Flow Mechanism of Sand-Water Mixtures in Pipelines

Václav Matoušek
Delft, 15 December 1997
I.
Een slurrie is niet gewoon vuil water.

II.
De mechanismen van een slurriestroming kunnen beschreven worden als de ontwikkeling van de inwendige opbouw (concentratie- en snelheidsprofiel) bekend is. De modellen die de stroming voorspellen op basis van de bepaling van de slurrie-stromingsmechanismen moeten getest worden middels experimenten die ook voorzien in de gegevens omtrent de inwendige opbouw van de stroming.

III.
Een fysisch twee-lagenmodel om de stroming te voorspellen is een geschikt instrument ter vervanging van de in de praktijk gebruikelijke empirische modellen. Het brengt de voornaamste mechanismen die de stroming van zand-watermengsels beheersen in rekening en het beschrijft het stromingsgedrag waarbij gebruik gemaakt wordt van parameters die geschikt zijn voor praktische evaluatie van het hydraulisch transport proces.

IV.
Een heterogene stroming van slurrie te beschouwen als een twee-lagenstroming is niet minder nauwkeurig dan Europa te beschouwen als Oost- en West-Europa. Zowel Europa als een heterogene stroming hebben een overgangszone die zich apart gedraagt en in belangrijke mate het gedrag van de gehele eenheid beïnvloedt.

V.
De overtuiging van het merendeel der baggeraars dat het probleem van het hydraulisch transport van vaste stof is opgelost is alleen begrijpelijk wanneer men zich realiseert dat er van de meeste andere baggerprocessen nog minder bekend is.

VI.
Experimenten geven soms inspiratie maar zijn altijd dodelijk voor de illusie.

VII.
De wetenschappelijke disciplines "sedimenttransport" en "hydraulisch transport" hebben meer gemeen dan literatuur en conferenties over beide vakgebieden doen vermoeden.

VIII.
Des te minder natuur beschikbaar is des te minder natuur men nodig heeft.

IX.
Het schrijven van stellingen in een vreemde taal is als vrijen met rubber handschoenen aan.
I.
Slurry is not just dirty water.

II.
The mechanism of a slurry flow can be evaluated if the development of the internal structure (concentration/velocity profile) of the slurry flow is known. The flow predictive models based on the determination of a slurry flow mechanism should be tested by experiments that also provide the data on the internal structure of slurry flow.

III.
A physical two-layer model is a suitable predictive tool to replace the empirical models used in practice. It takes into account the major mechanisms governing a flow of sand-water slurries and describes the slurry flow behaviour using parameters suitable for practical evaluation of the hydraulic transport process.

IV.
Considering a heterogeneous flow of a settling slurry as a two-layer flow is not less accurate than considering Europe as the Eastern- and the Western Europe. Both Europe and a heterogeneous flow have a mid-zone which behaves specifically and considerably influences the entire behaviour of the entity.

V.
The conviction of most dredging engineers that the hydraulic transport of solids is a problem solved is comprehensible only if one realises that even less is known about most other processes of the dredging cycle.

VI.
Experiments sometimes bring inspiration but always kill illusion.

VII.
The scientific disciplines called "sediment transport" and "hydraulic transport" overlap more than is suggested by their literature and conferences.

VIII.
As far as nature is concerned, the less man has the less he needs.

IX.
Writing propositions (stellingen) in a foreign language is like making love in rubber gloves.
Flow Mechanism
of
Sand-Water Mixtures
in
Pipelines
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PROEFSCHRIFT

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Summary

Flow Mechanism of Sand-Water Mixtures in Pipelines

Václav Matoušek

This doctoral thesis addresses the hydraulic transport of solids in pipelines. It summarises the results of an investigation carried out to describe the flow of sand-water mixtures in pipelines and to propose a method of a mixture flow prediction using a theoretically-based model. Special attention is paid to the phenomena occurring in the mixture flow in a pipeline during a dredging operation.

In dredging practice, the models used to predict the behaviour of mixture flow in pipelines are based on an empirical approach rather than on a physical description of the flow mechanisms. Although the physical approach to the modelling of the pipeline flow of settling mixtures was introduced in the form of a two-layer model, it has rarely been used in practice. This is because the model was considered applicable only to the special flow conditions obtained when transporting very coarse solids in pipelines. Moreover, there was little experimental evidence of the validity of the model principles. To verify these principles experiments are required, including measurements of the internal structure of mixture flow in various mixture flow conditions.

The most popular empirical models now in use date from the 1950's and 1960's (Durand et al., Führböter, Jufin-Lopatin). These are correlations of dimensionless groups containing the most important flow parameters. The values of the empirical coefficients of the models are determined from experiments. In 1970's a theoretical concept was introduced (Wilson et al.) based on the principle of force balances in the two layer pattern of a mixture flow stratified into a bed (contact) load and a suspended load. The two-layer model appeared to be a valuable tool for the prediction of the fully-stratified flow (all solid particles transported as a contact load) but problems arose with the application of the model to a partially-stratified flow. The proposed solutions for the division of transported solids into a contact load and a suspended load were not supported experimentally. A credible solution requires a knowledge of the concentration distribution and/or the velocity distribution of solids across the mixture stream. Measurements of the internal structure of the mixture flow have very rarely been done.

In this thesis the study of mixture flow mechanisms is supported with experiments which include measurements of either the velocity profiles or the concentration profiles in partially-stratified flows through a dredging pipeline and through a laboratory circuit. The research was based on a data base collected during extensive field measurements on a dredging installation. The dredging-pipeline measurements indicated a surprising phenomenon of a transformation of density fluctuations in the
mixture flow at the pipeline inlet into density waves along the long dredging pipeline. In this study this phenomenon of solids aggregation in an unsteady solids flow was analysed, explained and its effect on the pipeline operation evaluated in this study. The force-balance concept for two-layer flows was found appropriate for the description of the aggregation phenomenon in a dredging pipeline. The principles of the aggregation phenomenon were verified by additional measurements in a laboratory circuit.

Both the field and the laboratory experiments also made it possible to evaluate the division of transported solids into a contact load and a suspended load for the purpose of the two-layer model. It was found that the solids concentration gradient in the cross section of a mixture flow was not always caused exclusively by a turbulent mixing process. The behaviour of mixture flows under varying flow velocities or pipeline inclinations was different in relatively fine (medium sand) mixtures and relatively coarse (coarse sand, fine gravel) mixtures. The suspension process in the flow of relatively fine mixtures can be described by an equation which evaluates the interaction between the turbulent carrier flow and the suspended solid particles. In a flow of coarser mixtures, however, the concentration gradient is given by the interfacial interaction between the sliding bed and the faster flow above it rather than by the turbulent suspension process. For such a flow the turbulent-suspension equation is not valid. An interfacial shear stress between the sliding bed and the upper flow is responsible for the development of a transition zone (called a shear layer) in which particles are not predominantly supported by the turbulent eddies and still lose their permanent mutual contact. A partially-stratified flow with a developed shear layer can be evaluated, for the purposes of the two-layer model, as having the position of a virtual interface in the point of inflexion of the concentration profile. This assumption, based on a quantification of mutual particle interactions within the shear layer, is supported by laboratory observations. It is characteristic for flows with a shear layer that the position of the point of inflexion is not sensitive to mean flow velocity in the pipeline. Further the influence of the particle size is found to be negligible so that the position of the point of inflexion, i.e. the position of the virtual interface in a pipeline of a certain diameter, is given only by the volumetric concentration of solids. For the two-layer model applied to the stratified flow with a shear layer, the portion of solid particles considered to contribute to the suspended load is found to be directly related to the shear stress at the virtual interface.

Both the laboratory and the field experiments provided data suitable for an analysis of the mechanisms of mixture flow and for testing the two-layer model principles. The observed phenomena occurred as a result of stratification of flow of settling mixture in pipelines. The principles of the two-layer model were found capable of describing the flow of mixture containing particles, most of which are not finer than a medium sand. The two-layer model predicts the pressure gradient, the velocity of the sliding bed, the mean mixture velocity at which the bed stops its sliding and the slip ratio, the parameter giving the relation between velocities of the solid phase and the liquid phase in the mixture. To predict these parameters for the partially-stratified flow, rules were proposed for the two-layer model to determine the division of the solids into two layers of mixture flow. These were calibrated by the experimental data, including the concentration profiles, processed to the two-layer flow pattern.
In addition, the two-layer model was formulated for the partially-stratified flows in inclined pipes and verified by the experimental data. Experiments in a laboratory pipeline inclined at various angles showed a profound effect of a pipeline inclination on a behaviour of a partially-stratified flow. This effect was associated with the presence of a contact layer in inclined flows.

The research also revealed a fact which is of a direct practical importance: when sand is being transported in relatively highly concentrated mixtures the concentration has a positive effect on the parameters determining economy and safety of the pipeline operation.
Samenvatting

Stromingsmechanismen van zand-watermengsels in pijpleidingen

Václav Matoušek

Dit proefschrift behandelt het hydraulisch transport van vaste stoffen in pijpleidingen. Er wordt een samenvatting gegeven van de resultaten van een onderzoek uitgevoerd om het stromingsgedrag van zand-watermengsels in pijpleidingen te beschrijven en een methode op te stellen voor het voorspellen van het stromingsgedrag, met gebruikmaking van een theoretisch model. Speciale aandacht wordt geschonken aan de verschijnselen die optreden in de mengselstroming in een baggerpijpbedeling.

In de praktijk van het baggeren zijn de modellen die gebruikt worden om het gedrag van mengselstroming in pijpleidingen te voorspellen meer gebaseerd op een empirische benadering dan op een fysieke beschrijving van de stromingsmechanismen. Hoewel de fysische benadering voor het modelleren van heterogene mengselstromen in pijpleidingen werd geïntroduceerd in de vorm van een tweelagen-model, is dit in de praktijk nauwelijks toegepast. Dit komt omdat het model werd geacht alleen toepasbaar te zijn in geval van de speciale stromingscondities die optreden bij transport van zeer grote korrels door pijpleidingen. Bovendien was er weinig experimenteel bewijs voor de geldigheid van de uitgangspunten van het model. Om deze uitgangspunten te verifiëren zijn experimenten nodig waarbij de interne opbouw van de mengselstroom bij verschillende stromingscondities wordt gemeten.

De tegenwoordig meest gebruikte empirische modellen zijn uit de vijftiger- en zestigerjaren (Durand et al., Fürböter, Jufin-Lopatin). Dit zijn empirische vergelijkingen van dimensioze kentallen, samengesteld uit de belangrijkste stromingsparameters. De waarden van de empirische coëfficiënten van de modellen zijn uit experimenten bepaald. In de zeventigerjaren werd er een theoretisch model geïntroduceerd (Wilson et al.), gebaseerd op het krachtenevenwicht in de twee lagen van een mengselstroom verdeeld in een "bed load" en een "suspended load". Het tweelagen-model bleek waardevol bij het voorspellen van een volledig gestratificeerde stroming (alle vaste deeltjes worden getransporteerd in een bedlaag) maar er ontstonden problemen bij de toepassing van het model op een gedeeltelijk gestratificeerde stroming. De voorgestelde oplossingen om de getransporteerde korrels te verdelen in een bedlaag en een laag in suspensie werden niet door experimenten gestaafd. Een betrouwbaar oplossing vereist kennis van de concentratie- en/of de snelheidsverdeling van de korrels loodrecht op de mengselstroom. Metingen aan de interne opbouw van de mengselstroming zijn nog maar zelden uitgevoerd.

In dit proefschrift wordt de studie naar de mengselstromingsmechanismen ondersteund door experimenten waarbij metingen gedaan zijn van de snelheidsprofielen of van de
concentratieprofielen van gedeeltelijk gestratificeerde stromingen in een baggerpijpleiding en in een laboratoriumcircuit. Een database die verzameld is bij uitgebreide veldmetingen aan een baggerinstallatie was de basis van het onderzoek. De metingen aan de baggerleiding lieten het verrassende verschijnsel zien van de verandering van dichtheidsfluctuaties van de mengselstroom aan de intree van de pijpleiding naar geaccumuleerde dichtheidssgolven verderop in de lange baggerleiding. Dit verschijnsel van het opeenhopen van vaste delen in een niet-stationaire stroming wordt in dit proefschrift geanalyseerd, verklaard en de invloed ervan op de werking van de baggerinstallatie bediscussieerd. Voor de beschrijving van het opeenhopingssediment in een baggerleiding bleek het krachtenbalans-concept voor tweelagen-stromingen geschikt te zijn. De principes van het agregatie-verschijnsel zijn geverifieerd door het uitvoeren van aanvullende metingen in het laboratorium-circuit.

Zowel de veld- als de laboratorium-proeven maakten het ook mogelijk een verdeling aan te brengen van de getransporteerde vaste stof in een "contact load" deel en een "suspended load" deel te behoeve van het tweelagen-model. Het bleek dat de concentratie-gradiënt van de vaste stof in de dwarsdoorsnede van een mengselstroom niet altijd alleen maar door een turbulent mengproces wordt veroorzaakt. Het gedrag van mengsels onder wisselende stroomsnelheden of leiding-inclinaaties bleek verschillend bij relatief fijne mengsels (middelfijn zand) en relatief grove mengsels (grof zand, fijn grind). Het suspensie-proces in de stroming bij relatief fijne mengsels kan worden beschreven door een vergelijking die uitgaat van een interactie tussen de turbulente draagvloeistof en de zwevende vaste deeltjes. In een stroming van grovere mengsels wordt echter de concentratie-gradiënt meer bepaald door de grenslaag interactie tussen het glijdende bed en de sneller stromende draagvloeistof hierboven zodat de turbulent-suspensie vergelijking niet langer geldig is. Een grenslaag-schuifspanning tussen het schuwende bed en de bovenliggende vloeistof is verantwoordelijk voor de ontwikkeling van een overgangszone (schuiflaag genoemd) waarin de deeltjes niet voornamelijk door de turbulentie-wervels worden gedragen en toch hun permanente onderlinge contact verliezen. Een gedeeltelijk gestratificeerde stroming met een ontwikkelde schuiflaag kan voor het tweelagen-model beschouwd worden alsof het denkbeeldige grensvlak tussen de twee lagen, zoals aangenomen in het model, zich bevindt in het buigpunt van het concentratie-profiel. Dit uitgangspunt, gebaseerd op een quantificatie van onderlinge deeltjes-interacties in de schuiflaag, wordt ondersteund door laboratorium waarnemingen. Het is kenmerkend voor stromingen met een schuiflaag dat de positie van het buigpunt nauwelijks afhankelijk is van de gemiddelde stroomsnelheid in de leiding. Verder werd gevonden dat de invloed van de korrelgrootte verwaarloosbaar is, zodat de positie van het buigpunt, d.w.z. de positie van het denkbeeldige grensvlak, in een leiding van een zekere diameter alleen wordt bepaald door de volumetrische concentratie van de vaste stof. Wanneer het tweelagen-model toegepast wordt op de gestratificeerde stroming met een schuiflaag blijkt het deel van de vaste stof dat bijdraagt aan de "suspended load" direct gerelateerd aan de schuifspanning in het denkbeeldige grensvlak.

Zowel de laboratorium- als de veldmetingen gaven gegevens die geschikt zijn voor een analyse van de mechanismen van de mengselstroming en voor het testen van de grondslagen van het tweelagen-model. De geconstateerde verschijnselen traden op als gevolg van stratificatie van de mengselstroming in pijpleidingen. Op basis van de principes van het tweelagen-model bleek het mogelijk de stroming te beschrijven van
mengsels die deeltjes bevatten waarvan de meesten niet fijner zijn dan middelfijn zand. Het tweelagen-model voorspelt de drukval, de snelheid van het schuivende bed, de gemiddelde mengselsnelheid waarbij het bed stil komt te liggen en de slipfactor. De slipfactor is de parameter die de verhouding aangeeft tussen de stromingsnelheid van de vaste- en de vloeistoffase in het mengsel. Om deze parameters te voorspellen voor een gedeeltelijk gestratificeerde stroming, zijn regels opgesteld voor het tweelagen-model voor de verdeling van de vaste stof in twee lagen. Deze regels zijn gekalibreerd met de gegevens uit de experimenten, inclusief de concentratie profielen, waarbij de gegevens zijn omgerekend naar het tweelagen-stromingspatroon.

Bovendien is een formule opgesteld voor het tweelagen-model voor gedeeltelijk gestratificeerde stromingen in hellende leidingen en geverifieerd met de experimentele gegevens. Experimenten in een laboratoriumpijpliciding, die onder verschillende hellingshoeken gezet kon worden, liet een sterk effect zien van de hellingshoek op het gedrag van een gedeeltelijk gestratificeerde stroming. Dit effect werd in verband gebracht met de aanwezigheid van een contactlaag in een hellende stroming.

Het onderzoek liet ook een zaak zien van direct praktische betekenis: wanneer zand wordt getransporteerd in mengsels met een relatief hoge concentratie heeft dit een positief effect op de economische- en veiligheidsaspecten van de baggeroperatie.
List of symbols

\[ \begin{align*}
a & \quad \text{empirical coefficient in Eqs. 3.13, 3.14 and 3.54} \quad [-] \\
A & \quad \text{area of the pipe cross section} \quad [m^2] \\
A_b & \quad \text{part of } A \text{ occupied by granular bed} \quad [m^2] \\
A_f & \quad \text{part of } A \text{ occupied by fluid/liquid/water} \quad [m^2] \\
A_s & \quad \text{part of } A \text{ occupied by solids} \quad [m^2] \\
A_1 & \quad \text{cross section area of upper layer of two-layer model} \quad [m^2] \\
A_2 & \quad \text{cross section area of lower layer of two-layer model} \quad [m^2] \\
\alpha & \quad \text{Ar} \text{chimedes number} \quad [-] \\
A' & \quad \text{coefficient in Eqs. 3.7 and 3.8} \quad [-] \\
b & \quad \text{empirical coefficient in Eq. 3.54} \quad [-] \\
B & \quad \text{coefficient in Eq. 3.8} \quad [-] \\
B_1 & \quad \text{coefficient in Eq. 3.34} \quad [-] \\
B_2 & \quad \text{coefficient in Eq. 3.37} \quad [-] \\
c & \quad \text{empirical coefficient in Eq. 3.54} \quad [-] \\
c_b & \quad \text{local volumetric concentration of solids at the bottom of pipe} \quad [-] \\
c_v & \quad \text{local volumetric concentration of solids at some position in pipe} \quad [-] \\
c_{vc} & \quad \text{local volumetric concentration of solids contributing to} \text{ interparticle stress} \quad [-] \\
C_c & \quad \text{spatial volumetric concentration of solids in contact load} \quad [-] \\
C_{cd} & \quad \text{delivered volumetric concentration of solids in contact load} \quad [-] \\
C_D & \quad \text{drag coefficient of spherical particle} \quad [-] \\
C_L & \quad \text{lift coefficient of solid particle} \quad [-] \\
C_r & \quad \text{relative concentration } C_{vd}/C_{vb} \quad [-] \\
C_s & \quad \text{spatial volumetric concentration of solids in suspended load} \quad [-] \\
C_{sh} & \quad \text{spatial volumetric concentration of solids in shear layer} \quad [-] \\
C_{sh,c} & \quad \text{spatial volumetric concentration of solids in contact in shear layer} \quad [-] \\
C_{vb} & \quad \text{volumetric solids concentration in loose-poured bed} \quad [-] \\
C_{vd} & \quad \text{delivered volumetric solids concentration in pipe cross section} \quad [-] \\
C_{vi} & \quad \text{spatial (in situ) volumetric solids concentration in pipe} \text{ cross section} \quad [-] \\
C_1 & \quad \text{volumetric solids concentration in upper layer of two-layer model} \quad [-] \\
C_2 & \quad \text{volumetric solids concentration in lower layer of two-layer model} \quad [-] \\
C_{2,e} & \quad \text{volumetric concentration of solids in contact in lower layer of} \text{ two-layer model} \quad [-] \\
C_{2,fc} & \quad \text{volumetric solids concentration in fictive bed of thickness} \text{ given by } y_{\text{inflex}} \quad [-] \\
d & \quad \text{particle diameter} \quad [m] \\
d_i & \quad \text{particle diameter of i-fraction in Eq. 4.1} \quad [m] \\
d_s & \quad \text{mean particle diameter according to Eq. 4.1} \quad [m] \\
d_{12} & \quad \text{particle diameter at the interface in Eq. 3.32} \quad [m] \\
d_{50} & \quad \text{mass-median particle diameter} \quad [m] \\
d_{85} & \quad \text{diameter for which 85\% (by mass) of the particles are finer} \quad [m] \\
D & \quad \text{internal diameter of a pipe} \quad [m] \\
\end{align*} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_h)</td>
<td>hydraulic diameter of a pipe cross sectional area</td>
<td>[m]</td>
</tr>
<tr>
<td>(f_f)</td>
<td>Fanning friction factor defined by Eq. 2.21</td>
<td>[-]</td>
</tr>
<tr>
<td>(F_D)</td>
<td>drag force on a solid particle</td>
<td>[kg·m/s²]</td>
</tr>
<tr>
<td>(F_{Gp})</td>
<td>gravitational force on a solid particle</td>
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<td>(F_L)</td>
<td>lift force on a solid particle</td>
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<tr>
<td>(F_N)</td>
<td>normal intergranular force against pipe wall</td>
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<tr>
<td>(F_W)</td>
<td>submerged weight of granular bed</td>
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<tr>
<td>(F_{WP})</td>
<td>submerged weight a solid particle</td>
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<td>Froude number for pipe flow</td>
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<tr>
<td>(Fr_{vt})</td>
<td>particle Froude number</td>
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<tr>
<td>(g)</td>
<td>gravitational acceleration</td>
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<td>(Ga)</td>
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<tr>
<td>(h)</td>
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<td>hydraulic gradient</td>
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<td>value of I for flow of liquid</td>
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<td>(L)</td>
<td>length measured along pipe</td>
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<td>exponent in model of Wilson &amp; GIW</td>
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<tr>
<td>(Q)</td>
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<td>mean volumetric flow rate of mixture</td>
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<td>mean volumetric flow rate of solids</td>
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<td>hydraulic radius in layered flow</td>
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<td>(S_f)</td>
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<tr>
<td>(S_m)</td>
<td>relative density of mixture</td>
<td>[-]</td>
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<tr>
<td>(S_s)</td>
<td>relative density of solids</td>
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<td>(t)</td>
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<td>u*</td>
<td>shear velocity</td>
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<tr>
<td>v</td>
<td>local velocity at some position in pipe cross section</td>
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<tr>
<td>v'</td>
<td>fluctuating component of local velocity</td>
<td>[m/s]</td>
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<tr>
<td>( v' )</td>
<td>root-mean-square velocity in the y-direction, i.e. turbulent pulsative velocity in the y-direction</td>
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<td>v_b</td>
<td>local velocity of solids at the bottom of pipe</td>
<td>[m/s]</td>
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<tr>
<td>v_f</td>
<td>local velocity of liquid</td>
<td>[m/s]</td>
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<tr>
<td>v_r</td>
<td>relative local velocity between phases, ( v_p - v_s )</td>
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<tr>
<td>v_s</td>
<td>local velocity of solid particle</td>
<td>[m/s]</td>
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<td>v_t</td>
<td>terminal settling velocity of a single particle</td>
<td>[m/s]</td>
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<tr>
<td>v_th</td>
<td>hindered settling velocity</td>
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<td>v_ts</td>
<td>terminal settling velocity of spherical particle</td>
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<td>v_x</td>
<td>local velocity in the x-axis direction</td>
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<td>V</td>
<td>mean velocity</td>
<td>[m/s]</td>
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<tr>
<td>V_f</td>
<td>mean velocity of liquid in pipe cross section</td>
<td>[m/s]</td>
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<tr>
<td>V_d1</td>
<td>value of ( V_m ) at limit of stationary deposit, i.e. deposition-limit velocity</td>
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<tr>
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<td>relative velocity between layers, ( V_1 - V_2 )</td>
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<td>mean velocity of mixture in pipe cross section</td>
<td>[m/s]</td>
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<tr>
<td>V_min</td>
<td>value of ( V_m ) at minimum point of resistance curve, i.e. minimum velocity</td>
<td>[m/s]</td>
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<td>V_pg</td>
<td>velocity of the particulate core (plug) in pipe</td>
<td>[m/s]</td>
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<td>V_s</td>
<td>mean velocity of solids in pipe cross section</td>
<td>[m/s]</td>
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<tr>
<td>V_sm</td>
<td>maximum value of ( V_d1 )</td>
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<td>value of ( V_{sm} ) in inclined pipe</td>
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<td>mean velocity of solids in lower layer of two-layer model</td>
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<td>V_r</td>
<td>relative velocity ( V_m/V_{sm} )</td>
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<td>V_tt</td>
<td>value of ( V_m ) at threshold of turbulent suspension</td>
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<td>V_1</td>
<td>mean velocity of mixture in upper layer of two-layer model</td>
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<td>mean velocity of mixture in lower layer of two-layer model</td>
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<td>value of ( V_m ) given by Eq. 3.11</td>
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<td>X</td>
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<td>y</td>
<td>vertical distance from pipe wall defining a position in pipe cross section</td>
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<td>y_{inf}</td>
<td>value of ( y ) determining a position of a point of inflexion on a concentration profile curve</td>
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<td>y*</td>
<td>ordinate for velocity distribution</td>
<td>[m]</td>
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<tr>
<td>Y_b</td>
<td>thickness of granular bed</td>
<td>[m]</td>
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<tr>
<td>Y_sh</td>
<td>thickness of shear layer</td>
<td>[m]</td>
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<tr>
<td>Y_{12}</td>
<td>thickness of lower layer in two-layer model</td>
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<tr>
<td>Symbol</td>
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<td>Unit</td>
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<td>angle defining a position of interface in two-layer model</td>
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<td>shear rate in Eq. 2.33</td>
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<td>$\Delta D$</td>
<td>Durand deposition parameter in Eq. 3.60</td>
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<td>solids dispersion coefficient</td>
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<td>particle mobility number defined by Eq. 7.6</td>
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<td>von Kármán constant</td>
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<td>Darcy-Weisbach friction coefficient defined by Eq. 2.22</td>
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<td>value of $\lambda_f$ in lower layer of two-layer model</td>
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<td>dynamic viscosity of liquid</td>
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<td>mechanical friction coefficient of solids against pipe wall</td>
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<td>kinematic viscosity of liquid</td>
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<td>coefficient of proportionality equal to $V_{\text{min}}/V_{\text{dl}}$</td>
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<td>$\rho$</td>
<td>density</td>
<td>[kg/m$^3$]</td>
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<td>$\rho_f$</td>
<td>density of fluid/liquid/water</td>
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<td>$\rho_{\text{fines}}$</td>
<td>density of mixture of liquid and particles smaller than 0.074 mm</td>
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<td>density of mixture</td>
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<td>$\rho_S$</td>
<td>density of solids</td>
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<td>density of mixture in lower layer of two-layer model</td>
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<td>density of suspension in lower layer of two-layer model</td>
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<td>shear stress</td>
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<td>value of $\tau$ at the bottom of pipe</td>
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<td>$\tau_f$</td>
<td>liquid shear stress</td>
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<td>$\tau_0$</td>
<td>bed shear stress</td>
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<tr>
<td>$\tau_{0,cr}$</td>
<td>critical bed shear stress according to Shields</td>
<td>[kg.m$^{-1}$/s$^2$]</td>
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<td>$\tau_S$</td>
<td>intergranular shear stress</td>
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<td>$\tau_{sh,bot}$</td>
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<td>$\tau_{0}$</td>
<td>value of $\tau$ at pipe wall</td>
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<td>$\tau_{12}$</td>
<td>value of $\tau$ at interface in the two-layer flow pattern</td>
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<tr>
<td>$\phi'$</td>
<td>angle of internal friction of cohesionless solids (dynamic)</td>
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<td>variable in model of Durand et al.</td>
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<td>variable in model of Durand et al.</td>
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<td>angle defining an inclination of a pipe</td>
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Abbreviations:

DFDu  Density and Flow measuring location Duinjager
DFFr  Density and Flow measuring location Groningen
DFJa  Density and Flow measuring location Jagersplas
DIM   Data Interpretation Model
DN    Nominal Diameter
DTI   Data Transfer and Interpretation
Du    Duinjager
Gr    Groningen
Ja    Jagersplas
MeaVli Measurements Vlietlanden
PSD   Particle Size Distribution
PSVD  Particle Settling Velocity Distribution
RP    Reference Position
SEC   Specific Energy Consumption
SRC   Saskatchewan Research Council
TM    Testing Model
2LMi  Two - Layer Model for inclined flows
Chapter 1

Introduction

The research described in this thesis was motivated by a request to explain the effects observed during the extensive field measurements on a long pipeline connected with a dredger several years ago. An explanation of the phenomena observed in the dredging pipeline required the detection of the mechanisms governing the mixture flow in pipelines and a physical description of these mechanisms in a model.

1.1 Slurry pipelining applications

Hydraulic transport in pipelines is a means of conveying material that is widely used by various branches of industry. It plays an important role in the chemical and waste-disposal industries, for example in mining and power engineering, and particularly in dredging. During the execution of extensive dredging projects huge amounts of soil are transported hydraulically often to rather distant construction or disposal sites. For example during recent large dredging work, an extension of the major airport in Hong Kong, the total amount of solids to be moved by dredging between 1992 and 1995 was estimated to 256 million cubic metres. The cost of the power consumed in pumping the soil-water mixture, called slurry, was an important item in the budget of the entire project.

1.2 Definition of the problem studied

The effective design, control and operation of a hydraulic transport system composed of a pipeline and pumps requires the successful prediction of slurry flow behaviour in the pipeline. Insight into the physical phenomena involved, however, is still limited, as is the ability to describe the slurry flow mechanism in a pipeline. In practice, predictive methods based on an empirical correlation rather than on an analysis of a slurry flow behaviour are still used. Doubtful reliability of the predictive models leads to conservatism in the design and operation of hydraulic transport installations and results in unnecessarily high capital and operating costs. The excessive capital costs might be due to unnecessarily high number of pumps installed to the conveying system and the high operating costs due to unnecessarily high energy consumption by pumps when transporting solids. Prediction of the capacity of solids pumping is important to estimation of the cost of completing a work.

Empirical models lack a theoretical background based on a description of acting flow mechanisms. Thus the correlations may reflect the phenomena occurring in a slurry
pipeline but they do not reflect the physical mechanisms governing those phenomena. Measurements of the integral flow parameters on which the correlations are based are not adequate to allocate and to describe the flow mechanism because the mechanism governing slurry flow behaviour can be deduced only from the observed internal structure of a slurry flow. The empirical models are difficult to generalise and internally inconsistent. The majority of the models work properly only in the specific flow conditions for which they have been calibrated against the experimental data. The models reduce the slurry flow problem to predictions of frictional head loss and of deposition-limit velocity. These predictions are made by using different, mutually inconsistent correlations. Such correlations are not adequate for the on-line control of a conveying system composed of pumps and pipeline. Empirical correlations cannot be used for a dynamic simulation of the effects of unsteady flow occurring during a dredging operation. The simulation of dynamic effects demands an ability to handle changes in the internal structure of slurry flow.

Microscopic models describe the mechanisms governing the slurry flow by using a set of basic differential equations of conservation of mass, momentum and energy. In future these models might provide a powerful solution for two-phase flow in pipelines but at their present stage of development they exhibit continuing uncertainties in the theoretical description of the processes on microscopic scale in the solid-liquid flow and a continuing lack of experimental evidence for the verification of both the particular model components and the final model outputs. The need to describe the flow mechanisms at a local level leads to the introduction of many coefficients which are difficult to determine experimentally. Moreover, the rather complex computational scheme of a microscopic model makes it unattractive for use in practice.

Macroscopic modelling is a suitable compromise between the microscopic and empirical approaches. Despite simplification of the real internal structure of slurry flow, a macroscopic model may be capable of simulating the most important processes governing slurry flow behaviour in a pipeline. All the required slurry flow parameters are predicted by using one unified model. In comparison with the microscopic model, the macroscopic model reduces a number of necessary coefficients by simplifying a real slurry flow structure to derive a model pattern. Empirical coefficients for a macroscopic model can be determined from experiments which include observations of the solids concentration distribution and/or the solids velocity distribution across a pipeline cross section. The macroscopic model can be calibrated and verified by using the data available from laboratory or industrial pipelines. An application of the macroscopic approach is suitable in flows of settling slurries which exhibit some degree of stratification in a pipeline. A two-layer model has been developed for fully-stratified flow, thus for extreme flow conditions which in practice may occur only when very coarse solids are transported. Although the model appears to be a valuable design tool, its verification and adaptation to a wider range of slurry flow conditions (heterogeneous flow) and various pipeline geometries (pipeline inclination, pipeline diameter) is still under investigation.

There is a lack of experimental data suitable for investigating the slurry flow mechanisms and for the verification of the principles of the two-layer model. This is particularly valid for the slurry pipeline geometries usually not obtained in a laboratory
(pipeline inclination, large pipeline diameter) and for the specific flow conditions in field slurry pipelines. Laboratory conditions are characterised by short pipeline length, small pipeline diameter and steady flow conditions maintained by the slurry circulation in the laboratory loop. The field conditions are usually more complex, particularly in dredging practice. A dredging pipeline is composed of horizontal and inclined sections and it is of large diameter. As a result of the excavation and mixing processes in the suction mouth at the bottom of the waterway or the borrowing pit from which the material is dredged, solids flow along a dredging pipeline is unsteady.

1.3 Research objectives

Objectives of the research presented in this thesis are as follows:
- to observe slurry flow behaviour in both a laboratory circuit and a field dredging pipeline; to analyse the phenomena detected in both pipelines in order to detect prevailing mechanisms governing the flow of the slurry in a pipeline
- to test the basic principles of the two-layer model originally developed for the fully stratified flow and to adapt the two-layer model to the partially stratified flows
- to examine the effects of pipeline inclination on slurry flow behaviour and to implement their description to the model
- to apply the model to the unsteady flow conditions in the dredging pipeline.

1.4 Outline

The subject of the investigation presented in this thesis is a flow mechanism for sand-water mixtures in pipelines. This requires a literature review of the basic mechanics of two-phase flow and of the concepts for the settling slurry flows in pipelines, experiments on the slurry flow behaviour in a pipeline, an explanation of the behaviour resulting from the action of physical mechanisms and a description of the behaviour by a model. Basic principles of fluid and particle mechanics are described in Chapter 2. This summarises the theoretical background and the tools used to describe flow mechanisms. In Chapter 3 different approaches to describing the slurry flow behaviour in pipelines are discussed and compared with our experimental experience. Special attention is focused on Wilson’s two-layer model for fully-stratified flow, which has been computationally tested. Chapter 4 gives information about experimental tests carried out on two different slurry pipelines: a small laboratory pipeline in which steady flow conditions were maintained and a large dredging pipeline with unsteady flow conditions such as those usually occurring during a dredging operation. In Chapter 5 the phenomena observed in partially-stratified flow in the laboratory horizontal pipeline are described and analysed, using the principles of the two-layer model. Chapter 6 describes and models the effects of pipeline inclination on partially-stratified flows. In Chapter 7 the suspension mechanisms in partially-stratified flows are modelled in terms of the two-layer model. In Chapter 8 the slurry flow behaviour observed in the dredging pipeline is analysed, together with the effects of pipeline length on the unsteady character of solids flow in the pipeline. The two-layer model is applied to the flow conditions in a dredging pipeline. Chapter 9 gives conclusions and
recommendations, based on the research presented in this thesis, for an operation of slurry pipelines in practice.
Chapter 2

General principles of the flow of liquid and particles in a pipeline

In this chapter the basic slurry flow characteristics handled by the models describing the flow behaviour in slurry pipelines are defined and a general theory for the flow of liquid and solid particles in a pipeline is reviewed. A mathematical description of the physical principles of flow in pipelines is given to provide a basis for the description and improvement of a macroscopic model discussed in the following chapters.

2.1 Definition of basic characteristics of pipeline flow of slurry

Important parameters for the design and operation of a slurry pipeline are those which provide information about the safety and economy of slurry pipeline operation.

Mean velocity in a pipeline is a basic parameter characterising pipeline flow. It is defined as the bulk velocity, $V$, of a matter (liquid, solids, mixture) obtained from the volumetric flow rate, $Q$, of a matter passing a pipeline cross section of the area, $A$. The equation $V = Q/A$ is for a circular pipe of an inner diameter $D$ written as

$$V = \frac{4Q}{\pi D^2} \quad (2.1).$$

The determination of an appropriate mean slurry velocity, $V_{\text{m}}$, is crucial to safe and low-cost pipeline operation.

Solid particles of sand/gravel size and density tend to settle in a flowing carrier. Usually, these solid particles are distributed non-uniformly in a pipeline flow. If the carrier velocity is too low for the carrier lift forces to suspend all solid particles, a portion of the particles forms a bed at a bottom of a slurry pipeline. With extremely large particles and/or extremely low mean velocities in a pipeline all particles occupy a bed. The threshold velocity at the initiation of turbulent suspension, $V_{\text{tt}}$, is the mean velocity of the mixture in a pipeline cross section at which the first solid particles leave the bed, being supported by the diffusive effect of carrier turbulent eddies. This velocity is used in the evaluation of a measure of a flow stratification, but for practical pipeline operation the threshold velocity at which solid particles occupying a bed at the bottom of a pipeline stop their sliding and start to form a stationary deposit, i.e. a stationary bed, is more important. Operation below this threshold velocity might be
inefficient and potentially dangerous. Under certain circumstances a stationary deposit may be transformed into a solid plug which blocks the pipeline. The mean slurry velocity at the limit of stationary deposition is called the deposition-limit velocity, \( V_{dl} \), or, less accurately, the critical velocity. Slurry flow at velocities above the deposition-limit value is free of a stationary deposition.

The mean slurry velocity at which the least energy is dissipated in slurry flow is called the minimum velocity, \( V_{\text{min}} \). In the majority of slurry pipeline conditions the deposition-limit and the minimum velocities are not identical. The minimum slurry velocity determines the velocity at which the slurry flow is most economical of energy. This is the optimal transport velocity for slurry of a given slurry density. It is well known, however, that the minimum velocity is not a viable operation velocity in a conveying system. In practice the operation velocity, a result of an interaction between a pipeline and a pump, is taken as \( V_m > V_{\text{min}} \). This avoids an unstable transport regime in a conveying system experiencing a variation in \( V_m \). Some authors have tried to relate the minimum and the critical velocity by using coefficient of proportionality \( \xi \) as

\[
V_{\text{min}} = \xi V_{dl} \quad \text{(e.g. } \xi = 0.64/\sqrt{D} \text{ according to Jušk & Lopatin, 1966)}
\]

Flow resistance is given by the amount of mechanical energy dissipated in a slurry flow when flowing through a pipeline. The mechanical energy balance along a pipeline section - expressed by the Bernoulli equation - shows that energy dissipation in a steady slurry flow is characterised by the pressure difference along a horizontal pipeline section of constant diameter. The resistance is evaluated as

- the pressure drop \( \Delta P = P_1 - P_2 \) (differential pressure over a pipeline section confined at the inlet by the pipeline cross section 1 and at the outlet by the pipeline cross section 2) [Pa],

- the pressure gradient \( \Delta P/L \) (pressure drop over a pipeline section divided by the length \( L \) of a pipeline section between cross sections 1 and 2) [Pa/m] or

- the hydraulic gradient \( \frac{\Delta P}{\rho g L} \) due to friction, also termed the frictional head loss

\( (I_m) \) (head that is lost owing to friction is divided by the length of a pipeline section), which is dimensionless and expresses the pressure gradient by the ratio of metre liquid column and metre pipeline length.

The sum of the hydraulic gradient due to friction, the minor losses and the geodetic gradient (the potential energy gain or loss in a slurry from different geodetic heights at an inlet and an outlet of the pipeline) determine the amount of energy which has to be fed by the pumps to slurry flow in the pipeline.

The flow rate of solids, in the dredging practice termed the production of solids, is an important parameter from the economic point of view. It gives the amount of dry solids (in volume or mass) delivered by a pipeline at an outlet in a certain time period. It is defined as the flow rate (either volumetric, \( Q_s \), in \( m^3/s \) or mass in \( kg/s \)) of solids at the outlet of a slurry pipeline.

The solids flow rate may be calculated by using either the delivered or the spatial concentration of the solids in a slurry. The delivered concentration gives a fraction (by mass or volume) of solids delivered from a slurry pipeline and it is calculated as the
ratio between solids and slurry flow rates. Thus the volumetric delivered concentration
\[ C_{vd} = \frac{Q_S}{Q_m} \]
The spatial concentration gives the fraction (by mass or volume) of solids actually resident in a slurry pipeline, the volumetric spatial concentration, \( C_{vi} \), is calculated as the ratio between solids and slurry volumes in a pipeline section.

The difference between the spatial and the delivered concentration indicates slip (hold-up) within the slurry flow caused by the different velocities of the carrier and that of the solid phase within a slurry stream. Govier & Aziz (1972) described this phenomenon as follows:
"When the phases of two-phase flow differ in density and/or viscosity, one of them - usually the less dense phase - tends to flow at a higher in situ average velocity than does the other. This gives rise to an all-important characteristic of two-phase flow, the existence of "slip" of one phase past the other, or "holdup" of one phase relative to the other."

Govier and Aziz also summarised the factors influencing slip:
- the existence of a velocity profile across the pipeline cross section
- the existence of a concentration profile across the pipeline cross section
- the local relative velocity between phases \( (V_s - V_f) \) caused by gravitational effects.

The slip between two phases in a cross section of slurry pipeline can be quantified by the mean slip velocity in a pipeline cross section, \( V_s - V_f \) or by the ratio of the mean velocities of solids and mixture in a pipeline cross section, \( V_s/V_m \). This latter parameter, called the slip ratio, is exceptionally suitable for the evaluation of slip in a pipeline. This ratio is also equal to the ratio of mean concentrations in pipeline cross sections \( C_{vd}/C_{vi} \) since

\[ C_{vd} = \frac{Q_S}{Q_m} = \frac{V_s A_s}{V_m A_m} = \frac{V_s C_{vi} A}{V_m A} \]

in which \( A_s \) is the part of the cross sectional area of the pipeline occupied by solids, thus

\[ \frac{V_s}{V_m} = \frac{C_{vd}}{C_{vi}} \]

The slip phenomenon may influence the accuracy of the determination of the solids flow rate from the measurements on a dredging pipeline. Solids flow rate through a slurry pipeline connected with a dredger is often determined from measurements of the mean solids concentration and the mean slurry velocity in a horizontal pipeline section. The radiometric measurement of the concentration in a horizontal pipeline gives the value of the spatial concentration (not the delivered concentration that might be lower) so that \( Q_s \) calculated as \( C_{vi} V_m A \) overestimates the real solids flow rate unless the slip in a pipeline is negligible. Generally, it is desirable to take the slip into account during slurry flow calculations.
2.2 Principles of flow in a pipeline

The principles of the flow of a substance in a pressurised pipeline are governed by the basic physical laws of conservation of mass, momentum and energy. For the purposes of slurry flow modelling, the conservation laws of mass and momentum are expressed mathematically below by means of equations for the mass balance and the linear momentum balance. In the most general case, these are the differential equations which describe the flow process in general conditions in an infinitesimal control volume. Simpler equations may be obtained by implementing the specific flow conditions characteristic of a chosen control volume.

2.2.1 Conservation of mass

Conservation of mass in a control volume is written in the form: the rate of mass input = the rate of mass output + the rate of mass accumulation. In the general case of unsteady flow of a compressible substance of density $\rho$, the differential equation evaluating mass balance (or continuity) is (e.g., Shook & Roco, 1991)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (2.4)$$

in which $t$ denotes time and $\vec{V}$ velocity vector.

In solid-liquid flow the continuity equation is written for both phases. For the solids occupying the volume fraction $c_s$ of the total control volume

$$\frac{\partial}{\partial t} [c_s \rho_s] + \nabla \cdot [c_s \rho_s \vec{V}_s] = 0 \quad (2.5)$$

and for the liquid

$$\frac{\partial}{\partial t} [(1 - c_s) \rho_f] + \nabla \cdot [(1 - c_s) \rho_f \vec{V}_f] = 0 \quad (2.6).$$

Equation 2.4 is written in Cartesian co-ordinates as

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_x)}{\partial x} + \frac{\partial (\rho v_y)}{\partial y} + \frac{\partial (\rho v_z)}{\partial z} = 0 \quad (2.7).$$

For incompressible (\rho = const.) fluid and steady ($\partial \rho / \partial t = 0$) flow the equation is given in its simplest form

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \quad (2.8).$$
The physical explanation of the equation is that the mass flow rates for steady flow at the inlet and outlet of the control volume are equal. Expressed in terms of the mean values of quantities at the inlet and outlet of the control volume, given by a pipeline length section, the equation is

\[ \rho VA = \text{const., i.e.} \quad (\rho VA)_{\text{inlet}} = (\rho VA)_{\text{outlet}} \]

In a discrete environment, such as a two-phase mixture composed of liquid and solids, the conservation of mass is separately evaluated for each phase,

- liquid phase \[ \rho_l V_f A_f = \text{const.} \]
- solid phase \[ \rho_s V_s A_s = \text{const.} \]

### 2.2.2 Conservation of momentum

A momentum equation is an application of Newton's second law of motion. The summation of all external forces on a control volume filled with a substance is equal to the rate of change of momentum of the substance in the control volume. The sum of the external forces acting on the control volume is counterbalanced by the inertial force proportional to the momentum flux of the control volume

\[
\frac{d(\text{momentum})}{dt} = \sum F_{\text{external}}.
\]

The external forces are

- body forces due to external fields (gravity, magnetism, electric potential) which act upon the entire mass of the matter within the control volume,
- surface forces due to stresses on the surface of the control volume which are transmitted across the control surface.

Gravity is the only body force relevant to the description of the flow of a substance in a conduit. Surface forces are represented by the force from the pressure gradient and by friction forces from stress gradients at the control volume boundary.

In an infinitesimal control volume filled with a substance of density \( \rho \) the force balance between inertial force, on one side, and pressure force, body force, friction force, on the other side, is given by a differential linear momentum equation in vector form (e.g., Shook & Roco, 1991)

\[
\frac{\partial}{\partial t} (\rho \vec{V}) + \nabla \cdot (\rho \vec{V} \vec{V}) = -\nabla P - \rho g \vec{h} - \nabla \vec{T} \tag{2.9}
\]

where \( \vec{h} \) denotes the elevation above a datum, \( \vec{V} \) the velocity vector and \( \vec{T} \) the stress tensor with nine components \( \tau_{ij} \). In Cartesian co-ordinates the equation 2.9 becomes for the incompressible substance
\[ \rho \left( \frac{\partial V_i}{\partial t} + V_j \frac{\partial V_i}{\partial x_j} \right) = - \frac{\partial p}{\partial x_i} - \rho g \frac{\partial h}{\partial x_i} - \frac{\partial \tau_{ji}}{\partial x_j} \]  

(2.10)

in which \( \tau_{ji} \) is the shear stress acting on the surface of constant \( j \) in the direction \( i \). If the convention is abandoned that a repeated subscript implies summation, the Eq. 2.10 is written for the \( x \)-direction (given by direction of the pipeline axis) as

\[
\rho \left( \frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} \right) = - \frac{\partial p}{\partial x} - \rho g \frac{\partial h}{\partial x} - \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right).
\]

To apply the momentum equation to pipeline flow it is convenient to replace the infinitesimal control volume by a macroscopic one. The momentum equation written for the macroscopic control volume is simpler because quantities in the equation are averaged over the control volume, i.e. over the pipeline cross section when a section of pipeline provides the control volume. The momentum equation is obtained by integrating the differential linear momentum equation (Eq. 2.10) over the pipe cross section. For the \( x \)-direction it has the form (Longwell, 1966 in Shook & Roco, 1991)

\[
\rho \left( \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial h}{\partial x} \right) + \frac{\partial p}{\partial x} + 4 \tau_0 \frac{\tau}{D} = 0
\]

(2.11)

in which \( \tau_0 \) is the shear stress at the pipe wall (defined below by Eq. 2.22) and \( V \) the mean velocity in the pipe cross section. Averaging is appropriate as long as the flow can be considered one-dimensional. This condition is fulfilled when flow properties are nearly uniform over the pipe cross section and pipe-axis component of flow velocity is dominant. A uniform distribution of flow properties is anticipated in a straight pipe filled with single-phase fluid or a homogeneous mixture.

Let the selected control volume be a straight piece of pipe of the differential distance \( dx \), measured in the downstream direction. The pipe is filled with a single-phase fluid. Additional conditions (incompressible fluid, steady and uniform flow in a horizontal straight pipe) make it possible to obtain a simple form of the linear momentum equation for fluid flow. Under the chosen conditions, the momentum flux at the control volume inlet is equal to that at the control volume outlet and the inertial force in the control volume is zero. In this case the integrated form of the linear momentum equation relates the driving force generated by the pressure gradient over the pipe distance \( dx \) and the cross section area \( A \) (and the perimeter \( O \)) to the resisting force due to viscous friction at the flow boundary, which is a pipe wall. The balance is

\[
- \frac{dP}{dx} A = \tau_0 O
\]

(2.12),

i.e. for a pipe of a circular cross section and internal diameter \( D \)

\[
- \frac{dP}{dx} A = \frac{4 \tau_0}{D}
\]

(2.13).
This equation shows that the wall shear stress must be correlated with the flow conditions to solve the pressure drop due to friction in pipeline flow.

### 2.2.3 Friction in pipeline flow of liquid

The equation 2.13 is not only valid for a pipe flow boundary, it can also be generalised to flow within each cylinder of radius \( r \) coaxial with a cylindrical pipe. It then provides an equation for shear stress distribution in the pipe cross section that is valid for both laminar and turbulent fluid flow. This is

\[
\frac{dP}{dx} = \frac{\tau}{r} \quad \text{(2.14)}.
\]

Newton's law of fluid viscosity is

\[
\tau = \mu_f \left( -\frac{dv_x}{dr} \right) \quad \text{(2.15)},
\]

where \( v_x \) is the local fluid velocity in the pipe-axis direction at the position given by the radius \( r \) in a pipe cross section and \( \mu_f \) is the dynamic viscosity of fluid.

In laminar flow, the equation for a shear stress distribution (Eq. 2.14) and Newton's law of fluid viscosity (Eq. 2.15) determine a velocity profile \( v_x(r) \) of fluid flow. Its integration over a pipe cross section

\[
V_f = \frac{1}{A} \int_A v_x \, dA = \frac{8}{D^2} \int_0^{D/2} v_x \, r \, dr \quad \text{(2.16)}
\]

provides a relationship between pressure drop \( dP/dx \) and mean velocity \( V_f \)

\[
V_f = \frac{D^2}{32 \mu_f} \left( -\frac{dP}{dx} \right) \quad \text{(2.17)}.
\]

Shear stress at the pipe wall is thus determined as

\[
\tau_0 = \mu_f \frac{8V_f}{D} \quad \text{(2.18)}.
\]

This procedure cannot be used for turbulent flow because the relation between shear stress and strain rate in the turbulent flow is not fully described by the Newtonian viscous law. In a turbulent stream, the local velocity of the fluid fluctuates in magnitude and direction. This causes a momentum flux between fluid laminae in the stream. The momentum exchange has the same effect as a shear stress applied to the flowing fluid. These additional stresses set up by the turbulent mixing process are
called apparent shear stresses or Reynolds stresses. They predominate over the Newtonian, purely viscous stresses in the turbulent core of the fluid flow. In a fully developed turbulent flow the turbulent core usually occupies almost the entire pipe cross section, excepting only the near-wall region. A turbulent flow regime is typical for pipelines of an industrial scale.

Thus shear stress \( \tau_0 \) for turbulent flow cannot be determined directly from Newton's law of viscosity and the force balance equation 2.14. Instead, it is formulated by using dimensional analysis. A function

\[
\tau_0 = \text{fn}(\rho_f, V_f, \mu_f, D, k)
\]  

(2.19)

is assumed in which \( k \) denotes the absolute roughness of the pipeline wall. This provides a relation between dimensionless groups

\[
\frac{\tau_0}{\frac{1}{2} \rho_f V_f^2} = \text{fn}\left(\frac{\text{Re}}{D}, \frac{k}{D}\right)
\]  

(2.20).

The dimensionless group \( \text{Re} \) is Reynolds number of the pipeline flow, \( \text{Re} = \frac{\rho_f V_f D}{\mu_f} \). The dimensionless parameter on the left side of the equation 2.20 is called the friction factor. It is the ratio between the wall shear stress and kinetic energy of the fluid in a control volume in a pipeline

\[
f_f = \frac{\tau_0}{\frac{1}{2} \rho_f V_f^2}
\]  

(2.21).

The parameter \( f \) is known as Fanning friction factor. Darcy obtained a friction coefficient

\[
\lambda_f = \frac{\tau_0}{\rho_f V_f^2}
\]  

(2.22).

Thus the Darcy friction coefficient \( \lambda_f = 4f_f \).
The equation for the Darcy friction coefficient, combined with the integrated linear momentum equation for pipeline flow (Eq. 2.13), gives the equation first published by Weisbach in 1850

\[
-\frac{dP}{dx} = \frac{\lambda_f \rho_f V_f^2}{D} \frac{1}{2}
\]  

(2.23)
that is for \( \frac{dP}{dx} \) written as \( \frac{\Delta P}{L} = \frac{P_1 - P_2}{L} \) (see the definition of the hydraulic gradient above)

\[
I_f = \frac{\lambda_f V_f^2}{D \cdot 2g}
\]

(2.24).

