)\right)\right]_{w0}}$$

5   which is a function of the simplified filter coefficients, in order to obtain the inter-
polated signal value from the digital signal with fractional delay d.

This method has the advantage that each filter coefficient globally requires only
three operations, namely an addition, a subtraction and a division. This allows the
10   fabrication of finite impulse response filters which can be very rapidly adjusted to the
fractional delay d and consequently have a small total filter operation time of the size
of 4M operation times.

In one embodiment, the window weights of the symmetric stepped window
function are chosen such that the resulting finite impulse response filter exhibits a fre-
15   quency response having a ripple-shaped deviation of the normalised level (equiripple
filter). In a further embodiment, the window weights of the symmetric stepped window
function are chosen such that the resulting frequency response is maximally flat
(Lagrange filter). This means that a choice can be made from a number of classes of
finite impulse response filters by choosing the window weights without this having
20   consequences in terms of the way in which the filter coefficients are calculated.

As a result of, in one embodiment of the method according to the invention, an
additional common factor ($\frac{1}{4}$-d^2) being added when the simplified filter coefficients
$(h_j)_{w0}$ and the common normalisation factor $(F(d))_{w0}$ are determined, the method can
also be applied to a fractional delay d equal to $\pm\frac{1}{2}$. In the embodiment mentioned ear-
25   lier this value was excluded to prevent a division by zero from determining the simpli-
fied filter coefficients. To this end, in fact, two critical divisions are replaced by two
multiplications, the total number of operations increases slightly.

In one embodiment of the method according to the present invention, two finite
impulse response filtering operations are used to effect filtering which allows all fre-
30   quencies to pass ("all pass filter"), the simplified filter coefficients being determined

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solely from the window weights and the desired fractional delay **d**. The transfer function of the all pass filter is given by

$$H_{allpass,FD}(z) = \frac{h_0 + h_1 * z^{-1} + \dots + h_M * z^{-M}}{h_0 * z^{-M} h_1 * z^{-(M-1)} + \dots + h_M}$$

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Hence it can be deduced that both the multiplication by the common normalisation factor $(F(d))_{w0}$ and the addition of the factor $(\frac{1}{4}-d^2)$ can be dispensed with for this filter.

The above-described embodiments of the method according to the invention

10   have the drawback that one of the operations in determining the simplified filter coefficients and the common normalisation factor is a division. A division is an operation which, both when carried out by software and carried out by hardware, generally requires much more operation time than a multiplication operation.

In a further embodiment of the method according to the invention, the division

15   in the course of determining the filter coefficients and the common normalisation factor is replaced by an approximation with the aid of one or more multiplications. Thus, the finite impulse response filtering with fractional delay **d** can be carried out more rapidly, compared with the previously described embodiment, with an acceptable approximation error.

20   In a further embodiment of the method according to the invention, the approximation for a main lobe of the impulse response and the approximation for side lobes of the impulse response take place separately in such a way that the resulting impulse response retains the exact values for the extreme values of the fractional delay (i.e. for **d**=0 and **d**=±½).

25   In a first approximation, the finite impulse response filtering with a fractional delay **d**, where the simplified filter coefficients and the common normalisation factor are calculated with the aid of multiplications, can be described by a third-order polynomial (PO=3) of the fractional delay, the maximum approximation error in the impulse response being $4 \cdot 10^{-3}$. The complexity of this embodiment can, based on the approxi-

30   mation by a third order polynomial, be reduced in a further approximation to an embodiment using a second-order polynomial (PO=2) with a maximum approximation error remaining almost the same at $6 \cdot 10^{-3}$. In an alternative approximation, the filtering

can be described by a fifth-order polynomial (PO=5), the maximum approximation error being $2 \cdot 10^{-5}$. This, however, is at the cost of doubling the complexity of the filtering.

In a further embodiment of the method according to the invention use is made of the inverse of the finite impulse response filtering with fractional delay **d** as described above, to determine those values of the fractional delay at which the original signal has zero crossings and/or extremes. Not only the point in time and the corresponding amplitude, but also the conditions in which these phenomena occur can be determined accurately if the order of the polynomial is less than or equal to 3 (PO≤3). In the case of a third-order polynomial approximation, zero crossings can be determined via a solution of a third-order equation and extreme values according to a simpler solution of a second-order equation. If the inverse of the second-order polynomial approximation of the fractional delay filter according to the present invention is used, zero crossings can be determined by solving a second-order equation, and extremes can be determined by solving a linear equation.

A second aspect of the present invention relates to a device for implementing the method according to any one of Claims 1 to 7 inclusive. To this end, the device is provided with delay elements, addition elements, multiplication elements and inverter elements for filtering a digital signal with a fractional delay **d** which is a fraction of the sampling period associated with the digital signal,

predetermined window weights w_k of a window function which is applied to the infinite impulse response of an ideal interpolator being implemented in multiplication elements, simplified filter coefficients $(h_j)_{w0}$ being implemented as a combination of addition elements, multiplication elements and inverter elements and

a common normalisation factor $(F(d))_{w0}$ being implemented as a multiplication element,

in order to obtain the interpolated signal value from the digital signal with fractional delay **d**.

As this device is capable of hardware implementation, much more rapid operation of the filter can be obtained, compared with a software implementation.

In a further embodiment of the device according to the present invention, the device is further provided with computing means in which the inverse of the finite impulse response filtering with fractional delay **d** is employed to determine the values of the fractional delay **d** at which the original signal has zero crossings and/or extremes.

A possible application of the method for filtering a digital signal in order to re-construct a value of a sampled signal at any time between two samples of the digital signal is a numerically controlled oscillator (NCO). Known NCOs (for example the type HSP45116 from Harris Corporation) comprise a K-bit phase accumulator and an

5   electronic table containing 2^K numbers having a word size of 16 bits. In the case of a 20-bit phase accumulator this leads to a table of 16 Mbit. With the aid of the phase accumulator, any frequency can be chosen in the range from 0 to $f_s/2$ in steps of $2^{-K}*f_s$.

