Definition, transformation-formulae and measurements of tipvane angles

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Summary

This report contains the theoretical background of the different angle systems used to define the attitude of the tipvane in the 3-dimensional space.

The angle system is based on the Euler system.

Different Euler angle types were used for the various wind tunnel-, towing tank- and full-scale tipvane models.

For these different angle types the definitions are given in vector notation. From these vector notations the transformation formulae could easily be derived. The influence of a rotor blade flapping angle on the tipvane angles is included.

Also the method to measure the attitude of the tipvane and transformation from these measured angles to the Euler angle system is outlined.

Finally some side effects are described on the angle of attack of the tipvane due to rotation, translation and curving of the tipvane.
1. Symbols

$b$ vector in the B-direction of the local tipvane coordinate system (span wise direction)

$b''$ vector in the B-direction of the local tipvane coordinate system that is rotated over $\Lambda_w$ and $\theta_w$

$b_m$ vector in the B-direction of the local tipvane coordinate system that is rotated over $\gamma_m, \theta_m$ and $\Lambda_m$ or $\Lambda_a, \gamma_a, \theta_a$ and $\Lambda_m$.

$B$ x-axis of the local tipvane coordinate system

$c$ vector in the C-direction of the local tipvane coordinate system (chord wise direction)

$c^*$ vector in the C-direction of the local tipvane coordinate system that is rotated over $\theta_c$

$c'$ vector in the C-direction of the local tipvane coordinate system that is rotated over $\Lambda_a$ and $\gamma_a$

$c'''$ vector in the C-direction of the local tipvane coordinate system that is rotated over $\gamma_p$ and $\theta_p$

$c_m$ vector in the C-direction of the local tipvane coordinate system that is rotated over $\gamma_m', \theta_m$ and $\Lambda_m$ or $\Lambda_a', \gamma_a', \theta_a$ and $\Lambda_m$. 


c  
z-axis of the local tipvane coordinate system

i  
unit vector in the X-direction of the tipvane reference coordinate system

j  
unit vector in the Y-direction of the tipvane reference coordinate system

k  
unit vector in the Z-direction of the tipvane reference coordinate system

n  
vector in the N-direction of the local tipvane coordinate system (normal on the tipvane surface)

n*  
vector in the N-direction of the local tipvane coordinate system that is rotated over $\theta_c$

n'  
vector in the N-direction of the local tipvane coordinate system that is rotated over $\alpha_a$ and $\gamma_a$

n''  
vector in the N-direction of the local tipvane coordinate system that is rotated over $\Lambda_w$ and $\theta_w$

n'''  
vector in the N-direction of the local tipvane coordinate system that is rotated over $\gamma_p$ and $\theta_p$

N  
Y-axis of the local tipvane coordinate system

R  
tip radius of the rotor (m)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
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<tr>
<td>$U$</td>
<td>undisturbed wind velocity</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$v_{b_{hor}}$</td>
<td>tipvane measurement angles defined in section 7.1</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$v_{b_{vert}}$</td>
<td>tipvane measurement angle defined in section 7.1</td>
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<tr>
<td>$v_{c_{hor}}$</td>
<td>tipvane measurement angle defined in section 7.1</td>
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</tr>
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<td>$v_{c_{vert}}$</td>
<td>tipvane measurement angle defined in section 7.1</td>
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<tr>
<td>$X$</td>
<td>x-axis of the tipvane reference coordinate system</td>
<td></td>
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<td>y-axis of the tipvane reference coordinate system</td>
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<tr>
<td>$\beta$</td>
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<td>(degrees)</td>
</tr>
<tr>
<td>$\gamma$</td>
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<tr>
<td>$\Delta l$</td>
<td>displacement of the tipvane due to the wagging effect</td>
<td>(m)</td>
</tr>
<tr>
<td>$\Delta v_{c_{hor}}$</td>
<td>difference in the tipvane measurement angle $v_{c_{hor}}$; denotes accuracy, see section 7.3</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\Delta \theta$</td>
<td>change of the incidence angle $\theta$, see fig. 17</td>
<td>(degrees)</td>
</tr>
</tbody>
</table>
\( \theta \) incidence angle of the tipvane (degrees)

\( \theta_{\text{correction}} \) \( \left( = \frac{\Delta \theta}{R} \cdot \frac{180}{\pi} \right) \) correction of the incidence angle of the tipvane due to shifting of the tipvane (degrees)

\( \Lambda \) sweep angle of the tipvane (degrees)

\( \psi \) tipvane offset angle, see fig. 16 (degrees)

\( \Omega \) angular speed of the rotor (rad/s)

Indices

a indicates the aerodynamic angles

c indicates the construction angles

k indicates the Kolibrie mounting part angles

m indicates the wind tunnel model angles

R = 0.36 m and R = 0.468 m

w indicates the (towing tank) model angles

\( \beta \) indicates flapping angle
2. Introduction

The position of the tipvane in a coordinate system is fixed by three angles: the sweep, the tilt and the incidence angle.

The system used to define the angles is the Euler system. In the Euler definition the coordinate system is rotated over the first angle. The second angle is defined in this rotated coordinate system. The third angle is then defined in a coordinate system that is rotated over the first two angles.