This equation is known as the Darcy-Weisbach equation for the determination of the frictional head loss \( I_f \) in liquid flow in a pipeline.

For laminar flow, an equation for friction coefficient \( \lambda_f \) (or \( f_f \)) is calculated theoretically from the equation for pressure drop (Eq. 2.17) giving \( \lambda_f = 64/\text{Re} \). In turbulent flow there is no simple expression linking the velocity distribution with the shear stress (and so with the pressure gradient) in the pipe cross section. Over the years an empirical approach has provided a number of correlations \( \lambda_f = f(\text{Re}, k/D) \) for different pipe flow regimes. These have been derived from empirical expressions for a velocity profile in the turbulent flow in a pipeline.

2.3 Solid particles in a carrying liquid

Forces acting on solid particles submerged in a liquid have their origin either in a particle-liquid interaction or in a particle-particle interaction. Particles moving in a conduit may also interact with a conduit boundary. The forces acting on a single particle in a dilute suspension are the body forces. The particle-liquid body forces are the buoyancy force, drag force and lift force. When a solid particle is transported in the turbulent flow of a carrying liquid the turbulent diffusive force from carrier eddies is an additional particle-liquid force. Forces acting on solid particles due to particle-particle interaction are transmitted as the interparticle stress via the particle contacts. Coulombic stresses occur in a granular body occupied by particles in continuous contact. When a granular body is sheared and interparticle contacts are only sporadic, Bagnold stresses are transmitted through the granular body.

2.3.1 Gravitational and buoyancy force

The body force due to gravitational acceleration is determined from the solid particle volume and density. The gravitational force on a spherical solid particle of diameter \( d \) is

\[
F_{Gp} = \rho_g g \pi \frac{d^3}{6}
\]

(2.25).

According Archimedes law, a solid particle immersed in a liquid obeys a buoyancy effect, which reduces its weight in the carrying medium. The submerged weight of the solid particle is a result of gravitational and buoyancy effects on the solid particle
immersed in the liquid. For a spherical particle the submerged weight is determined by the expression

$$F_W = (\rho_s - \rho_f)g\pi \frac{d^3}{6}$$  \hspace{1cm} (2.26)

2.3.2 Drag force

When the surrounding liquid moves relative to a solid particle, an additional force is exerted from the liquid onto the submerged particle. The drag force, $F_D$, acts in the direction of the relative velocity $v_r = v_f - v_s$ between the liquid and the solid particle. The magnitude of the drag force is expressed in terms of the drag coefficient $C_D$. This comes from dimensional analysis of the function

$$F_D = \text{fin}(\rho_f, \mu_f, d, v_r)$$  \hspace{1cm} (2.27)

It provides two dimensionless groups of parameters:

**drag coefficient**

$$C_D = \frac{8F_D}{\pi d^2 v_r |v_r| \rho_f}$$  \hspace{1cm} (2.28)

and particle Reynolds number

$$Re_p = \frac{\rho_f |v_r|d}{\mu_f}$$  \hspace{1cm} (2.29)

giving $C_D = \text{fin}(Re_p)$.

An experimental determination of the drag coefficient is based on measurement of the terminal settling velocity of a spherical particle, $v_{ts}$, in a quiescent liquid. Measured $v_{ts}$ is the relative velocity $v_r$. The submerged weight of a solid particle in the liquid is balanced by the drag force of the liquid

$$\frac{C_D}{8} \pi d^2 v_{ts}^2 \rho_f = \frac{\pi d^3}{6} (\rho_s - \rho_f)g$$  \hspace{1cm} (2.30)

The drag coefficient is the only unknown parameter in the Eq. 2.30 for a particle of known particle size and terminal settling velocity. A relationship $C_D = \text{fin}(Re_p)$ is sensitive to a regime of the liquid flow round the settling solid particle. In a laminar regime (obeying Stokes' law) the relation is hyperbolic and in a turbulent regime (obeying Newton's law) the drag coefficient is no longer dependent on $Re_p$ ($C_D = 0.445$ for spherical particles). These two regimes are connected via a transition regime.
When a cloud of solid particles is settling in a quiescent liquid additional hindering effects influence its settling velocity. These are the increased drag caused by the proximity of particles within the cloud and the upflow of liquid as it is displaced by the descending particles. The hindering effects are strongly dependent on the volumetric concentration of solids in the cloud, $C_V$, (see Richardson & Zaki, 1954)

$$v_{th} = v_t (1-C_V)^m$$ (2.31)

in which $v_{th}$ is the hindered settling velocity of solid particle, $v_t$ is the terminal settling velocity of the solid particle and $m$ is the empirical exponent related to the solid particle properties.

2.3.3 Lift force

The lift force, $F_L$, on a single solid particle is a product of simultaneous slip (given by relative velocity $v_r = v_f - v_s$) and particle rotation. The force (sometimes called the Magnus force) acts in a direction normal to both the relative velocity $v_r$ and the particle rotation vector. A particle rotation combined with a slip results in a lower hydrodynamic pressure in flow above the particle than in that below the particle. Lift force is due to this pressure gradient.

Generally, the lift force magnitude is given by the lift coefficient

$$C_L = \frac{F_L}{\frac{1}{2} \pi d^2 v_f |v_r| \rho_f}$$ (2.32)

Saffman (1965) proposed an equation for lift acting under the condition that the particle spin is induced by liquid shear as a function of shear rate $\gamma = \frac{dv_f}{dy}$ (x-direction in the direction of a carrier liquid flow)

$$F_L = 1.615 d^2 \sqrt{\mu_f \rho_f} \gamma (v_s - v_f)$$ (2.33)

The lift force is most active near a pipeline wall where the velocity gradient is high. Hsu et al. (1989) included the Saffman force in the force balance of their microscopic model for solid-liquid flow in a pipeline. Ling (1995) evaluated the influence of the Saffman lift force on incipient motion of particles in a sediment bed. According to Roco & Cader (in Shook & Roco, 1991), however, the lift forces due to particle spin play a minor role in the majority of mixture flow regimes compared to the Bagnold and Coulombic forces.

2.3.4 Turbulent diffusive force

Solid particles are also subject to additional liquid-solids interactions when they are transported in a turbulent stream of the carrying liquid. An intensive exchange of
momentum and random velocity fluctuations in all directions are characteristic of the turbulent flow of the carrying liquid in a pipeline. Scales of turbulence are attributed to properties of the turbulent eddies developed within the turbulent stream. According to Prandtl's picture of turbulence, the length of the turbulent eddy is given as the distance over which the lump of liquid transports its momentum without losing its identity, i.e. before the lump is mixed with liquid in a new location. This distance is called the mixing length and since it is supposed to represent a mean free path of a pulse of liquid within a structure of turbulent flow it is considered a length scale of turbulence. A turbulent eddy is responsible for the transfer of momentum and mass in a liquid flow. The instantaneous velocity of liquid at any point in the flowing liquid and in arbitrary direction (x, y or z) is given by \( \mathbf{v} = \mathbf{\bar{v}} + \mathbf{v}' \) where \( \mathbf{\bar{v}} \) is the time-averaged velocity and \( \mathbf{v}' \) is the instantaneous fluctuation velocity. The turbulent fluctuating component \( \mathbf{v}' \) of the liquid velocity \( \mathbf{v} \) is associated with a turbulent eddy.

It is well known that turbulent eddies are responsible for solid particle suspension. The intensity of liquid turbulence is a measure of the ability of a carrying liquid to suspend the particles. The size of the turbulent eddy and the size of the solid particle are also important to the effectiveness of a suspension mechanism. The characteristic size of turbulent eddies is assumed to depend on the pipeline diameter.

A low concentration suspension is described by using a classical turbulent diffusion model of Schmidt and Rouse. The model was constructed as a flux balance per unit area perpendicular to the vertical direction in a flow balancing the volumetric settling rate (characterised by settling velocity \( v_t \)) in a quiescent liquid and the diffusion flux (characterised by the liquid velocity fluctuation in a vertical direction \( v'_y \), associated with the length of a turbulent eddy, \( l \)). A characteristic value of the turbulent pulsative velocity \( v'_y = \sqrt{v'^2_y} \), i.e. the root mean square of velocity fluctuations in the y-direction. As derived e.g. in Shook & Roco (1991) the balance gives an equation

\[
-\varepsilon_S \frac{dc_v}{dy} = v_t c_v
\]

(2.34)

when solids dispersion coefficient \( \varepsilon_S = 0.5v'_y l \).

Integration of Eq. 2.34 with \( \varepsilon_S \) considered constant gives an exponential concentration profile \( c_v(y) \) as

\[
c_v = C_{vb} \exp\left[ -\frac{v_t}{\varepsilon_S} (y - y_b) \right]
\]

(2.35)

i.e. an exponential concentration variation with height, \( y \), in a flow above a boundary characterised by a position \( y_b \) and a concentration \( C_{vb} \).

Wilson & Pugh (1988) expressed a turbulent diffusive force exerted on particles by turbulent eddies by rewriting the Eq. 2.34 as a force balance between the turbulent
diffusive force and the submerged weight of the particles in a unit volume of slurry in a horizontal pipe. The submerged weight was \( \rho_f g (S_s - 1)c_v \) so the turbulent dispersive force

\[
F_t = -(\rho_s - \rho_f)g \varepsilon_s \frac{dv}{d\gamma} v_t \frac{dc_v}{dy} \tag{2.36}
\]

How to determine the solids dispersion coefficient, \( \varepsilon_s \), is a major problem connected with the application of the turbulent diffusive model. The effect of distance from a boundary and of the presence of solid particles in a turbulent stream on a local value of \( \varepsilon_s \) cannot be neglected. Further, the neighbouring particles also affect the particle settling velocity handled in the model.

### 2.3.5 Coulombic contact force

Stress distribution in a granular body occupied by non-cohesive particles in continuous contact is a product of the weight of grains occupying the body. The intergranular pressure (or stress) from the weight of grains is transmitted within the granular body via interparticle contacts. The stress has two components: an intergranular normal stress and an intergranular shear stress. According to Coulomb's law these two stresses are related by the coefficient of friction. Du Boys (1879) applied Coulomb's law to sheared river beds. He related the intergranular normal stress, \( \sigma_s \), and intergranular shear stress, \( \tau_s \), at the bottom of a flowing bed by a coefficient

\[
\tan \phi = \frac{\tau_s}{\sigma_s} = \frac{\tau_s}{\rho_f g (S_s - 1)c_v b Y_{sh}} \tag{2.37}
\]

in which \( \phi \) is an angle of repose of the grains, \( S_s = \rho_s/\rho_f \) is the specific gravity of solids, \( Y_{sh} \) is the thickness of the sheared bed. \( c_v b \) denotes the maximum solids volume fraction of solids in the granular bed and it is considered to be valid for the sheared bed. The angle of repose, \( \phi \), is considered to be the angle at the internal failure of a static granular body. The value of this internal-friction coefficient is basically dependent on the nature of the surface over which the grains start to move, i.e. primarily on a grain size.

When the granular bed motion takes place over a pipe wall, the value of the bed-wall friction coefficient can be determined by a tilting tube test (Wilson, 1970). In this

\[
\mu_s = \tan \omega_{tilt} F_W/F_N,
\]

where \( \mu_s \) is called the mechanical friction coefficient, \( \omega_{tilt} \) is an angle of the tilting tube inclination at the incipient motion of the bed, \( F_W \) the submerged weight of the granular bed and \( F_N \) the normal force exerted by the bed grains against the tube wall. The ratio \( F_W/F_N \) approaches unity for thin beds. The granular bed is considered to be sliding en bloc, so shear of grain layers does not take place within the bed. The normal granular stress and the Coulombic granular shear stress in the en bloc sliding bed are independent of the shear rate and the shear is confined to bed boundaries. For sand and gravel the coefficient of sliding friction was found to be virtually independent of the particle size during tests reported by Shook et
al. (1982), Gillies (1993) considered a lubrication effect reducing the coefficient of sliding friction if particles and the pipe wall are separated by a liquid layer. The value of the coefficient is independent of the mean slurry velocity in a pipeline provided that the lubrication effect is small.

Sheared-bed particles flowing in the region of high shear rate maintain sporadic, rather than continuous contact with each other, provided that solids concentration in the sheared bed is considerably lower than the loose-poured bed concentration \( C_{\nu b} \). The nature of an interparticle contact influences the relationship between the intergranular stress components. It is appropriate to relate the particulate shear and normal stresses in a granular body experiencing the rapid shearing by using a coefficient of dynamic friction \( \tan \phi' \) instead of its static equivalent \( \tan \phi \). Bagnold (1954) measured and described the normal and tangential stresses in mixture flows at high shear rates.

### 2.3.6 Bagnold dispersive force

Bagnold's dispersive force is a product of intergranular collisions (particle - particle interactions) in a sheared layer rich in particles. The direction of the force is normal to the layer boundary on which it is acting. The force increases with increasing solids concentration and shear rate in the sheared layer.

Bagnold (1954) measured this force (an effect of which he called the granular dispersive pressure) in the annular space between two concentric drums. The distance between the walls of the inner and outer drums was 10.8 mm. The annular space was filled with liquid (water and glycerine) and wax particles of diameter 1.32 mm. To avoid the effects of radial acceleration on the phase interactions within the mixture, a liquid and particles of very similar specific gravity were chosen. When the outer drum rotated a normal dispersive stress \( \sigma_S \) was exerted between the solid particles in the sheared mixture. This was recorded as an increase of static pressure on the deformable surface of the inner stationary drum. The intergranular shear stress \( \tau_S \) was deduced from the measured torque on the inner drum. Shear rate in the mixture was controlled by the imposed revolutions of the outer drum.

Based on these measurements, Bagnold recognised two regimes of shear in the mixture, the regime in which liquid viscosity dominated and the regime of grain inertia domination. He observed that:

- in the macro-viscous region (characterised by low shear rates) the intergranular shear stress \( \tau_S \) and normal dispersive stress \( \sigma_S \) varied with the first power of the shear rate,
- \( \tau_S \) and \( \sigma_S \) were a quadratic function of the shear rate in the grain-inertia region of high shear rates, i.e. in flow regime where the particulate stress arise as a result of interparticle collisions within sheared mixture.

Bagnold suggested that the stresses \( \sigma_S \) and \( \tau_S \) at the flow boundary were related by a dynamic friction coefficient

\[
\tan \phi' = \frac{\tau_S}{\sigma_S}
\]  

(2.38)
where $\phi'$ is the angle of the dynamic friction for the sheared granular body. The relationship 2.38 was assumed to be valid in both viscous and inertial shear regimes.

In his further work, Bagnold (1956) applied his dispersive stress concept to the horizontal flow of mixtures within the sheared layer at the top of a stationary granular bed. The intergranular normal stress at any gravity boundary within the shear layer must be equal to the submerged weight of all bed-load grains moving above the boundary

$$
\sigma_s = \frac{y_{sh}}{y} \rho_f g (S_0 - 1) C dy
$$

(2.39)

where $y$ determines a position within the shear layer, $y_{sh}$ a position of the top of the shear layer and $C$ is the mean volumetric concentration of bed-load particles in a region delimited by $y_{sh}$ and $y$. The Eq. 2.39 suggests that the submerged weight of the bed-load particles (particles are of density higher than is that of the carrying liquid) in the sheared granular body is balanced by the particulate dispersive force arising from the interparticle collisions. The submerged weight of bed-load particles is transmitted through the interparticle normal stress to the stationary boundary, i.e. to the grains in the stationary bed. The submerged weight of the suspended-load particles is balanced by liquid stresses from turbulent eddies.

Shook & Roco (1983) implemented the Bagnold dispersive force in their microscopic model for slurry flow in a horizontal pipe. Unfortunately, the Bagnold effect was not verified experimentally, since it was impossible to separate the effect of the Bagnold dispersive force from other interactive mechanisms during the horizontal-pipe experiments. Recently, the attempts have been made to evaluate the Bagnold effect experimentally in a vertical pipe, where flow conditions are simpler and the Coulombic intergranular stress is avoided (Bartosik & Shook, 1992 and Bartosik & Shook, 1995).

Shook & Roco (1991) and Gillies & Shook (1994) expressed the particle interaction force (combining Coulombic and Bagnold effects) in terms of the gravitational force $-(\rho_S - \rho)g$ and an empirical coefficient $0 < k < 1$. In extreme cases, the particle interaction effects are negligible ($k = 0$) or they are so large that the interparticle force offsets the submerged weight of the granular body and produces a uniform concentration distribution ($k = 1$) in the pipe cross section.

2.4 References


Chapter 3

Literature review of and commentary on slurry flow concepts

This chapter presents a general literature review of concepts for the modelling of slurry flow and a more detailed review of the empirical and the macroscopic modelling approaches. Two widely used empirical models are described in detail and commentaries based on the literature and our own practical experience are given. Furthermore, the principles and their mathematical description of a macroscopic two-layer model, originally developed as a tool to predict the deposition-limit velocity and the hydraulic gradient in fully-stratified flows in a slurry pipeline, are discussed. The review includes the recent implementation of the effect of a shear layer on a friction mechanism within a stratified flow to the two-layer model. For a practical application of the model nomographs are used to predict the hydraulic gradient and deposition-limit velocity for different slurry flow conditions. Computational testing of the two-layer model provides several comments on the application of the two-layer model to fully-stratified flow in its mathematical and nomographic version. In addition, the attempts to extend the two-layer model application to the partially-stratified flows are reviewed. All information summarised above relates to horizontal slurry flows. At the end of the chapter, the literature on the modelling of effects of pipeline inclination on the slurry flow behaviour is reviewed.

3.1 General literature review

Almost 50 years ago, substantial progress in the exploitation of the hydraulic transport of solids in pipelines initiated systematic investigations in this field. With the design of new industrial pipelines, some of which were of considerable length, the demand for the reliable models capable of predicting slurry flow behaviour grew. Over the years, as experimental, theoretical and computational techniques have progressed, the predictive models have been gradually improved.

The first predictive tools were developed in the 1950's and 1960's. The tools were empirical correlations constructed to predict the basic slurry pipeline characteristics - the frictional head loss and the deposition-limit velocity - for various slurry flow conditions in a pipeline. The correlations were based on the experimental measurements of integral parameters of slurry flow in pipelines. Usually these parameters were mean slurry velocity, volumetric delivered concentration and pressure in flows of slurry containing particles of certain diameter. Some of the models have become popular and are still used in practice (e.g. Durand & Condolios, 1952; Führböter, 1961; Jufin & Lopatin, 1966). They are simple to use and easy to modify to the user's own data. The growing database from various experimental installations world-wide encouraged Turian & Yuan (1977) to apply a statistical approach to the
development of a set of empirical correlations for different specific slurry flow patterns. Recently, a semi-empirical model for a heterogeneous flow in slurry pipelines which is calibrated by using the integral-parameter data has been introduced by Wilson et al. (1992).

Since the mid 1980's attempts have been made to construct a general model for solid-liquid flow in pipelines by using a microscopic approach. A microscopic model defines the laws governing a slurry flow for an infinitesimal control volume of slurry. A slurry flow mechanism is described by using a set of differential equations for conservation of mass, momentum and energy in the solid-liquid flow. A microscopic model provides a numerical solution to the equations in local positions of a pipeline cross section. As a result, the model predicts the concentration and velocity profiles in a pipeline cross section, together with the pressure drop over a pipeline length section. Despite progress in the development of sophisticated experimental techniques which enable reasonably accurate measurements of the internal structure of the flow (concentration and velocity profiles) in a slurry pipeline, there is still not enough information on the slurry flow mechanism at microscopic level. The models of Roco & Shook (1983), Roco & Mahadevan (1986), Rasteiro et al. (1988), Hsu et al. (1989) or Shook & Roco (1991) differ by emphasising the different mechanisms used to describe the interactions between phases within a slurry stream.

A suitable compromise between the microscopic and empirical approaches is an approach using the principles of macroscopic modelling. This approach applies the balance (conservation) equations to a larger control volume of slurry given, for instance, by a pipeline cross sectional area of approximately uniform concentration of solids in a unit length of a pipeline. In a chosen control volume, the balance equations are formulated by using mean quantities, i.e. quantities averaged in the control volume. Newitt et al. (1955) were the first to apply the balance formulations to a macroscopic control volume to obtain the friction loss equations for different slurry flow regimes in a slurry pipeline. Wilson (1970) introduced the concept of a mechanistic force-balance model to predict the velocity at the limit of stationary deposition in a fully-stratified flow. Wilson (1976) developed the model further to provide a unified predictive tool, called a two-layer model, to predict both the limiting deposition velocity and the frictional head losses in fully-stratified and partially-stratified flows in a horizontal slurry pipeline. Since then further models based on the principle of force balances in the two-layer flow pattern have been developed (Doron et al., 1987, Shook & Roco, 1991). The latest improvement to the two-layer model has been made by Gillies et al. (1991). In this the empirical components of the model have been determined by using a large data base including data from several slurry pipelines of industrial sizes.

3.2 Empirical modelling

Different approaches to empirical modelling of slurry flow in pipelines are described by using two widely used models as examples: the model of Laboratoire Dauphinois d'Hydraulique (called sometimes the Durand model and dated to 50's) and the Wilson & GIW model (developed in 80's and 90's.). Experimental data from our laboratory circuit are used to demonstrate a modification of model coefficients based on the user's own data. Basic parameters used in the models are defined in Appendix 3.
3.2.1 Model of Laboratoire Dauphinois d’Hydraulique

The empirical model of Laboratoire Dauphinois d’Hydraulique to predict the pressure drop due to friction in the pipeline flow of slurry was constructed by using techniques for dimensional analysis. Durand and his co-workers sought an empirical relationship among the dimensionless groups of quantities anticipated to be of major importance for a description of slurry flow in a pipeline. Experimental data were collected for a reasonably wide range of slurry flow conditions including several pipeline sizes and sorts of sand and gravel. Investigators recognised two slurry flow patterns; heterogeneous for settling slurries and homogeneous, exhibited by non-settling slurries composed of liquid and solid particles smaller than approximately 40 microns.

Based on the experimental results (for low concentration slurries with delivered concentration \( C_{vd} \) up to 22%), the following issues were proposed for the pressure loss, represented by the hydraulic gradient \( I_m \) in the heterogeneous slurry flow characterised by constant particle size \( d \) and pipeline size \( D \):
- the solids effect \( I_m-I_f \) decreases gradually with increasing mean slurry velocity \( V_m \) in the flow of constant delivered volumetric concentration of solids \( C_{vd} \)
- the solids effect \( I_m-I_f \) increases approximately linearly with increasing \( C_{vd} \) at constant \( V_m \).

The latter condition was written as \( I_m-I_f=\text{const.}C_{vd} \). To eliminate the direct influence of the properties specific to one experiment (such as the pipe roughness and the slurry temperature) the ratio of the solids effect and the hydraulic gradient for liquid flow, \( I_f \) was introduced in place of the solids effect alone. Then the condition was written

\[
\frac{I_m - I_f}{I_f C_{vd}} = \text{const.} \quad (3.1).
\]

This dimensionless group, marked \( \Phi \), is not constant for slurry flows of different pipeline size, solids size or slurry flow rate. Durand and his co-workers proposed correlating the flow coefficient \( \Phi \) with the Froude number for slurry flow \( \text{Fr}_2^2 = \frac{V_m^2}{gD} \).

The Froude number is a criterion of dynamic similarity for flows with a dominant effect of inertia and gravity in different flow conditions. It relates slurry flows of different \( V_m \) in different pipeline sizes \( D \). The plot \( \Phi \) versus \( \text{Fr}_2^2 \) for Durand's experimental data provided a set of interpolated power-law curves different for various transported solids. To get one general \( \Phi \) correlation for different transported solids the Froude number for a solid particle was introduced in the form \( \text{Fr}_{vt}^{-1} = \frac{\sqrt{gd}}{v_t} \).

The new dimensionless group was marked \( \Psi \):

\[
\Psi = \text{Fr}_2^2\text{Fr}_{vt}^{-1} \quad (3.2).
\]

Different equations for \( \Psi \) were given in other publications (Durand, 1953; Gibert, 1960) replacing \( \text{Fr}_{vt}^{-1} \) with modified dimensionless group characterising particle settling process. A plot \( \Phi \) versus \( \Psi \) was proposed as an unified pattern for the
Figure 3.1. Relationship between dimensionless groups \( \Phi \) and \( \psi \).

\[
\frac{1}{\psi} \frac{d^2 \psi}{d \Phi^2} = \Phi
\]
evaluation of experimental data for solids of \( d = 0.18 - 22.5 \text{ mm} \) and pipelines of \( D = 40 - 580 \text{ mm} \). A general empirical relationship was established for a resistance due to friction in a heterogeneous slurry flow (Durand & Condolios, 1952) as

\[
\Phi = K \left( \frac{\sqrt{gD}}{V_{\text{in}}} \right)^3 \left( \frac{V_t}{\sqrt{gd}} \right)^{1.5}
\]

(3.3)

thus

\[
\Phi = K \left( Fr^2 Fr_t^{-1} \right)^{-1.5}
\]

(3.4).

Originally Durand & Condolios (1952) did not propose a value for the empirical coefficient \( K \) and preferred to use a published \( \Phi-\Psi \) curve (Fig. 3.1) for friction loss prediction. However, a hyperbolic curve in the \( \Phi-\Psi \) plot was later approximated by

\[
\Phi = K \Psi^n
\]

and the values for the \( K \) and \( n \) coefficients were determined by using the experimental data base. Different existing expressions for \( \Psi \) make it necessary to ensure that a proposed \( K \) value is correctly attached to a corresponding \( \Psi \) version. Gibert (1960) and Condolios & Chapus (1963) proposed \( K = 180 \) for sand/gravel slurries and \( \Psi \) according to Eq. 3.2 (they kept \( n = -1.5 \)). Over the years the Durand type of correlation has been tested by using different experimental data bases and a considerable number of values has been proposed for \( K \) and \( n \) by various investigators (see survey in Kazanski, 1978).

An advantage of the Durand method is that it provides a simple relationship which covers a wide range of slurry flow conditions and correlates all basic parameters influencing the behavior of slurry flow in a pipeline. A major disadvantage is the inaccuracy of the determination of \( \Phi \) for the extreme values of the coefficient \( \Psi \) (see Fig. 3.1). The \( \Phi - \Psi \) curve is very steep at low \( \Psi \). A small difference in \( \Psi \) may create a big difference in \( \Phi \) (so predicted \( V_{\text{in}} \) may differ by from ten per cent to several hundred per cent). At high \( \Psi \) values the coefficient \( \Phi \) decreases very slowly with increasing \( \Psi \). The regions of the extreme \( \Psi \) values represent a transition from heterogeneous flow to the extreme slurry flow patterns: fully-stratified for the lowest values of \( \Psi \) and fully-suspended (pseudo-homogeneous) for the highest values of \( \Psi \). The insensitivity of the correlation at its extremes reveals the fact that the model does not reflect different slurry flow patterns.

Doubts about the applicability of the correlation to a wide range of slurry flow conditions had already arisen in the early years, soon after its introduction. Silin & Kobernik (1962) tested the correlation against their own experimental data acquired in the laboratory and on dredging installations (DN100 - DN900). They found that the correlation could be used within the range \( 4 < \Psi < 15 \). Doubts were raised when Zandi & Govatos (1967) showed a large discrepancy between Durand's prediction and experimental data for a coarse slurry flow exhibiting considerable stratification. The
Figure 3.2a. Experimental data for flow of the 0.2 - 0.5 mm sand in the 150 mm pipeline plotted in Durand's dimensionless groups.

Figure 3.2b. Experimental data for flow of the 3.0-5.0 mm gravel in the 150 mm pipeline plotted in Durand's dimensionless groups.
Durand correlation seemed to be invalid for the solids transported by saltation, therefore Zandi & Govatos suggested a criterion for separation of the "heterogeneous" and "saltation" regimes in a slurry pipeline. Furthermore, they found that the coefficients K and n were not constant within the heterogeneous regime and stated two ranges (\(\Psi<10\), \(\Psi>10\)) for the heterogeneous regime described by different K and n coefficients. Babcock (1971), using experimental data for solids of different size and specific gravity, showed that the solids effect (\(I_m-I_f\)) on a frictional head loss was virtually independent of particle size for coarse slurry flow (flow transporting solids of a gravel size). The Durand correlation demonstrates this by using \(C_D\) (which denotes the drag coefficient for the particle and is constant for coarse particles settling in Newton's regime) as a solids size characteristic, but Babcock's analysis indicates that this is apparently not sufficient. Data presented by Babcock for coarse particles of different \(C_D\) produced a large scatter in Durand's coordinates but could be well correlated in the abscissa \((I_m-I_f)/(C_vd[S_S-1])\) versus \(V_m\) used by Newitt at al. (1955) for stratified flows.

Our tests, described in the next chapter, confirm that the Durand-type of correlation may provide a reasonable prediction of frictional head losses in a horizontal slurry pipeline for solids of medium sand size (as is sand 0.2-0.5 mm) but fails for the flow of coarse particles such as gravel 3.0-5.0mm (see Figs. 3.2a and 3.2b). The determination of model coefficients K and n from experimental data indicates a considerable variation in the coefficient values for sand-water flow of different solids concentrations (see Table III.1A and corresponding Fig. 3.2c). For gravel flow the exponent n gets a value -1.0 rather than -1.5 (see Table III.1B and corresponding Fig. 3.2d), thus the relative solids effect tends to be independent of mean slurry velocity \(V_m\).

**Table III.1A.** Sand 0.2 - 0.5 mm (\(d_{50} = 0.42\) mm) in a 150 mm pipeline.

<table>
<thead>
<tr>
<th>(C_vd)</th>
<th>K</th>
<th>n</th>
<th>Correl. coeff.*</th>
<th>Experim. run</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11 - 0.12</td>
<td>1098</td>
<td>-2.13</td>
<td>0.99</td>
<td>13/12/94</td>
</tr>
<tr>
<td>0.15-0.185</td>
<td>710</td>
<td>-1.85</td>
<td>0.91</td>
<td>14/12/94</td>
</tr>
<tr>
<td>0.205-0.25</td>
<td>366</td>
<td>-1.66</td>
<td>0.97</td>
<td>15/12/94-20/12/94</td>
</tr>
<tr>
<td>0.26-0.305</td>
<td>202</td>
<td>-1.42</td>
<td>1.00</td>
<td>21/12/94</td>
</tr>
<tr>
<td>0.295-0.365</td>
<td>182</td>
<td>-1.36</td>
<td>1.00</td>
<td>22/12/94</td>
</tr>
</tbody>
</table>

*Correlation coefficient is defined in Appendix 3.

**Table III.1B.** Gravel 3.0 - 5.0 mm (\(d_{50} = 4.20\) mm) in a 150 mm pipeline.

<table>
<thead>
<tr>
<th>(C_vd)</th>
<th>K</th>
<th>n</th>
<th>Correl. coeff.</th>
<th>Experim. run</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09-0.10</td>
<td>181</td>
<td>-1.17</td>
<td>0.97</td>
<td>12/09/95</td>
</tr>
<tr>
<td>0.16-0.17</td>
<td>132</td>
<td>-1.00</td>
<td>0.99</td>
<td>13/09/95</td>
</tr>
<tr>
<td>0.21-0.23</td>
<td>116</td>
<td>-0.87</td>
<td>0.95</td>
<td>14/09/95</td>
</tr>
<tr>
<td>0.26-0.285</td>
<td>222</td>
<td>-1.13</td>
<td>1.00</td>
<td>15/09/95-18/09/95</td>
</tr>
</tbody>
</table>
Figure 3.2c. Correlation of data using Durand's dimensionless groups for slurry flow of the 0.2-0.5 mm sand in the 150 mm pipeline. Experimental runs: 13/12/1994 - 09/01/1995.
Figure 3.2d. Correlation of data using Durand's dimensionless groups for slurry flow of the 3.0-5.0 mm gravel in the 150 mm pipeline.

Plotting experimental data to a graph of the Durand's abscissa may mask the slurry flow effects which can be indicated from the shape of the \( I_m \) versus \( V_m \) curve and which may play an important role in the description of slurry flow as discussed further in Chapters 5 and 7. The sensitivity of hydraulic gradient \( I_m \) to mean slurry velocity \( V_m \) under various slurry flow conditions is better demonstrated in a plot using the Newitt et al. abscissa. This plot is used to correlate the slurry flow characteristics in the Wilson & GIW model.

### 3.2.2 Wilson & GIW model

The semi-empirical *Wilson & Georgia Iron Works model* for heterogeneous flow in slurry pipelines is based on considering the heterogeneous flow as a transition between two extreme flows governed by different mechanisms for support of a solid particle in the stream of the carrying liquid: the fully-stratified flow and the fully-suspended flow.

Resistance in the *fully-stratified flow* is predominantly due to mechanical friction between solid particles and the pipeline wall. The frictional head loss is predicted by using a two-layer model. It can be computed in its original shape (a set of mass and force balance equations) by iteration. To avoid these computations Wilson et al. (1992) proposed the nomograph constructed as an interpolation of typical outputs from the original two-layer model. The nomograph curves can be approximated by the expression (Wilson, 1996)

\[
\frac{I_m - I_f}{S_m - 1} = \left( \frac{V_m}{0.55V_{sm}} \right)^{-0.25}
\]

(3.5)

in which \( S_m \) is the specific gravity of the slurry, giving \( S_m - 1 = C_V d (S^-1) \), and \( V_{sm} \) is the maximum value of deposition-limit velocity for different solids concentrations in slurry flow of certain \( S, d \) and \( D \). The \( V_{sm} \) is determined (Wilson et al., 1992) by

\[
V_{sm} = \frac{8.8 \left[ \mu \left( S_m - \frac{S^*}{1} \right) \right]^{0.55}}{0.66 \frac{D^{2.7}}{d_{50}^{1.75}} + 0.11D^{0.7}}
\]

(3.6)

in which \( \mu \) is the coefficient of mechanical friction between the solid particles and the pipeline wall, \( d_{50} \) is in millimetres and \( D \) in metres.

Friction loss in the *fully-suspended* (pseudo-homogeneous) *flow* is predicted (Clift et al., 1982) as

\[
\frac{I_m - I_f}{S_m - 1} = A'I_f
\]

(3.7).
An increase in the frictional head loss due to the presence of solid particles in a carrying liquid forming a Newtonian pseudo-homogeneous slurry is attributed to increased viscous friction at the pipeline wall. A measure of the slurry density effect on shear stress at the pipeline wall is given by an empirical coefficient $A'$. The Eq. 3.7 gives the "equivalent liquid" model $I_m = S_m I_f$ for $A' = 1$. According to this model the pseudo-homogeneous slurry flow behaves as a flow of a single-phase liquid having the density of the slurry. If $A' = 0$, solids do not affect a flow friction at all according to the model for fully-suspended flow. Such behaviour in horizontal pseudo-homogeneous sand-slurry flows was reported by Carstens & Addie (1981). Determination of the solids effect on friction in Newtonian pseudo-homogeneous slurry flows is still subject to investigation.

A relationship between the relative solids effect $(I_m - I_f)/(S_m - 1)$ and the mean slurry velocity $V_m$ was found appropriate to correlate the parameters for all flow regimes. The experimental data for heterogeneous flows showed a linear relationship between these two parameters when plotted in logarithmic co-ordinates (Fig. 3.3a, 3.3b).

![Figure 3.3a](image)

**Figure 3.3a.** Relationship between relative solids effect and mean slurry velocity for masonry sand slurry ($d_{50} = 0.42$ mm), after Chiff et al. (1982).
Figure 3.3b. Relationship between relative solids effect and mean slurry velocity for crushed granite slurry (d$_{50}$=0.68 mm), after Clift et al. (1982).

The partially-stratified (heterogeneous) flow was described in the first version of the heterogeneous model, considered as a mechanistically-based coalescing method, by combining the models for both extreme flow patterns (Clift et al., 1982)

$$\frac{I_m - I_f}{S_m - I} = (1 - Z)A'I_f + ZB$$ (3.8).

The empirical coefficient B represented the considered constant value of relative solids effect in a fully-stratified flow and coefficient Z, called the stratification ratio, was proposed to determine the proportion to which both mechanic and viscous friction mechanisms affect the total friction loss in a partially-stratified flow. The stratification ratio was proposed originally for the two-layer model (Wilson, 1976) to estimate the solids division into two layers.
\[ Z = \left( \frac{V_{tt}}{V_m} \right)^M \quad \text{for } V_m \geq V_{tt} \quad (3.9) \]

in which \( V_{tt} \) was mean slurry velocity at the threshold of turbulent uplift for solid particles and \( M \) was the empirical exponent.

Later (Wilson et al., 1990), an evaluation of the influence of a wide particle size distribution in the transported solids was incorporated into the model. A recent version of the model operates with a parameter \( V_{50} \) expressing the mean slurry velocity at which one half of the transported solid particles contribute to a suspended load and one half to a contact load (Wilson et al., 1992). An equation for this velocity expresses the influence of suspension mechanisms from the carrier turbulent diffusion and the hydrodynamic lift acting on particles larger than the sub-layer thickness in the near-wall region. The relationship between the relative solids effect and mean slurry velocity is given as

\[ \frac{I_m - I_f}{S_m - 1} = 0.5 \mu_s \left( \frac{V_m}{V_{50}} \right)^{-M} = 0.22 \left( \frac{V_m}{V_{50}} \right)^{-M} \quad (3.10) \]

in which the coefficient of mechanical friction between solids and the pipeline wall \( \mu_s \) is proposed to be equal to 0.44. According to the latest proposal (Wilson, 1996), \( V_{50} \) should be obtained experimentally or estimated roughly by the approximation

\[ V_{50} \approx 393(d_{50})^{0.35} \left( \frac{S_s - 1}{1.65} \right)^{0.45} \quad (3.11) \]

in which the particle diameter \( d_{50} \) is in mm and the resulting \( V_{50} \) in m/s. The exponent \( M \) is given by the approximation

\[ M \approx \left[ \ln \left( \frac{d_{85}}{d_{50}} \right) \right]^{-1} \quad (3.12) \]

\( M \) should not exceed 1.7, the value for narrow-graded solids, nor fall below 0.25.

The Wilson & GIW model gives a scale-up relationship for friction loss in slurry pipelines of different sizes transporting solids of different sizes at different concentrations. It is based on the assumption that there is a power-law relationship between the relative solids effect and the mean slurry velocity that is valid in all slurry flow conditions. The exponent \( M \) of this relationship is assumed to be dependent on the particle size distribution only. As seen on Figs. 3.3a, 3.3b the model was verified by experimental data for flows of relatively low concentrations. Recently, Sundqvist (1996) tested the power-law relationship by experimental data acquired in the
Figure 3.4a. Relationship between relative solids effect and mean slurry velocity for the 0.2 - 0.5 mm sand and the 3.0-5.0 mm gravel in the 150 mm pipeline. Experimental runs: 13/12/94 - 09/01/95 (sand), 12/09/95 - 18/09/95 (gravel).
laboratories of Georgia Iron Works and Luleå University of Technology for different pipeline and solids sizes and wide ranges of solids concentrations in pipelines. She found the effects of pipeline diameter and solids concentration on the relative solids effect.

The influence of solids concentration on the relative solids effect is also indicated by our experimental data (Fig. 3.4a). The Wilson & GIW model for heterogeneous flow has been tested by using our data (see Tab. III.2A) and by using the data from Sundqvist (1996) (see Tab. III.2B) for sand-slurry flows. Both flows have similar parameters $d_{50}/D$ and $C_{vd}$ and both are of a narrow particle size distribution. The relationship, derived from Eq. 3.10 as

$$\frac{1_m - 1_f}{C_{vd}(S_S - 1)} = aV^{-1}_m$$

is calibrated with the experimental data to get values for the coefficients $a$ and $M$. The coefficients provide the model velocity $V_{50}$ in terms of the Eq. 3.10 as

$$V_{50} = \left(\frac{a}{0.5\mu_S}\right)^{1/M}$$

$V_{50}$ is determined for $\mu_S$ values used in the Wilson & GIW model ($\mu_S = 0.44$) and measured in our laboratory ($\mu_S = 0.55$). The results of calibrating the model with experimental data from flows are given in Figs. 3.4a to 3.4c and corresponding Tabs. III.2A to III.2C.

**Table III.2A.** Sand 0.2 - 0.5 mm ($d_{50} = 0.42$ mm) in a 150 mm pipeline ($d_{50}/D = 0.0028$).

<table>
<thead>
<tr>
<th>$C_{vd}$</th>
<th>$M$</th>
<th>$a$</th>
<th>Correl. coeff.</th>
<th>$\mu_S$</th>
<th>$V_{50}$ Eq. 3.14</th>
<th>$V_{50}$ Eq.3.11</th>
<th>Experim. run</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11 - 0.12</td>
<td>2.25</td>
<td>1.90</td>
<td>0.99</td>
<td>0.55/0.44</td>
<td>2.36/2.60</td>
<td>2.90</td>
<td>13/12/94</td>
</tr>
<tr>
<td>0.15-0.185</td>
<td>1.71</td>
<td>1.07</td>
<td>0.99</td>
<td>0.55/0.44</td>
<td>2.22/2.53</td>
<td>2.90</td>
<td>14/12/94</td>
</tr>
<tr>
<td>0.205-0.25</td>
<td>1.32</td>
<td>0.68</td>
<td>0.93</td>
<td>0.55/0.44</td>
<td>2.00/2.37</td>
<td>2.90</td>
<td>15/12/94-20/12/94</td>
</tr>
<tr>
<td>0.26-0.305</td>
<td>0.84</td>
<td>0.39</td>
<td>0.97</td>
<td>0.55/0.44</td>
<td>1.53/1.99</td>
<td>2.90</td>
<td>21/12/94</td>
</tr>
<tr>
<td>0.295-0.36</td>
<td>0.72</td>
<td>0.36</td>
<td>0.99</td>
<td>0.55/0.44</td>
<td>1.44/1.96</td>
<td>2.90</td>
<td>22/12/94</td>
</tr>
</tbody>
</table>
Figure 3.4b. Correlation of data using Wilson’s model parameters for slurry flow of the 0.2-0.5 mm sand in the 150 mm pipeline.
Figure 3.4c. Correlation of data using Wilson’s model parameters for slurry flow of the 0.45-0.85 mm sand in the 203 mm pipeline. Experimental data from Sundqvist (1996).
Table III.2B. Sand 0.45 - 0.85 mm ($d_{50} = 0.63$ mm) in a 203 mm pipeline ($d_{50}/D = 0.0031$), experimental data from Sundqvist (1996).

<table>
<thead>
<tr>
<th>$C_{VD}$</th>
<th>$M$</th>
<th>$a$</th>
<th>Correl. coeff.</th>
<th>$\mu_S$</th>
<th>$V_{50}$ Eq. 3.14</th>
<th>$V_{50}$ Eq. 3.11</th>
<th>Experim. run</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12 - 0.15</td>
<td>1.24</td>
<td>1.09</td>
<td>0.98</td>
<td>0.55/0.44</td>
<td>3.02/3.61</td>
<td>3.34</td>
<td>M43-93</td>
</tr>
<tr>
<td>0.175-0.20</td>
<td>1.08</td>
<td>0.93</td>
<td>0.91</td>
<td>0.55/0.44</td>
<td>3.07/3.77</td>
<td>3.34</td>
<td>M38-93</td>
</tr>
<tr>
<td>0.25-0.275</td>
<td>0.55</td>
<td>0.51</td>
<td>0.97</td>
<td>0.55/0.44</td>
<td>3.09/4.63</td>
<td>3.34</td>
<td>M39-93</td>
</tr>
<tr>
<td>0.30-0.315</td>
<td>0.71</td>
<td>0.79</td>
<td>0.96</td>
<td>0.55/0.44</td>
<td>4.38/5.99</td>
<td>3.34</td>
<td>M40-93</td>
</tr>
</tbody>
</table>

According to the results obtained by calibrating the model against data of medium-sand flow at different concentrations the relative solids effect may be considered concentration-independent only in the narrow $V_m$ range just above the deposition limit threshold. At higher mean slurry velocities the increase in the relative solids effect with the increasing $C_{VD}$ is no longer negligible (see Figs. 3.4b and 3.4c). The variation in the relative solids effect is due to both $V_{50}$ and $M$ variation with mean solids concentration $C_{VD}$ in the slurry flow. No common trend is observed in the relationship between $V_{50}$ and $C_{VD}$ but the exponent $M$ seems to decrease with the increasing $C_{VD}$ in slurry flow (see Tabs. III.2A and III.2B).

The correlation for fully-stratified flow (Eq. 3.5) is more appropriate than the heterogeneous model for a 3.0-5.0 mm gravel flow. This flow tended to be fully stratified, but as a result of hydrodynamic interaction between a sliding bed and water stream above the bed the entire bed did not slide en bloc. Only at the highest measured concentrations ($C_{VD}$ between 0.26 and 0.28) did almost all transported particles occupy the en bloc sliding bed. For this regime the regression coefficient $M$ approaches the value given by the model equation (Eq. 3.5) for fully-stratified flow. The $V_{SM}$ value 1.6 m/s obtained for this regime by substituting the Eq. 3.13 to the Eq. 3.5 and taking coefficient $a = 0.96$ (see Tab. III.2C) is also in accordance with observations in the 150 mm pipeline. This result indicates that prediction of a fully-stratified flow using the Eq. 3.5 and thus using the two-layer model is appropriate.

Table III.2C. Gravel 3-5 mm ($d_{50} = 4.20$ mm) in a 150 mm pipeline ($d_{50}/D = 0.028$)

<table>
<thead>
<tr>
<th>$C_{VD}$</th>
<th>$M$</th>
<th>$a$</th>
<th>Correl. coeff.</th>
<th>$\mu_S$</th>
<th>$V_{50}$ Eq. 3.14</th>
<th>$V_{50}$ Eq. 3.11</th>
<th>Experim. run</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09 - 0.10</td>
<td>0.34</td>
<td>0.80</td>
<td>0.46</td>
<td>0.55/0.44</td>
<td>23.1/44.6</td>
<td>6.49</td>
<td>12/09/95</td>
</tr>
<tr>
<td>0.16-0.17</td>
<td>-0.02</td>
<td>0.49</td>
<td>0.07</td>
<td>0.55/0.44</td>
<td>-</td>
<td>-</td>
<td>13/09/95</td>
</tr>
<tr>
<td>0.21-0.23</td>
<td>-0.27</td>
<td>0.44</td>
<td>0.47</td>
<td>0.55/0.44</td>
<td>-</td>
<td>-</td>
<td>14/09/95</td>
</tr>
<tr>
<td>0.26-0.28</td>
<td>0.27</td>
<td>0.96</td>
<td>0.82</td>
<td>0.55/0.44</td>
<td>102.5/234.3</td>
<td>6.49</td>
<td>15/09/95-18/09/95</td>
</tr>
</tbody>
</table>
3.3 Macroscopic modelling

3.3.1 Flow stratification

When solids such as sand or gravel are transported in a slurry pipeline some degree of slurry flow stratification usually occurs. This is the effect of the tendency of solid particles in the carrying liquid to settle. Stratified slurry flow forms a particle-rich zone and a particle-lean zone in the pipeline cross section. According to the shape of its concentration profile, the slurry flow may be considered fully-stratified or partially-stratified.

3.3.2 Principles of two-layer model

The macroscopic two-layer model takes into account the slurry flow stratification and transforms a real concentration profile in a pipeline cross section into a simplified two-layer pattern. When the slurry flow is fully stratified solid particles transported in the carrying liquid are all accumulated in a granular bed sliding at the bottom of the pipeline (Fig. 3.5). All particles in this lower layer are in mutual contact. The volumetric concentration of solids in the lower layer of the fully-stratified flow approaches the concentration value of a loose-poured bed. The stream of the carrying liquid above the granular bed is particle-free. The position of an interface between two layers is determined by the angle $\beta$.

![Figure 3.5. Definition sketch for two-layer model of stratified flow, after Wilson et al. (1992).](image)

In a partially-stratified flow a considerable fraction of the total transported solids mass is suspended in the carrier stream. Suspended particles are assumed not to be in contact with other particles and the flow boundaries. Velocity distribution - as well as the concentration distribution - is idealised as being uniform within both the upper and the lower layers (Fig. 3.6). The distribution of the suspended particles within an idealised two-layer pattern has been subjected to investigation. Early versions of the model anticipated a suspension only in the upper layer. For an idealised flow pattern
the recent modification of the model by Gillies et al. (1991) assumes a uniform distribution of suspended particles along the entire pipeline cross section.

![Schematic cross section for two-layer model](image)

**Figure 3.6. Schematic cross section for two-layer model.**

The model is based on the assumption that there are two physical mechanisms for solid particle support in a pipeline - interparticle contact and particle suspension in a carrying liquid. Thus solids are transported as both suspended and contact loads. According to Bagnold (1956), the suspended particles transfer their submerged weight directly to the carrier, while the submerged weight of the non-suspended particles is transferred via interparticle contacts to the pipeline wall.

According to the model the behaviour of the flow is governed by the principle of force balance between driving and resisting forces in the flow in two layers. The driving force in the flow in a pressurised horizontal pipeline is produced by the pressure gradient over a pipeline length section. The resisting force is represented by shear stress exerted by flowing matter at a flow boundary. The same formulation of the force balance between the driving and resisting forces, combined with a friction coefficient equation, gives the Darcy-Weisbach equation (Eq. 2.24) for friction losses in a water pipeline. The Darcy-Weisbach equation is obtained from a two-layer model for the limiting case in which the particle-free upper layer occupies the whole pipeline cross section.

### 3.3.3 Mathematical description of flow by two-layer model

The model is composed of a set of equations expressing the conservation of mass and momentum in a mixture flow in both layers in the pipe section. A set of conservation equations is computed by iteration. The layer occupying a differential pipe length $dx$ is considered to be a control volume. Flow in the control volume is steady and uniform. The quantities describing the properties of the layer are given by values averaged in the control volume. This can be seen in Fig. 3.6 where $V_1$ and $V_2$ denote the mean velocity of mixture in the upper (lower respectively) layer. The same is valid for mean volumetric concentrations $C_1$ and $C_2$ in the layers. Slip between solid phase and liquid phase is considered negligible within both the suspension flow and the flow of contact.
particles. The model-equation parameters defining the geometry of the schematic cross section for a two-layer model are described in Fig. 3.7.

![Fully-stratified flow and Heterogeneous flow](image)

**Figure 3.7.** Geometry of schematic cross-section for two-layer model.

**Mass balance for flow in two layers**

The application of the mass conservation law to a two-layer pattern gives the mass balances

for slurry flow rate: \[ Q_m = Q_{m1} + Q_{m2} = \text{const.} \]

\[ V_{m1}A = V_1A_1 + V_2A_2 \]  \hspace{1cm} (3.15),

for solids flow rate: \[ Q_s = Q_{s1} + Q_{s2} = \text{const.} \]

\[ A_sV_s = C_{vi}AV_s = C_1A_1V_1 + C_2A_2V_2 \]  \hspace{1cm} (3.16),

and for liquid flow rate: \[ Q_f = Q_{f1} + Q_{f2} = \text{const.} \]

\[ A_fV_f = (1-C_{vi})AV_f = (1-C_1)A_1V_1 + (1-C_2)A_2V_2 \]  \hspace{1cm} (3.17).

The solids volume balance is written as

\[ C_{vi}A = C_1A_1 + C_2A_2 \]  \hspace{1cm} (3.18).

Since \( C_{vd} = Q_s / Q_m \) the Eq. 3.16 for solids flow rate can be written as

\[ C_{vd}AV_m = C_1A_1V_1 + C_2A_2V_2 \]  \hspace{1cm} (3.19).
Momentum balance for flow in two layers

A law of conservation of momentum is formulated as force balance between driving and resisting forces acting on the flow boundaries of each layer in a horizontal pipeline.

The force balance for the upper layer is written as

$$-\frac{dP}{dx}A_1 = \tau_1 O_1 + \tau_{12} O_{12}$$  (3.20)

and for the lower layer as

$$-\frac{dP}{dx}A_2 = (\tau_{2f} + \tau_{2s})O_2 - \tau_{12} O_{12}$$  (3.21).

A summation of these two equations gives a force balance in the whole pipeline section

$$-\frac{dP}{dx}A = \tau_1 O_1 + (\tau_{2f} + \tau_{2s})O_2$$  (3.22).

Resistance forces against flow, expressed in the right sides of force balance equations 3.20 - 3.22, are due to viscous or mechanical friction at flow boundaries.

3.3.4 Friction mechanisms by two-layer model

Mechanical friction between solids and pipeline wall

Solid particles in contact with each other and with the pipeline wall transmit their submerged weight to the pipeline wall. This is the source of the resisting force exerted by the contact load solids against the flow driving forces. The force is due to the solids stress acting at the pipeline wall.

The boundary frictional resistance depends on the radial interparticle stress, $\sigma_s$. In a horizontal pipeline the stress $\sigma_s$ between the solids grains and the pipeline wall acts in a radial direction in the pipeline cross section, so that it is normal to the pipeline wall (see Fig. 3.8). Considering that the accurate determination of $\sigma_s$ in the stress analysis may not be attainable, Wilson (1970) and Wilson et al. (1972) made the simple assumption for the $\sigma_s$ determination. A change in the normal intergranular stress $\sigma_s$ with a change in the vertical position in a pipeline cross section, $y$, was determined in a granular bed submerged in liquid by the "hydrostatic-type distribution of the normal pressure"

$$-\frac{d\sigma_s}{dy} = g(\rho_s - \rho_f )C_{vb}$$  (3.23).
The bed was assumed to be occupied exclusively by the particles in continuous contact and having a spatial solids concentration equal to the loose-poured bed concentration $C_{vb}$. This concept suggested that the solids pressure against the wall was approximately the same as the solids pressure on the horizontal plane at the same level in a bed of indefinitely large breath. This might be acceptable if the contact layer is shallow compared to its width. In this case the interparticle stress increases linearly with depth, as shown in Shook & Roco (1991). Shook & Roco submitted a theoretical explanation of the method used to determine the solids stress according to Eq. 3.23 and generalised it for lower layers of the partially-stratified flow.