A further aspect of the present invention relates to a numerically controlled oscillator for generating digitised band-limited periodic signals, for example sinusoidal

10 signals, comprising a K-bit phase accumulator and an addressable table characterised in that the addressable table comprises twice the oversampling (=2*b) of groups of P0+1 polynomial coefficients which are addressed by the $^2\log(2*b)$ most significant bits of the K-bit phase accumulator,

the addressed P0+1 polynomial coefficients being used for a P0th order approximation

15 of a filter with a fractional delay d, formed by P0 multiplication elements and P0 addition elements, the fractional delay d being determined by the $^2\log(2*b)$ least significant bits of the K-bit phase accumulator. Since only 2*b groups of P0+1 polynomial coefficients need be stored in the table, a much smaller table is sufficient. In the case of, for example, a sine signal oversampled by a factor of 16 (2b=32) and a third-order poly-

20 nomial approximation (P0=3) of a 5th order (M=5) fractional delay filter, only 128 numbers need be stored in the table. Simulations have shown that for a second-order (P0=2) polynomial approximation about 1000, and for a first-order (P0=1) polynomial approximation about 8000 numbers should be stored for comparable quality.

The present invention will now be explained in more detail with reference to

25 examples and the appended drawings, in which

Figure 1 shows a finite impulse response of a seventh-order filter, together with a window function which is applied to the infinite impulse response of the ideal low pass filter;

Figure 2 shows a general schematic for the fractional delay filter, implemented

30 using a finite impulse response filter;

Figure 3 shows an example of a schematic of a fifth-order fractional delay filter, in which the range of the fractional delay d has been extended by the value $|d| = \frac{1}{2}$;

Figure 4 shows an example of a schematic of a third-order fractional delay filter

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which passes all frequencies to a same degree;

Figure 5 gives the general schematic for a ninth order filter with 10 filter coefficients according to an embodiment of the present invention;

Figure 6 shows the schematic for a design of an integrated circuit for the first approximation (third-order polynomial) of the filter shown in Figure 5;

Figure 7 shows the schematic for a design of an integrated circuit for the second approximation (fifth-order polynomial) of the filter shown in Figure 5;

Figure 8 shows the schematic for a design of an integrated circuit of the filter shown in Figure 5 according to a second-order polynomial approximation, derived from the embodiment using a third-order polynomial approximation;

Figure 9 shows the schematic for a design of an integrated circuit for determining times of zero crossings and extremes and the corresponding amplitudes with the aid of the second-order polynomial approximation showing Figure 8;

Figure 10 shows the schematic for a numerically controlled oscillator according to a further aspect of the present invention.

The frequency response of a fractional delay filter, which has been implemented as a finite impulse response, can generally be described as

$$H(z) = h_0 + h_1 * z^{-1} + h_2 * z^{-2} + \ldots\ldots + h_M * z^{-M} = \sum_{j=0}^{M} h_j * z^{-j}$$

Figure 1 shows a finite impulse response of a seventh-order filter (a fragment of an infinite sinc function), together with the limiting window function which, in accordance with the method according to the invention, is applied to the infinite impulse response associated with the ideal interpolation filter. The symmetric window weights are given by $w_1 \ldots w_4$, the symmetry being round the centre of gravity of the impulse response. The filter coefficients $h_0 \ldots h_7$ are indicated by the vertical lines at times 0, T, ..., 7T, where T is the sampling period. The total filter delay time is indicated in Figure 1 by the phase delay D and the fractional delay by the letter **d**.

To filter a digital signal with the aid of the method according to the invention, the symmetric stepped window function is first applied to the infinite impulse response of the ideal low pass filter to produce the Mth-order interpolation filter. Because of the symmetry of the window function it is possible to use the parameters i and k, thus

allowing the filter coefficients to be divided into two equal parts $j=\{i; M-i\}$. For the example in Figure 1, the values of t(s), j, k and i are shown below the graph. Consequently, the filter coefficients $\mathbf{h_j}$ are derived by calculating the simplified filter coefficients $(\mathbf{h_j})_{w0}$ according to the equation

$$\left.\begin{array}{l}\left(\begin{array}{c}\mathbf{h\,(k,d)}\\ \tiny{j=i}\end{array}\right)_{w0} = (-1)^{k-1} * \dfrac{\mathbf{w_k}}{(k-\frac{1}{2})+\mathbf{d}}\\[3em] \left(\begin{array}{c}\mathbf{h\,(k,d)}\\ \tiny{j=M-i}\end{array}\right)_{w0} = (-1)^{k-1} * \dfrac{\mathbf{w_k}}{(k-\frac{1}{2})-\mathbf{d}}\end{array}\right\} \quad \begin{array}{l}\mathbf{i} = \left\{0,1,...,\frac{M-1}{2}\right\}\\[1em] \mathbf{k} = \left\{\frac{M+1}{2},...,2,1\right\} = \frac{M+1}{2}-\mathbf{i}\\[1em] |\,\mathbf{d}\,| < \frac{1}{2}\end{array}$$

whereupon the total result of the digital signals filtered therewith are multiplied by the common normalisation factor $(\mathbf{F(d)})_{w0}$, which is calculated using the equation

$$\left(\mathbf{F(d)}\right)_{w0} = \cfrac{1}{\displaystyle\sum_{k=1}^{\frac{M+1}{2}}\left[\left(\begin{array}{c}\mathbf{h\,(k,d)}\\ \tiny{j=1}\end{array}\right)_{w0} + \left(\begin{array}{c}\mathbf{h\,(k,d)}\\ \tiny{j=M-1}\end{array}\right)_{w0}\right]}$$

The window weights $\mathbf{w_k}$ are determined as a function of the desired design type of the filter. For an equiripple filter, the window weights are given by

$$\mathbf{W_k} = \left|(2k-1)*\dfrac{(\mathbf{h_k})_{mr}}{(\mathbf{h_1})_{mr}}\right|$$

where the coefficients $(\mathbf{h_k})_{mr}$ are determined via existing design software for equiripple filters.

For a maximally flat filter or Lagrange filter, the window weights are given by

$$(\mathbf{W_k})_{Lagr} = \dfrac{(\frac{M-1}{2})!*(\frac{M+1}{2})!}{(\frac{M+1}{2}-k)!*(\frac{M-1}{2}+k)!} \qquad \mathbf{k} = \{1,2,...,\frac{M+1}{2}\}$$

Shown below as an example are the window weights which were calculated with the aid of a computer program for a finite impulse response filter where the phase delay ripple remains below about 0.1% and where the frequency response ripple remains below a value of 0.7%, based on the normalised frequency response. M is the filter order, and N=M+1 is the number of filter coefficients.

