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The extreme force on an offshore structure and its variability

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Abstract

For the design or re-assessment of an offshore structure it is essential to have an estimate of the expected value and the associated uncertainty of the extreme force on the structure in a given oceanographic environment. These force characteristics are usually estimated for a large (e.g. 100 yr) crest occurring at the structure. The subject of this paper focuses on how the variability of the design wave force should be interpreted. Basically, there are two possible interpretations of the variability of the design wave force: (1) the uncertainty of the wave force peak that is associated with the largest wave height occurring at a particular point; (2) the uncertainty of the largest wave force occurring in a particular sea state, both within a given duration. For structural reliability purposes, the specific coupling between a wave crest at a particular location and the associated peak force on the structure is not required; only the statistics of the extreme force on a structure in a given extreme environment (sea state) is of interest. In this paper it will be demonstrated through numerical experiments that the Coefficient of Variation (CoV) of the global extreme wave force in a given sea state and period is much smaller than the CoV of the force associated with the extreme wave height occurring at a given location for the same period. For the numerical experiments a recently developed technique based on constrained random time domain simulations has been adopted.

Keywords: Variability; 'Wave-by-wave' force; Extreme wave force; Offshore structures; Tern; Constrained random time domain simulations

1. Introduction

1.1. Estimating the accuracy of wave load models

For estimating the probability of failure of an offshore structure through a reliability analysis, detailed information on the following aspects should be available:

- environmental description;
- wave loading model;
- ultimate structural strength estimator.

The results produced by a reliability analysis are strongly dependent on the accuracy of the modelling of these aspects. The uncertainties present in the modelling can be divided into two categories: physical and modelling uncertainties. To obtain meaningful results one should aim at incorporating physical modelling uncertainties only. A clear and comprehensive discussion of the treatment of uncertainties in reliability analyses is given by Efthymiou *et al.* [1]. This paper focuses only on the second aspect, i.e. on methods to determine wave loads on space frame structures.

It is common practice in offshore engineering to predict the extreme wave load on an offshore structure using a deterministic 'design wave' approach (see e.g. API [2]). In order to perform a reliability analysis an estimate of the (physical) uncertainty of this extreme wave load is needed. Such an estimate cannot be obtained using a deterministic input wave. Therefore full scale measurements of wave forces on real offshore structures have been performed to estimate the accuracy of the design wave force recipe. Several extensive measurement programmes have been performed, e.g. on the Magnus and Tern structures in the northern North Sea.

The accuracy of the design wave force model is usually verified on a 'wave-by-wave' basis, i.e. from a measured surface elevation signal individual deterministic wave parameters (wave height and period) are determined which are subsequently used to predict a force which is compared with a measured force. By comparison of predicted and measured wave forces for many individual waves the bias in the fluid loading model and its degree of variability can be estimated. Two studies [3,4] which analysed measured time series of surface elevations and wave loads on the Tern platform in this way both found typical CoV values of 25–30%. CoV is defined as the standard deviation divided by the mean value and is used as an expression of variability. It is useful to

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point out that this type of comparison determines *the uncertainty of the wave force peak that is associated with the wave height occurring at a particular measuring point.*

This type of analysis should be interpreted very carefully as the results are strongly influenced by a hidden parameter: the location relative to the structure of the wave sensor which records surface elevation. This has been demonstrated analytically by Jonathan and Taylor [4] and will be demonstrated numerically in this paper (see Section 4). Consequently, incorporating these CoV values in a reliability analysis should be avoided as they reflect a model uncertainty rather than a physical uncertainty.

From a structural reliability point of view a specific coupling between a wave crest and the associated peak force is not required; only the statistics of the extreme force on a structure in a given extreme sea state are of interest [1]. This is a different interpretation of variability than the one determined by 'wave-by-wave' force analysis. The *uncertainty of the largest wave force occurring in a particular sea state* is typically obtained through a 'sea state-by-sea state' analysis. In such an analysis the prediction of the most probable maximum force in a given sea state is compared with the maximum measured force in the same sea state. The sea state can be adequately described by a directional wave spectrum which is based on measured time series of surface elevation. Consequently, the results of a 'sea state-by-sea state' analysis are not influenced by the location of the wave sensor relative to the structure under the condition that both the wave sensor and the platform are in the same wave field. The bias and spread of the predicted wave forces are now estimated by analysing a large number of sea states. If the short-term variability of the predicted forces should overestimate the variability in measured forces (i.e. physical uncertainty), this would be a reflection of model uncertainties in the wave load recipe. Analysing wave force measurements in a large number of sea states is, however, subject to practical limitations as large numbers of sea states of identical characteristics are rare, especially in severe conditions.