Integration of the Eq. 3.23 using a cylindrical coordinate $y = 0.5D(1-\cos\alpha)$ gave the following equation for a granular layer delimited by $\alpha$ and $\beta$

$$\sigma_s = g(\rho_s - \rho_f)C_{vb} \frac{D}{2} (\cos\alpha - \cos\beta)$$  \hspace{1cm} (3.24).

$\sigma_s$ is the intergranular stress normal to the pipeline wall at a wall position defined by an angle $\alpha$ below a top of a granular bed determined by $\beta$.

![Figure 3.8. Intergranular stress at wall of horizontal pipeline.](image)

The normal stress $\sigma_s$ produces the (Coulombic) intergranular shear stress at the pipeline wall $\tau_s = \mu_s \sigma_s$. In this relationship $\mu_s$ is the coefficient of mechanical friction between solid particles and the pipeline-wall material. The total resistance force exerted by the sliding granular bed is

$$\mu_s F_N = \tau_2 \sigma_s \text{ for } O_2 = D\beta \text{ and } \sigma_s \text{mean} = \frac{1}{\beta} \int_0^\beta \sigma_s d\alpha.$$
The total normal force, $F_N$, exerted by the normal intergranular stress against the pipeline wall is obtained by integrating the normal stress over the pipeline perimeter $O_2$ given by $D$ and $\beta$. Thus $F_N = D \int_0^\beta \sigma_s d\alpha$. The result of integrating is

$$F_N = g(\rho_s - \rho_f)C_v b \frac{D^2}{2} (\sin \beta - \beta \cos \beta)$$

(3.25).

The force $F_N$ differs from $F_W$, which is the submerged weight of the granular bed. The force $F_W$, which represents the gravitational effect on a granular body, is integrated from the intergranular stress component $\sigma_W$. Only this component can act to support the bed weight. At each local pipeline-wall position, given by angle $\alpha$, the stress $\sigma_W = \sigma_s \cos \alpha$. By integrating over the perimeter $O_2$ of the interface between a bed and a pipeline wall

$$F_W = g(\rho_s - \rho_f)C_v b \frac{D^2}{4} (\beta - \sin \beta \cos \beta)$$

(3.26)

where $\frac{D^2}{4} (\beta - \sin \beta \cos \beta) = A_2$ and therefore

$$F_W = g(\rho_s - \rho_f)C_v b A_2$$

(3.27).

A validity of a described determination of $F_N$ and $F_W$ was confirmed by measurements on a dense-phase flow in which solid particles formed a particulate plug occupying the whole area of the pipeline cross section (Wilson et al., 1972).

For a dense-phase flow $F_N = 2F_W$. Force balance at an initial motion of a plug flow is written as

$$- \frac{dP}{dx} A = \mu_s 2F_W$$

that is

$$- \frac{dP}{dx} A = 2\mu_s g(\rho_s - \rho_f)C_v b A$$

Hydraulic gradient required to initiate a plug sliding is then

$$I_{pg} = - \frac{dP}{dx} \frac{1}{\rho fg} = 2\mu_s (S_s - 1)C_v b$$

(3.28).

The shear stress, $\tau_{2S}$, due to mechanical friction between granular bed forming a contact layer and pipeline wall is velocity-independent. It is determined from $\sigma_s$, the normal intergranular stress at the pipeline wall. A resisting force due to mechanical friction between a contact layer and a pipeline wall is perpendicular to the normal
intergranular force $F_N$ exerted against the pipeline wall and it is related with $F_N$ by $\mu_s F_N$. Considering the simplifications in the determination of $\sigma_s$, it is always advisable to determine the $\mu_s F_N$ experimentally in a tilting tube also. The value of $\mu_s F_N$ is equal to the value of $F_N \tan \omega$ if a tilting tube reaches an inclination angle $\omega$ at which a bed of solids fraction $C_\beta$ and thickness given by $\beta$ starts to move. Tilting-tube experiments are discussed in Chapter 4.

Viscous friction at flow boundaries

Viscous friction between the flowing carrying liquid and the flow boundary is a velocity-dependent process described by the boundary shear stress. Shear stress is related to the velocity gradient between the flowing carrier and the flow boundary by a friction coefficient expressing flow conditions at the boundary. The conditions are given by the flow regime and the boundary roughness. The Darcy-Weisbach friction coefficient is defined (Eq. 2.17) as

$$\lambda = \frac{8 \tau_0}{\rho V^2}.$$ 

The friction coefficient is related to the Reynolds number of the flow and/or the boundary roughness. Thus the determination of the viscous shear stresses in the two-layer model requires the values of
- the friction coefficient
- the velocity difference at the boundary ($V_1$ at the pipeline wall in contact with the upper layer, $V_2$ at the pipeline wall in contact with the lower layer, $V_1 = V_1 - V_2$ at the interface between layers)
- the density of the carrier.

The friction coefficient for water flow in a pipeline is obtained from the Moody diagram or its computational version (Churchill, 1977). The Reynolds number characterising the flow in the layer is calculated from the hydraulic diameter, $D_h$, of the layer ($D_h = 4A_1/O_1$, $D_{h2} = 4A_2/O_2$) by

$$Re = \frac{VD_h \rho_f}{\mu_f}$$

(3.29).

Friction coefficients
- $\lambda_1$ for flow in the upper layer over the pipeline wall of perimeter $O_1$,
- $\lambda_2$ for flow in the lower layer over the pipeline wall of perimeter $O_2$ and
- $\lambda_{12}$ for flow in the upper layer over the interface between two layers (the perimeter $O_{12}$) has to be determined in the two-layer model.

Suspended particles in flow in the upper layer might change the density of the carrying liquid but do not have a direct influence on the flow structure near a flow boundary. Thus it is appropriate to use the Moody diagram to determine the friction coefficient
\( \lambda_1 \) at a pipeline wall of perimeter \( O_1 \). The viscous friction at the flow boundary \( O_2 \) is generated by the carrier, occurring among contact-load grains flowing over the pipeline wall. Velocity of grains and of the interstitial carrier within the sliding bed is assumed to be identical and is \( V_2 \).

Wilson & Brown (1982) assumed that there was an effect on the viscous friction law caused by the presence of sliding-bed particles near flow boundary, and proposed a method for the determination of \( \lambda_2 \) for the interstitial liquid flow over the pipeline wall. From experiments with plug flow (dense-phase slurry flow) in a 26 mm pipe they learned that the friction coefficient for the velocity-dependent component of total friction loss was significantly larger than \( \lambda_f \) for flow of the carrying liquid alone. Their analysis of viscous friction in the annulus between the granular plug and the pipe wall was based on the assumption that a plug core occupied by particles in continuous contact moved without being sheared, i.e. en bloc, in the pipe. The thickness of the annulus between the plug core and the pipe wall was assumed to be proportional to the particle diameter \( d \). An analysis of the liquid velocity distribution within the annulus transformed the theoretical velocity profile into the relationship between a friction coefficient \( \lambda_2 \) and a dimensionless number \( \text{Re}_d \). The dimensionless \( \text{Re}_d = \frac{V_p d}{\nu_f} \), where \( V_p \) was velocity of the particulate core and \( \nu_f \) the kinematic viscosity of liquid. Analytical results gave two regimes for liquid flow friction in the annulus: laminar regime for \( \text{Re}_d < 335 \):

\[
\lambda_2 = \frac{22}{\text{Re}_d} \tag{3.30}
\]

and turbulent regime for \( \text{Re}_d \geq 335 \):

\[
\lambda_2 = 0.033 \left( 1 + \frac{138}{\text{Re}_d} \right)^2 \tag{3.31}
\]

As an alternative to the method of Wilson & Brown, the friction coefficient \( \lambda_2 \) may be determined by using the Moody diagram for the Reynolds number \( \text{Re}(V_2, D_{h2}, \nu_f) \) and pipe roughness \( k \).

The friction coefficient \( \lambda_{12} \) is determined for the interface between two layers in the stratified flow. When the interface is represented by a clearly identifiable flat surface of a contact bed it can be considered to have a roughness proportional to the diameter of the particles occupying the bed surface. The interfacial friction law is given by a formula for turbulent liquid flow over a fully-rough boundary, \( \lambda_{12} = \text{fn}(d/D) \). Several interfacial friction equations of this type have been submitted (Wilson, 1976; Televantos et al., 1979; Eyler & Lombardo, 1982; Noda et al., 1984; Shook et al., 1986, Gillies et al., 1991).

Wilson (1976) proposed using Nikuradse's friction equation for a rough boundary
\[
\lambda_f = \frac{1}{\left[2\log\left(\frac{D}{k}\right) + 1.138\right]^2} 
\]  
(3.32)

to determine \( \lambda_{12} \) by replacing the effective roughness \( k \) (k is originally the "equivalent sand roughness" by Nikuradse) in Eq. 3.32 with \( d_{50} \) and by multiplying the resulting \( \lambda \) by an empirical coefficient equal to 2.

Barnhart et al. (1982) proposed substitution of the effective roughness \( k \) by \( 1.72d_5 \) in Nikuradse friction equation to get \( \lambda_{12} \) and Eyler & Lombardo (1982) tested the same substitution in the Colebrook-White equation.

Gillies et al. (1991) proposed

\[
\sqrt{\frac{8}{\lambda_{12}}} = \frac{4\log\left(\frac{D}{d_{12}}\right) + 3.36}{\sqrt{0.5 + X}} 
\]  
(3.33)

in which

\[ X = 5 + 1.86\log\left(\frac{d_{12}}{D}\right) \text{ for } d_{12}/D > 0.002 \text{ and } X = 0 \text{ otherwise.} \]

In the Eq. 3.33, \( d_{12} \) is the diameter of particle at the interface. This is determined by assuming that all particles larger than the particle of the \( d_{12} \) size are below the interface.

**Effect of shear layer**

The interfacial friction law formulated for an interface considered as a hydraulically-rough boundary requires a flat and sharp interface, a condition more likely to be fulfilled in pipeline flow containing very coarse particles. Even at a sharp interface a thin transition zone may occur in which a few layers of particle-size thickness are sheared and individual particles roll and jump at the bed surface. This behaviour at the stationary bed surface under a shear action is described in Yalin (1972). An interface exhibiting this kind of behaviour may still be considered to have a roughness size represented by the particle diameter. According to Yalin (1992) an interfacial roughness, \( k \), is characterised by the particle size \( (k = 2d) \) if the dimensionless shear stress at the interface, \( \theta \) defined by Eq. 3.36, is lower than or equal to unity. For higher interfacial \( \theta \) the top of a contact bed is sheared off and a sharp interface is replaced by a transition zone, called shear layer, with concentration and velocity gradient. Thus an interface becomes virtual rather than real. For the virtual interface the particle-size roughness is no longer a parameter determining interfacial friction. An analysis of a flow at high shear stress above a stationary granular bed revealed (Wilson, 1984-95 with a contribution of Nnadi, 1990-95) that the thickness of the shear layer is a crucial parameter determining the interfacial friction. The boundary roughness is represented by the relationship \( k/d = f_\text{in}(\theta) \).
The general friction law for a flow with a rough boundary can be applied at the virtual interface in the shape (Nnadi & Wilson, 1995)

\[
\frac{V_1}{u_{*12}} = \frac{1}{\kappa} \ln \left( \frac{R_{hb}}{k} \right) + B_1 \quad \text{i.e.} \quad \sqrt{\frac{8}{\lambda_{12}}} = \frac{1}{\kappa} \ln \left( \frac{R_{hb}}{k} \right) + B_1 \quad (3.34)
\]

for von Kármán constant \( \kappa = 0.4 \) and constant \( B_1 = 6.0 \). In the Eq. 3.34, \( R_{hb} \) is the hydraulic radius of area associated with the surface of the bed (for its determination see Wilson, 1966), \( u_{*12} \) is the shear velocity at the interface and \( V_1 \) is the mean velocity of the flowing layer above an interface relative to the velocity of particles at the interface. \( V_1 \) is interpreted as \( V_1 - V_2 \) by the two-layer model.

The velocity distribution within the shear layer must be analysed to solve the interfacial friction law equation. This is written as \( v/u_{*12} = f(y/Y_{sh}) \), where \( Y_{sh} \) is the thickness of the shear layer, \( y \) the local height within the shear layer measured from the bottom of the shear layer and \( v \) the local velocity at the height \( y \). Wilson (1984) determined that local velocity has a specific value at the top of the shear layer \( (v = 8.2u_{*12} \text{ for } y = Y_{sh}) \) and estimated the vertical position at which the local velocity equals the mean velocity \( V_1 \) above the virtual interface as \( y = D_{h1}/8 \) when \( D_{h1} \) was the hydraulic diameter of the upper zone above a virtual interface. Integrating a velocity profile in the whole discharging cross section led to an expression for the mobile-bed friction equation. Comparison of this equation with Nikuradse friction equation for a hydraulically-rough boundary indicated that the effective roughness size of a mobile bed should be approximately half of the shear-layer thickness \( (k = 0.56Y_{sh}) \). This analysis showed that the interfacial friction did not obey either a smooth-boundary or a rough-boundary friction law. Friction at the interface was related to the thickness of the shear layer considered as roughness size.

Wilson (1989) determined the relative roughness size for friction law as a function of a particle mobility number (called also Shields parameter), \( \theta \), for the bed. A combination of the modified Du Boys equation (Eq. 2.37) for the shear layer thickness

\[
Y_{sh} = \frac{\tau_{12}}{g(\rho_s - \rho_f)C_{sh}\tan\phi'} \quad (3.35)
\]

in which \( \tau_{12} \) was the interfacial shear stress and \( C_{sh} \) was the mean solids concentration in the shear layer, and the equation for the Shields parameter \( \theta \)

\[
\theta = \frac{\tau_{12}}{g(\rho_s - \rho_f)d} \quad (3.36)
\]

gave \( \frac{Y_{sh}}{d} = \frac{\theta}{C_{sh}\tan\phi'} \approx 10\theta \). By substituting \( k = 0.56Y_{sh} \) in this equation the relationship for the interfacial roughness was obtained as \( \frac{k}{d} = 50 \).
At this stage of the investigation, the variation of shear stress within the shear layer was neglected. Further improvement was attained by the incorporation of the $\tau$ variation across the shear layer. Wilson & Nnadi (1990) determined the variation by using the parameter $Y_{sh}/R_{hb}$. Substitution of $\tau_{12} = \rho_g R_{hb} I_m$ in the equation for the shear layer thickness (Eq. 3.35) gave:

$$\frac{Y_{sh}}{R_{hb}} = \frac{I_m}{(S_S - 1) C_{sh} \tan \phi}.$$  

When a combination of $\frac{k}{d} = 50$ with the $Y_{sh}/R_{hb}$ equation was implemented in the general friction law for a rough boundary (Eq. 3.34), the friction equation obtained was:

$$\sqrt{\frac{8}{\lambda_{12}}} = 2.5 \ln \left[ B_2 \left( \frac{S_S - 1}{I_m} \right) \right]$$  \hspace{1cm} (3.37)

showing that the friction coefficient characterising flow in a shear layer is dependent on $S_S$ and $I_m$ and not directly on parameters of the boundary. Coefficient $B_2 = 2.2$ for $C_{sh} \tan \phi = 1/10$ was proposed by Wilson (1995) and $B_2 = 2.9$ for $C_{sh} \tan \phi = 1/7.5$ by Nnadi & Wilson (1995).

The relation was also approximated by the power-law function and the friction coefficient determined (Wilson & Pugh, 1995) as:

$$\lambda_{12} = 0.87 \left( \frac{I_m}{S_S - 1} \right)^{0.78} \quad \text{for } I_m > 0.0167(S_S - 1)$$  \hspace{1cm} (3.38)

The equation $\lambda_{12} = \text{fn}(I_m, S_S)$ can be implemented in the two-layer model. Since $I_m$ is not an input to the predictive two-layer model, its computation using the $\lambda_{12}$ for the shear layer requires an additional iteration cycle.

Comparison of the interfacial friction laws for a sharp flat interface, $\lambda_{12} = \text{fn}(d/D)$, and for a transition shear layer, $\lambda_{12} = \text{fn}[I_m(S_S - 1)]$, reveals that $\lambda_{12} = \text{fn}[I_m(S_S - 1)]$ tends to give higher values than the law for a fully-rough boundary in flow conditions typical for slurry pipelines either in laboratory or in field. The values are of the same magnitude only if $I_m$ is low, e.g. in flows of the lowest solids concentrations or for large pipeline sizes.

### 3.3.5 Computational testing of the two-layer model

In the literature, predictive charts were proposed as a graphical presentation of the two-layer model for fully-stratified flow. The aim was to make it possible for the practising engineers to predict the frictional head loss and the deposition-limit velocity in a pipeline by using the two-layer model, without the need to compute a set of model balance equations. The users should be aware, however, that the chart predictions match the computational model outputs only for certain values of the parameters $\mu_S$ and $C_2$ (considered as $C_{vb}$ in the charts). These parameters are usually inputs to the computational model but the nomographs are based on model outputs for certain
values of these parameters. In practice, the values of the mechanical-friction coefficient $\mu_s$ and the sliding bed concentration $C_2$ may vary significantly from the constant values used in the charts and so the computational model results may not agree with the chart predictions. The model outputs are also sensitive to the model configuration. Different outputs from the computational model due to the use of different friction equations are of more academic than practical interest but some friction equations may produce a difference in outputs which is not negligible in a flow prediction.

The predictive charts are described below and several comments are made on the application of both the computational model and the charts. These comments are based on a computational reconstruction of the charts, using the model in its original mathematical shape, for different model configurations and values of input parameters.

**Prediction of frictional head loss**

A nomograph on Fig. 3.9 (Wilson et al., 1992) provides the values of $I_M$ in the fully stratified flow for various combinations of input $d$, $D$, $S_s$, $C_{vd}$ and $V_m$. The nomograph is composed of a locus curve, determining the boundary of the stationary deposit zone, and of a set of fit-function curves relating the relative excess pressure gradient $(I_M - I_p)/I_{pg}$ with the relative velocity $V_r = V_m/V_{sm}$ for different relative concentrations $C_r = C_{vd}/C_{vb}$.

![Figure 3.9. Curves of relative excess pressure gradient, from Wilson et al. (1992).](image)

Computational reconstruction of the nomograph indicates that values of relative excess pressure gradient are not solely dependent on relative concentration and relative velocity, they are also sensitive to input $D$, $d$, $\mu_s$ and $C_2$ ($C_{vb}$).

The shape of the nomograph constructed from the computational model outputs further differs with a chosen model configuration. Van Riet et al. (1995) showed that using the friction coefficient $\lambda_7$ according to Wilson & Brown (1982) was not appropriate for cases other than the plug flow investigated by those authors. For cases
Figure 3.10. Influence of chosen friction coefficient Lambda2 on two-layer model outputs.
other than plug flow, the model computation indicated that the curve of the relative excess pressure gradient \( \left( \frac{I_{1m} - I_p}{I_{pg}} \right) \) for the constant relative delivered concentration \( C_r = \frac{C_{Vd}}{C_{Vb}} \) in a pipeline did not converge to a horizontal asymptote when \( V_m \) increased. Instead, the model curve reached a minimum and then increased. The model curve had a horizontal asymptote when the friction coefficient \( \lambda_2 \) was determined by using the Moody diagram for liquid flow alone.

The curve divergence is explained by using the computation model outputs plotted in Fig. 3.10. The Wilson & Brown equation (Eq. 3.31) predicts a considerably higher value for \( \lambda_2 \) than those given by the Moody diagram [see Friction coefficient (\( \lambda_2 \)) versus \( R_{eq} \) on Fig. 3.10]. Higher \( \lambda_2 \) means that there is higher viscous shear stress and so higher resistance at the boundary between the bed and the pipeline wall. Consequently the lower bed velocity \( V_2 \) is produced by the balance between driving and resisting forces acting on the sliding bed (see \( V_2/V_{sm} \) versus \( V_m/V_{sm} \) on Fig. 3.10) when the viscous friction obeys the Wilson & Brown friction law rather than the law for water flow alone. Thus a thicker bed (represented by a position of the interface \( \beta \) in Fig. 3.10) is necessary to maintain the required delivered concentration of solids in the slurry flow at a certain \( V_m/V_{sm} \). The difference between the bed thickness obtained from the force balance calculated with \( \lambda_2 \) by Wilson & Brown and \( \lambda_2 \) by Moody increases at higher mean slurry velocity [see Position of interface (\( \beta \)) versus \( V_m/V_{sm} \) on Fig. 3.10]. The thicker bed produces a higher total slurry flow resistance and so a larger relative excess pressure gradient than the thinner bed derived from the force balance with Moody’s \( \lambda_2 \). For flows other than plug flows, the overestimation of the viscous shear stress by the Wilson & Brown method becomes more evident with increasing slurry velocity. For typical slurry flows, the friction coefficient \( \lambda_2 \) according to Eq. 3.31 tends to overestimate liquid friction and should not be used in the two-layer model. In these cases it is recommended that the friction coefficient \( \lambda_2 \) should be evaluated by using the Moody diagram.

**Prediction of deposition-limit velocity**

The computation of the force balance at incipient motion of the bed in fully-stratified flow gives a locus curve - the curve relating the deposition-limit velocity, \( V_{dl} \), with the position of an interface between the layers in the pipeline cross section. The locus curve has a maximum (see Fig. 3.10) which determines the maximum velocity at the limit of stationary deposition \( V_{sm} \). Wilson (1979) processed \( V_{sm} \) values obtained as the model outputs for a variety of values of input parameters \( d_{50}, D, S_s \) to the nomographic chart (Fig. 3.11), sometimes called the demi-McDonald.

The demi-McDonald curve has a turbulent branch (for small particles of diameter less than approximately 0.5 mm) and a fully-stratified branch (for particles larger than approximately 0.5 mm). The threshold particle size delimits two branches at the peak of the nomographic curve. The fully-stratified branch of the demi-McDonald curve was constructed from outputs of the model for fully-stratified flow. According to this part of the demi-McDonald curve, \( V_{sm} \) decreases with increasing particle size in a pipeline of a certain \( D \). Thus a lower \( V_m \) is required to initiate motion in a coarser bed.
A computational determination of $V_{sm}$ is obtained by solving the force balance (Eqs. 3.20 - 3.22) in flow at an initial bed motion. It is written as

\[
\frac{\tau_{1}O_{1}}{A_{1}} + \frac{\tau_{12}O_{12}}{A_{1}} = -\frac{\tau_{12}O_{12}}{A_{2}} + \frac{h_{s}F_{N}}{A_{2}}
\]

(3.39).

Since $V_{2} = 0$ and $V_{1} = V_{d}l/A/A_{1}$, the deposition-limit velocity $V_{dl} = fn(\beta, D, \mu_s, C_{vb}, S_s, \lambda_{12}, \lambda_{1})$ according to Eq. 3.39. A computational generation of the locus curves by solving Eq. 3.39 for fully-stratified flow shows that the locus curve has its maximum at almost identical position $\beta$, even for very different values of the input parameters. Thus the maximum velocity at the limit of stationary deposit, $V_{sm}$, is not sensitive to $\beta$; and $V_{sm} = fn(D, S_s, h_s, C_{vb}, \lambda_{12}, \lambda_{1})$. Considering $\lambda_{12} = fn(d_{50}/D)$ for fully-stratified flow and constant $\lambda_{1}$, the simplest relationship is reached, $V_{sm} = fn(d_{50}, D, S_s, \mu_s, C_{vb})$. Compared with the nomographic $V_{sm} = fn(d_{50}, D, S_s)$ this shows that the nomograph was constructed for specific values of $\mu_s$ and $C_{vb}$ (as was indicated by its authors). Thus the $V_{sm}$ values in the demi-McDonald chart correspond with force-balance outputs for certain combinations of values of these parameters. A sensitivity analysis shows that there is a good agreement between the nomographic and the computed $V_{sm}$ values for $\mu_s = 0.40$, $C_{vb} = 0.60$, $\lambda_{12} = 2.75\lambda_{w}$ $[\lambda_{w}$ according to Nikuradse (Eq. 3.32) for $k = d_{50}$] and $\lambda_{1} = \lambda_{w}$ ($\lambda_{w}$ for $k$ and Re from the Moody
Figure 3.12. Reconstruction of the chart for maximum velocity at limit of stationary deposition.
diagram) as demonstrated in Fig. 3.12 for a pipeline DN150. Values of deposition-limit velocities obtained in our laboratory pipeline DN150 for different sands are reasonably matched by computational outputs for the following combination of model inputs:

\[ \mu_S = 0.53, C_{vb} = 0.51 \text{ and } \lambda_{12} = 2.75 \lambda_w \text{ (Eq. 3.32 for } k = d_{50}) \] as can also be seen in Fig. 3.12. These values for \( \mu_S \) and \( C_{vb} \) were very similar to those obtained from our experiments (see Chapters 4 and 5).

\( V_{sm} \) in turbulent branch is affected by a variable thickness of bed at an incipient motion under different flow conditions. The bed thickness diminishes owing to a turbulent suspension process which picks up the particles from the bed surface and suspend them in flow above the bed. The turbulent branch of the demi-McDonald was constructed from the outputs of the two-layer model for partially-stratified flow by Wilson (1976). Later, the computational outputs were correlated (Wilson & Judge, 1980) by a simple equation suitable for practical applications

\[
\frac{V_{sm}}{\sqrt{2gD(S_s - 1)}} = 2.0 + 0.3 \log \left( \frac{d}{C_D D} \right)
\]  

(3.40)

for the range \( 10^{-5} < \frac{d}{C_D D} < 10^{-3} \). In the equation \( C_D \) is the drag coefficient for the particle and \( D \) is the pipeline diameter. The computational outputs correlated to get the Eq. 3.40 were obtained for \( \mu_S C_{vb} = 0.24 \) (\( \mu_S = 0.40, C_{vb} = 0.60 \)) and \( \lambda_f = 0.013 \) (see Wilson & Judge, 1980). The Eq. 3.40 is compared with the demi-McDonald curve on Fig. 3.12.

The entire demi-McDonald chart can be approximated (Wilson et al., 1992) by the equation (see Eq. 3.6 in this chapter)

\[
V_{sm} = 8.8 \left[ \frac{\mu_S (S_s - S_f)}{0.66} \right]^{0.55} D^{0.7} d_{50}^{1.75} d_{50}^2 + 0.11 D^{0.7}
\]

This approximation includes an effect of \( \mu_S \) on the \( V_{sm} \) value. Plotted results from Eq. 3.6 for \( \mu_S = 0.40 \) match the demi-McDonald curve (see Fig. 3.12).

Effect of shear layer on deposition-limit velocity

The incorporation of a shear-layer-flow equation for \( \lambda_{12} \) in the two-layer model has recently led to a modification of the demi-McDonald nomograph (Wilson, 1992). The interfacial friction coefficient \( \lambda_{12} \) for the interface replaced by the shear layer is independent of the particle size and so is \( V_{sm} \). Wilson (1992) proposed that the \( V_{sm} \) for fully-stratified flow with the shear layer (marked \( V_{sm, \max} \)) should be determined by an approximation.
Figure 3.13. Effect of shear layer on maximum velocity at limit of stationary deposition.
\[
\frac{V_{sm,\text{max}}}{\sqrt{2gD(S_S - 1)}} = \left( \frac{0.018}{\lambda_f} \right)^{0.13}
\] (3.41).

The \(V_{sm,\text{max}}\) by the approximation is used when its value is lower than the \(V_{sm}\) value from the nomograph.

The best accordance between the outputs from a computational model with \(\lambda_{12}\) by Eq. 3.38 and the outputs from the \(V_{sm,\text{max}}\) approximation (Eq. 3.41) is obtained for \(\mu_s = 0.33\) and \(C_{vb} = 0.60\). Computational outputs for \(\mu_s = 0.40\) and \(C_{vb} = 0.60\) also match those from Eq. 3.41 very well (see Fig. 3.13). A computation with the values \(\mu_s = 0.53\) and \(C_{vb} = 0.51\) (the combination for which the model outputs match the experimentally observed deposition-limit velocities) gave a \(V_{sm,\text{max}}\) that was not significantly lower than the nomographic \(V_{sm}\), even for the grain size (of about 0.5 mm) considered as a threshold between the turbulent and fully-stratified branches of a nomographic curve (see Fig. 3.13). The fact that a nomographic curve peak need not to be cut off by the Eq. 3.41 line is in accordance with the trend in \(V_{sm}\) development observed in a 150 mm pipeline. However, for larger pipeline diameters it may be necessary to consider Eq. 3.41.

More \(\lambda_{12}\) equations have been tested by Van Riet et al. (1996) for various values of input parameters and compared with the \(V_{sm}\) approximation (Eq. 3.41). The simulations have shown that all recently published friction laws for sheet flow (Nnadi & Wilson, 1995; Wilson, 1995; Wilson & Pugh, 1995) reasonably fit the \(V_{sm}\) correlation (Eq. 3.41) for \(\mu_s = 0.40\) and \(C_{vb} = 0.60\).

### 3.3.6 Two-layer model for partially-stratified flow

For fully-stratified flow, the two-layer model considers the upper layer as particle-free and the lower layer as occupied by particles, all of which are in continuous contact. In a partially-stratified flow the solids are transported in a carrying liquid both as a contact load and as a suspended load. The amount of solids occupying a slurry pipeline is given by the volumetric spatial concentration \(C_V\) which is the sum of a solids fraction in suspension, \(C_S\), and a solids fraction in contact, \(C_C\). A suitable method must be used to predict the amounts of suspended solids \(C_S\) or of solids in contact \(C_C\). Assuming a two-layer pattern according to Fig. 3.14, the \(C_C\) determines the concentration of solids in contact within the lower layer, \(C_{2C}\), by recalculating of \(C_C\) from the cross-sectional area of the entire pipeline, \(A\), to the cross-sectional area of the lower layer, \(A_2\), using \(C_{2C} = C_C A / A_2\).
Threshold between fully-stratified and partially-stratified flow

The boundary between two basic slurry flow patterns:
- the fully-stratified flow and
- the partially-stratified flow,

is given by the mean slurry velocity at the beginning of solid particle suspension in a mixture. Wilson (1972) and Wilson & Watt (1974) derived an equation for this threshold velocity from an analysis of the interaction between settling particles and turbulent carrier flow. The condition was formulated for the initiation of particle suspension: the length scale of turbulence had to be larger than the particle size. Only particles smaller than a certain portion of the mixing length characterising the liquid turbulent eddies could be supported by the eddies, otherwise the turbulent dispersive mechanism was not effective in suspending transported particles. The turbulent length scale was considered to be dependent on the local position within a pipeline flow and thus the average mixing length depended on the pipeline diameter. Adopting the statistical picture of turbulence, Wilson & Watt quantified the condition at which liquid turbulence becomes effective to suspend a solid particle by an exponential relationship between the d/D ratio and the ratio of the mean velocity in the pipeline and the terminal settling velocity of solid particle. The threshold mean slurry velocity at the beginning of the turbulent suspension, $V_{tt}$, was related to the terminal settling velocity of solid particle $v_t$, the particle diameter $d$, the pipeline diameter $D$ and the friction coefficient for liquid flow $\lambda_f$ by the equation (Wilson & Watt, 1974)

$$V_{tt} = 0.6v_t \frac{8}{\lambda_f} \exp \left[ 45 \frac{d}{D} \right]$$

(3.42)

which was calibrated by experimental data from the 25 mm and 50 mm laboratory pipelines. This correlation has been recently replaced by a correlation that is intended to include the effects observed for relatively fine slurries in large pipelines of GIW Hydraulic Laboratory. When compared with the exponential relationship (Eq. 3.42), a new hyperbolic-cosine function by Wilson (1992)
\[ V_{tt} = 0.8 \nu_t \left( \frac{8}{\lambda_f} \cosh \left( \frac{60 \cdot d}{D} \right) \right) \]  

(3.43)

provides higher \( V_{tt}/\nu_t \) values for small \( d/D \) ratios. For \( d/D \) values approaching zero (\( d/D < 0.003 \)) the \( V_{tt}/\nu_t \) ratio according to Eq. 3.43 becomes virtually independent of \( d/D \). For \( 0.012 < d/D < 0.025 \) are results by both correlations almost identical.

On the basis of the concept for the initiation of a turbulent suspension described above, Wilson et al. (1992) proposed a simple indicator of the flow pattern given by the ratio between particle size and pipeline size. The flow of slurry would be fully stratified for \( d/D > 0.018 \) and partially stratified for \( d/D < 0.015 \). In the region \( 0.015 < d/D < 0.018 \) both types of the slurry flow behaviour could occur.

**Solids division into two layers**

Wilson (1976) proposed a simple power-law function to express the fact that the contact-load fraction \( C_{cd} \) of transported solids (represented by volumetric delivered concentration \( C_{vd} \) of solids in the slurry) diminishes with increasing mean slurry velocity \( V_m \) in a pipeline

\[ \frac{C_{cd}}{C_{vd}} = \left( \frac{V_{tt}}{V_m} \right)^M \]  

(3.44).

The lowest velocity for the function application is \( V_m = V_{tt} \) (\( V_{tt} \) is the threshold velocity at the beginning of particle suspension determined from Eq. 6.1 or 6.2). The coefficient \( M \) was assumed to be equal 2.0. In later works it was specified that the coefficient was dependent on the particle size distribution with the maximal value \( M = 1.7 \) for monodisperse solids (Wilson et al., 1990, 1992).

Shook et al. (1986) introduced a tentative correlation for \( C_C/C_{vi} \) to predict the contact-load fraction for the two-layer pattern of the partially stratified flow as a function of chosen dimensionless groups

\[ \frac{C_C}{C_{vi}} = \exp \left[ -\alpha_1 \text{Ar}^{\alpha_2} \left( \frac{V_m^2}{g d_{50}} \right) \alpha_3 \left( \frac{d_{50}}{D} \right)^{\alpha_4} (S_s - 1)^{\alpha_5} \right] \]  

(3.45),

in which \( \text{Ar} \) is the Archimedes number for a settling particle

\[ \text{Ar} = C_D \cdot Re \cdot \frac{\mu_f^2}{\rho_f^2} = \frac{4 \cdot g d_{50}^2 (S_s - 1) \rho_f^2}{\mu_f^2} \]
The correlation was calibrated with the experimental data base collected in the laboratory of the Saskatchewan Resource Council (SRC), Canada. The experimental data from the DN53, DN263 and DN495 pipelines provided the coefficient values \( \alpha_1 = -0.124, \alpha_2 = -0.061, \alpha_3 = -0.028, \alpha_4 = 0.43, \alpha_5 = -0.27 \). The correlation was proposed for Archimedes number smaller than 3.10^5 and \( C_{V_i} \) lower than 38%.

Doron et al. (1987) assumed a theoretical concentration profile (obtained by the well-known Schmidt-Rouse diffusion model) in an upper suspended layer and linked it directly to a uniform concentration profile \( (c_V = C_{V_b}) \) in the lower contact layer.

Gillies et al. (1991) exploited the extended SRC data base containing data for fine and coarse particle slurries (clay-free fine sands and coarse sands of particle size 0.17 - 2.40 mm and coal 0.8 - 3.1 mm) from horizontal pipelines of different sizes (DN53, DN159, DN263, DN495) to establish the \( C_{c}/C_{V_i} \) equation. When the \( C_{c}/C_{V_i} \) values inferred from the experimentally determined frictional head losses were plotted in semilogarithmic \( C_{c}/C_{V_i} \) versus \( V_m/V_t \) coordinates (see Fig. 3.15) a correlation was found

\[
\frac{C_c}{C_{V_i}} = \exp \left( -0.0184 \frac{V_m}{V_t} \right)
\]

(3.46).

![Graph showing the relationship between \( \frac{C_c}{C_{V_i}} \) and \( \frac{V_m}{V_t} \).](image)

**Figure 3.15.** Effect of \( V_m/V_t \) on the stratification ratio \( C_{c}/C_{V_i} \) for slurry flow of different particle and pipeline sizes, from Gillies et al. (1991).

**Mechanical friction from contact-load particles**

Modification of the two-layer flow pattern for the partially-stratified flow (Shook & Roco, 1991; Gillies et al., 1991; see Fig. 3.14) required modification of the method used to determine the normal intergranular force against the pipeline wall \( F_N \). The
buoyancy effect associated with the presence of suspended coarse particles \((d > 0.074 \text{ mm})\) and fine particles \((d < 0.074 \text{ mm})\) in the lower layer was included to the equation for the normal solids stress at the pipeline wall, \(\sigma_s\), so that

\[
- \frac{d\sigma_s}{dy} = g(\rho_s - \rho_{2f}) C_{2c}
\]  

(3.47).

In the lower layer the suspended coarse particles, the fine particles smaller than 0.074 mm and the liquid form a mixture of density \(\rho_{2f}\) determined as

\[
\rho_{2f}(1-C_{2c}) = \rho_s C_1 + \rho_{\text{fines}}(1-C_{2c}-C_1),
\]

so that

\[
\rho_{2f} = \frac{\rho_{\text{fines}}(1-C_2)+\rho_s C_1}{1-C_2+C_1}
\]  

(3.48).

in which \(\rho_{\text{fines}}\) is density of a mixture composed of the liquid and fine particles smaller than 0.074 mm and \(C_2 = C_1 + C_{2c}\). The normal force \(F_N\) is integrated from Eq. 3.47 as

\[
F_N = g(\rho_s - \rho_{\text{fines}}) \frac{C_{2c}(1-C_2)}{(1-C_{2c})^2} \frac{D^2}{4} (\sin\beta - \beta \cos\beta)
\]  

(3.49).

3.4 Modelling of the pipeline inclination effects

The literature on the hydraulic transport of solids in inclined pipelines is not abundant and it is focused on the effects of the pipeline inclination on the hydraulic gradient and the deposition-limit velocity in slurry flow.

3.4.1 Frictional head loss in a vertical pipeline

Uniform distribution of solids across a pipeline cross section is characteristic of slurry flow in a vertical pipeline. The homogeneous character of slurry makes prediction of vertical flows easier than prediction of horizontal and inclined flows. Coulson et al. (1996) summarised their conclusions for the prediction of frictional pressure drop in a vertical slurry pipeline as follows:
- for non-settling suspensions the standard equation for a single phase fluid is used with the physical properties of the suspension in place of those of the liquid;
- for a suspension of coarse particles the value calculated for the carrying fluid alone, flowing at the mixture velocity, is used.

Thus according to Coulson et al. it is assumed that transported particles do not affect the friction process in coarse-particle slurry flow in a vertical pipeline. This assumption is justified in the model for fully-suspended flow by Clift et al. (see Eq. 3.7) if the coefficient \(A' = 0\). Considering that \(A' = 1\) in the Eq. 3.7 fulfils Coulson's condition for
"non-settling suspensions" if the suspensions are the "equivalent liquids". According to Clift et al., the "equivalent liquid" has the density of the slurry but other properties (as viscosity) remain the same as in the liquid alone.

3.4.2 Frictional head loss in an inclined pipeline

Worster & Denny (1955) suggested a simple equation for the energy loss in settling slurries flowing in inclined pipelines

\[ I_{\text{mho}} = I_f + (I_m - I_f)\cos\theta + C_{vd}(S_S - 1)\sin\theta \]  

(3.50)

where \( I_{\text{mho}} \) is the manometric gradient in slurry flow in the inclined pipeline, \( I_m \) is the hydraulic gradient in the same slurry flow through the pipeline installed to the horizontal position, \( I_f \) is the hydraulic gradient in horizontal liquid flow and \( \theta \) is a pipeline inclination angle. The angle \( \theta \) is considered to have positive values in an ascending pipeline and the negative values in a descending pipeline.

![Figure 3.16. Pressure drops in inclined pipelines, after Worster & Denny (1955).](image)

The manometric gradient is a rearranged manometric pressure differential over a pipe section, sensed by a differential pressure transmitter. The tapping points in the pipe are linked to the transmitter by hoses containing water. This arrangement causes the transmitter to give a total pressure differential over the pipe section, diminished by the pressure differential due to the water column in the hoses. The total head loss (resulting from the total pressure differential) is composed of the head loss due to friction \( I_{\text{mho}} \) and the head loss due to the potential energy change. The head loss due to the potential energy change registered by a differential pressure transmitter is represented by the last term in Eq. 3.50. This is the hydrostatic effect on the pressure
differential measured over a section of an inclined slurry pipe. The ratio between solids effect on the frictional head loss in the inclined pipeline and solids effect on the frictional head loss in the horizontal pipeline for the same slurry flow parameters is then given as

$$\frac{I_{m_0} - I_f}{I_m - I_f} = \cos \omega$$

(3.51).

Gibert (1960) adapted the Durand & Condolios correlation (Eq. 3.3) to inclined pipelines by using a simple assumption that only the gravitational acceleration component perpendicular to an inclined-pipeline axis (g.cos\(\omega\)) influences the solids effect on the frictional head loss

$$\frac{I_{m_0} - I_f}{C_{vd} I_f} = K \left( \frac{V_m^2 \sqrt{gd}}{ggD v_t \cos \omega} \right)^{-1.5}$$

(3.52).

This gives

$$\frac{I_{m_0} - I_f}{I_m - I_f} = (\cos \omega)^{1.5}$$

(3.53).

The correlation was found appropriate when applied to the Durand & Condolios experimental data for medium sand (\(d_{50} = 0.89\) mm) in a small pipe, which was the DN41 pipe with inclination angles 15, 30, 45 deg. Gibert noticed, however, that the correlation did not match the Durand & Condolios data from the inclined DN150 pipe, where friction loss in the descending pipe section was observed to be lower than in the ascending pipe section for the pipe inclination angle \(\omega\) bigger than 30 deg.

According to models of Worster & Denny and Gibert, the solids effect is always lower in inclined pipelines (for both the negative and positive slopes) than in horizontal pipelines. Furthermore, the friction loss is the same in pipe sections of the negative and the positive slope when the pipe inclination angle and flow parameters \(V_m, C_{vd}, d\) are identical.

Rai (1972) measured consistently larger values of \(I_{m0}\) for the flow of coarse-sand slurries in an inclined DN50 pipe than were predicted by Eq. 3.50. Other investigators also found discrepancies between their measurements and Eq. 3.50 and suggested their own correlations for the \(I_{m0}\) prediction. These empirical correlations, however, are produced from a rather limited number of data. A survey of the empirical methods has been submitted by Kao et al. (1980).

Static pressure difference caused by pipe elevation is considered by Worster & Denny to be produced by a slurry column of concentration \(C_{vd}\). Accepting the fact that all solids present in inclined pipe contribute to slurry column weight, the actual spatial concentration \(C_{v1}\) should determine the solids concentration in the slurry column.
Correct determination of $I_{mh\omega}$ from measured manometric pressure differential demands understanding of the difference between $C_{vd}$ and $C_{vi}$ in a measuring pipe section.

Kao and his co-workers (Kao & Hwang, 1979; Kao et al., 1980) investigated the variation of $C_{vi}$ with the pipe inclination in the ascending and the descending pipes and found large differences in this quantity at different pipe inclinations. $C_{vi}$ measured in the descending pipe was always lower than that measured in the ascending pipe when the DN50 pipeline loop was inclined gradually from 0 deg to 90 deg. $C_{vi}$ reached the maximum value at inclination angles of 20-30 deg in the ascending pipe and then gradually dropped to the $C_{vd}$ value in the vertical position of the pipe. $C_{vi}$ in the descending pipe decreased monotonically to the $C_{vd}$ value with the increasing inclination angle. The same trend was observed for friction losses. These were determined from a measured manometric pressure drop by subtraction of the static pressure drop due to the $C_{vd}$ slurry column. Friction losses were always higher in the ascending pipe than in the descending pipe and again the variation in friction losses with the pipe inclination angle was similar to the $C_{vi}$ variation.

Unfortunately the Kao et al. determination of $C_{vi}$ in the pipe cross section by measurements (a radioactive beam in a cross-pipe direction from the top to the bottom of a pipe) cannot be considered sufficiently accurate. Although the measured $C_{vi}$ values may not be accurate, the incorrect application of the radiometric density meter did not result in misinterpretation of the trends in a $C_{vi}$ variation with a pipe inclination that were detected during the experiments.

For the inclined pipelines, Wilson & Byberg (1987) modified a heterogeneous-flow model based on a stratification-ratio concept (Clift et al., 1982). $C_{vd}$ was again used to determine the elevation-change ratio part of the manometric gradient for heterogeneous flow in the measuring pipe section. It was suggested that manometric gradient $I_{mh\omega}$ could be determined by the equation

$$I_{mh\omega} = I_f + C_{vd}(S_s - 1)\left[\sin \omega + 0.7 \left(\frac{a}{V_{mix}}\right)^b (\cos \omega)^c\right] \quad (3.54)$$

in which coefficients $a$, $b$, $c$ were calibrated against experimental data. The data were obtained from the 52.5 mm pipe at 0, 10, 20, 30 and 40 deg inclinations. Four sorts of sand (0.17 mm, 0.32 mm, 0.55 mm, 1.1 mm) were tested. Coefficients $a$, $b$, $c$ varied with the particle size of the tested solids.

A revised version of the heterogeneous-flow model (Wilson et al., 1992) produced the following modification of the Worster & Denny formula

$$I_{mh\omega} = I_f + (I_m - I_f)(\cos \omega)(1 + M_f) + C_{vd}(S_s - 1)\sin \omega \quad (3.55)$$

giving
\[
\frac{I_{m0} - I_f}{I_m - I_f} = (\cos \theta)(1 + M) \gamma \tag{3.56}
\]

The power \( \gamma \) had a lower limit of 0.333 for very fine particles and, hypothetically, an upper limit of unity for very coarse particles. The power \( M \) was PSD-dependent and it gained a value 1.7 for a uniform PSD. Lower values of \( M \) are obtained for a well graded PSD according to Eq. 3.12.

The two-layer model for partially-stratified flow was configured for slurry flow conditions in inclined pipelines by Shook & Roco (1991). However, the computational outputs have not been verified by experiments. The configuration was

\[
- \frac{d(P + \rho_1 gh)}{dx} A_1 = \tau_1 O_1 + \tau_{12} O_{12} \tag{3.57}
\]

\[
- \frac{d(P + \rho_2 gh)}{dx} A_2 = \tau_2 f O_2 + \mu_s F_N \cos \theta - \tau_{12} O_{12} \tag{3.58}
\]

\[
- \frac{d(P + \rho_m gh)}{dx} A = \tau_1 O_1 + \tau_2 f O_2 + \mu_s F_N \cos \theta \tag{3.59}
\]

in which \( \rho_1 = \rho_f + C_1(\rho_s - \rho_f) \), \( \rho_2 = \rho_f + C_2(\rho_s - \rho_f) \), \( \rho_m = \rho_f + C_m(\rho_s - \rho_f) \) and \( F_N \) was determined according to Eq. 3.49. This two-layer model, called the SRC model, considers that the static pressure difference is due to spatial concentration in each layer and that the gravitational acceleration component perpendicular in an inclined-pipe cross section determines the interparticle normal force responsible for a mechanical friction between a sliding bed and a pipeline wall.

### 3.4.3 Deposition-limit velocity in an inclined pipeline

Hashimoto et al. (1980) observed the effect of pipe slope on the deposition-limit velocity in slurry flow of three sorts of crushed rock (0.77 mm, 1.33 mm, 2.18 mm) in a glass 55.8 mm pipeline loop. Deposition-limit velocity decreased considerably in the descending pipe when the pipe was inclined from 0 deg to -10 and to -20 deg. In the ascending pipe an increase in the deposition-limit velocity was detected with a pipe slope increasing from 0 deg to 20 deg.

The same trend in the relation between the deposition-limit velocity and the pipe inclination angle was observed during experiments carried out at Queen's University, Canada (Wilson & Tse, 1984) with four coarse solids (particle sizes between 1.1 mm and 5.8 mm) in the aluminium 76 mm pipeline loop, inclinable up to 40 deg from a horizontal position. Measured deposition-limit velocity reached its maximum at an angle of about 30 deg in an ascending pipe section.
This effect was also measured by Roco (in Shook & Roco, 1991) for the 0.36 mm sand and the 0.078 mm fly ash in the DN100 pipeline. His measurements, however, gave a lower inclination angle at which the maximum value of deposition-limit velocity was reached (about 15-20 deg).

Wilson & Tse detected a decrease in the deposition-limit velocity with increasing particle size in pipes. This trend was in accordance with the $V_{sm}$ prediction using the demi-McDonald nomograph (Fig. 3.11). The same effect was also indicated by the measurements of Hashimoto et al. (1980). To extend the application of maximum deposition-limit velocity $V_{sm}$ by demi-McDonald to inclined pipelines Wilson & Tse correlated $V_{sm}$ in a horizontal pipeline and an inclined pipeline, using the dimensionless velocity called Durand deposition parameter $\Delta D = V_{dl}/[2g(S_S - 1)D]^{0.5}$, in which $V_{dl}$ is the deposition-limit velocity and $D$ the pipeline diameter. The deposition-limit velocity in an inclined pipeline $V_{sm\omega}$ was given as

$$V_{sm\omega} = V_{sm} + \Delta D \sqrt{2gD(S_S - 1)}$$  \hspace{1cm} (3.60)

and the relationship between the deposition parameter $\Delta D$ and the pipeline inclination angle $\omega$ was presented in a graph (Fig. 3.17).

![Figure 3.17. Effect of angle of pipe inclination on Durand deposition parameter. after Wilson & Tse (1984).](figure)

3.5 Conclusions

Comparison of empirical models with experimental data for three sorts of sand and one gravel in a 150 mm pipeline shows that:
A. The empirical model of Durand et al. cannot be used throughout the entire range of slurry flow conditions. It might be appropriate to predict the heterogeneous flow but it fails in flows which are very strongly stratified or very weakly stratified. Furthermore, the model based on correlating the flow parameters in a wide range of flow conditions tends to mask the existence of phenomena which might occur for certain flow conditions and which might significantly affect the flow behaviour during the slurry pipeline operation.

B. The modelling approach of Wilson et al. recognises different slurry flow behaviour in flows with a different degree of flow stratification. However, the assumptions made in the formulation of the semi-empirical model for the heterogeneous flow do not seem to hold in flows with a wide range of solids concentrations.

The macroscopic two-layer model for a fully-stratified flow is composed of the force balance equations and the mass balance equations written for both layers. The model has four coefficients:
- the coefficient of mechanical friction, $\mu_s$, between a granular bed and a pipe wall,
- the viscous friction coefficient, $\lambda_{12}$, for liquid flow at the interface between two layers,
- the viscous friction coefficient, $\lambda_2$, for liquid flow at the boundary between a granular bed and a pipe wall,
- the volumetric spatial concentration, $C_2$, in the contact layer.

Computational testing of the two-layer model for fully-stratified flow shows that:
A. the predictive nomographs are based on one specific combination of input values of the model coefficients $\mu_s$, $C_{vb}$ ($C_2$) and $\lambda_{12}$, different combinations of the input values for these coefficients may give rather different values for predicted quantities $l_m$ and $V_{sm}$
B. for flows other than plug flows, the model coefficient $\lambda_2$ should be determined by the Moody diagram rather than by the Wilson & Brown method.

3.6 References


Appendix 3

**Basic relationships used in predictive models**

Specific gravity of solids and mixture:

\[
S_s = \frac{\rho_s}{\rho_f} \quad S_m = \frac{\rho_m}{\rho_f}
\]

Volumetric concentration of solids:

\[
C_v = \frac{\rho_m - \rho_f}{\rho_s - \rho_f} = \frac{S_m - 1}{S_s - 1}
\]

Hydraulic gradient for flow of liquid:

\[
I_f = \frac{\lambda_f}{D} \frac{V^2}{2g}
\]

Solids effect:

\[
I_m - I_f
\]

Relative solids effect:

\[
\frac{I_m - I_f}{S_m - 1} = \frac{I_m - I_f}{C_v d (S_s - 1)}
\]

Relative excess pressure gradient:

\[
\frac{I_m - I_f}{I_{p_g}} = \frac{I_m - I_f}{2\mu_s C_{vb} (S_s - 1)}
\]

Relative velocity:

\[
V_r = \frac{V_m}{V_{sm}}
\]

Relative concentration:

\[
C_r = \frac{C_v d}{C_{vb}}
\]
Correlation (regression) coefficient $R_c$

for the correlation $y_n = f_n(x_o)$ determined from a set of points defined by coordinates $(y_o, x_o)$ is given as

$$
R_c = r_c^2 = \frac{1}{N} \frac{\sum (y_o - y_n)^2}{\sum (y_o - \bar{y}_o)^2}
$$

for $o = 1, \ldots, N$ and $\bar{y}_o = \frac{1}{N} \sum y_o$. 
Chapter 4

Experiments in slurry pipelines

In this chapter the laboratory and field experiments are described. The laboratory tests were carried out in a 150 mm pipeline with an inclinable section. The field experiments were carried out on a dredging installation composed of a long 650 mm pipeline and a set of pumps in series.