The sequence in which the angles are introduced is very important. For the same position of the tipvane the values of the three Euler angles are different if the sequence of rotation is different.

The choice of the systems depends on the specific purpose. For example, for aerodynamic calculation a different choice is convenient than for the construction of models.

Different Euler tipvane angle systems that have been used for tipvane measurements are described in this report. Also the relations between the different types of angles are given.
3. Coordinate systems

All coordinate systems are Cartesian and right hand orientated.

3.1 The tipvane reference coordinate system

The definition of the reference coordinate system is given in fig. 1.
The axis of this Cartesian coordinate system are X, Y, Z with the unit
vectors \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k} \). ( \( \mathbf{k} = \mathbf{i} \times \mathbf{j} \) )
The origin of the X, Y, Z reference system is in the middle of the
mid span chord of the tipvane.
The X-axis is parallel with the rotor shaft and positive in the down
wind direction. The Y-axis is radially orientated and positive in the
outward direction.

3.2 The local tipvane coordinate system

The definition of the local tipvane coordinate system is given in fig.
2.
This is together with the tipvane rotated over the three Euler angles.
The axis are B, N and C with the unit vectors \( \mathbf{b}, \mathbf{n}, \) and \( \mathbf{c} \). ( \( \mathbf{c} = \mathbf{b} \times \mathbf{c} \) )
The origin of the local system is the same as the origin of the
tipvane reference coordinate system.
When the three tipvane angles are zero, i.e.:
\[
\begin{align*}
\alpha &= 0 \\
\gamma &= 0 \\
\theta &= 0
\end{align*}
\]
then the local tipvane coordinate system coincides with the tipvane
reference coordinate system.
The B-axis is in span wise direction and positive in down wind
direction.
The N-axis as perpendicular to the B and C axis and positive in the
outward direction (over pressure side of the tipvane).
The C-axis is in chord wise direction, positive towards the trailing
edge.
4. Tipvane angle types

Fig. 3 gives the positive directions of the 3 tipvane angles $\Lambda$ (sweep), $\gamma$ (tilt) and $\theta$ (incidence).

The positive directions of $\Lambda$ and $\gamma$ are in agreement with a right hand orientated coordinate system. A positive increment of $\theta$ gives a positive increment of $\alpha$, the angle of attack of the tipvane.

Table 1 gives an overview of the different angles types. (pag. 34)

4.1 The aerodynamic angles

The sequence of rotation for the 3 Euler angles is in the case of the "aerodynamic" angles:

1. $\Lambda_a$, i.e. rotation around the N-axis of the local system
2. $\gamma_a$, i.e. rotation around the C-axis of the local system
3. $\theta_a$, i.e. rotation around the B-axis of the local system.

The last rotation is $\theta_a$. This has the advantage that a change in $\theta_a$ corresponds with the same change in $\alpha$.

For all the aerodynamic calculations this set of aerodynamic tipvane angles is used.

After the rotation over $\Lambda_a$ and $\gamma_a$ the components of the vectors $b$, $n'$ and $c'$ (see fig. 4) are, expressed in the $i$, $j$ and $k$ vectors of the tipvane reference coordinate system:

\[
\begin{align*}
  b &= i \cos \gamma_a \cos \Lambda_a \\
  &= j \sin \gamma_a \\
  &= k \cos \gamma_a \sin \Lambda_a
\end{align*}
\]

\[
\begin{align*}
  n' &= i \sin \gamma_a \cos \Lambda_a \\
  &= j \cos \gamma_a \\
  &= k \sin \gamma_a \sin \Lambda_a
\end{align*}
\]

and
\[ i \sin \Lambda_a \]
\[ c' = j 0 \]
\[ k \cos \Lambda_a \]

The last rotation is the rotation \( \theta_a \) about the work line of \( b \) in the \( n', c' \) plane (see fig. 5).

The vectors \( n \) and \( c \) expressed in \( n', c' \) become:

\[ n = n' \cos \theta_a - c' \sin \theta_a \]
\[ c = n' \sin \theta_a + c' \cos \theta_a. \]

And expressed in \( i, j \) and \( k \):

\[ \begin{align*}
\hat{n} &= i \sin \gamma_a \cos \Lambda_a \cos \theta_a - \sin \Lambda_a \sin \theta_a \\
&\quad + k \sin \gamma_a \sin \Lambda_a \cos \theta_a - \cos \Lambda_a \sin \theta_a \\
\hat{c} &= i \sin \gamma_a \cos \Lambda_a \sin \theta_a + \sin \Lambda_a \cos \theta_a \\
&\quad + k \sin \gamma_a \sin \Lambda_a \sin \theta_a + \cos \Lambda_a \cos \theta_a
\end{align*} \]

4.2 Construction angles

4.2.1 Construction angles general

The only difference with the aerodynamic angles is the different sequence of rotation of the Euler angles. The sequence of rotation of the 3 Euler angles is:

1. \( \theta_c \), rotation around the B-axis of the local system
2. \( \Lambda_c \), rotation around the N-axis of the local system
3. \( \gamma_c \), rotation around the C-axis of the local system.

