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behorende bij het proefschrift

"Forces on Ships in a Navigation Lock Induced by Stratified Flows"

A. Vrijburcht
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1. Indien voor de civiele praktijk merkbare dichtheidsverschillen in het water aanwezig zijn door de aanwezigheid van zout en zoet water, dient bij het hydraulisch ontwerp van een schutsluis niet alleen rekening gehouden te worden met de zoutindringing in de voorhaven maar ook met de krachten op de schepen in de sluis.

2. Voor het ontwerp van een schutsluis kan - rekening houdend met de variaties in stroming en dichtheidsverdeling - de langskracht op een afgemeerd schip met een één-dimensionale rekenwijze aangevuld met experimentele gegevens, voldoende nauwkeurig bepaald worden.

3. De hydraulische condities, zoals de stroomsnelheids- en dichtheidsverdelingen en de waterstand, ter plaatse van de boeg en het hek van het schip bepalen de langskracht op het afgemmerge schip in de sluis.

4. De langskracht op een afgemeerd schip in een sluis ten gevolge van dichtheidsstromen kan tijdens het vulproces nauwelijks beïnvloed worden bij toepassing van een vlissysteem in het bovenhoofd.

5. Een luchtbellenscherm, dat toegepast wordt om de uitwisseling van zout en zoet water tussen een sluiskolk en een voorhaven te vertragen, heeft als tweede functie het beperken van de langskracht op een in de sluis afgemeerde schip tijdens het uitwisselingsproces.

6. Als middelen voor de beperking van de zoutindringing bij schutsluizen valt te denken aan een in hoogte verstelbare drempel aan de zijde van het zoete pand, of aan in de sluis afgemmerge pontons.

7. Duidelijke tijdwinst in de passage van een schutsluis kan bereikt worden door het afstemmen van de hefssnelheid van de schuiven op het verval over de deur, de waterdiepte in de sluiskolk en de afmetingen en de ligging van de schepen in de sluiskolk.
8. De kielspeling van schepen tijdens het in-, uit- en doorvaren van sluizen kan bepaald worden met een rekenmodel van de waterbeweging (één-dimensionaal in langsrichting van de vaarweg) en de scheepsbeweging (in verticale richting en in langsrichting) als functie van de schroefkracht. Dit rekenmodel is van belang voor het optimaal benutten van de beschikbare vaarwegen.

9. Bij het ontwerpen van een remmingwerk moet behalve de veerstijfheid ook de demping van het remmingwerk beschouwd worden, aangezien de demping de krachten op het remmingwerk reduceert.

10. Een kunststof kering (zoals een balgstuw of een spinakerkerking) in een waterloop kan een goed en goedkoop alternatief zijn voor de traditionele kering met schuiven. Met name met het oog op de waterkwaliteit (zoals beperken gevolgen van calamiteiten) zouden kunststof keringen op veel ruimere schaal dan tot nu toe het geval is toegepast moeten worden.

11. Minder produktieve landbouwgebieden moeten met subsidies omgevormd kunnen worden tot natuurgebieden. Dit vanwege de afnemende hoeveelheid natuurgebieden, de overproduktie van landbouwgewassen en de onevenredig grote inspanning om minder produktieve gebieden toch vruchtbaar te maken.

12. Vanwege de schade die vrachtvervoer per vrachtauto aan het milieu toebrengt en het grote ruimtebeslag van autosnelwegen, dient dit vervoer over lange afstanden per trein plaats te vinden.

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INDUCED BY STRATIFIED FLOWS

Proefschrift

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Prof.dr.ir. J.P.Th. Kalkwijk

Toegevoegd promotor: Dr.ir. C. Kranenburg
Abstract

In many cases a navigation lock situated near the sea connects water bodies of different salt contents and, as a consequence, of different densities. If density differences are present, gravity will cause stratified flows when the lock is operated. These flows in particular arise during the filling of the lock chamber and after the opening of the lock gate, when the water in the lock is exchanged with that in the approach harbour. The stratified flows generate appreciable forces on a ship moored in the lock. The ship partially blocks the stratified flows and thus exerts a certain influence on them.

The investigation reported deals with these stratified flows, the related density distributions, and the longitudinal forces on the ship. The physical phenomena have been investigated by means of laboratory and field experiments. The flows and density distributions are calculated using simplified mathematical models that contain empirical coefficients of various kinds. The coefficients are derived from the experiments. The longitudinal force on the ship is calculated using the momentum equation for a two-layer flow and compared with experimental results.

Acknowledgements

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1 Introduction

1.1 General Remarks

A navigation lock is the link between two sections of a fairway with different water-levels. A navigation lock makes it possible for ships to move from one section of the fairway to the other by the operation of movable gates in the heads, and devices for the levelling of the water of the lock.

The progress of a ship through a lock is fairly simple. When a ship reaches the approach harbour of a navigation lock and the way in is open the ship will slowly enter the lock. The ship is moored in the lock chamber to prevent large horizontal motions during the levelling. The gate(s) is (are) closed. Subsequently the water in the lock is brought to the level of that of the following section of the fairway, by means of a filling or emptying system. Finally the gate(s) is (are) opened, the mooring lines are taken away and the ship leaves the lock.

During the filling process the ship encounters hydraulic forces due to the intruding water. The water intrudes in the form of concentrated filling jets, and causes transatory waves in the lock chamber. The transatory waves generate an oscillating force on the ship because of the reflections against the ship and the gates. The concentrated filling jets induce a more or less quasi-static force on the ship due to the lowering of the water-level above the jets and the impact of the jets against the bow of the ship. The hydraulic forces on the ship increase with a more rapid filling of the lock. During the emptying process the ship mainly encounters hydraulic forces due to the transatory waves.

In a lock in the vicinity of the sea, differences in salinity and hence in density can be present between the water of the navigation lock and that of the approach harbour. This lock may, for example, be the link between the sea and a canal, a harbour or a basin.

If differences in density between the water of the lock and that of the approach harbour are present, these density differences influence the flow patterns and therefore the hydraulic forces on the ship during the filling process.

An intruding layer is generated in the lock during the filling process. This intrusion induces an extra force on the ship. Though the relative differences in density are small (for example 2 %), this force can be considerable.

Another important phenomenon occurs after the filling process, i.e. when the gate is opened. The fresh (or salt) water of the lock is exchanged with the salt (or fresh) water of the approach harbour. During this exchange process a salt wedge intrudes at the bottom of a lock with initially fresh water, or a fresh wedge intrudes at the free surface of a lock with initially salt water. The ship is present in the lock during
a part of this exchange process and encounters considerable hydraulic forces due to this exchange flow.

This thesis concentrates on the influence of differences in density on the longitudinal forces on the ship during the filling and exchange processes.

1.2 Formulation of the Problem

The design of a lock must meet the principal requirements related to the capacity and the safety of the lock. The time necessary for levelling the water of the lock must be as short as possible and the hydraulic forces on the ship must be permissible.

The hydraulic forces on the ship usually increase with increasing ship dimensions with respect to the dimensions of the lock. In view of this fact, it is interesting to observe the following present trends for locks with sea-going navigation where density differences often play a role:
- New locks are usually designed so that the lock is only a little larger than the largest ship anticipated, to limit the intrusion of salt water (and sometimes the loss of fresh water) during the exchange process.
- The level of the bottom of a newly-designed lock is usually based on the required minimum keel clearance. A lower bottom level decreases the forces on the ship but is more expensive.
- The ships which are permitted to pass through existing locks nowadays are larger than they used to be, because of the improved manoeuvring techniques of the ships.
- The permissible forces of the mooring equipment on board the ships increases more slowly than the water displacements of the ships.
- In many cases a minimum number of mooring lines are used to reduce the time spent in the lock.

In view of the above mentioned requirements and trends it is important for the design of the lock to know the hydraulic forces on the ship. The hydraulic forces during the filling and emptying processes caused by water of constant density have been investigated extensively by many authors; these forces can nowadays be partially computed. The hydraulic forces caused by the differences in density (the non-homogeneous aspects) have only been incidentally measured in scale models. However, up to now calculations regarding this subject are not available.

An important question for the designer of the lock is as follows. What force is exerted on the ship by the flow due to the differences in density during the passage of the ship through the lock, and what measures can be taken to limit these forces? This question is partially answered in the investigation reported here. The possible measures to reduce this type of force are only mentioned briefly. The influence of
the differences in density on a ship entering or leaving the lock, has not been investigated. This might be a subject for further investigation.

1.3 Aim and Restrictions of the Investigation

The aim of the investigation presented here is the determination of the forces on a moored ship induced by stratified flows during the filling and exchange processes in a navigation lock. Large as well as small ships (relative to the lock dimensions) are considered. An initially fresh lock and a salt approach harbour, as well as an initially salt lock and a fresh approach harbour, are examined.

The determination of the longitudinal force on the ship has been carried out by means of experiments and calculations. Much attention is given to the hydraulic phenomena observed.

The investigation has been restricted to the dominant hydraulic phenomena which determine the force. The mathematical model to compute the forces is relatively simple and aims at an accuracy that is sufficient for practical applications. An extreme accuracy of the calculated forces is not required in view of the large variety in the dimensions of the ships.

Some other restrictions concern:
- Influence of differences in density
  The aspect of the density effects is investigated; little attention is given to the flow without differences in density.
- Filling and exchange process
  Only the filling and exchange processes are considered. The emptying process and the movement of the gate do not contribute to the forces caused by the differences in density.
- Initially homogeneous situations
  The water in the lock, like that in the approach harbour, always has a homogeneous density before the filling process and the exchange process. It appears that an initially stratified situation yields lower forces.
- Lock
  The lock has a filling system with openings in the upstream gate(s) or culverts in the upstream head. Filling systems with openings over the length of the lock or at the downstream head are not taken into consideration. Locks with devices to reduce salt intrusion are also not taken into consideration, with the exception of air bubble screens
- Longitudinal force
  The longitudinal force on the ship is measured and computed, transverse forces are only measured.
  No attention is given to forces caused by wind, wind waves, long waves from the approach harbour or motion of the gate. Moreover, no attention is given to the forces in the mooring lines, horizontal
motions of the ship or the use of the ship's propellers.

A remark must be made about the terms salt / fresh water used in the text. "Salt" water is the more dense fluid, and "fresh" water is the less dense fluid. This means that "fresh" water can be brackish.

1.4 Outline of the Investigation

The forces on the ships can be calculated with the help of the flow and density patterns in the lock. Therefore it is important to understand these patterns.

The thesis commences with a description of the hydraulic phenomena in relation to the differences in density. The flow and density patterns are strongly dependent on the situation considered (filling or exchange process) and the blockage of the ship. The sequence of presentation is tuned to the complexity of the flow and density patterns. Firstly, the hydraulic phenomena related to the exchange process are considered, and after that the filling process. Results of hydraulic scale models and measurements in situ are presented (Chapters 2 and 3).

Subsequently the force relations are derived (Chapter 4). The method of calculation of the differences in water-level and the longitudinal force assumes known flow and density patterns around the ship. These force relations are used in the next two chapters.

The calculations of the exchange process and the resulting longitudinal force are discussed in chapter 5. Firstly, the flow and density patterns of the exchange process are determined, making use of the results of the experiments of chapter 3. After that, the longitudinal forces are determined, by means of simplified calculations and some empirical coefficients.

The calculations of the longitudinal force caused by differences in density during the filling process are performed in chapter 6. The flow and density patterns are derived and schematized using the experimental results of chapter 3, and subsequently the longitudinal force is determined.

Finally the summary and conclusions are presented in chapter 7.
2 Physical Description of the Exchange Process

2.1 Introduction

We consider a navigation lock with a difference in density between the water in the lock and that in the approach harbour. A ship is present in the lock. The gate of the lock is opened after the filling or emptying process.

A system of extending wedges with fronts as leading edges develops after the moment of the opening of the gate. A wedge of salt water intrudes into the fresh water near the bottom of the lock, while a wedge of fresh water intrudes into the salt water near the water-surface. The wedge in the lock exchanges the water of the lock for that of the approach harbour. The ship moored in the lock retards this exchange flow and at the same time is subjected to hydraulic forces arising from the exchange flow.

A complete exchange of the water of the lock and that of the approach harbour can take a long time (e.g. up to one hour) in the case of a large ship. The ship usually departs soon after the opening of the lock gate, therefore only the first phase of the exchange process is of interest.

The flow and density patterns during the exchange process are complex and are considerably influenced by the presence of a ship in the lock. It appears that the available literature is mainly restricted to the exchange flow in a prismatic, rectangular channel without reflections. There are no mathematical models available for the calculation of the flow and density patterns and the forces on the ship during the exchange process. Therefore, as a part of this study, we have carried out experiments firstly to describe the hydraulic phenomena during the exchange process of the water of a lock and that of an approach harbour.

The purpose of this chapter is to describe the principal hydraulic phenomena in the lock during the exchange process with the influence of a ship, based on these experiments. Furthermore the measured forces on the ship are considered. The results of this chapter are used for the development of a schematized mathematical model in chapter 5.

In this chapter the exchange process is described with the aid of:
- flow and density patterns in the lock (height and velocity of the front, levels of the interface, flow velocities, etc.)
- water-level differences, density profiles and flow velocities near the ship.

The force on the ship can be explained by means of these phenomena.

The flow and density patterns are influenced by turbulence, mixing and friction at the interface of the exchange flow. These details of the
exchange flows are hardly considered in this chapter because of the lack of detailed measurements.
The literature, which mainly concerns the exchange flow without reflections, is not referred to in this chapter, but is dealt with in chapter 5 together with the schematized mathematical models. Formulae are also not presented in this chapter but in chapter 5.

Before the influence of differences in density is considered, a short survey of the data of the experiments is given (section 2.2) and the sources of external transitory waves are described (section 2.3). We look at the travelling wedge in its simplest form without reflections in section 2.4. Then the reflections against the discontinuities of the lock (section 2.5) and the ship (section 2.6) are considered. After that the flow and density patterns near the ship (section 2.7) and the forces on the ship (section 2.8) are presented. The influence of an air bubble screen on the exchange flow is shown in section 2.9. Finally some practical measures are presented to limit the influence of the differences in density on the force exerted on the ship (section 2.10).
The chapter is illustrated with a general survey of the beginning of the different exchange situations (figures 1 and 2). (The corresponding filling situations of chapter 3 are also shown.)

2.2 Experiments

The study of the general pattern of the exchange process has been carried out by means of a series of prototype and hydraulic scale model investigations.

The initial situation of the tests was as follows. The water of the lock chamber and the approach harbour each had an almost homogeneous density distribution. A difference in density between the water in the lock and that in the approach harbour was present. If a ship was present, it was moored to one side of the lock. The exchange process started when the lock gate was opened.

The prototype investigation concerns a lock (length * width = 154 * 16.1 m²) for yachts (figure 3, Volkerak lock). The water-depth was about 7 m and the relative difference in density varied between 0.013 - 0.016. (The relative difference in density is the quotient of the difference in density and the density of the fresh water). The lock has mitre gates which are opened in 180 s (photo 1). The first tests were concerned with the flow pattern and the density pattern around the air bubble screen without the presence of a ship. Then a laden barge (length * width * draught = 76.50 * 11.80 * 3.80 m³) was moored in the lock (photo 2). The other tests were carried out to measure the forces on the moored barge and the differences in density around the ship. The lock investigated can be considered for instance as a scale model 1:3 of a large sea lock.
The hydraulic model investigation is concerned with a lock for sea-going navigation with a scale 1:70 and with prototype dimensions 500 * 42 m² (figure 4). The water-depth was about 14 m (prototype) and the relative difference in density 0.02. The lock had a roller gate which was opened in 134 s (prototype). A bulk carrier (177 * 25.90 * 7 m³, prototype) had been moored in this lock. The forces on the moored ship and the water-level differences were measured. Various parameters were changed from one experiment to another.

2.3 External Translatory Waves during the Exchange Process

Before the exchange flow is considered, a survey of external translatory waves is given in this section. The external translatory waves are dealt with briefly, because the calculation scheme of these forces is known (Kalkwijk 1973, lit. [11]) and it is not the topic of this study.

External translatory waves in a channel are caused by variations of the discharge in time and are visible as a travelling deformation of the water-surface. The velocity of propagation in a lock is high (e.g. 7 - 12 m/s in prototype). This velocity decreases if a ship is present in a lock because the wet cross-section has been decreased and because the ship moves vertically together with the water-level. The periods of the external translatory waves in an half-open navigation lock are of the order of 0.5 - 2 minutes. The flow velocities induced by the waves are low and uniformly distributed over the wet cross-section.

External translatory waves are present in a lock with and without differences in density. External translatory waves can occur for different reasons during the start of the exchange process:
- Some time after the filling or emptying process external translatory waves caused by this process are still present because of the low damping of these waves.
- With the use of culverts for filling or emptying overtravel occurs with oscillations of the water-level after the filling or emptying.
- The gate is usually opened before the water-levels on both sides of the gate are equal. Translatory waves are then generated in the lock. Even with a pressure equilibrium over the valves external translatory waves arise in the case of differences in density, because there is no hydrostatic equilibrium between the cross-sections on both sides of the gate.
- With mitre gates translatory waves arise through the motion of these gates.
- During the exchange process long waves can penetrate into the half open lock from the approach harbour.

The result of the external translatory waves is a fluctuating longitudinal force on the ship. This force may be considerable in the case of a large blockage of the wet cross-section and cannot be ignored in any calculation of the total force on the ship. Examples of the
influence of external translatory waves on the forces of the ship during the exchange process are shown in section 2.8 (with figures 16 up to and including 19).

2.4 Exchange Flows without the Influence of Reflections

General

Internal wedges are generated in the case of differences in density between the water on both sides of the gate during and after the opening of this gate. In a lock with initially fresh water a salt wedge with a front as the leading edge propagates near the bottom of the lock. A reverse current of fresh water is present above this wedge. A wedge with a front as a leading edge also intrudes into a lock with initially salt water. However, the wedge is near the water-surface and the salt water flows beneath the wedge in the reverse direction. The water in front of the wedge will be at rest. A small deformation of the water-surface travels with the wedges and the fronts.

Fig. T1: The internal wedge after the opening of the lock gate

The origin of the internal wedges is the conversion of potential energy into kinetic energy. The loss of potential energy is due to the exchange of a volume of salt water at a higher level with a volume of fresh water which was at a lower level. The gain of kinetic energy results in an increase in the length of the flow.

Height and Velocity of the Fronts

The internal wedge has a front as the most advanced boundary between the water of the approach harbour and the water of the lock. The front consists of a head and a mixing zone near the interface behind the head. The mixing zone is caused by the large differences in flow velocities in and near the head.
Fig. T2: The front of an internal salt wedge

The velocity of propagation of the front is low (e.g. 0.5 – 1 m/s in prototype) and hardly changes in time. The front velocity of an internal salt wedge is somewhat lower than for an internal fresh wedge as a result of the friction at the bottom of the lock. The height of the front in the lock is roughly between 0.25 and 0.35 times the water-depth and hardly changes in time.

The gate is opened with a limited velocity. The gate is completely open after, for instance 2 to 4 minutes. The exchange process starts gradually with a low, circular front near the edge of the gate. Then the front becomes almost perpendicular to the axis of the lock at a short distance from the lock gate (e.g. twice the width of the lock).

Interface

The interface is the place where the vertical density gradient is at a maximum and can be considered as the separation between the fresh upper and the salt lower layer. The interface is at an angle to the horizontal plane and turns around a point at mid-depth at the opened gate. The heights of the fronts do not change much, so the angle of the interface to the horizontal decreases in time.

Actually the interface is not a plane but consists of a rather thin mixing layer between the salt and fresh water. The finite thickness of this layer is mainly caused by the mixing behind the front. The friction at the interface caused by the two opposite flows produces hardly any mixing.

Exchange Discharge

The exchange discharge is defined as the discharge of the salt or the fresh layer at the opened gate. This discharge is somewhat higher than the product of the front velocity, height of the front and width of the lock. This discharge is nearly constant in time because of the almost constant velocity and height of the front. Only increasing friction losses near the interface of salt and fresh water, the walls and the bottom cause a small decrease in the exchange discharge.
The discharge in each layer is almost equal to the exchange discharge because there is no resulting discharge in the lock. The mean flow velocities in the upper and lower layer are about inversely proportional to the thickness of the layer from the point of view of continuity.

A series of available literature, and the calculation of front velocities and heights in the exchange flow in a rectangular, prismatic channel are presented in section 5.2.

2.5 Influence of the Discontinuities in the Lock

Fig. T3: The salt wedge in a lock without the influence of a ship

When the front of the wedge which leaves the lock reaches the approach harbour, an internally critical flow situation occurs at the transition. The thickness of the layer which flows into the approach harbour diminishes considerably because the layer is radially spread over the larger width of the harbour. The wedge in the lock itself is hardly influenced by the approach harbour. This is in contrast to the external translatory waves with a negative reflection of the wave at an open boundary.

The internal wedge which propagates in the lock reflects completely against the closed gate, with about doubling of the thickness of the layer. After the reflection of the leading edge of the wedge, an internal wave travels to the approach harbour and a thin residual layer (e.g. 0.1 - 0.2 of the water-depth) will remain. This residual layer is near the bottom for an intruding salt wedge and near the water-surface for an intruding fresh wedge. After reflection the resulting flow velocities in both layers will be very low.

A certain period can be distinguished for a half-open lock. This period corresponds to four times the travelling time of the front from the closed gate to the approach harbour. When the internal wave reaches the approach harbour, the first half period is completed (e.g. after 20
minutes in prototype). After that, the residual layer will be exchanged with the water of the approach harbour. After about two or three half-periods the water in the lock will be completely exchanged with the water from the approach harbour.

The above-mentioned situations are presented in figure 1, situation I (lock with fresh water and an intruding salt wedge) and figure 2, situation X (lock with salt water and an intruding fresh wedge). A review of the available literature, and the calculation of exchange flows in a lock without the influence of a ship is presented in section 5.3.

2.6 Reflections against the Ship

Salt Wedge

First of all the salt wedge which intrudes into a lock with fresh water and a ship is considered (figure 6). The internal salt wedge, which starts at the opened lock gate, partially reflects against the bow of the ship (photos 3 - 9). It appears that the incident salt wedge produces three wave components at the reflection.

![Diagram showing salt wedge reflection](image)

Fig. T4: The flow situation near the bow after the passage of the front of the internal salt wedge

The first wave component consists of the component transmitted next to the bow. It is hardly hindered and propagates along the ship to the closed gate. This component is comparable to a situation without a ship. A fresh flow above this component flows to the approach harbour.

The second wave component is the component transmitted under the bow, which propagates to the closed gate in the space of the keel clearance. The height of the intruding salt layer is only a part of the height of
the keel clearance, because above this salt layer a fresh layer occurs, which flows in the opposite direction. The reason for the presence of this fresh layer under the ship is that the space beside the ship is usually insufficient for the fresh water to flow in the direction of the approach harbour, with a view to continuity. The fresh water below the underside of the ship escapes upwards next to the bow upwards and flows back in the direction of the approach harbour.

The third wave component is a reflected wave in front of the bow. This component propagates to the approach harbour and heightens the original salt wedge. The height of this reflected wave component depends on the possibility of transmission of the salt layer under the ship (second wave component).

The result of the reflection is a high level of the interface in front of the bow, a low level of the interface under the ship and an intermediate level of the interface next to the ship.

Near the stern, the heights of the salt layers under and beside the ship are already levelled across the width of the lock. The interface will be under the underside of the ship (photo 10). The greater part of the fresh water flows beside the ship in the direction of the approach harbour.

**Fresh Wedge**

![Diagram](image)

**Fig. T5:** The flow situation near the bow after passage of the front of the internal fresh wedge

Secondly the internal fresh wedge intruding into a lock with salt water is considered (figure 7). The reflection of an internal fresh wedge against the bow differs from the reflection of an internal salt wedge because of the different position of the ship with respect to the wedge. However, the reflection is also partial and the incident wedge also
produces three wave components.

The first wave component is the component of the fresh wedge transmitted beside the bow (figure T4). This component is hardly hindered by the ship and flows beside the ship in the direction of the closed gate. The second wave component is the continuous component of the fresh wedge at the bow. This second component travels along the bow to the space beside the ship and then travels, together with the first wave component, along the ship. The third wave component is the component of the fresh wedge reflected in front of the bow. This third component propagates to the approach harbour and lowers the original level of the interface at the bow.

The result of the reflection is a low level of the interface in front of the bow and a slightly higher level of the interface beside the bow. In the case of a ship with a small draught the interface can be just under the underside of the ship. A salt layer flows to the approach harbour at the bottom of the lock.

![Diagram](image)

**Fig. T6:** The flow situation near the stern after the front of the internal fresh wedge has passed

The interface at the side of the ship that is close to the wall of the lock will be somewhat higher than at the other side of the ship because of the influence of the wall friction in the narrow space between ship and wall. When the fresh layer passes the stern the flow separates from the ship because of the blunt shape of the stern. The result is an unequal level of the interface in transverse direction at the stern. The flow will be levelled across the width of the lock at some distance from the stern (e.g. twice the width of the lock). Then the height of the fresh layer will decrease.

**Front at the Ship**

The velocity of the front beside the ship decreases slightly because of
the friction at the walls, at the bottom of the lock and at the ship.

The position of the ship moored in the transverse direction in the lock is asymmetric, therefore the flow situation is asymmetric. The front which travels along the ship is usually somewhat oblique in transverse direction. The front in the narrow space beside the ship remains behind relative to that in the wide space beside the ship, because of the larger influence of the friction in the narrow gap.

**Reflection near the Stern**

The internal salt or fresh wave passes the stern and is subsequently completely reflected against the closed gate. An internal wave arises, which propagates in the direction of the approach harbour. This internal wave meets the stern of the ship and a reflection occurs similar to that at the bow (photo 11). The internal wave partially reflects against the stern which results in a transmitted and a reflected wave component.

**Exchange Time**

The quantity of water which must be exchanged in a lock with a ship is less than that in a lock without a ship. A ship in the lock partially blocks the wet flow profile, which results in reflections of the internal wedges against the ship. The blockage of the flow profile usually has a greater influence on the exchange time of the lock than on the smaller quantity of water which must be exchanged. The result is that the exchange process will be delayed by the presence of a ship.

The beginning of the exchange process of a lock with fresh water and an intruding salt wedge is shown in figure 1, situation II (small ship) and situation III (large ship). The reverse situation of the exchange process of a lock with salt water and an intruding fresh wedge is shown in figure 2, situation XI (small ship) and situation XII (large ship). The calculation of the exchange flows in a lock with a ship is presented in section 5.4.

**2.7 Flow and Density Distributions around the Ship**

**Interface**

We consider the intrusion of an internal salt wedge into a lock with fresh water and the experimental results of this situation (density profiles at the bow and the stern in figure 8).

An interface in front of the bow occurs from the moment the front of the wedge meets the bow. This interface rises further from the moment the small reflected wave from the approach harbour meets the bow. An interface will also be present behind the stern when the internal salt wedge passes the stern. This level will be much lower than that of the interface in front of the bow because a large part of the internal salt
wedge is reflected against the bow. The level of the interface behind the stern rises further from the moment the internal wedge meets the stern for the second time.

The result is that the level of the interface in front of the bow will continually be higher than the level of the interface behind the stern.

A similar situation occurs during the intrusion of an internal fresh wedge into an initially salt lock (figure 9).

The interface in front of the bow arises from the moment the internal fresh wedge meets the bow. When the internal fresh wedge passes the stern an interface will also be present behind the stern, but will be near the water surface because of the flow separation at the stern. The interface behind the stern descends much further from the moment the reflection from the closed gate meets the stern.

The result is thus that the level of the interface in front of the bow will continually be lower than the level of the interface behind the stern.

Differences in Density

It appears that the densities of the layers hardly deviate from the initial densities of the water of the lock and the approach harbour because of the limited mixing between the internal wedges. This results in more or less sharp interfaces (figures 8 and 9).

![Diagram](image)

Fig. T7: Examples of the mean density near the bow and the stern

\( t_1 = 1 \text{st passage bow, } t_2 = 1 \text{st passage stern} \)

\( t_3 = 2 \text{nd passage stern, } t_4 = 2 \text{nd passage bow} \)

The pattern of the cross-sectionally averaged densities can be derived from the level of the interface and the densities of the layers (figures T7, 12 and 13). After a short time the mean density of the water in front of the bow almost reaches the density of the water of the approach harbour. The mean density of the water near the stern remains the same as the initial density of the lock for a long time, especially for the
initially salt lock chamber. The moments of reflection are clearly visible in the mean density.

In the case of an intruding salt wedge in an initially fresh lock the mean density of the water in front of the bow is continuously higher than behind the stern. A similar conclusion can be drawn about the opposite situation of an intruding fresh wedge in an initially salt lock: the mean density of the water in front of the bow is continuously lower than at the stern.

Differences in Water-Level

The differences in water-level are important for the magnitude of the forces on the ships. It appears that the lowest water-level normally occurs near the salter cross-sections. The differences in water-level increase with a greater blockage of the ship. The explanation for this is given in chapters 4 and 5 with the aid of the momentum and continuity equations.

Flow Velocities

The flow velocities in the layers are also important for the forces on the ship. Unfortunately these velocities have hardly been measured during the experiments. It can be assumed that, until reflection occurs, the flow velocities are of the order of the front velocities. On the other hand the flow velocities of the layers are much smaller after the wave reflection against a strongly blocking ship or the closed gate.

2.8 Forces on the Ship

General

Fig. T8: Positive directions of the forces

The horizontal mooring force is composed of a longitudinal force, a transverse force on the bow and a transverse force on the stern. The longitudinal force is positive in the direction of the closed gate and negative in the direction of the approach harbour. The transverse force is positive in the direction of the widest space between lock wall and ship. The components of the force are made dimensionless with the weight
of the ship and are expressed in per mil (o/oo).

Longitudinal Force

![Diagram showing longitudinal force over time]

Initially fresh lock

Initially salt lock

Fig. T9: Examples of the longitudinal force

\( t_1 = 1\text{st passage bow, } t_2 = 1\text{st passage stern} \)

\( t_3 = 2\text{nd passage stern, } t_4 = 2\text{nd passage bow} \)

In the case of an intruding salt wedge in an initially fresh lock (figure 16, prototype investigation and 20, hydraulic model investigation), the longitudinal force is continuously negative. The force starts to increase from zero at the moment the front first passes the bow and increases very rapidly to its maximum value, which is caused by the reflection of the wedge against the bow. The force decreases a little after a short time because the front passes the stern. The force decreases considerably from the moment that the reflection against the closed gate reaches the stern. After this moment a small force remains for a long time.

A similar situation occurs in the case of an intruding fresh wedge in an initially salt lock (figure 17, prototype investigation and 21, hydraulic model investigation). There are two differences. The value of the force is continuously positive. The force hardly decreases at the moment that the front first passes the stern, which is caused by the flow separation at the stern.

It appears that the direction of the force is directed towards the saltier part of the lock, which corresponds to the part of the lock with the lowest water-level (explanation in chapters 4 and 5).
Homogeneous effects on the longitudinal force are also measured during the prototype investigation. These effects concern the equalizing of the water-levels just after the beginning of the opening of the gate and, later on, the penetration of long waves out of the approach harbour (figures 16 and 17 with oscillations in the force with a period of about 2 minutes).

The measured values of the longitudinal force caused by differences in density are considerable. The maximum value of the relative longitudinal force is - 0.8 o/oo in the case of an intruding salt wedge in an initially fresh lock and 0.6 o/oo in the case of an intruding fresh wedge in an initially salt lock. (Relative differences in density 0.013 to 0.016, figures 16 and 17, prototype investigation).

The permitted value of the relative longitudinal force is maximal 1 o/oo in the case of ships for inland navigation (water-displacement between 600 and 10,000 tons). The permitted value for large sea-going ships is much lower and decreases with the water-displacement (for instance 0.25 o/oo for 40,000 DWT). (Vrijer, 1977, lit.[31])

It appears that the longitudinal force increases with a greater difference in density between the lock and the approach harbour and sometimes with a greater blockage. The maximum longitudinal force hardly changes if the ship is at the back of the lock instead of just behind the opened gate; only the various moments change. The longitudinal force hardly changes in the case of a lower velocity of the gate. The force only decreases with extremely low velocities of the gate.

Transverse Forces

The wider space between ship and lock wall is more quickly exchanged than the narrower space between ship and the other lock wall. In the case of an intruding salt wedge in an initially fresh lock (figure 12) the wider space will be more salt than the narrower space . The measurements show positive transverse forces (figure 16 and 20). The direction of these forces is towards the saltier part of the space beside the ship. The passages of the fronts are clearly visible in the time history of the forces. The peak in the force during the period that the internal wedge passes the bow for the second time is remarkable. The measured maximum transverse forces are 0.25 o/oo each (figure 16 and 20). These values are quite large with respect to the permitted values. The permitted transverse force is limited for a lock for inland navigation (e.g. 0.3 o/oo) because of the method of mooring with springs in the longitudinal direction. The permitted value of the transverse force for a sea lock is also limited.

The initially salt lock gives similar results; only the density situation is opposite (figure 17). The transverse forces are now negative (in the direction of the narrower space between ship and lock wall) and possess the same maximum values. The negative force pushes the
ship towards the lock wall and is irrelevant for the mooring system of the ship.

2.9 Effect of an Air Bubble Screen

Purpose

The usual purpose of the air bubble screen is to delay the exchange process in the lock thus decreasing the intrusion of salt water into the fresh water canal. (Abraham, 1973, lit. [2]). In this investigation the purpose of the air bubble screen is different, namely to decrease the forces on the ship due to the differences in density.

The openings of the air bubble screen are situated in the bottom of the lock near the initial separation of the salt and fresh water, which is at the opened gate. A picture of the prototype measurements is shown in photo 2.

Flow Pattern and Differences in Density

If no differences in density are present, an air bubble screen generates two eddies with horizontal axes on both sides of the screen because of the vertical flow of the air discharge. The eddies turn in opposite directions. The screen generates a great deal of mixing.

![Diagram of flow pattern around the air bubble screen]

Figure T10: Flow pattern around the air bubble screen

An exchange flow in combination with an air bubble screen generates the following flow pattern with sufficient air discharge by the screen. The eddy at the fresh side turns against the direction of the exchange flow. The eddy at the salt side disappears because it turns in the same direction as the exchange flow. The result is that the water of the exchange flows must flow via the eddy at the fresh side. The salt flow passes the eddy along the water-surface, the fresh flow passes the eddy along the bottom. Meanwhile the exchanging water is mixed.

The effect of the air bubble screen is that the internal salt wedge, which intrudes into an initially fresh lock, has a lower density than the salt water in the approach harbour. The intruding internal "fresh" wedge in an initially salt lock has a higher density than the fresh
water of the approach harbour. (prototype investigation: figures 10 and 11, density profiles and figures 14 and 15, mean density). Due to the mixing the thickness of the interface becomes considerable.

After some time part of the contents of the lock has been exchanged and the differences in density become less. Then the homogeneous flow pattern (without differences in density) will dominate and two eddies will turn on both sides of the screen. Hence the possibility of exchange is diminished further on and a certain small difference in density over the screen remains (figure 14 and 15). The whole contents of the lock are gradually mixed by the eddy at the open lock side with a diminishing of the stratification of the lock (figures 10 and 11).

The patterns of the beginning of the exchange process of a lock with the use of an air bubble screen are presented in figure 1, situations IV, V and VI (initially fresh lock and salt approach harbour) and figure 2, situations XIII, XIV and XV (initially salt lock and fresh approach harbour).

Forces

The forces on the ship with the application of an air bubble screen are lower than it (figures 18 and 19 with respect to figures 16 and 17). The measured forces are about halved, which is due to the reduced differences in density between the intruding wedge and the original water of the lock. The calculation of the exchange flows of a lock with a ship is presented in section 5.4.

2.10 Investigation and Situation in Practice

The starting points and the results of the investigation can be compared to the situation in practice.

Permitted Forces

The forces on the ship induced by the exchange flows which are determined in this investigation are considerable. The forces can be more than those permitted, especially in the case of large sea-going vessels, because these ships have mooring lines with a (relatively) small break strength. Therefore it is important to develop measures to limit the forces.

Initial Differences in Density in the Lock

The investigated situation comprises the exchange of the water of a lock with homogeneous density with the water of an approach harbour. There is a maximum difference in density between the water of the lock and that of the approach harbour.
The initial situation under investigation in the lock does not occur frequently in practice. There are various reasons for this:
- The gate is only opened for a ship to enter or leave, to limit the salt intrusion or the loss of fresh water. This implies an incomplete exchange of the lock with a residual stratified situation in the lock.
- The stratification increases during the filling of the lock.
- The differences in density decrease because of the mixing brought about by the propellers of the ship which is leaving or entering.

The situation is usually less extreme in practice than that under investigation. Therefore the forces on the ship will mostly be lower in practice than those determined in this investigation.

Other Phenomena

A combination of the exchange process with other phenomena can occur:
- The forces not caused by density differences during the exchange process (section 2.3) must be taken into account in many cases.
- The internal, long waves which are generated during the filling process (next chapter) interact with the intruding wedges of the exchange process.

A combination could perhaps yield higher forces on the ship. In practice, a combination can occur. However, such combinations are not considered in this investigation.

Measures for Reducing the Force

It appears from the measurements that the best measure to reduce the forces on the ship during the exchange process is the application of an air bubble screen.

Forces can also be limited if the gate is opened for as short a time as possible. Then the exchange of the water of the lock with that of the approach harbour is reduced.

Lowering of the velocity of the gate hardly influences the forces on the ship. Mooring the ship at the back of the lock only delays the forces.
3 Physical Description of the Filling Process with Density Differences

3.1 Introduction

The filling of a navigation lock begins with the gradual opening of the filling openings by means of valves. The water of the approach harbour flows into the lock as separate jets through the openings. The intruding water mixes with the original water in the lock and the jets are broken down in the downstream direction. External translatory waves caused by the filling discharge are generated, propagate in the lock and reflect against the gates and the ship. The water-level rises during the filling process. When the water-level of the approach harbour corresponds with the water-level in the lock, the lock process is completed.

A ship moored in the lock experiences hydraulic forces caused by the external translatory waves and the jets. The filling system is so designed that the time necessary for the filling process is as small as possible and at the same time the hydraulic forces on the ship do not exceed those permissible. There is a great deal of literature, as well as experimental results and mathematical models available for the design of the filling systems without the influence of differences in density.

The description of the above-mentioned hydraulic situation in the lock must be extended if a difference in density exists between the water of the lock and that of the approach harbour. An intruding layer is generated, which propagates in the lock and which reflects at the ship and the gates. The intruding layer is partially blocked by the ship in the lock. The consequence is that considerable differences in density are present over the length of the ship. These differences in density induce forces on the ship. This implies an adjustment of the filling system to prevent the forces on the ship from exceeding those permissible.

The flow and density patterns in the lock during the filling process are complex, just like during the exchange process. These patterns, and the forces on the ship induced by differences in density, cannot be derived in a simple way using mathematical models. There is almost no literature available concerning the influence of differences in density during the filling process. Therefore experimental investigations have been done first to understand the physical phenomena caused by the differences in density.

As in chapter 2, the aim of this chapter is to describe and to explain the principal hydraulic phenomena and the forces on the ship during the filling process, with the help of experiments. After that, it will be possible to develop a mathematical model in chapter 6. The description pertains to the experiments, therefore other situations may yield other results.
This chapter is presented with the help of the particulars of a prototype investigation and a scale model investigation which are briefly described in section 3.2. After that, the principal flow aspects under homogeneous conditions (section 3.3) are considered. Then the flow under non-homogeneous conditions without the influence of a ship is presented: with the jet zone in section 3.4 and with the intruding layer in section 3.5. The influence of the ship is discussed in sections 3.6 and 3.7 respectively for ships with a limited and a large blockage of the cross-section. Finally the density aspects of the filling process are compared to other aspects (section 3.8).

3.2 Experiments

The investigation into the hydraulic phenomena has been carried out using prototype and scale model examinations. During these measurements various parameters have been varied. The principal data of the investigations are as follows.

Tests have been carried out with a lock with initially fresh water and an approach harbour with salt water, with a lock with initially salt water and an approach harbour with fresh water and without differences in density between lock and approach harbour. The difference between the filling situation with and without differences in density yielded the influence of the differences in density.

The prototype investigation concerns the same lock and ship as used for the prototype investigation of the exchange process (section 2.2, figure 3, length * width = 154 * 16.1 m²). The initial head differences varied between 1.15 and 1.70 m, the relative differences in density varied between 0.013 and 0.014. The value of the initial blockage of the cross-section of the lock by the ship varied between 0.39 and 0.49. The horizontal distance between the bow and the gate measured about 15 m. The filling time of the lock was about 9 minutes. The shape of the filling openings is shown in figure 22.

The hydraulic model investigation concerns the lock near Hansweert (the Netherlands) for push-tow units with a scale 1:40 and with prototype dimensions 280 * 24 m² (figure 5). The initial head difference measured 1.50 m and the relative difference in density 0.02. The filling time of the lock was about 9 minutes. A "4+1" push-tow unit was used (153 * 22.80 * 3.90 m³, prototype) with a distance between the gate and the bow of 2.60 m (prototype). The initial blockage of the transverse profile of the lock by the ship was 0.81. The shape of the filling openings is given in figure 23.
3.3 Filling Systems and Homogeneous Flow Aspects

Filling Systems

The filling system is usually designed in such a way that the filling time will be short and the maximum permissible forces on the ships will not be exceeded. The filling systems often used for navigation locks in tidal areas in the Netherlands are the gate-filling system, the culvert system and the system with bottom- and side-wall filling.

The lock can be filled by means of the gate filling system with openings in the upstream (i.e. filling) gate which are gradually opened with valves. This lock is meant for inland and small sea-going vessels with limited head differences. In the openings of the gate vertical or horizontal beams or guiding vanes are placed to reduce the influence of the filling jets on the ship (e.g. figure 23).

The system with culverts in the heads of the lock is mostly applied for large sea-going vessels (e.g. sea-lock IJmuiden, the Netherlands). For the filling of the lock the culverts in one head (the upstream head) are used. Much attention is paid to a good shape of the outflow of the culverts in the lock. The influence of filling jets is small.

The use of the culverts over the length of the lock is another possibility as regards the filling system. This is applied in situations with large head differences and/or differences in density. The water enters the lock via both heads (sea lock Baalhoek, the Netherlands) or via two perforated outlets (grids) in the bottom (sea lock Terneuzen, the Netherlands). Another system with separate bottom- and side-wall filling is used as a salt/fresh water separation system (Krammer locks, the Netherlands, with system Dunkerque). The latter systems are not the theme of this study, because it is restricted to head-filling systems (filling from one head).