N	M	w_1	w_2	w_3	w_4	w_5
2	1	1	0	0	0	0
4	3	1	0.4	0	0	0
6	5	1	0.6	0.18	0	0
8	7	1	0.74	0.38	0.1	0
10	9	1	0.84	0.57	0.3	0.11

The method according to the invention is also suitable for implementing a Lagrange filter. A Lagrange filter has a maximally flat filter response, but a smaller frequency range compared with the above-mentioned equiripple filter. The window weights in this case are given by the following table:

N	M	w_1	w_2	w_3	w_4	w_5
2	1	1	0	0	0	0
4	3	1	1/3	0	0	0
6	5	1	1/2	1/10	0	0
8	7	1	3/5	1/5	1/35	0
10	9	1	2/3	2/7	1/14	1/126

Figure 2 shows a general schematic for the fractional delay filter, implemented by means of a finite impulse response filter. The schematic incorporates delay elements, indicated by Z^{-1}, multiplication elements, indicated by $\triangleright$, and a summation

element, indicated by +. The filter coefficients $(h_j)_{w0}$ and the common normalisation factor $(F(d))_{w0}$ are calculated in the above-described manner. Once the filter order and the window weights have been chosen, the filter coefficients depend solely on the fractional delay d.

Figure 3 shows a schematic of an example of a fifth-order functional delay filter where the range of the fractional delay d has been extended by the value $|d| = \frac{1}{2}$. This is possible by introducing an additional common factor $(\frac{1}{4}-d^2)$ into the computation of the filter coefficients. The total number of operations is increased slightly, because the implementation in practice of twice this additional factor in total requires four more multiplications plus two more additions. It should be noted in this context, that at the same time two critical divisions (critical because of division by zero) have been replaced by two safe multiplications.

Figure 4 shows an example of a third-order fractional delay filter which allows all frequencies to pass. These filters are used for specific applications requiring a completely flat frequency response. The filter coefficients are calculated in the same way as for the previously described filters, whilst the multiplication by the common normalisation factor $(F(d))_{w0}$ can be dispensed with, as can the above-mentioned provision to avoid a division by zero. This last-mentioned advantage is a result of the fact that, compared with FIR filters, the fractional delay in all pass filters is increased by a factor of two, thus allowing the range of d to be halved and the critical division to be reliably avoided. Consequently, the third-order filter shown as an example in Figure 4 requires only three delay elements, eight multiplication elements, five addition elements and one subtraction element.

According to a further embodiment of the method according to the invention, the divisions carried out to calculate the simplified filter coefficients and the common normalisation factor are replaced by approximations using multiplications. In a further embodiment of the method according to the invention, the approximations for the side lobes of the impulse response $(k > 1)$ and the approximations for the main lobes $(k = 1)$ of the impulse response are performed separately. Hereinafter, the filter coefficients for the approximation will be indicated by $\hat{h}$.

A first approximation (indicated by subscript 3 because of the third-order polynomial approximation) of the filter coefficients of the side lobes of the impulse response is given by

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$$\left(\hat{h}_{i;M\text{-}i}(k,d)\right)_3 = (-1)^{k-1} * \left((k-\tfrac{1}{2}) \mp d\right) * \left(\tfrac{1}{4} - d^2\right) * C_k \qquad \text{for } k \ge 2$$

where subscript i represents the rank of the coefficients from the "left-hand" term of the
impulse response to which only the "upper" part of the $\mp$, i.e. a minus sign, applies and
where subscript k represents the rank of the coefficients from the "right-hand" term of
the impulse response to which only the "bottom" part of the $\mp$, i.e. a plus sign, applies
and where also

$$C_k = \frac{w_k}{(k-\tfrac{1}{2})^2 * F_M(0)}$$

is a constant and the common normalisation factor $F_M(0)$ is given by

$$F_M(0) = w_1 + \tfrac{1}{4} * \left\{ \sum_{kk=2}^{\frac{M+1}{2}} (-1)^{kk-1} * w_{kk} * \left(\tfrac{2}{kk-\tfrac{1}{2}}\right) \right\}$$

$$= \frac{w_1}{1} - \frac{w_2}{3} + \frac{w_3}{5} - \frac{w_4}{7} + \ldots + (-1)^{\frac{M-1}{2}} * \frac{w_{\frac{m+1}{2}}}{M}$$

A second approximation (indicated by subscript 5 because of the fifth-order
polynomial approximation) of the side lobes of the impulse response can be given by
adding to the first approximation of the filter coefficients a term $(1-B_k*d^2)$, the calcula-
tion of the filter coefficients consequently being given by

$$\left(\hat{h}_{i;M\text{-}i}(k,d)\right)_5 = (-1)^{k-1} * \left((k-\tfrac{1}{2}) \mp d\right) * \left(\tfrac{1}{4} - d^2\right) * \left(1 - B_k * d^2\right) * C_k$$

Approximations can also be derived for the main lobes, resulting in a first
approximation of the filter coefficients given by

$$\left(\hat{h}_{k=1}(d,M)\right)_3 = \left(\tfrac{1}{2} \mp d\right) * \left(1 + \left(\tfrac{1}{4} - d^2\right) * CC_1\right)$$

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where only the "upper" part of the $\overline{+}$, i.e. a minus sign, applies to the coefficients from the left-hand term of the impulse response and where only the "bottom" part of the $\overline{+}$, i.e. a plus sign, applies to the coefficient from the right-hand term of the impulse response and where also $CC_1 = C_1 - 4$.

A second approximation is provided by the addition of the factor $(1-B_1*d^2)$, resulting in the formula

$$\left(\hat{h}_{k=1}(d,M)\right)_5 = \left(\tfrac{1}{2} \mp d\right)*\left(1+\left(\tfrac{1}{4} - d^2\right)*\left(1 - B_1 *d^2\right)* CC_1\right)$$

The correction factors B_k can be calculated almost exactly for a specific value of the fractional delay, namely $d = 0.361$. At this value of the fractional delay, the first approximation error reaches a maximum as the filter order M increases.

