The curved high rise building

Structural investigation, part II
Preface

This report is the third in a series that will complete the thesis. The structural investigation part II compares several structural alternatives, before focusing on the most suitable alternative. Emphasis lies on the consequences of the curve to the structural behaviour of the building and on the question how the most eye-catching curved building can be constructed.

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### Symbols

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<thead>
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<td>$GFA$</td>
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<td>$z$</td>
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1 Introduction

In part I of the structural investigation five structural configurations for a curved building have been investigated. The curved core and curved tube structure resulted in very large bending moment in the structural system, up to six or seven times the moment caused by the wind load. Of course, almost everything is possible and thus even these kind of structural demands can be met. But a feasible and more realistic structural system should be found in straight and vertical types of primary structures. In this report, part two of the structural investigation, the structural principle that will be investigated more closely is the central core structure. This structure can be used to provide the necessary stiffness and to fold a curved shape around it.

1.1 General building parameters

Since the function of the building will be that of an office building one has to cope with the restrictions considering the availability of day light for the occupants of the building. An often used distance from a window to an office desk is 7.2 m. This figure is used in the standard floor plan of the building shown below. The offices are locate at the outside perimeter of the building and can be reached by an internal hallway (2 m). This hallway is placed around the central square core with a height to width ratio of 7.

![Diagram of standard floor plan]

Figure 1.1 Standard floor plan
The left and right hand side of the floor plan have a variable width. This parameter changes with the height of the building. The primary plan is the floor plan presented above, with the following parameters:

- Building height $h$: $150 \text{ m}$
- Core width (1:7): $21.4 \text{ m}$
- Office length: $7.2 \text{ m}$
- Hallway width: $2 \text{ m}$
- Building width $b$: $39.8 \text{ m}$
- Story height: $4 \text{ m}$

The mathematical formula by which the shape of the building is defined is:

$$y = A \sin \left( \frac{-2\pi}{h} \cdot x \right); \text{ Where } y = \text{Horizontal position}$$

$A = \text{Amplitude}$

$x = \text{Vertical position}$

**Wind pressure**

The wind pressure will be, independently of the applied amplitude, calculated by the formula stated below.

If one considers the wind load to be a static load (NEN 6702), the load can be determined by the formula stated below:

$$p_{rep} = C_{dim} C_{index} C_{eq} \phi_1 p_w$$

Where: $p_{rep} = \text{wind load by wind pressure}$

- $C_{dim} = \text{shape factor, for building dimensions}$
- $C_{index} = \text{wind shape factor}$
- $C_{eq} = \text{pressure factor}$
- $\phi_1 = \text{multiplication factor, for dynamic behaviour}$
- $p_w = \text{extreme value of stagnation pressure}$

- In case of a rectangular projection of the building, $C_{dim}$ depends on the height and width of the building. With the parameters presented above $C_{dim} = 0.86$
- The wind shape factor and the pressure factor can be neglected at this stage (low pressure near the roof): $C_{index} = 1$ and $C_{eq} = 1$
- The multiplication factor for dynamic behaviour $\phi_1 = 1.2$
- Stagnation pressure for a building located in the west of the Netherlands ($h = 150$), $p_w = 1.86 \text{ kN/m}^2$

---

$^1$ See Appendix E
\[ p_{re} = 0.86 \cdot 1.1 \cdot 1.2 \cdot 1.86 \approx 2 \text{ kN/m}^2 \]

Since the width of the building is 39.8 m, the continuous load on the building is:

\[ q_{re} = 2 \cdot 39.8 = 79.6 \text{ kN/m} \]

**Construction weight**

An important load is the gravity load of the building itself. In the case of a standard orthogonal building, the gravity load will be bared by, for example, a core structure and vertical supporting columns. The gravity load is led through the building and passed to the foundation. With a curved building, the gravity load will not only cause vertical loads on the building. Due to the shape of the building, horizontal forces will occur as well. These lateral forces will affect the minimal needed moment capacity and the minimal strength of the building.

Some rule of thumb figures that will be used for the weight of the structure are:

- **Floor height:** \( \frac{1}{30} \) \( \text{span} \)
- **Floor weight:** 15 kN/m\(^2\) (concrete hollow-core slab)

Since the building has a curved shape, the loads on the floors will be located eccentric from the core structure. This eccentricity of loads could cause an extra bending moment to the foundation. With the Dutch soil conditions in mind, this is not preferable. Therefore, the shape of the building will be that of a full sine curve. This implicates that the same gravity load will appear on both sides of the central structure (see figure 1.3).

Still, an eccentric variable load on the floors can cause an extra bending moment to the foundation.

![Figure 1.3 Gravity loads](image-url)
2 Structural design alternatives

2.1 Straight core and cantilevered floors

The 150 m high building is supported by a central core structure. The straight central core is the primary structure. Floors are cantilevering out of the core and the curved shape of the building is achieved by varying the length of the floors from story to story. A disadvantage of the changing length of the floors is that the floor plan will change from floor to floor as well. When a large amplitude is applied to the building the height of the cantilever floors may grow out of proportion and cause mechanical and functional problems as well.

Force flow
The force flow through the building is drawn in the structural scheme to the left. The floor will bare a permanent (green) and variable (red) load. Since the floors are fixed to the core structure by a moment-fixed connection, the loads will cause a bending moment to the core.
The magnitude of the bending moment differs from floor to floor, depending on the length of the floor:

\[ M_i = (F_p + F_v) \cdot (l_l - l_r) \]

Where:
- \( M_i \) = Moment at floor level \( i \) (kNm)
- \( F_p \) = Permanent floor load (kN)
- \( F_v \) = Variable floor load (kN)
- \( l_l \) = Floor length at level \( i \) on the left side (m)
- \( l_r \) = Floor length at level \( i \) on the right side (m)

**Standard building**

In order to be able to compare the various structural alternatives a standard building configuration will be used. The floor plan is already presented in chapter 1. The applied amplitude factor will be \( \eta = 0.5 \). The amplitude factor is used to calculate an amplitude by multiplying it with the total length of the office and hallway (\( l_o \)):

\[ A = \eta \cdot l_o \]

The shape of the standard building is presented in figure 2.3.

![Figure 2.3 Standard building shape](image)

Parameters for the standard building:
- Length of office and hallway \( l_o \): 9.2 m
- Amplitude factor \( \eta \): 0.5
- Amplitude: 4.6 m
- Foundation rotation stiffness: \( 9 \cdot 10^3 \) kNm/rad
The vertical loads and the gravity load of the floor itself will cause the bending moments to the central core. With this building configuration, the story height is set on 4 m, which means 37 stories. All these floors will generate a vertical force, linear to their length. In the diagrams of figure 2.4, the vertical forces, on the left and right side of the building, coming out of the floors are shown. The magnitude of the forces resembles the shape of the building (figure 2.3).

The bending moments on the central core structure are presented in figure 2.5. The cumulative effect of the moments coming from every floor is clearly visible. On the left side of the building, the larger vertical forces are located in the lower part of the building. In the upper part, the forces are smaller and accordingly, the cantilever arms are smaller as well. On the other hand, on the right side of the building, the larger forces are located in the upper part of the building. One can see the bending moment on the right side grow relatively fast before the curve flattens.

Figure 2.5 Bending moments left and right
The total bending moment coming from the floors on the left and right side of the building is presented in figure 2.6. The maximum bending moment is located halfway at 75 m. Because of the opposing bending moments on the left and right hand side of the building, the total bending moment at ground level is reduced to 0.

**Figure 2.6 Total bending moment**

Of course, the wind load cannot be neglected. The total bending moment including the moment caused by the wind load is shown in figure 2.7. The two extreme bending moments are drawn, with wind from the left side of the building (orange) and from the right (black).

**Figure 2.7 Total bending moment, incl. wind**
Deformation

The deflection of the standard building is drawn in figure 2.8. The core structure comes to a relatively large top deflection of 0.37 m. With a building height of 150 m the maximum deflection should be 0.30 m. The overall stiffness of the building will have to be improved to meet the requirement of the maximum top deflection.