Discharge and Water-Level

![Graph showing discharge, water-level, and area of the opening as functions of time](image)

Figure T11: Example of the discharge, water-level in the lock and area of the opening as functions of time
The discharge through the openings is controlled by the lift programme of the valves. The area of the openings increases in time until the maximum area is reached. The discharge through the openings is the product of the area of the openings, the discharge coefficient of the openings and the flow velocity at the maximum contraction. The flow velocity is determined by the local, instantaneous head difference over the openings. The result is a filling discharge which increases in time until a certain maximum value is reached and then decreases to zero when the lock has been filled.

The rising of the water-level is determined by the discharge through the openings and the horizontal area of the lock.

**Longitudinal Force**

The ship moored in the lock undergoes hydraulic forces. The hydraulic force on the ship is mainly a longitudinal force in the case of a gate filling system because the principal flow direction is in the longitudinal direction of the lock. The longitudinal force is caused by the one-dimensional, external translatory waves and by the three-dimensional effects of the flow (figure 24). (Schijf, 1936, lit. [21] and Kolkman, 1973, lit.[14])

The one-dimensional external translatory waves are generated by the unsteady filling discharge. This wave is determined by a number of contributing factors:

- The change of the filling discharge in time generates disturbances of the water-surface which propagate with the wave velocity in the lock. These disturbances reflect against the ship and the gates. The disturbances are coupled with adaptations of the flow velocities (and discharges) in the cross-sections. The result is an alternating contribution to the force on the ship.

- The mean discharge in time in a cross-section decreases in the direction of the downstream gate because a smaller part of the lock must be filled. This gives a negative contribution to the force of the ship (i.e. a force directed to the upstream gate).

- The friction of the water flowing along the wall and bottom of the lock and the ship's hull yields a positive contribution to the force (i.e. a force directed to the downstream gate).

The one-dimensional, external translatory waves with the influence of the ship can be computed (Kalkwijk, 1973, lit.[11] and de Jong/Vrijer, 1981, lit.[10]).

The three-dimensional flow effects are caused by the filling jets and the separation of the flow at the stern of the ship:

- The water intrudes into the lock by means of concentrated filling jets. The velocities in the filling jet decrease in the downstream direction. The result is a negative contribution to the force because of the lowering of the water-level above the filling jets and a positive contribution to the force because of the direct force of the
jet due to the impingement on the bow.
- The flow usually separates at the stern. This generates a positive
contribution in the force.
Calculations of the three-dimensional flow effects with the force
generated on the ship have yielded some success up to now.

The actual differences in water-level in the lock and the total
longitudinal force on the ship result from the above-mentioned
contributions. In the first part of the filling process the force is
mainly positive, in the second part mainly negative. If the ship is
outside the area of influence of the filling jets the three-dimensional
filling jet forces are nearly absent. An example of a time history of
the (relative) longitudinal force is shown in figure 36 for the scale
model investigation without differences in density and with a push-tow
unit in the lock. The schematized mathematical model LOCKFILL which
includes all the flow effects has been developed for the preliminary
design of a lock (Appendix B).

It appears that the most important parameters for the relative
longitudinal force are:
- initial head difference
- lift programme of the valves and the maximum area of the openings
- blockage of the wet cross-section caused by the ship
- distance between the upstream gate and the bow of the ship

3.4 Jet Zone

General

The intrusion of the water into the lock takes place with concentrated
filling jets. This section describes the hydraulic phenomena in the jet
zone between the filling openings and the place where the jets change
into a stratified flow. The behaviour of the jets in this jet zone is
important for the description of the intruding layer in the next
sections. The patterns presented are based on the literature and the
observations from the experiments of section 3.2.

Entrainment

Let us first consider the situation where a turbulent water jet intrudes
into still water with unlimited dimensions. The jet creates turbulence
due to the large differences in flow velocities with the ambient water
and mixes violently with the ambient water. The jet itself grows
thicker, the flow velocities of the jet decrease and the discharge of
the jet increases in the downstream direction. The effect of the
engulfment of ambient water by the jet due to turbulence is called
"entrainment". (Rajaratnam, 1976, lit.[19])
A different situation occurs in our investigation. A number of turbulent jets are situated beside each other in the lower part of the wet cross-section, the water is confined in the lock, differences in density are present between the water of the jets and the lock and the flow is unsteady.

The entrainment is reduced because of the presence of the boundaries of the cross-section and the nearest jets. The upper boundary is the water-surface. A gyre with a horizontal axis rotates above the jets. The water entrained into the upper side of the jets originates from this gyre. However, after passing the axis of the gyre, the jets give a part of their mixed water back to the gyre which results in a recirculation in the gyre. (Rajaratnam/Subramanya, 1968, lit.[18])

Entrainment of ambient water from the lower side of the jets is hardly possible because of the presence of the bottom. The filling jets are usually attracted to the bottom by the lack of entrainment. Downstream of the attachment point the flow resembles a plane wall jet. Entrainment in transverse direction is also not possible because of the proximity of the other jets.

The length of the turbulent gyre above the jets depends on the density of the water in the jets relative to the water of the lock. The gyre
will be longer for salt filling jets in an initially fresh lock than for fresh filling jets in an initially salt lock. The salt filling flow remains at the bottom as a non-buoyant flow. Fresh water from the downstream side flows to the upper side of the gyre whereby the gyre remains fresher than the jets (figure T12). On the contrary, a fresh filling flow is uplifted to the water-surface as a buoyant plume. (Wood, 1980, lit.[32], Sobey, 1988, lit.[29], Baddour, 1989, lit. [5]). The point of attachment at the water-surface determines the length of the gyre. Recirculation occurs in the salt gyre above the fresh jets which results in an almost fresh gyre. Salt water from the downstream side mixes with the rising current and cannot flow to the gyre (figure T13).

The differences in density between the water of the filling jets and that of the original water in the lock decrease considerably in the downstream direction because of entrainment and the turbulence of the jets.

Flow Velocities

The flow velocities are at a maximum at the point of maximum contraction in the filling openings. The velocity distribution is more or less uniform at that point.

A grid of beams is usually fixed at the downstream side of the filling openings. The flow impacts against these beams and generates a great deal of turbulence with related energy losses. Subsequently the flow leaves the openings with a very high level of turbulence, whereby the flow is divided over the entire area of the downstream side of the openings.

The core of the jets and the flow velocities decrease in the downstream direction due to the entrainment of the surrounding water. At a given distance from the origin the velocity profile can be represented as a more or less Gaussian curve. The level of turbulence remains high.

The flow meets with the bottom at a short distance from the filling gate. From that point the flow is only spread in the direction of the water-surface. The velocity profile obtains the form of a half Gaussian curve with the highest flow velocities near the lower part of the cross-section. At the bottom the flow velocities are zero.

Direction of the Jets

The direction of the filling jets with respect to the horizontal plane is determined by the shape of the openings and the attraction of the bottom because of the lack of entrainment at the lower side of the jets. The flow usually meets the bottom unless the openings give a strong upward lift. The initial direction of the jets is hardly influenced by differences in density because of the great momentum of the jets.
The Length of the Jet Zone

The momentum transport of the jets is determined by the product of discharge and flow velocity. The flow velocities start with a maximum value and decrease gradually in time because of the decreasing head difference. The discharge has its maximum value about half-way through the filling process and is small at the beginning and at the end of the filling process. Therefore until the moment of the maximum discharge the momentum transport is more or less constant, and later on the momentum transport decreases.

The end of the jet zone can be defined as the place where a fully developed flow profile is reached by which the velocity differences in a cross-section are limited. The length of the jet zone is proportional to the momentum transport of the filling jets. Therefore the influence of the jets is at a maximum with the longest jet zone occurring from the beginning of the filling process until the moment of the maximum discharge. However, a fully developed flow profile is never reached in a lock with differences in density because a two-layer current exists.

In our investigation the end of the jet zone is determined by the differences in density as well as the momentum of the jets. The flow velocities and the differences in density of the jets decrease in the downstream direction. At a certain point the mean flow velocities in the jets are so low that they correspond to the velocity of propagation of an internal wave. At this point the flow changes into an internally subcritical stratified flow, and from it a layer intrudes into the lock. This point can be considered as the end of the jet zone. The length of the jet zone is equal to the length of the gyre above the jets.

It is not possible to determine the length of the jet zone exactly. Lengths of twice or four times the water-depth have been observed.

3.5 Intruding Layer without the Influence of a Ship

Evolution of an Intruding Layer

This section is based on observations from the experiments of section 3.2. The flow in the downstream direction at the end of the jet zone is considered. The following remarks can be made:
- the flow consists of water from the filling discharge and from the
  entrainment of the jets
- the difference in density between the water of the flow and the
  original water of the lock is smaller than the initial difference in
  density between the water of the approach harbour and the lock.
- the discharge is larger than the filling discharge owing to the
  entrainment.
- the flow is more or less homogeneous.
At the end of the jet zone the flow rises to the water-level in an initially salt lock, and remains at the bottom in an initially fresh lock. At that point the flow changes into a two-layer flow. The water of the intruding layer flows in the downstream direction. The intruding layer has a front as leading edge which propagates in the downstream direction.

**Thickness of the Intruding Layer at the End of the Jet Zone**

The discharge and the flow velocity in the downstream direction at the end of the jet zone determine the thickness of the intruding layer at the end of the jet zone. The discharge at the end of the jet zone is proportional to the filling discharge and is a function of time. The flow velocity at the end of the jet zone is mainly determined by the differences in density of the water, and changes less in time. The result is a thickness of the intruding layer at the end of the jet zone which is comparable to the time history of the filling discharge.

In the jet zone the differences in density decrease and the discharge increases in the direction of the flow. The result is that the thickness of the intruding layer is considerable. Maximum thicknesses of 0.7 times the water-depth can occur.

**Flow Velocities**

The flow velocities in the intruding layer are roughly equal to the velocity of propagation of the front of the intruding layer. The flow velocity in the layer above or beneath the intruding layer will be restricted because a resulting discharge is present in each cross-section due to the filling of the lock. This is in contrast to an exchange flow and to the time after the filling process has been completed.

**Propagation in the Lock**

![Figure T14: Shape of the intruding layer](image)

Initially fresh lock       Initially salt lock

The length of the intruding layer is determined by the filling time of the lock and the velocity of propagation of the front. This length can be considerable, e.g. the length of the lock chamber. The total volume of the intruding layer is equal to the volume of the filling water plus
that of the entrainment. The shape of this intruding layer in the lock roughly coincides with the time history of the discharge. A residual layer remains behind the intruding layer.

The intruding layer reflects against the downstream gate causing, roughly, a doubling of the layer thickness, and reflects later on against the filling gate. Long after filling is completed the intruding layer is still visible as a deformation of the interface. (figures 25 and 28).

The beginning of the filling process of a lock without a ship is shown in figure 1, situation VII (lock with fresh water and approach harbour with salt water) and figure 2, situation XVI (lock with salt water and approach harbour with fresh water).

3.6 Influence of a Ship with a Limited Blockage

Flow in front of the Bow

This section is based on observations from the experiments in prototype and in the scale model (section 3.2).

![Diagram of Bow outside and within the jet zone](image)

Bow outside the jet zone  Bow within the jet zone

Figure T15: Flow pattern in front of the ship (initially fresh lock filled with salt water)

Firstly we consider the situation with a distance between the bow and the filling gate which is greater than e.g. 4 times the water-depth. The bow will be outside the jet zone. An intruding layer arises at the end of this zone. The initial flow and the entrainment in the jet zone correspond to the situation without a ship (section 3.4).

In the case of an intruding salt layer and the bow outside the jet zone, the layer partially reflects against the lower side of the bow (figure 26). The layer reflects at the filling gate, travels again in the downstream direction to the bow and partially reflects once more at the bow. Consequently the interface in front of the ship rises.
The case of an intruding fresh layer and the bow outside the jet zone shows a somewhat different pattern (figure 29). The amplitude of the wave reflected at the bow is larger than in the above-mentioned situation because the ship is near the water-surface. The reflected layer reflects at the filling gate, reflects again at the bow, etc. The whole volume of the zone between the filling gate and the bow gradually becomes fresher.

Subsequently we look at the situation with the bow within the jet zone. The flow differs from that mentioned above. A zone can be distinguished between the filling gate and the bow which acts more or less as a mixing zone. The filling jets intrude into this zone. A turbulent gyre is present at the upper side of the zone. Water from beside the ship is entrained into the jets. The highest flow velocities occur within the zone between the filling gate and the bow. An intruding layer leaves the zone at the bow. The entrainment of water into the filling jets is hindered by the presence of the ship.

Similar intruding layers develop in the cases where the bow of the ship is in the jet zone or downstream of it. The intruding layer occurs at the bow and the front of the intruding layer travels in the downstream direction. The difference in density between this layer and the ambient water will be larger if the bow is in the jet zone because the entrainment is reduced in that case.

Flow behind the Bow

The transmitted salt layer in an initially fresh lock propagates along the bottom of the lock in the downstream direction, passes the stern, reflects at the downstream lock gate and reflects at the stern. The interface behind the stern then rises. At the end of the filling process, when the filling discharge decreases, fresh water flows from the back to the front of the ship and the interface in front of the bow drops. After filling the intruding layer continues to propagate in the lock and an oscillating interface arises in the lock (figure 26).

The transmitted fresh layer in an initially salt lock propagates mainly beside the ship. After passing the stern the thickness of the intruding layer behind the stern decreases, because the layer is spread over the width of the lock. Subsequently, this layer reflects at the downstream gate and the stern. At the end of the filling time salt water flows along the bottom in the direction of the filling gate, because the filling discharge has decreased (figure 29).

The beginning of the filling process of a lock with a small ship is shown in figure 1, situation VIII (lock with fresh water and approach harbour with salt water) and figure 2, situation XVII (lock with salt water and approach harbour with fresh water).
Differences in density

Initially fresh lock

Initially salt lock

Figure T16: Mean density near the bow and the stern as functions of time

\( t_1 = \text{1st passage bow}, \quad t_2 = \text{1st passage stern}, \quad t_{ef} = \text{filling time}, \quad t_3 = \text{2nd passage stern} \)

In the case of an initially fresh lock the intruding lower layer is brackish because of the entrainment in the jet zone (figure 31, density profiles prototype investigation with data in section 2.3). The interface in front of the bow will be high from the beginning of the filling process because of the reflections at the bow and the filling gate. The interface behind the stern, after passing of the front, is near the bottom and rises slowly. The mean density of the water near the bow is higher than that near the stern (figure 33).

After the filling process is completed the situation is just the opposite. The interface in front of the bow has dropped and that behind the stern has risen. The largest mean density is near the stern. Subsequently, between the bow and the stern, an oscillating difference in density arises that diminishes in time.

The opposite case shows a similar pattern. In the case of an initially salt lock the intruding upper layer also becomes brackish because of the entrainment in the jet zone (figure 32, density profiles prototype investigation). All the water in the zone in front of the bow will be brackish soon after the beginning of the filling process. The interface near the stern descends slowly after some time. The mean density of the water near the bow is lower than that near the stern (figure 34).

After the filling process is completed the situation is the opposite. The interface near the stern has risen and that near the bow has dropped. The greatest mean density is near the bow. An oscillating difference in density between bow and stern arises, that diminishes in time.
Relative Longitudinal Force

The influence of the differences in density on the longitudinal force can be obtained by determining the difference between the force under non-homogeneous conditions and that under homogeneous conditions.

The relative longitudinal force under homogenous conditions which was measured during the prototype tests (data in section 3.2) is very low ($\approx 0.1 \text{ o/oo}$). This is due to the limited blockage of the ship, the rather great distance between the filling gate and the bow and the relatively small filling discharge.

![Diagram](image)

Initially fresh lock Initially salt lock

Figure T17: Longitudinal force due to differences in density ($t_{ef} = \text{filling time}$)

In the case of an initially fresh lock the relative longitudinal force caused by the differences in density is negative during the whole filling process. This component of the force is directed to the salter part of the lock, namely to the bow, and the time history of the force is similar to that of the filling discharge (figure 35, maximum force of about $-0.30 \text{ o/oo}$). After the filling process is completed the force becomes positive and fluctuates with the period of an internal wave in the lock. The amplitude of the force decreases quickly.

The case of an initially salt lock shows the same picture. However, the component of the relative longitudinal force caused by differences in density is now positive during the filling process and negative thereafter (figure 35, maximum value of about $+0.35 \text{ o/oo}$).

The transverse forces are very low (less than $0.1 \text{ o/oo}$).
3.7 Influence of a Ship with a Large Blockage

Flow

This section is based on observations from the hydraulic scale model (section 3.2).

Initially fresh lock  Initially salt lock

Figure T18: Mixing zone between the filling gate and the bow

The flow pattern in front of the ship with a large blockage is different from the situation without a ship in the lock. The filling jets impinge on the bow of the ship and are bent to the sides and the bottom of the ship and to the turbulent gyre above the jets. The water behind the bow flows in the downstream direction. No water from beside the ship is entrained into the jets. From the beginning of the filling process the original water in the zone between the gate and the bow of the ship is mixed with the intruding water of the approach harbour. After a short time this zone is almost completely filled with water from the approach harbour. This is true for the fresh lock filled with salt water (photo 12), the homogeneous situation (photo 14) and the salt lock filled with fresh water (photo 16).

From this mixing zone between the gate and the bow, the mixed water flows past the ship. For an initially fresh lock the salt water flows mainly under the keel, but also beside the ship. For an initially salt lock the fresh water flows mainly beside the ship but also in the zone of the keel clearance (figures 27 and 30).

The front of the intruding layer propagates further past the ship and passes the stern (density situations, photos 13 and 17 and homogeneous situation, photo 15). The intruding salt layer in the keel clearance passes the stern unchanged. On the other hand in the case of an intruding fresh layer this layer flows upwards and decreases in thickness because it spreads over the width of the lock after it has passed the stern. Hereafter reflection at the downstream gate and the stern takes place. In most cases the filling process has then already ended.
For an initially fresh lock, at the end of the filling process, fresh water flows in the direction of the filling gate, and a fresh layer appears near the bow (figure 27). For an initially salt lock the situation is just the opposite: a salt layer develops at the bottom near the bow (figure 30).

The beginning of the filling process of a lock with a large ship is shown in figure 1, situation IX (lock with fresh water and approach harbour with salt water) and figure 2, situation XVIII (lock with salt water and approach harbour with fresh water).

**Differences in Density**

The differences in density in the lock will be great because entrainment into the filling jets is largely suppressed.

We look at the filling of an initially fresh lock with salt water. A short time after the beginning of the filling of the lock the density of the water in front of the bow almost reaches the density of the approach harbour throughout the whole water-depth. The density of the water behind the stern remains the same as the initial density of the lock for a long time. The difference in density over the length of the ship is then at a maximum. After that a layer of salt water appears behind the stern.

At the end of the filling process fresh water flows towards the zone in front of the bow, resulting in a layer of fresh water. The thickness of this layer increases in time, and consequently the interface drops. Meanwhile the thickness of the layer of salt water behind the stern also increases and consequently the interface rises. The difference in density over the length of the ship continues to change for a long time after filling is completed.

The distribution of salt and fresh water in an initially salt lock is exactly the opposite.

**Relative Longitudinal Forces**

The relative longitudinal force on the ship in the homogeneous situation which was measured during the scale model tests (data in section 3.2) is considerable. This is caused by the large blockage and the small distance between the filling gate and the bow (figure 36, between 0.83 and -0.37 o/oo).
Initially fresh lock

Initially salt lock

Figure T19: Longitudinal force due to differences in density
\( t_{ef} = \text{filling time} \)

In the scale model with an initially fresh lock which was filled with salt water, the longitudinal force caused by differences in density increased gradually from zero to a maximum negative value (figure 36, about -0.3 o/oo). This force is directed to the salter part of the lock, namely to the bow. The maximum value remains for a very long time. After the filling has ended the force decreases very slowly.

The case of an initially salt lock which was filled with fresh water shows almost the same picture. The longitudinal force caused by differences in density is, however, positive (figure 36, maximum value about 0.4 o/oo).

3.8 Additional Remarks

Effects of Density Differences with Respect to Homogeneous Conditions

The same initial head difference for situations with and without differences in density yields somewhat different filling discharges because the pressure differences across the openings in the gate are not equal. This gives rise to a small adaptation of the longitudinal force under homogeneous conditions (section 2.3).

The wave velocity of the external translatory wave is much greater than the wave velocity of the internal wave. Therefore the effects of the external translatory wave can be separated in first approximation from the effects of the internal wave.

The breaking up of the filling jets is largely determined by the momentum of the jets and by the possibility of entraining surrounding water. The direction of the jets is largely determined by the shape of the filling openings. Therefore the lowering of the water-level above the filling jets and the impingement of the jet on the bow are almost the same in situations with and without differences in density.
Comparison of Exchange Process and Filling Process

The exchange discharge is practically constant over a long time and is determined by the differences in density and the water-depth. A salt or a fresh wedge intrudes into the lock. The wedge has an almost constant front height and an almost constant level of the interface at the opened gate (chapter 2).

The filling discharge is a function of time and is determined by the head difference, the area of the filling openings and the horizontal dimensions of the lock. The intruding layer which travels in the lock has about the shape of the discharge as a function of time (about a half sinus).

During the exchange process the differences in density in the lock are at a maximum because mixing between the layers is negligible. The differences in density between the bow and the stern and consequently the longitudinal force increase very rapidly when the front arrives at the bow.

During the filling process the differences in density in the lock and the longitudinal force caused by differences in density are limited if the blockage of the cross-section is small. If the blockage of the lock is considerable the difference in density of the water between bow and stern and the longitudinal force gradually increase to a maximum value.

In the case of the exchange process the component of the intruding wedge that is reflected against the bow practically vanishes when entering the approach harbour. However, in the case of the filling process the components of the internal wave which are reflected at the bow reflect completely at the filling gate.

Consequences for the Design Process

It appears that the longitudinal forces arising from the differences in density must be taken into account when designing the filling system of a lock. Their contribution to the total longitudinal force can be considerable.

In practice a stratified situation often occurs as the initial situation in the lock. Then the force induced by differences in density is less.

The entrainment into the filling jets decreases the differences in density in the lock. Considerable entrainment can be achieved by means of a limited blockage of the cross-section by the ship and a large distance between the bow and the gate. A smaller difference in density usually implies a larger thickness of the intruding layer. A smaller difference in density yields a lower force on the ship while a larger thickness of the intruding layer causes a higher force. Therefore it is not always true that an increase in the entrainment generates a lower force caused by differences in density. However, a small blockage or a large distance from the gate up to the bow will limit the force caused
by the filling jets.

Opening the valves of the filling openings more slowly decreases the filling discharge. In the case of a small blockage, the intruding layer which is generated is lower due to the lower filling discharge, and the longitudinal force on the ship decreases. In the case of a large blockage of the cross-section by the ship, the propagation of the intruding layer is hindered and large differences in density will occur over the length of the ship. Therefore the forces induced by density flows are hardly lower with a lower velocity of the valves.
4 Equations for the Longitudinal Force and the Water-level Difference

4.1 Introduction

We consider the situation of a stratified flow in a lock with a moored ship. The aim of this chapter is to calculate the longitudinal force on the ship as a function of the levels of the interface and the densities and the velocities in the two layers. The expressions derived are used for the determination of the longitudinal force during the exchange and filling processes (chapters 5 and 6).

This chapter is organised as follows. First of all the hydrostatic, longitudinal force on the moored ship is derived as a function of the levels of the interface, the densities of the layers and the difference in water-level around the ship (section 4.2). Subsequently the momentum equation is applied for the control volume of water surrounding the ship using the force function of section 4.2. The momentum equation yields the difference in water-level as a function of the level of the interface and the densities and the velocities in the two layers. After that the longitudinal force is calculated (section 4.3).

In section 4.4 some examples are presented to show the influence of the parameters. The summary and conclusions are stated in section 4.5.

4.2 Hydrostatic Longitudinal Force

4.2.1 Aim and Assumptions

The aim of this section is to derive the longitudinal force on the ship as a function of the known levels of the interface and the water-surface.

The density flows in a lock vary slowly and the related water velocities are relatively small. The deviations of the real pressure distribution around the ship from a hydrostatic pressure distribution will be small. Therefore it is assumed that the forces on a ship caused by differences in density can be determined with the help of a hydrostatic pressure distribution around the ship. The assumption is not quantitatively verified in this report because of the limited accuracy of the available measurements of the flow and density distributions around the ship.

In this section we consider the stratified case A according to figure T20. The interface is beside the ship. The levels of the interface and the water-surface have a step, the height and position of which are assumed to be known. The step in the water-surface is just above the step in the interface. Apart from this, the levels of the interface and the free surface are horizontal. The horizontal position of the ship is fixed. In vertical direction the ship can move freely but is at rest.
Figure T20: Case A: Both bow and stern immersed in lower layer

### 4.2.2 Vertical Position of the Ship

First of all the vertical position of the ship is determined. The inclination (or trim angle) of the ship in longitudinal direction is $\gamma$. The local draughts of the ship are $d_b$, $d_r$, $d_e$ and $d_h$ for the cross-sections at the bow, in front of the step of the interface, behind the step of the interface and at the stern (respectively). The horizontal distances in longitudinal direction in a fixed co-ordinate system are $x_b$, $x_r$ and $x_h$ for the bow, the step and the stern. The step in the water-level is $\Delta h_{bs}$ (respectively).

The geometry relations are:

\begin{align*}
    d_r - d_e &= \Delta h_{bs} \tag{4.1} \\
    d_e &= d_h - \gamma (x_h-x_r) \tag{4.2} \\
    d_b &= d_h + \Delta h_{bs} - \gamma l_s \tag{4.3}
\end{align*}

The hydrostatic pressures against the bottom of the ship are $p_b$, $p_r$, $p_e$ and $p_h$ for the bow, just in front of the step, just behind the step and the stern according to (respectively):

\begin{align*}
    p_b &= \rho_2 g d_b + (\rho_1-\rho_2) g (d_b-a_{2b}) \tag{4.4} \\
    p_r &= \rho_2 g d_r + (\rho_1-\rho_2) g (d_r-a_{2b}) \tag{4.5}
\end{align*}
\[ P_e = \rho_2 g d_e + (\rho_1 - \rho_2) g (d_e - a_{2s}) \]  
\[ P_h = \rho_2 g d_h + (\rho_1 - \rho_2) g (d_h - a_{2s}) \]  
with \( g \) = acceleration of gravity 
\( \rho_1 \) = density of the water of the salt layer 
\( \rho_2 \) = density of the water of the fresh layer 
\( a_{2b} \) = thickness of the fresh layer at the bow 
\( a_{2s} \) = thickness of the fresh layer at the stern 
\( a_{2b} < d_s \) and \( a_{2s} < d_s \)

The ship has a constant draught \( d_s \) over the length of the ship in still, homogeneous water with a density \( \rho_2 \). The vertical equilibrium of the hydrostatic force against the bottom and the weight of the ship can be described as (per unit of width of the ship):

\[ \frac{1}{4} (p_b + p_r) (x_t - x_b) + \frac{1}{4} (p_e + p_h) (x_h - x_t) = \rho_2 g d_s l_s \]  
with \( l_s \) = length of the ship

The equilibrium of the moments is derived relative to the centre of the ship. It is assumed that the moment is only determined by the vertical hydrostatic forces against the bottom of the ship according to:

\[ p_r (x_t - x_b) (x_m - x_t + \frac{1}{3} (x_t - x_b)) + \frac{1}{4} (p_b - p_r) (x_t - x_b) (x_m - x_t + 2/3 (x_t - x_b)) = \]
\[ p_e (x_h - x_t) (\frac{1}{3} (x_h - x_t) - (x_m - x_t)) + \frac{1}{4} (p_h - p_e) (x_h - x_t) (2/3(x_h - x_t) - (x_m - x_t)) \]

with \( x_m = x_b + \frac{1}{3} l_s \)

The nine equations (4.1) - (4.9) have nine unknowns \( d_b \), \( d_e \), \( d_h \), \( p_b \), \( p_r \), \( p_e \), \( P_h \) and \( \gamma \). These equations can easily be solved using a computer program.

### 4.2.3 Longitudinal Force

Subsequently the horizontal force on the ship can be derived. The horizontal force \( F_r \) is determined by the horizontal components of the forces on the bow, the bottom and the stern of the ship, according to:

\[ F_r = \left\{ \frac{1}{2} \rho_2 g d_b^2 + \frac{1}{2} (\rho_1 - \rho_2) g (d_b - a_{2b})^2 + \rho_2 g l_s d_s \gamma - \right\} b_s \]

with \( b_s = \) length of the ship

This expression (4.10) can be simplified with the help of the solution of equations (4.1) up to and including (4.9) and with the following
conditions. The inclination of the ship $\gamma$ will be small, and the difference in water-level $\Delta h_{bs}$ between the bow and the stern, will be small relative to the draught. This yields the following force equation:

$$F_r \approx \rho_2 \, g \, d_s \, \Delta h_{bs} + \frac{1}{2} \left( \rho_1 - \rho_2 \right) \, g \, (d_s - a_{2b})^2 - \frac{1}{2} \left( \rho_1 - \rho_2 \right) \, g \, (d_s - a_{2s})^2 \, b_s$$

(4.11)

The simplified equation (4.11) shows that the force is determined by the levels of the interface and the free surface at the bow and the stern, the cross-section of the ship and the densities of the water. The conclusion can be drawn that the longitudinal force is independent of the inclination of the ship. Calculations have been carried out with realistic examples which show that the force according to equation (4.10) agrees closely with the force according to equation (4.11).

A similar conclusion can be drawn with arbitrary levels of the interface and free surface between bow and stern. The conclusion for the homogeneous situation is presented by Kalkwijk (1973, lit.[11]).

The longitudinal force is made dimensionless by the weight of the ship and is expressed in per mil ($\circ/\circ$) according to:

$$F_r' \approx \frac{d_s \, \Delta h_{bs} + \frac{1}{2} \, \epsilon \, (d_s - a_{2b})^2 - \frac{1}{2} \, \epsilon \, (d_s - a_{2s})^2}{l_s \, d_s \, c_b}$$

(4.12)

with $F_r' =$ dimensionless longitudinal force (usually expressed in per mil)

$\rho_2 \, g \, c_b \, l_s \, d_s \, b_s =$ weight of the ship

$c_b =$ block coefficient of the ship

$\epsilon = (\rho_1 - \rho_2)/\rho_2 =$ relative density difference

4.2.4 Case B and C

Figure T21: Case B: Only the bow immersed in the lower layer; Case C: Bow and stern in the upper layer

Equations similar to equation (4.12) for the longitudinal force can be obtained for Case B with only the bow immersed in the lower layer (level of the interface against the bow and below the bottom of the ship) and for Case C with the bow and the stern in the upper layer (level of the
interface below the bow and below the stern, the homogeneous situation. These forces are also only determined by the situations at the bow and the stern:

\[ F' \approx \frac{d_s \Delta h_{bs} + \frac{1}{2} \varepsilon (d_s-a_{2b})^2}{l_s d_s c_b} \]  
\text{(case B)} \quad (4.13)

with \( a_{2b} < d_s \) and \( a_{2s} > d_s \)

\[ F' \approx \frac{\Delta h_{bs}}{l_s c_b} \]  
\text{(case C)} \quad (4.14)

with \( a_{2b} > d_s \) and \( a_{2s} > d_s \)

The case with only the stern immersed in the lower layer (interface below the bow and against the stern) corresponds to case B, if the bow and the stern are exchanged.

### 4.3 Difference in Water-level

#### 4.3.1 Aim and Assumptions

The aim of this section is to calculate the difference in water-level between the bow and the stern as a function of the thicknesses, the densities and the flow-velocities of the two layers.

![Figure T22: Case A: Both bow and stern immersed in lower layer](image)

We consider a schematized, stratified flow situation according to figure T22. The lock contains a two-layer fluid and a moored ship. The interface is beside the ship. A front (or step, jump) of the interface propagates past the ship. The levels of the interface on both sides of the front are horizontal. The fluid comprises two homogeneous layers of
different densities, separated by a sharp interface without mixing between the layers. External translatory waves are absent.

The following data are assumed to be known beforehand:
- levels of the interface beside the ship
- height and velocity of the front
- water-depth and flow velocity of the lower layer in front of the ship
- width of the lock
- dimensions of the ship

It is assumed that the flow is one-dimensional (horizontal and in the longitudinal direction of the lock) without flow separation from the bow or the stern. This implies flow velocities which are uniformly distributed over the layers. Furthermore it is assumed that the water-level does not rise. This is the situation in an exchange flow without any net flow rate in a cross-section.

The flow velocities are determined by means of equations for continuity. Subsequently the momentum equation is applied to the control volume without the ship. The term representing the hydrostatic longitudinal force of section 4.2 is incorporated into this equation. The momentum equation yields the difference in water-level between the bow and the stern.

4.3.2 Continuity Equations

The relevant symbols are defined as follows:
- \( v \): flow velocity of a layer
- \( a \): thickness of a layer

The flow-velocity of the lower layer is positive to the right and the flow-velocity of the upper layer is positive to the left.

The following subscripts are defined:
- \( b \): in front of the bow, \( r \): in front of the front, \( e \): behind the front, \( s \): behind the stern
- \( 1 \): lower layer 1, \( 2 \): upper layer

The subscript \( s \) is also used for the dimensions of the ship. The subscript \( k \) is used for the dimensions of the lock.

The equations for continuity are applied to the control volume without the ship (figure T22) for an exchange flow. It is assumed that the draught \( d_s \) of the ship is constant.

- Lower layer between \( b \) and \( r \):

\[
v_{1b} \ a_{1b} \ b_k = v_{1r} \ (a_{1b} \ b_k - (d_s - a_{2r}) \ b_s)
\]

with 
- \( b_k \): width of the lock
- \( d_s \): draught of the ship
- \( b_s \): width of the ship

\( (4.15) \)
\[ a_{1b} = a_{1r} \]

Lower layer between e and s:
\[ v_{1e} \left( a_{1s} b_k - (d_s - a_{2e}) b_s \right) = v_{1s} \left( a_{1s} b_k \right) \]
with \( a_{1s} = a_{1e} \)

- Lower layer between b and s:
\[ v_{1b} a_{1b} b_k = v_{1s} a_{1s} b_k + c_r \left( a_{1s} - a_{1b} \right) (b_k - b_s) \]

- Cross-section b:
\[ v_{2b} a_{2b} = v_{1b} a_{1b} \]

- Cross-section r:
\[ v_{1r} \left( a_{1b} b_k - (d_s - a_{2r}) b_s \right) = v_{2r} a_{2r} (b_k - b_s) \]

- Cross-section e:
\[ v_{1e} \left( a_{1s} b_k - (d_s - a_{2e}) b_s \right) = v_{2e} a_{2e} (b_k - b_s) \]

- Cross-section s:
\[ v_{2s} a_{2s} = v_{1s} a_{1s} \]

### 4.3.3 Momentum Equation

The momentum equation is applied in the horizontal direction to the control volume of the water. This control volume contains the contours of the ship (but not the ship itself). The boundaries of the control volume are cross-section b (just in front of the bow) and cross-section s (just behind the stern) (figure T22).

The momentum equation contains terms concerning the result of the external forces on the control volume, the transport of momentum through the boundaries of the control volume and the change of momentum in time of the control volume itself \((\delta (mv)/\delta t)\).

- **Hydrostatic force** \( F_{bs} \) against the boundaries of the control volume:
\[ F_{bs} = \frac{1}{2} \rho_2 g (a_{1b} + a_{2b})^2 b_k + \frac{1}{2} (\rho_1 - \rho_2) g a_{1b}^2 b_k - \]
\[ \frac{1}{2} \rho_2 g (a_{1s} + a_{2s})^2 b_k - \frac{1}{2} (\rho_1 - \rho_2) g a_{1s}^2 b_k \]

- **Force** \( F_s \) of the ship against the water of the control volume (derived from equation (4.11) with \( F_r = \) force of the water against the ship):
\[ F_s = -F_r = -\left( \rho_2 \, d_s \, (a_{1b} + a_{2b} - a_{1s} - a_{2s}) + \frac{1}{2} \, \left( \rho_1 - \rho_2 \right) \, (d_s - a_{2b})^2 \right) \, g \, b_s \]  

(4.23)

- Transport of momentum \( I_{bs} \) in longitudinal direction (through the boundaries of the control volume):

\[ I_{bs} = \rho_1 \, a_{1b} \, v_{1b}^2 \, b_k + \rho_2 \, a_{2b} \, v_{2b}^2 \, b_k - \rho_1 \, a_{1s} \, v_{1s}^2 \, b_k - \rho_2 \, a_{2s} \, v_{2s}^2 \, b_k \]  

(4.24)

- Change of the momentum \( I_t \) of the control volume in time: (the draught of the ship is assumed to be constant for this term)

\[ I_t = + \rho_1 \, v_{1e} \, c_r \, (a_{1s} \, b_k - (d_s - a_{2s}) \, b_s) - \rho_1 \, v_{1r} \, c_r \, (a_{1b} \, b_k - (d_s - a_{2b}) \, b_s) - \rho_2 \, v_{2e} \, c_r \, a_{2e} \, (b_k - b_s) + \rho_2 \, v_{2r} \, c_r \, a_{2r} \, (b_k - b_s) \]  

(4.25)

The momentum equation is as follows:

\[ F_{bs} - F_r + I_{bs} = I_t \]  

(4.26)

4.3.4 Result of the Solution

The following parameters in the above mentioned equations must be known beforehand:

\[ a_{1b}, a_{2b}, a_{1s}, v_{1b}, \text{and} c_r \]

To solve the equations the thicknesses of the fresh layers along the ship must be estimated according to:

\[ a_{2r} \approx a_{2b} \quad \text{and} \quad a_{2s} \approx a_{2s} \]

The 12 equations (4.15) up to and including (4.26) have as unknowns:

\[ a_{2s}, v_{1r}, v_{1e}, v_{1s}, v_{2b}, v_{2r}, v_{2e}, v_{2s}, F_{bs}, F_r, I_{bs}, \text{and} \ I_t. \]

The solution yields the difference in water-level between the bow and the stern according to:

\[ \Delta h_{bs} = a_{1b} + a_{2b} - a_{1s} - a_{2s} \]  

(4.27)

It appears that the change of momentum in time \( I_t \) is very small relative to the other terms of the momentum equation \( F_{bs}, F_r \) and \( I_{bs} \). This can be shown if the continuity equations (4.19) and (4.20) are substituted in equation (4.25). \( I_t \) then practically vanishes. Therefore the contribution of the change of momentum in time to the difference in water-level \( \Delta h_{bs} \) can be ignored.
The result is that the difference in water-level $\Delta h_{bs}$ is determined by the terms $F_{bs}$ (equation (4.22)), $F_r$ (equation (4.23)) and $I_{bs}$ (equation (4.24)). The first term $F_{bs}$ pertains to the hydrostatic force against the boundaries of the control volume. The second term $F_r$ comprises the hydrostatic force against the ship, which only depends on the situations at the bow and the stern. The third term is the transport of momentum through the boundaries of the control volume.

It can be concluded that the water-level difference $\Delta h_{bs}$ is practically only determined by the flow and density distribution at the bow and the stern. The flow situation beside the ship between the bow and the stern hardly influence this difference in water-level.

A similar conclusion can be drawn if the shape of the interface and the front between the bow and the stern are arbitrary. This is not taken into consideration in this report.

Two assumptions have previously been made. The first assumption comprises the one-dimensional two-layer flow without flow separation from the bow or the stern. Flow separation from the bow or the stern will hardly influence the flow between the bow and the stern. On the other hand flow separation considerably influences the transport of momentum at the boundaries of the control volume. This influence will be dealt with in the next chapters. The second assumption comprises the absence of a resulting discharge in each cross-section as an exchange flow without a raising of the water-level. It is shown in section 6.2 that an analogue conclusion can be drawn for the two-layer flow during the filling of a lock.

4.3.5 Simplified Solution

The solution can be simplified by deleting the equations concerning the flow situation between the bow and the stern (equations (4.15), (4.16), (4.19), (4.20) and (4.25)).

A further simplification can be achieved with flow velocities at the bow and the stern which are derived without a difference in water-level between the bow and the stern. Then the flow velocities can be calculated separately from the momentum equation.

The flow-velocities are derived from equations (4.17), (4.18) and (4.21) according to:

$$v_{1s} = \left( v_{1b} a_{1b} b_k - c_r (a_{1b} - a_{1s}) (b_k - b_s) \right) / (a_{1s} b_k)$$ \hspace{0.5cm} (4.28)

$$v_{2b} = v_{1b} a_{1b} / a_{2b}$$ \hspace{0.5cm} (4.29)

$$v_{2s} = v_{1s} a_{1s} / (h_k - a_{1s})$$ \hspace{0.5cm} (4.30)

with $h_k = a_{1b} + a_{2b}$
The difference in water-level $\Delta h_{bs}$ is derived from equations (4.22), (4.23), (4.24), (4.26) and (4.28). The term $I_c = 0$ and the second order terms are not taken into account. This yields:

$$\Delta h_{bs} \approx \frac{\frac{1}{2} \varepsilon \left[ \left( h_{k} - a_{2b} \right)^2 - (h_{k} - a_{2s})^2 \right] b_k - \left[ (d_s - a_{2b})^2 - (d_s - a_{2s})^2 \right] b_s + (I_{bs})/(\rho g)}{(b_k b_k - d_s b_s)}$$  \hspace{1cm} (4.31)

where $a_{2s} = h_{k} - a_{1s}$

$a_{2b} < d_s$ and $a_{2s} < d_s$

$I_{bs}$ according to equation (4.24)

The relative longitudinal force $F_r'$ can be derived by substitution of the difference in water-level $\Delta h_{bs}$ (equation (4.31)) in equation (4.12). The following is concluded from a lot of realistic calculations. The difference in water-level $\Delta h_{bs}$ which is determined without simplifications (solution of equations (4.15) up to and including (4.26)) corresponds very closely to the difference in water-level according to (4.31). Hence equation (4.31) is applied.

### 4.3.6 Cases B and C

![Diagram showing Cases B and C](image)

Figure T23: Case B: Only the bow immersed in the lower layer;
Case C: Both bow and stern in the upper layer

The difference in water-level for Cases B and C can be derived in the same way as mentioned in section 4.3.5 (Case A with the simplified solution).

