Estimation of medium parameters by acoustic echo measurements

PROEFSCHRIFT ter verkrijging van
de graad van doctor in de
technische wetenschappen
aan de Technische Hogeschool Delft,
op gezag van de Rector Magnificus,
prof. dr. J.M. Dirksen,
in het openbaar te
verdedigen en overstaan
van het College van Doktoren op
dinsdag 3 december 1983
te 16.00 uur door

PETER ROBERT MEKDAG

geboren te Zeedijk
nauwkeurig ingenieur

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Special thanks are due to professor Lukehouts and Jan Valdor, who were my tutors throughout my studies.

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Rik
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PREFACE

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CHAPTER I

1.1 INTRODUCTION

The interest in the physical phenomena of mechanical vibrations and sound dates back many centuries. The earliest experiments with strings are attributed to the Greek philosophers Pythagoras and Aristotle, but the basic elements for imaging with sound have become available only in the last few decades. For many centuries all the experiments were performed to try to discover the basic concepts of sound and vibration, and it wasn't until the late 18th and 19th century that a sound mathematical basis was given to the theory of sound propagation. Pioneers in this field were a.o. Huygens, Kirchhoff and Rayleigh.

Once the theory of sound propagation was introduced, the reflection of sound from obstacles could be studied, and the road was open to non-invasive investigation by means of reflected sound waves. The idea of visualizing structures in an optically opaque environment by mechanical excitation at the surface is not new. Palpation of patients was recognized as a diagnostic tool long before medical ultrasound was introduced. A blind person can also find his way without knowing Huygens' principle.

In the early days the researchers relied on their senses and used only their ears and eyes to measure vibrations. Thus it was principally impossible to measure and quantify an entire sound field. In the 1870's Bell and Curie discovered how to transform vibrations into electrical current by electro-magnetic and piezo-electric elements respectively.
This meant that all the basic elements of modern non-invasive sound investigation systems had then been discovered. The seismic field was first to benefit from these new discoveries, soon followed by sonar (S0und NAvigation and Ranging). For practical reasons the field of medical imaging is more closely related to sonar applications than to seismics, and its development was seriously impeded by the fact that most research was classified by naval intelligence. Only after the second world war, with the introduction of modern electronics, did high frequency sound become widely recognized as an imaging tool. In all applications of sound as an imaging tool a trade off must be made between penetration depth and required resolution. The penetration depth depends on the rate at which sound is attenuated. The greater the attenuation, the smaller the penetration depth. The attenuation in most cases increases monotonically as a function of frequency, while the resolution, as a rule, is inversely proportional to the frequency. Roughly, the areas where sound is applied may be subdivided into three classes, depending on the frequency range. Historically these classes have been fixed in relation to the audible frequencies, and we speak of ultrasound, sound and infrasound. In the seismic field, where large penetration depths are required, frequencies of 1-100 Hz are applied, limiting the resolution to tens of metres.

On the high frequency side of the scale we find medical imaging and N.D.E (Non-Destructive Evaluation of materials), where frequencies of 1-100 MHz are used with a resolution in the sub-millimetre range. Actually in the field of acoustic microscopy even higher frequencies are used, into the GHz region, but this can no longer be seen as a non-invasive technique.

The intermediate frequencies are mainly used for sonar, sub-bottom profiling and sodar (S0und Detection And Ranging in the lowest layers of the atmosphere).

Though the only principal difference between e.g. seismics and medical ultrasound is a scaling factor of $10^6$, their historical development has been quite different.

In medical ultrasound the first transducers were single, hand-held, piezo-electric elements, which were used simultaneously as a source and receiver. These transducers would emit what is often referred to as a pencil beam and could be used in a single scan mode (A-mode) or in a
two dimensional scan mode (B-mode). After the hand-held single element scanners, mechanical sector scanners were developed to eliminate operator movement, soon followed by phased array sector scanners and linear array scanners, where the transducer is subdivided into many small elements. Until recently the development of ultrasonic equipment was limited only by problems of technological origin, but now, as the resolution of the imaging systems is moving towards the theoretical half wavelength limit, we find that basic physical limitations are beginning to play a role. Consequently the improvement of image quality has been levelling off and in the seventies an impulse was given to a whole new field: tissue characterization. Since then medical ultrasound has been struggling to quantify and overcome basic system limitations inherent to the use of focused (pencil beam) transducers. Adequate compensation schemes for e.g. the so-called "diffraction effect" have yet to be developed.

In the meantime the seismic field was developing in a different direction. Owing to the fact that very low frequency and high energy sources are needed to penetrate into the earth, it is not possible to build transducers which emit and receive a pencil beam. Here the approach was adopted of using one source and many receivers for one experiment. As both source and receivers were omnidirectional, the entire sound field returning at the surface was sampled and stored without distortion due to the "diffraction effect". The focussing, which in medical imaging is done in transmission and reception (real time), now takes place later in a computer, by means of a synthetic aperture technique ("migration").

In the past, ideas in medical ultrasound were often derived from other fields. The first research was performed in the field of underwater acoustics. Later, transmission tomography was developed, based on X-ray tomography. The arithmetic reconstruction technique (A.R.T.) for sound propagation velocity and absorption imaging was implemented directly from X-ray tomography. Unfortunately the straight line approximation of A.R.T. is not applicable to media with a varying sound velocity, so the first arrival was not always the pulse which had travelled the shortest path between source and receiver. Thus, it was in transmission tomography that, for the first time in medical ultrasound, inverse wave field extrapolation techniques were attempted, similar to those in the seismic field.
There is an essential difference between the seismic method and acoustic transmission tomography. In the first case only the scattered wave is measured, while in the second case the (perturbed) incident and the scattered wave are measured at the same time. Thus in transmission tomography the information about scatterers is obscured by the much stronger incident wave field.

The reconstruction algorithms used in this thesis are also based on the seismic approach of inverse wave field extrapolation; but here the algorithms, which have been developed over the past years in the Delft laboratory of Seismics and Acoustics, are mainly applied to reflection measurements. It is the conviction of the author that this cross-fertilization between seismics and medical ultrasound will overcome some of the difficulties which are holding back the advancement of medical ultrasound tissue characterization. The new developments may also prove interesting to other fields of echo acoustics because a more general theory will evolve.

1.2 FOCUSING

A large part of this thesis is based on an inversion and imaging technique called synthetic focussing. We will try here to explain why focussing is an essential aspect in the estimation of medium parameters.

The reflection data that reach an imaging system have travelled for some distance between the object and the imaging system. The fundamental purpose of the imaging system is to reconstruct the object. Essentially a reconstruction or focussing technique removes the influences of propagation, allowing us to recover as much information as possible about the object, without the distortions due to propagation.

If the propagation influences have not been compensated for correctly, this can be seen straight away as a blurring of the image of the small inhomogenieties. In this case improvement of the focussing process will also improve the image quality. Even if all available data is treated correctly, the image will not show infinite detail. This is partly due to the limitations of the measurement system but also, as will be shown later on, there exists on physical grounds a theoretical limit to the resolution of any imaging system of half a wavelength of the central
frequency of the applied pulse. We are assuming here that we have no prior knowledge concerning the object.

The range over which the information from an infinitely small object is projected after passing through the imaging system is called the ambiguity function or point spread function (p.s.f.).

Two sets of parameters may be extracted from measurements with reflected sound.

- Firstly there are the bulk parameters or propagation parameters. For our purposes the most important bulk parameters are the spatially averaged sound propagation velocity and absorption. Correct imaging is only possible by inserting these propagation parameters into the focussing process, enabling these parameters to be assessed during focussing.

It is important to realize that the effect of these parameters will be averaged in some way over the path travelled between the object and the imaging system. Consequently any algorithm to estimate detail from bulk parameters will be of limited accuracy. Typically the resolution of these parameters is far worse than the image resolution.

- After removal of the propagation influences we may study a second set of parameters. These local reflectivity parameters describe how the incident wave is affected by the scattering objects. These parameters contain detailed information about the size, shape, distribution and substance of the scattering objects.

In conclusion we see that two sets of parameters may be extracted from an acoustical imaging system: the bulk parameters and the local reflectivity parameters. Focussing represents the step in which the separation of these parameters is achieved.

1.3 ON THIS THESIS

The basic pre-processing tool used throughout this thesis is, as mentioned above, the focussing or inversion technique for the removal of propagation effects from the measured data. Before any propagation effects may be compensated for, we must first determine our acoustic model. In the following chapter we will see that the acoustic model may be subdivided into separate subsystems.
Typically an acoustic reflection measurement with active sources and detectors may be described by a model with five subsystems:
- generation of the acoustic wave field by a source
- forward propagation into the medium
- reflection from the object
- forward propagation to the detector
- detection

In this thesis emphasis will be laid on the zero offset data acquisition technique. This type of data acquisition, where source and detector are in the same position, is pre-eminently suited for high resolution imaging. The zero offset data acquisition technique, which is essentially a reflection or backscatter technique, may under certain conditions be approximated by an "exploding reflector" model. Once this simplification has been made by means of the half velocity substitution, we are able to devise a computationally efficient inversion or focussing algorithm. This focussing algorithm was first developed in the field of seismics, and is described in detail in chapter III. Also in chapter III, the basic limitations of the data acquisition system are defined as the limits of an acoustic "window" in the wavenumber-frequency space. In the second half of chapter III examples will be given, showing some applications of the theory. In chapter IV the concept of zero offset data acquisition is extended to some non-zero offset techniques. Here the wavenumber-frequency space will also play a central role, and its limitations will become evident. Once the theoretical basis of modelling and inversion has been established, the analysis of medium parameters is treated in chapters V-IX. As already pointed out in the previous section, the medium parameters may be divided into two subsets: the local parameters and the bulk parameters.

As local parameters we will define the spatial distribution of reflectivity and the "character" of reflectivity (i.e. frequency and angle dependence). In chapters V and VI the spatial distribution of scattering objects is analysed, starting from a one dimensional model. From this analysis we may conclude that the scatterer distribution is best studied by using the envelope detected data. Angle dependent backscattering results from scattering objects which are not small in relation to the wavelength of the acoustic pulse, and can best be studied by means of the focused r.f. data. In chapter V we will also
see that Bragg diffraction analysis is in essence no different from the
analysis of the spatial distribution of reflectivity, and that the two
are related by Fourier transformation. The character of reflectivity is
discussed in chapter VIII. Here an attempt will be made to relate the
theory of acoustic inversion to the theory of scattering from small
objects by taking a closer look at the inhomogeneous wave equation. As
the theory of scattering was originally developed in the field of
optics its mathematical description seems quite different from the one
used in acoustic inversion. We will see that, after acoustic inversion,
many optical assumptions also apply to acoustic scattering, making it
possible to link the two theories.
In the remaining two chapters the bulk parameters attenuation and sound
propagation velocity are treated.
The measurement of attenuation is studied by two techniques, the
"spectral difference" and the "spectral shift" technique. Both rely on
an accurate estimation of the changes in the amplitude spectrum as a
function of depth, so the first part of chapter VII is devoted to the
treatment of frequency estimation techniques. Here the instantaneous
frequency estimation will prove to be a powerful tool for the
characterization of a medium. In chapter VII also one section will
treat the problem caused by "diffraction effects". These system
distortions are the result of beam focussing by large transducers.
The estimation of the sound propagation velocity, in the final chapter
of this thesis, is made by means of a computerized minimum entropy
technique. Basically an optimal estimation of the average sound
propagation velocity will produce a "spiky" image, and the minimum
entropy technique is used as a measure for the "spikyness" of that
image.
The minimum entropy technique will be shown to be a promising technique
for tissue characterization, as it is able to distinguish between
scattered information, reflected information and noise.
In chapters V and VI, the spatial distribution of scattering objects is analyzed, starting from a one dimensional model. From this analysis we may conclude that the scattering distribution is best studied by using the envelope detected data. Angle dependent backscattering results from scattering objects which are not small in relation to the wavelength of the acoustic pulse, and can best be studied by means of the focused time data. In chapter V we will...
CHAPTER II
DESCRIPTION OF THE ACOUSTIC MODEL

In this chapter a mathematical description will be given of an acoustic measurement system. Starting from the generalized Rayleigh integrals, the wave propagation will be discussed as a subsystem of the total acoustic measurement system. Wave propagation will be described in terms of spatial convolution and, as most acoustic data are available to us in sampled form, it is convenient to adopt a matrix notation. After the wave field has been divided into a downgoing "source" wave field and an upgoing "reflected" wave field the remaining acoustic elements of the measurement system will be discussed.

2.1 WAVE PROPAGATION IN TERMS OF SPATIAL CONVOLUTION

If a pressure distribution \( P(x,y,z_0,\omega) \) is known on a reflection-free surface \( S(z=z_0) \), or if the distribution of the normal component of the particle velocity \( V_n(x,y,z_0,\omega) \) is known on the same surface, then under the Rayleigh-Sommerfeld conditions (see fig. 2.1 and appendix A) we may write for any point A in the source free lower half space \((z>z_0)\):

\[
P(x_A,y_A,z_A,\omega) = \frac{1}{2\pi} \int_{S_0} \alpha P(x,y,z_0,\omega) \frac{\partial g}{\partial z} \, ds_0 \quad (2.1.1)
\]

or
Figure 2.1: The Rayleigh-Sommerfeld boundary conditions, see also appendix A.

\[ P(x_A, y_A, z_A, \omega) = \frac{1}{2\pi} \int_{S_0} \alpha \left[ j\omega \rho_o \nabla \cdot \n(x, y, z_o, \omega) \right] G dS_o \quad (2.1.2) \]

with
\[ \alpha = \rho(x_A, y_A, z_A) / \rho_o(x, y, z_o), \]

where \( x_A, y_A, \) and \( z_A \) are the spatial coordinates of point A, \( G \) is the Greens function describing the impulse response of the medium, and \( \rho_o(x, y, z_o) \) represents the density of the medium on surface \( z = z_o. \) Equations (2.1.1) and (2.1.2) formulate the generalized Rayleigh integrals [1]. Rayleigh integral II describes a pressure to pressure extrapolation, while Rayleigh integral I describes a velocity to pressure extrapolation (see also Berkhout [2]). Note that only one type of wave is considered. If the medium is fluid-like, only longitudinal waves are supported and no wave conversion takes place. If, in addition, the medium between surface \( S_0 \) and point A is homogeneous and absorption free, then (2.1.1) and (2.1.2) simplify to the classical Rayleigh integrals

\[ P_A = \frac{1}{2\pi} \int_{S_o} P \left( \frac{1 + k\Delta r}{\Delta r^2} \right) \cos \psi \, e^{-jk\Delta r} dS_o \quad (2.1.3) \]

and

\[ P_A = \frac{1}{2\pi} \int_{S_o} j\omega \rho_o \nabla \cdot \n e^{-jk\Delta r} dS_o, \quad (2.1.4) \]

where
\[ k = \omega/c \] is the wavenumber,
\[ \Delta r = \sqrt{(x_A - x)^2 + (y_A - y)^2 + (z_A - z_o)^2} \]

and

\[ \cos \phi = \frac{(z_A - z_o)}{\Delta r} \]

Note that in equation (2.1.3) the term \( \frac{1 + jkr}{r^2} e^{-jkr} \cos \phi \) represents the response of an acoustic dipole, while in (2.1.4) \( \frac{e^{-jkr}}{r} \) represents the response of an acoustic monopole.

These dipole and monopole responses can mathematically be seen as operators for the acoustic field on surface \( S \). As these operators solely depend on \( \vec{r} = \vec{r}_A - \Delta \vec{r} \) (see figure 2.2) and on the wavenumber \( k \), the homogeneous, monochromatic case may also be rewritten as a two-dimensional convolution integral:

\[ P(x_A, y_A, z_A, \omega) = \frac{1}{2\pi} \int\int w(x_A - x, y_A - y, \Delta z, \omega) P(x, y, z_o, \omega) dx dy \]  

(2.1.5)

and

\[ P(x_A, y_A, z_A, \omega) = \frac{1}{2\pi} \int\int w'(x_A - x, y_A - y, \Delta z, \omega) V_n(x, y, z_o, \omega) dx dy, \]  

(2.1.6)

where

\[ w' = j\omega \rho_o \frac{e^{-jkr}}{\Delta r} \]

and

\[ w = \frac{1 + jkr}{r^2} e^{-jkr} \cos \phi \]

are the Rayleigh I and Rayleigh II operators respectively. Symbolically
equations (2.1.5) and (2.1.6) may be written as

\[ P(x, y, z, \omega) = W(x, y, \Delta z, \omega) \ast P(x, y, z_0, \omega) \]  
(2.1.7)

and

\[ P(x, y, z, \omega) = W'(x, y, \Delta z, \omega) \ast V_n(x, y, z_0, \omega), \]  
(2.1.8)

where \ast denotes the convolution along the spatial coordinates \( x \) and \( y \) of surface \( S_0 \).

2.2 WAVE PROPAGATION IN TWO DIMENSIONS

To date most data acquisition systems acquire their acoustic data along one single line, so the sound field is not known over the entire surface and in all previous equations the variable \( y \) must be dropped (2D assumption). This inevitably leads to discrepancies between the real (three-dimensional) world and its mathematical description. For this reason one must constantly be on guard for errors due to the two dimensionality of the model. In section 3.6 experiments are discussed, which evaluate the effect of the three dimensionality of a medium on a two-dimensional image of a cross section of that medium.

Dropping one dimension in the mathematical model implies that the simulated acoustic system must be truly two-dimensional, so points are actually lines and all wave propagation must be described in cylinder coordinates. Thus equations (2.1.7) and (2.1.8) become

\[ P(x, z, \omega) = W(x, \Delta z, \omega) \ast P(x, z_0, \omega) \]  
(2.2.1)

and

\[ P(x, z, \omega) = W'(x, \Delta z, \omega) \ast V_n(x, z_0, \omega), \]  
(2.2.2)

where

\[ W' = -j \pi H_n^{(2)}(kr), \ W = -j k \pi \cos \phi H_n^{(2)}(kr) \] and where \( r \) is defined as \( r = \sqrt{x^2 + y^2} \). As we will mainly be dealing with the two-dimensional problem in the following, \( r \) will be defined in this manner. In these expressions of the two-dimensional Rayleigh operator \( H_n^{(m)} \) represents a Hankel function of the \( n \)th order and the \( m \)th kind. The derivation of \( W' \) and \( W \) in cylinder coordinates may be found in [2].
Generally, the acoustic field is not measured continuously over the surface $S$, but some spatial sampling always takes place. In this case, the sampled information may be contained in the elements of a vector, and the convolution is readily described by a matrix multiplication. Restricting ourselves to the pressure to pressure extrapolation we see that equation (2.2.1) for one single frequency becomes

$$\hat{P}(z_A) = W(z_A, z_0)^T \hat{P}(z_0),$$  \hspace{0.5cm} (2.2.3)

where one column of matrix $W$ represents the sampled point source response at the surface $z=z_0$, due to a dipole at $z=z_A$. If the medium between the surfaces $z=z_0$ and $z=z_A$ is homogeneous, the columns of matrix $W$ are shifted versions of each other and the diagonals all contain the same numbers. Lateral changes of the propagation properties may be included by limiting the operator angle (subcritical) and allowing the columns of matrix $W$ to vary.

Equation (2.2.3) describes one physical experiment, where the wave field travels from surface $z=z_0$ to surface $z=z_A$. If the parameters of the intervening medium do not vary between two experiments the same equation may be extended to describe a series of physical experiments:

$$P(z_A) = W(z_A, z_0) P(z_0),$$  \hspace{0.5cm} (2.2.4)

where each column of the $P$ matrix describes one single physical experiment. For instance if a point source, which in terms of pressure would be a dipole, were moved along surface $z=z_0$, matrix $P(z_0)$ would be a unity matrix and the columns of the $P(z_A)$ would contain the shifted versions of the dipole response at $z=z_A$.

2.3 **WAVE PROPAGATION IN THREE DIMENSIONS**

In principle it is possible to extend the description of section 2.2 to three dimensions. In fact the mathematics involved become considerably simpler, as Hankel functions (eqs. (2.2.1) and (2.2.2)) transform into exponentials (eqs. (2.1.5) and (2.1.6)).

The problem here is that we cannot apply a matrix notation for a two-dimensional convolution operator (eqs. (2.1.7) and (2.1.8)). If we wish to stick to matrix notation, then one of the convolution integrals
must be written as a summation. Let the source plane \( z = z_0 \) be divided into \( i \) lines, where \( i \in \{1, \ldots, N\} \) - the pressure distribution on plane \( z = z_0 \) may now be represented by a series of vectors \( \vec{p}_i(z_0) \). Likewise the pressure distribution on the object plane \( z = A \) may be represented by a series of vectors \( \vec{p}_j(z_A) \), where \( j \in \{1, \ldots, M\} \). The lines composing both planes must be parallel. By replacing the vectors by matrices we are able to describe a series of experiments, where the source is moved parallel to the lines in the planes; so now the forward-propagated pressure in the object plane may be derived by calculating for all \( j \):

\[
P_j(z_A) = \sum_i W_{ij}(z_A, z_0) P_i(z_0),
\]

(2.3.1)

where \( W_{ij} \) is the three dimensional extrapolation matrix.

---

**Figure 2.3:** Schematic and block diagram description of an echo acoustic measurement system. Here the response of one depth level \( z = z_m \) is considered.
2.4 THE FORWARD MODEL

In the previous sections we described the propagation of an acoustic wave from one plane surface to another in terms of spatial convolution. Now we will introduce our complete model which describes the response of an echo acoustic measurement system. In figure 2.3 the diagram is shown of such a model.

If the source is situated on the plane $z = z_o$ then, in terms of pressure, the downward propagating wave may be described by a source vector $\hat{S}(z_o)$. The elements of $\hat{S}$ contain the complex amplitudes of the sampled pressure field of the source at surface $z = z_o$ for one temporal frequency component:

$$\hat{\mathbf{p}}_d(z_o) = \hat{S}(z_o). \quad (2.4.1)$$

As in equation (2.2.4) this may be extended to describe a measurement sequence by replacing the vectors $\hat{\mathbf{p}}_d$ and $\hat{S}$ in (2.4.1) by matrices. Again, every column would represent one single acoustic experiment. Using (2.2.3) the pressure distribution at depth level $z = z_m$ may be calculated:

$$\hat{\mathbf{p}}_d(z_m) = W(z_m, z_o) \hat{\mathbf{p}}_d(z_o) \quad (2.4.2)$$

where $\hat{\mathbf{p}}_d(z_m)$ describes the pressure distribution at $z = z_m$ for the downward propagating wave. For accurate estimation of $\hat{\mathbf{p}}_d(z_m)$ matrix $W$ should be known accurately. $W$ must contain all information about

a. The main velocity and density boundaries,

b. The average velocities and velocity gradients within each layer,

c. The density gradients within each layer and

d. The average absorption within each layer.

Berkhout [3] named the model of the medium containing these propagation parameters the "macro subsurface model". In figure 2.4 an example of such a model is shown. The macro acoustic model will also be used later on as a reference medium for the inversion process (chapter III). In the field of medical ultrasound the reference medium is often chosen as homogeneous with a sound velocity of 1540 m/s. The fast fluctuations as
Figure 2.4: A macro subsurface model for a sound propagation velocity distribution.

a: the main velocity boundaries
b: a cross section AA' for the sound propagation velocity as a function of depth.

Figure 2.5: Example of a macro subsurface model for the sound propagation velocity in medical ultrasound (a) and the possible local fluctuations (b).

depicted in figure 2.5 need not be taken into account for the propagation matrix $W$, therefore, the parameters sound propagation velocity and absorption are referred to here as bulk parameters.

If there are inhomogeneities at depth level $z = z_m$, part of the downward propagating wave field will be reflected. The upward propagating reflected wave may be described in terms of $\hat{P}_d(z_m)$ by defining a reflectivity matrix $R(z_m)$ at depth $z = z_m$:

$$\hat{P}_u(z_m) = R(z_m)\hat{P}_d(z_m).$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} (2.4.3)
If the inhomogeneities at \( z=z_m \) are "locally reacting", then the downward propagating wave field only needs to be multiplied by a series of complex numbers. In terms of the reflectivity matrix, \( \mathbf{R}(z_m) \) would be a diagonal matrix. The rows of \( \mathbf{R}(z_m) \) contain one non-zero sample only. If a surface is "locally reacting" this implies that the plane wave reflection coefficient does not vary with the angle of incidence. This statement must be true within the spatial bandwidth of the incident wave field. Within this spatial bandwidth the Fourier spectrum of a row of \( \mathbf{R}(z_m) \) defines a reflectivity function with a white spectrum. In figure 2.6 \( r(\alpha_0) \) may be taken as an angle independent reflection coefficient within the spatial bandwidth of the incident wave. Contrarily, if the plane wave reflection coefficient shows some angle dependence, matrix \( \mathbf{R}(z_m) \) will no longer be a diagonal matrix and the rows of \( \mathbf{R}(z_m) \) will contain the sampled version of a reflectivity function.

\( \mathbf{R}(z_m) \) contains information about the local character of a medium, a.o. the fast fluctuations in figure 2.5. Thus the parameters derived from \( \mathbf{R}(z_m) \) are local parameters.

The upward propagating wave field of equation (2.4.3) will arrive back at surface \( z=z_0 \). Now, making use of equation (2.2.3) again, we find

![Diagram](image)

**Figure 2.6:** 

*a:* An arbitrary source-scatterer configuration, showing the incident wave field.

*b:* Spatially narrow band incident wave around angle of incidence \( \alpha_0 \) shown as a shaded area, and the angle dependent reflection coefficient \( r(\alpha) \) depicted as a dashed line.
\[ \tilde{p}_m(z_o) = W(z_o, z_m) \tilde{p}_u(z_m), \quad (2.4.4) \]

where \( \tilde{p}_m(z_o) \) denotes the pressure distribution at surface \( z=z_o \), resulting from reflection of the down going wavefield at depth level \( z=z_m \). The total pressure arriving back at surface \( z=z_o \) will be the sum of \( \tilde{p}_m(z_o) \) for all \( m \).

At \( z=z_o \) the upward travelling wavefield is detected by a series of detectors, which may be described by a detector matrix \( D(z_o) \),

\[ \tilde{p}(z_o) = D(z_o) \sum_m \tilde{p}_m(z_o). \quad (2.4.5) \]

If the detectors are small in relation to the wavelength of the sound field they will not show any angular dependence, and \( D(z_o) \) will be a diagonal matrix.

Combining equations (2.4.1) - (2.4.5) we find for one acoustic experiment:

\[ \tilde{p}(z_o) = D(z_o) \left[ \sum_m W(z_o, z_m) R(z_m) W(z_m, z_o) \right] \tilde{s}(z_o). \quad (2.4.6) \]

When a series of different experiments need be described, then the \( \tilde{p}(z_o) \) and \( \tilde{s}(z_o) \) vectors become matrices too

\[ P(z_o) = D(z_o) \left[ \sum_m W(z_o, z_m) R(z_m) W(z_m, z_o) \right] S(z_o). \quad (2.4.7) \]

For the derivation of equations (2.4.6) and (2.4.7) an important basic assumption has been made:

- Only the primary responses from all depth levels \( z=z_m \) are considered, so we assume weak scattering. If multiple scattering may be neglected and the propagation properties are not affected by the local inhomogeneities this is known as the Born approximation [4].

2.5 THE ZERO OFFSET MODEL

It is very common, especially in the field of medical ultrasound to acquire the data in a zero offset mode. In this mode of data acquisition the source and detector are in the same position and only this one detector is used to register the returning pressure field. The
scan is built up by moving the source/detector along the surface of the medium. In medical ultrasound this mode of scanning is called a B-scan. Here we will modify equation (2.4.7) to suit the zero offset data acquisition technique.

For a single depth level \( z = z_m \), equation (2.4.7) may be rewritten as

\[
P(z_o) = D(z_o)W(z_o, z_m)R(z_m)W(z_m, z_o)S(z_o). \tag{2.5.1}
\]

For simplicity we will assume that the sources and detectors are small in relation to the wave length. Then we may assume that they have no angular preference so \( D(z_o) \) and \( S(z_o) \) become diagonal matrices. The assumption of point source/detectors is not essential as we shall see in section 3.4, but it simplifies our argument at this point. If in addition the sensitivity of the sources and detectors does not change along the surface, then

\[
P(z_o) = SW(z_o, z_m)R(z_m)W(z_m, z_o), \tag{2.5.2}
\]

where \( S \) defines the amplitude and phase of the source detector combination for the frequency of interest. In zero offset data acquisition only the backscattered wave is measured. The angle of incidence on the scattering objects is always perpendicular to the reflecting surface, so, in addition to the discussion given in the previous section, the spatial bandwidth is always about \( \alpha = 0 \). Around the normal angle of incidence the reflection coefficient depends least on the angle. Consequently for zero offset data acquisition the medium may be considered as locally reacting and \( R(z_m) \) may be replaced by a diagonal matrix with zero offset reflection coefficients.

Using \( W = W^T \) and the fact that the zero offset data are represented by the main diagonal of matrix \( P(z_o) \), we may write

\[
P_{mm} = S \sum_i W_{mi}^2 R_{ii}, \tag{2.5.3}
\]

or, including all \( z = z_m \)

\[
P_{Z0} = S \sum_m W_{Z0}(z_o, z_m)R_{Z0}(z_m), \tag{2.5.4}
\]

where the subscript 'Z0' denotes zero offset data acquisition and the
elements of $W_{Z0}(z_o, z_m)$ are the squared elements of the one way matrix $W$.

2.6 THE HALF VELOCITY SUBSTITUTION

As an example of equation (2.5.4) we will take the two-dimensional Rayleigh operator of equation (2.2.1). The far field approximation, where $kr >> 1$ of $W$ is

$$W = \sqrt{\frac{jk}{2\pi}} \frac{\cos \phi}{\sqrt{r}} e^{-jkr}. \quad (2.6.1)$$

As the elements of $W_{Z0}(z_o, z_m)$ are $W^2$, we may write

$$W^2 = \frac{jk \cos^2 \phi}{2\pi r} e^{-2jkr}. \quad (2.6.2)$$

In practice, the two way operator $W^2$ is often approximated by the one way operator $W$, with the substitution of half the propagation velocity

$$W_{1/2} = \sqrt{\frac{jk}{\pi}} \frac{\cos \phi}{\sqrt{r}} e^{-2jkr}. \quad (2.6.3)$$

Figure 2.7 shows the amplitude and the phase of the two operators $\widetilde{W}^2(k_x, k)$ and $\widetilde{W}_{1/2}(k_x, k)$ as a function of $k_x$ for one frequency. In figure 2.8 the phase of figure 2.7 has been unwrapped for $k_x \geq 0$. In all figures the floating time reference has been applied, so the linear phase shift as a function of frequency $k \Delta z$ has been removed.