Again, two situations can be distinguished, viz. the correction factors B_k ($k > 1$) for the side lobes of the impulse response and the correction factor B_1 for main lobes of the impulse response. This results in

$$B_k = \frac{1}{(0.361)^2} * \left(1 - \frac{(k - \tfrac{1}{2})^2 * F_M(0)}{\left((k - \tfrac{1}{2})^2 - (0.361)^2\right)* F_M(0.361)}\right) \qquad k \geq 2$$

and

$$B_1 = \frac{1}{(0.361)^2} * \left(1 - \frac{1 - F_M(0.361)}{\left(1 - 4*(0.361)^2\right)* F_M(0.361)*\left(\dfrac{1}{F_M(0)} - 1\right)}\right)$$

where the normalisation factor $F_M(0.361)$ is found by substituting $d = 0.361$ into the equation

$$F_M(d) = w_1 + \left(\tfrac{1}{4} - d^2\right)* \sum_{kk=2}^{\frac{M+1}{2}} (-1)^{kk-1} * w_{kk} * \left(\frac{2*kk-1}{(kk-\tfrac{1}{2})^2 - d^2}\right)$$

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The second approximation provides a more accurate approximation at the expense of a number of additional multiplication operations.

Figure 5 gives the general schematic for a ninth-order filter with 10 filter coefficients. The section surrounded by a dotted line relates to the correction factors used for the second approximation. This form is suitable to be implemented, for example, in a software program for a digital signal processor.

A topology for both a software implementation and for the design of an integrated circuit can be found by rearranging the equations for the filter coefficients, so that a minimum number of multiplication operations using the fractional delay $\mathbf{d}$ are required.

The equations for the filter coefficients in the first approximation ($\mathbf{B_k} = 0$) for the filter from Figure 5 are then given by a third-order polynomial in $\mathbf{d}$:

$$\left(\hat{\mathbf{h}}_1\right)_3 = \tfrac{1}{8}\mathbf{C}_1 \mp \tfrac{1}{4}\mathbf{C}_1 * \mathbf{d} - \tfrac{1}{2}(\mathbf{C}_1 - 4) * \mathbf{d}^2 \pm (\mathbf{C}_1 - 4) * \mathbf{d}^3$$

$$\left(\hat{\mathbf{h}}_2\right)_3 = \left(-\tfrac{3}{8} \pm \tfrac{1}{4}\mathbf{d} + \tfrac{3}{2}\mathbf{d}^2 \mp \mathbf{d}^3 \right) * \mathbf{C}_2$$

$$\left(\hat{\mathbf{h}}_3\right)_3 = \left(\tfrac{5}{8} \mp \tfrac{1}{4}\mathbf{d} - \tfrac{5}{2}\mathbf{d}^2 \pm \mathbf{d}^3 \right) * \mathbf{C}_3$$

$$\left(\hat{\mathbf{h}}_4\right)_3 = \left(-\tfrac{7}{8} \pm \tfrac{1}{4}\mathbf{d} + \tfrac{7}{2}\mathbf{d}^2 \mp \mathbf{d}^3 \right) * \mathbf{C}_4$$

$$\left(\hat{\mathbf{h}}_5\right)_3 = \left(\tfrac{9}{8} \mp \tfrac{1}{4}\mathbf{d} - \tfrac{9}{2}\mathbf{d}^2 \pm \mathbf{d}^3 \right) * \mathbf{C}_5$$

The schematic for a design of an integrated circuit which comprises delay elements, multiplication elements, addition elements and inversion elements, for the first approximation (third-order polynomial), is shown in Figure 6. The maximum approximation error in the impulse response of this finite impulse response filter with fractional delay is about $4 \cdot 10^{-3}$.

The equations for the filter coefficients in the second approximation ($\mathbf{B_k} \neq 0$) for the filter from Figure 5 are then given by a fifth-order polynomial in $\mathbf{d}$:

15

$$\left(\hat{h}_1\right)_5 = \tfrac{1}{8}C_1 \mp \tfrac{1}{4}C_1 * d - \tfrac{1}{2}(C_1 - 4)*(1+\tfrac{1}{4}B_1)*d^2 \pm (C_1 - 4)*(1+\tfrac{1}{4}B_1)*d^3$$
$$+ \tfrac{1}{2}(C_1 - 4)*B_1*d^4 \mp (C_1 - 4)*B_1*d^5$$

$$\left(\hat{h}_2\right)_5 = \left(-\tfrac{3}{8} \pm \tfrac{1}{4}d + \tfrac{3}{2}(1+\tfrac{1}{4}B_2)*d^2 \mp (1+\tfrac{1}{4}B_2)*d^3 - \tfrac{3}{2}B_2*d^4 \pm B_2*d^5\right)*C_2$$

$$\left(\hat{h}_3\right)_5 = \left(+\tfrac{5}{8} \mp \tfrac{1}{4}d - \tfrac{5}{2}(1+\tfrac{1}{4}B_3)*d^2 \pm (1+\tfrac{1}{4}B_3)*d^3 + \tfrac{5}{2}B_3*d^4 \mp B_3*d^5\right)*C_3$$

$$\left(\hat{h}_4\right)_5 = \left(-\tfrac{7}{8} \pm \tfrac{1}{4}d + \tfrac{7}{2}(1+\tfrac{1}{4}B_4)*d^2 \mp (1+\tfrac{1}{4}B_4)*d^3 - \tfrac{7}{2}B_4*d^4 \pm B_4*d^5\right)*C_4$$

$$\left(\hat{h}_5\right)_5 = \left(+\tfrac{9}{8} \mp \tfrac{1}{4}d - \tfrac{9}{2}(1+\tfrac{1}{4}B_5)*d^2 \pm (1+\tfrac{1}{4}B_5)*d^3 + \tfrac{9}{2}B_5*d^4 \mp B_5*d^5\right)*C_5$$

The schematic for a design of an integrated circuit for the second approximation (fifth-order polynomial) is shown in Figure 7. The maximum approximation error in the impulse response for this finite impulse response filter with fractional delay is about $2 \cdot 10^{-5}$.