Results of measurements at Tern are presented as an illustration of the 'sea state-by-sea state' force analysis. Tern is an oil production platform located north-east of the Shetland Islands in 167 m of water. For several years, it has been the subject of an extensive monitoring programme using strain gauges, two wave elevation meters and a water velocity sensor. The data obtained have already been used extensively for the testing of modern wave models [5,6]. The results presented here are based on the analysis of time series of surface elevation and wave loads during the passage of a severe storm of 8-h period. During the full duration of the storm the wave spectrum did not significantly change, which implies that the recordings of waves and forces in this storm can be regarded as measurements obtained from eight independent 1-h sea states with the same underlying statistical properties. From each 1-h period, the largest observed wave crest and largest observed wave force were selected (see Table 1). The last two rows in the table give estimates of the mean and CoV of both variables. The variability expressed as CoV reflects the inherent uncertainty associated with each quantity.

The obtained CoV values are surprising in two ways. Firstly, the CoV value for the largest wave force is small compared to the typical values of 25–30% which have been found on a 'wave-by-wave' basis. Secondly, it appears that the uncertainty of the largest wave force in a period is almost identical to the uncertainty of the largest crest elevation in the same period. This observation will be discussed further in Section 5 of this paper.

1.2. Interpretation of wave force uncertainties

The large difference in CoV estimates obtained through 'wave-by-wave' force analysis and 'sea state-by-sea state' force analysis is introduced by the way in which one interprets extreme wave force variability. The following two interpretations are possible:

- the uncertainty of the wave force peak that is associated with the largest wave height occurring at a particular point;
- the uncertainty of the largest wave force occurring in a particular sea state,

both within a given duration. The second interpretation is the one that should be used in a reliability analysis.

In this paper we demonstrate that the variability of the largest wave force occurring in a particular sea state is much smaller than the variability of the wave force peak associated with the largest surface elevation occurring at a particular point. Furthermore, we confirm that the results of a 'wave-by-wave' force analysis are strongly influenced by the location of the wave sensor relative to the platform. In order to perform all these analyses correctly, one requires wave and force data obtained from many sea states with the same underlying process statistics, preferably in severe conditions. Unfortunately, we do not have this kind of data. Consequently, it was decided to perform numerical

Table 1
Relating largest measured hourly crest to largest measured hourly force

Hour	Largest measured crest at EMI wave sensor (m)	Largest measured force (MN)
1	9.9	16.8
2	9.8	17.9
3	11.2	19.2
4	9.9	19.7
5	8.5	13.8
6	10.4	18.5
7	10.2	16.8
8	8.2	19.1
Mean	9.8	17.7
CoV	9.4%	10.1%

experiments by simulating the ocean surface and its loading on Tern. This provided the additional advantage that the ocean surface can be 'measured' at any position of interest. In this way the influence of wave sensor location in a 'wave-by-wave' force analysis can easily be investigated.

2. Set-up of experiments

The Tern platform comprises a piled steel space frame substructure and a module support frame which carries the topside facilities. Tern is a considerably elongated structure: at the seabed the legs enclose a rectangle of 90 m by 70 m, whilst at deck level the outer legs form a rectangle of 71 m by 27.5 m. The effective spatial distribution of members for the force calculations for waves broad-side and end-on to the structure is therefore clearly different. Hence, differences are to be expected in the magnitude of the loading for these two directions.

As the locations of the wave sensors play an important role in this work, the locations considered (three in total) are shown in Fig. 1 and described below:

1. *CENTRE*: The geometric centre of Tern at mean sea level, an imaginary location for a wave sensor. This is also the origin of the global axes: *CENTRE*$(x, y, z) = (0.0, 0.0, 0.0)$; *EMI*: The position of the actual EMI laser sensor for surface elevation: *EMI*$(x, y, z) = (-20.0$ m, -30.0 m, $0.0)$; *MAREX*: The position of the actual MAREX radar sensor for surface elevation: *MAREX*$(x, y, z) = (+15.0$ m, -45.0 m, $0.0)$.

The wave force calculations have been performed for three sea states (see Table 2), varying from drag dominance to inertia dominance. The main analyses of the maximum wave force variability will be based on a storm which passed over Tern in January of 1993, labelled in previous publications by Jonathan and coworkers [4–6] as storm 1993[a]. This storm is one of the most severe storms seen in the last 30 years in the northern North Sea. The largest waves in this storm result in a mixed inertia/drag loading regime on Tern.

In the numerical experiments it was decided to look at the variability of the forces arising solely from the natural variability of the wave field. The wave kinematics were determined using a full multi-directional wave model which is based on linear wave theory and Wheeler stretching to account for the

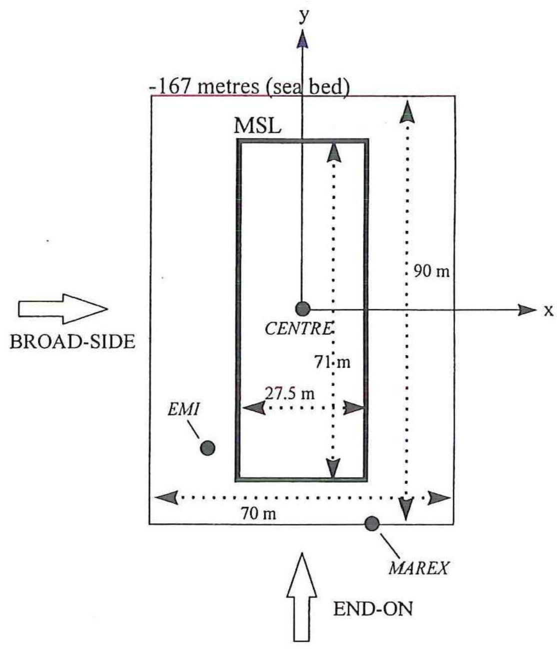

Fig. 1. Plan of Tern with possible location of wave sensors.