In the wavenumber-frequency domain the one way operator becomes a phase shift operator (see section 3.3):

$$\widetilde{W}(k_x, k) = e^{-jk \Delta z} = e^{-j\sqrt{k^2-k_x^2} \Delta z}, \quad (2.6.4)$$

where $\Delta z$ is the extrapolation depth. When floating time reference is applied the one way operator transforms to

$$\widetilde{W}^t(k_x, k) = e^{j(k-k_x)\Delta z}. \quad (2.6.5)$$

In the model of figures 2.7 and 2.8 a frequency of $f = 1.47$ MHz, a sound propagation velocity of $c = 1500$ m/s and an extrapolation depth of $\Delta z = 10\lambda$ were chosen. So we find
Figure 2.7: Amplitude (a) and phase (b) of the two-way extrapolation operator $\hat{\mathbf{W}}(\cdot)$, of the half velocity operator $\hat{\mathbf{W}}_\frac{1}{2}(\cdot)$ and cosine (---).

Figure 2.8: Unwrapped phase of two-way extrapolation operator $\hat{\mathbf{W}}(\cdot)$ and of the half velocity operator $\hat{\mathbf{W}}_\frac{1}{2}(\cdot)$.

\[ a: \text{full scale}, \quad b: \text{enlarged} \]

or in the half velocity substitution $\overline{k} = 1.96$ [mm$^{-1}$]. From the figures we see that the amplitude of the half velocity extrapolation operator differs from the true extrapolation operator $\hat{\mathbf{W}}$, whilst the phase, after compensation for the linear shift, differs by a constant $\frac{k\pi}{2}$ radian or $45^\circ$. The phase shift may be explained by taking a closer look at equations (2.6.2) and (2.6.3).

\[ \frac{\mathbf{W}^2}{\mathbf{W}} = \frac{\sqrt{jk \cos \phi}}{2N} = A(x,k)e^{j\pi/4}, \quad (2.6.6) \]
where with $\cos\phi = \Delta z/r_o$

$$A(x,k) = \sqrt{\frac{k}{\pi}} \frac{\Delta z}{2(x^2 + \Delta z^2)^{3/2}}$$  \quad (2.6.7)

$A(x,k)$ is a real function and symmetric around $x=0$. Thus after Fourier transformation from $x$ to $k_x$ we find that the difference between $\tilde{W}_2^2$ and $\tilde{W}_2$, apart from a phase shift of $\pi/4$ radian, may be written as a real function

$$\tilde{A}(k_x,k) = \Delta z \sqrt{\frac{k}{\pi}} \int_0^\infty \frac{\cos k_x x}{(x^2 + \Delta z^2)^{3/2}} \, dx,$$

or according to Abramowitz and Stegun [5]

$$\tilde{A}(k_x,k) = k \sqrt{\frac{k}{\pi}} K_1(k_x \Delta z),$$  \quad (2.6.8)

where $K_1$ is a modified Bessel function of the first order. So we find that $\tilde{A}(k_x,k)$ is a bounded, real and positive function. This means that the difference between $\tilde{W}_2^2$ and $\tilde{W}_2$ in the $k_x,k$ domain is described by a symmetric amplitude around $k_x=0$ and by a phase of $45^\circ$, as seen in figures 2.7 and 2.8.

2.7 REPRESENTATION OF A POINT SCATTERER IN THE ZERO OFFSET MODEL

In the zero offset data acquisition mode a dataset is built up by registering one single (zero offset) trace of many different wave fields. In the previous section we have seen that, by the half velocity substitution, the two-way zero offset extrapolation operator may be approximated by a one way extrapolation operator. In this substitution the scattering objects in the medium have been replaced by secondary sources, and this substitution is thus often referred to as the "exploding reflector model".

Here we will consider the problem of the kind of sources best suited to replace small scattering objects (a $<< \frac{1}{2} \lambda$) in the half velocity substitution. In the zero offset acquisition mode the response of all point scatterers to a plane incident wave is independent of the angle of incidence (see also section 4.2). So we may write
\[ \tilde{P}_u(k_x, k_z) = R(\omega)\tilde{P}_d(k_x, k_z), \]  

(2.7.1)

where \( \tilde{P}_u \) and \( \tilde{P}_d \) represent the amplitude and phase of, respectively, the up and downgoing plane pressure waves, and \( R(\omega) \) is the normal incidence (backscatter) reflection coefficient of the point scatterer. If in wavenumber-frequency space a point scatterer is best represented by a source with a 'white' spectrum, then our source must be a pressure dipole source, orientated towards the measurement surface:

\[ r(x, z) = R(\omega)\delta(x-x_n)\delta(z-z_m). \]  

(2.7.2)

References chapter 2


According to Abramovitz and Stegun [15], the gamma function \( \Gamma(z) \) is defined as:

\[
\Gamma(z) = \int_0^\infty \frac{t^{z-1}}{e^t} dt
\]

where \( \Re(z) > 0 \). It arises in various physical problems, such as the evaluation of moments of the normal distribution and the Gamma distribution. The function has properties that make it useful in many areas of mathematics and physics. In the context of electromagnetic theory, the gamma function is often used in the formulation of the Green's function for the wave equation.
CHAPTER III
INVERSION

In chapter II the forward model for the response of an acoustic measurement system was discussed. In the modelling problem it is assumed that the reflectivity distribution is known for all depths viz. the reflectivity matrices $\mathbf{R}(z_m)$ in equations (2.4.6) and (2.4.7) are known for all $z=z_m$. From the reflectivity distribution we calculated the response of the medium at the surface $z=z_0$. In this chapter we will consider the inverse problem, where the reflectivity distribution for all $z=z_m$ is to be estimated from a measured response at the surface of the medium.

3.1 \hspace{1cm} \textbf{INVERSE EXTRAPOLATION OF THE ACOUSTIC WAVE FIELD}

\textbf{(after Berkhout [1])}

Before anything may be concluded quantitatively about local reflectivity properties, the distorting effects of the measurement system and the propagation operator must be removed from the measured data. For simplicity we rewrite equation (2.4.7) for one depth level and for one single frequency $\omega$:

$$\mathbf{P}_m(z_0) = \mathbf{W}_d(z_0, z_m) \mathbf{R}(z_m) \mathbf{W}_s(z_m, z_0), \quad (3.1.1)$$

where

$\mathbf{W}_d(z_0, z_m) = \mathbf{D}(z_0) \mathbf{W}(z_0, z_m)$

and

$\mathbf{W}_s(z_m, z_0) = \mathbf{W}(z_m, z_0) \mathbf{S}(z_0)$. 

An estimate of the reflectivity at depth level \( z = z_m \) may now be obtained by matrix inversion

\[
\langle R(z_m) \rangle = F_d(z_m, z_o) P_m(z_o) F_s(z_o, z_m) \\
= [F_d(z_{m}, z_{o}) W_d(z_{o}, z_{m})] R(z_m) [W_s(z_{m}, z_{o}) F_s(z_{o}, z_{m})] \\
= W_d(z_m) R(z_m) W_s(z_m), \quad (3.1.2)
\]

where

\[
W_d(z_m) = F_d(z_{m}, z_{o}) W_d(z_{o}, z_{m}),
\]

\[
W_s(z_m) = W_s(z_{m}, z_{o}) F_s(z_{o}, z_{m})
\]

and where \( \langle .. \rangle \) denotes an estimate. Now, if in a noiseless situation the entire scattered wave field could be measured by extending the measurement plane from minus infinity to plus infinity and if we could measure all wavenumbers along the measurement plane \( (-\infty < k_x, k_y < \infty \) and \( -\infty < k_z < \infty \) ) then we would be able to invert \( W_d(z_o, z_m) \) and \( W_s(z_{m}, z_{o}) \) perfectly. Under these two extreme conditions \( W_d(z_m) \) and \( W_s(z_{m}, z_{o}) \) would be unity matrices and thus \( \langle R(z_m) \rangle = R(z_m) \) would be a perfect estimate. However, both extreme conditions cannot be met. Any measurement system is limited by the following factors

1) The wavenumbers along the measurement plane are on physical grounds band limited. The propagating field cannot contain useful spatial frequencies which are higher than the temporal frequency.

2) The data acquisition system limits the available spatial bandwidth even further by its limited aperture and the directivity of the individual elements.

3) The sampling of the data restricts the frequencies and wavenumbers which can be measured correctly.

4) Noise may limit the available bandwidth even further.

The system limitations mentioned here are shown in the wavenumber-frequency domain in figure 3.1, and will be treated in more detail in section 3.4. These limitations all result in the matrix inversion of equation (3.1.2) being an ill-posed problem, so the best we can obtain is a band limited inversion [1].
Figure 3.1: Measurement system limitations in $k_x$-$k$ space:

1) Principal frequency limitation $|k_x| < k$.
2) Transducer limitations.
3) Sampling limitations.

In the following examples zero offset data acquisition plays an important role. In section 2.5 the zero offset model was discussed.

According to equation (2.5.4) the response from one depth level $z = z_m$ may be represented by the zero offset pressure distribution

$$\hat{P}_{Z0} = sW_{Z0}(z_o, z_m)R_{Z0}(z_m),$$

(3.1.3)

where $W_{Z0}(z_o, z_m)$ contains the squared elements of the one way matrix $W(z_o, z_m)$ and $R_{Z0}(z_m)$ contains the zero offset reflection coefficients at depth level $z = z_m$. Here the double matrix inversion of equation (3.1.2) becomes a single zero offset inversion (section 3.3), and our estimate $\hat{R}(z_m)$ will be a spatially band limited version of $\hat{S}_{RZ0}(z_m)$.

Note that the frequency-dependent amplitude and phase distortion of the source and detector combination $S$ is usually removed during a deconvolution process prior to focusing.
To illustrate the principle of inversion we will use the model of a single point scatterer in a homogeneous background. In the proposed zero offset model this may be replaced by a single dipole point source in a homogeneous medium (section 2.7).

The computer simulation of the zero offset response is given by the $x$--$t$ frame in figure 3.2a. The thick line at the top shows the time $t=0$ for each source-detector combination. Every vertical line denotes the time

Figure 3.2: a) Zero offset response of a point scatterer in a homogeneous medium.

b) As a) after extrapolation over $\Delta x$.

c) As a) after extrapolation over $2\Delta x$.

d) Envelope of zero offset response, extrapolated over $2\Delta x$ (= point spread function or ambiguity function).
response of the source-detector at that x-position. If there had been any inhomogeneity at the surface, it would have produced a response at time $t=0$ in figure 3.2a.

Now, if we perform inverse wave field extrapolation over a distance $\Delta z$, where $\Delta z$ is half the depth at which we have placed the inhomogeneity in our model, then the extrapolated result is shown in figure 3.2b. Here again the x-t frame is shown and the thick line denotes the new $t=0$ for each virtual source-detector pair at depth $\Delta z$. Clearly we see that the scatterer is coming into focus and the diffraction hyperbola is starting to concentrate at its apex.

Figure 3.2c shows the result of a second extrapolation step over a distance $\Delta z$. This time one virtual source-detector pair is placed exactly at the scatterer and we see the response is completely in focus.

Notice how the edge artefacts in figure 3.2b develop into the well known "focussing cross" or "bow tie" in figures 3.2c and 3.2d. It clearly illustrates the effect of spatial band limitation due to a limited aperture angle.

In section 3.6 more examples will be given of applications of inverse wave field extrapolation. In section 3.6A an example is shown of one-way inverse extrapolation of a monochromatic dataset measured over a surface. Here the propagation effects are removed from the three dimensional version of equation (2.4.2) and a reconstruction is made of the source distribution. In sections 3.6B and C more examples are given of the high resolution zero offset focussing.

3.2 THE IMAGING PRINCIPLE

In section 3.1 the response from a single depth level was considered. However, in practical situations, responses from all depth levels $z=z_m$ are measured

$$P(z_0) = \sum_m W_d(z_0,z_m) R(z_m) W_s(z_m,z_0).$$  \hspace{1cm} (3.2.1)

Consequently, after inverse wave field extrapolation $<R(z_m)>$ must be selected from the extrapolation result

$$P(z_m) = F_d(z_m,z_0) P(z_0) F_s(z_0,z_m).$$  \hspace{1cm} (3.2.2)
Bearing in mind that inverse wave field extrapolation compensates for all propagation effects in the subsurface, we see that $F_d(z_m, z_o)$ and $F_s(z_o, z_m)$ compensate for all travel times between $z=z_o$ and $z=z_m$. Thus, after inverse extrapolation to level $z=z_m$, the reflectivity matrix $R(z_m)$ can be found in the time domain at zero traveltime ($t=0$), or

$$<R(z_m)> = p(x,z=z_m,t=0)$$
$$= \frac{1}{\pi} \Re \int_0^\infty P(x,z=z_m,\omega)d\omega. \quad (3.2.3)$$

Note that $<R(z_m)>$ is an estimate of $R(z_m)$, which has been averaged in the frequency domain. Due to band limitation of the acoustic pulse ($\omega_{\text{min}} \neq 0$ and $\omega_{\text{max}} < \infty$) the reconstruction of $R(z_m)$ is also temporally band limited. A perfect reflector will be imaged as a "package" of reflectors, of which $<R(z_m)>$ is one sample.

### 3.3 NON RECURSIVE MAPPING IN THE WAVENUMBER-FREQUENCY DOMAIN

If we may assume a homogeneous subsurface model (section 2.4) for the inversion process, then we may apply a computationally advantageous inversion process, which combines the inverse extrapolation and the imaging principle of the previous sections. The algorithm discussed here may be applied to data collected in the zero offset mode (sections 2.5 and 2.6) and to transmission data, which includes the primary wave field. In chapter IV non recursive mapping is treated in more general terms.

When we Fourier transform the measured data from the space-frequency domain to the wavenumber-frequency domain, the wave propagation effect discussed in chapter II may be described as a multiplication instead of a convolution. Wave field extrapolation in the wavenumber-frequency domain simplifies to multiplication with a phase shift operator [2]:

$$\tilde{P}(k_x,\omega) = \tilde{P}(k_x,z,\omega) e^{-jk_z z}, \quad (3.3.1)$$

or for the inversion process
\[ p(k_x, z, \omega) = \bar{p}(k_x, 0, \omega) e^{jk_x z}. \]  
(3.3.2)

Here \( k = \sqrt{\frac{\omega^2}{c^2} - k_x^2} \) is the wavenumber in \( z \)-direction, where

\[ k_x^2 = \frac{\omega}{c}. \]

The data, measured at the surface of the medium may be described as \( p(x, z=0, t) \) and our objective is to calculate an estimate of the reflectivity at all depth levels as given in equation (3.2.3):

\[
p(x, z, t=0) = \frac{1}{\pi} \text{Re} \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} P(x, z, \omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(x, z, \omega) d\omega. \quad (3.3.3)
\]

The right hand side of this equation may now be Fourier transformed into the wavenumber-frequency domain

\[
p(x, z, t=0) = \left( \frac{1}{2\pi} \right)^2 \int_{\omega} d\omega \int_{k_x} \tilde{p}(k_x, z, \omega) e^{-jk_x x} dk_x. \quad (3.3.4)
\]

Using equation (3.3.2) we find after wave field extrapolation

\[
p(x, z, t=0) = \left( \frac{1}{2\pi} \right)^2 \int_{\omega} d\omega \int_{k_x} \tilde{p}(k_x, 0, \omega) e^{jk_z z -jk_x x} dk_x. \quad (3.3.5)
\]

Using \( k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2} \), we may define a coordinate transformation from \( \omega \rightarrow k_z \), as

\[
d\omega = \frac{ck_z}{\sqrt{k_x^2 + k_z^2}} dk_z
\]

and equation (3.3.5) can be rewritten as

\[
p(x, z, t=0) = \left( \frac{1}{2\pi} \right)^2 \int_{k_x} \left. e^{jk_z z} \right|_{k_x} \int_{k_z} \tilde{p}(k_x, 0, k_z) e^{-jk_x x} dk_x. \quad (3.3.6)
\]
\( \tilde{P}(k_x,0,k_z) \) may be found from the measured data \( p(x,z=0,t) \) by two dimensional Fourier transformation along \( x \) and \( t \) and by coordinate transformation from \( \omega \rightarrow k_z \). The mapping of \( \tilde{P}(k_x,0,\omega) \) to \( \tilde{P}(k_x,0,k_z) \) in the wavenumber-frequency domain is shown in figure 3.3.

Summarizing, inversion and imaging of zero offset data may be done for a homogeneous reference medium in four steps [3].

1) Two dimensional Fourier transformation of the measured data.
2) Mapping of the data from the \( k_x-\omega \) to the \( k_x-k_z \) domain.
3) Multiplication of all data by the weighting factor \( ck_z/\sqrt{k_x^2 + k_z^2} \).
4) Inverse two dimensional Fourier transformation.

3.4 THE INFLUENCE OF THE PROPERTIES OF SOURCES AND DETECTORS

In this section the influence of the properties of sources and detectors on the inversion result will be studied for the zero offset data acquisition mode. Once the basic concepts of the limitations of the data acquisition system are made clear, examples of measurements will be given in section 3.6.

It is easiest to consider all source and detector influences on the data in the spectral domain. In the case of a two dimensional dataset this is the two dimensional spectral domain, thus, either the \( k_x-k \) or the \( k_x-k_z \) domain, respectively before or after focussing. In these domains we may consider both the amplitude spectrum and the phase spectrum.

The influence of the sources and detectors in the time \( (t) \) or the temporal frequency \( (k = 2\pi f/c) \) direction may be compensated for by
deconvolution of the received time signal. By deconvolution any phase or amplitude distortion may be removed within a given frequency band; so, in the following we will assume that the temporal phase and amplitude distortion of the source signature and the detector transfer function have been corrected. For the zero offset model this means that in equation (3.1.3) the source and detector influence $S=1$ within the given frequency band.

The model we will use to illustrate the various limitations of an acoustic measurement system is that of one single point scatterer in a homogeneous background. In the zero offset data acquisition technique this model may be replaced by a single pressure point-source (dipole) in a homogeneous medium with half the propagation velocity. The source strength of the dipole is given by the zero offset reflectivity. In two dimensional space this point source may be represented by a spatial deltapulse at $x=x_n$ and $z=z_m$:

$$r(x,z) = \delta(x-x_n)\delta(z-z_m), \quad (3.4.1)$$

where we have chosen $R(\omega)=1$ in equation (2.7.2). In matrix notation only one element of the main diagonal of $R(z_m)$ is non zero. The two dimensional Fourier spectrum of $r(x,z)$ yields a white amplitude spectrum

$$|\tilde{R}(k_x,k_z)| = 1.0 \quad \text{for all } k_x \text{ and } k_z \quad (3.4.2)$$

and a linear dependency of the phase on $k_x$ and $k_z$

$$\arg \tilde{R}(k_x,k_z) = -j(k_x x_n + k_z z_m) \quad (3.4.3)$$

If the measurement system has no limitations at all, we should be able to find an estimate for the reflectivity

$$\langle \tilde{R}(k_x,k_z) \rangle = \tilde{R}(k_x,k_z) \quad \text{for all } k_x \text{ and } k_z \quad (3.4.4)$$

a) Temporal band limitations
Inherent to any physical system the available frequencies which carry information about the medium are limited. The usable signal can be found in some frequency band $f_L \leq f \leq f_H$, where $f_L$ and $f_H$ are the lower and upper frequency limits respectively.
A solution to the Helmholtz equation [4] states that

\[ f = \frac{c}{2\pi} k = \frac{c}{2\pi} \sqrt{k_x^2 + k_z^2}, \]

meaning that the frequency is related to the temporal wavenumber, and can be decomposed into two orthogonal spatial wavenumbers \( k_x \) and \( k_z \). Equation (3.4.5) also states, that in a homogeneous medium the spatial wavenumbers cannot become larger than the temporal wavenumber:

\[ k_x^2 \leq k^2 \text{ and } k_z^2 \leq k^2. \]

If at any interface, due to a sound velocity increase, any of the spatial wavenumbers becomes larger than the temporal wavenumber, these spatial wavenumbers will support an exponentially attenuating wave field. This attenuating wave field is generally referred to as the evanescent wave field [5]. If the medium supporting the evanescent wave field is homogeneous, there will be no transport of energy into the medium at these spatial frequencies and the evanescent wave field will become negligibly small within a couple of wavelengths from the interface.

From equation (3.4.5) we see that our window, within which we may estimate \( R(k_x,k_z) \) will be limited by \( k_{\text{min}} = 2\pi f_L/c \) and \( k_{\text{max}} = 2\pi f_H/c \). These frequency limitations are visible as circles in the \( k_x-k_z \) domain. Figure 3.4 shows the area of the \( k_x-k \) domain and the \( k_x-k_z \) domain, outside of which we are, principally, not able to reconstruct any information, due to the temporal band limitations of the sources and detectors.

![Figure 3.4: Temporal band limitations in the wavenumber-frequency domain (a) and in the two dimensional wavenumber domain (b).](image-url)
b) Aperture limitations

If, in a noise free situation we were able to measure all angles of incidence, then we would be able to fill up the shaded area in figure 3.4 completely. Unfortunately, this would require an infinitely large measurement plane $-\infty < x < \infty$. In practice, the measurement plane will be limited. The part of the measurement plane in which the wave field has been measured is called the aperture. A typical measurement setup with our single point scatterer is depicted in figure 3.5. From this figure it is clear that waves coming from the point scatterer are not measured if their angle of incidence becomes larger than $\alpha$. The maximal angle $\alpha$ depends on the distance $z$ between scatterer and measurement plane as well as on the aperture size. It is therefore more convenient to speak in terms of aperture angle limitations of the measured data.

In the model the aperture angle limitation is $\alpha$. A plane wave entering the measurement plane at an angle $\alpha$ will be projected on a line through the origin in the two-dimensional Fourier space. In the $k_x-k_z$ domain the angle with the $k_z$ axis will also be $\alpha$. From figure 3.3 we see that the relationship between a plane monochromatic wave before and after focusing is

$$\sin \alpha = \frac{k_x}{k} = \tan \beta,$$  \hspace{1cm} (3.4.6)

a well-known relationship in the field of seismic migration. Thus, before focusing the same plane wave will be projected at an angle $\beta$ to the $k$-axis.

![Figure 3.5: Measurement setup with a limited aperture.](image-url)
Figure 3.6: Aperture limitations in the wavenumber–frequency domain (a) and in the two dimensional wavenumber domain (b).

Figure 3.7: Spatial and temporal band limitations for a frequency and aperture limited measurement.

The shaded area in figure 3.6 shows the further restriction due to aperture limitations of the measurable data in the two dimensional Fourier space. Outside this area we cannot give an estimate of the reflectivity. Combining the aperture limitations with the temporal band limitations we can say that for small apertures the lateral resolution (in the $x$-direction) is inversely proportional to the central frequency of the emitted pulse, as $\frac{-2\pi f_c \sin \alpha}{c} \leq k_x \leq \frac{2\pi f_c \sin \alpha}{c}$. The axial resolution is inversely proportional to the temporal bandwidth $f_H-f_L$ (see figure 3.7). As the aperture grows larger the two limitations will become the same and the p.s.f. will become circularly symmetric [6].

c) Spatial band limitations of source and detector

In appendix B the spatial band limitations of source and detector are discussed for a plane strip transducer which is operated in the zero offset mode. The true extrapolation matrix elements $W_d^2$ are compared with a convolution of some detector distribution and the omni-
directional response matrix elements $W^2$. In the appendix some ways of calculating $W_{z0}$ are compared. Here it was found that the correct, but time consuming method

$$W_{z0} = W_d^2 = (D \ast W)^2 \quad (3.4.7)$$

may be approximated with some accuracy by the convolutional models

$$W_{z0} = S \ast D \ast W^2 \quad (3.4.8)$$

and

$$W_{z0} = S \ast D \ast W_{1/2} \quad (3.4.9)$$

where $D$ represents the detector distribution function, $S$ the source distribution function and $W_{1/2}$ the half velocity extrapolation operator (section 2.6). Equation (3.4.8) was found to be a good approximation for large transducers and apertures if we are able to measure in the far field of the transducer. Moving closer to the transducer (3.4.8) produces a less accurate approximation, until in the near field any comparison with (3.4.7) is invalid. The same conclusions may be drawn for the half velocity substitution of equation (3.4.9), only here sensitivity at larger spatial wavenumbers is over estimated, as already predicted in section 2.6. Due to phase errors in the side lobes only the main lobe of the sensitivity function is normally used for further analysis, so with directional source-detector combinations it is usually quite acceptable to apply equation (3.4.9). It is also interesting to see that in the convolutional models of (3.4.8) and (3.4.9) not the true source and detector distributions must be substituted, but their sizes must be divided by two.

In appendix B only examples were given of sources and detectors which have a uniform pressure distribution across their surfaces, and do not introduce an $x$-dependent time delay. In those examples $S$ may be represented by a boxcar function in the $x$-direction. Now we will consider a system where the small sources and detectors are given a time delay which is linearly dependent on the $x$-position:

$$\tau(x) = \frac{1}{c_x} \cdot x \quad (3.4.10)$$
\[ \tau = \frac{x \sin \alpha}{c} \]

Figure 3.8: A linear time shift as a function of \( x \) (a) corresponding to an angle of maximal sensitivity (b).

Such time delays are used for beam steering in e.g. sector scanners. In this equation the phase velocity in the \( x \)-direction \( c_x \) depends on the direction of maximal sensitivity \( \alpha \) (see figure 3.8)

\[ c_x = \frac{c}{\sin \alpha}, \quad (3.4.11) \]

where \( c \) is the sound propagation velocity in the medium. Now \( D(x, \omega) \) in equation (3.4.7) represents a frequency dependent phase shift.

\[ D(x, \omega) = S(x, \omega) = e^{-j\tau(x)\omega} = e^{-jx \cdot \omega \cdot \sin \alpha / c} \quad (3.4.12) \]

or, after Fourier transformation \( \tilde{D}(k_x, \omega) \) is a shifted version of the \( \tilde{S}(k_x, \omega) \) without time delays. The shift is linear with frequency

\[ \Delta k_x = \sin \alpha \cdot \omega / c = \sin \alpha \cdot k. \quad (3.4.13) \]

In the \( k_x \)-\( k \) domain the maximal sensitivity of the transducer which was at \( k_x = 0 \) has been rotated by an angle \( \beta \), where

\[ \tan \beta = \frac{\Delta k_x}{k} = \sin \alpha. \quad (3.4.14) \]

Using equation (3.4.6) we see that after focussing, the sensitivity in the \( k_x \)-\( k_z \) domain is rotated by an angle \( \alpha \).

d) Sampling limitations

As mentioned before, in a multi-channel acoustic measurement system sampling takes place along the surface \( z = z_0 \); also, the measured time
series are usually sampled for storage into a digital system. Thus, the output of a two dimensional measurement system may be seen as a two dimensional sampled field with a sample interval \( \Delta x \) along the surface and a sample interval \( \Delta z = c \Delta t \) in the axial direction (for reflection measurements half the true sound velocity must be substituted here). The sampling criteria dictate which part of the two dimensional Fourier space may be measured correctly; so, even if we had no other limitations, our estimate of the reflectivity can only be correct within the boundaries 

\[-k_{xNyq} < k_x < k_{xNyq} \quad \text{and} \quad -f_{Nyq} < f < f_{Nyq}\]

where \( k_{xNyq} = \pi/\Delta x \) and \( f_{Nyq} = 1/2\Delta t \) are the Nyquist frequencies. In terms of resolution this means that two point spread functions (p.s.f.'s) cannot be resolved if the distance between their maxima becomes less than one sample interval. This seems a trivial statement, but in practical situations where one tries to limit the data acquisition time and the amount of data, the spatial sampling interval \( \Delta x \) is usually a limiting factor. In general great care must be taken that the sampling limitations do not unacceptably limit the measurement. If the acoustic wave field contains (spatial) frequencies beyond the Nyquist frequencies aliasing will occur and if the aliased data mix with the correctly measured data both will become useless for further processing.

In figure 3.1 the sampling limitations have been included with the other limitations for the case where the highest frequencies may show aliasing.

### 3.5 THE INFLUENCE OF PROPAGATION

The ultimate goal in echo acoustical experiments is to extract as much information as possible from the measured data. We have already stated that two kinds of parameters may be extracted from an acoustical experiment, namely bulk parameters and local parameters. In chapter 1 the point was stressed, that for the characterization of local parameters the propagation influences must be removed.

For the inversion process, which is used to remove propagation effects, it is necessary to make some assumption for a reference medium. After inversion the accuracy of the inversion process may be tested. If the inversion fails the test, the reference medium may be updated and the process can be repeated. In this manner we can arrive at an estimate of
the "macro acoustic subsurface model", described in section 2.4. This model for the reference medium will describe the bulk parameters absorption and sound propagation velocity. An important step in the iterative process of finding a correct estimate of the reference medium is the testing of the accuracy of inversion. The propagation affects both the phase and the amplitude of the signal. The sound propagation velocity mainly affects the phase, while absorption mainly affects the amplitude. The spatial phase characteristics in the image due to propagation effects are generally referred to as diffraction. The unfocused image of a point scatterer will show a maximal amount of phase distortion due to propagation effects; a point scatterer is therefore often referred to as a (point) diffractor. Even if we have no prior knowledge of the medium, the removal of diffraction energy by the inversion process is a measure of the accuracy of the estimate of the sound propagation velocity. The minimum entropy criterion [7], a measure for the sparsity of a time series, is an objective technique to assess the change in the amount of diffraction energy. In chapter IX the minimum entropy criterion in conjunction with the inversion process will be used to estimate the sound propagation velocity in scattering media. In principle it is also possible to estimate the absorption of the medium by looking at the diffracted (= depth dependent) signal from one point scatterer. Unfortunately this method is very sensitive to other effects, e.g. beamforming, scatterer shape, size, depth and distribution. The use of the half velocity substitution (section 2.6) even inhibits correct absorption estimation from the response of a single scatterer, as the amplitude of the zero offset measurement is treated incorrectly. As we shall see later on, the estimation of absorption is best accomplished after inversion by using some prior knowledge about the scatterer size and distribution (chapter VII). If we have some prior knowledge of the medium from other sources (e.g. well logging, biopsy, X-ray, experience) it is possible to make a good initial estimate of the "macro acoustic subsurface model". This prior knowledge will also help us to describe local parameters more accurately, which in turn will lead to better inversion techniques.
3.6 EXAM PLES

A. Three dimensional one way extrapolation

During a collaboration project with the department of electrical engineering of the university of Rochester continuous wave (single frequency) measurements were made of the acoustic wave field from a plane circular transducer. One of the objects was to find out whether it was possible to extrapolate the measured wave field and what the limitations were of the measurement setup on the extrapolation procedure. The theory and the measurement setup will be discussed, subsequently a computer simulation of the measurement will be compared with the real measurement.