The construction angles were used during the manufacturing process of the mounting parts of the FACT-rotor.
The first rotation is the $\theta_c$ rotation about the $i$ vector: in the $j, k$ plane. This gives the $b, n, c$ vector. Vector $b$ coincides with vector $i$. See fig. 6.

The vectors $n$ and $c$ are:

$$n^* = i \cos \theta_c - k \sin \theta_c$$
$$c^* = i \sin \theta_c + k \cos \theta_c$$

After the rotation $\theta_c$ the next rotation $\Lambda_c$ is carried out in the $i, n$, and $c$ system. The last rotation is $\gamma_c$. See fig. 7.

The vectors $b, n$ and $c$ become, expressed in $i, n$ and $c$ (comparable with $b, n'$ and $c'$ in the aerodynamic tipvane angle system):

$$b = n \cos \gamma_c$$
$$c = - \sin \gamma_c \cos \Lambda_c$$

$$n = n \cos \gamma_c$$
$$c = \sin \gamma_c \sin \Lambda_c$$

and

$$c = n \sin \Lambda_c$$
$$c = n \cos \Lambda_c$$

Expressed in the $i, j$ and $k$ components of the reference coordinate system the vectors $b, n$ and $c$ of the local tipvane coordinate systems are:

$$b = i \cos \Lambda_c \cos \gamma_c$$
$$b = j \cos \theta_c \sin \gamma_c - \sin \theta_c \sin \Lambda_c \cos \gamma_c$$
$$k = - \sin \theta_c \sin \gamma_c - \cos \theta_c \sin \Lambda_c \cos \gamma_c$$
$$i = \sin \gamma_c \cos \Lambda_c$$
\[ n = \begin{bmatrix} 1 & \cos \theta_k \cos \gamma_k + \sin \theta_k \sin \gamma_k \sin \Lambda_k \\ 1 & \sin \theta_k \cos \gamma_k + \cos \theta_k \sin \gamma_k \sin \Lambda_k \end{bmatrix} \]

and

\[ c = \begin{bmatrix} 1 & \sin \Lambda_k \\ 1 & \sin \theta_k \cos \Lambda_k \end{bmatrix} \]

4.2.2 Angles of the Kolibrie mounting parts

The sequence of the Euler angles is the same as for the construction angles:

1. \( \theta_k \)
2. \( \Lambda_k \)
3. \( \gamma_k \)

Due to a mistake the rotation \( \gamma_k \) is not introduced around the swept chord vector \( c \) but around the non swept \( c \) (i.e. around an axis which does not include rotation over the angle \( \Lambda_k \)).

This gives for the vectors \( b, n \) and \( c \), including the rotation over the non swept vector \( c \):

\[ b = \begin{bmatrix} 1 & \cos \Lambda_k \cos \gamma_k \\ 1 & \cos \theta_k \cos \Lambda_k \sin \gamma_k - \sin \theta_k \sin \Lambda_k \end{bmatrix} \]

\[ n = \begin{bmatrix} 1 & \sin \gamma_k \\ 1 & \cos \theta_k \cos \Lambda_k \sin \gamma_k \end{bmatrix} \]

\[ c = \begin{bmatrix} 1 & \sin \Lambda_k \cos \gamma_k \\ 1 & \cos \theta_k \sin \Lambda_k \sin \gamma_k + \sin \theta_k \cos \Lambda_k \end{bmatrix} \]
4.3 Wind tunnel model angles

4.3.1 Towing tank model $\theta = 0.18$ m tunnel model

These angles are indexed with $w$.

The sequence of rotation of the three Euler angles is:

1. $\Lambda_w$
2. $\theta_w$
3. $\gamma_w$

This set of angles is used for the towing tank / wind tunnel model. The difference with the aerodynamic angles is that the sequence of $\gamma$ and $\theta$ is interchanged. After the setting of the first two angles the components of the vectors $b''$, $n''$, $c$ (see fig. 8) are, when expressed in the $i$, $j$, $k$ vectors of the tipvane reference coordinate system:

\[
\begin{align*}
    b'' &= i \cos \Lambda_w + j \sin \Lambda_w \\
    n'' &= i \cos \theta_w \sin \Lambda_w + j \cos \theta_w \cos \Lambda_w \\
    c &= i \cos \gamma_w \sin \Lambda_w + j \cos \gamma_w \cos \Lambda_w
\end{align*}
\]

The last rotation is the rotation over the angle $\gamma_w$ about the $c$ vector in the $b''$, $n''$ plane (see fig. 9).

