Dikes constructed from sand generally have a sand core and clay layers on the slopes and the crest to protect the core against erosion. In extreme hydraulic conditions, several failure mechanisms can lead to destruction of the clay layers, exposing the sand core to water. When water overtops the dike and the protective cover of the land-side slope or the crest fails, water flows over the core and erodes the sand. The dike starts to breach, and eventually the land behind the dike is flooded. The rate at which the dimensions of the breach grow influences the rate of inundation of the polder. Reduction of the inundation rate may be achieved by retarding the breaching process. This may reduce the number of casualties, resulting in increasing safety for the inhabitants. In order to achieve, for instance, a safety level ten times higher, mortality has to decrease by a factor of 10. The breaching process can be retarded by reducing the erosion velocity of the sand core. Experiments were executed to investigate the effect on the erosion velocity of adding bentonite to sand. The results of these experiments showed a significant reduction of the permeability and erosion velocity of the sand–bentonite mixtures compared with those of pure sand. The effect of adding bentonite on the breaching process was investigated by applying the Bres model (breach erosion in sand dikes model) to a sand dike tested in a large-scale field experiment (Zwin94). It was found that adding a small percentage of bentonite reduces the rate of breach growth and the inundation rate in the polder. For the Zwin’94 dike it was determined that 5-4% of bentonite is sufficient to reduce the inundation rate below a threshold value of 0.5 m/h, leading to a significant increase in safety.

### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_b$</td>
<td>near-bed concentration (dimensionless) (volume concentration)</td>
</tr>
<tr>
<td>$D_*$</td>
<td>particle parameter (dimensionless)</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>particle diameter (m)</td>
</tr>
<tr>
<td>$f$</td>
<td>bulk friction (dimensionless)</td>
</tr>
<tr>
<td>$f_b$</td>
<td>bed friction (dimensionless)</td>
</tr>
<tr>
<td>$f_{evacuation}$</td>
<td>fraction of inhabitants evacuated (dimensionless)</td>
</tr>
<tr>
<td>$f_w$</td>
<td>wall friction (dimensionless) (curve fitting function)</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity (m/s²)</td>
</tr>
<tr>
<td>$h$</td>
<td>water depth (m)</td>
</tr>
<tr>
<td>$k$</td>
<td>permeability (m/s)</td>
</tr>
<tr>
<td>$k_{sand}$</td>
<td>permeability of sand (m/s) (calculated in the Bres model)</td>
</tr>
<tr>
<td>$k_{sb}$</td>
<td>permeability of sand–bentonite mixture (m/s)</td>
</tr>
<tr>
<td>$M_A$</td>
<td>mortality, casualties as a fraction of all inhabitants of the flood zone</td>
</tr>
<tr>
<td>$M_L$</td>
<td>mortality, casualties as a fraction of inhabitants left in the flood zone</td>
</tr>
<tr>
<td>$n$</td>
<td>porosity (dimensionless)</td>
</tr>
<tr>
<td>$n_i$</td>
<td>maximum porosity of the sand (dimensionless)</td>
</tr>
<tr>
<td>$P_f$</td>
<td>probability of inundation of the relevant area</td>
</tr>
<tr>
<td>$R$</td>
<td>hydraulic radius of the total flow area (m)</td>
</tr>
<tr>
<td>$S$</td>
<td>energy slope (m/m)</td>
</tr>
<tr>
<td>$U$</td>
<td>depth-averaged flow velocity (m/s)</td>
</tr>
<tr>
<td>$v_e$</td>
<td>erosion velocity (m/s)</td>
</tr>
<tr>
<td>$v_i$</td>
<td>inundation velocity (m/h)</td>
</tr>
<tr>
<td>$v_{i-sand}$</td>
<td>inundation velocity for the sand dike (m/h)</td>
</tr>
<tr>
<td>$w$</td>
<td>width of the flow (m)</td>
</tr>
<tr>
<td>$w_s$</td>
<td>hindered settling velocity of sand (m/s)</td>
</tr>
</tbody>
</table>
1. Introduction