The difference in water-level for case B is:

$$\Delta h_{bs} \approx \frac{\frac{1}{2} \varepsilon \left[ \left( h_{k} - a_{2b} \right)^2 - (h_{k} - a_{2s})^2 \right] b_k - \left[ (d_s - a_{2b})^2 - (d_s - a_{2s})^2 \right] b_s + (I_{bs})/(\rho g)}{(b_k b_k - d_s b_s)}$$  \hspace{1cm} (4.32)
where $a_{zb} < d_s$ and $a_{zs} > d_s$

The difference in water-level for case C is:

$$\Delta h_{bs} \approx \frac{\frac{1}{2} \epsilon ((h_k-a_{zb})^2 - (h_k-a_{zs})^2)b_k + (I_{bs})/(\rho_2 g)}{(h_k b_k - d_s b_s)}$$ (4.33)

where $a_{zb} > d_s$ and $a_{zs} > d_s$

The relative longitudinal force for Case B can be derived by substitution of equation (4.32) into equation (4.13). For Case C the relative longitudinal force can be derived by inserting equation (4.33) into equation (4.14).

### 4.4 Examples and Influence of Parameters

#### 4.4.1 Vertical Position and Hydrostatic Longitudinal Force

The vertical position of a ship is determined by the situation alongside the ship. This position can be calculated by means of the equations (4.1) up to and including (4.9) if the levels of the interface and the free surface alongside the ship and the densities of the layers are known.

The hydrostatic longitudinal force is determined by the situation at the bow and the stern. This force (e.g. equation 4.11) can be calculated if the levels of the interface and the free surface at the bow and the stern and the densities of the layers are known.

![Diagram](image)

**Figure T24: Example 1**

Example 1 is concerned with Case A with both the bow and the stern immersed in the lower layer (level of the interface against the bow and the stern). The position of the front is varied. The data are:

- fresh layer: $a_{zb} = 4 \text{ m}$ and $a_{zs} = 0$
- densities: $\rho_1 = 1020 \text{ kg/m}^3$, $\rho_2 = 1000 \text{ kg/m}^3$
- water-level: $\Delta h_{bs} = 0.10 \text{ m}$
- ship: $l_s = 150 \text{ m}$, $b_s = 20 \text{ m}$, $d_s = 6 \text{ m}$, $C_b = 0.8$, $x_b = 0$
Figure 37 shows the draughts of the ship at the bow, at the front and at the stern as functions of the horizontal distance from the front up to the bow. The inclination and the longitudinal force are shown in Figure 38. The inclination becomes zero if the front is just behind the bow or just in front of the stern. The inclination is at a maximum with the front in the centre of the ship ($x_r = 75 \text{ m}$). This maximum is given by:

$$
\gamma = \frac{3 (\Delta h_{bs} - \epsilon (a_{2b} - a_{2s}))}{2 \, l_s}
$$

(4.34)

The longitudinal force (Figure 38) is independent of the horizontal distance of the front.

### 4.4.2 Influence of Transport of Momentum

**Examples 2 and 3**

The influence of the transport of momentum on the difference in water-level between the bow and the stern and on the relative longitudinal force is demonstrated with the help of examples 2 and 3. There is no resulting discharge in any cross-section. The joint data are:

- densities of the water: $\rho_1 = 1020 \text{ kg/m}^3$, $\rho_2 = 1000 \text{ kg/m}^3$
- lock with water: $h_k = 10 \text{ m}$, $b_k = 25 \text{ m}$
- ship: $l_s \ast b_s \ast d_s = 150 \ast 20 \ast 6 \text{ m}^3$, $c_b = 0.8$

In example 2 a front propagates in the direction of the stern. The largest thickness of the lower layer is present at the bow. The height of the front is constant. The flow velocities in the lower layer are equal: the flow velocity of the salt layer at the bow corresponds to the value at the stern. The applied thicknesses and flow velocities of the salt layer are:

- $a_{1b} = 4 \text{ m}$ and $a_{1s} = 0$; $a_{2b} = 6 \text{ m}$ and $a_{2s} = 2 \text{ m}$; $a_{1b} = 8 \text{ m}$ and $a_{1s} = 4 \text{ m}$
- $v_{1b} = v_{1s}$ between 0 and 0.5 m/s

In contrast to example 2, for example 3 the most salt situation is present at the stern. The height of the front is constant. The front propagates in the direction of the bow. The flow velocities in the upper layer are equal: the flow velocity of the fresh layer at the bow corresponds to the value at the stern. The applied thicknesses and flow velocities of the fresh layer are:

- $a_{2b} = 8 \text{ m}$ and $a_{2s} = 4 \text{ m}$; $a_{2b} = 6 \text{ m} / a_{2s} = 2 \text{ m}$; $a_{2b} = 4 \text{ m}$ and $a_{2s} = 0$
- $v_{2b} = v_{2s}$ between 0 and 0.5 m/s
The flow velocities in the upper layer at the bow and at the stern are calculated with equations (4.29) and (4.30). The velocity of the front can be derived by means of equation (4.28) or similar equations; this velocity is not presented here. The difference in water-level between the bow and the stern is calculated by means of equations (4.31), (4.32) and (4.33) dependent on the level of the interface relative to the level of the bottom of the ship. The relative longitudinal force on the ship is calculated by means of equations (4.12), (4.13) or (4.14).

Result of the Calculation

The result of the calculation is presented in figure 39 (example 2) and figure 40 (example 3). The difference in water-level between the bow and the stern and the relative longitudinal force are shown as a function of the flow velocity of the salt layer for three combinations of thicknesses of the salt layer.

In example 2 with the largest thickness of the lower layer at the bow, the difference in water-level is negative, which corresponds with a lower water-level at the bow. The longitudinal force is negative, which corresponds with a force directed towards the bow.

The difference in water-level and the longitudinal force increase considerably with a greater flow velocity in the salt layer. The greater flow velocity implies a greater transport of momentum.

The difference in water-level and the relative longitudinal force also increase with greater thicknesses of the salt layer. For a low flow velocity the increase is limited, for higher flow velocities the increase is considerable. This increase is mainly caused by the greater transport of momentum due to the greater thickness of the salt layer.

In example 3 with the largest thickness of the lower layer at the stern, the difference in water-level between the bow and the stern and the longitudinal force are positive for a low flow velocity of the upper layer. The difference in water-level and the longitudinal force decrease and even become negative if the flow velocity of the upper layer increases. The difference in water-level and the relative longitudinal force are also influenced by the thickness of the fresh layer. This is
caused by the thickness itself and the changed transport of momentum.

It can be concluded that the difference in water-level between the bow and the stern and the (relative) longitudinal force are considerably influenced by the transport of momentum and also by the thicknesses of the layers. It depends on the case considered whether the difference in water-level and the longitudinal force increase or decrease with a larger flow velocity or momentum.

4.4.3 Influence of the Thickness of the Salt Layer

Example 4

The influence of the thickness of the salt layer on the difference in water-level and the relative longitudinal force is demonstrated with example 4. This example concerns a stagnant situation. A stratified fluid is present in front of the bow whereby the thickness of the salt layer varies. A homogeneous fresh layer is present behind the stern without a salt layer. There are no flow velocities. Two values are used for the draught of the ship. The particulars are as follows:

- Densities of the water: \( \rho_1 = 1020 \text{ kg/m}^3 \), \( \rho_2 = 1000 \text{ kg/m}^3 \)
- Lock with water: \( h_k = 10 \text{ m} \), \( b_k = 25 \text{ m} \), \( a_{ls} = 0 \)
- Ship: \( l_s = 150 \text{ m} \), \( b_s = 20 \text{ m} \), \( d_s = 2 \text{ m} \) and \( 8 \text{ m} \), \( c_b = 0.8 \)

![Diagram](image)

Figure T26: Example 4

Result of the Calculation

The result of the calculation is presented as a function of the height of the salt layer in front of the bow in figure 41. The lowest water-level occurs at the bow and the longitudinal force is directed towards the bow.

The difference in water-level and relative longitudinal force increase with a greater height of the salt layer in front of the bow because of the greater mean difference in density between the bow and the stern.
A larger draught of the ship generates a greater difference in water-level and a greater (absolute) longitudinal force (not visible in figure 41) due to the increased blockage of the wet cross-section. However, the relative longitudinal force can be greater or smaller.

4.4.4 Influence of the Water-depth

Example 5

The influence of the water-depth is demonstrated with example 5. This example comprises a stagnant situation with homogeneous water in front of the bow ($a_{2b} = 0$) and homogeneous fresh water behind the stern ($a_{1s} = 0$). Flow velocities are absent.

- Densities of the water: $\rho_1 = 1020 \text{ kg/m}^3$, $\rho_2 = 1000 \text{ kg/m}^3$
- Lock with water: $b_k = 25\text{m}$, $a_{2b} = 0$, $a_{1s} = 0$
- Ship: $l_s = 150\text{m}$, $b_s = 10$ and $20\text{m}$, $d_s = 5\text{m}$, $c_b = 0.8$

![Diagram](image)

Figure T27: Example 5

Result of the Calculation

The result of the calculation is presented in figure 42 which shows the difference in water-level and the relative longitudinal force as a function of the water-depth.

The water-level near the bow is lower than that near the stern because of the salt water near the bow and the fresh water near the stern. The longitudinal force is negative, the force is directed to the salt water near the bow. It appears that the value of the relative longitudinal force can be considerable in this example (up to -1 0/o).

In this example the thickness of the salt layer in front of the bow equals the water-depth; the water-depth varies. The difference in water-level and the relative longitudinal force increase with a larger water-depth. Two effects occur:
- The difference of the hydrostatic pressure forces between the cross-sections at the bow and the stern increases with the water-depth. This results in a larger longitudinal force.
- The blockage of the wet transverse profile decreases with a larger water-depth which results in a smaller longitudinal force. In this example the former effect is dominant.

4.4.5 First Estimation of the Longitudinal Force

It will be shown in the following chapters that in the case of a very large blockage of the wet cross-section of the lock by the ship the transport of momentum is small. This situation is comparable to a ship in stagnant water with an imaginary vertical separation between salt and fresh water:

![Diagram of imaginary gate separating the water near the bow and the stern](image)

**Figure T28: Imaginary gate separating the water near the bow and the stern**

In this case equations (4.31), (4.32) and (4.33) for the difference in water-level can be simplified by neglecting the transport of momentum term ($I_{bs} = 0$, equation (4.24)).

A first estimation of the maximum relative longitudinal force can be obtained by means of example 5 (section 4.4.4). The difference in water-level between the bow and the stern can be derived from equation (4.32), substituting $a_{2b} = 0$, $a_{1s} = 0$ and $I_{bs} = 0$:

$$\Delta h_{bs} = \frac{h_k^2 b_k - d_s^2 b_s}{h_k b_k - d_s b_s}$$

(4.35)

The substitution of equation (4.35) into equation (4.13) yields the relative longitudinal force:

$$P' = \frac{\frac{1}{2} \varepsilon}{1_s c_b} \left( d_s - \frac{h_k^2 b_k - d_s^2 b_s}{h_k b_k - d_s b_s} \right)$$

(4.36)

These equations can be used as first estimates of the difference in water-level and the relative longitudinal force.
4.5 Summary and Conclusions

In a two-layer situation with the interface alongside the ship the vertical position of the ship is determined by the levels of the interface and of the free surface alongside the ship, and by the difference in density between the two layers.

The hydrostatic longitudinal force on the ship is determined by the levels of the interface and of the free surface in front of the bow and behind the stern and by the difference in density between the two layers.

It can be demonstrated that the hydrostatic longitudinal force on the ship is not influenced by the vertical position of the ship - only the situations at the bow and the stern are important. The longitudinal force can be derived as a function of the levels of the interface and of the free surface at the bow and the stern and the difference in density.

In a stratified flow, the difference in water-level between the bow and the stern can be computed with the help of the momentum equation if the pressure distribution is assumed to be hydrostatic. The flow velocities, the levels of the interface and the difference in density must be known beforehand.

The momentum equation is applied to a control volume of water between the cross-sections just in front of the bow and just behind the stern. The ship itself is not included in the control volume. The equation includes four terms: the hydrostatic force against the boundaries of the control volume, the hydrostatic longitudinal force of the ship, the transport of momentum through the boundaries and the change of momentum of the control volume in time.

It can be proved for an exchange flow that the difference in water-level is not influenced by the term comprising the change of momentum of the control volume in time; the difference is determined by the three other terms. Therefore, the difference in water-level is only determined by the flow situation at the bow and the stern; the flow situation between the bow and the stern has no influence.

After the calculation of the difference in water-level caused by the stratified flow, the hydrostatic longitudinal force on the ship can be calculated.

In a stratified flow, both the difference in water-level between the bow and the stern and the longitudinal force are considerably influenced by the flow velocities in the layers and hence by the transport of momentum. The longitudinal force can increase, decrease or change direction dependent on the change in the flow velocities (or transport of momentum).
In a stagnant, stratified situation the lowest water-level is present near the most salt profile at the bow or the stern. The relative longitudinal force is directed to this most salt profile. The difference in water-level and the relative longitudinal force increase in the case of a greater mean difference in density between the profiles near the bow and the stern. This can be caused by another level of the interface or another difference in density. The difference in water-level and the relative longitudinal force are also influenced by the water-depth and the draught and the blockage of the ship.
5 Calculation of the Exchange Process

5.1 Introduction

The longitudinal force on the ship moored in the lock is determined by the flow and density patterns around the ship (as mentioned in the previous chapter) and the variations in the water-level. These flow and density patterns concern the position of the interface, the density differences and the flow velocities of the upper and the lower layers at the bow and the stern of the ship.

The first aim of this chapter is to determine the flow and density patterns in the lock during the exchange process. The ultimate aim is to calculate the difference in water-level between the bow and the stern and the longitudinal force on the ship during the exchange process.

The flow and density patterns during the exchange process in a prismatic channel are very complicated. Wedges with fronts as leading edges intrude into the ambient water. The flow and density pattern is three-dimensional and unsteady, and mixing occurs at the fronts. In the case of a lock containing a ship, the exchange process becomes even more complex owing to the presence of discontinuities in the channel. Reflections of the exchange flow occur at the closed gate and the approach harbour and at the bow and the stern of the ship.

At the moment there are no mathematical models available for the calculation of the flow and density patterns of the exchange flow in a lock with a ship. The following approach is applied to solve the problem. The physical phenomena are investigated by means of experiments (chapter 2) and literature. The phenomena are schematized in such a way that one-dimensional momentum-, continuity- and energy-equations can be derived for the two layers. Empirical coefficients are introduced to simulate all the effects which cannot be included in the equations. The coefficients are determined by adjusting the calculated results to the experimental values. Mathematical models, based on these equations, can be applied within the measuring scope of the experiments.

The problem is too complicated to develop in one step a schematized mathematical model which includes all the aspects of the exchange flow considered. Therefore simpler sub-problems are first examined, to show that good results can be achieved with the applied approach. After that, the ultimate situation of the exchange flow in a lock containing a ship is examined. The result of each step is used as input for the next step.

First of all we look at the flow and density pattern pertaining to the exchange of two sections of a rectangular, prismatic channel without reflections (section 5.2). Then we consider the exchange of a lock influenced by reflection against the closed gate and the approach harbour but without the influence of a ship. The flow and density
patterns and the water level are determined (section 5.3). Then we investigate the exchange of a lock influenced by the presence of a ship. The flow and density patterns are schematized and the water-levels and the longitudinal force are calculated (section 5.4). The chapter is summarized and the conclusions are drawn in chapter 7.

5.2 Exchange Flow in a Rectangular, Prismatic Channel

5.2.1 General Description and Literature

5.2.1.1 General

A stratified flow comprises the flow of layers with different densities. The directions of the flow velocity in the layers can be the same as each other or reversed. Mixing between the layers at the interface may occur. The flow can be caused by a resulting discharge, difference in density, wind, etc.

An example of a stratified flow with two layers (or two-layer flow) is the intrusion of a salt wedge into an estuary. Another example is the filling of a lock with a different water density from that in the approach harbour.

An exchange flow is also an example of a two-layer flow. The only driving force is the difference in specific weight of the two fluids. The net flow rate is in any cross-section is zero. An example of an exchange flow is the flow between two stratified basins with different levels of interface. Another example is the exchange flow which develops after the removal of a gate separating a salt and a fresh section of a channel. This exchange flow consists of a fresh upper layer which propagates in the direction of the salt section of the channel and a lower salt layer which propagates in the direction of the fresh section of the channel.

The first aim of this section is to present a short description of the equations of a stratified flow with two layers (or two-layer flow) and a review of the existing literature relating to exchange flows in a rectangular, prismatic channel (5.2.1). The second aim is to present a method of calculating the exchange flow in a rectangular, prismatic channel (5.2.2). The results of this section are used as a starting point for the more complicated situations to be discussed in the sections to follow.

5.2.1.2 Equations for the Two-Layer Flow

Schijf and Schönfeld (1953, lit. [22]) presented theoretical considerations on the motion of salt and fresh water. They introduced, among other things, the one-dimensional equations for two-layer flow. The flow comprises two homogeneous layers separated by a sharp
interface, without mixing between the layers. Only small disturbances of the interface are allowed. The vertical pressure distribution is assumed to be hydrostatic.

![Diagram of two-layer flow](image)

Figure T29: The two-layer flow

**Notations:**

- $x$ = horizontal co-ordinate
- $t$ = time
- $a_1$ = thickness of lower layer (layer 1)
- $a_2$ = thickness of upper layer (layer 2)
- $u_1$ = flow velocity of lower layer
- $u_2$ = flow velocity of upper layer
- $h_b$ = level of the bottom
- $\rho_1$ = density of lower layer
- $\rho_2$ = density of upper layer
- $\tau_b$ = shear stress between bottom and layer 1
- $\tau_{12}$ = shear stress between layer 1 and layer 2

The one-dimensional momentum equations for the lower layer (layer 1) and the upper layer (layer 2) are as follows:
\[ \frac{\partial}{\partial t} (\rho_1 a_1 u_1) + \frac{\partial}{\partial x} (\rho_1 a_1 u_1^2) + (\rho_2 g a_1 \frac{\partial}{\partial x} (a_2) + \rho_1 g a_1 \frac{\partial}{\partial x} (h_b + a_1)) - \left( \tau_{12} - \tau_b \right) = 0 \]  \hspace{1cm} (5.1)

\[ \frac{\partial}{\partial t} (\rho_2 a_2 u_2) + \frac{\partial}{\partial x} (\rho_2 a_2 u_2^2) + \rho_2 g a_2 \frac{\partial}{\partial x} (h_b + a_1 + a_2) + \tau_{12} = 0 \]  \hspace{1cm} (5.2)

The momentum equations comprise the following terms. The first term represents the change of momentum in time. The second term represents the transport of momentum in longitudinal direction. The third term concerns the change in hydrostatic pressure in the longitudinal direction. The fourth term represents the friction between the layers and, for equation (5.1), also that between the bottom of the channel and the lower layer.

The continuity equations for the lower layer (1) and the upper layer (2) are as follows:

\[ \frac{\partial}{\partial t} (a_1) + \frac{\partial}{\partial x} (u_1 a_1) = 0 \]  \hspace{1cm} (5.3)

\[ \frac{\partial}{\partial t} (a_2) + \frac{\partial}{\partial x} (u_2 a_2) = 0 \]  \hspace{1cm} (5.4)

The friction caused by the bottom is determined by the velocity of the lower layer and the roughness of the bottom. The friction caused by the interface is determined by the difference in the velocities of the two layers. The expressions for the friction are:

\[ \tau_b = \rho_1 k_b \frac{|u_1|}{u_1} \]  \hspace{1cm} (5.5)

\[ \tau_{12} = - \rho_1 k_{12} \frac{|u_1 - u_2|}{(u_1 - u_2)} \]  \hspace{1cm} (5.6)

where \( k_b \) = bottom friction coefficient
\( k_{12} \) = interfacial friction coefficient

The characteristics of the equations for the two-layer system yield two kinds of waves with the following velocities of propagation (assuming small density differences):

\[ c_e = \frac{a_1 u_1 + a_2 u_2}{a_1 + a_2} \pm \sqrt{(g (a_1 + a_2))} \]  \hspace{1cm} (5.7),(5.8)

\[ c_i = \frac{a_1 u_2 + a_2 u_1}{a_1 + a_2} \pm \frac{\epsilon g a_1 a_2}{(a_1 + a_2)} \left( 1 - \frac{(u_1 - u_2)^2}{\epsilon g (a_1 + a_2)} \right) \]  \hspace{1cm} (5.9),(5.10)

where \( c_e \) = velocity of propagation of external waves
\[ c_i = \text{velocity of propagation of internal waves} \]
\[ \epsilon = \text{relative density difference} = (\rho_1 - \rho_2) / \rho_2 \]

There are four possible wave velocities, two external and two internal. The internal wave velocities are of the order of \( \sqrt{\epsilon} \) less than the external wave velocities. A small disturbance of the water-level or the interface propagates with these velocities of propagation.

The internal Froude number \( F_i \) is defined as:

\[ F_i^2 = \frac{u_1^2}{\epsilon g a_1} + \frac{u_2^2}{\epsilon g a_2} \]  \hspace{1cm} (5.11)

Externally critical flow occurs if one of the external wave velocities becomes zero. Internally critical flow occurs if one of the internal wave velocities (equation (5.9) and (5.10)) becomes zero. Substituting \( c_i = 0 \) in (5.9) or (5.10) yields for internally critical flow:

\[ F_i = 1 \]

Internally critical flow is considered with the whole vertical profile including both layers.

If \( F_i \) is less than 1 the flow is internally sub-critical and disturbances on the interface propagate in both directions. If \( F_i \) becomes greater than 1 super-critical flow occurs and disturbances on the interface can only propagate in one direction. Near the change in sub- to super-critical flow \( F_i \) is equal to 1 and the angle of the interface with the horizontal plane becomes large. An example of internally critical flow is in an estuary with an arrested salt wedge. The flow becomes critical near the mouth of the river if the sea is completely salt.

Vreugdenhil (1970, lit. [30]) has applied the equations for two-layer flow in computations of stratified flows in an estuary.

5.2.1.3 Literature on Experiments

The situation of the exchange flow in two sections of a rectangular, prismatic channel in still water is considered. Initially the two parts are separated by a gate, have different densities and are homogeneous. The exchange flow is generated after the removal of the gate.

The exchange flow consists of wedges of fluid of different densities, the length of which increase continually. At the fresh side a wedge of heavy fluid intrudes at the bottom into the lighter fluid, while at the salt side a wedge of light fluid intrudes near the water-surface into heavier fluid.
A front occurs as the leading edge of each of the wedges. The front is caused by the fact that waves of small heights are overtaken by higher waves. The fronts propagate and can be considered as the moving boundaries of the exchange flow. The fronts determine to a high degree the whole exchange flow. The water-surface above the exchange flow deviates from a horizontal plane. If the differences in density are small the differences in water-level are also small, relative to the water-depth. (However, differences in water-level are important for the calculation of the force on the ship, sections 5.4).

![Diagram of exchange flow](image)

**Figure T30: Exchange flow**

**Notations:**

- \(a_{f1}\) = height of the salt front
- \(a_{fu}\) = height of the fresh front
- \(a_{11}\) = thickness of the salt layer behind the salt front
- \(a_{2u}\) = thickness of the fresh layer behind the fresh front
- \(c_1\) = velocity of the salt front
- \(c_u\) = velocity of the fresh front
- \(v_{11}\) = flow velocity of the salt layer behind the salt front
- \(v_{2u}\) = flow velocity of the fresh layer behind the fresh front

The height of a salt front is about twice the height of the salt wedge at the rear of the front. Above the salt wedge a mixing layer arises which is generated at the upper edge of the front. The front velocity and the salinity of the salt wedge behind the front diminish slowly as the front travels further. (Keulegan '58, lit. [13]).

The flow pattern in the salt front and the ambient fresh water is three-dimensional. Within the front a circulating flow occurs. There are Kelvin-Helmholtz instabilities at the top of the front because of the great differences between the velocities of the front and the ambient fluid. This causes local turbulence and mixing at the interface of the front, which results in the mixing layer behind the front above the salt wedge. The foremost part of the front (the nose as a stagnation point of the flow) is above the bottom of the channel because of the influence of the bottom friction on the front. The result is that instabilities occur on the underside of the front at the bottom of the channel with small
lobes and clefts adjacent to each other (Allan, 1971, lit. [3] and Simpson, 1972, lit. [24]).

A fresh front travels along the water-surface of the salt water. Just like a salt front, the instabilities at the interface generate mixing behind the front. The foremost part of the front is at the water surface (Anwar, 1977, lit. [4]).

The interface at the initial separation is at about mid-depth. The interface between the fronts deviates from a straight line. (Keulegan 1958, lit. [13]).

The following data are derived from experimental investigations reported in the literature.

Just after the beginning of the exchange process the height of a salt front equals about half the water-depth. The length of the salt front is more than twice the height of the front. The velocity of propagation of the salt front \( c_1 \) varies between 0.42 and 0.48 \( \sqrt{(\varepsilon g h)} \) and, for a fresh front, between 0.48 and 0.52 \( \sqrt{(\varepsilon g h)} \). The height of the salt wedge behind the front varies between 0.25 and 0.33 times the water-depth.

The velocity of propagation of the front, the difference between its density and that of the ambient flow and the height of the front decrease slowly during propagation, because of the friction at the bottom and at the interface and the mixing at the front. After a distance of propagation of 40 times the water depth, the velocity and the density difference may, for example, have decreased by 10 % and the front height by 20 %. Therefore the flow rates at the initial separation also decrease slowly during the exchange process. (Keulegan 1957, lit. [12]).

<table>
<thead>
<tr>
<th></th>
<th>( \varepsilon )</th>
<th>( a_{11}/h )</th>
<th>( a_{fl}/h )</th>
<th>( c_1/\sqrt{(\varepsilon gh)} )</th>
<th>( c_u/\sqrt{(\varepsilon gh)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keulegan 1957</td>
<td>.037</td>
<td></td>
<td></td>
<td>.48/.43</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.060</td>
<td></td>
<td></td>
<td>.45/.42</td>
<td></td>
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<tr>
<td></td>
<td>.020</td>
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<td>.53</td>
</tr>
<tr>
<td>Barr 1967</td>
<td></td>
<td></td>
<td></td>
<td>.465</td>
<td>.59</td>
</tr>
<tr>
<td>Abraham 1971</td>
<td>.020/.028</td>
<td>.27/.35</td>
<td></td>
<td>.45/.46</td>
<td>.48/.52</td>
</tr>
<tr>
<td>Simpson 1979</td>
<td>.0074</td>
<td>.33</td>
<td>.50</td>
<td>.46</td>
<td></td>
</tr>
<tr>
<td>Rottman 1983</td>
<td>.010/.040</td>
<td>.25</td>
<td></td>
<td>.44</td>
<td>.53</td>
</tr>
</tbody>
</table>

Huppert and Simpson (1980, lit. [27]) presented an internal Froude number of one layer at the salt front for \( a_{11}/h > .075 \) according to:

\[ F_1 = \frac{c_1}{\sqrt{(\varepsilon g a_{11})}} = 0.5 \left( \frac{a_{11}}{h} \right)^{-0.33} \]
In general, the existence of fronts as leading edge depends on the flow pattern of the ambient flow and the level of the interface. A front occurs if the ambient fluid is at rest or has a uniform velocity profile (figure T31). An exchange flow fulfils this condition. In contrast, an arrested salt wedge in a river meets a logarithmic velocity profile and has no front (figure T32). (Britter and Simpson, 1978, lit. [26]).

If the initial difference in level of the interface between two sections of a channel is less than half the water-depth no front occurs after the removal of the separation (figure T33). (Barr, 1963, lit. [6 and 7])

Figure T31 : An arrested front behind a moving bottom
Figure T32 : An arrested salt wedge without a front in a river
Figure T33 : Wedges without fronts in a initially stratified situation

There are no experimental results available on the variations in the water-level above an exchange flow. (The differences in water-level are important in determinating the force on the ship (section 5.4))

5.2.1.4 Literature on Theory

The flow in the fronts is three-dimensional as mentioned in the previous section. A detailed calculation of the flow and density distributions will therefore be very difficult.

The flow between the fronts is controlled by the fronts, and can be modelled as a one-dimensional two-layer flow. Calculation of the flow
between the fronts is possible only if the one-dimensional two-layer flow equations ((5.1) - (5.4)) are extended with fronts as boundaries of the flow. For this, front relations are needed. (Abraham and Vreugdenhil 1971, lit. [1]).

The front relation gives the velocity of propagation of the front as a function of the height of the front. This relation can be derived by applying momentum, continuity and energy equations for the two layers and the introduction of energy loss coefficients. The energy losses are caused by the circulation flow in the front itself, the deceleration of the fresh water behind the front and the friction caused by the bottom. These coefficients must be determined by model investigations. (Abraham and Vreugdenhil, 1971, lit. [1]).

![Diagram of a salt front]

Figure T34: The salt front

Kranenburg (1978, lit. [16]) presented the following front equation:

$$c_1 = \sqrt{\frac{\frac{a_{11} (h_k - a_{11}) (2 h_k - a_{11})}{h_k (h_k + a_{11} + k_1 (h_k - a_{11}))}}{(\epsilon g a_{21})}}$$  \hspace{1cm} (5.12)

where $a_{11}$ = thickness of the salt layer behind the salt front  
$c_1$ = velocity of the salt front
$h_k$ = water-depth
$\epsilon$ = $(\rho_2 - \rho_1) / \rho_2$ = relative density difference
$k_1$ = energy loss coefficient at the salt front

The front relations are only valid the quotient of the thickness of the salt layer behind the salt front and the water-depth is not too large. From the point of view of the conservation of energy the maximum thickness of the intruding layer can be half the water-depth (Benjamin, 1968, lit. [8]). The stability of the front of the exchange flow requires that the two-layer flow behind the front must be internally sub-critical, or critical at most, with respect to the front velocity. The internal Froude number (with a co-ordinate system moving with the front) has a maximum value of $1$ according to Kranenburg (1978, lit. [16]) (and experimentally affirmed by Wright (1987)):

$$F_1 = \frac{v_{21} + c_1}{\sqrt{\frac{\epsilon g a_{21}}{2}}} \leq 1$$  \hspace{1cm} (5.13)
where $a_{21} =$ thickness of the fresh layer behind the salt front

$v_{21} =$ flow velocity of the fresh layer behind the salt front

Computations of the exchange flow have been executed by Rottman and Simpson (1983, lit. [28]). They used the one-dimensional two-layer equations, the front relations and energy loss coefficients from experimental investigation and solved these equations with a finite-difference method. The results of these computations appear to be in agreement with experimental investigation for stratified situations only if the maximum initial difference in the level of the interfaces is half the water-depth. This corresponds to the situation less influenced by fronts. The results of the computation of the exchange flow of the water of two homogeneous, unstratified parts of the channel did not agree with their experiments.

5.2.1.5 TWOLAY Calculations for Exchange Flows

Within the framework of our investigations calculations have been executed with the mathematical model TWOLAY which has been developed by Delft Hydraulics as a sequel to Vreugdenhil's investigation (1970, lit. [30]). TWOLAY is a model which makes use of the one-dimensional equations for the two-layer flow (equations (5.1) up to and including (5.4)). These equations are extended with terms for the entrainment through the interface. Moreover, two salinity equations have been added. This system of hyperbolic differential equations is solved numerically by means of a finite-difference method. TWOLAY has been specially developed for the calculation of the intrusion of salt wedges into an estuary with a tide.

TWOLAY has been applied in this study for the calculation of the exchange flow in a rectangular channel. However, the finite difference scheme of TWOLAY does not allow jumps in the interface. Therefore the interfacial friction coefficient was considerably heightened instead of the introduction of front relations. The results of the calculations do not show fronts and the interface remains smooth everywhere. The calculated levels and slopes of the interface deviate too much from the experimental results for practical use. Therefore TWOLAY was considered unsuited to the present purpose.

5.2.2 Schematized Calculations

5.2.2.1 Set-up of the Schematized Calculations

The aim of the calculations presented here is to determine the front heights, the velocities of the front and the water-level differences at the fronts for an exchange flow in a rectangular channel. Our study is a sequel to the studies of Abraham and Vreugdenhil (1971, lit. [1]), Kranenburg (1978, lit. [16]) and Denton (1990, lit. [9]) who derived the relationships between the height and the velocity of a salt front.
The flow pattern of the exchange flow, as observed in experimental studies (section 5.2.), is schematized as follows:
- The fronts are vertical
- The interface is a straight line between the salt and the fresh fronts
- The energy losses are concentrated near the fronts
- Mixing between the layers is not taken into consideration; a mixing layer between the upper and lower layer is absent
- Interfacial and bottom shear stresses are not taken into account
- The net flow rate is zero and there are no external transitory waves

One-dimensional momentum and continuity equations and energy equations are applied at the discontinuities of the flow. Cross-sections 1 and 5 (figure T35) are just outside the exchange flow, cross-sections 2 and 4 are just behind the fresh and the salt fronts and cross-section 3 is at the cross section of the initial separation. The flow velocities are positive defined in the direction of the flow as shown in the figure below.

![Diagram](image)

Figure T35: Schematized exchange flow

**Notations:**

- $h_k$ = water-depth
- $a_{2u}$ = height of the fresh front
- $a_{1u}$ = thickness of the salt layer beneath the fresh front
- $a_{1l}$ = height of the salt front
- $a_{2l}$ = thickness of the fresh layer above the salt front
- $a_{1g}$ = thickness of the fresh layer at the opened gate
- $a_{1g}$ = thickness of the salt layer at the opened gate
- $c_l$ = velocity of the salt front
- $c_u$ = velocity of the fresh front
- $c_l'$ = velocity coefficient of the salt front
- $c_u'$ = velocity coefficient of the fresh front
\[ v_{21} = \text{flow velocity of the fresh layer above the salt front} \]
\[ v_{1u} = \text{flow velocity of the salt layer beneath the fresh front} \]
\[ v_{2g} = \text{flow velocity of the fresh layer at the opened gate} \]
\[ v_{1g} = \text{flow velocity of the salt layer at the opened gate} \]
\[ \Delta h = \text{total difference in water-level} \]
\[ \Delta h_1 = \text{water-level elevation above the salt front} \]
\[ \Delta h_u = \text{water-level elevation above the fresh front} \]
\[ \Delta h_g = \text{water-level elevation at the opened gate} \]
\[ \rho_1 = \text{density of lower layer} \]
\[ \rho_2 = \text{density of upper layer} \]
\[ \epsilon = \frac{(\rho_2 - \rho_1)}{\rho_2} = \text{relative density difference} \]
\[ k_1 = \text{energy loss coefficient at the salt front} \]
\[ k_u = \text{energy loss coefficient at the fresh front} \]

5.2.2.2 Equations for a Control Volume Containing the Exchange Flow

The momentum equation is applied for a fixed co-ordinate system between cross-sections 1 and 5. The terms in the equation are concerned with the hydrostatic pressure forces at the boundaries of the control volume and the change of momentum in time within the control volume. The change of momentum is estimated from the schematized flow situation. The result is:

\[ \frac{1}{2} \rho_1 g \frac{h_k^2}{2} - \frac{1}{2} \rho_2 g (h_k + \Delta h)^2 \]
\[ - \rho_1 \frac{(a_{1u} + a_{1l})}{2} (v_{1u} + c_1) \]
\[ + \rho_2 \frac{(a_{2u} + a_{2l})}{2} (c_u + v_{21} + c_1) = 0 \]

(5.14)

The continuity equation accounts for the displacements of the fronts, and the rotations of the interface and the free surface between the fronts.

\[ (a_{2u} + \frac{1}{2} \frac{c_u}{c_u + c_1}) (a_{21} - a_{2u}) \]
\[ c_u = \]
\[ (a_{1l} + \frac{1}{2} \frac{c_u}{c_u + c_1}) (a_{21} - a_{2u}) + (\Delta h - \Delta h_1) c_1 \]

(5.15)

5.2.2.3 Equations for the Fresh Front

We consider the control volume between cross-sections 1 and 2. The momentum, continuity and energy equations are applied for a co-ordinate system moving with the velocity of the fresh front.
Figure T36: The schematized fresh front in a moving co-ordinate system

The momentum equation consists of terms for the hydrostatic pressure forces at the boundaries and the momentum transport through the boundaries of the control volume:

\[ \frac{1}{2} \rho_1 g h_k^2 - \frac{1}{2} \rho_2 g (a_{1u} + a_{2u})^2 - \frac{1}{2} (\rho_1 - \rho_2) g a_{1u}^2 + \rho_1 c_u^2 h_k - \rho_1 (c_u + v_{1u})^2 a_{1u} = 0 \]  
(5.16)

The continuity equation consists of terms for the water flowing through cross-sections 1 and 2:

\[ c_u h_k = (c_u + v_{1u}) a_{1u} \]  
(5.17)

Figure T37: The water-level at the fresh front

The energy equations are applied at the fresh front along streamlines A-B and D-C (Benjamin, 1968, lit.[]). Energy losses are included in order to account for the real fluid effects. The flow velocities at the tip of the front (points B and C) are almost zero with respect to the moving front (the stagnation point of the flow). The flow velocities with respect to the moving front are \( c_u \) at point B and \( v_D \) at point D. Energy losses occur between the points A and B and between D and C.
because of the influence of turbulence.

Points A - B:

\[
\frac{c_u^2}{(2g)} = \Delta h_B + \Delta H_{AB} \tag{5.18}
\]

where \(\Delta h_B\) = water-level elevation above point B, \(\Delta H_{AB}\) = height of the energy loss between points A and B

Points D - C:

\[
\Delta h_C + \frac{v_D^2}{(2g)} = \Delta h_C + \Delta H_{DC} \tag{5.19}
\]

where \(\Delta h_C\) = water-level elevation above point C,
\(\Delta H_{DC}\) = height of the energy loss between points D and C,
\(v_D\) = flow velocity at point D (with respect to front velocity)

The energy loss at the front is defined as:

\[
\Delta H_{DC} - \Delta H_{AB} = \frac{v_D^2}{(2g)} = k_u \frac{c_u^2}{(2g)} \tag{5.20}
\]

where \(k_u\) = energy loss coefficient at the fresh front.

The water-levels at points B and C are almost equal according to \(\Delta h_B \approx \Delta h_C\). The equations (5.18), (5.19) and (5.20) (with \(h_k + \Delta h_u = a_{1u} + a_{2u}\)) yield:

\[
a_{1u} + a_{2u} = h_k + (1 + k_u) \frac{c_u^2}{(2g)} \tag{5.21}
\]

5.2.2.4 Equations for the Salt Front

We consider the control volume between cross-sections 4 and 5. The momentum and continuity equations are applied for a co-ordinate system moving with the velocity of the salt front.

![Diagram of salt front in a moving coordinate system](image)

Figure T38: The schematized salt front in a moving co-ordinate system
The momentum equation consists of terms for the hydrostatic pressure forces and the momentum transport:

\[
\frac{1}{2} \rho_2 \ g \ (a_{11} + a_{21})^2 + \frac{1}{2} (\rho_1 - \rho_2) \ g \ a_{11}^2 - \frac{1}{2} \rho_2 \ g \ (h_k + \Delta h)^2 \\
+ \rho_2 \ (c_1 + v_{21})^2 \ a_{21} - \rho_2 \ c_1^2 \ (h_k + \Delta h) = 0
\]  

(5.22)

The continuity equation consists of terms for the water flowing through cross-sections 4 and 5:

\[
(c_1 + v_{21}) \ a_{21} = c_1 \ (h_k + \Delta h)
\]  

(5.23)

Figure T39: The pressures at the salt front

The energy equations are applied for the energy losses at the salt front along streamlines E-F and H-G. As with the fresh front, the flow velocities at the nose of the front (stagnation points F and G) are almost zero with respect to the moving front. The flow velocities with respect to the moving front are \(c_1\) at point H and \(v_E\) at point E.

Points H - G:

\[
P_H / (\rho_2 g) + c_1^2 / (2g) = P_G / (\rho_2 g) + \Delta H_{HG}
\]  

(5.24)

where

- \(P_H / (\rho_2 g) = h_k + \Delta h\)
- \(P_H = \) pressure at point H
- \(P_G = \) pressure at point G
- \(\Delta H_{HG} = \) height of the energy loss between points H and G

Points E - F:

\[
P_E / (\rho_1 g) + v_E^2 / (2g) = P_F / (\rho_1 g) + \Delta H_{EF}
\]  

(5.25)
where \( P_E / (\rho_1 g) = a_{11} + (\rho_2 / \rho_1) a_{21} \)
\( P_E = \) pressure at point E
\( P_F = \) pressure at point F
\( \Delta H_{EF} = \) height of the energy loss between points E and F
\( v_E = \) flow velocity at point E (with respect to front velocity)

The energy loss over the whole front is defined as:

\[
\Delta H_{EF} - \Delta H_{FG} - v_E^2 / (2g) = k_1 c_1^2 / (2g) \tag{5.26}
\]

where \( k_1 = \) energy loss coefficient at the salt front

The pressures at points G and F are almost equal with \( P_G \approx P_F \). The equations (5.24), (5.25) and (5.26) yield:

\[
(\rho_1 / \rho_2) a_{11} + a_{21} = h_k + \Delta h + (1 + k_1) c_1^2 / (2g) \tag{5.27}
\]

### 5.2.2.5 Continuity Equations

The deformation of the water-surface consists of the displacements of the jumps in the water-surface near the fronts and the rotation of the water-surface between the salt and fresh fronts:

\[
\frac{c_u}{c_u + c_1} \left( \frac{\Delta h_1 - \Delta h_u}{c_u + c_1} \right) \Delta h_u + \frac{c_1}{c_u + c_1} \left( \frac{\Delta h - \Delta h_1}{c_u + c_1} \right) \Delta h_u = \tag{5.28}
\]

The water-depths at cross sections 2 and 4 are as follows:

- **Cross-section 2:** \( a_{1u} + a_{2u} = h_k + \Delta h_u \)
- **Cross-section 4:** \( a_{11} + a_{21} = h_k + \Delta h_1 \)

### 5.2.2.6 Solution to the Equations

The eleven above-mentioned equations (5.14), (5.15), (5.16), (5.17), (5.21), (5.22), (5.23), (5.27), (5.28), (5.29) and (5.30) have eleven unknowns:

\( a_{2u}, a_{1u}, a_{11}, a_{21}, \Delta h, \Delta h_1, \Delta h_u, c_u, u_{1u}, c_1, u_{21} \)

These equations have the following quantities as input:

\( h_k, \rho_1, \rho_2, k_u, k_1 \)

The equations (5.14) up to and including (5.24) have been solved with the equation solver Eureka.
5.2.2.7 Results

Calculations have been carried out for the following example:

\[ h_k = 10 \text{ m} \]
\[ \rho_1 = 1020 \text{ kg/m}^3, \quad \rho_2 = 1000 \text{ kg/m}^3 \]
\[ k_1 \text{ varies between 0 and 1.5} \]
\[ k_u \text{ varies between 0 and 1} \]

In figure 43 up to and including 45 the non-dimensional results are presented, according to:

- coefficients of the front velocity:

\[ c_1' = c_1 / \sqrt{(\epsilon \ g \ (a_{11} + a_{21}))} \]
\[ c_u' = c_u / \sqrt{(\epsilon \ g \ (a_{1u} + a_{2u}))} \]

- relative front heights:

\[ a_{11} / h_k \]
\[ a_{2u} / h_k \]

- relative head difference across the front:

\[ (\Delta h - \Delta h_1) / \Delta h \]
\[ \Delta h_u / \Delta h \]

where \[ \Delta h = (1/(\rho_1/\rho_2) - 1) \ h_k \]

The head difference \( \Delta h \) is the total head difference over the exchange flow as a whole and follows from equation (5.14) neglecting the change in momentum with time.

