Observing the stiffness change of a soil-structure system
by shifts in eigenfrequencies

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Abstract

Monitor (geotechnical) constructions is often based upon displacement measurements. However, these measurements do not offer information about the stiffness behaviour of a soil-structure system. A loss of stiffness might be observed as a decrease of the system’s eigenfrequencies. This research investigates if monitoring of ambient vibrations can be used to observe a change in the system’s stiffness.

Stiffness monitoring of structural parts (e.g. steel and concrete beams) using vibrations is already common. These implementations are based on measuring natural frequencies and mode shapes. Any change in structural stiffness results in a change in these modal characteristics of the structure.

A technique similar to this, but operating in the lower frequency range (i.e. below 300 hertz), is already used to derive the shear elasticity of soil. These techniques are known as seismic methods, and they record body and surface waves. The denser and stiffer the layer of the strata is, the faster it vibrates and the faster the phase velocity of the recorded waves will be. This provides an estimate of the strength of the soil and its ability to resist permanent deformation (i.e. its elastic behaviour). It is also used to find boundaries between different soil layers.

In this research, the possibility of monitoring a relative change in stiffness during construction works is investigated. By a relative change is meant the change in stiffness with respect to the initial stiffness, expressed as a percentage. The initial stiffness will be coupled to the initial eigenfrequency of the system. A changed eigenfrequency can then be coupled to a percentage of this initial stiffness.

The soil-structure system used for the analytical and empirical part of this research is part of a railway bridge in Nijmegen. In Nijmegen, diaphragm walls are constructed to a depth of more than 20 meters, surrounding the old pillars of this bridge. It is assumed that, during construction works, there will be a change in the system’s stiffness due to the installation of the diaphragm walls.

The eigenfrequencies of the soil-structure system are determined by continuous vibration monitoring of ambient vibrations, where the ambient vibrations are caused by the railway traffic.

The formula to calculate the eigenfrequency of a system is:

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \text{ [Hz]} \]

This implies that when the stiffness \( k \) decreases, the eigenfrequency of the system should also decrease.

Two models have been analysed to simulate the bridge: a single and a double mass-spring model. Multiple parameters of these mass-spring models are modified in order to determine which parameter influences the eigenfrequency of both the soil and the structural part of the system.
From the mass-spring model it follows that the dominant frequency in the lower frequency range, between 5 and 15 hertz, represents the eigenfrequency of the soil. The dominant frequency in the higher frequency range, between 40 and 50 hertz, represents the eigenfrequency of the structure. With a changing stiffness of the construction, the eigenfrequency between 40 and 50 hertz changes significantly while the change in eigenfrequency around 10 hertz is insignificant. When the stiffness of the soil decreases, the eigenfrequency around 10 hertz decreases significantly while the eigenfrequency between 40 and 50 hertz remains almost unchanged.

With the mass-spring model it is also concluded that only a change in stiffness relative to the initial stiffness can be monitored.

A Fast Fourier Transform is used to convert the measured data into a frequency spectrum. When there is a phase difference between the first and the last data point a so called leakage occurs. Since it is impossible to determine the phase of the signal when dealing with ambient vibrations, a phase difference cannot be avoided. Due to leakage, the velocities in the frequency spectrum do not correspond well to the real velocities. Actual velocities may be more than 30% higher than the velocities obtained after a Fast Fourier Transform.

With the recorded datasets of both the author, in cooperation with the Municipality of Rotterdam, and Fugro GeoServices B.V. it is investigated if the eigenfrequencies are changing during the construction works. The initial data is compared with the data recorded during and after the construction works.

The dataset recorded by the author contains continuous vibration measurement in three directions, recorded by 10 geophones with a sampling frequency of 1000 hertz. The datasets are recorded on two different days. The first day represents the initial phase. The second day represents the construction phase.

From these measurements it can be concluded that different types of trains do not have an influence on the observed eigenfrequencies. On the other hand, they do have an influence on the magnitude of the recorded vibration.

In between the two days of measurement, hydraulic jacks were installed in between the girders and the pillar of the bridge to correct the settlements that occurred during the construction of the diaphragm walls. These jacks have made the joint in between the girder and the pillar more rigid. Due to this, the girder is acting stiffer than before. From the dataset it can be concluded that the eigenfrequency of the structure increases significantly after installation of the jacks. The frequency peak values representing the pillar and the girder increase. The eigenfrequency of the soil remains almost unchanged after installation of the jacks.

This conclusion is consistent with the results that followed from the analytical model, as described above.
With the dataset recorded by Fugro GeoServices B.V. it is possible to analyse and compare the results of the initial phase, the construction phase and the post phase. For the analysis only the recorded traces are used. These traces contain continuous vibration measurements during 2 seconds, with a sampling frequency of 1024 hertz. The lower frequency range, between 0 and 25 hertz, of multiple monitored traces is analysed and compared. From these results, it can be concluded that a change in stiffness of the soil can be observed by a shift in eigenfrequencies. A decrease in eigenfrequency compared to the initial measured eigenfrequency is observed during the construction works. The decrease is small, but comparable to the decrease that was expected beforehand.

In the post phase, when the construction works are finished, the eigenfrequency increases again. This leads to the conclusion that the stiffness has recovered again.

Observing a change in stiffness of a soil-structure system by shifts in eigenfrequencies is possible, but only a relative stiffness change can be observed (i.e. the change in stiffness with respect to the initial stiffness). It is possible to monitor the shifts in eigenfrequencies by measuring ambient vibrations. It should be noted that with this monitoring system it is not possible to monitor settlements.

The outcome of this research is relevant for stiffness monitoring of constructions. For projects where the deformation of a construction is rather irrelevant if the stiffness is not being influenced significantly, a vibration monitoring system which monitors shifts in eigenfrequency can offer information about the dynamic response (i.e. stiffness behaviour) of the construction.


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\[ a \] = acceleration [mm/s^2]
\[ A \] = amplitude [-]
\[ c \] = damping [Ns/m]
\[ c_{cr} \] = critical damping [Ns/m]
\[ D \] = damping coefficient [-]
\[ df \] = frequency resolution [Hz]
\[ dt \] = time step [s]
\[ E \] = Young's modulus [Pa]
\[ E_k \] = kinetic energy [J]
\[ F \] = force [N]
\[ f \] = frequency [Hz]
\[ F_d \] = damping force [N]
\[ f_e \] = eigenfrequency [Hz]
\[ f_n \] = new frequency [Hz]
\[ f_o \] = old frequency [Hz]
\[ F_p \] = the pulse load acting on the system [N]
\[ f_s \] = sampling frequency [Hz]
\[ F_s \] = spring force [N]
\[ G \] = shear modulus [Pa]
\[ k \] = stiffness / spring constant [N/m]
\[ k_0 \] = initial stiffness [N/mm]
\[ k_1 \] = stiffness / spring constant pillar [N/mm]
\[ k_2 \] = stiffness / spring constant soil [N/mm]
\[ k_{dyn} \] = dynamic stiffness [N/m]
\[ k_{stat} \] = static stiffness [N/m]
\[ l \] = length [m]
\[ m \] = mass [kg]
\[ m_1 \] = mass m1 [kg]
\[ m_2 \] = mass m2 [kg]
\[ N \] = number of analysed data points [-]
\[ p \] = constant, depending upon Poisson's ratio [-]
\[ sens \] = sensitivity of the geophone [Vs/m]
\[ T \] = oscillation time [s]
\[ t \] = time with maximum velocity [s]
\[ T_s \] = total time of data collection [s]
\[ \nu \] = Poisson's ratio [-]
\( v \) = velocity [mm/s]
\( V \) = voltage [V]
\( v_F \) = velocity after a Fast Fourier Transform [mm/s]
\( v_{\text{max}} \) = maximum velocity [mm/s]
\( v_r \) = real velocity calculated with formulas [m/s]
\( V_R \) = velocity of Rayleigh waves [m/s]
\( V_S \) = velocity of shear waves [m/s]
\( v_w \) = velocity of the wave [m/s]
\( x \) = displacement [m]
\( x_0 \) = initial displacement [m]
\( \varepsilon \) = tensile strain [-]
\( \lambda \) = wavelength [m]
\( \rho \) = bulk density of the soil [g/cm\(^3\)]
\( \sigma \) = tensile stress [Pa]
\( \varphi \) = phase [rad]
\( \omega \) = angular frequency [rad/s]
\( \omega_n \) = angular eigenfrequency [rad/s]
\( \zeta \) = damping ratio [-]
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0. Introduction

Monitoring (geotechnical) constructions is often based upon (static) displacement measurements. However, these measurements do not offer information about the stiffness behaviour of a construction. In some cases, for example for dynamically loaded constructions such as bridges, the deformation of a construction is rather irrelevant if the stiffness of the construction is not being influenced significantly. In such cases it would be interesting if another type of measurement could offer information about the dynamic response (i.e. stiffness behaviour) of the construction, additionally to the traditional deformation monitoring.

The dynamic response of a system depends on its stiffness, where the natural frequencies of a structure are indicators for the conditions of a structure. A loss of stiffness can be observed as a decrease of a system’s eigenfrequencies, which creates possibilities for analysing this change of frequency to monitor structural conditions. This is already being done in structural damage detection, where vibration based damage identification is used as a non-destructive testing technique for corrosion damage in reinforced or pre-stressed concrete.

The complete system may consist of different parts, not only the structure but the surrounding soil as well. This research will investigate if it is possible to monitor soil conditions by using vibration measurements. Besides, it will be investigated to what extent the stiffness of the subsoil contributes to the total response of the soil-structure system.

The soil-structure system used for the analytical and empirical part of this research is part of a railway bridge in Nijmegen. The pillars underneath this bridge were built in 1870 and are placed on a raft foundation. One of these pillars and its foundation are the structural part of the system, the surrounding subsoil is the soil part of the system.

The bridge crosses the river Waal and its flood plains. Construction works around the pillars are part of the so-called ‘Room for the River’ project. To prevent future flooding and to protect the inhabitants of the city of Nijmegen and the village Lent against the water, a side channel will be constructed in the flood plains of the river. An excavation of approximately 10 meters will take place to construct this side channel, which will cause undermining of the pillars since the new ground level will be 3 meters beneath the foundation level of the pillars. To preserve the pillars and to ensure stability of the railway bridge, the Municipality of Rotterdam designed a construction of diaphragm walls surrounding the foundation of the pillars. The concept of this box of diaphragm walls is that the stresses in the subsoil surrounding the foundation of the pillars, inside the box, remain intact. Besides this, the stiffness of the soil-structure system as a whole and the stresses in the masonry upper part pillars remain the same and the superstructure of the bridge does not need to be adapted while no traffic obstruction takes place.

To ensure the stability of the soil-structure system mentioned above during installation of the diaphragm walls, the stiffness is an important factor. It is assumed that during construction works
there will be a change in stiffness. As soon as the stiffness of the soil-structure system changes, a change in velocity and frequency of the vibrations measured is expected. Therefore, this project can be used as a test site for the monitoring system with ambient vibrations, where the ambient vibrations in this case are caused by trains.
1. Outline of the research

The aim of this research is to investigate the possibility of monitoring stiffness changes by using vibration measurements.

1.1 Problem statement

The problem statement of this thesis can be formulated as:

*Monitoring systems, such as traditional deformation monitoring, do not give information about the stiffness behaviour of a soil-structure system*

1.2 Main objective

In order to find a solution to this problem, the main objective of this study can be described as:

*Investigating the possibilities of using shifts in eigenfrequencies to observe the changing stiffness of a soil-structure system*

This will be done by both theoretical and analytical analysis, and by coupling this to results obtained by measuring ambient vibrations during construction works.

1.3 Research questions

Three research questions have been defined to optimize the literature review and to achieve the main objective of this research.

1. *To what extent does the stiffness of the subsoil contribute to the total response of the system, and can the frequency components caused by the soil and the frequency components caused by the structural part of the system be distinguished from each other?*

2. *Are there visible changes in the measured frequencies of the system when comparing measurements of the initial state with measurements made during the construction and post phase and if so, do the observed changes correspond to the changes predicted with the analytical model?*

3. *Is it possible to set up a monitoring system that monitors the relative stiffness change of the soil-structure system during construction works and, if so, is the monitoring system that is in operation in Nijmegen able to monitor a stiffness change?*

Answering these research questions has been a guideline for accomplishing this report.
1.4 Scope
Because the main topic of this research was complex and the time span was limited, restrictions were defined for this thesis:

- The analytical mass-spring model, used to model the soil-structure system, is a relatively simple model with only one degree of freedom. In reality the soil-structure system is assumed to be more complex, with multiple degrees of freedom. Unfortunately a more complex model was not available. Developing a complex model was outside the scope of this research.
- The dataset recorded by the author contains vibration measurements made during (1) the initial state and (2) the construction works. Unfortunately there was no time to record vibrations after construction works. Therefore, the post phase has been investigated with the vibration measurements executed on site just by Fugro GeoServices B.V.

1.5 Outline of the research
The research started with a literature study, of which the acquired knowledge is later used for the analytical calculations and measurements in the field. The results of the study are presented in chapter 2.

In chapter 3 an analytical mass-spring model is presented. The construction was modelled as a mass on springs with a spring constant depending on the stiffness of the soil. If it is possible to model the soil-structure system of the pillar, then with the mass-spring system the influence of the soil stiffness in the system can be investigated by changing the spring constant of the model.

Measurements from two different measurement set-ups in Nijmegen were used to compare with the results of the analytical model.

Analysis of the first dataset can be found in chapter 4. This has been recorded by the author during this study, with the highly appreciated assistance of Don Zandbergen. This dataset contains continuous vibration measurements of the full spectrum in three directions (X, Y, Z) for the duration of approximately two hours during: (1) one day before the construction, representing the initial state of the site, and (2) one day during construction. The measurements have been recorded on ten different locations on site; on the girders and pillar of the bridge and in the surrounding subsoil on surface and in depth.

Analysis of the second dataset can be found in chapter 5. This dataset originates from the vibration measurements executed on site by Fugro GeoServices B.V. with the Profound Vibra system. These measurements were required in the contract of the project in Nijmegen, and they are kindly provided by ProRail. These measurements contain continuous vibration measurements in three directions (X, Y, Z) where the highest measured velocity with corresponding frequency is saved every ten minutes. Additionally, the Profound Vibra system records a so-called trace six times per hour. In such a trace,
the full spectrum of the measured signal is recorded during two seconds. The measurements are recorded at one location on site; at the top of the pillar.

In chapter 6, conclusions will be drawn and recommendations will be formulated.

1.6 Possible results
If there is a relation between the analytical model and the measurements, this could be helpful to other projects. It is likely that vibration measurements will show changes sooner than normal displacement measurements do, so the monitoring system might be able to function as an early warning system. With the results from initial vibration measurements an estimation can be made of the frequency of vibrations at critical stiffness values. With monitoring vibrations during construction works, possible changes in frequency can be noticed making it possible to take measures before loss of stability becomes unacceptable. It might be a cost-effective and simple way to monitor and secure the conditions of a soil-structure system during different kinds of future construction works.

In the case of this research, when the measurements show a change in frequency, a relation between the vibrations measured and the stiffness of the subsoil can be found as it is known that the stiffness will change due to the construction of the diaphragm walls. This can be an outcome of the research even if there is no relation between the analytical model and the measurements.

If in the field a different frequency change is measured than predicted with the analytical model, apparently the real soil-structure system reacts differently than with the model is expected. When changes in frequency are larger, in reality the soil reacts less favourable than expected. When changes are smaller, the system reacts more favourable than expected and will remain stiffer.

In any case, the concern is not what the exact value of the stiffness of the system is. It is about the comparison of initial and relative stiffness i.e. the change of the stiffness with respect to the initial stiffness, and whether that is retrievable from ambient vibration measurements.
2. Literature review

This chapter outlines the literature which was used for the theoretical basis of the research.

The literature study starts by looking at structural damage detection systems, where vibration based damage identification is used. Definitions of vibrations and waves will be summarized as well as the equations which are considered to be most important for this research. Next, soil stiffness monitored by vibrations in the form of seismic waves will be covered. Related to this, vibration monitoring with ambient vibrations and the monitoring equipment will be discussed.

Finally, to be able to set up a simple, analytical model in chapter 3, the theory concerning mass-spring models will be outlined.

2.1 Stiffness monitoring in constructions

The idea that vibration measurements can be used to monitor the soil-structure stiffness, is based on similar systems that are already being used in other monitoring fields. Structural conditions, for example corrosion damage in reinforced or pre-stressed concrete, are monitored using methods based on measuring natural frequencies and mode shapes. These non-destructive testing techniques became more important in recent years.

2.1.1 Waves

A (mechanical) wave is a disturbance that travels through a medium. They are everywhere in nature (e.g. sound waves, light waves, water waves). The motion of waves moves energy from one location to another, often with no permanent displacements of particles of the medium. A wave can be transverse, with oscillations perpendicular to the propagation of the wave, or longitudinal, with oscillations parallel to the direction of propagation.

Vehicle-bridge interaction will cause vibrations induced in the construction and subsoil that can be described as sine waves, also known as sinusoids. Sinusoids are defined for all times and distances. In physical situations usually waves occur that exist for a limited span and duration. Fortunately, an arbitrary wave shape can be decomposed into an infinite set of sinusoidal waves by the use of Fourier analysis, as will be shown later in paragraph 2.6.

The most basic form of a wave as a function of time is:

\[ y(t) = A \sin(\omega t + \varphi) \]  

Where

- \( A \) = amplitude [-]
- \( \omega \) = angular frequency [rad/sec]
- \( \varphi \) = phase [rad]

The frequency of a vibration, the number of oscillations each second, can be found by:
\[ f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{\lambda}{v_w} \text{ [Hz]} \]  

Where

\( T \) = period, the time needed for one complete cycle of an oscillation of a wave [s]
\( \lambda \) = wavelength, the distance between two equivalent points [m]
\( v_w \) = velocity of the wave [m/s]

### 2.1.2 Eigenfrequencies and Mode Shapes

The frequency in which a system vibrates with no outside interference is called its natural frequency or eigenfrequency. It represents absolute quantities, provides global information and can be measured accurately. Frequencies at which the (forced) response amplitude is a relative maximum are known as the system's resonant frequencies, or resonance frequencies.

If all the conditions are met, structural stiffness damage can be detected if it causes shifts of eigenfrequencies of at least 1-2%, if the environmental influences are properly taken into account (De Roeck & Reynders, 2007).

The configurations in which a structure will naturally displace are called mode shapes. They can provide useful information about local changes (like joints and support stiffness). But the level of detail will be reduced due to the unfeasibility of measuring the higher modes accurately and due to the lack of absolute scaling of the mode in case of ambient vibrations. Also, often fewer mode shapes are available that can be measured accurately than eigenfrequencies (Papadrakakis et al., 2006).

A mechanical system absorbs more energy than it does at other frequencies when the frequency of its oscillations matches its natural frequency. The natural frequency is the frequency of the first mode. Frequencies of higher modes, at which the response amplitude is a relative maximum, are known as the system's resonant or resonance frequencies, also called harmonics, see Figure 1. Resonance occurs when there is a dynamic (periodic) load with a frequency spectrum containing the frequencies of one or more harmonics of the system, forcing the system to vibrate. It is relatively easy to let a system vibrate at its resonant frequencies and hard to let it vibrate at other frequencies.

A vibrating object will absorb frequencies close to its normal and/or resonance frequencies from a complex excitation like that of a train passage and it will vibrate at those frequencies, essentially 'filtering out' other frequencies that are present in the excitation (Berrett, 2007).

![Mode shapes of a vibrating string](image)

**Figure 1: Mode shapes of a vibrating string**
2.1.3 Methods

Vibration-based damage identification methods are non-destructive and utilise the fact that any change in structural stiffness leads to a change in modal characteristics of the structure. This means that when structural stiffness reduces due to, for example, corrosion, its natural frequencies and mode shapes are also affected. Monitoring these dynamic properties can therefore be used to perform a structural diagnosis. In structural damage detection the measured modal parameters of the damaged structure as well as the original undamaged structure are required. Damages are detected by assessing resonant frequencies that fall outside the mean standard deviations (Adewuyi & Wu, 2009).

Vibration monitoring proved that it can identify the changes in magnitudes, frequency and amplitude of high frequency peaks. These frequencies are better indicators of such degradation rather than low frequency peaks (Torres-Acosta, Fabela-Gallegos, Vazquez-Vega, & Martinez-Madrid, 2003).

An advantage of vibration-based methods is the possibility to evaluate damage at its earliest stage, which will reduce the risk of unexpected structural failures and which will increase the life-span of structures. Also, it increases the overall efficiency of operation. It is therefore beneficial to a cost effective maintenance strategy (Maung, Chen, & Alani, 2013).

2.2 Stiffness monitoring in the subsoil

A technique similar to the previously described technique, but operating in the lower frequency range (i.e. below 300 hertz), is used to derive the shear elasticity of soil. It provides an estimate of the strength of the soil and its ability to resist permanent deformation. With this monitoring technique, the soil is treated as a perfectly elastic, isotropic and homogeneous medium. It is extending downwards to an infinite depth (i.e. semi-infinite case) or it is composed into a number of finite layers overlaying a semi-infinite base (i.e. stratified soil). In reality, soil is not perfectly elastic. But under small vibratory forces, soil behaves elastic and elastic theory can be applied.

Figure 2: Stiffness-strain relation soil
Methods to estimate the stiffness of the soil by recording surface waves, also known as seismic methods, use waves inducing shear strains with an absolute magnitude that is thought to be very small (i.e. less than 0.001%). The stiffness becomes constant and reaches the maximum stiffness. Figure 2 shows the stiffness-strain relation for most soils. The effective elastic moduli do not change with frequency within the range of 15 hertz to 300 hertz (Jones, 1958).

2.2.1 Subsurface and surface methods

There is a growing appreciation of the value of measuring the shear modulus, \( G \), using seismic methods as part of a site investigation. Seismic methods have the advantage of not being affected by sampling disturbance and insertion effects. The two kinds of methods to observe stiffness in the subsoil using seismic waves, are subsurface and surface methods, see Figure 3.