The measurement setup (see figure 3.9). The beam pattern of a transducer with a diameter of .25 inch was registered in a plane perpendicular to the axis of the transducer. A narrow band signal was transmitted into the medium and the envelope and instantaneous phase at a frequency of 2.9 MHz were measured on a grid of 181 x 181 sample points. The number 181 squared was chosen to have the largest possible aperture within the limitations of an integer control system. The separation of the points on the grid was .46 mm in both the x- and the y-directions. According to the sampling limitations (section 3.4) the maximal spatial frequency in the measurement plane must satisfy the anti-aliasing criterion

\[ c_0 = 1502 \pm 2 \text{ m/s} \]

![Figure 3.9: Measurement setup and its dimensions for the one way extrapolation experiment.](image)
\[ k_{x\text{max}} < k_{xNyq} = \frac{\pi}{\Delta x}, \] (3.6.1)

or taking the angle of incidence into account

\[ k_{x\text{max}} = k \sin \alpha_{\text{max}} < \frac{\pi}{\Delta x}, \]

so we find for the maximal aliasing-free angle of incidence

\[ \alpha_{\text{max}} = \arcsin \frac{c}{2 \Delta x} = \arcsin \left( \frac{1502}{2 \cdot 2.91 \times 10^6 \cdot 4.61 \times 10^{-4}} \right) = 34^\circ, \] (3.6.2)

where \( c = 1502 \text{ m/s} \) is the sound propagation velocity in water. The measurement probe was a circular microprobe with an active diameter of approximately 1.4 mm.

The distance between the source and the measurement plane was 75 mm. By performing a two dimensional Fourier transformation of the data along the \( x^- \) and \( y^- \) directions, the pressure on plane \( z = 75 \text{ mm} \) may be represented by

\[ \tilde{P}(k_x, k_y, z, \omega) = \tilde{P}(k_x, k_y, 0, \omega) \cdot e^{-jk_z z}, \] (3.6.3)

which is the three dimensional equivalent of equation (3.3.1) and where \( k_z = \sqrt{\omega^2/c^2} - k_x^2 - k_y^2 \). In equation (3.3.1) and (3.6.3) the source and detector planes have been reversed. In equation (3.6.3) the measurement probe has not been taken into account. In real space the effect of the measurement probe may be described by a two dimensional convolution along \( x \) and \( y \) with a circular disc with a diameter of 1.4 mm. In Fourier space the convolution becomes a multiplication:

\[ \tilde{P}_m'(k_x, k_y, z, \omega) = \tilde{D}(k_x, k_y, \omega) \tilde{P}(k_x, k_y, 0, \omega) e^{-jk_z z}, \] (3.6.4)

where \( \tilde{P}_m' \) is the measured pressure and \( \tilde{D}(k_x, k_y, \omega) \) is the two dimensional Fourier transformation along the \( x \) and \( y \) coordinates of the weighting function of the 1.4 mm circular disc.

The simulation

The measurement depicted in figure 3.9 was simulated by computer. Firstly a source was defined in the source plane \( z=0 \). The complex pressure on the source plane was defined as
\[ P(x,y,o,\omega) = 1 \quad \text{for} \quad \sqrt{x^2 + y^2} \leq r \]

and

\[ P(x,y,o,\omega) = 0 \quad \text{for} \quad \sqrt{x^2 + y^2} > r, \quad (3.6.5) \]

where \( r = 3.2 \) mm is the radius of the disc transducer. By means of equation (3.6.3) the wave field was calculated at a distance \( z = 75 \) mm from the source plane. Figure 3.10a shows the amplitude of \( P(x,y,z,\omega) \) as a function of \( x \) and \( y \) in a grey scale image. A cross section through the centre of figure 3.10a is depicted in figure 3.11a. In the panels

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure310.png}
\caption{a) Simulated beam profile on the measurement plane of a 6.4 mm transducer. 
\hspace{1cm} b) Simulated beam profile of a 1.4 mm measurement probe. 
\hspace{1cm} c) Beam profile of transducer, as measured by the probe. 
\hspace{1cm} d) Measured beam profile. 
All grey level plots show a dynamic range of approximately 60 dB.}
\end{figure}
Figure 3.11: a) Cross section through the centre of 3.10a.
b), c) and d) Spectral component $k_x=0$ from 3.10a, 3.10b and 3.10c respectively.
e) Cross section through the centre of 3.10c.

a, b, d and e of figure 3.11 the sidelobe levels are given in decibels relative to the main lobe. In panel c the amplitude is given at the positions of the sidelobes in the other panels.

After two dimensional Fourier transformation from space $(x,y)$ to $k$-space $(k_x, k_y)$ every complex number represents the amplitude and phase of a plane wave component of the incident wave field. In figure 3.11b all plane waves normally incident on the $x$-axis are shown as a function of $k_y$. The cut off at the fourth side lobe is due to truncation of the extrapolation operator at an angle of 30°. The effect of truncation is also clearly observed in figure 3.11a as additional noise around the fourth side lobe. Truncation of the extrapolation is necessary to avoid aliasing (equation (3.6.2)).
In the second step of this simulation we define the measurement probe. Keeping as close as possible to the real measurement situation we define a disc probe with an active aperture of 1.4 mm as in equation (3.6.5). The beam profile this probe would have at a depth of 75 mm for 2.9 MHz is given in figure 3.10b. Following equation (3.6.4) we may also calculate \( \tilde{D}(k_x, k_y, \omega) \). Figure 3.11c shows \( |\tilde{D}(k_x, k_y, \omega)| \) for \( k_x = 0 \) and all \( k_y \). Note that figure 3.11c was derived only by Fourier transformation of the 1.4 mm disc. There is no limitation to the \( k_y \)-values, as was the case in figure 3.11b.

The measured pressure \( \tilde{p}_m(k_z, k_y, z, \omega) \) now follows by multiplication according to (3.6.4). For \( k_x = 0 \) the amplitude of the result is shown in figure 3.11d. Two dimensional Fourier transformation back to real space yields figure 3.10c, and in cross section figure 3.11e. In this particular measurement the measurement probe causes tapering of the sidelobes of the transducer beam profile. The result of the real measurement is shown in figure 3.10d. Though the data in the real measurement are more irregular, the similarities between the simulated and the measured data are clearly observed. The irregularities in the real measurement can be explained by performing inverse extrapolation of the measured data back to the source plane. Figure 3.12 is an enlarged grey scale map of the reconstructed pressure amplitude distribution on the source plane. The dynamic range here is approximately 10 dB. Clearly, the source was not a perfect disc radiator, as the reconstruction shows a distinct area of decreased intensity.

![Figure 3.12: Reconstructed pressure distribution at the transducer surface. Approximately 10 dB dynamic range.](image)
B. Model verification

In section 3.4 the influence of sources and detectors on the measured data was discussed for the zero offset data acquisition technique. Here four computer simulations will be given, demonstrating the different kinds of limitations of the data acquisition system. Two of these examples will also be conducted as watertank experiments to verify the simulations. Table 3.1 summarizes the input parameters of the simulation programme and also the parameters of the experimental setup.

Table 3.1: Input parameters for the model verification examples.

<table>
<thead>
<tr>
<th></th>
<th>Simulation I</th>
<th>Simulation II Experiment I</th>
<th>Simulation III</th>
<th>Simulation IV Experiment II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture</td>
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<td>102.3 mm</td>
<td>102.3 mm</td>
<td>102.3 mm</td>
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<td>40 mm</td>
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<td>.1 mm</td>
<td>.1 mm</td>
<td>.1 mm</td>
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<tr>
<td>Temporal sample interval</td>
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<td>$10^{-7}$ sec</td>
<td>$10^{-7}$ sec</td>
<td>$10^{-7}$ sec</td>
</tr>
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<td>.3 mm</td>
<td>1.8 mm</td>
<td>.3 mm</td>
</tr>
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<td>Frequency spectrum</td>
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<td>2-4MHz,$\cos^2$</td>
<td>2-4MHz,white</td>
<td>2-4MHz,$\cos^2$</td>
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<td>1480 m/s</td>
<td>1480 m/s</td>
<td>1480 m/s</td>
</tr>
</tbody>
</table>

Simulation I, frequency and aperture limitations

The first simulation was devised to study the response of an "ideal" zero offset measurement system. As in section 3.4, the response of a point scatterer is studied. An "ideal" measurement system introduces no spatial or temporal spectral colouring. The sources and detectors are sufficiently small to show no angular preference, the sources emit a pulse with a white spectrum between 2 and 4 MHz, and the detectors have no temporal frequency preference.

Due to the aperture limitation, discussed in section 3.4, a scatterer situated at a distance $z$ from the centre of an aperture with length $L$ will show a maximal angle of incidence
The $k_x^2 - k_z^2$ representation of the focused result of simulation 1 is shown in figure 3.13. Here the amplitude spectrum $|\tilde{P}(k_x, k_z, t=0)|$ is given as a function of $k_x$ and $k_z$. From figure 3.13 we can clearly see that $\tilde{P}(k_x, k_z, t=0)$ cannot be reconstructed outside the acoustic window defined by $2 \text{ MHz} \leq f \leq 4 \text{ MHz}$ and $-52^\circ \leq \alpha \leq 52^\circ$, also, therefore, our estimate of the local reflectivity distribution at the position of the scatterer is limited to this acoustic spectral window (sections 3.1 and 3.4).

The total area covered by figure 3.13 is defined by the spatial and temporal sample intervals $-10^4 \pi \leq k_x \leq 10^4 \pi$ and $0 \leq k_z \leq 1.35 \times 10^4 \pi$, where for the calculation of $k_z$ the half velocity substitution must be used: $\Delta z = \frac{1}{2}c\Delta t$.

Simulation II, realistic spectrum with slight beamforming

In the first simulation no spectral colouring was introduced. In practice however, it is not possible to produce sources and detectors in the megahertz region, which are much smaller than the wavelength. The state of the art in the fabrication of ceramic material is an element size of approximately .3 mm. If the ceramic material has a resonance frequency at 3 MHz, the temporal amplitude spectrum of one element used in backscatter (zero offset) mode will be in reasonable approximation tapered by a squared cosine between 2 and 4 MHz. The amplitude spectrum $|\tilde{P}(k_x, k_y, t=0)|$ of the simulation of this more realistic situation is given in figure 3.14a. The beamforming due to the finite element size results in a more rapid fall off towards the
Figure 3.14: Simulated a) and measured b) two dimensional amplitude spectrum of a focused point scatterer, as measured by finite sized transducer elements (.3 mm), emitting a cosine squared amplitude spectrum.

higher $k_x$ values, whilst the cosine squared tapering gives a more rapid fall off in the radial direction than in the first simulation. Due to the finite element size of .3 mm the first zeroes in the $k_x$-direction occur at $k_x = -1.3 \times 10^4 \pi$ and $k_x = 1.3 \times 10^4 \pi$, so the beam pattern of one element shows no sidelobes below a frequency of 5 MHz and the first sidelobes occur at an angle of 52° for a frequency of 9.5 MHz.

If one should use this element size to build up a linear array the minimal sampling interval along the array would be $\Delta x = .3$ mm. In this case the sampling criteria state that only the frequencies below 1.25 MHz will be measured correctly for all angles.

From this we may conclude that if a linear array of transducer elements is operated below the spatial aliasing frequency, the individual elements in good approximation show no angular preference.

Figure 3.15: Two dimensional amplitude spectrum of a focused point scatterer, as measured by 1.8 mm transducer elements.
Simulation III, large flat elements
If in simulation I the element size should be increased to 1.8 mm, the
first zeroes in the $k_x$-direction would occur at $k_x = -2.2 \times 10^3 \pi$ and
$k_x = +2.2 \times 10^3 \pi$. This implies beamforming for the whole frequency
range from 2 to 4 MHz. (Here the convolution model of equation (3.4.8)
is used).

Figure 3.15 shows the amplitude spectrum $|\tilde{p}(k_x, k_z, t=0)|$, where we see
the sinc-squared weighting function in the $k_x$ direction. As the
element size increases the usable area of the $k_x$-$k_z$ spectrum
decreases. The usable area of the $k_x$-$k_z$ spectrum is defined as the
main lobe in figure 3.15, as the side lobes are not in phase with the
main lobe. With an infinitely large element only $k_x = 0$ will be
represented. Large flat elements have the undesirable effect of
reducing the lateral resolution. In the limit, where only one spatial
frequency is measured no more lateral resolving power exists.

Simulation IV, beamforming and aperture limitations
In simulation II a realistic situation was sketched, showing the effect
of beamforming of a .3 mm element on the measured data. By decreasing
the aperture angle the aperture limitations can also be demonstrated.
The maximal aperture angle was decreased to 32.6° by moving the
scatterer to a depth of 80 mm. Figure 3.16a shows the result of this
simulation on the two dimensional amplitude spectrum.

![Image of Figure 3.16](image)

**Figure 3.16:** Simulated a) and measured b) two dimensional amplitude
spectrum as in figure 3.14, introducing aperture
limitations.
Experiments I and II

The computer simulations II and IV, discussed above, were set up to simulate two watertank experiments. For the watertank experiments the same procedure was followed, only the original dataset was not generated by computer, but measured by reflection from a wire in a watertank. The two dimensional amplitude spectra of the focused results, depicted in figures 3.14b and 3.16b, bear a good resemblance to the simulated spectra of figures 3.14a and 3.16a respectively. There are some minor differences in the order of a few decibels, which are all dependent on the angle of incidence $\alpha$ of the wave field. There is an asymmetry in relation to $k_x = 0$ in figure 3.14b, and also the experimental data show small fluctuations as a function of $\alpha$. These discrepancies between measurement and simulation are the result of using an incomplete model to describe the transducer characteristics.

We may rule out the wire target as a source of the discrepancies, as the wire was much smaller than one wavelength of the highest frequency in the pulse, and also the wire was circularly symmetric. Even though the fluctuations may be of interest from the point of view of physics, their effect on the estimation of medium parameters is only a minor one, and a detailed research into their origin will be omitted here.

C. Resolution of the zero offset algorithm

The resolution of the zero offset algorithm was measured by computer simulations and by watertank experiments. In the computer simulations a spatial deltapulse was introduced at a varying distance from the middle of a 51.2 mm aperture. In the watertank a thin tungsten wire with a diameter of 100 $\mu$m was used as a target. As transducer one element of a multi-element linear array transducer was used. This element had an effective aperture of .4 mm and a bandwidth of 1.5-3.5 MHz at the -20 dB points. The watertank data were acquired by moving the element along the water surface perpendicular to the direction of the wire. Both the simulated and the measured data were subsequently treated by the zero offset focusing algorithm. The -6 dB lateral width of the point spread function of the images were measured for depths varying from 20-140 mm in the watertank and from 2.5-140 mm in the computer simulation. The lower limit of 20 mm in the watertank measurements is
Figure 3.17: Resolution of the zero offset algorithm (1.5-3.5 MHz).

- - - = zero phase pulse, no spectral colouring,
  omnidirectional element, simulated data
-.-. = zero phase pulse, cosine\(^5\) spectral tapering,
  omnidirectional element, simulated data
--- = zero phase pulse, cosine\(^8\) spectral tapering, .4 mm
  element, simulated data
x-x- = measured data, optimal at -1.3 μs rotation and c =
  1480 m/s
o-o- = measured data, matched filter deconvolution, no
  rotation and c = 1480 m/s

due to the fairly long recovery time (± 20 μs) of the applied detector
pre-amplifier. The pre-amplifier is completely saturated by the
transmission pulse. The measured values of the lateral width are given
in figure 3.17 as a function of 1/\sin α, where α is the maximal aperture
angle for a given depth.
The dotted line in figure 3.17 represents a computer simulation where a
zero phase pulse with a white spectrum between 1.5 and 3.5 MHz has been
applied, and where the transducer element is small enough (.1 mm) not to show any angular preference. Theoretically therefore, the dotted line represents the maximally obtainable lateral resolution within the given frequency band. Note that for large aperture angles there is a lower limit to the lateral resolution of approximately .20 mm. This corresponds to half a wavelength at 3.5 MHz, the upper limit in the temporal frequency spectrum. The crosses in figure 3.17 represent the result of the watertank experiment. The resolution found with the measured data is considerably worse than with the simulated data, especially at large aperture angles, also, for optimal focusing the data had to be rotated over 13 samplepoints (-1.3 μs). After some time it was discovered that three sources cause these discrepancies:

1) The measured pulse is not zero phase, so the maximal energy of the pulse does not arrive at t=0 (see section 3.2).

2) The measured pulse does not have a white spectrum.

3) The transducer elements do not have an omnidirectional sensitivity. The first source of discrepancy was removed by applying a matched filter to the measured watertank data prior to focusing. The matched filter produces a zero phase pulse. As the amplitude spectrum of the filter is the same as the original pulse the filtered result will show a narrower frequency band than the original pulse. The result is shown as circles in figure 3.17.

The computer simulation was then repeated, applying a tapering to the temporal amplitude spectrum, which resembled the output of the filtering process. Also an element width of .4 mm was introduced, to simulate the angular preference of the transducer elements. The results of this simulation are depicted by the dashed line in figure 3.17. As can be observed the dashed line follows the measured circles very well (+ .02 mm). The spatial band limitations due to the finite element size cause the resolution limit for large aperture angles to move up to approximately .30 mm. This corresponds to half a wavelength at 2.5 MHz, the central frequency of the temporal frequency spectrum.

D. Some considerations concerning the two dimensionality of the theory

In section 2.2 the step was made from the three dimensional real world to a two dimensional mathematical model. This was necessary as most data acquisition systems acquire their data over one line of the
measurement surface. The result of this data reduction is that we only have information of one cross section of the three dimensional dataset. For instance, if we were measuring the three dimensional dataset of a single point source in a homogeneous background, the arrival times of the responses would lie on a hyperboloid in the $x,y,t$ space. The apex of the hyperboloid defines the position of the point source marked by a cross in figure 3.18. If the receiver has no angular preference and the source is a monopole, the received signal amplitude would be inversely proportional to the distance $\Delta r$ between receiver and source (equation 2.1.4). Any angular preference of the receiver will be superimposed on this amplitude distribution over the hyperboloid.

Now if the data is only acquired over one line of the measurement surface, the measured cross section of the hyperboloid would be defined by the plane parallel to the $t$-axis, containing the data acquisition line. Such a cross section is also shown in figure 3.18. Any cross section of a hyperboloid parallel to the axis of rotation is itself a hyperbola of which the asymptotes are at the same angle to the axis as the asymptotes of the hyperboloid. This last observation implies that any wave field extrapolation technique which correctly compensates for travel times, will do so on any measured two dimensional dataset of the
Figure 3.19: Cross section of a measurement plane with a three dimensional line source response, where the line source is positioned in the y-t plane.

point source response. The imaged result of the two dimensional measurement will be projected in the apex of the hyperbola. When we measure the hyperboloid as a series of parallel cross sections all apices will form a hyperbola in the orthogonal plane. Thus, we see in homogeneous media containing (secondary) point sources that inverse wave propagation in three dimensions may be split up into two two-dimensional extrapolation and imaging steps.

Defining the measurement plane as the x-t plane we shall consider a line source in the y-t plane. In figure 3.19 an attempt has been made to draw the arrival times of the responses of such a line source in the three dimensional x,y,t space. The arrival times form two hyperboloids from the diffraction at the ends of the line source and these hyperboloids are enveloped by the arrival times of the direct line source response. Again the amplitude of the received signal is tapered due to the divergence of the acoustic wave and any angular preference of the receiver. In all x-t cross sections of the line source response the arrival times form hyperbolas with the same asymptotes. Therefore our wave field extrapolation technique applied to the x-t data will give a sharp focus in the apices of the hyperbolas. As the apices of all parallel x-t cross sections are situated in one y-t plane we may
Figure 3.20: Cross section of a measurement plane with a three-dimensional line source response, where the line source is arbitrarily positioned.

also split the three dimensional wave field extrapolation and imaging technique into two two-dimensional ones. Both two-dimensional extrapolation steps will optimally compensate for traveltimes in the planes of interest.

In the case of a two-dimensional line source at an arbitrary angle to the measurement plane the diffraction hyperboloids will also optimally focus in the two two-dimensional extrapolation and imaging steps, as can be seen in figure 3.20. The outer response in the measurement plane, corresponding to the specular reflection of the line source itself does not form a hyperbola, and will not focus in one two-dimensional extrapolation and imaging step. Another way of explaining the bad focus of the line source response is by considering the origin of the measured response as one moves across the measurement plane. The origin of diffraction energy is always one single point in space, wherever the receiver is located. The origin of the specular line source response depends on the receiver position. For the case sketched in figure 3.20 the origin of the outer response in the measurement
plane moves from one end of the line source to the other as the receiver position changes.

References chapter 3


CHAPTER IV
DATA ACQUISITION AND INFORMATION CONTENT

In this chapter we will take a closer look at the local reflectivity parameters. In the foregoing examples on modelling (chapter II) and on inversion (chapter III) we mainly concentrated on zero offset data acquisition and point scatterers. In this chapter this concept will be extended to data acquisition systems which have a certain offset between source and detector and also an extension will be made to larger scatterers.

Throughout this chapter we will assume that the position of the scattering region (Region Of Interest) is known in relation to the source and receiver. This implies that either the scattering region is isolated in a homogeneous background, or some inversion has taken place to remove distorting propagation influences.

4.1 FAR FIELD CONDITIONS

In many experiments, described in the literature, the assumption is made that the acoustic measurements have been conducted under far field conditions. In this chapter we also make use of this assumption, however in an indirect way. For far field conditions we must assume a plane incident wave at the r.o.i. for every source position, i.e. the phase changes over the incident wave front are negligible. Likewise the scattered wave arriving at any individual receiver must be a plane wave - this second requirement is relaxed if the scattered wave field is known on an entire measurement surface. Now plane wave decomposition
may take place by two dimensional Fourier transformation:

$$\tilde{\mathbf{p}}(k_x, z = z_0, \omega) = \int_x \int_t p(x, z = z_0, t) e^{j k_x x - j \omega t} \, dx \, dt. \quad (4.1.1)$$

All complex points in wavenumber-frequency space define the amplitude and phase of one plane wave component arriving at the detector plane $z = z_0$.

We will assume for a moment the hypothetical case in which all scattering objects are sources which simultaneously emit an acoustic pulse at time $t = 0$. By one way inverse wave field extrapolation and imaging we are able to reconstruct the source distribution (chapter III):

$$p(x, z, t = 0) = \frac{1}{4\pi^2} \int_{k_x} \int_{k_z} \tilde{p}'(k_x, k_z, t = 0) e^{-j k_x x} e^{j k_z z} \, dk_x \, dk_z, \quad (4.1.2)$$

where $\tilde{p}'(k_x, k_z, t = 0)$ is related to the measured pressure distribution at $z = z_0$ by the mapping procedure

$$\tilde{p}'(k_x, k_z, t = 0) = \frac{c k}{k_z^2} \tilde{p}(k_x, z = 0, k). \quad (4.1.3)$$

Note that the Fourier transformation $x \rightarrow k_x$ on the one hand, and the Fourier transformations $t \rightarrow \omega$ and $z \rightarrow k_z$ on the other, are of opposite polarity. This is because a plane wave travelling in the $+x$-direction is defined as $P = \exp(-j k_x x + j \omega t)$, so a phase related to positive $k_x$ and $x$ has an opposite sign to a phase related to positive $\omega$ and $t$.

After focussing the sign is retained in $k_z$ and $z$ (see also section 8.2). For this reason we will define the dot product

$$\hat{k} \cdot r = k_x x - k_z z. \quad (4.1.4)$$

Substituting (4.1.4) in (4.1.2) and adding the subscript $s$ to indicate the scattered wave field we find
\[ p(x, z, t=0) = \frac{1}{4\pi^2} \int \int \hat{p}'(k_x, k_z, t=0) e^{-jk_s \cdot \hat{r}} \, dk_x \, dk_z. \quad (4.1.5) \]

With the zero offset data acquisition technique the half velocity substitution is often used, so \( k_s = 2k \) and we find

\[ p(x, z, t=0) = \frac{1}{4\pi^2} \int \int \tilde{p}'(k_x, k_z, t=0) e^{-jk2k \cdot \hat{r}} \, dk_x \, dk_z. \quad (4.1.6) \]

As mentioned earlier equation (4.1.5) is only correct for the hypothetical case where all scattering objects simultaneously emit a pulse at \( t=0 \). Generally the scatterers are "illuminated" by some incident wave field. Consequently equation (4.1.5) must be modified to compensate for the time lapse or phase shift due to the sequential illumination. If we may assume that the r.o.i. is in the far field of the source, as defined above, then we must substitute

\[ \tilde{p}'(k_x, k_z, t=0) = \tilde{p}(k_x, k_z, t=0) e^{jk_1 \cdot \hat{r}}, \quad (4.1.7) \]

where the subscript \( i \) indicates the incident wave field. Note that the phase shift due to the incident wave and that due to the scattered wave are of opposite polarity. Now we find a modified Fourier relationship between the measured pressure field and the scatterer distribution

\[ p(x, z, t=0) = \frac{1}{4\pi^2} \int \int \tilde{P}(k_x, k_z, t=0) e^{-j(k_s - k_1) \cdot \hat{r}} \, dk_x \, dk_z. \quad (4.1.8) \]

By substituting equation (4.1.7) we have introduced a different mapping procedure than the one described in (4.1.3). This necessitates a recalculation of the weighting factor. For an incident plane wave we find \( k_z = k + \sqrt{k^2 - k_x^2} \), resulting in a new weighting factor \( c(k_x^2 - k_z^2) / 2k_z \) (see section 3.3).

From this we may draw the following conclusions:
Figure 4.1: Construction of the k-vector at the region of interest.

- Mapping the measured data in the wavenumber domain may be done correctly only if the source wave may be assumed to be a plane wave at the r.o.i.
- Mapping in the wavenumber domain must be done with the "imaged" data as defined in section 3.3, only assuming one way propagation.
- Mapping in the wavenumber domain must be done by using a modified k-vector defined by (see also figure 4.1):

$$\hat{k} = \hat{k}_s - \hat{k}_i,$$

where $\hat{k}_s$ may be found by Fourier transformation of the measured wave field, and where $\hat{k}_i$ is the incident wave vector, which may only be constructed if the position of the r.o.i. is known in relation to the source.
- The half velocity substitution for zero offset data is essentially incorrect in amplitude as the incident wave field is not a plane wave.

If the measurement is conducted with finite sized transducers, and no sophisticated inversion takes place, we may assume a plane wave only on axis in the far field and in the focal zone of the transducer. Only in these two areas does the transducer give a true "time shifted" image of the r.o.i. This approach sets severe constraints on the measurement setup and often necessitates dubious assumptions. For instance it is impossible to unambiguously select the signal coming from a r.o.i. in the far field, unless that r.o.i. is placed in a homogeneous background. In the focal zone a spatially broadband signal is assumed to be a plane wave; also the focal zone is never spherically symmetric, so it is hard to define one r.o.i. as the transducers are "rotated around the object".
These considerations have in the past led to large scale measurement setups and to elaborate mathematical compensation schemes [1, 2]. With respect to this k-space mapping procedure single shot record inversion [3] must also be mentioned. In single shot record inversion the incident wave need not be a plane wave. This generalization is possible as the correlation of the incident and scattered waves is not done in k-space. Summarizing the computation scheme for single shot record inversion the following steps are defined [3]
1) Forward extrapolation of the source wave to a depth level \(z^wz_m\).
2) Inverse extrapolation of the reflected wave field to depth level \(z^wz_m\).
3) Correlation of the extrapolated source wave field with the extrapolated reflected wave field.
4) Application of the imaging principle.
5) Combination of all migrated shot records.
The similarity between k-space mapping and the steps defined here is unmistakable. Compare e.g. step 1 with equation (4.1.7), steps 2 and 4 with equation (4.1.2) and finally step 3 with equation (4.1.8).

4.2 POINT SCATTERERS

In this section we will consider the scattering of small inhomogeneities in a homogeneous background. Here, small is related to the wavelength of the applied pulse, i.e. \(a \ll \frac{1}{2} \), or \(ka \ll \pi\), where \(a\) is the radius of the particle. A particle may scatter an acoustic wave if its acoustic properties differ from the surrounding medium. The acoustic impedance is a product of the density and sound propagation velocity, but usually the more fundamental physical properties, density and compressibility, are considered. As both properties give a different reaction to mechanical stress we will consider a density inhomogeneity

\[
\gamma_\rho = \frac{\Delta \rho}{\rho} = \frac{\rho - \rho_0}{\rho},
\]

where \(\rho\) is the particle density and \(\rho_0\) is the density of the surrounding medium and we will consider a compressibility inhomogeneity

\[
\gamma_K = \frac{\Delta K}{K} = \frac{K - K_0}{K},
\]
where $K$ and $K_0$ are the compression moduli of the particle and of the surrounding medium respectively.

The relationship between the sound propagation velocity and the compression modulus is given by

$$c = \sqrt{K/\rho}.$$  \hfill (4.2.3)

A density inhomogeneity will 'behave' as a rigid mass, so there will be a pressure gradient over the particle. Consequently a density inhomogeneity will react as an acoustic pressure dipole, which is orientated by the incident acoustic wave.