Results

Figures 43 up to and including 45 show the relations as functions of the energy loss coefficients of the fronts. The front velocities and the front heights agree with the experimental values from the literature (table 5.1).

If the energy loss coefficient \( k_1 \) of the salt front is increased while the energy loss coefficient \( k_u \) of the fresh front is kept constant, it can be said that:

- the velocity of the salt front decreases
- the front height of the salt front and the head difference over the salt front as a whole increase

It appears that it is chiefly the energy loss coefficient of the salt front which determines the properties of the salt front such as its velocity and height, and the head difference over the salt front as a whole. The fresh front is hardly influenced by the energy loss coefficient of the salt front. Comparable conclusions can be drawn about the energy loss coefficient of the fresh front.

Kranenburg (1978, lit. [16]) recommends for the energy loss coefficient \( k_1 = .6 \) (salt front) and \( k_u = 0 \) (fresh front); this yields:

- salt front: \[ c_1' = .47, \quad a_{11} / h_k = .33, \quad (\Delta h - \Delta h_1) / \Delta h = .32 \]
- fresh front: \( c_u' = 0.52 \), \( a_{2u} / h_k = 0.27 \), \( \Delta h_u / \Delta h = 0.27 \)

If the calculated data are averaged (figure 43 up to and including 45) the following relations can be derived if \( 0 < k_1 < 1.5 \) and \( 0 < k_u < 1 \):

- coefficients of front velocity:
  \[ c_{1u}' = -0.072 k_1 + 0.52 \]  
  \[ c_{u}' = -0.072 k_u + 0.52 \]  
  \( (5.31) \)  
  \( (5.32) \)

- relative front heights:
  \[ a_{11} / h_k = 0.091 k_1 + 0.27 \]  
  \[ a_{2u} / h_k = 0.091 k_u + 0.27 \]  
  \( (5.33) \)  
  \( (5.34) \)

- relative head difference across the front (with \( \Delta h \approx 0.5 \epsilon g h_k \))
  \[ (\Delta h - \Delta h_1) / \Delta h = 0.082 k_1 + 0.27 \]  
  \[ (\Delta h_u) / \Delta h = 0.082 k_u + 0.27 \]  
  \( (5.35) \)  
  \( (5.36) \)

**Internal Froude Number**

The internal Froude numbers according to three definitions were derived from the calculations (with \( 0 < k_1 < 1.5 \) and \( 0 < k_u < 1 \)):

- The internal Froude numbers at the fronts with a co-ordinate system which moves with the front according to (application of equation (5.13)):
  \[ F_1 = \sqrt{\left( (u_{21}+c_1)^2 / (\epsilon g a_{21}) \right)} \]  
  \[ F_1 = \sqrt{\left( (u_{1u}+c_u)^2 / (\epsilon g a_{1u}) \right)} \]  
  These internal Froude numbers vary between 0.8 and 0.9 (figure 46). The values are in agreement with the requirement that the flow just behind the front should be sub-critical, with \( F_1 \leq 1 \) (Kranenburg, 1978, lit. [16]).

- The internal Froude number at the front for one layer with a fixed co-ordinate system according to:
  \[ F_1 = c_1 / \sqrt{\left( \epsilon g a_{11} \right)} \]  
  \[ F_1 = c_u / \sqrt{\left( \epsilon g a_{1u} \right)} \]  
  These internal Froude numbers vary between 0.6 and 1.0. This is in agreement with the measured values of Huppert and Simpson (1980, lit. [27]).

- The Internal Froude number at the opened gate with a fixed co-ordinate system according to equation (5.11):
  \[ F_1 = \sqrt{\left( u_{1g}^2 / (\epsilon g a_{1g}) + u_{2g}^2 / (\epsilon g a_{2g}) \right)} \]  
  These internal Froude numbers vary between .75 and .78 and meet the requirement \( F_1 < 1 \) for sub-critical flow.

**Exchange Flow Rate**

The exchange flow rate is the flow rate in each layer at the initial separation (the gate). The exchange flow rate of the lower layer follows from:
\[ q_{ex} = a_{11} c_1 + \frac{1}{2} (a_{1g} - a_{11}) c_1 \]  

\[
\text{where } a_{1g} = a_{11} + \frac{c_1}{c_u + c_1} (a_{1u} - a_{11})
\]

The relative exchange flow rate near the initial separation (the gate) is presented in figure 46. This relative exchange discharge is defined as:

\[ q_{ex}' = q_{ex} / (h_k \sqrt{(\epsilon g h_k)}) \]

This relative exchange flow rate \( q_{ex}' \) varies between .19 and .20 for energy loss coefficients \( 0 < k_1 < 1.5 \) and \( 0 < k_u < 1 \). The values of the flow rate are in agreement with the requirement that the maximal exchange flow rate is 0.25 if \( F_1 \leq 1 \).

Application of the Results

It can be concluded that the calculated values agree with the experiments from the literature and meet the requirements of internally sub-critical flow.

The equations (5.31) up to and including (5.36) describe the relation between the velocity and the height of the fronts and the energy loss coefficients of the fronts. These equations were derived from the calculations presented here. In the following sections these equations are applied to the exchange of the water in a lock with that in the approach harbour. The values of the energy loss coefficients are then determined by means of the available experiments.

5.3 The Exchange in a Lock without the Influence of a Ship

5.3.1 General

The exchange situation which is investigated in this section concerns the exchange of the water in a lock with that in an approach harbour, without the influence of a ship. Referring to the previous section, the exchange flow is extended with the reflection against the lock end (closed gate) and the influence of the sudden widening at the approach harbour.

A number of situations are examined with the help of the literature. The literature on the subject is, however, scant.

The first situation is that of a lock and a channel with the same rectangular cross section. The influence of the reflection against the lock end is dealt with in section 5.3.2.

The second situation concerns the exchange of the water of a channel with that of a sea (or wide approach harbour). The influence of the
expansion is presented in section 5.3.3. The third situation concerns the exchange of the water in a lock with that in an approach harbour to determine of the combined influence of the lock end and the approach harbour (section 5.3.4).

A schematized calculation of the exchange of the water in a lock with that in a channel is made (section 5.3.5), making use of the results of the previous section, the results of the literature study (sections 5.3.2 up to and including 5.3.4) and our measurements (chapter 2). The differences in water-level can be calculated using this schematized mathematical model. These water-level differences are needed for the calculation of the longitudinal force on a ship, if this ship does not influence the flow and density distribution in the lock.

The ultimate aim of this section is to show that reliable results can be obtained with the schematized mathematical model. The approach of the mathematical model is used in the next section (5.4), which incorporates the influence of the ship on the exchange flow.

5.3.2 The Influence of the Lock End

Measurements of Barr (1963, 1967)

Barr presented an experimental study (1963, lit. [7] and 1967, lit. [8]) concerning the exchange flow of an initially homogeneous salt lock and a fresh channel. The lock and channel have the same, rectangular cross-sections. The exchange flow is generated after the sudden removal of the gate between the lock and the channel.

The exchange of the water in a salt lock with that in a fresh channel is shown in figure T40. The lower layer propagates with a front in the channel. The front of the upper layer propagates towards the lock end and reflects against this lock end. The front of the reflected upper layer is visible as an internal wave (a jump in the interface), which propagates away from the lock end and eventually overtakes the salt front (the speed of the reflected wave is about 12 % larger than that of the salt front). A small residual layer remains behind the propagating wave.

It appears from the measurements that the velocity of the salt front is practically independent of the length of the lock, when the front travels distances between 0 and 6 to 10 times the lock length. This is true for a lock length greater than twice the water-depth. The salt front propagates independently of the fresh front until the salt front is reached by the internal wave. The velocity of the salt front corresponds to that for the exchange flow in a prismatic channel \( C_1 \approx 0.465 / (\epsilon \, g \, h_k) \). When the front travels longer distances, its velocity diminishes.
Measurements of Rottman and Simpson

As a sequel to Barr's study, Rottman and Simpson (1983, lit. [28]) reported a study concerning the exchange flow produced by instantaneous releases of an amount of heavy fluid in a rectangular channel. The experimental results presented here, concern the situation of an initially homogeneous salt lock and a fresh channel.

![Diagram of exchange flow](image)

Fig. T40: The exchange of the water of a salt lock and that of a fresh channel at four successive moments after release. (a), (b), and (c) are the initial phase, (d) is the beginning of the self-similar phase. (Rottman and Simpson, 1983, lit. [28])

Three phases are distinguished:
- Initial phase
  Wedges are generated after the removal of the gate, the length of which increase continually (a). When the fresh wedge meets the lock end an internal wave is generated, which propagates away from the lock end ((b) and (c)) and eventually overtakes the salt front (d). The fronts have a constant velocity and height during the first phase. The velocity of the salt front is \(0.44 / (\epsilon g h_k)\). The velocity of the fresh front until the reflection, and the velocity of the internal wave (jump in the interface) due to the reflection, are \(0.54 / (\epsilon g h_k)\). The front heights and the height of the salt wedge behind the front do not deviate from those of the exchange in a prismatic channel.
- Self-similar phase
  The second phase is reached when the reflection of the upper layer against the lock end overtakes the salt front (d). This occurs after about 10 times the lock length. Then the front velocity decreases abruptly by a factor of \(t^{-1/3}\) (fig. T41, \(t\) = time). Inertia effects dominate the viscous effects of the flow during this phase.
- Viscous phase
  The third phase is reached when the viscous forces become dominant; the front velocity decreases further on. This viscous phase is quickly reached with small flow velocities caused by small scale models or low
The time interval in our application corresponds to distances travelled by the front of only twice to three times the length of the lock. Therefore, only the first phase is important for our study.

--- salt front --- fresh front, internal wave

\[ \frac{x}{l_k} \quad \frac{t}{t_c} \]

Fig. T41: The fronts and the internal wave as functions of time after release (Rottman and Simpson, 1983, lit. [28]). (where \( l_k \) = length of lock, \( t_c \) = travelling time of front between opened and lock end = \( l_k/c_u \))

It may be supposed that the exchange flows of a fresh lock and a salt channel are comparable during the first phase. In that case the velocities of the fresh front, the salt front and the internal wave caused by reflection are equal during the first phase. However, the fresh front propagates more rapidly than the salt front. Hence it is doubtful that the fresh front is overtaken.

Results

The following can be concluded from the literature if the maximum distance travelled by the front is six times the length of the lock.

The velocity and the height of the front which propagates into the channel are practically constant. The velocity and the height of the front which propagates towards the lock end are also constant. This front changes into a propagating internal wave (a jump in the interface) after reflection against the lock end and propagates in the direction of the channel with about the same velocity as, and with a height comparable to, that before reflection. A residual layer remains behind the internal wave.
5.3.3 Influence of the Approach Harbour

Measurements of Keulegan

Keulegan presented an extensive hydraulic model study (1958, lit. [13]) concerning the exchange flow of a fresh channel and a salt sea. This study is interesting because of the flow process at the mouth.

The channel is rectangular and long so that no reflection is noticeable near the mouth. The sea is stagnant and is very wide. The sea can be compared with an approach harbour. A gate is present at the mouth and separates the salt and the fresh water. A salt wedge intrudes into the fresh channel with a front as the leading edge of the current after the sudden removal of the gate, in the same way as during the exchange in a prismatic channel (section 5.2). A fresh layer flows out of the channel into the sea as a thin layer at the surface.

![Diagram of flow process](image)

Fig. T42: The exchange of the fresh water in a channel and the salt water in a stagnant sea

The data mentioned in the following table concern a channel with a width / water-depth ratio of 1 ($b_k/h_k = 1$) and a relative difference in density $\epsilon$ of 0.022.

Table 5.2: Data of the exchange flow between a fresh water channel and a salt sea according to Keulegan (1958, lit. [13])

<table>
<thead>
<tr>
<th>$x_r/h_k$</th>
<th>$a_{1a}/h_k$</th>
<th>$v_{1a}/(\epsilon g h_k)$</th>
<th>$q_{ex}'$</th>
<th>$c_1'$</th>
<th>$a_{11}/h_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.62</td>
<td>0.34</td>
<td>0.21</td>
<td>0.49</td>
<td>0.30</td>
</tr>
<tr>
<td>40</td>
<td>0.66</td>
<td>0.28</td>
<td>0.18</td>
<td>0.44</td>
<td>0.24</td>
</tr>
<tr>
<td>80</td>
<td>0.70</td>
<td>0.22</td>
<td>0.15</td>
<td>0.37</td>
<td>0.15</td>
</tr>
</tbody>
</table>

where $x_r$ = distance travelled by the front

$a_{1a}$ = thickness of the salt layer at the mouth

$v_{1a}$ = flow velocity of the salt layer at the mouth

$q_{ex}' = q_{ex}/(h_k/(\epsilon g h_k))$

$c_1' = exchange flow rate at the mouth$

$c_1' = exchange flow rate at the mouth$

$a_{11}$ = thickness of the salt layer behind the salt front
The length of a lock is e.g. 20 times the water-depth. Therefore the most interesting length for our investigation is less than 40 times the water-depth.

\[ a_{1a}/h_k \ [\text{-}] \]

*Fig. T43:* The shape of the saline wedge

\[ b_k/h_k = 0.5, \ \epsilon = 0.022 \] (after Keulegan, 1958, lit. [13])

**Interface and Exchange Discharge at the Mouth**

The thickness of the salt layer \( (a_{1a}) \) at the mouth increases from 0.62 up to 0.70 times the water-depth during a travelling time of the front of 80 times the water-depth. The high level of the interface at the mouth and the steep gradient of the interface just behind the mouth in the sea are caused by the spreading of the fresh layer in the sea. The measured level of the interface at the mouth is higher than that at the open gate during the exchange in a prismatic channel (section 5.2).

The flow velocity \( (v_{1a}) \) of the salt layer at the mouth decreases as the travelling time of the front increases. The exchange discharge \( q_{ex} \) is the product of the thickness and flow velocity of the salt layer at the mouth. The exchange discharge decreases during the travelling time of the front because of the increased energy losses in the salt wedge. The value of the exchange discharge between \( x_r/h_k = 0 \) and 40 (our applications) is comparable with that in a prismatic channel.

**Internal Froude Number \( F_I \) at the Mouth**

The internal Froude number at the mouth \( (F_I) \) is determined from equation (5.11):
\[ F_i^2 = \frac{v_{1a}^2}{\varepsilon g a_{1a}} + \frac{v_{2a}^2}{\varepsilon g a_{2a}} \]  

(5.37)

where \( F_i \) = internal Froude number at the mouth
\( a_{2a} \) = thickness of the fresh layer at the mouth \( \approx (h_k - a_{1a}) \)
\( v_{2a} \) = flow velocity of the fresh layer at the mouth \( \approx \frac{v_{1a} a_{1a}}{a_{2a}} \)

Substitution of the measured values yields an internal Froude number \( F_i \) \( \approx 1 \) at the mouth: the flow is internally critical at the mouth. The internal Froude number at the mouth is higher than that at the opened gate of a prismatic channel \( (F_i \text{ between } 0.75 \text{ and } 0.78 \text{ (section 5.2)}) \).

A free overflow with internally super-critical flow in the sea occurs due to the radial spreading in the sea. Friction influences in sea are not perceptible in the canal and the mouth. The exchange discharge are nearly the same in both situations.

The flow at the mouth is comparable with the stationary flow situation above an arrested salt wedge in a river at the mouth of a tideless sea (Schijf and Schönfeld, 1953, lit. [22]) and figure T32). The salt water in the salt wedge is at rest, if the mixing through the interface is not taken into account. The fresh water of the river is spread over the sea in a thin layer behind the mouth, as an internally super-critical flow. The transition of the internally sub-critical flow in the river into the super-critical flow is located at the mouth. The flow at the mouth is internally critical with \( F_i = \frac{v_{2a}}{\sqrt{(\varepsilon g a_{2a})}} = 1 \). One internal wave speed is zero at the mouth and the other internal wave speed is directed seaward. Small disturbances of the interface at the mouth, which propagate in the direction of the sea, are not reflected back to the river.

The transition of a steady flow from internally sub-critical via critical into super-critical flow occurs at the cross section with \( F_i = 1 \). This internal Froude number criterion cannot be applied for the transition from internally sub- to super-critical flow, if the flow is unsteady.

In the case of an exchange flow, the changes in time at the mouth are slow relative to the changes in time of the complete flow. Therefore the behaviour of the flow at the mouth can be considered as quasi-stationary.

Front

The velocity \( (c_1) \) of the salt front and the height \( (a_{11}) \) of the front are about equal to those with a prismatic channel (section 5.2) at the beginning of the exchange process.
The velocity and height of the salt front decrease as the travelling time of the front increases. This decrease is partially caused by the high level of the interface at the mouth. The salt exchange discharge equals the increase (per unit of time) of the volume of the salt layer caused by the translation of the front and the rotation of the interface. If the exchange discharge is constant and if the level of the interface at the mouth rises, the height of the salt front must decrease (figure T44). Another reason for the decrease in the height and the velocity of the front is the decrease in time in the exchange discharge caused by the increased friction.

Conclusions

The level of the interface at the discontinuity of the lock and the approach harbour rises gradually. In our applications, the thickness $a_{la}$ of the salt layer at the mouth varies e.g. between 0.62 and 0.66 times the water-depth. These values are somewhat greater than those at the opened gate during the exchange flow in a prismatic channel.

The internal Froude number $F_i$ at the mouth is equal to 1. The outflow at the mouth is comparable with the internally critical outflow at the mouth of a river into a salt sea. Small disturbances of the interface in the channel which propagate past the mouth will not be reflected. The internal Froude number at the mouth is slightly higher than that at the opened gate during the exchange in a prismatic channel.

The velocity $c_1$ and the height $a_{11}$ of the salt front decrease gradually as the travelling time of the front increases. In our applications $c_1'$ varies between 0.49 and 0.44 and $a_{11}/h_k$ between 0.30 and 0.24. These values are comparable with those during the exchange flow in a rectangular channel.

The coefficient $q_{ex}'$ of the exchange flow rate of a lock varies between 0.21 and 0.18. The decrease in the exchange discharge during the observed travelling time of the front is small. The exchange discharge
is comparable with that during the exchange flow in a prismatic channel. Some calculations are presented in appendix A.

5.3.4 Combined Influence of the Lock end and the Approach Harbour

Measurements of Abraham, van der Burgh and Vos (1973, lit. [2])

The study reported in lit. [2] concerns the calculation and the measurements of the salt intrusion through locks as a function of the air discharge of an air-bubble screen. As a part of this study, measurements concerning the extent of water exchange in the lock chamber are presented as functions of time.

The measurements presented here concern the prototype measurements of the exchange of the water of two fresh locks with a salt approach harbour near IJmuiden (the Netherlands). The approach harbour is very wide relative to the locks, the air-bubble screen is not in use, the opened gate is near the approach harbour and there are no ships in the lock. The extent of exchange of the contents of the lock is presented as a function of time. Unfortunately no measurements of the level of the interface as a function of time or distance are available.

From the measurements the following variables as functions of time are derived:
- The mean, unmixed thickness of the salt layer in the lock which is divided by the water-depth: $a_1 / h_k$. This agrees with the extent of exchange of the contents of the lock.
- The coefficient of the front velocity: $c_1' = c_1 / \sqrt{\epsilon g h_k}$
- The coefficient of the exchange discharge: $q_{ex}' = q_{ex} / (h_k / (\epsilon g h_k))$

Time is made dimensionless by dividing by the travelling time $t_c = l_k / c_1$ of the front between the opened gate and the lock end.

Table 5.3 : Measured values of the exchange of the water in a fresh lock with that in a salt approach harbour (without air-bubble screen) (Abraham, 1973, lit. [2])

<table>
<thead>
<tr>
<th></th>
<th>$t/t_c$</th>
<th>$c_1'$</th>
<th>$q_{ex}'$</th>
<th>$\bar{a}_1/h_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle Lock IJmuiden</td>
<td>0 - 2</td>
<td>0.45</td>
<td>0.19</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 - 4</td>
<td></td>
<td>0.10 - 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>Northern Lock IJmuiden</td>
<td>0 - 2</td>
<td>0.45</td>
<td>0.20</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 - 3.5</td>
<td></td>
<td>0.10 - 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>
The mean thickness of the salt layer and the exchange discharge are presented as functions of time in figure T45 (dimensionless variables). The results for the situation of the exchange of the water in a salt lock with that in a fresh approach harbour are similar.

\[ \frac{a_1}{h_k} \quad \text{and} \quad q_{ex}' \]

\[ \frac{a_1}{h_k} \quad [-] \quad q_{ex}' \quad [-] \]

Fig. T45: The mean thickness of the salt layer and the exchange discharge (after Abraham, van der Burgh and Vos, 1973, lit. [2])

**Mean thickness of the salt layer**

The mean thickness \( a_1 \) of the salt layer increases linearly during the period \( 0 < t < 2t_c \) because of the constant exchange discharge. After a period of \( t = 2t_c \), when the reflected wave reaches the opened gate for the first time, the interface in the lock is almost horizontal. The flow velocity in the lock then has vanished. At this stage, the thickness of the salt layer is between 0.8 and 0.85 times the water-depth.

After \( t = 2t_c \) there is a decrease in the growth of the mean thickness. The situation can be considered in two ways. The first possibility is that a new exchange flow arises in a stratified lock with a salt layer of 0.8 and 0.85 times the water-depth and the salt sea. Another possibility is that the internal wave is partially reflected against the approach harbour. After \( t = 3.5 \) or \( 4t_c \) the contents of the lock are completely exchanged.

**Exchange flow rate**

The exchange flow rate \( q_{ex}' \) is between 0.19 and 0.20 during the period \( 0 < t < 2t_c \). This corresponds to the situation of the exchange flow in a prismatic, rectangular channel (section 5.2).

The exchange flow rate \( q_{ex}' \) decreases markedly when the reflected front
reaches the approach harbour for the first time. The flow rate decreases from \(0.10\) to \(0\) between \(t = 2\ t_c\) and \(4\ t_c\).

Results

The measured values of the mean thickness of the salt layer and the exchange discharge correspond to the situation of the exchange of the water of a lock with that of a channel with the same cross-section as the lock (section 5.3.2). Therefore it can be concluded that the influence of the approach harbour and of the distance between the gate and the approach harbour are imperceptible.

The coefficient \(q_{ex}'\) of the exchange discharge in a lock varies between \(0.19\) and \(0.20\) until the reflected front reaches the opened gate (at \(t \approx 2\ t_c\) where \(t_c\) is the travelling time of the front between opened gate and lock end). The thickness of the salt layer \(a_1\) is between \(0.15\) and \(0.20\ h_k\) at \(t \approx 2\ t_c\).

The exchange discharge decreases immediately when the reflected front reaches the approach harbour (at \(t \approx 2\ t_c\)). The mean value of the coefficient \(q_{ex}'\) is about \(0.045\) between \(t = 2\) and \(4\ t_c\). The lock is completely exchanged after \(t = 4\ t_c\).

5.3.5 Schematicized Calculations

5.3.5.1 Set-up of the Schematicized Calculations

![Diagram showing schematicized situation of the exchange flow after reflection](image)

Fig. T46: Schematicized Situation of the Exchange Flow after Reflection

The following subscripts are used: (figures T35 and T46)

- \(l\) = lower front (before reflection occurs)
- \(u\) = upper front
\[ g = \text{opened gate} \]
\[ c = \text{lock end} \]
\[ e = \text{just in front of the internal wave (after reflection has occurred)} \]
\[ r = \text{just behind the internal wave (after reflection has occurred)} \]

The exchange situation which is calculated is as follows. The lock and the channel have the same rectangular cross sections. The lock has a lock end at one end and a gate which is opened in a given time interval at the other end. The channel is so long that, in the time and length considered, no reflection occurs. The gate which is opened is the initial separation between the lock and the channel. An initial difference in density is present between the water in the lock and the channel. The initial difference in water-level between the lock and the channel is so small that no external translatory waves are generated. There is no ship present in the lock or the cross section of the ship is so small that it does not influence the exchange flow.

The results of the schematized calculation of the exchange in a rectangular channel without reflection (section 5.2) are used up to the instant of the reflection of the salt front against the lock end. These results concern the height and the velocity of the front and the water-levels at the fronts. Assuming there is a straight interface between the salt and fresh fronts, the water-level at the opened gate is calculated by means of the momentum equation.

The determination of the exchange flow after the reflection of the salt front against the lock end is done in the following way. Contrary to the preceding approach, the layer thicknesses and the wave speed after reflection are determined empirically. This approach is based on the observations from the experiments. The reason for this is that simplified calculations were not successful.

The observations showed that the height of the salt layer at the lock end is doubled and the velocity of the front of the internal wave (or jump in the interface) is equal (but opposite) to the velocity of the front before the reflection. The water-level is calculated with the help of the empirical level of the interface and with continuity and momentum equations. The part of the exchange flow which is not influenced by the reflection is determined in the same way as in the absence of the reflection.

The interface level and the water-level are determined as functions of time and distance in the lock during one inward and return movement of the salt front in the lock (two times the travelling time). The calculations of the water-level are compared with model investigations.

The calculation of a longitudinal force on a ship in the lock as a sequel to the above-mentioned calculation is done for a ship cross-section which is very small compared to the cross-section of the lock. With this condition, the longitudinal force is calculated using the method of section 4.2.
5.3.5.2 Assumptions

Influence of the Approach Harbour

The influence of the presence of an approach harbour on the exchange flow in the lock is not taken into consideration. The theory of the exchange flow in a prismatic channel (section 5.2) is applied. If an approach harbour is present the level of the interface at the approach harbour and the gradient of the interface in the lock will therefore deviate in some degree.

Interface and Free Surface

The level of the interface and the free surface at the discontinuities are determined. These discontinuities are the salt and the fresh fronts, the closed and opened gate and the reflected internal wave. The discontinuities in the interface and the water-level are linked by straight lines.

Before the Reflection at the Lock End

The same assumptions for the exchange flow in the channel (section 5.2) are now valid. The velocity and the height of the front do not change during the exchange process and are not influenced by increased friction at the bottom and the interface and energy losses around the front. The leading edge of the front is vertical.

After the Reflection at the Lock End

The interface and water-levels between the fresh front and the reflected internal wave are treated in the same way as before the reflection. The level of the interface between the lock end and the internal wave is determined by:

* the complete reflection of the salt layer against the lock end, with doubling of the salt layer thickness at the lock end.
* the velocity of the internal wave, equal to the original velocity of the front
* the height of the internal wave, equal to the original height of the salt front

The original velocity of the salt front is used for the velocity of the propagating internal wave. Instead of this velocity of the front, the velocity of a small disturbance in the interface according to equation (5.9) can be considered. If equation (5.9) is simplified, the velocity of the internal wave is then:

\[
  c_i \approx \sqrt{\frac{\epsilon g a_1 a_2}{(a_1 + a_2)}}
\]  

(5.38)
Denton (1990, lit. [9]) presented a relation for the velocity of an internal bore as a function of the thicknesses of the layers with the introduction of energy loss coefficients.

These methods are not used because the disturbances in the interface are too large, and because the experiments show that the velocity of the internal wave is equal to the velocity of the front.

Flow velocities

The flow velocities in the lower layer are derived by means of continuity equations using the calculated velocities, the heights of the fronts and the assumed straight interfaces. The velocities in the upper layer are estimated by neglecting the deformations of the water-level.

5.3.5.3 Level of the Interface and Flow Velocities

Fronts

The heights $a_{11}$ and $a_{21}$ of the fronts are determined according to section 5.2.6 with the equations (5.33) and (5.34).

The front velocities $c_1$ and $c_u$ of equations (5.31) and (5.32) are simplified according to:

$$c_1 = c_1' \sqrt{(g h_k)}$$

$$c_u = c_u' \sqrt{(g h_k)}$$

where $c_1'$ and $c_u'$ are taken from equations (5.31) and (5.32)

The flow velocity $v_{21}$ above the salt front is estimated by means of the continuity equation (5.23) and the substitution of $(h_k - a_{11})$ for $a_{21}$.

The flow velocity $v_{1u}$ beneath the fresh front is estimated in the same way with equation (5.17) and the substitution of $(h_k - a_{2u})$ for $a_{1u}$.

Opened Gate

The thickness $a_{1g}$ of the lower layer at the opened gate is determined by the assumption of a straight line between the fronts and the substitution of $(h_k - a_{2u})$ for $a_{1u}$ according to (figure T35):

$$a_{1g} = a_{11} + \frac{c_1}{(c_u + c_1)} (a_{1u} - a_{11})$$

The flow velocity of the lower layer $v_{1g}$ at the opened gate can be derived from the following continuity equation for the lower layer between the salt front and the opened gate:
\[ v_{1g} a_{1g} = c_1 \left( a_{11} + \frac{1}{2} (a_{1g} - a_{11}) \right) \quad (5.42) \]

The flow velocity \( v_{2g} \) of the upper layer at the opened gate is estimated by the continuity equation for the lower and upper layers at the opened gate:

\[ v_{2g} (h_k - a_{1g}) = v_{1g} a_{1g} \quad (5.43) \]

**Lock End**

The salt layer thickness \( a_{1c} \) at the lock end is determined by the doubling of the height of the incoming salt layer according to: (figure T46)

\[ a_{1c} = 2 \left( a_{11} + (c_1 t - x_c) \right) \left( a_{1u} - a_{11} \right) / \left( (c_1 + c_u) t \right) \quad (5.44) \]

where \( x_c \) = horizontal distance between opened and lock end.

At the lock end flow velocities in both the upper and the lower layer vanish.

**Just in front of the Internal Wave**

The position \( x_r \) of the internal wave is as follows: (figure T46)

\[ x_r = 2 x_c - c_1 t \quad (5.45) \]

The thickness \( a_{1r} \) of the lower layer just in front of the internal wave can be estimated with the assumption of a straight line for the interface between the fronts:

\[ a_{1r} = a_{11} + \frac{c_1 t - x_r}{(c_1 + c_u) t} \left( a_{1u} - a_{11} \right) \quad (5.46) \]

The flow velocity of the lower layer \( v_{1r} \) can be derived from the continuity equation (5.47) between the opened gate and just in front of the internal wave. The flow velocity of the upper layer \( v_{2r} \) can be estimated by means of the continuity equation (5.48) for the upper and lower layers.

\[ v_{1r} \left( a_{1r} + \frac{1}{2} (a_{1g} - a_{1r}) \right) = c_1 a_{11} + \frac{c_1^2}{c_1 + c_u} \left( a_{1u} - a_{11} \right) \quad (5.47) \]

\[ v_{2r} = \frac{v_{1r} a_{1r}}{(h_k - a_{1r})} \quad (5.48) \]
Just behind the Internal Wave

The difference between the thickness $a_{1e}$ of the lower layer just behind the internal wave and the thickness $a_{1r}$ of the lower layer just in front of the internal wave is the front height $a_{1l}$: (figure T46)

$$a_{1e} = a_{1r} + a_{1l} \quad (5.49)$$

The flow velocities of the lower layer $v_{1e}$ and the upper layer $v_{2e}$ can be estimated by means of continuity equations:

$$v_{1e} a_{1e} = v_{1r} a_{1r} - c_1 a_{1l} \quad (5.50)$$
$$v_{2e} (h_k - a_{1e}) = v_{1r} a_{1r} - c_1 a_{1l} \quad (5.51)$$

5.3.5.4 Equations for the Water-level at the Discontinuities

The thickness of the lower layer and the flow velocities of the upper and lower layers are now derived with the help of the previous section. The water-levels of the different cross-sections are defined as the elevations of the water-level with respect to the water-level of the salt water outside the exchange flow. These elevations are determined by the application of momentum equations.

The differences in water-level are calculated because they are needed for the determination of the longitudinal force on a small ship which does not influence the flow and density distribution.

Water-level outside the Salt Front in front of the Reflection

In section 5.2.2.2 the momentum equation (5.14) of the whole control volume is derived by means of terms for the change of momentum in time (figure T35). The contribution of this change of momentum in time is very small in relation to the hydrostatic forces and can be ignored. The total difference in water level $\Delta h$ is then:

$$\Delta h = \left(\frac{\rho_1}{\rho_2} - 1\right) h_k \approx \frac{1}{2} \varepsilon h_k \quad (5.52)$$

Salt Front

In section 5.2.2.4 the equations for the salt front are presented. The water level elevation $\Delta h_1$ above the salt front can be derived from equations (5.22), (5.23) and (5.52), neglecting second order terms: (figure T35)

$$\Delta h_1 = \Delta h - \frac{1}{2} \varepsilon \frac{a_{1l}^2}{h_k} = \frac{c_1^2 a_{11}}{g (h_k - a_{1l})} \quad (5.53)$$
The thickness of the fresh layer \( a_{21} \) can be derived with equation (5.30).

**Fresh Front**

In a similar way the water-level elevation \( \Delta h_u \) above the fresh front can be derived from equations (5.16), (5.17) and (5.52): (figure T35)

\[
\Delta h_u = \frac{1}{2} \epsilon \left( h_k - \frac{(h_k - a_{2u})^2}{h_k} \right) - \frac{c_u^2 a_{2u}}{g (h_k - a_{2u})}
\]  

(5.54)

**Opened Gate**

The momentum equation for the control volume between the cross-sections of the opened gate (3) and just outside the salt front (5) yields the thickness of the upper layer \( a_{2g} \) (figure T35)

\[
\frac{1}{2} \rho_2 g (a_{1g} + a_{2g})^2 + \frac{1}{2} (\rho_1 - \rho_2) g a_{1g}^2 - \frac{1}{2} \rho_2 g (h_k + \Delta h)^2 + \\
\rho_1 v_{1g}^2 a_{1g} + \rho_2 v_{2g}^2 a_{2g} - \\
\frac{1}{4} \rho_1 c_1 (a_{1l} + a_{1g}) (c_1 + v_{1g}) + \frac{1}{4} \rho_2 c_1 (a_{2l} + a_{2g}) (v_{2l} + v_{2g}) = 0
\]

(5.55)

The water-level elevation \( \Delta h_g \) at the opened gate is derived by means of:

\[
\Delta h_g = a_{1g} + a_{2g} - h_k
\]

(5.56)

**Lock End**

The momentum equation can be derived for the control volume between the cross-sections of the lock end (c) and just outside the fresh front (1), after the reflection of the salt front against the lock end: (figure T46)

\[
\frac{1}{2} \rho_1 g h_k^2 - \frac{1}{2} \rho_2 g (a_{1c} + a_{2c})^2 - \frac{1}{2} (\rho_1 - \rho_2) g a_{2c}^2 \\
- \frac{1}{4} \rho_1 (a_{1u} + a_{1g}) (v_{1u} + v_{1g}) c_u + \frac{1}{4} \rho_2 (a_{2u} + a_{2g}) (v_{2g} + c_u) c_u \\
+ \frac{1}{4} \rho_1 (a_{1g} + a_{1r}) (v_{1g} + v_{1r}) c_1 - \frac{1}{4} \rho_2 (a_{2g} + a_{2r}) (v_{2g} + v_{2r}) c_1 \\
- \frac{1}{4} \rho_1 (a_{1e} + a_{1c}) v_{1e} c_1 + \frac{1}{4} \rho_2 (a_{2e} + a_{2c}) v_{2e} c_1 = 0
\]

(5.57)

The thickness of the fresh layer \( a_{2c} \) can be derived from this equation. The water-level elevation \( \Delta h_c \) at the lock end is then:

\[
\Delta h_c = a_{1c} + a_{2c} - h_k
\]

(5.58)
Just in front of the Internal Wave

The water-level elevation $\Delta h_r$ just in front of the internal wave is determined by means of a straight water-surface between the cross-sections of the opened gate (g) and just in front of the internal wave in the interface (r): (figure T46)

$$\Delta h_r = \Delta h_g + x_r / (c_1 t) \left( \Delta h_1 - \Delta h_g \right)$$  \hspace{1cm} (5.59)

The thickness of the fresh layer $a_{2r}$ is then:

$$a_{2r} = h_k + \Delta h_r - a_{1r}$$  \hspace{1cm} (5.60)

Just behind the Internal Wave

The thickness of the fresh layer $a_{2e}$ just behind the internal wave is also derived by means of the momentum equation. This momentum equation concerns the control volume just in front of (r) and behind the internal wave (e) in the interface, and is applied with a co-ordinate system moving with the internal wave: (figure T46)

$$\frac{1}{2} \rho_2 \ g \ (a_{1r} + a_{2r})^2 + \frac{1}{2} (\rho_1 - \rho_2) \ g \ a_{1r}^2 - \frac{1}{2} \rho_2 \ g \ (a_{2e} + a_{1e})^2$$
$$- \frac{1}{2} (\rho_1 - \rho_2) \ g \ a_{1e}^2 + \rho_2 \ (c_1 - v_{2r})^2 a_{2r} - \rho_2 \ (c_1 - v_{2e})^2 a_{2e}$$
$$+ \rho_1 (c_1 + v_{1r})^2 a_{1r} - \rho_1 (c_1 + v_{1e})^2 a_{1e} = 0$$  \hspace{1cm} (5.61)

The water-level elevation $\Delta h_e$ just behind the internal wave is then:

$$\Delta h_e = a_{1e} + a_{2e} - h_k$$  \hspace{1cm} (5.62)

5.3.5.5 Computer Model

The computer model LOCKDENS 1 has been developed to describe the exchange process in the fresh lock as a function of time and distance. (LOCKDENS 2, which is not dealt with, concerns the salt lock)

Two empirical coefficients are introduced in the model:

The first coefficient concerns the decrease in the difference in density during the exchange process. The fronts of the exchange current generate mixing. The local differences in density decrease through mixing and therefore the exchange process is delayed. The difference in density is adapted according to:

$$\epsilon = B (\rho_1 - \rho_2) / \rho_2$$

where $B =$ coefficient for the decrease of the relative difference in density between 0 and 1 (e.g. 0.9)
The second coefficient concerns the time of the beginning of the exchange process. The gate is opened within a given interval of time and the exchange process starts gradually. Therefore the beginning of the exchange process is shifted in the computer model, by means of the following time step:

\[ P_t \cdot t_g \]

where \( t_g \) = the opening time of the gate
\( P_t \) = coefficient for shifting the beginning of the exchange process between 0 and 1 (e.g. 0.5)

The computer model LOCKDENS1 calculates time histories of:
- the position of the fronts and the internal wave
- the levels of the interface and the flow velocities at the discontinuities (salt and fresh fronts, opened and lock end, just in front of and behind the internal wave in the interface) according to section 5.3.5.3
- the water-level at the discontinuities according to section 5.3.5.4
- the levels of the interface, the flow velocities and the water-level at the bow and the stern of a small ship with the assumption of a straight interface and a straight water-surface between the discontinuities
- the longitudinal force of a small ship according to section 4.2 with the assumption that the ship does not influence the flow pattern

5.3.5.6 Results of the Schematized Calculations

Input

The results of the computer model are presented together with results of the hydraulic model tests (section 2.2) with the following input data:
- the lock: \( l_k \cdot b_k \cdot h_k = 7.30 \cdot 0.60 \cdot 0.20 \text{ m}^3 \)
- the densities: \( \rho_1 = 1020 \text{ kg/m}^3 \) (channel), \( \rho_2 = 1000 \text{ kg/m}^3 \) (lock)
- the opening time of the roller gate: \( t_g = 16 \text{ s} \)

The lock can be compared with a push-tow lock with a scale factor 40 or a sea-lock with a scale factor 70. The length co-ordinates of the lock are between \( x = 0 \) (opened gate) and \( x = 7.30 \text{ m} \) (lock end).

The coefficients which are used for the calculation are as follows:
- the coefficients: \( k_1 = 1.00 \), \( k_u = 0.25 \), \( \beta = 0.90 \), \( P_t = 0.60 \)

Results

In figures 47 up to and including 50 the thickness of the lower layer, the flow velocities of the lower and the upper layer and the water-level are presented as functions of the distance up to the opened gate, for time intervals of 20 s. The initial line is marked with +, then box (20 s), \( x \) (40 s) up to box cross (160 s).

As mentioned in section 5.3.5.1, the calculated shape of the interface
is determined with the help of the observations in the hydraulic model. The greater height and the lower velocity of the salt front as compared to the fresh front are clearly visible (figure 47, thickness of the lower layer). The shape of the fresh layer after reflection is also visible.

The flow velocity in the lower layer decreases very strongly (almost to zero) behind the reflected wave (figure 48, flow velocity lower layer). The flow velocity in the upper layer above the reflected wave is not negligible because of the small thickness of the layer. This flow velocity increases with distance as far as the lock end, because of the decreasing layer thickness. (figure 49, flow velocity upper layer).

The shape of the water-surface is opposite to that of the lower layer (figure 50, water-level). The water-level at the opened gate is roughly on a straight line between the water-levels at the fronts. The fall in water-level above the (reflected) internal wave in the interface is somewhat smaller than the fall in the water-level at the (non-reflected) salt front. The water-level above the reflected part of the salt layer is almost horizontal.

The water-levels and the differences in water-level are also calculated as functions of time (figure 52 up to and including 55). The water-levels concern the following cross-sections in the lock: cross-sections 1 (x = 0.40 m), 2 (x = 2.40 m), 5 (x = 4.93 m) and 7 (x = 6.80 m). The differences in water-level are derived from these water-levels, for cross-sections 1 and 7 and cross-sections 2 and 5.