The deflection of the core is calculated by using MatrixFrame.

Figure 2.8 Deflection of standard building
2.2 Straight core and supporting inclined columns

With this alternative the primary structural system is formed by a straight central core. The floors are supported by the core and by inclined perimeter columns in the façade of the building. Extra (straight) columns can be placed, depending on the applied amplitude. If the amplitude equals the total length of the office area no columns can be placed.

Force flow

The external forces on the building are shown in figure 2.10 (l). The floors bear the gravity (green) and variable loads (red). The floors are supported by the central core structure on one side and by inclined columns on the other. These inclined columns cause a lateral component that flows through the floors onto the central core (see figure 2.10 (r)). The magnitude of this lateral force depends on the angle of
the inclined column at that particular spot. In figure 2.11 part of the structural system is shown. If one analyses the central joint $i$, it is clear that the vertical load on floor $i-1$ is led to the central core and to the inclined column. The force in the inclined column $i-1$ will be:

$$F_{ci-1} = \frac{1}{2} \cdot (q_{fi-1} + q_{gi-1}) \cdot l \cos(\alpha_{i-1})$$

This force is led to joint $i$. At this point the following forces play a role:

- $F_{ci-1}$ horizontal component: $F_{ci-1H} = \tan(\alpha_{i-1}) \cdot \frac{1}{2} \cdot (q_{fi-1} + q_{gi-1}) \cdot l_{i-1}$

- $q_{fi-1} + q_{gi-1}$ (only vertical)

The total vertical force that has to be supported by inclined column $i$ is:

$$F_{ci} = F_{ci-1V} + \frac{1}{2} \cdot (q_{fi} + q_{gi}) \cdot l_i$$

Force in inclined column $i$:

$$F_{ci} = \frac{F_{ciV}}{\cos(\alpha_i)}$$

Since column $i$ is under an angle $\alpha_i$, a horizontal component will arise as well:

$$F_{ciH} = \tan(\alpha_i) \cdot F_{ciV}$$

![figure 2.11 Force flow through inclined columns and floors](image)

The resulting horizontal force in floor $i$ is:

$$F_{bi} = F_{ciH} - F_{ci-1H}$$

If one assumes that the floors have an infinite stiffness, this resulting lateral force $F_{bi}$ works directly on the core at level $i$ and cause a bending moment to the core of $M_i = F_{bi} \cdot h_i$.

The magnitude of the lateral force in the floors is dependent on the applied amplitude of the curve. When a large amplitude is used, the angle of the inclined columns will be more extreme and a larger
horizontal component has to be bearcd by the core structure. If the applied amplitude is smaller than the office length, there is room for a straight vertical column in the building (see figure 2.9). This vertical column will bare a part of the floor loads and thereby reduce the force that has to be bared by the inclined columns in the façade. Consequence of an extra row of vertical columns is that the part of the load that originally was bared by the core now flows through the vertical columns. This effect of less load on the central structure will result in lower compressive stresses or even tensile stresses.

**Standard building**

The same standard building configuration as with the previous alternative will be used to investigate the behaviour of the straight core combined with inclined perimeter columns.

**Figure 2.12 Standard building shape**

Parameters for the standard building:
- Length of office and hallway $l_o$: 9.2 m
- Amplitude factor $\eta$: 0.5
- Amplitude: 4.6 m

As mentioned before, with this alternative the lateral forces out of the floors onto the core may play an important role regarding the building's behaviour. These lateral forces act like shear forces which the core will have to deal with. The diagrams on the left show the influence of the left and right hand side of the building. Clear is that on the left side (blue), where most of the mass is in the lower part of the building, the lateral forces are smaller than on the right side of the building (pink).

Due to the cumulative effect of the lateral forces, the two diagrams do not show the same pattern. The lateral forces do not form a continuous load on the core. A peak load will occur at every floor level, in this case every 4 m.

**Figure 2.13 Lateral forces on left and right side of building**
A negative figure stands for a pulling force on the core pointing to the left whereas a positive number resembles a pulling force pointing to the right.

The total lateral force load is presented in figure 2.14 (†). One can see that the lateral forces will switch from left to right two times. The maximum lateral force will occur in the lower part of the building at $h = 28$ m, $F_L = 5851$ kN. And the lateral forces at ground level come close to zero: $F_L(0) = 1392$ kN

To provide better a better view of the lateral forces figure 2.14 (†) is added. This is a diagram of the interpolated lateral forces.

**Figure 2.14 Total lateral forces / Interpolated lateral forces**

Of course, besides the shear forces generated by the inclined columns, the wind load will cause a shear force as well. The continuous wind load of 2 kN/m² on the building will cause a shear force as presented in the diagram below.

**Figure 2.15 Wind induced shear force**
Although the lateral forces on both sides of the building do not present the same pattern (see figure 2.13), the similar pattern of the resulting bending moments on both sides of the building is clear. The total bending moment is mainly determined by the floors on the right side of the building. This, because of the larger weight that is present at the right side of the upper part of the building. The resulting total bending moment has its maximum at 37.5 m and is reduced to 0 at ground level.

The resulting total bending moments including the moment caused by the wind load are presented in figure 2.17. One can see that with this building configuration the extra moment that the core will have to bare is minimal. The resulting bending moment is mainly determined by the wind load.
Deformation

The deflection of the standard building is drawn in figure 2.18. The core structure deforms as one can expect from the bending moment pattern. The deflection of the core is calculated by using MatrixFrame.

Figure 2.18: Deflection of standard building
2.3 Curved core and cantilevered floors

To solve the problem of an altering floor plan the alternative of the curved core will be investigated. The curved core functions as the primary structure, with the floors cantilevering out of the core. A problem that arises when employing the curved core is the lack of or limited space available for vertical transport. Elevators can be constructed under a small angle (see report literature investigation) but the change in horizontal movement may cause serious problems to the users of the elevator. A solution may be a non-structural outside elevator shaft. By placing the elevator shaft outside the square floor plan of the building, there will be no interference of the elevators with the functional layout.

**Force flow**

The permanent (gravity) load and the variable load on the floors are transferred to the curved core by the moment-fixed connection. The result is an extra load on the core at every floor level. Together with the gravity load of the core itself these extra loads cause a bending moment to the curved core.

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*Figure 2.19 Structural system: curved core and cantilevering floors*

*Figure 2.20 External forces / Force flow*
In figure 2.21 the upper part of the building is shown (from point Q upwards). The forces out of the floors are coloured red and the gravity load of the core itself green. The bending moment in point Q consists of:

1. \[ M_1 = q_x \cdot l \cdot \frac{1}{2} \cdot A \]
2. \[ M_2 = \sum F_i \cdot a_i \]

\[ M_Q = M_1 + M_2 \]

This of course is besides the moment caused by the wind load on the building.

*Standard building*

The same standard building configuration as with the previous alternatives will be used to investigate the behaviour of the straight core combined with inclined perimeter columns.

To provide quick inside in the global behaviour of this configuration the curved core is modelled as a straight, inclined core with a change in angle at 37.5 m and at 112.5 m (figure 2.23). The results will not be exact but they may provide enough information to see whether the system is feasible.
Because of the curved shape of the core, bending moments caused by the core itself will arise (blue). The major part of the total bending moment is caused by the effect of the floor loads on the core (pink).

When analyzing the bending moments one should keep in mind that the system is modelled as shown in figure 2.23. Because of the used model the real maximum bending moment is slightly smaller than the peak moment presented in the diagram on the left. Still, one can clearly see that the maximum moment at 37.5 m is substantially larger than the moment caused by the wind load.