Dikes have failed in the past all over the world, and will continue to do so in the future. Failures can happen during extremely high river or sea levels. In most cases, dikes are protected by protective covers or revetments, but if these fail, a dike may start to breach. This is especially the case for sand dikes, in which a major breach can form within a few hours after the failure of the protective cover (Visser, 1998). In such an event, water will enter the polder behind the dike and the area will be inundated. This flooding causes casualties and economic damage. Most casualties of such a flood are caused by relatively large flow velocities and rapidly rising water levels in the polder. These inundation parameters depend on the development of the discharge through the breach, and thus on the speed of the breach growth process and the area of the polder (Rijkswaterstaat, 2006). Usually, dikes are constructed such that they are sufficiently high and strong enough to minimise the chance of an inundation. This can be costly and space-consuming, and, in the case of breaching of the dike, the dike failure is generally still brittle so that there is little time to battle the failure to reduce the consequences. Instead of increasing the crest height and the related dimensions of a dike, there are other ways to strengthen a dike and to reduce the number of casualties if it fails.

One of these ways is to take measures that will retard the breaching process so that the dike will fail in a more gradual way. The breaching process can be slowed down by reducing the erosion velocity of the sand in the sand core. Due to this ducile failure, the breaching process will be more gradual and the consequences will be reduced.

2. Breaching process and safety level

The breaching process often starts with erosion of the inner (land-side) slope. This inner slope has a protective cover of clay and grass to prevent this erosion. The dike will not erode as long as the cover stays intact. It is, however, possible that the cover is damaged (e.g. by wave overtopping, piping, macro- or micro-instability) and the sand core of the dike is then exposed to water and waves. When that happens, the dike core starts to erode and a breach will develop.

In breach models, it is often assumed that the breaching process starts (at time $t = t_0$) with an initial breach in the crest of the dike (made on purpose as an initial breach or pilot channel in experiments or representing damage to the protective cover during a real breaching event). Water will flow through this initial breach (in critical flow conditions), initiating the breaching process of the core. Five stages (Figure 1) can be distinguished in this process (Visser, 1998).

- **Stage I** (from $t_0$ until $t_1$). The (supercritical) flow velocity along the inner slope increases. As a result, erosion increases along the slope, increasing the angle of the inner slope from an initial value of $\beta_0$ to a critical value of $\beta_1$ at $t = t_1$ (the critical angle may be the internal friction angle, although Visser (1998) notes that steeper slopes have been observed).
- **Stage II** (from $t_1$ until $t_2$). The erosion of the inner slope continues at the critical angle $\beta_1$ (retrograde erosion), reducing the width of the dike crest in the breach. This continues until the crest width has become zero at $t_2$.
- **Stage III** (from $t_2$ until $t_3$). During this stage, the remainder of the dike core in the breach is eroded down to the base of the dike (or the berm on the outer slope (Visser, 1998)). Consequently, the width of the breach also increases, as the side slopes of the breach maintain a critical angle $\gamma_1$. This stage ends at $t_3$ when the bottom of the breach has reached the level of the base of the dike.
- **Stage IV** (from $t_3$ until $t_4$). The critical flow through the breach continues to erode the breach, mainly horizontally but also vertically. The vertical erosion depends on the erodibility of the base of the dike and is usually small compared with the lateral erosion. As in stage III, the side slopes of the breach remain at the critical angle $\gamma_1$. This stage ends at $t_4$ when the flow through the breach becomes subcritical due to the rising water level in the polder.
- **Stage V** (from $t_4$ until $t_5$). In this subcritical flow stage, the breach continues to grow (mainly) laterally, with breach side slopes remaining at the critical angle $\gamma_1$. In this stage, the water level in the polder continues to rise and the difference between the outside water level and the inside water level decreases. This reduces the flow velocity through the breach, and eventually the velocity becomes too low to pick up sand particles, stopping the erosion process and thus the breaching process at $t_5$. The flow through the breach stops at $t_5$ when the inside water level equals the outside water level.