The vectors $b$ and $n$ expressed in $b''$ and $n''$ become:

\[
\begin{align*}
    b &= b'' \cos \gamma_w + n'' \sin \gamma_w \\
    n &= -b'' \sin \gamma_w + n'' \cos \gamma_w
\end{align*}
\]
\[ b_m = b \cos \Lambda_m - n' \sin \theta_a \sin \Lambda_m - c' \cos \theta_a \sin \Lambda_m \]
\[ n = n' \cos \theta_a + c' \sin \theta_a \]
\[ c_m = b \sin \Lambda_m + n' \sin \theta_a \cos \Lambda_m + c' \cos \theta_a \cos \Lambda_m. \]

And expressed in the \(i, j\) and \(k\) components:
\[ i \cos \gamma_a \cos \Lambda_a \cos \Lambda_m + \sin \gamma_a \cos \Lambda_a \sin \theta_a \sin \Lambda_m + \sin \Lambda_a \cos \theta_a \sin \Lambda_m - \sin \Lambda_a \cos \theta_a \sin \Lambda_m \]
\[ b_m = i \sin \gamma_a \cos \Lambda_m - \cos \gamma_a \sin \theta_a \sin \Lambda_m \]
\[ k = - \cos \gamma_a \sin \Lambda_a \cos \Lambda_m - \sin \gamma_a \sin \Lambda_a \sin \theta_a \sin \Lambda_m \]
\[ - \cos \Lambda_a \cos \theta_a \sin \Lambda_m \]
\[ i = - \sin \gamma_a \cos \Lambda_a \cos \theta_a - \sin \Lambda_a \sin \theta_a \]
\[ n = i \cos \gamma_a \cos \theta_a \]
\[ k = \sin \gamma_a \sin \Lambda_a \cos \theta_a - \cos \Lambda_a \sin \theta_a \]

and
\[ i \cos \gamma_a \cos \Lambda_a \sin \Lambda_m - \sin \gamma_a \cos \Lambda_a \sin \theta_a \cos \Lambda_m \]
\[ + \sin \Lambda_a \cos \theta_a \cos \Lambda_m + \sin \Lambda_a \cos \theta_a \cos \Lambda_m \]
\[ c_m = i \sin \gamma_a \sin \Lambda_m + \cos \gamma_a \sin \theta_a \cos \Lambda_m \]
\[ k = - \cos \gamma_a \sin \Lambda_a \sin \Lambda_m + \sin \gamma_a \sin \Lambda_a \sin \theta_a \cos \Lambda_m \]
\[ + \cos \Lambda_a \cos \theta_a \cos \Lambda_m \]

\(\Lambda_a = 0\) for the \(R = 0.36\) m wind tunnel model. The expressions then become:
\[ b_m = i \cos \gamma_a \cos \Lambda_m + \sin \gamma_a \sin \theta_a \sin \Lambda_m \]
\[ n = i \sin \gamma_a \cos \Lambda_m - \cos \gamma_a \sin \theta_a \sin \Lambda_m \]
\[ k = - \cos \theta_a \sin \Lambda_m \]
\[ i - \sin \gamma_a \cos \theta_a \]
\[ n = i \cos \gamma_a \cos \theta_a \]
\[ k = -\sin \theta_a \]

and

\[ i \cos \gamma_a \sin \Lambda_m - \sin \gamma_a \sin \theta_a \cos \Lambda_m \]
\[ c_m = i \sin \gamma_a \sin \Lambda_m + \cos \gamma_a \sin \theta_a \cos \Lambda_m \]
\[ k \cos \theta_a \cos \Lambda_m \]

And written with the correct index \( m \) of the \( R = 0.36 \) model (the Euler sequence is 1: \( \gamma_m \), 2: \( \theta_m \), 3: \( \Lambda_m \)):

\[ i \cos \gamma_m \cos \Lambda_m + \sin \gamma_m \sin \theta_m \sin \Lambda_m \]
\[ b_m = i \sin \gamma_m \cos \Lambda_m - \cos \gamma_m \sin \theta_m \sin \Lambda_m \]
\[ k = \cos \theta_m \sin \Lambda_m \]

\[ i - \sin \gamma_m \cos \theta_m \]
\[ n = i \cos \gamma_m \cos \theta_m \]
\[ k = -\sin \theta_m \]

\[ i \cos \gamma_m \sin \Lambda_m - \sin \gamma_m \sin \theta_m \cos \Lambda_m \]
\[ c_m = i \sin \gamma_m \sin \Lambda_m + \cos \gamma_m \sin \theta_m \cos \Lambda_m \]
\[ k \cos \theta_m \cos \Lambda_m \]

4.4 Production angles

These angles are indexed with \( p \). The sequence of rotation of the 3 Euler angles is:

1. \( \gamma_p \)
2. \( \Lambda_p \)
3. \( \theta_p \)

The difference with the aerodynamic angles is that \( \gamma \) and \( \Lambda \) are interchanged. Accidentally this sequence was used by the production of the tipvanes for the FACT rotor.
After the first two angles have been set, the components of the vectors \( \mathbf{b}, \mathbf{n}'''', \mathbf{c}'''' \) (see fig. 11) expressed in the \( i, j \) and \( k \) vectors of the tipvane reference coordinate system are:

\[
\begin{align*}
\mathbf{b} &= i \cos \Lambda_p \cos \gamma_p \\
&\quad + j \cos \Lambda_p \sin \gamma_p \\
&\quad - k \sin \Lambda_p \\
\mathbf{i}' &= - \sin \gamma_p \\
\mathbf{n}''' &= i \cos \gamma_p \\
&\quad + j \sin \gamma_p \\
&\quad - k 0 \\
\mathbf{c}''' &= i \sin \Lambda_p \cos \gamma_p \\
&\quad + j \sin \Lambda_p \sin \gamma_p \\
&\quad - k \cos \Lambda_p 
\end{align*}
\]

The last rotation is \( \theta_p \) about the \( b \) in the \( n'''', c'''' \) plane (see fig. 12).