Figure 2.24 Bending moment: Gravity load core (11), Floor load (11) and wind (1)
The deflection of the standard building is drawn in figure 2.25. The core structure deforms as one can expect from the bending moment pattern. The "sweep-effect" due to the large bending moment at $h = 37.5$ m is clearly visible. The smaller bending moment at $h = 112.5$ m causes the structure to bend a little bit backwards. The deflection of the core is calculated by using MatrixFrame.
2.4 Curved core and supporting inclined columns

The primary structural system of this alternative is formed by a central curved core. The floors are no longer cantilevering out of the core but are supported by the core and by inclined perimeter columns in the façade of the building. Like with the previous alternative problems will occur regarding the place of the elevators.

Force flow
This alternative will show a more efficient and better force flow than the curved core with cantilevered floors. However, because of the clearly better performance of the straight core alternatives in structural behaviour, this variant will not be further analyzed in detail.
2.5 Comparison of alternatives

To be able to chose between the alternatives that are investigated in the previous chapters the advantages and disadvantages of the structural systems will have to be weighed up against each other. The structural alternatives can be compared within two different fields:

- Structural behaviour
- Functional behaviour

2.5.1 Structural behaviour

The overall structural behaviour of a high rise structure is determined by the bending moments, the shear forces and the resulting deformation. These aspects are summarized and compared below.

- Bending moments

The structural behaviour of the three alternatives will be identical when the wind load is investigated. Of course, the bending moment caused by the wind load is not dependent on the structural system of the building. The diversity in structural behaviour will be caused by the extra moments on the central core. These extra moments are initiated by vertical floor loads and the gravity load of the building itself. In the figure below the extra bending moments that will appear in the cores of the three alternatives are drawn.

![Figure 2.27 Bending moments (excl. wind moment) on core](image)

<table>
<thead>
<tr>
<th>Maximum moment (kNm)</th>
<th>Location x (m)</th>
<th>Percentage of wind moment (%)</th>
</tr>
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<td>1082927</td>
<td>75</td>
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<tr>
<td>Curved core</td>
<td>11835206</td>
<td>37.5</td>
</tr>
<tr>
<td>Core and inclined columns</td>
<td>141268</td>
<td>37.5</td>
</tr>
</tbody>
</table>

*Table 2.1 Maximum bending moments (excl. wind moment) on core*

Analyzing the extra bending moment, it is clear that the combination of a straight core and inclined columns provides the most effective system. The curved core system will create a massive extra
bending moment at 37.5 m and will only be feasible with a minimal applied amplitude. A straight core combined with cantilevered floors results in a rather large extra bending moment as well, although not as extreme as the curved core system. The maximum bending moment will be 1.21 times the maximum wind moment, but it is located at a convenient location (75 m). At this spot, the available moment capacity is substantial because of the relatively small moment caused by the wind.

With the core and inclined columns combination, the maximum extra bending moment is only 16% of the maximum wind moment. Furthermore, the maximum extra moment is not located at ground level, where it would cause an extra moment on the foundation, but at a quarter of the total height of the building (37.5 m). Some moment-capacity is still available at this height.

* Shear forces

![Shear force pattern on core](image)

<table>
<thead>
<tr>
<th>Core and inclined columns</th>
<th>Location x</th>
<th>Percentage of wind moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum shear force</td>
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<td>31</td>
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<tr>
<td>Minimum shear force</td>
<td>6187</td>
<td>59</td>
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<table>
<thead>
<tr>
<th>Curved core</th>
<th>Location x</th>
<th>Percentage of wind moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum shear force</td>
<td>-89327</td>
<td>748</td>
</tr>
<tr>
<td>Minimum shear force</td>
<td>86913</td>
<td>727</td>
</tr>
</tbody>
</table>

If a curved core is chosen as the primary structural system, very large shear forces will occur in the core. These shear forces are caused by the gravity load of the core itself and the load of the floors (permanent and variable), see figure 2.21.

The shear forces created by the lateral forces coming out of the floors in the core and inclined columns alternative are drawn in blue above. The diagram shows that the extra shear force is not excessive. The maximum total shear force will be located at ground level with wind from the right (18135 kN) or at 52 m when the wind blows from the left (10573 kN). These local shear forces should not result in major design problems.
In the diagram above, the initial deflection pattern of the three structural systems is presented. The initial deformation is caused by the loads on the floors and the gravity load of structure itself. The wind load is not included. Although the curved core system will react differently from the other two alternatives to a continuous wind load, it is clear from the pattern that this system will show the poorest behaviour.

The core and floors alternative shows a much better performance, with an initial top deflection of 0.12 m. Although the initial top deflection already forms 40% of the allowed maximum deflection (0.30 m), there is still space for extra deformation of the building due to wind and rotation of the foundation of the building.

The best behaviour is clearly shown by the core and inclined columns combination. With an initial top deflection of only 0.011 m, which is only 3.7% of the maximum allowed deflection, the deformation is minimal. The smaller lateral forces in the upper part of the building and the larger lateral forces in the lower part seem to counterbalance each other due to the respectively longer and shorter cantilever arms. The end result is a core construction that will show a minor top deflection (with the standard building configuration).
2.5.2 *Functional behaviour*

Besides the structural consequences of an uncommon curved building, one should review the functional behaviour of the building as well. Points of attention when checking the functional performance of a curved building are:

- **Floor plan**

If the standard floor plan can be applied to all stories of the building, it would create a high level of efficiency. A logical floor layout has to be designed only once and can be repeated at every floor level, as it is often done in a straight orthogonal building. This, in contrast with a constantly changing floor plan where a unique functional design has to be made for every single story (figure 2.30).

Both the core and floor alternative and the core and inclined columns have a straight core with the curved shape “folded” around it. Due to this straight core the floor plan will change from story to story, no repetition can be used in the design of these floor plans.²

![Figure 2.30 Floor plan](image)

The curved core alternative, on the other hand, has a core that follows the shape of the curve. The floor plan keeps the same layout at every floor level. From this perspective, the curved core could be a well working alternative.

An aspect that is will be similar for every curved building, irrespective of the applied structural system is the loss of useful floor space. Due to the inclination of the façade, useless office space will be created when the angle of the façade is too large for people to stand, irrespective of the employed structural system.

- **Vertical transport**

A great part of the available space in a building is covered by ducts and elevators for vertical transportation purposes. With a core in the centre of the building it would be logical to locate the vertical transport system there. The space inside the core can only be used for activities that do not require direct daylight, like an elevator system. In case of a curved building with a straight core, the allocation of the elevator system will not differ from a regular straight building. But if the core of the building follows the curved shape of the façade, serious problems regarding the vertical transport will occur. Consequence of the curved core will be that the vertical transport, in particular the elevator system, will have to be curved as well or at least has to be placed under an angle. It is not impossible to apply an inclined elevator system up to about $10^\circ$ inclination. If a larger angle is required, the lateral acceleration of the elevator cabin will become too large and endanger the safety of the people inside the elevator cabin [16]. The fact that the inclined elevator has to make a change in horizontal direction makes the problem even bigger. Not only will there be a lateral acceleration component, but the lateral acceleration will move from left to right and vice versa as well. A possible solution may be the allocation of the elevator shafts outside the building. By doing so, the shafts can be vertical and do not

² In the Case Study report functional behaviour of the building regarding repetitive floor plans will be analyzed more closely (chapter 3).
interfere with the floor plan. If the shafts are to be placed outside the building, that will change the
look of the building but it will result in a better functional behaviour.
The curved core building is not a preferable primary structural system from the vertical transport point
of view if the shafts have to be placed inside the building. By locating the shafts outside the building,
the curved core alternative can be an option as well. Alternatives with a straight and vertical core will
not cause extra problems to the vertical transport facilities inside the building.

2.5.3 Best performing alternative

In table 2.3 the criterions that were used to compare the alternatives are listed. The scores have a range
from -- to ++ (from bad to good behaviour). From the previous paragraphs it is clear that from a
structural point of view, the core and inclined columns alternative shows the best overall behaviour,
this is shown in the first part of the score table. In particular, the small extra bending moment and the
minor deformation contribute to the best building performance.
From a functional perspective, the curved core alternative can be considered to be an option as a
primary structural system. The repetitive floor plan will result in a better use of the available floor
space. If the non-curved elevator shafts are moved to the outside perimeter of the building,
interference with the curved core or the repetitive floor plan will not occur. On the other hand, the
curved core system is not a feasible alternative because of the poor structural behaviour.