The breaching process described above was modelled on the Bres model (breach erosion in sand dikes model; ‘bres’ is also the Dutch word for breach (Visser, 1998)). Besides the Bres model, there are several breaching models available for sand, clay and composite dikes and dams (Morris et al., 2009). The Bres model was chosen due to its relevance to sand dikes,

---

\[ \begin{align*}
\beta & \quad \text{angle of bed slope (degrees)} \\
\Delta & \quad \text{Shields parameter (dimensionless)} \\
\theta & \quad \text{critical Shields parameter (dimensionless)} \\
\phi & \quad \text{angle of internal friction (degrees)} \\
\nu & \quad \text{kinematic viscosity of water (m}^2\text{/s)} \\
\rho & \quad \text{water density (kg/m}^3\text{)} \\
\tau & \quad \text{shear stress (Pa)} \\
\tau_0 & \quad \text{total shear stress (Pa)} \\
\tau_b & \quad \text{bed shear stress (Pa)} \\
\end{align*} \]
the SWOT (strengths, weaknesses, opportunities and threats) analysis of Peeters et al. (2011), its good performance in engineering practice and its availability to the authors. For a more detailed explanation of the set-up and limits of the Bres model, reference is made to Visser (1998).

In the Netherlands, inundation probability ($P_I$) is determined as the probability of exceedance of a maximum water level in front of the dike ($P_E$) multiplied by the probability of failure of the dike ($P_F$):

$$P_I = P_E \times P_F.$$  

A drawback of this definition is that it does not take mitigating measures into account. Therefore, the definition was changed to the probability of the loss of life of a person as a result of a dike failure. This new definition makes it possible to define a more accurate and local level of safety. It includes all relevant failure mechanisms, evacuation, mitigating measures and so on. It is then possible to define the desired decrease of mortality to increase the level of safety. A formula used to derive the safety level is the localised individual risk (LIR) of loss of life due to a flood (Deltares, 2011)

$$LIR = P_F(1 - f_{evacuation})M_A = P_F M_L$$

1. in which $f_{evacuation}$ is the fraction of inhabitants evacuated, $M_A$ is mortality in terms of casualties as a fraction of all inhabitants of the flood zone, $M_L$ is the mortality in terms of casualties as a fraction of inhabitants left in the flood zone and $P_F$ is the probability of inundation of the relevant area.

To increase safety by a factor of 10, the LIR has to be reduced by a factor of 10. This can be achieved by decreasing mortality by a factor of 10 as well. Research into loss of life due to flood events (Jönkman, 2007) shows that mortality increases with

Figure 1. Schematic overview of the breaching process of a sand dike (Visser, 1998)
inundation speed. This rise rate has an estimated threshold value of 0·5 m/h for mortality, as shown in Figure 2. Mortality, and thus the LIR, can be reduced by slowing down the breaching process.

The breaching process of a sand dike can be slowed down by reducing the erosion velocity of sand. There are several methods available to reduce the erodibility of sand – such as grouting or biological cementation of the sand in a dike – which have been compared by means of a literature study (Lemmens, 2014). The study concluded that adding bentonite is the best method, based on adequate substantiated scientific studies. Bentonite reduces the permeability and therefore the erodibility of sand (Gailani et al., 2001). During erosion, water needs to flow into the pores to pick up grains. By reducing this inflow, the erosion velocity can be reduced.

3. Experiments
Experiments were conducted by Lemmens (2014) to test whether bentonite can reduce the erosion velocity of sand and retard the breaching process. The experiments were executed for clean sand and sand mixed with bentonite. The main goal of the experiments was to determine whether there is a difference in erosion velocity between sand and different sand–bentonite mixtures. The secondary goal of the experiments was to determine whether the additives influence relevant soil parameters such as shear strength and permeability.

3.1 Materials
The sand used for the tests and for mixing is a pure quartz sand ($D_{50} = 208 \mu m$). The coefficient of uniformity ($C_u$) and coefficient of curvature ($C_c$) are 1·7 and 1·0, respectively, hence the sand can be classified as poorly graded according to Holtz and Kovacs (1981).

The sand–bentonite mixtures were prepared by mixing sand and bentonite using a concrete mixer. The materials were dry mixed in batches of ~25 kg (weight of one bag of sand and additive).

3.2 Geotechnical tests
The shear strength of the sand and the two mixtures was determined by direct drained shear tests (Table 1). It can be concluded from Table 1 that the (apparent) cohesion of both mixtures was very small and that only the mixture with 2·4% bentonite had a small increase in the friction angle. Therefore, it can be concluded from this test that both mixtures behave like sand and thus can be modelled like sand.

