ADDITIONAL DAMPING FOR TALL STRUCTURES

by

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SUMMARY

A study is made of a method of vibration reduction in tall civil engineering structures in which flexibility is introduced artificially, but with passive dampers to absorb energy during any motion which results. In particular, an examination is carried out of the effect of introducing an articulation part-way up a tall tower, with passive dampers and springs to restrict the rotation that results. The method involves assigning three degrees of freedom, and the use of Lagrangian equations of motion. Power spectral density concepts are exploited using wind turbulence as a stationary random forcing function. In-wind vibrations only are studied. The optimal spring-stiffness/damper combination is sought for two specific tower designs.
ACKNOWLEDGEMENTS

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NOTATION

The following notation is used in this paper where all units are in ft., lb., and sec.

A, B, C functions which are included in self weight terms

C_{\text{VIS}} effective linear damping coefficient of spring-damper system

C_D coefficient of drag

D function appearing in potential energy terms

D_0, D_s diameter of tower at base, and rate of change of this with height

D_3, D_4 diameters of lower and upper mast segments

D_{ij} coefficients in dissipative function, terms in damping matrix

D(x), D(z) general diameters, functions of height

E_c, E_s moduli of elasticity of concrete and of steel

E_{ij} complex matrix terms, equations of motion

F effective stiffness of spring system

F_D Lagrangian dissipative function

F_i, F_j Lagrangian generalised forces

g acceleration of gravity

H height of spring-damper system above mast bearing

H_{ij} complex frequency response functions

L' scale factor, wind expression

M_{ij} coefficients in kinetic energy expression, terms in mass matrix

M_{11}^* function forming part of M_{11}

\{p(x,t), P(x), p(x,t)\} total, steady and turbulent wind pressures

I_1, I_4 moments of inertia of lower and upper mast segments

I, I_0, I_1, I_2, I_3 moment of inertia of tower, and constant coefficients in expression for this

i, j, k general subscripts

K surface roughness coefficient, wind velocity expression.
Notation continued

$L_1, L_2, L_3, L_4$ length of various components, see fig.1.

$Q_1, Q_2, Q_3$ non-dimensional amplitudes of modes of deflection

$S_{u'}(z_1, z_2, \omega)$ cross-spectral density of wind at two heights

$S_u(\omega)$ fully correlated wind spectral density

$S_{F_i F_j}(\omega)$ cross spectral density of generalised forces $i$ and $j$

$S_{F_i}(\omega)$ power spectral density of generalised force $i$

$t$ time

$T$ kinetic energy

$u_{(x,t)}$ turbulent wind velocity

$V$ total potential energy

$V_{ij}$ coefficients in potential energy expression, terms in stiffness matrix

$V^*_{11}$ function forming part of $V_{11}$

$V_{rel}(x,t), \{V(x)\}$ relative and steady velocities of wind

$V_{10}$ mean wind velocity at 10 metre height

$V_G$ gradient wind at height $Z_G$

$W_2$ total weight of mast

$W_3, W_4$ total weight of lower and upper mast segments

$W_F$ weight of tower per unit foot at base

$W_e$ external work by wind

$x_1$ height of tower points measured from the ground

$x_2$ height of mast points measured from the tower top

$x$ general measure of height from ground

$y_1, y_2$ elastic deflection of tower and of mast (functions of $x_1, x_2$)

$y^1$ total lateral movement of mast

$Z$ rate of change of weight per unit foot of tower, with height
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FIGURES

1. STRUCTURE AND MODES OF DEFLECTION
2. POSITION OF SPRING-DAMPER SYSTEM
3. SIMPLIFIED FLOW-CHART OF COMPUTER PROGRAM
4. FREQUENCY RESPONSE FUNCTIONS
5. FREQUENCY RESPONSE FUNCTIONS
6. P.S.D. CURVES
7. RESULTS, STRUCTURE I
8. RESULTS, STRUCTURE II.
1. INTRODUCTION

As materials and methods of design improve, civil engineering structures become increasingly slender. Bridges have longer spans, buildings and towers are taller, cross sections and weights are smaller. This has meant that stability and vibration studies have to be performed in great depth as a standard part of the structural design procedure.

The problem of vibrations in tall buildings and some of their harmful effects were emphasised by Waller (Ref.1). Fatigue life, architectural finish, comfort, even safety, all may be adversely affected by excessive vibrations. Various methods of vibration reduction in structures have been proposed in the past. The problem is generally seen as one of energy dissipation, since it is rarely possible in Civil Engineering structures to isolate the structure from the source of excitation; usually the wind, earthquakes, or the loads themselves which the structure supports. A standard engineering undergraduate problem consists of the reduction of vibration of a dance floor by the suspension of a properly 'tuned' weight supported by springs and dashpots beneath the floor. Waller (Ref.1) proposes a rolling weight, with dampers, installed on an upper service floor of a tall building as a variation on this. Some success is claimed, but efficiency will vary with frequency of excitation, and the weight itself must be added to the dead weight the structure must support. To return to the undergraduate problem, the beat of the music may change and the floor must be strengthened to support the weight!

Roorda (Ref.2) has proposed a more promising method of dynamic response reduction, involving the use of 'active' damping. This consists of a system of vibration sensors, electro-hydraulic servo-mechanisms, and forcers. The reduction of dynamic response of a cantilever column, by the suspension of a feedback-controlled pendulum near the tip is successfully demonstrated. It is suggested that the method might be applicable to more complex structures with several modes of vibration. In a practical case, however, the question of electro-mechanical reliability must arise, and it must be admitted that common Civil Engineering reliability figures (unity minus the probability of failure) are much higher than those generally obtained for electro-mechanical devices. One suspects for instance, that a malfunction causing badly phased feedback signals might cause a structure to self-destruct.

The investigation of yet another method of vibration reduction in structures was thought to be warranted, especially in view of the shortcomings of existing proposals. Examination of a simple single degree of freedom bar and spring mechanism showed that it was actually possible, under certain circumstances, to reduce dynamic response by the introduction of an additional degree of freedom, for example a hinge, equipped with finite stiffness and viscous damping.
Additional flexibility is introduced, but the energy lost to the damper seems to outweigh this, and dynamic response is reduced. The idea is not obvious, but not unattractive either, since no extra weight is added, and passive viscous dampers, being unsophisticated, seem reliable enough. In the case of a tall building, for example, some of the upper floors might be carried on a flexible mounting which would allow passive dampers to be effective, and, should the principle hold, the structure might vibrate less in gusty wind or during an earthquake.

It was decided to investigate tower-type structures because of their relative simplicity, and because some recent designs consist of two obvious main components, between which an articulation might logically be introduced. A common design involves a reinforced concrete tower of some considerable height, supporting a steel-framed aerial mast. Total heights are of the order of 1000 feet and more.

The aim of the investigation was to compare performance of structures with and without the added 'flexibility'. Certain approximations in the analysis were therefore thought justified, since the relative magnitude of the output was considered most important. The structures were assigned three degrees of freedom, and Lagrange's energy method was used to get the equations of motion, which yield the frequency response functions. A mean-square spectral density approach was then used, with Harris' (Ref. 3) modified version of Davenport's (Ref. 4) spectrum of stationary random wind excitation as input. Earthquake excitation was not considered.

2. TOWERS AND VIBRATION MODES

A recent paper (Ref. 5) has discussed the details of the design of a specific television broadcasting tower and presented a comparison with eleven other towers in the world of comparable height (600 ft to 1600 ft approximately).

A schematic diagram of the type of structure examined here appears in figure 1. It consists of a circular tapering, hollow concrete tower L1 in height, surmounted by a steel mast L2 high. "Tower" will be used to refer only to the concrete supporting structure, and "mast" to the steel superstructure. The tower tapers according to a linear relationship between diameter, D, and x1, measured from the ground;

\[ D = D_0 - D_s x_1, \]  

... (1)

and the moment of inertia, I, varies according to a third order polynomial in x1

\[ I(x_1) = I_0 + I_1 x_1 + I_2 x_1^2 + I_3 x_1^3. \]  

... (2)
The weight per unit length, \( W(x_1) \), over the tower height varies according to a linear relationship:

\[
W(x_1) = W_F - Z x_1. \tag{3}
\]

These relationships apply only over the interval \( 0 \leq x_1 \leq L_1 \).

The mast would commonly be steel lattice with geometry dictated by electronic (aerial) requirements. It might be shrouded with a circular plastic resin shield. The mast considered here presents two external diameters to the wind, \( D_3 \) over length \( L_3 \) and diameter \( D_4 \) over upper length \( L_4 \). Moment of inertia is constant over \( L_3 \), equal to \( I_3 \), and again constant over length \( L_4 \), equal to \( I_4 \). Weight per unit foot is also assumed constant over the two intervals. Total weight of the lower part of the mast is \( W_3 \) and of the upper part, \( W_4 \). Total mast weight is \( W_2 \).

The arbitrarily chosen modes of vibration also appear on figure 1. The tower deflects according to the shape

\[
y_1 = Q_1 L_1 \left[ 1 - \cos \frac{\pi x_1}{2L_1} \right] \tag{4}
\]

where \( Q_1 \) is a nondimensional deflection amplitude. The deflection of the mast consists of two parts, measured from the tangent to the top of the supporting tower. One is the rigid body rotation allowed because of an artificial articulation introduced here. A rotation \( Q_2 \) radians arises and at any point the deflection is \( Q_2 x_2 \) provided \( Q_2 \) is small. A further elastic deflection

\[
y_2 = Q_3 L_2 \left[ 1 - \cos \frac{\pi x_2}{2L_2} \right] \tag{5}
\]

arises, in which \( Q_3 \) is a non-dimensional amplitude.