![Figure 3: Seismic methods (Sawangsuriya, 2012)]

Subsurface seismic methods require boreholes, which add cost and time. Surface methods permit the determination of a modulus-depth profile without the aid of boreholes.

Methods using surface waves are little used but promising, and according to multiple papers (Moxhay, Tinsley, & Sutton, 2001) (Matthews, Hope, & Clayton, 1996) they give similar results as subsurface methods. The tests on the surface are performed in situ and are therefore unaffected by disturbed/non-representative samples. Besides, they are non-invasive and the most cost-effective of all direct stiffness measurement methods.

2.2.2 Types of waves that propagate through soil

If a material has the property of elasticity and the particles in a certain region are set in vibratory motion, an elastic wave will be propagated through the material. Considering the subsoil as an elastic medium, two types of elastic waves are produced due to vibrations (see also Figure 4):
- body waves, which propagate into the ground. This type of waves can be divided into two sub types:
  - compressional waves (P-waves), with particle motion in the direction of propagation (longitudinal);
  - shear waves (S-waves), with particle motion which is perpendicular to the direction of propagation.
Shear waves are slower than compressional waves.

- surface waves, which propagate and cause deformations near the ground surface. Surface waves consist of two sub types as well:
  - Rayleigh waves (R-waves), in which the soil particles have displacements at right angles to the surface and also in the direction of propagation of the wave. This type of waves occur when a source produces vibrations with greatest amplitude normal to the surface;
  - Love waves (L-waves), which are basically horizontal polarized shear waves (SH-waves). They have a particle displacement parallel to the surface and transverse to the direction of propagation. They occur when vibrations are produced in a horizontal direction transverse to the line of measurement.

*Figure 4: Waves in the subsoil (Athanasopoulos, Pelekisa, & Anagnostopoulos, 2000)*

In general, body waves are faster than surface waves with P-waves having the highest velocity before S-waves, L-waves and R-waves respectively.

In this research, the vibrations due to trains passing over the railway bridge are induced into the pillars, and by its shallow foundation into the surrounding soil. The vibrations that will be recorded along the surface of the soil will be almost exclusively Rayleigh waves, because Rayleigh waves are produced by a vibrator normal to the structure and travel along the surface of a semi-infinite solid. Shear and compressional waves are radiated into the entire volume of the medium and therefore suffer much larger attenuation (Jones, 1960). Love waves are produced when a source produces
vibrations in a horizontal direction transverse to the line of measurement. In the case of this research, this is not or at least submissive to the amount of Rayleigh waves.

2.2.3 Rayleigh waves

Rayleigh wave particle motion is only found in the vertical plane with no tangential motion (Ketter, 1999). The velocity does not vary with frequency but they are dispersive waves, meaning that the velocity and frequency (i.e. wavelength) of a wave are not independent. Rayleigh wave velocity varies with frequency in a layered medium where there is a variation of stiffness with depth. It is this dispersive behaviour that can be exploited by geotechnical engineers.

Related to the frequency of the vibrations, Rayleigh waves have a wavelength, see Figure 5. High frequency corresponds to a short wavelength, where low frequency corresponds to a long wavelength. Waves with a short wavelength penetrate the shallower zone of the near surface. The Rayleigh waves then travel at speed dependent on the soil properties in the upper layer. Waves with a long wavelength penetrate deeper into the soil, and the velocity of the Rayleigh waves will then be affected principally by the lower layer or by a combination of layers.

A soft impact is generating a frequency spectrum which includes much more lower range frequencies, covering bigger depths (Godlewski & Szczepański, 2012).

![Figure 5: Wavelengths and surface wave velocity (Brown, Diehl, & Nigbor, 2000)](image)

In a layered medium the velocity of Rayleigh waves depends on the frequency of the induced vibrations and the thickness, density and elastic properties of the strata. The denser and stiffer the layer of the strata is, the faster it vibrates and the faster the phase velocity of the Rayleigh waves will be. The dispersive relation between phase velocity and frequency enables the depth to be found at which there is a marked change in elastic properties, such as a transition between a clay layer and a gravel layer. Phase velocity of Rayleigh waves increases with decreasing frequency, and vice versa. For example, when low frequencies give a steep rise in phase velocity, this indicates that below the
upper layer of soil there is a medium of considerable higher rigidity (Jones, 1958). This makes Rayleigh waves a valuable tool for determining the upper crustal structure of a region.

According to elastic theory, the velocity of a Rayleigh wave is a function of, inter alia, the shear modulus of the host medium. In the case of this research, when the stiffness of the soil starts to change or has changed, the velocity of the measured vibrations will also show a change.

![Diagram of Rayleigh wave velocity measurement.](image)

**Figure 6: Determination Rayleigh wave velocity (Rosyidi, Taha, & Nayan, 2004)**

Rayleigh wave velocity can be measured as in Figure 6. In addition, by using the theory of elasticity, shear wave velocity and shear modulus $G$ can be determined from these velocity measurements. The amplitude of the particle motion diminishes exponentially with distance from the free surface. The majority of the wave energy is contained within a zone that extends to a depth of approximately one wavelength. When referring to equation 2.2, for example a velocity of Rayleigh waves of 150 meters per second, found in Figure 5, and a frequency of the vibration of 10 hertz, one wavelength corresponds to a depth of:

$$\lambda = \frac{v}{f} = \frac{150 \text{ [m/s]}}{10 \text{ [Hz]}} = 15 \text{ [m]}$$

The velocity of shear wave propagation is related to the velocity of Rayleigh waves as:

$$V_S = p \cdot V_R \text{ [Pa]}$$  \hspace{1cm} (2.3)

Where

- $V_S = \text{velocity of shear waves [m/s]}$
- $V_R = \text{velocity of Rayleigh waves [m/s]}$
- $p = \text{a constant, depending upon Poisson's ratio (v). For soils v is usually about 0.4/0.45, so p is about 0.95 which indicates that the Rayleigh wave travels about 5% slower than the shear wave (Jones, 1958).}$

The shear modulus is related to $V_S$ as:

$$G = \rho \cdot V_S^2 \text{ [Pa]}$$  \hspace{1cm} (2.4)
Where

\( G \) = shear modulus \([\text{Pa} = \frac{\text{N}}{\text{m}^2}]\)
\( \rho \) = bulk density of the soil \([\text{kg/m}^3]\)

Which leads to:

\[ G = \rho \cdot p^2 \cdot V_R^2 \, [\text{Pa}] \]  \hspace{1cm} (2.5)

This will give a straightforward conversion from a Rayleigh wave velocity–depth profile to a stiffness–depth profile (Matthews, Hope, & Clayton, 1996).

Another measure of the stiffness of an elastic material and a quantity used to characterize materials is the Young's modulus, also known as the tensile modulus or elastic modulus (Verruijt & van Baars, 2005).

\[ E = \frac{\sigma}{\epsilon} \, [\text{Pa}] \]  \hspace{1cm} (2.6)

Where
\( \sigma \) = tensile stress \([\text{Pa}]\)
\( \epsilon \) = tensile strain \([-\]]

When considering the soil as a homogeneous, isotropic material a simple relation between the elastic constants can be made. Combined with the formula for the shear modulus \( G \), this will result in:

\[ E = 2 \cdot G \cdot (1 + v) = 2 \cdot \rho \cdot V_S^2 \cdot (1 + v) = 2 \cdot \rho \cdot V_R^2 \cdot (1 + v) \, [\text{Pa}] \]  \hspace{1cm} (2.7)

In the case of this research, finding the boundary between layers is not important. The stiffness of the upper layers as a whole is assumed to change. Recording the phase velocity of waves in these upper layers, a change in velocity can indicate a change in stiffness when frequencies remain the same. An increase in wave velocity \( V_R \) indicates an increase in stiffness (i.e. shear modulus \( G \)), while a decrease in wave velocity indicates a decrease in stiffness.

In the near fields of a source, the Rayleigh waves are not yet well developed. They are developed at about two wavelengths from a point source.

In general, reliable data was obtained to a depth of 8 meters in heavily overconsolidated soils, and 20 meters in weak rock, with frequencies greater than 6 hertz (Matthews, Hope, & Clayton, 1996). This is applicable to the Waal bridge.

The phase velocity of vibrations travelling along the surface provides an estimate of the dynamic shear elasticity of the soil. However, if there are considerable local variations in the soil properties along the line of measurement, no values are obtained for the extreme values of the shear modulus. For this research this is not a problem, since initial values of the upper soil layers as a whole will be used to compare with values recorded during the construction phase.
2.2.4 Ambient vibrations as an input source

In this research, the recorded Rayleigh waves are caused by trains passing the railway bridge. The trains cause repeating, transient vibrations, and they are assumed to introduce a continuous spectrum of waves (i.e. a spectrum with all wavelengths and therefore frequencies). As mentioned earlier, the easiest way for a construction to vibrate is in its eigenfrequency or in one of the harmonics of its eigenfrequency (i.e. $n$ times the eigenfrequency). This means that the vibration spectrum of the pillar will be a line spectrum; a spectrum consisting of its fundamental frequency and its harmonics (Rosyidi, Taha, & Nayan, 2004).

Ambient Vibration Monitoring (AVM) makes use of the natural vibrations of a structure when it is in service or due to wind loads (De Roeck & Reynders, 2007). Natural vibrations are for example vibrations caused by traffic. These vibrations can be used for periodic vibration monitoring as well as for continuous vibration monitoring when robust equipment is used.

One of the big advantages of AVM is that no traffic obstruction takes place, which will reduce the costs of the test and the impact on the economy will be low as there is no need to close the bridge. Besides, the conditions of the measurements are real operating conditions, and therefore the levels of the acting forces and vibrations are real. Furthermore, AVM requires a short set up time, and there will be no destruction of the structure.

Disadvantages of AVM could be that monitoring needs to be done during a longer period of time. Therefore more data will be acquired which leads to higher computational costs.

Because of different vibration levels during one test, it is recommended to use sensors with a high sensitivity and resolution (i.e. a very low internal noise level and a high A/D conversion rate) of at least 24 bits (De Roeck & Reynders, 2007). The minimum scanning or sampling rate has to be the maximum identified target frequency in fivefold. This means that when the highest noticeable eigenfrequency is 10 hertz, the minimum scanning rate should be at least 50 hertz. A scanning rate of 100 hertz has proved appropriate for registering individual events (Wenzel & Pichler, 2005).

The monitoring system used in this research has to monitor only the stiffness, where the strength is no issue. The exact stiffness is not important either, but rather the relative change in stiffness is. An approximation of the vibration frequencies introduced due to the train traffic can be made by placing a geophone on top of the bridge/pillar.

When monitoring ambient vibrations there are several things that have to be taken into account in order to create a useful dataset of the recorded vibrations. There are several environmental conditions that can have a significant influence on the eigenfrequencies of a structure and therefore on the outcome of the ambient vibration measurements. When effects of the environment are not removed, eigenfrequencies cannot be used in order to detect damage. In this case mode shapes and peaks from modal filters could be used.

Temperature is one of the environmental conditions that has to be taken into account. A temperature change affects the global material properties which has a uniform influence on the
eigenfrequency. Normal environmental changes in temperature can cause changes in eigenfrequencies of as much as 3-4% during the year. The relation between eigenfrequencies and temperatures can be obtained by taking measurements during a period of at least one year. The first eigenfrequency can vary 5% during a 24 hour period, due to for example different temperatures at night and during the day. The influence of these different temperatures can be studied by making a 24 hour measurement.

Because of the relatively short period of this research these long term measurements are not possible, however changes are expected to be really small and therefore irrelevant. Besides, in this research it is not about numbers, it is about relative change.

Besides temperature, the ground water table has to be taken into account which is possible with the aid of the ground water table. Previous tests have shown that, in soil, Rayleigh wave velocity and shear modulus decreased as the soil became wetter, and increased as the soil dried. It is obvious that also the strength/stiffness of the soil decreased and increased during the same period (Baker, Steeples, & Schmeissner, 2002).

As mentioned earlier, the media (i.e. the soil) is assumed to be uniform and isotropic. In reality, this is not the case, but anisotropy is likely to be small in relation to other imperfections (Stoneley, 1948). Also, the soil is not perfectly elastic. But, as mentioned before, under small vibratory forces soil behaves elastic. The effective elastic moduli do not change with an input force having a frequency within the range of 15 to 300 hertz (Jones, 1958).
No relation is found between the eigenfrequencies of a system and the wind, rainfall and humidity of the environment (Peeters & De Roeck, 2001).

### 2.3 The railway bridge

The railway bridge in Nijmegen is placed on a shallow foundation. This type of foundation is generally sensitive to vibrations, except on really strong compacted sands.

The railway bridge was built more than 100 years ago. The big load of the concrete construction of the bridge and the passing cargo trains in the past, compacted the sand as the years passed. Results of cone penetration tests show that the sand underneath and around the foundation of the bridge is heavily compacted. Theoretically, the foundation would not be sensitive to vibrations. Therefore, initially, the velocity of the measured vibrations is assumed to be small.

The sand below and around the pillar will be less compacted during the construction of the diaphragm walls due to, for example, excavations. Because of this, the structure will be more sensitive to vibrations during the construction phase. It is assumed that the measured vibrations will then be larger.

#### 2.3.1 Supports

There are different kinds of support to place the girders of a bridge on its pillars.
When the support is able to move in one direction and is free to rotate, this is called a roller support. The only reaction of the roller support is normal to the surface on which the roller rolls without friction.

At the hinged support the beam does not move either along or normal to its axis. The beam, however, may rotate. The total support reaction has a horizontal and vertical component. Since the beam is free to rotate, no resisting moment will exist.

At the fixed support, the beam is not free to rotate or slide along the length of the beam or in the direction normal to the beam. Therefore, there are three reaction components; a vertical one, a horizontal one and a moment. A fixed support is also known as a built-in support.

![Figure 7: Types of support](image)

It is assumed that there will be more force due to the weight of the train than due to braking or accelerating. This means that there will be more input force acting in the vertical plane than in the horizontal plane.

In the case of the railway bridge in Nijmegen, rubber blocks are placed in between the girders and the pillars. In this way, the vertical forces on the bridge are evenly divided to each pillar.

The horizontal forces however are being transported to pillar 2, see Figure 8. At the support of pillar 2 steel pins are placed in the vertical direction of the support to create more of a hinged support. Also, the girders of the bridge are divided in such a way that ¾ of the girders is transporting its horizontal force to pillar 2, and only ¼ is being transported to pillars 1 and 3.

![Figure 8: Pillar 1, 2 and 3 respectively](image)
2.4 Mass-spring-damper system

A model that can be used to investigate the problem in an analytical way, is a mass-spring-damper system. A mass-spring-damper system is a good way to model the vibrations of a construction depending the stiffness of a soil. It is a linear system, with a single degree or multiple degrees of freedom. It consists of a vibrating mass \( m \) [kg], a spring with spring constant \( k \) [N/m] and a damper with damping \( c \) [Ns/m].

2.4.1 Determining the eigenfrequency

From a mass-spring-damper system its eigenfrequency and its damping ratio can be determined;

\[
\omega_n = \sqrt{\frac{k}{m}} \ [rad/s]
\]  
\[ \zeta = \frac{c}{2\sqrt{km}} \]  

Where
\( \omega_n \) = eigenfrequency [rad/s]
\( \zeta \) = damping ratio [-]
\( k \) = stiffness [N/m]
\( m \) = mass [kg]
\( c \) = damping [Ns/m]

From the radian frequency \( \omega_n \) the eigenfrequency \( f_n \) (i.e. the natural frequency), can be found by dividing \( \omega_n \) by \( 2\pi \). This gives the following equation for \( f_n \):

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \ [Hz]
\]

Where
\( k \) = spring constant of the soil = \( \frac{E}{\text{A1}} \) [N/m]
\( m \) = mass of the structure [kg], where the construction is assumed to be a rigid element which means that elasticity will be neglected (Becedas, Mamani, Feliu-Batlle, & Sira-Ramírez, 2007).

As can be seen from equation 2.10, the natural frequency depends on two system properties: mass and stiffness. Since damping is not in the equation, it has no influence on the value for the eigenfrequency. Damping only influences the amplitude of oscillations. Therefore, for a first estimation of the eigenfrequency, it can be assumed that the system is undamped. Instead of a mass-spring-damper system, a mass-spring system can be used.

A mass-spring system is described in equations 2.11 and 2.12, where equation 2.11 contains the oscillatory force and equation 2.12 contains Newton’s law of motion.
Where
\[ a = \text{acceleration of the mass } [\text{m/s}^2] \]
\[ x = \text{displacement of the mass } [\text{m}] \]
Combining these equations gives the formula for the total forces acting on the system:
\[ F = ma + kx = m\ddot{x} + kx \]  

2.4.2 Dynamic behaviour of the soil-structure system

The dynamic behaviour of a mass-spring system is dominantly influenced by the bearing material. The stiffness and the frequency and amplitude of excitation are the most important dynamic characteristics.

The static stiffness \( k_{\text{stat}} \) is responsible for the deflection of the MS system under dead and live loads, whereas the dynamic stiffness \( k_{\text{dyn}} \) is the key-parameter for the insertion loss (i.e. damping). Therefore, \( k_{\text{dyn}} \) is responsible for the vibration attenuation (Geier & Wenzel, 2003). In this case, it is the dynamic stiffness \( k_{\text{dyn}} \) which will be used for modelling the soil stiffness, because of the dynamic input loads of the trains.

Changes of the dynamic behaviour of the bearing material over time should be considered. In this case the railway bridge has been constructed in 1875, so trains have been running over it for more than 100 years. Besides, in the past there were even more heavy trains than nowadays. Therefore, it may be assumed that the soil is at its most compact state and that the dynamic behaviour will not change if the conditions stay unchanged; long time stability is ensured for the relevant applied load combinations.

The soil will become less compact and in response less stiff due to the excavations of the diaphragm walls. In the MS system this can be modelled as a decreasing value of the spring constant \( k \). When settlements or deflections are calculated, static values will lead to the lower limit of the natural frequencies, as the dynamic stiffness will lead to the upper limit. The measured values have to be situated between these two limits.

Frequency as well as velocity of ground transmitted vibrations depend on the weight and the velocity of the train as well as on the quality (i.e. roughness) of the wheel surface (Geier & Wenzel, 2003). When these vibrations are used for monitoring, a differentiation for different train types has to be made.

2.5 Monitoring equipment

Geophones will be used to monitor the ambient vibrations in the pillar and the surrounding subsoil. A geophone sensor exists of a coil and a magnet. When the sensor is moving, the magnet stays in place
due to inertia, and the coil moves with respect to the magnet. In that way it generates voltage [V] which is proportional to the velocity [mm/s] of the movement of the sensor.

2.5.1 Sensitivity
The velocity of the vibrations measured can be calculated with the sensitivity of the geophone, which can be found on the calibration certificate:

\[
\frac{V}{\text{sens}} \times 1000 = v
\]

Where

\[ V = \text{voltage [V]} \]
\[ \text{sens} = \text{sensitivity of the geophone [Vs/m]} \]
\[ v = \text{velocity [mm/s]} \]

2.5.2 Axilog
An Axilog sensor contains 3 geophones that measure the particle velocity in the X and Y-direction in the horizontal plane, and in the Z-direction in the vertical plane. The sensors have a high nominal sensitivity; they measure vibrations up to 100 hertz, with a resolution of 1 hertz (i.e. one sample every second). The sensor can be mounted on the wall or placed on the floor or ground and is watertight.

The natural frequency of the geophone should be less than the smallest input frequency, which limits their usefulness at low frequency input (Matthews, Hope, & Clayton, 1996).

2.5.3 RoDo-system
The RoDo-system is a spectrum analyser. Like the Axilog, it measures vibrations in three different directions, however, it measures vibrations up to 500 hertz with a resolution of 1000 hertz (i.e. one sample every 1/1000 of a second). The system exists of a maximum of 12 geophones, where each geophone can measure in three different directions X, Y and Z. From spectral data the phase difference between the signals at each geophone and the coherence of the cross-correlated signals can be determined. Because of the high resolution, the RoDo-system will be used for the measurements in Nijmegen performed by the author.

2.6 Fast Fourier Transform
As mentioned earlier, vibrations consist of a combination of multiple basic waves in the form of sine and cosine functions, see 2.1.1. The Fourier Transform describes a mathematical technique to split a vibration signal into all its ground frequencies. The Fourier Transform changes the domain of the function from the time domain, see Graph 1, to the frequency domain. The frequency domain shows the intensity of all frequencies as shown in Graph 2. This graph is called a frequency spectrum. In this spectrum, the y-axis contains the intensity in Volts, the same unit as with the input signal on the y-axis. The voltage can be easily converted into a velocity in millimetres per second by the sensitivity factor of the equipment. The x-axis contains the frequency in hertz. The width of the spectrum
depends on the amount of samples per second. According to the Nyquist criterion, reliable information about frequencies is only obtained for frequencies less than half the sampling frequency (Nyquist, 1928). This means that with a sampling rate of 1000 hertz like the RoDo-system, the spectrum reaches to 500 hertz.

Graph 1: Time domain

Graph 2: Frequency domain

The equipment measures vibrations in 3 different directions, whereby the intensity of the vibrations is recorded as millivolts in time. The graph of these results shows a function $f(t)$.

Every function, also this function $f(t)$, can be reproduced as a combination of different sine and cosine functions of the form $A \cos(\omega t) + j B \sin(\omega t)$, with which function $f(t)$ can be multiplied (van Dam, 1998). The integral over the function, which follows after multiplication, provides information on the periodicity of the original $f(t)$. When the integral of a frequency $\omega$ gets close to 0, this particular frequency is not in the spectrum. When the integral is greater than 0, the frequency is present. In fact the signal is being unravelled into a summation of multiple sine and cosine functions with different frequencies $\omega$ and amplitudes $A$ and $B$. The summation of all possible combinations of sine and cosine waves with the right amplitudes gives (an approach of) $f(t)$. The amplitudes $A$ and $B$, corresponding to the different frequencies $\omega$, show how much of that frequency $\omega$ is present in the function $f(t)$.