The vectors \( \mathbf{n} \) and \( \mathbf{c} \) expressed in \( n'''', c'''' \) become:

\[
\begin{align*}
\mathbf{n} &= - \mathbf{c}'''' \sin \theta_p + \mathbf{n}''' \cos \theta_p \\
\mathbf{c} &= \mathbf{c}'''' \cos \theta_p + \mathbf{n}''' \sin \theta_p 
\end{align*}
\]

And expressed in \( i, j \) and \( k \):

\[
\begin{align*}
\mathbf{i} &= - \sin \Lambda_p \cos \gamma_p \sin \theta_p - \sin \gamma_p \cos \theta_p \\
\mathbf{n} &= i \sin \Lambda_p \sin \gamma_p \sin \theta_p + \cos \gamma_p \cos \theta_p \\
&\quad - k \cos \Lambda_p \sin \theta_p \\
\mathbf{c} &= i \sin \Lambda_p \cos \gamma_p \cos \theta_p - \sin \gamma_p \sin \theta_p \\
&\quad + k \cos \Lambda_p \cos \theta_p 
\end{align*}
\]
5. Aerodynamic angles with blade flapping angle included

The flapping angle $\beta$ rotates the $i$ and $j$ vectors of the tipvane reference coordinate system. The $k$ vector is only translated a little down wind parallel to itself. See fig. 13.

The vectors in the flapped tipvane reference coordinate system are called $i_\beta$, $j_\beta$ and $k_\beta$.

The components $i_\beta$, $j_\beta$ and $k_\beta$ of the vectors $b_\beta$, $n_\beta$ and $c_\beta$ in the flapped tipvane coordinate system $X_\beta$, $Y_\beta$, $Z_\beta$ have the same length as the components $i$, $j$ and $k$ of $b$, $n$ and $c$ in the unflapped tipvane reference coordinate system $X$, $Y$, $Z$. See fig. 14.

The components $i$, $j$ and $k$ of the flapped vectors $b_\beta$, $n_\beta$ and $c_\beta$ in the unflapped tipvane reference coordinate system are given by the relations:

$$i = i_\beta \cos \beta + j_\beta \sin \beta$$

$$j = -i_\beta \sin \beta + j_\beta \cos \beta$$

$$k = k_\beta$$

Vector $b$ has the components, expressed in the aerodynamic tipvane angles:

$$b = i \cos \gamma_a \cos \lambda_a$$

$$= j \sin \gamma_a$$

$$= k - \cos \gamma_a \sin \lambda_a$$

and with the flapping angle $\beta$:

$$b_\beta = i_\beta \cos \gamma_a \cos \lambda_a \cos \beta + \sin \gamma_a \sin \beta$$

$$= j - \cos \gamma_a \cos \lambda_a \sin \beta + \sin \gamma_a \cos \beta$$

$$k_\beta = \cos \gamma_a \sin \lambda_a \cos \beta - \cos \gamma_a \sin \lambda_a$$
Similarly for $g_\beta$ and $c_\beta$:

\[
i ( - \sin \Lambda a \sin \theta a - \sin \gamma a \cos \Lambda a \cos \theta a ) \cos \beta a a + ( \cos \gamma a \cos \theta a ) \sin \beta a a
\]

\[
k = i - ( - \sin \Lambda a \sin \theta a - \sin \gamma a \cos \Lambda a \cos \theta a ) \sin \beta a a + ( \cos \gamma a \cos \theta a ) \cos \beta a a
\]

\[
i ( - \sin \Lambda a \cos \theta a - \sin \gamma a \cos \Lambda a \sin \theta a ) \cos \beta a a + ( \cos \gamma a \sin \theta a ) \sin \beta a a
\]

\[
k = i - ( - \sin \Lambda a \cos \theta a - \sin \gamma a \cos \Lambda a \sin \theta a ) \sin \beta a a + ( \cos \gamma a \sin \theta a ) \cos \beta a a
\]

\[
i \quad \cos \Lambda a \cos \theta a + \sin \gamma a \sin \Lambda a \sin \theta a
\]

\[
k = \cos \Lambda a \cos \theta a + \sin \gamma a \sin \Lambda a \sin \theta a
\]
6. Conversion between the different tipvane angle types

The expressions for the conversion between the different tipvane angle types can be obtained by comparing the \( i \), \( j \) and \( k \) components of the \( b \), \( n \) and \( c \) vectors. These components for the different angle types are derived in section 4. In this way 9 equations are obtained with 3 unknown angles. Of these 6 equations are redundant, because the axes are orthogonal. Therefore the 3 most convenient equations are used.