It was felt that these modes would be close enough to reality in form, and sufficient in number to achieve the aims of the investigation. Higher modes are excluded both for simplicity, and because most of the energy from wind excitation is concentrated at the lower frequencies which correspond to these modes.

3. **EQUATIONS OF MOTION**

It is necessary to calculate the total potential energy stored due to the deflections allowed for, as well as the energy dissipated in the dampers and the kinetic energy developed.

Beginning with the total potential energy, the flexural energy in the tower is

\[
\frac{1}{2} \int_0^{L_1} E_c I(x_1) \left( \frac{\partial^2 y}{\partial x_1^2} \right)^2 \, dx_1 \tag{6}
\]

which is

\[
Q_1^2 \left[ \frac{E_c \pi^4}{32L_1^2} \int_0^{L_1} \left( I_0 + I_1 x_1 + I_2 x_1^2 + I_3 x_1^3 \right) \cos^2 \frac{\pi x_1}{2L_1} \, dx_1 \right] \tag{7}
\]
where $E_c$ is the modulus of elasticity of concrete. This integral and many which follow are evaluated numerically in the computer program developed. The coefficient multiplying $Q_i^2$ in (7) is later referred to as $\alpha V_{11A}$. The flexural energy in the mast is, similarly

$$Q_3^2 \left\{ \frac{E_c \pi^4}{32L_2^2} \left[ \int_0^{L_3} I_3 \cos^2 \frac{\pi x}{2L_2} \, dx_2 + \int_{L_3}^{L_2} I_4 \cos^2 \frac{\pi x}{2L_2} \, dx_2 \right] \right\}. \quad (8)$$

which is

$$\frac{D}{2} Q_3^2 = \frac{E_c \pi^4}{64L_2^2} \left[ I_3' L_3 + I_4' L_4 \right] + \frac{L_2}{\pi} \sin \frac{\pi L_3}{L_2} \left( I_3' - I_4' \right) \quad ... \quad (9)$$

where $E_s$ is the modulus of elasticity of steel and $D$ is defined implicitly.

A sketch of a possible mounting for springs and dampers at the mounting of mast to tower appear as fig.2. The deflection at the spring-damper system, height $H$ above the hinge is

$$Q_2 H + Q_3 L_2 \left( 1 - \cos \frac{\pi H}{2L_2} \right).$$

If $F$ is the spring stiffness, the energy absorbed in the springs is

$$\frac{FH^2}{2} Q_2^2 + FHL_2 \left( 1 - \cos \frac{\pi H}{2L_2} \right) Q_2 Q_3 + \frac{FL_2^2}{2} \left( 1 - \cos \frac{\pi H}{2L_2} \right)^2 Q_3^2 \quad ... \quad (10)$$

In a structure which is to be made flexible, such as these are, it was thought prudent to include terms accounting for decreases in total potential energy due to lowering of the self-weight of the structure during deflections. In the event, these proved of small significance, but were nonetheless retained. Their calculation appears in Appendix I.

It will also be necessary to have the kinetic energy as a function of the assigned coordinates, $Q_1, Q_2, Q_3$. If a dot over a variable signifies a time derivative, then, for the tower, velocity squared is

$$\dot{Q}_1^2 = \dot{Q}_1^2 L_1^2 \left( 1 - \cos \frac{\pi x}{2L_1} \right)^2 \quad ... \quad (11)$$

and total kinetic energy of the tower is

$$\dot{Q}_1^2 \cdot \frac{M_{11}}{2} = \dot{Q}_1^2 \left[ \frac{L_1^2}{2g} \int_0^{L_1} (W_p - Zx_1) \left( 1 - \cos \frac{\pi x}{2L_1} \right)^2 \, dx \right] \quad ... \quad (12)$$
where $M^*$ is defined implicitly, and $g$ is the acceleration of gravity.

The displacement laterally of any point on the mast is the sum of

$$y_2(x) = Q_1 L_1 + \frac{Q_1 \pi}{2} x_2 + Q_2 x_2 + Q_3 L_2 \left(1 - \cos \frac{\pi x_2}{2L_2}\right).$$

...(13)

For the mast, therefore,

$$\left(\dot{y}_2(x)\right)^2 = \left(L_1^2 + \frac{\pi}{4} x_2^3\right) \dot{Q}_1^2 + 2 \left(L_1 x_2 + \frac{\pi}{2} x_2^3\right) \dot{Q}_2 \dot{Q}_1 + 2 L_2 (1 - \cos \frac{\pi x_2}{2L_2}) \dot{Q}_3 \dot{Q}_2 + 2 x_2 L_2 (1 - \cos \frac{\pi x_2}{2L_2}) \dot{Q}_3^2 + 2 L_2^2 (1 - \cos \frac{\pi x_2}{2L_2}) \dot{Q}_3^2$$

...(14)

The kinetic energy of the mast is

$$\int_0^{L_3} \frac{1}{2g} \frac{W_3}{L_3} \left(\dot{y}_2(x)\right)^2 dx_2 + \int_{L_3}^{L_2} \frac{1}{2g} \frac{W_4}{L_4} \left(\dot{y}_2(x)\right)^2 dx_2$$

...(15)

where (24) must be substituted and the integrations carried out for any given structure.

The energy dissipated at the dampers depends on the velocity of movement of the mast at the point of attachment. This is

$$\dot{y}_{2r}(H) = H \dot{Q}_2 + Q_3 L_2 \left(1 - \cos \frac{\pi H}{2L_2}\right).$$

...(16)

If the effective damping constant of the system of dampers arranged at this height is $C_{VISC}$, then the dissipative function is

$$\frac{C_{VISC}}{2} \left[ H^2 \dot{Q}_2^2 + 2 H L_2 (1 - \cos \frac{\pi H}{2L_2}) \dot{Q}_2 \dot{Q}_3 + L_2^2 (1 - \cos \frac{\pi H}{2L_2}) \dot{Q}_3^2 \right]$$

...(17)

Considering the free vibrations of the system, Lagrange's equations appear as

$$\frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{Q}_i}\right) + \frac{\partial F_D}{\partial \dot{Q}_i} + \frac{\partial V}{\partial Q_i} = 0$$

...(18)
where $T$ is kinetic energy, $t$ is time, $F_D$ is dissipative function and $V$ is total potential energy. These yield a set of matrix equations;

$$
\begin{bmatrix}
M_{ij}
\end{bmatrix}
\begin{bmatrix}
\dot{Q}_i
\end{bmatrix}
+
\begin{bmatrix}
D_{ij}
\end{bmatrix}
\begin{bmatrix}
\ddot{Q}_i
\end{bmatrix}
+
\begin{bmatrix}
V_{ij}
\end{bmatrix}
\begin{bmatrix}
Q_j
\end{bmatrix}
= 0
$$

\hspace{1cm} i, j = 1, 2, 3 \ldots \ (19)

These matrices are 3 by 3, and have the following elements where only 6 of the 9 are given, since they are symmetric:

$$
M_{11} = M_{11}^* + \frac{W_4}{gL_4}\left( L_1^2L_3 + \frac{\pi}{2}L_1L_2^2 + \frac{\pi^2L_2^3}{12} \right)
$$

$$
+ \frac{W_4}{gL_4}\left( L_1^2L_3 + \frac{\pi}{2}L_1L_2^2 + \frac{\pi^2L_2^3}{12} \right)
$$

$$
M_{12} = \frac{1}{g} \begin{bmatrix}
W_3L_2 \\
L_3 g \\
\int_0^{L_3} (L_1 + \frac{\pi}{2}x_2)(1 - \cos\frac{\pi x_2}{2L_2}) dx_2 \\
+ \frac{W_4}{L_4g} \int_0^{L_3} \frac{2L_2}{L_1} \left( L_1 + \frac{\pi}{2} \right)(1 - \cos\frac{\pi x_2}{2L_2}) dx_2
\end{bmatrix}
$$

$$
M_{13} = \frac{W_3L_2}{L_3 g} \int_0^{L_3} \frac{L_2}{L_1} \left( L_1 + \frac{\pi}{2} \right)(1 - \cos\frac{\pi x_2}{2L_2}) dx_2
$$

$$
+ \frac{W_4}{L_4g} \int_0^{L_3} \frac{L_2}{L_1} \left( L_1 + \frac{\pi}{2} \right)(1 - \cos\frac{\pi x_2}{2L_2}) dx_2
$$

$$
M_{22} = \frac{W_3L_2}{3g} \int_0^{L_3} \frac{L_3}{L_1} x_2 \left( 1 - \cos\frac{\pi x_2}{2L_2} \right) dx_2
$$

$$
+ \frac{W_4}{gL_4} \int_0^{L_3} \frac{L_2}{L_1} \left( L_1 + \frac{\pi}{2} \right)(1 - \cos\frac{\pi x_2}{2L_2}) dx_2
$$

$$
M_{33} = \frac{W_3L_2}{gL_3} \int_0^{L_3} \left( L_1 + \frac{\pi}{2} \right)(1 - \cos\frac{\pi x_2}{2L_2}) dx_2
$$

$$
+ \frac{W_4}{gL_4} \int_0^{L_3} \frac{L_2}{L_1} \left( L_1 + \frac{\pi}{2} \right)(1 - \cos\frac{\pi x_2}{2L_2}) dx_2
$$