The Fast Fourier Transform is an efficient form of the Discrete Fourier Transform. It needs less calculation time and less computer memory. But because it is a finite method it is an approach of the
function $f(t)$. The better the approach, the more sine and cosine functions thus the more calculation time and memory are needed.

When a Fast Fourier Transform is performed, it is assumed that the last data point is identical to the first data point. However, most of the times the fundamental period of the signal is not known. Sampling of the signal may stop at a different phase than it started. This will cause that the last data point is not identical to the first point. This phase difference causes inaccuracies in the amplitude of the frequencies when a Fast Fourier Transform is performed over the data (Cimbala, 2010).

For most Fast Fourier Transform analysis, the computer algorithm restricts the amount of data points to a power of 2. For example Microsoft Excel. For Matlab, this restriction does not apply.

After the Fast Fourier Transform is performed, there is more information available about the build-up of the signal. It is known which frequencies are present in the signal, and which of those frequencies predominate. Also specific frequencies, like noise (i.e. 50 hertz), can be filtered out of the signal easily.
3. **Analytical model**

In this chapter an analytical model in the form of a mass-spring model will be presented. The construction will be modelled as a mass on springs, with a spring constant depending on the stiffness of the soil. Initially, damping will be neglected since damping does not have an influence on the eigenfrequency of a system, see paragraph 2.4.

A model with only one degree of freedom, displacement in the Z-direction, will be chosen for the first estimation. This model is chosen because the largest input force acting on pillar 1 acts in the Z direction, as discussed earlier in paragraph 2.3.1.

3.1 **Single-degree-of-freedom one-mass-spring model**

The model used to provide a first estimation of the changes of frequencies and velocities is a one-mass-spring model with one degree of freedom, as shown in Figure 9.

A mass-spring system is a good way to model vibrations of a construction depending the stiffness of a soil, as discussed in paragraph 2.4. The chosen model is a very simple linear, single-degree-of-freedom system consisting of a vibrating mass \( m \) [kg] and a spring with spring constant \( k \) [N/m].

In this model, the mass represents the total mass of the structure. It consists of the mass of the girders resting on one pillar, the mass of the concrete block on top of the pillar, the mass of the masonry part of the pillar and the mass of the foundation block. A drawing of the pillar can be found in appendix A. The spring represents the soil, where the spring constant is the bedding constant of the soil.

The mass-spring system can be described by equations 2.11 – 2.13.

3.1.1 **Vibrating mass \( m \)**

As said before, the vibrating mass \( m \) contains all dead loads. Besides, the mass due to slow traffic on the slow traffic bridge is included. The mass of a passing train, on the other hand, is included in the dynamic force \( F \) acting on the system.
<table>
<thead>
<tr>
<th></th>
<th>20.000</th>
<th>kN</th>
<th>2.000.000</th>
<th>Kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Railway bridge</td>
<td>1.200</td>
<td>kN</td>
<td>120.000</td>
<td>Kg</td>
</tr>
<tr>
<td>Slow traffic</td>
<td>1.600</td>
<td>kN</td>
<td>160.000</td>
<td>Kg</td>
</tr>
<tr>
<td>Slow traffic bridge</td>
<td>7.200</td>
<td>kN</td>
<td>720.000</td>
<td>Kg</td>
</tr>
<tr>
<td>Soil</td>
<td>8.000</td>
<td>kN</td>
<td>800.000</td>
<td>Kg</td>
</tr>
<tr>
<td>Concrete block on pillar</td>
<td>3.000</td>
<td>kN</td>
<td>300.000</td>
<td>Kg</td>
</tr>
<tr>
<td>Footing</td>
<td>14.000</td>
<td>kN</td>
<td>1.400.000</td>
<td>Kg</td>
</tr>
<tr>
<td><strong>Total mass m</strong></td>
<td><strong>55.000</strong></td>
<td><strong>kN</strong></td>
<td><strong>5.500.000</strong></td>
<td><strong>Kg</strong></td>
</tr>
</tbody>
</table>

Table 1: Total mass m

### 3.1.2 Spring constant k

The spring constant $k$ for the initial situation is calculated using the outcome of Plaxis calculations made by the Municipality of Rotterdam in the pre-phase of the project in Nijmegen. More information about the calculation can be found in the internal report ‘Waalbrug Nijmegen – referentieontwerp’. In these calculations, a train with a load corresponding to the maximum dynamic force is placed on top of the soil-structure system as a dead load. The maximum dynamic force amounts to 14.000 kN. This force is caused by a freight train which is suddenly put to a halt on one track, while on the other track a freight train is accelerating at full power.

After 10.000 days (i.e. 30 years) the settlement is calculated to be 8 millimetres. Soil reacts 5 to 10 times as stiff when subject to dynamic loads in comparison to being subject to static loads, see also Figure 2. From site investigations in Nijmegen it is known that the soil surrounding the pillar was heavily compacted. Therefore, a dynamic stiffness 10 times as large as the static stiffness is chosen. This can be seen as a settlement of 0,8 millimetres.

By knowing the settlement and the dynamic force, this gives a spring constant $k$ of:

$$
\frac{F}{\Delta L} = \frac{1400000 \times N}{0.008 \text{ m}} = 1.75 \cdot 10^{10} \text{ [N/m]}
$$

### 3.1.3 Dynamic input force

Ambient vibrations caused by trains will be used as a source for the vibration measurements. To model these input vibrations, an input force derived from a real passage of a train is used. This input force is provided by Paul Hölscher of Deltares and derived from the CUR Report ‘Eindrapport Trillingshinder’ (CUR, 2000). The model used can be found in the CUR report section about the emission of railway traffic.

The force is composed out of small input forces that change every 0,005 seconds (i.e. 200 times per second) in order to simulate a real train passage. The input force contains eight wheel axes (i.e. two wagons) from a train passing one particular point. The summation of these forces is shown in Graph 3. It shows a logical pattern of a train passage, since forces add up until approximately half of the train passed. Afterwards the forces decrease. A passage of two wagons causes noticeable vibrations for about 6 seconds.
3.2 The one-mass-spring model in Excel

The single-degree-of-freedom one-mass-spring model is constructed in Excel. More details of the model can be found in appendix B. It is expected that the frequency of the mass-spring system will be in the lower frequency range, since heavy structures have a relatively low eigenfrequency. In addition, the eigenfrequency of soils is assumed to be in the lower frequency range, around 5-15 hertz.

3.2.1 Results

The model gives an approximation of the velocity of the vibrations. To convert this velocity into a frequency spectrum, a Fast Fourier Transform analysis is performed. With the Analysis ToolPack of Excel an FFT analysis can be made. More details about this FFT Analysis can be found in appendix C.

The frequency-velocity graph of the initial situation is shown in Graph 4 and Graph 5. In both graphs, the frequency in hertz is plotted on the x-axis and the corresponding magnitudes are plotted on the y-axis.
The peak in the graph is assumed to be the eigenfrequency of the mass-spring system. As can be seen, this frequency is in the lower frequency area, as assumed. It amounts to 8.98 hertz.

### 3.3 Changing parameters of the one-mass-spring model

The following parameters will be adapted to check whether or not a parameter is contributing to a shift of the peak frequency:

- The mass of the structure (i.e. the mass \( m \))
- The stiffness of the soil (i.e. the spring constant \( k \))
- The dynamic input force \( F \)
- The damping to the system (i.e. the damping constant \( c \))

#### 3.3.1 Mass of the structure \( m \)

According to formula 2.10, the eigenfrequency of a system has to go up when the mass of the system goes down. In this section the mass of the structure is decreased with a value of 50% of the initial mass to check its contribution in the above described mass-spring system. The frequency spectrum is shown in Graph 6. On the x-axis is plotted the frequency in hertz, and on the y-axis are plotted the corresponding magnitudes (i.e. velocities in millimetres per second). Indeed, as can be seen, the peak frequency goes up when the mass goes down. It now amounts to 12.74 hertz.

The amount of increase of the frequency with a decreasing mass of 50% is therefore:

\[
\text{Frequency increase} = \left( \frac{f_n - f_o}{f_o} \right) \times 100\% = \left( \frac{12.74 - 8.98}{8.98} \right) = +41.9\%
\]

With

\( f_n = \) new frequency
\( f_o = \) old frequency
Graph 6: Frequency spectrum with a decreased mass

Graph 7 shows the progress of the peak frequency when the mass decreases from 100% to 10% of the initial mass. On the x-axis, the mass as a percentage of the initial mass is plotted. On the y-axis to the left, the corresponding eigenfrequency of the system in hertz is plotted. On the y-axis to the right, the magnitude of the eigenfrequency in millimetres per second is plotted to show why this magnitude is not taken into account.

Graph 7: Progress of the eigenfrequency and velocity with a decreasing mass

As can be seen from the graph, the development of the eigenfrequency shows a logical pattern. According to formula 2.10, the eigenfrequency develops exponentially when the mass changes.

However, the magnitude of the peak frequency (i.e. the velocity) does not show a logical progress. It seems to be a random value for each frequency. The velocity should develop exponentially as well as will be further discussed in paragraph 3.4. An assumption that can be made is that the magnitudes after the Fast Fourier Transform do not correspond with the real magnitudes, since the transform is an approach to reality. This can be caused by the 200 hertz input signal, which might be of too low a quality.
### 3.3.2 Stiffness of the soil $k$

According to formula 2.10, the eigenfrequency of a system has to decrease when the stiffness of the system decreases. In this section the stiffness of the structure is decreased with a value of 50% of the initial stiffness to check its contribution to the mass-spring system. Graph 8 shows the corresponding frequency spectrum. On the x-axis the frequency in hertz is plotted, and on the y-axis the corresponding magnitudes (i.e. velocities in millimetres per second) are plotted.

Indeed, as can be seen, the peak frequency goes down when the stiffness goes down.

![Frequency spectrum with decreased stiffness](image)

**Graph 8: Frequency spectrum with a decreased stiffness**

The decrease in frequency, with a decreasing stiffness of 50%, is:

$$\text{Frequency decrease} = \frac{(f_n - f_o)}{f_o} = \frac{(6.35 - 8.98)}{8.98} = -29.3\%$$

A decreasing stiffness has a smaller influence on the eigenfrequency in comparison with a decreasing mass. This is due to the square root in formula 2.10. A decrease in the mass of the structure (i.e. a decrease of the denominator) has a larger influence on the shift of eigenfrequency than a decrease in stiffness of the soil (i.e. a decrease of the numerator).

The peak frequency of the system is also observed with a decreasing stiffness from 100% to 10% of the initial stiffness of the soil. The result is shown in Graph 9, with on the x-axis the decrease of the stiffness as a percentage of the initial stiffness. On the y-axis to the left, the corresponding peak frequency of the system is plotted in hertz. On the y-axis to the right, the magnitude of the frequency (i.e. velocity in millimetres per second) is plotted.
Like in paragraph 3.3.1 the progress of the eigenfrequency of the system shows a logical progress since the eigenfrequency should decrease exponentially, according to formula 2.10.

The corresponding magnitudes show again random values, also different from the values with a decreasing mass. The assumption, that the Fast Fourier Transform influences the magnitudes of the peaks, seems to be right. This will be further investigated in paragraph 3.4, as mentioned before.

### 3.3.3 Dynamic input force $F$

The weight and speed of a train are important factors for the value of the dynamic input force $F$. These factors can vary according to the type of train and its driving direction. In this section, the dynamic input force $F$ is multiplied by an amount of 50% of the initial force to investigate its influence. The corresponding frequency spectrum is shown in Graph 10, where the frequency in hertz is plotted on the x-axis and the corresponding magnitudes are plotted on the y-axis.
The peak frequency of the system is still 8.98 hertz, which means the eigenfrequency of the system did not change. However, the corresponding magnitude (i.e. the velocity) is significantly higher. This is a logical result referring to kinetic energy. When the input energy is higher, the total energy in the system is higher as well. Since the mass of the system remains unchanged, the velocity has to go up when there is only kinetic energy present in the system (i.e. when the mass passes its equilibrium position), see formula 3.2.

\[ E_k = \frac{1}{2} \cdot m \cdot v^2 \ [J] \]  
\[ (3.2) \]

### 3.3.4 Damping c

Damping is the capacity of soils to absorb energy during cyclic and dynamic loading. It can be viscous (i.e. frequency dependent) or hysteretic (i.e. frequency independent). It is assumed that damping is hysteretic in nature.

Damping of soils is rarely measured. In soils, damping can consist of both material damping and geometric damping. Material damping occurs when the energy is dissipated by deformation of the soil. Geometric damping occurs when the energy is radiated into the surrounding soil. The total damping of the soil can be derived by comparing the amplitude of a wave at one point, and at another point further away from the source.

Damping in soils is defined as:

\[ D = \frac{c_{cr}}{2\sqrt{m/k}} \]  
\[ (3.3) \]

Where

- D = damping coefficient [-]
- \( c_{cr} \) = critical damping [Ns/m]

The formula for the damping force \( F_d \) acting on the mass-spring-damper system is expressed by:

\[ F_d = -c \cdot v = -c \cdot \dot{x} = -c \frac{dx}{dt} \ [N] \]  
\[ (3.4) \]

Where

- c = damping [Ns/m]
- v = velocity of the mass [m/s]

Combining equation 3.3 with equations 2.11 and 2.12 gives:

\[ F = ma + cv + kx = m\ddot{x} + c\dot{x} + kx \ [N] \]  
\[ (3.5) \]

The mass-spring-damper system will look like Figure 10.
It is very difficult to predict the damping of soil. Primarily, damping is affected by strain level and creep. When the shear strains are small, the damping ratio is also small. No significant effects on damping were observed when looking at stress level or frequency of loading (Thandava Murthy, 1990). The damping ratio used for the model in this research is derived from the graph shown in Graph 11.

Graph 11: Damping ratio of soils (Zhang & Aggour, 1996)

With small strains (i.e. smaller than 0.001) the damping ratio is assumed to be around 1 to 3%. The lower limit of the damping ratio is chosen, since the soil of the soil-structure system considered in this research is very stiff. Graph 12 shows the frequency spectrum with 1% damping ratio. On the x-axis the frequency in hertz is plotted, and on the y-axis the corresponding magnitudes are plotted. To compare different frequency spectra with damping, also the spectrum with 0.1% damping ratio is shown in Graph 13.

With a 0.1% damping ratio, the frequency spectrum is the same as the frequency spectrum without damping shown in Graph 5, however the magnitude of the peak frequency is lower. A 1% damping ratio gives a slightly lower dominant frequency, namely 8.59 instead of 8.98 hertz. This 8.59 hertz was the second highest frequency peak when looking at the frequency spectrum without damping. Although this frequency now has the highest magnitude, it is not assumed to be the eigenfrequency of the system. It can be seen that, due to damping, the only magnitude that changes is the magnitude corresponding with a frequency of 8.98 hertz, when comparing Graph 12 with Graph 13. The other peaks have the same magnitudes, with or without damping. This means they are not
affected by the changing damping parameter. Therefore, it is assumed that these frequencies are not related to the eigenfrequency of the system.

![Graph 12: Frequency spectrum with damping ratio 1%](image1)

![Graph 13: Frequency spectrum with damping ratio 0.1%](image2)

Since the value of the eigenfrequency of the system is not affected by the added damping, the assumption to use an undamped system as a first estimation seems correct. From Graph 12 and Graph 13 it can be concluded that when damping is relatively small, the eigenfrequency of the system stands out more. Therefore, it is even more effective to use a system without damping to find the value of the eigenfrequency of the system, when the corresponding magnitude is of no importance.

### 3.4 Corresponding magnitude of the eigenfrequency

According to paragraphs 3.3.1 and 3.3.2, the corresponding magnitude (i.e. the velocity) of the eigenfrequency does not seem to have a logical progress after a Fast Fourier Transform has been performed. According to formulas, the velocity change should show an exponential increase when stiffness is decreasing. In this paragraph this phenomenon is further investigated.
The mass-spring model remains unchanged. The input force is taken differently, since a pulse load is more suitable for this part of the research. Besides, the irregular dynamic input force may be the cause of the odd velocity outcome. A pulse load of 2.000.000 Newton, derived from the mass of a long and heavy intercity train, is used as an input load. With this input load the initial displacement $x_0$ is calculated by using formula 3.6. Removing the pulse load causes a pure sine wave vibration of the mass. The formulas 3.6 till 3.9, shown below, are used for the derivation of the maximum velocity of the mass during the sinusoidal vibration, shown in formula 3.10. The velocity depends on the stiffness of the spring and the mass, even as on the eigenfrequency.

Formula 3.6 calculates the force acting on the system, which is equal to the displacement of the mass times the stiffness of the spring (i.e. the stiffness of the soil). Since the force and the spring constant are known, the displacement $x_0$ can be calculated.

Formula 3.7 calculates the time required to complete one oscillation. The mass and spring constant of the system are known.

Formula 3.8 calculates the frequency of the vibration of the systems in radians per second. This frequency also depends on the mass and stiffness of the system.

The oscillation of the system can be described with a cosine function as shown in formula 3.9. The time is necessary at which the system has its maximum velocity (i.e. when the mass passes its equilibrium position). This particular time, indicated with ‘t’ can be calculated with formula 3.9, with a displacement equal to zero. This results in $\frac{T}{4}$ or $\frac{3T}{4}$. Also, since a cosine function is zero when the cosine is $\frac{\pi}{2}$ or $1\frac{1}{2}\pi$, $\omega$ times $t$ can be taken $\frac{\pi}{2}$ or $1\frac{1}{2}\pi$.

Formulas 3.6 till 3.9 can be combined to calculate the maximum velocity of the system on the eigenfrequency. As can be seen in formula 3.10, the frequency $\omega$ of the system is in the formula of the velocity. This means that the velocity of the system is dependent on the eigenfrequency of the system, and not the other way around.

$$F_p = -k \cdot x_0$$ (3.6)

$$T = 2\pi \sqrt{\frac{m}{k}}$$ (3.7)

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$ (3.8)

$$x = x_0 \cdot \cos(\omega \cdot t)$$ (3.9)

$$v = \frac{dx}{dt} = -x_0 \cdot \omega \cdot \sin(\omega \cdot t)$$ (3.10)
Where
\( F_p \) = the pulse load acting on the system [N]
\( k \) = stiffness of the system [N/m]
\( x_0 \) = initial displacement of the mass [m]
\( x \) = displacement of the mass [m]
\( \omega \) = frequency [rad/s]
\( T \) = oscillation time [s]
\( t \) = time with maximum velocity [s]

The calculation can be seen in appendix D. For example, an input force of 2.000.000 Newton causes a displacement \( x_0 \) of 0.1 millimetres with the initial stiffness \( k \). As a response, after removing the load, the mass will start vibrating as a pure sine wave. The maximum velocity is 6.45 millimetres per second.

### 3.4.1 Varying stiffness \( k \)

In this section, the stiffness \( k \) has decreased from 100 to 10%. This is done to investigate the progress of the eigenfrequency and its corresponding velocity according to the above described formulas 3.6 till 3.10. Again, it concerns the velocities calculated with a pulse force instead of a dynamic force. This means the graphs shown in this section differ from the graphs shown earlier in paragraphs 3.2 and 3.3.

As an input for the mass-spring system, the initial displacement caused by a force of 2.000.000 Newton is used.

\[
x_0 = -\frac{F_p}{k} = \frac{2.000.000}{1.75 \times 10^{10}} = 0.114 \text{ mm}
\]

This initial displacement is the value for \( x_t \) for the first time step \( t_0 \) in the mass-spring system, see appendix E. This displacement increases with a decrease of the stiffness \( k \) and a constant pulse load. In the Excel file the acceleration, the velocity and the displacement of the mass are calculated as follows:

\[
a = \frac{d^2x}{dt^2} = \frac{\sum F}{m} [m/s^2] \quad (3.11)
\]

\[
v = \frac{dx}{dt} = v_{t-1} + a_t \cdot dt [m/s] \quad (3.12)
\]

\[
x = x_{t-1} + v_t \cdot dt [m] \quad (3.13)
\]
Where \( dt \) is the time step in between two points of measurement. This time step depends on the sampling frequency \( f_s \). For example, when the sampling frequency is 200 hertz, the time step in between two points is:

\[
dt = \frac{1}{f_s} = \frac{1}{200 \text{[Hz]}} = 0.005 \text{[s]}
\]

(3.14)

The velocity graph of the mass-spring system is shown in Graph 14. For comparison, the velocity graph of the system with a stiffness \( k \) of 50% of the initial stiffness is shown in Graph 15. What can be seen is that the frequency of the vibrations decreases (i.e. the number of vibrations during one second decreases). On the other hand, the velocity of the vibrations increases.

![Graph 14: Velocity graph initial situation](image)

![Graph 15: Velocity graph when stiffness decreased with 50%](image)

The result of changing the stiffness from 100% to 10% can be seen in Graph 16. On the x-axis the decrease of the stiffness as a percentage of the initial stiffness is plotted. For the sake of completeness, the eigenfrequency of the system is shown in hertz on the y-axis to the left. On the y-axis to the right, the corresponding magnitude (i.e. the velocity in millimetres per second) of the eigenfrequency is shown. The whole calculation has been performed with Excel and is shown in appendix F.

To make a fair comparison, the total time in which samples have to be taken has to be equal for every sampling frequency. This means that, for a higher sampling frequency, a higher number of data has to be analysed. Since Excel restricts the amount of data points to a power of 2, the observed
sampling frequencies have to be each other’s multiplication. In this particular example, 200 hertz is observed with 1024 data points. This amounts to 5.12 seconds. For comparison, 400 hertz is sampled with 2048 data points and 800 hertz is sampled with 4096 data points.

Graph 16: Progress of the eigenfrequency and velocity with a decreasing stiffness

As can be seen in Graph 16, the four lines, representing the frequency change with different sampling frequency, show the same development. It does not matter which sampling frequency is chosen, the decrease in frequency is well approached.

The velocity change however shows a different pattern. The fuchsia line represents the velocity change calculated by the formulas. The progress is exponential. The green, blue and purple lines show the corresponding velocity calculated after a Fast Fourier Transform, when the sampling frequency is 200, 400 and 800 hertz respectively. As can be seen, these lines are not equal to the real velocity of the mass. But, they almost do not differ from each other. The assumption is correct that the velocity corresponding to the eigenfrequency after a Fourier Transform is not the real corresponding velocity. The assumption, that 200 hertz might be of too low quality, was not correct.