6.1 Conversion to the aerodynamic angles

6.1.1 Conversion from the construction angles to the aerodynamic angles (general)

The \( j \) component of vector \( b \) gives:

\[
\sin \gamma_a = \sin \gamma_c \cos \theta_c - \sin \theta_c \sin \Lambda_c \cos \gamma_c
\]

\( j \) component of \( c \):

\[
\sin \theta_a = \sin \theta_c \cos \Lambda_c / \cos \gamma_a
\]

\( j \) component of \( b \):

\[
\cos \Lambda_a = \cos \Lambda_c \cos \gamma_c / \cos \gamma_a
\]

6.1.2 Conversion from the Kolibri mounting part angles to the aerodynamic angles

The \( j \) component of vector \( b \) gives:

\[
\sin \gamma_a = \cos \theta_k \cos \Lambda_k \sin \gamma_k - \sin \theta_k \sin \Lambda_k
\]

\( j \) component of \( c \):

\[
\sin \theta_a = ( \cos \theta_k \sin \Lambda_k \sin \gamma_k + \sin \theta_k \cos \Lambda_k ) / \cos \gamma_a
\]
The \( j \) component of vector \( b \) gives:

\[
\sin \gamma_a = \sin \gamma_w \cos \theta_w
\]

\( k \) component of \( b \):

\[
\sin \Lambda_a = \left( \cos \gamma_w \sin \Lambda_w + \sin \theta_w \cos \Lambda_w \sin \gamma_w \right) / \cos \gamma_a
\]

\( j \) component of \( c \):

\[
\sin \theta_a = \sin \theta_w / \cos \gamma_a
\]

6.1.4 Conversion from the \( R = 0.36 \text{ m} \) wind tunnel model angles to the aerodynamic angles

The \( j \) component of vector \( b \) gives:

\[
\sin \gamma_a = \sin \gamma_m \cos \Lambda_m - \cos \gamma_m \sin \theta_m \sin \Lambda_m
\]

\( j \) component of \( n \):

\[
\cos \theta_a = \cos \gamma_m \cos \theta_m / \cos \gamma_a
\]

\( k \) component of \( b \):

\[
\sin \Lambda_a = \cos \theta_m \sin \Lambda_m / \cos \gamma_a
\]
6.1.5 Conversion from the production angles to the aerodynamic angles

The \( j \) component of vector \( b \) gives:

\[
\sin \gamma_a = \cos \lambda_p \sin \gamma_p
\]

k component of \( b \):

\[
\sin \lambda_a = \sin \lambda_p / \cos \gamma_a
\]

i component of \( c \):

\[
\sin \theta_a = (\sin \lambda_p \cos \theta_p - \sin \gamma_p \cos \gamma_p) / \cos \gamma_a
\]

6.2 Conversion from the aerodynamic angles

6.2.1 Conversion from the aerodynamic angles to the construction angles

The \( i \) component of vector \( c \) gives:

\[
\sin \lambda_c = \sin \lambda_a \cos \theta_a \sin \gamma_a \cos \lambda_a \sin \theta_a
\]

j component of \( b \):

\[
\cos \gamma_c = \cos \gamma_a \cos \lambda_a / \cos \lambda_c
\]

i component of \( c \):

\[
\sin \theta_c = \cos \gamma_a \sin \theta_a / \cos \lambda_c
\]

6.2.2 Conversion from the aerodynamic angles to the production angles

The \( k \) component of vector \( b \) gives:

\[
\sin \lambda_p = \cos \gamma_a \sin \lambda_a
\]
6.3 Conversion from the unflapped aerodynamic angles to the aerodynamic angles including a flapping angle $\beta$

Comparison of the expressions for $b$, $n$ and $c$ with $b_\beta$, $n_\beta$, and $c_\beta$ gives the conversion formulae.

The $j$ component of vector $b$ gives:

$$\sin \gamma_{a_\beta} = \sin \gamma_a \cos \beta - \cos \gamma_a \cos \Lambda_a \sin \beta$$

$k$ component of $b$:

$$\sin \Lambda_{a_\beta} = \sin \Lambda_a \cos \gamma_a / \cos \gamma_{a_\beta}$$

$i$ component of $c$:

$$\sin \theta_{a_\beta} = (\sin \theta_a \cos \gamma_a \cos \beta - \sin \Lambda_a \cos \theta_a \sin \beta$$

$$+ \sin \gamma_a \cos \Lambda_a \sin \theta_a \sin \beta) / \cos \gamma_{a_\beta}$$

These 3 expressions give for $\beta = 0$ the unflapped angles:

$$\gamma_{a_\beta} = \gamma_a$$

$$\Lambda_{a_\beta} = \Lambda_a$$

$$\theta_{a_\beta} = \theta_a$$
7. Measurement of the aerodynamic tipvane angles

The aerodynamic angles cannot be measured directly in a convenient way, because the tipvane reference coordinate system is not fixed with respect to the ground. It rotates with the rotor.
For this purpose an indirect method was developed. Via a simple conversion the aerodynamic angles can be calculated.

7.1 Definition of the measurement angles

The line from the chord point of the tipvane at the mounting part towards the rotor shaft is positioned exactly horizontal. The angle between the vertical vector \( v \) and the span vector \( b \) is then called \( \psi_{\text{hor}} \). The angle between \( v \) and the chord vector \( c \) is \( \psi_{\text{hor}} \). The same angles are measured once again with the line from the chord point of tipvane at the mounting part now positioned exactly vertical. These angles are denoted \( \psi_{\text{vert}} \) and \( \psi_{\text{vert}} \) respectively. See fig. 15. \( \psi_{\text{hor}}, \psi_{\text{vert}}, \psi_{\text{hor}}, \psi_{\text{vert}} \) are called the measurement angles.