\hspace{1cm} (20a)

and

$$
V_{11} = V_{11}^* - 2\left( \frac{WL_2}{8} + \frac{W_2L_1}{16} + C \right)
$$

$$
V_{12} = -\frac{WL_2}{2}
$$

$$
V_{13} = -\frac{B\pi}{2}
$$

$$
V_{22} = FH^2 - WL_2
$$

$$
V_{23} = FH_2 \left( 1 - \cos\frac{\pi H}{2L_2} \right) - B
$$

$$
V_{33} = D + \frac{FL_2}{2} \left( 1 - \cos\frac{\pi H}{2L_2} \right)^2 - 2A
$$

\hspace{1cm} (20b)
where A, B, C are defined in Appendix 1, and
\[
D_{11} = \frac{0.01 M_{11}}{M_{11}} \sqrt{\frac{V_{11}}{M_{11}}}
\]
\[
D_{12} = D_{13} = 0
\]
\[
D_{22} = C_{\text{visc}} H^2
\]
\[
D_{23} = C_{\text{visc}} H L_2 \left( 1 - \cos \frac{\pi H}{2L_2} \right)
\]
\[
D_{33} = C_{\text{visc}} L_2^2 \left( 1 - \cos \frac{\pi H}{2L_2} \right)^2
\]

The expression for \( D_{11} \) is the result of an approximate and arbitrary attempt to account for some structural, or material damping in the structure, especially the concrete tower. Taking the tower as a single degree of freedom structure for a moment, its natural frequency \( \omega_n \) will be
\[
\sqrt{\frac{V_{11}}{M_{11}}}
\]
and its equation of motion appear as
\[
\dddot{Q}_1 + \frac{D_{11}}{M_{11}} \ddot{Q}_1 + \frac{V_{11}}{M_{11}} Q_1 = 0
\]
or
\[
\dddot{Q}_1 + 2\beta \omega_n \dot{Q}_1 + \omega_n^2 Q_1 = 0 \quad \ldots \quad (21)
\]
where \( \beta \), the structural damping constant can be taken as, say 2% and
\[
D_{11} = \frac{0.04 M_{11}}{M_{11}} \sqrt{\frac{V_{11}}{M_{11}}} \quad \ldots \quad (22)
\]

Assuming a solution for the vector \( \hat{Q}_j \) of the form of the real part of
\[
\begin{bmatrix}
Q_1 \\
Q_2 \\
Q_3
\end{bmatrix} = \begin{bmatrix}
Q_1^0 \\
Q_2^0 \\
Q_3^0
\end{bmatrix} e^{i\omega t}
\]
where \( i \) is the usual imaginary number and \( \omega \) is angular velocity, and substituting in equation (19), it becomes, dividing through by \( e^{i\omega t} \);
\[
-\omega^2 \begin{bmatrix}
M_{ij} \\
D_{ij}
\end{bmatrix} \begin{bmatrix}
Q_j^0 \\
Q_j^0
\end{bmatrix} + i\omega \begin{bmatrix}
D_{ij} \\
V_{ij}
\end{bmatrix} \begin{bmatrix}
Q_j^0 \\
Q_j^0
\end{bmatrix} = 0
\]
or
\[
\begin{bmatrix}
E_{ij}(\omega)
\end{bmatrix} \begin{bmatrix}
Q_j^0
\end{bmatrix} = 0 \quad \ldots \quad (24)
\]
where \( E_{ij}(\omega) \) is a complex matrix whose elements are
\[
E_{ij}(\omega) = -\omega^2 M_{ij} + i\omega D_{ij} + V_{ij} \quad \ldots \quad (26)
\]
The inverse of this matrix is the matrix of complex frequency response functions, since for any forcing vector of the form of the real part of $[F_j]e^{i\omega t}$,

$$
\begin{bmatrix}
E_{ij}(\omega)
\end{bmatrix}
\begin{bmatrix}
Q_j^0
\end{bmatrix}
e^{i\omega t}
=
\begin{bmatrix}
F_j
\end{bmatrix}
e^{i\omega t}
\quad \ldots \quad (27)
$$
or

$$
\begin{bmatrix}
Q_j^0
\end{bmatrix}
=
\begin{bmatrix}
E_{ij}(\omega)
\end{bmatrix}
\begin{bmatrix}
F_j
\end{bmatrix}
\quad \ldots \quad (28)
$$

which is usually written

$$
\begin{bmatrix}
Q_i
\end{bmatrix}
=
\begin{bmatrix}
H_{ij}(\omega)
\end{bmatrix}
\begin{bmatrix}
F_j
\end{bmatrix}
\quad \ldots \quad (29)
$$

where the $H_{ij}$ are complex frequency response functions giving response as a function of $\omega$ in mode $i$ to the periodically varying generalised force related to mode $j$.

4. WIND EXCITATION

It is now necessary to derive expressions representing the generalised forces acting on the structure. The horizontal motion of the tower is given by expression (4) and that of the mast by (13). Incremental deflections are therefore

$$
\delta y_1(x) = \delta Q_1 L_1 \left(1 - \frac{\cos \pi x_1}{2L_1}\right)
$$

$$
\delta y_2(x) = \delta Q_1 \left(L_1 + \frac{\pi x_2}{2}\right) + \delta Q_2 x_2 + \delta Q_3 L_2 \left(1 - \frac{\cos \pi x_2}{2L_2}\right)
\quad \ldots \quad (30)
$$

If $D(x)$ represents diameter of the tower as a function of height, and $p(x,t)$ pressure on the tower from the wind (net in the direction of the wind), then incremental external work, $\delta W_e$, is,

$$
\delta W_e = \int_{0}^{L_1} D(x_1)p(x_1,t)\delta y_1(x)dx_1
+ \int_{0}^{L_2} D(x_2)p(x_2,t)\delta y_2(x)dx_2
\quad \ldots \quad (31)
$$

and the generalised force, $F_i$, associated with mode of vibration $i$ in Lagrange's equation is

$$
F_i = \frac{\partial (\delta W_e)}{\partial (\delta Q_i)}
\quad \ldots \quad (32)
$$

This gives
Two comments are worthwhile at this point. These expressions are all of the form

\[ F_1 = \int_{\text{length}} D(x)p(x,t)\phi_1(x)dx \] ... (34)

where the \( \phi_1 \) are the relevant displacement shapes and are distinguishable in expressions (33).

Further, the integrals in (33) which are over length \( L_2 \) must be performed in two steps, with \( D(x_2) \) set constant at \( D_3 \) and \( D_4 \).

The wind pressure variation with height is

\[ P(x,t) = \frac{1}{2} \rho_0 C_D (V_{\text{rel}}(x,t))^2 \] ... (35)

where \( \rho_0 \) is air density, \( C_D \) is coefficient of drag, taken here as constant over height, and \( V_{\text{rel}} \) is relative velocity of structure and wind. The structures studied are relatively tall, heavy and flexible, and will therefore have low natural frequencies and negligible velocities compared to the wind. \( V_{\text{rel}} \) is then approximately

\[ V_{\text{rel}}(x,t) = \vec{V}(x) + u(x,t) \] ... (36)

where \( \vec{V}(x) \) is the steady state component of wind superimposed on which is the turbulent, time-varying component \( u(x,t) \), comparatively smaller, and with zero mean. Pressure may be written as approximately

\[ P(x,t) = \frac{1}{2} \rho_0 C_D \vec{V}(x)^2 + \rho_0 C_D \vec{V}(x)u(x,t) \]
\[ = \vec{P}(x) + p(x,t) \] ... (37)
where a term in $u(x,t)$ squared has been deemed to be negligible, and $p(x)$ is the steady pressure, while $p(x,t)$ is the turbulent, zero mean, time-variant pressure. Only the time-variant pressure will be of interest, since periodic vibrations of the structure are to be studied. A check on all structures examined showed that there was negligible error in assuming this alternating force applied to the undeflected geometry. The vibrations with zero mean are, of course, superimposed on the steady wind deflection.

The generalised forces now appear in the form

$$F_i = \int \left( \rho_0 C_D \bar{v}(x) u(x,t) \phi_1(x) \right) dx \quad \cdots (38)$$

after a substitution from (37) in (34). Unfortunately $u(x,t)$ has been observed to be a random variable, with only certain statistical properties as measureable characteristics. A key measure of intensity of turbulence is the mean-square value or its root, the r.m.s. value. The spectral distribution of this mean square value with frequency, or with angular velocity, $\omega$, has also been measured, and a number of functions, largely empirical, are available to express this. This distribution, or spectral density of mean squared value is also called power spectral density, p.s.d., from the electronic terminology, in which field early use of the concept was made. The integral of p.s.d. over all frequencies gives the mean square value. We are virtually obliged to accept these statistical averages of wind input as all that is usefully available, and to use these to obtain statistical averages describing the output vibrations.