Likewise a compressibility inhomogeneity will behave as a massless spring and, therefore, it will react as an acoustic pressure monopole. Using equation (2.1.3) we find for a density inhomogeneity

$$P_A = \frac{1+jk\Delta r}{\Delta r} \frac{e^{-jk\Delta r}}{\Delta r} \cos \psi \cdot S/\rho,$$  \hfill (4.2.4)

or in the far field, where $k\Delta r >> 1$

$$P_A = \frac{jkS}{\rho} \frac{e^{-jk\Delta r}}{\Delta r} \cos \psi,$$  \hfill (4.2.5)

where $\psi$ is the angle between the incident and scattered waves. For a compressibility inhomogeneity we find

$$P_A = \frac{jkS}{\gamma} \frac{e^{-jk\Delta r}}{\Delta r}.$$  \hfill (4.2.6)

In these equations the scattering strength is contained in $S$; it is proportional to the incident field strength, the volume of the scatterer and $\gamma$. Scattering will be treated in more detail in chapter VIII.
4.3 PLANE BOUNDARY BETWEEN TWO MEDIA

A plane boundary between two semi-infinite half spaces has been amply discussed elsewhere [4]. The ratio between the reflected pressure wave field amplitude and the incident wave field amplitude is the reflection coefficient. The angle dependent reflection coefficient may be derived from the boundary conditions at the interface:

\[ r(\alpha) = \frac{\rho_2 c_2 \cos \alpha - \rho_1 \sqrt{c_1^2 - c_2^2 \sin^2 \alpha}}{\rho_2 c_2 \cos \alpha + \rho_1 \sqrt{c_1^2 - c_2^2 \sin^2 \alpha}} \]  

(4.3.1)

where \( \alpha \) is the angle between the incident wave and the normal on the boundary, \( \rho_1 \) and \( c_1 \) are the density and sound propagation velocity of the first medium and \( \rho_2 \) and \( c_2 \) are those of the second medium. In wavenumber \( (k_x - k_z) \) space equation (4.3.1) translates into

\[ R(k_z) = \frac{\rho_2 k_{z1} - \rho_1 k_{z2}}{\rho_2 k_{z1} + \rho_1 k_{z2}}, \]  

(4.3.2)

where \( k_{z1} \) and \( k_{z2} \) are the wavenumbers perpendicular to the reflecting surface in the two media. The boundary condition in wavenumber space is \( k_{x1} = k_{x2} \). From the same boundary condition the transmission coefficient may be derived

\[ T(\alpha) = \frac{2\rho_2 c_2 \cos \alpha}{\rho_2 c_2 \cos \alpha + \rho_1 \sqrt{c_1^2 - c_2^2 \sin^2 \alpha}}, \]  

(4.3.3)

or

\[ T(k_z) = \frac{2\rho_2 k_{z1}}{\rho_2 k_{z1} + \rho_1 k_{z2}}. \]  

(4.3.4)
From equation (4.3.1) we may calculate the normal incidence reflection coefficient, as we would measure with a zero offset data acquisition system (far field assumption):

$$r(\varnothing) = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1}$$  \hspace{2cm} (4.3.5)

and also we can find the angle at which total reflection occurs ($r(\alpha) = 1$)

$$\alpha = \pm \arcsin \left( \frac{c_1}{c_2} \right).$$  \hspace{2cm} (4.3.6)

Beyond this angle the reflection coefficient becomes complex and the second medium can only support an evanescent wave field.

4.4 SCATTERING FROM AN OBJECT WITH A FINITE SIZE

In the two previous sections an infinitely small object (scatterer) and an infinitely large object (reflector) were discussed. In real life however, most scattering objects are an intermediate form between these two extremes. As soon as the object becomes larger than half a wave length the character of the scattering changes from the omni-directional, diffractive response to the highly directional reflective response. In the far field of the scatterer the scattered amplitude may be given as a scalar quantity or directivity function [5], depending on the angle of the incident field and the scattered angle. If the directivity function of a scattering object is known for all angles of incidence and all observer angles, it is, in principle, possible (within the available bandwidth) to reconstruct not only the size and shape, but also the angle dependent acoustic parameters of that object. To understand better how a directivity function may reveal scatterer information it is essential to be able to map the available information into the wavenumber-frequency domain. In the next sections a few examples will be given of data acquisition techniques, and how they relate to the wavenumber-frequency domain. In the final section of this chapter we will return to the question of the angle dependent acoustic parameters.
4.5 MAPPING IN THE WAVENUMBER DOMAIN

Zero offset
In the zero offset data acquisition technique the acoustic source and receiver are placed in the same position. For one temporal frequency component the k-vector of the incident wave and the k-vector of the scattered wave are pointing in opposite directions. According to equation (4.1.9) the resulting k-vector at the region of interest is directed towards the receiver and has twice the length of the incident k-vector (figure 4.4). The measured amplitude and phase of this plane wave component may now be mapped into the $k_x - k_z$ domain retaining the angle of the k-vector in relation to the r.o.i. To complete the zero offset measurement sequence the source/detector combination is moved in relation to the r.o.i. and the measured amplitude and phase of the monochromatic plane wave component are mapped onto the same arc in the $k_x - k_z$ domain as the previous measurement. If the incident pulse is band limited between $f_{\text{min}}$ and $f_{\text{max}}$ the area of the k-space which is covered by a zero offset measurement sequence will be as shown in the shaded area of figure 4.4.

Plane wave
Another way of exciting the medium is by plane wave insonification. In practice a large plane transducer is used as a source and it is kept in a fixed position during the experiment, while a small receiver is used to measure the scattered wave. Due to practical considerations two types of plane wave measurements are distinguished. In plane wave

![Figure 4.4: Mapping in the wavenumber space of a zero offset measurement.](image)
Figure 4.5: Mapping in the wavenumber space of a plane wave reflection measurement.

Figure 4.6: Mapping in the wavenumber space of a plane wave transmission measurement.

Reflection measurements the scattered wave field is measured at the near side of the r.o.i. in relation to the source, so the incident and scattered k-vectors are at an angle larger than 90°, while in plane wave transmission measurements the scattered wave field is measured at the far side and the angle is smaller than 90°. At a scattering angle of 90° the transmission and reflection measurements give the same results (figures 4.5 and 4.6).

Due to the fixed position of the source the incident wave field may be described by a fixed vector \( \vec{k}_i \) per frequency. The scattered vector \( \vec{k}_s \) describes a circle around \( \vec{k}_i \) (see figs. 4.5 and 4.6), this circle is well known in the field of X-ray tomography as an Ewald circle, or in the three dimensional case as the Ewald sphere [6].
Note that the coverage of k-space with the plane wave method is not so good as in the case of zero offset measurements. This has two consequences: the lateral resolution of the plane wave method is almost twice as bad as that of the zero offset method, secondly, the sampling criteria are relaxed in relation to the zero offset method. Generally in X-ray tomography, but also in acoustic plane wave transmission tomography, the poor coverage of k-space is compensated for by repeating the experiment for many different source positions.

Common midpoint
As a final example the common midpoint measurement technique is considered. Here the source and detector are moved in opposite directions away from a common midpoint. The resulting k-vector at the region of interest is thus kept pointing in one direction during the whole measurement. The length of the k-vector varies from $2k_\perp$ if the source and receiver are at the midpoint (zero offset) to zero if the source and receiver are facing each other.

In a broad band experiment the same single line in k-space is covered many times.

From figure 4.7 it is clear that the lateral resolution of a common midpoint measurement is non-existent. Such a measurement cannot be used as an imaging technique, but due to the angle information it can be used to study angle dependent acoustic features.

![Diagram](image)

*Figure 4.7: Mapping in the wavenumber space of a common midpoint measurement.*
4.6 ANGLE DEPENDENT PARAMETERS

From the previous section we see that the two dimensional wavenumber space of a region of interest may be covered many times by acoustic experiments. Therefore we must extend our concept of k-space with an extra label, namely the angle between the incident and the scattered wave $\psi$. This extra label gives us the information we need as to how the data were acquired. In the zero offset data acquisition technique this angle $\psi = 180^\circ$, while in other techniques $\psi$ may vary between $0^\circ$ for transmission measurements and $180^\circ$ for backscatter measurements. In figures 4.8 and 4.9 an attempt has been made to sketch the cross sections of the $k_x$-$k_z$-$\psi$ space which are covered by band limited zero offset and common midpoint measurements. Unfortunately such a sketch for a plane wave measurement was beyond the capacities of the author. Please note that the angle $\psi$ is not an extra dimension and the length of the k-vector is given by its projection on the $k_x$-$k_z$ plane.

Now if we had all the possible information from the r.o.i. at our disposal we could ask ourselves two kinds of questions:

![Diagram](image)

*Figure 4.8: Three dimensional representation of the $k_x$-$k_z$-$\psi$ space for a zero offset and a common midpoint data acquisition technique.*
1) Are we interested in the spatial distribution of reflectivity events or 2) Are we interested in the angle dependent parameters of one reflectivity event?

In the first case we would like a maximal coverage of \( k_x - k_z \) space – a zero offset experiment or a cross section of our measurement space at \( \psi = 180^\circ \) would give us the best information. In the second case we would prefer to have information containing as many angles \( \psi \) as possible – a common midpoint experiment would be most suitable. All other experiments are a mixture between these two extremes.

The angle \( \psi = 0^\circ \) for transmission measurements can only be obtained for \( k_x = k_z = 0 \), so for \( \psi = 0^\circ \) we have no lateral or axial resolving power.

References chapter 4


CHAPTER V
ANALYSIS OF THE REFLECTIVITY DISTRIBUTION

1) Deterministic models

Chapters V and VI are closely related, as they both treat the analysis of the spatial distribution of reflectivity. In this chapter deterministic models will be discussed. Here the reflectivity distribution may be regarded as a regular lattice, so a.o. the Bragg diffraction criterion is satisfied. In chapter VI stochastic models are treated, the reflectivity distribution is then regarded as a perturbed lattice. In both chapters the time and frequency domain analysis are compared. Criteria will be given for when the raw r.f. data should be analysed and when envelope detection must take place prior to the analysis. Also the multi-dimensional auto correlation function will be discussed as a means to extract any periodicities in the reflectivity distribution.

5.1 ENERGY SPECTRAL DENSITY FUNCTION VERSUS AUTO CORRELATION FUNCTION

Any energy bounded time function f(t) may be rewritten in terms of its Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt, \quad (5.1.1)$$

where the complex function $F(\omega)$ can also be given in terms of the
amplitude spectral density $|F(\omega)|$ and the phase delay angle $\phi(\omega)$:

$$F(\omega) = |F(\omega)|e^{j\phi(\omega)}. \quad (5.1.2)$$

Since $f(t)$ is a real function, the following relation holds

$$F^*(\omega) = F(-\omega). \quad (5.1.3)$$

Hence $F(\omega)$ is often given for $0 \leq \omega < \infty$ only and $f(t)$ may be defined as

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t} d\omega = \frac{1}{\pi} \text{Re} \int_{0}^{\infty} F(\omega)e^{j\omega t} d\omega. \quad (5.1.4)$$

The energy spectral density function, which is defined as the squared amplitude spectral density

$$X(\omega) = F^*(\omega)F(\omega) = |F(\omega)|^2 \quad (5.1.5)$$

can be written as the Fourier transform of the autocorrelation function

$$R(t) = \lim_{T \to \infty} \int_{-T}^{T} f(\tau)f(\tau + t) d\tau \quad (5.1.6)$$
as

$$X(\omega) = \int_{-\infty}^{\infty} R(t)e^{-j\omega t} dt, \quad (5.1.7)$$
or

$$R(t) = \frac{1}{\pi} \text{Re} \int_{0}^{\infty} X(\omega)e^{j\omega t} d\omega = \frac{1}{\pi} \int_{0}^{\infty} X(\omega)\cos\omega t \, d\omega. \quad (5.1.8)$$

The plot of the energy spectral density $X(\omega)$ versus frequency is called the Energy Spectrum, just as the plot of the amplitude spectral density $|F(\omega)|$ versus frequency is called the Amplitude Spectrum. As can be seen from equations (5.1.5) and (5.1.8) there is principally no
difference between the information contained in the amplitude spectrum and the auto correlation function, except that the first is defined in the Fourier domain, while the second is defined in the time domain. A time series with a periodicity of $\tau_0$ will reveal this periodicity in its auto correlation function $R(t)$.

With a two dimensional distribution $f(x,z)$ we can perform a two dimensional Fourier transformation

$$\tilde{F}(k_x, k_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,z)e^{j(k_x x - k_z z)} \, dz. \quad (5.1.9)$$

Then we may define the amplitude spectral density $|\tilde{F}(k_x, k_z)|$ and the phase delay angle $\phi(k_x, k_z)$ according to

$$\tilde{F}(k_x, k_z) = |\tilde{F}(k_x, k_z)| e^{j\phi(k_x, k_z)}. \quad (5.1.10)$$

The two dimensional energy spectral density function may now be defined as

$$\tilde{X}(k_x, k_z) = \tilde{F}^*(k_x, k_z) \tilde{F}(k_x, k_z) = |\tilde{F}(k_x, k_z)|^2, \quad (5.1.11)$$

and the two dimensional auto correlation function as its Fourier counterpart

$$R(x,z) = \frac{1}{2\pi^2} \text{Re} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} \tilde{X}(k_x, k_z) e^{j(k_z z - k_x x)} dk_x. \quad (5.1.12)$$

5.2 AMPLITUDE SPECTRUM OF A FINITE SERIES OF DIRAC PULSES

If we assume a model of weak point scatterers distributed on a regular lattice in a homogeneous background, we may represent this medium as a series of delta pulses at the positions of the scatterers (section 2.7). The image produced by an acoustic system of this medium can be found by convolution of the delta pulses with the point spread function of the measurement system.
In this section we will study the broad band frequency response (no system limitations) of a series of deltapulses. For simplicity we will start with a one dimensional model, where

\[ f(t) = \sum_{n=-\frac{1}{2}(N-1)}^{\frac{1}{2}(N-1)} \delta(t - nt_0), \quad \text{for } N = 2, 3, 4, \ldots \]  

(5.2.1)

With equation (5.1.1) we find

\[ F(\omega) = \sum_{n=-\frac{1}{2}(N-1)}^{\frac{1}{2}(N-1)} e^{-j\omega nt_0}, \quad \text{for } N = 2, 3, 4, \ldots \]  

(5.2.2)

or

\[ F(\omega) = 2\cos \frac{1}{2} \omega t_0, \quad \text{for } N = 2 \]

\[ F(\omega) = 2\cos \omega t_0 + 1, \quad \text{for } N = 3 \]

\[ F(\omega) = 2\cos \frac{3}{2} \omega t_0 + 2\cos \frac{1}{2} \omega t_0, \quad \text{for } N = 4 \]

etc.

Figures 5.1 - 5.4 depict the real part of the Fourier spectrum \( F(\omega) \) and the amplitude spectrum \(|F(\omega)|\) for \( N = 2, 3, 4 \) and 16 for \( t_0 = 1.6 \times 10^{-6} \) s. The amplitude spectra show maxima for \( \omega = 0, 2\pi/t_0, 4\pi/t_0 \), or for \( f = \omega/2\pi = 0, .6 \) MHz, 1.2 MHz, ... These maxima

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**Figure 5.1:** a) Real part of the Fourier spectrum for two deltapulses, \( \Delta t = 1.6 \) μs.

b) Amplitude spectrum of a) = deltapulses; ... = bandlimited; --- = envelope.
Figure 5.2: a) Real part of the Fourier spectrum for three delta pulses,\[ \Delta t = 1.6 \, \mu s. \]
b) Amplitude spectrum of a) ___ = delta pulses; \ldots = bandlimited; \ldots = envelope.

Figure 5.3: a) Real part of the Fourier spectrum for four delta pulses,\[ \Delta t = 1.6 \, \mu s. \]
b) Amplitude spectrum of a) ___ = delta pulses; \ldots = bandlimited; \ldots = envelope.

become more pronounced and narrower as the number of delta pulses increases. The amount of submaxima in between two maxima is \(N-2\), and their amplitude is approximately \(1/N\) times that of the maxima. The ratio between the amplitude of a maximum and the amplitude of the adjacent submaxima (here approximately \(N\)) we will define as the modulation depth. The significance of the modulation depth will become clear in the next chapter.
Figure 5.4: a) Real part of the Fourier spectrum for sixteen
delpulses, $\Delta t = 1.6$ $\mu$s.
b) Amplitude spectrum of a) ___ = deltpulses; ... = 
bandlimited; --- = envelope.

5.3 FREQUENCY BAND LIMITATIONS AND ENVELOPE DETECTION

Any practical measurement system will introduce frequency band limitations. The series of deltpulses will be convolved with the system response. In figures 5.1b - 5.4b a band limitation has been introduced (the dotted lines) between 2 and 4 MHz. Between these frequencies a cosine shaped tapering was applied. After frequency band limitation envelope detection was performed, and subsequently the amplitude spectrum of the envelope detected signal was plotted as the dashed lines in figures 5.1b - 5.4b.

From these experiments it seems that envelope detection of a band limited signal only shifts the part of the amplitude spectrum which can be measured to a lower frequency. Both the envelope detected signal and the original r.f follow the notches and maxima of the unlimited spectrum accurately. The frequency band of the envelope is always situated around zero frequency, whilst the original band limited signal is always situated around a non zero centre frequency. Later on this will be shown to be of great importance with the analysis of real measured data.

From the above experiments it appears that no information is lost by envelope detection. This is only true if we are not interested in recovering the instantaneous phase of the pulse (see chapter VIII) and if the original pulses are sufficiently far apart.

If the pulses are so close together that their main lobes overlap the
envelope detected signal is no longer a good representation of the real reflectivity distribution (see also Berkhout [1]). That the character of the pulse is not preserved can easily be seen if we realize that the envelope rectifies the signal, so it always shows a maximum for $f=0$. A simple example showing that the envelope is not a good representation of the reflectivity distribution is given in figure 5.5, where the amplitude spectrum is shown of two pulses with opposite polarity. Here the amplitude spectrum of the deltapulses is zero for $f=0$, while the amplitude spectrum of the envelope is not, as shown in figure 5.5b by the dashed line.

5.4 TWO DIMENSIONAL DETERMINISTIC MODELS

In this section two examples will be discussed of two dimensional deterministic models.

The first example consists of three scatterers situated in a triangle, as shown in figure 5.6a. If there are no system limitations, and we could reconstruct three spatial deltapulses, then the two dimensional amplitude spectrum would be filled from $-k_{xNyq} < k_x < k_{xNyq}$ and $0 < k_z < k_{zNyq}$, where the subscript Nyq denotes the Nyquist frequency. In these experiments the spatial sample interval $\Delta x = 0.1$ mm and the temporal sample interval $\Delta t = 10^{-7}$ s, while $c = 1500$ m/s. From these parameters the Nyquist frequencies may be calculated.
Figure 5.6: a) Relative positions of the three scatterers in mm.

b) Relative positions of the maxima of the two dimensional amplitude spectrum in mm⁻¹.

![Diagram of scatterers and k vectors](image)

Figure 5.7: Two dimensional amplitude spectrum of three delta pulses as shown in figure 5.6.

\[
\bar{k}_x N_{\text{Nyq}} = 0.5 \times 10^4 \text{ m}^{-1} \quad \text{and} \quad \bar{k}_z N_{\text{Nyq}} = 6.6 \times 10^3 \text{ m}^{-1}, \quad \text{where} \quad \bar{k} = k/2\pi.
\]

Figure 5.7 shows the two dimensional amplitude spectrum in this situation. Introducing the measurement system limitations, as discussed in chapter III we arrive at figure 5.8a. From this figure we can clearly observe the temporal frequency band limitations and the aperture limitations. Note that the lateral bandwidth is larger than the axial bandwidth, which causes a better lateral than axial resolution of the resulting point spread function. The experiment described above was repeated in a watertank with three submerged 100 μm tungsten wires (see also section 3.6B). The resulting two dimensional amplitude spectrum of the focused image is given in figure 5.8b.

These experiments prove that the theory introduced in the previous chapters, and verified for single scatterers in chapter III, also holds for multiple scatterers. Consequently we may conduct the following
Figure 5.8: a) Two dimensional amplitude spectrum of the simulated three scatterer model.
b) Two dimensional amplitude spectrum of the measurement of three wires in a watertank.

Computer simulation involving a matrix of scatterers, situated on a two dimensional lattice. In this simulation a matrix of 16x16 scatterers was chosen with a characteristic distance of 1.2 mm both axially and laterally. As in the previous examples the lateral sample interval $\Delta x = .1$ mm, and the temporal sampling frequency $f_s = 10$ MHz. Figure 5.9 gives the simulated results before and after focussing. As can be seen in figure 5.9b the scatterers were placed on a completely regular lattice. We see as well, that the lateral resolution is again better than the axial resolution, which also follows from the two dimensional amplitude spectrum of the focused image in figure 5.10a. The two dimensional auto correlation function of the focused image is depicted.

Figure 5.9: Matrix of 16x16 point scatterers $\Delta x = \Delta z = 1.2$ mm.
a) modelling output  
b) focused output
Figure 5.10: a) Two dimensional amplitude spectrum of the focused image of 5.9b.

b) Two dimensional auto correlation function of 5.9b.

Figure 5.11: a) Two dimensional amplitude spectrum of the envelope detected image of 5.9b.

b) Two dimensional auto correlation function of the envelope detected 5.9b.

in figure 5.10b. The characteristic distance in figure 5.10b of 1.2 mm translates into a characteristic distance in figure 5.10a of $\Delta k = 1/1.2 = 0.83 \text{ mm}^{-1}$. As all the scatterers in the model were chosen to have a positive reflection coefficient $k_x = k_z = 0$ shows a maximum. Due to various aberrations in the reproduction of images from a screen, the scales in these and subsequent figures may only be treated as an indication. The two dimensional amplitude spectrum and the two dimensional auto correlation function of the envelope detected focused image (actually shown in figure 5.9b) are given in figure 5.11. The lateral cross section at $z=0$ and the axial cross section at $x=0$ of
Figure 5.12: a) Lateral cross section of figs. 5.10b (...) and 5.11b (...) at z=0.

b) Axial cross section of figs. 5.10b (...) and 5.11b (...) at z=0.

figures 5.10b and 5.11b are depicted as an amplitude plot in figure 5.12. From both figures 5.12a and b we see that envelope detection worsens the performance of the two dimensional auto correlation function; hence in case of a completely regular lattice it is always preferable to analyse the focused r.f. data.

5.5 BRAGG DIFFRACTION

In the field of electromagnetic radiation Bragg observed the scattering of X-rays from a crystal lattice. When a plane wave hits such a lattice the scattered wave shows an angular periodicity which depends on the angle of incidence, the scattered angle, the applied frequency and the lattice constant. For a one dimensional lattice with a characteristic distance d it is simple to show that the backscattered intensity is maximal for

\[ \theta = \arcsin \frac{\lambda_n}{2d} = \arcsin \frac{\pi n}{kd} \quad \text{for} \quad n = 0,1,2,\ldots \]

(5.5.1)

where \( \theta \) is the angle between the incident wave and the normal vector on the lattice.

The two dimensional amplitude spectrum of a backscattered signal may also be seen as the plane wave decomposition of that signal.

- Bearing in mind that the zero offset registration may be translated into a "true" wave field registration by means of the half velocity substitution (section 2.6). -
Figure 5.13: Isobar plot of the two dimensional amplitude spectrum of three deltapulses oriented along a line in the x-direction. The cross sections along $k_B=0$, arc AB and line CD are also shown.

Figure 5.13 shows the two dimensional amplitude spectrum for a "lattice" of three deltapulses oriented in the x-direction. The characteristic distance between the deltapulses $d = 1.5$ mm. If we now observe the backscattered signal amplitude as a function of angle of incidence for e.g. a frequency of 2 MHz, equation (5.5.1) predicts a maximum for $\theta = \arcsin n/4$, where a sound propagation velocity $c = 1500$ m/s was substituted.

In figure 5.13 this experiment may be simulated by following the arc $AB$. The radius of the arc is $k/2\pi = 2f/c = 2.67 \times 10^3$ m$^{-1}$. The back-scattered signal has maximal intensity for $\theta = 14.5^\circ$, $30^\circ$, $48.6^\circ$ and $90^\circ$ as predicted by equation (5.5.1).

Similarly an experiment can be conducted keeping the angle of incidence $\theta$ constant and sweeping the frequency. Equation (5.5.1) can be rewritten as

$$\frac{k}{2\pi} = \frac{f}{c} = \frac{n}{2d \sin \theta} \quad \text{for} \quad n = 0,1,2,... \quad (5.5.2)$$
Line C-D in figure 5.13 simulates an experiment where $\theta = 45^\circ$ and the frequency is swept from 1 to 4 MHz. Equation (5.5.2) correctly predicts the backscattered signal intensity to be maximal for $f = .707$ n MHz. In conclusion we may say that the two dimensional amplitude spectrum of a focused acoustical image shows great similarity to a Bragg diffraction experiment. Therefore coherent acoustical scattering from a lattice is often referred to as Bragg diffraction [2, 3].

5.6 ANGLE DEPENDENT SCATTERING

Until now, in all the examples and experiments the scattering objects were assumed to be small in relation to the wavelength of the applied acoustic pulse. Consequently the scatterers were treated as point scatterers, meaning that the intensity of the scattered wave was independent of the angle of incidence.

In this section we will study the backscattered signal when the above assumption is no longer true.

In section 3.4 it was argued that the ideal image of a point scatterer would be a spatial (band limited) deltapulse, so $|\tilde{R}(k_x,k_z)| = 1.0$ within the band limitations of the measurement system. In section 2.6 we also found that due to the half velocity assumption in the inversion process the focused result of a point scatterer was not spectrally white, but shows some tapering in the lateral direction (figure 3.13). The angle dependence of the backscattered (zero offset) signal will be studied by averaging the focused response for all received temporal frequencies and by plotting the result as a function of angle of incidence, so in figure 3.13 the data are added together along radial lines from $\theta = -90^\circ$ to $\theta = 90^\circ$. The result of this process is shown as the solid line in figure 5.15a.

Two models were chosen. In the first model a small plane scatterer of .2 mm was placed at an angle of 45° to the x-axis. In the second model the scatterer size was increased to .4 mm. With the temporal frequency limitations of 2 MHz $\leq f \leq 4$ MHz the length of the smaller scatterer was approximately $\lambda/4$ at the lowest frequency and $\lambda/2$ at the highest. For the larger scatterer these figures were $\lambda/2$ and $\lambda$. 
Figure 5.14: Two dimensional amplitude spectrum of a small strip oriented at an angle of $45^\circ$ to the x-axis.

a) width of strip = .2 mm
b) width of strip = .4 mm.

Figure 5.15: a) Average signal amplitude as a function of angle

- point scatterer = .2 mm strip = .4 mm strip
b) As a), but compensated for amplitude errors by using the point scatterer data.

After focussing, the two dimensional amplitude spectrum $|\tilde{R}(k_x,k_2)|$ of the two models is shown in figures 5.14a and b. If these data are now added together for $0 < k < k_{Nyq}$ and plotted as a function of angle $\theta$, the result is shown in figure 5.15a. The dotted line is for the .2 mm scatterer, while the dashed line is for the .4 mm scatterer. For direct comparison the maximal scattering amplitude was kept constant by choosing the reflectivity inversely proportional to the scatterer size. Ideally we would expect the dotted and dashed lines in figure 5.15a to show a maximum for $\theta = 45^\circ$, but due to the fact that the combination of...
modelling and inversion is not a spectrally white process a bias is introduced. The focused result may be "whitened" for any particular reference model, by dividing the two dimensional amplitude spectrum by the two dimensional amplitude spectrum of the reference model. In our case we will choose the point scatterer as a reference model. For signal to noise reasons a threshold of 5% was introduced in the division process.
As a result the whitening effect is not perfect and some tapering remains in the solid line of figure 5.15b. The whitening process has removed the bias, and the maximal amplitude of the backscattered signal may be correctly estimated at 45°.

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After focussing, the two dimensional amplitude spectrum $|R(x, y)|$ of the two models is shown in figures 3.1a and b. If these data are now added together for $0 \leq x \leq L$ and plotted as a function of angle $\theta$, the result is shown in figure 3.1b. The dotted line is for the $\frac{1}{2}$ wave scatterer, while the dashed line is for the $\frac{1}{4}$ wave scatterer. For direct comparison the maximal scattering amplitude was kept constant by choosing the reflectivity inversely proportional to the scatterer size. Ideally we would expect the dotted and dashed lines in figure 3.1b to show a maximum for $\theta = 45^\circ$, but due to the fact that the combination of
CHAPTER VI
ANALYSIS OF THE REFLECTIVITY DISTRIBUTION
2) Statistical models

In chapter V deterministic models were discussed. The models either contained single scatterers, or scatterers arranged according to a regular lattice. In nature such regular arrangements only occur in crystalline structures. Generally any regular structure shows a certain degree of randomness. In this chapter the basic structure is still assumed to be a lattice, but the scatterers will be arranged according to some statistical perturbation of that lattice.

6.1 GAUSSIAN DISTRIBUTION

When introducing a degree of randomness in a computerized model a choice must be made as to what kind of probability function to use. In nature, and certainly in a biological process, the size, shape and distribution of particles is generally governed by a number of independent processes.

According to the central limit theorem a large number of independent random variables acting together will often result in a Gaussian distribution. Therefore it seems acceptable to use this distribution for our purposes. The Gaussian probability function is defined as

\[ p(t) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{(t-\mu)^2}{2\sigma^2} \right\} \]  

(6.1.1)

where \( \mu \) is the mean of \( p(t) \), and where \( \sigma \) is the standard deviation. If two independent Gaussian stochastic processes with the same variance
are added or subtracted, as is the case when calculating the
distance between two random points in one dimensional space, then the
combined variance of the distance is equal to the sum of the individual
variances, or

\[ \sigma' = \sqrt{\sigma_1^2 + \sigma_2^2} = \sigma \sqrt{2}. \] (6.1.2)

### 6.2 AMPLITUDE SPECTRUM OF A SERIES OF DIRAC PULSES

In section 5.2 pulses were positioned at regular intervals \( \Delta t = t_0 \).
Now a model will be introduced, where the positions of the pulses are
distributed randomly around these regular intervals. Rewriting equation
(5.2.1) we find

\[ f(t) = \sum_{n=-\frac{1}{2}(N-1)}^{\frac{1}{2}(N-1)} \delta(t - t_0(n + p(t))) \text{ for } N = 2, 3, 4, \ldots \] (6.2.1)

where \( p(t) \) is a random variable with a Gaussian probability function
with mean \( \mu = 0 \). The Fourier transform of \( f(t) \) is

\[ F(\omega) = \sum_{n=-\frac{1}{2}(N-1)}^{\frac{1}{2}(N-1)} e^{-j\omega t_0(n + p(t))} \text{ for } N = 2, 3, 4, \ldots (6.2.2) \]

For \( N = 2 \) we find

\[ F(\omega) = e^{-j\omega t_0 \frac{1}{2}(p_1 - p_2)} + e^{-j\omega t_0 \frac{1}{2}(p_1 + p_2)}, \] (6.2.3)

where \( p_1 \text{ and } p_2 \) are two realizations of the stochastic process \( p(t) \).
From this equation we can derive

\[ F(\omega) = 2e^{-j\omega t_0 \frac{1}{2}(p_1 + p_2)} \cos\{\frac{1}{2}\omega t_0 (1 - p_1 + p_2)\}. \] (6.2.4)

The amplitude spectral density function shows maxima for \( \omega = 0, \)
\( 2\pi/t_0(1 - p_1 + p_2), 4\pi/t_0(1 - p_1 + p_2), \text{ etc.} \) If we regard the
distance between the two pulses as a new stochastic process with
realizations \( p' = p_1 - p_2 \), the new mean will still be \( \mu' = 0 \) and the
standard deviation \( \sigma' = \sigma \sqrt{2} \).