Falling head tests were executed to determine the permeability of the sand and the two mixtures. The results are given in Table 2.

Table 2 shows that adding bentonite to sand decreases the permeability significantly. For 1·2% of bentonite, the permeability reduced to about 1/3 of that of sand without bentonite; 2·4% of bentonite gave a reduction with a ratio of about 1/6. Figure 3 shows an empirical relation to derive the relative permeability ($k/k_{sand}$) of a sand–bentonite mixture. The reduced permeability of the sand–bentonite mixtures is expected to decrease erosion velocities, which were tested in the erosion experiments.

3.3 Erosion experiment
The erosion tests were performed in a flume at the Laboratory of Fluid Mechanics of Delft University of Technology.

<table>
<thead>
<tr>
<th>$\phi$: degrees</th>
<th>c: kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>34·6</td>
</tr>
<tr>
<td>Sand–bentonite (1·2%)</td>
<td>34·3</td>
</tr>
<tr>
<td>Sand–bentonite (2·4%)</td>
<td>36·7</td>
</tr>
</tbody>
</table>

Table 1. Friction angle ($\phi$) and (apparent) cohesion (c) of the mixtures

<table>
<thead>
<tr>
<th>$k$: m/s</th>
<th>$k/k_{sand}$: dimensionless</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>$1·91 \times 10^{-4}$</td>
</tr>
<tr>
<td>Sand–bentonite (1·2%)</td>
<td>$7·07 \times 10^{-5}$</td>
</tr>
<tr>
<td>Sand–bentonite (2·4%)</td>
<td>$3·08 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 2. Permeability coefficients ($k$) and reduction ratio ($k/k_{sand}$) of permeability of the mixtures relative to sand

For mixing, Cebogel Sealfix bentonite (Cebo Holland BV, 2014) was used. This bentonite is already being used in practice to create sand–bentonite seals. It is a powder of activated sodium bentonite, which has a high swell capacity. Two mixtures with different weight percentages were tested during these experiments, 1·2% and 2·4% mass of bentonite of the total dry mass.
The flume has a length of 4·4 m, a width of 0·105 m and a height of 0·4 m. Flow into the flume is driven by gravity, from a reservoir in the laboratory, through a pipe into the flume. A magnetic flow meter was used to measure the discharge through the pipe. In each test, the rate of flow over the bed was constant. The side walls of the flume were made from glass panels. The water levels and bed levels were observed through one of these panels (Figure 4) with a video camera, from the start until the end of each test. Figure 4 also shows the grid drawn on the panel and the measurement area with nine measurement locations. The video recordings allowed determination of the erosion velocities of the bed and, with the known discharge and the flume width, determination of the depth-averaged flow velocities. The erosion velocity, which is defined as the vertical velocity of the bed level, was determined by measuring the bed level at set intervals (by means of the video recording) and then dividing the difference in bed levels by the set time interval.

The bed of sand or mixture was prepared at a dry bulk density of \(1600 \text{ kg/m}^3\) (resulting in a porosity of 0·40). The bed, with a length of 3·4 m, was compacted by hand and by means of a wooden tool and hammer. Due to inconsistency in the compaction method, a small spread in the dry bulk densities (between 1482 and 1667 kg/m\(^3\)) and therefore the porosities (between 0·37 and 0·40) occurred over the tests. After preparation of the bed, water was kept on top of the bed for an extended period of time to allow saturation of the bed. For the
sand–bentonite mixtures, this was particularly important, since the bentonite needs time to swell to its full capacity. The tests were executed at two different flow rates ($Q = 10$ l/s and $Q = 20$ l/s) for each mixture. Due to differences between the tests (e.g. bed density), the depth-averaged flow velocities varied over the tests, as shown in Figure 5. Turbulence in the inflow section yielded inaccurate measurements in measurement locations 1, 2 and 3 and these data were not included in Figure 5.