To get the p.s.d. of generalised forces, the cross-correlation at different heights of any force must be accounted for. The cross-correlation of $F_i$ and $F_j$ at heights $z_1$ and $z_2$ at times $t$ and $(t+\tau)$ gives the expectation

$$\langle \delta F_i(z_1,t) \cdot \delta F_j(z_2,t+\tau) \rangle$$

$$= \phi_1(z_1) \phi_j(z_2) D(z_1) D(z_2) \rho_0^2 C_D^2 \bar{v}(z_1) \bar{v}(z_2)$$

$$\langle u(z_1,t) \cdot u(z_2,t+\tau) \rangle$$

$$\cdots (39)$$

the cross spectral density of $F_i$ and $F_j$ will be the Fourier transform of the double integral of this over the structure's height, that is, the cross-spectral density (or p.s.d. if $i = j$), $S_{F_iF_j}$ is,

$$S_{F_iF_j}(\omega) = \int \int \rho_0^2 C_D^2 \bar{v}(z_1) \bar{v}(z_2) S_u(z_1,z_2,\omega) dz_1 dz_2$$

$$\quad \text{height} \quad \cdots (40)$$
where
\[ S'_u(z_1, z_2, \omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \langle u(z_1, t) u(z_2, t+\tau) \rangle e^{-i\omega \tau} d\tau \] ... (41)

The latter Fourier transform is the cross spectral density of wind velocity between two points at differing heights \( z_1 \) and \( z_2 \). In fact only the real part of (41) will interest us.

Harris (Ref.3) has suggested an expression for this cross-spectral density which consists of a coherence function multiplied by a spectral density independent of height. This is
\[ S'_u(z_1, z_2, \omega) = e^{-n|z_1-z_2|} S_u(\omega). \] ... (42)

where
\[ S_u(\omega) = \frac{2KL'\bar{V}_{10}}{\pi(2+\omega^2)^{5/6}} \] ... (43)
\[ n = \frac{8\bar{V}}{L'\bar{V}(z)} \sqrt{2 + \omega^2} \] ... (44)

and reduced angular velocity \( \bar{\omega} \), is
\[ \bar{\omega} = \frac{\omega L'}{2\pi\bar{V}_{10}} \]

In these \( K \) is a surface roughness coefficient, \( L' \) is a scale factor of 5900 feet, \( \bar{V}_{10} \) is the mean wind at 10 metres and \( S_u(\omega) \) is the spectral density of velocity. \( K \) varies from .005 for open terrain to .015 for terrain with 30 to 50 foot obstacles, to .050 for built-up central urban areas. An expression for \( \bar{V}_{10} \) is given later. The cross spectral density of forces \( i \) and \( j \) can be written as
\[ S_{F_i F_j}(\omega) = \int_{\text{height}} \phi_i(z_1) \phi_j(z_2) D(z_1) D(z_2) \rho_0^2 C_D^2 \]
\[ \bar{V}(z_1) \bar{V}(z_2) e^{-n|z_1-z_2|} S_u(\omega) dz_1 dz_2 \] ... (45)

These will be onerous to evaluate unless a simplification is made, since diameter varies under three regimes discontinuously with height. Even the simplest of the expressions, the p.s.d. of generalised force \( F_2 \) has to be written as
\[ S_{F_2}(\omega) = \rho_0^2 C_D^2 S_u(\omega) \left\{ \begin{array}{c} \int_0^{L_1} \int_0^{L_2} \bar{V}(z_1) \bar{V}(z_2) e^{-\eta|z_1-z_2|} dz_1 dz_2 \\
+ 2D_3 D_4 \int_0^{L_1} \int_0^{L_2} z_1 z_2 \bar{V}(z_1) \bar{V}(z_2) e^{-\eta|z_1-z_2|} dz_1 dz_2 \\
+ D_4^2 \int_0^{L_1} \int_0^{L_3} z_1 z_2 \bar{V}(z_1) \bar{V}(z_2) e^{-\eta|z_1-z_2|} dz_1 dz_2 \end{array} \right\} \]

and the expression for \( S_{F_1}(\omega) \) requires six double integrals of this type. The double integrals, further, are not easy to evaluate numerically. It was decided at this stage to admit an approximation to the study, that of turbulence fully correlated with height. This is equivalent to saying that the structure will be buffeted by turbulence in the same sense at all heights at any given instant. The coherence function is taken as unity, and \( S'_i(z_1, z_2, \omega) \) becomes \( S_i(\omega) \) as defined in expression (43). A welcome simplification of (45) is available;

\[ S_{F_i F_j}(\omega) = \rho_0^2 C_D^2 S_u(\omega) \left\{ \begin{array}{c} \int_0^{L_1} \phi_i(z_1) D(z_1) \bar{V}(z_1) dz_1 \\
\int_0^{L_2} \phi_j(z_2) D(z_2) \bar{V}(z_2) dz_2 \end{array} \right\} \]  

where these integrals may be evaluated separately and simply multiplied together. Each integral will be evaluated in two or three stages, following the discontinuous variations of \( D(z) \). Again, if \( i = j \), these expressions simply give p.s.d. functions. For those wishing to follow the computer program in detail (Appendix II) a sample cross spectral density function would be

\[ S_{F_1 F_2}(\omega) = \rho_0^2 C_D^2 S_u(\omega)(RF_1(RF_2); \ ... \ (48) \]

where \( RF_1 = F_{1 \text{low}} + F_{1 \text{mid}} + F_{1 \text{top}} \)

\[ RF_2 = F_{2 \text{mid}} + F_{2 \text{top}} \]

\[ F_{1 \text{low}} = \int_0^{L_1} (D_0 - Zx) L_1 (1 - \frac{\cos \pi x}{2L_1}) \bar{V}(x) \ dx \]

\[ F_{1 \text{mid}} = \int_0^{L_1} \int_0^{L_2} D_3 \left( \frac{\pi}{2} (x-L_1) + L_3 \right) \bar{V}(x) \ dx \]

\[ F_{1 \text{top}} = \int_0^{L_1} \int_0^{L_3} D_4 \left( \frac{\pi}{2} (x-L_1) + L_3 \right) \bar{V}(x) \ dx \]

\[ F_{2 \text{mid}} = \int_0^{L_1} \int_0^{L_2} D_3 \left( x-L_1 \right) \bar{V}(x) \ dx \]

\[ F_{2 \text{top}} = \int_0^{L_1} \int_0^{L_3} D_4 \left( x-L_1 \right) \bar{V}(x) \ dx \]  

\( ... \ (49) \)
The expression used for mean wind is

\[ \bar{V}(x) = \bar{V}_G(x/Z_G)\gamma \quad \ldots (50) \]

where \( \gamma \) is a power exponent appropriate to \( Z_G \). \( \bar{V}_G \) is the average gradient wind at the gradient height \( Z_G \), and \( x \) throughout (49) and (50) is a uniform measure of height from the ground. The value of \( \bar{V}_{10} \) used in expression (43) is therefore

\[ \bar{V}_{10} = \bar{V}_G(10/Z_G)\gamma \quad \ldots (51) \]

A list of these constants appropriate to different conditions is given by Davenport (Ref.4):

Open terrain, few obstacles  \( \gamma = .16, Z_G = 900 \text{ ft} \)

Uniform 30-50 foot obstacles  \( \gamma = .28, Z_G = 1300 \text{ ft} \)

Large irregular obstacles  \( \gamma = .40, Z_G = 1700 \text{ ft} \)

Expressions such as (48) and (49) are readily calculable on a digital computer, giving spectral density functions for the generalised forces acting on the structure.

5. RESPONSE SPECTRAL DENSITY

It is now necessary to see how the frequency response functions defined earlier may be used in conjunction with this spectral density of input force, to give spectral density of output vibration displacement.

Looking at equation (29) it is clear that, for the present three degree of freedom system, any of the output amplitudes, say \( Q_1 \), may be written

\[ Q_1 = H_{11} F_1 + H_{12} F_2 + H_{13} F_3 \quad \ldots (52) \]

Unfortunately it has been necessary to settle for obtaining only the mean square densities of the generalised forces, \( F_1, F_2, F_3 \), and it will be necessary to accept this type of information regarding the displacements obtained. The mean square value of any output, say of \( Q_1 \) again, takes the following form, where \( H_{ij}(\omega) \) is written as \( H_{ij} \) for compactness:

\[ <Q_1^2> = <(H_{11} F_1 + H_{12}^* F_2 + H_{13}^* F_3)(H_{11} F_1 + H_{12} F_2 + H_{13} F_3)> 
= H_{11}^2 <F_1^2> + H_{11}^* H_{12} <F_1 F_2> + H_{11}^* H_{13} <F_1 F_3> 
+ H_{12}^* H_{11} <F_2 F_1> + H_{12}^* H_{12} <F_2^2> + H_{12}^* H_{13} <F_2 F_3> 
+ H_{13}^* H_{11} <F_3 F_1> + H_{13}^* H_{12} <F_3 F_2> + H_{13}^* H_{13} <F_3^2> 
\quad \ldots (53) \]
Now \( \langle Q_1^2 \rangle = \int_0^\infty S_{Q_1}(\omega) d\omega \) \( \ldots \) (54)

\( \langle F_1 F_j \rangle = \int_0^\infty S_{F_1 F_j}(\omega) d\omega \) \( \ldots \) (55)

where \( S_{Q_1}(\omega) \) is the p.s.d. of output \( Q_1 \), and the asterisks denote complex conjugates.