If the overall behaviour of the three alternatives is compared, the “core and inclined columns”
alternative generates the highest score. This alternative clearly shows the best structural behaviour.
When the functional behaviour is analyzed the best alternative is the curved core system, that is if
outside elevator shafts are accepted. The functional advantage of a repetitive floor plan is clear.
Since it is the intention of the thesis to design a structural system, the structural behaviour of the
alternatives is considered to be more important than the functional behaviour at this point. The curved
core is not feasible when the structural behaviour is analyzed. Although, the functional behaviour of
the core and inclined columns alternative is not desirable, the structural behaviour is clearly the best.
The functional behaviour of the building will have to be improved in order to present an acceptable
building for the future occupant. In the next report, the functional behaviour will be analysed more
closely.

<table>
<thead>
<tr>
<th></th>
<th>Core and floors</th>
<th>Core and inclined columns</th>
<th>Curved core</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending moment at ground level</td>
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<td>++</td>
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<tr>
<td>Maximum extra bending moment</td>
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<td>++</td>
<td>--</td>
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<tr>
<td>Shear forces</td>
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<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Deformation</td>
<td>+</td>
<td>++</td>
<td>--</td>
</tr>
<tr>
<td><strong>Functional</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Floor plan</td>
<td>-</td>
<td>-</td>
<td>++</td>
</tr>
<tr>
<td>Vertical transport</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td><strong>Overall score</strong></td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2.3 Score table

Since the “core and inclined columns” alternative is the best performing alternative, this option will be
looked upon more closely in the next chapters. The structural system behaviour will be investigated in
more detail and later on the curved shape will be stretched to the limit.
3 Straight core and inclined façade columns alternative

Gravity loads and lateral force action are the primary loads that a building must resist, irrespective of its shape. The weight (gravity load) of a building's structure increases more or less linearly with the height of the building. This as a result of the roughly linear bond between the axial stresses and the size of the vertical members, and the more or less constant weight of the floors. The lateral force action becomes an important design factor for buildings higher than about 30 stories. The higher and more slender buildings are the more dominant will be the lateral force they are submitted to. The size of the material needed to resist this lateral force is not proportional to the height of the building but increases as the square of the height. This means that the dominant design factor for high rise buildings is no longer the strength of the building (as with low-rise buildings), but it is the stiffness of the building that controls the design. The primary structural requirements for a high rise building will be:

1. \( w < \frac{h}{500} \)
2. \( \varphi < \frac{1}{500} \)

When investigating a curved building the question arises whether the structural requirements presented above will cover the total list of demands. The structural system will be subjected to extra lateral forces due to the shape of the building. These lateral forces will affect the deflection and rotation of the building and will cause an additional bending moment to the primary structural system. The primary structural system must be able to resist this bending moment. An additional structural requirement will be:

3. \( M_{\text{core}} < \mu \cdot M \)

The factor \( \mu \) is dependent on the bending moment that is acceptable, on the maximum size of the primary structural system and on the concession that the architect is willing to make.

From the previous chapters it is clear that the "straight core and supporting inclined columns" alternative shows the best behaviour when used in a curved building with the selected parameters. The overall performance of this alternative regarding deflection and moment capacity is better than the other alternatives. The straight core with supporting inclined columns alternative will be stretched to the limit and investigated in more detail in the remainder of this report.

\[ \text{Figure 3.1 Possible curve configurations of the core & inclined columns alternative} \]
3.1 Structural behaviour

3.1.1 Stability

The straight core will have to provide sufficient stiffness to the whole building. A regular, straight building will undergo a deformation due to lateral wind load. In this case, with extra lateral loads at every floor level, the deformation is caused by both the wind load and (indirectly) by the vertical floor loads.

The resulting deflection will activate the buildings gravity load which causes a second deflection, this deflection causes a third deflection, etc, etc. This phenomenon is called the 2\textsuperscript{nd} order effect.

If the 2\textsuperscript{nd} order effect is progressive the construction is not stable, the effect should be regressive so the deflection is finite. It may very well be that the extra deformation caused by the lateral loads at every floor level cause the 2\textsuperscript{nd} order effect to grow out of control. The 2\textsuperscript{nd} order effect may prove to be a decisive factor in the design process of a curved building.

The standard building configuration in chapter 2 will be used to calculate the 2\textsuperscript{nd} order displacement of the building. Because of the curved lateral load pattern, the deflection will be calculated by using MatrixFrame software. The lateral loads\(^3\) are calculated as explained in chapter 2.2 and then transferred to the MatrixFrame file. The behaviour of the building is dependent on the loads employed on the building; the applied load combination. The load cases that are tested on the structure are listed below.

1. Wind load left side of the building
   As mentioned in chapter 1, the wind load \(q_{\text{rep}} = 2 \cdot 39.8 = 79.6\ \text{kN/m}\).

2. Wind load right side of the building;
   \(q_{\text{rep}} = 2 \cdot -39.8 = -79.6\ \text{kN/m}\)

\(^3\) The lateral loads coming from the floors are calculated with an MS-Excel program, see CD-rom
3. **Permanent lateral load left side of the building**

The permanent lateral load is caused by the permanent vertical load of the floors. The weight of the floors is partially bared by the inclined columns in the façade of the building. These inclined columns cause the lateral forces on the core structure through the floors. To show the influence of the curve on both sides of the building, the permanent load is split in a left and right (4) side of the building. The part of the permanent load that is actually bared by the inclined columns is presented in green.

![Figure 3.3 Permanent lateral load left](image)

4. **Permanent lateral load right side of the building**

The permanent lateral load on the right side of the building is presented in figure 3.4. The load pattern is clearly different from the pattern on the left side of the building.

![Figure 3.4 Permanent lateral load right](image)

5. **Variable lateral load left side of the building**

Of course, the variable load on top of the floors will cause lateral loads on the core as well. The load pattern is the same as for the permanent load (3), see figure 3.3.

6. **Variable lateral load right side of the building**

The load pattern resembles that of the permanent load on the right side of the building, see figure 3.4.
7. **Permanent vertical load**
   The permanent vertical loads in this structure are caused by the part of the floors that is carried by the core and the weight of the core itself. The part of the permanent vertical load that is supported by the core structure is marked by the dashed blue line in figure 3.5.

8. **Variable vertical load**
   The variable load on the floors is partially bared by the core structure as well. The load pattern is the same as with the permanent vertical load.

9. **Variable lateral load upper part**
   To check the response of the structure under various load conditions the building will be subjected to a variable load only in the upper half (green). The lateral load pattern is presented in figure 3.6.

10. **Variable lateral load lower part**
    When the variable load is reduced to the lower part of the building only, the lateral load pattern will look like figure 3.7.
Several load combinations are composed out of these different load cases in order to test the behaviour of this structural system (see table 3.1).