In all the tests, the depth-averaged flow velocities increased in the flume, thus the flow accelerated. This acceleration was induced by the adaptation distance of the flow being longer than the length of the flume, resulting in the flow still reaching equilibrium in the flume. The depth-averaged flow velocities increased irregularly (Figure 5), most likely caused by turbulence in the flow. This turbulence in the flow was partly caused by the water depth reduction due to the presence of the bed at the beginning of the flume. This was particularly noticeable during the higher velocity tests ($U > 1$ m/s). A part of the spread in the results can be explained by this turbulence.

4. Results
With the data of water level and bed level measurements, the depth-averaged flow velocities and erosion velocities were determined. These were used to create linear regression lines, which were used to calculate the erosion velocity as a function of depth-averaged flow velocity for each mixture. Figure 6 shows these linear regression lines for all tests, including the erosion velocity at the depth-averaged flow velocity $U$ during each test.

Figure 6 shows the effect of adding 1.2% bentonite to the sand on the erosion velocity. Adding a small amount of...
bentonite (1.2%) decreases the erosion velocity slightly for depth-averaged flow velocities $U$ larger than about 0.8 m/s (for $U = 1.2$ m/s the erosion velocity reduces with 30%), but increases erosion velocities for $U < 0.8$ m/s. Adding 2.4% of bentonite, however, reduces the erosion velocity significantly – that is, with a ratio of about 0.12 (Table 3) for all depth-averaged flow velocities. It should be noted that (a) in the present experiment only two mixture ratios were tested, and other mixture ratios might give better results and (b) the present tests were executed at relatively low depth-averaged flow velocities, so the mixtures need to be tested at higher flow velocities to determine the effect at these flow velocities.

For all tests, linear regression lines for the erosion velocity as a function of $U^2$ were also determined. The $U^2$-axis crossings at $v_e = 0$ m/s were also calculated, as shown in Figure 7. These $U^2$ values are proportional to the critical shear stress and are an indication of the critical shear velocity.

Since both the critical $U$ values and the critical $U^2$ values were similar for the different mixtures and sand, the erosion processes were considered to be similar. If this was not the case, large deviations in the critical velocities should be noticeable – for example, those caused by a significant cohesion of a mixture.

### 5. Analysis

The bed shear stress was derived from measurements of the total pressure and energy loss during the erosion tests. In order to derive the effect of bed shear stress, the measured pressure gradient was corrected for the acceleration of the flow and the influence of the friction along the wall of the flume.
For each test, the water depth at six different locations was measured (measurement locations 4–9). These water depths were used to calculate the depth-averaged flow velocities during each test. For each measurement location, the time-averaged and depth-averaged flow velocity and average water depth were known, and were used to calculate the hydraulic radius ($R$), energy height ($H$) and pressure ($p$).

The pressure gradient ($\frac{dp}{dx}$) is corrected for the acceleration of the flow using the method described by Van Rhee and Talmon (2010):

\[ \frac{dp}{dx} = \frac{dh}{dx}\rho g = -\tau_a \frac{1}{R} - \frac{d}{dx} \left( \rho U^2 \right) \]

in which $h$ is the water depth (m), $R$ is the hydraulic radius of the total flow area (m), $U$ is the depth-averaged flow velocity (m/s), $\rho$ is the water density (kg/m$^3$) and $\tau_a$ is the total shear stress (Pa).

Equation (2) also comprises a term for the density gradient ($\frac{d\rho}{dx}$). This gradient was assumed to be zero for the executed tests because the effect of erosion on the increase of the density of the eroding flow was negligible. By rewriting Equation (2) and neglecting the term ($\frac{d\rho}{dx}$), an expression for the total shear stress ($\tau_a$ (Pa)) is found

\[ \tau_a = -R \left( \frac{d\hat{h}}{dx} \right) \]

with

\[ \frac{d\hat{h}}{dx} = \frac{dh}{dx}\rho g - \rho \frac{dU^2}{dx} \]

in which $\hat{h}$ is the corrected water depth ($m$) and $\hat{U}$ is the depth-averaged flow velocity corrected for acceleration (m/s).

The total shear stress was determined based on the pressure gradient ($\frac{dp}{dx}$) corrected for the acceleration of the flow, which is the corrected pressure gradient ($\frac{dh}{dx} - \rho \frac{dU^2}{dx}$).