Substituting (54) and (55) back into (53), removing the common integral signs from both sides, and taking the modulus of the complex function which results from the use of the \( H_{1j} \), gives

\[
S_{Q_1}(\omega) = |H_{11}^* H_{11}| S_{F_1}(\omega) + 2 |H_{11}^* H_{12}| S_{F_1 F_2}(\omega) + 2 |H_{11}^* H_{13}| S_{F_1 F_3}(\omega) \\
+ |H_{12}^* H_{12}| S_{F_2}(\omega) + 2 |H_{12}^* H_{13}| S_{F_2 F_2}(\omega) \\
+ |H_{13}^* H_{13}| S_{F_3}(\omega)
\]

Similar expressions exist for \( S_{Q_2}(\omega) \) and \( S_{Q_3}(\omega) \). These are easily evaluated for a sequence of values of \( \omega \) by digital computer. Again for those wishing to examine the program in Appendix II in detail, a convenient way to write the above appears as:

\[
S_{Q_1}(\omega) = \rho_o^2 C_D^2 S_u(\omega) \left\{ |H_{11}^* H_{11}| (RF1)^2 + 2 |H_{11}^* H_{12}| (RF1)(RF2) \\
+ |H_{12}^* H_{12}| (RF2)^2 + 2 |H_{12}^* H_{13}| (RF2)(RF3) + 2 |H_{11}^* H_{13}| \\
(RF1)(RF3) + |H_{13}^* H_{13}| (RF3)^2 \right\}
\]

It must be remembered that each of the \( H_{1j} \) are complex functions of \( \omega \).

An integration of \( S_{Q_1}(\omega) \) for all values of \( \omega \) will give the mean square value of \( Q_1 \). This is easily done at the same time by the computer. A simplified flow chart for the program developed appears as figure 3.

6. PARAMETRIC STUDY

Two particular practicable structural designs were studied. The first is similar to the specific structure mentioned earlier. This structure was considered in some detail. The second structure is quite hypothetical and was used to verify whether behaviour would be generally similar or not, i.e. whether anything "peculiar" had been chosen by chance in the first instance.

A list of dimensions and other properties follows in the table below. Reference may be made to figure 1 for the definitions of the various symbols. It will be seen that structure I is a 900 ft concrete tower supporting a mast nearly 200 ft high. The tower tapers as stated earlier. Structure II has a tower only 700 ft high, but a longer mast of 240 ft. The greater proportion of the heights of the second structure lies beyond the articulation.
<table>
<thead>
<tr>
<th>Structure I</th>
<th>Structure II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lengths (ft)</strong></td>
<td></td>
</tr>
<tr>
<td>L₁</td>
<td>900</td>
</tr>
<tr>
<td>L₂</td>
<td>199</td>
</tr>
<tr>
<td>L₃</td>
<td>115</td>
</tr>
<tr>
<td>L₄</td>
<td>84</td>
</tr>
<tr>
<td><strong>Weights (lb)</strong></td>
<td></td>
</tr>
<tr>
<td>W₂</td>
<td>157,000</td>
</tr>
<tr>
<td>W₃</td>
<td>102,000</td>
</tr>
<tr>
<td>W₄</td>
<td>55,000</td>
</tr>
<tr>
<td>W₅</td>
<td>65,973.5/ft</td>
</tr>
<tr>
<td>z</td>
<td>61.4492/ft²</td>
</tr>
<tr>
<td><strong>Diameters (ft)</strong></td>
<td></td>
</tr>
<tr>
<td>D₃</td>
<td>12.0</td>
</tr>
<tr>
<td>D₄</td>
<td>5.0</td>
</tr>
<tr>
<td>D₀</td>
<td>80.0</td>
</tr>
<tr>
<td>Dₛ</td>
<td>0.067/ft</td>
</tr>
<tr>
<td><strong>Second Moments (ft⁴)</strong></td>
<td></td>
</tr>
<tr>
<td>I₁'</td>
<td>8.380</td>
</tr>
<tr>
<td>I₄'</td>
<td>0.589</td>
</tr>
<tr>
<td>I₀</td>
<td>351858</td>
</tr>
<tr>
<td>I₁</td>
<td>-1364.49/ft</td>
</tr>
<tr>
<td>I₂</td>
<td>+1.85476/ft²</td>
</tr>
<tr>
<td>I₃</td>
<td>-.000854/ft³</td>
</tr>
</tbody>
</table>

H for both structures was constant at 15 ft, and the value $E_c$, elastic modulus for concrete, was taken to be one tenth that for steel. The goal of the parameter study was to vary the spring stiffness, $F$, at the articulation and the viscous damping constant, to find an optimum combination of the two. The criteria of optimality were the minimisation of displacement at the top of the mast and angular rotation. The first criterion is necessary in order to minimise structural dynamic stresses and the second is required for the purpose of reducing signal scatter.

The wind parameters used were those appropriate to an open rural site with surface drag coefficient $K = .005$, gradient wind $V_G = 80$ m.p.h., gradient height $Z_G = 900$ ft., and exponent $\gamma = 0.16$. 

A typical mean square spectral density (p.s.d.) curve for one of the system's three modes appears in figure 6A, and the p.s.d. curve for movement at the mast tip appears in figure 6b. The integral of this curve over all (effective) frequencies is 18.2 ft², giving an r.m.s. amplitude of 4.26 ft.

This r.m.s. amplitude applies only for this spring stiffness, $F$, and value of damping, $C_{visc}$. What is required is a survey of all r.m.s. displacement values for the complete series of combinations over a reasonable range of values of parameters $F$ and $C_{visc}$. Figure 7 presents such a survey, and as such, is the main result of this study. Each curve in the figure is for a specific value of damping constant. The extreme right-hand ordinates have stiffness values so high that they approach the 'rigid' case. There appears to be a local 'optimum' region around $F = 350,000$ lb/ft, and an optimally bad region in the vicinity of $F = 1,000,000$ lb/ft. The local 'optimum' however, represents higher deflections than the rigid case. This would appear to suggest that the proposed method of amplitude reduction is a failure, and that the rigid case is, in fact, the best solution. This is partially true. Some advantages accrue however to the local 'optimum.' Superposed on figure 7 is a dashed curve of concrete tower-top r.m.s. displacements, for the 60,000 lb/ft/sec damping values. They are lower in the region of the optimum and dynamic bending moments will be smaller. This is however an undergraduate dance-floor type of solution, since displacements of the suspended mass, the mast in our case, are quite large. However, in some designs, lower tower bending moments may be an advantage.

The local optimum solution may have a further advantage for some systems. The p.s.d. curves for this solution show much of the energy concentrated at lower frequencies than that of the rigid case, since it is a more 'flexible' system. Much research remains to be done on the environmental issue of which vibrations are most tolerable to persons in a structure but velocity and more particularly acceleration are both as important as amplitude. Thus, in the case of a tall building, the 'optimal' system may well prove preferable, since it involves only slightly greater displacements with much of the energy concentrated at an appreciable lower frequency.

The extrema of the curves in figure 7 are explained by an examination of the mean square value of the individual modes, and of the orthogonal vibration modes. It seems that in the optimally 'good' systems the dominant mode of vibration (at 1.4 rad/sec) involves motion of the tower to the right while the mast tilts in mode 2 to the left (an vice versa). This not only reduces total motion
at the mast tip, it also allows a lot of dissipation of energy at the damper. In the optimally bad case, the dominant mode is at 1.3 rad/sec, and in this mode, both mast and tower move in the same direction in phase, with small $Q_z$ amplitudes. This exaggerates motion at the tip and the dampers are least effective.

Results for Structure II, which has characteristics listed above, are similar. The concrete tower in this case has a higher natural frequency, and the taller mast requires greater spring stiffnesses to give the same results. The overall behaviour is summarised in figure 8. Again, there is an optimally good region of spring stiffnesses, this time at about $F = 900,000$ lb/ft, and an optimally bad region near $3,400,000$ lb/ft. R.m.s. displacements are however, even less for values of $F$ nearing $4,800,000$ lb/ft., and decrease monotonically to even better values for higher $F$. The stiffest case is again the best, and the optimum at 900,000 is only a local one.

All the above deals only with dynamic vibratory response. The rigid case is therefore even more attractive, since dynamic deflections will be super posed on much lower values of steady-state wind deflection.

7. CONCLUSIONS

The random vibrations of a tall un-guyed television-broadcasting-type structure, subject to random wind excitation have been studied. Only the in-wind vibration caused by turbulent wind velocity fluctuations is considered. Vibrations due to vortex shedding and due to earthquake motion are not considered.

The scheme of introducing an articulation, with added springs and dampers, as an attempt to reduce r.m.s. response proved unsuccessful, in the sense that response was never as small as for the rigid case, without the articulation. This conclusion is valid, at least for the range of spring stiffness and damping values studied. This range was felt to represent something of what was available commercially.

This touches however, on the major limitation of the study, the assumption of linear springs and dampers. The springs arranged radially about the mast, at the position shown in figure 2, will have an effective system stiffness which is non-linear and hardening. This is easy to show and is due to the 'tightrope' effect of the pair of springs perpendicular to any motion. This will have a beneficial effect on the local optimum r.m.s. displacements, almost certainly reducing them. By how much it is not possible to say, without a much more complicated formulation. Again, a linear damping constant was used as in most conventional dynamics calculations, mainly because this renders the differential equations of motion tractable. An added justification is present here however, in that low natural frequencies are involved with small displacements. This makes velocities small, and linear damping quite desirable. It would not be surprising however, if in fact, a commercially available damper were able to develop some portion of 150,000 lbs/ft/sec; if this damper were also to display some non-linearity. One would indeed almost expect this. Certainly a system of synthetic rubber
blocks in shear able to dissipate this much energy linearly for such a relatively low linear stiffness would seem difficult to develop.