7.2 Measurement method

In general it is impossible to position exactly the line between the chord point of the tipvane and the shaft horizontally and vertically, because the tipvane is a little bit shifted forwards. Therefore, the trailing edge of the rotor blade or tube are used for the positioning. The angle that is introduced by this offset is called \( \psi \). See fig. 16. \( \psi_{\text{hor}}, \psi_{\text{hor}}, \psi_{\text{vert}} \) and \( \psi_{\text{vert}} \) of the tipvanes of the experimental wind turbine in Hoek van Holland are measured with the assistance of a wooden clamp and a plumb line.

7.3 Calculations of the aerodynamic tipvane angles from the measurement angles

When all the vectors are expressed in the tipvane reference coordinate system the angle between two vectors can be calculated by the dot product.
Vector \( \mathbf{b} \) and \( \mathbf{c} \) are expressed in \( \Lambda_a \), \( \gamma_a \) and \( \theta_a \):

\[
\mathbf{b} = j \cos \Lambda_a \sin \gamma_a - \sin \gamma_a \cos \Lambda_a \sin \theta_a
\]

\[
\mathbf{c} = j \cos \gamma_a \sin \theta_a + \sin \gamma_a \sin \Lambda_a \sin \theta_a
\]

In the horizontal position of the blade, the vector \( \mathbf{v} \) is:

\[
\mathbf{v} = j \sin \psi - k \cos \psi
\]

\[
(v.b)_{hor} = |v| |b| \cos (vb_{hor}) = \sin \psi \sin \gamma_a + \cos \psi \cos \gamma_a \sin \Lambda_a
\]

\[
(v.c)_{hor} = |v| |c| \cos (vc_{hor}) = \sin \psi \cos \gamma_a \sin \theta_a - \cos \psi (\cos \Lambda_a \cos \theta_a + \sin \gamma_a \sin \Lambda_a \sin \theta_a)
\]

In the vertical position of the blade, the vector \( \mathbf{v} \) is:

\[
\mathbf{v} = j \cos \psi + k \sin \psi
\]

and the angles:

\[
(v.b)_{vert} = |v| |b| \cos (vb_{vert}) = \cos \psi \sin \gamma_a - \sin \psi \cos \gamma_a \sin \Lambda_a
\]

\[
(v.c)_{vert} = |v| |c| \cos (vc_{vert}) = \cos \psi \cos \gamma_a \sin \theta_a + \sin \psi (\cos \Lambda_a \cos \theta_a + \sin \gamma_a \sin \Lambda_a \sin \theta_a)
\]

There are 4 relations with 4 unknowns: \( \psi \), \( \Lambda_a \), \( \gamma_a \) and \( \theta_a \).

In the case that \( \psi \) can be measured or calculated from the geometry 1 relation can be used for checking the measurement of the angles. If \( \psi \) is non-zero an simple iterative procedure is used to calculate \( \Lambda_a \), \( \gamma_a \).
and $\theta$. The angle $(v_c)_{\text{hor}}$ is used for checking by comparing the measured value of $(v_c)_{\text{hor}}$ with the calculated value of $(v_c)_{\text{hor}}$ from $\Lambda_a$, $\gamma_a$ and $\theta_a$ ($\psi$ is known). The difference is denoted with $\Delta(v_c)_{\text{hor}}$.

### 7.4 FACT rotor angles

The geometry of the FACT-rotor blades implies a value of $\psi = 0$. This gives very simple relations. $\Lambda_a$, $\gamma_a$ and $\theta_a$ are calculated by:

\[
\sin \gamma_a = \cos (v_b_{\text{vert}})
\]

\[
\sin \theta_a = \cos (v_c_{\text{vert}}) / \cos \gamma_a
\]

\[
\sin \Lambda_a = \cos (v_b_{\text{hor}}) / \cos \gamma_a
\]

The accuracy of the measured and calculated angles is in the order of 0.5 degrees.
8. Wagging effect

When the mounting position of the tipvane shifts forward or backward with no rotation the effective incidence angle changes, because the oncoming flow is circular. See fig. 17. If the tipvane shifts forward or backward the angle of attack decreases or increases respectively.

The shifting of the tipvane occurs as an unavoidable by product when $\theta$ is changed by a rotation around a centre that is not located on the chord of the tipvane but which is somewhat closer to the rotor axis. This gives a reduction of the $\theta$ change. If the rotation point of $\theta$ should coincide with the rotor axis line, there would be no effective change of $\theta$ at all.

If the displacement, generated by the rotation of the tipvane, is $\Delta l$ than the correction on $\theta$ due to shifting of the tipvane is:

$$\theta_{\text{correction}} = \frac{\Delta l}{R} \times \frac{180}{\pi} \quad \text{(degrees)}$$

with $\Delta l$: amount of shift (backwards = positive).

$\theta_{\text{correction}}$ and $\psi$ are the same kind of angles. But the difference is that the offset angle $\psi$ is used in section 7.3 for the effect of the displacement of the tipvane by the measuring method for the aerodynamic angles.
9. Airfoil section curving

9.1 Introduction

The airfoil section of the tipvane should be adapted to the rotating situation.