---

Figure 7. Crossing of $U^2$-axis at $v_e = 0$ m/s of the linear regression lines (with mean results of experiments)
The method used for calculating the bed shear stresses of the tests was the Vanoni and Brooks formula (Cheng and Chua, 2005). Equation (5) was selected since it applies the necessary corrections for wall friction

5. \[
    \tau_b = \frac{w}{w + 2h} \frac{f_b}{\rho g h S} \frac{1}{C_0}
\]

6. \[
    f = \frac{8g RS}{U^2}
\]

7. \[
    f_b = f + \frac{2h(f - f_w)}{w}
\]

8. \[
    f_w = \left(20 \left(\frac{4UR/v}{f}\right)^{0.4} - 39\right)^{-1}
\]

in which \(f\) is the bulk friction (dimensionless), \(f_b\) is the bed friction (dimensionless), \(f_w\) is the wall friction (curve fitting function (dimensionless)), \(g\) is acceleration due to gravity (m/s²) (= 9.81 m/s²), \(R\) is the hydraulic radius of the total flow area (m), \(S\) is the energy slope (m/m), \(v\) is the kinematic viscosity of water (m²/s), \(\rho\) is the water density (kg/m³) and \(\tau_b\) is the bed shear stress (Pa).

The bed shear stresses were converted to a Shields parameter \(\theta\) (Van Rijn, 1984)

9. \[
    \theta = \frac{\tau}{\rho g \Delta D_{50}}
\]

in which \(D_{50}\) is the particle diameter (m), \(\Delta\) is the relative density (dimensionless) (= 1.65) and \(\tau\) is the shear stress (Pa).

Due to the decreasing permeability, the erosion velocity decreases. The flow conditions during the erosion tests were in the sheet flow regime \(\theta > 0.3 - 1\). In this regime, the erosion process is dominated by the shearing of layers of sand. Sand layers will dilate before shearing, and water has to flow between the grains to fill the voids. This effect is evident from the results of the tests. From the permeability tests it can be concluded that the permeability of sand decreases with increasing bentonite content, while the erosion tests showed that the erosion rate decreased with increasing bentonite content. The lower permeability of sand with a high bentonite content hinders the inflow of water into the sand bed, hence reducing the erosion rate. The resulting higher pressure gradient pushes grains into the bed, making it more difficult to erode the sand.

5.1 Erosion formula

The test results were compared with the erosion formula of Van Rhee (2010) (Figure 8). The Van Rijn–Van Rhee formula (Van Rhee, 2010) was used since it has corrections for bed slope and near-bed concentration, porosity and permeability, of which the latter is necessary to model the effect of sand–bentonite mixtures. Also, this formula is valid in the dilatancy-restricted erosion regime, which is the regime in which these experiments were executed. When implemented in the Bres model, the model produces results that are in good agreement with the experimental data (Bisschop et al., 2010). Thus, for the sand used in the experiments, with or without bentonite, the erosion velocity can be calculated using the Van Rijn–Van Rhee erosion formula, given by Van Rhee (2010)

10. \[
    v_e = \frac{1}{1 - n - c_b} \left( \phi_p \sqrt{g \Delta D_{50} - c_b w_s} \right)
\]

in which

11. \[
    \phi_p = 0.00033 D_e^{0.3} \left( \theta - \theta_1 \right)^{1.5}
\]

12. \[
    \theta_1 = \theta_s \left( \sin(\phi - \beta) \sin(\phi) + \phi \frac{v_e}{K} \right)
\]

13. \[
    \delta_p = \left( n_i - n \right) \frac{1}{1 - n_i \Delta(1 - n)}
\]

14. \[
    D_e = D_{50} \left( \frac{\Delta g}{v^2} \right)^{1/3}
\]

where \(c_b\) is the near-bed concentration (dimensionless) (volume concentration), \(D_e\) is the particle parameter (dimensionless), \(D_{50}\) is the particle diameter (m), \(k\) is the permeability (m/s) (= 1.91 x 10⁻⁴ m/s for the used sand), \(n\) is the porosity (dimensionless) (during sand tests ~0.38), \(n_i\) is the maximum porosity of the sand (dimensionless) (= 0.48), \(v_e\) is the erosion velocity (m/s), \(w_s\) is the hindered settling velocity of sand (m/s),
ϕ is the angle of internal friction (degrees) (= 34°), β is the angle of bed slope (degrees) (= 0°), Δ is the relative density (dimensionless) (= 1·65) and ν is the kinematic viscosity (m²/s).