From the formulae for the filter coefficients using the third-order polynomial approximation, filter coefficients can be derived for a second-order polynomial approximation. The filter coefficients for the side lobes of the filter using a second-order polynomial approximation (where subscript 2 is used to indicate the second-order polynomial approximation) are given by the formulae

$$\left(\hat{h}_{i;M-i}(k,d)\right)_2 = (-1)^{k-1} * \left(\tfrac{1}{4}(k-\tfrac{1}{2}) \mp \tfrac{3}{8}d + (\pm\tfrac{3}{4} - (k-\tfrac{1}{2})) * d^2\right) * C_k$$

for $0 \le d \le \tfrac{1}{2}$, and

$$\left(\hat{h}_{i;M-i}(k,d)\right)_2 = (-1)^{k-1} * \left(\tfrac{1}{4}(k-\tfrac{1}{2}) \mp \tfrac{3}{8}d + (\mp\tfrac{3}{4} - (k-\tfrac{1}{2})) * d^2\right) * C_k$$

for $-\tfrac{1}{2} \le d \le 0$. For the main lobes, the filter coefficients are given by

$$\left(\hat{h}_{k=1}(d,M)\right)_2 = \tfrac{1}{8}C_1 \mp (\tfrac{3}{8}C_1 - \tfrac{1}{2})d + (2 \mp 3 + (-\tfrac{1}{2} \pm \tfrac{3}{4}) * C_1) * d^2$$

for $0 \le d \le \tfrac{1}{2}$, and

$$\left(\hat{h}_{k=1}(d,M)\right)_2 = \tfrac{1}{8}C_1 \mp (\tfrac{3}{8}C_1 - \tfrac{1}{2})d + (2 \pm 3 + (-\tfrac{1}{2} \mp \tfrac{3}{4}) * C_1) * d^2$$

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for $- \frac{1}{2} \leq \mathbf{d} \leq 0$. The maximum approximation error in the impulse response is less than $6 \cdot 10^{-3}$ (in the equiripple embodiment), which is virtually no greater than in the case of the third-order polynomial approximation.

Figure 8 shows a schematic for a design of an integrated circuit of the filter shown in Figure 5 according to a second-order polynomial approximation, derived from the embodiment using a third-order polynomial approximation. The equations for the filter coefficients in this approximation for the filter from Figure 8 are then given by a second-order polynomial in $\mathbf{d}$:

for positive $\mathbf{d}$:

$$\left(\hat{\mathbf{h}}_1\right)_2 = \tfrac{1}{8}\mathbf{C}_1 \mp (\tfrac{3}{8}\mathbf{C}_1 - \tfrac{1}{2})*\mathbf{d} \quad + \left(-(\tfrac{1}{2} \mp \tfrac{3}{4})*\mathbf{C}_1 + 2 \mp 3\right)*\mathbf{d}^2$$

$$\left(\hat{\mathbf{h}}_{2.}\right)_2 = \left(-\tfrac{3}{8} \quad \pm \tfrac{3}{8} \quad *\mathbf{d} \quad + (\tfrac{3}{2} \mp \tfrac{3}{4})*\mathbf{d}^2 \right)*\mathbf{C}_2$$

$$\left(\hat{\mathbf{h}}_3\right)_2 = \left(\tfrac{5}{8} \quad \mp \tfrac{3}{8} \quad *\mathbf{d} \quad - (\tfrac{5}{2} \mp \tfrac{3}{4})*\mathbf{d}^2 \right)*\mathbf{C}_3$$

$$\left(\hat{\mathbf{h}}_4\right)_2 = \left(-\tfrac{7}{8} \quad \pm \tfrac{3}{8} \quad *\mathbf{d} \quad + (\tfrac{7}{2} \mp \tfrac{3}{4})*\mathbf{d}^2 \right)*\mathbf{C}_4$$

$$\left(\hat{\mathbf{h}}_5\right)_2 = \left(\tfrac{9}{8} \quad \mp \tfrac{3}{8} \quad *\mathbf{d} \quad - (\tfrac{9}{2} \mp \tfrac{3}{4})*\mathbf{d}^2 \right)*\mathbf{C}_5$$

and for negative $\mathbf{d}$:

$$\left(\hat{\mathbf{h}}_1\right)_2 = \tfrac{1}{8}\mathbf{C}_1 \mp (\tfrac{3}{8}\mathbf{C}_1 - \tfrac{1}{2})*\mathbf{d} \quad + \left(-(\tfrac{1}{2} \pm \tfrac{3}{4})*\mathbf{C}_1 + 2 \pm 3\right)*\mathbf{d}^2$$

$$\left(\hat{\mathbf{h}}_2\right)_2 = \left(-\tfrac{3}{8} \quad \pm \tfrac{3}{8} \quad *\mathbf{d} \quad + (\tfrac{3}{2} \pm \tfrac{3}{4})*\mathbf{d}^2 \right)*\mathbf{C}_2$$

$$\left(\hat{\mathbf{h}}_3\right)_2 = \left(\tfrac{5}{8} \quad \mp \tfrac{3}{8} \quad *\mathbf{d} \quad - (\tfrac{5}{2} \pm \tfrac{3}{4})*\mathbf{d}^2 \right)*\mathbf{C}_3$$

$$\left(\hat{\mathbf{h}}_4\right)_2 = \left(-\tfrac{7}{8} \quad \pm \tfrac{3}{8} \quad *\mathbf{d} \quad + (\tfrac{7}{2} \pm \tfrac{3}{4})*\mathbf{d}^2 \right)*\mathbf{C}_4$$

$$\left(\hat{\mathbf{h}}_5\right)_2 = \left(\tfrac{9}{8} \quad \mp \tfrac{3}{8} \quad *\mathbf{d} \quad - (\tfrac{9}{2} \pm \tfrac{3}{4})*\mathbf{d}^2 \right)*\mathbf{C}_5$$

The schematic shown in Figures 6, 7 and 8 can serve as a basis both for software implementations of the filters and for the design of integrated circuits.

In a further embodiment of the present invention, the inverse of the finite response filtering with fractional delay $\mathbf{d}$ is used for determining the values of the fractional delay $\mathbf{d}$, at which the original signal has zero crossings and/or extremes. In a

fractional delay filter, the fractional delay parameter **d** (within the total phase delay) determines to what extent input samples are shifted in time by interpolation. For each clock pulse, a polynomial function in **d** is calculated. If the design of the fractional delay filter is such that the polynomial coefficients are provided directly, this filter is

5   suitable to serve as an inverse fractional delay filter. The polynomial function and its derivative then directly enable the positions in time of zero crossings (d_{zero}) and/or of extremes (d_{extr}) to be calculated. If the delay found at an extreme (d_{extr}) is introduced as the input of a fractional delay filter, this directly gives the corresponding amplitude value as the output signal of the filter. Zero crossings can only occur between two suc-

10   cessive sample values of opposite signs. To find the zero crossings it is first necessary, therefore, to determine where in the series of sample values this condition is met.