At some points, the velocity calculated after a Fourier Transform approaches the real velocity. But at other points, the differences cannot be neglected. The maximum difference in this example occurs at a decrease of 50% of the initial stiffness:

\[
\text{Maximum difference after FFT} = \frac{(v_F - v_r)}{v_r} = \frac{5.83 - 9.16}{9.16} = -36.4%
\]

With

\(v_F = \text{Velocity after a Fast Fourier Transform}\)

\(v_r = \text{Real velocity calculated with formulas}\)
What attracts the most attention is the fact that a higher sampling frequency, which improves the resolution of a signal, does not improve the reliability of the velocity outcome after a Fast Fourier Transform.

**3.4.2 What does the Fourier Transform do?**

Paragraph 3.4.1 concluded that the velocity after a Fast Fourier Transform does not correspond with the real velocity of the mass calculated with formulas. The question remains why the frequency spectrum after a Fourier Transform shows these odd values.

As can be seen from Graph 16, the velocities after the Fast Fourier Transform are always lower than the real velocities. The first hypothesis that can be made according to this, is that there is some leakage of information with respect to the velocities. The Fourier Transform samples at a certain frequency. It is possible, or rather realistic, that it does not exactly sample on the eigenfrequency of the system. This can cause a peak frequency that is not exactly the eigenfrequency, with a magnitude that is not exactly the magnitude that corresponds with the eigenfrequency either.

When a Fast Fourier Transform is performed, it is assumed that the last data point is identical to the first data point with respect to its phase, see paragraph 2.6. But, in general, the period of the signal is not known ahead of time. In the case of natural vibrations from trains, the signal is composed out of multiple sine and cosine functions. Since all these functions have a different phase, it is impossible to find a phase which is applicable to all functions. This means that the fundamental period of the signal is not known, and sampling of the signal may stop at a different phase than where it started at. This will cause a last data point that is not identical to the first data point.

A phase difference causes inaccuracies, called leakage, when a Fast Fourier Transform is performed (Cimbala, 2010). The following example shows the amount of leakage. Since the pulse load used above gives a pure sine wave as a vibration, the same pulse load will be used. The eigenfrequency, the corresponding amplitude and the phase of the signal are then known. The eigenfrequency of the signal is, calculated with formulas:

\[
f_e = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{1.75 \times 10^{10}}{5.500.000}} = 8.978 \text{ Hz}
\]

The amplitude of the sine wave is equal to the maximum velocity \(v_{\text{max}}\) of the mass:

\[
v_{\text{max}} = -x_0 \cdot \omega \cdot \sin(\omega \cdot t) = -0.114 \cdot \sqrt{\frac{1.75 \times 10^{10}}{5.500.000}} \cdot \sin\left(\frac{1}{2}\pi\right) = 6.447 \text{ [m/m/s]}
\]

The maximum velocity amounts to 6,447 millimetres per second. So an ideal Fourier transform would have a peak frequency at 8,978 hertz, with a maximum velocity of 6,447 millimetres per second. Besides, no other frequency should contain any magnitude (i.e. energy).
The outcome of a Fast Fourier Transform depends on the following factors:

- the sampling frequency, $f_s$
- the number of data points analysed, $N$
- the total time of data collection, $T_s$, where $T = N/f_s$

For this example, $f_s$ is chosen to be 200 data points per second (i.e. 200 hertz). The number of analysed data points $N$ is chosen to be 4096 data points. This means, the vibration is observed for a total time $T_s$ of 20.48 seconds.

![Graph 17: Phase difference start and end of a vibration](image)

As can be seen in Graph 17, the phase of the vibration at the end of the measurement is not exactly equal to the phase at the start of the measurement.

When the Fast Fourier Transform is performed, it shows the following frequency spectrum.

![Graph 18: Frequency spectrum after a Fast Fourier Transform](image)

While the peak of the frequency should point at a velocity of 6.447 millimetres per second, instead it shows 4.354 millimetres per second. The approximation of the value for the frequency is good: 8.984 hertz.

As previously concluded (see paragraph 3.4.1), a higher sampling frequency does not improve the velocity outcome after a Fast Fourier Transform. Generally, leakage is worse when the frequency
resolution of the spectrum (i.e. the interval of the frequencies on the x-axis) is poorer. The smaller the frequency resolution, the better. The frequency resolution can be calculated with formula 3.15:

\[ df = \frac{1}{T_s} \text{ [Hz]} \]  

(3.15)

With
\( df \) = frequency resolution
\( T_s \) = sampled time

For example, with a sampling frequency of 200 hertz and 4096 data points, 20,48 seconds are analysed. The frequency resolution amounts to:

\[ df = \frac{1}{T_s} = \frac{1}{20,48} = 0.049 \]

When for example the sampling frequency is 200 hertz but analysed with only 512 points, the sampling time \( T_s \) amounts to \( 512/200 = 2.56 \) seconds, which gives a frequency resolution of:

\[ df = \frac{1}{T_s} = \frac{1}{2.56} = 0.39 \]

Which is poorer. The frequency spectrum of this case is shown below, in Graph 19.

Graph 19: Frequency spectrum with poor frequency resolution

The value for the magnitude better approaches the real velocity. This is a coincidence, since apparently the phase difference after 2.56 seconds is different but more favourable from the phase difference after 20.48 seconds. There is more leakage, which can be seen because of the wider peak. A wider peak means the energy has been split over more frequencies than just the eigenfrequency. The frequency is still approached well (i.e. 8.984 Hz). But, as can be seen on the x-axis, the number
of points in between frequencies has decreased. This will cause a less accurate approximation of the frequency.

The following can now be concluded. As shown earlier, with respect to the approximation of the real velocity of the mass, a different sampling frequency does not make the difference. The difference is caused only due to a different phase from the starting and final point of the analysed vibration. When the analysed time is the same, the phase difference will be practically the same as well, no matter which sampling frequency is used. With respect to approaching the eigenfrequency, a smaller frequency resolution will give a better approximation. This means that the more data points are analysed, the better the approximation of the eigenfrequency will be.

### 3.4.3 The perfect Fast Fourier Transform

It is not as simple to create a frequency spectrum, with both a good approximation of the eigenfrequency and a good approximation of the corresponding peak velocity. Leakage has to be reduced, and the frequency resolution has to be improved.

According to the Nyquist criterion, a sampling frequency is needed of at least two times the eigenfrequency. Since the eigenfrequency is 8,978 hertz, the sampling frequency should be at least 18 hertz. With this, the first data point must have the same phase as the last data point. Therefore, a time \( T_s \) is needed that shows a whole amount of vibrations (i.e. waves). The oscillation time \( T \) of one wave is:

\[
T = 2\pi \frac{m}{k} = 2\pi \frac{5.500.000}{1.75 \cdot 10^{10}} = 0.11 \text{ [s]}
\]

To improve the frequency resolution, the time \( T_s \) that should be observed should be as large as possible. But since the mass-spring system in Excel calculates with a time step \( dt \) for the integration, \( dt \) should not be too large. Otherwise, this will cause infinite displacements and therefore velocities.

The golden mean should be found. By trial and error, an amount of 50 oscillations is chosen which gives an investigated time \( T_s \) of:

\[
T_s = 50 \cdot 0.11 = 5.57 \text{ [s]}
\]

This gives a sampling frequency, when still using 4096 data points, of:

\[
f_s = \frac{N}{T_s} = \frac{4096}{5.57} = 735.44 \text{ Hz}
\]

And a time step \( dt \) of:

\[
dt = \frac{1}{f_s} = \frac{1}{735.44} = 0.0014 \text{ [s]}
\]

This gives the following frequency spectrum, shown in Graph 20.
This ‘perfect’ frequency spectrum shows a peak at exactly 8,978 hertz, with a corresponding velocity of 6,449 millimetres per second (a negligible difference). Note that it is only possible to plot this ‘perfect’ frequency spectrum because the eigenfrequency is known in advance. Because of that, it is possible to choose a time span which accounts for an exact amount of vibrations. The phase of the first data point is then the same as the phase of the last data point. When the period of the vibration is not known, as is the case in nature, it is impossible to know the oscillation time of the vibrations. It is then impossible to have the same phase at the start and end of the measurement.

3.4.4 Conclusions with respect to the magnitude

A conclusion that can be drawn now is that, if the Fast Fourier Transform is used, it is not possible to investigate or monitor the exact magnitude of the vibrations corresponding with the eigenfrequency. Therefore, in this research, the exact decrease of the stiffness of the soil cannot be monitored. Only a relative decrease, or increase, in stiffness with respect to the initial stiffness can be derived. Luckily for this research, there is no need to know the exact value of the magnitude of the eigenfrequency. It is about the shift in eigenfrequency. This shift can be analysed from the graphs after a Fast Fourier Transform. It can be seen that the peak frequency and the surrounding mound of small peaks (i.e. the leakage) are moving in a particular direction. In this way, it can be seen that the eigenfrequency is changing due to a stiffness change.

3.5 Conclusion one-mass-spring model

As can be concluded from sections 3.1 to 3.4, it is possible to model the soil-structure system of the pillar with the single-degree-of-freedom one-mass-spring model described in paragraph 3.1.

The response of the model corresponds to formula 2.10 for the eigenfrequency of a system. When the stiffness decreases, the peak frequency decreases as well. When the mass decreases, the peak frequency increases. Also, the peak frequency does not change when the input energy changes. The corresponding magnitude however increases with an increasing input energy, which can be explained.
with kinetic energy. Taking all this into account, it is assumed that indeed the peak frequency of all frequency spectra shown in this chapter is the eigenfrequency of the mass-spring system.

With regards to damping it can be said that the eigenfrequency of the system stands out more when damping is relatively small. Therefore, when the corresponding magnitude of the eigenfrequency is not important, it is more convenient when damping is neglected.

With respect to the magnitude of the eigenfrequency, the exact value of the velocity of the mass will not be shown after a Fast Fourier Transform, as can be found in paragraph 3.4.4. Therefore, only a relative change in stiffness with respect to the initial stiffness can be derived.

3.6 Double-degree-of-freedom two-mass-spring model

A disadvantage of the single-degree-of-freedom one-mass-spring model is that there is only one degree of freedom. The frequency spectrum of the system therefore shows only one frequency peak.

The research is about the eigenfrequency of the soil part of the system and not about the eigenfrequency of the structure. Therefore, it has to be investigated if the peak frequency visible is the eigenfrequency of the soil part, as assumed, or the eigenfrequency of the structure part, or maybe even a combination of those two. It is assumed that with a two-mass-spring model (i.e. with two degrees of freedom), two frequency peaks will be visible in the frequency spectrum. The lower peak frequency is assumed to represent the eigenfrequency of the soil part of the system. The higher peak frequency is assumed to represent the eigenfrequency of the structural part. A two-mass-spring model will be investigated in this paragraph.

When dividing the system into two parts, it is assumed that the lowest part will vibrate with the eigenfrequency of the soil. At the same time, the upper part of the system will vibrate with the eigenfrequency of the structure. The model is shown in Figure 11.

![Two-mass-spring model](image)

Figure 11: Two-mass-spring model

3.6.1 Mass $m_1$ and $m_2$

In the new model, the total mass of $m_1$ and $m_2$ remains the same as the total mass of the previous mass-spring model described in paragraph 3.1. For the first estimation of the two-mass-spring model,
the total mass of the soil-structure system is divided in two equal masses \( m_1 \) and \( m_2 \). This means that the spring \( k_1 \) will be located somewhere in the middle of the pillar, representing the stiffness of the pillar.

<table>
<thead>
<tr>
<th>Total mass ( m )</th>
<th>5.500.000</th>
<th>kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper mass ( m_1 )</td>
<td>2.750.000</td>
<td>kg</td>
</tr>
<tr>
<td>Lower mass ( m_2 )</td>
<td>2.750.000</td>
<td>kg</td>
</tr>
</tbody>
</table>

**Table 2: Total mass 1 and 2**

### 3.6.2 Spring constant \( k_1 \) and \( k_2 \)

In the two-mass-spring model, the spring constant \( k_2 \) represents the stiffness of the soil part. This spring constant is taken equal to the spring constant in the previous mass-spring model.

Assuming the other spring to be somewhere in the middle of the pillar, as described above, the spring constant \( k_1 \) represents the stiffness of the pillar (i.e. the stiffness of the structural part). Since the pillar consists of both concrete and masonry, the pillar should be stiffer than the soil.

A reasonable value for the spring constant of the structural part has to be determined. The E-modulus of the soil can be calculated by:

\[
E = \frac{\sigma}{\varepsilon} \text{[MPa]}
\]

Where

\[
\sigma = \frac{F}{A} \text{[N/m²]}
\]

\[
\varepsilon = \frac{\Delta L}{L} [-]
\]

With a force of 14.000 kN (i.e. the dead load of the structure) acting on a 200 m² footing, and assuming the ground is stiff with a settlement of 0,8 millimetres over a depth of 10 meters, the E-modulus of the soil amounts to:

\[
E = \frac{\sigma}{\varepsilon} = \frac{14.000.000}{200 \times 10^{-3}} = 870 \text{ MPa}
\]

Concrete of good quality is assumed to have an E-modulus of 10.000 to 30.000 MPa. The pillar is made of concrete which is assumed to be of bad quality since it dates from the 19th century. Therefore, the E-modulus of the pillar is assumed to be less, 5.000 MPa, which is approximately 5 times as stiff as the soil. Therefore, the choice has been made to make the pillar five times as stiff as the soil for the first approximation with the two-mass-spring model.

<table>
<thead>
<tr>
<th>Spring constant pillar - ( k_1 )</th>
<th>8,75 * 10^10</th>
<th>N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring constant soil - ( k_2 )</td>
<td>1,75 * 10^10</td>
<td>N/m</td>
</tr>
</tbody>
</table>

**Table 3: Spring constants \( k_1 \) and \( k_2 \)**
3.7 The two-mass-spring model in Excel

Like the single-degree-of-freedom one-mass-spring model, also the double-degree-of-freedom two-mass-spring model is constructed in Excel. More details of the model can be found in appendix G. The upper part of the model is assumed to vibrate in the eigenfrequency of the structure. The lower part of the model is assumed to vibrate in the eigenfrequency of the soil. Besides, it is assumed that the eigenfrequency of the structural part is higher than the eigenfrequency of the soil part. This is due to the fact that the soil is less stiff than the structure (i.e. sand is less stiff than brick and concrete).

3.7.1 Results

The same Fast Fourier transform is performed over the velocity results of this two-mass-spring model. Because there are now two different velocities, the velocities of mass 1 and mass 2, the Fast Fourier Transform can be performed over two different sets of data.

The parameters determined in paragraph 3.6 give a frequency spectrum as shown Graph 21 and Graph 22.

Graph 21: Frequency spectrum mass 1, initial situation two-mass-spring model

Graph 22: Frequency spectrum mass 2, initial situation two-mass-spring model
The only difference comparing the two graphs is the magnitude of the vibrations. The frequency peaks are located at the same frequency. Paragraph 3.4 showed that the magnitude of the vibrations is not in reality corresponding with the velocities. Therefore, in the following sections of this chapter, only the frequency spectrum of the lower mass will be shown.

As assumed, there are two frequency peaks visible. The two peak values are 8.79 hertz in the lower frequency range, and 44.77 hertz, with a lower magnitude, in the higher frequency range. The assumption is that the peak in the lower frequency range represents the eigenfrequency of the soil, and that the peak in the higher frequency range represents the eigenfrequency of the structure. This can be verified by changing different parameters of the system, which will be treated in the following paragraph.

3.8 Changing parameters of the two-mass-spring model

As for the previous mass-spring model, parameters will be changed to check whether a parameter is contributing to a shift of the frequency of the system or not. The following parameters will be changed:

- The stiffness of the structure, $k_1$
- The stiffness of the soil, $k_2$
- Distribution of the load over mass 1 and mass 2

Because the mass of the system does not change during construction works, and because the contribution of a changing mass is already investigated with the one-mass-spring model, changing the amount of the total mass will not be repeated for the two-mass-spring model.

Besides, the parameter study in paragraph 3.1.3 showed that another input load will not change the eigenfrequency of the system. Therefore, changing this parameter will also not be repeated.

Because the one-mass-spring model already proved that adding damping to the system doesn't influence the eigenfrequency, damping is not taken into account.

3.8.1 Stiffness of the pillar, $k_1$

To see whether the high frequency peak represents the eigenfrequency of the structure, the stiffness $k_1$ of the structure part is varied. It is assumed that with the change of this parameter, the peak in the higher frequency range will move to the left (i.e. to a lower frequency) as the pillar becomes less stiff. This should cause a drop in the eigenfrequency of the structural part.

A decrease in stiffness $k_1$ of 50% is chosen. The graph of the frequency spectrum is shown in Graph 23.
As expected, the peak in the higher frequency range moves to the left, while the peak in the lower frequency range remains almost the same. The amount of change of the peak in the higher frequency range is:

$$\text{Frequency decrease} = \frac{(f_n - f_o)}{f_o} = \frac{(31,15 - 44,77)}{44,77} = -30.4\%$$

While the peak in the lower frequency range changes only:

$$\text{Frequency decrease} = \frac{(f_n - f_o)}{f_o} = \frac{(8,54 - 8,79)}{8,79} = -2.8\%$$

### 3.8.2 Stiffness of the soil, $k_2$

To see whether the peak in the lower frequency range represents the eigenfrequency of the soil, the stiffness $k_2$ of the soil is varied. It is assumed that, with changing this parameter, the peak will move to the left (i.e. to a lower frequency) as the soil becomes less stiff. This should cause a drop in the eigenfrequency of the soil part.

A decrease of stiffness $k_2$ of 50% is chosen. The graph of the frequency spectrum is shown in Graph 24.
The peak in the lower range moves to the left as assumed, while the peak in the higher range stays almost the same. The amounts of change in eigenfrequencies are:

\[
\text{Frequency decrease} = \left( \frac{f_n - f_o}{f_o} \right) \left( \frac{100}{100} \right) = \frac{6.3 - 8.79}{8.79} = -28.3\%
\]

and

\[
\text{Frequency decrease} = \left( \frac{f_n - f_o}{f_o} \right) \left( \frac{100}{100} \right) = \frac{44.09 - 44.77}{44.77} = -1.5\%
\]

respectively.

The assumption seems to be correct that the peak in the lower frequency range represents the eigenfrequency of the soil, and the peak in the higher frequency range represents the eigenfrequency for the structure.

The progress of the eigenfrequency in the lower and higher range with, for the sake of completeness, the progress of the velocity is shown in Graph 25 and Graph 26. The stiffness \(k_2\) of the soil is changed. As can be seen, the progress of the eigenfrequency in the lower range (i.e. the soil part) is exponential and significant. The progress of the eigenfrequency in the higher range (i.e. the structure part) is linear and small.
3.8.3 Different mass distribution

Initially, the total mass of the structure was divided into two masses with an even mass distribution. The mass ratio is varied in order to be able to see the difference when this distribution is not even. A varied mass distribution can be useful for the coupling of the model with the real situation.

As a first variation, the upper mass consists of 1:4 of the total mass, while the lower mass consists of 3:4 of the total mass. The assumption is that the heavier mass will get a lower eigenfrequency, while the mass that is less heavy will get a higher eigenfrequency. The frequency spectrum is shown in Graph 27.
Graph 27: Frequency spectrum with a mass distribution of 1:3

It is remarkable that both frequency peaks are moved to the right (i.e. both frequencies became higher), although the changes are small.

\[
Frequency \ increase = \frac{(f_n - f_o)}{(f_o)} = \frac{(8.94 - 8.79)}{(8.79)} = +1.7\%
\]

\[
Frequency \ increase = \frac{(f_n - f_o)}{(f_o)} = \frac{(52.39 - 44.77)}{(44.77)} = +17.0\%
\]

A second variation is the opposite, which means that the upper mass consists of 3:4 of the total mass and the lower mass consists of 1:4 of the total mass. The assumption is that the eigenfrequency of the soil goes up, because the soil is less heavy than at the beginning. Besides, as the structure is now heavier, its eigenfrequency should go down.

Graph 28: Frequency spectrum with a mass distribution of 3:1
What is remarkable in this situation, when looking at Graph 28, is that both frequency peaks are moving in the opposite direction. The eigenfrequency of the soil goes down, while the eigenfrequency of the structure goes up.

\[
\text{Frequency decrease } = \frac{(f_n - f_0)}{(f_0)} = \frac{(8,54 - 8,79)}{(8,79)} = -2,8\%
\]

\[
\text{Frequency increase } = \frac{(f_n - f_0)}{(f_0)} = \frac{(55,91 - 44,77)}{(44,77)} = +24,9\%
\]

This remarkable shift of the eigenfrequency also occurs when the derivation of the masses is changed in another way, see Graph 29.

An explanation of this phenomenon is probably that both masses are not independent of each other. Because the change of the eigenfrequency of the upper mass is much more dependent on the change in mass distribution than the lower mass, it can be said that the behaviour of the upper mass is submissive to the behaviour of the lower mass.
3.9 Conclusion two-mass-spring model

With a two-mass-spring model, two peak frequencies of the system are found. Those frequencies are proven to be the eigenfrequencies of the system, see paragraph 3.8.1 and 3.8.2. The peak in the lower frequency range is the eigenfrequency of the soil part, and the peak in the higher frequency range is the eigenfrequency of the structural part.

As can be seen from Graph 25 and Graph 26, when varying the soil stiffness $k_2$, both frequencies decrease. However, there is only a small decrease in structural eigenfrequency. The decrease in eigenfrequency of the soil has a parabolic gradient while the decrease in eigenfrequency of the structure has a linear gradient. The form of Graph 25 has the same form as Graph 9. Also, the values of the eigenfrequencies are the same.

For the sake of completeness, the change in velocity is also shown in the graphs. Again, there is no logical variation. The cause of this variation has been discussed in paragraph 3.4.

3.10 Conclusion analytical modelling of the situation

In this chapter, both a one-mass-spring model and a two-mass-spring model were evaluated. This was done in order to check if an analytical model could be used to analyse the situation in Nijmegen. Due to this evaluation an answer can be found to the first three research questions.