The water-level falls after the passage of the front and the internal wave. The height and the time of the fall in water-level agree rather closely with the model investigations (figure 52 and 53). The chosen coefficients for the energy losses (k₁ and k₉) at the fronts, for the decreased difference in density (β) and the shifted beginning (p₀) are correct. The differences in water-level between cross-sections 1 and 7 (almost at the opened and lock end) and between cross-sections 2 and 5 (almost at 1/3 and 2/3 of the lock length) correspond closely to the measurements (figure 54 and 55). The results of the calculation of the longitudinal force on a small ship are presented in figure 56. Unfortunately there are no measurements available. The dimensions of the ship are: lₙ * bₙ * dₙ = 0.96 * 0.125 * 0.055 m³. This agrees with the "Spits" type of ship in a push-tow lock, with a model scale of 40. The longitudinal force is negative while the front passes the bow, decreases after the front has passed the stern, becomes positive after the internal wave passes the stern and vanishes when the internal wave passes the bow. In the next section it is found that the calculated forces for the "Spits" are rather small when compared to the forces on a large ship.
5.4 The Exchange in a Lock with the Influence of a Ship

5.4.1 Introduction

The exchange flow of the water of a lock with that of an approach harbour is investigated in this section. In contrast with the previous section a ship is present in the lock, which hinders the exchange flow. As mentioned in chapter 2 the pattern of the exchange flow undergoes important changes, because the exchange of the water behind the ship with the water of the approach harbour is much delayed. The levels of the interface, the flow velocities and the water-levels adapt to the changed situation. The ship encounters hydraulic forces because of the exchange flow, while the ship generates a pressure distribution on the surrounding water.

There is practically no literature available concerning this flow pattern and the longitudinal force in the above-mentioned exchange situation. Only Ringers (Ringers, 1927, lit. [20]) measured longitudinal forces in the prototype. Kolkman (Kolkman, 1986, lit. [14]) described an investigation into the methods to limit salt water / fresh water exchange in locks. As part of the study he looked at the difference in water-level in transverse direction of a ship during the transverse exchange of locks with a salt/fresh separation system. This exchange occurs via openings in the walls and in the bottom of the lock. The flow pattern differs widely from that in our situation.

The aim of this section is to develop computer models to estimate the longitudinal force on the ship during the exchange process. The computer model is only approximate, because of the very complicated flow situation. The model is fitted to the measurements, therefore the computer model can only be applied around the range of the measurements. We have not attempted to develop a more precise mathematical model within the framework of this study.

Firstly we describe the computer model for the exchange of the water in a fresh lock with that in a salt approach harbour (section 5.4.2) and the results of this model (section 5.4.3). After that, the computer model for the opposite situation of a salt lock and a fresh approach harbour and the results of this model are shown.

In this section the emphasis is laid on the derivation of the density and flow pattern in the lock around the ship and on the presentation of the longitudinal force on the ship. We refer to chapter 4 for the calculation of this force.

5.4.2 Computer Model of a Fresh Lock and a Salt Approach Harbour

5.4.2.1 Modelling

The longitudinal force on the ship is derived from the level of the
interface, the densities of the layers and the transport of momentum at
the bow and the stern of the ship (chapter 4). These variables must
therefore be determined before the longitudinal force can be calculated.

The computer model is based on the following starting points:
- Exchange flow before reflections
  The results of the calculation of the exchange in a rectangular
  channel without the influence of a ship are used for the initial
  exchange flow (section 5.2).
- Reflection against the ship
  The partial reflection of the intruding salt wedge against the bow is
  schematized as separate wave components. This scheme is based on the
  observations of chapter 2.
- Reflection at the lock end and the approach harbour
  The assumptions of the complete reflection of the exchange flow at the
  lock end and the partial reflection at the approach harbour originate
  from the results of section 5.3.
- The longitudinal force
  The calculation of the differences in water-level and longitudinal
  force are based on the equation of momentum with a ship (chapter 4).
  The results of the mathematical model are fitted to the measured
  values (chapter 2) by means of coefficients.

The mathematical modelling is as follows:
- The front heights and velocities of the salt wedge without the
  influence of reflections are determined.
- The salt wedge is separated into different wave components at the bow.
  These components consist of incoming, transmitted and reflected waves.
  All the wave components are followed in time during their propagation.
  The components reflect against the lock end, the approach harbour, the
  stern and/or the bow.
- The different wave components determine the thickness of the lower and
  the upper layer at the bow and at the stern as functions of time.
  Furthermore, the flow velocities and the transports of momentum at the
  bow and the stern are determined by the wave components.
- The difference in water-level between the bow and the stern and the
  longitudinal force are determined.

5.4.2.2 Exchange Parameters without Reflection

The intruding salt wedge without the influence of reflection against the
ship (sections 5.2 and 5.3) is as follows: (figure T35)

- the front velocities:
  salt front: \( c_1 = c_1' \times (\epsilon g h_k) \) where \( c_1' = -0.072 \ k_1 + 0.52 \) (5.31)
  fresh front: \( c_u = c_u' \times (\epsilon g h_k) \) where \( c_u' = -0.072 \ k_u + 0.52 \) (5.32)

- the thicknesses of the salt layer:
  at the salt front: \( a_{11} = (0.091 \ k_1 + 0.27) h_k \) (5.33)
  at the fresh front: \( a_{1u} = h_k - (0.091 \ k_u + 0.27) h_k \) (5.34)
at the opened gate: \( a_{1g} = a_{11} + c_1 / (c_u + c_1) (a_{1u} - a_{11}) \)  

- the coefficients for the exchange process:
  - \( k_1 \) = energy loss coefficient for the salt front
  - \( k_u \) = energy loss coefficient for the fresh front
  - \( \beta \) = coefficient for the decrease in relative difference in density because of mixing at the front
  - \( p_c \) = coefficient for the delay in the exchange process caused by the gradual opening of the lock gate

5.4.2.3 Thickness of the Salt Layer and the Transport of Momentum in front of the Bow

Assumptions for the Reflection of the Salt Wedge against the Bow

![Diagram](image)

beside the bow  
in front of the bow

Figure T47: Schematized wave reflection at the Bow

The salt wedge which intrudes into the lock is influenced by the presence of the ship. This influence is visible as partial reflections of the salt wedge against the bow and the stern.

The incoming salt wedge at the bow is separated into three wave components:
- wave component 1: The salt wedge beside the bow propagates undisturbed in the direction of the lock end as a transmitted wave.
- wave component 2: The continuing component of the salt wedge in front of the bow propagates in the keel clearance in the direction of the lock end as a transmitted wave.
- wave component 3: The reflected component of the salt wedge in front of the bow propagates back in the direction of the opened gate as a reflected wave.

So wave components 1 and 2 propagate in the direction of the lock end, wave component 3 propagates back to the approach harbour.

The ratio between the discharge of wave component 1 and the sum of wave components 2 and 3 is determined by the ratio between the space beside
the ship and the width of the ship itself.

The ratio between the discharges in wave components 1 and 2 is determined by the maximum thickness of the transmitted salt wave in the keel clearance. A maximum layer thickness exists, because fresh water must flow in the direction of the opened gate from the point of view of continuity. This maximum thickness of the transmitted wave in the keel clearance is:

\[ p_{kc} (h_k - d_s) \]

where \( p_{kc} \) = keel clearance coefficient for the maximum thickness of the transmitted wave in the keel clearance with \( 0 < p_{kc} < 1 \)

The front speed and the flow velocity of the lower layer decreases in the keel clearance, because of increased friction and the decreased thickness of the salt layer. The measurements show that equations (5.64) and (5.31) yield good results (equation (5.64) is comparable to the velocity of propagation of external transitory waves):

\[ c_{1b} = c_1' \sqrt{\frac{\epsilon g (h_k b_k - b_s d_s)}{b_k}} \]

where \( c_1' = -0.072 k_1 + 0.52 \)

The exchange flow under the ship only remains undisturbed if the keel clearance is about equal to the water-depth.

The schematized calculation is based on one-dimensional flow. Therefore the incoming and reflected components of the salt wedge in front of the bow are replaced by one thickness of the salt layer at the bow, which is constant in transverse direction. The resulting flow velocity of the salt layer in front of the bow is determined by consideration of the total discharge of the incoming salt wedge and the reflected wave components. The resulting flow velocity of the fresh layer in front of the bow is derived by the application of the continuity equation in the cross section at the bow.

Assumptions for the Reflection at the Approach Harbour

In the situation investigated here, the distance between the opened gate and the approach harbour is not negligible with respect to the length of the lock. A fresh wedge with a front height of about 0.30 times the water-depth propagates in the direction of the approach harbour. The exchange discharge pertains to the situation of the exchange in a prismatic channel (section 5.2). After some time the fresh wedge propagates into the wide approach harbour, with internally critical flow at the transition of the approach harbour. The thickness of the fresh layer at the discontinuity of the approach harbour is at least equal to the height of the front (0.30 times the water-depth, section 5.3.3).
Figure T48: Schematized reflections at the approach harbour and the bow

In the situation of an exchange flow of the water in a channel with that in a sea with the opened gate just at the mouth (section 5.3.3), the thickness of the fresh layer measures between about 0.34 and 0.38 times the water-depth, while the flow is internally critical at the mouth. The exchange discharge is practically the same as for the exchange flow in a prismatic channel (section 5.2).

The two situations of the outflow of the fresh layer into the approach harbour are for the greater part similar. It is therefore assumed that the outflow of the fresh layer into the approach harbour causes no disturbances in the interface. This is affirmed by the measurements of Abraham (lit. [2], section 5.3.4).

The situation changes after some time, when the wave which is reflected against the bow ("wave component 3") reaches the approach harbour. Wave component 3 is partially reflected back in the direction of the lock and wave component 4 arises. The reason for this is that the disturbance of the interface ("wave component 3") meets a stratified situation at the discontinuity of the approach harbour. This generates a reflection in the direction of the lock. Subsequently, wave component 4 partially reflects against the bow to produce wave component 5, and so on.

Reflection coefficients are introduced:

\[ r_a = \text{coefficient for reflection of wave component 3 at the approach harbour (e.g. 0.2)} \]
\[ r_b = \text{coefficient for reflection of wave component 4 against the bow (between 0 and 1)} \]

The velocities of the disturbances in the interface between the bow and the transition at the approach harbour are assumed to be equal to the velocity of the salt front. The interface remains horizontal in transverse direction.
Assumptions for the Transport of Momentum

The transport of momentum at the bow and the stern must be known in order to determine the difference in water-level and the longitudinal force.

The flow velocities in front of the bow are calculated as the mean velocity of the lower and upper layers. In reality the flow velocity is not uniformly distributed across a layer. If these mean velocities are used for the calculation, the transport of momentum will be too low. Therefore the transport of momentum calculated with the mean velocity is artificially increased by multiplying by the following coefficient:

\[ p_I = \text{momentum coefficient for the deviation of the mean velocities (e.g. between 1 and 2)} \]

Determination of the Thickness of the Salt Layer in front of the Bow

Figure T49: Notations

The intruding salt wedge reaches the bow at the time \( t_b \):

\[ t_b = \frac{x_b}{c_1 + p_c t_g} \]

The time between the reflection at the bow and the approach harbour is \( \Delta t_{ba} \):

\[ \Delta t_{ba} = \frac{(x_b - x_a)}{c_1} \]
The salt layer in front of the bow is composed of:

- the incoming salt wedge over the width of the lock $b_k$:
  \[ a_{10}(t) = a_{11} + (c_1 t - x_b) / ((c_1 + c_u) t) (a_{1u} - a_{11}) \]

- three components of the reflected salt wedge over the width of the ship $b_s$:
  \[ a_{13}(t) = a_{10}(t) - c_{lb} / c_1 p_{KC} (h_k - d_s) \text{ if } a_{10} > p_{KC} (h_k - d_s) \]
  \[ a_{10}(t) \text{ if } a_{10} < p_{KC} (h_k - d_s) \]

- wave component 3, $a_{13}$, caused by the reflection of the incoming salt wedge ($a_{10}$) against the bow:

- wave component 4, $a_{14}$, caused by the reflection of wave component 3 ($a_{13}$) at the approach harbour:
  \[ a_{14}(t) = r_a a_{13}(t - 2t_{ba}) \]

- wave component 5, $a_{15}$, caused by the reflection of wave component 4 ($a_{14}$) against the bow:
  \[ a_{15}(t) = r_b a_{14}(t) \]

The salt layer contributions present in front of the bow are:

- from $t_b$: $a_{10}, a_{13}$
- from $(t_b + 2\Delta t_{ba})$: $a_{10}, a_{13}, a_{14}, a_{15}$

The resulting thickness of the salt layer is $a_{1b}$:

\[ a_{1b}(t) = (a_{10}(t) h_k + (a_{13}(t) + a_{14}(t) + a_{15}(t)) b_s) / b_k \tag{5.65} \]

**Determination of the Transport of Momentum in front of the Bow**

The flow velocity $v_{10}$ of the incoming salt wedge in front of the bow is derived from the continuity equation of the salt layer between the opened gate and the bow (figure T35):

\[ v_{10}(t) = c_1 (a_{1g} + a_{11}) / (a_{1g} + a_{10}(t)) \]
The resulting flow velocity \( v_{1b} \) in the salt layer in front of the bow can be derived from the contributions of the different wave components:

\[
v_{1b}(t) \ a_{1b}(t) \ b_k = v_{10}(t) \ (a_{10}(t) b_k - (a_{13}(t) - a_{14}(t) + a_{15}(t)) \ b_s)
\] (5.66)

The flow velocity \( v_{2b} \) in the fresh layer in front of the bow can be derived from the continuity equation of the cross-section at the bow:

\[
v_{2b}(t) \ (h_k - a_{1b}(t)) = v_{1b}(t) \ a_{1b}(t)
\] (5.67)

The transport of momentum \( I_b \) in front of the bow is:

\[
I_b(t) = p_I \ (\rho_1 \ a_{1b}(t) \ v_{1b}(t)^2 + \rho_2 \ a_{2b}(t) \ v_{2b}(t)^2) \ b_k
\] (5.68)

5.4.2.4 The Thickness of the Salt Layer and the Transport of Momentum behind the Stern

Assumptions for the Passage of the Exchange Flow past the Stern

Component 1 of the salt wedge which is the wave transmitted beside the bow, reaches the stern somewhat earlier than component 2, which is the wave transmitted in the space of the keel clearance. This is owing to the different front velocities. Both wave components of the salt layer will be spread over the width of the lock at the stern (constant thickness in transverse direction).

Assumptions for the Reflection of the Exchange Flow at the Stern

![Diagram](image_url)

Figure T51: Schematized reflections at the lock end and the stern

The salt wave, consisting of components 1 and 2, reflects completely against the lock end and propagates (as wave components 6 and 7) as an internal wave in the direction of the stern. After some time, wave components 6 and 7 partially reflect against the stern, and wave components 8 and 9 arise, which propagate back to the lock end. The coefficient for reflection against the stern is:

\[ r_s = \text{coefficient for reflection of wave components 6 and 7 against the stern} \]
The velocity of the internal waves is taken to be equal to the velocity of the initial salt front.

**Determination of the Thickness of the Salt Layer behind the Stern**

The salt wave beside the ship reaches the stern at the instant \( t_n \):
\[
  t_n = x_s / \frac{c_1}{P_c} t_g
\]

The salt wave beneath the ship reaches the stern at the instant \( t_s \):
\[
  t_s = x_s / \frac{c_{1b}}{P_c} t_g
\]

The time period between the passage of the stern and the reflection against the lock end is \( \Delta t_{sc} \):
\[
  \Delta t_{sc} = (x_c - x_s) / c_1
\]

![Diagram of salt wave components at the back of the stern](image)

**Figure T52: Schematized salt wave components at the back of the stern**

The salt layer behind the stern is composed of a maximum of 6 wave components in the computer model:

- three wave components beside the ship over the width \((b_k - b_s)\):
  - wave component 1, \( a_{11} \), the transmitted salt wave:
    \[
    a_{11}(t) = a_{11} + \frac{(c_1 t - x_s)}{((c_1 + c_u) t)} (a_{1u} - a_{11})
    \]
  - wave component 6, \( a_{16} \), caused by the reflection of wave component 1 \( a_{11} \) against the lock end:
    \[
    a_{16}(t) = a_{11}(t-2\Delta t_{sc})
    \]
  - wave component 8, \( a_{18} \), caused by the reflection of wave component 6 \( a_{16} \) against the stern:
    \[
    a_{18}(t) = r_s a_{16}(t)
    \]
- three wave components beneath the bottom of the ship over the width of the ship \( b_s \):
  - wave component 2, \( a_{12} \), the transmitted salt wave
    \[
    \text{if } a_{10} > P_{kc} (h_k - d_s): \quad a_{12}(t) = c_{1b} / c_1 P_{kc} (h_k - d_s)
    \]
    \[
    \text{if } a_{10} < P_{kc} (h_k - d_s): \quad a_{12}(t) = a_{11} + \frac{(c_1 t - x_s)}{((c_1 + c_u) t)} (a_{1u} - a_{11})
    \]
  - wave component 7, \( a_{17} \), caused by the reflection of wave component 2 \( a_{12} \) against the lock end:
    \[
    a_{17}(t) = a_{12}(t-2\Delta t_{sc})
    \]
  - wave component 9, \( a_{19} \), caused by the reflection of wave component 7 \( a_{17} \) against the stern:
    \[
    a_{19}(t) = r_s a_{17}(t)
    \]
The wave components present are:
- from \( t_n \): \( a_{11} \)
- from \( t_s \): \( a_{11} \) and \( a_{12} \)
- from \((t_n + 2\Delta t_{sc})\): \( a_{11}, a_{16}, a_{18}, a_{12} \)
- from \((t_s + 2\Delta t_{sc})\): \( a_{11}, a_{16}, a_{18}, a_{12}, a_{17}, a_{19} \)

The resulting thickness of the salt layer at the stern is \( a_{18} \):

\[
a_{18}(t) = \frac{(a_{11}(t) + a_{16}(t) + a_{18}(t)) (b_k - b_s) + (a_{12}(t) + a_{17}(t) + a_{19}(t)) b_s}{b_k}
\]  \( (5.69) \)

Determination of the Transport of Momentum behind the Stern

The flow velocity of the salt wave transmitted beside the stern is:

\[ v_{11}(t) = c_1 \frac{(a_{1g} + a_{11})}{(a_{1g} + a_{11}(t))} \]

The flow velocity of the salt wave transmitted beneath the stern is:

\[ v_{12}(t) = c_1 \frac{(a_{1g} + a_{11})}{(a_{1g} + a_{12}(t))} \]

The resulting flow velocity \( v_{1s} \) in the salt layer at the stern can be derived from the contributions of the different wave components:

\[
v_{1s}(t) a_{1s}(t) b_k = v_{11}(t) (a_{11}(t) - a_{16}(t) + a_{18}(t)) (b_k - b_s) + v_{12}(t) (a_{12}(t) - a_{17}(t) + a_{19}(t)) b_s
\]  \( (5.70) \)

The flow velocity \( v_{2s} \) in the fresh layer at the stern can be derived from the continuity equation of the cross-section behind the stern:

\[ v_{2s}(t) (h_k - a_{1s}(t)) = v_{1s}(t) a_{1s}(t) \]  \( (5.71) \)

The transport of momentum \( I_s \) at the stern is:

\[ I_s(t) = -p_I (\rho_1 a_{1s}(t) v_{1s}(t))^2 + \rho_2 a_{2s}(t) v_{2s}(t)^2) b_k \]  \( (5.72) \)

5.4.2.5 Computer Model

The computer model LOCKDENS 3 has been developed to calculate the longitudinal force on a large ship as a function of time, during the exchange process.

Firstly LOCKDENS 3 calculates the exchange parameters without the influence of the ship, according to section 5.4.2.2. After that the thicknesses of the salt layer and the transports of momentum at the bow and the stern are calculated as functions of time, according to sections 5.4.2.3 and 5.4.2.4. Then the differences in water-level between the bow and the stern and the longitudinal force on the ship are calculated according to section 4.3. The calculation is completed when the bow is reached by the wave reflection that has reflected against the lock end.
5.4.3 Results of Calculations for a Fresh Lock and a Salt Approach Harbour

5.4.3.1 Test 3 of the Prototype Investigation

The computer model LOCKDENS 3 is used for the calculation of the exchange situation of test 3 of the prototype measurements (chapter 2).

The data of test 3 are:
- the lock: \( l_k \times b_k \times h_k = 154.00 \times 16.10 \times 7.00 \, \text{m}^3 \), \( x_c = 154 \, \text{m} \)
- the ship: \( l_s \times b_s \times d_s = 76.50 \times 11.80 \times 3.80 \, \text{m}^3 \), \( x_b = 12 \, \text{m} \), \( x_s = 88.50 \, \text{m} \), \( C_b = 0.80 \)
- the densities: \( \rho_1 = 1015 \, \text{kg/m}^3 \) (approach harbour), \( \rho_2 = 999 \, \text{kg/m}^3 \) (lock)
- the mitre gates: \( t_g = 180 \, \text{s} \) with \( x_g = 0 \)
- the approach harbour: \( x_a = 58 \, \text{m} \) and very wide
- blockage: \( A_g/A_k = 0.40 \)

The chosen coefficients for test 3 are as follows:
- the fronts: \( k_1 = 1.00 \), \( k_d = 0.25 \), \( \beta = 0.90 \)
- the initial time: \( p_t = 0.40 \)
- the keel clearance: \( p_{kc} = 0.40 \)
- the reflections: \( p_{kc} = 0.40 \), \( r_a = 0.20 \), \( r_b = 0.20 \), \( r_s = 0.20 \)
- the transport of momentum: \( I_p = 1.50 \)

A similar calculation without the ship is executed with the help of LOCKDENS 1 (section 5.3). The same inputs are used, with the exception of the ship, the coefficients for reflection and the coefficient for the transport of momentum.

The horizontal position of the front is shown in figure 57 as a function of time. From the moment the salt front reaches the bow (\( t_b = 102 \, \text{s} \) with \( x_b = 12 \, \text{m} \)), the salt wedge is separated into three wave components, each with its own front.

Wave component 1 beside the ship reflects as the first wave component against the lock end (\( t_c = 417 \, \text{s} \) and \( x_c = 154 \, \text{m} \)) and then propagates in the direction of the opened gate. After about \( t = 770 \, \text{s} \) the bow is reached for the second time. At that moment the calculation is completed.

Wave component 2, beneath the ship, undergoes delayed propagation in the keel clearance and reaches the lock end, the stern and the bow for the second time somewhat later (about 48 s) than wave component 1.

Wave component 3 propagates from the bow in the direction of the approach harbour and partially reflects against the approach harbour (\( t_a = 255 \, \text{s} \) and \( x_a = 58 \, \text{m} \)). The reflected component propagates subsequently in the direction of the lock end.

The thickness of the salt layer at the bow and the stern is shown in figure 58 as a function of time for the situations with and without a
ship.
The following can be said of the situation with a ship. From the moment
the salt front reaches the bow a thick salt layer is present, which
thickens further on, after the arrival of the wave component reflected
at the approach harbour. The interface at the stern arises later and
remains lower than that in front of the bow.
The salt layer is much thinner at the cross-section of the "bow" in the
situation without a ship than in that with a ship. This is true until
the front reaches the cross-section of the "bow" for the second time.
The salt layer at the cross-section of the "stern" is much thicker in
the situation without a ship than in that with a ship. The reason for
this is that the ship blocks the exchange flow.

The flow velocities of the lower and upper layers at the bow and the
stern in the situation with a ship are presented in figure 59.
The flow velocity of the lower layer in front of the bow is limited
because of the influence of the reflection, which is accompanied by a
great layer thickness. The flow velocity of the upper layer is high and
increases further on, after the moment that the reflection from the
approach harbour reaches the bow, because of the thinness of the layer.
The flow velocities behind the stern are high from the moment that the
salt layer reaches the stern, because the salt layer passes the stern
undisturbed, without reflection. The flow velocity decreases after the
moment that the reflection against the lock end reaches the stern. The
flow velocity of the fresh layer behind the stern is restricted.

The transport of momentum for the situation with a ship is presented in
figure 60. The transport of momentum in front of the bow is mainly
determined by the high flow velocity of the upper layer. The shape of
the time function of the transport of momentum largely agrees with the
flow velocity of the upper layer (figure 59). In the same way the
transport of momentum behind the stern is determined by the high flow
velocity of the lower layer behind the stern.

The difference in water-level between the bow and the stern for the
situation with and without a ship is shown in figure 61.
After the moment that the salt front reaches the bow, the water is
continually lower at the bow than at the stern for the situation with a
ship.
The difference in water-level for the situation without a ship is much
smaller, owing to the absence of the blockage due to the ship. After
some time the front reaches the "ship" for the second time and the
water-level at the "bow" becomes higher than at the "stern".

Both the calculated and the measured longitudinal forces are shown in
figure 62. After the moment that the salt front reaches the bow the
longitudinal force is continually negative. Both the measured and the
calculated values agree well, with the exception of the oscillations in
the measured longitudinal force. These oscillations are caused by
external transatory waves.
The calculated and the measured mean densities over the cross-section in front of the bow and behind the stern can be seen in figure 63. The calculated values follow the measured values rather closely.

The close similarity between the calculated and the measured forces and the mean densities at the bow and at the stern is partly due to the choice of the coefficients in the schematized mathematical model. The influence of the coefficients is shown in the following section 5.4.3.2.

5.4.3.2 Sensitivity Analysis concerning the Coefficients for the Influence of the Ship

In the last section coefficients are so chosen that the calculated values are comparable with the measured values. In this section, the sensitivity of the coefficients for the influence of the ship on the exchange flow is investigated for the same example (Test 3).

Coefficient for the Transport of Momentum

The transports of momentum at the bow and the stern are important for the determination of the longitudinal force on the ship (Chapter 4, force-relations). In the computer model, the transport of momentum is multiplied with a coefficient \( P_L \), to introduce the non-uniformities in the flow velocity of the cross-section. The coefficients \( P_L = 1, 1.5 \) and 2 are used in the example of figure 64. The longitudinal force increases with the coefficient \( P_L \).

Coefficient for the Keel Clearance

The coefficient \( P_{kc} \) determines which height of the salt wave is transmitted in the keel clearance. In figure 65, the coefficient \( P_{kc} \) is varied between \( P_{kc} = 0.2 \) and 0.6.

The thickness of the salt layer in front of the bow and behind the stern is influenced by \( P_{kc} \). It appears that the longitudinal force is hardly influenced until the front reaches the stern. The reason for this is that the influence of the change in thickness of the salt layer at the bow is compensated for by the change in the transport of momentum at the bow.

In the case of a greater coefficient \( P_{kc} \), the longitudinal force decreases after the front has reached the stern, because more salt water flows to the stern, which implies a higher salt layer at the bow.

Coefficients for the Reflection

The influences of the reflection coefficients \( r_a, r_b \) and \( r_s \) on the longitudinal force are shown in figure 66.

Coefficient \( r_a \) concerns the reflection at the approach harbour. The
influence of this coefficient is noticeable from the moment that the reflection from the approach harbour reaches the bow (412 s). The longitudinal force increases considerably in the case of a greater coefficient $r_a$, because of the increased thickness of the salt layer at the bow. This implies a decreased thickness and an increased flow velocity of the upper layer, and an increased transport of momentum at the bow.

Coefficient $r_b$ concerns the reflection against the bow. A variation in the coefficient $r_b$ hardly influences the longitudinal force. The effect of this coefficient is just like that of coefficient $p_{kc}$. The adjustment to the salt layer thickness is compensated for by the adjustment to the transport of momentum at the bow.

Coefficient $r_s$ concerns the reflection against the stern. The longitudinal force is influenced from the moment the internal wave (jump in the interface) reaches the stern (570 s). The influence is not large.

5.4.3.3 Test 5 with an Air Bubble Screen

The longitudinal force can be decreased by the application of an air bubble screen at the opened gate. Test 5 was therefore carried out as part of the prototype measurements, to show the decrease of the longitudinal force (chapter 2). Test case 5 corresponds to test case 3 (section 5.4.3.1), with the exception of the application of an air bubble screen.

The influence of an air bubble screen on the exchange flow is described in chapter 2. The principal difference in the lock itself is the decreased density of the salt wedge which intrudes into the lock, with respect to the density of the approach harbour.

The calculation of test 5 is carried out in a much simplified way. The coefficient $\beta$, which is intended for the adjustment of the difference in density because of mixing at the front, etc., is decreased to account for the effect of the air bubble screen. With reference to the measured densities the coefficient $\beta$ has been chosen: $\beta = 0.5$.

In figures 67 and 68 the results are presented. The calculated longitudinal force coincides rather well with the measured value. The oscillations in the force are caused by the external translatory waves. The mean densities at the bow and the stern differ somewhat.

5.4.3.4 Tests of the Hydraulic Model Investigation

In this section, test cases originating from the model investigation (as mentioned in chapter 2) are considered. In addition we look at an example of the prototype investigation (Ringers, 1927, lit.[20]). The same coefficients as were applied for test 3 (section 5.4.3.1) are used for these test cases.
Test 26, "Rupel"

The data of the hydraulic model investigation (test 26) are:
- lock: \( l_k \cdot b_k \cdot h_k = 7.30 \cdot 0.60 \cdot 0.20 \text{ m}^3 \), \( x_c = 7.30 \text{ m} \)
- ship ("Rupel"): \( l_s \cdot b_s \cdot d_s = 2.53 \cdot 0.37 \cdot 0.10 \text{ m}^3 \),
  \( x_b = 1.05 \text{ m}, \ x_s = 3.58 \text{ m}, \ c_b = 0.80 \)
- densities: \( \rho_1 = 1020 \text{ kg/m}^3 \) (approach harbour), \( \rho_2 = 1000 \text{ kg/m}^3 \) (lock)
- roller gate: \( t_g = 16 \text{ s} \) with \( x_g = 0 \)
- approach harbour: \( x_s = 20 \text{ m} \)
- blockage: \( A_s/A_k = 0.31 \)

The calculated longitudinal force agrees rather closely with the measured longitudinal force (figure 69). The measured difference in water-level between the bow and the stern differs somewhat from the measured value (figure 70). The oscillations in the measured difference in water-level are external disturbances of the water-level, which are not calculated.

Increased Water-depth, Test 41

This only differs from test 26 in the water-depth: \( h_k = 0.25 \text{ m} \) (with blockage \( A_s/A_k = 0.25 \)). The agreement between the calculated and measured value of the longitudinal force is quite close (figure 71).

Decreased Water-depth, Test 40

This only differs from test 26 in water-depth: \( h_k = 0.15 \text{ m} \) (with blockage \( A_s/A_k = 0.41 \)). The agreement between the calculated and measured value of the longitudinal force is quite close (figure 72).

Push-Tow Unit, Test 80

This differs from test 26 both in a lower water-depth and in the sort of ship:
  water-depth: \( h_k = 0.12 \text{ m} \)
  ship "4+1": \( l_s \cdot b_s \cdot d_s = 3.82 \cdot 0.56 \cdot 0.098 \text{ m}^3 \),
  \( x_b = 0.07 \text{ m}, \ x_s = 3.89 \text{ m}, \ c_b = 0.90 \)
  blockage: \( A_s/A_k = 0.76 \)

The agreement between the calculated and measured values of the longitudinal force is moderate (figure 73).

Ringers and Josephus Jitta, 1927

Ringers and Josephus Jitta carried out prototype investigations in 1927 for the design of the large sea-lock in IJmuiden, The Netherlands. (1927, lit. [20]). They carried out the measurements in the small sea-lock, with a ship with a water displacement of 2500 m³ and measured a maximum longitudinal force of \( F_x = 27 \text{ kN} \) during the exchange process. This value strongly exceeded the longitudinal force during the filling
process of the lock. With the computer model LOCKDENS 3 the same situation is calculated with the above-mentioned coefficients. In figure 74 the calculated longitudinal force is presented. The maximum force agrees with the measured value.

5.4.4 Computer Model of a Salt Lock and a Fresh Approach Harbour

The computer model LOCKDENS 4 has been developed to calculate the longitudinal forces on a ship as functions of time, during the exchange of the water of a salt lock with that of a fresh approach harbour. The difference between this and the previous sections, 5.4.2 (LOCKDENS 3) and 5.4.3, is that in this section the density situation is changed. The approach of LOCKDENS 4 is the same as that of LOCKDENS 3. Therefore only the principal differences between the computer models are dealt with in this section.

Most of the assumptions and coefficients introduced for the schematizing of the computer model LOCKDENS 3 are also used in LOCKDENS 4. The only differences concern:
- The intruding wedge
  A fresh wedge intrudes into the lock at the water-surface instead of the salt wedge at the bottom
- The front velocity beside the ship
  The fresh wedge passes mainly alongside the ship instead of beneath it. The front velocity is hardly delayed and the same front velocity is applied in front of, alongside and behind the ship.
- The wave components at the bow
  This point is explained below.

The intruding fresh wedge at the bow is separated into three wave components:
- wave component 1: The wave transmitted beside the bow propagates undisturbed in the direction of the lock end
- wave component 2: The component transmitted in front of the bow propagates beside and possibly beneath the ship, in the direction of the lock end.
- wave component 3: The component of the fresh layer reflected in front of the bow propagates back in the direction of the opened gate

The ratio between the discharge of wave component 1 and the sum of the discharges of wave components 2 and 3 is determined by the ratio between the difference of the widths of the lock and the ship and the width of the ship.

The ratio between the thicknesses of wave components 2 and 3 is determined by the draught of the ship and the thickness of the intruding fresh wedge. The ratio proceeds from the idea that the bow can reflect a thickness of the intruding wedge equal to at least half the draught of the ship. There are two possibilities:
Figure T53: Schematized Division of the Fresh Layer at the Bow

- Twice the height of the intruding wedge is smaller than the draught; The wedge is completely reflected against the bow. The reflected component 3 has the same thickness as the intruding fresh layer; the transmitted wave component 2 is missing:

\[ a_{22}(t) = 0 \]
\[ a_{23}(t) = a_{20}(t) \]

where \((2 a_{20}) < d_s\)
- \(a_{20}(t)\) is the thickness of the intruding fresh layer at the bow
- \(a_{22}(t)\) is the thickness of continuing component 2
- \(a_{23}(t)\) is the thickness of reflected component 3

- Twice the height of the intruding wedge is greater than the draught. The wedge is partially reflected. The transmitted component 2 has a thickness equal to the difference between the double intruding wedge and the draught. The reflected wave component 3 has a thickness equal to the difference between the intruding wedge and wave component 2.

\[ a_{22}(t) = p_b \ (2 \ a_{20} - d_s) \quad (5.73) \]
\[ a_{23}(t) = a_{20}(t) - p_b \ (2 \ a_{20} - d_s) \]

where \((2 a_{20}) > d_s\)
- \(p_b\) = bow coefficient

The bow coefficient \(p_b\) replaces the keel clearance coefficient \(p_{kc}\) of section 5.4.3.

5.4.5 Results of the Calculation for a Fresh Lock and a Salt Approach Harbour

Results of Test 4

The computer model LOCKDENS 4 is used for the calculation of the exchange situation of test 4 of the prototype measurements (chapter 2).

The data of test 4 are the same as mentioned for test 3 (section 5.4.3), with the exception of the water-depth and the densities of the water:
\[ h_k = 7.30 \text{ m} \]
\[ \rho_1 = 1014 \text{ kg/m}^3 \text{ (lock)}, \quad \rho_2 = 1001 \text{ kg/m}^3 \text{ (approach harbour)} \]

The chosen coefficients for test 4 are exactly the same as for test 3 (section 5.4.3.1), with the exception of the bow coefficient:
\[ p_b = 0.25. \]

The horizontal position of the front is shown in figure 75 as a function of time. The fresh front reaches the bow at \( t_b = 102 \text{ s} \). Wave components 1 and 2 then propagate in the direction of the lock end. These components reach the stern for the first time at \( t_s = 264 \text{ s} \), and for the second time at \( t_{ss} = 552 \text{ s} \). The calculation is concluded at \( t_{bb} = 717 \text{ s} \), when the bow is reached for the second time. Component 3 propagates from the bow in the direction of the approach harbour and partially reflects against the approach harbour. This reflection reaches the bow at about \( t = 405 \text{ s} \). The exchange process is a bit quicker than test 3 (section 5.4.3.1), because of the greater water-depth and the greater front velocity of a fresh wedge.

The thickness of the salt layer at the bow and the stern is shown in figure 76 as a function of time. The thickness of the fresh layer at the bow becomes considerable after the arrival of the fresh front at the bow, and increases further after the reflection from the approach harbour reaches the bow. The salt layer at the stern arises later and its thickness remains limited owing to the blockage of the ship.

The flow velocities of the lower and upper layers at the bow and the stern in the situation with a ship are presented in figure 77. The flow velocity of the fresh layer at the bow is small because of the reflection of the fresh layer against the bow, while the flow velocity of the salt layer is high because of the small thickness of the layer. The flow velocity of the fresh layer at the stern is considerable, because the fresh layer flows undisturbed along the stern. This flow velocity decreases from the moment that the front reaches the stern for the second time. Figure 78 shows the transports of momentum at the bow and the stern.

The difference in water-level between the bow and the stern is shown in figure 79. From the moment at which the salt front reaches the bow, the water-level is continually higher at the bow than at the stern. The difference in water-level decreases after the moments at which the fresh layer reaches the stern for the first and then the second time.

The calculated and the measured longitudinal forces are shown in figure 80. From the moment at which the salt front reaches the bow, the longitudinal force is continually positive. The measured and the calculated values agree closely. The measured oscillations in the longitudinal force originate from the external transitory waves, and are not calculated here.
The calculated and the measured mean density in the cross-sections in front of the bow and behind the stern can be seen in figure 81. The calculated values follow the measured value rather closely.

**Sensitivity Analysis concerning the Coefficients for the Influence of the Ship**

In the computer model, the transport of momentum is multiplied by a momentum coefficient $p_I$, to introduce the non-uniformity of the flow velocity of the cross-sections at the bow and the stern. The coefficients $p_I = 1, 1.5$ and 2 are used in the example of figure 82. The longitudinal force increases with a greater coefficient $p_I$ and so a greater transport of momentum.

The **bow coefficient** $p_b$ in the computer model determines which part of the fresh wedge at the bow is reflected towards the approach harbour, and which part continues to propagate in the direction of the lock end. The coefficients $p_b = 0, 0.25$ and 0.5 are used in the example of figure 83. The longitudinal force decreases considerably with a greater coefficient $p_b$. The reason for this is that, in the case of a large value of $p_b$, the thickness of the fresh layer in front of the bow decreases, while that of the fresh layer behind the stern increases. The differences in the thicknesses of the layers between the bow and the stern are then reduced, and so are the longitudinal forces.

The influence of the **reflection coefficients** $r_a$, $r_b$ and $r_s$ on the longitudinal force are shown in figure 84. The influence is small, because of the small layer thicknesses of the reflected waves.

**Test 6 with an Air Bubble Screen**

The calculation of test 6 is carried out in the same way as mentioned in test 4. The coefficient $B$ for the adjustment to the difference in density is changed. With reference to the measured densities, the coefficient $B$ has been chosen as test 4: $B = 0.5$.

The oscillations in the measured longitudinal force (figure 85) are large and originate from the external disturbances of the water-level of the approach harbour. These oscillations are not calculated in this study.

The calculated and the mean measured longitudinal force agree rather closely until 300 s. After 300 s the calculated force remains too great. The calculated and the measured mean densities at the bow and the stern (figure 87) agree closely.

**Test 32 of the Hydraulic Model Investigation**

In this section test case 32 is investigated, which originates from the model investigation as mentioned in chapter 2. The same particulars as in test 26 are used, with the exception of the difference in density:
- 115 -

- the densities: $\rho_1 = 1020 \text{ kg/m}^3$ (lock), $\rho_2 = 1000 \text{ kg/m}^3$ (approach harbour)

The calculated longitudinal force agrees quite closely with the measured longitudinal force (figure 87). The measured difference in water-level between the bow and the stern differs somewhat from the measured value (figure 88). The oscillations in the measured difference in water-level concern external disturbances of the water-level, which are not calculated.
6 Calculation of the Influence of Density Differences during the Filling Process

6.1 Introduction

The longitudinal force on a ship during the filling process has various causes. The ship is subject to the gradients of the water-level caused by the external transitory waves, and the filling jets exert forces on the bow. An extra force is exerted in the case of density differences between the water in the lock and that of the filling discharge. This extra force can be an important part of the total longitudinal force, as mentioned in chapter 3.

The purpose of this chapter is to calculate the longitudinal force on the ship caused by density differences during the filling process. There is no literature available on this subject. Therefore the observations of our investigations of chapter 3 have been used to set up computations to attain this goal.

The flow and density patterns in the lock during the filling process are very complicated and depend strongly on the method of filling and the dimensions of the lock and the ship. The presentation of a single mathematical model which includes all possible flow and density situations is difficult to derive. Therefore a number of mathematical models are developed, which are tuned to the different flow and density situations.

All the mathematical models which have been developed proceed from the following principle. The observed flow and density patterns are schematized, and the water-level differences and the longitudinal force caused by the density differences are subsequently calculated by means of one-dimensional continuity and momentum equations.

First of all the various contributions to the total longitudinal force are described. The method of separating the contribution of the density differences from the other factors contributing to the longitudinal force is presented in section 6.2. Subsequently, the situations of three different flow and density patterns are considered. The first situation is the (apparently) simple case of the influence of the density differences on the water-level difference in a lock without a ship (section 6.3). Then the second situation of a lock with a ship with a limited blockage of the wet cross-section is determined (section 6.4). In section 6.5, the third situation of a lock with a ship with a large blockage is investigated.

The chapter is summarized and the conclusions are drawn in chapter 7.
6.2 General Description and Approach to the Calculation

6.2.1 Description of the Filling Process

We look at the general pattern in the lock during the filling process. This pattern concerns the filling discharge, the water-level, the flow and density distribution and the longitudinal force. Both situations without and with density differences are considered. This section is a sequel to chapter 3, and is intended as a preparation for the calculations.

6.2.1.1 Filling Discharge and Mean Water-level

The area of the filling openings in the gate increases gradually in time during the lifting of the valves. This area reaches a maximum value after a certain time. The water of the approach harbour flows into the lock through the filling openings. Owing to this, the mean water-level in the lock rises and the head difference over the filling gate decreases. The filling discharge increases until a maximum value is reached, after which the discharge decreases. When the discharge becomes zero, the lock is filled. (More detailed information in Appendix B).