The experiments were focused to the observation of both the integral flow characteristics and the local flow characteristics in slurry pipelines. Measurement of local flow characteristics is crucial to the identification of mechanisms governing the process of slurry flow in a pipeline. Advanced measuring techniques were used to observe the development of the internal structure of slurry flow in the pipelines under the various slurry flow conditions.

Installations and measuring techniques used for experiments are presented together with measured data for solids characteristics. Experimental data for slurry flow are presented and discussed in further chapters in conjunction with an analysis and a modelling of processes observed during experiments.

4.1 Experiments in a 150 mm laboratory pipeline

4.1.1 Aim and programme of experiments

One aim of the experimental work was to observe the behaviour of slurry flow in a pipeline in a way that would show not only the effects of slurry flow as energy dissipation and solids deposit formation but also the reasons for the effects. This made it necessary to measure the flow characteristics appropriate to the detection and analysis of mechanisms governing the process of slurry flow in the pipeline. Furthermore, the measured characteristics had to be appropriate for the verification of the components of a two-layer model. Two types of flow parameters were measured:

- the integral flow characteristics; the measured values represent the mean values of a quantity in a pipeline cross section (slurry velocity, delivered and spatial concentration of solids, pressure)
- the local flow characteristics in a pipeline cross section (local concentrations at different vertical positions in the pipeline cross section and local solids velocity near the bottom of the pipeline)

A further aim of the experimental work was to collect a sufficiently representative database to verify a two-layer model over a wide range of slurry flow conditions. The experimental programme was focused on slurries and pipeline configurations typically
Figure 4.1. Laboratory circuit with the 150 mm pipe.

1. Centrifugal pump
2. Sump tank
3. Vertical tube (inclined)
4. Test loop (inclined)
5. Differential pressure measuring section
6. Flexible rubber section
7. Drain valve
8. Slide valve
9. Vent valve
10. Magnetic flow meter
11. Water-filled observation section
12. Piggyback observation section
13. Rotameter density meter
14. Bev. velocity meter
handled in dredging, i.e. on aqueous sand mixtures and gravel mixtures flowing in horizontal and inclined pipelines. Steady flow conditions were characteristic for tests in the laboratory circuit.

The experiments performed on slurry flow behaviour in a laboratory pipeline are comparable with those of the Saskatchewan Research Council, Canada which is considered the best equipped slurry pipeline laboratory in the world. The measurements of the internal structure of slurry flow in an inclined pipeline are unique. The SRC laboratory contains only horizontal pipeline loops.

4.1.2 System layout

The circuit DN150 (Fig. 4.1) in the laboratory of the Chair of Dredging Technology of Delft University of Technology consists of a 24 metre long test loop which can be inclined from horizontal to vertical positions, an 18 metre long vertical U-tube, the connecting pipes and the sump tank by means of which solids are introduced into the pipeline and in which solids are stored at the end of each experimental run. During measurements the tank can be bypassed. The entire pipeline circuit has a diameter of 150 mm and is 65 metres long. The system is served by a centrifugal pump driven by a 164 kW MAN diesel engine with variable revolutions.

The test loop is composed of a wide U-bend and two 10.55 m long straight pipes. These two pipes of the test loop are here called the ascending pipe and the descending pipe to indicate the flow direction in the pipes when the test loop is inclined. Each pipe contains one measuring section. Both measuring sections are 3 metres long and are equipped with a differential pressure transmitter and a radiometric density meter. This permits simultaneous measurement of slurry flow characteristics in both the ascending and the descending pipes. Measuring sections are placed in the straight pipes in such way that the slurry flow structure in the sections is not affected by bends and other sources of flow disturbance. In the ascending pipe the measuring section (termed Section I) starts 6.55 m (i.e. in the 44 times D distance) behind the 45 deg flexible rubber bend. The Section I ends 1 metre (i.e. 6.7 D) before the U-bend at the top of the test loop and 0.5 metre before the flange of the pipe section in which the bed velocity meter is mounted. In the descending pipe, the measuring section (termed Section II) starts at 44 D distance behind the U-bend at the top of the test loop. The Section II ends 1 metre before a flange of a 45 deg rubber bend and 0.5 metres before the flange of a Plexiglass observation section, 0.5 metres long, mounted in the descending pipe.

The vertical U-tube also contains 3-metre long measuring sections in both the ascending and descending limbs of the U-tube. To avoid the effects of bends on the measured pressure drop over the measuring section the measuring sections start 4.70 metres (i.e. 31D) behind the wide 90 deg. bends (of the radius 5 D). Bends distort the velocity and concentration profiles in the slurry flow. A flow meter is installed in the descending limb 0.5 metres behind the end of the measuring pipe section and 0.3 metres before a 90 deg. bend of a 5 D radius. The height of the U-tube is 9.45 m when measured from the axis of a horizontal pipe on which the U-tube is vertically mounted.
1. Sump tank inlet
2. Sump tank outlet
3. Sump tank overflow
4. Funnel shaped overflow pipe
5. Overflow edge of the funnel
6. Anti-vortex plate
7. Drain valve
8. Slide valve

Figure 4.2. The sump tank of the laboratory circuit.
The sump tank is open to the atmosphere. It is equipped with a funnel-shaped overflow pipe which can be lifted within the tank (Fig. 4.2). The elevation of the pipe regulates the path of the slurry flow through the tank. At the end of an experimental run the movement of the funnel permits the separation of solids from slurry and their storage in the tank. Two different positions of the funnel determine two different flow modes in the tank. When the funnel is lifted up so that its overflow edge is above the slurry level in the tank, the slurry flows directly through the tank leaving it by an outlet in the tank bottom. In this case slurry does not flow through the funnel. The second position of the overflow pipe is reached by letting the funnel sink to the tank bottom so that the bottom of the overflow pipe rests on the tank outlet. The funnel is entirely submerged in the slurry and slurry can reach the tank outlet only via the overflow at the top of the funnel. The overflow edge is positioned above the level of the tank inlet. Therefore the solids, tending to settle in the slurry, are collected in the tank and they are quickly separated from the carrier streaming through the overflow back to the pipeline circuit. The feeding of the pipeline circuit with solids at the beginning of an experimental run is achieved by lifting the funnel to open the tank outlet for the sediment deposited on the bottom of the tank. When the tank inlet is closed and water circulates in the pipeline circuit through a bypass, there is no flow through the tank and the solids from the bottom of the tank flow only gravitationally to the circuit. Feeding is slow (it takes about 10 minutes) and steady, and this prevents the creation of unsteady slurry-flow conditions in the circuit. Constant delivered concentration is reached along the entire circuit.

4.1.3 Measured parameters and measuring techniques

Manometric pressure gradients and concentration distributions were measured for different sorts of solids, various slurry flow conditions (controlled by measured mean slurry velocity, $V_m$, and delivered concentration, $C_{vd}$) and pipe inclinations. In the final stage of the experimental programme a bed velocity meter was developed and incorporated into the measuring system. The temperature of the water in the circuit was maintained within a narrow range (round 21°C) by regulation of the gland-water flow rate at the centrifugal pump. A small amount of gland water always entered the system in the centrifugal pump and the same amount of water left the system via the overflow at the edge of the sump tank.

Differential pressure over a pipeline section

Pressure differences over the 3 metre long measuring sections are measured by Rosemouth differential pressure transmitters (Model 1151DP). The differential pressure is sensed as pressure-induced deflection of a diaphragm in the 6-Cell of the instrument (Rosemouth Product Manual, 1990). The pressure is transmitted from the measuring point to the diaphragm of the transmitter via a medium, e.g. clear water, in a hose. The pressure transmission must not be disturbed by impurities or air bubbles in the hose. For this reason we use transparent PVC hoses to connect the differential pressure transmitters to sedimentation pots in order to detect the presence of air bubbles or sediment deposits. A sedimentation pot is mounted on the slurry pipe wall to cover a tap (of diameter 3 mm) in the pipe wall. The taps are located on the upper part of the pipe perimeter. The sedimentation pots prevent the penetration of solid
particles from the slurry pipe into the PVC hoses and to the δ-Cell. A valve at the top of the pot permits the venting of air from the pot and the hose. Vent valves are also available at the pressure transmitter. The sedimentation pots are connected to a pressurised water-supply circuit so that the pressurised water can be admitted to the pots to remove any incidental air bubbles, to clean the sedimentation pots and to flush the taps. Calibration of each pressure transmitter was carried out by adjusting zero differential pressure and known differential static pressure on the transmitter diaphragm. Static pressures from water columns of known heights in a transparent PVC hose were exerted against an atmospheric pressure at the diaphragm. There is a linear relationship between differential static pressure and the transmitter output current (in mA). The instruments appeared to be very reliable, accurate and stable in all conditions that occurred in the circuit during the tests.

**Slurry velocity**

The Krohne magnetic-inductive flow meter Altometer TIV 50 is used to measure the mean slurry velocity in the laboratory circuit. The magnetic flow meter is a practical application of Faraday's law of electromagnetic induction. When an electrically conducting liquid moves through a magnetic field, a voltage proportional in magnitude to liquid velocity is induced at 90 deg. to both the field and direction of the motion. The induced voltage is proportional to the flow velocity only and is unaffected by density, viscosity, pressure or temperature of the liquid (Altometer Installation and Operating Instructions). The measured velocity is the mean velocity in a pipe cross section. Mounting the instrument on a vertical pipe allows the interpretation of the measured velocity as the slurry velocity because the slip velocity between phases is considered, and also verified by experiments, to be negligible in the vertical pipe for all solids tested during our experiments. Furthermore, the accuracy of measurement in a vertical pipe is not affected by distortion of velocity distribution in the pipe cross section. Calibration of the instrument was carried out and certified by the manufacturer.

**Delivered concentration**

The inverted, vertically mounted, U-tube is used as the counter flow meter to determine the slurry density in the pipeline. This device is often used in laboratory and field installations because it is simple to construct and operate. It was originally proposed by Hagler (1956) and adopted for dredging pipelines by van der Veen (1972). A careful description of the measuring principle, together with an analysis of a measurement accuracy under different conditions is given by Cliff & Cliff (1981). Differential pressure is measured over the equally-long sections in the ascending and the descending limbs of the vertical U-tube. Pressure drop due to friction is considered independent of solids concentration and equal in both pipe sections. Averaging measured differential pressures from both sections eliminates the influence of wall shear stress (and thus of friction) and the average pressure drop can be attributed to the hydrostatic pressure exerted by a slurry column in a pipe section. The calculated slurry density of the slurry column is the average slurry density for both limbs of a U-tube. It is interpreted as the average spatial concentration in a vertical U-tube. When the absolute value of slip velocity is assumed to be identical in the ascending and descending limbs, the slip effect is also eliminated by averaging the measured
differential pressures. The mean delivered concentration in a pipeline is then obtained by the counter-flow meter.

Our tests showed that slip is practically negligible in a vertical pipe for all solids used for our experiments. Almost identical values for concentration were obtained from measured concentration distributions in both the ascending and descending sections of a vertical U-tube and from the counter-flow meter. Thus the concentration obtained from the vertical U-tube is considered to be the mean delivered concentration of solids in the experimental circuit.

**Concentration distribution in a pipeline cross section**

The local value of slurry density in a pipeline cross section is sensed by a radiation density meter Berthold LB 367 with a Cs-137 source. The absorption of a radiation beam passing through a pipeline between a radioactive source and a transmitter is represented by the ratio of beam intensity at the source and at the transmitter. It is an exponential function of the absorption coefficients of the media and the lengths of the beam for each medium through which the beam is passing. The values of absorption coefficients for liquid and solids are nearly proportional to their densities thus the attenuation of the radiation beam is a function of slurry density in the beam path. For details see e.g. Shook & Roco (1991).

A two-point calibration (Berthold Operating Manual, 1989) was carried out in the pipeline with a beam directed to the centre of a pipeline cross section. The change in radiation intensity was measured for a water-filled pipeline and a water-filled pipeline with glass plates of known volume and specific gravity. The specific gravity of the glass was very similar to that of sand and gravel. The results were processed by the instrument software and the absorption coefficients were determined automatically.

Furthermore, the instrument was calibrated for a water-filled pipeline at each vertical position in the measuring pipe cross section in which it was also planned to measure slurry density. To eliminate the influence of pipe wall wear on the values of measured local concentrations, the instruments were recalibrated for each position several times during a long period of the experimental work.

A special support, in which the Berthold radiometric density meter is mounted, enables vertical positioning of a radioactive source and a transmitter in a pipeline cross section. The radiation beam is collimated by a hole in a shield of a lead lined chamber locking a radioactive source. The radiation beam is directed horizontally in the pipeline cross section. By traversing the beam in a vertical direction across the pipeline cross section the chord-averaged density profiles are measured. Values for slurry density are converted to values of the local spatial volumetric concentration of solids in the pipeline cross section. The concentration profiles are measured in pipeline cross sections located approximately in the middle of the measuring pipe section in both the ascending and the descending pipes of the test loop.

**Slip ratio**

The slip ratio is determined from simultaneous measurements of concentration profiles and the mean volumetric delivered concentration \( C_{Vd} \) in the pipeline. Integration of the concentration profile \( c_V(y) \) gives the mean volumetric spatial concentration \( C_{Vl} \), hence slip ratio \( C_{Vd}/C_{Vl} \) is determined directly from the measured parameters in the pipeline cross sections.
Local solids velocity at the bottom of a pipeline

An instrument to measure solids velocity at the bottom of a pipeline is installed in the ascending pipe of the test loop. It is a modified version of the Wiedenroth & Wetzlar prototype, constructed and tested a few years ago in the laboratory of GHS Paderborn in Meschede, Germany (Dreex & Asal, 1989). The measuring principle (described in Wetzlar et al., 1991, Wetzlar, 1992) is based on the cross-correlation of impedance signals generated by two electrodes mounted in a pipe wall at the bottom of a pipe. The impedance of an electrical field occupying a small control volume of slurry above an electrode is sensed. The impedance varies with the amount of solids passing the electrical field. Some particles or clusters of particles generate characteristic peaks in the impedance signal generated on the electrode.

From the cross-correlation of the characteristic peaks of two similar signals, the time is determined that a solid-particle cluster needs to move from a control volume above the first electrode to that above the second electrode. The velocity of solid particles at the bottom of the pipe is calculated from this time and a known distance between two electrodes.

The development and testing of the instrument for our experimental set-up are described in detail in Gerrits & Stok (1996). Adapted spark plugs are used as the electrodes generating the impedance signals. The signals are modulated by a Wheatstone bridge and digitised by an A/D card in the PC. A LabView program enables the PC to be a physical measuring unit processing the signals, carrying out their cross correlation and presenting the results. The very short response time of the system permits on-line measurement of the local solids velocity.

The technique is practically non-invasive. The surface of the electrode is smooth and it forms part of the surface of the pipe wall. Testing of the system showed that the deviation in velocities sensed by this instrument is within 10% for all sorts (fine and coarse) of tested solids. This is acceptable when it is realised that the sliding of solids at the bottom of a pipeline is often a very unsteady process.

4.1.4 Solids used for experimental slurries

Particle size distribution and particle settling velocity

Two techniques are used to determine particle size distribution (PSD) in tested solids - the sieving and the sedimentation tests.

a. Sieving

A 550 gram sample of dry solids is sieved through a series of sieves with standard sieve meshes. The sample fraction remaining in each sieve is weighed on a 0-800 gram balance with a 0.5 milligram resolution. The fractions by mass are recalculated to obtain a percentage of the mass of the entire sample and plotted against particle diameter to the summation PSD curves (% mass passed versus particle diameter represented by sieve opening).

b. Sedimentation tests

A 50 gram sample of solids is collected in a cup at the top of a sedimentation column filled with water. The cup is opened and the time is measured which solid particles of
the sample need to reach a plate at the bottom of the sedimentation column. The Plexiglass sedimentation column is of 50 mm diameter. The distance between the cup and plate is measured before each experimental run, it is typically 950 mm. The water level in the sedimentation column touches the bottom of the cup. The plate is immersed in water and it hangs in a position a few centimetres above the bottom of the column. Thus each solid particle is submerged in water along its entire trajectory between the cup and the plate. The measuring principle is the sensing of the progression of the deflection in time of a thin metal membrane connected with both the cup and the plate. The membrane senses an impulse determining the time at which the cup is opened and the time-dependent increase in the weight of the plate as the particles settle on its surface. A voltage signal is generated in the sensor connected with the membrane. The signal is processed in a computer connected with the instrument via a Keithley A/D converter. A computer program recognises the voltage impulse at the opening of the upper cup and starts to collect data of voltage in a short time sequence controlled by the internal clock of the PC. Collected data are converted and plotted as a summation curve of % solids mass passed versus particle settling velocity.

The sedimentation method has the advantage of direct provision of the settling velocity of the particles, which is the parameter characterising the solids impact on the slurry flow behaviour rather than the particle size. The effect of the concentration of solids in a settling cloud on the particle settling velocity in a sedimentation column is negligible because of small solids samples and thus the low concentration of solids in the settling cloud. The disadvantage of this method is that the length of the sedimentation column is too short to test very coarse particles which need only a few seconds to reach the plate at the bottom of the sedimentation column.

Three sorts of sand and one sort of quartz gravel were used for the laboratory experiments:
the 0.2 - 0.5 mm sand (Sand 1)
the 0.5 - 1.0 mm sand (Sand 2)
the 1.4 - 2.0 mm sand (Sand 3)
the 3.0 - 5.0 mm quartz gravel (Gravel).

The PSD curves and curves for the particle settling velocity distribution (PSVD) obtained from tests in our laboratory are shown on Figs. 4.3 and 4.4. The tests confirmed that the solids were very narrowly graded. The grading of each type of solid was measured before its introduction into the circuit and after its removal from the circuit at the end of a set of experimental runs. Tables IV.1 and IV.2 show the characteristic particle sizes for all solids tested. The mean diameter of particles in a type of solid with a certain degree of grading is determined as

\[ d_s = \frac{\sum d_ip_i}{\sum p_i} \]  

(4.1)

\[ \sum p_i = 100 \]

where \( i \)
Figure 4.3. Particle size distribution of four sorts of solids used for the 150 mm pipe tests. Results from a set of sieves.

Figure 4.4. Particle settling velocity distribution of four sorts of solids used for the 150 mm pipe tests. Results from a sedimentation column.
Table IV.1. The sieve tests for solids before their introduction to the circuit.

<table>
<thead>
<tr>
<th>Solids sort</th>
<th>( d_{s0} ) [mm]</th>
<th>( d_{g5} ) [mm]</th>
<th>( d_s ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand 1</td>
<td>0.42</td>
<td>0.47</td>
<td>0.38</td>
</tr>
<tr>
<td>Sand 2</td>
<td>0.70</td>
<td>0.83</td>
<td>0.59</td>
</tr>
<tr>
<td>Sand 3</td>
<td>1.85</td>
<td>1.95</td>
<td>1.69</td>
</tr>
<tr>
<td>Gravel</td>
<td>4.20</td>
<td>4.70</td>
<td>3.74</td>
</tr>
</tbody>
</table>

Table IV.2. The sieve tests for solids after their removal from the circuit.

<table>
<thead>
<tr>
<th>Solids sort</th>
<th>( d_{s0} ) [mm]</th>
<th>( d_{g5} ) [mm]</th>
<th>( d_s ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand 1</td>
<td>0.40</td>
<td>0.46</td>
<td>0.35</td>
</tr>
<tr>
<td>Sand 2</td>
<td>0.70</td>
<td>0.83</td>
<td>0.59</td>
</tr>
<tr>
<td>Sand 3</td>
<td>1.68</td>
<td>1.88</td>
<td>1.45</td>
</tr>
<tr>
<td>Gravel</td>
<td>3.60</td>
<td>4.00</td>
<td>2.99</td>
</tr>
</tbody>
</table>

The solid portion became gradually finer and more broadly-graded when it had circulated in the laboratory circuit for a long time. The wear and crushing of the solid particles were primarily due to the contacts of particles with the rotating parts of the centrifugal pump. The pipeline did not contain sharp bends and intrusive fittings which would have accelerated the particle disintegration process. It was observed that coarser particles were subject to higher disintegration rates than finer particles. Even for the coarsest solids, however, the effects of the disintegration process were not so significant that they could have influenced the slurry flow characteristics measured during different experimental runs. The slurry flow tests always started with the lowest solids concentration and portions of new material were added to increase the concentration for the next test run. The first portion might circulate in the system for up to 25 hours and the last-added portion for about 5 hours.

**Coefficient of mechanical friction**

The mechanical friction coefficient, \( \mu_s \), of solids against the pipeline wall should be determined by using a tilting tube according to the proposal of Wilson (1970). Part of the descending pipe in the test loop was adapted to serve as a tilting tube. The Plexiglass observation section near the end of the descending pipe of the test loop was removed and the connection between the test loop and the rubber bend dismantled, so that an inclinable 0.6 metre long steel pipe section was separated. The pipe section could be tightened to make it water-proof by mounting a Plexiglass cover and a small Plexiglass tank at the inlet and the outlet of the pipe section. The Plexiglass tank was a 0.35 metre long cylinder of 0.19 metres inner diameter. It was equipped with a water-filling valve and an air-vent valve.

The pipe section was placed in a horizontal position and partially filled with a granular bed of loose-poured concentration and chosen thickness. The section was then water-proof tightened and filled with water of inlet velocity low enough not to disturb the geometry of an installed bed. A valve at the top of the tank enabled all air to escape from the pipe section, then the pipe section was slowly inclined to an angle at which the bed began to slide. The tangent of this angle determined the coefficient \( \mu_s \) provided that the bed thickness was small and its effect on the \( \mu_s \) value was negligible.
A value of 29 deg was typically measured as the angle of the pipe inclination which initiated sliding of the thin bed. This gave the mechanical friction coefficient $\mu_s = 0.55$. It should be noted that this value is not valid for all types of sand and gravel so it is recommended that the coefficient for the solids handled in pipelines should always be determined experimentally.

### 4.1.5 Data acquisition system

The data acquisition system (DAS) provides on-line observations of sensed quantities and their presentation on a PC screen. In chosen time periods DAS generates the data for sensed quantities and records them in data files on a computer disk. The differential pressure transmitters, radiometric density meters and magnetic flow meter are connected to the 286AT/25MHz computer via the Keithley A/D converter. The data are collected at the frequency of 1 Hz within a 30 second interval.

### 4.1.6 Collected experimental database

DAS produces two types of data files:
- "friction files" which contain the values of the mean slurry velocity, the differential pressure over the measuring pipe sections, local concentrations at the lowest position in measuring pipe cross sections of the test loop and the slurry density (considered to provide delivered concentration) (see Tab. A.IV.1 in Appendix 4); if the bed velocity is measured it is added to the "friction files" as channel 9;
- "concentration profile files" containing the values of the local volumetric concentration and the vertical position of the radioactive beam within the measuring pipe cross section.

### 4.1.7 Data processing

The processing and presentation software, installed on the computer hard disk produces data files containing data converted from volts to integer numbers. Further data processing comprising the data conversion to physical units, data averaging and the graphical presentation is carried out by additional software. The data acquired at 1 Hz frequency are averaged in the 30 second time interval so that the measured values of parameters for steady slurry flow can be handled as the average values of the measured parameters for the 30 second measuring interval.
4.2 Experiments in a 650 mm dredging pipeline

4.2.1 Aim and programme of experiments

The effect of slurry density fluctuation on the working conditions of a dredging installation has been of scientific interest since the first on-line measurements on a pipeline connected with dredger pumps were carried out (de Koning, 1968). In 1980 a research project was set up at Delft University of Technology to investigate this effect in detail. A research team headed by Prof. de Koning (Beulink, de Groot, de Vries) prepared a plan for extensive measurements on a field dredging installation (Beulink, 1980a,b,c). The field measurements on a long conveying system, composed of the dredger and a long slurry pipeline with three booster stations in series, were carried out by the Chair of the Technology of Soil Movement of Delft University of Technology and the dredging company Royal Boskalis Westminster N.V. during works on the project "de Vlietlanden" in February and March 1981 (Beulink, 1982).

During a twelve-day experimental program, information was collected about slurry flow behaviour in a pipeline under conditions of permanently fluctuating slurry density and on the development of the flow behaviour along a long pipeline. The operation of the pumps was also carefully monitored, by measuring significant pump and drive characteristics. A high frequency of data acquisition was chosen to record all flow instabilities and their effects on the parameters characterising slurry flow behaviour in the pipeline and pump.

The collected database, named MeaVli (Measurements Vlielanden), is unique experimental material, monitoring the real processes in a full-scale long slurry pipeline with unsteady flow of a sand-water mixture. It covers time-continuous on-line observations of the most important parameters along the entire long conveying system during a dredging operation.

4.2.2 System layout

The conveying system was fed by the deep-dredger (Groningen, Gr) and driven by one floating (Zaandam) and two land-based booster stations (Jagersplas, Ja; Duinjager, Du) (see Fig. 4.5). Identical engines and pumps were used at the booster stations Ja and Du. The major part of the long pipeline was horizontal and of DN650. At a few places short pieces of pipeline of different pipeline diameter (DN600 and DN500) were incorporated. These occurred on board of the deep-dredger Gr (DN500), at the discharge outlets of the booster pumps (DN500) and in the descending limb of a vertical U-tube behind the Ja booster (DN600). The total length of the conveying pipeline varied according to the chosen deposit site and usually exceeded 9 kilometres.

The place where the floating pipeline from the dredger was connected with the land-base pipeline was determined as the reference position (RP) for the length profile of the entire conveying system. The suction mouth of the deep-dredger was 438.5 metres distant from RP and the submerged pump of the dredger was 407 metres from the RP. The pump of the booster station Ja was at a distance of 1700 metres behind the RP and the booster Du pump was 6520 metres behind the RP.
Figure 4.5. Sketch of the dredging installation for the 1981 works "de Vlielanden".
Integral slurry flow parameters were measured at three locations (DFGr [Density meter and Flow meter in the location Groningen], DFJa [Density meter and Flow meter in the location Jagersplas], DFDu [Density meter and Flow meter in the location Duinjager]) along the entire pipeline. One radiometric density meter was installed in each measuring location together with a magnetic flow meter. Two land-based measuring locations (DFJa, DFDu) were positioned on the descending sections of the inverted, vertically mounted, U-tubes (of the height 6 metres) incorporated in the long pipeline behind the boosters Jagersplas and Duinjager. The density meters were placed at 5 times D (pipeline diameter) distance behind the 90 deg bend of the U-tubes. DFJa was mounted to the DN600 pipe 186 metres behind the booster pump and DFDu to the DN650 pipe 18 metres behind the Du pump. The DFGr location was on the descending inclined DN500 pipe (inclination angle 30 deg) 40 meters behind a pump of the dredger Groningen. The pipeline length between the Groningen density meter (location DFGr) and the Jagersplas density meter (location DFJa) was 2253 metres and the DFJa-DFDu pipeline length was 4652 metres. Local solids velocities were measured at the Jagersplas measuring site in a horizontal pipeline before the Jagersplas pump, approximately 250 m in front of the Ja density meter. The velocity meter sensors were mounted at different positions on the pipeline wall and were defined by the angle from a vertical axis in the pipeline cross section. To avoid mutual interference of ultrasonic frequencies from different sensors, each sensor was placed in a separate 6 metre long pipeline section connected with the neighbouring sections by flanges. The pipeline distance between different ultrasonic sensors and the Jagersplas pump is in Table IV.3.

<table>
<thead>
<tr>
<th>Position of sensor [deg]</th>
<th>0</th>
<th>20</th>
<th>40, 45</th>
<th>60</th>
<th>90</th>
<th>135</th>
<th>180</th>
<th>30</th>
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<td>05/03/1981</td>
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<td>52</td>
<td>58</td>
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<td></td>
</tr>
<tr>
<td>17-18/03/81</td>
<td>40</td>
<td>46</td>
<td>52</td>
<td>58</td>
<td>64</td>
<td>70</td>
<td>76</td>
<td>82</td>
</tr>
</tbody>
</table>

The geodetic height of the pipeline varied slightly along its entire length. The reference point RP, at which the floating and land-based pipelines were connected, had a geodetic height 0.07 metres above N.A.P. (Amsterdam Ordnance Datum). The geodetic height of the Jagersplas pump was -0.60 metres and of the Duinjager pump -2.23 metres. The geodetic profile of the entire pipeline was carefully recorded, together with the length position of all incorporated fittings. This permits the determination of friction loss from measured pressure differences in long pipeline section between two boosters. Detailed information about the geometry of the entire dredging installation is given in Duizendstra (1987c).
4.2.3 Measured parameters and measuring techniques

The measurements of parameters important for an analysis of the slurry flow behaviour in a pipeline are discussed below. A survey of all measured parameters is given in Tab. IV.4. The measuring technique used is described only briefly as it is the same as that used in the laboratory circuit and described above in paragraph 4.1.3.

Pressure difference over a pipeline section

The pressures were measured at the suction and discharge pipes of Groningen, Jagersplas and Duinjager pumps by pressure transducers Druck PTX/110 D. They sensed the absolute pressure in the slurry flow at the pipeline locations where they were mounted. No measurements were made on the floating booster station Zaandam. Druck transducers were calibrated by the Tradinco test bench in the Boskalis instrumentation laboratory.

The difference between discharge pressure at the Jagersplas pump and suction pressure at the Duinjager pump provides a measure of the mechanical energy dissipation in slurry flow over the pipeline section between the pump locations. The total differential pressure is composed of a hydrostatic pressure due to a difference in the geodetic positions of two locations (expressed by a height of slurry column) and a pressure drop due to mechanical energy dissipation in the slurry flow over the pipeline section. Dissipation caused by minor losses (local dissipation of mechanical energy in fittings) can be subtracted to get the friction loss in slurry flow over the pipeline section. Measured values of absolute pressure are influenced by differing pipeline diameters at the suction pipe (DN650) and discharge pipe (DN500) at the pumps. However, the effect of pipeline diameter reduction on the value of the total pressure difference over a long pipeline section is negligible.

Slurry velocity

A magnetic-inductive flow meter (Altoflux TIV-30) was mounted in the pipeline on board of the dredger Groningen and Altoflux TIV-50 flow meters were mounted in the vertical descending limbs of the U-tube bridges at the Jagersplas and Duinjager locations. The flow meters provided a mean value of liquid velocity in pipeline cross section considered as the mean slurry velocity when the slip velocity in the pipeline was neglected. Generally, the mean slip velocity \( V_f - V_s \) can be assumed to be equal to particle settling velocity \( v_t \) in vertical pipeline sections. The typical settling velocity was \( v_t = 0.07 \) m/s for solids transported in the MeaVlii pipeline. Thus the difference between mean slurry velocity (typically \( V_m = 3.5 \) m/s) and settling-velocity-dependent mean solids velocity in the measuring locations was about 2 per cent and was neglected. Measured values of slurry velocity were influenced by different pipeline diameters in DFGr (DN500), DFJa (DN600) and DFDu (DN650) locations. In the MeaVli database the values from all flow meters are recalculated to \( V_m \) in pipeline DN650. The gland water added to the system at each centrifugal pump increased the flow rate (and so mean slurry velocity) along the long pipeline. Thus phenomena occurring in one pipeline section must be evaluated for the mean slurry velocity measured in that pipeline section. The flow rate of gland water was 0.024-0.025 m\(^3\)/s at the pump Groningen, 0.018-0.019 m\(^3\)/s at the pump Zaandam, 0.042-0.043 m\(^3\)/s at the pump Jagersplas and 0.035 m\(^3\)/s at the pump Duinjager. This gave an increase in
mean slurry velocity in the DN650 pipeline of about 0.18 m/s at DFJa and 0.29 m/s at DFDu as compared with DFGr.

Delivered concentration

The slurry density was measured by a radiometric method, using the IHC radiometric density meter in the inclined pipeline section on the board of the dredger Groningen and the γ-ray radiometric density meter Berthold LB 370 (with a Co-60 radiation source) in the descending vertical section of the U-tubes at the Jagersplas and Duinjager locations. A radiation beam was directed to the centre of the pipeline cross section so therefore measured values were considered the mean values for the pipeline cross section. Since a uniform concentration profile was hardly established in the -30 deg inclined pipeline at the Groningen density meter location, the recorded values might differ from the real average density value for the Groningen pipeline cross section. In the vertical pipeline section the concentration profile may be distorted by the effect of the bend, since the length of straight pipeline between the bend and the density meter was rather short at the Jagersplas and Duinjager locations. This may cause certain inaccuracies in the density values when they are interpreted as mean values for pipeline cross section.

Values of slurry density are recalculated to values of volumetric concentration of solids in a pipeline cross section. This concentration is considered to be the mean delivered concentration \( C_{\text{vd}} \) in a pipeline cross section, because the effect of the slip velocity \( V_F \cdot V_s = V_t \) is negligible.

Solids velocity distribution in a pipeline cross section

The solids velocity profiles were acquired from local solid velocity measurements at the measuring site on the horizontal pipeline in front of the Jagersplas pump. Profiles were constructed from signals of local solids velocities sensed in different vertical positions (\( \alpha = 0, 20, 30, 40(45), 60, 90, 135 \) and 180 deg) in a horizontal pipeline by a set of Doppler ultrasonic flow meters Uniflow UNIF-1.

A beam of ultrasounds was directed at a certain angle to the flowing mixture of liquid and solid particles. Solid particles in the liquid reflected the sound back to the receiver with a Doppler shifted frequency. The frequency shift was a function of the frequency of the transmitted wave, the angle of the transmitted beam, the velocity of sound in the liquid and the velocity of solid particles.

A beam was transmitted by the UNI meter at an angle of 149° to the flow in the MeaVli pipeline. The frequency of the transmitted wave was 634 kHz, giving the transmitted wave a length of approximately 2.37 mm. A control volume, in which the solids velocity was measured, was characterised by a distance approximately 10 - 30 mm inside the slurry flow from the pipeline wall. Calibration of the instruments was done by using the formulas provided by the manufacturer (Flowmeter Company, USA). The accuracy of the instruments was estimated as better than 4 per cent by the manufacturer (see Flowmeter Co. manual). Measurements of solids velocity distributions in the MeaVli pipeline are described in detail in de Vries (1981) and Beulink (1983).
Figure 4.6. Particle size distribution of solids transported in the dredging pipeline. Results from MeaVli data base: 17/03/81 - samples from location Jagersplas.
4.2.4 Solids transported in the dredging pipeline

Samples of transported solids were collected frequently (approximately every 15 minutes) by soil samplers mounted in the pipeline behind the pumps at the Groningen, Jagersplas and Duinjager locations. The soil sampler was a 1 litre tube closed at both ends by valves. The sampler was connected with a nipple covering a tap bored in the pipeline wall. When both sampler valves were opened, slurry flow was established through the sampler tube. A one-litre sample was collected in the sampler by closing both valves. A tap was bored at approximately 75 deg position from the vertical axis in the pipeline cross section 10-15 metre distant from the pump outlet. Previous tests on dredging pipelines suggested that at this distance behind a pump the slurry flow was still not stratified and a sample collected there would realistically represent all fractions in transported solids.

Coarser fractions (sand) of the collected samples were sieved in a set of standard sieves. Finer fractions (silt) were pre-processed and sieved at the Retsch vibration table. This determination of the silt particle size distribution was carried out in the laboratory Grondmechanica, Delft. Detailed information about the sample processing are given in Duizendstra (1987a). The PSD curves thus obtained are collected in a set of 10 books named MEAVLI - Samples. According to the curves of the particle size distribution for samples collected during the MeaVli measurements, the PSD of dredged solids is rather wide (Fig. 4.6). The material dredged from the bottom of the Vlielanden lake was sand 0.02-0.8 mm. A typical sample of the transported solids had a particle diameter \(d_{50}\) of about 0.25 mm and was composed of silt and fine to medium sand. It contained a portion of sand coarser than 0.6 mm (less than 10% of the total amount of transported solids) but also silt finer than 0.075 mm (about 10% of the total amount of transported solids).

4.2.5 Collected experimental database

The database thus collected is very large, with its almost 2 GB of information in a mainframe storage. The database contains measured data of pumps and pipeline operational parameters. To get valuable experimental material for an analysis of the phenomena detected, the measurements of solids velocity profiles in the pipeline were finally added to the experimental programme and separate data files were created. Thus two types of experimental measurements were carried out on the dredging installation and two types of data files were acquired during the field measurements:

Data files (A): time continuous on-line measurement of pumps and pipeline operational parameters of the entire system of a dredger, pipeline sections and three booster stations in series (acquired data files have 45 channels, see Tab. A.IV.2. and A.IV.4 in Appendix 4)

Data files (B): time continuous on-line measurement of local solids velocities in the pipeline cross section at one measuring place on the slurry pipeline and of basic integral slurry flow parameters in the pipeline (acquired data files have 14 channels, see Tab. A.IV.3 in Appendix 4). From the data files (B) the solids velocity profiles at the measuring place in front of the pump Jagersplas were obtained, together with mean slurry velocities and slurry densities from a measuring place behind the Jagersplas pump.
4.2.6 Database processing

During the measurements the analogue signals were recorded to the magnetic tapes. These signals were later digitised and calibrated. A description of the data conversion process is given in Duizendstra (1987b). The organisation of converted data to data files in the magnetic tapes is described by Duizendstra (1987d,e). Originally the magnetic tapes were deposited in a mainframe library and catalogued (see Tab. A.IV.5 in Appendix 4). They could be allocated by the Fortran program available on the mainframe and loaded to a data set in a mainframe storage. Recently the contents of the tapes were copied to the mainframe storage so that the names originally attributed to magnetic tapes are now the names of data sets in the mainframe storage. The technique used to handle the data base is described in Matoušek (1994).

During experiments all on-line measured parameters were sensed four times in one second (so with the frequency 4 Hz). They were recorded, converted and saved to data files with the same time step. For a purpose of data analysis, the sets of parameters have been selected with a time step of 2 seconds (so frequency 0.5 Hz) from the mainframe data sets. Data files of this time step have been loaded to a personal computer (PC) and then processed and analysed by the developed PC program called DTI (Data Transfer and Interpretation). The measured signals plotted in the graphs in Chapter 8 are from the data files loaded to the PC.

During the data processing by DTI in the computer, the signals can be averaged within the chosen time interval. Typically the step 4 has been chosen (four following values in each channel have been averaged), so that the averaged values have been produced for the 8 second period. These values have been handled by the data interpretation model DIM and plotted as the data interpretation outputs. DTI & DIM operations are discussed in the Appendix 8 of the Chapter 8.

4.3 References

Altometer Installation and Operating Instructions. TIV 50. p.28.


Experiments in slurry pipelines


Flowmeter Co. Some Basic Facts on Ultrasonic Doppler Flowmetering. p.6.


MEAVLI - Samples. Serial I (Sample 1-636). Serial II (Sample 1-636).


Appendix 4

Laboratory database - structure of data file

Table A4.1. "friction files" (8 channels)

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>$V_m$</td>
<td>$\Delta P_{\text{desc}}$</td>
<td>$\Delta P_{\text{asc}}$</td>
<td>$\Delta P_{\text{ver,up}}$</td>
<td>$\Delta P_{\text{v,dow}}$</td>
<td>$c_v_{\text{asc}}$</td>
<td>$c_v_{\text{desc}}$</td>
<td>$\rho_m$</td>
</tr>
</tbody>
</table>

List of symbols:

- $V_m$: mean slurry velocity in the pipe of the laboratory circuit
- $\Delta P_{\text{desc}}$: differential pressure in measuring section of descending pipe
- $\Delta P_{\text{asc}}$: differential pressure in measuring section of ascending pipe
- $\Delta P_{\text{ver,up}}$: differential pressure in measuring section of upgoing pipe
- $\Delta P_{\text{v,dow}}$: differential pressure in measuring section of downcoming pipe
- $c_v_{\text{asc}}$: local concentration of solids in measuring section of ascending pipe
- $c_v_{\text{desc}}$: local concentration of solids in measuring section of descending pipe
- $\rho_m$: mean slurry density in the pipe
**MeaVli database - structure of data files**

**Table A.IV.2. Type (A) (45 channels):**

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<tr>
<td>Parameter</td>
<td>Tq2</td>
<td>Et1</td>
<td>Et2</td>
<td>Ra1</td>
<td>Ra2</td>
<td>Pm</td>
<td>Vm</td>
<td>rpm</td>
</tr>
<tr>
<td></td>
<td>Du</td>
<td>Du</td>
<td>Du</td>
<td>Du</td>
<td>Du</td>
<td>Gr</td>
<td>Gr</td>
<td>Gr</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>41</th>
<th>42</th>
<th>43</th>
<th>44</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Gp</td>
<td>GV</td>
<td>P44</td>
<td>soil</td>
<td></td>
</tr>
</tbody>
</table>

Remark: Channels 8 and 43 of a source data file - type (A) are empty.
Symbols are explained in Table A.IV.4.
Table A.IV.3. *Type (B)* (14 channels):

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow1CC</td>
<td>t</td>
<td>v135</td>
<td>v90</td>
<td>v180</td>
<td>v45</td>
<td>v0</td>
<td>PS</td>
<td>PD</td>
<td>PM</td>
<td>VM</td>
<td></td>
<td></td>
<td></td>
<td>soil</td>
</tr>
<tr>
<td>Flow2&amp;3</td>
<td>t</td>
<td>v180</td>
<td>v135</td>
<td>v90</td>
<td>v60</td>
<td>v40</td>
<td>v20</td>
<td>v0</td>
<td>PM</td>
<td>VM</td>
<td>PS</td>
<td>PD</td>
<td>v30</td>
<td>soil</td>
</tr>
</tbody>
</table>

List of symbols:

- **v0**: local solids velocity in vertical position 0 deg. measured before Ja pump
- **v20**: local solids velocity in vertical position 20 deg.
- **v30**: local solids velocity in vertical position 30 deg.
- **v40**: local solids velocity in vertical position 40 deg.
- **v45**: local solids velocity in vertical position 45 deg.
- **v60**: local solids velocity in vertical position 60 deg.
- **v90**: local solids velocity in vertical position 90 deg.
- **v135**: local solids velocity in vertical position 135 deg.
- **v180**: local solids velocity in vertical position 180 deg.
- **V_m**: mean slurry velocity measured behind Ja pump
- **PM**: mean slurry density measured behind Ja pump
- **PS**: suction pressure measured on Ja pump
- **PD**: discharge pressure measured on Ja pump
- **soil**: code for soil sample
Table A.IV.4. Survey of parameters in data files - type (A) (45 channels).

<table>
<thead>
<tr>
<th>Characteristics for</th>
<th>Measured parameters:</th>
<th>Measuring site</th>
</tr>
</thead>
<tbody>
<tr>
<td>pipeline:</td>
<td>( V_m ) (mean slurry velocity)</td>
<td>Gr Ja Du</td>
</tr>
<tr>
<td></td>
<td>( \rho_m ) (mean slurry density)</td>
<td>Gr Ja Du</td>
</tr>
<tr>
<td>centrifugal pumps:</td>
<td>( P_s ) (suction pressure)</td>
<td>Gr Ja Du</td>
</tr>
<tr>
<td></td>
<td>( P_d ) (discharge pressure)</td>
<td>Gr Ja Du</td>
</tr>
<tr>
<td></td>
<td>( \text{rpm} ) (revolutions per minute)</td>
<td>Gr Ja Du</td>
</tr>
<tr>
<td></td>
<td>( Gp ) (gland water pressure)</td>
<td>Gr Ja</td>
</tr>
<tr>
<td></td>
<td>( GV ) (gland water velocity)</td>
<td>Gr Ja</td>
</tr>
<tr>
<td></td>
<td>( Jp ) (jet pressure)</td>
<td>Gr</td>
</tr>
<tr>
<td></td>
<td>( JV ) (jet velocity)</td>
<td>Gr</td>
</tr>
<tr>
<td>diesel engines:</td>
<td>( Tq1, Tq2 ) (torque of engine 1 &amp; 2)</td>
<td>Ja Du</td>
</tr>
<tr>
<td></td>
<td>( Bp1, Bp2 ) (blower pressure)</td>
<td>Ja Du</td>
</tr>
<tr>
<td></td>
<td>( Et1, Et2 ) (exhaust temperature)</td>
<td>Ja Du</td>
</tr>
<tr>
<td></td>
<td>( Ra1, Ra2 ) (rack position)</td>
<td>Ja Du</td>
</tr>
<tr>
<td>Miscellaneous:</td>
<td>( t ) (time)</td>
<td>Gr</td>
</tr>
<tr>
<td></td>
<td>( LD ) (ladder depth)</td>
<td>Gr</td>
</tr>
<tr>
<td></td>
<td>( STd ) (suction tube depth)</td>
<td>Gr</td>
</tr>
<tr>
<td></td>
<td>( STp ) (suction tube pressure)</td>
<td>Gr</td>
</tr>
<tr>
<td></td>
<td>( Vdi ) (diluting water velocity)</td>
<td>Gr</td>
</tr>
<tr>
<td></td>
<td>( P02 ) (pressure within suction mouth)</td>
<td>Gr</td>
</tr>
<tr>
<td></td>
<td>( P44 ) (pressure outside suction mouth)</td>
<td>Gr</td>
</tr>
<tr>
<td></td>
<td>soil sample coding</td>
<td>Gr</td>
</tr>
</tbody>
</table>
**MeaVli database - data storage**

Table A.IV.5. Data location on tapes and mainframe storage.

<table>
<thead>
<tr>
<th>Measuring day</th>
<th>Tape / New data set</th>
<th>Data set on tape</th>
</tr>
</thead>
<tbody>
<tr>
<td>05/02/1981 Th</td>
<td>WB9251.L0003</td>
<td>WBTG.RWDA01CC</td>
</tr>
<tr>
<td>09/02/1981 Mo</td>
<td>WB9251.L0004</td>
<td>WBTG.RWDA02CC</td>
</tr>
<tr>
<td>10/02/1981 Tu</td>
<td>WB9252.L0003</td>
<td>WBTG.RWDA03CC</td>
</tr>
<tr>
<td>11/02/1981 We</td>
<td>WB9252.L0004</td>
<td>WBTG.RWDA04CC</td>
</tr>
<tr>
<td>12/02/1981 Th</td>
<td>WB9253.L0003</td>
<td>WBTG.RWDA05CC</td>
</tr>
<tr>
<td>16/02/1981 Mo</td>
<td>WB9253.L0004</td>
<td>WBTG.RWDA06CC</td>
</tr>
<tr>
<td>17/02/1981 Tu</td>
<td>WB9254.L0003</td>
<td>WBTG.RWDA07CC</td>
</tr>
<tr>
<td>18/02/1981 We</td>
<td>WB9254.L0004</td>
<td>WBTG.RWDA08CC</td>
</tr>
<tr>
<td>19/02/1981 Th</td>
<td>WB9255.L0002</td>
<td>WBTG.RWDA09CC</td>
</tr>
<tr>
<td>05/03/1981 Th</td>
<td>WB9256.L0004</td>
<td>WBTG.FLOW01CC</td>
</tr>
<tr>
<td>17/03/1981 Tu</td>
<td>WB9256.L0005</td>
<td>WBTG.FLOW02CC</td>
</tr>
<tr>
<td>18/03/1981 We</td>
<td>WB9256.L0006</td>
<td>WBTG.FLOW03CC</td>
</tr>
</tbody>
</table>

Remarks:

Data sets listed in the table contain calibrated data. Non-calibrated versions of data sets are stored under the same name (but with the extension NC instead of CC) on the tapes with the same name (but with labels L0001, L0002 instead of L0003 and L0004). For example the non-calibrated data from 12/02/1981 are on tape WB9253.L0001 in a data set WBTG.RWDA05NC.

Six data tapes WB9257 - 9262 are the back-up copies of WB9251 - 6.

A tape WB8400 contains additional text files and programs concerning MeaVli measurements.
Chapter 5

Horizontal steady flows: observation and analysis

This chapter presents and analyses the experimental results obtained in the laboratory circuit DN150 with a test loop installed in the horizontal position. The phenomena observed during the tests are described and analysed in order to detect the prevailing mechanisms governing the behaviour of slurry flow in a pipeline. Some results presented in this chapter were recently published in Matoušek (1995, 1996a, 1996b).

5.1. Introduction

Steady flow was maintained in a laboratory circuit during all experimental runs. Tested settling slurries exhibited the concentration and velocity gradients across a pipeline cross section. The tests were focused on the observation of the influence of different particle size, solids concentration and slurry velocity on the slurry flow behaviour.

5.2 Observations

5.2.1 Forming of a stationary bed

Experiments revealed that the mean slurry velocity at which solid particles first stopped their sliding and a stationary deposit started to be formed cannot be characterised by one concrete $V_m$ value. The sliding bed was very unstable (particularly in coarser slurries) at mean slurry velocities round the threshold for a stationary bed. A transition between the regime with a stable stationary deposit and the regime with a steadily sliding bed was given by the velocity range of approximately 0.15 m/s. It may be anticipated that the accuracy of the determination of the deposition-limit velocity, $V_{dl}$, by experiments will be of the same order. It was found that the $V_{dl}$, that is the deposition-limit value of $V_m$, was influenced by the particle size and solids concentration in the pipeline.

The results of visual observations (see Table V.1) showed that
- $V_{dl}$ decreased with increasing particle size $d_{50}$
- $V_{dl}$ decreased with increasing solids concentration $C_{vd}$.
<table>
<thead>
<tr>
<th>solids size [mm]</th>
<th>C_{vd} [-]</th>
<th>deposition-limit velocity V_{dl} [m/s]</th>
<th>Experim. run</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2-0.5</td>
<td>0.11</td>
<td>2.70-2.80</td>
<td>13/12/94</td>
<td></td>
</tr>
<tr>
<td>Sand 1</td>
<td>0.18</td>
<td>2.60-2.65</td>
<td>14/12/94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>2.55-2.60</td>
<td>15/12/94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.31</td>
<td>2.40-2.45</td>
<td>21/12/94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>2.25-2.35</td>
<td>22/12/94</td>
<td></td>
</tr>
<tr>
<td>0.5-1.0</td>
<td>0.09</td>
<td>2.60</td>
<td>15/02/95</td>
<td></td>
</tr>
<tr>
<td>Sand 2</td>
<td>0.17</td>
<td>2.50</td>
<td>16/02/95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>2.45-2.50</td>
<td>10/03/95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.23</td>
<td>2.35-2.45</td>
<td>21/02/95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>2.10-2.15</td>
<td>31/03/95</td>
<td></td>
</tr>
<tr>
<td>1.4-2.0</td>
<td>0.08</td>
<td>2.00-2.10</td>
<td>07/04/95</td>
<td>unstable bed, bed forms</td>
</tr>
<tr>
<td>Sand 3</td>
<td>0.21</td>
<td>2.00</td>
<td>25/04/95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>1.90</td>
<td>27/04/95</td>
<td></td>
</tr>
</tbody>
</table>

Instabilities occurred at the top of the bed in slurry flow of mean slurry velocities below and around the deposition-limit value, the bed surface was no longer plane. The instabilities were very weak in the flow of the finest tested material (Sand 1), but in the coarsest Gravel flow they led to the development of bed forms occupying a considerable area of the bed. This made observation of the deposition-limit velocity impractical because the bed forms developed might easily block the pipeline near the critical components of the circuit such as the bends and other fittings. Unfortunately, this effect was experienced during the measurements in the Gravel-slurry flow. The V_{m} value at which only small clusters of grains interrupted sliding occasionally at the bottom of the pipeline was set by regulating the rpm of the pump. A visual observation carried out within the first five minutes of the imposed constant V_{m} operation showed some degree of bed instability but no actual formation of a steady stationary bed. However, after approximately 20 minutes of unattended operation the circuit was blocked. Further careful observations of this critical stage of the flow of a coarse-particle slurry showed that an unsteady solids flow might gradually develop from an initially steady flow. The unsteady character of the solids flow was also indicated by a considerable fluctuation of slurry density sensed in the vertical U-tube of the circuit. It was also learned that a small increase in V_{m} (of about 0.15 m/s) above the deposition-limit value prevented development of bed forms and enabled the maintenance of a steady solids flow in the circuit.

Very weak instabilities in the flow of the finest measured solids (the 0.2-0.5 mm sand) made it possible to measure the flow parameters for V_{m} below the deposition-limit value (see measurements for V_{m} < 2.40 m/s in December 1994 - January 1995 on Figs. A5.1-1a and A5.2-1a in Appendix 5). These measurements showed that the solids concentration in the bed increased rapidly when the bed stopped sliding. Also the frictional head loss, I_{m}, increased rapidly within a narrow velocity range in which the stationary bed began to be formed. A comparison of the shapes of the I_{m}-V_{m} resistance curves for slurry of different solids concentrations indicates a gradual decrease of V_{dl} value with an increasing solids concentration in the pipeline.
5.2.2 Energy dissipation in slurry flows

The total amount of solids in the circuit was constant during each experimental run, producing a $I_m - V_m$ curve. Thus in the test loop installed in a horizontal position the delivered concentration $C_{vd}$ might increase (and the spatial concentration $C_{vi}$ might decrease) slightly with increasing $V_m$ during one experimental run [see the slight increase in slurry density (determining $C_{vd}$) with the increasing $V_m$ in Figs. A5.1 in Appendix 5]. This was true except during the earliest tests (December 1994 - January 1995, Fig. A5.1-1a) in which the sump tank had not yet been by-passed and the amount of solids in the pipeline was influenced by the mixing process in the tank.

Measured hydraulic gradients showed the following general trends:

- Compared with the flow containing finer solids at the same $V_m$ and $C_{vd}$, the slurry flow containing coarser particles provides higher resistance.
- When other parameters (as $V_m$ and $d_{50}$) remain unchanged, the friction loss in the slurry flow increases with the slurry density.