A one mass spring system is a good way to model a situation where the stiffness of the soil is changing or about to change. Both the one-mass-spring model and the two-mass-spring model are suitable for this analysis.

A one-mass-spring model proves to be sufficient when only the lower stiffness is of importance. The frequencies of the different parts of the system can be distinguished by modelling the problem as a two-mass-spring system, with two degrees of freedom. This results in a frequency spectrum with two frequency peaks. The peak frequency in the lower range represents the soil part of the system. The peak frequency in the higher range represents the structural part of the system. This has been proven in chapter 3.8.1 and 3.8.2.

The stiffness of the subsoil mostly contributes to the value of the peak frequency in the lower range (i.e. the eigenfrequency of the soil part). As can be seen from Graph 25 and Graph 26, the eigenfrequency of the soil decreases with a parabolic gradient as a response to a decreasing soil stiffness $k_2$. On the other hand, the decrease in eigenfrequency of the structure is minimal and has a linear gradient.

A decrease in stiffness of the soil leads to a significant decrease of the eigenfrequencies of the system. This decrease of eigenfrequencies is most visible in the lower frequency range. This indicates that measuring the frequency of vibrations in the field is useful to monitor the stiffness of the soil.

An important conclusion follows from paragraph 3.4. By monitoring ambient vibrations and analysing the results after a Fast Fourier Transform, the exact decrease of the stiffness of the soil cannot be monitored. Only a relative decrease, or increase, in stiffness with respect to the initial stiffness can be derived. This is due to the fact that it is impossible to determine the phase of the
signal when dealing with ambient vibrations. Sampling of the signal may stop at a different phase than when it started, and a different phase of the first and last data point will cause leakage when performing a Fast Fourier Transform. Also, a higher sampling frequency does not imply a better approximation of the real velocity. The amount of leakage remains the same, since the phase difference does not change with a higher sampling frequency.
4. Measurements recorded by the author

The following chapters 4 and 5 will describe the empirical part of the research. This chapter will start with the elaboration of the dataset recorded by the author, in cooperation with the Municipality of Rotterdam. At first, the monitoring equipment will be outlined briefly. A more detailed description of the equipment can be found in paragraph 2.5. Afterwards, the monitoring setup and the days of measurement will be outlined. Processing of the datasets and performing the Fast Fourier Transform, which is necessary to split the recorded wave signal into its ground frequencies, is performed with Matlab. The Matlab codes written will be outlined briefly in this chapter. More information can be found in the mentioned appendices. Finally, the results and difficulties of the first and second measurement day will be discussed.

4.1 Measurement setup

In this paragraph the measurements on site in Nijmegen, performed by the author in cooperation with the Municipality of Rotterdam, will be discussed. The RoDo-system, described in more detail in paragraph 2.5, is used to monitor the vibrations in Nijmegen. The system measures vibrations up to 500 hertz, with a resolution of 1000 hertz (i.e. one sample every 1/1000 of a second). The system used in Nijmegen exists of ten geophones. The geophones measure the frequencies of the vibrations, with the corresponding magnitude measured in Volts. This magnitude can be converted into the velocity with the sensitivity factor of the geophones.

The vibration measurement in Nijmegen is performed at the location of pillar 1. An outline of the situation in Nijmegen can be found in appendix H. Almost all geophones are installed on the south side of the pillar. The south side is the side of pillar 1 which faces pillar 2. All geophones measure the vibrations in 3 directions (i.e. X, Y and Z), with a sampling frequency of 1000 hertz. The X-direction, in the horizontal plane, measures vibrations in the direction perpendicular to the rail tracks (i.e. east-west). The Y-direction, also in the horizontal plane, measures vibrations parallel with the rail track (i.e. north-south). The Z-direction measures vibrations in the vertical plane. Geophones on the surface of the structure and the soil will mostly record surface waves (i.e. Rayleigh waves). The two geophones in depth of the soil will record body waves (i.e. compressional and shear waves). A drawing of the measurement setup can be found in appendix A. Initially, the setup described in this paragraph will be applied for both measurement 1 and 2. When there is a modification of the setup due to circumstances, it will be mentioned in the relevant section.

It has to be noted, that an extensive and detailed vibration measurement like this is not often performed. Therefore, some geophones are placed to be able to investigate phenomena which are outside the scope of this research. This means that not all information that is present in the recorded dataset will be used for this research.
4.1.1 The structure part

Geophone 1 is attached to the girder of the bridge, at the downside of the girder. When comparing the results from this geophone with other geophones located at the pillar, it can be examined if the girder is vibrating different with respect to the pillar. Besides, the frequency spectrum of geophone 1 can be seen as the input signal caused by the rail traffic. The input signal could not be measured more accurately, since it was not allowed to put a geophone on or nearby the rail track.

The pillar is supposed to move in three different directions (i.e. X, Y and Z-direction). Therefore, geophones 2, 3 and 4 are placed on the top, middle and downside of pillar 1 respectively. Tilting in the horizontal plane can be detected by comparing the amplitudes (i.e. velocities) of vibrations in the X and Y-direction. Tilting in the Z-direction will be detected by geophone 5, which is placed on the north downside of the pillar. A difference in amplitude on the north side (i.e. geophone 5) and south side (i.e. geophone 4) indicates tilting of the pillar. It has to be noted that the exact amount of tilting cannot be distinguished from the data after a Fast Fourier Transform, since those velocities do not correspond with the real velocities, see paragraph 3.4. For a detailed investigation of tilting, which is outside the scope of this research, the raw recorded data should be analysed.

Besides, with geophones 4 and 5, the difference in vibrations of trains going in one direction or the other can be compared. It can be investigated if the construction reacts different with rail traffic coming from different directions.

Geophones 1 till 4 will make it also possible to investigate how the frequencies and velocities of the recorded vibrations change during their travel through the construction. For example, from the girder to the concrete block on the pillar, and from the concrete block to the brickwork of the pillar. It can be studied which part of the structure causes the most attenuation of the vibrations. Besides, it can be studied if various materials react different when exposed to the same amount of energy. A detailed study to this phenomenon is also outside the scope of this research.

Assumed is that the measurements will record frequencies in the higher and in the lower frequency range. The higher frequencies will represent the eigenfrequencies of the structure. The lower frequencies will represent the eigenfrequencies of the soil surrounding the structure. The frequencies in the lower range will be used to look into the shift of frequencies when the stiffness of the soil is decreasing. It can be investigated if the higher frequencies may also show shifts that can be explained with, or coupled to, the decreasing stiffness of the soil. Referring to the analytical model in paragraph 3.8.2, there should be a small change in the high frequencies as well. The question is if these shifts are large enough to derive from the vibration measurements.

4.1.2 The soil part

Besides geophones attached to the structure, there are also geophones placed on ground surface and in depth. These geophones will be used to investigate Rayleigh waves travelling from pillar over the ground surface, and to investigate body waves going into the subsoil.

Geophones 8, 9 and 10 are placed in a row on ground surface. These three geophones will be used to examine how much attenuation the waves suffer (i.e. how much the velocity of vibrations...
decreases with respect to the distance to the pillar). The information gathered by three geophones is the minimum amount of information for a good comparison.

Geophones 6 and 7 are placed in depth on the south side of the pillar, directly next to its foundation block. These two geophones will be used to examine how the velocity of the waves develops when vibrations radiate into the subsoil. This can be done by comparing the magnitudes of the frequency peaks. The change of frequency of the vibrations can be also investigated, assuming the soil will be stiffer in deeper soil layers. With geophone 8 on top, again three geophones are put in a row.

### 4.1.3 Measurement day 1

Measurement day 1 takes place at the 21st of June 2013, from 12.45 till 15.00. The temperature is in between 16 and 22 degrees, there is no rain (<2 millimetres) and the humidity is 84% (WeatherOnline, 2013).

This measurement represents the initial state of the soil. The construction works around pillar 1 have not yet started. But, unfortunately, some preparation works already started. The upper soil layer around the pillar, approximately 0.5 metres of soil, was excavated. This meant that now the surface level of the soil is located at approximately +10 NAP.

During the measurements, there are no construction works on or around pillar 1. There are some excavators driving along the site. This kind of traffic causes no significant vibrations. The vibrations caused by rail traffic can be recognized easily.

The trains passing the bridge are photographed. In this way, particular vibrations can be coupled to the time of the passage. Later on, with Matlab, the particular time can be retrieved easily. In this way, a particular frequency spectra can be coupled to a time of passage and its corresponding type of train.

The outline of sensors during measurement day 1, which can be also found in appendix A, was as follows:

<table>
<thead>
<tr>
<th>Spot</th>
<th>Geophone</th>
<th>Channel</th>
<th>Place</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00602</td>
<td>1</td>
<td>At the bridge girder</td>
</tr>
<tr>
<td>2</td>
<td>00599</td>
<td>2</td>
<td>At the top of the pillar, at the concrete block</td>
</tr>
<tr>
<td>3</td>
<td>00597</td>
<td>3</td>
<td>In the middle of the pillar</td>
</tr>
<tr>
<td>4</td>
<td>00600</td>
<td>4</td>
<td>At the bottom of the pillar</td>
</tr>
<tr>
<td>5</td>
<td>00605</td>
<td>5</td>
<td>At the bottom of the pillar, on the other side</td>
</tr>
<tr>
<td>6</td>
<td>TDS 00142</td>
<td>6</td>
<td>In the subsoil, 4.5 meters deep</td>
</tr>
<tr>
<td>7</td>
<td>TDS 00143</td>
<td>7</td>
<td>In the subsoil, 3 meters deep</td>
</tr>
<tr>
<td>8</td>
<td>00598</td>
<td>8</td>
<td>At the surface, 2.1 meters from the pillar</td>
</tr>
<tr>
<td>9</td>
<td>00937</td>
<td>9</td>
<td>At the surface, 8.7 meters from the pillar</td>
</tr>
<tr>
<td>10</td>
<td>00936</td>
<td>10</td>
<td>At the surface, 14.8 meters from the pillar</td>
</tr>
</tbody>
</table>

Table 4: Setup during measurement day 1
4.1.4 Measurement day 2

Measurement day 2 takes place at the 12th of September 2013, from 12.00 till 14.00. The weather is almost the same as on measurement day 1, it is just a little colder. The temperature is in between 12 and 19 degrees, there is no rain (<1 millimetre) and the humidity is 86% (WeatherOnline, 2013). These variations are negligible for the comparison of the vibrations.

This measurement is supposed to represent the construction phase. The work that has to be carried out around pillar 1 is, inter alia, the construction of 16 diaphragm wall panels. All panels reach to a depth of approximately 23 metres. Because of multiple panels that are already constructed before measurement 2 took place, the stiffness of the soil should have changed. The works that already took place were:

- excavating and filling of diaphragm wall panels 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14 and 15
- excavation of diaphragm wall panel 5. During the measurement, this panel is open and filled with bentonite.
- only diaphragm wall panel 13 remains to be constructed

See also appendix I for a drawing of the situation.

At the time of measurement, there are no construction works around pillar 1. The construction works during the day take place around pillar 2. Some passing excavators cause vibrations, but these vibrations are negligible in comparison to the vibrations caused by trains.

Compared to the first day, the setup of measurement day two differed slightly (shown in red in Table 5). Mostly, the setup remains the same, in order to exclude differences in results because of a difference in the sensors. Although there are now diaphragm walls constructed in between the pillar and sensor spot 9, this spot is maintained to see what influence the diaphragm has on the vibrations.

<table>
<thead>
<tr>
<th>Spot</th>
<th>Geophone</th>
<th>Channel</th>
<th>Place</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00602</td>
<td>1</td>
<td>At the bridge girder</td>
</tr>
<tr>
<td>2</td>
<td>00599</td>
<td>2</td>
<td>At the top of the pillar, at the concrete block</td>
</tr>
<tr>
<td>3</td>
<td>00597</td>
<td>3</td>
<td>In the middle of the pillar</td>
</tr>
<tr>
<td>4</td>
<td>00936</td>
<td>4</td>
<td>At the bottom of the pillar</td>
</tr>
<tr>
<td>5</td>
<td>00605</td>
<td>5</td>
<td>At the bottom of the pillar, on the other side</td>
</tr>
<tr>
<td>6</td>
<td>TDS 00142</td>
<td>6</td>
<td>In the subsoil, 4.5 meters deep</td>
</tr>
<tr>
<td>7</td>
<td>TDS 00143</td>
<td>7</td>
<td>In the subsoil, 3 meters deep</td>
</tr>
<tr>
<td>8</td>
<td>00598</td>
<td>8</td>
<td>At the surface, 2.1 meters from the pillar</td>
</tr>
<tr>
<td>9</td>
<td>00600</td>
<td>10</td>
<td>At the surface, 2.1 meters from the pillar, next to 8</td>
</tr>
<tr>
<td>10</td>
<td>00937</td>
<td>9</td>
<td>At the surface, 8.7 meters from the pillar</td>
</tr>
</tbody>
</table>

Table 5: Setup during measurement day 2
4.2 Matlab

In this research the Fast Fourier Transform is performed over the recorded signals with Matlab. Different codes are written for the analysis of the data, and they will be explained in the different sections of this paragraph.

4.2.1 From measurements to matrix in Matlab

The used geophones record millivolts in time. The computer starts recording when vibrations are larger than a specific amount of millivolts characteristic for the passage of a train. In this way, vibrations outside the scope of this research do not occupy useless data space. Besides, the data is better to handle. A disadvantage of this recording method is that all the data is put directly after each other when the recorded vibrations are extracted and transposed into a matrix in Matlab. This means that in Matlab it seems like there is no gap in between two recorded vibrations, although in reality there can be.

The Matlab code shown in appendix J converts the data into a matrix with the actual time gaps. With this matrix it is possible to generate a 3D graph presenting vibrations against real time. The code first checks if the time steps in between two recorded measurements are 1/1,000 of a second (i.e. the sampling frequency). When the time step in the matrix is larger, the code adds a value of time with a measured vibration equal to zero. Besides, it converts the measured millivolts into a velocity by multiplying with the sensitivity factor. In the end, the newly generated matrix contains the real time with the measured vibrations in millimetres per second. This new matrix is called DataWithTime.

4.2.2 From matrix to 3D graph in Matlab

After the matrix DataWithTime is saved, the 3D graph of all trains that past during the measurement is plotted. The Matlab code for this operation can be found in appendix K. With a 3D graph it is easier to find a train passage and its corresponding time in the large amount of recorded data.

The matrix DataWithTime is first divided into parts of 1,000 samples (i.e. one second). A Fast Fourier Transform is then performed over every second, which results in 2D frequency spectra. All these 2D frequency spectra are put after each other in time, creating a 3D frequency spectrum. More information about the Fast Fourier Transform can be found in paragraph 2.6.

For example:

`plotPeriodOfFile (1,3600,DataWithTime)`
Graph 30: 3D graph of vibration measurement day 1, in Z-direction

This command creates a 3D graph of second 1 till second 3600 (i.e. the first hour) of the matrix DataWithTime. Above, in Graph 30, the 3D graph of the first hour of the measurements during day 1 in Z-direction is shown. To make a comparison between different types of trains and the vibrations they caused, the passed trains are shown in Table 6. Also, the table shows in which direction the train was going (i.e. from Nijmegen to Lent (NL) or from Lent to Nijmegen (LN)).

<table>
<thead>
<tr>
<th>Passage in graph</th>
<th>Train type</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passage 1</td>
<td>Not recorded</td>
<td>-</td>
</tr>
<tr>
<td>Passage 2 (train 1)</td>
<td>Intercity Train, 1 deck, 7 wagons + locomotive</td>
<td>NL</td>
</tr>
<tr>
<td>Passage 3 (train 2)</td>
<td>Sprinter Light rail train, 1 deck, 3 wagons</td>
<td>LN</td>
</tr>
<tr>
<td>Passage 4 (train 3+4)</td>
<td>2 Intercity Trains at the same time</td>
<td>Both</td>
</tr>
<tr>
<td></td>
<td>Both decks, 4 wagons</td>
<td></td>
</tr>
<tr>
<td>Passage 5 (train 5)</td>
<td>Sprinter Light rail train, 1 deck, 3 wagons</td>
<td>NL</td>
</tr>
<tr>
<td>Passage 6 (train 6)</td>
<td>Intercity Train, 1 deck, 7 wagons + locomotive</td>
<td>LN</td>
</tr>
<tr>
<td>Passage 7 (train 7)</td>
<td>Intercity Train, 2 decks, 4 wagons</td>
<td>LN</td>
</tr>
<tr>
<td>Passage 8 (train 8)</td>
<td>Intercity Train, 2 decks, 4 wagons</td>
<td>NL</td>
</tr>
<tr>
<td>Passage 9 (train 9)</td>
<td>Intercity Train, 1 deck, 7 wagons + locomotive</td>
<td>NL</td>
</tr>
<tr>
<td>Passage 10 (train 10)</td>
<td>Sprinter Light rail train, 1 deck, 3 wagons</td>
<td>LN</td>
</tr>
<tr>
<td>Passage 11 (train 11)</td>
<td>Intercity Train, 2 decks, 4 wagons</td>
<td>NL</td>
</tr>
<tr>
<td>Passage 12 (train 12)</td>
<td>Sprinter Light rail train, 1 deck, 3 wagons</td>
<td>NL</td>
</tr>
<tr>
<td>Passage 13 (train 13)</td>
<td>Intercity Train, 1 deck, 7 wagons + locomotive</td>
<td>LN</td>
</tr>
<tr>
<td>Passage 14 (train 14)</td>
<td>Intercity Train, 2 decks, 6 wagons</td>
<td>LN</td>
</tr>
</tbody>
</table>

Table 6: Train passages in 3D graph

Some conclusions can already be drawn by comparing Graph 30 and Table 6. It can be seen that the vibrations are always a bit different. Nevertheless, the peaks are located at more or less the same frequency values. So the input force does not seem to have an influence on the peak frequencies of the system. This was also concluded before with the mass-spring model, in 3.1.3.
However, a different type of train does have an influence on the magnitude of the vibrations. For example, the highest peaks in the lower frequency range are all caused by a sprinter light rail train. This type of train does not have high peaks in the higher frequency range.

The peak frequencies are assumed to be the eigenfrequencies of the system. There are three peak frequencies visible every train passage. It is assumed that the peak in the highest frequency range represents the eigenfrequency of the girder of the bridge. The peak in the middle frequency range is assumed to represent the eigenfrequency of the pillar. The frequency range, around 40 to 50 hertz, has a comparable value to the value for the eigenfrequency found with the mass-spring system which was 44.7 hertz, see section 3.7.1. Finally, the peak in the lowest frequency range represents the eigenfrequency of the soil. This eigenfrequency seems to be around 10 to 15 hertz, which is a bit higher than expected with the mass-spring model (i.e. 8.98 hertz) but which is still a plausible value for the eigenfrequency of soil.

What also can be concluded from this 3D graph is that frequencies higher than approximately 120 hertz do not exist in the signal. Therefore, all graphs used in this chapter will be limited with respect to the y-axis to a frequency of 200 hertz, instead of the possible 500 hertz.

4.2.3 From 3D plot to 2D plot of every train passage

The times of all train passages are predicted with the created 3D graph. With the Matlab code shown in appendix L, a 2D plot of the frequency spectrum can be made for every train passage, for all sensors, in all directions.

First, a comparison between a Fast Fourier Transform with Matlab and an Fast Fourier Transform with Excel has been made to ensure the Matlab code for the Fast Fourier Transform is working correctly. In Excel, the amount of input data for an Fast Fourier Transform is limited. The limit of 4,096 samples means that only 4 seconds of data can be compared with a sampling frequency of 1,000 hertz.

![Figure 12: Comparison Excel and Matlab frequency spectrum](image)

The comparison has been made with 5 seconds of recorded data (i.e. 5,000 samples), of which 4,096 will be used. The recorded vibrations during 5 seconds from passage 1 are chosen for this
comparison. The graphs are shown in Figure 12. As can be seen, the graphs are similar. Therefore, it is assumed that the Matlab code functions properly.

![Graph 31: 2D graph for sensor 1 in Z-direction](image)

Graph 31: 2D graph for sensor 1 in Z-direction

This means that 2D graphs can now be plotted. For example, the 2D graph for train passage 5 (i.e. a sprinter light rail with 3 wagons, driving from Nijmegen to Lent) is shown in Graph 31. This train passage took place from 14:08:11 till 14:08:28, as shown in the title. The graph shows the frequency spectrum for the total passage of the train (i.e. 17 seconds). Every second is plotted with a different colour. On the x-axis, the frequency is plotted in hertz. On the y-axis, the corresponding magnitude is plotted in millimetres per second. Remind that this is not the real velocity because a Fast Fourier Transform has been performed, see paragraph 3.4.

With the three Matlab codes described in this paragraph, the most important elaborations and comparisons of the data are made. Results will be discussed in the next paragraph.

### 4.3 Results

In this paragraph, the results will be discussed from measurement day 1 and measurement day 2. Also, a comparison between both days will be made. For most measurements performed on day 1, train passage 5 will be used for the analysis. This train passage consists of a sprinter light rail train driving from Nijmegen to Lent, and this type of train driving in that particular direction is showing the most clear low frequency peaks.

#### 4.3.1 Measurement day 1 - comparison three directions

The geophones were measuring vibrations in three directions. In this paragraph it is investigated what the difference is in outcome of the measured vibrations comparing these three directions. The three used graphs are presented in appendix M. They are all derived from the same train passage but in the three different directions (i.e. X, Y and Z-direction).
What can be seen when comparing the graphs, is that there are 5 different mound that can be distinguished. In the lower frequency range, there is a visible peak frequency around 12 Hertz. A smaller peak is present around 20 Hertz. These peaks are visible in all three directions of the measurement.

In the higher frequency range, the most clear peak which is also visible in all directions is the frequency peak around 67 Hertz. Besides, there is a mound around 100 Hertz, mostly visible in the X-direction. In the Y and Z-direction on the other hand, there is a peak visible around 45 Hertz, which in the X-direction is almost invisible.