The exemplary schematics of Figures. 6, 7 and 8 have been elaborated in such a way that the separate polynomial coefficients a_0, a_1, a_2 ... are calculated directly. From these coefficients, the values of the fractional delay at zero crossings can be determined

15   directly and without iterations via the inverse fractional delay filter concept, and after differentiation, extremes of the signal such as maxima and minima can be determined. The calculations of these values of the fractional delay are the simpler, the lower the order of the polynomial approximation. The above-discussed second-order polynomial approximation is consequently the most suitable for use in the inverse filter with frac-

20   tional delay. Zero crossings can therefore be found by solving the equation

$$f_2(d) = a_0 + a_1 * d + a_2 * d^2 = 0$$

The extremes can be found by solving the equation

25

$$f_2'(d) = a_1 + 2 * a_2 * d = 0$$

where the type of extreme can be found by a calculation of the second derivative if $f_2''(d) = 2 * a_2$, the extreme relating to a minimum if $f_2''(d) > 0$, a maximum if $f_2''(d) < 0$

30   and a point of inflection if $f_2''(d) = 0$.

Such determinations of zero crossings can also be carried out with the aid of the third-order polynomial approximation, as discussed with reference to Figure 6. Figure 9 shows the schematic for a design of an integrated circuit for determining the times of

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zero crossings and extremes and the corresponding amplitudes with the aid of the second-order polynomial approximation shown in Figure 8. This embodiment is provided with computation means 10 which perform the above-mentioned calculations of the zero crossings and extremes. If the fractional delay at which an extreme occurs

5   (d_{extr}) is directly fed back to the filter, the amplitude value at the extreme is supplied at the output of the filter.

Hereinafter a number of practical applications will be discussed for using the method or device according to the invention.

Hilbert transformers in the ideal case are based on infinite sinc functions. This

10   opens the possibility of applying the concept of filtering with fractional delay according to the invention, resulting in a finite impulse response by using the windowing technique with predetermined window weights. The ideal Hilbert transformer consists of two filters having a flat frequency response with output signals phase-shifted by precisely 90 degrees for common real input signals. The two output signals form a single

15   complex output signal. The windowing technique according to the invention cannot be applied directly to the two impulse responses of the ideal Hilbert transformer which are currently found in the literature, as there is an odd number of samples of the sinc impulse response within the even number of window weights which has to be used. This can be solved by causing the impulse responses of the two filters in the Hilbert trans-

20   former to be mirror-antisymmetric and of the same odd order. The filter coefficients of the two filters are then found by substituting the value $d = \frac{1}{4}$ (resulting in the real impulse response) and $d = -\frac{1}{4}$ (resulting in the imaginary impulse response) into the interpolation formulae according to the invention. Real bandwidth-limited signals (for example coming from an antenna) are thereby directly converted into complex band-

25   width-limited signals, half-band filtering being used in the process. Both time errors (or phase errors) and the spurious signals introduced thereby, which occur in conventional Hilbert transformers, are thus eliminated.

A further application can be found for numerical controlled oscillators (NCO) like, for example, those used in software programs. Known NCOs, such as, for

30   example, the type HSP45116 from Harris Corporation, consist of two parts, namely a digital K-bit phase accumulator linked to an electronic table which comprises stored sample values of a sine wave signal. The table contains 2^{20} (about one million) numbers (for K=20) having a word size of 16 bits, (i.e. a total of 16 Mbit of data). By using the

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phase accumulator it is possible to select any frequency value in the range from 0 to $f_s/2$ at steps of $2^{-20} * f_s$.

The embodiments of the fractional delay filters, as discussed in the present application, all have the characteristic that they exhibit very low distortion both in

5   phase delay and in the level of the frequency response for signal frequencies close to zero. As a result, these are eminently suitable for constructing all sinusoidal signal values by interpolation of a relatively small number of sample values of a constant sine wave which, for example, is oversampled by a factor of 16 (b=16). Starting from a set of at least 2*b=32 stored sample values of a sine wave (amply sufficient for charac-

10   terising the entire period), it is possible to construct a numerical controlled oscillator which is able to generate a very accurate digital sine wave of any desired frequency. Computer simulations have shown that the best results are achieved by using a third-order polynomial approximation (PO=3) and a filter order of M = 5. In this case, only 32 sine signal samples need be stored for a better quality, compared with the known

15   NCO (error of 10^{-7} instead of error of $1.5*10^{-5}$). If use is made of filters employing a lower-order polynomial approximation, more sine signal samples have to stored for comparable quality. Simulations have shown that about 250 sine signal samples should be stored for a second-order (PO=2) approximation and about 2000 with a first-order (PO=1) approximation.

20   If the approach is based on an interpolator with fractional delay using a third-order polynomial approximation, which is fed a periodic sine wave which, for example, is oversampled by a factor of 16 (b=16), a set of four polynomial coefficients $\{a_0, a_1, a_2, a_3\}$ is calculated at each sampling time. Owing to the periodicity of the digitised sine wave, not only the 32 samples of the sine wave but also the 32 related groups of

25   four polynomial coefficients will be repeated periodically. This implies that instead of storing 32 sine wave samples it is better to store the 32 related groups of polynomial coefficients. As a result, a large number of operations of the filtering with fractional delay can be dispensed with. Only the calculation of the polynomial value at each sampling time remains. A possible set up for a numerical controlled oscillator is shown

30   in Figure 10. The NCO comprises a K-bit phase accumulator 20 and an addressable table 22, for example in the form of a read-only memory (ROM). The K-bit phase accumulator 20 converts the desired frequency F into an addressing word and a delay word. The way in which this is done is assumed to be known to those skilled in the art

20

and will not be explained here in more detail. The addressable table 22 comprises 2*b sets of P0+1 polynomial coefficients, which are addressed by the addressing word which comprises the $^2\log(2*b)$ most significant bits of the K-bit phase accumulator 20. The addressed P0+1 polynomial coefficients (in the examples shown a_0, a_1, a_2, a_3) are