The hydrostatic pressure differences across the filling openings and the area of the openings determine the filling discharge. For the same initial head difference across the filling gate, the filling discharge changes slightly in the case of density differences if compared to the case without density differences. The change in the discharge is very small because the hydrostatic pressure differences across the openings change only slightly due to the small difference in densities of the water.

6.2.1.2 Flow Pattern

The flow pattern in the lock during and shortly after the filling of the lock involves three aspects (figure 24):

One-dimensional Flow Effects

External, translatory waves are generated by the inflowing water at the filling gate. These waves are visible as travelling deformations of the free surface. The waves transport the water in the longitudinal direction of the lock. They propagate rapidly, but slacken and heighten beside the ship because the wet cross-section has decreased and because the ship moves vertically, together with the free surface. The waves reflect against the bow and the stern of the ship and against the downstream and the filling gates.

The water-level as a function of time has various harmonic periods in a closed lock (Kalkwijk, 1973, lit.[1]). The basic harmonic period \( T_e \) of the external translatory waves is determined by the length of the lock,
the water-depth and the ship according to:

\[ T_c \approx \frac{2 \left( l_k - l_s \right)}{\sqrt{\left( gh_k \right)}} + \frac{2 l_s}{\sqrt{\left( gh_k b_k - b_s d_s \right)}} \]

where \( \sqrt{\left( gh_k \right)} \) = wave velocity without ship
\( \sqrt{\left( gh_k b_k - b_s d_s \right)} \) = principal wave velocity beside the ship
\( l_k \) = length of the lock
\( h_k \) = water-depth
\( b_k \) = width of the lock
\( l_s \) = length of the ship
\( d_s \) = draught of the ship
\( b_s \) = width of the ship

The discharge in an arbitrary cross-section in the lock has two components:

- A rapidly varying component
  The rapidly varying component is caused by disturbances which propagate with the velocity of external transitory waves. The harmonic period and the passages of fronts and reflections are visible in the time history of this component.

- A slowly varying component
  The slowly varying component varies in a similar way to the filling discharge as a function of time. Furthermore, this component decreases in the longitudinal direction of the lock, because a smaller part of the lock must be filled (maximum at the filling gate, zero at the downstream gate).

The flow velocities pertaining to the external waves are low and uniformly distributed across the wet cross-sections.

Three-dimensional Flow Effects

The filling water flows into the lock in the form of jets. A great deal of turbulence is present in these filling jets, caused by the beams at the downstream side of the filling openings. Ambient water is entrained into the filling jets. However, a ship can hinder this entrainment. In the zone of entrainment, the discharge of the jet increases, the flow velocities in the jet decrease and the possible density differences relative to the ambient water decrease in the longitudinal direction. The filling jets can impinge on the bow of a ship which is near the filling gate.

The flow pattern beside the ship will deviate less from a one-dimensional flow situation. The flow at the stern of the ship usually separates from the ship. The flow velocities at the stern are low, because the discharge decreases in the downstream direction.
Intruding Layer due to the Density Differences

At a certain distance behind the filling gate the flow velocity of the jets is diminished in such a way that a stratified flow arises. A layer with a front as leading edge intrudes into the lock. This intruding layer consists of the water of the filling discharge and the entrainment. The front propagates with a low velocity in the downstream direction because of the small density differences. The intruding layer is at the bottom or at the water-surface, depending on the densities of the water in the intruding layer and the ambient water. The intruding layer reflects against the bow and the stern of the ship and against the gates of the lock, and continues to propagate long after the filling has ended.

The basic harmonic period $T_1$ of the internal wave is (with neglecting of the influence of the ship):

$$T_1 \approx \frac{2 \lambda_k}{c_1'/(\epsilon \ g \ h_k)}$$

where $c_1'/(\epsilon \ g \ h_k) = $ velocity of the internal front (e.g. equation (5.39))

6.2.1.3 Longitudinal Force on the Ship

The longitudinal force on the ship is positive in the downstream direction (direction of the filling discharge) and negative in the upstream direction. The force depends on the same aspects of the flow pattern, namely:

One-dimensional External, Translatory Wave

The water-levels at the bow and the stern and the longitudinal force on the ship oscillate rapidly with a period $T_e$, caused by the external translatory waves, around a slowly varying value.

The slowly varying component of the force is determined by the slowly changing discharge. The decrease of the discharge in the downstream direction continually yields a negative contribution to the force. The change of the filling discharge in time yields a positive contribution to the force until the instant of the maximum filling discharge. When the filling discharge decreases, this contribution becomes negative. The result is a positive component of the force just after the beginning of the filling process, which soon changes into a negative component of the force.

Three-dimensional Effects in the Flow

If the concentrated filling jets are present at the bow and directed
towards it, they will impinge on it. This results in a component of the longitudinal force in the downstream direction. Above the filling jets the water-level is lowered. If the bow is near this lower water-level, a component of the longitudinal force in the direction of the filling gate arises. If the flow separates at the stern, the water-level at the stern is lowered because of the energy-loss; the ship encounters a component of the longitudinal force in the downstream direction. The contribution to the longitudinal force caused by the three-dimensional effects is at a maximum just before the discharge has reached its maximum value.

Intruding Layer Due to the Density Differences

The component of the longitudinal force caused by the density differences is determined by the levels of the interface against the bow and the stern, and also by the difference in water-level between the cross sections of the bow and the stern. This component of the force is considered in the following sections (6.3, 6.4 and 6.5).

An example of the one- and three-dimensional effects of the flow in the longitudinal force under homogeneous conditions is presented in figures 89a and 89b (Rozenburg Lock, the Netherlands).

Considerations concerning the filling process without density differences can be found in the literature of Schijf (1936, lit. [21]), Kalkwijk (1973, lit. [11]), Kolkman (1973, lit. [14]), and de Jong and Vrijer (1980, lit. [31]) and in Appendix B.

6.2.2 Approach to the Calculations

In this section we present a method of calculating the difference in water-level and the longitudinal force caused by density differences in the case where the flow and density distribution around the ship are known. The added difference in water-level is determined by subtracting the difference in water-level in the case of absence of density differences from the difference in water-level in the case of presence of density differences. The added longitudinal force is determined in the same way.

6.2.2.1 Assumptions

The following assumptions underlie the calculations of the influence of the density differences on the longitudinal force:

- one-dimensional flow effects
  The rapidly varying flow effects can be separated from the slowly varying flow effects because of the different time-scale. The density differences influence the slowly varying flow effects, but hardly influence the rapidly varying flow effects of the external,
translatory waves. The influence of density differences on the friction force is not taken into account, because of the minor importance of friction.

- three-dimensional flow effects
The density difference in the lock between the water of the intruding layer and the original water in the lock is determined by the filling jets, and the possibility of entraining ambient water into these jets. The density difference hardly influences the three-dimensional effects of the filling jets themselves. The reason for this is that the behaviour of the jets is mainly determined by their momentum, as long as this momentum is high.

In sections 6.3, 6.4 and 6.5 schematized computer models are developed on the basis of these assumptions.

In the following section (6.2.2.2) the consequence of the first assumption is investigated. This concerns the influence of the density differences on the slowly varying one-dimensional flow in the lock.

The consequence of the second assumption, concerning the value of the density differences, is discussed in sections 6.3, 6.4 and 6.5.

6.2.2.2 Approach
As a sequel to the last section, we assume that the influence of density differences is determined by subtracting the slowly varying effects with density differences from the slowly varying effects without density differences. Friction is not taken into account.

Figure T54: Filling discharge as a function of time and slowly varying discharge as a function of distance

The slowly varying one-dimensional flow concerns the following discharge in the cross-sections:
- Discharge as a function of time
The discharge in a cross-section varies with the filling discharge as a function of time: from zero to a maximum value when the filling discharge is at a maximum, and subsequently back to zero when the lock has been filled.

- Discharge as a function of distance

The discharge in a cross-section decreases linearly in the longitudinal direction of the lock. The discharge is equal to the filling discharge at the upstream gate, and is zero at the downstream gate.

The one-dimensional momentum and continuity equations are applied twice: for the case without density differences and the case with density differences. The control volume considered contains the water in the lock between the cross-sections just in front of the bow and just behind the stern minus the volume of the ship. In the case of density differences, a two-layer current has been assumed with the levels of the interface below the bow and the stern.

The momentum equation concerns the following terms:
- the difference between the horizontal components of the hydrostatic forces against the boundaries of the control volume
- the transport of momentum through the boundaries of the control volume
- the change of the momentum in the control volume in time.

Figure T55: Control volumes for the case without density differences and the case with density differences

Momentum Equation in the Case of Absence of Density Differences

Firstly the resulting hydrostatic force is determined. It can be seen from figure T55 that:

\[ h_b - h_s = \Delta h_h \quad \text{and} \quad h_k = \frac{1}{2} (h_b + h_s) \]

The resulting hydrostatic force \( F_{hh} \) is:

\[ F_{hh} = \frac{1}{2} \rho_2 g \, b_k \, (h_b^2 - h_s^2) - \frac{1}{2} \rho_2 g \, b_s \, ((d_s + h_b - h_s)^2 - d_s^2) \]

\[ \approx \rho_2 g \, (b_k \, h_k - b_s \, d_s) \, \Delta h_h \quad (6.1) \]
Then the net transport of momentum $I_{dh}$ is derived. The slowly varying discharge decreases linearly in the longitudinal direction. The transport of momentum $I_{dh}$ is as follows:

$$I_{dh} = b_k \rho_2 \left( h_b v_b - h_s v_s \right)$$  \hspace{1cm} (6.2)

Subsequently, the time derivative momentum is determined. The change of momentum in time is estimated with the help of the mean velocity in the control volume. The mean flow velocity and discharge in the control volume are as follows:

$$v_m \approx \frac{l_k - \frac{1}{2} (x_b + x_s)}{l_k (b_k h_k - b_s d_s)} Q$$

$$Q_m \approx v_m (b_k h_k - b_s d_s)$$  \hspace{1cm} (6.3)

The change of momentum in time $I_{th}$ is the derivative of the product of the mass and velocity of the water of the control volume:

$$I_{th} \approx d(\rho_2 l_s (b_k h_k - d_s b_s) v_m)/dt$$

Substituting from (6.3), this yields:

$$I_{th} \approx d(\rho_2 l_s Q_m)/dt$$

$$\approx \rho_2 l_s dQ_m/dt$$  \hspace{1cm} (6.4)

**Momentum Equation in the Case of Presence of Density Differences**

A two-layer current has been assumed with the levels of the interface below the bow and the stern. The geometry yields (figure T55):

$$\Delta h_r = (a_{1b} + a_{2b}) - (a_{1s} + a_{2s})$$

$$h_k \approx \frac{1}{2} (h_b + h_s)$$

$$\approx \frac{1}{2} (a_{1b} + a_{2b} + a_{1s} + a_{2s})$$

The resulting hydrostatic force $F_{hr}$ is determined by the pressures against the boundaries of the control volume and the ship:
The total discharge of the two layers in a cross section in the case without density differences corresponds to that in the case with density differences. The continuity equations at the boundaries are as follows:

\[ F_{hr} = \frac{1}{2} \, g \, b_k \left[ \rho_2 \left( a_{1b} + a_{2b} \right)^2 + \left( \rho_1 - \rho_2 \right) a_{1b} \left( a_{1s} + a_{2s} \right)^2 - \left( \rho_1 - \rho_2 \right) a_{1s}^2 \right] \]

\[ - \frac{1}{2} \, g \, b_s \, \rho_2 \left( \left( d_s + a_{1b} + a_{2b} - a_{1s} - a_{2s} \right)^2 - d_s^2 \right) \]

\[ \approx \mu \rho_2 \left( b_k \, h_k - b_s \, d_s \right) \Delta h_k + \frac{1}{2} \, g \, b_k \left( \rho_1 - \rho_2 \right) \left( a_{1b}^2 - a_{1s}^2 \right) \]  
\[ (6.5) \]

The net transport of momentum \( I_{dr} \) through the boundaries is:

\[ I_{dr} = b_k \left( \left( \rho_1 \, a_{1b} \, v_{1b}^2 + \rho_2 \, a_{2b} \, v_{2b}^2 \right) - \left( \rho_1 \, a_{1s} \, v_{1s}^2 + \rho_2 \, a_{2s} \, v_{2s}^2 \right) \right) \]

\[ (6.6) \]

The mean discharge in the control volume (with the interface below the ship and with a resulting discharge which is equal to the case without density differences) is as follows:

\[ Q_m = a_{1m} \, b_k \, v_{1m} - \left( a_{2m} \, b_k - b_s \, d_s \right) \, v_{2m} \]

\[ (6.7) \]

where \( Q_m \) is according to equation (6.3)

The change of momentum in time \( I_{tr} \) is the derivative of the product of the mass and velocity of the water of the control volume:

\[ I_{tr} = \frac{d}{dt} \left( l_s \, \rho_1 \, a_{1m} \, b_k \, v_{1m} - l_s \, \rho_2 \left( a_{2m} \, b_k - d_s \, b_s \right) \, v_{2m} \right) \]

If the small term \( l_s \, \left( \rho_1 - \rho_2 \right) \, a_{1m} \, b_k \, v_{1m} \) is neglected with respect to the terms \( l_s \, \rho_2 \, a_{1m} \, b_k \, v_{1m} \), then the equation can be written as:

\[ I_{tr} \approx \frac{d}{dt} \left( l_s \, \rho_2 \, a_{1m} \, b_k \, v_{1m} - l_s \, \rho_2 \left( a_{2m} \, b_k - d_s \, b_s \right) \, v_{2m} \right) + l_s \, \left( \rho_1 - \rho_2 \right) \, a_{1m} \, b_k \, v_{1m} \]/dt

With the help of equation (6.7), this yields:

\[ I_{tr} \approx \rho_2 \, l_s \, dQ_m / dt \]

\[ (6.8) \]

It appears that the density differences do not influence the change of the momentum in time. \( I_{tr} \) (with density differences (equation (6.8)) is comparable to \( I_{th} \) (without density differences equation (6.4)). This yields the following equation:
\[ \text{I}_\text{tr} \approx \text{I}_\text{th} \]  \hspace{1cm} (6.9)

**Momentum Equations**

The momentum equation for the case without density differences is as follows:

\[ F_{hh} + I_{dh} - I_{th} = 0 \]  \hspace{1cm} (6.10)

The momentum equation for the case with density differences (with the friction force again not taken into account) is:

\[ F_{hr} + I_{dr} - I_{tr} = 0 \]  \hspace{1cm} (6.11)

Substitution of equations (6.9) and (6.10) yields:

\[ F_{hh} + I_{dh} - F_{hr} - I_{dr} = 0 \]  \hspace{1cm} (6.12)

Substitution of equations (6.1) and (6.4) in equation (6.12) yields the added difference in water-level caused by density differences:

\[ \Delta h_r - \Delta h_h \approx \left\{ - \frac{1}{2} g \ b_k \ (\rho_1 - \rho_2) \ (a_{1b}^2 - a_{1s}^2) + I_{dh} - I_{dr} \right\} / \]

\[ \left\{ \rho_2 \ g \ (b_k \ h_k - b_s \ d_s) \right\} \]  \hspace{1cm} (6.13)

where:

\[ I_{dh} = \rho_2 \ b_k \ (h_b \ v_b^2 - h_s \ v_s^2) \]  \hspace{1cm} (6.2)

\[ I_{dr} = b_k \ ((\rho_1 \ a_{1b} \ v_{1b}^2 + \rho_2 \ a_{2b} \ v_{2b}^2) - (\rho_1 \ a_{1s} \ v_{1s}^2 + \rho_2 \ a_{2s} \ v_{2s}^2)) \]  \hspace{1cm} (6.6)

The added longitudinal force caused by density differences is:

\[ F_r - F_h \approx \rho_2 \ g \ b_s \ d_s \ (\Delta h_r - \Delta h_h) \]  \hspace{1cm} (6.14)

A similar derivation holds for the case where the bow and/or the stern are immersed in the lower layer. This possibility is not taken into consideration in this section.

### 6.2.2.3 Summary

The influence of density differences on the difference in the water-level can be determined from equation (6.13). This equation has the following terms:

- the resultant hydrostatic force of the cross-sections at the bow and the stern in the case of density differences
- the difference between the transport of momentum in the case of density differences and the transport of momentum in the case without density differences
These terms are determined by the slowly varying discharges, the flow velocities and levels of the interface at the bow and the stern. The term concerning the change of the momentum in time of the slowly varying discharge vanishes, because these terms coincide in the cases with and without density differences (equation (6.9)).

The following aspects are not taken into account in determining the influence of the density differences:
- friction
- effect of the rapidly varying discharge in the cross-sections because of the different time scale
- three-dimensional effects of the filling jets and the separation of the flow at the stern

In the next sections we will see how this approach can be applied to the calculation of the influence of the density differences.

6.3 The Water-level Differences Caused by Density Differences without the Influence of a Ship

6.3.1 General

The aim of this section is to show the usefulness of the schematized calculations according to the approach of paragraph 6.2.2. Furthermore, the calculations are used as the preparation for the next sections, which include the influence of a ship.

The flow and density patterns in the lock during and after the filling process are needed for the calculation of the water-level differences caused by density differences. There is no literature available relating to these patterns and water-levels, as mentioned in chapter 3. Therefore the observations of the experiments of chapter 3 are used to describe the patterns.

On the basis of these observations the flow and density patterns in the lock are dealt with (section 6.3.2). Subsequently the thicknesses, the flow velocities and the densities of both layers are derived as functions of time for two arbitrary cross sections. After that, the water-level differences caused by the density differences are calculated with the help of one-dimensional momentum and continuity equations, according to the approach of section 6.2.2 (section 6.3.3).

Finally, two examples are presented with a comparison between the calculations using the semi-empirical model and the measurements.

6.3.2 Observations and Modelling of the Flow and Density Pattern

We consider the situation of an initially fresh lock and a salt approach harbour.
A jet zone occurs behind the filling gate. The following can be said relating to this zone (figure T12 and T13):
- The outflow of the filling jets in the case of density differences occurs as in the case without density differences.
- The filling jets entrain ambient fresh water circulating in the gyre above the jets.
- The density and flow velocities in the jets decrease in the downstream direction. The mixing yields an almost homogeneous density in the jets.
- The entrainment of fresh water into the jets and the length of the jet zone depend on the momentum of the jets. An important parameter is the ratio of the areas of the filling jets to the area of the cross-section of the lock.

The jet changes into an intruding layer at a certain distance from the filling gate. The density of the water of the filling discharge with the entrained water at the end of the jet zone is also the density of the water of the intruding layer. The flow rate in the intruding layer has increased with respect to the filling discharge, because of the entrained water.

A intruding layer arises at the bottom, behind the jet zone (figure T14, chapter 3). This layer has the following properties:
- The leading edge of the intruding layer has a front. The velocity of the front is comparable to the front velocity of an exchange current (chapter 5).
- The density of the water in the intruding layer is presumed to be constant and is less than the density of the water of the approach harbour. Hardly any mixing with the ambient water occurs any more.
- The shape of the intruding layer as a function of distance corresponds in some degree to the discharge as a function of time. The maximum height of the layer can be considerable, and diminishes in time.
- Two parts of the layer can be distinguished: a propagating one and residual one. The propagating part of the intruding layer is moving (with a flow velocity which corresponds to the front velocity), the residual part is at rest.
- The intruding layer reflects against the downstream gate.

The mechanism causing the residual layer, which was observed in the hydraulic model, is not clear.

The situation of the filling of an initially salt lock with fresh water from an approach harbour comprises the same elements. The principal difference is that an intruding layer occurs at the free surface, instead of at the bottom.

Only the differences in water-level above the intruding layer (i.e. outside the zone of the filling jets) are considered.
6.3.3 Approach to the Calculations

The schematized calculations comprise the following steps. Two cross-sections are chosen in the lock. The flow velocities at these cross-sections are examined for the situation without density differences situation. Then the thickness of the salt layer and the flow velocity of the fresh layer are modelled for the situation with density differences, according to the observations in the previous section. Finally the added difference in water-level between the two cross-sections caused by density differences is calculated and compared with the measured values.

6.3.3.1 Flow Velocity in the Case without Density Differences

If the filling openings are rectangular and the velocity of the valves is constant the net area of the filling openings can be described according to:

\[
A_n = b_h \, v_h \, t \quad \text{if } t < t_h \quad (6.15)
\]

\[
A_n = A_h \quad \text{if } t \geq t_h
\]

where \( A_n \) = net area of the filling openings
\( b_h \) = total width of the filling openings
\( v_h \) = velocity of the valves
\( t_h \) = time needed for the opening of the valves
\( A_h \) = maximum filling opening

The filling discharge is determined by the net area of the filling openings and the head difference over the gate:

\[
Q = \mu \, A_n \, / (2 \, g \, (h_u - h_k)) \quad (6.16)
\]

where \( Q \) = filling discharge
\( \mu \) = discharge coefficient
\( h_u \) = upstream water-level in the approach harbour
\( h_k \) = mean water-level in the lock with respect to the bottom of the lock (\( h_k \) is also used for the water-depth in the lock)

The rise of the mean water-level is given from:

\[
dh_k/\, dt = b_k \, l_k \times Q \quad (6.17)
\]

where \( dh_k/\, dt \) = vertical velocity of the mean water-level in the lock
\( b_k \) = width of the lock
\( l_k \) = length of the lock

The initial water-level in the lock is \( h_k = h_d \) (with \( h_d \) as the downstream water-level).
The substitution of equations (6.15) and (6.16) in (6.17) and the solution of this differential equation yield the mean water-depth $h_k$ and the filling discharge $Q$ as a function of time. (See Appendix B). The filling time of the lock is $t = t_e$ and is reached when the mean water-level of the lock is equal to the upstream water-level $h_k \approx h_u$. The filling discharge remains zero from that moment.

The two cross-sections where the flow is investigated have the subscripts $b$ and $s$. The flow velocities in these cross-sections can be derived from the continuity equations:

$$v_b \ h_k \ b_k = Q \ (l_k - x_b) / l_k$$  \hspace{1cm} (6.18)

$$v_s \ h_k \ b_k = Q \ (l_k - x_s) / l_k$$  \hspace{1cm} (6.18a)

where $v_b$ = flow velocity at cross-section $b$

$x_b$ = horizontal distance filling gate / cross-section $b$

$v_s$ = flow velocity at cross-section $s$

$x_s$ = horizontal distance filling gate / cross-section $s$

The flow velocities $v_b$ and $v_s$ vanish after the end of the filling time ($t > t_e$).

6.3.3.2 Density Difference and Front Velocity

The relative density difference between the water of the intruding layer and the ambient flow, which is assumed to be constant, is:

$$\epsilon = \beta (\rho_1 - \rho_2)/\rho_2$$  \hspace{1cm} (6.19)

where $\beta$ = coefficient for the decrease in the relative density difference because of entrainment and mixing by the filling jets

The density $\rho$ of the water of the intruding layer is:

$$\rho = \rho_2 \ (1 + \epsilon)$$  \hspace{1cm} (6.20)

The velocity $c_1$ of the front of the intruding layer is:

$$c_1 = c_1' \ / (\epsilon \ g \ \frac{1}{2} \ (h_u + h_d))$$  \hspace{1cm} (6.21)

where $c_1' = \text{coefficient for the front velocity (section 5.2)}$

The front of the intruding layer reaches cross-section $b$ for the first time at $t = t_b$ and for the second time at $t = t_{bb}$, according to:

$$t_b = x_b / c_1$$  \hspace{1cm} (6.22)

$$t_{bb} = (2 \ l_k - x_b) / c_1$$  \hspace{1cm} (6.22a)
Similar instants can be derived for cross-section s.

6.3.3.3 Thickness of the Salt Layer and Flow Velocity of the Fresh Layer

The situation at cross-section b is considered.

The salt layer at cross-section b consists of a propagating part (thickness $a_{10b}$) with a flow velocity $c_1$ and a stagnant part (thickness $z_b$). This approach with two components arises from the observations in the hydraulic model, the explanation of which is not clear.

No salt layer is present at cross-section b, before the front reaches it:

$$a_{10b} = z_b = a_{1b} = 0 \quad \text{if } t < t_b$$  \hspace{1cm} (6.23)

During the passage of the intruding layer the discharge of the layer at cross-section b consists of the filling discharge, which is both increased by a factor $1/\beta$ because of entrainment, and decreased by an empirical decay term. This results in the following continuity equation for the propagating part of the layer (which yields the thickness $a_{10b}$):

$$c_1 b_k a_{10b} = \frac{Q(t-t_b)}{\beta} \left( -\delta c_1 \frac{t}{l_k} \right)$$  \hspace{1cm} (6.24)

if $t_b < t < (t_b + t_b)$ with $(t_e + t_b) < t_b$

where $\delta$ = empirical coefficient for decay

$a_{1b}$ = thickness of the propagating part of the layer

The stagnant layer extends from the filling gate as far as the front according to the following equation (which yields the thickness $z_b$):
\[
\begin{align*}
\frac{(h_k - h_d) l_k}{\beta} &= \frac{(-\delta c_1 t/l_k)}{(1 - e t/b)} \quad (6.25)
\end{align*}
\]

if \( t > t_b \)
where \( z_b \) = thickness of the residual part

After the passage of the intruding layer at cross-section \( b \), and before the front reaches cross-section \( b \) for the second time, the thickness \( a_{1b} \) of the salt layer consists of the stagnant layer \( z_b \). The propagating component \( a_{10b} \) is absent:
\[
a_{10b} = 0 \quad \text{if} \quad (t_e + t_b) < t < t_{bb} \quad (6.26)
\]
The total thickness of the salt layer at cross-section \( b \) is:
\[
a_{1b} = a_{10b} + z_b \quad (6.27)
\]
where \( a_{10b} \) is derived from equation (6.23), (6.24) or (6.26)

\( z_b \) is derived from equation (6.25)
The flow velocity \( v_{2b} \) of the upper layer with fresh water can be derived with the help of the continuity equation for the whole cross-section \( b \):
\[
\begin{align*}
v_{2b} (h_k - a_{1b}) &= -(l_k - x_b) Q / (b_k l_k) + a_{10b} c_1 \\
\end{align*}
\]
where \( v_{2b} = \) flow velocity in the upper layer
\( a_{10b} \) is derived from equation (6.23), (6.24) or (6.26)
The parameters of cross-section \( s \) can be derived in the same way.

6.3.3.4 Difference in Water-level

The net transport of momentum \( I_{dh} \) through the boundaries of the cross-sections \( b \) and \( s \) in the case without density differences can be written as:
\[
I_{dh} = \rho_2 b_k h_k (v_{b}^2 - v_{s}^2) \quad (6.29)
\]
where \( v_{b} \) and \( v_{s} \) are derived from equations (6.18) and (6.18a)
The difference \( \Delta h_h \) in water-level between cross-sections \( b \) and \( s \) in the case without density differences can be derived from the momentum equation:
\[
\rho_2 g b_k h \Delta h_h + I_{dh} = 0 \quad (6.30)
\]
The net transport of momentum \( I_{bs} \) through the boundaries of the cross-sections \( b \) and \( s \) in the case of density differences can be written
as:

\[ I_{bs} = \rho_2 \left( h_k - a_{1b} \right) \left( c_1 + v_{2b} \right)^2 b_k - \rho_2 \left( h_k - a_{1s} \right) \left( c_1 + v_{2s} \right)^2 b_k \]  
(6.31)

where \( a_{1b} \) is according to equation (6.27)
\( v_{2b} \) is derived from equation (6.28)

The difference \( \Delta h_{bs} \) in water-level with density differences between cross-sections b and s can be derived from the following momentum equation:

\[ \rho_2 g b_k h \Delta h_{bs} + \frac{1}{2} \left( \rho - \rho_2 \right) g \left( a_{1b}^2 - a_{1s}^2 \right) b_k + I_{bs} = 0 \]  
(6.32)

The added difference in water-level \( \Delta h_r \) between cross-sections b and s caused by the density differences is then:

\[ \Delta h_r = \Delta h_{bs} - \Delta h_h \]  
(6.33)

The water-level difference caused by density differences for the filling of an initially salt lock with fresh water comprises the same elements. This situation is omitted.

### 6.3.4 Results

#### Examples

The calculations are applied to the following examples of the hydraulic scale model of the Hansweert lock (the Netherlands) without a ship (section 3.2):

- test D1-3: initially fresh lock, salt approach harbour, \( c_1' = 0.42 \)
- test D2-3: initially salt lock, fresh approach harbour, \( c_1' = 0.46 \)

The other data are:
- the water-depths: \( h_u = 6.10 \text{ m} \) and \( h_d = 4.60 \text{ m} \)
- the lock: \( l_k \times b_k = 280 \times 24 \text{ m}^2 \)
- the filling openings: \( \mu = 0.65, b_h = 16.50 \text{ m}, v_h = 0.003 \text{ m/s}, A_h = 20.64 \text{ m}^2 \)
- the densities: \( \rho_1 = 1020 \text{ kg/m}^3, \rho_2 = 1000 \text{ kg/m}^3 \)
- the coefficients: \( B = 0.75, \delta = 0.70 \)
- the cross-sections: \( x_b = 10 \text{ m} \) and \( x_s = 160 \text{ m} \)

The coefficients \( B \) and \( \delta \) are chosen in such a way that the calculated flow and density patterns correspond to the observed patterns as much as possible. The density of the intruding layer is \( \rho = 1015 \text{ kg/m}^3 \) for test D1-3 and \( \rho = 1005 \text{ kg/m}^3 \) for test D2-3.

The water-level and the discharge are presented as functions of time in figure 90. The lock is filled at \( t_e = 470 \text{ s} \), the maximum discharge
measures 32.5 m³/s at \( t = 280 \) s.

The thicknesses of the salt layer at cross-sections b and s are shown as functions of time in figure 91. The thickness at cross-section b becomes considerable with respect to the water-depth, during the passage of the intruding layer. After the passage of the layer the residual layer is visible. The maximum thickness of the salt layer at cross-section s is lower and later.

The flow velocities in the upper layer are also shown in figure 91. These flow velocities are low during the filling of the lock, because the resulting discharge in the lock has the same direction as the intruding layer. After the filling time the resulting discharge is zero and the flow velocity of the upper layer above the intruding layer increases.

The water-level difference caused by the density differences between cross-sections b and s is shown in figure 92. The water-level difference is negative while the intruding layer passes cross-section b (i.e. lowest water-level at cross-section b). The water-level difference becomes positive from the moment the layer passes cross-section s. The calculated values agree closely with the measured values.

The thicknesses of the salt layer and the flow velocities of the salt layer for test D2-3 (reverse density situation of D1-3) are shown in figure 93. A similar picture is visible to that in figure 91. The difference in water-level caused by the density differences is shown in figure 94. The computed values also correspond closely to the measured values.

Summary

The flow and density patterns are determined according to the observations during the experiments. The computations concern the water-level differences in the lock caused by the density differences, using the approach outlined in section 6.2.2.

It can be concluded on the basis of the examples that the approach involving the calculations according to section 6.2.2 is applicable for further computations of the influence of the density differences on the differences in water-level in the lock.

The calculations of the water-level differences can also be used for the determination of the longitudinal force caused by density differences on a small ship which does not influence the flow and density patterns in the lock.
6.4 The Longitudinal Force Caused by Density Differences on a Ship with a Limited Blockage

6.4.1 General

The aim of this section is to develop a computer model for the longitudinal force on a ship caused by density differences during the filling process. The blockage of the cross-section of the lock by the ship is limited. The blockage is such that a great deal of ambient water which originates from the zone beside the ship can be entrained into the jets.

The flow and density patterns are examined (section 6.4.2), according to the observations in the scale model and prototype investigations (as presented in chapter 3). Then the computer model is developed (section 6.4.3). Finally the computer model is applied to some examples (section 6.4.4).

6.4.2 Observations and Schematizing of the Flow and Density Pattern

We consider the filling situation of a lock with initially fresh water and a salt approach harbour.

![Diagram](image)

Figure T57: The mixing zone in front of the bow

The turbulence of the filling jets, which intrude into a lock without a ship, collapses in the jet zone and an intruding layer arises at the end of the jet zone. If a ship with a limited blockage is present in this jet zone, the flow pattern in the jet zone changes. Between the filling gate and the bow, a zone can be distinguished, which acts as a kind of mixing zone. The jets intrude into this zone at the filling gate. The jets mix with the water of the zone itself and with the water which is entrained into the jets. This water originates from the zone beside the ship. The density of the water in the mixing zone is lower than that of the water of the approach harbour, owing to mixing with the ambient water. An intruding salt layer originates near the underside of the bow.

The underside of the mixing zone is the bottom of the lock, the upper side of the zone is the interface with the fresh water. The water which is entrained into the jets flows through this "interface". The level of
the interface changes during the filling process, because the filling discharge changes in time. To begin with, the level rises until the moment of the maximum filling discharge, and after that the level falls. The interface is almost horizontal.

If the interface reaches the water-surface, the entrainment of water from beside the ship is hindered and the density of the water in the mixing zone increases. The blockage of the ship is then too large for the entrainment process to continue. This situation is examined in the following section (6.5).

A two-layer current is present just in front of the ship. The lower layer is the water of the intruding layer, with a flow velocity in the downstream direction. The upper layer consists of the water from the lock which is entrained by the jets, and the flow is in the upstream direction. The total discharge in the two layers must fulfil the condition that the part of the lock which lies behind the bow is filled.

The intruding salt layer originates at the lower side of the mixing zone, below the bow. The intruding layer propagates in the downstream direction below the ship. A front is present as the leading edge of the layer. The density of the water of the layer is lower than that of the water of the approach harbour. The layer passes the stern after which a two layer current is also present at the stern, which implies an increased transport of momentum at the cross-section of the stern relative to the case without density differences. The layer reflects against the downstream gate. Subsequently the layer reaches the stern for the second time.

The situation of the filling of an initially salt lock with fresh water comprises similar elements. The mixing zone in front of the ship consists of the zone between the filling gate, the bow, the water-surface and the interface. The outflow of the intruding layer takes place beside the ship in the downstream direction. Relative to the reverse density situation, the height of the mixing zone increases more quickly because the ship blocks a larger part of the intruding layer.

The presence of a residual layer, as mentioned in the last section, is not taken into consideration.

The set-up of the calculations of the previous chapter (the exchange process) consisted of the tracking separate salt or fresh layers which reflect against the bow and the gates. This method is not used here, because many reflections against the bow and the filling gate arise within a short time. These reflections provide an almost horizontal interface between filling gate and bow.
6.4.3 Schematized Calculations

6.4.3.1 Parameters in the Case without Density Differences

The part of the calculation without the influence of density differences is similar to that of section 6.3. The water-level $h_k$ and the filling discharge $Q$ are determined as functions of time in this section (equations (6.15), (6.16), and (6.17)). The flow velocities ($v_b$ and $v_s$) and the transport of momentum $I_{dh}$ through the cross-sections in front of the bow and behind the stern are also presented for the one-dimensional situation without density differences (equations (6.18), (6.18a) and (6.29)).

6.4.3.2 Density Differences

It is assumed that the density of the salt water in the lock, i.e. in the mixing zone and in the intruding layer, has a constant value which is lower than the density of the water in the approach harbour. This constant value is given by the approach of section 6.3, with the relative density difference $\epsilon$ (equation (6.19)), the density of the salt water $\rho$ (equation (6.20)) and the front velocity $c_1$ (equation 6.21). The coefficient $\beta$ has been introduced for the lowering of the density differences due to mixing.

6.4.3.3 Salt Layer and Flow Velocities in front of the Bow

The water which flows into the mixing zone comprises the filling discharge at the gate and the entrainment from the zone beside the ship. The water which flows out of the mixing zone is the intruding salt layer below the bow. The mixing zone expands as the interface rises. It is assumed that the level of the interface of the mixing zone between the filling gate and the bow remains horizontal.

The discharge of fresh water which is entrained by the jets can be derived from equation (6.19) and is $Q (1 - \beta) / \beta$. It is assumed that the flow velocity of the water in the intruding layer is equal to the front velocity $c_1$. 
Figure T58: The mixing zone in front of the bow (level of the interface above the bottom of the ship)

If the level of the interface is below the bottom of the ship, the continuity equation for the mixing zone is as follows:

\[ Q + Q \frac{(1-\beta)}{\beta} = \frac{da_{1b}}{dt} x_b b_k + c_1 a_{1b} b_k \]  \hspace{1cm} (6.34)

if \( a_{1b} < (h_k - d_s) \)

where \( \frac{da_{1b}}{dt} \) = vertical velocity of the interface
\( a_{1b} \) = thickness of the salt layer of the mixing zone (and so in front of the bow)
\( c_1 \) = flow velocity of the intruding layer according to equation (6.21)
\( x_b \) = distance between the filling gate and the bow

This first order differential equation can be solved with known values of the filling discharge \( Q \) and the coefficient \( \beta \).

If the level of the interface is above the bottom of the ship, the continuity equation (6.34) is adjusted, because of the blockage of the ship, according to:

\[ Q + Q \frac{(1-\beta)}{\beta} = \frac{da_{1b}}{dt} x_b b_k + c_1 ((h_k - d_s) b_s + a_{1b} (b_k - b_s)) \]  \hspace{1cm} (6.35)

if \( a_{1b} > (h_k - d_s) \)

It is assumed that the flow velocity \( v_{1b} \) of the salt layer just in front of the bow corresponds to the front velocity of the intruding layer \( (v_{1b} = c_1) \). The flow velocity \( v_{2b} \) of the upper layer can then be derived from the continuity equation at the cross-section in front of the bow:

\[ c_1 a_{1b} b_k - v_{2b} (h_k - a_{1b}) b_k = Q (l_k - x_b) / l_k \]  \hspace{1cm} (6.36)
6.4.3.4 Salt Layer and Flow Velocities just behind the Stern

The variations in the velocity of the front are not taken into account. The front starts from the bow at \( t = t_b \):

\[
t_b = x_b / c_1
\]  (6.22)

The front of the intruding layer reaches the stern for the first time at \( t = t_s \) and for the second time at \( t = t_{ss} \), according to:

\[
t_s = x_s / c_1
\]  (6.37)

\[
t_{ss} = (2 l_k - x_s) / c_1
\]  (6.37a)

where \( x_s \) = distance between the filling gate and the stern

![Diagram of flow velocities and layers](image)

Figure T59: The situation at the stern

No salt layer is present at the stern (\( a_{1s} = 0 \) and \( v_{1s} = 0 \)) before the front reaches it (\( t < t_s \)). The flow velocity \( v_{2s} \) of the upper layer at the stern is derived from the continuity equation, according to:

\[- v_{2s} h_k b_k = Q (l_k - x_s) / l_k \quad \text{if} \quad 0 < t < t_s\]  (6.38)

After the front of the intruding layer has passed the stern for the first time, a salt layer is present. The thickness of this salt layer \( a_{1s} \) is derived from the earlier situation at the bow with a time shift equal to \( (t_b - t_s) \). The separate situations with the interface below and against the bow must be distinguished:

\[
a_{1s}(t) = a_{1b}(t-t_s + t_b)
\]  (6.39)

if \( t_s < t < t_{ss} \) and \( a_{1b}(t-t_s + t_b) < (h_k - d_s) \)

\[
a_{1s}(t) = ((h_k - d_s) b_s + a_{1b}(t-t_s + t_b) (b_k - b_s)) / b_k
\]  (6.39a)

if \( t_s < t < t_{ss} \) and \( a_{1b}(t-t_s + t_b) > (h_k - d_s) \)

During the time between the first and second passing of the stern the flow velocity \( v_{1s} \) of the salt layer corresponds to the front velocity \((v_{1s} = c_1)\). The flow velocity \( v_{2s} \) of the upper layer is derived from the
continuity equation according to (if \( t_s < t < t_{ss} \)):

\[
a_{ls} c_1 b_k - v_{2s} (h_k - a_{ls}) b_k = Q (l_k - x_s) / l_k
\]

(6.40)

After the front of the intruding layer has passed the stern for the second time (\( t > t_{ss} \)), the thickness of the salt layer \( a_{ls} \) is derived from the earlier situations at the bow with time shifts (\( t_b - t_s \)) and (\( t_b - t_{ss} \)) earlier:

\[
a_{ls}(t) = a_{lb}(t-t_s+t_b) + a_{lb}(t-t_{ss}+t_b)
\]

if \( t > t_{ss} \) and \( a_{lb}(t-t_s+t_b) < (h_k - d_s) \)

(6.41)

For the same conditions, equations can be presented for the flow velocity \( v_{ls} \) of the lower layer and \( v_{2s} \) of the upper layer, according to:

\[
(a_{lb}(t-t_s+t_b) - a_{lb}(t-t_{ss}+t_b)) c_1 = v_{ls} a_{ls}
\]

(6.42)

\[
v_{ls} a_{ls} b_k - v_{2s} (h_k - a_{ls}) b_k = Q (l_k - x_s) / l_k
\]

(6.42a)

6.4.3.5 Transport of Momentum

The transport of momentum \( I_{dh} \) through the boundaries of cross-sections \( b \) and \( s \) in the case without density differences can be written as:

\[
I_{dh} = \rho_2 b_k h_k (v_b^2 - v_s^2) p_I
\]

(6.43)

where \( v_b \) and \( v_s \) are derived from equations (6.18) and (6.18a)

where \( p_I \) = coefficient for the increase of momentum owing to the deviations from uniform flow velocities in the cross-sections

The transport of momentum through the cross-sections of the bow and the stern in the case of density differences can be determined as follows:

\[
I_{bs} = (\rho a_{lb} v_{lb}^2 + \rho_2 (h_k - a_{lb}) v_{2b}^2 - \rho a_{ls} v_{ls}^2 - \rho_2 (h_k - a_{ls}) v_{2s}^2) b_k p_I
\]

(6.44)

where \( a_{lb} \) is derived from equations (6.34) and (6.35)

\( v_{2b} \) is derived from equations (6.36)

\( a_{ls} \) is derived from equations (6.39), (6.39a) and (6.41)

\( v_{2s} \) is derived from equations (6.38), (6.40) and (6.43)

6.4.3.6 Water-Level Differences and Longitudinal Force caused by Density Differences

The differences in water-level and the longitudinal force are determined by means of the force relations which have been derived in chapter 4.
The following data are needed:
- the water-depth
- the thicknesses of the salt layer at the bow and the stern
- the densities of both layers
- the transport of momentum through the cross-sections at the bow and the stern

In addition, the width of the lock and the dimensions of the ship must be known.

Computations are done for the case without density differences. Then computations are made for the case with density differences. Finally the values of the case with density differences are subtracted from those of the case without density differences. This yields the added difference in the water-level and the added longitudinal force on the ship caused by density differences.