What is also notable is the fact that in the X-direction (i.e. perpendicular to the rail track) the vibrations with the highest frequency have the smallest magnitude. In the lower frequency range, the vibrations in all directions are almost equal with respect to the magnitude. This means that the construction vibrates heavier in the Y and Z-direction than in the X-direction, while the soil vibrates evenly in all directions. This sound logical. The construction will vibrate the most in the Y and Z-direction, since these directions are the directions of the largest dynamic input forces caused by the trains. The soil, in turn, will spread out the vibrations evenly into all possible directions.

4.3.2 Measurement day 1 - comparison different sensors in Z-direction

In this section, all sensors are compared to each other, to see what happens with the vibration traveling from sensor 1 to sensor 10. The comparison is made in Z-direction only, since earlier analysis during this research were also in Z-direction.

It is interesting to observe which eigenfrequencies of the system are visible at which sensor (i.e. at which location of the soil-structure system). The graphs that are used for the comparison can be found in appendix N. A small resume has been made in Table 7. The thick numbers are the prominent frequency peaks.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Frequency peak(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input signal - all frequency peaks – 6, 12, 22, 40/45, 55, 67, 100</td>
</tr>
<tr>
<td>2</td>
<td>6, 12, 45, 80</td>
</tr>
<tr>
<td>3</td>
<td>6, 12, 45, 100</td>
</tr>
<tr>
<td>4</td>
<td>6, 12, 45, 50, 67, 90</td>
</tr>
<tr>
<td>5</td>
<td>12, 22, 45, 67</td>
</tr>
<tr>
<td>6</td>
<td>6, 12, small 45</td>
</tr>
<tr>
<td>7</td>
<td>6, 12, 22</td>
</tr>
<tr>
<td>8</td>
<td>12, 22, 45</td>
</tr>
<tr>
<td>9</td>
<td>Short circuit, but 12, 22, 45</td>
</tr>
<tr>
<td>10</td>
<td>12, 22, 45</td>
</tr>
</tbody>
</table>

Table 7: Frequency peaks at different sensors

As mentioned earlier, the result of sensor 1 consists of almost all frequencies which are occurring at the results of the other sensors. Therefore, the vibration at sensor 1 can be seen as the input signal.

Furthermore, it can be concluded that the 12 hertz frequency is visible at all sensors. This frequency is supposed to be the eigenfrequency of the soil.
The frequency peak at 45 hertz is also visible at almost all sensors, but does not stand out really clear in the results of sensors which are in contact with the soil. Therefore, this frequency is supposed to be the eigenfrequency of the pillar.

The highest frequency mound, around 80 to 100 hertz, is supposed to be the eigenfrequency of the girders. This eigenfrequency is only visible in the results of sensors which are placed on the construction and mostly on the upper side. This means the vibration, caused by the girder, attenuates when approaching the soil.

A 20 hertz frequency peak is only visible in the input signal, and it returns in the results of the sensors placed on surface of the soil. It cannot be seen in the results of the sensors placed in depth. Therefore, it is assumed to be not the eigenfrequency of the soil.

The frequency peak at 67 hertz is visible in the input signal, and therewith only at the lower side of the pillar.

Since the peak of 50 hertz is only visible at sensor 4, and also not in the input signal, it is possible that sensor 4 is influenced by a 50 Hertz power cord.

It can be concluded that the eigenfrequency of the soil is visible at all sensors, and that the eigenfrequency of the construction is not (especially not at the sensors placed on the surface of the soil and in depth). Therefore, the eigenfrequency of the soil can be of good use when the stiffness of a soil-structure system has to be monitored.

4.3.3 Comparison day 1 and day 2 – Z-direction

For the comparison of day 1 and day 2, 4 sensors have been selected. Sensor 1 has been selected because of the comparison of the input signal. Sensor 2 has been selected to compare the vibrations of the construction, just underneath the installed hydraulic jacks. Sensor 5 has been selected to compare the reaction of the construction near the soil surface. Sensor 8 has been selected to compare the vibrations in the soil. See appendix O.

In between day 1 and day 2, hydraulic jacks were installed. These hydraulic jacks should have an influence on the dynamic input signal. When comparing the results of sensor 1 of both days, the influence is clearly visible. The signal is disturbed, and what attracts the most attention is the higher frequencies around 160 hertz. This indicates a higher eigenfrequency for the construction part of the system. Since there are still low frequencies around 12 hertz, it seems the eigenfrequency of the soil did not change much.

When comparing the results from sensor 2, the influence of the hydraulic jacks is even more visible. The frequency peak in the lower range, the eigenfrequency of the soil, did not change. The peak frequency is still located around 12 hertz. But both frequency peaks in the higher range, around 47 and 80 hertz, seem to be moved to around 65 and 90 hertz respectively. So both frequency peaks corresponding to the structure part of the system increased.
The results from sensor 5 and sensor 8 seem to be unchanged. Changes in the lower frequency range cannot be obtained on such a large scale. The changes will be further investigated in the next section, when the lower frequency range will be zoomed in.

4.3.4 Comparison day 1 and day 2 – low frequency range Z-direction
To make a better comparison between the low frequency range, the range of the above described sensors has been zoomed in. The graphs of both measurement day 1 and 2 can be found in appendix P. The upper graphs of a page refer to measurement day 1 (i.e. from 14.08.11 till 14.08.28), the lower graphs refer to measurement day 2 (i.e. from 12.43.24 till 12.43.41).

What attracts the first attention, is that there is no clarity. The lines of multiple observed seconds do all point at different values, for both frequency and velocity. Also, there are lines visible which consist out of two small peaks. Such lines are created when the leakage is very large. Because the Fast Fourier Transform does not exactly sample on the eigenfrequency of the signal at that second, the signal is has been split over two frequencies, and so is the velocity.

Also, when looking at the results more superficial and when trying to determine the average, it seems that the eigenfrequency indeed decreased. But with these measurements, a clear conclusion cannot be drawn.

4.4 Difficulties
Most of the times, during measurements in the field, not everything goes as planned. The results from the measurements made in Nijmegen were also not all satisfactory. In this paragraph the problems and difficulties during the measurements are discussed, including their influences on the results.

4.4.1 Sensors in depth
A problem occurred during the installation of the geophones in depth. The surface level of the soil is located at +10,5 NAP. The bottom of the foundation of the pillar is located at +5 NAP. It was planned to put geophone 6 half a meter deeper than the foundation level at +4,5 NAP (i.e. a depth of 6 meters). This was desired to be able to not only investigate the vibrations in a horizontal direction, but also in the vertical direction underneath the footing. Unfortunately, the deeper soil was too stiff to push the cone of geophone 6 to the desired depth. It broke, but luckily it still worked. Instead of the desired depth of +4,5 NAP, it reached to a depth of +6 NAP (i.e. a depth of 4,5 meters). Because of the stiff soil, decided was to put geophone 7 not deeper than the planned +7,5 NAP (i.e. a depth of 3 meters). The chance of destroying the geophone by pushing it to a deeper level, was too big.

4.4.2 Sensor 4
While all sensors showed the same frequency peaks in X, Y and Z-directions, as could be seen in section 4.3.1, there was one sensor which did not show this. This sensor, sensor 4, is showed in appendix Q.
As can be seen in the graphs, sensor 4 (i.e. sensor number 00600) showed in the Y and Z-direction a large peak at the frequency of 50 hertz. Sensor 5 is located at the same place on the pillar, only on the other side. But this sensor does not show any frequency peak around 50 hertz.

Since this peak is only visible in the frequency spectra of sensor 4, it is probably caused by an alternating current voltage. To investigate this, other possible causes have to be investigated as well to exclude them.

- A broken sensor

The easiest cause is that sensor number 00600 is broken. To investigate this, the sensor has been switched with another sensor during the measurement day 2. Decided was to change the sensor number 00600 with the sensor used as sensor 10 during measurement day 1 (i.e. sensor number 00936). This means that the spot of sensor 10 of the first measurement will lapse. Due to installation of the diaphragm walls in between the pillar and sensor spots 9 and 10, the spot of sensor 10 was already not so interesting any more. So sensor spot 4 now consists of sensor number 00936. Sensor number 00600 was placed next to sensor number 00598 at sensor spot 8, to see if both sensors would give the same results which also would mean that sensor number 00600 was not broken.

The results of both measurements at spot 4 turned out to be the same. During measurement day 2, the shift of frequencies in the different directions was still visible. Also, sensor number 00600 was showing the same frequency spectra as sensor number 00598 now. This means it can be concluded that it is not the sensor to blame.

- A broken connection from sensor cable to ‘measuring box’

Another cause to investigate is the ‘box’ with sockets where all the sensors are connected. When there is a broken one at channel 4, this can explain the 50 hertz peak. To investigate if this is the cause of the different measurement outcome, a comparison with measurements of past projects has been made. The result of another measurement with sensor 4 can be found in appendix R. The graphs of the other measurement show the same outcome in all three directions. There is no difference in 50 hertz peak visible in the direction X compared with the directions Y and Z.

- A power cord, which operates at voltages of 50 hertz

So the last, logical cause of the 50 hertz peak is a power cord. The only inexplicable thing is that when there will be a power cord close to the measuring point of sensor 4, the peak frequency of 50 hertz should be visible in all three directions X, Y and Z.

### 4.5 Conclusions

The first thing that can be concluded from the measurements carried out by the Municipality of Rotterdam, is that the results of the analysed vibrations are always a bit different with respect to the magnitudes. This is caused by different types of trains. A different input vibration has its influence on the magnitude of the recorded vibrations, as also concluded with the mass-spring system.
The frequency peaks during all train passages are located at more or less the same values. So, as also concluded with the mass-spring system, the input force does not seem to have an influence on the value of the eigenfrequencies of the system.

It can be also concluded that the eigenfrequency of the structure increases significantly after the installation of hydraulic jacks. The jacks make the joint in between the girder and the pillar stiffer, which means they change the stiffness of the construction part of the system. This results in an increase of both the frequency peak values representing the pillar and the girder. The change in eigenfrequency of the soil due to the jacks is negligible.

Because the lower frequency range, in between 5 and 15 hertz, is visible at the results of all sensors, it can be said that it is sufficient to measure at only one location when focusing on the lower frequency range. With respect to the measurements recorded by Fugro, which are recorded at only one location, it can be concluded that also these measurements can be used to determine the change in stiffness of the soil-structure system. Because these measurements contain more data than the measurements described in this chapter, they will be used for further analysis of the changing stiffness. The measurements recorded by the author will not be explored into more detail.
5. Measurements required by the contract

In the contract of the project in Nijmegen it was required that vibrations should be monitored before, during and after the construction works. Fugro GeoServices B.V. is responsible for these vibration measurements. The measurements are provided for this research by Prorail.

In this chapter, the setup of the system of Fugro GeoServices B.V. will be outlined, followed by a description and interpretation of the information that is used for this research. In this research, the dataset of Fugro is not used to compare with the dataset made by the Municipality of Rotterdam. This is because, unfortunately, the trace function of the monitoring system of Fugro was not working during the days the Municipality of Rotterdam was performing the measurements due to a full memory. Therefore, specific train passages could be compared one to one. Since every train passage differs from each other, comparing different train passages would add nothing to this research. Both datasets are therefore used to observe different phenomena.

5.1 Setup Fugro GeoServices B.V.

The Fugro GeoServices B.V. dataset has been recorded with a Profound Vibra+ system. The system is located at the top of the pillars 1, 2, 3 and 4, as can be seen in appendix A.

The dataset contains continuous vibration measurements in three directions (X, Y, Z). The highest measured velocity with corresponding frequency is saved every 10 minutes. Additionally, the Profound Vibra system records a so called ‘trace’ 6 times per hour. The full spectrum of the measured signal, based upon a 1.024 hertz sampling frequency, is then recorded during 2 seconds.

The Fast Fourier Transform, which is performed by the computer program of Profound Vibra 2.75, processes the results of the trace into a frequency spectrum. These spectra are used in this research to observe possible changes in frequencies during the different construction phases. The measurements are continuous (that is, when the battery of the system is not low and the memory is not full). Therefore, comparisons can be made between datasets made during the initial state and made during various construction phases.

5.2 Selection of data

The dataset of Fugro GeoServices B.V. contains a lot of data, due to the continuous measured vibrations in three directions. Therefore, a selection of data will be made to facilitate the analysis and comparison. This paragraph will describe the way the selection is performed.

5.1.1 Type of vibration

After observing the data, it can be concluded that not all traces are useful for the analysis and comparison in this research. Some traces contain different kind of vibrations, showing no clear train passage. See Graph 32 and Graph 33. Graph 32 shows a train passage, consisting of a clear,
continuous vibration. Graph 33 shows a different kind of velocity graph. These kind of vibrations can be caused by different sources than a train passage, which may cause that this vibration contains other frequencies. Therefore, the graphs chosen for the analysis contain only graphs similar to Graph 32.

Graph 32: Trace with train passage

Graph 33: Trace with other vibration

5.1.2 Time table

Another selection criterion is the time that the traces are made. Earlier research has concluded that the passage of a different type of train can cause a slightly different frequency spectrum (Spruit, 2012). The research made use of the vertical vibrations. One of the conclusions was that a different speed of the train influences the velocity of the measured vibrations. Generally, in Nijmegen, the velocity of trains is higher when they travel from south to north than from north to south. The higher the velocity of the train, the higher the velocity of the vibrations. A different weight of the train does not have that much influence. These conclusions have been verified with the measurements made by the author. With respect to the frequency of the vibrations, it can be said that the type and velocity of the train do not influence the measured frequencies. This is shown in paragraph 4.2.2.

Based on these conclusions, it is tried to observe traces which were made when the same type of train was crossing the bridge. Also, when it was possible, the observed trains travelled in the same
direction. This will ensure a more or less equal input vibration based upon the type and velocity of the train. The times are obtained with the train time table between Nijmegen and Lent. This means that trains with delays can’t be taken into account, and may influence the results.

The selected traces date from minute 14, 29, 44 or 59 of every hour. An intercity with two decks then runs from Nijmegen to Lent.

5.1.3 Initial situation

For the analysis of the initial situation, measurements which are made before construction works are used. The first measured vibrations by Fugro GeoServices B.V. date from March 2013. The construction works started at the end of July. March had a high water table compared to July, which is assumed to make a difference in frequencies and velocities of the vibrations measured. Due to the short term (dynamic) loading of passing trains, an undrained situation occurs. In an undrained situation, no water movement takes place which means that excess pore pressures build up, and the soil will react stiffer. This means that the soil may react more stiff in March than in July due to the high water table. To investigate this, both datasets made in March and July are compared with each other. For the comparison with the other phases, July will be chosen to represent the initial situation since this is the situation right before construction works start.

For both datasets, an average of velocities and corresponding frequencies is taken over a period of 3 days. This is done in order to exclude uncertainties and results from outstanding trains. Over these 3 days, 9 train passages are observed for both the initial situation in March and in July. The analysed train passages occur around the same time each day. The result of this analysis can be seen in appendix S.

5.1.4 During construction works

A selection of data is also made out of the dataset recorded during the construction works. To make a fair comparison between the initial phase and the construction phases, the data is selected as described in this section.

As a first attempt, it is tried to compare the frequency spectra made during different types of construction works, and the initial situation. However, this comparison is not possible due to a couple of reasons. In the first place, the comparison is not fair. The initial situation is observed over a period of three days, while the different types of construction works only last a couple of hours. Besides, not all construction works have multiple traces available. This means no average can be made, and outstanding train passages can have a big influence on the graph.

As a reaction to this, an overview is created containing the graphs of March, July and every day during construction works. This overview can be seen in appendix T. In this overview, the shift of lines is more clear. A disadvantage is that every day during construction works contains multiple types of work, which makes the comparison a bit unrealistic.

Hence, a different approach is chosen. During the construction phase, small settlements and large settlements occurred. Chosen is to analyse 3 days in a row with small settlements, and 3 days in a
row with large settlements. In this way two averages are made over the observed data, which are comparable with the average of the initial situations.

A settlement curve can be found in appendix U. During 2 till 5 August, almost no settlements occurred. From 3 to 4 August, the settlements even decreased. Therefore, this situation is seen as a period in which the construction was at rest. After the 5th of August, the construction starts to settle again. During the 6th, 7th and 8th of August, the pillar settled with a maximum of 0,5 millimetres a day. These days are therefore chosen as the days when small settlements occurred. After these three days, the settlements became larger. During the 9th, 10th and 11th of August, the pillar settled with a maximum of 2 millimetres a day. These three days are therefore chosen to observe the construction during large settlements. For the average of each day, 5 to 8 train passages are analysed.

5.1.5 After construction works

After all diaphragm walls around pillar 1 are constructed, the vibration measurements continue. The data that is used for the analysis is selected in the same way as the data for the initial phase. So over 3 days, 9 train passages are analysed.

5.1.6 Resume

So finally, the data that is used for the analysis and comparison are:

<table>
<thead>
<tr>
<th>Situation</th>
<th>Month</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial situation 1</td>
<td>March</td>
<td>27th, 28th and 29th of March 2013</td>
</tr>
<tr>
<td>Initial situation 2</td>
<td>July</td>
<td>18th, 19th and 20th of July 2013</td>
</tr>
<tr>
<td>During small settlements</td>
<td>August</td>
<td>6th, 7th and 8th of August 2013</td>
</tr>
<tr>
<td>During large settlements</td>
<td>August</td>
<td>9th, 10th and 11th of August 2013</td>
</tr>
<tr>
<td>Post situation</td>
<td>October</td>
<td>3rd, 4th and 5th of October 2013</td>
</tr>
</tbody>
</table>

Table 8: Data used for the analysis

5.2 Analysis of the results

The analysis phase will be briefly discussed in this paragraph.

All frequency spectra made by Fugro GeoService B.V. are more or less identical to the one showed in Graph 34. On the x-axis the frequency of the vibrations is plotted in hertz. On the y-axis, the velocity is plotted in millimetres per second, divided by the square root of the corresponding frequency. Probably, this is done in order to compare the higher frequencies with the lower frequencies in a glance. Since the higher frequencies have a higher velocity, dividing by the square root of the corresponding frequency makes the values of the y-axis more equal.

The red, blue and green line show the vibrations in X-, Y- and Z-direction respectively.
Graph 34: Frequency spectrum of a trace, measured by Fugro GeoServices B.V.

To observe the change in stiffness of the soil, only the lower frequency range, from 0 to 25 hertz, is considered important. Therefore the frequency spectrum is zoomed in, as can be seen in Graph 35. The frequency spectrum has multiple peak frequencies in the lower frequency range. It is assumed that all vibrations recorded in a particular direction have an average of 4 clear frequency peaks in the range below 15 hertz. The value for these peak frequencies with the corresponding velocity is noted for every graph. An example can be seen in Graph 35, where the 4 peak values for the vibration in the Z-direction are defined.

Graph 35: Analysis of the frequency spectrum

In this way all graphs are analysed and put in Excel. Finally, a graph is made showing the five different phases of measurement. See Graph 36, Graph 37 and Graph 38. The velocities of the vibrations are shown on the y-axis for the sake of completeness, although conclusions have already
been made about the incorrect velocities after a Fast Fourier Transform. But in this part of the research, the values of the frequencies and the velocities are an average of multiple vibrations. Therefore, there may also be a useful trend visible in the progress of the vibrations.

Graph 36: Comparison frequencies and velocities during different phases, X-direction

Graph 37: Comparison frequencies and velocities during different phases, Y-direction
5.2.1 Analysing the frequency peaks

The average frequency of each peak is now known. Next, a selection can be made. The peaks that will be analysed further are the ones that may have a relation to the eigenfrequency (i.e. stiffness) of the soil. To determine which peaks are important to observe, the conclusions from the previous chapters are taken into account. It is assumed that the eigenfrequency of the soil is in the zone between 5 and 15 hertz. According to this, the two peak frequencies around 8 and 10 hertz are analysed. The frequencies below 5 hertz are not further investigated.

The values of the third and fourth peak frequency are summarized in the following tables Table 9, Table 10, and Table 11. The tables show the frequencies of the vibrations with the corresponding velocities in X, Y, and Z-direction. The moment of measurement is shown in the first column, in chronological order. The frequencies and velocities shown in the second and third column are the values associated with the third peak of the frequency spectrum. The frequencies and velocities shown in the fourth and fifth column are the values that are associated with the fourth peak. The frequencies are shown in hertz. The velocities are converted to millimetres per second. The plus and minus signs indicate if the value is decreased (minus sign) or increased (plus sign) with respect to the value from the previous phase.

<table>
<thead>
<tr>
<th>X-direction</th>
<th>Values of 3rd peak</th>
<th>Values of 4th peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>When</td>
<td>Frequency [Hz]</td>
<td>Velocity [mm/s]</td>
</tr>
<tr>
<td>Initial March</td>
<td>7,44</td>
<td>0,28</td>
</tr>
<tr>
<td>Initial July</td>
<td>7,66 (+)</td>
<td>0,34 (+)</td>
</tr>
<tr>
<td>Small settlements</td>
<td>6.93 (-)</td>
<td>0,23 (-)</td>
</tr>
<tr>
<td>Large settlements</td>
<td>7.08 (+)</td>
<td>0,21 (-)</td>
</tr>
<tr>
<td>Post phase</td>
<td>7,33 (+)</td>
<td>0,33 (+)</td>
</tr>
</tbody>
</table>

Table 9: Peak frequency and velocity values, X-direction
In advance, a decrease in stiffness should mean a decrease in frequency of the soil according to formula 2.10. As a response, a decrease in frequency should mean an increase in velocity of the vibration, according to formula 3.10. Initially, the peak frequency is observed. Remind that the velocities may not correspond with the real velocities, as proven in paragraph 3.4.

A logical progression of the frequencies should be a decrease of all values, except the last value. The soil should be stiffer in March than in July due to the high water table (as discussed earlier in paragraph 5.1.3). In August, the construction works start, which should cause a decrease in stiffness of the soil. Finally, in the post phase, the diaphragm walls are installed and the soil is ‘locked’. It can only compact due to the vertical forces of the trains. This means, the stiffness of the soil should recover (i.e. increase) again.