The positions of the maxima in the amplitude spectral density function
are now defined as
Figure 6.1: a) Amplitude spectra of two pulses with $\Delta t = 1.5$ $\mu$s ($\cdots$) and $\Delta t = 1.7$ $\mu$s ($\cdots$).

b) Real part of Fourier spectrum ($\cdots$) and amplitude spectrum ($\cdots$) of three pulses with $\Delta t_1 = 1.5$ $\mu$s and $\Delta t_2 = 1.7$ $\mu$s.

$$\omega = 2\pi(n-1)/\Delta t_0 \{ 1 - p'(t) \} \quad \text{for } n = 1, 2, \ldots \quad (6.2.5)$$

Adding the Fourier spectra of two realizations of equation (6.2.4), where we choose $p_1' + p_2' = 0$ (the mean is zero), we can calculate that the maxima will cancel for $n = 1 + 1/\Delta p'$. Here $n$ is as defined in (6.2.5) for $p'(t) = 0$. Figure 6.1a shows the amplitude spectra of two realizations of equation (6.2.4). The solid line is for a distance of $\Delta t = 1.5$ $\mu$s between the pulses, while the dotted line represents a distance of $\Delta t = 1.7$ $\mu$s. The maxima in figure 6.1a occur for $f = (n-1)/1.6 \times 10^{-6}(1-1/16)$ and $f = (n-1)/1.6 \times 10^{-6}(1+1/16)$ respectively. These maxima cancel for $n = 9$, where $n$ is related to $\Delta t = 1.6$ $\mu$s. Placing three pulses in line with $\Delta t = 1.5$ and $1.7$ $\mu$s, the real part of the Fourier transform changes into the solid line of figure 6.1b. The maxima in the real part of the Fourier transform correspond to the maxima in figure 5.2b, and thus to $\Delta t = 1.6$ $\mu$s. The dotted line in figure 6.1b depicts the amplitude spectrum of the three pulses. We see that the modulation depth, as defined in section 5.2 becomes smaller than one for

$$n = 1/2\Delta p' + 1/2. \quad (6.2.6)$$
Figure 6.2: a) Amplitude spectrum of a series of 10 pulses $\Delta t = 1.6 \mu s$ and $\sigma = 0.1 \mu s$.
b) Band limited version of a) (…) and the amplitude spectrum of its envelope (…).
c) Average of the amplitude spectra of 32 realizations of a).

In this example the modulation depth becomes one at $n = 4.5$. Now we will increase the amount of pulses. As an example we will take a series of 10 deltapulses. The characteristic distance is again $\Delta t = 1.6 \mu s$, while the standard deviation of the Gaussian distribution of the error in $\Delta t$ is $.1 \mu s$. In terms of the Gaussian distribution defined in equation (6.1.1) we find $\mu=0$ and $\sigma = 1/16$. The expected value of $n$ in equation (6.2.6) now becomes

$$n = \frac{1}{2\sigma} + \frac{1}{2}. \quad (6.2.7)$$

Figure 6.2a shows the amplitude spectrum of this series of 10 deltapulses. The peaks of the true maxima and those of the submaxima have been traced by dotted lines. These lines intersect at $n = 4.5$, where the frequency is $f = 3.5/1.6 \times 10^{-6} = 2.19$ MHz. Below this frequency the modulation depth is larger than one, and above this frequency it is smaller than one. When we measure this series of pulses by means of a band limited measurement system, e.g. 2-4 MHz, we find we are measuring in the incoherent part of the amplitude spectrum. This is shown as the dotted line in figure 6.2b. If we are given only this part of the amplitude spectrum, we will not be able to estimate the characteristic
distance between the pulses. By envelope detection we are able to transform the given information to a lower frequency region. If our original band width was sufficiently large we now will be able to estimate the characteristic distance, as shown in the solid line of figure 6.2b.

Ensemble averaging of many realizations of the amplitude spectra of such experiments will only show maxima related to the characteristic distances for frequencies where the expected modulation depth is larger than one. Above the frequency where the modulation depth is one averaging will cancel out the maxima and minima in the spectrum. This effect is shown in figure 6.2c, where the amplitude spectra of 32 realizations of the experiment in 6.2a have been averaged. Note that a completely random process is defined by $\sigma = 0.5$. The modulation depth will now become smaller than one at $n = 1.5$, meaning that $f = 0$ is the only coherent maximum in the spectrum.

6.3 TWO DIMENSIONAL STATISTICAL MODELS

In section 5.4 a model was discussed with a matrix of 16 x 16 scatterers, resulting in figures 5.9 to 5.12. That same experiment will be repeated here, introducing a small aberration in the positions of the scatterers. Again the positioning error is chosen to have a Gaussian distribution with $\mu = 0$ and $\sigma = 1/16$, but now the error is introduced both laterally and axially. Figure 6.3 shows both the

![Figure 6.3: a) Modeled response of an array of 16 * 16 point scatterers. b) Focused result of a).](image-url)
Figure 6.4: a) Two dimensional Fourier transform of the r.f. data of the focused 16 * 16 array.  
b) Envelope detected auto correlation function of the r.f. data.

Figure 6.5: a) Two dimensional Fourier transform of the envelope detected image of the 16 * 16 array.  
b) Auto correlation function of the envelope detected image.

Figure 6.6: Lateral cross section at $z=0$ of the normalised auto correlation functions of figures 6.4b (___) and 6.5b (___).
modeled and the focused data after envelope detection. The two
dimensional amplitude spectrum and the two dimensional auto correlation
function of the r.f. and the envelope detected data are given in
figures 6.4 and 6.5 respectively. A lateral cross section of the auto
correlation functions is shown for z=0 in figure 6.6. Substituting
\( \sigma = 1/16 \) in equation (6.2.7) we find that the modulation depth becomes
smaller than one for \( n = 4.5 \), corresponding to a spatial wave number
\( \bar{k}_x = \bar{k}_z = 3.5/1.2*10^{-3} = 2.9*10^3 \). Indeed a scrutinious look at
figure 6.4a reveals some weak coherent maxima in the lower part of the
spectrum. After envelope detection the two dimensional amplitude
spectrum clearly reveals the periodicity in the image (figure 6.5a).
The Fourier counterpart of the two dimensional energy spectrum, the two
dimensional auto correlation function, shows the same character. The
characteristic distances are best determined after envelope detection.
Figure 6.6 shows that the auto correlation function separates the
coherent information from the random information. The random infor-
mation (or noise) only correlates for \( x = z = 0 \), while the coherent
information may be characterized by a correlation distance.
In section 5.6 angle dependent scattering was discussed, using a single
scatterer model. The dimensions of the scatterer were chosen to be in
the order of one wavelength so the backscattering strength would show
some angle dependence. Here the same model of .4 mm scatterers will be
used, only these scatterers are positioned on a random matrix. A 10 x
10 matrix is used with a characteristic distance of .5 mm in both
lateral and axial directions. The .4 mm scatterers are all set at an
angle of 45° to the x-axis. Both the positioning and the angle show a
degree of random error. The error has a Gaussian distribution with a
standard deviation of 10% of distance and angle.
Due to the resolution, the scatterer size and the random positioning
the image of this matrix of scatterers shows an incoherent character
(figure 6.7a). For the same reasons neither does the two dimensional
auto correlation function reveal the characteristic periodicity of
.5 mm. In this particular example only the two dimensional amplitude
spectrum in figure 6.7c gives any information on the nature of the
scattering. In figure 6.7c the two dimensional amplitude spectrum has
been corrected for the response of a point scatterer. After averaging
over all frequencies the amplitude is given as a function of angle by
the solid line in figure 6.7d. The dotted line in figure 6.7d has been
Figure 6.7: a) Focused image of a 10 x 10 array of .4 mm scatterers.
   b) Auto correlation function of the envelope detected image.
   c) Two dimensional Fourier transform of the r.f. data, "whitened" by means of a point scatterer response.
   d) Frequency averaged response of c) (...) and the single scatterer response of figure 5.15b (...).

copied from figure 5.15b. From the resemblance between the two curves in figure 6.7d we may draw some conclusions as to both the scatterer size and orientation. The width of the maximum in figure 6.7d is indicative of the scatterer size, though we must take into account that the degree of randomness in the orientation is superimposed on this measure. If sufficient averaging takes place (not the case here) the position of the maximum in figure 6.7d gives the scatterer orientation.
CHAPTER VII
MEASUREMENT OF ATTENUATION

When an acoustic wave travels through a medium its amplitude will usually change. On the one hand this amplitude change may be due to propagation effects, described in earlier chapters as focussing and defocussing of the acoustic wave. These propagation effects do not influence the total energy contained in the acoustic wave, where the total energy is defined as the integral over the entire space of the product of pressure and particle velocity. In a homogeneous, loss-free medium the energy contained in the incident wave is preserved and the attenuation in the medium is zero. On the other hand the amplitude change may be due to energy loss. Here we will define three mechanisms which cause energy loss or attenuation of the incident wave. Firstly there are viscous losses, where mechanical vibration energy is transformed into heat. Secondly there are thermal relaxation processes, where also energy is lost in the form of heat. Both these loss mechanisms may be incorporated in the wave equation by modifying the equation of motion and Hookes law respectively [1]. The contribution to attenuation, which may be described in the wave equation is called absorption. Apart from absorption the incident wave also looses energy by scattering. Part of the energy of the incident wave is dispersed in all directions. Clearly this third loss mechanism is dependent on the size, shape and impedance mismatch of the scattering particles, and cannot be incorporated in the macroscopic wave equation as described in this work.

In many cases the contribution of scattering to attenuation is small
compared to absorption [2, 3], certainly in cases where the Born approximation is valid. As it is also often impossible to separate the effects of absorption and scattering in limited angle acoustic measurements, it is common to consider only attenuation, the sum of all three loss mechanisms.

Usually the higher frequencies attenuate more than the lower frequencies. Consequently the centre frequency of signals returning from a medium will decrease as the arrival time increases. As there are many more mechanisms influencing the amplitude of the returned signal than those influencing the frequency content, the frequency shift of the returned signal is often a more accurate measure for the attenuation.

In this chapter therefore, we will first consider some frequency estimators, next the theory of absorption will be treated, and finally some measurement techniques will be discussed.

7.1 FREQUENCY ESTIMATORS

In the next two chapters one of the steps in the estimation of medium parameters from echo acoustic data will be the estimation of the frequency content of the returned signal. We can approach the estimation of this frequency content in two ways.

Firstly we can make use of the entire frequency spectrum by means of Fourier transformation and analyse this spectrum as a function of the position in the echo acoustic image. For every image point we now may calculate the amplitude spectrum of a portion of the signal around that image point. Generally only a one dimensional Fourier transformation in the axial direction is considered, so from a two dimensional image we calculate a three dimensional dataset. This approach is used a.o. in the "spectral difference" technique, for the measurement of the frequency dependence of attenuation, as described in section 7.6.

Secondly, if the shape of the amplitude spectrum of the emitted pulse is known, an estimate of the mean frequency as a function of position may be used. As the frequency spectrum of the reflected signal is represented by one single number here, we must first make some assumptions for the spectral colouring of scattering and propagation. The counterpart of the "spectral difference" technique using the mean frequency is called the "spectral shift" technique and will be discussed in section 7.7.
Accurate and unbiased frequency estimation is also necessary in Doppler techniques for calculating the flow velocity. Doppler techniques will not be treated here.

In chapter VIII the frequency estimators will be used to enable us to say something about the "character" of the reflecting objects. In the following sections some frequency estimators will be discussed, emphasizing their individual advantages and disadvantages. Here a distinction is made between mean frequency estimators and instantaneous frequency estimators. With mean frequency estimators a time window is involved and the resulting estimate is an average value over that time window. The advantage of instantaneous frequency estimators, as discussed in section 7.4, is that local variations in the carrier frequency may be studied.

7.2 SOME MEAN FREQUENCY ESTIMATORS

In the literature we find many ways of estimating the mean frequency within a time window. Generally speaking, these estimators fall into three classes. Firstly there are the simple time domain techniques, where the frequency is estimated by counting the number of periods within a window. Two of these simple time domain techniques are the "zero crossing" counter, where the number of zero crossings of the signal is counted, and the "level crossing" counter, where a threshold is added to reduce the effect of noise [4, 5]. These techniques are often used because they are simple to implement and very robust. Unfortunately they are also inaccurate.

The second class of estimators is based on Fourier transformation. Apart from the two spectral moment estimators discussed in the following, we may a.o. also use prior knowledge of the spectral shape and feed the amplitude spectrum through a set of correlators, each correlator containing a shifted version of the known spectral shape, so the maximal correlation gives the best estimate [6]. The disadvantage of these estimators based on Fourier transformation is, that they are very time consuming. For every estimate one must perform a Fourier transformation.

The third class of mean frequency estimators, like the first, also operates in the time domain. The difference being, that under ideal conditions these time domain techniques may be mathematically derived
from the Fourier domain techniques. Two of these estimators will be discussed in the following. The first one uses the auto correlation of the time series, and is closely related to the covariance estimator, used in astronomy and radar techniques [7, 8]. The second one is based on envelopes, and cannot be found in literature.

First spectral moment by Fourier transformation

The first mean frequency estimator is derived by estimating the true mean of the amplitude spectrum. For every estimate the amplitude spectrum is calculated by Fourier transformation of a portion of the original time signal

$$\langle f_m \rangle_1 = \frac{\int_{f_{\min}}^{f_{\max}} f |S(f)| df}{\int_{f_{\min}}^{f_{\max}} |S(f)| df}, \quad (7.2.1)$$

where $|S(f)|$ is the amplitude spectrum of a portion of the signal $s(t)$. In the presence of noise the biasing effect, which will be discussed later, may be reduced by setting $f_{\min}$ and $f_{\max}$ where the signal-to-noise ratio is sufficient. Thus the contribution to $\langle f_m \rangle_1$ of noise at higher and lower frequencies is eliminated.

Squared first spectral moment by Fourier transformation

The second mean frequency estimator is derived by calculating the square of the amplitude spectrum

$$\langle f_m^2 \rangle_2 = \frac{\int_{f_{\min}}^{f_{\max}} f^2 |S(f)|^2 df}{\int_{f_{\min}}^{f_{\max}} |S(f)|^2 df}. \quad (7.2.2)$$

If we want to compare the estimators (7.2.1) and (7.2.2) we must take the square root of equation (7.2.2).

The accuracy of these two mean frequency estimators depends on the accuracy with which we are able to calculate the amplitude spectrum $|S(f)|$. The length of the time window will prove to be important.
Squared first spectral moment by auto correlation

Here a mean frequency estimator will be derived, which uses the auto correlation of the original time series. The advantage of an auto correlation technique over the Fourier spectral moments is, that there is no need for the time consuming Fourier transformations. As we shall see we only need two convolutions per frequency estimate. The auto correlation function $R(\tau)$ is defined as the inverse Fourier transformation of the energy spectrum $|S(f)|^2$:

$$R(\tau) = \int_{-\infty}^{\infty} |S(f)|^2 e^{j2\pi f \tau} df. \quad (7.2.3)$$

As the energy spectrum is symmetric in $f$ ($R(\tau)$ is a real function) we may rewrite (7.2.3) as

$$R(\tau) = 2 \int_{0}^{\infty} |S(f)|^2 \cos(2\pi f \tau) df. \quad (7.2.4)$$

For small $f \tau$ the cosine may be written as a series expansion

$$R(\tau) = 2 \int_{0}^{\infty} |S(f)|^2 df - 4\pi^2 \tau^2 \int_{0}^{\infty} f^2 |S(f)|^2 df + O(\tau^4). \quad (7.2.5)$$

With

$$R(0) = 2 \int_{0}^{\infty} |S(f)|^2 df \quad (7.2.6)$$

we find

$$\int_{0}^{\infty} f^2 |S(f)|^2 df = \frac{1}{2\pi^2} \frac{R(0) - R(\tau)}{R(0)} + O(\tau^2 \tau^2). \quad (7.2.7)$$
So we see that under ideal circumstances (no noise, sufficient time window length, small \( f_t \)) equation (7.2.2) may be approximated by equation (7.2.7).

Mean frequency by envelopes

In this section we will derive a mean frequency estimator based on the envelope of the time signal and the envelope of its derivative. First we define a complex time signal \( s'(t) \) [9]

\[
s'(t) = \frac{1}{2}[s(t) + jH\{s(t)\}]. \tag{7.2.8}
\]

The Fourier transform of \( s'(t) \), \( S'(f) = 0 \) for \( -\infty < f < 0 \) and \( S'(f) = S(f) \) for \( 0 < f < \infty \). The envelope of \( s(t) \) is defined as

\[
\text{Env}\{s(t)\} = \sqrt{s^2(t) + H^2\{s(t)\}} = 2s'(t)s'^*(t), \tag{7.2.9}
\]

where \( s'^*(t) \) is the complex conjugate of \( s'(t) \). Using equation (7.2.8) and the fact that Hilbert transformation and differentiation are commutative

\[
\frac{dH\{s(t)\}}{dt} = H\left\{\frac{ds(t)}{dt}\right\} \tag{7.2.10}
\]

we find

\[
\text{Env}\left\{\frac{ds(t)}{dt}\right\} = \sqrt{\frac{ds(t)}{dt}^2 + H^2\left\{\frac{ds(t)}{dt}\right\}} = 2\frac{ds'(t)}{dt}\frac{ds'^*(t)}{dt} \tag{7.2.11}
\]

Finally Parseval's theorem for complex time series [10] states

\[
\int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 df \tag{7.2.12}
\]

Now the numerator of equation (7.2.2) may be rewritten

\[
\int_{0}^{F} f^2 |S(f)|^2 df = \int_{0}^{\infty} |fS(f)|^2 df = \int_{-\infty}^{\infty} |fS'(f)|^2 df.
\]
Substituting equations (7.2.11) and (7.2.12) we find

\[ F \int_{*}^{t} |S(f)|^2 df = \frac{1}{4\pi} \int_{*}^{t} \frac{ds(t)}{dt} \left| \frac{ds(t)}{dt} \right|^2 dt = \frac{1}{16\pi^2} \int_{*}^{t} Env^2 \left\{ \frac{ds(t)}{dt} \right\} dt. \]  
(7.2.13)

Substituting eqs. (7.2.9) and (7.2.12) in the denominator of equation (7.2.2)

\[ F \int_{*}^{t} |S(f)|^2 df = \int_{-\infty}^{\infty} |S'(f)|^2 df = \int_{*}^{t} \left| s'(t) \right|^2 dt = \frac{1}{4} \int_{*}^{t} Env^2 \{s(t)\} dt. \]  
(7.2.14)

Combining these last two equations the squared first spectral moment of equation (7.2.2) may be replaced by

\[ \frac{\int_{*}^{t} F^2 |S(f)|^2 df}{\int_{*}^{t} |S(f)|^2 df} = \frac{\int_{*}^{t} Env^2 \left\{ \frac{ds(t)}{dt} \right\} dt}{\int_{*}^{t} Env^2 \{s(t)\} dt}. \]  
(7.2.15)

So once we have calculated the envelope of a time series and the envelope of its time derivative, the central frequency simply follows from two integrations over a time window.

### 7.3 Comparison of the Mean Frequency Estimators

In this section we will compare the four mean frequency estimators derived in the previous section. The comparison will be made using a chirp signal, the frequency of which increases linearly with time (figure 7.1). The sample frequency \( f_s = 10 \) MHz, and the frequency of the chirp varies between 0 MHz at time \( t=0 \) and the Nyquist frequency at \( t=51.2 \) \( \mu s \), so if we fed this signal through an ideal frequency estimator it would show a linear, monotonically increasing, response from zero to 5 MHz.
Figure 7.1: Chirp with a linearly increasing frequency from zero to 5 MHz.

Figure 7.2: a) First spectral moment by Fourier transformation.
   b) Squared spectral moment by Fourier transformation.
   c) Squared spectral moment by auto correlation.
   d) Squared spectral moment by envelopes.

___ = ideal ... = 6.3 μs window
--- = 1.5 μs window
The effect of the time window length

As mentioned earlier all mean frequency estimators use a portion of the original time series to produce one single estimate. In the Fourier techniques the signal within this time window is first Fourier transformed. To reduce edge effects inherent to Fourier transformation a cosine shaped taper was used. The length of the running time window was varied between 1.5 µs and 6.3 µs. Figure 7.2 shows the results of the four estimators on the linear chirp for 1.5 µs as the dashed lines and 6.3 µs as the dotted lines. The solid lines in the curves delineate the ideal case if the estimators were perfect.

From figure 7.2 we may draw the following conclusions:

- The Fourier techniques become unstable for short window lengths because the amplitude spectrum can, in principle, only be calculated correctly over infinite time windows. From figures 7.2a and 7.2b we see that time window lengths of 6.3 µs or longer are in order.

- The auto correlation technique shows a structural bias related to the truncation error 0{f^2_τ} in equation (7.2.7). As figure 7.2c shows the square root of <f^2_m> this error becomes 0{f^2_τ}, so the error is proportional to τ and to f^2. For the calculation of figure 7.2c τ was set at one sample interval, and further error reduction is only possible if the original time series is resampled on a finer grid.

- The time domain techniques remain stable, even at window lengths smaller than 1.5 µs, where the envelope technique shows least error. From other (unpublished) studies on band limited pulsed signals it was found that the envelope technique performs at least as well or better than the squared first spectral moment of equation (7.2.2).

The effect of white noise

The effect of white noise was studied by repeating the previous experiment with a time window length of 3.1 µs, but now adding white noise to the linear chirp signal. The amplitude distribution of the white noise had a Gaussian probability function with zero mean. The standard deviation of the probability function defined the average noise amplitude. The signal-to-noise ratio was defined as the maximal value of the amplitude spectrum of the signal (|S(f)|) divided by the average value of the amplitude spectrum of the noise.
Figure 7.3: As figure 7.2 with a 3.1 μs window

- = no noise ... = 25% r.m.s. noise (-12 dB)
--- = 50% r.m.s. noise (-6 dB)

The results produced by the four estimators are shown in figure 7.3 for a signal-to-noise ratio of ∞ as the solid lines, for a signal-to-noise ratio of four (12 dB) as the dotted lines and for a signal-to-noise ratio of two (6 dB) as the dashed lines. From figure 7.3 we may draw the following conclusions:

- All mean frequency estimators show a bias if the first spectral moment of the signal is not equal to the first spectral moment of the noise. As the first spectral moment of the white noise is at 2.5 MHz, the estimators are only unbiased at this frequency.

- The estimators based on the squared first spectral moments all show a comparable bias, which is smaller than that of the estimator based on the first spectral moment.

Looking back at equations (7.2.1), (7.2.2), (7.2.7) and (7.2.15) we may say

- The computational efficiency increases as we move from estimators one to four. Estimators one and two both need a Fourier transformation
and two summations per estimate, estimator three needs two 
correlations and estimator four needs two summations per estimated 
value. For the envelope technique we must first calculate the 
envelope of the time series and the envelope of its derivative to 
time, which makes this technique less efficient for short time 
series.

It seems as if the envelope technique has absolutely no disadvantages, 
but unfortunately there is another consideration we must make.

- The flexibility of the time domain algorithms is less than that of 
the frequency domain algorithms.

This flexibility may be used e.g. to eliminate the bias due to noise. 
If the spectral content of the noise is known its effect may be 
eliminated in the Fourier domain techniques by subtracting this known 
spectrum from the calculated amplitude spectrum prior to using 
equation (7.2.1) and (7.2.2). This technique is described in [7].

Finally we must point out that all mean frequency estimators described 
here become useless if the signal-to-noise, as defined above, is in the 
order of one or less. All estimates of single realizations will give 
the mean frequency of the noise only, so in poor signal-to-noise 
conditions it is preferable to remove some of the random noise by 
averaging several realizations prior to frequency estimation.

7.4 INSTANTANEOUS FREQUENCY

In section 5.1 the Fourier transform of a time series was split up into 
an amplitude spectral density \(|S(\omega)|\) and a phase delay angle \(\phi(\omega)\). Here 
we will do the same for the original time series:

\[
s(t) = |s(t)|e^{j\phi(t)} = a(t)e^{j\phi(t)}, \tag{7.4.1}
\]

where \(a(t) = |s(t)|\) is the envelope, or instantaneous amplitude of the 
time series, and \(\phi(t)\) is the instantaneous phase. If the amplitude 
spectrum of a real time series is symmetric around a given \(\omega_c\), then 
this spectrum may be written in terms of two shifted components

\[
S(\omega) = \frac{1}{2}A'(\omega + \omega_c) + \frac{1}{2}A'(\omega - \omega_c) \\
= \frac{1}{2}A'(\omega) * \{\delta(\omega + \omega_c) + \delta(\omega - \omega_c)\}, \tag{7.4.2}
\]
where \* denotes a convolution, and where \( \delta \) is a dirac pulse. After inverse Fourier transformation the convolution becomes a multiplication, and the time series may be represented by

\[
s(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} S(\omega)e^{j\omega t}d\omega
\]

\[
= \frac{1}{2}a'(t)(e^{-j\omega_c t} + e^{j\omega_c t}) = a'(t)\cos(\omega_c t), \quad (7.4.3)
\]

where \( a'(t) \) is the Fourier transform of \( A'(\omega) \) and is a real function. The rectified \( a'(t) \) represents the envelope of the time signal, so in equation (7.4.1) \( a(t) = |a'(t)| \).

In the following we will assume that a time series may be described by (7.4.3). In appendix C, where non-symmetric spectra are also treated, we will look back at the more general case of (7.4.1). In equation (7.4.3) the central frequency \( f_c = \omega_c/2\pi \) may vary with time. For various reasons we might want to follow this variation. A necessary step in the calculation of \( f_c \) is quadrature detection. If quadrature detection is performed by multiplying with a sine and a cosine of carrier frequency \( 2\pi\omega \)

\[
f_c(t) = \frac{1}{2\pi} \frac{\partial}{\partial t} \arctan\left( \frac{Q_2}{Q_1} \right) + \omega/2\pi, \quad (7.4.4a)
\]

or when Hilbert transformation is applied

\[
f_c(t) = \frac{1}{2\pi} \frac{\partial}{\partial t} \arctan -\left( \frac{H[s(t)]}{s(t)} \right). \quad (7.4.4b)
\]

Here \( f_c(t) \) is defined as the instantaneous frequency. Note that before equation (7.4.4a) is applied a low pass filtering of \( Q_1 \) and \( Q_2 \) must be performed. So in case of broad band sampled signals aliasing will occur if the bandwidth \( b \) satisfies the inequality

\[
b \geq 2(f_{Nyq} - f_c), \quad (7.4.5)
\]

where \( f_{Nyq} \) is the Nyquist frequency and \( f_c \) is the central frequency of the signal. If aliasing does occur it is impossible to separate the high frequency component from \( Q_1 \) and \( Q_2 \). For this reason it is
preferable to make use of Hilbert transformation to calculate the quadrature components of a broadband sampled time series.

The use of the instantaneous frequency as a local frequency estimator

In sections 7.2 and 7.3 some mean frequency estimators were evaluated. The shortest time windows which could be used there were in the order of a few microseconds, meaning that there was always some averaging taking place. Now, with the instantaneous frequency we have, in principle, a tool which will give us a samplepoint to samplepoint estimate of the centre frequency. For the linear chirp, discussed in section 7.3, and for the chirp with added noise of a signal-to-noise ratio of 4 the instantaneous frequency is shown in figure 7.4. An important feature of the instantaneous frequency is seen here: the noise effect is added to the unbiased instantaneous frequency. This means that low pass filtering or averaging will remove the effect of noise, making this estimator even more flexible than the Fourier techniques. Thus, in principle, we have a local estimator with more flexibility than any mean frequency estimator - assuming the required output of the instantaneous frequency estimator has a narrower frequency band than the original time signal -. Unfortunately nothing in nature comes free.

One of the basic limitations of the instantaneous frequency as a frequency estimator is, that it becomes principally unstable when the envelope of a time series is zero. This is easiest seen when the frequency spectrum of a pulse is a zero phase top hat function. The envelope of the corresponding time signal then becomes a rectified sinc function. A sinc function changes polarity at every zero crossing, so

\[ \text{Figure 7.4: Instantaneous frequency of chirp.} \]

\[ \text{--- = no noise ... = 25\% r.m.s. noise} \]
Figure 7.5: Envelope and r.f. of a band limited zero phase pulse with a symmetric amplitude spectrum (a) and its instantaneous frequency (b).

s(t) in equation (7.4.3) must also have a ± 180° phase shift at these points. The instantaneous frequency, which is the time derivative of these stepwise phase changes, is undefined ±ω at these points (see figure 7.5).

For this reason an algorithm was derived, which uses the envelope information to stabilize the instantaneous frequency estimation. The stabilization is done in a way comparable to the Wiener inverse filter technique, the difference being that stabilization is done in the time domain and not in the frequency domain. In the following examples this stabilized algorithm was used as described in appendix D, equation (D.7).

7.5 THE THEORY OF ABSORPTION

As already mentioned in the introduction to this chapter attenuation may be split up into two parts, scattering and absorption. In the literature we find many references to the theory behind absorption. Two derivations will be given here. One derivation starts from the mathematical introduction of the Q-factor, while the other follows a more acoustical approach. With some simplifications both derivations lead to the same mathematical representation of absorption.