Using Equation (10), the erosion velocities were calculated for the same conditions as for those during the tests with pure sand. First this was calculated for clear water erosion, which means assuming a near-bed concentration of zero (c₉ = 0). The Van Rijn–Van Rhee function does not match the measurements of the sand tests (Figure 9). A possible cause is the near-bed concentration, which was not zero during the tests. By adapting the near-bed concentration (c₉ ≈ 0·03) and dividing the permeability by 20 (k = k/20), the calculated and experimental results match quite well, as Figure 9 shows (sand and fitted lines). The reduction in permeability is necessary to fit Equation (10) to the measurements and is not representative of the real value of the permeability. This reduction ratio is an empirical coefficient.

This fit was used to adapt Equation (10) to the sand–bentonite tests. This was achieved by dividing the permeability values of the sand–bentonite mixtures (1·2% or 2·4%), 7·07 × 10⁻⁵ or 3·08 × 10⁻⁵ m/s, respectively, by 20. By only adapting the permeability of the fitted formula, the calculated erosion velocities show a similar reduction as the test results (Figure 9). This makes the function suitable for modelling the erosion during the breaching process of a sand–bentonite dike.

6. Application
As described above, mixing sand with bentonite can significantly reduce the erosion velocity of sand. To investigate the effect of the reduced erosion velocity on the breaching process of sand dikes, the Bres model was applied to the sand dike of the Zwin’94 experiment (Visser, 1998). The data of the Zwin’94 experiment were used to calibrate the Bres model (Visser, 1998). Figure 10 shows a cross-section of this dike.
The effect of bentonite on the erosion velocity was modelled with the Van Rijn–Van Rhee erosion formula (Equation (10)) (Van Rhee, 2010), by adapting the permeability according to Figure 3. The reduction of permeability as a function of the weight percentage shown in Figure 3 can be approximated as

\[ k_{sb} = \frac{k_{sand}}{C_0^{0.5} \theta^{4}} \]

Figure 9. Fitting of the Van Rijn–Van Rhee formula (Equation (10)) (for \( C_b \approx 0.03 \) and \( k = k/20 \)) to the results of the sand–bentonite tests (1.2% and 2.4%), by using permeability to the values measured.

Figure 10. Cross-section of the Zwin’94 dike (Visser, 1998)
in which $k_{\text{sand}}$ is the permeability of the sand (m/s) (calculated in the Bres model), $k_{\text{sb}}$ is the permeability of the sand–bentonite mixture (m/s) and $\zeta_b$ is the weight fraction of the added bentonite (dimensionless).

By implementing Equation (15) in the Bres model, the model simulates the breaching process for the different weight percentages of the added bentonite. This was done for the Zwin’94 dike for sand with 0, 1.2 and 2.4% of added bentonite. The effect of the different percentages of added bentonite on the breach width ($B$), breach discharge rate ($Q_{\text{br}}$), depth-averaged flow velocity ($U$) and inundation velocity ($v_i$) is shown in Figures 11 and 12. These figures show that the breach width, breach discharge rate and inundation velocity decrease with increasing bentonite percentage.

The model results show that a weight percentage of 2.4% of bentonite added to sand decreases the maximum values of the
inundation parameters (breach width, breach discharge rate and inundation velocity) for the Zwin’94 dike by half. This is, however, not sufficient to lower the inundation velocity below the threshold value of 0·5 m/h. Based on the model and experimental results, it is possible to determine what weight percentage of bentonite would lower the inundation velocity below this threshold value for a normal dike.

6.1 Necessary mixture

On the basis of the results given by the Bres model for 1·2% and 2·4% of added bentonite, a relation between the weight fraction of added bentonite and a reduction ratio for the inundation velocity can be formulated

\[ \frac{v_i}{v_{i-\text{sand}}} = e^{-30 \xi_b} \]

in which \( v_i \) is the inundation velocity (m/h), \( v_{i-\text{sand}} \) is the inundation velocity for the sand dike (m/h) and \( \xi_b \) is the weight fraction of added bentonite (dimensionless).