What can be concluded from the tables above is that the values corresponding to the third peak are not showing this progression. What is most striking is the lower frequency in March than in July in all three directions of measurement. Although it has to be noted that changes are small, it seems that the third peak frequencies do not have a relation to the stiffness of the soil.

On the other hand, the values corresponding to the fourth peak frequencies do show a logical progression. Especially during construction and during the post phase, the frequencies develop as expected. It is therefore assumed that these frequencies do have a relation to the stiffness of the soil.

Comparing the measured frequency in March and July in X-direction, some variation can be seen. But in Y and Z-direction, the eigenfrequency in March and July is the same. Therefore, it can be said that the difference in groundwater table does not have a significant influence on the eigenfrequency of the system. This means that an observed change in eigenfrequency is caused by a change in stiffness of the system, and not by a change in environmental conditions.
When comparing the changes in eigenfrequencies in the X, Y and Z-direction, it can be noted that the changes in X-direction are the smallest. Since the distance from the pillar to the constructed diaphragm walls is smaller in Y than in X-direction, the stiffness in Y-direction should indeed decrease more than in X-direction. Therefore, this result also implies that the monitoring system is working correctly.

When observing the corresponding velocities of the vibrations, it can be concluded that the progression is the opposite of what was expected. In the Y-direction, the velocity values are more or less the same during all measurements. But in the X and Z-direction, the velocity decreases when the stiffness decreases and vice versa. Since the changes of the velocities are really small, it is possible that this change in velocity is a coincidence and due to leakage, see paragraph 3.4.

To see whether or not these results give a realistic value for the change in stiffness of the system, the results are compared with results following from the analytical model. Afterwards, they are compared with an approximation of the stiffness change during construction works. This is done by coupling the settlements that occurred in Nijmegen to an approximated change in stiffness of the soil.

The analytical model only analysed vibrations in the Z-direction. Therefore, also in this part of the research vibrations in Z-direction are analysed to be able to make a good comparison. The change in frequency in the Z-direction according to the dataset of Fugro GeoServices B.V., when comparing the initial state with the large settlement state, is:

\[ \text{Frequency decrease} = (f_n - f_o) = (9.67 - 10.17) = -0.5 \text{ Hz} \]

\[ \text{Frequency decrease} = \left( \frac{f_n}{f_o} \right) = \left( \frac{9.67}{10.17} \right) = -4.9\% \]

This change in frequency can be coupled to a change in stiffness by rewriting formula 2.10 for \( k \). The exact mass of the system is irrelevant, since this mass remains the same before and after construction of the diaphragm walls. Therefore, the mass does not have an influence on the change in stiffness, when the change is expressed as a percentage.

The frequency decrease of -4.9% as calculated above, gives the following relative change in stiffness:

\[ \text{Old stiffness } k = \left( \frac{f}{\frac{1}{2\pi}} \right)^2 \cdot m = \left( \frac{10.17}{\frac{1}{2\pi}} \right)^2 \cdot m = 4.1 \cdot 10^3 \cdot m \]

\[ \text{New stiffness } k = \left( \frac{f}{\frac{1}{2\pi}} \right)^2 \cdot m = \left( \frac{9.67}{\frac{1}{2\pi}} \right)^2 \cdot m = 3.7 \cdot 10^3 \cdot m \]
Which seems to be a very small, but plausible decrease.

The small change in stiffness can be explained in two ways. On beforehand, only a slight change in stiffness was expected. The first explanation is that the stiffness during construction indeed did not change that much. This would mean that the monitoring system is working. A second explanation can be that a change in stiffness of the soil cannot be observed by monitoring the change in eigenfrequencies of the soil, because the changes are too small.

The first explanation can be further investigated by estimating the decrease in stiffness of the soil. This is done by coupling the change in stiffness of the soil to the settlements that occurred during construction. Of course, the exact change in stiffness cannot be calculated in this way. But, the calculation is used only as an approximation of the change in soil stiffness. It is used to check the order of magnitude, and to compare this order with the order calculated with the measured change in eigenfrequency.

During construction works, the pillar settled. These settlements can be related to a stiffness change of the soil. The construction works around pillar 1 started at the 22nd of July. The last measurement of the pillar’s settlement dates from October 22nd. The maximum settlement that occurred during this period of time contained 41,4 millimetres at the east side of the pillar, and 33,8 millimetres at the west side of the pillar. The profound system of Fugro GeoServices B.V. is located at the west side of the pillar. Therefore, the 33,8 millimetres of settlement will be used to calculate the change in stiffness of the soil.

The stiffness is calculated with formula 3.1. The initial stiffness of the soil was calculated with Plaxis and amounts 17,500,000,000 N/m. It was then assumed that the soil was already fully settled by the dead weight of the construction. Only the passing trains caused an extra settlement of 8 millimetres. Now, the soil is being rearranged due to the construction works. This means that not only the dynamic load of the passing trains will cause settlements, but also the dead load of the construction. Since the dynamic stiffness of the soil is much larger than the static stiffness, the dynamic load will cause small settlements compared to the dead load. Therefore, this dynamic load is ignored for the approximation of the change in soil stiffness. Only the dead load is considered responsible for the 33,8 millimetres of settlement of the pillar.

The dead load amounts 5,500,000 kilograms (i.e. 55,000,000 Newton) as calculated in paragraph 3.1.1. This gives a change in stiffness of the soil of:

\[
\Delta k = \frac{F}{\Delta} = \frac{55,000,000 \text{ N}}{0,0338 \text{ m}} = 1,63 \cdot 10^9 [\text{N/m}]
\]

This means that the soil stiffness during construction works has been reduced with:
This change of \(-9.3\%\) is comparable with the \(-9.6\%\) calculated from the analysed measurements of Fugro GeoServices B.V. So, it seems that the measured values in the field and the calculated values from the settlements are comparable. Therefore, it can be said that there was indeed only a small change in stiffness during the construction works. And it is possible to observe this small change in stiffness with the results from the vibration measurements that have been made.

\[
\frac{\Delta k}{k_0 \cdot \frac{100}{100}} = \frac{-1.63 \cdot 10^9}{-1.75 \cdot 10^{10}} = -9.3\%
\]

5.3 Conclusion

A decrease in soil stiffness is detectable by monitoring the decrease in eigenfrequency of the soil. In this particular case, the changes were very small but comparable with expected and calculated values.

With this research it is proven that a decrease in eigenfrequency occurs when settlements occur, so when the stiffness of the soil is already decreased. Because during this particular project the construction almost immediately started to settle, it was not possible to conclude if vibration monitoring shows changes before settlements occur. It can therefore not be said if vibration monitoring can act as an early warning system. But there is reason to believe it can, since the decrease in frequency of 0.5 hertz in the Z-direction was already visible during small settlements of the construction, as can be seen from the tables Table 9, Table 10, and Table 11. After these small settlements, the settlements increased further while the frequency did not further decrease. This means that the frequency was at its minimum value before settlements were at their maximum value. Further investigation is required with respect to this.

When comparing the initial eigenfrequency (July) and the eigenfrequency in the post phase (October), it seems plausible to conclude that the stiffness of the soil has recovered or is recovering to the original conditions. This is as expected. After installation of the diaphragm walls, the stiffness is supposed to increase again since the soil is being compacted in the box of diaphragm walls.
6. Conclusions and recommendations

In this final chapter, the conclusions and recommendations with respect to this Master's thesis will be outlined. The problem statement of the research was:

*Monitoring systems, such as traditional deformation monitoring, do not give information about the stiffness behaviour of a soil-structure system*

In order to find a type of monitoring which can offer information about the dynamic response (i.e. stiffness behaviour) of a soil-structure system, a mass-spring model is used to model a change in soil stiffness and to observe the shifts in eigenfrequencies of the soil-structure system. To verify the model, ambient vibrations are measured in the field after which both results are coupled.

6.1 Conclusions

It is possible to observe a change in stiffness of a soil-structure system by shifts in eigenfrequencies, using rail traffic induced vibrations as a vibration source. Even though the ambient vibrations are in the higher frequency range (i.e. in the range of 50 to 100 hertz), vibrations around the soil stiffness eigenfrequencies are recognizable in the measurements. A restriction that has to be made is that only a relative stiffness change with respect to the initial stiffness can be observed when a Fast Fourier Transform is used to analyse the data. By a relative change is meant the change in stiffness with respect to the initial stiffness, expressed as a percentage.

6.1.1 Research question 1

*To what extent does the stiffness of the subsoil contribute to the total response of the system, and can the frequency components caused by the soil and the frequency components caused by the structural part of the system be distinguished from each other?*

The situation can be modelled with an analytical mass-spring model. When using a two-mass-spring model, the peak frequency in the lower frequency range represents the eigenfrequency of the soil. The peak frequency in the higher frequency range represents the eigenfrequency of the structure. With a changing stiffness of the construction, the eigenfrequency of the construction changes significantly while the change in eigenfrequency of the soil is not worth mentioning. The opposite is also true. When the stiffness of the soil decreases, the eigenfrequency of the soil decreases significantly while the eigenfrequency of the structure remains almost unchanged.

By knowing this it can be concluded that for observing a change in soil stiffness, only changes in the lower frequency range are of importance.
6.1.2 The Fast Fourier Transform

During analysis of the model, attention was also concentrated on the Fast Fourier Transform. When analysing data after a Fast Fourier Transform, only a relative change in stiffness can be monitored. Since it is impossible to determine the phase of the signal when dealing with ambient vibrations, a phase difference between the first and last data point cannot be avoided. This will cause leakage during a Fast Fourier Transform. Due to leakage, the velocities in the Fast Fourier frequency spectrum do not correspond to the real corresponding velocities of the eigenfrequency. The differences can be even more than 30%, where the velocities after a Fast Fourier Transform are always lower than the velocities in reality.

With respect to the approximation of the real velocity of the mass, the difference is caused only by a different phase from the starting point and final point of the analysed vibration. A different sampling frequency does not make a difference. On the other hand, a higher sampling frequency (i.e. a smaller frequency resolution) will give a better approximation of the eigenfrequency.

6.1.3 Research question 2

Are there visible changes in the measured frequencies of the system when comparing measurements of the initial state with measurements made during the construction and post phase and if so, do the observed changes correspond to the changes predicted with the analytical model?

From the dataset recorded by the author in cooperation with the Municipality of Rotterdam it can be concluded that the eigenfrequency of the structure increases significantly after the installation of hydraulic jacks. Both the frequency peak values representing the pillar and the girder increase. The eigenfrequency of the soil-part of the system remains almost unchanged after installation of the jacks.

This also follows from the analytical mass-spring model, as described above. The hydraulic jacks make the joint in between the girder and the pillar stiffer, which means they are changing the stiffness of the construction part of the system. According to the mass-spring model, this should indeed cause a significant change in eigenfrequency of the structure, while the change in eigenfrequency of the soil can be neglected.

After installation of the jacks, the structure of the input signal (i.e. geophone 1) changes. But the reaction of the soil remains the same. Therefore, it can be said that the reaction of the soil depends on the amount of energy that is put into the system, and not so much on the structure of the input signal. The amount of energy remains the same, since the type of train and its velocity during construction works do not change.

The eigenfrequency of the soil cannot be determined really precise. But when observing the results from more closely, it seems indeed that the eigenfrequency of the soil decreased.

With the traces recorded by Fugro GeoServices B.V. it is possible to analyse and compare the results from the initial phase, the construction phase and the post phase. It can be concluded that a change in stiffness of the soil can be observed by a shift in eigenfrequencies. A decrease in eigenfrequency
with respect to the initial measured eigenfrequency is observed during the construction works. The decrease in eigenfrequency is small but comparable to the estimated amount calculated with the formulas and mass-spring model.

In the post phase, the eigenfrequency increases again. This too is as expected, since after installation of the diaphragm walls the stiffness is supposed to recover again because of soil compaction inside the diaphragm walls.

When comparing the changes in eigenfrequencies in the X, Y and Z-direction, it can be noted that the changes in X-direction are the smallest. This implies that the monitoring system works correctly. Since the distance from the pillar to the construction works is larger in X than in Y-direction, the stiffness of the soil-structure system in X-direction should indeed decrease less than in Y-direction.

6.1.4 Research question 3

Is it possible to set up a monitoring system that monitors the relative stiffness change of the soil-structure system during construction works and, if so, is the monitoring system that is in operation in Nijmegen able to monitor a stiffness change?

The system that is in operation in Nijmegen proved to be able to observe a change in stiffness of the soil. It is sufficient to measure at one location when focussing on the lower frequency range.

However, some adjustments can be made. For example, when only the stiffness of the soil is of interest, a low pass filter can be used since only the low frequencies are of importance. The higher frequencies are of importance for observing the stiffness of the construction. They do not show a significant decrease when the stiffness of the soil is changing.

6.2 Relevance

The relevance of this research for the project in Nijmegen is the conclusion that the stiffness of the system did not change significantly during construction works. Also, the stiffness recovered after the construction works. This means that the construction does not have to be adapted, and costs for repairing the construction can be saved.

This can also be relevant for other projects where the deformation of a construction is rather irrelevant if the stiffness of the construction is not being influenced significantly. For these projects a vibration monitoring system, which monitors shifts in eigenfrequency, can offer information about the dynamic response (i.e. stiffness behaviour) of the construction, additionally to the traditional deformation monitoring.
6.3 Recommendations

Overall, when using a vibration monitoring system to observe a change in stiffness, it is recommended to first check the frequency range in which the frequencies of interest are situated. A simple mass-spring system can be of good use for a first approximation. Also, measuring a larger frequency range will give useful information. When the frequency range of interest is known, a filter can be used to minimize the amount of data, the time needed to analyse the data, and therefore the costs of the measurement.

Because of a limited time span, some interesting phenomena with respect to this research were not investigated into detail. Therefore, some further investigations will be suggested in this paragraph.

6.3.1 Recommendations related to the research

Because of the leakage occurring after Fast Fourier Transform, only a relative stiffness change can be monitored with vibration measurements. In this research, leakage is only briefly investigated. But the Fast Fourier Transform is very much used to generate the frequency spectrum during all kinds of vibration measurements, all over the world. When leakage is not taken into account, this can have an enormous impact on the implementation of vibration measurements. For a better understanding of the impact of leakage, and to get a better grip on this phenomenon, it is recommended to investigate leakage more into detail. This can for instance be done by a mathematician.

It was suggested that when vibration measurement would show changes sooner than measurements like normal displacement measurements do, a monitoring system based on vibration measurements might be able to function as an early warning system. During this particular project in Nijmegen, the construction almost immediately started to settle. Therefore, it was not possible to conclude if a shift in eigenfrequencies was visible before settlements occurred. But there is a reason to believe that eigenfrequencies decrease before settlements occur, since the eigenfrequency was already decreased during small settlements of the construction, and since it did not further decrease while the settlements increased a lot further. Further investigation is required with respect to this.

6.3.2 Recommendations only partly related to the research

During the empirical part of the research, vibrations were measured in three different directions on ten different locations on and around the structure of the railway bridge. For this research, only particular results were used which were interesting with respect to the stiffness of the soil. But it would be interesting to use the other available information to investigate if tilting of the construction and attenuation of different materials could also be monitored by vibration measurements. When investigating this, the raw data have to be used. The data after a Fast Fourier Transform will not provide the information that is necessary about the real velocity of the vibrations.
7. References


8. Appendices

Appendix A. Drawing of the pillar including sensor setup
| t (s) | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.005 | 1.05E-06 | 1.554E-07 | -1.102E-09 | 1.04E-09 | -3.336E-13 | 3.480E-07 | 2.014E-17 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Name | Symbol | Format | Value | Unit |
| 0.01 | 2.02E-06 | -4.757E-07 | -3.194E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Mass | m | 5.0000E+00 |
| 0.015 | 3.03E-06 | -5.032E-07 | -3.010E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Spring constant | k | 1.7E+00 |
| 0.02 | 4.05E-06 | -5.954E-07 | -3.923E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Critical Damping | c | 2.00E-09 |
| 0.025 | 5.07E-06 | -6.854E-07 | -4.823E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Damping | c | 2.50E-09 |
| 0.03 | 6.20E-06 | -7.686E-07 | -5.760E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Damping coefficient | c | 3.00E-09 |
| 0.035 | 7.34E-06 | -8.546E-07 | -6.658E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Time step | dt | 1.00E-09 |
| 0.04 | 8.50E-06 | -9.346E-07 | -7.564E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | S | 3.750E-12 |
| 0.044 | 9.70E-06 | -1.027E-06 | -8.504E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Start value | S | 1.75E+00 |
| 0.05 | 1.05E-05 | -1.200E-06 | -9.404E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Difference (%) | S | -1.75E+00 |
| 0.055 | 1.14E-05 | -1.394E-06 | -1.036E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | New value | S | 0.00E+00 |
| 0.06 | 1.24E-05 | -2.095E-06 | -1.596E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Mass | m | 5.0000E+00 |
| 0.065 | 1.36E-05 | -2.894E-06 | -2.176E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Mass | m | 5.0000E+00 |
| 0.07 | 1.49E-05 | -3.685E-06 | -2.768E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | Mass | m | 5.0000E+00 |
| 0.075 | 1.63E-05 | -4.494E-06 | -3.370E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | New value | c | 0.00E+00 |
| 0.08 | 1.78E-05 | -5.305E-06 | -4.008E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | New value | c | 0.00E+00 |
| 0.085 | 1.95E-05 | -6.125E-06 | -4.850E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | New value | c | 0.00E+00 |
| 0.09 | 2.12E-05 | -6.950E-06 | -5.706E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | New value | c | 0.00E+00 |
| 0.095 | 2.30E-05 | -7.786E-06 | -6.606E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | New value | c | 0.00E+00 |
| 0.10 | 2.50E-05 | -8.640E-06 | -7.532E-09 | 1.00E-09 | 9.06E-09 | 1.00E-09 | 3.410E-12 | 8.893E-18 | 1.778E-12 | New value | c | 0.00E+00 |

Note:
- A positive difference will add a value to the start value.
- A negative difference will subtract a value from the start value.
<table>
<thead>
<tr>
<th>Row</th>
<th>What?</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>t</td>
<td>Time steps dt [s]</td>
<td>0.005</td>
</tr>
<tr>
<td>B</td>
<td>F(t)</td>
<td>Input force from the train [N]</td>
<td>Given values for every time step dt</td>
</tr>
<tr>
<td>C</td>
<td>Fkt</td>
<td>Sum of spring forces [N]</td>
<td>$-x(t-1) \times k$</td>
</tr>
<tr>
<td>D</td>
<td>Fct</td>
<td>Sum of damper forces [N]</td>
<td>$-c(t-1) \times v$</td>
</tr>
<tr>
<td>E</td>
<td>SumF</td>
<td>Sum of all forces [N]</td>
<td>Sum row B, C and D</td>
</tr>
<tr>
<td>F</td>
<td>at</td>
<td>Acceleration of the mass [m/s²]</td>
<td>$at = \text{sum } F / m$</td>
</tr>
<tr>
<td>G</td>
<td>vt</td>
<td>Velocity of the mass [m/s]</td>
<td>$v_t = v_t(t-1) + at \times dt$</td>
</tr>
<tr>
<td>H</td>
<td>xt</td>
<td>Displacement of the mass [m]</td>
<td>$x_t = x_t(t-1) + v_t \times dt$</td>
</tr>
<tr>
<td>I</td>
<td>v(t-mm/s)</td>
<td>Velocity of the mass in [mm/s]</td>
<td>$v_t \text{ [mm/s]} = v_t \text{ [m/s]} \times 1000$</td>
</tr>
</tbody>
</table>
Appendix C. Fast Fourier Transform in Excel

Description of creating an FFT in Excel

Make 2 blocks
- **Sampling rate** = Fugro samples at 200 hertz (i.e. 200 times per second), which means the sampling rate is 200.
- **Data points** = The maximum amount of data points that can be analysed in Excel is 4096. The more data points, the more accurate the result. So chosen are 4096 data points.

Make 4 columns
- **Numbering** = 0:2048
- **FFT Frequency** = the column with the values for the frequency axis (i.e. the x-axis) in the frequency spectrum. To calculate this value:
  \[ \text{Corresponding numbering} \times (\text{sampling rate} / \text{data points}) \]
  Since the sampling frequency of Fugro is 200, the frequency axis goes till 200/2 = 100.
- **FFT Magnitude** = the magnitude that corresponds with the frequency (i.e. the y-axis of the frequency spectrum). This magnitude can be calculated with the formula:
  \[ 2/\text{data point} \times \text{C.ABS('corresponding FFT Complex')} \]
  C.ABS takes the absolute value of the complex number that is in the FFT Complex column
- **FFT Complex** = The real and complex value that comes out of the Fourier Analysis as described below.