The same procedure of this section (6.4.3) can be applied for the reverse density situation of an initially salt lock and a fresh approach harbour.

The above-mentioned procedure for the calculation of the influence of the density differences on the longitudinal force has been developed as part of the computer model LOCKDENS 1. This part consists of the calculation during the filling process for both density situations (initially fresh and initially salt locks).

6.4.4 Results

6.4.4.1 Tests 8 and 7

Input Data

The calculations are applied to tests 8 and 7 of the prototype investigation of the Volkerak Lock (the Netherlands) for yachts, which is mentioned in chapter 3:

- test 8: initially fresh lock, salt approach harbour
  - the water-depths: $h_u = 8.55$ m and $h_d = 7.30$ m
  - the densities: $\rho_1 = 1015$ kg/m$^3$, $\rho_2 = 999$ kg/m$^3$
  - the coefficients: $\beta = 0.45$, $c_1' = 0.42$, $p_1 = 1$
  - initial blockage 0.37

- test 7: initially salt lock, fresh approach harbour
  - the water-depths: $h_u = 7.40$ m and $h_d = 5.80$ m
  - the densities: $\rho_1 = 1013$ kg/m$^3$, $\rho_2 = 999$ kg/m$^3$
  - the coefficients: $\beta = 0.70$, $c_1' = 0.46$, $p_1 = 1$
  - initial blockage 0.46

The data these tests have in common are:
. the lock: \( l_k \times b_k = 154 \times 16.1 \text{ m}^2 \)

. the filling openings: \( \mu = 0.6 \), \( b_h = 3.50 \text{ m} \), \( v_h = 0.005 \text{ m/s} \), \( A_h = 5.25 \text{ m}^2 \)

. the cross-sections: \( x_b = 14 \text{ m} \) and \( x_s = 90.50 \text{ m} \)

. the ship: \( l_s \times b_s \times d_s = 76.50 \times 11.40 \times 3.80 \text{ m}^3 \)

The coefficients \( c_1 \), \( c_\mu \), and \( \beta \) have been chosen in such a way that the measured value of the front velocity and the density of the intruding layer correspond to the observed values.

Results of the Calculations

The results of the calculations are presented as functions of time.

The water-level and the filling discharge are shown in figures 95 (test 8) and 99 (test 7). The lock was filled in 530 s (test 8) and 590 s (test 7).

The thicknesses of the salt layer at the bow and the stern are shown in figures 96 (test 8) and 100 (test 7). The thicknesses of the lower layers are considerable. This is especially true for test 7 (initially salt lock), because the intruding layer reflects against the ship. The delay in the arrival of the salt layer at the stern in relation to its arrival at the bow is clearly visible.

The calculated and the measured relative, longitudinal forces caused by the density differences are presented in figures 97 (test 8) and 101 (test 7). The calculated and the measured forces agree quite closely. The maximum forces measure about -0.3 o/oo for test 8 (initially fresh lock) and 0.4 o/oo for test 7 (initially salt lock).

The calculated and the measured mean values of the density at the cross-sections of the bow and the stern are presented in figures 98 (test 8) and 102 (test 7). These values also agree rather closely.

6.4.4.2 The Influence of the Coefficients

The coefficient \( \beta \) concerns the mixing of the jets with the ambient water. Mixing is absent with \( \beta = 1 \). The coefficient \( \beta \) was varied between 0.35 and 0.55 for test case 8 in figure 103. The influence of this on the longitudinal force caused by density differences is shown to be considerable. The force increases with a smaller coefficient \( \beta \), because the influence of the increased thickness of the salt layer in front of the bow is greater than that of the decrease in the density difference. The force with \( \beta = 0.35 \) yields a completely different curve, because the interface in front of the bow reaches the water-surface. The possibility of entrainment is hindered and the computer model switches over to the approach which is dealt with in the following section (6.5).
The coefficient $\beta$ was varied between 0.60 and 0.80 for test case 7 (initially salt lock) in figure 105. The same curve is visible. The coefficient has a considerable influence on the longitudinal force caused by density differences.

The value of the coefficient $\beta$ depends strongly on the situation under consideration. A good choice for test 8 is $\beta = 0.45$, while for test 7 it would be $\beta = 0.70$. The reason for this is the difference in water-depth, filling discharge and density situation. The coefficient $\beta$ will also be dependent on the shape of the filling openings, the beams at the downstream side of the openings, etc.

It may be concluded that the choice of the coefficient $\beta$ is very difficult and can only be determined with a prototype or scale model investigation. This reduces the predictive value of the computer model for limited blockages.

The coefficient $p_I$ concerns the influence of the transport of momentum on the longitudinal force. The coefficient $p_I = 1$ belongs to the one-dimensional flow situation. A coefficient $p_I$, which is higher than 1, comprises flow velocities which deviate from the mean flow velocities. Figures 104 (test 8) and 106 (test 7) show the influence of the variation of $p_I$. The influence is small.

6.4.4.3 Tests D1-23 and D2-23

The calculations are also carried out for tests D1-23 and D2-23 of the hydraulic scale model of the "Hansweert Lock" of chapter 3. These tests pertain to a ship of 2000 tons for inland navigation in a push tow lock. The data of the lock and the water are given in section 6.3.4. Test D1-23 pertains to an initially fresh lock and coincides with test D1-3, test D2-23 is an initially salt lock and coincides with test D2-3.

The other data are:
- ship: $l_s \cdot b_s \cdot d_s = 95 \cdot 11.20 \cdot 2.80 \, \text{m}^3$ (initial blockage 0.28)
- coefficients: $\beta = 0.9$ (test D1-23 and D2-23), $p_I = 1, c_1' = 0.42$ (test D1-23) and $c_u' = 0.46$ (test D2-23)

The differences in water-level between the bow and the stern caused by the density differences are presented in figures 107 and 108. The agreement between the calculated and the measured values appears to be moderate. The calculated values are too small.
6.5 The Longitudinal Force Caused by Density Differences on a Ship with a Large Blockage

6.5.1 General

The purpose of this section is to calculate the longitudinal force caused by density differences during the filling process of a lock containing a ship with a large blockage of the cross-section.

The flow and density patterns in the lock are described in chapter 3. These patterns are schematized in section 6.5.2. The mathematical model is developed in section 6.5.3. The application of the mathematical model is presented in section 6.5.4 by means of a number of examples.

6.5.2 Observations and Schematizing of the Flow and Density Pattern

In contrast to section 6.4, we consider the filling situation with a large ship with a considerable blockage of the cross-section. This blockage hinders the entrainment by the filling jets of fresh water from the downstream side of the lock. The jets only mix with the water between the filling gate and the bow.

The situation of an initially fresh lock is investigated.

![Figure T60: The mixing zone in front of the bow](image)

The zone between the filling gate, the bow, the bottom of the lock and the water-surface is considered. This zone is filled with the water of the filling discharge. The upper side of the zone rises with the water-surface. Beside and below the bow, the water leaves the zone as an intruding layer which propagates in the downstream direction.

The filling jets are mixed with the water originally present in the zone. The density of the mixed water in this zone increases in time. The rate of increase depends on the filling discharge, the size of the zone and the initial density difference. Eventually the zone has the same density as the approach harbour.

The water which leaves the zone flows as a bottom layer in the
downstream direction. The velocity in the intruding layer will probably be equal to the front velocity of the intruding layer. Part of the water initially present in the lock is pushed in the downstream direction. As with the situation with a limited blockage, the intruding layer passes the stern, reflects against the downstream gate and reaches the stern for the second time.

The result is that, over a long period of time, a large density difference is present between the water at the bow and that at the stern. It is possible that, for a certain period, the density of the water in front of the bow is equal to the density of the water of the approach harbour, while the water behind the stern has the original density of the lock.

6.5.3 Schematized Calculations

The approach is presented for an initially fresh lock which is filled with salt water. The opposite situation is not taken into consideration.

6.5.3.1 Density of the Zone in front of the Bow

The initial density of the water in the zone in front of the bow is that of the fresh water in the lock. This density increases up to that of the water from the approach harbour. The density of the water in the mixing zone can be determined by means of the mass balance. This equation consists of terms relating to the filling discharge, the change of density of the water of the zone itself and the water which flows out as an intruding layer (figure T60):

\[ \rho_1 Q = h_k x_b b_k d\rho/dt + \rho_1 Q (1_k - x_b) / l_k \]  \hspace{1cm} (6.45)

where \( d\rho/dt = \) rate of the increase of the density of the water in the mixing zone
\( \rho = \) density of the water in the zone
\( \rho(0) = \rho_2 \)

The relative density difference is:

\[ \epsilon = (\rho - \rho_2) / \rho_2 \]  \hspace{1cm} (6.46)

The front velocity of the intruding layer is estimated as:

\[ c_1 = c_1' / (\epsilon g \frac{1}{2} (h_u + h_d)) \]  \hspace{1cm} (6.21)

The front reaches the stern for the first time at \( t = t_s \) and for the second time at \( t = t_{ss} \) (equations (6.37) and (6.37a)).

6.5.3.2 Situation at the Bow and the Stern

The thickness of the salt layer \( a_{lb} \) in front of the bow corresponds to
the water-depth \( h_k \), while the fresh layer is absent. The flow velocity of the salt layer \( v_{1b} \) in front of the bow agrees with the one-dimensional situation without density differences; the continuity equation yields this velocity:

\[
v_{1b} h_k b_k = Q \left( l_k - x_b \right) / l_k
\]

Before the front of the intruding layer reaches the stern for the first time, the whole water-depth at the stern is fresh (\( a_{2s} = h_k \) and \( a_{1s} = 0 \) if \( t < t_s \)). The flow velocity of the fresh layer \( v_{2s} \) agrees with the situation without density differences according to equation (6.38).

After the front has passed the stern for the first time, a salt layer is present at the stern with a thickness \( a_{1s} \) and a flow velocity \( c_1 \). The thickness of the salt layer \( a_{1s} \) is determined by the situation at the bow at the earlier moment \( t = t_s + t_b \) (if \( t_s < t < t_{ss} \)):

\[
c_1 a_{1s}(t) b_k = Q(t-t_s+t_b) \left( l_k - x_b \right) / l_k
\]

The flow velocity of the fresh layer \( v_{2s} \) is determined according to the continuity equation (6.40).

After the front passes the stern for the second time, the thickness of the salt layer \( a_{1s} \) is determined (if \( t > t_{ss} \)) according to:

\[
c_1 a_{1s}(t) b_k = Q(t-t_s+t_b) \left( l_k - x_b \right) / l_k + Q(t-t_{ss}+t_b) \left( l_k - x_b \right) / l_k
\]

The flow-velocity of the salt layer then diminishes.

6.5.3.3 The Water-level Difference and the Longitudinal Force Caused by Density Differences

The transport of momentum \( I_{dh} \) through the boundaries at the cross-sections of the bow and the stern in the case without density differences can be written as equation (6.43).

The transport of momentum \( I_{dr} \) through the boundaries at the cross-sections of the bow and the stern in the case of density differences can be written as:

\[
I_{dr} = (\rho h_k v_{1b}^2 - \rho a_{1s} v_{1s}^2 - \rho_2 (h_k - a_{1s}) v_{2s}^2) b_k p_l
\]

The water-level differences and the longitudinal force caused by density differences as functions of time are computed according to section 6.4.3.6. The results with density differences are subtracted from the results without density differences.

The computer model LOCKDENS 1 has been developed to calculate the
water-level differences and the longitudinal force as a function of time for the period of the filling process.

6.5.4 Results

6.5.4.1 Tests D1-2 and D2-2

Tests D1-2 and D2-2 of the hydraulic scale model of the "Hansweert Lock", which are mentioned in chapter 3, are simulated. Test D1-2 has an initially fresh lock, test D2-2 has an initially salt lock. The data concerning the dimensions of the lock, the water-levels, the initial density difference and the filling system correspond to those of tests D1-3 and D2-3, which are mentioned in section 6.3.4.

Additional data are:
- ship (push tow "4+1"): \( l_s \times b_s \times d_s = 153 \times 22.80 \times 3.90 \text{ m}^3 \), initial blockage of the lock: 0.81
- coefficients: \( p_1 = 1 \), \( c'_1 = 0.42 \) (test D1-2) and \( c'_u = 0.46 \) (test D1-2)

The results of the calculations are presented as functions of time. The water-level and the filling discharge of both tests are shown in figure 109.

The thickness of the salt layer at the stern, the density at the bow and the mean density at the stern for test D1-2 are presented in figure 110. After about \( t = 240 \text{ s} \), the water in the zone in front of the bow reaches the density of that in the approach harbour. The density of the water at the stern increases just after \( t = 380 \text{ s} \), when the front reaches the stern. The water-level difference and the longitudinal force caused by the density differences are shown in figure 111. The calculated values agree closely with the measured values.

A similar result is shown for test D2-2. The thickness of the salt layer and the densities are shown in figure 112, and the water-level differences and the longitudinal force caused by the density differences are shown in figure 113. The calculated and the measured values of figure 113 also agree very closely.

6.5.4.2 The influence of the coefficients

The coefficient \( p_1 \) comprises the influence of the transport of momentum on the longitudinal force caused by the density differences. A uniform flow situation implies uniform flow velocities and a coefficient \( p_1 = 1 \).

The momentum at the bow for the case with density differences is roughly equal to the case without density differences during the whole filling time. The momentum at the stern for the non-dimensional situation differs from that in the case without density differences after the front has passed the stern. Therefore the influence of the coefficient \( p_1 \) on the longitudinal force is perceptible after the front passes the
stern (figures 114 and 115, data of tests D1-2 and D2-2). The influence is small, because the front does not reach the stern before the end of the filling process.

The influences of the coefficients $c_1'$ and $c_u'$ of the front velocity are not presented. The influence is small for realistic values of these coefficients.

6.5.4.3 Tests R1, R2, H1 and H2

Tests R1 and R2 of the scale model investigation of the "Rozenburg Lock" (M 950, Delft Hydraulics) are simulated. The tests concern a lock with a push tow unit "4+1" (initial blockage 0.53 and 0.70). The data of the tests are given in appendix C. The longitudinal force due to the density differences is presented in figures 116 and 117. The calculated forces agree well with the measured forces.

Tests H1 and H2 of the scale model investigation of the "Haringvliet Lock" (M 662, Delft Hydraulics) are also computed. The tests concern a lock with an inland navigation ship of 2,000 tons (initial blockage 0.40 and 0.63). The data of the tests are mentioned in appendix C. The longitudinal forces due to the density differences are presented in figures 118 and 119. The calculated forces also agree well with the measured forces.
7 Summary and Conclusions

General Approach

Density differences caused by salt and fresh water are usually present in the water of navigation locks in the vicinity of the sea. The water in the approach harbour on the side of the sea will be salt and at the channel side it will be fresh or brackish.

If density differences are present in the water, gravity will cause stratified flows in the lock. These flows are generated during the filling of the lock when the water of the approach harbour intrudes into the lock. Stratified flows also occur when the gate of the lock is opened and the water in the lock, the density of which differs from that in the approach harbour, is exchanged with the water in the approach harbour. Stratified flows generate forces on a ship moored in the lock. The ship partially blocks the stratified flows and in this way influences the stratified flows.

In many cases the forces on the ship caused by the density differences are considerable during the filling and exchange process of the lock. Their magnitude is comparable to the forces which occur under homogeneous conditions. Therefore the influence of density differences cannot be ignored when designing a navigation lock which is in an area with both salt and fresh water. However, until now hardly any attention has been paid in the literature to the forces on the ship induced by density differences in locks and no useful calculation methods are available.

The aim of this investigation was to determine the forces on the moored ship caused by the stratified flows.

The stratified flows in the lock largely depend on the following aspects:
- whether the exchange process or the filling process is concerned
- whether there is fresh water in the lock with salt water in the approach harbour, or salt water in the lock with fresh water in the approach harbour
- the presence (or absence) of a ship and its blockage

The investigation was divided according to these aspects. The sequence of the presentation was determined by the available literature, the possibility of calculation and the progression from a simple to a more complex approach.

The approach was as follows. Firstly the physical phenomena during the exchange process (chapter 2) and the filling process (chapter 3) were investigated with the help of experiments. Subsequently a relation was derived between on the one hand the flow and density distributions, and on the other hand the difference in water-level and longitudinal force on the ship (chapter 4). Then the flow and density distributions were
derived for the exchange process using experimental results and schematized calculations. The differences in water-level and the longitudinal force on the ship were calculated (chapter 5). After that, calculations for the filling process (chapter 6) were made in the same way as those for the exchange process.

The principal restrictions of the investigation are as follows. The lock is filled from one head of the lock. The water in the lock, like that in the approach harbour, has a homogeneous density before both the filling and exchange processes. The only force calculated is the longitudinal force on the horizontally fixed ship. The emptying of the lock is not dealt with because in this case no stratified flows are generated in the lock.

**Physical Description of the Exchange Process** (Chapter 2)

During and after the opening of the gate the water in the lock is exchanged with that in the approach harbour, which has another density. At the opened gate fresh and salt internal wedges are formed which extend in opposite, longitudinal directions.

An internal salt wedge at the bottom intrudes into a lock with initially fresh water. A reverse flow of fresh water is present above this salt wedge.

In the case of a lock with initially salt water an internal fresh wedge intrudes into the lock at the free surface. A reverse flow of salt water is present beneath the fresh wedge.

A small deformation of the free surface travels with the internal wedges. The wedges have fronts as leading edges, which propagate with an almost constant velocity.

The internal wedge which intrudes into the lock reflects against the lock end (the closed gate). The reflection results in an internal wave which propagates in the reverse direction. It appears that the thickness of the intruding layer is approximately doubled at the lock end. The velocity of the internal wave is about equal to the front velocity prior to the reflection. A thin layer of water originally present in the lock remains.

When the front of the wedge which leaves the lock after the opening of the gate reaches the wider approach harbour, an internally critical flow situation occurs at the transition. The thickness of the layer which flows into the approach harbour diminishes considerably because the layer is radially spread over the width of the harbour. The layers in the lock are hardly influenced by the outflow into the approach harbour.

The influence of a ship in the lock on the exchange flow is dependent on the blockage of the ship. An incoming salt wedge partially reflects against the bow. The reflected wave component propagates to the approach harbour. The salt wedge intruding in the space of the keel clearance is
hampered because the reverse flow of fresh water is hindered by the
presence of the ship. A similar situation is present with a fresh wedge
which intrudes into the lock. On meeting the bow a reflected wave starts
to travel to the approach harbour. The transmitted surface layer
continues to spread alongside the ship.

The zone in front of the ship is filled with water from the approach
harbour. After a short time, due to the reflection at the bow, the
density of the water at the bow will become equal to the density of the
water of the approach harbour. This is in contrast what happens at the
stern, where the density adjusts only slowly to the density of the water
of the approach harbour.

The result is that density differences across the length of the ship,
and a longitudinal force, occur for a long time after the gate has been
opened. It appears that this force can exceed the forces caused by the
filling of the lock. The longitudinal force is directed to the saltier
part of the lock.

In addition transverse forces on the ship occur because the wider space
between the ship and the one lock wall is more quickly exchanged than
the narrower space between the ship and the other lock wall.

An air bubble screen at the opened gate reduces the longitudinal force
since it decreases the density differences between the intruding wedge
and the water originally in the lock.

**Physical Description of the Filling Process with Density Differences**
(Chapter 3)

The filling of a lock begins when the valves in the filling openings are
gradually opened. The water of the approach harbour flows into the lock
as separate jets through the openings. The water-level rises, while the
head difference over the gate and the flow velocities in the filling
jets decrease. The filling discharge increases until a maximum value is
reached, after which it decreases. The intruding water mixes with the
water originally in the lock and the jets decay in the downstream
direction. The shape of the filling openings with the beams at the
downstream side is important for the influence of the jets on the flow
pattern in the lock.

An external translatory wave and - at a certain distance from the
filling gate - an intruding salt or fresh layer are generated, which
propagate in the lock and reflect against the gates and the ship. The
filling jets may impinge on the bow and the water-level above the
filling jets is lowered. The longitudinal force on the ship is mainly
determined by the effects of these waves and filling jets.

The flow situation in the lock without the influence of a ship is as
follows. The filling jets entrain water from the gyre above these jets.
The flow velocity and the density difference between the water in the
jets and the ambient water in the lock decreases in the downstream direction because of the entrainment. A stratified flow with an intruding layer arises at a certain distance from the filling gate because of the decreased momentum of the jets. An intruding salt layer at the bottom (or an intruding fresh layer at the free surface) moves with a low velocity and little mixing in the downstream direction and reflects against the downstream gate. The thickness of this layer can be considerable.

If a ship with a large blockage of the cross-section is present in the lock a mixing zone can be distinguished between the filling gate, the bow, the bottom and the free surface. The water of the filling jets is mixed with the water of this zone. No water from beside the ship is entrained into this zone. The density of the water in the zone becomes equal to that of the approach harbour. The mixed water leaves the zone at the downstream side (at the bow), flows as an intruding layer in the downstream direction and reflects against the downstream gate.

The result is that density differences over the length of the ship occur during and after the filling process. These density differences induce a longitudinal force on the ship which differs from the force under homogeneous conditions. It appears that the added force due to the density differences may be of the same order as the force arising under homogeneous conditions.

**Equations for the Difference in Water-level and the Longitudinal Force (Chapter 4)**

The stratified flow in the lock is conceived as a two-layer flow, with a sharp interface between the fresh upper layer and the salt lower layer. Each layer has a homogeneous density, while the velocity is uniformly distributed across the layer.

In a two-layer situation with an interface alongside the ship, the position of the ship in the vertical plane is determined by the levels of the interface and the free surface alongside the ship, and by the density difference between the two layers. The longitudinal force is made dimensionless by the weight of the ship.

The longitudinal force on the ship is calculated assuming a hydrostatic pressure distribution. The longitudinal force on the ship is not influenced by the position of the ship in the vertical plane - only the situations at the bow and the stern are relevant. The hydrostatic longitudinal force can be derived as a function of the levels of the interface and the free surface at the bow and the stern, and the difference in density.

The difference in water-level between the bow and the stern of the ship can be computed using the momentum equation if the flow velocities and the densities of the layers and the levels of the interface are known. The momentum equation is applied in the horizontal direction again
assuming a hydrostatic pressure distribution.
The momentum equation is applied to a control volume of water, excluding
the ship, between the cross-sections just in front of the bow and just
behind the stern. The equation includes four contributions:
- hydrostatic force against the boundaries of the control volume
- force of the ship on the control volume
- transport of momentum through the boundaries
- time derivative of the momentum of the control volume.
The first and second contributions are functions of the levels of the
interface and the free surface at the bow and the stern, the third and
fourth contributions are determined by the flow velocities and the
thicknesses of the layers.

It appears that the difference in water-level between the bow and stern
is not influenced by the fourth term, which comprises the time
derivative of the momentum of the control volume. Therefore, the
difference in water-level is only determined by the flow and density
distributions at the bow and at the stern; the flow and density
distributions between the bow and the stern have no influence. After a
calculation of the difference in water-level, the longitudinal force on
the ship can be determined.

In a stratified flow, both the difference in water-level between the bow
and the stern and the longitudinal force are considerably influenced by
the flow velocities of the layers at the bow and the stern, and so by
the transport of momentum.

In a stratified situation with small flow velocities, the lowest
water-level is present at the salter part of the lock. The longitudinal
force is directed to this part of the lock.
In the case of a greater mean difference in density between the cross-
sections near the bow and the stern, the difference in water-level and
the relative longitudinal force increase. This greater mean density can
be caused by a higher level of the interface or an increased difference
in density.
The difference in water-level and the longitudinal force are also
influenced by the water-depth, the draught and the blockage of the ship.
These influences depend on the situation.

The conclusion is that the difference in water-level between the bow and
the stern can be calculated if the flow velocities, the thicknesses and
the densities of the layers at the bow and the stern are known. The
longitudinal force can be derived from this difference in water-level.

**Calculations of the Exchange Process** (Chapter 5)

Before the calculations for the exchange in a lock with the presence of
a ship are presented, the exchange in a rectangular, prismatic channel
and the exchange in a lock without a ship are considered.
Exchange Flow in a Rectangular, Prismatic Channel

The exchange flow between two sections of a channel with different densities but the same water-level begins after the removal of an initial separation. Upper and lower layers are formed with fronts as leading edges which propagate in the channel in opposite directions. Experiments show that the flow pattern in and near the front is three-dimensional, with a certain mixing at the interface just behind the front. The flow between the fronts may be approximated as a one-dimensional, two-layer flow.

A calculation of the exchange flow in a rectangular, prismatic channel was carried out. The calculation is based on one-dimensional momentum, continuity and energy equations. It is assumed that the fronts are vertical, the interface is a straight line between the fronts and no mixing occurs. The energy losses are concentrated near the fronts and are parameterized introducing empirical coefficients. The calculation yields the height, the velocity and the difference in water-level of the fronts and the exchange flow rate. Good results are obtained with results from the literature which concern one front.

Exchange Flow in a Lock without the Influence of a Ship

The exchange of the water in a lock with that in a channel with the same cross section has been considered. The stratified flow before the reflection against the lock end is based on the calculations of the exchange in a rectangular, prismatic channel. The calculation of the stratified flow after the reflection is based on laboratory observations. The thickness of the layer at the lock end is doubled and the height and the velocity of the internal wave are equal to the values of the front prior to the reflection. The water-level as a function of time and distance in the lock was calculated by applying one-dimensional momentum and continuity equations. The water-level variations derived agree with the experiments.

Exchange Flow in a Lock with a Ship

A schematized mathematical model was developed. In this model the stratified flow is deduced from the experimental observations. The intruding wedge calculated previously reflects at the ship, the lock end and the approach harbour. Empirical coefficients for the partial reflections at the bow, the stern and the approach harbour are introduced. Furthermore, coefficients are introduced for the transport of momentum at the bow and the stern (caused by the non-uniformity of the flow velocities in the cross-sections) and for the time delay and the reduction of the density differences (because of the gradual opening of the gate). The influence of an air-bubble screen is modelled by reducing the density differences.
The one-dimensional momentum and continuity equations yield the difference in water-level between the bow and the stern and the longitudinal force on the ship.

It is apparent that a fixed set of coefficients can be derived for a number of different test cases that result in a rather close agreement between the calculations and the experiments. The agreement relates to the calculated and measured longitudinal forces as functions of time. Furthermore, the mean density and the difference in water-level between the bow and the stern also agree quite well.

The conclusion is that the mathematical model, together with the set of coefficients presented, can be applied to the calculation of the longitudinal force on a ship caused by the exchange flow in a lock.

Calculation of the Influence of Density Differences during the Filling Process (Chapter 6)

Approach to the Calculations

Rapid and slow flow phenomena can be distinguished during the filling process under homogeneous conditions. The rapidly varying discharge pertains to the external transitory waves. The slowly varying discharge varies with the filling discharge as a function of time and decreases in the longitudinal direction of the lock.

The influence of density differences during the filling process may be considered as a slowly varying phenomenon which does not influence the rapid variations of the external transitory waves. This can be applied because the differences in density, the Froude number and the influences of friction are small.

The calculation of the water-level differences between the bow and the stern is based on the one-dimensional momentum and continuity equations. These equations are applied for the slowly varying flow phenomena in the cases with and without density differences. The influence of the density differences is equal to the difference in the values of these two cases. In this application the discharge in a cross-section varies in time with the filling discharge, and decreases linearly in the longitudinal direction of the lock.

Small Blockage of the Ship

The flow and density patterns in the lock are strongly dependent on the shape of the filling openings with the downstream beams, the filling discharge as a function of time, the presence and blockage of a ship, the density differences, etc. Until now the flow and density patterns could not be calculated reliably and had to be inferred from experiments.

A schematized mathematical model was developed for the influence of the
density differences on the longitudinal force. An empirical coefficient is introduced in this model to account for the reduction of the density difference due to the entrainment into the jets. It is assumed that the density and the flow velocity of the intruding layer do not depend on time. The added differences in water-level caused by the density differences are computed using the momentum equation in the cases with and without density differences. After that the added longitudinal force can be derived.

It is apparent that the differences in water-level and the longitudinal force can be computed well if the coefficient for the reduction of the density difference is known from experiments. The computed values are found to be very sensitive to variations in this coefficient. Therefore only a qualitative prediction of the longitudinal force can be made if no experimental results are available.

Large Blockage of the Ship

The maximum longitudinal force increases with a larger blockage of the wet cross-section by the ship. This is caused by the decreased velocity of the external translatory waves and the increased influence of the filling jets. Furthermore, the density differences over the length of the ship increase with a larger blockage due to the absence of entrainment into the filling jets. Therefore the situation of a ship with a large blockage is in many cases decisive for the filling system of the lock.

A mathematical model was developed for the situation of a ship with a large blockage. The continuity equation of the mixing zone in front of the ship zone is applied with the filling discharge as the inflow. The water-level of the zone in this equation agrees with the mean water-level in the lock. An intruding layer as the outflow leaves this zone in the downstream direction with assumed flow and front velocities. The equation yields the mean density of the water in front of the ship as a function of time. The intruding layer is determined as a function of time at the stern by assuming coefficients of reflection at the gates and the stern.

It is apparent that the mean density of the water in the mixing zone in front of the ship can be computed very well because hardly any entrainment from the water beside the ship occurs. The intruding layer can be determined with a good estimation of the flow and front velocities of the intruding layer. It has been verified by experiments that the differences in water-level between the bow and the stern and the longitudinal force caused by density differences can be calculated reliably.

The conclusion is that the influence of the density differences on the longitudinal force can be computed in the case of a large blockage of the lock during the filling of the lock.
KRACHTEN OP SCHEPEN IN EEN SCHUTSLUIS DOOR GELAAAGDE STROMINGEN

SAMENVATTING

Algemene opzet studie

Bij schutsluizen in de omgeving van de zee treden vaak dichtheidsverschillen in het water op door de aanwezigheid van zout en zoet water. Het water in de voorhaven aan de zeezijde is zout en aan de kanaalzijde is dit zoet of brak.

Door deze dichtheidsverschillen worden in de sluiskolk gelaagde stromingen van zout en zoet water opgewekt tijdens het vulproces waarbij het water van de voorhaven de sluiskolk binnendringt. Gelaagde stromingen treden ook op wanneer de deur van de sluis wordt geopend en het water in de sluis met een andere dichtheid dan dat van de voorhaven wordt uitgewisseld met het water in de voorhaven. Deze stromingen oefenen krachten uit op het in de sluis afgemeerde schip. Het schip blokkeert gedeeltelijk de stromingen en beïnvloedt hierdoor deze stromingen.

De krachten op het in de sluis afgemeerde schip ten gevolge van de dichtheidsverschillen in het water kunnen aanzienlijk zijn. De grootte van deze krachten is vergelijkbaar met de grootte van de krachten in sluizen zonder dichtheidsverschillen. De krachten op de schepen door de dichtheidsverschillen kunnen daarom niet verwaarloosd worden bij het hydraulisch ontwerp van een schutsluis. Tot nu toe wordt in de literatuur nauwelijks aandacht besteed aan de krachten op schepen door gelaagde stromingen. Bruikbare rekenmodellen zijn ook niet aanwezig.

Dit onderzoek heeft tot doel de krachten op de schepen door dichtheidsverschillen en de hieraan gerelateerde stroom- en dichtheidsverdelingen en waterstandsverschillen te bepalen.

De gelaagde stromingen in de sluis hangen voornamelijk af van de volgende aspecten:
- uitwisselingsproces of vulproces
- zoet water in de sluis met zout water in de voorhaven of andersom
- de aanwezigheid van een schip en de mate van blokkering

Het onderzoek is afgestemd op deze aspecten. De volgorde van het onderzoek is bepaald door de beschikbare literatuur en de mogelijkheid berekeningen uit te voeren voor eenvoudige en vervolgens voor meer ingewikkeld gevallen.

De opzet van de studie was als volgt. De fysische verschijnselen gedurende het uitwisselingsproces (hoofdstuk 2) en het vulproces (hoofdstuk 3) zijn onderzocht met behulp van model- en prototype-metingen. Vervolgens is de relatie opgesteld tussen de stroom- en dichtheidsver-
delingen enerzijds en de waterstandsverschillen en langskracht anderzijds. (hoofdstuk 4)
De stroom- en dichtheidsverdelingen zijn afgeleid voor het uitwisselingsproces waarbij gebruik gemaakt is van experimentele gegevens en van geschematiseerde berekeningen. De waterstandsverschillen en de langskrachten op de schepen zijn berekend (hoofdstuk 5). Tenslotte zijn berekeningen uitgevoerd van het vulproces (hoofdstuk 6) op soortgelijke wijze als van het uitwisselingsproces.

De voornaamste beperkingen van het onderzoek zijn de volgende. Het vulsysteem betreft alleen het vullen van de sluis vanuit één hoofd. Het water in de kolk en de voorhaven heeft een homogene dichtheid bij het begin van zowel het vul- als het uitwisselingsproces. De enige berekende kracht is de langskracht op het schip dat zodanig afgemeerd is dat alleen verticale bewegingen mogelijk zijn. Het ledigen van de sluis wordt niet behandeld, omdat hierbij geen gelaagde stromingen in de sluis opgewekt worden.

**Fysische beschrijving van het uitwisselingsproces** (hoofdstuk 2)

Na het openen van de deur (of deuren) wordt het water in de sluis uitgewisseld met het water in de voorhaven, dat een andere dichtheid heeft. Bij de geopende deur vormen zich interne zoet- en zoutwigggen die zich in tegenovergestelde richtingen uitbreiden.

Een interne zouttwig aan de bodem dringt een sluis met zoet water binnen, waarbij een tegengestelde stroming zoet water boven deze zouttwig aanwezig is.

In het geval van een zoute sluis dringt een interne zouttwig bij het wateroppervlak de kolk binnen. Een tegengestelde stroming zout water is onder deze zouttwig aanwezig.

Een kleine vervorming van het wateroppervlak is boven de wiggen aanwezig. De wiggen bezitten fronten als voorste begrenzing. De frontsnelheid is vrijwel constant.

De interne wig die de kolk binnendringt reflecteert tegen het sluisseind (gesloten deur). De reflectie resulteert in een interne golf die zich voortplant in tegenovergestelde richting. Het blijkt dat de dikte van de binnendringende laag bij het sluisseind ongeveer wordt verdubbeld. De snelheid van de interne golf is ongeveer gelijk aan de snelheid van het front voor de reflectie. Een dunne laag oorspronkelijk water blijft achter.

Een schip in een sluis beïnvloedt de uitwisselingsstroom afhankelijk van de blokking van het schip. Een binnendringende zouttwig reflecteert gedeeltelijk tegen de boeg. De gereflecteerde golfcomponent plant zich voort naar de voorhaven. De zoutwig die de ruimte van de kielspeling binnendringt wordt afgeremd omdat de tegengestelde zoetstroom wordt gehinderd door de aanwezigheid van het schip.
Een soortgelijke situatie treedt op met een zoetwig die een zoute sluis binnendringt. Bij het reflecteren van de zoetwig tegen de boeg plant zich een interne golf voort in de richting van de voorhaven. De doorgaande laag aan het oppervlakte stroomt langs het schip.

De zone voor het schip wordt gevuld met water vanuit de voorhaven. Na korte tijd zal de dichtheid van water bij de boeg overeenkomen met de dichtheid van het water van de voorhaven door de reflecties bij de boeg. Dit is in tegenstelling tot de dichtheid van het water bij het hek, die zich slechts langzaam aanpast aan de dichtheid van het water van de voorhaven.

Het gevolg is dat dichtheidsverschillen in het water over de lengte van het schip aanwezig zijn. Deze veroorzaken een langskracht op het schip die gedurende lange tijd na het openen van de deur optreedt. Het blijkt dat deze kracht groter kan zijn dan die tijdens het vullen van de kolk. De langskracht is steeds gericht naar het zoutste deel van de kolk.

Ook dwarskrachten treden tijdens het uitwisselingsproces op omdat de brede ruimte tussen schip en sluiswand sneller wordt uitgewisseld dan de nauwe ruimte tussen schip en de andere sluiswand.

Een luchtbellenscherm bij de geopende deur reduceert de langskracht omdat de dichtheidsverschillen tussen de binnendringende interne wig en het oorspronkelijke water in de kolk kleiner zijn dan in het geval zonder luchtbellenscherm.

**Fysische beschrijving van het vulproces** (hoofdstuk 3)

Het vullen van de sluis begint met het openen van de schuiven in de vulopeningen. Het water van de voorhaven stroomt in de sluis in de vorm van afzonderlijke stralen. De waterspiegel rijst, terwijl het verval over de deur en de stroomsnelheden in de vulstralen afnemen. Het vuldebiet neemt toe tot een maximum waarde bereikt is, waarna het afneemt. Het binnendringende water mengt zich met het oorspronkelijke water in de kolk waarbij de stralen in benedenstroomse richting afbreken. De vorm van de vulopeningen met de breekbalken hierachter bepaalt in belangrijke mate de invloed van de stralen in de sluis.

In de sluiskolk worden externe translatiegolven opgewekt en op zekere afstand van de vuldeur ontstaat een binnendringende zoet- of zoutlaag. De golven planten zich in de kolk voort en reflecteren tegen de deuren en het schip. De vulstralen kunnen tegen de boeg botsen en de waterspiegel boven de vulstralen daalt. De langskracht op het schip wordt voornamelijk bepaald door het effect van de golven en de vulstralen.

De stromingssituatie in de sluis zonder schip is als volgt. De vulstralen nemen omringend water op van de neer boven de vulstralen (aanzuiging). Dit gaat gepaard met veel menging in de stralen. De stroomsnelheid en de dichtheidsverschillen met het omringende water nemen af in benedenstroomse richting. Op zekere afstand van de vulopeningen ontstaat een
gelaagde stroming omdat de impuls van de stralen afgenomen is. Een bin-
nendringende zoutlaag bij de bodem (of een binnendringende zuil laag bij
het vrije oppervlak) stroomt met lage snelheid en weinig menging in
benedenstroomse richting en reflecteert tegen de benedenstroomse deur.
De dikte van deze laag kan aanzienlijk zijn.
Wanneer een schip met een grote blokkering voorin de sluis aanwezig is
zal een mengzone voor het schip ontstaan tussen de vuldeur, de boeg, de
bodem en het wateroppervlak. In deze zone mengt het water uit de vul-
stralen zich met het oorspronkelijke water van de zone. Water naast het
schip kan nauwelijks aangezogen worden naar deze zone vanwege de aanwe-
zigheid van het schip. Het water in de zone verkrijgt na enige tijd de
dichtheid van de voorhaven. Het gemengde water verlaat de zone naast en
onder het schip en plant zich als een gelaagde stroming voort in bene-
denstroomse richting en reflecteert tegen de benedenstroomse deur.
Het resultaat is dat tijdens en ook na het vulproces dichtheidsverschil-
len in het water over de lengte van het schip aanwezig zijn. Deze dich-
theidsverschillen wekken een langskracht op die verschilt van de langskracht
tijdens het vullen zonder dichtheidsverschillen. Het blijkt dat
de extra langskracht ten gevolge van dichtheidsverschillen van dezelfde
orde van grootte kan zijn als de kracht tijdens het vullen zonder dich-
theidsverschillen.

Vergelijkingen voor het waterstandsverschil en de langskracht
(hoofdstuk 4)

De gelaagde stroming in de sluis wordt beschouwd als een twee-lagen
stroming met een scherp grensvlak tussen de zoete bovenlaag en de zoute
onderlaag. Elke laag heeft een homogene dichtheid en de stroomsnelheid
is gelijkmatig over de laag verdeeld.

In een twee-lagen situatie met het grensvlak van zout en zoet water ter
hoogte van het schip wordt de stand van het schip in het vertikale vlak
bepaald door het verloop van de waterspiegel en van het grens vlak tussen
boeg en hek en de dichtheidsverschillen tussen de twee lagen.

De langskracht op het schip wordt berekend door uit te gaan van een
hydrostatische drukverdeling. De langskracht wordt niet bepaald door de
voornoemde vertikale stand van het schip, alleen de situaties bij de
boeg en het hek zijn van belang. De hydrostatische langskracht wordt
uitgedrukt als functie van de niveaus van het grens vlak en de waterspie-
gel bij de boeg en het hek en het dichtheidsverschil. Het waterstands-
verschil is nodig voor de bepaling van de langskracht.

Het waterstandsverschil tussen boeg en hek kan berekend worden door het
toepassen van de impulsvergelijking. Hierbij moeten de stroomsnelheden
van de lagen, de niveaus van de grens vlakken en het dichtheidsverschil
bekend zijn. De impulsvergelijking wordt toegepast in horizontale rich-
ting waarbij een hydrostatische drukverdeling wordt aangenomen.
De impulsvergelijking wordt afgeleid voor een balansgebied voor het water tussen de raaien net voor de boeg en net achter het hek, waarbij het schip zelf juist buiten het balansgebied ligt. De vergelijking omvat vier bijdragen:
- hydrostatische drukverdeling tegen de grenzen van het balansgebied
- kracht van het schip op het balansgebied
- impulstransport door de grenzen van het balansgebied
- afgeleide in de tijd van de impuls in het balansgebied

De eerste en tweede bijdrage zijn functies van het niveau van het grensvlak en het vrije oppervlak bij de boeg en het hek, de derde en de vierde bijdrage worden bepaald door de stroomsnellenheden en de dikten van de lagen.

Het blijkt dat het waterstandsverschil tussen boeg en hek niet beïnvloed wordt door de vierde bijdrage, die de verandering van de impuls in de tijd van het balansgebied weergeeft. Daarom wordt het waterstandsverschil alleen bepaald door de snelheids- en dichtheidsverdelingen bij de boeg en het hek; de situatie tussen boeg en hek is van geen belang. Uit het waterstandsverschil kan de langskracht op het schip bepaald worden.

In een twee-lagen stroming worden het waterstandsverschil tussen boeg en hek en de langskracht op het schip sterk bepaald door de stroomsnellenheden in de lagen en dus door het impulstransport.

In een vrijwel stilstaande, gelaagde situatie zal de laagste waterspiegel steeds bij de zoutste raai van boeg of hek aanwezig zijn. De langskracht is steeds gericht naar de zoutste raai.

De langskracht neemt toe bij het toenemen van het gemiddelde dichtheidsverschil tussen boeg en hek, bijvoorbeeld ten gevolge van een gewijzigd niveau van het grensvlak.