The development of $I_m$ with the increasing $V_m$ is shown by the shape of the $I_m - V_m$ curve (called also the resistance curve) obtained during one experimental run. Resistance curve plots for different tested solids in mixtures of different solids concentrations are given on Figs. A5.1 in Appendix 5. The curve shapes were observed to be different for flows of solids of different particle sizes. The shape of the curve for the flow of certain material was also found to be dependent on the solids concentration. The typical measured resistance curves for different solids (and similar concentrations) are plotted in Fig. 5.1.

![Figure 5.1. Resistance curves for flows of different solids in the horizontal 150 mm pipeline.](image)
The figure illustrates different trends in the development of \( I_m \) with an increasing \( V_m \) observed in our laboratory circuit:

- for a flow of relatively fine solids (the 0.2 - 0.5 mm sand, Sand 1) the solids effect \((I_m - I_p)\) decreased gradually with increasing \( V_m \) and reached a limit value for the high slurry velocities; the limit value was dependent on the solids concentration in the flow (see also Figs. A5.1-1a to A5.1-1c),

- for a flow of very coarse solids the same trend was indicated, only the \((I_m - I_p)\) limit value was considerably higher than for the fine particle flow and it was reached at considerably lower \( V_m \); this trend was obtained in a flow of the coarsest measured material (the 3.0 - 5.0 mm gravel, Gravel) for the highest measured solids concentrations (see also Fig. A5.1-4a),

- for the medium to coarse sand (the 0.5 - 1.0 mm sand, Sand 2), the coarse sand (the 1.4 - 2.0 mm sand, Sand 3) and the Gravel (the 3.0 to 5.0 mm gravel, transported at low concentrations) there was no monotonic decrease of the solids effect with the increasing \( V_m \). Instead, the solids effect might rise within the range of \( V_m \). This caused a characteristic "two-peak" shape of the resistance curve detected within the measured range of slurry velocities \( V_m \) (see also Figs. A5.1-2a and A5.1-2b, Figs. A5.1-3a and A5.1-3b and Fig. A5.1-4a). The peaks became more pronounced in a denser slurry flow. Two maxima and two minima occurred on the resistance curve. The positions of the maxima and the minima on a resistance curve tended to be shifted to lower slurry velocities when the slurry flow was denser.

The efficiency of a slurry pipeline is evaluated by means of a parameter called specific energy consumption (SEC). The SEC determines the energy required to move a given quantity of solids over a given distance in a pipeline. It is given as \( \text{SEC} = \frac{0.2778 I_m g}{S_s C_{vd}} \) in units [kWh/tonne·km] and plotted against solids throughput (the amount of dry solids delivered at the pipeline outlet over a time period) in Figs. 5.2. The lowest values of the specific energy consumption in the laboratory pipeline (Fig. 5.2) were found in slurries with the volumetric concentrations of solids higher than 25%, flowing at mean velocities not far above the deposition-limit velocity \( V_{dl} \). However, the operation of a pump-pipeline system in this regime of slurry flow is not always practical. When compared with the flow of dilute slurries, the deposition-limit velocity of the flow of highly concentrated slurries is shifted to the lower values or the mean slurry velocity \( V_m \) and may become lower than the velocity \( V_{min} \) at the minimum of the resistance curve \( I_m \) versus \( V_m \) for slurry flow of the constant delivered concentration. Operation in the velocity range between \( V_{dl} \) and \( V_{min} \), in which the flow resistance increases with decreasing mean slurry velocity, is inherently unstable and, in practice, is usually avoided. During our observations in the 150 mm pipeline, the deposition-limit velocity dropped below the minimum velocity in sand slurries of the highest measured concentrations (\( C_{vd} \) round 35%).

The specific energy consumed to transport coarse sand is approximately twice that needed to transport medium sand in the 150 mm pipeline.

### 5.2.3 Solids distribution in slurry flows

The measurements of solids concentration profiles showed that the solids effect \((I_m - I_p)\) was strongly dependent on the shape of the concentration profile in a pipeline cross section. Less stratified flow exhibited less resistance. Generally, \( V_m \) and \( C_{vi} \)
Figure 5.2. Specific energy consumption (SEC) in a 150 mm slurry pipeline. Exp. data: Section II of laboratory testing loop.

Legend: Cvd [%]

- 9-12
- 15-18
- 21-25
- 26-29
- 31-35

$V_{dl} < V_m < 6.00 \text{ m/s}$. 
Low concentrated slurry flow.

<table>
<thead>
<tr>
<th>No bed</th>
<th>Station. bed to y/D = 0.20</th>
<th>Station. bed to y/D = 0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
</tr>
</tbody>
</table>

High concentrated slurry flow.

<table>
<thead>
<tr>
<th>Sliding bed to y/D = 0.40</th>
<th>Sliding bed to y/D = 0.45</th>
<th>Station. bed to y/D = 0.60</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4" alt="Graph" /></td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
</tbody>
</table>

Figure 5.3a. Stationary and en bloc sliding beds in the horizontal 150 mm pipe. A bed development in flow of Sand2 slurry under the condition of decreasing Vm and maintained constant Cvd.
are the major parameters determining the measure of slurry flow stratification in a pipeline cross section if $\rho_s$ and $\rho_f$ are constant. The stream carrying the larger particles requires a higher $V_m$ to overcome the particle settling tendency and keep the particle in suspension. Thus the flow at a certain $V_m$ tends to be more stratified when coarser particles are transported.

The local concentrations near the bottom of the pipeline may approach a limiting value given by the solids concentration in a loose-poured bed. The measurements showed that this was the case in the stationary beds if they occurred in the test pipeline. Moving beds, however, tended to be less concentrated than stationary beds, even if solids within moving beds remained uniformly distributed. Moving beds with a uniform concentration distribution also exhibited a negligible solids velocity gradient over their height so that they slid en bloc. The solids concentration in a stationary bed had values of about 0.60 considered a loose-poured value. This is seen in concentration profiles measured in the Sand 1 slurry (for $V_m < 2.40$ m/s) on Fig. A5.2-1a and in the Sand 2 slurry on Fig. 5.3a. Fig. 5.3a gives results of a test in a specially adapted test loop during which the bed development in the pipeline was observed under the condition of a gradually decreasing $V_m$ and constant $C_{vd}$; the constant value of $C_{vd}$ was maintained by adding sand to the circuit when the lower $V_m$ was installed. The en bloc sliding beds had concentrations of about 0.55 or even less (see Fig. 5.3a), indicating that even uniformly distributed beds exhibit certain dilation. The local concentrations near the bottom of a pipeline were further found to be strongly dependent on mean solids concentration in the pipeline. Low concentration slurries did not produce a bed of a concentration approaching the loose-poured value even for the slurry velocities only slightly above the deposition-limit value. At higher velocities (e.g. $V_m = 3.5$ m/s) the local pipe-bottom concentrations remained higher in concentrated slurries than in diluted slurries (see Figs. A5.2).

An increase in the flow velocity should decrease the flow stratification, i.e. the flow should become more homogeneous. However, this trend was not observed in slurry flows of medium and coarse solids (Sand 2, Sand 3 and Gravel). Measured concentration profiles revealed that a slurry flow containing medium or coarse grains was gradually homogenised if the $V_m$ increased gradually from the deposition-limit value, but above a certain $V_m$ value the flow exhibited a gradual restratification under the further increasing $V_m$. Experiments also indicated that the restratified flow would be gradually homogenised again if the $V_m$ should increase above the maximum values used during the tests. The development described in the solids distribution within a slurry stream under the continuously increasing slurry velocity can be interpreted as a process of a gradual erosion of an originally uniform bed followed by a process of a gradual bed restoration. The process of the gradual erosion of an originally uniform bed is characterised by the gradual increase in the thickness of the layer exhibiting a concentration gradient at the top of the bed. The area occupied by the uniform bed diminishes. The uniform bed restoration is the reverse process characterised by a gradual diminishing of the concentration-gradient layer and thus by an expansion of the area occupied by a uniformly-distributed bed at the bottom of a pipeline. The flow restratification effect is shown on Fig. 5.3b for coarse slurry flow under the increasing $V_m$. 
The typical concentration profile of a partially-stratified flow was composed of a convex ("left-hand" bend) curve in the upper region of a pipe and a concave ("right-hand" bend) curve in the lower region. The concave curve might approach the course of a vertical line near the bottom of the pipe. The upper and lower regions were delimited by the position of the point of inflexion on the profile curve. In the vicinity of the point of inflexion the profile curve tended to maintain a constant concentration gradient. Its value varied with integral flow parameters.

5.2.4 Slip between phases within slurry flows

The slip ratio ($C_{\text{vd}}/C_{\text{vi}} = V_S/V_M$) is a parameter used to describe the slip between the flowing solid and liquid phases in a test pipeline. It was determined experimentally as $C_{\text{vd}}/C_{\text{vi}}$ from the measurements of the slurry density $\rho_M$ in the vertical U-tube and concentration profiles in the test loop. Measured $\rho_M$ gave $C_{\text{vd}}$ and the $C_{\text{vi}}$ values were obtained by integrating the measured profiles over the pipeline cross section area. The slip was found to be strongly related to the degree of flow stratification in a pipeline. The slip ratio approached unity when the concentration distribution became less stratified. Tests in the laboratory circuit revealed that
A. the slip was always negligible when slurry flow was pseudo-homogeneous (the concentration profile in the pipeline cross section was uniform). This was observed in slurry flows of all tested solids through both the ascending and the descending pipes (see Table V.2) of the test loop installed in the vertical position. In the horizontally-positioned test loop negligible slip was detected in flows of the finest tested sand (Sand 1) at the highest testing slurry velocities ($V_m > 5 \text{ m/s}$). At these velocities a low concentration gradient across the pipeline cross section was measured so the flow could not be considered to be stratified.

Table V.2. Slip ratio in the ascending pipe (Section I, +90 deg.) and the descending pipe (Section II, -90 deg.) of the 150 mm test loop installed in a vertical position.

<table>
<thead>
<tr>
<th>Solids size [mm]</th>
<th>$C_{vd}$ [-]</th>
<th>$V_m$ [m/s]</th>
<th>$C_{vi}$ [-] (+90)</th>
<th>$C_{vi}$ [-] (-90)</th>
<th>Slip r. [-] (+90)</th>
<th>Slip r. [-] (-90)</th>
<th>Experim. run</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2-0.5</td>
<td>0.232</td>
<td>3.50</td>
<td>0.231</td>
<td>0.229</td>
<td>1.00</td>
<td>1.01</td>
<td>29/09/95</td>
<td>C_{vi} by integrating the conc. profile over a pipe area</td>
</tr>
<tr>
<td></td>
<td>0.185</td>
<td>3.51</td>
<td>0.184</td>
<td>0.182</td>
<td>1.01</td>
<td>1.02</td>
<td>04/10/95</td>
<td></td>
</tr>
<tr>
<td>0.5-1.0</td>
<td>0.246</td>
<td>2.50</td>
<td>0.247</td>
<td>0.243</td>
<td>1.00</td>
<td>1.01</td>
<td>12/08/96</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.249</td>
<td>3.51</td>
<td>0.250</td>
<td>0.248</td>
<td>1.00</td>
<td>1.00</td>
<td>12/08/96</td>
<td></td>
</tr>
<tr>
<td>1.4-2.0</td>
<td>0.245</td>
<td>2.59</td>
<td>0.250</td>
<td>0.239</td>
<td>0.98</td>
<td>1.03</td>
<td>30/07/96</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.241</td>
<td>3.52</td>
<td>0.248</td>
<td>0.238</td>
<td>0.97</td>
<td>1.01</td>
<td>30/07/96</td>
<td></td>
</tr>
<tr>
<td>3.0-5.0</td>
<td>0.232</td>
<td>2.53</td>
<td>0.238</td>
<td>0.218</td>
<td>0.97</td>
<td>1.06</td>
<td>18/09/95</td>
<td>C_{vi} = c_{v} (centre)</td>
</tr>
<tr>
<td></td>
<td>0.230</td>
<td>3.49</td>
<td>0.238</td>
<td>0.225</td>
<td>0.97</td>
<td>1.02</td>
<td>18/09/95</td>
<td></td>
</tr>
</tbody>
</table>

B. the slip increased with an increasing degree of flow stratification in a horizontal pipe, i.e. the less-uniform concentration profile provided a bigger difference between mean velocities of phases in a pipe cross section. In Table V.3 the slip ratio values for different solids in the horizontal 150 mm pipe transported in mixture flows of similar $C_{vd}$ are compared. The slip ratio values higher than one are due to inaccuracy of the measurements.
Table V.3. Slip ratio in the ascending pipe (Section I, +0 deg) and the descending pipe (Section II, -0 deg) of the 150 mm test loop installed in a horizontal position.

<table>
<thead>
<tr>
<th>Solids size [mm]</th>
<th>C_{vd} [-]</th>
<th>V_{m} [m/s]</th>
<th>C_{vi} (+0)</th>
<th>C_{vi} (-0)</th>
<th>Slip r. (+0)</th>
<th>Slip r. (-0)</th>
<th>Experim. run</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2-0.5</td>
<td>0.230</td>
<td>3.01</td>
<td>0.245</td>
<td>0.260</td>
<td>0.94</td>
<td>0.89</td>
<td>28/09/95</td>
<td>sliding bed, gradual flow homogenisation</td>
</tr>
<tr>
<td></td>
<td>0.235</td>
<td>3.50</td>
<td>0.239</td>
<td>0.251</td>
<td>0.98</td>
<td>0.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.233</td>
<td>4.24</td>
<td>0.231</td>
<td>0.242</td>
<td>1.01</td>
<td>0.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.237</td>
<td>5.49</td>
<td>0.229</td>
<td>0.236</td>
<td>1.03</td>
<td>1.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5-1.0</td>
<td>0.230</td>
<td>3.00</td>
<td>0.274</td>
<td>0.263</td>
<td>0.84</td>
<td>0.88</td>
<td>06/03/95</td>
<td>restratific. effect</td>
</tr>
<tr>
<td></td>
<td>0.237</td>
<td>3.50</td>
<td>0.293</td>
<td>0.267</td>
<td>0.81</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.239</td>
<td>3.89</td>
<td>0.292</td>
<td>0.280</td>
<td>0.82</td>
<td>0.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.241</td>
<td>4.26</td>
<td>0.279</td>
<td>0.278</td>
<td>0.86</td>
<td>0.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4-2.0</td>
<td>0.212</td>
<td>2.50</td>
<td>0.261</td>
<td>0.291</td>
<td>0.81</td>
<td>0.73</td>
<td>25/04/95</td>
<td>restratific. effect</td>
</tr>
<tr>
<td></td>
<td>0.218</td>
<td>2.99</td>
<td>0.297</td>
<td>0.263</td>
<td>0.73</td>
<td>0.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.220</td>
<td>3.49</td>
<td>0.293</td>
<td>0.252</td>
<td>0.75</td>
<td>0.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.215</td>
<td>4.24</td>
<td>0.296</td>
<td>0.289</td>
<td>0.73</td>
<td>0.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.218</td>
<td>5.50</td>
<td>0.266</td>
<td>0.278</td>
<td>0.82</td>
<td>0.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.0-5.0</td>
<td>0.229</td>
<td>2.70</td>
<td>0.302</td>
<td>0.319</td>
<td>0.76</td>
<td>0.72</td>
<td>14/09/95</td>
<td>weak restratific. effect</td>
</tr>
<tr>
<td></td>
<td>0.226</td>
<td>3.01</td>
<td>0.298</td>
<td>0.302</td>
<td>0.76</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.221</td>
<td>3.50</td>
<td>0.294</td>
<td>0.280</td>
<td>0.75</td>
<td>0.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.214</td>
<td>4.25</td>
<td>0.281</td>
<td>0.269</td>
<td>0.76</td>
<td>0.80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C. the slip tended to be less important (i.e. the slip ratio tended to approach unity) in the slurry flow when solids concentration increased. This was particularly evident in Sand1 slurry flows where the comparison of slip ratio values for a specific $V_{m}$ and different slurry densities was not affected by a restratification effect.

Table V.4. Slip ratio in the ascending pipe (Section I, +0 deg) and the descending pipe (Section II, -0 deg) of the 150 mm test loop installed in a horizontal position.

<table>
<thead>
<tr>
<th>Solids size [mm]</th>
<th>C_{vd} [-]</th>
<th>V_{m} [m/s]</th>
<th>C_{vi} (+0)</th>
<th>C_{vi} (-0)</th>
<th>Slip r. (+0)</th>
<th>Slip r. (-0)</th>
<th>Experim. run</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2-0.5</td>
<td>0.119</td>
<td>3.00</td>
<td>0.137</td>
<td>0.148</td>
<td>0.87</td>
<td>0.80</td>
<td>25/09/95</td>
<td>no restratific. effect</td>
</tr>
<tr>
<td></td>
<td>0.174</td>
<td>3.00</td>
<td>0.203</td>
<td>0.213</td>
<td>0.86</td>
<td>0.82</td>
<td>03/01/95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.230</td>
<td>3.01</td>
<td>0.245</td>
<td>0.260</td>
<td>0.94</td>
<td>0.89</td>
<td>28/09/95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.306</td>
<td>3.00</td>
<td>0.317</td>
<td>0.331</td>
<td>0.97</td>
<td>0.92</td>
<td>02/10/95</td>
<td></td>
</tr>
<tr>
<td>3.0-5.0</td>
<td>0.090</td>
<td>2.74</td>
<td>0.134</td>
<td>0.142</td>
<td>0.67</td>
<td>0.63</td>
<td>12/09/95</td>
<td>weak restratific. effect</td>
</tr>
<tr>
<td></td>
<td>0.170</td>
<td>2.78</td>
<td>0.219</td>
<td>0.248</td>
<td>0.78</td>
<td>0.69</td>
<td>13/09/95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.229</td>
<td>2.70</td>
<td>0.302</td>
<td>0.319</td>
<td>0.76</td>
<td>0.72</td>
<td>14/09/95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.271</td>
<td>2.77</td>
<td>0.354</td>
<td>0.363</td>
<td>0.77</td>
<td>0.75</td>
<td>15/09/95</td>
<td></td>
</tr>
</tbody>
</table>
5.2.5 Bed sliding

The effects observed in the slurry flow during the first experiments (from December 1994 to April 1995) indicated a crucial influence of the granular bed on the slurry flow behaviour. To analyse a flow restratification effect or a slip phenomenon in the pipeline the measurement of the velocity of solids in the bed appeared to be essential. These measurements were carried out in autumn 1995 and summer 1996. Local solids velocity $v_b$ and concentration $c_b$ were measured at the bottom of the test loop section together with integral parameters (pressure gradient, mean slurry velocity and mean slurry density) in the test loop section. Concentration profiles were measured only for several chosen mean slurry velocities. During each experimental run the parameters were measured over a wide $V_m$ range covering different stages of the development in the internal structure of the tested slurry flow. $V_m$ was always higher than the deposition-limit velocity, so no stationary bed occurred in the pipeline during the experimental run.

Tests showed the same trends in the development of the solids effect ($I_m - I_f$), the local bed concentration, $c_b$, and the local bed velocity, $v_b$, under the increasing mean slurry velocity in the pipeline. A typical development of these parameters for two different solids is illustrated on Fig. 5.4. For all solids tested, the bed velocity, $v_b$, tended to increase with an increasing mean slurry velocity, $V_m$, in the pipeline. However, different developments of the relative bed velocity (expressed as $v_b/V_m$) with an increasing $V_m$ were observed for beds composed of finer particles (Sand 1) and of coarser particles (Sand 2, Sand 3). In a flow of a fine-particle slurry a monotonic increase in $v_b/V_m$ corresponded with a monotonic decrease in both the bed concentration $c_b$ and the solids effect $I_m - I_f$. In a coarse-particle slurry flow the $v_b/V_m$ first decreased within a $V_m$ range not far above the deposition-limit value (till $V_m = 3.25$ m/s on Fig. 5.4). Comparison of the course of the $v_b/V_m$ curve with courses of the $c_b$-$V_m$ curve and the $I_m$-$V_m$ curve within this $V_m$ range indicates that the $v_b/V_m$ drop was associated with the process of uniform bed erosion detected during the concentration-profile tests and discussed above. At $V_m > 3.25$ m/s the bed was gradually restored and this process was associated with a strong increase in $v_b/V_m$.

A granular bed composed of coarser particles slid faster than a bed composed of finer particles in pipeline flows of comparable solids concentrations and mean velocities (see Fig. 5.5a). Exceptions to this trend were caused by the interference of a flow restratification effect within a certain $V_m$ range, as seen on Fig. 5.4 for $3.1$ m/s < $V_m$ < $4.0$ m/s. The difference in $v_b$ for different solids diminished for high mean slurry velocities at which a fine-particle bed gradually dissolved, forming a turbulent suspension. A significant difference in the velocity of a sliding bed was restricted to an approximate range $V_{dl} < V_m < 1.75V_{dl}$.

Grains near the bottom of the pipeline slide faster within a highly concentrated slurry flow than within a flow of lower slurry density but the same slurry velocity. This trend was observed in the flows of all materials tested. For Sand 1 slurry flow the trend is illustrated in Fig. 5.5b.
Figure 5.4. Bed parameters and their impact on the hydraulic gradient.
Figure 5.5a. Solids velocity at the bottom of the 150 mm pipe for slurries of different transported solids.

Figure 5.5b. Solids velocity at the bottom of the 150 mm pipe for various mean slurry velocities and solids concentrations.
5.2.6 Observed phenomena in a light of experiments in other laboratory

The Saskatchewan Research Council (SRC) is the only research centre producing and publishing the slurry pipeline data comparable with those from our tests, i.e. flow data including information about an internal structure of the slurry flow. The most complete data set from the SRC laboratory pipelines of internal diameters of 53.2 mm, 263 mm and 495 mm was published in Gillies (1993). Slurry flow tests in SRC were carried out for a wide range of solids concentrations \(0.15 \leq C_{v, d} \leq 0.40\) presenting a good opportunity to compare the concentration effects on slurry flow parameters detected in our pipeline with those detected in SRC pipelines. For the majority of situations tested the SRC data indicate the same trends in the development of slurry flow parameters with the increasing spatial volumetric concentration \(C_{v, i}\), as those detected in our laboratory pipeline. The SRC data for two sands (0.1 mm to 0.3 mm and 0.3 mm to 1.0 mm) and gravel (2 mm to 6 mm) showed that

- the deposition-limit velocity \(V_{dl}\) decreased with \(C_{v, i}\) in flows of different particle and pipeline sizes
- the slip ratio increased with \(C_{v, i}\) in flows of different particle and pipeline sizes
- the local solids velocity at the bottom of a pipeline increased with \(C_{v, i}\) in flows of different particle and pipeline sizes for mean slurry velocities \(V_{m}\) only slightly above \(V_{dl}\).

5.3 Analysis

All phenomena detected in our laboratory circuit are mutually connected and provide a picture of the slurry flow behaviour in a pipeline. The slurry flow observations can be summarised by the following conclusions:

A. The solids contribution to the total resistance of mixture flow is predominantly due to the resistance of particles transported in the contact layer above the bottom of the pipeline. The solids effect \(f_{m} - f_{l}\) is strongly dependent on the degree of flow stratification in a pipeline, high values of the solids effect are associated with a strong stratification of the slurry flow.

B. The mean slip velocity between solids and liquid in a pipeline cross section is predominantly due to the flow stratification. Tests in pseudo-homogeneous flows indicated a negligible mean slip velocity within the suspension flow. This is because the local slip velocity for individual grains in suspension is negligible. From the literature it appears that according to experience of seepage flow through a granular plug, the local slip velocity between particles and interstitial liquid within a sliding submerged bed is also negligible. The dense-phase flow tests in Wilson & Brown (1982) had a relative velocity to the order of mm/s. If the mean slip velocity occurs in a pipeline cross section occupied by a partially-stratified flow, it is due rather to a mutual shift between the highly concentrated layer and the dilute layer of different velocities than to integrating the local slip velocities over a pipeline cross section. The movement of grains occupying a highly concentrated layer is hindered by their mutual contacts and contacts with the pipeline wall. Grains suspended in the dilute layer are not affected by contact hindrance. As a result of this a highly concentrated
layer above the bottom of a pipeline moves more slowly than a dilute layer near the top of the pipeline. Experiments showed a strong influence of the degree of flow stratification on the slip phenomenon in a pipeline. A measure of slip given by the slip ratio was further found strongly dependent on the local bed velocity in partially-stratified flows.

The observed phenomena indicate the different impacts of suspended particles and contact particles on a slurry flow behaviour. The friction and the slip in slurry flows exhibiting some degree of stratification are highly sensitive to the exchange of particles between the contact bed and the suspension if the exchange occurs as a result of the change in slurry flow conditions. The slurry flow conditions in the laboratory circuit are controlled by setting the mean slurry velocity and concentration. This suggests that an idealised two-layer pattern for a partially-stratified flow can be applied to the partially-stratified flows that have been tested and the principle of a force balance for flow in a two-layer pattern can be used as a suitable basis for an analysis of the phenomena observed during the experiments.

5.3.1 Application of two-layer model principles

The principles of the force balance in the two-layer flow pattern explain the observed faster sliding of a coarser bed, compared to a finer bed of a comparable thickness and solids concentration, by the fact that the coarser granular bed is subjected to a higher driving force than the bed composed of finer particles. This is due to the higher shear stress exerted at the coarser bed surface in stratified flows. The shear stress increases with the roughness of the granular bed surface and the roughness size is derived from the size of the particles occupying the bed surface. Consequently, as a result of this mechanism a coarser bed requires a lower mean slurry velocity to maintain its sliding motion than a finer bed of the same thickness and concentration. This may explain the lower values of the deposition-limit velocity, \( V_{dl} \), observed in flows of finer slurries in the horizontal 150 mm pipeline. At the initiation of motion the bed was observed to be slightly thicker in the flow of coarser solids than in the flow of finer solids at the same concentration in slurry because more particles were suspended in the finer-solids slurry. However, the effect of the higher resisting force produced by the thicker bed in coarser flows was overcome by the higher driving force from the pressure gradient and the bed surface roughness.

The higher density of the sand-water mixture in the partially-stratified flow promotes bed sliding. As observed in our laboratory pipeline, this is the case even if the increase of the slurry density causes formation of a denser and larger contact bed. The increment in the bed driving force due to an increase in the slurry density exceeds the increment in the bed resisting force due to the larger and denser bed. The increment of the driving force on the bed is due to an increase in solids concentration of the suspension above the bed. A denser suspension stream exerts a higher interfacial shear stress, so that a higher driving force acts on the bed via the interface. If the impelling effect of the increased driving force exceeds the resisting effect arising from a thicker contact bed, the contact bed is accelerated. The observed shift of the resistance curve extremes (minima and maxima) to the lower \( V_{m} \) for the denser slurry flows, compared with the extremes at the resistance curve for the less concentrated slurry, is related to
the bed acceleration in the denser slurries. The friction loss is related to the bed velocity in the pipeline as can be seen in Fig. 5.4.

The slip velocity variation in the partially-stratified flow is caused predominantly by a different mutual shift between two layers of different solids concentration, velocity and thickness. If the particle-rich layer is accelerated relatively to the particle-lean layer the mean slip velocity in the flow decreases and the slip ratio approaches the value one. This was the case, for instance, in the flow of Sand1 slurry under the condition of an increasing $C_{\text{vd}}$ and constant $V_{\text{m}}$. The measured slip ratio and relative bed velocity increased in the denser Sand1 slurry flow. The effect of the bed acceleration apparently exceeded the effect of the contact bed growth in the flow when the slurry density increased.

Principles of the two-layer model are based on the assumption that there are two physical mechanisms for the solid particle support in a pipeline - an interparticle contact and a particle suspension in the carrying liquid. These mechanisms are analysed below for partially-stratified flows observed in the laboratory circuit DN150.

### 5.3.2 Transition zone in partially-stratified flows

In an ideal fully-stratified flow all particles occupy the bed. The bed exhibits a uniform concentration distribution and the particles occupying the bed are in permanent contact with the neighbouring particles. At the top of the bed the concentration drops to zero so the flow above the bed is particle-free. In an ideal partially-stratified flow the ceiling of the contact layer is still a sharp interface but a considerable portion of solid particles is found also above the interface. All these particles are suspended in the carrying liquid. In real slurry flows the sharp interface is often replaced by a transition zone between the region in which particles in continuous contact (Coulombic interparticle contact) form an en bloc sliding, or a stationary, bed, and the region in which solid particles are supposed to have no contact with each other and are supported exclusively by the diffusive effect of the carrier turbulence. The character of solid particle support changes gradually across the transition zone in which the support mechanisms due to interparticle contact and particle suspension meet. A concentration profile within the transition zone typically exhibits the change in the course of the profile curve associated with a point of inflexion. The concentration gradient within the part of the transition zone below the point of inflexion decreases with the decreasing distance from the uniform bed.

The observations of slurry flows in the 150 mm test loop suggest that there are two types of transition zones differing basically in the character of flow within the zone. If the turbulent eddies of the carrier within the transition zone are capable of suspending transported solid particles, the flow in the transition zone has the character of a dense-phase turbulent-suspension flow, while in pipeline flow without an effective turbulent suspension mechanism the flow in the transition zone has the character of a sheet flow.

Sheet flow refers to the flow of slurry in the region of the high shear stress above the upper plane bed. The region occupied by the sheet flow is called the shear layer. To date, sheet flow has been reported only for a high-stress-region above the fixed
(stationary) bed (e.g. Nnadi & Wilson, 1992; Wilson & Pugh, 1995). Our experiments show that the shear layer may also occur in the slurry flows handled during a slurry pipeline operation. In slurry pipelines the formation of a stationary bed is usually avoided but the high-shear-stress flow might occur above an en bloc sliding bed of uniform concentration distribution.

The concentration-gradient drop within the transition zone occupied by the flow of the dense-phase turbulent suspension is associated with a hindering effect of the surrounding particles on particle settling in a highly concentrated mixture. At the bottom of the transition layer the interaction between the settling particles and the turbulent eddies does not produce any concentration gradient. The concentration gradient within the transition zone occupied by the sheet flow is due to collisions between particles travelling at different velocities in different vertical positions within the sheet flow. Both types of transition zones are usually of thickness of an order of magnitude larger than the particle diameter in the slurry pipeline.

Turbulent suspension may occur in the uppermost region of the shear layer, but the shear layer effect on slurry flow behaviour is most pronounced for flows in which liquid turbulence is not an effective suspension mechanism. In the literature the particles transported within the shear layer are considered to contribute exclusively to the contact load. At the top of the shear layer the solids concentration approaches the value zero. At the bottom of the shear layer the solids concentration reaches a value typical in a uniformly-distributed bed, i.e. a loose-poured value in a stationary bed or a slightly lower value in an en bloc sliding bed. Consequently, a steep concentration gradient is typical in the shear layer. It is accepted in the literature (e.g. Wilson, 1984 based on Turner, 1973) that the turbulent suspension mechanism is suppressed within a flow with a large concentration gradient. This is caused by a reduction in vertical turbulent mixing in flows with large concentration gradients. This supports the assumption that the effect of carrier turbulence on the processes within the shear layer can be neglected.

At the bottom of the shear layer, the uniform bed is sheared off due to high shear stress and sheared particle layers are in almost permanent mutual contact. Nearer to the top of the shear layer the particles have sporadic, rather than continuous, mutual contact. Solid particles are in sporadic contact in a major part of the shear layer, interparticle contacts occur less often near the top of the shear layer. In the absence of turbulent suspension, the solid particles are supported exclusively by repulsion forces caused by the high shear rate within the shear layer. Bagnold (1954) measured the repulsion effect of interparticle collisions in a region with a high shear rate and called it "dispersive pressure". His concentric-rotating-drum experiment is described in Chapter 2. In his application of the dispersive pressure concept to a horizontal slurry flow Bagnold (1956) stated that the dispersive granular stress at any level within the sheared granular layer must be equal to the submerged weight of particles dispersed above that level. Thus the dispersive granular stress balances the gravitational effect on solids within the shear layer.

5.3.3 Identification of the shear layer

The borders and the thickness of the transition layer can be estimated from the shape of the concentration profile. The recognition of the type of transition layer requires
more information. Experimental results show that the slurry flow containing the finest tested material (Sand 1) behaves differently than the flow containing the coarser materials tested (Sand 2, Sand 3 and Gravel). This is due to the different character of flow within the transition layer. The sheet flow within the transition layer of partially-stratified flows is identified according to the following phenomena detected in the test loop:

a. the concentration gradient across the shear layer is steeper than that across the transition layer occupied by the dense-phase turbulent suspension,
b. the position of the point of inflexion of the concentration-profile curve does not change with changing the mean slurry velocity in pipeline flows exhibiting the shear layer, while in flows containing a turbulent suspension the position of the point of inflexion approaches the pipeline bottom if mean slurry velocity increases,
c. the slurry flows containing a shear layer exhibit the flow restratification effect under the condition of an increasing mean slurry velocity in the pipeline, such an effect was not detected in the flows in absence of a shear layer,
d. very different degrees of the slurry flow stratification are observed in the ascending and the descending pipes of the inclined test loop for flows exhibiting a shear layer, the difference is weak in flows with the turbulent suspension.

ad a.

In the partially-stratified flow with turbulent suspension in the upper layer and the transition layer, the turbulent concentration profile should be linked to the concentration profile of the contact layer at the point where the turbulent suspension starts to be effective. Measured concentration profiles were examined by using an adapted turbulent diffusion model of Rouse and Schmidt (Eq. 2.31 in Chapter 2) to evaluate whether the observed concentration gradients were due to turbulent suspension.

The Rouse-Schmidt turbulent model reflects the fact that transfer of momentum, which is responsible for a solid particle suspension, is associated with the properties of the vertical turbulent eddies, i.e. with their length, l, and attributed turbulent fluctuation velocity, $v'_{y} = \sqrt{\bar{v}^2_y}$, i.e. the root mean square of velocity fluctuations in the y-direction. In the Rouse-Schmidt model the mixing length l and turbulent fluctuation velocity $v'_{y}$ form the group $0.5 v'_{y} l$ which is considered the solids dispersion coefficient, $\varepsilon_s$. An appropriate determination of $\varepsilon_s$ is a basic obstacle to the application of the model. Substituting $\tilde{v}'_{y}$ by the shear velocity, $u*$, (widely used in the literature and discussed further in Chapter 7) reduces the problem to the determination of the mixing length, l. The model is also sensitive to the settling velocity of the solids. In its original version it was proposed that the terminal settling velocity of a particle should represent the particle settling tendency. The model reflected the channel flows of low concentration suspensions and the influence of local concentration on the settling velocity was not considered. In a pipeline conveying slurry the concentration effect cannot be omitted. The local-concentration effect on the solids settling tendency within
the flowing liquid can be incorporated in the Rouse-Schmidt model by substituting the terminal settling velocity with the Richardson & Zaki equation (Eq. 2.28 in Chapter 2) for the hindered settling velocity. The solids settling generates a flow of displaced fluid around the settling particles in the quiescent fluid. The Richardson & Zaki equation expresses the influence of this flow on particle settling velocity. In turbulent flow the liquid momentum transfer in vertical direction by eddies substitutes the liquid-displacement effect considered by Richardson and Zaki for the quiescent fluid. Usually the suspended particles do not settle in flowing liquid so no displacement of liquid due to a particle-cloud settling occurs. However, the small mutual distance between particles in a concentrated cloud causes the velocity gradient between liquid and particle to be high as is the vertically-directed drag force exerted by liquid on particles. Therefore a reduction in the particle settling tendency, described by the Richardson-Zaki equation, must be taken into account in the evaluation of highly concentrated turbulent suspensions.

Three different configurations of the Rouse-Schmidt turbulent-diffusion model which differed in the determination of the mixing length and the particle settling velocity in a pipeline slurry flow were selected. These were:

Configuration I.: The mixing length, \( l \), was equal to \( \kappa y_w \) where \( y_w \) was a vertical distance from the pipeline wall. Taking \( y \) as the vertical distance from the pipeline bottom, \( y_w = y \) for \( y \leq D/2 \) and \( y_w = D - y \) otherwise. \( \kappa = 0.4 \) according to von Kármán. The terminal velocity of particle, \( v_t \) (i.e. taking its non-spherical shape into account), was considered to be the particle settling velocity in this model.

Configuration II.: In an accordance with the variation of the mixing length across a pipeline cross section which is discussed in Chapter 7, see Fig. 7.4, the mixing length, \( l \), was estimated to be equal to \( \kappa y_w \) in a region \( y_w \leq D/4 \) and to \( \kappa D/4 \) in the flow core. The terminal velocity of particle, \( v_t \), was used in the model.

Configuration III. This configuration used the same distribution of the characteristic mixing length across a pipeline as the configuration II. The hindered settling velocity of particle, \( v_{th} \), was used as the settling velocity in the model.

Theoretical concentration profiles derived by using the Rouse-Schmidt model were linked to measured concentration profiles at several characteristic points. In Figs. 5.6a and 5.6b the theoretical profiles are compared with measured profiles for two different slurries at very similar \( V_{III} \). In the Sand1-slurry flow (Fig. 5.6a) an almost constant concentration gradient was detected along the entire pipeline cross section, thus the curve of a theoretical turbulent profile was linked to the lowest measured position (\( y/D = 0.10 \)). In the Sand2-slurry flow (Fig. 5.6b) a theoretical profile was linked to two different levels above the en bloc sliding bed, one given by the point of inflexion (\( y/D = 0.62 \)) and second by the point of a sudden decrease of the concentration gradient (\( y/D = 0.50 \)).
Figure 5.6a. Comparison of theoretical profile due to turbulence with measured concentration profile for Sand1-slurry flow.

A comparison between the theoretical curves and the measured profiles shows that the diffusive effect of liquid turbulence was responsible for a concentration gradient in the flow of the Sand1 slurry but not in the coarser slurry flow. While in Sand1-slurry flow the turbulent-model curve reasonably matches the measured profile (Fig. 5.6a), in the Sand2-slurry flow the theoretical turbulent profile tends to provide more suspension than was observed (Fig. 5.6b). Figure 5.6b shows that this is the case for each model configuration used and each chosen linking point in a measured concentration profile. Thus in flows of particles coarser than Sand 1 the turbulent suspension was not an effective particle support mechanism, at least till flow velocity of about 1.7 times higher than the deposition limit velocity $V_{dl}$. In these flows the deformation of the concentration profile curve within the transition zone was not due to the turbulent suspension mechanism.

ad b.

The shear layer is represented by an almost linear ramped portion of a concentration profile curve. The layer is delimited by the curve points at which the concentration gradient shows a considerable change. Thus the shear layer is characterised by an approximately constant concentration gradient. The point of inflexion is in the centre of the line-like portion of a concentration profile curve (Fig. 5.7).
Figure 5.6b. Comparison of the theoretical profile due to turbulence with the measured concentration profile for Sand2-slurry flow.
Figure 5.7. Concentration profile in the horizontal 150 mm pipeline. Shear layer above the sliding bed.

The positions of the points of inflexion, plotted in Fig. 5.8, are obtained by computing the second derivative of a measured-concentration-profile regression equation $c_V = f(y/D)$ for the condition

$$\frac{d^2 c_V}{d(y/D)^2} = 0$$

(5.1).

The $c_V = f(y/D)$ equation is produced by interpolating the measured profile by a polynomial regression of the fourth order

$$c_V = \chi_0 + \chi_1 \left( \frac{y}{D} \right) + \chi_2 \left( \frac{y}{D} \right)^2 + \chi_3 \left( \frac{y}{D} \right)^3 + \chi_4 \left( \frac{y}{D} \right)^4$$

(5.2).

For the finest slurries tested, the concentration distribution along the pipeline cross section tended to become gradually more uniform with the increasing slurry velocity $V_m$. At the same time, the position of the point of inflexion tended to approach the pipeline bottom (see Fig 5.8 for Sand 1) because even more particles were suspended by the carrier turbulence and the layer of particles in contact became thinner. This tendency became weaker in highly concentrated slurry flows, presumably due to the
Figure 5.8. Point of inflexion on concentration profile curves measured in the horizontal 150 mm pipe (Section II).
hinder effect of the high solids concentration on the particle settling tendency and thus on the shape of the concentration profile in a dense flow. In the coarser mixtures the shape of the concentration profiles was also deformed with the increasing $V_m$, although the position of the point of inflexion did not change in slurry flows of approximately constant mean concentration. The positions of the point of inflexion on curves of concentration profiles for coarse slurries processed in Fig. 5.8 were slightly sensitive to $V_m$ only for flows of $C_{v_d}$ higher than approximately 25%. Profiles show that the position of the point of inflexion, $y_{inflex}$, on concentration profiles of coarse flows is strongly dependent only on mean spatial concentration of solids, $C_{v_d}$.

The constant position of the point of inflexion on profile curves of coarse-slurry flow under the increasing $V_m$ reflects the fact that the mechanisms deforming the concentration curve are predominantly active within a transition zone, i.e. in the region associated with the top of the bed. The region may expand or contract according to flow conditions at this flow interface. Different behaviour is observed in the Sand1-slurry flow in which solids distribution is governed by the turbulent suspension mechanism acting in the entire discharge area of a pipeline cross section. If there is no stationary bed in the pipeline, the discharge area is equal to the area of a pipeline cross section.

The deformation of the concentration profile curve due to turbulent diffusive effect of liquid flow is not confined to a region associated with the top of the bed. Turbulent eddies capable of suspending solid particles often occur in the entire discharge area of a pipeline cross section thus also in the fluidized bed, of almost uniformly distributed solids at concentrations lower than the loose-poured concentration, flowing at the bottom of the pipeline. Therefore particles suspended by turbulence may be found virtually everywhere in the discharge area of the pipeline. Intensity of liquid turbulence increases with increasing $V_m$ in the pipeline and so increases the amount of particles supported by liquid diffusive eddies. The mixture flow is gradually homogenised under the increasing $V_m$. In low concentration mixtures the contact bed diminishes as the flow picks up the bed-particles and maintains them in a suspension. This process is reflected by a fall in the position of the point of inflexion on the concentration profile curve if $V_m$ increases in a pipeline. In highly concentrated flows a fluidized bed occupies the majority of the discharge area in a pipeline cross section. The exponential profile curve, characteristic of dilute turbulent suspensions, is restricted to the uppermost zone in the discharge area. Flow homogenisation in the discharge area under the increasing $V_m$ is due rather to a further expansion of the fluidized bed (an increase of a void fraction in the bed) than to the bed diminishing, thus a drop in a position of a point of inflexion becomes less, as shown for the dense Sand1 slurries in Fig. 5.8.

ad c.

The flow exhibiting the restratification effect, described earlier in connection with Fig. 5.4b, tends to be restratified at high slurry velocities. Thus the gradual flow homogenisation at the lower velocities is not due to liquid turbulence because its intensity at the low velocities is less than at the high velocities where the restratification indicated poor or no suspension. Instead, the gradual flow homogenisation is due to shear layer development at the top of the granular bed in
stratified flow. A steep velocity gradient between the granular bed at the bottom of the pipeline and the upper layer produces interfacial shear stress. If the interfacial shear stress rises above a threshold value for an upper-plane-bed regime, the bed undulations disappear. Shearing of the plane bed starts as a result of which the granular "carpets" of particle-size thickness at the bed surface start to move over each other with a velocity that decreases with increasing depth of the carpet below the bed surface. A shear layer is developed. Further increase in the interfacial shear stress causes the shear layer to become thicker and the particles in a major part of the layer lose their permanent mutual contact. This is replaced by interparticle collisions. The diminishing of the shear layer and the restoration of the uniform bed at higher slurry velocities is due to the gradual decrease in the velocity gradient between the bed and the upper stream, and thus in the lower interfacial shear stress in faster slurry flows.

The explanation of the flow restratification effect is illustrated by the results of parallel measurements of the pressure gradient, mean slurry velocity, local bed concentration \( c_b \), and local bed velocity \( v_b \), in the Sand3-slurry flow plotted in Fig. 5.4. The Sand3-slurry flow was of relatively low concentration \( C_{vd} = 15 - 16\% \) so the bed was thin. In such flows the bottom of the expanded shear layer may approach the bottom of the pipeline and the process of the expansion and the contraction of the shear layer can be measured as the variation in the \( c_b \) with the increasing \( V_{mf} \). Furthermore the variation in the velocity of flow in the layer above the thin bed does not differ significantly from the variation in velocity \( V_{m} \). Therefore the ratio \( v_b/V_{m} \) might be considered a suitably accurate measure of the velocity gradient at the top of the bed. The drop in the \( v_b/V_{m} \) indicates an increase in the velocity gradient and thus in the bed shear stress. The increase in the \( v_b/V_{m} \) indicates that the bed shear stress becomes smaller.

At the velocity \( V_{m} = 2.50 \text{ m/s} \) (i.e. approximately \( 1.25V_{d1} \)) bed sliding was already fully developed (the motion of the bed was steady without any disturbances, the bed velocity near the pipeline bottom was 1.4 m/s) and the majority of particles occupied a uniformly distributed bed with the solids fraction occupying about 53% of the bed volume. When the velocity \( V_{m} \) in the test pipeline was gradually increased, the shear layer at the top of the bed was gradually expanded (\( c_b \) decreased) taking over part of the area occupied earlier by the uniform bed. However, when the velocity \( V_{m} \) increased above the approximate value of 3.3 m/s, the shear layer gradually diminished again (\( c_b \) increased). Comparison of the \( c_b \) versus \( V_{m} \) plot with the \( v_b/V_{m} \) versus \( V_{m} \) plot on the Fig. 5.4 shows that the shear layer at the top of a sliding bed became thicker when the bed shear stress grew as indicated by the \( v_b/V_{m} \) ratio drop in the range 2.5 m/s < \( V_{m} < 3.3 \text{ m/s} \). At \( V_{m} \) of about 3.3 m/s the resisting force of the still more eroded bed became too small to continue preventing the faster acceleration of the bed sliding within the accelerated slurry flow and \( v_b/V_{m} \) started to rise with further increasing \( V_{m} \). Therefore the bed shear stress decreased gradually and consequently the shear layer diminished. The bed sliding en bloc at the bottom of a pipeline was gradually restored.

ad d.

The sharp flow stratification in the descending pipe and the gradual concentration change across the pipe cross section in the ascending pipe of the test loop inclined to the angle 25 deg and 35 deg was detected in flows of the slurries of Sand 2, Sand 3 and Gravel. This effect was due to the very different velocity gradient, and so caused
by the different shear stress at the top of sliding bed in pipes of different inclinations. The shear stress at the top of the slowly sliding bed in the ascending pipeline was bigger than that at the top of the fast sliding bed in the descending section of the test loop. Flow of the slurry of Sand 1 at the same pipe inclinations demonstrated a considerably smaller difference in the shapes of concentration profiles, suggesting that here the influence of the interfacial shear stress on the deformation of the concentration profile was much less important. The effect is described in more detail in Chapter 6.

A transition zone behaving as a shear layer was identified in the flow of Sand 3 slurry and the Gravel slurry flow in the entire range of tested $V_m$ and in the flow of Sand 2 slurry for $V_m$ lower than approximately 4.5 m/s. A transition zone behaving as a dense-phase turbulent-suspension layer was observed in the Sand 2 flow at $V_m \geq 4.5$ m/s and in the Sand 1 flow at all tested velocities $V_m$.

5.3.4 Solid particle support within the shear layer

The development of a shear layer is associated with the variation of the shear stress at the boundary at which the shear layer is linked to the uniformly distributed bed. The bed shear stress varies with the velocity gradient at the boundary and thus with the difference between the velocity of the *en bloc* sliding bed and the mean velocity of flow above the bed. In our data, a variation in the velocity gradient is indicated by a variation in the difference between the local bed velocity $v_b$ and mean slurry velocity $V_m$ or by a variation in the relative bed velocity, $v_b/V_m$. As can be seen in Fig. 5.4 an expansion of the shear layer, detected as a drop in the local bed concentration $c_p$, is associated with a drop in $v_b/V_m$. This indicates that the shear layer expansion is due to an increase in the velocity gradient between layers and so due to an increase of the shear stress at the bottom of the shear layer. Fig. 5.4 also reveals that the expansion of the shear layer leads to a decrease in the hydraulic gradient. Evidently, solid particles contribute much less to the slurry flow resistance when they occupy a shear layer than when they occupy an *en bloc* sliding bed.

The contribution of solid particles in a shear layer to the total flow resistance may be determined by using Bagnold's concept. Bagnold's dispersive stress is assumed to be the quantity exclusively balancing a gravity effect on solid particles within the shear layer. The Saffman lift force may play a role in particle support near a pipe wall (Gillies, 1993) where a combined effect of slip and high shear occurs, but it can be considered negligible within a shear layer. The dispersive normal stress, $\sigma_S$, equal to the submerged weight of particles in sporadic contact in a sheared granular body, contributes to the flow resisting force via a particulate shear stress, $\tau_S$, equal to the product of $\sigma_S$ and the dynamic friction coefficient for particles in mobile beds $\tan \phi'$. This coefficient is the dynamic equivalent of the coefficient given by the angle of an internal friction in a static granular body. The static friction coefficient is dependent only on properties of the granular body. Bagnold (1954) obtained an approximately constant value for the dynamic friction coefficient from measurements of $\sigma_S$ and $\tau_S$ in a grain-inertia regime at high shear rates. He recognised, however, that this constant value was higher in mixtures of volumetric concentrations higher than approximately 0.56 ($\tan \phi' = 0.40$) than for concentrations below 0.56 ($\tan \phi' = 0.32$). This suggests
that the dynamic friction coefficient is sensitive to the character of the interparticle contact. In mixtures of concentrations lower than approximately 0.56, particles have only sporadic contacts so particulate stresses are due to collisions between particles.

The relationship between \( \tan \phi' \) and the dimensionless bed shear stress \( \theta \) (for definition see Eq. 3.36) by Bagnold (1966) given in Fig. 5.9 shows that the wholly inertial conditions characteristic of the grain-inertia regime apply to sheet flow, providing that the size of particles occupying the bed and the shear layer is large enough.

\[
\frac{\tau}{P} = \tan \phi'
\]

**Figure 5.9.** Solid-friction coefficient \( \tan \phi' \) in terms of the dimensionless bed shear stress for quartz-density solids of various sizes in water, from Bagnold (1966).

The sheet flow occurs at the top of the granular bed if \( \theta > 0.8 \) (e.g. Wilson & Nnadi, 1992). According to the plot in Fig. 5.9, the wholly inertial conditions have already been reached at \( \theta = 0.8 \) when the sheared grains are of the coarse-sand size or coarser. The plot in Fig. 5.9 does not include the effect of solids concentration in the sheared mixture reported in Bagnold (1954).

The different values for the dynamic friction coefficient \( \tan \phi' \) found in a sheared bed in which particles maintain permanent contact and in a sheared bed in which the contacts are only sporadic indicates that \( \tan \phi' \) should be considered as the coefficient expressing both the solids strain-stress relation and the effectiveness of the contribution of the submerged weight of particles dispersed by interparticle collisions to the total resisting force exerted by contact load solids. This effectiveness depends on the
character of the interparticle contact. The transfer of the submerged weight of particles, which have only sporadic mutual contact, to the bed below the shear layer is restricted only to moments in which contacts actually occur. Thus transfer of the submerged weight of particles within the shear layer occurs both via the particle-particle interaction and the particle-liquid interaction. At each moment a repulsion force due to interparticle collisions (the force exerted by a particulate dispersive stress) is able to maintain a portion of solids in suspension within the shear layer. One portion of the particles transfers its submerged weight to the bed below the shear layer and the remaining particles transfer their submerged weight to the carrying liquid within the shear layer. At each instant the portion of solid particles in immediate contact contributes to the contact load and the remaining particles contribute to the suspended load. This particle-support approach to a shear layer is suitable to apply to a two-layer flow concept which recognises only a permanent interparticle contact and a permanent particle suspension in each layer.

5.3.5 Solid particle suspension in different slurry flow regimes

Two different mechanisms are capable of suspending solid particles in a pipeline occupied by partially-stratified slurry flow. These are
- the diffusive effect of the carrying liquid turbulence within the discharge area of a pipeline flow or
- the dispersive effect of interparticle collisions within the shear layer.

Turbulent suspension is a product of particle-liquid interaction in the flow. Turbulent flow of the carrier suspends particles fine enough to be supported by the diffusive effect of turbulent eddies. The dispersive (repulsion) force acting on particles flowing in a region of high shear rate is capable of suspending a portion of coarse solids in slurry flows. Suspension by interparticle collision may occur in combination with the turbulent suspension, but it may also act as an exclusive suspension mechanism in slurry flows where the carrying liquid turbulence is not capable of suspending transported solid particles. The two mechanisms should be distinguished when an equation to estimate the amount of suspended solids in slurry flow is being derived for the two-layer model.

Experiments showed that the shear layer may be rather thick in coarse particle slurries flowing at velocities not far above the deposition-limit value. The energy loss due to friction is considerably lower in a slurry flow with a thick shear layer than with a less well developed shear layer. Thus the force tending to disperse solid particles within a shear layer may play an important role in suspending solid particles in a pipeline.