So for example for row 2 the formulas are:

<table>
<thead>
<tr>
<th>Numbering</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT Frequency</td>
<td>=A2*($H$2/$G$2)</td>
</tr>
<tr>
<td>FFT Magnitude</td>
<td>=2/$G$2*C.ABS(D2)</td>
</tr>
<tr>
<td>FFT Complex</td>
<td>Data out of Fourier Analysis</td>
</tr>
</tbody>
</table>

To start the Analysis:
- Tools – data analysis – Fourier Analysis
  The input range of the data for the analysis is column I of the one-mass-spring system (figure appendix B) from 2:4097.
- This gives an outcome of real and complex numbers. This output has to be put in the FFT Complex column. All columns will now be filled.
- The Frequency spectrum can be drawn by creating a scatter plot graph. On the x-axis the FFT Frequency is plotted. On the y-axis the FFT Magnitude is plotted.
Appendix D. Frequency and velocity calculated with formulas in Excel

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>velocity of vibrations</td>
<td>With this the velocity of the mass can be calculated, with different stiffness and a fixed force</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>displacement x₀ due to force F</td>
<td>Red = to be filled in</td>
<td>F</td>
<td>2000000 N</td>
<td>k</td>
<td>f</td>
<td>v</td>
<td>decrease k</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>F = - k * x₀</td>
<td>x₀ = F / -k</td>
<td>k</td>
<td>1,75E+10 N/m</td>
<td>100</td>
<td>1,75E+10</td>
<td>8,98</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>With F = force</td>
<td>x₀</td>
<td>0,0001143 m</td>
<td>90</td>
<td>1,57E+10</td>
<td>8,32</td>
<td>10</td>
<td>20,29</td>
<td>-10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>k = stiffness of spring/soil</td>
<td>m</td>
<td>5500000 kg</td>
<td>70</td>
<td>1,22E+10</td>
<td>7,11</td>
<td>40</td>
<td>11,77</td>
<td>-50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>x₀ = displacement</td>
<td>k</td>
<td>1,75E+10 N/m</td>
<td>60</td>
<td>1,65E-10</td>
<td>6,95</td>
<td>40</td>
<td>10,19</td>
<td>-40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>With T = 2π/ω</td>
<td>ω</td>
<td>0,11338897 sec</td>
<td>50</td>
<td>875000000</td>
<td>6,35</td>
<td>50</td>
<td>9,12</td>
<td>-50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>W = frequency in rad/s</td>
<td>W</td>
<td>56,4076075 rad/sec</td>
<td>40</td>
<td>700000000</td>
<td>5,58</td>
<td>60</td>
<td>8,32</td>
<td>-60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>With T = 2π/k</td>
<td>k</td>
<td>0,02784724 sec</td>
<td>30</td>
<td>525000000</td>
<td>4,92</td>
<td>70</td>
<td>7,71</td>
<td>-70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>m = mass of the system</td>
<td>v</td>
<td>0,00644658 m/s</td>
<td>10</td>
<td>175000000</td>
<td>2,84</td>
<td>90</td>
<td>6,8</td>
<td>-90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>k = stiffness of spring/soil</td>
<td>6,4468371 mm/s</td>
<td>0</td>
<td>100</td>
<td>6,45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

By formulas

![Graph showing frequency and velocity change with stiffness change](image)

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### Excel file 1: Mass-Spring Model with Pulse Load

#### Data Table

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Mass</td>
<td>Start value</td>
<td>Difference</td>
<td>[%] New value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>l1</td>
<td>1.7321</td>
<td>0</td>
<td>0.7321</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>m</td>
<td>0.0500</td>
<td>0</td>
<td>0.0500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Notes

- A positive difference will add a value to the start value.
- A negative difference will subtract a value from the start value.

---

---

---

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### Appendix F

#### FFT analysis in Excel with pulse load

<table>
<thead>
<tr>
<th>FFT Frequency</th>
<th>FFT Magnitude</th>
<th>FFT Complex</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0150000000</td>
<td>0.100671785333999</td>
</tr>
<tr>
<td>0.35625</td>
<td>0.0020000000</td>
<td>0.384505547005623996</td>
</tr>
<tr>
<td>0.707125</td>
<td>0.0040000000</td>
<td>0.727297794114334966</td>
</tr>
<tr>
<td>1.0625</td>
<td>0.0060000000</td>
<td>1.06254112112112112</td>
</tr>
<tr>
<td>1.5625</td>
<td>0.0080000000</td>
<td>1.56254112112112112</td>
</tr>
<tr>
<td>2.0000</td>
<td>0.0100000000</td>
<td>2.00004112112112112</td>
</tr>
<tr>
<td>2.5000</td>
<td>0.0120000000</td>
<td>2.50004112112112112</td>
</tr>
<tr>
<td>3.0000</td>
<td>0.0140000000</td>
<td>3.00004112112112112</td>
</tr>
<tr>
<td>3.5000</td>
<td>0.0160000000</td>
<td>3.50004112112112112</td>
</tr>
<tr>
<td>4.0000</td>
<td>0.0180000000</td>
<td>4.00004112112112112</td>
</tr>
<tr>
<td>4.5000</td>
<td>0.0200000000</td>
<td>4.50004112112112112</td>
</tr>
<tr>
<td>5.0000</td>
<td>0.0220000000</td>
<td>5.00004112112112112</td>
</tr>
<tr>
<td>5.5000</td>
<td>0.0240000000</td>
<td>5.50004112112112112</td>
</tr>
<tr>
<td>6.0000</td>
<td>0.0260000000</td>
<td>6.00004112112112112</td>
</tr>
<tr>
<td>6.5000</td>
<td>0.0280000000</td>
<td>6.50004112112112112</td>
</tr>
<tr>
<td>7.0000</td>
<td>0.0300000000</td>
<td>7.00004112112112112</td>
</tr>
<tr>
<td>7.5000</td>
<td>0.0320000000</td>
<td>7.50004112112112112</td>
</tr>
<tr>
<td>8.0000</td>
<td>0.0340000000</td>
<td>8.00004112112112112</td>
</tr>
<tr>
<td>8.5000</td>
<td>0.0360000000</td>
<td>8.50004112112112112</td>
</tr>
</tbody>
</table>

#### Change Frequency and Velocity

- **Frequency Spectrum after FFT**
- **Velocity** (mm/s)

![Velocity vs Frequency Graph](image)

- **Frequency change Formula**
- **Velocity change Formula**

<table>
<thead>
<tr>
<th>Stiffness % as a percentage of the initial stiffness (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

---

Data points: 512
Sampling rate: 200
Sum area below graph: 23.6202
Velocity change Formula: $V = \frac{\Delta F}{\Delta t}$
What has been done?

4 different FFT graphs have been made
- 200 Hz
- 400 Hz
- 800 Hz
- ‘Perfect FFT’ (described in section 3.4.3)

An FFT has been performed over the data of the one-mass-spring model as described in appendix F (with pulse load). Depending on the sampling frequency, the time step \( dt \) has been modified. The time which is analysed has to be the same for each sampling frequency. Hence, when 200 Hz (with a \( dt \) of 0,005) has 1024 samples, 400 Hz (with a \( dt \) of 0,0025) needs 2048 samples and 800 Hz (with a \( dt \) of 0,00125) needs 4098 samples to analyse the same total time.

This has been done for each frequency for the stiffness \( k \) initial till \( k=10\% \) of the initial value, with steps of -10\%. The peak frequency and corresponding velocity are noted each time, and in the end plotted in a graph shown in the figure above (Change Frequency and Velocity).
Appendix G. Excel file two-mass-spring model
<table>
<thead>
<tr>
<th>Row</th>
<th>What?</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>t</td>
<td>Time steps dt [s]</td>
<td>0.005</td>
</tr>
<tr>
<td>B</td>
<td>F(t)</td>
<td>Input force from the train [N]</td>
<td>Train input force for every time step</td>
</tr>
<tr>
<td>C</td>
<td>Fkt1</td>
<td>Sum of spring forces [N]</td>
<td>$= - k_1 * x_t1(t-1) + k_1 * x_t2(t-1)$</td>
</tr>
<tr>
<td>D</td>
<td>Fct1</td>
<td>Sum of damper forces [N]</td>
<td>$= - c_1 * v_t1(t-1) + c_1 * v_t2(t-1)$</td>
</tr>
<tr>
<td>E</td>
<td>SumF1</td>
<td>Sum of all forces [N]</td>
<td>Sum row B, C and D</td>
</tr>
<tr>
<td>F</td>
<td>at1</td>
<td>Acceleration of the mass [m/s²]</td>
<td>$a_t1 = \frac{\text{somF1}}{m_1}$</td>
</tr>
<tr>
<td>G</td>
<td>vt1</td>
<td>Velocity of the mass [m/s]</td>
<td>$v_t1 = v_t1(t-1) + a_t1 \cdot dt$</td>
</tr>
<tr>
<td>H</td>
<td>xt1</td>
<td>Displacement of the mass [m]</td>
<td>$x_t1 = x_t1(t-1) + v_t1 \cdot dt$</td>
</tr>
<tr>
<td>I</td>
<td>vt1(mm/s)</td>
<td>Velocity of the mass in [mm/s]</td>
<td>$v_t1 \text{ [mm/s]} = v_t1 \text{ [m/s]} \times 1000$</td>
</tr>
<tr>
<td>P</td>
<td>Fkt2</td>
<td>Sum of spring forces [N]</td>
<td>$= - (k_2+k_1) * x_t2(t-1) + k_1 * x_t1(t-1)$</td>
</tr>
<tr>
<td>Q</td>
<td>Fct2</td>
<td>Sum of damper forces [N]</td>
<td>$= - (c_2+c_1) * v_t2(t-1) + c_1 * v_t1(t-1)$</td>
</tr>
<tr>
<td>R</td>
<td>SumF2</td>
<td>Sum of all forces [N]</td>
<td>Sum row P and Q</td>
</tr>
<tr>
<td>S</td>
<td>at2</td>
<td>Acceleration of the mass [m/s²]</td>
<td>$a_t2 = \frac{\text{somF2}}{m_2}$</td>
</tr>
<tr>
<td>T</td>
<td>vt2</td>
<td>Velocity of the mass [m/s]</td>
<td>$v_t2 = v_t2(t-1) + a_t2 \cdot dt$</td>
</tr>
<tr>
<td>U</td>
<td>xt2</td>
<td>Displacement of the mass [m]</td>
<td>$x_t2 = x_t2(t-1) + v_t2 \cdot dt$</td>
</tr>
<tr>
<td>V</td>
<td>vt2(mm/s)</td>
<td>Velocity of the mass in [mm/s]</td>
<td>$v_t2 \text{ [mm/s]} = v_t2 \text{ [m/s]} \times 1000$</td>
</tr>
</tbody>
</table>
Appendix H. Outline situation Nijmegen
Appendix I. Drawing of construction works during measurement 2

Pink = diaphragm wall finished
Blue = diaphragm wall open and filled with bentonite
White = construction works of the diaphragm wall did not start yet
Appendix J. Matlab code for matrix with all times

```matlab
function DataWithTime = saveCorrectMatrix(file)

load(file); %Loads the data as a matrix with the name ‘double_data’,
%with on the first row the times and on the second row the corresponding
millivolts measured
DataNoTime = double_data; %Renames ‘double_data’ to ‘DataNoTime’
start = DataNoTime(1,1); %The start value of time is the first value of the
first row of the matrix ‘DataNoTime’
last = DataNoTime(1,end); %The end value of time is the last value of the
first row of the matrix ‘DataNoTime’
Sensitivity = 23.3; %Recalculates the millivolts to velocity: Vs/m =
volt*second/meter

C = DataNoTime(1,:);
Volt = DataNoTime(2,:);

% K=0; %check; in the end K has to be the amount of values in the matrix
without time

B = (Volt/Sensitivity) * 1000; %Gives velocity vibration = mm/s

LastTime = last;
time = start; %Start value time
deltaTime = C(2)-C(1); %Normal time step in between samples

DataWithTime = zeros(2,((LastTime-time)/deltaTime)); %Fills the matrix
with zeros when there is no sample
place = 1; %Place of column in where it all starts

for n = 1 : length(C)
    if(time == C(n))
        DataWithTime(1,place) = C(n);
        DataWithTime(2,place) = B(n);
        %K = K + 1;
        place = place + 1;
        time = time + deltaTime;
    else
        while(time < C(n))
            if((time+deltaTime) >= C(n))
                DataWithTime(1,place) = C(n);
                DataWithTime(2,place) = B(n);
                %K = K + 1;
            else if(time <(C(n))
                DataWithTime(1,place) = time;
                DataWithTime(2,place) = 0;
            end
            place = place + 1;
            time = time + deltaTime;
        end
    end
end

%check; K has to be the amount of values in the matrix without time
%K
```

120
% double_dataLoc = 1;
% bool = 0;
% for n = 1 : length(DataWithTime)
  if (DataWithTime(1,n) == double_data(1,double_dataLoc))
    if (~(DataWithTime(2,n) == double_data(2,double_dataLoc)))
      bool = bool + 1;
    end
  double_dataLoc = double_dataLoc + 1;
% end
% bool

Procedure:
Appendix Matlab conversing data into data with correct time

To start the procedure for one of the measurements, the following command is typed in the
Command Window of Matlab:
DataWithTime = saveCorrectMatrix('..file..')
Where;
file = the name of the file (i.e. matrix) you want to convert.

For example:
DataWithTime = saveCorrectMatrix('S1Z1415matlab.mat')
This will convert the original matrix of Sensor 1 in the Z direction into a matrix were all the time steps
are 1/1000 second.
Appendix K. Matlab code for 3D graph

```matlab
% Function to retrieve the 3D-Plot for all frequencyvectors.
% Input 'seconds' is the count of seconds you want to see the vectors from.
% PRE: STARTSECOND > 0
function plotPeriodofFile(startsecond, endsecond, namegoodmatrix);

double_data = namegoodmatrix;

% TimeAxis = datestr(double_data(1:10000:(endsecond*1000)));

Fs = 1000; % Sampling frequency
L = 1000;  % Length of signal
NFFT = 2^nextpow2(L); % Next power of 2 from length of y
start = (startsecond-1)*1000; % Position where you get your first data from (in the loop)

% Creates the matrix where all frequencyvectors are saved in.
% Standard this gives for colf 513, but is here automated maybe for later.
% Creates the matrix where all frequencyvectors are saved in.
% Standard this gives for colf 513, but is here automated maybe for later.
f = Fs/2*linspace(0,1,NFFT/2+1);
[rowf colf] = size(f);
matrix = zeros((endsecond-startsecond+1),colf);

% Indicates the right row for the matrix with the Fourier data!
row = 1;
for second = startsecond : endsecond
    % Retrieves a vector of 1000 samples that represent the 'second'.
    vector = double_data(2, (start + 1) : second*1000);
    % Updates the starting point with one second (is 1000 samples).
    start = start + 1000;

    % Fourier function and extra function for scaling.
    vectorFFT = fft(vector, NFFT)/L;
    newVector = (2*abs(vectorFFT(1:NFFT/2+1)));

    % Saves the found frequencyvector in the total matrix.
    matrix(row,:) = newVector;
    row = row + 1;
end

matrix;

% Plots the 3D graph, and all settings for the Plot.
figure(2)
surf((1:colf), (startsecond : endsecond), matrix(:, 1:colf))
xlim([1 colf])
ylim([0 1.5])
xlabel('Frequency (hertz)')
ylabel('Time (seconds)')
set(gca, 'YTickLabel','TimeAxis')
%datetick('y','dd-mmm HH:MM:SS')
zlabel('Intensity (mm/s)')
ylim([startsecond endsecond])
shading interp
```
**Procedure:**

To start the 3D plot, the following line is typed in the Command Window of Matlab:

```
plotPeriodofFile = (startsecond,endsecond,'matrix')
```

Where;

- `startsecond` = the desired start second of the measurement (Note: because 1 is the first measured second, this `startsecond` has to be > 0)
- `endsecond` = the desired end second
- `matrix` = DataWithTime

The matrix DataWithTime is first divided into parts of 1000 samples (i.e. 1 second). A Fast Fourier Transform is then performed over every part, which results in a 2D graph for every second. All these 2D graphs are put after each other in time, which creates a 3D graph. More information about the Fast Fourier Transform can be found in paragraph 2.6.

For example:

```
plotPeriodofFile (1,3600,DataWithTime)
```

This command creates a 3D graph of second 1 till second 3600 (i.e. the first hour) of the matrix DataWithTime.

The figures below show the 3D graphs of the recorded measurements of sensor 1 in the Z direction. The first figure consists of the dataset made at day 1. The second figure consists of the dataset made at day 2. As already can be seen from these graphs, the frequency spectrum of both days differs a lot. The reason for this is discussed in the report.
Appendix L. Matlab code for 2D graph

% Step 1; Fill in the name of the data that has to be loaded
% Step 2; Change Name for title figure and name saved file
% Step 3; Change num to one higher! Because otherwise the 'hold function' holds also the previous plot
% Step 3; Change datenum in the for loop for desired plotted data

load('S1Z1214.mat');
C = double_data(1,:);
Volt = double_data(2,:);
Sensitivity = 23.3;
B = (Volt/Sensitivity) * 1000;
DataWithTime = [C;B];

Name = 'Sensor 1 - low frequency range - 2 - 12.43.24 12.43.41';
num = 2172;

% Take uneven steps for 'n', otherwise the total graph does not have gridlines
for n = 1:17
% Seconds can be + more than 60. Matlab makes a new minute automatically
Sec(n) = datenum(2013,9,12,12,43,(24+(n-1)));
Place(n) = find(DataWithTime==Sec(n));
Sec(n+1) = datenum(2013,9,12,12,43,(24+n));
Place(n+1) = find(DataWithTime==Sec(n+1));

x = DataWithTime(1,(((Place(n)/2)+0.5):(((Place(n+1)-2)/2)+0.5)));
% -2, makes the value for n+1 for the next n only, and not double used
y = DataWithTime(2,(((Place(n)/2)+0.5):(((Place(n+1)-2)/2)+0.5)));
% /2 + 0.5, because it has to be a place in the matrix, not only in the row

Fs = 1000;
L = ((Place(n+1)-Place(n))/2); % Sampling frequency
NFFT = 2^nextpow2(L); % Length of signal
Yfft = fft(y,NFFT)/L;

f = Fs/2*linspace(0,1,NFFT/2+1);

% This plots a single-sided amplitude spectrum for second n
figure(n);
plot(f,2*abs(Yfft(1:NFFT/2+1)));
set(gcf,'Visible','off'); % Does not show the graph that has been made
grid on;
set(gca,'GridLineStyle','-');
grid(gca,'minor');
xlim([0 200]);
ylim([0 0.5]);
title([Name,' - Second ', num2str(n)]);
xlabel('Frequency (Hz)');
ylabel('Velocity (mm/s)');
saveas(gcf,[Name,' - Second ', num2str(n),'.jpg']); % Saves the graph
% This plots a single-sided amplitude spectrum for all seconds in one graph.

figure(num);
hold all;
plot(f,2*abs(Yfft(1:NFFT/2+1)));  % Plot the amplitude spectrum
set(gcf, 'Visible', 'off');
grid on;
set(gca, 'GridLineStyle', '-');
grid(gca,'minor');
xlim([0 25]);
ylim([0 1]);
title([Name,' All Seconds']);
xlabel('Frequency (Hz)');
ylabel('Velocity (mm/s)');
saveas(gcf,[Name,' All Seconds.jpg']);
end

% Makes the sound of a train when everything is ready
load train
sound(y,Fs)
Appendix M. Graphs to compare the three directions X, Y, Z
Appendix N. Graphs to compare all sensors
Appendix O. Graphs to compare day 1 and day 2

Comparison day 1 and day 2 measurements municipality of Rotterdam – Z-direction

Day 1 sensor 1
Day 1 sensor 2
Day 1 sensor 5
Day 1 sensor 8
Day 2 sensor 1
Day 2 sensor 2
Day 2 sensor 5
Day 2 sensor 8
Appendix P. Graphs to compare low frequency range of day 1 and day 2
Sensor 7 - low frequency range - Z - 14.06.11 14.06.28 All Seconds

Sensor 7 - low frequency range - Z - 12.43.24 12.43.41 All Seconds
Appendix Q. Results Sensor 4

Sensor 4 – measurement day 1 – sensor number 00600
Sensor 4 – measurement day 2 – sensor number 00936
Appendix R. Other measurement sensor 4

4Y

4X

4Z
|   | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | AA |
| 1 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 2 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 3 | X | Temporal 2.5 | 0.03 | 2.5 | 0.072 | 2.5 | 0.051 | 2.5 | 0.068 | 3 | 0.07 | 3.5 | 0.064 | 7.7 | 0.082 | 2.5 | 0.043 | 1 Temporal 2.5 | 0.03 | 2.5 | 0.04 |
| 4 | Y | 2nd peak 4.5 | 0.03 | 4.5 | 0.046 | 4.5 | 0.028 | 4.5 | 0.03 | 4.5 | 0.023 | 4.5 | 0.031 | 6.5 | 0.034 | 4.5 | 0.031 | 2nd peak 4.5 | 0.03 | 4.5 | 0.03 |
| 5 |   | 3rd peak 7 | 0.11 | 7.5 | 0.123 | 7.25 | 0.117 | 6 | 0.043 | 7 | 0.027 | 7.75 | 0.106 | 7.325 | 0.113 | 6.875 | 0.101 | 3rd peak 7 | 0.023 | 6.5 | 0.035 |
| 6 |   | 4th peak 0.5 | 0.01 | 0.5 | 0.02 | 0.5 | 0.02 | 0.5 | 0.02 | 0.5 | 0.02 | 0.5 | 0.02 | 0.5 | 0.02 | 0.5 | 0.02 | 4th peak 0.5 | 0.02 | 0.5 | 0.02 |
| 7 |   |   | 11 | 0.02 | 10 | 0.002 | 10.5 | 0.057 | 11 | 0.02 | 10.5 | 0.045 | 11.5 | 0.11 | 10.5 | 0.09 | 10.67 | 0.052 | 4th peak 1 | 0.023 | 10 | 0.023 |
| 8 |   |   |   | 11 | 0.095 | 11 | 0.095 | 11 | 0.095 | 11 | 0.095 | 11 | 0.095 | 11 | 0.095 | 11 | 0.095 | 11 | 0.095 | 11 | 0.095 |
| 9 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 10 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 11 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 12 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 13 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 14 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 15 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 16 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 17 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 18 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 19 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 20 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 21 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 22 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 23 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 24 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 25 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 26 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 27 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

Appendix S. Overview selected data analysis Fugro
Appendix T. Overview graphs made with data Fugro

At first, different train passages during different construction works have been selected with the criterion described in paragraph 5.2. In the Excel overview in appendix S, the selection of for example the 6th of August can be seen. All peak values (frequencies and corresponding velocities) were noted and in the end averaged.

Afterwards different graphs were plotted to see which method would be the most suitable method to compare the results. Below, the results of the first two attempts are showed in the Z-direction, also described in paragraph 5.2. These graphs give an impression of the ‘mess’ of information. Because of this mess, the selection of data has been further averaged as described in paragraph 5.2. The results of the chosen method (i.e. the initial situation compared to small and big settlements and the post situation) have been showed in paragraph 5.2.

![Changes during types of construction works](image)
Changes during different days

Astitel

Maart Z
Juli Z
August 6 Z
August 7 Z
August 8 Z
August 9 Z
August 10 Z
August 11 Z
Appendix U. Settlement curve

This settlement curve has been produced and provided by Arie van den Heerik.