Figures (4a) and (5a) show the modulii of the complex frequency response functions for the optimally 'good' case, and figures (4b) and (5b) refer to the optimally 'bad' case. In the good case the frequency of the articulated mast system coincides with that of the tower without the damping system. The resonant peaks present in these figures correspond to the following modal configurations.

Figure (4a). At 1.4 rad/sec motion of mast and tower are 180 degrees out of phase. At 0.95 rad/sec the mast and tower are in phase.

Figure (4b). At 1.3 rad/sec the tower and mast are in phase whereas at 1.8 rad/sec both are out of phase.

Clearly the system equations of motion are highly coupled and the response functions are very sensitive to changes in stiffness and damping at the mast articulation point.

In addition to the r.m.s. mast top deflection being greater in the optimally 'good' case than in the 'rigid' case, the r.m.s. rotation or slope of the transmitter mast was also greater. It is essential in T.V. transmission to minimise mast rotation in order to reduce signal scatter. Thus the 'rigid' system gives the least rotation as well as the least tip deflection.

A suggestion for further work in this area would appear to be toward a study of manufactured damping systems and their exact characteristics. Some of the other work in the Cranfield group is quite compatible with this direction, especially the forthcoming work on non-linear systems by Kirk (Ref.7). At the same time it is far from certain that the initial idea (of added flexibility with added dampers) would not in fact be a good idea for the reduction of earthquake response in some structures. This appears to be another possible subject for study.
REFERENCES


APPENDIX I

SELF-WEIGHT TERMS

The change in height of the top of the tower, due to the variable slope of the tower with respect to its original vertical centre line causes a lowering of the mast. The accompanying change in potential energy is

\[-W_z \int_0^{L_1} \frac{1}{2} \left( \frac{dy_1}{dx_1} \right)^2 dx_1 = \frac{\pi^2 W_2 L_1}{16} Q_1^2 \ldots (A1.1)\]

The mast is also lowered because of the rotation, \( \theta \), at its mounting, equal to

\[\left( Q_1 \frac{\pi}{2} + Q_2 \right).\]

If \( \bar{W}L_2 \) is the first moment of the weight of the mast with respect to the mast base, then this change in potential energy is

\[-\bar{W}L_2 \left( Q_1 \frac{\pi^2}{4} + Q_1 Q_2 \pi + Q_2^2 \right) \ldots (A1.2)\]

provided the movements are small.

General deflections of the mast in its elastic mode will be from an initially sloped position, angle \( \theta \) from the vertical. Elastic deflections of the mast will therefore have a vertical component. If \( R_M \) and \( R_N \) are distances above the hinge of the centres of gravity of the lower and upper parts of the mast, then this change of potential energy is

\[-\sin \theta \left[ W_3 Q_3 L_2 \left( 1 - \cos \frac{\pi}{2} \frac{R_M}{2L_2} \right) + W_4 Q_3 L_2 \left( 1 - \cos \frac{\pi}{2} \frac{R_N}{2L_2} \right) \right] \]

\[\ldots (A1.3)\]

Further, the slope of the elastically deflected mast with respect to its position at angle \( \theta \) draws the centre of gravity back toward the hinge. The change in potential energy here is

\[-\cos \theta \left[ \frac{W_3 \pi^2}{8} Q_3^2 \int_0^{R_M} \sin^2 \frac{\pi x_2}{2L_2} dx_2 + \frac{W_4 \pi^2}{8} Q_3 \right.\]

\[\left. \int_0^{R_N} \sin^2 \frac{\pi x_2}{2L_2} dx_2 \right] \]

\[\ldots (A1.4)\]
Approximately \( \sin \theta \) by \( \theta \), and \( \cos \theta \) by unity to eliminate fourth order terms and higher, and adding (A1.3) and (A1.4) gives,

\[
-(\frac{\pi}{2}BQ_1Q_3 + BQ_2Q_3 + AQ_3^2)
\]

... (A1.5)

where

\[
B = W_3L_2\left(1 - \cos \frac{\pi RM}{2L_2}\right) + W_4L_2\left(1 - \cos \frac{\pi RN}{2L_2}\right)
\]

... (A1.6)

and

\[
A = \frac{\pi^2}{16} (W_{RM} + W_{RN} - \frac{W_3L_2}{\pi} \sin \frac{\pi RM}{L_2} - \frac{W_4L_2}{\pi} \sin \frac{\pi RN}{L_2})
\]

... (A1.7)

A more significant loss of potential energy is the lowering of self-weight of the concrete tower during flexure. The weight above any point on the tower at height \( x_0 \), excluding the mast, is

\[
\begin{aligned}
L_1 \\
(W_F - Zx_1)dx_1 = W_FL_1 - \frac{ZL_1^2}{2} - W_Fx_0 + \frac{4\omega x_0^2}{2}
\end{aligned}
\]

... (A1.8)

the downward movement at any point is

\[
\frac{1}{2}(dy_1/dx_1)^2 = \frac{Q_1^2}{8}\pi^2 \sin^2 \frac{\pi x_1}{2L_1}
\]

... (A1.9)

so that the change in potential energy is

\[
-CQ_1^2 = -Q_1^2\left(\frac{\pi^2}{8}\int_{0}^{L_1} \left(W_FL_1 - \frac{ZL_1}{2} - W_Fx_1 + \frac{Zx_1^2}{2} \sin^2 \frac{\pi x_1}{2L_1}\right)dx_1
\]

... (A1.10)

where \( C \) is implicitly defined.
APPENDIX II

COMPUTER PROGRAM

The following pages comprise the computer program used to obtain numerical results. Most of the notation is as it appears in the report text, with minor changes with which most readers will have no difficulty. The following equivalences between text and program apply, as well as some others which are obvious.

<table>
<thead>
<tr>
<th>Text</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{ij}$</td>
<td>DAMP(I,J)</td>
</tr>
<tr>
<td>$M_{ij}$</td>
<td>EM(I,J)</td>
</tr>
<tr>
<td>$V_{ij}$</td>
<td>V(I,J)</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>EQIJ</td>
</tr>
<tr>
<td>$I_{j'}$</td>
<td>EIJ</td>
</tr>
<tr>
<td>$\omega$</td>
<td>W</td>
</tr>
<tr>
<td>$V^*_{ll}$</td>
<td>V11A</td>
</tr>
<tr>
<td>$M_{ll}$</td>
<td>EM11A</td>
</tr>
</tbody>
</table>
LIBRARY (SUBGROUPFSCFSE)
PROGRAM(D11A)
INPUT 1=CR0
OUTPUT 2=LPO
COMPRESS INTEGER AND LOGICAL
END

MASTER TOWER PROB COMPLETE

UNITS THROUGHOUT ARE FT., LBS., SEC.
DIMENSION EM(3,3), DAMP(3,3), V(3,3)
REAL L1,L2,L3,L4,13,14
REAL PRIME

COMPLEX E422,E423,E432,E433,DELTA, H11,H12,H13,H22,H23,H31,H32
COMPLEX H53,H54,H55,SH11,SH12,SH13,SH21,SH22,SH31,SH32,SH33

REAL L1,L2,L3,L4

AFLOW(X) = (D0-D5*X)*L1*(1.-COS(PI*L*X))*VGZG*X**GAMMA
AF1MID(X) = D5*(.5*PI*(X-L1)+L1)*VGZG*X**GAMMA
AF1TOP(X) = D4*(.5*PI*(X-L1)+L1)*VGZG*X**GAMMA
AF2MID(X) = D5*(X-L1)*VGZG*X**GAMMA
AF2TOP(X) = D4*(X-L1)*VGZG*X**GAMMA
AF3MID(X) = L2*D5*(1.-COS(PI*L*(X-L1)))*VGZG*X**GAMMA
AF3TOP(X) = L2*D4*(1.-COS(PI*L*(X-L1)))*VGZG*X**GAMMA

CX(X) = (PI*SQR/8.)*(WF*L1-Z*L1*L1*.5-WF*X+Z*X*X*.5)*(SIN(PTL0*X))**2

V11AX(X) = (EC*PI**4/(16.*L1*L1))*(EINOT+E11*X+E12*X*X+EI3*X**X**X)

EM3AX(X) = (1./G)*((WS/L3)*(L1*L2*(1.-COS(PTLT*X)) + .5*L2*PI*X*(1.-COS(PTLT*X))))
EM3BX(X) = (1./G)*((W4/L4)*(L1*L2*(1.-COS(PTLT*X)) + .5*L2*PI*X*(1.-COS(PTLT*X))))
EM33AX(X) = (1./G)*W3*L2*L2*(1.-COS(PTLT*X))**2/L3
EM33BX(X) = (1./G)*W4*L2*L2*(1.-COS(PTLT*X))**2/L4