5   then used for a P0th order approximation of a filter with a fractional delay **d**, formed by P0 multiplication elements and P0 addition elements. The fractional delay **d** is determined by the delay word which is formed by the K-$^2\log(2*b)$ least significant bits of the K-bit accumulator 20. Clearly, in this embodiment the ROM 22 can be much smaller than in the known NCO. In the known NCO, about one million numbers need

10 to be stored. If use is made of a third-order (P0=3) polynomial approximation of a fifth order (M=5) filter with fractional delay, only 128 polynomial coefficients need to be stored to achieve comparable quality. The inverter 24 in the schematic shown in Figure 10 changes the binary representation of the fractional delay **d** to a two-complement representation. The multiplexer 26 shown in Figure 10 has been added to allow

15 the choice between generating a sine or a cosine signal by feeding the two most significant bits of the addressing word directly to the ROM 22, or via a second inverter 28 and an EXOR-gate 30. The frequency of the sine/cosine is defined, in the example shown, by the relationship:

frequency = $(F/2^K)*f$, where 0 F 2^{K-1}-1.

20 In the case of a signal oversampled by a factor of 16, a 5th order filter with fractional delay and a third-order polynomial approximation (b=16, M=5, P0=3 implying four polynomial coefficients), the polynomial coefficients are then given by:

$$a_0(n) = \tfrac{1}{8}\{5C_3(x_n + x_{n-5}) - 3C_2(x_{n-1} + x_{n-4}) + C_1(x_{n-2} + x_{n-3})\}$$

25 $$a_2(n) = -4a_0 + 2(x_{n-2} + x_{n-3})$$

$$a_1(n) = \tfrac{1}{4}\{C_3(-x_n + x_{n-5}) + C_2(x_{n-1} + x_{n-4}) + C_1(-x_{n-2} + x_{n-3})\}$$

$$a_3(n) = -4a_1 + 4(x_{n-2} - x_{n-3})$$

where x_n (not delayed) to x_{n-5} (delayed by 5 clock pulses) inclusive form a set of six

30 successive sample values from a set of 2*b sine wave sample values which are calculated according to

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$$x_n = \sin((n + \tfrac{M+1}{2}) * \tfrac{\pi}{b}) \qquad n = \{0,1,2,\ldots,(2*b-1)\}$$

It will be obvious that instead of periodic digitised and oversampled sine wave shapes it is also possible to use other arbitrarily chosen periodic wave shapes which are limited in bandwidth and have been oversampled.

Many more fields than the two for which examples have been elaborated hereinabove lend themselves to the application of the method for filtering with a fractional delay according to the present invention, including applications requiring time-continuous adjustable delay lines, synchronisation of receivers by using inverse filtering with fractional delay, etc. All these elaborations of the general concept of filtering with fractional delay are deemed to be within the scope of the appended claims.

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<u>C l a i m s</u>

1.   Method for filtering a digital signal with the aid of an Mth order finite impulse
response filtering with a fractional delay **d** which is a fraction of the sampling period

5   associated with the digital signal, in order to obtain an interpolated value, comprising
the step of predetermining (M+1)/2 window weights w_k of a window function which is
applied to the infinite impulse response of the ideal interpolator, <u>characterised in that</u>
the window function is a symmetric stepped window function and in that the method
further comprises the following steps:

10   - determining simplified filter coefficients $(h_j)_{w0}$ as a function of the fractional delay **d**
according to the formula

$$\left. \begin{array}{c} \left(\begin{matrix} \mathbf{h(k,d)} \\ \scriptstyle j=i \end{matrix} \right)_{w0} = (-1)^{k-1} * \dfrac{\mathbf{w_k}}{(\mathbf{k}-\frac{1}{2})+\mathbf{d}} \\ \\ \left(\begin{matrix} \mathbf{h}\ \ \mathbf{(k,d)} \\ \scriptstyle j=M-i \end{matrix} \right)_{w0} = (-1)^{k-1} * \dfrac{\mathbf{w_k}}{(\mathbf{k}-\frac{1}{2})-\mathbf{d}} \end{array} \right\} \qquad \begin{array}{c} \mathbf{i} = \left\{ 0,1,...,\frac{M-1}{2} \right\} \\ \mathbf{k} = \left\{ \frac{M+1}{2},...,2,1 \right\} = \frac{M+1}{2} - \mathbf{i} \\ |\,\mathbf{d}\,| < \frac{1}{2} \end{array}$$

15

and subsequently applying the finite impulse response filtering to the digital signal;
- multiplying the weighted result of the finite impulse response filtering by a common
normalisation factor $(F(d))_{w0}$, given by the formula:

20     $$(\mathbf{F(d)})_{w0} = \dfrac{1}{\displaystyle\sum_{k=1}^{\frac{M+1}{2}} \left[\left(\mathbf{h(k,d)} \atop {\scriptstyle j=1} \right)_{w0} + \left(\mathbf{h}\ \ \mathbf{(k,d)} \atop {\scriptstyle j=M-1} \right)_{w0} \right]}$$

which is a function of the simplified filter coefficients, in order to obtain the inter-
polated signal value from the digital signal with fractional delay **d**.

25   2.   Method according to Claim 1, <u>characterised in that</u> the window function is
optimised for equiripple filtering, the window weights w_k being given by the formula:

$$W_k = \left| (2k-1) * \frac{(h_k)_{mr}}{(h_l)_{mr}} \right|$$

3. Method according to Claim 1, characterised in that the window function is optimised for Lagrange filtering, the window weights w_k being given by the formula:

$$(W_k)_{Lagr} = \frac{(\frac{M-1}{2})! * (\frac{M+1}{2})!}{(\frac{M+1}{2} - k)! * (\frac{M-1}{2} + k)!} \qquad k = \{1,2,...,\frac{M+1}{2}\}$$

4. Method according to Claim 1, 2 or 3, characterised in that the method is also suitable for $|d| = \frac{1}{2}$ by adding an additional common factor ($\frac{1}{4}$-d^2) when the simplified filter coefficients $(h_j)_{w0}$ and the common normalisation factor $(F(d))_{w0}$ are determined.

5. Method according to any one of the preceding claims, characterised in that two finite impulse response filtering operations are employed to effect filtering which allows all frequencies to pass, simplified filter coefficients being determined solely from the window weights and the desired fractional delay d.