The Q-factor (after Aki and Richards [11])

If a volume of material is cycled in stress at a frequency \( f = \omega/2\pi \), then a dimensionless measure for the internal friction is given by
\[ Q^{-1}(\omega) = -\frac{\Delta E}{2\pi E^4} \]  

(7.5.1)

where \( E \) is the peak strain energy stored in the volume and where \(-\Delta E\) is the energy loss at each cycle because of imperfections in the elasticity of the material. For a medium with a linear stress-strain relationship the wave amplitude \( A \) is proportional to \( \sqrt{E} \), so for a small energy loss per cycle we may write \( \Delta A/A = \Delta E/2E \). Substitution in equation (7.5.1) yields

\[ Q^{-1}(\omega) = -\frac{\Delta A}{\pi A}. \]  

(7.5.2)

Now if we follow a particular wave peak along a distance \( dx \), the gradual decay of its amplitude can be described as \( \Delta A = (dA/dx)\lambda \). Here we have assumed that the direction of propagation is along the \( x \)-axis (plane wave). So from equation (7.5.2) we find the differential equation

\[ \frac{dA}{dx} + \frac{\pi}{\lambda Q} A = 0 \]  

(7.5.3)

with its exponentially decaying solution

\[ A(x) = A(o) \exp(-\frac{\pi x}{\lambda Q}). \]  

(7.5.4)

The acoustical approach

Acoustic absorption may be due to two kinds of physical phenomena, viscous effects and thermal relaxation effects. A.o. Kinsler and Frey [12] assumed in first approximation that the dissipative forces were proportional to the particle velocity, which led to a modified equation of motion

\[ -\nabla p = \rho_o \frac{\partial \vec{v}}{\partial t} + \gamma \vec{v}. \]  

(7.5.5)

From this equation of motion, together with Hooke's law, a modified wave equation was derived [12]. Berkhout [1] pointed out, that equation (7.5.5) only describes instantaneously reacting viscous effects. He stated the more general case, where friction forces are described by
\[-\nabla \mathbf{p} = \rho \frac{\partial \mathbf{v}}{\partial t} + r_v(t) \times \mathbf{v}. \quad (7.5.6)\]

Thermal relaxation effects are described by a modified Hooke's law. For a non-adiabatic process Hooke's law may be rewritten as

\[-\nabla \cdot \mathbf{v} = \frac{1}{K} \frac{\partial p}{\partial t} + r_p(t) \times p. \quad (7.5.7)\]

Fourier transformation of these two equations yields

\[-\nabla \tilde{\mathbf{P}} = (R_v(\omega) + j\omega p) \tilde{\mathbf{V}} \quad (7.5.8)\]

\[-\nabla \cdot \tilde{\mathbf{V}} = (R_p(\omega) + j\omega K^{-1}) \tilde{p}, \quad (7.5.9)\]

which leads to a modified wave equation

\[\hat{\beta} \nabla \cdot \left( \frac{1}{\hat{\beta}} \nabla \tilde{p} \right) + k^2 \tilde{p} = 0, \quad (7.5.10)\]

where

\[\hat{\beta} = \rho - jR_v(\omega)/\omega\]

and where for small losses \( k \) may be written as \[1\]

\[\hat{k} = k - j\frac{R_v}{\rho c} + \rho c R_p = k - jk_1. \quad (7.5.11)\]

In equation (7.5.11) \( k \) represents the wavenumber in the loss-free situation. If for the frequencies of interest \( R_v \) and \( R_p \) are independent of frequency, then \( r_v \) and \( r_p \) in equations (7.5.6) and (7.5.7) become constant coefficients and

\[k_1 = \frac{r_v}{\rho c} + \frac{\rho c r_p}{p}. \quad (7.5.12)\]

For a plane wave in the \( x \)-direction the solution to the wave equation is given by

\[P(x,t) = P(0,t) e^{-jkx} = P(0,t) e^{-k_1x - jkx}. \quad (7.5.13)\]
Using equation (7.5.4) we find the relation between the Q-factor and \( k_1 \):

\[
k_1 = \pi / \lambda Q. \tag{7.5.14}
\]

Usually in the medical field the absorption is given in dB/MHz·cm, yielding for the amplitude decay

\[
A(x) = A(o) 10^{-0.05\alpha fx}, \tag{7.5.15}
\]

where \( f \) is in MHz and \( x \) is given in cm. For small losses the relation between \( \alpha \) and the Q-factor is given by

\[
\alpha = 20 \frac{\pi}{cQ} \log e = 27.3/cQ. \tag{7.5.16}
\]

The absorption may also be formulated per unit wavelength, which yields

\[
a_{dB} = 27.3/Q. \tag{7.5.17}
\]

### 7.6 THE SPECTRAL DIFFERENCE METHOD

In this chapter two ways of estimating the attenuation coefficient \( \alpha \) (equation (7.5.15)) will be discussed. The first method is called the spectral difference method. If a medium is insonified with a plane wave, and we are able to detect the backscattered signals coming from two interfaces, then the transfer function between the two interfaces is given by

\[
H(\Delta x,f) = P(x_2,f)/P(x_1,f), \tag{7.6.1}
\]

where \( x_1 \) is the distance between the transducer and the nearest interface, and \( x_2 \) is the distance to the farthest interface. Substituting equation (7.5.13) we find

\[
H(\Delta x,f) = e^{-k_1 \Delta x - jk \Delta x}, \tag{7.6.2}
\]

where \( \Delta x = 2(x_2-x_1) \) to compensate for the two-way propagation of the
plane wave. In the spectral difference method we take the real part of
the log of the transfer function, or the difference of the log
amplitude spectra of $P(x_2,f)$ and $P(x_1,f)$:

$$\text{Re}\{20 \log H(\Delta x, f)\} = 20 \log |P(x_2,f)| - 20 \log |P(x_1,f)|$$

$$= -20 k \Delta x \log e = -\alpha f \Delta x. \quad (7.6.3)$$

From the slope of the log spectral difference versus frequency we may
now calculate the desired attenuation coefficient $\alpha$ in dB/MHz.cm. This
model assumes a linear dependence of $\alpha$ on frequency. This seems a good
assumption for band limited experiments in biological soft tissues [13]
though other models have been suggested [3, 14].

For the derivation of equation (7.6.3) a few assumptions have been
made:

- The attenuation is small, an assumption made in the previous section.
- The incident and the reflected wave are both plane waves. In real
  life, where plane waves are rare, this sets a severe constraint on
  the accuracy of the estimation of absorption. The errors due to
  diverging and converging acoustic wave fields are generally referred
to as the "diffraction effect" and will be discussed in section 7.8.
- The "character" of the reflection at the two interfaces is the same.
  ("Character" will be discussed in some detail in the next chapter).
  This means that the size and shape of the reflecting objects must be
  identical. It seems a trivial statement in this example of two plane
  interfaces, but in real media it might become a problem.
- The spatial distribution of scattering objects within the time
  windows is the same. This last assumption will be treated here in
  somewhat more detail.

In order to calculate the spectra of $P(x_1,f)$ and $P(x_2,f)$ we must
define two time windows in the received signal. If the response from
more than one scatterer is present within the time window, then the
true amplitude spectrum will show interference effects (see chapters V
and VI). The error due to these interference effects in the estimation
of $\alpha$ can be substantial [15]. Obviously, if we could measure over an
infinite bandwidth we would be able to measure $\alpha$ correctly, but given a
band limited set of data, Kuc [16] suggested four ways to reduce the
error in the estimation of $\alpha$. 
1) Increase the distance between the two time windows. This increases $\Delta x$ in equation (7.6.3), while the absolute error in the estimation of the log spectral difference does not change. Consequently the relative error in $\alpha$ decreases.

2) Acquire additional independent signals and average the individual estimates for the attenuation. As the mean of the interference effects is zero, averaging will reduce the error in the estimation of $\alpha$.

As a rule of thumb we may say that signals from different parts of a medium are statistically independent if their responses do not overlap. Some authors take the full width half maximum of the system point spread function as a minimal distance for statistical independence (see section 8.5). If the region of the medium in which we want to estimate the attenuation coefficient has only a limited size, there is only a limited number of statistically independent estimations of $\alpha$ possible. Kuc and Schwartz [17] combined 1) and 2) and calculated the optimal distance between the time windows for a given thickness of the r.o.i. They argued that when the distance between the time windows was increased, the possible number of statistically independent measurements of $\alpha$ would decrease. Thus the variance in the estimation of $\alpha$ will increase when the distance between the time windows is too large. A minimal variance of $\alpha$ was found to occur at a distance between the centres of the time windows of $2/3$ of the thickness of the r.o.i.

3) Increase the time window length. A longer signal will contain the responses of an increased number of uncorrelated scatterers, so the phase cancellation due to the interference effects will be reduced. Unfortunately increasing the time window length has a negative effect on both 1) and 2), meaning that a window length as short as possible is preferable.

4) Incorporate prior knowledge into the estimation procedure. If e.g. the shape of the amplitude spectrum of the signal is known, this may be fitted to the measured data in a least squares sense to reduce the effect of phase cancellation.

7.7 THE SPECTRAL SHIFT METHOD

The second method for the estimation of attenuation is called the "running spectrum" or "spectral shift" method. In this method the mean
frequency, or instantaneous frequency is plotted as a function of distance from the transducer. If the shape of the amplitude spectrum of the returned pulse is known, as well as the frequency dependence of the attenuation, then the shift in the mean frequency of the returned pulse can be directly related to the attenuation coefficient. The interrogating pulse used in most medical ultrasonic imaging systems can be approximated by using a band limited Gaussian modulated amplitude spectrum. It can be shown [18] that such a pulse, propagating in a medium with an attenuation coefficient that is linearly dependent on frequency, retains its Gaussian shape. The mean frequency of the pulse is down shifted by

$$\Delta f = \beta \sigma^2 \Delta x,$$

(7.7.1)

where $\beta$ is the attenuation coefficient, $\sigma$ is the standard deviation of the Gaussian shape - a measure for the bandwidth of the pulse - , and where $\Delta x$ is the distance travelled through the medium. In equation (7.7.1) $\beta$ is given in Np/MHz.cm. To convert from Nepers to decibels a conversion factor 20 log $e$ should be used. In the spectral shift method the same assumptions are made as in the spectral difference method, only here we also assume that the reflection coefficients are frequency independent or linearly dependent on frequency, so the Gaussian spectral shape is retained. A major advantage of the spectral shift method over the spectral difference method is that the problem of interference of reflections in the time window can be overcome by using an instantaneous frequency estimator.

7.8 DIFFRACTION EFFECTS

One of the assumptions made for the calculation of attenuation is that the medium is one dimensional. This requirement can be relaxed as long as for the attenuation-free case the shape of the amplitude spectrum of the returned pulse does not vary with distance. So if the geometrical spreading of both the incident and reflected wave field is independent of frequency the theory will still hold. Unfortunately this is only true for transducers and scatterers which are either infinitely small (points) or infinitely large (planes) in relation to all applied wavelengths. As most models assume either point scatterers or plane
interfaces the problem narrows down to the transducer characteristics, but as a rule any error due to geometrical spreading of the acoustic wavefield is loosely referred to as "diffraction effect" [19]. In most clinical settings the effect of geometrical spreading on the amplitude spectrum of the signal is in the order of the effect attenuation has on that spectrum. To date most people active in the field have recognized the problem and elaborate callibration schemes are being devised to compensate for the diffraction effect [19, 20]. These callibration schemes must be elaborate, as the diffraction effect does not only depend on the source and receiver, but also on propagation parameters such as the sound velocity. Diffraction effects due to the measurement setup may be easily eliminated if we use synthetically focused data as described in chapter III. In section 3.4 all system limitations were treated in detail. All we have to do is filter out all parts of the $k_x$-$k_z$ domain, where the data may be affected by the system limitations anywhere within the region of interest. If necessary one may also compensate for amplitude errors in the inversion process or apply severe angle limitations to estimate a true plane wave attenuation.

7.9 EXAMPLE

In this chapter two methods were discussed for the estimation of the attenuation coefficient from reflection measurements, the spectral difference and the spectral shift method. In medical ultrasound much work has been done on these methods by a.o. Kuc et al. and Fink et al. respectively. Both researchers based their techniques on Fourier analysis of the received signals.

Here an example will be given of attenuation estimation from a real time measurement, based on the spectral shift method. In this example the instantaneous frequency will be used as a centre frequency estimator. The advantage of the use of the instantaneous frequency over Fourier analysis is not only the computational efficiency, but also an instantaneous frequency image can be produced. As will be shown in this example, such an image makes it possible to exclude regions where the character of the reflectivity (chapter VIII) differs from its surrounding.
The measurement

For the measurement a tissue mimicking phantom was used, made in the physics department of the Royal Marsden Hospital in Sutton. This measurement was performed as part of an E.E.C. effort to compare state of the art ultrasound techniques of measuring attenuation in real time in various research institutes throughout Europe. The measurement was performed by means of a linear array transducer, which recorded the zero offset responses at 512 positions, .2 mm apart. The total data acquisition time was less than 150 ms.

To avoid diffraction effects in the plane of scanning the data entering the transducer at a greater angle than 15° was not used for the reconstruction. This angle of 15° ensured constant aperture angle limitations over the entire region of interest. Figure 7.6a depicts the measurement setup and figure 7.6b shows the focused image. The dynamic range of this envelope detected grey scale image is approximately 40 dB, and the image covers an area of approximately 10x15 cm. The

![Image of measurement setup and focused image]

Figure 7.6: a) Measurement setup for the attenuation measurement. b) Focused image of approximately 10 x 15 cm.
Figure 7.7: Part of the focused image of figure 7.6b, showing an area of 10 x 7.5 cm (a) and the stabilized instantaneous frequency (50% stabilisation factor) (b).

Figure 7.8: Close up of the tissue mimicking phantom (a) and the stabilized instantaneous frequency (b).

region of interest is the tissue mimicking phantom, covering an area of 5.5x4 cm. From the data of figure 7.6b a sample volume was chosen as shown in figure 7.7a. The stabilized instantaneous frequency was calculated as described in appendix D, eq. (D.7), with a stabilization factor of 50% of the maximal signal amplitude. The result is shown in figure 7.7b. In the r.o.i. (figure 7.8) we can see two areas, where the character of the reflectivity differs from the background. In these areas, which may be attributed to the inclusion of gas, the instantaneous frequency shows a marked drop of 100 kHz. Also the specular reflections from the mylar foil (figure 7.7) give this 100 kHz drop in frequency. These effects will be studied in more detail in the next chapter. In figure 7.9a the laterally averaged instantaneous
Figure 7.8: a) Laterally averaged instantaneous frequency as a function of traveltime and the straight line fit.

b) Average amplitude spectrum within the region of interest (figure 7.8) and the Gaussian shaped fit.

Frequency is shown as a function of depth. The averaging was performed over the r.o.i. Note that the 100 kHz dips due to the mylar foil and the smaller dips due to the gas inclusions are clearly visible (see the arrows). Excluding these areas in our estimation, a straight line fit can be made to figure 7.9a and the frequency shift in the r.o.i. may be assessed.

The second variable to be entered into equation (7.7.1) is the standard deviation $\sigma$ of the Gaussian spectral shape. As no callibration measurements were performed at the time to estimate the spectral shape of the pulse, the average Fourier amplitude spectrum of the r.o.i. was taken. This average amplitude spectrum is given as the solid line in figure 7.9b. The truncation of the noise at 1 and 4 MHz is a standard procedure during the focussing of the data. Using equation (6.1.1) a Gaussian distribution was fit to the measured amplitude spectrum. The parameters entered to form the dotted line in figure 7.9b were a mean $\mu = 2.45$ MHz and a standard deviation $\sigma = 0.30$ MHz.

Finally for the calculation of $\Delta x$ in equation (7.7.1) a sound propagation velocity of 1505 m/s was used. This velocity was based on the bulk velocity found for an optimal focusing result (Chapter IX).

Now we can calculate an attenuation coefficient $\alpha = 1.0$ [dB/MHz·cm].
Discussion

To complete the picture some sources of error must be discussed. Assuming that in an operational system the pulse amplitude spectrum is known more accurately than in this example, a better fit to a Gaussian distribution is possible e.g. by deconvolution of the measured data. As the standard deviation is squared in equation (7.7.1) this will reduce a large source of error in this example.

Another large source of error is the accuracy of the straight line fit in figure 7.9a. This error again depends on how closely the amplitude spectrum fits a Gaussian distribution, but also on the size and homogeneity of the r.o.i., on the spectral width and the magnitude of the attenuation coefficient. For this particular example an error in the estimation of the frequency shift may be expected of 5-10%.

Near the transducer, another substantial source of error was found to be due to diffraction effects, viz. beamforming in the direction perpendicular to the plane of scanning. This source of error can only be removed by extensive calibration of the transducer, and application of a frequency and depth dependent correction table. In conclusion we may make the following remarks

- Although care must be taken to remove some substantial sources of error it has been shown that it is possible to calculate an attenuation coefficient from the stabilized instantaneous frequency.
- The instantaneous frequency technique allows the operator to remove suspect data of local inhomogeneities from the estimation procedure.
- Compared to Fourier-based techniques, this technique is computationally efficient.

References chapter VII


In chapter I a "macro subsurface model" was introduced, which was to be used as a reference medium for the inversion process. Two of the most important parameters for this reference medium are the sound propagation velocity and absorption. These two bulk parameters are treated in chapters IX and VII respectively. In this chapter we will take a closer look at the parameters contained in the reflectivity matrix, the local reflectivity parameters.

In the literature we can distinguish two areas of acoustic scattering. The first area is mainly concerned with inversion of propagation effects, while the second area is mainly directed to the scattering properties of inhomogeneities. The approaches followed in the two areas differ on a few essential points. In this thesis we have been concentrating on the first area and now we will try and relate this to the acoustic scattering literature. In the acoustic scattering literature the assumption is always made that measurements take place in the far field. In the far field the behaviour of scattering objects is very much dependent on their size relative to the wavelength of the incident pulse. The relative scatterer size is often described in terms of the dimensionless number \( ka = 2\pi a/\lambda \), where \( a \) is the characteristic radius of the scatterer or half its diameter. Typically, in acoustic measurements the \( ka \) values of the reflecting objects may vary anywhere between \( 10^{-2} \) and \( 10^{4} \). The higher region, where \( ka > 50 \), is called the geometric region as the portion of the acoustic wave which is scattered is small and the optical approximation of straight ray paths is valid.
If the ka-value of the reflecting object becomes small \((ka < 1)\) then the incident wave is scattered more or less uniformly in all directions. This is referred to as Rayleigh scattering. As the size of the resolution cell of an imaging system, expressed in a ka-value, may vary between 2 and 40, the geometric region is generally "well resolved" and the Rayleigh scattering region is "unresolved". The local reflectivity distribution in the "well resolved" region has been amply discussed in chapters V and VI.

Here we will concentrate on the influence of inhomogeneities which are typically on the same scale as the resolution cell or smaller. To be able to understand the available literature on this topic we will first derive the wave equation for inhomogeneous media and relate that to the wave equation which is used in the literature on scattering. Next we will define some basic concepts such as scattering cross sections, followed by a discussion on scattering from single objects. Then scattering from a cloud of inhomogeneities will be treated. Finally some methods will be mentioned for the estimation of scattering parameters from sub-resolution scattering.

8.1 THE INHOMOGENEOUS WAVE EQUATION

In seismic literature the inhomogeneous wave equation is derived from the equation of motion \([1, 2]\)

\[
\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \nabla \mathbf{p} = 0 \quad (8.1.1)
\]

and Hooke's law

\[
\frac{\partial \mathbf{p}}{\partial t} + \rho c^2 \nabla \mathbf{v} = 0, \quad (8.1.2)
\]

where both \(\rho\) and \(c\) may vary with the spatial coordinates. After Fourier transformation we easily find

\[
\rho \mathbf{v} \cdot \left[ \frac{1}{\rho} \nabla \mathbf{p} \right] + \frac{\omega^2}{c^2} \mathbf{p} = 0. \quad (8.1.3)
\]

or

\[
\nabla^2 \mathbf{p} + \frac{\omega^2}{c^2} \mathbf{p} = -\rho \nabla \frac{1}{\rho} \nabla \mathbf{p}. \quad (8.1.4)
\]
Introducing a homogeneous reference medium with velocity \( c_0 \) and density \( \rho_0 \)

\[
\nabla^2 p + k_0^2 p = -\left(\frac{c_0^2}{c^2} - 1\right)k_0^2 p - \nabla \ln \beta \cdot \nabla, \quad (8.1.5)
\]

where \( \beta = \rho_0/\rho \) and \( k_0 = \omega/c_0 \). In the literature on scattering the following wave equation is often used [3, 4]

\[
\nabla^2 p + k_0^2 p = -k_0^2 \gamma_\kappa p + \nabla \cdot (\gamma_\rho \nabla p), \quad (8.1.6)
\]

where \( \gamma_\kappa = \kappa - \kappa_0/\kappa_0 \) and \( \gamma_\rho = \rho - \rho_0/\rho_0 \). Note that here \( \kappa \) is the compressibility as defined in \( c = \sqrt{1/\rho \kappa} \), which makes this \( \kappa \) the inverse of the compression modulus as defined in equation (4.2.3). The difference between (8.1.5) and (8.1.6) arises from the fact that in the literature on scattering equation (8.1.3) is first divided by \( \rho \) to find a separation of the more fundamental parameters, compressibility and density

\[
\nabla \left( \frac{1}{\rho} p \right) + \omega^2 \kappa p = 0 \quad (8.1.7)
\]

Again introducing a reference medium with a reference \( c_0 \) and \( \rho_0 \) equation (8.1.6) is simply derived. From the point of view of scattering, wave equation (8.1.6) is the more fundamental of the two, and will be used in this chapter.

### 8.2 SCATTERING CROSS SECTIONS

If a region of interest is illuminated by a plane wave of unit amplitude, defined by its \( k \)-vector and angle of incidence, the scattered wave field in the far field of the r.o.i. will behave as a spherical wave, given by

\[
P_s = f(\vec{s}, \vec{t}) e^{-jkR/R}, \quad (8.2.1)
\]

where \( f(\vec{s}, \vec{t}) \) represents the amplitude and phase of the scattered wave in the far field in the direction \( \vec{s} \), related to the scattered wave from a point scatterer. \( R \) defines the distance between the centre of the scattering volume and the receiver.
In figure 4.1 the incident and scattered waves are depicted in relation to the scattering region of interest. \( f(\hat{s}, \hat{l}) \) is called the scattered amplitude or the angle distribution factor, and does not depend on the distance \( R \) between the r.o.i. and the receiver. The right hand side of equation (8.1.6) generates a scattered wave \( P_s \), where the first term acts as a pressure monopole and the second term as a pressure dipole (see also section 4.2):

\[
P_s = \frac{1}{4\pi} \int_V \left\{ 2k_o \gamma_P \frac{e^{-jk\Delta r}}{\Delta r} + \gamma_P \nabla_P \cdot \nabla \left( \frac{e^{-jk\Delta r}}{\Delta r} \right) \right\} dV, \tag{8.2.2}
\]

where \( \Delta r \) defines the position of the scatterers in volume \( V \) in relation to the receiver. In the far field of the scatterers

\[
\nabla \left( \frac{e^{-jk\Delta r}}{\Delta r} \right) = jk \cos \varphi \left( \frac{e^{-jk\Delta r}}{\Delta r} \right). \tag{8.2.3}
\]

In the far field of a limited scattering volume \( \Delta r \) may be approximated by \( R - \hat{r} \hat{s} \), where \( \hat{r} \) defines the position within the scattering volume and \( \hat{s} \) defines the direction of the scattered wave as shown in figure 8.1.

Now equation (8.2.2) transforms into

\[
P_s = \frac{e^{-jkR}}{4\pi R} \int_V \left\{ k_o^2 \gamma_P + j\gamma_P k_s \cdot \nabla_P \right\} e^{jk_s \cdot \hat{r}} dV, \tag{8.2.4}
\]

or with equation (8.2.1)

![Figure 8.1: Definition of various vectors in relation to the detector D scattering volume V.](image)
\[ f(s, i) = \frac{1}{4\pi} \int \left\{ k_0^2 \gamma_k^2 + j \gamma \rho_s \cdot \nabla P \right\} e^{jk \cdot s} dV. \quad (8.2.5) \]

If the Born approximation is valid the pressure field \( P \) equals the incident pressure field. If the scattering volume, in addition, is in the far field of the source, \( P \) may be approximated by a plane wave. With a plane wave of unit amplitude we find

\[ P = e^{-jk \cdot s} \]

and

\[ \nabla P = -jk \cdot e^{-jk \cdot s}. \quad (8.2.6) \]

Substituting this into equation (8.2.5) gives [3, 5, 6]

\[ f(s, i) = \frac{k_0^2}{4\pi} \int \gamma e^{jk \cdot s} dV, \quad (8.2.7) \]

where \( \gamma = \gamma_k + \gamma_\rho \cos \psi \) and \( \psi \) is the angle between the incident and the scattered waves \( \hat{s} \) and \( \hat{i} \). In equation (8.2.7) \( \hat{k} = \hat{s} - \hat{i} \) represents the modified \( k \)-vector as defined in section 4.1, while \( k_0 = \omega/c_0 \). As the source wave at the r.o.i. is assumed to be a plane wave of unit amplitude, all distortions of this wave due to the source distribution and propagation between source and r.o.i. have been compensated for. Thus some inversion process has taken place. At this moment we will return to the three dimensional version of the Fourier transformation of equation (4.1.8).

\[ \tilde{P}(k_x, k_y, k_z, t=0) = \int \int \int p(x, y, z, t=0) e^{jk \cdot r} dx dy dz. \quad (8.2.8) \]

By comparing the last two equations we see we may substitute

\[ f(s, i) = \frac{k_0^2}{4\pi} \tilde{P}(k_x, k_z, t=0), \quad (8.2.9) \]

bearing in mind that the dot product in (8.2.8) is defined differently than in (8.2.7). So we may conclude that there is a direct relationship
between the angle distribution factor of a scattering volume and the
two dimensional Fourier transform of the focused image of that volume.
This again illustrates the fact that correct focussing is equal to
creating far field conditions in the image, and also that there exists
a Fourier relationship between the far field response and the spatial
distribution of scatterers.

From the scattered amplitude the differential scattering cross section
may be defined

\[ \sigma_d(x, \hat{1}) = |f(x, \hat{1})|^2. \] (8.2.10)

The backscattering cross section \( \sigma_b \) is defined as

\[ \sigma_b = 4\pi \sigma_d(-\hat{1}, \hat{1}), \] (8.2.11)

where we assume that the observed backscattered power is uniformly
distributed over all angles. Finally we define the scattering cross
section as

\[ \sigma_s = \int \sigma_d(x, \hat{1})d\Omega = \int |f(x, \hat{1})|^2d\Omega, \] (8.2.12)

where \( \Omega \) is the solid angle around the scattering volume.

In this work we are mainly interested in reflection measurements where
the Born approximation is applicable; but for the sake of completeness
we must also consider the total power absorbed by the particle. The
cross section of a particle which would correspond to the absorbed
power is called the absorption cross section \( \sigma_a \), and the sum of the
scattering and the absorption cross sections is called the total cross
section \( \sigma_t = \sigma_s + \sigma_a \).

8.3 SCATTERING FROM SINGLE OBJECTS

Extensive studies have been made on the scattering of sound by single
objects. A.o. the scattering from rigid spheres and cylinders may be
found in [3]. As mentioned in the introduction to this chapter two
regions of scatterer sizes are usually defined, namely when the circumference of the scattering object $2\pi a$ is smaller than a wavelength, $2\pi a < \lambda$ or $ka < 1$, and when it is much larger than a wavelength. In the first case we are in the Rayleigh scattering region, while in the second case geometric effects dominate.

In the geometric region the frequency dependence of the backscattering cross section $\sigma_b$ varies with the shape of the scatterer. If we take the average over many measurements, the backscattering cross section, normalized to the geometrical cross section $\frac{\sigma_b}{\pi a^2}$, is proportional to $(ka)^p$, where $p=0$ for ellipsoidal shaped scatterers, $p=1$ for cylindrical scatterers and $p=2$ for plane scatterers [4]. The backscattered amplitude shows a $k^2$ dependence, where $p$ ranges from 0 for the ellipsoidal shape to $p=1$ for the plane.

In the Rayleigh scattering region the scatterers show a $\sigma_b/\pi a^2$ which is proportional to $(ka)^4$, independent of their shape. Using equation (8.2.5) an approximation may be derived for the scattering from a small sphere $(ka < 1)$ in a homogeneous background [3]

$$f(\hat{s}, \hat{l}) = f(\psi) = \frac{2}{3} \frac{3}{\kappa} \left( \frac{e^{-\kappa}}{3} \right) \left( \frac{2\rho -3\rho}{2\rho +\rho} \right) \cos\psi, \quad (8.3.1)$$

where $\kappa$ and $\rho$ are the compressibility (equation (8.1.6)) and density of the medium, and $\kappa_e$ and $\rho_e$ those of the scatterer.

For small scatterers the Born approximation usually is invalid, as the incident pressure field cannot be assumed to be constant over the scatterer, so this equation generally gives different but more exact solutions than equation (8.2.7). From equation (8.3.1) we may conclude that compressibility and density act as independent variables, whilst a compressibility inhomogeneity behaves like an acoustic monopole and a density inhomogeneity as a dipole. A typical example of Rayleigh scatterers is blood, where the scatterers are the red blood cells suspended in plasma. The scattering cross section, normalized to the geometrical cross section of Rayleigh scatterers may be calculated by equation (8.2.12),

$$\frac{\sigma}{\pi a^2} = \frac{4}{9} (ka)^4 \left[ \frac{\kappa -\kappa}{\kappa} \right]^2 + \frac{2}{3} \left( \frac{e^{-\kappa}}{2\rho +\rho} \right)^2. \quad (8.3.2)$$

Throughout this work the emphasis has been on sparsely placed scatterers and their distribution in space. In the computer simulations
the scatterers would not of themselves give any spectral colouring. Consequently the only frequency dependence was due to the far field approximations of the Rayleigh II operators. The two dimensional Rayleigh II extrapolation operator yields a linear frequency dependence (equation 2.6.2). From equation (8.3.1) we find that adjustments must be made if the computer simulations are to simulate the true frequency dependence of the scattering from point scatterers. Though not treated here, it should be mentioned that both turbulence and resonance can have a considerable effect on scattering. Particularly the resonance effects may occur at surprisingly low frequencies, e.g. for air bubbles in water $f_r = 3.22/a$, where $a$ is the bubble radius in metres. Thus, in the Megahertz region, resonance from micro bubbles may overshadow the scattering experiments. This effect presents a major problem e.g. when studying scattering from tissues in vitro [7].