<table>
<thead>
<tr>
<th></th>
<th>Wind load left</th>
<th>Wind load right</th>
<th>Permanent lateral load left</th>
<th>Permanent lateral load right</th>
<th>Variable lateral load left</th>
<th>Variable lateral load right</th>
<th>Permanent vertical load</th>
<th>Variable vertical load</th>
<th>Variable lateral load upper part</th>
<th>Variable lateral load lower part</th>
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<tbody>
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</tr>
</tbody>
</table>

Table 3.1 Load combinations

For the standard building configuration the results are listed below:

<table>
<thead>
<tr>
<th>Deflection w(h)</th>
<th>Rotation φ(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>rad</td>
</tr>
<tr>
<td>L.C.1 0.1779</td>
<td>0.0001</td>
</tr>
<tr>
<td>L.C.2 0.1823</td>
<td>0.0001</td>
</tr>
<tr>
<td>L.C.3 0.1888</td>
<td>0.0131</td>
</tr>
<tr>
<td>L.C.4 0.1895</td>
<td>0.0154</td>
</tr>
<tr>
<td>L.C.5 0.1848</td>
<td>0.0001</td>
</tr>
<tr>
<td>L.C.6 0.1921</td>
<td>0.0154</td>
</tr>
<tr>
<td>L.C.7 0.1874</td>
<td>0.0001</td>
</tr>
<tr>
<td>L.C.8 0.1948</td>
<td>0.0155</td>
</tr>
<tr>
<td>L.C.9 0.1797</td>
<td>0.0001</td>
</tr>
<tr>
<td>L.C.10 0.1868</td>
<td>0.0154</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deflection w(h)</th>
<th>Rotation φ(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>rad</td>
</tr>
<tr>
<td>L.C.11 -0.1779</td>
<td>0.0001</td>
</tr>
<tr>
<td>L.C.12 -0.1735</td>
<td>0.0001</td>
</tr>
<tr>
<td>L.C.13 -0.1797</td>
<td>0.0131</td>
</tr>
<tr>
<td>L.C.14 -0.1804</td>
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<td>L.C.15 -0.171</td>
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<td>L.C.21 0.0114</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Table 3.2 Deflection and rotation by various load combination, wind load left (L.C.1-10) and wind load right (L.C.11-20)
The 1\textsuperscript{st} order displacement caused by the wind load on the left side of the building and the permanent lateral forces is printed in red: 0.1823 m. This first order displacement is in including the initial displacement of the core structure caused by the lateral loads due to the permanent weight of the structure (see load case 3 & 4, L.C.21 above). In case of a small amplitude factor (in this case <1) is this initial displacement of the core relatively small so that its influence on the total deflection is minimal. In figure 3.2 the initial top displacement with different amplitude factors is shown.

![Figure 3.2 Initial top deflection and amplitude factor](image)

The maximum 2\textsuperscript{nd} order displacement is 0.1948 m (green). The 2\textsuperscript{nd} order multiplication factor for this building configuration is: \( v = \frac{0.1948}{0.1823} = 1.07 \).

The precondition for a stable construction: \( n = \frac{v}{v-1} > 10 \)

In this case the \( n \)-value is: \( n = \frac{1.07}{1.07 - 1} = 15.6 \).

This is more than sufficient to achieve a stable construction. For a curved building it seems to be the case that the 2\textsuperscript{nd} order effect is not as influential as one may consider in the first instance.

Check for \( n \)-value with wind load on the right side of the building:
1\textsuperscript{st} order displacement: -0.1735 m
2\textsuperscript{nd} order displacement: -0.1804 m
\( v = \frac{-0.1804}{-0.1735} = 1.04 \) and \( n = 26.1 \)

This shows that the critical value for the 2\textsuperscript{nd} order displacement is found if the wind load is employed on the left side of the building.

The assumption is made that the rotation stiffness will cause a deflection
\( w_f = \frac{1}{2} \frac{w_{\text{max}}}{h} = \frac{1}{2} \frac{h}{500} = 0.15 \text{m}. \)

By this assumption the rotation stiffness of the foundation is set on: \( k = \frac{0.5 \cdot q \cdot h^3}{w_f} = 9 \cdot 10^8 \text{ kNm/rad} \)

The core height-to-width ratio is set on 7 (as mentioned before). With a height of 150 m, the core width is \( b_c = 21.43 \text{ m} \). The core wall width is set on 1 m.

With the standard building configuration, the 2\textsuperscript{nd} order effect will not endanger the overall stability of the structure. In the table below, the 2\textsuperscript{nd} order effect for a curved building with various building amplitudes is listed. From the table it is clear that the 2\textsuperscript{nd} order effect causes instability of the system when an amplitude factor close to 1 or larger is applied to the building. An amplitude factor of 1 means that façade of the building moves over the total length of the office and hallway (in case of the
standard building: 9.2 m). In table 3.3 the n-value of the 2\textsuperscript{nd} order effect is listed for various amplitude factors.

<table>
<thead>
<tr>
<th>Amplitude factor</th>
<th>Amplitude [m]</th>
<th>n-value</th>
<th>Multiplication factor</th>
</tr>
</thead>
<tbody>
<tr>
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<td>15.6</td>
<td>1.07</td>
</tr>
<tr>
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<td>5.52</td>
<td>13.4</td>
<td>1.08</td>
</tr>
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<td>0.7</td>
<td>6.44</td>
<td>11.9</td>
<td>1.09</td>
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<td>0.8</td>
<td>7.36</td>
<td>10.7</td>
<td>1.10</td>
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<td>0.9</td>
<td>8.28</td>
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<td>1.0</td>
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<tr>
<td>1.5</td>
<td>13.80</td>
<td>5.72</td>
<td>1.21</td>
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</table>

*table 3.3 n-value, 2nd order effect*
3.2 Amplitude variation

The standard building configuration has been investigated. At first sight, the central core construction combined with inclined perimeter columns seems to be a proper structural system to apply to a curved building. The lateral loads do not alter the behaviour of the building significantly. Both the bending moment and the top deflection stay within reasonable margins and the 2\textsuperscript{nd} order deflection does not cause instability of the system.

Now, the question arises whether a more extreme building may be possible as well. What is the maximum amplitude that can be achieved? In the diagrams below the amplitude grows gradually from 4.6 m (standard building configuration) to 9.2 m, the total length of office and hallway. The diagrams show the bending moment on the core (l) and the compression (r) on the left and right side of the core.

The compression is calculated by: \( \sigma = \sigma_N \pm \frac{n}{n-1} \sigma_M = \frac{N}{A} \pm \frac{n}{n-1} \frac{M}{W} \)

\[ \begin{array}{ll}
\text{total lateral 1st order} & \text{sigma N} \\
\text{total (wind left) 1st order} & \text{sigma M} \\
\text{total (wind right) 1st order} & \text{sigma left 1st order} \\
\text{total lateral 2nd order} & \text{sigma right 1st order} \\
\text{total (wind left) 2nd order} & \text{sigma left 2nd order} \\
\text{total (wind right) 2nd order} & \text{sigma right 2nd order} \\
\end{array} \]

\[ \begin{array}{c}
\text{Amplitude factor} \\
\text{Amplitude} \\
\text{GFA} \\
\text{n-value} \\
M_{\text{max lateral}} \\
\sigma_{\text{max}} \\
W_{\text{top}}
\end{array} \begin{array}{c}
0.5 \\
4.6 \text{ m} \\
5596 \text{ m}^2 \\
15.6 \\
1.41 \times 10^7 \text{ kNm} \\
-2.63 \text{ N/mm}^2 \\
0.195 \text{ m}
\end{array} \]

In figure 3.9 the behaviour of the standard building is presented. The bending moments of 1\textsuperscript{st} (dashed) and 2\textsuperscript{nd} order are drawn. The compression (r) on the left and the right side of the building remain below zero in the whole core structure. The compression caused by the permanent vertical load on the core is drawn as the dashed blue line. The dashed pink line represents the absolute value of the compression and tension caused by the bending moment on the building.
The diagrams above show the building with amplitude factor 0.6 and 0.7. One can see the growing influence of the 2nd order effect. Because of the larger amplitude, less floor area will be supported by the core structure, this effect is can be found in the decreasing value for $\sigma_\text{max}$ when the amplitude grows larger.
The diagrams above show the building with amplitude factor 0.8 and 0.9. The 2nd order effect cannot be neglected and causes an increase of the maximum moment of more than 10%. Because of the larger amplitude, less floor area will be supported by the core structure, this effect can be found in the decreasing value for $\sigma_N$ when the amplitude grows larger. Noticeable is the constant gross floor area (GFA). Floor space that is lost on one side of the building due to a larger amplitude, is won on the other side.
In the diagrams above a central core structure and an amplitude factor of 1.0 and 1.1 is simulated. The 2nd order effect will cause instability to the structure (n < 10) which means that the core structure does not provide enough stability by itself and extra measures are required. When an amplitude factor of 1.0 (A = 9.2 m) is applied to the building the concrete core on the left side of the building is no longer fully under compression. At h = 37.5 m a little tension can be noticed σ = 0.06 N/mm².