6. Method according to any one of the preceding claims, characterised in that in determining the filter coefficients and the common normalisation factor the division is replaced by an approximation with the aid of one or more multiplications.
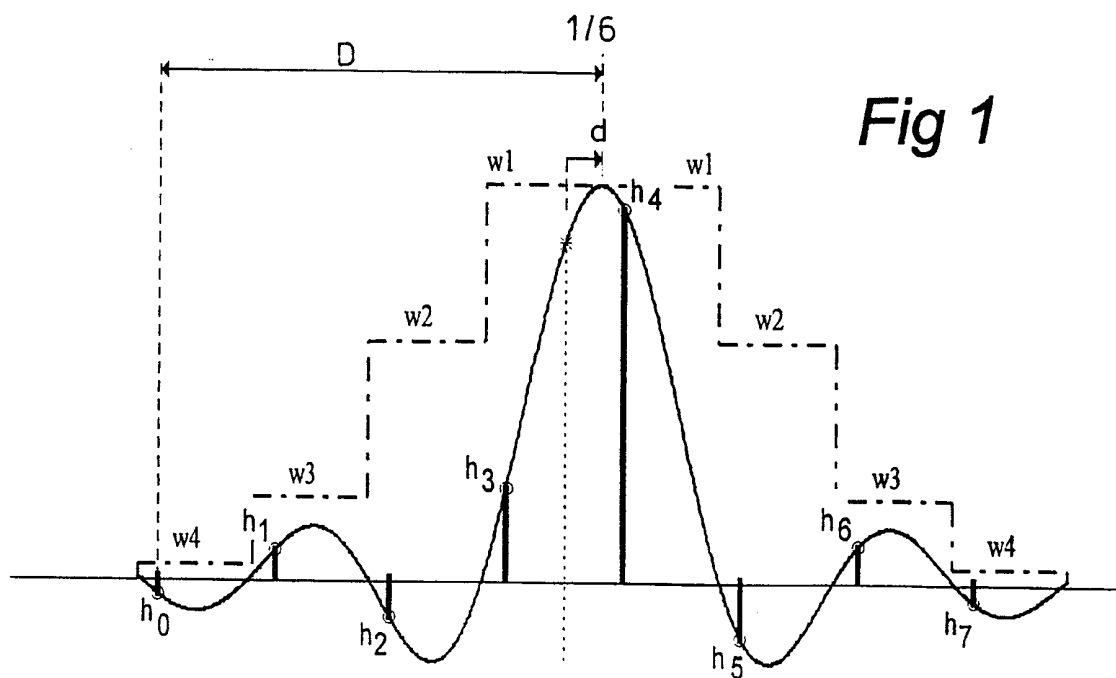
7. Method according to Claim 6, characterised in that the approximation for the main lobe of the impulse response and the approximation for the side lobes of the impulse response take place separately, so that the resulting impulse response retains the exact values for the extreme values of the fractional delay.

8. Method according to any one of the preceding claims, characterised in that the inverse of the finite impulse response filtering with fractional delay d is used for determining the values of the fractional delay d, at which the original signal has zero crossings and/or extremes.

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9.   Device for implementing the method according to any one of Claims 1 to 7 in-
clusive, comprising delay elements, addition elements, multiplication elements and
inverter elements for filtering a digital signal with a fractional delay **d** which is a frac-
tion of the sampling period associated with the digital signal,

5   predetermined window weights w_k of a window function which is applied to the infinite
impulse response of an ideal interpolator being implemented in multiplication elements,
simplified filter coefficients $(h_j)_{w0}$ being implemented as a combination of addition
elements, multiplication elements and inverter elements and
a common normalisation factor $(F(d))_{w0}$ being implemented as a multiplication ele-

10   ment,
in order to obtain the interpolated signal value from the digital signal with fractional
delay **d**.

10.   Device according to Claim 9, <u>characterised in that</u> the device is further provided
15   with computing means in which the inverse of the finite impulse response filtering with
fractional delay **d** is employed to determine the values of the fractional delay **d** at
which the original signal has zero crossings and/or extremes.

11.   Numerically controlled oscillator for generating digitised band-limited periodic
20   signals, comprising a K-bit phase accumulator and an addressable table <u>characterised in</u>
<u>that</u> the addressable table comprises twice the oversampling (=2*b) of groups of P0+1
polynomial coefficients which are addressed by the $^2\log(2{*}b)$ most significant bits of
the K-bit phase accumulator,
the addressed P0+1 polynomial coefficients being used for a P0th order approximation
25   of a filter with a fractional delay **d**, formed by P0 multiplication elements and P0 addi-
tion elements, the fractional delay **d** being determined by the $K{-}^2\log(2{*}b)$ least signifi-
cant bits of the K-bit phase accumulator.

*****

Fig 1

	0	T	2T	3T	3.5T	4T	5T	6T	7T	$\dfrac{t\,(s)}{}$ →
	0	1	2	3	4	5	6	7		j →
	4	3	2	1	1	2	3	4		k →
	0	1	2	3	3	2	1	0		i →

Fig 2

Fig 3

Fig 4

Fig 5

Fig 6

4/6

Fig 7

Fig 8

Fig 9

Fig 10

INTERNATIONAL SEARCH REPORT

A. CLASSIFICATION OF SUBJECT MATTER
IPC 6   H03H17/06

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 6   H03H

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	SANGIL PARK: "REAL-TIME PITCH (FREQUENCY) SHIFTING TECHNIQUES" PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON CONSUMER ELECTRONICS (ICCE), ROSEMONT, JUNE 5 - 7, 1991, no. CONF. 10, 5 June 1991 (1991-06-05), pages 296-297, XP000289043 INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS	1
A	LIU G -S ET AL: "PROGRAMMABLE FRACTIONAL SAMPLE DELAY FILTER WITH LAGRANGE INTERPOLATION" ELECTRONICS LETTERS, vol. 26, no. 19, 13 September 1990 (1990-09-13), pages 1608-1610, XP000106903	1

☐ Further documents are listed in the continuation of box C.   ☐ Patent family members are listed in annex.

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"A" document defining the general state of the art which is not considered to be of particular relevance

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"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
28 July 1999	04/08/1999

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