The high degree of stratification of the Sand3 flows and the Gravel flows at the highest tested \( V_m \) velocities in the 150 mm test loop indicated that turbulent suspension was not an active suspension mechanism in coarse particle slurries even in flows with high Reynolds numbers. In flows with low Re numbers (for \( V_m \) lower than approximately 5.0 m/s) the velocity in the layer above the bed was not higher than it was in flow with a high Re number. The velocity in the upper layer was also not significantly higher in the coarser slurry than in the finer slurry of the similar \( V_m \) and \( C_v \) as was found from a comparison of the measured local bed velocity \( v_b \) values and concentration profiles for flow of different solids within a wide range of tested \( V_m \) velocities. Thus coarse
particle suspension was caused by the high shear rate along a pipeline cross section rather than by high turbulent intensity in the flow in the upper layer.

Sand flow did not exhibit a restratification effect. The majority of particles occupying the bed at the lowest $V_m$ values was gradually suspended by turbulent eddies when $V_m$ grew. The sliding bed became thinner and its resistance to flow was reduced so the bed velocity increased enough to prevent an increase in the velocity gradient at the interface.

The flow restratification effect was responsible for a characteristic "two-peak" shape of the $I_m$-$V_m$ curve for a partially-stratified flow with a shear layer. Different sections of the curve characterise different slurry flow regimes occurring in a pipeline. These are illustrated on Fig. 5.10.

The flow restratification effect was measured in both the horizontal and the inclined test loop of the laboratory circuit DN150 described on Fig. 4.1. Furthermore the effect was also detected in a different lay-out of the test loop in the laboratory circuit. In the adapted test loop the ascending pipe was replaced by two parallel pipes and the entire test loop was prolonged. The Sand2-slurry tests carried out in January 1997 in the adapted test loop confirmed that the restratification effect measured during previous tests was not caused by the influence of a length of a straight pipe before and behind the measuring sections or by the shapes of fittings just before straight pipe sections on the slurry flow behaviour.
Figure 5.10. Different slurry flow regimes in a slurry pipeline.
5.4 Testing two-layer model to process experimental data

The testing model (TM) is a two-layer model adapted in such a way that it processes experimental data to transform the internal structure of the partially-stratified flow observed in an experimental pipeline into an idealised two-layer pattern. This transformation is done for flow conditions given by the measured integral slurry flow parameters \( V_m, I_m, C_{v4d} \) and \( C_{vi} \) and concentration profiles \( c_v(y) \). The testing model is composed of the mass balance and force balance equations of the two-layer model for the partially-stratified slurry flow. A set of the balance equations is rearranged for the testing model so that it uses the quantities measured during our experiments as the model inputs. The outputs from the testing model are used for two different tasks:

1. an examination of the ability of a two-layer model to reflect the phenomena observed in the laboratory pipeline; the TM outputs are compared with the values of independent slurry flow parameters which are measured \( (V_2, v_b) \) or calculated \( (\lambda_{12} \text{ as the TM output versus } \lambda_{12}) \) determined by various friction-law equations; the values or at least the trends (this in a case of the integral and local values of the observed parameters as \( v_b \) versus \( V_2 \)) should be the same under the variable slurry flow conditions;
   the compared parameters are considered to be mutually independent in the sense that the measured quantity which is a subject to a comparison with the TM quantity is not used as a TM input \( (v_b \text{ compared with } V_2) \) or it is not used for the computation of the TM outputs \( (\lambda_{12} \text{ equation}) \),

2. a verification of the theoretically-based equations proposed to determine the model components in a predictive version of the two-layer model and the determination of the empirical coefficients in the theoretically-based equations; the TM outputs are used for this purpose in following chapters.

### 5.4.1 Inputs and outputs

A computational scheme which exploited the maximum number of measured parameters as the inputs to a testing model was chosen:

#### Input parameters:

<table>
<thead>
<tr>
<th>Liquid:</th>
<th>( S_f, v_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solids:</td>
<td>( d, S_s, \mu_s )</td>
</tr>
<tr>
<td>Slurry flow:</td>
<td>( \Delta P, V_{m}, C_{vd}, c_v(y), C_{vi} )</td>
</tr>
<tr>
<td>Pipe:</td>
<td>( L, k, \omega, D )</td>
</tr>
</tbody>
</table>

#### Output parameters:

| \( V_{12}/D \) |
| \( C_1, C_2 (C_s, C_c) \) |
| \( V_1, V_2 \) |
| \( \lambda_{12} \) |
5.4.2 Computation scheme

5.4.3 Configuration

The configuration of the testing model was given by the chosen combination of equations for the three components of the two-layer model:

i. \( C_{2c} = C_2 - C_1 \) or \( C_{2c} = C_2 \)

ii. \( \tau_N = \text{m}(\rho_f) \) or \( \tau_N = \text{m}(\rho_2f) \)

iii. \( \tau = \text{f}_N(\rho_f) \) or \( \tau_1 = \text{f}_N(\rho_1) \), \( \tau_{12} = \text{f}_N(\rho_1) \), \( \tau_{2f} = \text{f}_N(\rho_2f) \).

The following combination of the model components was found a suitable configuration for the testing model:

ad i. \( C_{2c} = C_2 - C_1 \) (see Fig. 3.14)

Modification of the two-layer flow pattern for the partially-stratified flow is used as proposed in Shook & Roco (1991) or Gillies (1993). This distribution reflects the fact that suspended particles may also occur below the virtual interface of a stratified flow. This pattern is suitable for partially-stratified flow with turbulence as a predominant suspension mechanism. Particles suspended by carrier turbulence may occur virtually everywhere in the discharge area of a pipeline. The pattern is also suitable for a stratified flow with shear layer. For the purposes of the two-layer model a shear layer must be replaced by a sharp interface which splits the shear layer into two parts. Thus particles suspended within a shear layer may also occur below the virtual interface.
ad ii. \( F_N = fn(\rho_f) \)

Normal force exerted by the contact layer against a pipe wall is considered to be a product of the bed submerged weight in a carrier of density \( \rho_f \) (in place of \( \rho_{2f} \) in Eq. 3.47 of Gillies et al., 1991 resulting to Eq. 3.49). Thus the normal solids stress at the pipeline wall, \( \sigma_S \), is obtained from

\[
-\frac{d\sigma_S}{dy} = g(\rho_S - \rho_f)C_{2c}
\]  

(5.3).

The normal force \( F_N \) is integrated from Eq. 5.3 as

\[
F_N = g(\rho_S - \rho_f)C_{2c} \frac{D^2}{2} (\sin \beta - \beta \cos \beta)
\]  

(5.4).

The contribution of suspended sand or gravel particles to the density of a carrier exerting the buoyancy force on a submerged granular bed in the lower layer is considered negligible. Shook (1988) observed the effect of carrier density, increased by added fine clay, on the reduction of the tendency of sand to settle in a pipe. The addition of a small quantity of clay (of about 2 per cent of the total slurry volume) to the coarse-sand slurry significantly reduced friction and slip in the slurry flow. Thus fines of silt size (\( d < 0.074 \) mm) might contribute to the denser-carrier buoyancy effect on the submerged bed. However, such fine solids have not been investigated in this work. Sand-size particles are considered in this study. It has not been proved that the presence of the suspended particles of sand size and density in the carrying liquid could exert the buoyancy effect on the submerged weight of the contact bed.
ad iii. \( \tau_1 = \text{fn}(\rho_1), \tau_{12} = \text{fn}(\rho_1), \tau_{2f} = \text{fn}(\rho_2) \)

The introduction of the effect of the suspension density on boundary shear stress \( \tau \) reflects the fact that the solids effect \( (\text{Im}-\text{I}) \) values measured in our laboratory circuit for fully-suspended flows always exceeded zero. Thus solid particles suspended in the carrier flow affected the process of the mechanical energy dissipation in slurry flow. For the model purposes, the dissipation of energy due to the presence of suspended solid particles is considered to be realised through the action of the shear stresses exerted by the flowing suspension at the flow boundaries. These boundary shear stresses increase with the suspension density.

5.5 Conclusions

Experiments in a laboratory pipeline of a diameter 150 mm showed the effects of the particle size and the concentration of particles on characteristics of a mixture flow. Particle size and concentration affected the frictional head loss, the deposition-limit velocity, the velocity of bed sliding and the slip ratio in a partially-stratified flow.

Observations of mixture flows indicated different behaviour in flows of relatively fine (medium sand) mixtures and flows of relatively coarse (coarse sand, fine gravel) mixtures. The different behaviour was associated with the presence or absence of a shear layer in a partially-stratified flow of mixture. A shear layer was identified in flows of relatively coarse mixtures.

The force-balance principle applied to an idealised two-layer pattern of partially-stratified flow has been found to be capable of explaining the phenomena observed in a horizontal 150 mm pipeline.

An evaluation of the suspension mechanism for solids in a flowing mixture is of primary importance for the application of the two-layer model. An analysis of flow mechanisms based on the slurry flow observations suggests that there are two different suspension mechanisms acting in a slurry stream. Depending basically on its size, the solid particle may be suspended either by the turbulent diffusion forces within a carrying liquid or by the repulsion forces resulting from interparticle collisions within a shear layer.

The appropriate configuration of the two-layer model to interpret the measured flow characteristics in terms of the two-layer pattern of the partially-stratified flow has been proposed. This configuration includes an assumed effect of suspension density on the boundary shear stresses and does not consider any possible buoyancy effect of sand-size particles in suspension on the submerged weight of the contact layer.
5.6 References


Appendix 5

Figures: Plotted experimental data

A5.1 The $I_m - V_m$ curves

A5.1 - 1. Sand 1
A5.1 - 2. Sand 2
A5.1 - 3. Sand 3
A5.1 - 4. Gravel

A5.2 Concentration profiles

A5.2 - 1. Sand 1
A5.2 - 2. Sand 2
A5.2 - 3. Sand 3
A5.2 - 4. Gravel
A5.1 The $I_m-V_m$ curves
Figure A5.1-1a. Integral characteristics of Sand1 flow in the horizontal 150 mm pipe (Section II) (Experiments: December 1994 - January 1995).
Figure A5.1-1b. Integral characteristics of Sand1 flow in the horizontal 150 mm pipe (Section 1) (Experiments: September 1995 - October 1995).
Figure A5.1-1c. Integral characteristics of Sand1 flow in the horizontal 150 mm pipe (Section II) (Experiments: September 1995 - October 1995).
Figure A5.1-2a. Integral characteristics of Sand2 flow in the horizontal 150 mm pipe (Section I) (Experiments: February 1995 - March 1995).
Figure A5.1-2b. Integral characteristics of Sand2 flow in the horizontal 150 mm pipe (Section 1) (Experiments: August 1996).
Figure A5.1-3a. Integral characteristics of Sand3 flow in the horizontal 150 mm pipe (Section I) (Experiments: April 1995).
Figure A5.1-3b. Integral characteristics of Sand3 flow in the horizontal 150 mm pipe (Section 1) (Experiments: June 1996 - August 1996).
Figure A5.1-4a. Integral characteristics of Gravel flow in the horizontal 150 mm pipe (Section 1) (Experiments: August 1995 - September 1995).
Figure A5.1-4b. Integral characteristics of Gravel flow in the horizontal 150 mm pipe (Section 1) (Experiments: October 1995).
A5.2 Concentration profiles
Sand 0.2 - 0.5 mm

$C_{vd} = 0.11 - 0.12$

$C_{vd} = 0.16 - 0.18$

$C_{vd} = 0.22 - 0.24$

$C_{vd} = 0.28 - 0.31$

Legend: Mean slurry velocity $V_m$ [m/s].

Sand 0.2 - 0.5 mm

$V_m = 2.20 - 2.40 \text{ m/s}$

$V_m = 3.46 \text{ m/s}$

Legend: Delivered concentration $C_{vd}$ [-].

Figure A.5.2-1a. Concentration distribution in Sand1 flow in the horizontal 150 mm pipe (Section II).

Figure A5.2-1b. Concentration distribution in Sand1 flow in the horizontal 150 mm pipe (Section 1). (Experiments: September 1995 - October 1995).
Figure A5.2-1c. Concentration distribution in Sand1 flow in the horizontal 150 mm pipe (Section II).
Figure A5.2-2a. Concentration distribution in Sand2 flow in the horizontal 150 mm pipe (Section I).
Sand 0.5 - 1.0 mm

Cvd = 0.15 - 0.17

Vertical position y/D vs. Concentration

Cvd = 0.22 - 0.24

Vertical position y/D vs. Concentration

Cvd = 0.27 - 0.29

Vertical position y/D vs. Concentration

Legend: Mean slurry velocity Vm [m/s]

2.47  3.50  4.49
□ □ □

2.55  3.49  4.48
□ □ □

Sand 0.5 - 1.0 mm

Vm = 2.47 - 2.56 m/s

Vertical position y/D vs. Concentration

Vm = 3.48 - 3.50 m/s

Vertical position y/D vs. Concentration

Legend: Delivered concentration Cvd [-]

15-17  22-24  27-29
□ □ □

15-17  22-24  27-29
□ □ □

Figure A5.2-2b. Concentration distribution in Sand2 flow in the horizontal 150 mm pipe (Section I).
(Experiments: August 1995).
Figure A5.2-3a. Concentration distribution in Sand3 flow in the horizontal 150 mm pipe (Section I). (Experiments: April 1995).
Figure A5.2-3b. Concentration distribution in Sand3 flow in the horizontal 150 mm pipe (Section I).
(Experiments: June 1996 - August 1996)
Figure A5.2-4a. Concentration distribution in Gravel flow in the horizontal 150 mm pipe (Section 1). (Experiments: September 1995).
Chapter 6

Inclined steady flows: observation, analysis and modelling by the two-layer model

In this chapter the experimental and analytical results of the investigation of inclined flows are presented. The experimental tests, including measurements of the internal structure of slurry flow in a pipeline at different inclinations, revealed interesting effects of pipe inclination on the slurry flow behaviour. The mechanism for slurry flow behaviour is analysed to explain the observed phenomena. A two-layer model is formulated for inclined flows. Comparison of model outputs with experimental data shows that the model configuration proposed for inclined pipelines is appropriate. Some results of the investigation have been published in Matoušek (1996).

To date little research has been done on the effect of pipeline inclination on slurry flow behaviour. This is also because of the lack of experimental data suitable to detect the mechanisms governing slurry flow behaviour in a pipeline installed at different inclination angles. Understanding the development of the internal structure of slurry flow (concentration and velocity distributions in a cross section of a slurry pipeline) is crucial to the successful description and modelling of the effects of pipeline inclination on a slurry flow behaviour. Experimental results available in the literature are, however, all based on observations of integral parameters such as mean slurry velocity, density and manometric gradient in inclined slurry pipelines. These data were used to verify and calibrate published empirical formulas predicting the effect of pipeline inclination on friction loss and deposition-limit velocity in a slurry pipeline.

6.1 Observations

6.1.1 Inclined flows in a wide range of slopes

The observations of the pipeline inclination effect on the selected flow characteristics in a wide range of pipeline inclinations were carried out in the 150 mm laboratory circuit with the test loop inclinable from the horizontal to the vertical position (see Chapter 4). During experiments with inclined flows in a wide range of slopes each experimental run provided measurements for slurry of a constant system concentration (thus for a constant amount of solids in the entire circuit) flowing at velocity given by the same constant rpm of the pump. The first set of data was collected for the horizontal position of the test loop. The next set of data was collected in the loop
Figure 6.1a. Slurry flow characteristics in a 150 mm pipe of variable inclination.
Figure 6.1b. Slurry flow characteristics in a 150 mm pipe of variable inclination.
installed at the inclination angle $\omega$ of 15 deg and for each following measurement the pipeline inclination angle was increased in steps.

The measured characteristics from two experimental runs are plotted in Figs. 6.1a and 6.1b. The figures show that the bed at the bottom of an ascending pipe tended first to lag in a slurry flow when the pipe inclination increased, the relative bed velocity $v_b/V_m$ reached its minimum somewhere within the range $20 < \omega < 35$ deg, and then to increase its velocity when the pipe inclination angle was increased. Local solids concentration near the bottom of the ascending pipe tended to decrease with the increasing pipe slope. In the descending pipe the local concentration increased first when the pipe started to be inclined and gradually dropped as the slope of the pipe increased further. The variation in the mean slurry velocity, $V_m$, with the pipe inclination angle was a result of the variation in the total resistance in the entire circuit. This varied because the frictional head loss over the test loop changed with the inclination of the loop. As shown on the $V_m$ versus $\omega$ plots, the mean slurry velocity tended to rise with the pipe inclination angle and its growth was more pronounced at high inclination angles. Frictional head losses (hydraulic gradients) in Figs. 6.1a and 6.1b were obtained by subtracting the static pressure differential of a slurry column of concentration $C_{vd}$, and considering the static pressure differential of the hose water column, from the measured manometric pressure differentials. The friction losses in the ascending and descending sections of the test loop tended to differ.

The experiments indicated that slurry flow behaves differently in an ascending pipe and in a descending pipe if flow tends to be stratified, i.e. if there is a bed at the bottom of the pipe. The difference diminishes at angles above approximately 45 deg where the bed starts to be disintegrated owing to a continuously diminishing cross-pipe component of solid particle weight - the force component usually responsible for the flow stratification. In an ascending pipe the bed resisting forces are the highest at inclination angles similar to an angle of internal friction in the granular bed. High resisting forces produce low velocity of the bed. This may lead to the development of a shear layer. This is indicated in Figs. 6.1a and 6.1b by a drop in the local bed concentration in the ascending pipe, compared to the local bed concentration in the descending pipe.

6.1.2 Detailed observation of inclined flows in two slopes

In the literature no information about the effects of a pipeline inclination on the internal structure of slurry flow is available. No measurements of the concentration distribution and/or the velocity distribution in the inclined pipelines have been published. Such measurements are, however, necessary to verify the validity of the two-layer modelling approach in an inclined pipeline. The measurements must also provide information about the friction process in inclined flows.

Observations of the concentration profiles and local bed velocities were carried out in the horizontally-positioned test loop and in the test loop inclined to the 25 and 35 deg. As well as the profiles the manometric gradients, the mean slurry velocities and delivered concentrations were measured for a wide range of slurry flow conditions.
Pressure gradients and their comparison with existing models

The pressure differential between two pipeline cross sections 1 and 2 separated from each other by the pipeline length \( dx \) is measured as a manometric pressure differential \( P_1 + \rho_f \cdot g \cdot dh - P_2 \), i.e. \( -dP + \rho_f \cdot g \cdot dh \) (Fig. 6.2). The total pressure differential \( -dP \) is obtained by eliminating the static pressure differential due to the water column in the hoses of the differential pressure transmitter (manometer). The pressure differential due to friction is obtained from the total pressure differential \( -dP \) by subtracting (or adding) the static pressure differential \( \rho_m \cdot g \) due to a slurry column in the measuring pipe section from a pipe elevation. Existing methods for the prediction of friction losses in inclined pipelines assume that the slurry of the column has the density \( \rho_m = \rho_f + C_{V_i} (\rho_s - \rho_f) \). Determination of the pressure gradient due to friction from the manometric gradient is often simplified in the calculations by using \( C_{V_d} \) - an input parameter to the predictive methods - instead of \( C_{V_i} \) in the slurry column. Measurement of concentration profiles (and so \( C_{V_i} \)) within a pipe section over which a manometric gradient has been measured permits the testing of the relationship between the friction loss in an inclined pipeline and in a horizontal pipeline for both the \( C_{V_d} \) and the \( C_{V_i} \) slurry columns.

![Schematic length-section of inclined pipe.](image)

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**Figure 6.2.** Schematic length-section of inclined pipe.

Hydraulic gradients \( l_{\text{m0}} \), obtained from the measured manometric gradients by subtraction of either the \( C_{V_i} \) slurry column or the \( C_{V_d} \) slurry column, do not support the prediction of the empirical models (called here "cos" models) by Worster & Denny (1955), Gibert (1960) and Wilson et al. (1992), reviewed in Chapter 3, that the ratio of
Figure 6.3. Friction ratio in inclined 150 mm pipes.
frictional solids effects in inclined and horizontal pipelines, \( (I_{m0} - I_f)/(I_m - I_f) \), should decrease with an increasing pipeline slope as \( \cos \theta [\cos \theta^{1.5}, \cos \theta(1 + M \gamma)] \) respectively, in both the ascending and the descending pipeline.

This can be seen in Fig. 6.3, where the solids friction ratio \( (I_{m0} - I_f)/(I_m - I_f) \) is determined by using both \( C_{V_i} \) and \( C_{V_d} \) in the slurry column. No correlation has been found between this parameter and \( \cos \theta \). The values of \( (I_{m0} - I_f)/(I_m - I_f) \) were rather different in the ascending and the descending pipe sections. The predictions of the friction loss by the "cos" model matched the experimental data better for finer than for coarser particle slurries.

The values of the solids friction ratio \( (I_{m0} - I_f)/(I_m - I_f) \) on Fig. 6.3 suggest that the total friction should be smaller throughout the laboratory circuit when the test loop is inclined rather than horizontal (except for the finest slurry flow). This was indicated during measurements, since a lower rpm of the pump impeller had to be employed to reach the same \( V_m \) in the circuit for an inclined test loop than for a horizontal test loop.

To compare the experimental \( I_m \) and \( I_{m0} \), it must be considered that different \( C_{V_i} \) (and sometimes also \( C_{V_d} \)) may be reached during one experimental run at different pipe inclinations. This should not, however, obscure the fact that the "cos rule" is not generally acceptable. Another approach must be used to describe the effect of pipeline inclination on slurry flow behaviour.

**Internal structure of slurry flow**

The development of the internal structure of slurry flow in the measuring sections of the test loop at various inclinations was observed. Concentration profiles were measured in stratified flows with a sliding bed, that is in slurry flows at mean slurry velocities above the deposition-limit velocity. The mechanism which is assumed to cause the phenomena detected is described by applying the principles of the two-layer model. The validity of the principles is verified by analysing slurry flow parameters measured in both the ascending and the descending pipe sections of the test loop. Measured slip ratio and solids velocity at the bottom of a slurry pipe are of primary importance in explaining the acting flow mechanism.

The following effects of a pipe inclination on a slurry flow have been observed:

1. deformation of the concentration profiles
2. slip ratio variation with the pipe inclination angle
3. variation of bed sliding velocity
4. flow restratification effect.
Figure 6.4. Concentration profiles in ascending and descending 150 mm pipes at \( V_m = 3.00 \) m/s (3.50 m/s for Sand 3).
Shape of the concentration profiles

Different degrees of flow stratification have been observed in the ascending pipe (+\( \omega \)) and the descending pipe (-\( \omega \)) for the same slurry flow conditions (\( V_m \), \( C_{vd} \)). The difference is large for the coarse slurry flow (Sand 2, Sand 3, Gravel) and small for the relatively fine slurry flow (Sand 1) (see Fig. 6.4). Thus the effect occurs in slurry flow where the turbulent suspension mechanism plays a minor role (or it is not effective at all) and the majority of particles occupy either a granular body sliding en bloc at the bottom of a pipeline (the zone where all particles are in continuous mutual contact) or the transition zone at the top of the granular bed in which the contacts between solid particles are sporadic rather than continuous. This transition zone is called the shear layer, since it is established by high shear stress acting at the top of the sliding bed.

A sharp flow stratification in the descending pipe (Fig. 6.5a) and a gradual concentration change across the pipe cross section in the ascending pipe (Fig. 6.5b) for coarse slurries (Sand 2, Sand 3, Gravel) suggest that the small concentration gradient in the ascending pipe is the product of dispersive forces acting within the shear layer rather than of the turbulent mixing process in the liquid flow. In the ascending pipe the bed-submerged-weight component exerted against the flow direction has a resisting effect on the sliding bed and, owing to the steep velocity gradient between the sliding bed and the flow above it, a thick shear layer is developed. In the descending pipe, owing to the propelling effect of the submerged weight component in the flow direction, the velocity of the moving bed is sufficient to prevent the formation of a shear layer. Liquid turbulence alone is not able to suspend the coarse particles. Finer slurry flow (Sand 1) at the same pipe inclinations (+\( \omega \), -\( \omega \)) demonstrates a considerably smaller difference between the shapes of concentration profiles, suggesting that here the carrier turbulence is the prevailing suspension mechanism and the shear layer is not well developed. It should be remembered that these effects are of importance primarily in flows inclined to angles not far above 35 deg. At these angles the cross-pipe component of the submerged bed weight is still important and the pipe slope is not the main cause of bed disintegration.

The conclusion that coarse particles in an ascending pipe tend to be suspended by dispersive forces within the shear layer rather than by turbulent diffusion is supported by the existence of a restratification effect in the slurry flow. Flow restratification under the condition of increasing slurry velocity was detected in medium to coarse slurry flows (Sand 2, Sand 3, Gravel) in the ascending pipe section (see Fig. 6.5b and also 6.6b). This flow effect has been explained in Chapter 5.
Pipe inclination: -25 deg
Sand 1  Cvd=0.235

Pipe inclination: -35 deg
Sand 1  Cvd=0.235

Sand 2  Cvd=0.29

Sand 2  Cvd=0.29

Sand 3  Cvd=0.20

Sand 3  Cvd=0.20

Gravel  Cvd=0.25

Gravel  Cvd=0.24

Legend: Mean slurry velocity [m/s]

2.5 m/s  3.0 m/s  3.5 m/s  4.2 m/s  5.5 m/s

Figure 6.5a. Concentration profiles in descending 150 mm pipes.
Pipe inclination: +25 deg
Sand 1  $C_{vd}=0.235$

![Graph 1](Image)

Pipe inclination: +35 deg
Sand 1  $C_{vd}=0.235$

![Graph 2](Image)

Sand 2  $C_{vd}=0.29$

![Graph 3](Image)

Sand 3  $C_{vd}=0.20$

![Graph 4](Image)

Gravel  $C_{vd}=0.25$

![Graph 5](Image)

Gravel  $C_{vd}=0.24$

![Graph 6](Image)

Legend: Mean slurry velocity [m/s]

- 2.5 m/s
- 3.0 m/s
- 3.5 m/s
- 4.2 m/s
- 5.5 m/s

Figure 6.5b. Concentration profiles in ascending 150 mm pipes.
**Slip ratio**

The two-layer model assumes no slip within either layer. Variation in the slip ratio is due to the variable mutual shift between two layers of different concentration, velocity and thickness. This conclusion was reached for vertical and horizontal flows in Chapter 5. Experimental results for the partially-stratified flow in inclined pipes also support this conclusion. The following phenomena observed in inclined flows confirm that the variation in the slip ratio is primarily due to a variation in the shift between layers in a stratified flow:

I. The slip ratio increases when the slope of a descending pipe is increased (see Tables VI.1A and VI.1B in Appendix 6). This is particularly evident in sharply stratified flows (see Fig. 6.6a for Sand 3). The acceleration of the granular bed due to the growing axial component of the submerged weight causes an increase in the slip ratio. The same force acting in an ascending pipe against bed motion, decreases its velocity and so should decrease the slip ratio. However, the slip ratio value in an ascending pipe in certain flow conditions is influenced by the development of a shear layer which changes the amount of particles in the contact layer. If the shear layer develops further with an increasing pipe slope (here till a pipe angle 35 deg) the slip ratio in the pipe rises (Fig. 6.6a). In general, as seen from the tables in Appendix 6, no common trend has been observed in the relation between the slip ratio and the pipe inclination in an ascending pipe for the slurries tested.

II. Slip ratio values higher than one have been found in the descending pipe section at pipe slopes given by an inclination angle -35 deg (and -25 deg in some cases) and they have decreased towards one with the increasing $V_m$. This trend is illustrated in Fig. 6.6b. According to the force balance in a tilting tube (in which there is no flow) the slip ratio value exceeds one at angles $\alpha$ exceeding that for $\tan \alpha = \mu_s F_N/F_W$ in which $F_N/F_W = fn(C_{2c}, \beta)$. Thus the bed starts to slide when the submerged weight component $F_W \sin \alpha$ exceeds the bed resisting force $\mu_s F_N \cos \alpha$ in a tilting tube. The ratio $C_{vd}/C_{vi} = V_s/V_m > 1$ in a descending slurry pipeline is also caused by $F_W \sin \alpha$. This force component is able to accelerate the lower particle-rich layer relatively to the upper particle-lean layer and so the mean solids velocity $V_s$ relatively to $V_m$. The effect of the submerged weight component is negligible for suspended solid particles. The slip velocity between phases is given by the axial component of particle settling velocity $v_t$ ($v_t$ is produced by the balance between the particle submerged weight and the liquid drag). This velocity is negligible when it is compared with $V_m$. Slip ratio values higher than one decrease with the increasing $V_m$ in the descending pipe because the concentration profile becomes less stratified, so more solid particles become suspended and thus are not affected by the acceleration of the granular bed (see Fig. 6.6b). This can be also seen when the concentration profiles in Fig. 6.6a are compared with the slip ratio values in Tabs. VI.2A, VI.2B, VI.2C, VI.2D. The slip ratio value should increase towards one (Sand 3, Gravel in -25 deg pipe) with increasing $V_m$ when the granular bed is slower than the upper layer.

III. A decrease in the slip ratio is observed in some flow regimes of Sand 2, Sand 3 and Gravel slurry with increasing $V_m$ in an ascending pipe (Tabs. VI.3B, VI.3C, VI.3D). Comparison of the slip ratio values with the shapes of concentration
Figure 6.6a. Slip ratio and corresponding concentration profiles in inclined 150 mm pipe for different pipe inclinations.

Figure 6.6b. Slip ratio and corresponding concentration profiles in inclined 150 mm pipe for different mean slurry velocities.
profiles shows that the slip ratio values decrease because the profiles become more stratified at higher $V_m$ (see Fig. 6.6b and/or compare concentration profiles in Fig. 6.5b with $C_{vd}/C_{vi}$ in Tabs. VI.3A, VI.3B, VI.3C, VI.3D). This is caused by the process of slurry flow restratification. When no restratification effect is observed, the slip ratio increases with increasing $V_m$ in accordance with the observed gradual homogenisation of the concentration profiles. The increase in slip ratio values above one in fine slurry flow (Tab. VI.3A) is due to inaccuracy of the measurement.

Slip ratio is found to be strongly dependent on the shape of the concentration profile. The slip ratio value tends to approach unity when the flow becomes less stratified. If $F_{W\sin\theta}$ exceeds $\mu_k F_{N\cos\theta}$ in a descending pipe the slip ratio reaches a value higher than unity. In accordance with observations in a horizontal pipe described in Chapter 5 a link is found between the development of the slip ratio (and so in the shape of the concentration profile) and the changes in the solids velocity at the bottom of a pipe.

**Solids velocity at the bottom of an ascending pipe section**

It has been observed that, generally, the change in the solids velocity at the bottom of an ascending pipe is in accordance with that for the slip ratio in all conditions tested (a change under the condition of variable mean slurry velocity, concentration or pipe inclination). When the restratification process is detected in an ascending pipe the relation between the curves for the manometric gradient, the local concentration and the local bed velocity is identical to that in a horizontal pipe. In Fig. 6.7 the shear layer in the Gravel slurry flow gradually increases with the increasing $V_m$ first (it is the most developed at $V_m$ approximately 3.0 m/s) and then gradually diminishes (the flow restratification effect). The restratification effect is due to faster bed acceleration at the higher mean slurry velocities $V_m$.

Velocity $V_b$ has not been found to be significantly lower in the ascending pipe than in the horizontal pipe at the same $V_m$ and $C_{vd}$ of the Gravel slurry flow. Therefore the less stratified profile in an ascending pipe is not due to turbulent intensity in its carrier flow being higher than that in the upper layer of a horizontal pipe. Coarse particle suspension in an ascending pipe (of inclination angle not higher than 25 deg) is caused by dispersive forces generated by the high shear rate along a pipe cross section.

6.2 Analysis

6.2.1 Pressure differential

The total pressure gradient $(P_1-P_2)/dx = -dP/dx$ over a pipeline section of the length $dx$ (see Fig. 6.2) is composed of
- the static pressure gradient $d(pgh)/dx$, giving the potentially reversible effect of elevation change on the total pressure gradient in a slurry flow of the density $p$ gaining the height $h$ and
- the pressure gradient due to friction $-dP/dx = d(\rho gh)/dx$ that is the irrecoverable energy loss due to friction in inclined slurry flow over the pipe length $dx$. 
Figure 6.7. Manometric gradient of slurry flow and local characteristics of bed in ascending 150 mm pipes.
Ratio \( \frac{dh}{dx} = \sin \omega \). The inclination angle \( \omega \) gains a negative value in a descending pipeline. The static pressure gradient \( d(\rho g h)/dx \) is produced by a slurry column of the height \( dh \) in the pipeline section \( dx \). The density of the column, \( \rho \), is determined from the concentration of solid particles in the section \( dx \) which contribute to the weight of the slurry column.

According to Bagnold's concept for the solids support in a mixture flow, the contact-load particles transfer their submerged weight to the pipe wall via the interparticle contacts. The particles are supported by the interparticle contacts. Solid particles with no interparticle contacts (suspended particles) transfer their weight to the carrying liquid and increase the density of the suspension. Thus only the solid particles whose submerged weight is not transmitted to the pipeline wall contribute to the slurry column which exerts the static pressure differential over an inclined pipeline section.

The density of the slurry column is the density of the mixture of the carrying liquid and suspended particles in an inclined pipeline section. The spatial concentration \( C_{v1} \) in a pipeline section can be used to calculate slurry column density only when all particles are suspended. The delivered concentration \( C_{vd} \) determines the slurry column density only when all particles are suspended and, furthermore, the slip between phases in a pipeline section is negligible.

6.2.2 Forces exerted by the contact-load solids

The direction of the gravitational force acting on liquid and solid particles is not perpendicular to the axis of an inclined pipe. Gravitational acceleration components \( g \cdot \cos \omega \) and \( g \cdot \sin \omega \) represent the effect of pipe inclination on a mixture body force in two directions characteristic for mixture flow. The pipe-axis component of the body force is generated when the pipeline is inclined.

The body force of individual suspended particles contributes to the body force of the suspension composed of suspended particles and the carrying liquid in the pipeline section. Contact-load solids do not contribute to the body force of the suspension. Contact-load particles (consist of to be in permanent mutual contact) form a granular bed sliding at the bottom of the pipeline section. The submerged weight of the contact bed is responsible for two forces which must be taken into account in the force balance for inclined flows. The pipe-axis component of the submerged weight \( F_{ws} \sin \omega \) is the body force acting as an additional resistance (in an ascending pipeline) or an additional drive (in a descending pipeline) force in the lower layer of a stratified slurry flow. The submerged-weight-dependent normal intergranular force exerted by the contact bed against the pipeline wall is the surface force integrated from normal solids stress at the pipeline wall. At any point (defined by an angle \( \alpha \) lower than an angle defining the position of the top of the contact bed \( \beta \)) on the boundary between the contact bed and the pipeline wall, the two components of the intergranular stress [determined according to the method of Wilson (1970), Eq. 3.23 in Chapter 3] exerted by solids against the pipeline wall (see Fig. 6.8) contribute to the force balance:
- the cross-pipe component $\sigma_{st} = g \cos \omega (\rho_2 - \rho_1) C_2 c D / 2 (\cos \alpha - \cos \beta)$, normal to the pipe wall acting in a radial direction in a pipeline cross section. This component, integrated over the boundary perimeter $O_2$, as described in Chapter 3, gives $F_N \cos \omega$. $F_N$ is the normal intergranular force against the pipeline wall in a horizontal pipeline,
- the pipe-axis component $\sigma_{st} = g \sin \omega (\rho_2 - \rho_1) C_2 c D / 2 (\cos \alpha - \cos \beta) \cos \alpha$, tangential to the $\sigma_{st}$; integrating of $\sigma_{st}$ over the perimeter $O_2$ gives $F_W \sin \omega$.

![Figure 6.8. Intergranular stress in an inclined pipe.](image)

6.3 Modelling

6.3.1 Configuration of two-layer model for inclined flows

The effects of pipeline inclination are incorporated in the force balance of a two-layer model, assuming the flow pattern described on Fig. 3.14 and in this chapter shown on Fig. 6.2. The body forces $\rho_1 g \sin \omega A_1 dx$ and $\rho_2 g \sin \omega A_2 dx$ of the suspension in layers, the body force of the contact bed and the surface forces (from the boundary shear stresses) exerted by the suspension and the contact bed are incorporated in a linear momentum equation expressing a force balance in each layer of a partially-stratified flow in an inclined pipeline.

The force balance between the driving pressure force and phase-body forces on the left side of the equation and the resisting surface forces at the right side of the equation for the upper layer is given as
-dP.A1 - ρfg.sinω.dx.A1(1-C1) - ρsg.sinω.dx.A1C1 = τ1O1.dx + τ12O12.dx

and because ρf(1-C1) + ρsC1 = ρ1 that is rewritten as

-dP.A1 - ρ1.g.sinω.dx.A1 = τ1O1.dx + τ12O12.dx.

In the lower layer the volume dx.A2(1-C2c) is occupied by a suspension of density ρ2f and the rest of the total volume, dx.A2C2c, is occupied by the contact-load particles.

The force balance in the lower layer is written as

-dP.A2 - ρ2fg.sinω.dx.A2(1-C2c) - ρsg.sinω.dx.A2C2c = (τ2f + τ2s)O2.dx - τ12O12.dx.

in which ρ2f = [ρsC1 + ρf(1 - C2c - C1)]/(1 - C2c).

Substituting ρ2f(1-C2c) = C1(ρs·ρf) + ρf(1-C2c) in the force balance equation for the lower layer gives

-dP.A2 - ρ1.g.sinω.dx.A2 - (ρs - ρf)g.sinω.dx.A2C2c = (τ2f + τ2s)O2.dx - τ12O12.dx.

The term (ρs·ρf)g.sinω.dx.A2C2c gives the pipe-axis component of the submerged weight FWsinω of the bed of length dx occupied by particles in contact. The bed acts like an immersed solid body in contact with the pipeline wall. The force balance in the lower layer is thus

-dP.A2 - ρ1.g.sinω.dx.A2 - FWsinω = (τ2f + τ2s)O2.dx - τ12O12.dx.

Written in the form used by Shook & Roco (1991) for the SRC model, see Eqs. 3.57, 3.58 and 3.59 in Chapter 3, the linear momentum equation from the analysis above is obtained in the upper layer as

\[-\frac{d(P+ρ1gh)}{dx}A1 = τ1O1 + τ12O12\]  \hspace{1cm} (6.1)

and in the lower layer, for τ2sO2 = \int_{0}^{β} 1/\beta dα. O2 = μsFNcosω, as

\[-\frac{d(P+ρ1gh)}{dx}A2 - FW\sin ω = τ2fO2 + μsFN\cosω - τ12O12\]  \hspace{1cm} (6.2).

In the whole pipe section the balance is then

\[-\frac{d(P+ρ1gh)}{dx}A - FW\sin ω = τ1O1 + τ2fO2 + μsFN\cosω\]  \hspace{1cm} (6.3).

The model of this configuration for the inclined flows is termed 2LMi. This configuration differs from that proposed by Shook & Roco (1991) for the SRC model (Eqs. 3.57, 3.58 and 3.59 in Chapter 3) in taking the inclination effects of different solids-support mechanisms into account in the model configuration. The SRC model assumes that the entire spatial concentration of both layers contributes to the static pressure gradient exerted by the slurry column and therefore the model does not include the pipe-axis component of the submerged weight of a contact layer (FWsinω) to the force balance.
6.3.2 Comparison with SRC configuration

The difference between the new configuration and the SRC configuration of the two-layer model for inclined flows can be demonstrated by the following considerations.

The pipe-axis component of the suspension weight is supported by the static pressure differential over the pipeline section. However, the weight of the contact layer is not supported by the pressure differential in the carrying liquid. This can be illustrated by a simple example of an object of greater density than water that is immersed in water occupying a horizontal pipe section. The pipe is pressurised but there is no flow. The object lies at the bottom of the pipeline and is in permanent contact with the pipe wall. The pressure differential over the entire length dx of the pipe section is zero. If the pipeline section is gradually inclined and there is no water flow the pressure differential over the section at each inclination angle $\omega$ is equal to the static pressure $\rho_f g \Delta x \sin \omega$ exerted by the water column only. Thus only the weight of water is balanced by the pressure differential. The pipe-axis component of the body force of the object is opposed by the resisting force of mechanical friction between the object surface and the pipe wall. The object starts to move over the pipe wall if the pipe inclination angle reaches the value at which the axial component of the submerged weight of the object exceeds the resisting force dependent on the submerged weight and the coefficient of the mechanical friction between the glass surface and the steel wall. The conditions described might be obtained in the experimental tilting tube with contact layer occupied by the sand grains. The initial motion of the bed in the tilting tube occurs when the condition $\mu_s F_N \cos \omega = F_W \sin \omega$ is satisfied. This equation is Eq. 6.3 for the no-flow condition. The SRC model (Eq. 3.58) does not produce this force balance at the initial motion of the bed in the tilting tube.

In the linear momentum equation of the SRC model (Eq. 3.58), the weight of all particles in the lower layer contributes to the body force supported by the static pressure gradient over the pipe section. Thus it is anticipated that the weight of contact-load particles contributes to both the static pressure gradient and to the solids stress at the pipe wall giving the normal solids force $F_N$. This is not acceptable because each particle may transfer its weight either to the pipe wall or to the carrying liquid.

According to the SRC model, the pressure force due to the frictional gradient (left side of the Eq. 3.59) does not vary with the sign of the inclination angle $\omega$ on the right side of the equation. Thus the predicted value of the frictional pressure loss is identical in the ascending pipeline $+\omega$ and in the descending pipeline $-\omega$. Our laboratory observations revealed that this estimate is acceptable only for slurry flows in which the majority of the transported solid particles is suspended in the carrying liquid.

According to the 2LMi configuration of the two-layer model, as a result of the action of the pipe-axis component of the contact-bed submerged weight (Eq. 6.3) very different behaviour is anticipated in pipelines inclined to $+\omega$ and $-\omega$ angles. The differences in the shape of the concentration profiles and in the slip ratio values, observed in the laboratory test loop, are primarily caused by an axial component of the submerged weight of the contact layer $F_W \sin \omega$. 
6.3.3 Verification

The new configuration of the two-layer model (2LMi, Eqs. 6.1 to 6.3) uses the same flow pattern as the SRC model (Eqs. 3.57 to 3.59). Both model configurations have been tested by using experimental data from a 150 mm test loop installed at inclination angles of 25 deg and 35 deg and the results have been compared. The computational scheme used for the testing model TM, which processes the measured concentration profiles into the idealised two-layer patterns of partially-stratified slurry flow, is described in Chapter 5.

The TM outputs of a specific tested configuration of the two-layer model (either 2LMi or SRC) were compared with the values of independent slurry flow parameters which were measured (V₂ versus v_b) or calculated (λ₁₂ as the TM output versus λ₁₂ determined by the friction-law equations). The parameters are considered to be independent in the sense that they were not used as the TM inputs and they were not used for the computation of the TM outputs.

Figures 6.9a and 6.9b give examples of the simplified concentration profiles (C₁, C₂, Y₁₂/D) produced by the TM using the 2LMi configuration (Eqs. 6.1 to 6.3) from measured profiles which were the TM inputs. The position of the interface between layers is appropriate when it delimits the layers within the region of a sharp drop on the measured concentration curve (a sharp interface between the particle-rich and particle-lean zone in a descending pipe) or within the region occupied by a shear layer according to the measured concentration curve (in an ascending pipe).

The output λ₁₂ value is the most sensitive indicator of the quality of a chosen model configuration and of the consistency of experimental data used as model inputs. When experimental data are not consistent or a model configuration is not appropriate λ₁₂ values may easily fall outside an acceptable range. An acceptable range for the λ₁₂ values is given by the scatter of results from various existing friction equations for a stratified-flow interface. The following equations have been used to determine the acceptable range for the λ₁₂ outputs of the testing model TM:
- the friction equation for a hydraulically-rough boundary of turbulent flow (using Nikuradse's equation [see Eq. 3.32 in Chapter 3] for sand equivalent roughness equal to the particle diameter) multiplied by an empirical coefficient (equal to 2 according to Wilson, 1976), λ₁₂ = 0.11 for Gravel d₅₀/D = 4.2/150
- the modification of Nikuradse's friction law proposed by Shook & Roco (1991), see Eq. 3.33 in Chapter 3, λ₁₂ = 0.22
- the friction law based on a shear layer analysis (Wilson & Pugh, 1995), see Eq. 3.38 in Chapter 3.

These equations provide a rather narrow scatter when compared with that produced by the λ₁₂ outputs from TM of different configurations (see Figs. 6.10a and 6.10b).

Figs. 6.10a and 6.10b give an example of outputs from the testing model TM of both the 2LMi configuration (black points) and the SRC model configuration (blank points) for the Gravel flow, the concentration profiles of which are given in Figs. 6.9a and 6.9b. The outputs are compared with the independent parameters measured (crosses) and those calculated (lines). For the descending pipe the TM of the SRC configuration
has given much higher interfacial friction coefficient $\lambda_{12} = 0.4 - 0.8$ than the TM of the 2LMi configuration ($\lambda_{12} = 0.08 - 0.3$). The $\lambda_{12}$ values from the TM of the 2LMi configuration are more realistic. Very high $\lambda_{12}$ output values have been found for the input $C_{\text{Vd}}/C_{\text{Vl}}$ value approaching unity. In such cases a basic model condition - a mutual shift between two layers - is not fulfilled. The velocity difference $V_1-V_2$ is negligible and thus an unrealistically high $\lambda_{12}$ is required to produce the shear stress at the interface.

Testing for four different solids in slurry flows of various concentrations and velocities has confirmed that the 2LMi configuration using Eqs. 6.1 to 6.3 is appropriate to describe slurry flow behaviour in inclined pipes. When compared with the independent measured and calculated parameters, the outputs of the TM based on the 2LMi configuration show a good agreement for the flow conditions suitable to the flow modelling by the principles of the two-layer model. These conditions are a reasonable degree of flow stratification and a slip ratio value different from one.

6.3.4 Discussion

No pressure gradient is required to initiate granular bed sliding at pipe inclinations for which $\mu_{s} F_{N} \cos \omega < F_{W} \sin \omega$ in a descending pipe. Thus it follows that in a slurry flow with a sliding bed no pressure gradient is required to maintain the sliding when the bed is moving faster than the upper layer, so the condition $\mu_{s} F_{N} \cos \omega < F_{W} \sin \omega$ is valid. The granular bed is impelled gravitationally and only the viscous friction on the boundaries of the sliding bed contributes as a solids effect to the total friction pressure gradient in slurry flow. Therefore the frictional head loss should not significantly exceed the value for liquid flow alone in sharply stratified flows in descending pipelines at inclination angles near the angle of the initial motion of the bed in the tilting tube. This fact is reflected by the 2LMi configuration but not by the SRC model and the "cos" models. These models assign a much bigger part of the total pressure gradient to the static gradient than the 2LMi and consequently assume very similar, or identical, friction gradients in both the ascending ($+\omega$) and the descending ($-\omega$) pipe section.

Processed data from our measurements show a rather weak effect of the bed thickness on the friction loss in descending pipe at inclination 35 deg (when compared with horizontal pipe). This fact supports the validity of the friction mechanism described above. A significantly larger friction loss in an ascending pipe than in a descending pipe of the same slope by the 2LMi is in an accordance with our experimental observations.

Widely-used empirical "cos" models assuming a static pressure differential caused by a $C_{\text{Vd}}$ slurry column do not successfully describe the effect of a sliding bed on frictional head losses in inclined pipes. In either fully or partially stratified flow the "cos" models tend to overestimate the friction loss in descending pipes and underestimate the friction loss in ascending pipes. The SRC model can be considered as the "cos model" that assumes a static pressure differential caused by a $C_{\text{Vl}}$ slurry column. This model also overestimates friction losses in descending pipes by attributing a large portion of the pressure gradient to static pressure difference due to the slurry column and for the same reason underestimates friction losses in ascending pipes.
Figure 6.9a. Concentration profiles measured and simulated by the reservoir model for critical flow in inclined 150 mm pipes.

Pipe Inclination: 25° deg

Pipe Inclination: -25° deg
Pipe inclination: -35 deg

Pipe inclination: +35 deg

Figure 6.9b. Concentration profiles measured and simulated by the testing model for Gravel flow in inclined 150 mm pipes.
Pipe inclination: +/- 25 deg

Figure 6.10a. Testing model outputs for configurations 2LMi and SRC and their comparison with independent measured or calculated parameters.
Pipe inclination: +/- 35 deg

Legend: Gravel, Cvd=0.24; Model, inclination angle [deg]

2LMi,-35 2LMi,+35 SRC,-35 SRC,+35 Data,-35 Data,+35 Water

Figure 6.10b. Testing model outputs for configurations 2LMi and SRC and their comparison with independent measured or calculated parameters.
6.4 Conclusions

Mixture flows in which the majority of particles is transported in a bed are very sensitive to changes in a pipeline slope while mixture flows in which the majority of particles is suspended by carrier turbulence is affected insignificantly by a pipeline slope. At pipe inclinations close to the angle of internal friction of transported solids, the internal structure of the partially-stratified slurry flow in an ascending pipe (+α) differs greatly from that in a descending pipe (-α) for the same flow conditions (V_M, C_Vd) in relatively coarse slurry flows (Sand 2, Sand 3, Gravel) but remains almost unchanged in relatively fine slurry flows (Sand 1). This is because deformation of the concentration profile at different pipe inclinations is primarily due to the components of the submerged weight of the bed at the bottom of an inclined pipe. Flows with a thin bed exhibit only a slight concentration-profile deformation due to a change in a pipeline slope.

An empirical approach to the modelling of a pipeline inclination effect (the "cos" models) is not capable of evaluating the development of the internal structure of partially-stratified flow and seems to provide poor predictions, particularly for very stratified flows and flows with a developed shear layer.

The development of the internal structure of stratified slurry flow in inclined pipes can be described by applying the principles of the two-layer model. The proposed configuration of the two layer model (Eqs. 6.1-6.3) provides a description of slurry flow behaviour in inclined pipes. It has been verified by experiments which included measurements of the concentration profile and local solids velocity at the bottom of the 150 mm pipe at various angles of inclination.