The oncoming flow to the tipvane is circular. If the airfoil section has the same curvature as the flow, the situation will be comparable with the original airfoil section in the parallel flow.

Until January '84 all tipvane airfoil sections of the wind tunnel models and Kolibrie tipvane airfoils were corrected by curving the chord line. After January '84 the correction was made by curving the meanline.

A correction of the incidence angle $\theta$ is necessary - and through that also the angle of attack $\alpha$ is changed - if the mounting point of the tipvane does not coincide with the point which is kept fixed during the curving correction. The $\frac{1}{2}$ chord position is used as the fixed point, when curving the airfoil section. See fig. 18a and fig. 18b.

The correction of the incidence angle is given by:

$$\theta_{\text{correction}} = \frac{AL}{R} \times \frac{180}{\pi} \text{ (degrees)}$$

where $AL$ is the distance between the mounting point and the fixed point during curving.

If the mounting point is between the leading edge and the fixed point during curving, $\theta_{\text{correction}}$ is positive and the angle of attack $\alpha$ increases.

The fixed point during curving may be defined as the point where the chord, and after curving the tangent to the chord, is perpendicular to the radius.

The correction is of the same kind as is given by the wagging effect but it is introduced in a slightly different way.
<table>
<thead>
<tr>
<th>tipvane angles</th>
<th>sequence of rotation</th>
<th>description in section</th>
</tr>
</thead>
<tbody>
<tr>
<td>aerodynamic angles</td>
<td>( \Lambda_a \gamma_a \theta_a )</td>
<td>4.1</td>
</tr>
<tr>
<td>construction angles</td>
<td>( \theta_c \Lambda_c \gamma_c )</td>
<td>4.2.1</td>
</tr>
<tr>
<td>Kolibrie angles 1)</td>
<td>( \theta_k \Lambda_k \gamma_k )</td>
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<tr>
<td>towing tank model R = 18 cm.</td>
<td>( \Lambda_w \theta_w \gamma_w )</td>
<td>4.3.1</td>
</tr>
<tr>
<td>wind tunnel R = 36 cm. model angles</td>
<td>( \gamma_m \theta_m \Lambda_m )</td>
<td>4.3.2</td>
</tr>
<tr>
<td>wind tunnel R = 46.8 cm. model angles</td>
<td>( \Lambda_a \gamma_a \theta_a \Lambda_m )</td>
<td>4.3.3</td>
</tr>
<tr>
<td>production angles</td>
<td>( \gamma_p \Lambda_p \theta_p )</td>
<td>4.4</td>
</tr>
</tbody>
</table>

1) not Euler angles.

Table 1: Overview of the different tipvane angle types.
Fig. 1: Definition of the tipvane reference coordinate system.
Fig. 2: Definition of the local tipvane coordinate system (position before rotation over the Euler-angles).
Fig. 3: Definition of the 3 tipvane angles $\Lambda$, $\gamma$ and $\theta$ with their positive directions.
Fig. 4: The $b$, $n^1$, $c'$ vectors after rotation over $\Lambda_a$ and $\gamma_a$ for the aerodynamic tipvane angles.
Fig. 5: Rotation $\theta_a$ in the $n', c'$ plane for the aerodynamic tipvane angles.
Fig. 6: Rotation of the first angle $\theta_c$ of the construction tipvane angles.
Fig. 7: The vectors of the local tipvane coordinate system in the $i, \hat{n}^*, \hat{e}^*$ coordinate system ($i, \hat{n}^*, \hat{e}^*$ is rotated over $\theta_c$).
Fig. 8: The vectors $b''$, $n''$, $c$ after rotation over $\Lambda_w$ and $\theta_w$ for the wind tunnel model angles.
Fig. 9: Rotation $\gamma_w$ in the $n''$, $b''$ plane for the wind tunnel model angles.
Fig. 10: Rotation $\Theta_a$ and the fourth rotation $\Lambda_m$ in the $b$, $n'$, and $c'$ axis system of the wind tunnel model with $R = 0.468$ m.
Fig. 11: The vectors of the local tipvane coordinate system \( b, n'', c''' \) after rotation over \( \gamma_p \) and \( \Lambda_p \) for the production angles.
Fig. 12: Rotation $\theta_p$ in the $n'''$, $c'''$ plane for the production angles.
Fig. 13: The change of the tipvane reference coordinate system due to the flapping angle $\beta$. 
Fig. 14: The flapped and unflapped vectors in the X, Y plane.
Fig. 15: The horizontal and vertical position of the tipvane for measuring the tipvane angles.
Fig. 16: Vector $v$ and angle $\psi$ with a tipvane offset.
Fig. 17: The wagging effect of the tipvane if the $\theta$ "hinge" is moved to the rotor shaft.
Fig. 18a: Uncurved and curved airfoil section at \( \frac{1}{4} \) chord point

= curving point = mounting point.
Fig. 18b: Curved airfoil over $\Delta l$ shifted backwards.
Fig. 19: Increase of the local effective camber of the down wind tipvane airfoil section due to the $\gamma$-effect.
Fig. 20: Decrease of the local angle of attack of the down wind tipvane airfoil due to the A-effect.