Het waterstandsverschil tussen boeg en hek en de langskracht op het schip worden ook beïnvloed door de waterdiepte, de diepgang en de blokkering van het dwarsprofiel door het schip. De gevoeligheid voor deze grootheden hangt af van de situatie.

De conclusie is dat het waterstandsverschil tussen boeg en hek berekend kan worden indien de stroomsnellenheden, de dikte en dichtheden van de lagen bij de boeg en het hek bekend zijn. De langskracht kan afgeleid worden uit dit waterstandsverschil.

**Berekeningen van het uitwisselingsproces** (hoofdstuk 5)

Voordat de uitwisseling in een sluis met een schip behandeld wordt, wordt eerst ingegaan op de uitwisseling in een rechthoekig, prismatisch kanaal en de uitwisseling in een sluis zonder schip.

**Uitwisselingsstroom in een rechthoekig, prismatisch kanaal**

De uitwisseling tussen twee secties van een kanaal met verschillende dichtheid komt op gang na het wegnemen van de initiële scheiding. Een
zoutwieg met een front als voorste begrenzing dringt aan de bodem van de zoete sectie binnen en plant zich voort. Boven de zoutwieg stroomt zout water in tegengestelde richting met hetzelfde debiet als de zoutwieg. Evenzo dringt een zoutwieg met een front aan het oppervlak de zoute sectie binnen met een tegengestelde zoute stroming bij de bodem. Experimenten beschreven in de literatuur tonen aan dat het stroombeeld ter plaatse van het front drie-dimensionaal is. Menging tussen de lagen treedt ter plaatse van de fronten op. De stroming tussen de fronten is vrijwel een een-dimensionale twee-lagen stroming.

Een berekening van de uitwisselingsstroom in een rechthoekig, prismatisch kanaal is uitgevoerd. Gebruik is gemaakt van één-dimensionale impuls-, continuïteits- en energie-vergelijkingen. Hierbij is aangenomen dat de fronten verticaal zijn, het grensvlak tussen de fronten recht is en menging verwaarloosd mag worden. De energieverliezen worden gecentreerd bij de fronten en worden ingevoerd met behulp van empirische coëfficiënten. Uit de berekening volgt de hoogte, de snelheid en het waterstandsverschil van de fronten en het uitwisselingsdebiet. Goede resultaten worden verkregen met uit de literatuur bekende waarden voor de coëfficiënten voor een front.

Uitwisselingsstroom in een sluis zonder de invloed van een schip

De uitwisseling van het water in een sluis met het water in een kanaal met dezelfde dwarsdoorsnede is beschouwd. De gelaagde stroming vóór de reflectie is gebaseerd op de berekeningen van de uitwisseling in een rechthoekig, prismatisch kanaal. De stromingssituatie na de reflectie is gebaseerd op metingen. De dikte van de binnendringende laag is bij het sluiseind verdubbeld en de hoogte en de snelheid van de interne golf na reflectie zijn gelijk aan overeenkomstige waarden van het front voorafgaande aan de reflectie. De waterspiegel als functie van tijd en afstand in de sluis is berekend door het toepassen van de een-dimensionale impuls- en continuïteitsvergelijkingen. De berekende hoogten van de waterspiegel komen overeen met de metingen.

Uitwisselingsstroom in een sluis met schip

Een geschematiseerd rekenmodel van het uitwisselingsproces in een sluis met een schip is ontwikkeld. De structuur van de gelaagde stroming is in dit rekenmodel afgeleid van model- en prototype-metingen. De binnendringende interne laag reflecteert tegen het schip, het sluiseind en de voorhaven. Empirische coëfficiënten zijn ingevoerd voor de genoemde reflecties. Tevens zijn coëfficiënten ingevoerd voor het impulstransport in verband met afwijkingen van het uniforme snelheidsprofiel en voor het geleidelijk openen van de deur. De invloed van een luchtbellenscherm is gemodelleerd door het reduceren van de dichtheidsverschillen. Uit de impuls- en continuïteitsvergelijkingen volgen het waterstandsverschil tussen boeg en hek en de langskracht op het schip.
Het blijkt dat een vast stel coëfficiënten opgesteld kan worden uitgaande van een groot aantal verschillende model- en prototype-metingen. De langskracht als functie van tijd komt hierbij overeen met de gemeten waarde. Dit geldt eveneens voor de gemiddelde dichtheden bij de boeg en het hek en het waterstandsverschil tussen boeg en hek.

De conclusie is dat het geschematiseerde rekenmodel in combinatie met de set coëfficiënten toegepast kan worden bij de berekening van de langskracht op het schip tijdens het uitwisselingsproces in een sluis.

**Berekeningen van de invloed van dichtheidsverschillen tijdens het vulproces** (hoofdstuk 6)

**Algemene opmerkingen**

Er kan onderscheid gemaakt worden tussen snelle en langzame stromingsverschijnselen die tijdens het vulproces optreden. De snelle debietvariaties hebben betrekking op de externe translatiegolven. Het langzaam variërende debiet is evenredig met het tijdsafhankelijke vuldebet en neemt in langsrichting van de kolk af. De invloed van dichtheidsverschillen gedurende het vul-proces kan beschouwd worden als een langzaam variërend verschijnsel dat de externe translatiegolven niet beïnvloedt.

De berekening van de waterstandsverschillen tussen boeg en hek is gebaseerd op de een-dimensionale impuls- en continuïteitsvergelijkingen. Deze vergelijkingen worden toegepast voor de langzame stromingsverschijnselen in het geval met en zonder dichtheidsverschillen. De invloed van dichtheidsverschillen is het verschil tussen deze twee gevallen. In de toegepaste berekeningen varieert het debiet in een raai met het vuldebet als functie van tijd en neemt het debiet lineair af in langsrichting van de kolk.

**Kleine blokkering door het schip**

Het stroombeeld en de dichtheidssituatie in de sluis hangen sterk af van de vormgeving van de vulopeningen met de breekbalken, het vuldebet, de blokkering van het dwarsprofiel door het schip, de dichtheidsverschillen, en dergelijke. Tot op heden kunnen deze stroombeelden niet betrouwbaar berekend worden en moeten verkregen worden uit experimenteel onderzoek.

Een geschematiseerd rekenmodel is ontwikkeld voor de invloed van dichtheidsverschillen op de waterstandsverschillen en de langskrachten. Een empirische coëfficiënt is ingevoerd om de reductie van de dichtheidsverschillen ten gevolge van de aanzuiging van de stralen in rekening te brengen. Er is aangenomen dat de dichtheid en de stroomsnelheid van de binnendringende laag niet afhangen van de tijd. De extra waterstandsverschillen ten gevolge van de dichtheidsverschillen zijn berekend door toepassing van de impulsvergelijking met en zonder dichtheidsverschillen, waarna de langskracht is berekend.
Het blijkt dat de waterstandsverschillen en de langskracht goed berekend kunnen worden indien de coëfficiënt voor de reductie van de dichtheidsverschillen bekend is. De berekende waarden zijn zeer gevoelig voor deze coëfficiënt. Daarom kan alleen een kwalitatieve voorspelling gegeven worden van de langskracht wanneer experimentele gegevens ontbreken.

**Grote blokkering door het schip**

De situatie met een groot, blokkerend schip in de sluis is meestal bepalend voor het vulsysteem omdat dan de grootste langskrachten optreden. Dit geldt ook voor het deel van de langskracht ten gevolge van de dichtheidsverschillen aangezien de dichtheidsverschillen over de lengte van het schip dan groot zijn doordat vrijwel geen aanzuiging door de stralen plaatsvindt.

Een rekenmodel is ontwikkeld voor de situatie van een schip met een grote blokkering. De continuïteitsvergelijking van de mengzone voor het schip is opgesteld met het vuldebiet als instroming en een uitstromende zout- of zoetlaag met aangenomen stroom- en frontsnelheden. De vergelijking levert de gemiddelde dichtheid van het water voor het schip op als functie van tijd. Met de impulsvergelijking worden de waterstandsverschillen tussen boeg en hek en de langskracht ten gevolge van de dichtheidsverschillen berekend.

Het blijkt dat de gemiddelde dichtheid in de zone voor het schip goed berekend kan worden omdat nauwelijks aanzuiging optreedt. Uit vergelijkingen met metingen blijkt dat de waterstandsverschillen tussen boeg en hek en de langskracht ten gevolge van dichtheidsverschillen betrouwbaar berekend kunnen worden.

De conclusie is dat de invloed van dichtheidsverschillen op de langskracht berekend kan worden bij een grote blokkering van het schip. Voor een kleine blokkering van de sluis geldt dit alleen als experimentele gegevens over de aanzuiging ten gevolge van de stralen beschikbaar zijn.
Fig. 1: Overview of the density distributions for a lock with initially fresh water and an approach harbour with salt water.
Fig. 2: Overview of the density situations for a lock with initially salt water and an approach harbour with fresh water.
Fig. 3: Prototype lock for the investigation of the exchange and filling processes

Fig. 4: Hydraulic model for the investigation of the exchange process

Fig. 5: Hydraulic model for the investigation of the filling process
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Fig. 7: Exchange process in a lock with initially salt water and with a ship
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Fig. 9: Density profiles during the exchange in a lock with initially salt water and an approach harbour with fresh water, without an air bubble screen (prototype investigation)
Fig. 10: Density profiles during the exchange in a lock with initially fresh water and an approach harbour with salt water, with an air bubble screen (prototype investigation)
Fig. 11: Density profiles during the exchange in a lock with initially salt water and an approach harbour with fresh water, with an air bubble screen (prototype investigation)
Fig. 12: Mean density during exchange in a lock with initially fresh water without air bubble screen (prototype investigation)

Fig. 13: Mean density during exchange in a lock with initially salt water without an air bubble screen (prototype investigation)
Fig. 14: Mean density during exchange in a lock with initially fresh water with an air bubble screen (prototype investigation)

Fig. 15: Mean density during exchange in a lock with initially salt water with an air bubble screen (prototype investigation)
Fig. 16: Forces on the ship during exchange in a lock with initially fresh water without an air bubble screen (prototype)

Fig. 17: Forces on the ship during exchange in a lock with initially salt water without an air bubble screen (prototype)
Fig. 18: Forces on the ship during exchange in a lock with initially fresh water with an air bubble screen (prototype)

Fig. 19: Forces on the ship during exchange in a lock with initially salt water with an air bubble screen (prototype)
Fig. 20: Forces on the ship during exchange in a lock with initially fresh water without an air bubble screen (scale model)

Fig. 21: Forces on the ship during exchange in a lock with initially salt water without an air bubble screen (scale model)
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Fig. 23: Roller gate of the hydraulic scale model with filling openings.
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Fig. 26: Filling of a fresh lock with salt water and with a small blockage of a ship

Fig. 27: Filling of a fresh lock with salt water and with a large blockage of a ship
Fig. 28: Filling of a salt lock with fresh water without the influence of a ship

Fig. 29: Filling of a salt lock with fresh water and with a small blockage of a ship

Fig. 30: Filling of a salt lock with fresh water and with a large blockage of a ship
Fig. 31: Density profiles during the filling of a fresh lock with salt water (prototype investigation)
Fig. 32: Density profiles during the filling of a salt lock with fresh water (prototype investigation)
Fig. 33: Mean densities during the filling of a fresh lock with salt water (prototype investigation)

Fig. 34: Mean densities during the filling of a salt lock with fresh water (prototype investigation)
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Fig. 36: Longitudinal forces on the ship during the filling process (hydraulic scale model)
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**Density [kg/m³]**

- **bow (meas.)**
- **stern (meas.)**
- **bow (calc.)**
- **calc. (calc.)**

Time $t$ [s]

Fig. 64: Influence of coefficient of momentum $p_I$ on the longitudinal force, Test 3 (exchange in an initially fresh lock)

- $p_I = 1.0$
- $p_I = 1.5$
- $p_I = 2.0$
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Fig. 105: Influence of coefficient $\beta$ for mixing on the longitudinal force caused by density differences, Test 7 (filling process of an initially salt lock)
--- 220 ---

\[ p_I = 1.00 \quad \cdots \quad p_I = 1.50 \]

\[ F_r' \ [\circ/\circ] \]

Fig. 106: Influence of the momentum coefficient \( p_I \) on the longitudinal force caused by density differences, Test 7 (filling process of an initially salt lock)

--- measured \quad -- calculated ---

Difference in water-level \( \Delta h_r \) [m]

Fig. 107: Difference in water-level caused by density differences as a function of time, Test D1-23 (filling process of an initially fresh lock)
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Fig. 119: Longitudinal force caused by density differences as a function of time, Test R2, Haringvliet Lock (filling of a salt lock)
Photo 1:

The beginning of the exchange process with the opening of the mitre gates. The air bubble screen is functioning.

Photo 2:

The situation in the lock chamber during the exchange process. The air bubble screen is functioning.
Photos 3 - 7:
The internal salt wedge passes the bow.
(exchange process)
Photo 8:
Flow situation near the roller gate (beginning of the exchange process)

Photo 9:
The front passes the bow (exchange process).

Photos 10 - 11:
The internal salt wedge passes the stern (exchange process).
Photos 12 - 13:
Filling of an initially fresh lock with salt water. Flow situation near the bow (beginning of the filling process) and near the stern (front passes the stern).

for all the cases:
the intruding layer is blue, the original water in the lock is colourless and red)

Photos 14 - 15:
Filling process without density differences. Flow situation near the bow (beginning of the filling process) and near the stern.

Photos 16 - 17:
Filling of an initially salt lock with fresh water. Flow situations near the bow (beginning of the filling process) and near the stern (front passes the stern).
REFERENCES


[22] Schijf, J.B. and Schönfeld, J.C. (1953), "Theoretical Considerations on the Motion of Salt and Fresh Water, IAHR Congress Minnesota


LIST of SYMBOLS

\[ a_1 \] = thickness of lower layer \[ [\text{m}] \]
\[ a_{1a} \] = thickness of the lower layer at the mouth \[ [\text{m}] \]
\[ a_{1b} \] = thickness of the lower layer in front of the bow \[ [\text{m}] \]
\[ a_{1c} \] = thickness of the lower layer at the closed gate \[ [\text{m}] \]
\[ a_{1e} \] = thickness of the lower layer behind the jump \[ [\text{m}] \]
\[ a_{1g} \] = thickness of the lower layer at the opened gate \[ [\text{m}] \]
\[ a_{1l} \] = thickness of the lower layer behind the salt front or height of the schematized salt front \[ [\text{m}] \]
\[ a_{1r} \] = thickness of the lower layer in front of the jump \[ [\text{m}] \]
\[ a_{1s} \] = thickness of the lower layer behind the stern \[ [\text{m}] \]
\[ a_{1u} \] = thickness of the lower layer beneath the fresh front \[ [\text{m}] \]
\[ a_2 \] = thickness of upper layer \[ [\text{m}] \]
\[ a_{2a} \] = thickness of the upper layer at the mouth \[ [\text{m}] \]
\[ a_{2b} \] = thickness of the upper layer in front of the bow \[ [\text{m}] \]
\[ a_{2c} \] = thickness of the upper layer at the closed gate \[ [\text{m}] \]
\[ a_{2e} \] = thickness of the upper layer behind the jump \[ [\text{m}] \]
\[ a_{2g} \] = thickness of the upper layer at the opened gate \[ [\text{m}] \]
\[ a_{2l} \] = thickness of the upper layer above the salt front \[ [\text{m}] \]
\[ a_{2r} \] = thickness of the upper layer in front of the jump \[ [\text{m}] \]
\[ a_{2s} \] = thickness of the upper layer behind the stern \[ [\text{m}] \]
\[ a_{2u} \] = thickness of the upper layer behind the fresh front or height of the schematized fresh front \[ [\text{m}] \]
\[ a_{f1} \] = height of the salt front \[ [\text{m}] \]
\[ a_{fu} \] = height of the fresh front \[ [\text{m}] \]
\[ A_k \] = transverse area of the lock \((A_k = b_k h_k)\) \[ [\text{m}^2] \]
\[ A_s \] = transverse area of the ship \((A_s = b_s d_s)\) \[ [\text{m}^2] \]
\[ A_n \] = net area of the filling openings \[ [\text{m}^2] \]
\[ A_h \] = maximum filling opening \[ [\text{m}^2] \]
\[ b_k \] = width of the lock \[ [\text{m}] \]
\[ b_s \] = width of the ship \[ [\text{m}] \]
\[ b_h \] = total width of the filling openings \[ [\text{m}] \]
\[ c_b \] = block coefficient of the ship \[ [-] \]
\[ C_l \] = coefficient of Chézy \[ [\text{m/s}] \]
\[ c_e \] = velocity of propagation of external transitory waves \[ [\text{m/s}] \]
\[ c_i \] = velocity of propagation of internal waves \[ [\text{m/s}] \]
\[ c_r \] = velocity of internal jump \[ [\text{m/s}] \]
\[ c_l \] = velocity of the salt front \[ [\text{m/s}] \]
\[ c_u \] = velocity of the fresh front \[ [\text{m/s}] \]
\[ c_u' \] = coefficient of the velocity of the salt front \[ [-] \]
\[ c_u'' \] = coefficient of the velocity of the fresh front \[ [-] \]
\[ d_b \] = draught of the ship at the bow \[ [\text{m}] \]
\[ d_e \] = draught of the ship behind the front \[ [\text{m}] \]
\[ d_h \] = draught of the ship at the stern \[ [\text{m}] \]
\[ d_r \] = draught of the ship in front of the front \[ [\text{m}] \]
\[ d_s \] = draught of the ship \[ [\text{m}] \]
\[
dh_k/\text{dt} = \text{vertical velocity of the water-level in the lock} \quad \text{[m/s]}
\]
\[
d\alpha_{lb}/\text{dt} = \text{vertical velocity of the interface in front of the bow} \quad \text{[m/s]}
\]
\[
dv/\text{dt} = \text{velocity of the increase of the density of the water} \quad \text{[kg/m}^3\text{s]}
\]
\[
F_i = \text{internal Froude number} \quad [-]
\]
\[
F_b = \text{longitudinal force on the ship without density differences} \quad \text{[N]}
\]
\[
F_{hh} = \text{resulting hydrostatic force on control volume between bow and stern without density differences} \quad \text{[N]}
\]
\[
F_{hr} = \text{resulting hydrostatic force on control volume between bow and stern with density differences} \quad \text{[N]}
\]
\[
F_r = \text{longitudinal force on the ship due to density differences} \quad \text{[N]}
\]
\[
F_{r', r} = \text{relative longitudinal force on the ship} \quad [-]
\]
\[
F_{bs} = \text{hydrostatic force between the cross sections bow and stern} \quad \text{[N]}
\]
\[
g = \text{acceleration of the gravity} \quad \text{[m/s}^2\text{]}
\]
\[
h = \text{water-depth} \quad \text{[m]}
\]
\[
h_b = \text{water-depth at the bow or level of the bottom} \quad \text{[m]}
\]
\[
h_i = \text{initial water-level in the lock} \quad \text{[m]}
\]
\[
h_k = \text{water-depth in the lock or water-level in the lock} \quad \text{[m]}
\]
\[
h_g = \text{water-depth at the stern} \quad \text{[m]}
\]
\[
h_d = \text{upstream water-level in the approach harbour} \quad \text{[m]}
\]
\[
I_b = \text{momentum transport in front of the bow} \quad \text{[N]}
\]
\[
I_{bs} = \text{momentum transport between the bow and the stern} \quad \text{[N]}
\]
\[
I_{dh} = \text{transport of momentum without density differences} \quad \text{[N]}
\]
\[
I_{dr} = \text{transport of momentum with density differences} \quad \text{[N]}
\]
\[
I_s = \text{momentum transport at the stern} \quad \text{[N]}
\]
\[
I_{ct} = \text{change of momentum in time with density differences} \quad \text{[N]}
\]
\[
I_{ch} = \text{change of momentum in time without density differences} \quad \text{[N]}
\]
\[
I_{cr} = \text{change of momentum in time with density differences} \quad \text{[N]}
\]
\[
k = \text{interfacial friction coefficient} \quad [-]
\]
\[
k_1 = \text{energy loss coefficient at the salt front} \quad [-]
\]
\[
k_2 = \text{energy loss coefficient at the fresh front} \quad [-]
\]
\[
l_k = \text{length of lock} \quad \text{[m]}
\]
\[
l_s = \text{length of the ship} \quad \text{[m]}
\]
\[
P_L = \text{momentum coefficient for deviations of the uniform flow profile} \quad [-]
\]
\[
P_{Kc} = \text{coefficient for the maximum thickness of the lower layer in the keel clearance} \quad [-]
\]
\[
P_b = \text{bow coefficient} \quad [-]
\]
\[
P_c = \text{coefficient for the delay of the exchange process due to the gradual opening of the lock gate} \quad [-]
\]
\[
p = \text{pressure} \quad \text{[kg/m}^2\text{s}^2\text{]}
\]
\[
Q = \text{filling discharge} \quad \text{[m}^3\text{/s]}
\]
\[
q_{ex} = \text{exchange discharge (per m width)} \quad \text{[m}^3\text{/s]}
\]
\[
q_{ex'} = \text{relative exchange discharge} \quad [-]
\]
\[
r_a = \text{coefficient for reflection at the approach harbour} \quad [-]
\]
\[
r_b = \text{coefficient for reflection against the bow} \quad [-]
\]
\[
r_s = \text{coefficient for the reflection against the stern} \quad [-]
\]
\[
t = \text{time} \quad \text{[s]}
\]
\[
t_b = \text{time when front reaches the bow} \quad \text{[s]}
\]
\( t_c \) = time when front reaches the closed gate [s]  
\( t_e \) = filling time of the lock [s]  
\( t_g \) = opening time of the gate [s]  
\( t_h \) = opening time of the valves [s]  
\( t_n \) = time when front beside the ship reaches the stern [s]  
\( t_s \) = time when front beneath the ship reaches the stern [s]  
\( t_{ss} \) = time when front reaches the stern for the second time [s]  
\( T_e \) = harmonic period of closed lock for external waves [s]  
\( T_i \) = harmonic period of closed lock for internal waves [s]  
\( u_1 \) = flow velocity of lower layer [m/s]  
\( u_2 \) = flow velocity of upper layer [m/s]  
\( v_1 \) = flow velocity of the lower layer [m/s]  
\( v_{1a} \) = flow velocity of the lower layer at the mouth [m/s]  
\( v_{1b} \) = flow velocity of the lower layer in front of the bow [m/s]  
\( v_{1e} \) = flow velocity of the lower layer behind the jump [m/s]  
\( v_{1g} \) = flow velocity of the lower layer at the opened gate [m/s]  
\( v_{1l} \) = flow velocity of the lower layer behind the salt front [m/s]  
\( v_{1r} \) = flow velocity of the lower layer in front of the jump [m/s]  
\( v_{1s} \) = flow velocity of the lower layer behind the stern [m/s]  
\( v_{1u} \) = flow velocity of the lower layer beneath the fresh front [m/s]  
\( v_2 \) = flow velocity of the upper layer [m/s]  
\( v_{2a} \) = flow velocity of the upper layer at the mouth [m/s]  
\( v_{2b} \) = flow velocity of the upper layer in front of the bow [m/s]  
\( v_{2e} \) = flow velocity of the upper layer behind the jump [m/s]  
\( v_{2g} \) = flow velocity of the upper layer at the opened gate [m/s]  
\( v_{2l} \) = flow velocity of the upper layer above the salt front [m/s]  
\( v_{2r} \) = flow velocity of the upper layer in front of the jump [m/s]  
\( v_{2s} \) = flow velocity of the upper layer behind the stern [m/s]  
\( v_{2u} \) = flow velocity of the upper layer behind the fresh front [m/s]  
\( v_b \) = flow velocity in front of the bow (cross section b) [m/s]  
\( v_s \) = flow velocity behind the stern (cross section s) [m/s]  
\( v_h \) = velocity of the valves of the filling gate [m/s]  
\( x \) = length co-ordinate [m]  
\( x_a \) = horizontal distance between opened gate and the approach harbour [m]  
\( x_b \) = horizontal distance between opened gate and the bow of the ship [m]  
\( x_c \) = horizontal distance between opened and closed gate [m]  
\( x_r \) = horizontal distance between opened gate and the front [m]  
\( x_m \) = horizontal distance between opened gate and the centre of the ship [m]  
\( x_h \) = horizontal distance between opened gate and the stern of the ship [m]  
\( x_s \) = horizontal distance between opened gate and the stern of the ship [m]  
\( z_b \) = thickness of the residual layer at the bow [m]  
\( z_s \) = thickness of the residual layer at the stern [m]
\( \beta \) = coefficient for the decrease in relative density difference because of mixing at the front
\( \gamma \) = inclination of the ship [rad]
\( \delta \) = coefficient of damping wave height [-]
\( \epsilon \) = \((\rho_1 - \rho_2)/\rho_2\) = relative density difference [-]
\( \Delta h \) = difference in water-level outside the fronts [m]
\( \Delta h_a \) = water-level elevation at the approach harbour [m]
\( \Delta h_c \) = water-level elevation at the closed gate [m]
\( \Delta h_e \) = water-level elevation behind the jump [m]
\( \Delta h_g \) = water-level elevation at the opened gate [m]
\( \Delta h_l \) = water-level elevation above the salt front [m]
\( \Delta h_r \) = water-level elevation in front of the jump [m]
\( \Delta h_u \) = water-level elevation above the fresh front [m]
\( \Delta h_{bs} \) = difference in water-level between the bow and the stern [m]
\( \Delta t_{sc} \) = period between passage of the stern and the reflection closed gate [s]
\( \Delta t_{ba} \) = period between reflection against the stern and the approach harbour [s]
\( \rho \) = density of the water [kg/m³]
\( \rho_1 \) = density of the water of the lower layer [kg/m³]
\( \rho_2 \) = density of the water of the upper layer [kg/m³]
\( \mu \) = discharge coefficient [-]
\( \tau_b \) = frictional stress between bottom and lower layer [N/m²]
\( \tau_{12} \) = frictional stress between lower and upper layers [N/m²]

**PRINCIPAL SUBSCRIPTS**

1 = lower layer
2 = upper layer
a = approach harbour
b = bow
c = closed gate
e = just in front of the jump
g = opened gate
h = stern
k = lock
l = lower front
m = centre of the ship
r = just behind the jump
s = stern, ship
u = upper front
APPENDIX A

Schematized Calculations for the Exchange of a Channel with a Sea

A.1 General Remarks

The aim of this appendix is to determine the height and the velocity of the salt front and the related difference in water-level with the help of the measured values at the mouth of the channel and the sea. The measurements concern the model investigation of Keulegan (Keulegan, 1958, lit. [13]).

The schematized calculation of the exchange of a channel with a sea is similar to the calculation of the exchange of a prismatic channel (section 5.2) with the same assumptions. The momentum, continuity and energy equations are applied for the salt front. The measured value of the interface is used for the situation at the mouth and it is assumed the internal Froude number at the mouth is 1. The momentum and continuity equations are applied between the salt front and the mouth.

A.2 Equations

![Diagram](image)

Fig. T61  Exchange of a channel and a sea

Control Volume between Mouth and Front

The momentum equation between cross-sections 4 and 5 is as follows:

\[
\frac{1}{2} \rho_2 \ g \ (a_{1a} + a_{2a})^2 + \frac{1}{2} (\rho_1 - \rho_2) \ g \ a_{1a}^2 - \frac{1}{2} \rho_2 \ g \ (h_k + \Delta h)^2 + \rho_1 v_{1a}^2 a_{1a} + \rho_2 v_{2a}^2 a_{2a} - \frac{1}{2} \rho_1 (a_{1a} + a_{11}) (v_{1a} + c_1) c_1 + \frac{1}{2} \rho_2 (a_{2a} + a_{21}) (v_{2a} + v_{21}) c_1 = 0 \]  

(A.1)
The continuity equation of the salt layer is as follows:

$$c_1 \ a_{1l} + \frac{1}{2} \ c_1 \ (a_{1a} - a_{1l}) = a_{1a} \ v_{1a}$$ \hspace{1cm} (A.2)

**Mouth**

It is assumed the internal Froude number is 1 at cross-section a:

$$F_1^2 = \frac{v_{1a}^2}{(\epsilon \ g \ a_{1a})} + \frac{v_{2a}^2}{(\epsilon \ g \ a_{2a})} = 1$$ \hspace{1cm} (A.3)

The continuity equation for the upper and lower layer in cross-section a is:

$$a_{1a} \ v_{1a} \approx a_{2a} \ v_{2a}$$ \hspace{1cm} (A.4)

**Salt Front**

Equations (5.22), (5.23) and (5.27) are applied for the salt front.

**Geometry:**

- cross-section 2: \hspace{1cm} $$a_{1a} + a_{2a} = h_k + \Delta h + \Delta h_a$$ \hspace{1cm} (A.5)
- cross-section 4: \hspace{1cm} $$a_{1l} + a_{2l} = h_k + \Delta h + \Delta h_l$$ \hspace{1cm} (A.6)

**A.3 Results**

The nine above mentioned equations have the following nine unknowns:

- \( a_{2a}, a_{1l}, a_{2l}, \Delta h_a, \Delta h_l, v_{1a}, v_{2a}, c_1, a_{1a} \)

The equations have the following knowns as input:

- \( h_k + \Delta h), \rho_1, \rho_2, k_1, F_1 \) and \( a_{1a} \)

The equations (5.22), (5.23), (5.27) and (A.1) up to and including (A.6) have been solved. Calculations have been carried out for the following data:

- \( h_k = 10 \ \text{m} \)
- \( \rho_1 = 1020 \ \text{kg/m}^3, \rho_2 = 1000 \ \text{kg/m}^3 \)
- \( \Delta h = (/(\rho_1/\rho_2) - 1) \ h_k \)
- \( k_1 \) varies between 0.25 and 1
- \( F_1 = 1 \) (measured value)
- \( a_{1a}/h_k = 0.62, 0.66 \) and \( 0.70 \)

The results are presented in dimensionless form in figures A.1 and A.2
as functions of the energy loss coefficient of the salt front for the measured value $a_{1a}/h_k = 0.66$:

- coefficient of the front velocity:
  
  $$c_1' = c_1 / \sqrt{(\epsilon g (a_{1a} + a_{2a}))}$$

- relative front height:
  
  $$a_{1a} / h_k$$

- relative water-level difference over the front:
  
  $$d_1' = (\Delta h - \Delta h_1) / \Delta h$$

- relative exchange discharge:
  
  $$q_{i1} = a_{1a} v_{1a} / h_k \sqrt{(\epsilon g h_k})$$

The calculated velocity and the height of the front coincides with the experimental values (Keulegan, 1958, lit. [13], table 5.2) for the following combinations of the energy loss coefficient $k_1$ and the height of the lower layer at the interface $a_{1a}$:

<table>
<thead>
<tr>
<th>$a_{1a}/h_k$</th>
<th>$k_1$</th>
<th>$c_1'$</th>
<th>$a_{11}/h_k$</th>
<th>$d_1'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.62</td>
<td>0.50</td>
<td>0.46 (0.49)</td>
<td>0.29 (0.30)</td>
<td>0.25</td>
</tr>
<tr>
<td>0.66</td>
<td>0.75</td>
<td>0.42 (0.44)</td>
<td>0.25 (0.24)</td>
<td>0.18</td>
</tr>
<tr>
<td>0.70</td>
<td>1.00</td>
<td>0.38 (0.37)</td>
<td>0.20 (0.15)</td>
<td>0.11</td>
</tr>
</tbody>
</table>

The results of Keulegan's model show that the level of the interface at the mouth rises with an increasing distance travelled by the salt front (table 5.2). It appears from the calculations that the energy loss coefficient $k_1$ must increase as the front travels further. This is caused by the friction of the bottom and the interface also increase as the front travels further. However, these energy losses are concentrated at the front in the calculations.

The conclusions of the calculations are as follows:

- the velocity and the height of the salt front diminishes with an increasing distance travelled.

- the energy loss coefficient $k_1$ must be increased with a greater distance travelled by the front.
Fig. A1: Velocity and height of the front as functions of the energy loss coefficient of the front (exchange sea/channel)

Fig. A2: Water-level difference and exchange discharge as functions of energy loss coefficient of the front (exchange sea/channel)
APPENDIX B

Schematized Calculation of the Filling Process without Density Differences

B.1 Water-level and Filling Discharge

It is assumed that the velocity of the valves and the widths of the openings are constant. This implies a linear increase of the filling openings with a maximum value according to:

\[ A_n = b_h v_h t \quad \text{if } t < t_h \]
\[ A_n = A_h \quad \text{if } t > t_h \]

where \( A_n \) = filling opening
\( b_h \) = total width of the filling openings
\( v_h \) = velocity of the valves
\( t_h \) = time, needed for the opening of the valves
\( A_h \) = maximum filling opening

The filling discharge is determined by the net filling opening and the head difference over the gate:

\[ Q = \mu \frac{A_n}{(2 g (h_u - h_k))} \]

where \( Q \) = filling discharge
\( \mu \) = discharge coefficient
\( h_u \) = upstream water-level in the approach harbour
\( h_k \) = water-level in the lock with respect to the bottom of the lock (\( h_k \) is also used as the water-depth in the lock)

The rising of the mean water-level follows from:

\[ \frac{dh_k}{dt} \cdot b_k l_k = Q \]

where \( \frac{dh_k}{dt} \) = vertical velocity of the mean water-level in the lock
\( b_k \) = width of the lock chamber
\( l_k \) = length of the lock chamber

The water-level \( h_k \) and filling discharge \( Q \) can be solved from equations (B.1), (B.2) and (B.3); this yields the following analytical equations:

If \( t < t_h \):

\[ h_k = h_d + \frac{\mu v_h b_h}{2 b_k l_k} \left( \frac{2 g (h_u - h_d)}{t^2} \right) - \left( \frac{\mu v_h b_h}{8 (b_k l_k)^2} \right) t^4 \]
where \( h_d \) = initial water-level in the lock chamber

\[
Q = \mu \sqrt{\frac{v_h b_h}{2 (g (h_u - h_d))}} t - \frac{\left( \mu \sqrt{v_h b_h} \right)^2 g}{2 b_k l_k} t^3
\]  

(B.5)

If \( t > t_h \):

\[
h_k = h_h + \frac{\mu A_h \sqrt{(2 g (h_u - h_h))}}{b_k l_k} (t - t_h) - \frac{(\mu A_h)^2 g}{2 (b_k l_k)^2} (t - t_h)^2
\]  

(B.6)

where \( h_h = h_k(t_h) \)

\[
Q = \mu A_h \sqrt{(2 g (h_u - h_h))} - \frac{(\mu A_h)^2 g}{b_k l_k} (t - t_h)
\]  

(B.7)

(i.e. a linear decrease of the discharge)

The filling time can be derived from the condition with \( Q = 0 \), which gives:

\[
t_e = \frac{1}{2} t_h + \frac{b_k l_k \sqrt{(2 g (h_u - h_d))}}{g \mu A_h}
\]  

(B.8)

where \( t_e \) = filling time of the lock

Numerical solution of equations (B.2) and (B.3) may be needed for more complicated time functions of the area of the filling openings.

### B.2 Estimation of Longitudinal Force

The longitudinal force can be estimated by means of the following terms in the momentum equation for a control volume containing the ship:

#### 1 Horizontal, hydrostatic force \( F_h \)

The horizontal, hydrostatic force \( F_h \) against the boundaries of the control volume including the ship can be schematized as:

\[
F_h = \rho g (b_k d_k - b_s d_s) \Delta h_h
\]  

(B.9)

where \( \Delta h_h \) = water-level difference between bow and stern

\( \rho \) = density of the water
2 Time derivative of the momentum $I_t$

The change of momentum within the control volume varies strongly in time because of the propagation and the reflection of the transitory waves. This results in oscillations of the water-level around a mean value. It appears that in most cases the maximum change of momentum can be estimated as follows (for the cross-section at the bow):

$$I_t = \rho \ l_s \ \frac{dQ}{dt} \ (l_k - x_p) / l_k$$  \hspace{1cm} (B.10)

where \(dQ/dt\) = time derivative of discharge (equations (B.5) or (B.7))

The minimum value appears to amount to \(I_t = 0\).

3 Transport of momentum through the boundaries of the control volume $I_d$

It is assumed that the discharge decreases linearly in longitudinal direction of the lock and is zero at the downstream end of the lock. The flow-velocity $v_b$ at the bow is due to the velocity of the jets and the flow velocity $v_s$ at the stern is due to the flow separation at the stern. The mean transport of momentum through the boundaries is:

$$I_d = \rho \ (Q_b \ v_b - Q_s \ v_s)$$  \hspace{1cm} (B.11)

where

\begin{align*}
Q_b &= Q \ (l_k - x_b) / l_k \\
Q_s &= Q \ (l_k - x_s) / l_k \\
v_b &= Q_b \ A_s \\
v_s &= Q_s / (h_k \ b_k - d_s \ b_s) \\
A_s &= \text{area of the jets at the bow}
\end{align*}

4 Force of the jets against the bow $F_j$

The force of the jets depends on the area of cross-section of the jets at the bow and the angle of the bow with the jets.

$$F_j = \rho \ A_j \ v_b^2 \ C_j$$  \hspace{1cm} (B.12)

where $A_j$ = impact area against the bow

$C_j$ = drag coefficient

5 Friction force along ship, walls and bottom $F_w$

The friction force on the ship is due to the friction of the water along the ship's hull and to the gradient of the water-level caused by the friction along the walls and bottom of the lock.

$$F_w = \rho \ g \ v_m^2 \ (b_s + 2 \ d_s) \ l_s / C_s^2 + C_1 \ \rho \ g \ v_m^2 \ (b_k + 2 \ h_k) \ l_s / C_k^2$$  \hspace{1cm} (B.13)
where \( C_s \) = Chézy coefficient for friction along the ship's hull
\( C_k \) = Chézy coefficient for friction along the lock
\( C_l \) = coefficient for development of the boundary layer
\( V_m = Q (l_k - x_m) / (l_k (h_k b_k - d_s b_s)) \)

The momentum equation is:

\[ F_h + I_t - I_d + F_w + F_j = 0 \]  \hspace{1cm} (B.14)

where \( I_t \) varies between 0 and equation (B.10)

Substituting equations (B.9) up to (B.13) in equation (B.14) yields two values for \( \Delta h_h \). The relative longitudinal force is:

\[ F'_s = \Delta h_h / l_k \]  \hspace{1cm} (B.15)

In this way a maximum and a minimum longitudinal force are found. The principle of the above mentioned procedure is used for the mathematical model LOCKFILL, which was developed for a preliminary design of filling systems.

B.3 Example

Data of the example:
- water-depths: \( h_u = 6.10 \) m and \( h_d = 4.60 \) m
- density of the water: \( \rho = 1000 \) kg/m\(^3\)
- horizontal lock dimensions: \( l_k \times b_k = 280 \times 24 \) m\(^2\)
- filling openings: \( \mu = 0.65, b_h = 16.5 \) m, \( v_h = 0.003 \) m/s, \( A_h = 20.63 \) m\(^2\)
- distance filling gate / bow: \( x_b = 5 \) m
- ship: \( l_s \times b_s \times d_s = 153 \times 22.80 \times 3.90 \) m\(^3\)

The results are presented as functions of time. Figure B.1 shows the water-depth and the filling discharge and figure B.2 shows the various components of the longitudinal force. Figure B.3 shows the total relative longitudinal force. The measured force is between the calculated values of the maximum and minimum forces.
Fig. B1: Water-depth and discharge as functions of time (filling of a lock, appendix B)

Fig. B2: Components of the longitudinal force as functions of time (filling of a lock, appendix B)
Fig. B3: Total longitudinal force as a function of time (filling of a lock, appendix B)
APPENDIX C

Data of Tests

Rozenburg Lock

- test R1: initially fresh lock, salt approach harbour
  . the water-depths: \( h_u = 9.30 \text{ m} \) and \( h_d = 6.00 \text{ m} \)
  . the densities: \( \rho_1 = 1026 \text{ kg/m}^3 \), \( \rho_2 = 1000 \text{ kg/m}^3 \)
  . the coefficients: \( c_1' = 0.42 \), \( p_1 = 1 \)
  . initial blockage: 0.53

- test R2: initially salt lock, fresh approach harbour
  . the water-depths: \( h_u = 6.50 \text{ m} \) and \( h_d = 4.50 \text{ m} \)
  . the densities: \( \rho_1 = 1026 \text{ kg/m}^3 \), \( \rho_2 = 1000 \text{ kg/m}^3 \)
  . the coefficients: \( c_1' = 0.46 \), \( p_1 = 1 \)
  . initial blockage: 0.70

The joint data of these test are:
  . the lock: \( l_k \times b_k = 330 \times 24 \text{ m}^2 \)
  . the openings: \( \mu = 0.5 \), \( b_h = 14.40 \text{ m} \), \( v_h = 0.00325 \text{ m/s} \), \( A_h = 19.42 \text{ m}^2 \)
  . the cross-sections: \( x_b = 10 \text{ m} \) and \( x_s = 163 \text{ m} \)
  . the ship: \( l_s \times b_s \times d_s = 153 \times 23 \times 3.30 \text{ m}^3 \)

Haringvliet Lock

- test H1: initially fresh lock, salt approach harbour
  . the water-depths: \( h_u = 7.90 \text{ m} \) and \( h_d = 4.65 \text{ m} \)
  . the densities: \( \rho_1 = 1020 \text{ kg/m}^3 \), \( \rho_2 = 1000 \text{ kg/m}^3 \)
  . the coefficients: \( c_1' = 0.42 \), \( p_1 = 1 \)
  . initial blockage: 0.40

- test H2: initially salt lock, fresh approach harbour
  . the water-depths: \( h_u = 5.50 \text{ m} \) and \( h_d = 3.00 \text{ m} \)
  . the densities: \( \rho_1 = 1020 \text{ kg/m}^3 \), \( \rho_2 = 1000 \text{ kg/m}^3 \)
  . the coefficients: \( c_1' = 0.46 \), \( p_1 = 1 \)
  . initial blockage: 0.63

The joint data of these test are:
  . the lock: \( l_k \times b_k = 150 \times 16.10 \text{ m}^2 \)
  . the openings: \( \mu = 0.60 \), \( b_h = 6.50 \text{ m} \), \( v_h = 0.0068 \text{ m/s} \), \( A_h = 6.50 \text{ m}^2 \)
  . the cross-sections: \( x_b = 7.50 \text{ m} \) and \( x_s = 102.50 \text{ m} \)
  . the ship: \( l_s \times b_s \times d_s = 95 \times 11.20 \times 2.70 \text{ m}^3 \)
Curriculum Vitae