6.5 References


Appendix 6

Experimental data for the slip ratio in inclined 150 mm pipes

**Table VI.1A.** Slip ratio in the inclined 150 mm pipe at slurry velocity $V_m = 3.00$ m/s (slurry flow of different solids [Sand 1, Sand 2, Sand 3, Gravel])

<table>
<thead>
<tr>
<th></th>
<th>SAND 1</th>
<th>SAND 2</th>
<th>SAND 3</th>
<th>GRAVEL</th>
<th>$V_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{vd}=0.235$</td>
<td>$C_{vd}=0.29$</td>
<td>$(V_m=3.50)$ $C_{vd}=0.20-0.22$</td>
<td>$C_{vd}=0.24-0.27$</td>
<td></td>
</tr>
<tr>
<td>0.89</td>
<td>0.94</td>
<td>0.88</td>
<td>0.79</td>
<td>0.87</td>
<td>0.75</td>
</tr>
<tr>
<td>1.07</td>
<td>0.91</td>
<td>1.00</td>
<td>0.96</td>
<td>0.92</td>
<td>0.88</td>
</tr>
<tr>
<td>1.10</td>
<td>0.93</td>
<td>1.11</td>
<td>0.95</td>
<td>1.11</td>
<td>0.90</td>
</tr>
</tbody>
</table>

**Table VI.1B.** Slip ratio in the inclined 150 mm pipe at slurry velocity $V_m = 4.25$ m/s (slurry flow of different solids [Sand 1, Sand 2, Sand 3, Gravel])

<table>
<thead>
<tr>
<th></th>
<th>SAND 1</th>
<th>SAND 2</th>
<th>SAND 3</th>
<th>GRAVEL</th>
<th>$V_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{vd}=0.235$</td>
<td>$C_{vd}=0.29$</td>
<td>$(V_m=3.50)$ $C_{vd}=0.20-0.22$</td>
<td>$C_{vd}=0.24-0.27$</td>
<td></td>
</tr>
<tr>
<td>0.97</td>
<td>1.01</td>
<td>0.91</td>
<td>0.89</td>
<td>0.74</td>
<td>0.73</td>
</tr>
<tr>
<td>1.04</td>
<td>1.00</td>
<td>1.05</td>
<td>0.91</td>
<td>0.91</td>
<td>0.82</td>
</tr>
<tr>
<td>1.05</td>
<td>1.00</td>
<td>1.07</td>
<td>0.93</td>
<td>1.05</td>
<td>0.86</td>
</tr>
</tbody>
</table>
Table VI.2A. Slip ratio in the descending 150 mm pipe at different $V_m$
(slurry flow of 0.2 - 0.5 mm sand [Sand 1])

| $C_{vd}$ = 0.12 |
| Angle | $C_{vd}$ = 0.185 |
| Angle | $C_{vd}$ = 0.235 |
| Angle | $C_{vd}$ = 0.305 |
| Angle |
| -25  | -35  | -25  | -35  | -25  | -35  | $V_m$ |
| 1.10 | 1.15 | 1.07 | 1.13 | 1.07 | 1.10 | 1.00 | 1.03 | 3.00 |
| 1.04 | 1.09 | 1.05 | 1.09 | 1.05 | 1.07 | 1.00 | 1.01 | 3.50 |
| 1.01 | 1.03 | 1.04 | 1.05 | 1.03 | 1.04 | 1.00 | 1.01 | 4.25 |
| 0.99 | 0.99 | 1.03 | 1.04 | 1.03 | 1.04 | 1.01 | 1.01 | 5.50 |

Table VI.2B. Slip ratio in the descending 150 mm pipe at different $V_m$
(slurry flow of 0.5 -1.0 mm sand [Sand 2])

| $C_{vd}$ = 0.20 |
| Angle | $C_{vd}$ = 0.29 |
| Angle | $C_{vd}$ = 0.34 |
| Angle |
| -25  | -35  | -25  | -35  | -25  | -35  | $V_m$ |
| 1.06 | 1.14 | 1.00 | 1.11 | 1.03 | 1.09 | 3.00 |
| 1.07 | 1.10 | 1.03 | 1.09 | 1.03 | 1.06 | 3.50 |
| 1.06 | 1.05 | 1.05 | 1.06 | 1.03 | 1.04 | 4.25 |

Table VI.2C. Slip ratio in the descending 150 mm pipe at different $V_m$
(slurry flow of 1.4 -2.0 mm sand [Sand 3])

| $C_{vd}$ = 0.09 |
| Angle | $C_{vd}$ = 0.155 |
| Angle | $C_{vd}$ = 0.20 |
| Angle |
| -25  | -35  | -25  | -35  | -25  | -35  | $V_m$ |
| 0.88 | 1.08 | 0.92 | 1.12 | 0.92 | 1.11 | 2.50 |
| 0.92 | 1.05 | 0.92 | 1.09 | 0.91 | 1.05 | 3.50 |
| 0.98 | 1.03 | 0.94 | 1.03 | 0.97 | 1.02 | 5.50 |

Table VI.2D. Slip ratio in the descending 150 mm pipe at different $V_m$
(slurry flow of 3.0-5.0 mm gravel [Gravel])

| $C_{vd}$ = 0.09 |
| Angle | $C_{vd}$ = 0.14 |
| Angle | $C_{vd}$ = 0.19 |
| Angle | $C_{vd}$ = 0.24 |
| Angle |
| -25  | -35  | -25  | -35  | -25  | -35  | $V_m$ |
| 1.19 | 0.86 | 1.14 | 0.88 | 1.06 | 0.86 | 1.08 | 3.00 |
| 1.15 | 0.88 | 1.08 | 0.86 | 1.03 | 0.89 | 1.05 | 3.50 |
| 1.13 | 0.90 | 1.04 | 0.87 | 0.99 | 0.89 | 4.25 |
### Table VI.3A. Slip ratio in the ascending 150 mm pipe at different $V_m$
(slurry flow of 0.2 - 0.5 mm sand [Sand 1])

<table>
<thead>
<tr>
<th>$C_{vd} = 0.12$ Angle</th>
<th>$C_{vd} = 0.185$ Angle</th>
<th>$C_{vd} = 0.235$ Angle</th>
<th>$C_{vd} = 0.305$ Angle</th>
<th>$V_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25</td>
<td>+25</td>
<td>+25</td>
<td>+25</td>
<td>+25</td>
</tr>
<tr>
<td>0.84</td>
<td>0.87</td>
<td>0.88</td>
<td>0.91</td>
<td>0.94</td>
</tr>
<tr>
<td>0.89</td>
<td>0.91</td>
<td>0.93</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td>0.96</td>
<td>0.96</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.96</td>
<td>0.98</td>
<td>1.02</td>
<td>1.03</td>
<td>1.01</td>
</tr>
</tbody>
</table>

### Table VI.3B. Slip ratio in the ascending 150 mm pipe at different $V_m$
(slurry flow of 0.5 - 1.0 mm sand [Sand 2])

<table>
<thead>
<tr>
<th>$C_{vd} = 0.20$ Angle</th>
<th>$C_{vd} = 0.29$ Angle</th>
<th>$C_{vd} = 0.34$ Angle</th>
<th>$V_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25</td>
<td>+25</td>
<td>+25</td>
<td>+25</td>
</tr>
<tr>
<td>0.88</td>
<td>0.91</td>
<td>0.96</td>
<td>0.95</td>
</tr>
<tr>
<td>0.91</td>
<td>0.93</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td>0.91</td>
<td>0.94</td>
<td>0.91</td>
<td>0.93</td>
</tr>
</tbody>
</table>

### Table VI.3C. Slip ratio in the ascending 150 mm pipe at different $V_m$
(slurry flow of 1.4 - 2.0 mm sand [Sand 3])

<table>
<thead>
<tr>
<th>$C_{vd} = 0.09$ Angle</th>
<th>$C_{vd} = 0.155$ Angle</th>
<th>$C_{vd} = 0.20$ Angle</th>
<th>$V_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25</td>
<td>+25</td>
<td>+25</td>
<td>+25</td>
</tr>
<tr>
<td>0.89</td>
<td>0.85</td>
<td>0.93</td>
<td>0.95</td>
</tr>
<tr>
<td>0.90</td>
<td>0.88</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td>0.87</td>
<td>0.91</td>
<td>0.80</td>
<td>0.88</td>
</tr>
</tbody>
</table>

### Table VI.3D. Slip ratio in the ascending 150 mm pipe at different $V_m$
(slurry flow of 3.0 - 5.0 mm gravel [Gravel])

<table>
<thead>
<tr>
<th>$C_{vd} = 0.09$ Angle</th>
<th>$C_{vd} = 0.14$ Angle</th>
<th>$C_{vd} = 0.19$ Angle</th>
<th>$C_{vd} = 0.24$ Angle</th>
<th>$V_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25</td>
<td>+25</td>
<td>+25</td>
<td>+25</td>
<td>+25</td>
</tr>
<tr>
<td>0.81</td>
<td>0.82</td>
<td>0.84</td>
<td>0.84</td>
<td>0.80</td>
</tr>
<tr>
<td>0.90</td>
<td>0.93</td>
<td>0.87</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td>0.94</td>
<td>0.95</td>
<td>0.92</td>
<td>0.83</td>
<td>0.79</td>
</tr>
</tbody>
</table>
Chapter 7

Suspension in partially-stratified flows: modelling by the two-layer model

In this chapter attention is focused on the modelling of two different mechanisms for particle suspension in slurry flow in a pipe. The mechanisms have been described and explained in the Chapter 5. Correlations are proposed to divide the solids into a suspended load and a contact load in a simplified two-layer pattern of the partially-stratified slurry flow. These correlations are different for slurry flows in which the turbulent diffusion is a predominant suspension mechanism and for slurry flows in which suspension is due to interparticle collisions. Some results presented in this chapter were published in Matoušek (1996, 1997).

7.1 Introduction

A two-layer model was originally developed for fully-stratified flow. The experiments, including the observation of concentration distribution in a pipeline cross section and the velocity of solids at the bottom of a pipeline, have shown that model principles can also be used to describe phenomena occurring in partially-stratified (heterogeneous) slurry flow. However, to permit the use of the model as a predictive tool for partially-stratified flow in a pipeline a rule for the division of solids into two layers must be incorporated in the model.

For fully-stratified flow, the two-layer model considers the upper layer as particle-free and the lower layer as occupied by particles, all of which are in continuous contact. In a partially-stratified flow the solids are transported in a carrying liquid both as a contact load and as a suspended load. The amount of solids occupying a slurry pipe is given by the volumetric spatial concentration $C_{vi}$ which is the sum of the solids fraction in suspension, $C_S$, and the solids fraction in contact, $C_C$. The amount of suspended solids $C_S$, and the amount of solids in contact $C_C$, must be predicted by a suitable method. To model a partially-stratified flow assuming a two-layer pattern according to Fig. 3.14 the $C_C$ determines the concentration of solids in contact within the lower layer, $C_{2c}$, by recalculating $C_C$ from the cross-sectional area of the entire pipeline, $A$, to the cross-sectional area of the lower layer, $A_2$, using $C_{2c} = C_C A / A_2$. Gillies et al. (1991) found the correlation (Eq. 3.46 in Chapter 3)

$$\frac{C_C}{C_{vi}} = \exp \left( -0.0184 \frac{V_m}{V_t} \right)$$
in which \( V_m \) is mean slurry velocity in a pipeline and \( v_t \) terminal settling velocity of a solid particle. The correlation was based on experimental data plotted in Fig. 3.15

![Graph showing the effect of \( V_m/v_t \) on the stratification ratio \( C_C/C_{vi} \) for slurry flow of different particle and pipeline sizes, from Gillies et al. (1991).](image)

**Figure 3.15.** Effect of \( V_m/v_t \) on the stratification ratio \( C_C/C_{vi} \) for slurry flow of different particle and pipeline sizes, from Gillies et al. (1991).

Surprisingly low \( C_C/C_{vi} \) values were reached for "Fine Sand" in Fig. 3.15. It is rather debatable whether the two-layer pattern is applicable to flows in which the fraction of contact load particles is lower than approximately one tenth of the total solid fraction. Such a small fraction can hardly form a contact bed of any significance to the slurry flow behaviour. The testing model TM, configured and used as described in Chapter 5, did not find the force balance for the lower layer when it processed flow data for our finest tested sand (the 0.2 to 0.5 mm sand) at the highest measured velocities. At these velocities the degree of flow stratification detected by the measured concentration profiles was very low.

Experimental data successfully processed by the TM show a considerable scatter in the \( C_C/C_{vi} \) versus \( V_m/v_t \) plot as can be seen on Fig. 7.1 for flow of different sorts of sand in a horizontal 150 mm pipe. However, the Gillies type of equation proves to be a suitable tool for an initial estimation of the stratification ratio \( C_C/C_{vi} \) when there is no information about the regime of the handled slurry flow. The correlation calibrated against our experimental data for medium and coarse sand has the form

\[
\frac{C_C}{C_{vi}} = \exp\left(-0.024 \frac{V_m}{v_t}\right)
\]

(7.1).
Figure 7.1. Fraction of solids in contact load using Gillies et al. type of formula.

Equations for the solids division proposed for the two-layer model to date have been reached by an empirical approach, rather than by an analysis of the mechanisms active in solid particle suspension in a slurry flow. A new analysis of the division of the solids into suspended load and contact load is required, considering the assumption that there are two different mechanisms for solid particle suspension in a slurry stream. The analysis should provide stratification-ratio correlations suitable for use in the two-layer model. The analysis results should be verified and the correlations calibrated by experimental data processed by the testing model TM. The experimental C_C/C_Vi values produced by the TM have the advantage of taking into account the shape of the measured concentration profiles. The empirical correlations found in the literature were calibrated by using the integral quantities of slurry flow only.

7.2 Modelling of turbulent suspension

7.2.1 Analysis

The aim of the analysis is to propose a simplified relationship determining the amount of solid particles supported by the carrier turbulence in a pipeline. It is intended that this relationship should be incorporated in the two-layer model. Therefore the equation should correlate the integral parameters of a slurry flow which are handled by the two-layer model. The correlation should evaluate the effect of major parameters influencing the turbulent suspension process in the flow of a two-layer pattern which
simplifies a real solids distribution in a partially-stratified flow. In the two-layer pattern a uniform distribution of the solids supported by a carrier turbulence is assumed, there is no local concentration gradient; \( \frac{dc}{dy} = 0 \).

Suitable groups of parameters are sought to determine the stratification ratio for a simplified two-layer pattern of a partially-stratified flow. These groups can be further related using a correlation calibrated by the outputs of the testing model (TM) processing the experimental data inclusive measured concentration profiles.

Groups of parameters are obtained from a description of a liquid-solid interaction in a simplified flow pattern assuming a uniform distribution of suspended and contact particles across the slurry stream. In partially-stratified flow a portion of particles is supported by the turbulent eddies and the rest of the particles are supported by the interparticle contacts. Individual suspended particles follow random trajectories in a turbulent carrier stream but the local time-averaged concentration of suspended particles at each vertical position in a stream cross section might be considered constant providing that the solids flow is steady. Assuming the simplified two-layer flow pattern (see Fig. 3.14) for an evaluation of the dispersion of solid particles by a turbulent diffusion in a partially-stratified flow, the diffusive effect of the turbulent eddies is considered to be active in the entire cross section of the discharge area, i.e. in the entire pipeline cross section if there is no stationary bed at the bottom of a slurry pipeline.

To obtain a relationship between integral parameters which are of major importance in the determination of a portion of particles contributing to the turbulent suspension it is useful to compare the tendency of a cloud of solid particles in suspension to settle with the tendency of liquid flow to disperse the solid particles. Wilson (1972) pointed out that for the initiation of turbulent suspension two conditions must be satisfied. Following Prandtl's concept of particle support by a turbulent eddy, he stated that turbulent suspension is effective only if \( \frac{\bar{v}}{v_t} > 1 \) and the diameter of a turbulent eddy is larger than the particle diameter. Since the eddy diameter can be considered to be directly related to the eddy length, the particle size must be smaller than the mixing length. The classical approach to describing the mechanism for a solid particle interaction with a turbulent eddy is formulated in the turbulent diffusive model by Rouse and Schmidt (see Eq. 2.34 in Chapter 2). The model sets up the ratio of turbulent pulsative velocity and particle settling velocity, \( \frac{\bar{v}}{v_t} \), as a basic parameter for the effectiveness of a suspension process. This approach does not expect the presence of particles supported by any other support mechanism than the carrier turbulence in a slurry stream. Furthermore, it is considered valid only for flows of low concentrated turbulent suspensions.

In a partially-stratified flow only a fraction (represented by the volumetric concentration \( C_s \)) of all solid particles (represented by the volumetric concentration \( C_v \)) is supported by the carrier turbulence. A useful approximation for a stratification ratio (given by \( C_s/C_v \) or \( C_s/C_v \) when \( C_v = C_s + C_c \)) may be obtained by incorporating the effect of the presence of solid particles which are not supported by the turbulence on the basic parameter for the effectiveness of the turbulent suspension.
process. In low concentrated suspensions the effectiveness of the turbulent suspension process is represented by the ratio of terminal settling velocity and turbulent pulsative velocity. The effect of the concentration of suspended particles, $C_s$, and of the presence of contact-load particles in a slurry stream can be incorporated into the ratio as $(v_{th} C_s) / (\bar{v}_y [1-C_v])$. In this ratio $v_{th}$ is the hindered settling velocity of a suspended solid particle in suspension having concentration $C_s$. $\bar{v}_y$ gives a value of the turbulent fluctuating velocity representative for an entire stream cross section and $1-C_v$ is a fraction of the total volume of slurry occupied by the carrying liquid. The effectiveness of the suspension process might be further affected by the mixing length in a turbulent stream of a carrying liquid.

In the liquid flow the mixing length, $l$, increases with the distance from a pipe wall. Close to the pipe wall the relation is approximately linear. The constant of proportionality between the mixing length and the pipe-wall distance is determined by the von Kármán constant $\kappa$. Closer to the centre of a pipe the increase is smaller as can be seen on Fig. 7.2 where the ratio of the mixing length and the pipe radius, $l/R$, at a location within the flow is plotted for various relative distances of the location from the pipe wall, $y/R$. Generalising from Nikuradse's data in Fig. 7.2, the mixing length in the core of the turbulent flow has a representative value larger than $0.10R$, i.e. $0.05D$.

![Figure 7.2. Eddy mixing length $l$ as a function of a distance from wall for turbulent pipe flow, data by Nikuradse, plot from Davies (1972).](image)

Relationship between $l/R$ and $y/R$. Data for flows of $Re = 4000$ (x), $Re = 23.000$ (+) and $Re = 105.000$ to $3,240.000$ (.) in a smooth pipe.

Thus it may be anticipated that the representative value of the mixing length will be larger than the particle size in sand-slurry flows in pipelines of industrial size. Once the length scale of turbulence is larger than the particle size, momentum transfer from a turbulent eddy to a solid particle may occur. Further increase in the turbulent length scale due to a larger pipeline size is not of direct importance for the momentum
transfer and so for the particle suspension. The complex effects of the presence of solid particles in the stream on the mixing length distribution in a pipeline cross section are
anticipated. However, the effectiveness of the turbulent suspension in large pipelines
might still be considered to be primarily determined by the relation between the
intensity of turbulence and the particle settling velocity. The effect of the pipeline size,
if the pipeline is large, may be neglected.

The hindering effect of solids concentration on the particle settling velocity in a cloud
of suspended particles is described by Richardson & Zaki equation (see Eq. 2.31 in
Chapter 2)

\[ v_{th} = v_t (1 - C_s)^m \]

Exponent m is related to the Galileo number for particle (the Ga number is also called
Archimedes number and it can transformed to the particle Reynolds number, Re_p, as
\( Ga = C_D Re_p^2 \)). For extreme Ga values the exponent m becomes constant. The
exponent m = 4.8 for Stokes' law region of particle settling, i.e. for low Ga values, and
m = 2.4 for Newton's law region, i.e. for high Ga values (see e.g. Coulson et al.,
1996). Wallis (1969) suggested determining the exponent m by the equation

\[ m = \frac{4.7 \left( 1 + 0.15 Re_p^{0.687} \right)}{1 + 0.253 Re_p^{0.687}} \]  

(7.2)

in which \( Re_p = v_t d / \nu_f \).

The validity of the Richardson & Zaki equation (Eq. 2.31) is limited by the maximum
solids concentration which permits solid particle settling in a particulate cloud. This
maximum concentration corresponds with the concentration in an incipient fluidized
bed, i.e. with the solids concentration in a fluidized bed reached when the minimum
fluidizing velocity is equal to hindered settling velocity. Hence the maximum
concentration for the Eq. 2.31 application has a value of about 0.57 (Davidson et al.,
1985). Such a high value of local concentration was not measured anywhere in our
laboratory-pipe slurry flows except in the stationary beds and several en bloc sliding
beds. For flows with detected turbulent suspension, the maximal local concentrations
reached were considerably lower. Therefore Eq. 2.31 can be considered applicable at
any local position within the slurry flow in which solid particles are transported in the
turbulent suspension.

Substituting Eq. 2.31 to the ratio \((v_{th} C_s)/(\nu' Y [1-C_{vi}])\) gives
\( v_t [1-C_s]^m C_s / (\nu' Y [1-C_{vi}]) \). This suggests the existence of a relationship between the
dimensionless groups \( C_s [1-C_s]^m / C_{vi} \) and \( (1-C_{vi}) / C_{vi} \). This relationship appears to be a
suitable indicator of the presence of the turbulent suspension mechanism in a slurry
flow in our experimental pipelines. Initial plotting of the experimental data processed
by the testing model (for a description of TM see Chapter 5) showed a strong
relationship between these two concentration-containing dimensionless groups (see
Fig. 7.3) for slurries in which turbulent suspension in the pipe dominated and the shear
layer effect was negligible. This was the case for 0.2-0.5 mm sand (Sand 1) in a 150
mm pipe at different inclinations (0, 25 and 35 deg) and 0.05-0.70 mm sand (Sand 4) in a 650 mm pipeline. Data for 0.5-1.0 mm sand (Sand 2) in the 150 mm pipe approached an envelope of plotted data for high $V_m$, where the turbulent suspension mechanism became effective.

![Image](image.png)

**Figure 7.3.** Relationship between dimensionless groups.

The characteristic value of the turbulent pulsative velocity $\tilde{v}'_y = \sqrt{\langle v'^2 \rangle}$ for fluid flowing in a pipeline can be considered as being proportional to the shear velocity $u_\ast$. This substitution, which is widely used in modelling slurry flows both in open channels (see e.g. van Rijn, 1989) and in pipelines (see e.g. Wilson, 1972), has its basis in the classic experiments published in 1954 by Laufer. He used a hot-wire anemometer to measure the three components of fluctuating velocity in a pipe flow of air. In Fig. 7.4 Laufer's experimental data show the relation between the ratio of the time averaged value of the velocity fluctuations and the shear velocity $\tilde{v}'_y/u_\ast$ ($\tilde{v}'_x/u_\ast$ respectively) and the ratio of the vertical position above the pipe wall and the pipe radius $y/R$. The data are from a 247 mm circular pipe and flow at $Re = 4.105$.

The figure shows that the ratio $\tilde{v}'_y/u_\ast$ is almost constant and close to unity across a pipe cross section except near the pipe wall. Turbulence tends to be isotropic in the central region of a flow, i.e. in the region of small axial-velocity gradients.

Estimating $\tilde{v}'_y$ equal to shear velocity for fluid flow in a circular pipe leads to a direct relationship between the turbulent intensity and mean velocity of flow.
\[ \tilde{v}_y' = u_* = \frac{\tau_0}{\sqrt{\rho_f}} = V_m \sqrt{\frac{\lambda_f}{8}} \]  \hspace{1cm} (7.3).

**Figure. 7.4.** Laufer's data for turbulent pipe flow of air, from Davies (1972).

Relationship between \( \tilde{v}_y'/u_* \) (\( \tilde{v}_x'/u_* \)) and \( y/R \).

For slurry flow in a circular pipe it can be assumed that \( \tilde{v}_y' \) is a function of \( V_m \) and may be influenced by the presence of solids in fluid flow.

The ratio \( v_{th} C_s/(\tilde{v}_y' (1-C_{vi})) \), \( v_{th} C_s/(V_m (1-C_{vi})) \) respectively, was found to be independent of flow parameters additional to those already present in the ratio, so the dimensionless groups of parameters can be related to determine the \( C_s/C_{vi} \) ratio as \( C_s/C_{vi} = \text{fn}(V_m/v_{th}, [1-C_{vi}]/C_{vi}) \). These dimensionless groups of parameters representing an interaction between solids and liquid in a turbulent flow are linked in a correlation

\[ \frac{C_s(1-C_{vi})^{\beta_2 m}}{C_{vi}} = \beta_1 \left( \frac{V_m}{v_{th} \cos \omega} \right)^{\beta_2} \left( \frac{1-C_{vi}}{C_{vi}} \right)^{\beta_3} \]  \hspace{1cm} (7.4).

The correlation reflects the balance between the gravitational effect on the solid particles and the dispersion effect of the liquid turbulence in the direction perpendicular to the flow direction. In the inclined pipeline the cross-pipe component of the particle settling velocity diminishes so the particles can be more easily suspended by turbulent eddies in the direction perpendicular to the flow direction. In steep pipelines the pipe-cross component of the settling velocity is small compared to the pipe-axis component. This component interferes with the local carrier velocities in the flow direction. Their fluctuating parts are usually far smaller than the total time-averaged values of the local velocities, thus turbulent diffusion is of less importance for the
particle suspension. The correlation 7.4 was verified experimentally within the pipeline inclination range \( \alpha \leq 35 \) deg. The correlation 7.4 was tested by experimental data processed by the testing model TM (see Chapter 5) to get the experimental \( C_S \) values. The coefficients \( \beta_1, \beta_2 \) and \( \beta_3 \) were determined by regression of processed experimental data.

### 7.2.2 Determination of solids fraction suspended by turbulence

The correlation 7.4 provided a good agreement with data for flows in which turbulence was a dominant suspension mechanism. Although subject to modification in the light of an increased data base, the correlation with coefficient values \( \beta_1 = 0.108, \beta_2 = 0.346 \) and \( \beta_3 = 0.248 \) fits very well with the available data (regression coefficient is 0.91) for Sand 1 data for different slurry velocities, solids concentrations and pipeline inclinations (0, 25, 35 deg.) and Sand 2 data for the highest tested velocities. Fig. 7.5 shows a comparison of curves given by the equation

\[
\frac{C_S(1-C_S)^{0.346m}}{C_{VI}} = 0.108 \left( \frac{V_m}{V_{t \cos \alpha}} \right)^{0.346} \left( \frac{1-C_{VI}}{C_{VI}} \right)^{0.248}
\]

with the experimental data processed by the TM. The equation is also successful in predicting the suspended portion in flow of slurry of Sand 4 in a large dredging pipeline, as shown on an example comparing the model outputs with the dredging pipeline data in Chapter 8. Application of the equation is limited to the \( C_{VI} \) range given by approximate boundary values 0.07 and 0.39.

According to the correlation 7.4, the amount of solid particles supported by turbulence does not depend on pipeline diameter \( D \). Comparison of processed experimental data from a 150 mm pipeline (DN150) (Sand 1, Sand 2, Sand 3 and Gravel) and a 650 mm pipeline (DN650) (Sand 4) in Fig. 7.3 seems to confirm this prediction. Experimental data obtained in Saskatchewan Research Council test loops for a variety of pipeline diameters (DN53, DN159, DN263, DN495) also did not show any significant influence of \( D \) on the determined \( C_C \) [see the correlation by Gillies et al., 1991 (Eq. 3.46 in Chapter 3)]. This may be explained by the fact that the size of the representative turbulent eddies in pipes of industrial size (typically of \( D \) larger than 100 mm) is not significantly affected by the near-pipe-wall region in which the eddy size is much smaller than in the flow core.

The correlation 7.4 is not found in coarse particle slurries (the Sand 2 slurry except the highest mean slurry velocities \( V_m \) and the slurries of Sand 3 or Gravel). In flows of these slurries the dispersion due to lift forces within the shear layer predominates over turbulent suspension, as identified and discussed in Chapter 5. Even for velocities at which coarse-particle slurry flow exhibited its lowest degree of flow stratification the data did not fit the turbulent suspension correlation 7.4. This is also indicated by the large scatter of coarse-slurry-flow data in the plot of dimensionless groups occurring in the correlation (see Fig. 7.3).
Figure 7.5. Comparison of the correlation 7.5 with data processed by the testing model.
Gillies et al. (1991) observed a systematic deviation in their correlation (Eq. 3.46 in Chapter 3) for the "Fine Sand" data, particularly at high $C_{Vf}$. Presumably, the relationship $C_{C}/C_{Vf}$ versus $V_{m}/V_{f}$ does not sufficiently represent the effect of solids concentration on the process of turbulent suspension of solid particles in slurry flow. The non-linear relationship between $C_{S}$ and $C_{Vf}$ in equation 7.4 expresses the fact that the ability of a carrying liquid to suspend particles by turbulence is proportional to the amount of the carrying liquid in a pipeline.

The correlation 7.4 does not provide an indication whether the turbulent suspension is effective in a pipeline. The threshold velocity for turbulent suspension can be estimated from the shape of the resistance curve ($I_{m}$ versus $V_{m}$), if measured in the velocity range containing the threshold value, or by Wilson's equation for the threshold velocity $V_{tt}$ (Eq. 3.43 in Chapter 3). The effectiveness of the mechanism of turbulent suspension can also be roughly estimated by using the threshold $d/D$ ratio values as reported by Wilson et al. (1992) and reviewed in Chapter 3.

7.3 Modelling of solids dispersion within the shear layer

Only a complete analysis supported by experimental observations of sheet flow may lead to general conclusions about friction and transport laws for the shear layer. Determination of the solids fraction transported (and of the sub-fraction suspended) within the shear layer requires knowledge of layer dimensions, i.e. positions of its top and bottom, and of sheet flow characteristics, namely the velocity and concentration gradients over the shear layer and the shear stress at the bottom of the shear layer. Our experiments provided only a limited number of characteristics and were carried out in rather complex flow conditions (no stationary bed below the shear layer, no sharp top of the sliding bed).

7.3.1 Sheet flow conditions

Sheet flow occurs within the shear layer and contains particles in either sporadic or permanent contact with neighbouring particles. The sheet flow is initiated and developed by the action of the shear stress, $\tau_{O}$, at the surface of the granular bed. The initiation of the sheet flow requires a rather high bed-shear-stress value, much higher than that necessary to initiate the motion of the first grains at the bed surface. The dimensionless bed shear stress, called also the particle mobility number, $\theta$,

$$\theta = \frac{\tau_{O}}{(p_{S} - p_{f})gd} = \frac{u_{m}^2}{(S_{S} - S_{f})gd} \quad (7.6)$$

is a basic parameter used to evaluate interaction between the liquid flow over the granular bed and the granular bed surface. The shape of the granular bed surface and solids transport at the bed surface are effects of the interaction between the flow and the bed. The particle mobility number $\theta$ expresses the bed shear stress in a form considering a relationship between the shear and the gravitational effects on a solid
particle occupying the bed surface. Its critical value \( \theta_{cr} \), related with \( \text{Re}_{cr} = u*d/\nu \) by Shields (1936), gives the flow condition at the beginning of sediment transport, i.e. a stage at which the first grains of the bed surface begin to move. For flows of \( \text{Re}_{cr} \geq 400 \) Shields proposed the constant \( \theta_{cr} = 0.06 \). However, the upper plane-bed regime is not established until \( \theta \) exceeds the approximate value 0.8 (e.g. Wilson & Nnadi, 1990). Van Rijn (1989) qualified the bed forms by using the excess bed-shear stress parameter, \( T \), and predicted a plane bed surface for

\[
T = \frac{\tau_o^2 - \tau_{o,cr}^2}{\tau_{o,cr}^2} > 15-25
\]

(7.7).

In this equation the parameter \( \tau_{o,cr} \) is the bed shear stress at the incipient particle transport and it is obtained from the Shields \( \theta_{cr} \). The high-shear-stress flow over the bed is typically reached in closed pressurised conduits. Shear stresses at the surface of the granular bed are usually bigger by an order of magnitude in pressurised pipes than in open channels of comparable dimensions. In a pressurised pipe the bed shear stress \( \tau_o \) is high because of the high hydraulic gradient over the pipe length. In an open channel a large depth and/or a steep slope have to be available to reach the similar \( \tau_o \) values. Visual observation of stratified flows in our laboratory circuit showed a flat bed surface for all slurries tested at mean slurry velocities above the deposition-limit value.

Shear stress at the bed surface can be determined experimentally for flow over a stationary bed, provided that the hydraulic gradient, mean velocity and geometry of the discharge area are known (e.g. Wilson, 1966). However, the flow in pipelines transporting slurry often exhibits some degree of stratification although the flow is almost always stationary-deposit free. It delivers a considerable amount of solid particles as contact load and/or as suspended load. The determination of the bed shear stress \( \tau_o \) in such a complex flow is extremely difficult, not least because of the dubious identification of the interface at the top of the mobile bed. The interface is an additional flow boundary in the pipeline. The interface is either real, when the flow is sharply stratified at the top of the sliding bed, or virtual, when there is a developed transition region between the dilute stream and the high-concentrated sliding bed.

From our laboratory data, which include the measured concentration profiles in slurry flow, the positions of the virtual interface and the interfacial shear stresses may be reconstructed for a simplified two-layer flow pattern using the testing model TM described in Chapter 5. This approach to the interpretation of the flow conditions at the interface is discussed in the following section. The aim is to determine the position of the interface, \( Y_{12}/D \) and the stratification ratio, \( C_{v}/C_{vi} \), in slurry flow with a developed shear layer for the purposes of the predictive two-layer model.

7.3.2 Virtual interface and suspended solids fraction in flows with developed shear layer

The analysis, submitted in Chapter 5, revealed that a portion of the solids occupying a shear layer should be considered to be suspended in the carrying liquid. The existence
of a suspension mechanism within the shear layer can be also deduced from the shape of the concentration-profile curve within the shear layer. The centre of the shear layer is located at the point of inflexion of the profile curve. The convex curve \[ \left( \frac{d^2c}{d(y/D)^2} > 0 \right) \] and the concave curve \[ \left( \frac{d^2c}{d(y/D)^2} < 0 \right) \] of the concentration-profile curve express the fact that one particle-support mode (suspension or contact) is dominant in the flow region characterised by certain course of the curve. Thus for stratified flows with a developed shear layer the position of the point of inflexion on the concentration profile curve is a suitable position for the virtual interface if the internal structure of slurry flow is idealised to form a two-layer flow pattern.

In a fully-stratified flow the position of the interface between the layers varies only with the mean spatial concentration, \( C_{vi} \), in the pipeline, provided that a constant solids concentration in an en bloc sliding bed, termed \( C_2 \) in the two-layer flow pattern, is maintained at different bed velocities, \( V_2 \). The real interface is replaced by the shear layer if high-shear-stress flow occurs at the bed surface. The observations, discussed in Chapter 5, of coarse slurry flows in the 150 mm test loop, showed that the shear layer may change its thickness with \( V_m \) in the slurry flow but that the shear layer centre, corresponding with the position of the point of inflexion of the concentration profile, tends to remain constant provided that the solids concentration \( C_{vi} \) in the pipeline does not change with changing \( V_m \).

The position of the point of inflexion changes significantly only with \( C_{vi} \). This can be seen in Figs 7.6a and 7.6b for profiles in slurries of different particle sizes, solids concentrations and mean slurry velocities. The position of the point of inflexion can be considered as being the position a real interface would occupy if the slurry flow was fully stratified. The parameter \( C_{2,inf} \), plotted in Figs. 7.6a and 7.6b, gives the solids concentration in a fictive bed having its top at the measured position of the point of inflexion and containing all solids occupying the test loop. The \( C_{2,inf} \) values obtained are very similar to the values detected in the real en bloc sliding beds (\( C_2 \) between 0.50 and 0.55) in the test loop. This confirms that the position of the point of inflexion is a suitable position for a virtual interface in flows with a developed shear layer. The point of inflexion is positioned at the centre of the shear layer, as is the virtual interface.

The correlation based on our laboratory data (Fig. 7.6b) gives an equation determining the position of the virtual interface for slurry flows exhibiting the shear layer as

\[
\frac{Y_{inflex}}{D} = \frac{Y_{12}}{D} = 1.68 C_{vi}^{0.92}
\]

The mean \( C_{2,inf} \) value of 0.53 for data of \( C_{vi} > 0.15 \) (Fig. 7.6b) corresponds very well with observed concentrations within en bloc sliding beds. Although requiring a modification in pipelines of different sizes, the power-law relationship between \( Y_{inflex}/D \) and \( C_{vi} \) can be used in the predictive two-layer model to determine the position of the virtual interface, \( Y_{12}/D \).

It is known that the shear stress at the top of the granular bed is responsible for the development of the shear layer linked to the top of the bed. According to the analysis
Figure 7.6: Point of inflexion on concentration profile curves and have bed concentration.

**Graph 1:**
- Key: Coefficient: 0.992
- $q = 0.023$
- $q = 1.595$
- $y_{inflow} = a + b y_{outflow}$
- Regression

**Graph 2:**
- Key: Coefficient: 0.986
- $q = 0.886$
- $q = 1.596$
- $y_{inflow} = a + b y_{outflow}$
- Regression

**Graph 3:**
- Key: Coefficient: 0.952
- $q = 0.920$
- $q = 1.752$
- $y_{inflow} = a + b y_{outflow}$
- Regression

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Regression:
\[ y_{\text{inflex}}/D = a C_{\text{vi}}^b \]
- \( a = 1.682 \)
- \( b = 0.915 \)
- Regr. coefficient: 0.975

**Sand 2, Sand 3, Gravel**

![Graph showing regression data for different concentration levels.]

Mean value: \( C_{2, \text{inf}} = 0.560 \) for all points.
Mean value: \( C_{2, \text{inf}} = 0.534 \) for \( C_{\text{vi}} > 0.15 \).

Figure 7.6b. Point of inflexion on concentration profile curves and fictive bed concentration for Sand 2, Sand 3 and Gravel together.
submitted in Chapter 5, a certain portion of the solid particles within the developed shear layer may be considered to be suspended. Therefore it is encouraging to see the close correlation between the parameter associated with the development of the shear layer, $\tau_{12}$, and the parameter $C_C/C_{vi}$, quantifying the suspension process in the slurry flow, as obtained for the processed experimental data on Fig. 7.7a and Fig. 7.7b.

Slurry flow data for different particle sizes, mean slurry velocities and mean solids concentrations are processed by the testing model TM rearranged in such a way that the $Y_{12}/D$, determined by Eq. 7.8, became an input parameter for the TM. Thus the solids concentration of contact load in the lower layer, $C_{2c}$, became an independent variable in the TM, the $C_{2c}$ value was no longer considered equal to $C_2 - C_1$. Fig. 7.7a shows a strong relationship between the TM outputs for the shear stress $\tau_{12}$ at the virtual interface and the stratification ratio $C_C/C_{vi}$. This relationship seems to be virtually independent on particle size, mean slurry velocity or solids concentration in the slurry flow. The relationship can be approximated by a power law function (see Fig. 7.7b), this function provides the highest value of the regression coefficient from different tested approximations. The stratification ratio for slurry flow described by an idealised two-layer pattern in which the shear layer is split by the virtual interface so that the solid particles occupying the shear layer contribute to both model layers can be estimated by a correlation

$$\frac{C_C}{C_{vi}} = 0.03 \tau_{12}^{0.68} \quad (7.9).$$

Interfacial shear stresses $\tau_{12}$ from the experimental data tended to be higher in coarser slurry flows than in finer slurry flows (compare the range of the experimental $\tau_{12}$ values obtained for flow of Sand 2 and flow of Gravel in plots of Fig. 7.7a). For the particle mobility parameter at the interface (see Eq. 7.6)

$$\theta_{12} = \frac{\tau_{12}}{(\rho_s - \rho_f)gd_{50}}$$

the trend is opposite. The $\theta_{12}$ values, calculated from the output $\tau_{12}$ values, for Gravel flow lie within the range 0.9 - 1.4 while for Sand 2 flow the approximate range limits are 0.9 and 3.4 and for Sand 2 flow the approximate limits are 1.3 and 6.1. The values for $\theta_{sh,bot}$, given by the shear stress at the bottom of the shear layer $\tau_{sh,bot}$, may be higher because of the shear stress distribution within the shear layer (see Fig. 7.8). Thus the theoretical condition $\theta_{sh,bot} > 0.8$ for the upper-plane-bed flow was always satisfied and a plane bed was actually observed in the test loop for all measured flows. Yalin (1992) proposed that the interfacial friction coefficient should be determined from an interfacial roughness size given by the particle diameter instead of the shear-layer thickness for $\theta_{sh,bot} < 2$. Thus the shear layer can be considered to be poorly developed in flows satisfying this $\theta_{sh,bot}$ condition. The condition can be considered satisfied in the most stratified flows observed in our test loop, particularly in the Gravel flows of different mean velocities and concentrations.

As noted in the Chapter 5 the testing model output for the interfacial friction coefficient $\lambda_{12}$ is the most sensitive indicator of the quality of the testing model
configuration and of the consistency of the experimental data used as the testing model inputs. The values of interfacial friction coefficient $\lambda_{12}$ in Fig. 7.7a are realistic being of the same order of magnitude as the values given by the literature models reviewed in Chapter 3. This confirms that experimental data processing by the testing model also gave realistic output values for other quantities plotted on Fig. 7.7a. The $y_{\text{inflex/D}}$ values plotted in Fig. 7.7a illustrate that the experimental database used to establish the Eqs. 7.8 and 7.9 represented a wide range of positions of the shear layer within the stratified slurry flow.

The solids distribution in the two-layer flow pattern is not fully solved by using the Eqs. 7.8, 7.9 and mass balance equations for the lower and the upper layers. Additional information is required about the distribution of the solids fraction in the flow pattern. This information is either the value for mean solids concentration in the lower layer, $C_2$, or the value of the slip ratio $C_{vd}/C_{vi}$. The determination of the solids distribution by using the stratification-ratio equation 7.9 in the predictive two-layer model is possible only for a given value of the frictional head loss $I_m$. If the $I_m$ value is to be predicted by the model, the $\tau_{12}$ required to determine the stratification ratio in the computational scheme of the predictive model is the unknown parameter directly related to $I_m$.

An increase of $C_C/C_{vi}$ with $\tau_{12}$ in the Eq. 7.9 seems to contradict the consideration that higher bed shear stress causes a thicker shear layer and thus less contact among transported particles. It must be considered, however, that the interfacial shear stress $\tau_{12}$ is not equal to the shear stress at the bottom of the shear layer, $\tau_{sh,bot}$, which is responsible for the development of the shear layer. The value of $\tau_{12}$ approaches that of $\tau_{sh,bot}$ only when the thickness of the shear layer approaches zero, otherwise the $\tau_{sh,bot}$ value is larger than the $\tau_{12}$ value, as can be seen from the shear stress distribution within the shear layer in Fig. 7.8. The difference between the two shear stress values grows with the increase in shear layer thickness $Y_{sh}$. Comparison of the $\tau_{12}$ and $C_C/C_{vi}$ values with corresponding concentration profiles for measured flows showed that the highest $\tau_{12}$ values were reached in the most stratified flows, i.e. in flows with the thinnest shear layer for which $\tau_{12}$ approached $\tau_{sh,bot}$ and $C_C/C_{vi}$ reached the highest values. This can be seen also on the $C_2$ versus $\tau_{12}$ plot in Fig. 7.7a where $\tau_{12}$ gains the highest values for the highest $C_2$ values. The $C_2$ is the mean concentration of solids in the flow below the interface given by the position of the point of inflexion on the concentration-profile curve. The $C_2$ value is obtained by integrating the measured concentration profile over the area, $A_2$, of the pipeline cross section below the interface. Thus the highest $C_2$ values on Fig. 7.7a indicate the flow in which the sliding bed below the interface is the least sheared and the shear layer is the thinnest. Low $\tau_{12}$ values are associated with low $C_2$ values and thus with the thick shear layer. For such flow conditions the $\tau_{sh,bot}$ might be considerably higher than $\tau_{12}$. The relationship between $\tau_{12}$ and $\tau_{sh,bot}$ is a product of the shear stress distribution across the pipe height (see Fig. 7.8) and it is discussed in the following paragraph.

Low $C_C/C_{vi}$ values indicate that the mean concentration of contact load particles in the lower layer, $C_{2c}$, is low. The $C_{2c}$ is the $C_C$ recalculated for the area of the lower layer, $C_{2c} = C_C/A_2$. The $C_{2c}$ values obtained by processing the experimental data (resulting to Eqs. 7.8 and 7.9) were found to be lower than the integrated $C_2$ values.
Figure 7.7a. Stratification ratio and other parameters related to the shear stress at the virtual interface positioned to the height yinflex. Data are processed by the testing model.
Regression:
Cc/Cvi = a.T12^b
a = 0.029
b = 0.681
Regression coefficient: 0.899

**Sand 2, Sand 3, Gravel**

Figure 7.7b. Stratification ratio $\text{Cc/Cvi}$ as a function of interfacial shear stress $T12$. 
for flows exhibiting a developed shear layer. This indicated that not all particles below the virtual interface contributed to the contact load. A portion of particles occupying the shear layer below the interface was suspended. This portion is represented by the mean concentration $C_2 - C_{2c}$ in the lower layer.

![Schematic diagram of concentration and shear stress distribution in pipeline flow of slurry with the shear layer above the en bloc sliding bed.](image)

**Figure 7.8.** Schematic distribution of concentration and shear stress in pipeline flow of slurry with the shear layer above the *en bloc* sliding bed.

### 7.3.3 Shear stresses characterising the shear layer

The applied shear stress at a certain vertical position, $y$, in the cross section of the slurry pipe is exerted by the flow impelling force. This is generated by the pressure gradient in the carrying liquid flow over the pipe length. The applied-shear-stress distribution across the pipe vertical diameter is linear with the zero shear stress at the vertical position of the maximum local liquid velocity (Fig. 7.9). The applied shear stress, $\tau_a$, at any position $y$ is resisted by the shear stress, $\tau_R$, composed of the liquid component due to (turbulent eddy) viscous friction, $\tau_p$, and the solid component due to intergranular friction, $\tau_s$. The balance between the applied shear stress and the resisting shear stress at a local position $y$ is

$$\tau_a = \tau_R \quad (7.10).$$

In a one-dimensional flow picture (see Fig. 7.9) the balance components are written as

$$\tau_a = \rho g (R' - y) l_m \quad (7.11),$$

where $R'$ gives the distance between the position of zero shear stress and the bottom of the discharge area in the carrying liquid flow and $l_m$ is hydraulic gradient due to friction, and

$$\tau_R = \tau_s + \tau_f \quad (7.12).$$
where the liquid component is related to the local velocity gradient and the solids component is related to the intergranular normal stress, \( \sigma_s \). The \( \tau_s \) within the shear layer is determined according to Bagnold (1956), see Eq. 2.38 in Chapter 2, as

\[
\tau_s = \sigma_s \tan \phi'
\]

when \( \phi' \) is the angle of dynamic friction for the sheared granular body. The intergranular normal stress at the vertical position \( y \) is given by the equation

\[
\sigma_s = \rho g C_{YS,c}(S_S - S_f)(Y_{sh} - y) \quad (7.13)
\]

in which \( Y_{sh} \) is the thickness of the shear layer and \( C_{YS,c} \) is the mean concentration of particles in mutual contact within the layer of thickness \( Y_{sh} - y \).

The shear stress distribution in Fig. 7.9 shows that by definition the thickness of the shear layer cannot span the whole pipe cross section. Considering that the shear layer is generated by the applied shear stress, the shear-layer top cannot exceed the position in the pipe cross section at which the shear stress reaches the zero value.

![Figure 7.9. Schematic shear stress distribution in pipe occupied by a stratified slurry flow with a shear layer.](image)

At the bottom of the shear layer, the applied shear stress is offset by the intergranular shear stress exerted by the submerged weight of contact particles above this boundary. According to Eq. 7.11, the applied shear stress at the bottom of the shear layer is

\[
\tau_{sh,bot} = \rho g (R - Y_b) f_m
\]

where \( Y_b \) is the thickness of the uniform \textit{en bloc} sliding bed below the shear layer. The applied \( \tau_{sh,bot} \) is resisted by the stress determined by substituting the equation 7.13 for \( Y_{sh} \) and \( C_{sh,c} \) to the Bagnold's equation (Eq. 2.38) as

\[
\tau_{sh,bot} = \rho g C_{sh,c}(S_S - S_f)Y_{sh} \tan \phi'
\]

in which \( C_{sh,c} \) is the mean concentration of contact load particles in the shear layer.
The relationship between $\tau_{zh, bot}$ and $\tau_{12}$ at the virtual interface used in the two-layer flow pattern in the one-dimensional picture of pipe flow is given as

$$
\tau_{zh, bot} = \tau_{12} + 0.5 \rho g I_m Y_{zh}
$$

(7.15).

7.4 Conclusions

For the purposes of the two-layer model the amount of solids supported by liquid turbulence in a partially-stratified slurry flow is predicted by equation 7.5. No effect of the pipeline diameter on the stratification ratio determining the solids division into two layers of a simplified pattern for the partially-stratified flow has been found. The stratification ratio is affected by the ratio of mean slurry velocity and particle settling velocity and by the mean spatial concentration of solids in the turbulent stream. This concept determines the fraction of solids contributing to a suspended load for fine and medium sands.

Slurry flows exhibiting a shear layer behave differently from stratified flows with turbulent suspension in a pipeline. In a slurry flow exhibiting a developed shear layer a virtual interface between two idealised layers is positioned in the centre of the shear layer. The centre of the shear layer is given by the position of the point of inflexion on the concentration profile of slurry flow in the pipeline cross section. The position of the point of inflexion is dependent on the mean in-situ concentration $C_{vi}$ only (Eq. 7.8). The solids fraction transported as a contact load in the shear-layer-containing pipeline flows, modelled by the simplified two-layer flow pattern, is directly related to the shear stress at the virtual interface (Eq 7.9).

7.5 References


Chapter 8

Unsteady solids flow in a long pipeline connected with a dredger: observation, analysis and modelling by the two-layer model

In this chapter the two-layer model is applied to specific conditions of slurry flow in a pipeline connected with a dredger. The two-layer model analyses and explains the interesting phenomena detected in the slurry flow of the continuously fluctuating slurry density in a long conveying pipeline. These phenomena are associated with the development of an internal structure in the slurry flow due to an unsteady state of solids flow and result in a process of solids aggregation within slurry flow along the long pipeline.

The effects of slurry density fluctuation and solids aggregation on the behaviour of a granular bed at the bottom of a pipeline and on the process of mechanical energy dissipation in the pipeline are observed and analysed. Understanding the impacts of the solids aggregation process on friction loss and deposition-limit velocity is important for economic and safe pipeline operation. An explanation of the unsteady solids flow effects carried out on the basis of the two-layer model is confirmed by laboratory tests. Some results of the investigation presented in this chapter have been published recently (Matoušek, 1995a, b, c, 1996a, b).

8.1 Introduction

Extensive field measurements (described in Chapter 4) on a dredging installation with a pipeline that is approximately 10 km long and which has three booster stations in series, have shown that density fluctuations in the slurry flow generated in the system inlet are not flattened. Whilst passing along the pipeline with pumps in series, they are transformed into long density waves with a high amplitude.

This phenomenon is typical of dredging operations, because it appears only in long pipelines with slurry flow of continuously fluctuating density. Only on-line measurements at several measuring sites along the entire conveying system can detect the phenomenon. For this reason it was observed for the first time only during the MeaVli measurements (Beulink, 1982). The phenomenon has not been analysed and explained yet.

An aggregation process may have an influence on the efficiency and safety of the operation of the system. Consideration of the effects of the aggregation process on mechanical energy dissipation and granular deposit formation in a slurry pipeline may lead to a more effective control of a conveying system.
8.2 Observations

8.2.1 Solids aggregation process

During dredging operations slurry density varies in time and space along the entire long pipeline of the conveying system. The controlled global operational parameters of the system (slurry flow rate through the conveying system, rpm of pumps) are usually maintained at an approximately constant level throughout the operational period of the system. That was also the case during the MeaVli field measurements on the conveying system composed of the dredger Groningen (Gr) and a long pipeline equipped with three booster stations: Zaandam (Za), Jagersplas (Ja) and Duinjager (Du), in series (see the lay-out in Fig. 4.5). Three measuring sites along the long pipeline were positioned to the locations in certain distances behind the Groningen, Jagersplas and Duinjager pumps.

The fluctuation in the density of the slurry which entered the system and moved through the pipeline was detected at three measuring sites along the pipeline length by radioactive density meters and interpreted as moving density waves in the pipeline. The solids aggregation process, which was observed along the long pipeline, was demonstrated by the transformation of the density waves which were moving along the pipeline (Fig. 8.1a, 8.1b, 8.1c, 8.1d). Analysis of data from the system pumps and drives (torque and rpm measurements at the Ja and Du boosters) showed that the transformation of the density waves was not caused by the pump performance (possible inertia effects of the centrifugal forces on solid particles). Comparison of the torque and rpm signals from the booster with the signal from density meter behind the booster at the Jagersplas and Duinjager locations (Fig. 8.2a and 8.2b) showed that density waves were transformed between the location of the density meter Jagersplas and the booster station Duinjager. The booster drive reacted to the passing density waves which already had the same geometry as those measured by the density meter a short distance behind the booster. Thus an aggregation mechanism was active in the pipeline. The negligible effect of the pump performance on solids aggregation phenomenon is not surprising when we consider the dimensions of the density waves and the pumps. The detention time of slurry in a pump was much shorter than was the time a long density wave needed to pass the pump.

8.2.2 Further detected effects of unsteady solids flow

Besides the aggregation process, the following effects of the unsteady state of solids flow on the slurry flow parameters were detected in the MeaVli pipeline when the database was analysed:
- a considerable variation in the velocity of solid particles at the bottom of the pipeline as a result of the fluctuating slurry density in the pipeline (Fig. 8.3a, 8.3b)
- a weak variation in friction loss in the long pipeline section with mean slurry density (Fig. 8.4a, 8.4b).

The movement of a granular bed at the bottom of the pipeline is accelerated within a denser slurry flow. When the signal for local solids velocity at the bottom of the
pipeline cross section is compared with the signal for mean slurry density just passing the pipeline cross section, the reaction of bed behaviour to the fluctuating slurry density is clearly seen (Fig. 8.3b). The correlation between bed velocity and mean slurry density is the most pronounced when the mean slurry velocity variation has been eliminated by filtering of the data for a narrow range of velocities.

Pressure drop due to friction over a long pipeline section increases very slowly with the increasing average slurry density in the pipeline section. Values for the friction loss in the Ja-Du pipeline section (4820 m of pipeline length) can be determined as the difference between measured absolute pressure at the beginning and at the end of the pipeline section when minor losses in the pipeline section and a difference in geodetic heights of the pumps are subtracted. Pressures were measured at the inlet and the outlet of each pump, thus the determination of the friction loss from the measured pressures in the pipeline can be done in a long pipeline section for slurry density averaged over the long pipeline section. Instant friction loss in a long pipeline section is related to the average density of the slurry occurring in the entire pipeline section at the moment of measuring the differential pressure (Fig. 8.4a, 8.4b). Mutual comparison of friction losses obtained from the long pipeline section at an approximately constant mean slurry velocity for various mean slurry densities present in the section shows a weak relationship between friction loss and average slurry density.

8.3 Analysis

8.3.1 Solids aggregation process

Fluctuations in the mean slurry velocity in the MeaVli pipeline were of a much lower order than fluctuations of mean slurry density. Slurry density was a major variable parameter in the pipeline. In accordance with the flow conditions observed during MeaVli measurements, the slurry flow is assumed to be steady for the purpose of solids flow analysis in the dredging pipeline.

Density variation produces an unsteady solids flow (solids flow rate $Q_S = \text{const}$) along the pipeline, even when the slurry flow is considered steady ($Q_M = \text{const}$). For the simplified no-slip condition (mean solids velocity $V_S$ is equal to mean slurry velocity $V_M$) the continuity law states that $V_S$ in an arbitrary cross section of the pipeline (of a constant diameter) is constant, so independent of the slurry density passing that pipeline cross section. In this condition no solids aggregation is expected and solids flow can be considered stable although unsteady. The stability of solids flow in the steady slurry flow in a pipeline of constant diameter is characterised by the constant mean solids velocity in the pipeline cross section along the entire long pipeline.

When the simplified no-slip condition is accepted for the modelling of the unsteady solids flow in a slurry of constant flow rate in a long pipeline, and the flow behaviour is simulated by means of basic hydrodynamic equations, including description of transport and turbulent dispersion effects by diffusion-dispersion mechanism (Basco, 1977), the fluctuating slurry density entering the conveying system is assumed to be gradually flattened by the turbulent mixing process and to become almost constant in
Figure 8.1a. Density waves measured along the Rheinl. Pipeline.