55 FORMAT(2X,A8,15)

G = 32.2
PI=3.14159
PISQR= 9.86960
PIF0UR= 97.400
EC = .432E+09
EST = .432E+10
READ(1,1) L1,L2,L3,L4
1 FORMAT(4F0.0)
READ(1,2) W2,W3,W4
2 FORMAT(3F0.0)
READ(1,3) WF,Z
3 FORMAT(2F0.0)
READ(1,4) I3,I4
4 FORMAT(2F0.0)
READ(1,21) EINOT,E11,E12,E13
21 FORMAT(4F0.0)
READ(1,21) EINOT,E11,E12,E13
21 FORMAT(4F0.0)
W2BAR = W3*L3/2. + W4*(L3+L4/2.)
READ(1,14) D0,DS,D3,D4
14 FORMAT(4F0.0)
READ(1,15) RK,LPRI,VBARG,ZG,GAMMA
15 FORMAT(5F0.0)


```
WRITE(2,13)
13 FORMAT(1X,45HSTRUCTURAL/AEROSPACE DYNAMICS GROUP CRANFIELD,
     1/,2X,45H***************************************************************************)
WRITE(2,12)
12 FORMAT(1X,2X,45HSTEEL MAST ON CONCRETE TOWER WIND RESPONSE,
     1/,2X,33HKEN JOHNHS-COLIN KIRK SUMMER 1973 /)
WRITE(2,96)
96 FORMAT(1/,2X,16HTOWER PROPERTIES,/,2X,16H-------------------- )
WRITE(2,32)L1,L2,L3,L4,W2,W3,W4,WL2BAR,WF,Z,I3,I4
52 FORMAT(2X,3HL1=,F9.3,3X,3HL2=,F9.3,3X,3HL3=,F9.3,3X,3HL4=,F9.3,
     1//,2X,3HH4=,F9.0,3X,3HH5=,F9.0,3X,3HH6=,F9.0,3X,7WHL2BAR=,E12.6,
     2//,2X,3HHF=,F9.1,3X,2HH=,F8.4,/,2X,3HI3=,F6.5,3X,5HI4=,F6.3)
WRITE(2,36)E1N0,T1,E1,T2
56 FORMAT(1/,2X,6HE1N0=T,2X,6HE1=,F9.6,2X,6HE1=,F9.6,2X,6HE1=,F9.6,
     1/,2X,6HE1F=,F8.0,2X,6HE1=,F9.3,2X,4HE12=,F9.6,2X,4HE13=,
     1/,2X,6HE14=,F9.6)
WRITE(2,37)D0,DS,D3,D4
57 FORMAT(1//,2X,15HWIND PROPERTIES,/,2X,15H---------------------- )
WRITE(2,28)RK,LPRIME,V13ARG,Z6,GAMMA
37 FORMAT(1/,2X,3HDU=,F9.3,3X,3HD3=,F9.3,3X,3HD4=,F9.3)
WRITE(2,35)
5H FORMAT(1/,2X,15HWIND PROPERTIES,/,2X,15H---------------------- )
WRITE(2,39)RK,LPRIME,V13ARG,Z6,GAMMA
ALF= PI*L3/L2
BET= ALF/2
SALF = SIN(ALF)
CALF = COS(ALF)
S2ET = SIN(BET)
CBET = COS(BET)
RM = .5*L3
RN =L3 +.5*L4
PTLU = PI/(2.*L1)
PTLT = PI/(2.*L2)
A = .0625*PI*W*W*(W5*RM+W4*RN-W3*L2*SIN(P1*RM/L2)/PI
1-W4*L2*W3*L3+W3*L2*PI*RM/L2))/PI
B=W3*L2*(1.-COS((.5*PI/RM)*L2))/W4*L2*(1.-COS((.5*PI/L2)*RN))
D=(EST*PI*FOUR/(32.*L2*L2))*(I3*L3+I4*L4*(I3*L3/L2)+SIN(P1*L3/L2)
1-(14*L2/P1)*SIN(P1*L3/L2))
H0=0.
H0=0.
H1=L1
H2=L1+L3
H3=L1+L2
E = .01
IND = 3
CALL4INTSMP(H0,H1,CX,E,IND,C)
WRITE(2,5)
5 FORMAT(1/,1X,22HEVALUATING USEFUL ABCD)
WRITE(2,9)
6 FORMAT(1/,1X,22HEVALUATING USEFUL ABCD)
WRITE(2,7)A,B,C,D
7 FORMAT(1X,E12.6,3X,E12.6,3X,E12.6,3X,E12.6,3X)

RFI ARE GEN*ZD FORCES DIVIDED BY U(T)*KHO*CD*SQUARED
ALL VELOCITIES ARE IN FT./SEC.
ZM IS 2C IN METRES 

KHO=.07604/32.
```
CD = .00
PM = 5.0.
ZGM = 0.30524
V02G = VBARG/(ZG**GAMMA)
VBAH10 = VBARG*(10./ZGM)**GAMMA
PI1 = >PI/L2
F = .01
IND = 3
CALLF4INTSMP(H0,H1,AF1LOW,E,IND,F1LOW)
IND = 3
CALLF4INTSMP(H1,H2,AF1MID,E,IND,F1MID)
IND = 3
CALLF4INTSMP(H2,H3,AF1TOP,E,IND,F1TOP)
IND = 3
CALLF4INTSMP(H1,H2,AF2MID,E,IND,F2MID)
IND = 3
CALLF4INTSMP(H2,H3,AF2TOP,E,IND,F2TOP)
IND = 3
CALLF4INTSMP(H1,H2,AF3MID,E,IND,F3MID)
IND = 3
CALLF4INTSMP(H2,H3,AF3TOP,E,IND,F3TOP)
WRITE(2,31)F1LOW,F1MID,F1TOP,F2MID,F2TOP,F3MID,F3TOP
31 FORMAT(//,2X,7HF1LOW=E12.6,2X,7HF1MID=E12.6,2X,7HF1TOP=, 1E12.6,2X,7HF2MID=E12.6,2X,7HF2TOP=,E12.6,2X,7HF3MID=, 2E12.6,2X,7HF3TOP=,E12.6)
RF1=F1LOW+F1MID+F1TOP
RF2=F2MID+F2TOP
RF3=F3MID+F3TOP
WRITE(2,53)RF1,RF2,RF3
53 FORMAT(5X,4HRF1=,E12.6,5X,4HRF2=,E12.6,5X,4HRF3=,E12.6)