If the amplitude factor is set on 1.1, the office and hallway length is enlarged to 10.12 m. By doing so, the gross floor area is enlarged with 5570 m² to 61266 m². A larger part of the left side of the core has to resist tension instead of the ideal compression. But still, the magnitude of the tension will not cause major problems and can be resisted by pre-tensioning of the core.
3.3 Optimizing to the extreme curved building

3.3.1 Phase shifting

As mentioned before, the shape of the building that has been analyzed so far is formulated as:

\[ y = A \cdot \sin \left( -\frac{2\pi}{h} \cdot x \right) \]

Where: \( A = \text{Amplitude} \)
\( h = \text{Building height} \)

In chapter 3.2 amplitude \( A \) is stretched to its maximum of about 9.2 m. A larger amplitude causes tension in parts of the core. If one analyzes the compression pattern of the sine-shaped structure, there is some room left for extra tension (see figure 3.13). If this free capacity could be utilized, a more extreme curved shape may be possible. One way to change the compression pattern is to move the sine up or down. This can be achieved by shifting the phase of the sine.

Now, the shape of the building is formulated as:

\[ y = A \cdot \sin \left( -\frac{2\pi}{h} \cdot (x + \beta) \right) \]

Where: \( \beta = \text{phase shift} \)

The building configurations that have been analyzed so far all have \( \beta = 0 \). In that case the load case with the wind load from the left side of the building will determine the maximum compression in the core. If the sine curve is shifted up or down by altering the value of \( \beta \), wind load from the right side of the building may become the determining factor. In figure 3.14 the compression pattern due to wind load from the right side of the building is drawn as well.

---

**Figure 3.13** Compression diagram, \( A=9.2 \text{ m} \)

**Figure 3.14** Compression pattern due for wind load from left and right side of building (\( A=9.2 \text{ m}, \beta =0 \))
The diagrams above show that if the curve of the building is shifted up, the critical stress will move from the left side of the building to the right. The phase shifting of the curve will cause the lateral moment at ground level to grow. With $\beta = 30$, the lateral moment at $x=0$ is about the size of the bending moment caused by the wind solely. The shape of the building moves towards the shape of a cosine.
If the curve is shifted even further, one can see that the lateral moment at ground level starts to decrease again. All the critical stresses are now located on the right side of the building.
When factor $\beta$ is set on 75 m, the building is shifted half a period. The compression pattern is similar to the non-shifted sine curve, it only has switched from left to right.

Phase shifting of the sine curve does not provide a more extreme building than the original configuration with $\beta=0$. If one wants to design a cosine-like structure, phase shifting can be applied, but the consequence of such a shape is a relatively large bending moment at ground level.

Phase shifting will result in an increasing bending moment at ground level.
3.3.2 Damped sine curve

If one analyzes the curved shape of the perfect sine as it was investigated in the previous chapters, it may be logical to find a shape that is curved, but not a perfect sine. The objective, a curved building, is still achieved and a better building performance may be possible.

At first sight, the cantilevering mass located on the upper part of the core (see figure 3.18) is likely to cause the most problems.

By "damping" the sine, one can realise a shape that is still curved, but with less mass at the top of the building. The function of the perfect sine is there for multiplied with a linear function (figure 3.19):

\[ y = A \cdot \sin \left( -\frac{2\pi}{h} (x + \beta) \right) \cdot \left( C \frac{x}{h} + B \right) \]

Where:  
\( C \) = constant coefficient  
\( D \) = constant coefficient

<table>
<thead>
<tr>
<th>Amplitude factor</th>
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</tr>
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<tbody>
<tr>
<td>Amplitude</td>
<td>9.2  m</td>
</tr>
<tr>
<td>GPA</td>
<td>55696 m²</td>
</tr>
<tr>
<td>n-value</td>
<td>19.1</td>
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<tr>
<td>( M_{\text{max, lateral}} )</td>
<td>( 6.27 \times 10^5 ) kNm</td>
</tr>
<tr>
<td>( M(0) )</td>
<td>( -1.17 \times 10^5 ) kNm</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0 m</td>
</tr>
<tr>
<td>( B )</td>
<td>1.23 m</td>
</tr>
<tr>
<td>( \sigma_{\text{max}} )</td>
<td>-140 N/mm²</td>
</tr>
<tr>
<td>( W_{\text{sup}} )</td>
<td>m</td>
</tr>
</tbody>
</table>
In figure 3.20 the result of a damped sine curve is presented. Although the shape is less extreme than the perfect sine curved structure, the curve is still there. The sine function is multiplied by: \[ y_2 = C \frac{x}{h} + B = -1 \cdot \frac{x}{150} + 1.238 \]

The values for \( B \) and \( C \) are chosen this way to generate the same gross floor area as a perfect sine curved building. The effect on the behaviour of the building is clear:

- The n-value of the 2\(^{nd}\) order effect has increased from 8.37 to 19.1, the influence of the 2\(^{nd}\) order effect is less dominant.
- Tension in the core with the perfect sine has changed into compression. 
  \( \text{min.compr.} = 1.4 \text{N/mm}^2 \) 

The resulting building behaviour will improve if a damped sine curve is applied to the building. A negative side is that the original curve of the building will disappear, the curve is less impressive.

**Amplitude adjustment**

All parts of the core are under pressure with the applied building configuration (figure 3.20). When the amplitude of the damped sine is enlarged the influence of the 2\(^{nd}\) order effect will be larger and the lateral forces will grow as well. The figure below is found when the maximum amplitude (with no tension) is applied to the building and the 2\(^{nd}\) order effect is still under control.

![Diagram](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Amplitude factor</td>
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</tr>
<tr>
<td>Amplitude</td>
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</tr>
<tr>
<td>GFA</td>
<td>65421 m²</td>
</tr>
<tr>
<td>n-value</td>
<td>17.7</td>
</tr>
<tr>
<td>( M_{\text{max; lateral}} )</td>
<td>( 2.35 \times 10^7 ) kNm</td>
</tr>
<tr>
<td>( M(0) )</td>
<td>( -2.17 \times 10^4 ) kNm</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0 m</td>
</tr>
<tr>
<td>( B )</td>
<td>1.14 m</td>
</tr>
<tr>
<td>( C )</td>
<td>-1</td>
</tr>
<tr>
<td>( \sigma_{\text{max}} )</td>
<td>0.00 N/mm²</td>
</tr>
<tr>
<td>( w_{\text{kap}} )</td>
<td>m</td>
</tr>
</tbody>
</table>

*figure 3.21 Damped curved building, extreme*

The building configuration presented above shows the most extreme damped sine curve shape. With this shape, there will be no tension in the core and the amplitude is set on 11.96 m. It has to be mentioned that the effective amplitude reaches only 10.81 m, this because of the damping.
The gross floor area of the building is now 65421 m², larger than with the original configuration. The larger floor area is caused by the larger amplitude factor of 1.3. Due to this amplitude factor the maximum office and hallway length is now 10.81 m instead of 9.2 m.

Damping of the sine curve will result in a better overall building performance, but the original curved idea fades away.
### 3.3.3 Reverse damping

The effect of the damped sine curve is shown in the previous chapter. A larger amplitude can be applied, the 2nd order effect stays under control and the structural core is fully under compression. The disadvantage of the damped curve is that the shape is less extreme and not as impressive as the perfect sine curve. Not a damped curve would be impressive but the opposite, a constantly increasing curve would be a true extremely curved building.

In this chapter the possibility of a constantly increasing curve is analyzed and investigated. The mathematical function that describes the shape of the building is the same as with the damped function, the standard building will be multiplied by a linear function:

\[
y = A \cdot \sin \left( -\frac{2\pi}{h} (x + \beta) \right) \cdot \left( C \frac{x}{h} + B \right)
\]

An example of the linear function is presented in figure 3.22.