Density meter location: DINNAKER (6538m)

Density meter location: JACERPLAS (1886m)

Density meter location: GRONINGEN (367 m)

Flow meter location: JACERPLAS (1886 m)

Fine to Medium Sand

Steel Pipe DN560

Slurry density

Slurry density [kg/m³]

Time
Figure 8.1b. Density waves measured along the MeaVli pipeline.
Figure 8.1c. Density waves measured along the Mekhali pipeline.

Density meter location: DUNJAGER (6538m)

Density meter location: JACGERAS (1886m)

How meter location: JAGGERAS (1886 m)

Fine to Medium Sand
Steel Pipe DN650

MeaVli 11.02.1981

Fine to Medium Sand

Flow meter location: JAGERSPLAS (1886 m)

Density meter location: GRONINGEN (-367 m)

Density meter location: JAGERSPLAS (1886 m)

Density meter location: DUINJAGER (6538 m)

Figure 8.1d. Density waves measured along the MeaVli pipeline.
Figure 8.2b. Torque of the diesel engine and rpm of the centrifugal pump on the booster Duinjager. Reaction of the booster on the density waves.
Figure 8.3b. Local velocity of solids at the bottom of MeaVli pipeline, measured at the position 0 deg in a pipeline cross section in the location Jagersplas. Comparison of the local velocity signal with corresponding signals for global parameters.
Figure 8.4: Relationship between pressure drop and density of the measured fluid.

Hydraulic gradient (J)

Pressure (bar)

Time

Average density in the poplite section 1.63
Average density in the poplite section 0.74.5
Average density in the poplite section 0.74.5

Fine to Medium Sand

Meawil 02.1.931

Steel Pipe DNG0
Figure 8.4b. Relationship between pressure drop and slurry density in the Ja-Du section of MeaVli pipeline. Comparison of the measured relationship with the prediction by Durand model (for \( V_m = 3.40 \) m/s and \( d_{50} = 0.25 \) mm).
time and space along the long pipeline. This mechanism may be effective in short time and length scales and may cause a flattening of short-time density fluctuations in a pipeline behind a dredger pump (the Ja density signal is smoother than the Gr density signal in Figs. 8.1a, 8.1b, 8.1c and 8.1d). A longer time and length scale is more suitable for description of the process in a dredging pipeline, which is usually long and in which each particle needs a long time period to reach the deposit site from the borrowing pit. Over the longer time and length scales the effects of the different flow behaviour of the phases may prevail, although these effects are negligible in a short pipe.

The no-slip condition \( V_S = V_{m} \), which gives also the equality between the spatial and delivered concentrations \( C_{Vd} = C_{Vi} \), does not apply in the flow of a settling slurry, e.g. a mixture of sand and water. Slip may occur, even when the difference between the local velocity of a solid particle and the carrying liquid at an arbitrary vertical position in the pipeline cross section is negligible. Because a relationship between the slip velocity \( (V_f - V_S) \) and the slurry density can be expected (different level of mutual particle hindrance), the mean solids velocity varies in different pipeline cross sections along the whole long pipeline. This \( V_S \) variability along the pipeline causes a relative transfer of solids within the slurry stream and results in the transformation of the density waves.

To evaluate an aggregation mechanism it is necessary to determine \( V_S \) in the pipeline cross section for different slurry flow conditions. In practice, the accurate determination of \( V_S \) in the pipeline is difficult because it assumes a knowledge of the distribution of solids velocity, \( v_S(y) \), and concentration, \( c_V(y) \), in the pipeline cross section:

\[
V_S = \frac{1}{C_{Vi}A} \int_a \int_c c_V(y) v_S(y) dA
\]  

(8.1)

when

\[
C_{Vi} = \frac{1}{A} \int_c c_V(y) dA
\]  

(8.2)

In spite of attempts made to describe the internal structure of the slurry flow by mathematical models (e.g. Kril, 1990), it is still difficult to successfully predict \( v_S \) and \( c_V \) distribution in a pipeline cross section for various slurry flow conditions. To evaluate the internal structure of the slurry flow it seems to be more practical to approximate the real structure by the pattern handled by a macroscopic model than to apply a microscopic model. In this case the number of model coefficients is limited and the role of integral flow parameters in determining the quality of the model results is more pronounced.

The integral parameters of interest for the slip evaluation are mutually related by
\[
\frac{V_S}{V_m} = \frac{C_{vd}}{C_{vi}},
\]

that is Eq. 2.3 derived in Chapter 2. The slip ratio \(V_S/V_m\) (\(C_{vd}/C_{vi}\)) can be considered as a parameter describing mechanism for solids aggregation in slurry flow in a long pipeline.

Because of the lack of experimental data suitable for slip evaluation there is still no sufficiently verified tool for the determination of slip ratio in slurry pipelines. Sobota & Kril (1992) proposed an approximating correlation for the slip ratio

\[
\frac{C_{vd}}{C_{vi}} = 1 - f_t \left( \frac{C_{vb} - C_{vi}}{C_{vb}} \right)^{2.16} \left( \frac{V_{dl}}{V_m} \right)^{1.66}
\]

(8.3),

in which \(f_t\) was the coefficient and \(V_{dl}\) was the deposition-limit velocity. The correlation was based on outputs of Kril's microscopic model for the internal structure of slurry flow (Kril, 1990). Berman (1994) determined the coefficient \(f_t\) as

\[
f_t = 0.45 \left( 1 + \text{sign} \left( \ln R_e_p - 0.88 \right) \tanh \left[ 0.967 \left( \ln R_e_p - 0.88 \right)^{0.6} \right] \right)
\]

(8.4)

where particle Reynolds number \(R_e_p = v_t d/v_F\).

The correlation was verified by a limited number of laboratory data of Krivenko and Shook. This correlation was used to predict the slip ratio for slurry flow conditions in the MeaVli pipeline and compared with the slip ratio interpreted from MeaVli data using a macroscopic model. The results are discussed later in this chapter.

With respect to the specific flow conditions in a long slurry pipeline connected with a dredger, it is believed that the process of solids aggregation is caused by the variable slip velocity in an unsteady solids flow along the long pipeline. This is a result of the instability of the solids flow along the pipeline.

### 8.3.2 Application of the empirical and macroscopic approaches

All the phenomena observed in the MeaVli pipeline are mutually related and should be described by a single unified physical model. To analyse and explain the effects of unsteady solids flow in a pipeline, the internal structure of slurry flow must be handled by the model. Empirical models are unsuitable as they can only provide a variation in friction loss and deposition-limit velocity under the condition of fluctuating slurry density; evaluation of phenomena like slip or bed sliding is beyond the scope of empirical models. The appropriate model must reflect the continuous variation in the
internal structure of slurry flow and describe the immediate effect of such variations on the slurry flow behaviour.

A large majority of semi-empirical correlations anticipate a linear relationship between the friction loss and the mean slurry density with a rather steep slope of the $I_m-S_m$ line. The MeaVli data show that this estimate is not appropriate in a dredging pipeline. Higher delivered concentration of solids in the pipeline provides much lower additional friction loss than is predicted by models anticipating a linear relationship between the friction loss and the delivered concentration (see a comparison of measured friction losses with Durand model in Fig. 8.4a, 8.4b). Dušek & de Koning (1992) compared pressure gradients over the 4820 metre long Ja-Du section of the MeaVli pipeline with the Jufin-Lopatin empirical model in I-V curves for different mean concentrations of solids in the pipeline section. A very good agreement was found between the data and the empirical model for the low mean delivered concentration ($C_{vd} = 0.05$), but for the higher concentrations ($C_{vd} = 0.10, 0.15$) the model had a tendency to overestimate the pressure gradient, particularly for higher operational velocities.

The observed higher velocity of the sliding bed at the bottom of the pipeline in denser slurry flows than in more dilute flows indicates that the threshold mean slurry velocity at which the bed begins to stop sliding (the deposition-limit velocity) may be lower in denser slurry flows than in less dense slurry flows. This tendency is not reflected by the empirical formulas for the deposition-limit velocity. Application of semi-empirical correlations may lead to the overestimation of the design and operational parameters and to uneconomical operation of a conveying installation.

When assuming stratification of the slurry flow and accepting the simplifications of internal structure of slurry flow required by a macroscopic two-layer model (homogeneous velocity and concentration distribution and no slip velocity within each layer), the model pattern for heterogeneous flow can be used to describe the unsteady solids flow effects. The model predicts the simplified two-layer velocity and concentration distribution in the pipeline cross section and provides the slip ratio value, together with values of the pipeline operational parameters such as the friction loss and the velocity of the granular bed at the bottom of the pipeline. The limit mean slurry velocity at which stationary deposits start to be formed can be also determined.

Flow stratification with a very clear interface has been observed in the MeaVli pipeline, as is seen from the shape of the measured solids velocity profiles (Fig. 8.5a, 8.5b). The appearance of the flow stratification in the relatively fine particle slurry can be explained by the low $V_m$, which is near the critical value during the whole operation period, and by the relatively wide particle size distribution (PSD) of the transported solids. Thus the principles of the force balance between two layers can be applied to describe the mechanism governing the observed slurry flow and to explain the effects of the unsteady state of solids flow in a long pipeline.

The choice of a configuration for the two-layer model and the selection of a list of input parameters is crucial to a successful strategy for the description of the phenomena observed in the unsteady solids flow in the MeaVli pipeline. The development of the internal structure in the slurry flow is indicated by the measured solids velocity profiles. This is used, in combination with measured integral flow
Figure 8.5a. Solids velocity distribution in unsteady solids flow in the MeaVli pipeline.
Figure 8.5b. Solids velocity distribution in unsteady solids flow in the MeaVli pipeline.
parameters (mean slurry velocity, slurry density in pipeline cross section), as the inputs to the set of mass and force-balance equations of the two-layer model. Since slip is anticipated in long horizontal pipeline sections, the interpretation of measured mean slurry density in terms of solids concentration must be considered. Mean slurry density was measured by an \( \gamma \)-ray radiometric density meter directing its radiation beam to the centre of a cross section in a vertical descending section of a U-tube installed in the pipeline approx. 250 m behind the measuring site for solids velocity distribution. The mean slurry density measured in the vertical pipeline section represents the mean delivered concentration of solids in the entire pipeline. This is because the slip ratio is considered negligible in the location where the slurry density is measured, i.e. in homogeneous flow of relatively fine particle slurry in the vertical pipeline section. Experimental data are interpreted by the model equations to verify the analysis of the observed phenomena based on flow principles represented by the model.

8.3.3 Experimental data interpretation

Evidence that solids aggregation is caused by the slip phenomenon can be obtained by confirming the relationship between the slip ratio and the mean slurry density from the experimental database, i.e. by means of processing MeaVli data files (B). These 14-channel files contain the solids velocity profiles and related integral parameters of the slurry flow (mean slurry velocities and mean slurry densities) measured at the location Jagersplas. For details about the data files (B) see Chapter 4. To evaluate \( V_s \) from measured velocity profiles, the solids concentration distribution must be known. Since this was not measured it must be reconstructed by an appropriate data interpretation model (DIM).

The DIM must operate with some degree of simplification to estimate the local concentration distribution in the pipeline cross section where local solids velocities were measured. The structure of the stratified flow detected in the MeaVli pipeline is suitable for interpretation by the pattern of a macroscopic two-layer model. The model is considered appropriate to evaluate the phenomena observed in the MeaVli pipeline. Therefore the DIM adopts the model balance equations. The data processing technique and the DIM solution are described in the Appendix 8.

8.3.4 Effects of unsteady solids flow by the interpreted data

**Flow stratification in a pipeline cross section**

Processing of the shapes of solids velocity profiles shows that variation in the thickness of the contact layer, \( Y_{12} \), is confined to the lowest part of the pipeline, even for very different slurry densities in the pipeline cross section. The \( Y_{12} \) value is determined from the shape of the measured solids velocity profile as the height of the position at which the measured local velocity is considerably lower than the local velocity at the next measuring position above. \( Y_{12} \) varies between the extreme values of 44 mm and 162 mm, so \( 0.07 < Y_{12}/D < 0.25 \), but for most of the profiles (especially when \( V_m \) does not drop below 2.9 m/s) it is maintained at \( Y_{12}/D \leq 0.12 \). When the profiles are evaluated within a narrow slurry velocity range (3.60 < \( V_m \) < 3.80 m/s), only two
Figure 8.6a. Thickness and velocity of the contact layer in the 650 mm pipeline interpreted by DIM.
Experimental run: 17/03/81.
Figure 8.6b. Thickness and velocity of the contact layer in the 650 mm pipeline interpreted by DIM.
Experimental run: 18/03/81.
Figure 8.7a. Slip ratio in the 650 mm pipeline. Values interpreted by DIM and calculated using Sobota & Kri 1983. Experimental run: 17/03/81.
Figure 8.7b. Slip ratio in the 650 mm pipeline. Values interpreted by DIM and calculated using Sobota & Kiri equation 8.3. Experimental run: 18/03/81.
positions of the interface between the layers are detected (44 mm and 76 mm) from solids velocity profiles in a wide range of the mean slurry densities (Figs. 8.6a and 8.6b). A plot of the $Y_{12}$ values shows that the most of the solids velocity profiles exhibit the same detected position of the interface, $Y_{12} = 76$ mm. (Figs. 8.6a and 8.6b).

The relatively small variation of $Y_{12}$ is due to the wide PSD of transported material and small fraction of coarse particles in the transported solids. Only the coarsest portion of transported solids contributes to the contact load and the rest remains suspended above the bed or, in the case of the finest solids, contributes directly to the carrying liquid.

With a low degree of idealisation it can be predicted that the flow is segregated into two layers at the beginning of the pipeline and a stable situation is maintained along the entire pipeline, except in the short vertical pipeline sections. Coarser particles are transported within the bed which, despite the fluctuating mean slurry density, is of approximately constant thickness. Only a long lasting increase or decrease in the mean slurry density in a pipeline cross section would cause an increase or decrease of the bed thickness in the pipeline cross section. The bed is sliding slowly (with a variable velocity) at the bottom of the pipeline and it is passed by the faster upper layer containing suspended solids in a variable concentration.

**Slip ratio in a pipeline cross section**

Results of the data interpretation (Figs. 8.7a and 8.7b) verify the existence of a relationship between the slip ratio and the mean slurry density in a pipeline cross section and support the hypothesis that the solids aggregation along the long pipeline, characterised by density wave transformation, is due to the slip effect. While $V_m$ is considered constant in time, $V_s$ varies according to the slurry density in various pipeline cross sections along the long pipeline of constant diameter. The slip ratio values provided by the Sobota & Kril model show the same sort of relationship to slurry density (Figs. 8.7a and 8.7b).

**Bed velocity in a pipeline cross section**

The relation between $V_2$ and $\rho_m$ processed by the DIM and plotted (Figs. 8.6a and 8.6b) shows a regular increase in the lower-layer velocity, $V_2$, with mean slurry density, a trend already detected when the signals of the local solids velocity at the bottom of a pipeline, $v_s(0)$, were compared with $\rho_m$ and $V_m$. This trend is maintained when $V_2$ is integrated from local velocities measured within the bed. This indicates that the bed is sliding *en bloc*, and an increase in velocity of solids in denser slurry flow is due to an increase in velocity of the entire granular body of the contact layer.

**Frictional head loss**

The data interpretation model calculates the friction loss for the flow conditions determined by the solids velocity profile and related mean slurry density and mean slurry velocity, i.e. for the inputs provided by the data file (B). The calculated friction loss values are compared with the friction losses measured over the Ja-Du section of
the MeaVli pipeline and collected in data files (A). The data file (A) contains data for flow characteristics from all measuring sites along the conveying system as described in detail in Chapter 4. Values for the friction loss in the Ja-Du pipeline section are determined as the difference between the measured absolute pressure at the beginning and at the end of the Ja-Du pipeline section when minor losses in the pipeline section and a difference in geodetic heights of the pumps are subtracted. Pressures were measured at the inlet and the outlet of each pump. Instant friction loss in the pipeline section is related to the average density of the slurry occurring in the entire pipeline section at the moment of measuring the differential pressure.

The same trend in the relationship between the friction loss and the slurry density is obtained for both the interpreted and measured data (Fig. 8.8). This trend is much flatter than that predicted by empirical models. Concrete values for the friction loss calculated from the data file (B) are influenced by the estimated $C_2$, $\mu_s$ and interpreted $Y_{12}$. A comparison of model outputs with MeaVli data confirms that a 44 mm $< Y_{12} < 76$ mm interval provides good values of friction losses for various mean slurry densities in a pipeline section.

8.4 Explanation of the observed phenomena

Slip ratio

Stratified flow in a horizontal pipeline produces a different degree of mutual particle hindrance near the top and near the bottom of a pipeline. Particles with a permanent mutual contact and contact with a pipeline wall are more hindered than particles in turbulent suspension. Slip between phases in a pipeline cross section, represented by the slip ratio, is therefore predominantly a product of mutual shift between layers obeying two different mechanisms for particle motion in slurry flow. The shift is a product of force interaction between the contact and suspended layers in a pipeline. Different slip is caused by different mutual shifts between two layers in a slurry pipeline with fluctuating slurry density.

Bed velocity

The increase in velocity of the sliding bed in slurry flows of constant mean slurry velocity is explained by the mechanism of the force balance within a two-layer flow structure. The bed velocity is a product of the balance of driving and resisting forces acting on the bed. Increasing slurry density might have the following impact on the forces acting in a pipeline section with a partially-stratified flow:

- the higher total driving force acts on the bed layer and increases its velocity of sliding; this is because the denser suspended flow (higher $C_1$) in the upper layer produces higher shear stress at the interface between the two layers and so increases the driving force acting on the bed via the interface,

- the higher density of the carrier liquid may reduce the total resisting force exerted by the pipeline wall on the bed. The finest portion of the transported solids (particles of silt size) contributes directly to the carrying liquid and increases its density. An increase in the carrier (water + silt) density has the effect of increasing the buoyancy of the body of the granular bed submerged in the carrier liquid. The increased
Figure 8.8: Frictional head loss in the Ja-Du section of MeaVi pipeline. Comparison of measured and interpreted data.
buoyancy reduces the submerged weight of the granular body which is in mechanical
contact with pipeline wall and so, therefore, the bed resistance force caused by
mechanical friction between the granular bed and the pipeline wall also diminishes.
The effect of the resisting force reduction due to the contribution of the finest
particles to the carrier has been observed, for instance, during the SRC experiments
with slurry containing sand and clay in a DN315 pipeline (Shook, 1988).

Incorporation of the effect of increased carrier density due to the presence of silt in
slurry was tested in the DIM, but very small impacts on both the bed velocity and the
friction loss were found for slurry flow conditions in the MeaVli pipeline. This was
caused by too low concentration of the finest solids fraction in the MeaVli pipeline. Silt
particles represented usually less than 10 per cent of the total amount of transported
solids in the MeaVli pipeline (see the particle size distribution of the transported solids
in Fig. 4.6). Thus the increase in velocity of the bed was primarily due to the increasing
shear stress at the interface between the layers.

**Solids aggregation**

The smaller slip between the phases in a denser slurry flow in the cross section of a
horizontal pipeline is due to the increase of the velocity of the solids at the bottom of
the pipeline. This is caused by the higher driving force exerted on the bed layer by the
denser upper-layer flow via the interface between the layers. Solids aggregation occurs
in the lower region of the pipeline where the bed moving faster within the dense slurry
flow gradually accumulates the solids from the slower bed in front of the high density
wave.

**Frictional head loss**

Permanent or sporadic contacts with the pipeline wall by particles occupying the
contact layer establish mechanical sliding friction (due to Coulombic and Bagnold
stress) which is a major contributor to the process of mechanical energy dissipation in
slurry flows. When the thickness of the contact (bed) layer does not vary significantly
with slurry density in a pipeline section, there is very little variation in the friction loss.

8.5 **Data interpretation model**

8.5.1 **Sensitivity analysis on the data interpretation model**

The suitable configuration and computational scheme of the DIM are crucial to correct
evaluation of the processes in the MeaVli pipeline. The following configuration for the
DIM has been found appropriate for a description of slurry flow in the laboratory
pipeline as given in Chapter 5:

- only a part of solids in the lower layer contributes to Coulombic contact friction,
  \[ C_{2c} = C_2 - C_1 \]
- fluid shear stresses at the flow boundaries are dependent on the density of the
  suspension, \( \tau = f_{\ln}(\rho_1) \)
- the buoyancy effect on the interparticle force is negligible, \( F_N = f_{\ln}(\rho_f) \)
- the coefficient of mechanical sliding friction has a value 0.55 measured in the
inclinable pipe of the laboratory circuit.

Computational scheme for solution of the balance equations was determined according
to a chosen list of input parameters to the DIM. The following computational schemes
for the DIM have been tested:

**Scheme 1:** Input parameters: \( C_{vd}, V_m, V_{s1}, V_{s2}, Y_{12}, C_2 \).
This scheme uses all measured parameters as the inputs to the DIM. An additional
assumption must be made to equalise a number of unknowns and equations in the
DIM. This assumption is that there is slip between the phases within the layers. No
equation for mass balance in slurry flow is used in the set of equations because the
slurry velocity within a layer is not necessarily equal to the solids velocity \( (V_1 \neq V_{s1},
V_2 \neq V_{s2}) \). The equation is replaced by the relationship \( C_{vd}V_mA = C_{V1}V_{s1}A \).

**Scheme 2:** Input parameters: \( C_{vd}, V_m, V_{s2}, Y_{12}, C_2 \).
The scheme establishes a no-slip condition for either layer and considers \( V_{s1} = V_1 \) to
be the unknown variable to satisfy the "three equations - three unknowns" scheme.
This scheme provided the results plotted in Figs. 8.6 to 8.8.

**Scheme 3:** Input parameters: \( C_{vd}, V_m, V_{s1}, V_{s2}, C_2 \).
No-slip condition for a layer holds, \( Y_{12} \) is considered to be the unknown variable.

**Scheme 4:** Input parameters: \( C_{vd}, V_{s1}, V_{s2}, Y_{12}, C_2 \).
No-slip condition for a layer holds, \( V_m \) is considered to be the unknown variable.

**Sensitivity to a chosen computational scheme**

Comparison of outputs from Scheme 1 and Scheme 2 shows that both versions provide
the same trends in the relationships of the slip ratio and the bed velocity with the mean
slurry density. The values for the slip ratio obtained by Scheme 1 and Scheme 2 are,
however, rather different.

Scheme 1 gives rather low values for slip ratio, when compared with the laboratory
data or Sobota & Kri predictions. This is because the values of local solids velocities
measured in the entire pipeline cross section are always lower than mean slurry velocity
(see Fig. 8.5a, 8.5b). Mean solids velocity is obtained by integrating the solids velocity
profile using simplified two-layer concentration profile via the entire pipeline cross
section area.

Scheme 2 does not use the measured local solids velocities in the upper layer. It
replaces \( V_{s1} \), obtained by integrating the solids velocity profile over the upper layer,
with \( V_1 \) calculated from \( Y_{12}, V_{s2} = V_2 \) and \( V_m \). Since no slip is assumed within the
layers, \( V_{s1} \) is considered to be equal to \( V_1 \). More realistic values are reached for the
slip ratio than are produced by Scheme 1. The coefficient \( \lambda_{12} \) as the model output is
the most sensitive indicator of the quality of a chosen model configuration and of the
consistency of the input parameters. The \( \lambda_{12} \) values provide much lower scatter if the
interpretation model uses the Scheme 2 in place of the Scheme 1.

Scheme 3 is not applicable to the data interpretation. The no-slip condition for either
layer cannot be applied in parallel with the solids velocities and slurry velocity used as
the inputs when the measured \( V_{s1} \) is lower than the measured \( V_m \).
Scheme 4 relies more on the measured velocity profiles than on the measured $V_m$ and considers the $V_m$ as the unknown variable determined from solids velocity profile by the mass equation for slurry flow. This scheme does not provide credible values for the interpreted data because it is based on an incorrect premise. The $V_m$ measurements are more reliable than the $V_{s1}$ measurements. However, when applied, the scheme provides the same trends as given by Schemes 1 and 2.

The data analysis shows that the local solids velocities measured by ultrasonic Doppler flow meters in suspension layer always have lower values than the measured $V_m$. Scheme 1 attributes this discrepancy to the slip velocity within the suspended layer. The measurements detect the approximately uniform distribution of solids velocity in the suspension layer (see Fig. 8.5a, 8.5b). The slip velocity, particularly of the value indicated by measured $V_{s1}$ and $V_m$, is very unlikely to occur in pseudo-homogeneous flow observed in the suspended layer. It was realised that the $V_m$ value measured by the Ja flow meter might be up to 3.5% higher than that from the measuring site for the velocity profile owing to gland water inflow to the system at the Jagersplas pump placed between the measuring site and the flow meter. It is, however, more likely that the acoustic method did not provide representative values of local solid velocities in suspension. The diameter of particles is expected smaller in suspension than in the contact bed. The relationship between the wave length of an acoustic signal (approximately 2.37 mm by flow meter UNI/F-1) and the particle size affects the measured value of particle velocity. The values of the solid velocities detected in the contact bed are considered more reliable because the sensed solids velocity in a control volume of an ultrasonic sensor provides a better representation of the chord-average velocity at a certain vertical position in pipeline cross section for the sliding bed than for the suspension. The UNI/F-1 flow meter detects solid particles within approximately 10 - 30 mm distance of a pipeline wall (see Chapter 4).

The insufficiently accurate measurement of local solid velocities in the upper region of the pipeline does not detract from an evaluation of the phenomena observed in the pipeline. The trends are of interest in the relation between the internal structure of the flow and slurry density and these are not disturbed by insufficiently accurate absolute values of local solid velocities in the upper layer. Only the concrete values for the output parameters are affected. Finally, the incorrect input value of $V_{s1}$ results in a scatter in the output $\lambda_{12}$ value. The Scheme 2 is the most appropriate to MeaVli data interpretation because it avoids using the least accurate measured parameter as the DIM input.

**Sensitivity to a chosen $C_2$**

The volumetric concentration of solids in the bed, $C_2$, is a compulsory input to all computational schemes of the DIM. Measured solids velocity profiles must be linked to concentration profiles to obtain $V_s$. To make the link between profiles an initial condition must be given for a concentration profile in the pipeline cross section. The shapes of the measured velocity profiles indicate the existence of a granular bed at the bottom of the pipeline. Small solids velocity gradients across the bed area suggest that the contact layer is not sheared and that it slides en bloc.
The DIM sensitivity has been tested for the input parameter \( C_2 \) estimated as:
- a constant; the concentration is similar to that of a loose-packed bed \( (C_{\text{vb}}) \) and
- a variable changing linearly with \( C_{\text{vd}} \); a simple linear approximation has been used
based on values of the bed concentrations in slurries of medium sand measured in the
laboratory pipeline.

It has been found that the model outputs are influenced by the values of the input
parameter \( C_2 \), but that the observed trends in the relationship between the model
output parameters and the mean slurry density are not adversely affected by the chosen
method for the estimation of \( C_2 \). It is, however, preferable to anticipate a variation in
\( C_2 \) with mean slurry density, since at least the finest solids contributing to the carrying
liquid penetrate to the pores among particles in contact within the lower layer and
increase the total concentration of solids in the lower layer within a high density wave.

**Sensitivity to a chosen \( Y_{12} \)**

The position of the interface between layers, \( Y_{12} \), is determined from the shape of the
solids velocity profile. Accuracy of the \( Y_{12} \) estimation is related to the number of
locations in which local solids velocities are measured in a pipeline cross section.
A change in \( Y_{12} \) is sensed in steps equal to the distance between two neighbouring
measuring locations in the pipeline cross section. Resolution is rather rough for \( Y_{12} \) in
the 650 mm pipeline equipped by 9 sensors. The low resolution for \( Y_{12} \) influences the
values of the DIM output parameters, particularly those for friction loss (see Figs. 8.8).

The MeaVli data have been interpreted for both the constant \( (Y_{12} = 76 \text{ mm}) \) and the
variable \( Y_{12} \) determined by the data processing software DTI from the shape of
measured velocity profiles. The results obtained did not differ significantly and both
showed the same trends in the variation of the output parameters with the mean slurry
density.

**8.5.2 Scatter of the interpreted data**

The MeaVli database was acquired during the field operation of a dredging pipeline
installation. The database has the advantage of practical authenticity, but on the other
hand it also contains all the disadvantages of field measurements - uncontrolled
continuous variation of some parameters (as the particle size distribution) and
non-repeatability of the tests in identical conditions. There are three reasons for a slight
scatter of the output values of parameters interpreted by DIM:
- a fluctuation of parameters used as inputs to the DIM or uncontrolled in the system,
- an estimate of the time shift for the correlation of the signals measured at different
locations on a pipeline and
- an influence of inertia in the flow of dynamically changing conditions.

The force balance mechanism applied in the pipeline cross section has an inertia with
respect to correlated density variation in the pipeline cross section. This is a reason for
the slight scatter particularly of plotted relation between \( V_{s2} \) and \( \rho_m \). There is some
delay between density change and mechanism reaction. At the head of a density wave,
when the density in the suspended layer is increasing abruptly, some yield resistance
must be overcome to accelerate the bed. At the end of an isolated wave, the density drops but the bed still slides for a while because of its inertia.

8.6 Predictive two-layer model

The principles of the two-layer model were found to be appropriate for the interpretation of MeaVli data and for the description of the phenomena monitored in a long pipeline. Prediction of flow parameters in the 650 mm dredging pipeline (called the MeaVli pipeline) is suitable by the two-layer model configured for a partially-stratified flow regime as proposed in Chapter 5 (see the configuration of the two-layer testing model TM) and in Chapter 7 (Eq. 7.11 giving the stratification ratio). Prediction of slurry flow characteristics using this model is carried out for the model input parameters: $D = 650$ mm, $d = 0.42$ mm (characteristic particle diameter of a fraction of settling particles in the solids transported in the MeaVli pipeline), $C_{vd} = 0.20$ and $\mu_s = 0.55$ (estimated as equal to the value of mechanical friction coefficient determined for the similar type of sand in a laboratory pipe). Computation of a set of model equations provides the following results:

**Table VIII.1.** Prediction of slurry flow in a MeaVli pipeline using the proposed model

<table>
<thead>
<tr>
<th>$V_m$</th>
<th>$Y_{12}/D$</th>
<th>$I_m$</th>
<th>$C_{vi}$</th>
<th>$V_1$</th>
<th>$V_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>0.15</td>
<td>0.037</td>
<td>0.232</td>
<td>3.37</td>
<td>-0.42</td>
</tr>
<tr>
<td>3.25</td>
<td>0.11</td>
<td>0.028</td>
<td>0.218</td>
<td>3.46</td>
<td>0.15</td>
</tr>
<tr>
<td>3.50</td>
<td>0.08</td>
<td>0.022</td>
<td>0.209</td>
<td>3.60</td>
<td>0.84</td>
</tr>
<tr>
<td>3.75</td>
<td>0.04</td>
<td>0.018</td>
<td>0.203</td>
<td>3.77</td>
<td>1.76</td>
</tr>
<tr>
<td>4.00</td>
<td>too thin bed</td>
<td>no</td>
<td>no slip iteration</td>
<td>expected</td>
<td></td>
</tr>
</tbody>
</table>

A. **Deposition-limit velocity** $V_{dl}$ has a value of about $3.15 \text{ m/s}$ for slurry flow at $C_{vd} = 0.20$. This is in accordance with observations in the MeaVli pipeline (see Fig. 8.3a).

B. Values for the frictional head loss predicted by the model fit the experimental values very well. For $V_m = 3.50 \text{ m/s}$ the **hydraulic gradient** $I_m$ is predicted as 0.022 [-] for $C_{vd} = 0.20$, i.e. slurry density $1330 \text{ kg/m}^3$, (compare the predicted value with the trend in a development of the hydraulic gradient with slurry density on Figs. 8.4a, 8.4b). Literature models give considerably higher values for $I_m$.

C. **Predicted velocity** ($V_2$) of sliding bed reasonably matches the MeaVli data (compare the $V_2$ value for $V_m = 3.75 \text{ m/s}$ from Table VIII.1 with those on Figs. 8.6a and 8.6b for slurry density $1330 \text{ kg/m}^3$).

D. **Slip ratio** $C_{vd}/C_{vi}$ is predicted as $0.200/0.209 = 0.957$ for $V_m = 3.50 \text{ m/s}$ and $0.200/0.203 = 0.985$ for $V_m = 3.75 \text{ m/s}$. Comparison with the corresponding slip ratio values on Figs. 8.7a and 8.7b for slurry density $1330 \text{ kg/m}^3$ shows that the model prediction is appropriate.
8.7 Verification by the laboratory tests

Data interpretation using principles of the two-layer model confirmed that the slip ratio varies with the mean slurry density in a pipeline cross section. This should cause the solids aggregation phenomenon. It is, however, desirable to verify the relation between the slip ratio and the slurry density by experiment. It is also desirable to confirm that the slip-ratio variation is caused predominantly by a different shift between two layers in a stratified flow. This should prove that the simplifications accepted for the interpretation model do not misinterpret the results.

In the MeaVli database some important parameters were not measured directly and these had to be reconstructed from available measured parameters. These were the friction losses and the concentration profiles in a short pipeline section where the velocity profiles were measured. A sensitivity analysis showed that prediction of the slurry flow behaviour using the two-layer model was strongly dependent on the determination of the position of the interface between the layers. \( Y_{12} \) could be estimated only roughly from the shapes of measured velocity profiles. However it is very unlikely that the rough estimate of \( Y_{12} \) in the data interpretation model could cause misinterpretation of the slurry flow mechanism occurring in the dredging pipeline. The more accurate and controlled measurements in a laboratory should help to find a rule for the division of solids into two layers.

Laboratory experiments and data processing procedures are described in detail in Chapter 4. During the measurements in the 150 mm pipeline, the concentration profiles and the local solids velocity at the pipeline bottom \( (v_b) \) were measured, together with the integral slurry flow parameters \( (I_M, V_M, C_{V_d}) \). The slip ratio was determined from parallel measurements of concentration profiles and \( C_{V_d} \) in the pipeline. Integration of the concentration profile provided \( C_{V_i} \), hence the slip ratio \( C_{V_d}/C_{V_i} \) was obtained directly from measured parameters in the cross section of the laboratory pipe.

Laboratory tests for the solid (a 0.2 - 0.5 mm sand) that was similar to that transported in MeaVli pipeline show that
- the slip ratio increases with the increasing mean slurry density in the horizontal pipeline for all mean slurry velocities at which the slip ratio has been measured (Fig. 8.9);
- the solids velocity at the bottom of the horizontal pipeline increases with the increasing mean slurry density for all mean slurry velocities at which solids velocity has been measured (Fig. 8.10).

Laboratory tests verify a trend in the relation between slip ratio and mean slurry density identified in the dredging pipeline by the MeaVli data interpretation. The tests also confirm a trend in the relation between the solids velocity at the bottom of the pipeline and the mean slurry density. It can be concluded that solids are moving faster within denser slurry flow and that this is caused by increase in velocity of the contact layer due to the impelling effect of the denser suspension acting on the contact layer.

An increase of hydraulic gradient due to friction, \( I_M \), with mean slurry density was observed in the laboratory pipeline (Fig. 8.11). This was much steeper than the
Figure 8.9. Slip ratio in the horizontal 150 mm pipeline. Experimental runs: 25/09/95 - 04/10/95.

Figure 8.10. Local solids velocity at the bottom of the horizontal 150 mm pipeline. Experimental run: 06/10/95.
Figure 8.11. Friction loss measured in the horizontal 150 mm pipeline.
Experimental runs: 25/09/95 - 04/10/95.

Figure 8.12. Parameters of the two-layer model interpreted from sand flow data acquired in the horizontal 150 mm pipeline.
Experimental runs: 25/09/95 - 04/10/95.
increase observed in the dredging pipeline. Simultaneously, an increase of $Y_{12}$ with mean slurry density was obtained when measured concentration profiles were transformed into a simplified two-layer pattern by the testing model TM (Fig. 8.12). This is basically due to the narrower PSD of solids tested in a laboratory pipeline (medium sand 0.2-0.5 mm, compared with silt and sand 0.01-0.8 mm in the dredging pipeline) and the controlled steady conditions of slurry flow in the laboratory pipeline. Flow stratification is less developed in the slurry flow of the narrower PSD and of the steady state of solids flow. This means that there is a more gradual transition between the layers. Owing to steady flow conditions, bed properties ($Y_{12}, C_2$) are strictly dependent on mean slurry density installed in the laboratory circuit. In the dredging pipeline, however, it may happen that almost no solids are transported in the suspended layer in the pipeline cross section for a while (e.g. at the minimum of a density wave) but there is still a thick granular bed sliding at the bottom of the pipeline below the diluted suspended layer. The properties of a granular bed are therefore less dependent on the mean slurry density in the dredging pipeline than in the laboratory pipeline. In the laboratory circuit the different slurry densities are not generated by the time-continuous density fluctuations in the slurry flow but by a set of different test runs with slurry flow of different installed constant slurry densities. For instance, a low delivered concentration installed in the laboratory pipeline means only a small amount of solids in the whole system.

Comparison of results from both the pipelines tested indicates that the same mechanisms govern partially-stratified flow in the dredging pipeline and in the laboratory circuit. It also confirms the existence of a mechanism which is assumed to govern the solids aggregation process observed in the dredging pipeline. The simplifications chosen for the data interpretation model did not misinterpret the description of the behaviour of slurry flow in the dredging pipeline. The two-layer model correctly evaluates the phenomena occurring in both pipelines. The prevailing mechanisms governing the slurry flow behaviour occurring in both slurry pipelines are covered by the two-layer model.

8.8 Pipeline operation in case of unsteady solids flow

The solids aggregation phenomenon observed in a long slurry pipeline connected with a dredger is not dangerous for pipeline operation. The formation of high density waves does not produce moving dunes or a stationary deposit at the bottom of a pipeline, nor does it increase the friction loss in the slurry flow in the pipeline. This is valid for the relatively fine broadly-graded solids dredged from the bottom of the lake FleVli during the MeaVli measurements. This solid material is typical of that from dredging locations in Holland; it is a fine-to-medium sand with a small proportion of coarse sand and silt.

Frictional head loss in unsteady solids flow does not increase significantly with the mean slurry density in the pipeline cross section if the transported solids are relatively broadly graded. As a result the specific energy consumption (SEC) in the dredging pipeline decreases rapidly when the density of transported slurry increases (see Fig. 8.13).
Figure 8.13. Specific energy consumption (SEC) in a 650 mm slurry pipeline.
Exp. data: Ja-Du section of MeaVli pipeline.
Mean velocity range: 3.30 < Vm < 3.50 m/s.
Different phenomena may occur when coarse solids (like coarse sand and gravel) are pumped. In this case there is no impelling effect caused by the denser suspended layer, since the majority of particles occupy the bed in the flow of a mean slurry velocity not far above the deposition-limit value. The unsteady state of the solids flow causes that the thickness of the bed varies significantly along the pipeline. Further instabilities may occur owing to shear stress variation at the top of a bed of variable thickness. Instabilities may lead to the gradual development of dunes, their mutual separation and their transformation into plugs along the pipeline. Such plugs may block the pipeline. This was experienced in our laboratory loop as described in Chapter 5 in connection with the observed deposition-limit velocities. The gravel-slurry flow was observed in a transparent pipe section at mean slurry velocity just above the deposition-limit value. The initially steady flow with a flat sliding bed was gradually transformed to an unsteady flow characterised by the development of bed forms and their transformation to plugs. The plugs finally blocked the system. We decided not to repeat this interesting test.

The formation of density waves in a dredging pipeline has a considerable impact on the operation of slurry pumps and drives incorporated into a conveying system. Density waves passing through the slurry pumps cause the working point of a pump-pipeline system to vary in time during the operation of the system (de Koning, 1968). The situation is more complex in a system composed of a pipeline and a set of pumps. Analysis of the pump-pipeline interactions is not within the scope of the research presented here. The impact of slurry density fluctuation on the efficiency of a conveying system is an interesting subject for further research.

Slip occurs between solid and liquid phases in slurry pipelines as a result of a flow stratification. The slip must be taken into account when the solids throughput in a dredging pipeline is being determined. In horizontal pipelines occupied by the slurry exhibiting a considerable slip the solids concentration \( C_{vi} \) (a fraction of solids actually present in the a pipeline section) is higher than the delivered concentration \( C_{vd} \). During a dredging operation the solids throughput is usually determined on-line and displayed on the dredgemaster's control board. The solids throughput \( Q_s \) is calculated as \( Q_s = C_v \pi D^2 / 4 \) from on-line signals of the measured mean liquid velocity \( V_F \) by a magnetic flow meter) and mean spatial concentration (\( C_{vi} \), by a radiometric density meter) in a pipeline cross section. The measuring instruments are often installed in a horizontal pipeline section at some distance behind a dredge pump. If flow stratification resulting in slip occurs in this pipeline section, the values of \( Q_s \) obtained when using \( C_{vi}V_F \pi D^2 / 4 \) may be too high. Using \( C_{vd}V_m \pi D^2 / 4 \) would give the correct values. Thus the monitoring system for a dredging installation may overestimate the solids throughput in a pipeline connected with a dredger.

8.9 Conclusions

The process of solids aggregation was detected in a slurry pipeline during a dredging operation. Solids aggregation occurs as an effect of unsteady solids flow caused by fluctuating slurry density in a long pipeline. Transported solids are gradually accumulated into highly concentrated density waves along the pipeline. This effect is a
product of variable slip in different pipeline cross-sections along the slurry pipeline. Slip velocity in pipeline cross-section is related to slurry density passing the cross-section. The slip ratio $V_s/V_m$ increases with increasing slurry density in the pipeline cross-section. The variable slip ratio in partially-stratified flow is caused by variable mutual shift in the velocities of the contact load and the suspended load when there are fluctuations in the slurry density in the pipeline.

Fluctuations in slurry density in a flow with relatively fine broadly-graded solids do not adversely affect the safe operation of the pipeline. Denser slurry flow does not mean that there is a higher deposition-limit velocity and hence a higher required operational velocity. In fact, solid particles occupying the bed at the bottom of the pipeline move faster within the denser slurry owing to the higher impelling force exerted by denser suspension on the bed.

Slurry density fluctuation in flows of relatively fine broadly-graded solids does not impair the efficiency of a pipeline operation. The unsteady state of solids flow does not increase slurry flow resistance in a dredging pipeline. Furthermore, the higher average slurry density in the pipeline section produces only a weak additional friction loss in the usual range of operational conditions in dredging pipelines. The additional friction loss is much lower than that predicted by the empirical models. This is because the thickness of the contact layer does not vary significantly with mean slurry density along the pipeline so friction loss caused predominantly by mechanical friction between the contact layer and the pipeline wall does not vary significantly either.

All the phenomena described have been evaluated by a model which handles the two-layer flow pattern. The model has been found appropriate to describe and simulate processes occurring in stratified flow in a long pipeline connected with a dredger.

8.10 References


Appendix 8

Data Processing Technique (DTI & DIM)

The data interpretation model (DIM) has been implemented into a data processing program Data Transfer & Interpretation (DTI). DTI executes all operations required for the interpretation of data from an experimental database. It contains modules for the following operations:

I. the selection and loading of data of interest from a source database for further DTI processing
II. integration and averaging within a chosen time period per channel type in channels of the loaded data file
III. the correlation of mutually related parameters measured at different places in the pipeline
IV. the adaptation of the measured velocity profiles to the two-layer pattern
V. the determination of the simplified concentration distribution in the slurry flow and \( C_{v_i} V_s \)
VI. the determination of the slip ratio
VII. the repeating of steps III.-VI. for a new value of the slip ratio acquired from VI.
VIII. the comparison of the slip ratio value with an existing model (Sobota & Kril, 1992)
IX. the additional calculations with the two-layer model: the determination of the friction loss and the model coefficient - friction factor at the interface between layers
X. the creation and saving of output data files for further graphical and statistical processing.

The operation IV. provides the input parameters to the DIM. It transforms measured velocity profiles into a simplified two-layer pattern. In the two-layer pattern, the lower bed layer has a homogeneous local concentration and solids velocity distribution characterised by mean \( V_{s2} \) and \( C_2 \), the same as upper suspended layer with \( V_{s1} \) and \( C_1 \). No slip is anticipated within either layer, thus \( V_1 = V_{s1} \) and \( V_2 = V_{s2} \). The profile transformation is done in two steps:

i. the determination of an interface position in a pipeline cross section from the shape of the measured velocity profile

ii. the integration of \( V_1 \) and \( V_2 \) from the measured velocity profile over cross-section areas of the lower layer (\( A_2 \)) and upper layer (\( A_1 \)) respectively.

The thickness of the lower contact layer (\( Y_{12} \)) is estimated from the shape of the measured velocity profile.

A further step in the preparation of input parameters for the DIM is the correlation of the slurry density (\( \rho_m \)) value related to an evaluated velocity profile by means of a time shift determination between a signal from the density meter and a set of ultrasonic velocity meters.
DIM equations and their solution

The determination of a simplified concentration distribution for slurry flow, \( C_{vi}, V_s, \) slip ratio, friction loss and coefficients of the two-layer model (operations V., VI. and IX. in DTI) is performed by the DIM. Basically, the DIM is composed of a set of mass and force-balance equations for a two-layer slurry flow pattern. Two mass balance equations and one solids volume balance equation are sufficient to determine the slip ratio:

- Mass balance for slurry flow: \( V_mA = V_1A_1 + V_2A_2 \)
- Mass balance for solids flow: \( C_{vd}V_mA = C_1V_1A_1 + C_2V_2A_2 \)
- Solids volume balance: \( C_{vi}A = C_1A_1 + C_2A_2. \)

The relationship between the solids flow rate and slurry flow rate is

\[ C_{vd}V_mA = C_{vi}V_sA. \]

A set of three balance equations has to be solved to get \( C_1, C_2, C_{vi} \) and \( V_s \). The DTI operation IV. provides enough parameters to solve the set of equations. For pipeline of a cross section area marked as \( A \), these parameters are \( A_1, A_2, V_1 \) and \( V_2, V_m, C_{vd} \) (\( \rho_m \)).

Three balance equations contain eight unknown quantities - \( Y_{12}, \) defining \( A_1, A_2, V_1, V_2, V_m, C_1, C_2, C_{vi}, C_{vd} \). Thus five of the listed parameters must be delivered as inputs to the set of equations. Five input parameters are available from the database \( (Y_{12}, V_1, V_2, V_m, C_{vd}). \) However, the set of equations cannot be solved without an assumption for a concentration profile. Therefore \( C_1 \) or \( C_2 \) must be estimated as an input to the DIM and one of database quantities available as the DIM input parameter becomes an independent variable. The choice of a free variable determines the computational scheme for the DIM.

The force-balance equations are used in the DIM to determine the friction loss and the coefficients of the two-layer model. The same configuration is used for the DIM as is used for laboratory data evaluation.

The friction loss is calculated by the force-balance equations of the mechanistic model for each set of data (i.e. each measured velocity profile) using the processed parameters \( (V_{s1}, V_{s2}, C_1, C_2, Y_{12}). \)
Chapter 9

Conclusions and recommendations

In this final chapter general conclusions are formulated for the practical application of the research results that have been presented. Conclusions relating to the explanation and modelling of the specific effects of the flow of sand-water slurries in pipelines have been given at the end of the individual chapters. This chapter closes with recommendations for future research.

9.1 Conclusions

1. The flow mechanism of sand-water mixtures was studied by using both theoretical and experimental approaches. Phenomena associated with mixture flows in laboratory and field pipelines were observed and described. Some of these phenomena, such as the process of solids aggregation in a long dredging pipeline, the identification and development of a shear layer above an en bloc sliding bed in a slurry pipeline or the development in a concentration gradient across a slurry stream under the condition of a changing pipeline slope, had not been investigated before.

2. Observations of flows of sand-water mixtures were made in two pipelines of very different sizes. Measurements in the 650 mm dredging pipeline provided information about the integral and local parameters of medium-sand slurry flow under the condition of strongly fluctuating slurry density and almost constant mean slurry velocity. Laboratory experiments in the 150 mm pipeline were carried out for four different types of solids within the wide ranges of mean velocities and concentrations and for different pipeline inclinations. Both the integral and local parameters of slurry flow were measured.

3. The radiometric density meter adapted as the concentration profiler proved to be a sufficiently accurate tool to detect the solids concentration distribution in a pipeline cross section. The bed velocity meter used in the laboratory pipeline was adequate to detect the trends in the bed velocity changes under the various slurry flow conditions (different mean slurry velocities, slurry densities or particle sizes).

4. Laboratory tests in the 150 mm pipeline confirmed the existence of a mechanism which is assumed to govern the process of solids aggregation observed in flow of fine to medium sand in the 650 mm dredging pipeline (see Conclusions in Chapter 8). These experiments also detected the presence of sheet flow above a sliding bed in coarse-sand and gravel flows in the pipeline. It was shown that the slurry flow containing a portion of solids in turbulent suspension behaved differently from the
slurry flow with a developed shear layer (see Conclusions of Chapter 5). Modelling of both types of flow was proposed for the purposes of the two-layer model (see Conclusions of Chapter 7).

5. Observation of the internal structure of slurry flow in a laboratory pipeline installed to various inclination angles led to better understanding of pipe inclination effects on partially-stratified flow processes. It appeared that slurry flow was very sensitive to changes of pipeline inclination if a sliding bed was present. A physical description of inclination effects was obtained by formulating force balance equations for inclined flows in the two-layer model (see Conclusions of Chapter 6).

6. Laboratory experiments were carried out for a relatively wide range of slurry densities up to maximum density values (of about 1580 kg/m³) higher than are those usually reached in dredging pipelines. Important effects of the high solids concentration on the resistance and internal structure of slurry flows in pipelines were observed and described. The tests revealed that the transportation of sand in high concentrations in the slurry might be attractive.

7. The specific energy consumed to transport coarse sand is approximately twice that needed to transport medium sand in the 150 mm pipeline (see Fig. 5.2). This is because energy is primarily dissipated through mechanical friction between the solid particles and the pipeline wall as a result of flow stratification and the formation of a contact bed at the bottom of the pipeline. Flow is more stratified when transported particles are coarser. The granular bed is composed of particles which are not supported by turbulence of the carrying liquid. However, under suitable flow conditions, the top of the granular bed is subjected to shearing and a shear layer is developed in which the interparticle contacts are sporadic rather than continuous. The development of the shear layer is associated with a drop in the hydraulic gradient $I_m$ (see e.g. shapes of the $I_m$-$V_m$ curves in Fig. 5.10) and thus with diminishing of the specific energy consumption (SEC) in the flow. To profit from the SEC drop resulting from the suspension mechanism caused by interparticle collisions within the shear layer, the pump-pipeline system must be run at a velocity slightly higher than the $V_{dl}$, but not too close to it.

8. The transportation of relatively highly concentrated slurries is recommended when fine and medium sand is dredged. It is safe and efficient. A continuous fluctuation in slurry density in a dredging pipeline and the solids aggregation process resulting in the development of the highly-concentrated slurry masses along the long pipeline do not reduce the efficiency of a dredging pipeline operation. Owing to the faster sliding of the bed within highly-concentrated slurry flows in a pipeline the danger of a pipeline blockage is not increased by the formation of highly concentrated slurry masses. The pressure gradient due to friction in a pipeline section occupied by slurry masses does not seem to be higher than in steady solids flow of the same slurry density in a pipeline section.
9.2 Recommendations for further research

1. More information is required about the behaviour of slurry flows in large dredging pipelines. A more detailed analysis of the mechanisms of slurry flows in dredging pipelines requires the support of experiments providing parallel observations of the concentration and velocity distributions in slurry flows.

2. It would be worthwhile to detect the further development of the observed flow effects for even higher slurry densities up to the maximum density at which the sand-water slurry can still be pumped (i.e. till the slurry density of 1850 kg/m$^3$ approximately). In practice, the concentration limit to which the pumping of highly concentrated slurries is efficient is lower than the maximum pumpable concentration. This limit concentration should be determined, together with the possible effects of solids properties on its value.

3. Dredged solids are seldom uniformly graded. For dredging locations in the Netherlands a fine to medium sand with some proportion of silt is typical. The presence of silt in solids transported in dredging pipelines may positively influence the process of mechanical energy dissipation in slurry flows. The energy dissipation in the flow might be reduced owing to the decreased mechanical friction between particles in the bed and the pipeline wall. This may possibly be achieved as a result of a reduction in the submerged weight of the granular bed caused by increased buoyancy in the suspension of liquid and silt. The granular bed is submerged in the carrier suspension which has a density higher than is that of liquid alone. The conditions leading to the buoyancy effect must be determined experimentally before the effect can be implemented in the two-layer model predicting the slurry flow behaviour in a pipeline.
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Curriculum vitae

Václav Matoušek was born in Teplice, Czechoslovakia, on August 14, 1963. After completing his secondary education at the Gymnasium Chomutov in June 1981, he entered the Czech Technical University in Prague where he studied Water Engineering. He graduated as a Civil Engineer in June 1986. He joined the Institute of Hydrodynamics of the Czech Academy of Sciences as a junior researcher and was engaged in the research into two-phase flows. Since September 1992 he has been associated with the Delft University of Technology, Chair of Dredging Technology and Bulk Transport. He has supervised several students and has presented lectures to university students and at an industrial seminar. He has also contributed to two journals and a number of international conferences. In 1995 he received the Award of the International Association of Dredging Companies for the paper he presented at the 14th World Dredging Congress.