HAVING A,B,C,D FORM MATRICES OF EQNS OF MOTION
MATRIX NAMES- EM IS INERTIA,DAMP OBVIOUS, V IS STIFFNESS

E = .01
IND = 3
CALLF4INTSMP(H0,L5,EM13AX,E,IND,EM13A)
IND = 3
CALLF4INTSMP(L3,L2,EM13BX,E,IND,EM13B)
IND = 3
CALLF4INTSMP(H0,L5,EM23AX,E,IND,EM23A)
IND = 3
CALLF4INTSMP(L3,L2,EM23BX,E,IND,EM23B)
IND = 3
CALLF4INTSMP(L3,L2,EM33AX,E,IND,EM33A)
IND = 3
CALLF4INTSMP(L3,L2,EM33BX,E,IND,EM33B)
EM11A=(L1**2/6)*(.4F*L1*(.5-4./PI)-2*L1**2*(.75-4./PI+7./PI*SQRT))
EM11B=(W4/(6*L4))*(L1*L1*L2+0.5*PI*L1*L2*L2 1+0.5*3333*PI*SQRT*L2**3)+(1./6)*((W3/L3-W4/L4)*(L1*L1*L3+.5*PI*L1*L3 2*L1+0.5*3333*PI*SQRT*L3**3)
EM12=(1./6)*((W3/L3)*(.5*L1*L3*L3+.16667*PI*L3**3)+W4/L4)* 1.5*3333*PI*SQRT*L2**3*3.5*L1*L3*L3-166667*PI*L3**3))
EM(1,1) = EM11A+EM11B
EM(1,3) = EM13A+EM13B
EM(2,3) = EM23A+EM23B
EM(3,3) = EM33A+EM33B
EM(2,1) = EM(1,2)
EM(3,1) = EM(1,3)  
EM(3,2) = EM(2,3)  
IND = 6  
F = 10.  
CALL F4.NTSMPC(H0,H1,V11AX,E,IND,V11A)  
V(1,1) = V11A - 2.*(.125*WL2BAR*PI*SQRT(V11A) + .0625*PI*SQRT(WL2BAR*H1 + C)  
V(1,2) = -.5*WL2BAR*PI  
V(1,3) = -.5*H*PI  
V(2,1) = V(1,2)  
V(3,1) = V(1,3)  
F IS MASt SUPPORT SPRING STIFFS , LB./FT.  
VISC IS DAMPING AT MASt SUPPORT , LB./FT./SEC.  
H=15,  
DU 49 JJ = 4,48,4  
VEX=JJ  
F = 100000.*VEX  
DU 59 K = 60000,150000,90000  
VISC = K  
WRITE(c,34)F,VISC  
4 FORMAT(//,2X,4HF = ,F8.0,4X,8HCVISC = ,F8.0)  
WRITE(2,48)  
48 FORMAT(2X,29H............................ )  
CALL TIME(TIM)  
WRITE(2,55)TIM,IND  
DAMP(1,1) = .04*(EM(1,1))**SQRT((V(1,1))/(EM(1,1)))  
DAMP(1,2) = 0.  
DAMP(1,3) = 0.  
DAMP(3,1) = 0.  
DAMP(4,1) = 0.  
DAMP(2,2) = CVISC*H*H  
DAMP(2,3) = CVISC*H*L2*(1.-COS(P1*H/(2.*L2)))  
DAMP(3,2) = DAMP(2,3)  
DAMP(3,3) = CVISC*H*L2*(1.-COS(P1*H/(2.*L2)))  
V(2,2) = F*H*H - WL2BAR  
V(2,3) = F*H*L2*(1.-COS(P1*H/(2.*L2))) - B  
V(3,2) = V(2,3)  
V(3,3) = 0 + F*L2*L2*(1.-COS(P1*H/(2.*L2)))**2 -2.*A  
WRITE(2,4)  
8 FORMAT(// ,5X,17HMATRIX EM FOLLOWS)  
WRITE(2,4)((EM(I,J),J=1,3)I=1,3)  
9 FORMAT(//,5X,E12.6,5X,E12.6,5X,E12.6)  
WRITE(2,10)  
10 FORMAT(//,5X,19HMATRIX DAMP FOLLOWS)  
WRITE(2,4)((DAMP(I,J),J=1,3)I=1,3)  
WRITE(2,11)  
11 FORMAT(//,5X,28HMATRIX V(STIFFNESS) FOLLOWS )  
WRITE(2,4)((V(I,J),J=1,3)I=1,3)  
TUT IS A SUBTOTAL TO INTEGRATE SPECT. DENSE. OF WIND  
TUT1 =0.  
TUT2=0.  
TUT3 =0.  
TUTTOP=0.  
WRITE(c,92)  
92 FORMAT(//,4X,1HW,7X,3HH11,8X,3HH12,8X,3HH13,8X,3HH22,8X,3HH23,8X,3HH33,8X,3HSQ1,8X,3HSQ2,8X,3HSQ3,8X,4HSTOP)  
DU 99 I = 5,2000,5  
SEX=I  
- A7 -
W = .01*SEX
EQ11 = COEFF*T OF EQ*M EQ*N
EQ11 = -W*W*EM(1,1) + V(1,1)
EQ12 = -W*W*EM(1,2) + V(1,2)
EQ13 = -W*W*EM(1,3) + V(1,3)
EQ21 = EQ12
EQ22 = -W*W*EM(2,2) + V(2,2)
EQ23 = -W*W*EM(2,3) + V(2,3)
EQ31 = EQ13
EQ32 = EQ23
EQ33 = -W*W*EM(3,3) + V(3,3)
DELTA = EQ11*(EQ22*EQ33-EQ23*EQ23)-EQ12*(EQ21*EQ33-EQ23*EQ21) +
1*EQ13*(EQ21*EQ32-EQ22*EQ31)
H11 = (EQ22*EQ33-EQ23*EQ23)/DELTA
H12 = (EQ13*EQ32-EQ12*EQ33)/DELTA
H13 = (EQ12*EQ23-EQ13*EQ22)/DELTA
H21 = H12
H22 = (EQ11*EQ33-EQ13*EQ13)/DELTA
H23 = (EQ13*EQ21-EQ11*EQ23)/DELTA
H32 = H25
H31 = H13
H33 = (EQ11*EQ22-EQ12*EQ12)/DELTA
SH11 = CONJG(H11)
SH12 = CONJG(H12)
SH13 = CONJG(H13)
SH22 = CONJG(H22)
SH23 = CONJG(H23)
SH33 = CONJG(H33)
SH21 = SH12
SH31 = SH13
SH23 = SH23
HT1 = (L1 + .5*PI*L2)*H11 + L2*H21 + L2*H31
HT2 = (L1 + .5*PI*L2)*H12 + L2*H22 + L2*H23
HT3 = (L1 + .5*PI*L2)*H13 + L2*H32 + L2*H33
SHT1 = CONJG(HT1)
SHT2 = CONJG(HT2)
SHT3 = CONJG(HT3)
SU = 2.*RK*LR*VBAR10/(PI*(2.+(W*LP**10)/(2.*PI*VBAR1O))**2)**2
1PRU = W
RHC0 = (RHC0**2)
SU1 = RHC0*SU*(CABS(H11*SH11)*RF1*RF1+CABS(H12*SH12)*RF2*RF2)
1*CABS(H13*SH13)*RF3*RF3
2*K1*RE*SU*(CABS(H11*SH11)*RF1*RF2+Z.*CABS(H11*SH13)*RF1*RF3)
3*K1*RE*SU*(CABS(H12*SH12)*RF2*RF2+Z.*CABS(H21*SH21)*RF1*RF1)
1*CABS(H23*SH23)*RF3*RF3
2*K1*RE*SU*(CABS(H11*SH11)*RF1*RF2+Z.*CABS(H12*SH22)*RF1*RF3)
3*K1*RE*SU*(CABS(H13*SH13)*RF1*RF3+Z.*CABS(H12*SH23)*RF1*RF3)
1*CABS(H21*SH21)*RF1*RF1+Z.*CABS(H21*SH22)*RF1*RF1+Z.*CABS(H31*SH31)*RF1*RF1
SHTP = RHC0*SU*(CABS(H11*SH11)*RF1*RF1+CABS(H12*SH12)*RF2*RF2)
\[ -A9 - \]

\[ 1 + \text{CABS}(HT3*SHT3)*RF3^2 \]
\[ 2 + 2.\text{CABS}(HT1*SHT2)*RF1*RF2+2.\text{CABS}(HT1*SHT3)*RF1*RF3 \]
\[ 3 + 2.\text{CABS}(HT2*SHT3)*RF2*RF3 \]

\[ RH11 = \text{CABS}(H11) \]
\[ RH12 = \text{CABS}(H12) \]
\[ RH13 = \text{CABS}(H13) \]
\[ RH22 = \text{CABS}(H22) \]
\[ RH23 = \text{CABS}(H23) \]
\[ RH33 = \text{CABS}(H33) \]

\[ \text{WRITE}(2,95) \ W, RH11, RH12, RH13, RH22, RH23, RH33, SQ1, SQ2, SQ3, STP \]


\[ MSL \text{ ARE M.S. RESP. NON*DIM*SED} \]
\[ \text{SUBTOTALS WILL GIVE M.S. RESPONSE} \]

\[ TOT1 = TOT1 + SQ1 \]
\[ TOT2 = TOT2 + SQ2 \]
\[ TOT3 = TOT3 + SQ3 \]
\[ TOTTOP = TOTTOP + STP \]

\[ \text{CONTINUE} \]

\[ \text{M.S. RESPONSE BY //LLOGRAM RULE} \]
\[ ZMS1 \text{ IS SIMPLY AN ALIAS FOR MSL} \]
\[ ZMS1 = 0.5*TOT1 \]
\[ ZMS2 = 0.5*TOT2 \]
\[ ZMS3 = 0.5*TOT3 \]
\[ ZMSTOP = 0.5*TOTTOP \]

\[ \text{WRITE}(2,94) ZMS1, ZMS2, ZMS3, ZMSTOP \]

\[ \text{FORMAT}(2X, 5X, 6HMS1 = , E9.3, 3X, 6HMS2 = , E9.3, 3X, 6HMS3 = , E9.3, 13X, 6HMSSTOP = , E9.3) \]
\[ RMS1 = \text{SQRT}(ZMS1) \]
\[ RMSDT = L1*RMS1 \]
\[ RMSSTOP = \text{SQRT}(ZMSTOP) \]

\[ \text{WRITE}(2,95) RMSDT, RMSSTOP \]

\[ \text{FORMAT}(2X, 5X, 3HR, M.S. DEFLECTION AT TOWER TOP=} , F7.3, 140\text{FEET, AND R.M.S. DEFLECTION AT MAST TOP=} , F7.3, 12HFEET (TOTAL)) \]

\[ \text{CONTINUE} \]

\[ \text{CONTINUE} \]
\[ \text{STOP} \]
\[ \text{END} \]
\[ \text{FINISH} \]
FIGURE 1: STRUCTURE AND MODES OF DEFLECTION

FIGURE 2: POSITION OF SPRING-DAMPER SYSTEM
Calculate matrices of equations of motion.

Fix \( \omega \) (small).

Calculate complex matrices \( E_{ij} \) and invert, giving \( H_{ij} \).

Calculate \( s_0(\omega), s_{Q_1}(\omega) \).

Integrate by adding up \( s_{Q_1}(\omega) \Delta \omega \) subtotal.

Increase \( \omega \).

\( \omega \) still small.

\( \omega \) large enough.

Calculate mean square \( Q_i \), R.M.S. values.

**Figure 3: Simplified Flow-Chart of Computer Program**
STRUCTURE I
F = 300,000 lb/ft
C\text{VIS}C = 60,000 lb/ft/sec
LOCAL OPTIMUM:
FREQUENCY OF MAST
TUNED TO FREQUENCY
OF TOWER
OPTIMALLY GOOD CASE

APPROXIMATELY
NATURAL FREQUENCY OF
ORIGINAL TOWER

\( \omega_n = 0.95 \text{ rad/sec} \)
(approx frequency of
articulated mast only)

FIGURE 4 (a): FREQUENCY RESPONSE FUNCTIONS \( H_1, H_2, H_3, H_{11}, H_{12}, H_{13} \)

OPTIMALLY BAD CASE

\( \omega_n = 1.8 \text{ rad/sec} \)
(approx frequency of
articulated mast only)

FIGURE 4 (b): FREQUENCY RESPONSE FUNCTIONS
FIGURE 5(a) FREQUENCY RESPONSE FUNCTIONS $H_{22}, H_{23}, H_{33}$
FIGURE 6: POWER SPECTRAL DENSITY FUNCTIONS FOR STRUCTURE I
FIGURE 7: RESULTS, STRUCTURE I

FIGURE 8: RESULTS, STRUCTURE II