8.4 SCATTERING FROM A CLOUD OF SCATTERERS

When an acoustic wave traverses a cloud of small objects, each object produces a scattered wave and these wavelets reinforce in some directions and interfere destructively in others. The wave incident on each scatterer is affected by the presence of the other scatterers, which in turn affects the shape of the scattered wave. This gives rise to coherent, incoherent, and multiple scattering and, in cases where the scatterers are regularly spaced, to Bragg diffraction (section 5.5).

Apart from the assumptions made for Rayleigh scattering from single scatterers (equations (8.3.1) and (8.3.2)) we will suppose that the mean distance between the scatterers is not larger than $\lambda$, and that the r.o.i. contains $N$ scatterers. The number $N$ will later be specified by experiments. With the scatterers so closely spaced, the average compressibility and density of the medium has altered. The average value of the density in the r.o.i. is given by [3]

$$\rho_R = \left[ \frac{1}{\rho_o} + \sum_{n=1}^{N} \frac{4}{3} \pi a_n^3 \left( \frac{1}{\rho_n} - \frac{1}{\rho_o} \right) \right]^{-1}, \quad (8.4.1)$$

where $\rho_o$ is the medium density, $\rho_n$ is the density of the $n^{th}$ scatterer and $4/3\pi a_n^3$ is the volume of the $n^{th}$ scatterer. For the
average value of the compressibility in the r.o.i. we find

$$\kappa_R = \kappa_o + \sum_{n=1}^{N} \frac{4}{3} \pi a_n^3 (\kappa_n - \kappa_o). \quad (8.4.2)$$

The scattering, related to these average values of density and compressibility is the coherent scattered wave. The average scattered amplitude of this coherent scattering is proportional to the number of scatterers N. The scattering characteristics of the coherent scattered wave are related to the size of the area, where $\rho_R$ and $\kappa_R$ are constant. Coherent scattering will occur at the boundaries where the number of scatterers per volume suddenly changes. The variable parts of the density and compressibility give rise to incoherent scattering.

This incoherent scattering may be calculated by using equation (8.2.7), and inserting for

$$\gamma = \frac{\kappa_n - \kappa_R}{\kappa_R} + \frac{3\rho_n - 3\rho_R}{2\rho_n + \rho_R} \cos \psi$$

inside the $n^{th}$ scatterer, and

$$\gamma = \frac{\kappa_o - \kappa_R}{\kappa_R} + \frac{3\rho_o - 3\rho_R}{2\rho_o + \rho_R} \cos \psi \quad (8.4.3)$$

inside the r.o.i. and outside a scatterer. The incoherent scattering will depend strongly on the spatial distribution of the scatterers, and on the distribution of their sizes. It can also be shown [3] that the scattering amplitude is proportional to the square root of the number of scatterers $\sqrt{N}$.

The coherent and the incoherent scattering approach is closely related to the way the subsurface model in chapter II is divided into a macro subsurface model containing the bulk parameters, and the local reflectivity model.

8.5 ASSESSMENT OF THE REFLECTIVITY CHARACTER FROM BACKSCATTER MEASUREMENTS

The simplest way to assess the character of reflectivity is by calculating a grey scale histogram of an image. The grey scale of the envelope detected image is assumed to be proportional to the backscattered signal amplitude. If the approximate size and shape of
the scattering objects were known, an assessment could be made of their number. As mentioned in section 8.4 the backscattered signal amplitude is proportional to the square root of the number of scatterers. Due to various effects of wave propagation and measurement system the back-scattered signal amplitude is not a very reliable parameter to analyse. It is interesting to mention that, if the amount of scatterers per resolution cell is large, the grey scale image shows a granular structure, which is independent of the medium parameters [8]. This granular structure, which is a result of constructive and destructive interference of the band limited signals coming from the medium, is generally referred to as speckle. The size of the speckle is related to the resolution limit of the imaging system [9, 10]. The speckle amplitude has a modified Gaussian probability distribution, defined by

\[ p(A) = \frac{A}{\beta^2} \exp\left(-\frac{A^2}{2\beta^2}\right), \quad (8.5.1) \]

where \( A \) is the signal amplitude. This Gaussian probability distribution has a mean

\[ \mu_o = \int_0^\infty A p(A) dA = \frac{1}{\beta^2} \int_0^\infty A^2 \exp\left(-\frac{A^2}{2\beta^2}\right) dA = \beta \sqrt{\frac{\pi}{2}}. \quad (8.5.2a) \]

Its variance may be calculated as

\[ \sigma^2 = \int_0^\infty A^2 p(A) dA - \mu_o^2 = \frac{1}{\beta^2} \int_0^\infty A^3 \exp\left(-\frac{A^2}{2\beta^2}\right) dA - \frac{\beta^2}{2} = \frac{3}{2} \beta \sqrt{\frac{\pi}{2}}. \quad (8.5.2b) \]

So we see that for random speckle the average grey level divided by its standard deviation, often referred to as the signal-to-noise ratio of a B-scan image, reaches a limit of

\[ \text{S.N.R.} = \frac{\mu_o}{\sigma} = \sqrt{\frac{\pi}{4-\pi}} = 1.91, \quad (8.5.3) \]

a value verified experimentally by Thyssen et al. [8]. Though the statistics of the backscattered signal amplitude are not uniquely
determined by one single image texture, we may conclude, with some reserve, that if the S.N.R. as defined above reaches the value of 1.91 the envelope detected image does not contain any information about the medium.

The second way to assess the character of reflectivity is by auto correlation. If the scatterers are "well resolved", i.e. if their characteristic distances are larger than the system resolution, then the auto correlation of the envelope detected image will reveal these characteristic distances (Chapters V and VI). As the characteristic distance decreases to approximately the size of the resolution cell, we may take the full width half maximum (f.w.h.m.) of the auto correlation function as a measure for the distance between the scatterers or the scatterer density. Thyssen [8] showed that this f.w.h.m. is inversely proportional to the logarithm of the scatterer density. In the case of many scatterers for each resolution cell we may again speak of random speckle in the image. Here the auto correlation function is that of speckle and will show no dependence on the scatterer density.

The third way of assessing the character of reflectivity is by calculating the slope of the amplitude spectrum of the returned signal. This may be done either qualitatively or quantitatively. For a qualitative assessment of the character of the returned pulse we may follow the instantaneous frequency of the returned signal. Changes in reflectivity character will show up as sudden changes in the centre frequency of the pulses. The quantitative assessment, where the frequency power dependence of the spectral slope is calculated, is more complicated. Principally the spectral slope may be calculated by Fourier transformation, but as this calculation is very sensitive to any distortion of the spectrum, we must first get rid of the diffraction effects (section 7.8). The signals from a sample volume of a focused image must first be Fourier transformed to the wavenumber space. Then a two dimensional band limited inversion must be applied to compensate for any distortions due to the source distribution or propagation. The band limited inversion, which is only applied to the amplitude spectrum, is done by means of a measured or calculated reference signal. This spectral "whitening" process has already been described in section 5.6. After spectral "whitening" the data are averaged along
lines of constant frequency, which in the $k_x-k_z$ domain of backscatter measurements are circles with $k_x=k_z=0$ as centre. A plot of the average returned spectral amplitude as a function of frequency will show the differences between the "known" reference medium and the sample under investigation. Due to the interference of the different responses the amplitude spectrum shows maxima and minima. These distortions are minimized by averaging over lines of constant frequency and may be reduced even further by averaging the results from different sample volumes. If the range of measured frequencies is large enough we may find a transition point, where either the $ka$-value of the reference or of the sampled scatterers passes $ka=1$. Here the power of the frequency dependence of the scattered signal amplitude changes from two in the Rayleigh scattering region to its geometric optical value.

Ma et al. [11] observed this transition point and calculated the $ka$-value at which it occurred for different scatterer size distributions. They found the $ka$-value to vary between $0.6 < ka < 0.8$, where the lower boundary was found for a size distribution with a Rayleigh probability density function and where the higher boundary was found for a uniform probability density function. For single sized scatterers the transition should occur at $ka=1$ [12]. In principle, another transition point may be found, where either the sample or the reference start showing multiple scattering. If the power of the frequency dependence exceeds two, then we may assume this is the result of multiple scattering [13]. Sehgal [14] suggested that multiple scattering will occur when the attenuation due to scattering exceeds a certain value. Defining the attenuation due to scattering as $e^{-\tau}$ he drew an analogue between acoustical and optical scattering. For values of $\tau < 0.1$ single scattering is predominant, if $0.1 < \tau < 0.3$ double scattering becomes important, and for $\tau > 0.3$ multiple scattering is significant. These considerations imply that multiple scattering is likely in the higher frequency ranges for human soft tissue.

References chapter VIII


CHAPTER IX
SOUND VELOCITY ESTIMATION

If the sound propagation velocity in a homogeneous medium differs from the sound propagation velocity we assume for the imaging process, the depths at which we image the scattering objects will differ from their true depths. As only the travel time of the acoustic wave is known by measurement, both range and propagation velocity are variables. For a single backscattered signal therefore, it is, in principle, impossible to overcome this range ambiguity. It has long been assumed in medical ultrasound, where focused sound beams are mainly used, that it was impossible to estimate both range and propagation velocity from backscattered signals. If the sound propagation velocity in a medium varies as a function of the spatial coordinates, not only will depth distortion appear in the image, but due to refraction of the sound field at the velocity discontinuities the incident wave field will be distorted. At a velocity increase the wave will curve away from the normal on the boundary, whilst at a velocity decrease the transmitted angle will be smaller than the angle of incidence. In the event of a velocity increase critical angle effects may occur. When the angle of incidence is larger than the critical angle, the wave cannot penetrate any distance beyond the discontinuity, making it impossible to image parts of the medium. Acoustic inversion layers, where this total reflection of the incident wave field occurs, are encountered in practically all fields of acoustics. In this chapter we will show that we are able to estimate the sound propagation velocity from backscattered signals. When we have a correct estimate of the sound
propagation velocity it is possible to compensate correctly for the angle and range ambiguities. As a simple example of a sound velocity estimation technique we will start by discussing a method proposed by Robinson et al. [1]. Naturally there are far more sophisticated methods available, such as the parametric inversion techniques. Unfortunately these sophisticated methods are beyond the scope of this thesis. Robinson stated that, if the positions of the sound propagation discontinuities are known, only two backscattered signals taken from different angles are necessary to estimate the velocity changes. In the following sections velocity estimation from multi-trace records will be discussed. If the correct propagation parameters are inserted into the inversion process, an optimally focused image will be the result. The minimum entropy criterion will be introduced as a mathematical technique for the assessment of the quality of the focussing. Using an iterative process of focussing and entropy calculation any depth dependent velocity profile may be estimated without prior knowledge of the medium.

9.1 VELOCITY ESTIMATION FROM TWO BACKSCATTERED SIGNALS

For the estimation of the sound propagation velocity from backscattered signals a minimum of two signals from different angles is needed. To show that it is principally possible with only two signals a method will be discussed which was proposed by Robinson et al. [1]. He used a conventional imaging system, where a transducer transmits and receives a narrow beam, and an image is built up by scanning the image plane. Typically the sound propagation velocity in the medium directly adjacent to the transducer was known, because a waterbath coupling was used. Thus by scanning the image plane the true positions of the boundaries of the scattering objects could be assessed. If a sound velocity discontinuity occurred at the boundaries, the image behind the boundary would be out of focus due to the angle and range ambiguities discussed above. Ideally one should locate a single scattering object behind the sound velocity discontinuity, and aim the narrow beam at this scattering object from two different angles. For simplicity we will suppose the scattering object is a small point scatterer. As the imaging system assumes only one single sound velocity throughout the medium, the scattering object will be imaged in two different places,
neither of which coincide with its true position. Figure 9.1 depicts the real and imaged sound rays for a plane velocity discontinuity. In figure 9.1 the velocity in the lower halfspace was chosen to be exactly twice that of the upper half space. In his work Robinson did not restrict himself to a plane interface, also, more general shapes of scatterers could be used. For reasons of simplicity we will keep to this example. From the image, the lateral distance between the two projections of the scatterer, and the lateral distance between the points where the rays hit the interface may be assessed. These two distances are named $l_{12}$ and $R_{12}$ respectively. Using Snell's law and a few geometrical considerations (see figure 9.1) we can calculate, for this situation, that

$$c_2 = c_1 \sqrt{R_{12}/(R_{12} - l_{12})},$$

(9.1.1)

where $c_1$ is the known sound velocity in the upper half space and $c_2$ is the unknown sound velocity.

Though this sound velocity estimation technique shows some restrictions concerning the imaging system and the medium, it does elegantly show that only two backscattered signals are needed to estimate sound velocity by making use of imaging errors.

### 9.2 Entropy

Though the concept of entropy has been known for over a century in thermodynamics, its introduction into the signal processing field is relatively new. Shannon [2] introduced the function...
\[ H = - \sum p_i \log p_i \]  
(9.2.1)

as a measure of uncertainty about the realization of an event. In this equation \( p_i \) represents the probability of an event \( i \), and \( \sum p_i = 1 \).

Throughout this chapter a more general form of equation (9.2.1) will be used as a measure of the information content of an image. How the uncertainty measure works is demonstrated by the following examples. If a series of events, or a sampled time series shows maximal information, it will contain one non-zero sample. Applying equation (9.2.1) to this time series we find \( H = 0 \), the uncertainty or entropy is minimal.

Conversely if the sampled time series shows no information at all this will be represented as a series of \( N \) events with equal probability \( 1/N \). Applying (9.2.1) again, we find

\[ H = -N \frac{1}{N} \log \frac{1}{N} = \log N, \]

which is the maximal value of \( H \). So we see that maximal information content coincides with minimum entropy. When applying a minimum entropy criterion to sampled acoustical signals Shannons function may be rewritten in a more general form

\[ V = \frac{1}{N} \sum q_i F(q_i), \]  
(9.2.2)

where \( F(q_i) \) is a monotonically increasing weighting function of \( q_i \), and where \( q_i \) is a normalized amplitude parameter

\[ q_i = \frac{a_i}{\frac{1}{N} \sum a_i}. \]  
(9.2.3)

Here \( a_i \) is a positive definite parameter representing the time series, such as its envelope. In his thesis De Vries [3] evaluated the entropy norm \( V \) extensively. As monotonically increasing entropy functions \( F(q_i) \) he considered "weak" functions \( F(q_i) = \log (q_i) \) and \( \sqrt{q_i} \), but also the stronger dependencies on the amplitude parameter ranging from a linear function \( F(q_i) = q_i \) to a fourth power of \( q_i \), \( F(q_i) = q_i^4 \). He found that the best choice of entropy function depends on the application, the higher powers of \( q_i \) make \( V \) more sensitive to
peaks in the signal. De Vries also evaluated various parameters $a_i$, such as the absolute value of the data, the squared value, the RMS value and the envelope. Because of the flexibility of this definition of entropy, many other norms can be derived from these equations. When e.g. we choose the squared amplitude as parameter $a_i$, and a linear entropy function, we find a measure for the kurtosis of a time series.

$$V_{kur} = \frac{\sum_i y_i^4}{\left(\sum_i y_i^2\right)^2},$$

(9.2.4)

where $y_i$ is the amplitude of the $i^{th}$ sample. The kurtosis is normally used as a measure for the "peakedness" of a pulse, or as a measure for the "spikyness" of a time series [4, 5].

9.3 **FOCUSSING AND SPIKYNESS OF AN IMAGE**

Generally speaking focussing is applied to enhance the detail of an image. Focussing will increase the image resolution. In terms of echo acoustics the lateral resolution is chiefly affected. As the resolution of an image increases, the "well resolved" objects will show a better definition of their edges, also, increasing resolution will cause some unresolved objects to become well resolved, and more detail will be visible.

In chapter III it has been shown, that correct inversion of acoustic data produces pulses with a spatial zero-phase spectrum. The diffraction energy has collapsed into the focal point. For a given amplitude spectrum it can be shown [6] that a zero-phase spectrum produces the shortest pulse. If the focussing is not perfect, the spatial phase spectrum will no longer be zero-phase, causing the diffraction energy to spread out around the focal point and degrading the image resolution. Thus imperfect focussing, a result of an incorrect subsurface model, will cause mainly lateral dispersion. One of the parameters which is inserted into the inversion operator is the sound propagation velocity. Consequently any velocity errors will show up as "blurring" of the image. The effects of velocity errors on imaged results are discussed in detail elsewhere [7] and will not be treated here.
Correct assessment of the sound propagation velocity will produce an optimally focused image with sharp contours and fine detail, thus an optimally spiky image.

In the case of random speckle (section 8.5), where we have many scattering objects per resolution cell, the envelope detected image gives no information about the medium. Here, changing the sound propagation velocity in the inversion operator will not alter the spikiness of the image.

9.4 SOUND VELOCITY ESTIMATION BY MINIMUM ENTROPY

In the previous section the estimation of the correct sound propagation velocity was related to the spikiness of the image after inversion. And in section 9.2 the minimum entropy criterion was discussed, as a mathematical measure for the spikiness of an image. Putting two and two together we can use the minimum entropy criterion as a measure for the effectiveness of the inversion process, so, by varying the sound propagation velocity and applying a minimum entropy criterion after every inversion an optimal sound propagation velocity may be estimated. As the lateral resolution is the most sensitive to any focussing errors, one would naturally assume that the minimum entropy criterion applied in the lateral direction would be most effective. De Vries [3] argued that in scattering media lateral dispersion would also degrade the axial information content. He showed that correct application of the minimum entropy criterion was equally effective latterally as axially.

In the case of a homogeneous sound velocity the zero offset diffraction responses form hyperbolas (chapter III). By the inversion process with the correct sound velocity the diffraction responses collapse into the apices of the hyperbolas. If the sound velocity is only a function of depth \( c = c(z) \), then the diffraction responses for limited apertures retain approximately their hyperbolic shape and an optimal velocity may be found by applying the procedure mentioned above. For a system with \( N \) parallel layers where the velocity is constant within each layer Dix [8] calculated an optimal average velocity

\[
V_N^2 = \frac{1}{T_N(o)} \sum_{n=1}^{N} c_n^2 A_T n'
\]  
(9.4.1)

where \( V_N \) is the RMS (Root Mean Square) velocity for the \( N^{th} \) layer,
\( T_N(o) \) is the two way travel time to the apex of the hyperbola, \( c_n \) is the sound velocity in the \( n^{th} \) layer and \( \Delta T_n \) is the two way travel time within the \( n^{th} \) layer. This equation is known in seismics as the Dix formula. Note that for a multi-layered system the RMS velocity \( V_N \) does not represent the true average velocity. The RMS velocity only tells us which velocity will produce an optimal focus for the \( n^{th} \) layer. Consequently using the travel times and the estimated RMS velocities will not give us the true scatterer positions. The true sound velocities may be estimated by starting at the top and applying a recursive inversion formula:

\[
c_n^2 = \frac{V_N^2 T_N(o) - V_{N-1}^2 T_{N-1}(o)}{T_N(o) - T_{N-1}(o)},\tag{9.4.2}
\]

where the subscripts \( N \) and \( N-1 \) represent the numbers of the layers.

So when, within a given aperture, the sound propagation velocity is only a function of depth, we may choose a sample volume at any given depth and calculate the optimal RMS velocity for that depth, assuming the sample volume contains some scatterers which are well resolved.

When we apply a running time window for the sample volume as discussed in chapter VII, we may estimate the optimal RMS velocity as a function of depth. In the ideal case of a small time window any boundaries in the true sound velocity will show up as discontinuities in the RMS profile (see figure 9.2). By applying the recursive inversion formula (9.4.2) the true velocity profile may be calculated from the RMS velocity profile.

![Figure 9.2: a) True velocity profile as a function of the true depth. b) R.M.S. velocity profile as calculated with Dix' formula.](image-url)
9.5 Example

As an example of sound velocity estimation from backscattered signals a two velocity medium will be considered. To create the two velocity medium a piece of steak was submerged in alcohol. From the literature we find, that alcohol has a sound propagation velocity of approximately 1170 m/s at room temperature [9], while the skeletal muscle of a cow has a sound propagation velocity of 1560 - 1600 m/s [10]. A cross section of the submerged beef was scanned by means of a .3 mm transducer element. The cross section was scanned at 512 positions, .1 mm apart at a sample frequency of 10 MHz. The emitted pulse was band limited between 1.5 MHz and 3.5 MHz (-20 dB points). Figure 9.3 shows a grey scale representation of the envelope detected, focused image, covering an area of approximately 5x13 cm. This image was produced by applying the $k_x - k$ mapping procedure (section 3.3) with a sound propagation velocity of 1350 m/s. Consequently only one single depth is in focus in figure 9.3, depicted by the arrow at C.

![Figure 9.3: Envelope detected grey scale image of a piece of steak, submerged in alcohol.](image-url)
To verify the value of 1170 m/s for the sound propagation velocity in alcohol under these experimental conditions, a calibration in measurement was performed on a set of wires submerged in alcohol. By means of the minimum entropy method a bulk sound propagation velocity of $1195 \pm 5$ m/s was found, 25 m/s higher than the value found in [9], indicating that this alcohol probably was not pure. Applying this value and 1580 m/s in beef to equation (9.4.1) the solid curve in figure 9.4 was calculated. In figure 9.4 the RMS velocity is given as a function of the arrival time of the acoustic pulse. The arrival time of the pulse from the boundary between alcohol and meat was at approximately $T_1 = 76 \, \mu s$.

Figure 9.4: RMS velocity as a function of pulse arrival time.

- Measurement
- Calculated for $c_1 = 1195$ m/s and $c_2 = 1580$ m/s
- Calculated for $c_1 = 1230$ m/s and $c_2 = 1580$ m/s

Figure 9.5: Entropy as a function of the sound velocity for the areas, depicted by arrows in figure 9.3.
Next the RMS sound velocity was calculated for various areas in the left half of the image in figure 9.3. For three of these areas the entropy curves are given as a function of the sound velocity in figure 9.5. The areas are indicated by A-C in figures 9.3 and 9.5. The entropy curves were calculated in the lateral direction, with a quadratic entropy function \( F(q_i) = a_i^2 \) and a squared amplitude parameter \( a_i = y_i^2 \). The data in the sample volumes were rejected at 20 dB below the maximum to minimize the effect of noise. The measurements where a clear peak was found in the entropy curves are indicated with error bars in figure 9.4. Clearly the expected sound propagation velocity is over-estimated in the part of the meat nearest the transducer, and under-estimated in the part farthest from the transducer. For correct prediction of the measured values a velocity of 1230 m/s in alcohol and 1530 m/s in meat must be inserted in equation (9.4.1), producing the dashed line in figure 9.4.

9.6 DISCUSSION

From the experiments, discussed in the previous section, we may draw some interesting conclusions, firstly about the sound velocity estimation and secondly about the entropy curves themselves. The sound propagation velocity in alcohol seems to vary in this set of experiments, and also it differs from the values quoted from the literature. This effect may be explained if we consider that alcohol is a strongly hygroscopic fluid, and that any impurities in a fluid will alter its sound propagation properties. As suggested by the calibration measurement on the wires, the alcohol was not pure to start with. The presence of water in the alcohol may account for the 25 m/s increase in propagation velocity in relation to the 1170 m/s found in the literature. Once the meat was submerged in the alcohol all kinds of substances, e.g. blood and fat, started going into solution. It seems likely that the propagation velocity in the alcohol was affected by the presence of the meat, accounting for a further increase by 35 m/s. In retrospect a second calibration measurement should have been conducted after the scan had been completed, to verify this statement, but unfortunately this was not the case.

The sound propagation velocity in muscle of 1530 m/s, which was found experimentally, also differs from the values quoted from the
literature. Here not only the presence of alcohol, but also the freshness of the meat may account for the difference.

Consequently there is no reason to assume that the entropy method is giving us the wrong values for the propagation velocity, and an accuracy of ± 1% may be expected for the estimation of the RMS velocity. Unfortunately, in an inhomogeneous medium, the estimation of the interval velocities is less accurate. Even under these "ideal" laboratory conditions with a large velocity step (300 m/s) an error of ± 25 m/s may be expected for the estimation of the velocity in the second medium. In practice we would like to measure far smaller velocity changes (± 20 m/s) in smaller areas. This practical consideration makes the entropy method of velocity estimation in inhomogeneous media of limited value for diagnostic purposes.

A second important consideration follows from the entropy curves in figure 9.5. The curves, denoted by A, B and C, correspond to the areas indicated by arrows in figure 9.3. From both figures we observe the different character of A, B and C.

In area A the main echo is from the specular reflections of the tissue/alcohol interface. Here the entropy curve shows a low maximum at 1230 m/s, the propagation velocity in alcohol. The value of the entropy is moderately high over the range of the curve, and does not change much as a function of the sound propagation velocity. This indicates a minor effect of focussing on the spikyness of the data, which is characteristic for a series of specular reflectors.

From frame B of figure 9.5 we see that focussing has a great effect, as the entropy curve shows a large peak. Here we may assume a dominant influence of sparsely distributed small scatterers, which is confirmed by the image of figure 9.3. The sound propagation velocity at B is approximately 1265 m/s. Finally in frame C the entropy is low for the considered velocity range, suggesting the absence of sparsely spaced scatterers. Here the entropy method fails to give an estimate for the sound propagation velocity. In area C we may assume either that the scatterers are too densely packed, or that noise effects dominate.

De Vries et al. [11] studied the sensitivity of the entropy V (equation (9.2.2)) as a function of scatterer density. For the entropy function used here the results are shown in figure 9.6. In the first frame of figure 9.6 the entropy is shown as a function of the "sparsity"
Figure 9.6: Entropy and its derivative as a function of sparsity.

\[ \sigma = (1 - \frac{n}{N}) \times 100\% \]  

where \( n \) is the amount of spikes in a time signal and where \( N \) is the total amount of sample points. In both frames of figure 9.6 three curves are drawn, for a bandwidth of \( 1/40 f_s \), \( 1/20 f_s \) and \( 1/10 f_s \) respectively. In our case the bandwidth is \( 1/5 f_s \). In the second frame the derivative of the entropy to the sparsity is given as a function of the sparsity. Relating figures 9.5 and 9.6, both 9.5a and 9.5b should be placed on the steep part of the curves in 9.6, where the sparsity is large, and 9.5c should be placed where the curves of 9.6 are practically zero. The difference between 9.5a and 9.5b being, that the variation in the sparsity \( \sigma \) as a function of the sound velocity is smaller in the first case.

In conclusion the following remarks can be made:
- The entropy method is accurate to about 1% for a single velocity medium with sparsely spaced scatterers.
- For a multi-velocity medium the entropy method becomes too inaccurate to be of diagnostic value, especially if the second velocity medium is small, and at some distance from the transducer.
- The entropy method shows the difference between sparse specular reflectors, sparse diffractions and densely packed scatterers or noise.
References chapter IX


APPENDIX A
THE RAYLEIGH-SOMMERFELD BOUNDARY CONDITIONS

In the field of optics Kirchhoff [1,2] was the first to give a sound mathematical basis to the theory of diffraction. He stated that at a homogeneous, source free medium the amplitude of a wave at any point A in the medium is given by

\[ P_A = \frac{1}{4\pi} \oint_S \left[ \frac{\partial}{\partial n} \left( \frac{e^{-jk\Delta r}}{\Delta r} - \frac{e^{-jk\Delta r}}{\Delta r} \right) \right] dS, \tag{A.1} \]

where S is the surface, enveloping the medium, \( \hat{n} \) is the inward pointing unity vector on S and \( \Delta r \) is the distance between point A and S. Some fifteen years later Rayleigh in his work on the passage of waves through apertures [3,4] discussed the boundary conditions \( \partial\phi/\partial n = 0 \) and \( \phi = 0 \) on S, where \( \phi \) in (A.1) is the Green function \( e^{-jk\Delta r}/\Delta r \). These boundary conditions apply only in the simplest case when S is a plane surface. In terms of figure 2.1 the contribution of surface \( S_1 \) was not included. Sommerfeld [5] did include surface \( S_1 \) in his calculations by letting radius R of \( S_1 \) go to infinity and stating that at infinity the contribution of the wave will be zero. Sommerfeld's statement was treated more rigorously by Born [6]. He stated, that the contribution of a source in volume V to the field on surface S may be expressed in terms of the distance R between A and S (see figure A.1):
Figure A.1: Configuration used by Born [8] for the calculation of the contribution of surface $S$ to the amplitude in $A$.

$$P_n \text{(on } S) = Q_n(\omega) \frac{e^{-jk\Delta r_n}}{r_n} = Q_n(\omega) \frac{e^{-jk(R+\Delta r_n)}}{R+R\Delta r_n}$$

$$= e^{-jkR} \left( A_n + B_n \frac{R}{R} + C_n \frac{R^2}{R^2} + \ldots \right), \quad (A.2)$$

where

$$A_n = Q_n(\omega) e^{-jk\Delta r_n},$$

$$B_n = -Q_n(\omega) r_n e^{-jk\Delta r_n},$$

$$C_n = Q_n(\omega) r_n^2 e^{-jk\Delta r_n}$$

and $Q_n(\omega)$ is the source strength. For a conglomerate of (monopole) sources in $V$ the contribution on $S$ would be

$$P_S = \sum_n P_n = e^{-jkR} \left( A + B \frac{R}{R} + C \frac{R^2}{R^2} + \ldots \right), \quad (A.3)$$

where $A = \sum_n A_n$, $B = \sum_n B_n$, etc.

If we rewrite Kirchhoff's equation in cylinder coordinates, and substitute the above result we find
\[ P_A = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \left\{ \frac{e^{-jkR}}{R^5} \frac{\partial p}{\partial n} - \frac{e^{-jkR}}{R^3} \frac{\partial p}{\partial n} \right\} R^2 \sin \theta \sin \phi \, d\phi \, d\theta \]

\[ = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \frac{e^{-2jkR}}{R^2} \left[ (jk-1/R) (A + B/R + C/R^2 + \ldots) + \frac{B}{R^2} + \frac{2C}{R^3} \right] R^2 \sin \theta \sin \phi \, d\phi \, d\theta \]

\[ = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} e^{-2jkR} \left\{ \frac{B}{R^2} + \frac{2C}{R^3} + \ldots \right\} \sin \theta \sin \phi \, d\phi \, d\theta. \quad (A.4) \]

As \( e^{-2jkR} \) is a bounded function we may conclude from equation (A.4) that the contribution of surface \( S_1 \) in figure 2.1 to the pressure in \( A \) vanishes for \( R \to \infty \).