Equation (16) gives the reduction of the inundation velocity as a function of the weight fraction of added bentonite compared with the situation with no added bentonite. A reduction of the inundation velocity of more than 80% is necessary to reduce the inundation velocity for 0% bentonite to the threshold value of 0·5 m/h. Equation (16) predicts a bentonite percentage of 5·4% to obtain this reduction.

To verify that adding 5·4% of bentonite to sand would actually lead to an inundation velocity below the threshold value, the breaching process for this mixture was also simulated in the Bres model. However, in the Zwin’94 experiment, the measured outside water level (tide) dropped faster than the drop of the breach bottom in stage II predicted by the Bres model, thus stopping the breaching process in stage II. Stage III was never initiated since the outside water level was too low at that time step. For the Zwin’94 dike, this means it is possible to prevent a catastrophic breach of the dike by adding enough bentonite to the sand. This might be possible for other dikes as well, but will depend on case-specific circumstances. For each
dike, a bentonite percentage could be determined for which the breaching process does not pass stage II.

To show that the calculated bentonite percentage could reduce the inundation velocity sufficiently, a second simulation was run. This time, the outside water level was programmed to remain at a higher level for a longer period of time to ensure a fully developing breach. A comparison with the results of the other mixtures is shown in Figure 13. The inundation velocities of all mixtures are shown in Figure 14.

The results show that a sand–bentonite mixture with an added bentonite weight percentage of 5.4% reduces the inundation velocity below the threshold value of 0.5 m/h. Based on Jonkman (2004), this results in a mortality of <1%, which is a reduction by a factor of 10 compared with an unimproved sand dike.

Figure 13. Comparison of Bres model results for the Zwin’94 dike for different added weight fractions of bentonite, including added bentonite percentage of 5.4%
It should be noted that this result is only applicable for the dike, polder and circumstances of the Zwin’94 experiment.

7. Conclusions

The rate of breach development and the resulting rate of inundation in a polder will be significantly reduced when bentonite is added to the sandy core of a dike. Depending on the circumstances, this may result in a significant decrease in mortality and thus a significant increase in safety. A reduction of the erosion velocity of sand was achieved without changing the original behaviour of the sand. The direct shear tests and erosion tests indicate that the sand–bentonite mixtures still show sand-like behaviour; only the permeability is decreased by adding bentonite. This is favourable as it means that the stability of sand dikes will not change when the sand is mixed with small amounts of bentonite.

An added bentonite percentage of 2·4% will result in a significant reduction in erosion velocity. This is a promising result and could be used to slow down the breaching process of a sand dike. The model results show that just a small percentage of added bentonite achieves a significant decrease in important breach parameters and may even stop the breaching process completely. Retarding the breaching process reduces the inundation velocity in a polder. This results in decreased mortality or risk. The reduction of this risk means an increase in safety for the inhabitants of a polder. The necessary amount of bentonite to increase the safety to a desired level is different for each case.

An initial estimation of the cost of improving a dike using bentonite compared with traditional dike reinforcement shows that the cost of using bentonite is approximately the same as that of traditional dike reinforcement, while the same safety level is achieved (Lemmens, 2014). Also, compared with traditional dike reinforcement, a bentonite-improved dike does not require additional space relative to the original dike profile.

This study has shown that 5·4% of bentonite results in a reduction of mortality by a factor of 10. This percentage,
however, is based on extrapolation of the present data. Erosion tests with sand and a bentonite percentage of 5-4\% at higher flow velocities are necessary to confirm this behaviour. Further laboratory tests are also necessary to confirm that the strength characteristics of the sand core are not influenced by the presence of this amount of bentonite.

It may also be concluded that, besides adding bentonite, sand with an initial small percentage of clay can have a similar effect. Because of this, it is not necessary to use clean sand for dike construction, and slightly ‘polluted’ sand might actually be a better option, as long as the stability of the dike is not negatively affected.

REFERENCES

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