![Figure 3.22 Multiplication function (C=Z, B=0)](image)

![Figure 3.23 Reverse damped building](image)

<table>
<thead>
<tr>
<th>Amplitude factor</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>9.2 m</td>
</tr>
<tr>
<td>GFA</td>
<td>55696 m²</td>
</tr>
<tr>
<td>n-value</td>
<td>9.48</td>
</tr>
<tr>
<td>( M_{\text{max, lateral}} )</td>
<td>7.60 (10^7 ) kNm</td>
</tr>
<tr>
<td>( M(0) )</td>
<td>2.94 (10^5 ) kNm</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0 m</td>
</tr>
<tr>
<td>( B )</td>
<td>0 m</td>
</tr>
<tr>
<td>( C )</td>
<td>1.895</td>
</tr>
<tr>
<td>( \sigma_{\text{max}} )</td>
<td>-0.41 N/mm²</td>
</tr>
<tr>
<td>( W_{\text{eff}} )</td>
<td>0.265 m</td>
</tr>
</tbody>
</table>
The shape of the reverse damped building (figure 3.23) is certainly more spectacular than the damped curved building that was investigated in the previous chapter. A disadvantage of the reverse damped curve structure is the bending moment at ground level due to lateral forces out of the floors. This extra bending moment is so large that it cannot be absorbed by a proper foundation, but it will certainly increase the cost of the foundation works. Considering the 2nd order effect, the building finds itself on the edge of a stable or unstable construction. In a strict way, the building is unstable because of the $n$-value < 10. Nevertheless, the stiffness of the overall construction only has to be improved slightly to achieve a stable building with an $n$-value >10. If a larger factor $C$ of the multiplication function is applied to the building, the 2nd order effect will become more dominant and cause a more and more unstable building.

![Graphs showing lateral forces and moments](image)

**Figure 3.24 Lateral forces / Lateral moment**

In the diagrams of figure 3.24 the lateral forces out of the floors are shown. The dashed red line represents the pattern of the lateral forces of the perfect sine building (amplitude = 9.2 m as well). The diagrams show a more or less similar pattern at the top of the building but the patterns differentiate when moving downwards. The pattern of the lateral forces of the reverse damped curve building switches from side to side more often. The bending moment due to lateral forces is mainly caused by the lateral forces in the upper part of the building. Since the pattern in this area is similar to the pattern of the perfect sine building, the bending moments do not show great differences other than at ground level, where the bending moment of the reverse damped curve building is relatively large.
The stability of the building can be improved by choosing another combination of the factors $B$ and $C$ of the multiplication function (see figure 3.25). In the diagrams below the factors are chosen in a way that the building has the same floor area of 55696 m² and still no tension in the structural core, but a better performance considering the overall stability of the construction.

![Figure 3.25 Multiplication function (B=0.5, C=0.98)](image)

![Figure 3.26 Reverse damped curve building, improved stability](image)

<table>
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<tbody>
<tr>
<td>Amplitude</td>
<td>9.2</td>
<td>m</td>
</tr>
<tr>
<td>GFA</td>
<td>55696</td>
<td>m²</td>
</tr>
<tr>
<td>$n$-value</td>
<td>10.49</td>
<td>-</td>
</tr>
<tr>
<td>$M_{\text{max, lateral}}$</td>
<td>8.35 $10^5$</td>
<td>kNm</td>
</tr>
<tr>
<td>$M(0)$</td>
<td>1.91 $10^7$</td>
<td>kNm</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>$B$</td>
<td>0.5</td>
<td>m</td>
</tr>
<tr>
<td>$C$</td>
<td>0.9751</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$</td>
<td>-0.31</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$w_{\text{max}}$</td>
<td>0.249</td>
<td>m</td>
</tr>
</tbody>
</table>

With the reverse damped shape the curve of the building grows with the height of the building. At first sight, this configuration may look "counterproductive" and bound to result in a poor building performance. But when the system is analyzed more closely one discovers an improving instead of deteriorating building behaviour.

The effect can easily be clarified by using the moment-formula: $M = F \cdot z$. Moving from the top downwards, the force in the inclined columns will gradually increase at every floor level. In the upper part of the building, the inclined columns will only have to bear the load of a limited number of stories. This means that in order to utilize the existing moment capacity in the upper part of the core, the lever arm (in this case the curve) can be enlarged when moving upwards.
Although the performance of the structure has improved, the resulting shape of the building may be a little disappointing. The curve of the building grows larger when moving upwards, but the effect may be not as clear as one would have hoped for. The shape of the building does not differ too much from the original perfect sine shape. In the next paragraph an effort is made to make the curve of the building even more extreme and at the same time consolidate the improved building performance.

Reverse damping will give the building a more extreme curved shape and at the same time improve the overall building performance.
3.3.4 Multiple curves

In chapter 3.3.3 the reverse damped curve building was analyzed. The result of the analysis was that the shape of such a building is quite spectacular and more extreme than the perfect sine curved building. However, the building tends to become unstable with a reverse damped shape. When the building is stabilized by altering the multiplication function, the extreme effect of the reverse damped curve building gradually fades away.

In this chapter, the number of curves of the reverse damped structure is changed to see whether this has effect on the stability of the structure. The shape of the building is now described by the following formula:

\[ y = A \cdot \sin \left( \frac{-2\pi}{h/v} \cdot (x + \beta) \right) \left( C \cdot \frac{x}{h} + B \right) \]

Where \( v \) = number of curves

In the diagrams below the number of curves is set on two. The effect of the two curves is obvious from the bending moment and compression diagram. Because of the two curves, the lateral forces will switch from side to side more often than in case of the single curved structure (see figure 3.27). The switching of the lateral forces creates a better balance between the forces on the left and right of the building. The lateral bending moment cannot grow to the magnitude like with a single curved building, due to this better balance.

![Figure 3.27 Reverse damped curve building, 2 curves](image-url)
The 2nd order deflection of the building is no longer growing out of control, the n-value stays out of the danger zone. The stabilized bending moment pattern causes less tension on the left side of the core so that the whole core will be under compression at all times.
The shape of the building is even more extreme with the double curved building and reverse damping. The structure shows an unnatural extreme shape but still, the overall performance of the building meets the structural demands.

Figure 3.28 Lateral forces, reverse damped building, 2 curves
In the diagrams below, the number of curves is respectively set on 3 and 4. The effect on the overall behaviour of the building only gets better. The n-value increases, the bending moment is even more stable and the core will be totally under compression.

![Graphs showing bending moments and compression forces](image)

<table>
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<td>m²</td>
</tr>
<tr>
<td>ν</td>
<td>3</td>
<td></td>
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<tr>
<td>n-value</td>
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<td>$M_{max; lateral}$</td>
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<td>kNm</td>
</tr>
<tr>
<td>$M(0)$</td>
<td>$1.47 \times 10^4$</td>
<td>kNm</td>
</tr>
<tr>
<td>β</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>C</td>
<td>1.2014</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{max}$</td>
<td>-0.49</td>
<td>N/mm²</td>
</tr>
</tbody>
</table>

Figure 3.29 Reverse damped curve building, 3 curves / 4 curves
One can see that applying multiple curves to the reverse damped building improves the performance of the structure. The question is what the maximum number of curves should be. The criteria to answer this question will lie in the field of usable floor area because of the increasingly inclined façades. Another criterion may be the level of extremity that one wants to achieve. Is a building with eight reverse damped curves still attractive and inviting (see figure 3.30)?

![Graphs showing bending moment and compression](image)

<table>
<thead>
<tr>
<th>Amplitude factor</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>9.2 m</td>
</tr>
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<td>GFA</td>
<td>55696 m$^2$</td>
</tr>
<tr>
<td>ν</td>
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<td>n-value</td>
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<tr>
<td>β</td>
<td>0 m</td>
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<tr>
<td>$\sigma_{\text{max}}$</td>
<td>-0.31 N/mm$^2$</td>
</tr>
</tbody>
</table>

*figure 3.30 Reverse damped curve building, 8 curves*

**Applying multiple curves to the building leads to an improved building performance regarding:**
- Lateral bending moment
- Building deformation
- Stability
4 Curved building conclusions

4.1 Fine tuning

In the previous chapter, the curved building has been optimized in a way that the most extreme curved building is created. Starting with a perfect sine shape several modifications have been made to the structure to come to the reverse damped structure as the most extreme curved building with an acceptable building performance.