In later work Born and Wolf [7] stated that \( R \) need not go to infinity if the pulses from the sources were broadband, causal and of limited duration. In this practical case the radius \( R \) need only be large enough to ensure that the contribution from \( S_1 \) has not yet reached point \( A \).

References appendix A


APPENDIX B
SPATIAL BAND LIMITATIONS OF SOURCE AND DETECTOR

In section 2.5 the zero offset model was discussed. In the step from equation (2.5.1) to equation (2.5.2) it was simply stated that the sources and detectors showed no angular preference. Hence the source and detector matrices became unity matrices and were dropped from the equation. As a result of this operation zero offset data acquisition could be described as in equation (2.5.4)

\[ \tilde{P}_{ZO} = S \sum_m W_{ZO}(z_o,z_m) \tilde{R}_{ZO}(z_m), \]  

(2.5.4)

where the elements of \( W_{ZO}(z_o,z_m) \) are the squared elements of the one way extrapolation matrix. In section 2.6 we discussed the error which occurs when the true \( W_{ZO}(z_o,z_m) \) matrix is replaced by the one way matrix with the half velocity substitution, and

\[ W^2 \approx W^\frac{1}{2}. \]  

(B.1)

was evaluated. If the source distribution is the same as the detector distribution, as is usually the case with zero offset data acquisition, then the source and detector matrices are each others transposed. In terms of equation (3.1.1) we find

\[ W_d(z_o,z_m) = W_s^T(z_m,z_o). \]  

(B.2)

where

\[ W_d(z_o,z_m) = D(z_o)W(z_o,z_m) \]
and
\[ W_s(z_m, z_o) = W(z_m, z_o) S(z_o). \]

So now the elements of \( W_{Z0}(z_o, z_m) \) are the squared elements of \( W_d(z_o, z_m) \)
\[ W_{Z0} = W_d^2. \]  \hspace{1cm} (B.3)

In practice however, the convolution with the detector distribution is often applied after using equation (2.5.4)
\[ \hat{P}_{Z0} = D(z_o) \sqrt{2 \pi} \sum_m W_{Z0}(z_o, z_m) R_{Z0}(z_m), \]  \hspace{1cm} (B.4)

where the sensitivity of sources and detectors \( S \) is incorporated in \( D(z_o) \). In equation (B.4) \( W_{Z0} \) may contain the squared elements of the one way matrix, or the half velocity substitution may have been used. So the elements of the true operator \( W_d \) are often approximated by
\[ W_d^2 \approx D * W^2, \]  \hspace{1cm} (B.5)
or by
\[ W_d^2 \approx D * W_s. \]  \hspace{1cm} (B.6)

The substitution of the half velocity operator was treated in section 2.6 and in the following the approximation of equation (B.5) will be discussed in some detail.

As an example we will take a strip transducer with width \( a \) and source strength \( 1/a \). In the far field the one way extrapolation operator is given by equation (2.6.1):
\[ W = \sqrt{jk} \cos \phi e^{-jkr}. \]  \hspace{1cm} (B.7)

The response at a distance \( r \) from the strip transducer may be calculated by
\[ P(\hat{r}, \phi) = \frac{1}{a} \int_{-a}^{a} \sqrt{\frac{jk}{2\pi}} \frac{e^{-jkr}}{\sqrt{r}} dx, \]  \hspace{1cm} (B.8)
where \(-\frac{1}{2}a \leq x \leq \frac{1}{2}a\) is the coordinate along the surface of the transducer. In the far field of the transducer we may substitute for \(r\)

\[ r = r_0 + x \sin \phi. \]  

(B.9)

Also assuming for the amplitude that \(r_0 + x \sin \phi \approx r_0\) and that the angle \(\phi\) is constant over the integration we find (far field assumption)

\[
P(r, \phi) = \frac{1}{2\pi} \sqrt{\frac{k \cos \phi}{r_0}} e^{-jk r_0} \int_{-\frac{1}{2}a}^{\frac{1}{2}a} e^{-j k x \sin \phi} dx
\]

\[
= \sqrt{\frac{k \cos \phi}{2\pi r_0}} e^{-jk r_0} \sin(\frac{1}{2} k \sin \phi) \frac{1}{\frac{1}{2} k \sin \phi}.
\]  

(B.10)

So we see that in the far field of a transducer with width \(a\) the one way extrapolation operator of equation (B.7) is weighted by a sinc function

\[
W_d = W \cdot \frac{\sin(\frac{1}{2} k \sin \phi)}{\frac{1}{2} k \sin \phi},
\]  

(B.11)

and we find for the true \(W_{Z0}\) in equation (B.3)

\[
W_{Z0} = W_d^2 = W^2 \frac{\sin(\frac{1}{2} k \sin \phi)}{\frac{1}{2} k \sin \phi}. \]  

(B.12)

If the convolution is performed after two way extrapolation, as in equation (B.5), then

\[
P(r, \phi) = \frac{1}{2\pi} \int_{-\frac{1}{2}a}^{\frac{1}{2}a} \frac{k \cos^2 \phi}{2\pi} \frac{e^{-2j k r}}{r} dx.
\]  

(B.13)

Under the same far field assumptions made above we find

\[
W_{Z0} = W_d^2 = W^2 \cdot \frac{\sin(k \sin \phi)}{k \sin \phi}.
\]  

(B.14)

Consequently if we want to simulate the source and detector directivity in this example as defined in equation (B.5) we must substitute a transducer width of \(\frac{1}{2}a\) in the convolution operator. In figures B.1a and B.1b the sensitivity of a zero offset registration of a point scatterer is plotted as a function of the spatial wavenumber \(k_x\), where
**Figure B.1:** Sensitivity of a zero offset registration of a point scatterer as a function of \( k_x \).

\[
\text{--- } = \frac{w^2}{k_x^2}
\]
\[
\ldots = D \cdot \frac{w^2}{k_x^2}, \text{ where } D = .2 \text{ mm for } a \text{ and } .8 \text{ mm for } b
\]
\[
\ldots = (D \cdot \frac{w}{k})^2, \text{ where } D = .4 \text{ mm for } a \text{ and } 1.6 \text{ mm for } b
\]

\( k_x = 2 \kappa \sin \phi \). The solid lines represent the situation where the transducer sensitivity is not dependent on the angle. The dotted lines were calculated by convolution, while the dashed lines represent the true directivity patterns. Figure B.2 shows the entire \( k_x - k_z \) space of both the convolutional model and the true case for two different depths, while figure B.3 gives the zero offset sensitivity for four values of \( z \). From these figures we may conclude the following:

- Convolution with a box of half the transducer width gives a reasonable approximation of the true situation as long as the transducer is small, or as long as aperture limitations only allow us to measure the main lobe.

- Convolution with a box over-estimates the sidelobe levels by approximately a factor 2 (in dB).

- Near the transducer, where the far field approximation is not valid, the directivity pattern is not a true sinc function. This is most clearly observed at the \( k_x \)-values where the sensitivity should be zero.

To simulate the true situation of equation (B.12) more accurately the omnidirectional zero offset response \( w^2 \) must be convolved in the \( x \)-direction with the convolution of the source and the detector.
Figure B.2: Wavenumber space representation of beam forming of a 1.6 mm element in a 5.11 cm aperture.

a) $D \times W^2$ for depth $z = 1.0$ cm
b) $(D \times W)^2$ for depth $z = 1.0$ cm
c) $D \times W^2$ for depth $z = 12$ cm
d) $(D \times W)^2$ for depth $z = 12$ cm

Figure B.3: Sensitivity of a zero offset registration of a point scatterer as a function of $k_x$ for a 1.6 mm element.

- = depth $z = 1.0$ cm
...
--- = depth $z = 6.0$ cm
--- = depth $z = 12.0$ cm
distributions. Now we will approximate $w_d^2$ by

$$w_d^2 \approx S \times D \times w^2,$$  \hspace{1cm} (B.15)

where again for our example we substitute half the transducer width in $S$ and $D$. Thus, in our example $S \times D$ is a triangular function with width $a$. In figure B.4 the solid line depicts the true sensitivity, while the dotted line was calculated according to the convolutional model of equation (B.15).

Figure B.4 shows that this model is a good approximation because the sidelobe levels are predicted correctly. Again, due to the lack of far field conditions, the true sensitivity does not reach zero between the lobes. It can be shown that true far field conditions do not occur under normal circumstances. Even at depths of $z = 100a$, where $a$ is the width of the transducer, differences could be seen at low sensitivity.

Finally, to complete the picture, the half velocity substitution will be applied in equation (B.15)

$$w_d^2 \approx S \times D \times w_{\frac{1}{2}}^2$$  \hspace{1cm} (B.16)

The same convolution procedure as described above was followed, and the result is depicted as the dashed line in figure B.4. From figure B.4 we see that both equations (B.15) and (B.16) accurately describe the sensitivity of the main lobe, and that the half velocity substitution over estimates the sidelobe levels. This over estimation at large spatial wavenumbers was already predicted in section 2.6.
APPENDIX C
NON-SYMMETRIC AMPLITUDE SPECTRA

In section 7.4 the special case was considered, where the spectrum of a pulse was symmetric around a non-zero frequency $f_c$. Here we will discuss the more general case of non-symmetric amplitude spectra.

Let us assume that any amplitude spectrum $|S(\omega)|$ may be split up into a symmetric part and an anti-symmetric part around a given non-zero frequency $f_c = \omega_c / 2\pi$. If for the moment we take a zero-phase signal, then

$$S(\omega) = S_1(\omega) + S_2(\omega), \quad (C.1)$$

where

$$S_1(\omega) = \frac{1}{2} \{A(\omega + \omega_c) + A(\omega - \omega_c)\}$$

is symmetric around $\omega = \omega_c$, and where

$$S_2(\omega) = \frac{1}{2} j \{B(\omega + \omega_c) - B(\omega - \omega_c)\}$$

is anti-symmetric around $\omega = \omega_c$. Rewriting this equation as in section 7.4 we find

$$S(\omega) = \frac{1}{2} A(\omega) \ast \{\delta(\omega + \omega_c) + \delta(\omega - \omega_c)\} + \frac{1}{2} j B(\omega) \ast \{\delta(\omega + \omega_c) - (\omega - \omega_c)\},$$

or after inverse Fourier transformation
\[ s(t) = a(t) \cos_\omega t + b(t) \sin_\omega t, \quad (C.2) \]

where \( a(t) \) and \( b(t) \) are the Fourier transforms of \( A(\omega) \) and \( B(\omega) \) respectively. Both \( a(t) \) and \( b(t) \) are real time functions, so their amplitude spectra are symmetric around \( \omega = 0 \) and their phase spectra are anti-symmetric. So in addition to the relationship

\[ S(\omega) = S^*(\omega) \quad (C.3) \]

we find

\[ S(\omega_c - \omega) = S^*(\omega_c + \omega), \quad (C.4) \]

where * denotes the complex conjugate. Rewriting equation (C.2)

\[ s(t) = a(t)\left(\cos_\omega t + \frac{b(t)}{a(t)} \sin_\omega t\right) \]

\[ = a(t)\left(\cos_\omega t + \tan(\psi(t)) \sin_\omega t\right) \]

\[ = \frac{a(t)}{\cos(\psi(t))} \{\cos(\psi(t)) \cos_\omega t + \sin(\psi(t)) \sin_\omega t\} \]

\[ = \sqrt{a^2(t) + b^2(t)} \cos(\omega_c t - \psi(t)), \quad (C.5) \]

where we have used the relationships \( \tan(\psi(t)) = b(t)/a(t) \) and \( \cos(\psi(t)) = a(t)/\sqrt{a^2(t) + b^2(t)} \).

For a symmetric spectrum, where \( b(t) = 0 \) we find that \( \psi(t) = \arctan\{b(t)/a(t)\} = 0 \) (see section 7.4).

For a non-symmetric spectrum \( s(t) \) may be represented by an instantaneous amplitude \( \sqrt{a^2(t) + b^2(t)} \) and an instantaneous phase \( \phi(t) = \omega_c t - \psi(t) \), where \( \psi(t) = \arctan\{b(t)/a(t)\} \).

Substituting equation (C.2) in the closed form instantaneous frequency estimator of equation (D.4) we find that the asymmetry of the amplitude spectrum introduces a bias in the estimate

\[ \langle \omega_c(t) \rangle = \langle 2\pi f_c(t) \rangle = \omega_c(t) + \frac{b(t) a'(t)/\partial t - a(t) b'(t)/\partial t}{a^2(t) + b^2(t)}, \quad (C.6) \]

where \( \langle . \rangle \) denotes an estimate. This last equation may also be found by
Figure C.1: Any non-symmetric amplitude spectrum (---) may be divided into a symmetric part (---) and an anti-symmetric part (---) around a central frequency.

Taking the time derivative of the instantaneous phase in equation (C.5)

\[
\frac{d}{dt} = \frac{d\phi(t)}{dt} = \omega_c(t) - \frac{\partial}{\partial t} \arctan \left( \frac{b(t)}{a(t)} \right). \tag{C.7}
\]

As an example we will choose for \( S_1(\omega) \) a top hat function and for \( S_2(\omega) \) a ramp. The frequency spectrum is band limited

\[ \omega_{\text{min}} \leq 2\pi f \leq \omega_{\text{max}}, \text{ where } \omega_{\text{max}} - \omega_{\text{min}} = b \text{ is the band width. So now (see figure C.1)} \]

\[ S(\omega) = \pi a_1/b + \pi a_2/b (\omega - \omega_c) \text{ for } \omega_{\text{min}} \leq \omega \leq \omega_{\text{max}} \tag{C.8} \]

and \( S(\omega) = 0 \) elsewhere. Using the inverse Fourier transform

\[ s(t) = \frac{1}{\pi} \int_0^\infty S(\omega) e^{j\omega t} d\omega \tag{C.9} \]

we find

\[ s(t) = \frac{1}{b} e^{j\omega_c t} \int_{-\frac{1}{2}b}^{\frac{1}{2}b} \left( a_1 - a_2 x \right) e^{jxt} dx \]

\[ = e^{j\omega_c t} \left\{ a_1 \frac{\sin\frac{1}{2}bt}{\frac{1}{2}bt} + \frac{a_2}{t} \left( \cos\frac{1}{2}bt - \sin\frac{1}{2}bt \right) \right\}. \tag{C.10} \]

We measure only the real part of \( s(t) \):

\[ \text{Re}\{s(t)\} = a_1 \frac{\sin\frac{1}{2}bt}{\frac{1}{2}bt} \cos\omega_c t + \frac{a_2}{t} \left( \frac{\sin\frac{1}{2}bt}{\frac{1}{2}bt} - \cos\frac{1}{2}bt \right) \sin\omega_c t \tag{C.11} \]
Figure C.2: The signal and its envelope, relating to the symmetric part (a) and the anti-symmetric part (b) in the spectrum of figure C.1.

Figure C.3: The signal and its envelope of the non symmetric amplitude spectrum of figure C.1 (a) and of a non symmetric amplitude spectrum with a positive slope (b).

Figure C.4: Instantaneous frequency of a signal with a symmetric amplitude spectrum (---) and with a positive (---) and a negative (---) spectral slope.
The symmetric first term on the right hand side of equation (C.11) is shown in figure C.2a, while the anti-symmetric second term is shown in figure C.2b. The resulting time signal is given in figure C.3a for a negative ramp and in C.3b for a positive ramp. Notice that there is a marked decrease in the frequency moving away from the main lobe in figure C.3a, while figure C.3b shows an increase in the frequency. This effect is confirmed in figure C.4, where the instantaneous frequency is given of the signals in figure C.3 as the dotted and dashed lines respectively.

The solid line in figure C.4 depicts the case of a symmetric amplitude spectrum, discussed in chapter VII. From the input parameters in equation (C.8) the true mean frequency can be calculated. The true mean frequency of the signal with a negative spectral slope was 2 2/3 MHz, while the true mean frequency of the signal with a positive spectral slope was 3 1/3 MHz.

Using equation (C.6) the bias in the instantaneous frequency estimate may be calculated. For this particular example we find, after some calculations that the bias is \( \pm 1/3 \) MHz at time \( t=0 \). So for \( t=0 \) \( \langle \omega_c(t) \rangle \) shows the true mean frequency. For \( t \to \infty \) the bias is \( \pm 1/2 \) b, depending on whether the slope of the spectrum is positive or negative.

As the example given here may be considered as a worst possible case, the experiment was repeated for a more realistic amplitude spectrum with a slight asymmetry. Figure C.5 gives the amplitude spectrum of this zero phase pulse on a linear scale, while figures C.6a and C.6b give the time signal and instantaneous frequency respectively. In figure C.6b the solid line depicts the unstabilized instantaneous frequency while the dotted and dashed lines show the instantaneous

![Figure C.5: Amplitude spectrum with a slight asymmetry.](image-url)
Figure C.6: a) The signal and its envelope of the spectrum in figure C.5.

b) Instantaneous frequency of a) without stabilization (___) and with a stabilization of \( \varepsilon \) of 5% (…) and 25% (---).

stabilized to a prior estimate of \( f_c \) with a stabilization factor \( \varepsilon \) of 5% and 25% of the maximal signal amplitude (See also appendix D, equation (D.7)).

In conclusion we may say that the stabilized instantaneous frequency estimator can only be used as an estimator for the true mean frequency if the amplitude spectra of the individual pulses are approximately symmetric. Even when the amplitude spectra are approximately symmetric, as in the previous example, a stabilization factor \( \varepsilon \) of 25% or larger is needed to produce a stable estimate of the mean frequency.
Generally an arbitrary real time series may be described by

\[ s(t) = a(t) \cos \phi(t), \quad (D.1) \]

where \( a(t) \) is the instantaneous amplitude and \( \phi(t) \) is the instantaneous phase. The instantaneous frequency is now defined as the time derivative of the instantaneous phase

\[ f_c(t) = \frac{1}{2\pi} \frac{3\phi(t)}{3t} \quad (D.2) \]

For the stabilization criterion the envelope \( a(t) \) must first be calculated

\[ a(t) = \sqrt{s^2(t) + H^2(s(t))}, \quad (D.3) \]

where \( H(.) \) denotes a Hilbert transform. Now we find

\[ a(t)f_c(t) = -\frac{1}{2\pi} \left[ s(t) \frac{3}{3t} \left( \frac{H(s(t))}{a(t)} \right) - H(s(t)) \frac{3}{3t} \left( \frac{s(t)}{a(t)} \right) \right]. \quad (D.4) \]

The multiplication of signals in the right half of this equation may cause problems for broadband sampled time signals. If the low frequency component and the double frequency components overlap in the frequency domain, the double frequency components in equation (D.4) do not cancel out completely. Therefore a low pass filter was used and its cut-off
Figure D.1: a) 16 delta pulses and b) band limited version of a, where $f_c$ ranges from 0.5 MHz for the four pulses in window 1 to $f_c = 2$ MHz for the pulses in window 4.

Figure D.2: Instantaneous frequency of the signal in figure D.1b as calculated by equation (D.5) for a stabilization factor $\varepsilon$ of 0% in a), 1% in b), 10% in c) and 50% in d). The dotted lines denote the ideal case.
frequency was set at half the Nyquist frequency, thus making this stabilized algorithm accurate for $f_c < 0.5 f_{Nyq}$ only. An unstable, unbiased frequency estimator is found by dividing equation (D.4) by the envelope $a(t)$. A stable, but biased frequency estimator may be found by multiplying equation (D.4) by a stabilized inversion factor

$$<f_c(t)> = \frac{a(t)}{a^2(t) + \varepsilon^2} \{a(t)f_c(t)\}, \quad (D.5)$$

where $\varepsilon$ is a fraction of the maximal signal amplitude. In figure D.1 a signal is shown, containing 16 band limited, zero phase pulses. The centre frequency ranges from .5 MHz in 1 to 2 MHz in 4. Note that the envelopes of 1 and 4 and of 2 and 3 respectively are practically equal, while the centre frequency varies. Figure D.2 shows the effect of stabilization, as described by equation (D.5). The stabilization $\varepsilon$ is increased from 0 in figure D.2a to 50% of the maximal signal amplitude in figure D.2d. The stabilization introduces a bias, as $f_c$ drops to zero for large $\varepsilon$. If we may assume that $f_c$ does not vary too much in time, then the bias may be reduced considerably by not stabilizing to zero as in equation (D.5), but to some prior estimate of $f_c$:

$$<f_c(t)> = \frac{a(t)}{a^2(t) + \varepsilon^2} \{a(t)f_c(t) - a(t)F_c\} + F_c, \quad (D.6)$$

where $F_c$ is the prior estimate. To reduce the bias even further a combination of the unstabilized instantaneous frequency and the stabilized instantaneous frequency of equation (D.6) was applied.

$$<f_c(t)> = \frac{1}{a(t)} \{a(t)f_c(t)\} \quad \text{for } a(t) \geq \varepsilon$$

$$<f_c(t)> = \frac{2a(t)}{a^2(t) + \varepsilon^2} \{a(t)f_c(t) - a(t)F_c\} + F_c \quad \text{for } a(t) < \varepsilon. \quad (D.7)$$

This estimator was applied to the example of figure D.1. As the centre frequency varies considerably in this example the prior estimate was set at zero. Comparing the result in figure D.3 with the previous result of figure D.2, we see that for large signal amplitude the bias has been reduced considerably, while the stabilization is still effective.

Finally figures D.4 and D.5 show an example of 50 closely spaced
Figure D.3: Instantaneous frequency of D.1b, calculated by equation (D.7), where \( \varepsilon = 10\% \) in a) and \( \varepsilon = 50\% \) of the maximal signal amplitude in b). The dotted lines denote the ideal case.

Figure D.4: a) Time signal from 50 randomly spaced scatterers in an attenuating medium. Time variant gain added.

b) Amplitude spectrum of the reflected signal (—) and the emitted pulse (---).

scatterers in an attenuating medium, where the centre frequency of the emitted pulse was approximately 2.5 MHz. Figure D.4a shows the time signal from the 50 randomly placed scatterers, while figure D.4b gives the amplitude spectrum as the solid line. The Gaussian shaped amplitude spectrum of the emitted pulse is depicted as a dotted line in figure D.4b. The mean of the Gaussian, as defined in equation (6.1.1), is \( \mu = 2.5 \text{ MHz} \) and the standard deviation \( \sigma = 0.5 \text{ MHz} \). The attenuation coefficient was defined as \( \alpha = 0.5 \text{ dB/MHz cm} \). In figure D.5a the instantaneous frequency is given of the signal in D.4a, as calculated by equation (D.7) with \( F_c = 2.45 \text{ MHz} \) and \( \varepsilon = 10\% \) of the maximal signal.
amplitude. Figure D.5b shows the same for an $\varepsilon = 50\%$ of the maximal signal amplitude. For the Gaussian shaped amplitude spectrum the frequency shift may be calculated as a function of depth (section 7.7). The dashed line in figure D.5b gives the expected frequency according to equation (7.7.1). In figure D.5c 16 realizations of the experiment previously described were averaged to reduce the error in the final estimate. Again the dashed line gives the expected frequency. In conclusion we may say that, in randomly scattering media, stabilization and averaging must take place to produce an accurate estimate of the instantaneous frequency for attenuation measurements.
The relationship is expressed in terms of both, as indicated by equation
\[ y = ax + \beta \]
where \( y \) and \( x \) are the variables of interest. The constants \( a \) and \( \beta \) are determined through regression analysis.

In Figure 0.4, the time signal from the 50 randomly placed scatterers, while Figure 0.4b gives the amplitude spectrum of the signal. The Gaussian shaped amplitude spectrum of the emitted pulse is depicted as a dotted line in Figure 0.4b. The mean of the Gaussian, as defined in equation (6.1), is \( \mu = 2.5 \text{ MHz} \) and the standard deviation \( \sigma = 0.5 \text{ MHz} \). The attenuation coefficient was defined as \( a = 0.5 \text{ dB/kr} \). In Figure 0.5a, the instantaneous frequency is given of the signal in 0.4b, as calculated by equation (8.7) with \( F = 2.43 \text{ MHz} \) and \( \epsilon = 10\% \) of the maximal signal.
SUMMARY

Echo acoustic techniques may be subdivided into three separate parts: data acquisition, data processing and information extraction. Before data acquisition a choice must be made as to the kind of information needed regarding the target. If the zero offset or backscatter mode is chosen the target information will yield a good estimate of the product of local sound velocity and density variations. On the other hand, if a multi-offset mode of data acquisition is chosen, the velocity and density information may be separated. Once the data acquisition choice has been made the basic limitations of the measurement system are fixed. In this thesis these limitations are expressed in the form of an acoustic "window" in Fourier space.

One of the aims of data processing is to produce an undistorted, high resolution and "low noise" image of the target. In this thesis the basic tools in the processing are inverse wave field extrapolation and imaging. An estimation of the propagation parameters must be made prior to applying the wave field extrapolation technique. Correct estimation of these propagation parameters produces the optimal high quality image needed for the information extraction.

Current efforts in acoustic imaging are moving towards quantitative information extraction and medium characterization, where the high quality image is analysed to produce detailed information about the local reflectivity (or velocity and density) parameters. In this thesis techniques are discussed which extract both propagation and reflectivity parameters from measured acoustic echo data.

The results of this work not only apply to the traditional fields of acoustic inversion, but also form a link with the field of inverse scattering.
SAMENVATTING

Echo akoestische technieken kunnen worden onderverdeeld in drie aparte delen: - opname, verwerking en interpretatie van gegevens.

Bij de opname moet een keuze worden gemaakt ten aanzien van het soort informatie over het te meten object, waarover men wenst te beschikken. Als de bron en ontvanger samenvallen kan in praktische situaties alleen de reflectiviteit, het produkt van geluidsnelheids- en dichtheidsvariaties, worden bepaald. Wanneer daarentegen de bron en ontvanger ten opzichte van elkaar worden verschoven, dan is het mogelijk om de geluidsnelheids- en dichtheidsvariaties te scheiden. Is het opname systeem eenmaal gedefinieerd, dan zijn ook de grenzen ervan bepaald. In dit proefschrift worden de grenzen van het opname systeem beschreven in de vorm van een akoestisch venster in de Fourier ruimte.

Een van de doelstellingen van dataverwerking is een onvormd beeld van het object te maken, waarvan de resolutie hoog en het "ruisniveau" laag is. In dit proefschrift vormen inverse golfveld extrapolatie en afbeelding de basis van de verwerkingstechnieken. Voordat inverse golfveld extrapolatie kan worden toegepast moet een schatting worden gemaakt van de voortplantings parameters. Een goede schatting van deze parameters is noodzakelijk om een beeld van hoge kwaliteit te verkrijgen, welke nodig is voor de interpretatie van de gegevens.

Bij akoestische afbeeldingstechnieken wordt tegenwoordig steeds meer aandacht besteed aan de kwantitatieve interpretatie van gegevens en de typering van het medium. Hierbij wordt het beeld geanalyseerd om gedetailleerde informatie te verkrijgen over de plaatselijke reflectiviteits (of geluidsnelheids en dichtheids) parameters. In dit proefschrift worden technieken besproken, waarmee men zowel de voortplantings als de reflectiviteits parameters kan schatten uit de gemeten akoestische data.

De resultaten van dit onderzoek kunnen worden gebruikt bij de traditionele akoestische inversie technieken, maar bovendien wordt een brug geslagen naar het gebied van de akoestische verstrooing.
CURRICULUM VITAE

Surname : Medesag, given names Peter Robert
Date of birth : 23 July 1952
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1975 - 1976 : Physics teacher at the secondary school Christelijke Mavo in Waddinxveen - The Netherlands
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CURRICULUMVERTEGENW.

In het algemeen worden de onderwerpen in deze afdeling "Ontwikkeling van de reflexiviteit" en "Reflectiviteit van de wetenschap" besproken. De reflexiviteit is een belangrijke eigenschap van wetenschappelijk materiaal, wat betekent dat het materieel bewust en actief wordt geanalyseerd en bewerkt. Dit proces van reflexiviteit kan worden gedefinieerd als een interactie tussen de reflecterende persoon en het materiaal dat bekeken wordt. De reflexiviteit is een cruciaal element voor het begrijpen van de werking van wetenschap en van de betekenis van dat materiaal voor de reflecterende persoon.
STELLINGEN
behorende bij het proefschrift
Estimation of medium parameters by acoustic echo measurements

1. Als de investeringen in de seismische en in de medische
toepassingen van de echo akoestiek een maatstaf zouden zijn voor
het maatschappelijk belang, dan is het slecht gesteld met onze
gezondheidszorg.

2. Een van de problemen bij transmissie metingen is, dat men
informatie over verstrooiers probeert te halen uit metingen, waarin
het invallende veld overheerst.

3. Er bestaan meer verbanden tussen akoestische verstrooing bij bloed
en optische verstrooing bij Pernod dan men op het eerste gezicht
zou vermoeden.

4. Als een test mislukt leidt dit vaak tot belangrijkere conclusies
dan als dit niet het geval zou zijn geweest.

5. Zelfs met een hoge mate van standaardisering in computer software
is het meestal sneller om eigen software te ontwikkelen dan om
andermans programma's aan te passen.

6. Tijdsdruk is helaas dikwijls een belangrijk onderdeel van het
leveren van topprestaties.

7. In een democratie zijn splinterpartijen belangrijk om hun signaal-
functie, niet om hun stemgedrag.

8. Kinderen nemen of geen kinderen nemen is een beslissing vanuit een
zuiver egolstisch standpunt, maar maatschappelijk gezien is het
eerste een noodzaak.

9. Hoe verder de geneeskunst voortschrijdt, hoe ongeneeslijker de
ziekten.
10. Bij het verleggen van grenzen moeten onvoorspelbare risico's worden genomen.

11. De mooiste momenten in elke sport zijn die, waarop grenzen worden verlegd.

12. Hoe hoger het niveau van een examen, hoe meer de meester wordt getoetst en hoe zwaarder de beslissing tot afwijzen.

3 december 1985    Peter R. Mesdag