An option that has not been mentioned yet is the possibility of adding a part of a curve to a certain configuration instead of a whole extra curve. In the next chapters this fine tuning of the curved building is presented.

4.1.1 Perfect sine curve

If one wants to create a curved building based on a perfect sine, the question is how many curves and what amplitude one should apply to design a well performing structure. In figure 4.1 the relation between the top deflection caused by the lateral loads of the building itself, the bending moment (excl.

![Diagram showing top deflection and lateral moment](image-url)
wind moment) at ground level and the applied number of curves is presented. From this diagram it is clear that the optimum regarding the top deflection is not at all achieved by applying an integer number of curves. The optimum lies close to 1.4, 2.4, 3.4, etc. Furthermore, the conclusion can be drawn that an increasing number of curves contributes to a smaller initial top deflection.

If one analyzes the pattern of the bending moment at ground level the optimum lies at the same number of curves that was found with the initial top deflection. However, it is not possible to realize a bending moment of 0 kNm if more than 1.7 curves are applied. Figure 3.31 can be used as a guideline to find the right combination of curves and amplitude. Of course, the ideal curved building is created by an impressive curve but at the same time a minor extra bending moment at ground level. From the figure above it is clear that this ideal combination can be achieved by choosing the right number of curves and the right amplitude.

In the diagram below the \( \mu \)-factor related to the number of curves is presented. The \( \mu \)-factor stands for the ratio of the extra bending moment on ground level caused by the internal lateral forces and the wind moment at level 0. One can see that the extra bending moment can be controlled by applying a small number of curves combined with a small amplitude. But even with a large amplitude of the total office length (9.2 m) and five curves, the extra bending moment at ground level is not larger than 45% of the wind moment.

---

**Figure 4.2 \( \mu \)-factor**
4.1.2 Reverse damped sine curve

The reverse damped sine curve has come out of the investigation as the most spectacular shaped building, with an acceptable building behaviour. Like with the perfect sine shape, the investigation of the reverse damped curve is based on configurations with an integer number of curves. In the figure below, the influence of the number of curves on the initial top deflection and the bending moment at ground level is presented.

![Graph showing the influence of the number of curves on top deflection and bending moment.]

*Figure 4.3 Lateral moment at x=0 and top deflection*

If the initial top deflection is chosen as a starting point, the optimum for a configuration with no initial top deflection has 1.22, 1.72 2.22, 2.72, etc curves. These numbers of curves will provide the best behaviour regarding the initial top deflection. Similar to the perfect sine curve, the conclusion can be drawn that an increasing number of curves contributes to a smaller initial top deflection.

If one analyzes the pattern of the bending moment at ground level the optimum lies at the same number of curves that was found with the initial top deflection. Not only lies the optimum for the
lateral bending moment at the same number of curves, but the optimum comes close to zero as well. The diagram above can be used as a guideline to find the right combination of curves and amplitude for the reverse damped building shape.

From figure 4.3 it is clear that an increasing number of curves has a positive effect on both the initial deflection and the extra bending moment at ground level. Important is to keep in mind that a minor change in the number of curves could cause a relatively large change in both deflection and bending moment.

In the diagram below the μ-factor related to the number of curves for the reverse damped structure is presented. The μ-factor stands for the ratio of the extra bending moment on ground level caused by the internal lateral forces and the wind moment at level 0. One can see that the extra bending moment can be controlled by applying a large number of curves to the building. The amplitude of the curve only causes small differences in the μ-factor, especially when the number of curves increases. The conclusion regarding the extra bending moment at ground level for a reverse damped structure is that the extra bending moment decreases when the number of curves is enlarged.

Figure 4.4 μ-factor
4.2 Conclusions

The reverse damped building with multiple curves is found to be the best alternative to come to a extreme curved high rise design. If one compares the different building configurations in figure 4.5, the most impressive and attractive sight is clearly the multiple curved building on the right. The impression of its shape cannot be matched by any of the other alternatives.

![Figure 4.5: Alternative building shapes](image)

The reverse damped building with multiple curves does not only provide the most impressive view, but the mechanical behaviour of this configuration can compete with the other alternatives as well. The lateral bending moment, that is the bending moment in the central core exclusive of the wind moment, is drawn in figure 4.6. The maximum lateral bending moment of the multiple curved building is much smaller than with the other alternatives and even more important: the bending moment at ground level is not excessive and comes close to a value of zero.

![Figure 4.6: Lateral bending moment in central core](image)
Due to the multiple curves of the building, the bending moment switches from one side to the other several times. The resulting deformation pattern of the building shows a core that is bended back and forth due to this better balance between the forces on the left and right of the building. The lateral bending moment cannot grow to the magnitude like with one of the other alternatives and the top deflection of the building will be significantly smaller than with one of the four other alternatives, see.

![Graph showing deformation patterns](image)

**Figure 4.7 Deformation patterns**

An additional effect of the switching and thus smaller bending moment is that the stresses in the central core are likely to be compressive stresses. The bending moment does not reach to a level where it can eliminate the compression caused by the normal force in the core. When the core is under compression only, the needed reinforcement will be minimal.

![Graph showing compression curves](image)

**Figure 4.8 Compression curves**
The final conclusion is that a primary structural system based on a central core and inclined columns in the façade of the building should be combined with a reverse damped shape with multiple curves. This combination will show the best building behaviour from a structural point of view and will result in the searched for extreme curved building:

- Multiple curves, with larger amplitude at higher level, an extreme design
- Small bending moment at ground level
- Small initial deflection
- No tension in central core

One should keep in mind that a minor change in the applied number of curves may not have a great impact on the resulting shape of the building, but the impact on the structural behaviour cannot be underestimated, fine tuning is required.
### Appendix A: Literature

7. Form follows finance, *Carol Willis*, New York 1995
Appendix B: Internet

1  www.greatbuildings.com
2  www.hoogbouw.nl
3  www.hoogbouw.pagina.nl
4  www.skyscrapers.com
5  www.skyscraperphotos.com
6  www.twistscrapers.com
7  www.arup.com
8  www.som.com
9  www.kpf.com
10 www.elevator-world.com
11 www.otis.com
12 www.schindler.com
Appendix C: Address file

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Appendix D: Timetable
Appendix E: dynamic multiplication factor, wind load

\[ \phi_i = \frac{1 + 7 \cdot I(h) \cdot \sqrt{B + E}}{1 - 7 \cdot I(h) \cdot \sqrt{B}} \]

with:

\[ B = \frac{1}{0.94 + 0.021 \cdot h^3 + 0.029 \cdot b^3} \]

\[ E = \frac{0.0394 \cdot f_e^2}{D \cdot (1 + 0.1 \cdot f_e \cdot h) \cdot (1 + 0.16 \cdot f_e \cdot b)} \]

\[ I(h) = \frac{1}{\ln\left(\frac{h}{0.2}\right)} \]

Where:

\( I(h) \) = Turbulence intensity at height \( h \)
\( h \) = Height of the structure
\( b \) = Mean building width perpendicular to the wind direction
\( D \) = Damping
\( f_e \) = Natural frequency

\[ f_e = \sqrt{\frac{a}{\delta}} \]

Where:

\( \delta \) = Deflection
\( a \) = Acceleration

With parameters:

\( h = 150 \) m
\( b = 40 \) m
\( D = 0.02 \) (concrete)

We find \( \phi_i = 1.2 \)
Appendix F: Compression curves

Perfect curved structure

**1 curve**

![Graph 1](attachment:image1)

**2 curves**

![Graph 2](attachment:image2)
Reverse damped curved structure

1 curve

2 curves