free surface effects. The hydrodynamic forces were determined with the Morison equation using constant values for drag and inertia coefficients. The structure is assumed to be rigid and immobile so the question of relative velocity formulations of the Morison equation does not arise. The different wave force analyses on a model of Tern were performed with the finite element program NIRWANA [7]. For reference, Tern was originally designed to an extreme wave height of 30.5 m and a period of 17.5 s with a high current velocity.

3. Constrained random time domain simulations

3.1. Brief description of the procedure

For each sea state the computer simulated time series of surface elevation are considered to be as 'measured' and are analysed at the three wave sensors. They are analysed together with the time series of the simulated forces on the platform. Since for both the 'wave-by-wave' and the

Table 2
Description of data for three sea states and the wave force model as used in this work

	'10 000 years sea state'	Sea state 1993[a]	'Monthly sea state'
Parameters of Jonswap spectrum	$H_S = 19.0$ m; $T_P = 17.6$ s; $\gamma = 3.3$	$H_S = 11.5$ m; $T_P = 13.7$ s; $\gamma = 3.3$	$H_S = 7.0$ m; $T_P = 10.7$ s $\gamma = 3.3$
Spreading standard deviation, σ (°)	32	32	32

Notes: (1) The measured wave spectrum for the 1993[a] storm was used. The wave spectral shape is taken to be identical for all three sea states, which also are of the same steepness. The H_S and T_P values have been scaled up to an approx. 10 000-yr and down to a 1-month condition. (2) A simple frequency-independent wrapped normal distribution is used for spreading. (3) The influence of wind has been neglected, and in storm 1993[a], the current was very small and is also ignored. (4) Values for drag and inertia coefficients used in the calculations were: rough members: $C_D = 1.26$, $C_M = 1.30$ (C_D values have been increased to reflect anodes); smooth members: $C_D = 0.63$, $C_M = 1.70$.

'sea state-by-sea state' force analyses a large amount of computational effort is needed we have tried to decrease the simulation time. Therefore, we adopted the methodology of constrained random time domain simulations [8] in order to accurately estimate the statistics of the extreme wave force in a period. This technique is transparent and provides the whole statistical structure of the extreme response (being quasi-static or dynamic) in a period within a reasonable amount of simulation time. It should be noted that this technique is not based on the extrapolation of individual maxima of response time series and provides complete statistical information rather than an estimate of one measure of central tendency only.

The basic idea behind the application of this technique is the underlying assumption that large forces on a structure occur due to the presence of big wave crests in the vicinity of the structure. This assumption is used to determine the distribution of the maximum force peak, F_{max}, which is associated with a particular crest height occurring at a particular location X:

$$F(F_{max}|\text{crest of height } A_{crest} \text{ at location } X) \qquad (1)$$

The distribution function 1 can be determined 'experimentally' for a range of crest heights A_{crest} at location X. Using knowledge of the probability density function of crest heights, $f(A_{crest}$ at location $X)$, from theory the distribution of maximum force associated with the occurrence of a crest of unknown height at location X can be determined:

$$F(F_{max}|\text{crest of any height at location } X) = \int_{(A_{crest})_{min}}^{(A_{crest})_{max}} F(F_{max}|\text{crest of height } A_{crest} \text{ at location } X) \times f(A_{crest} \text{ at location } X)\cdot dA_{crest} \qquad (2)$$

where $(A_{crest})_{min}$ and $(A_{crest})_{max}$ are the lower and upper values of the simulated crest heights, respectively. The distribution of large crests within a random sea state is known to fit the tail of the Rayleigh distribution.

Next the distribution of the extreme force in a certain period, $F(F_{extr})$, can be determined using the assumption that the forces given consecutive crests are uncorrelated, which is the case for large crest heights:

$$F(F_{extr}) = F(F_{max}|\text{crest of any height at location } X)^{N_{crest} \text{ at location } X} \qquad (3)$$

where N_{crest} is the number of crests in the period considered.

The distribution function 1 can in principle be determined using many fully random simulations. Obviously, this is very time consuming as we have to wait for the random occurrence of (large) crest heights all of height A_{crest} at location X. Since we are able to produce time series of surface elevation of short length each of which is constrained to include a maximum of given height at a given position in time and place, but is otherwise completely random, a considerable reduction in simulation time is now achieved. Consequently, we avoid the necessity of extensive random simulation of the ocean surface and the searching for the random occurrence of a large crest elevation. In Fig. 2a, a completely random signal of 200 s length is given together with the same signal but then constrained to have a wave crest of 12 m at a time instant of 100 s. For the purpose of illustration Fig. 2b shows the force signal corresponding to the constrained simulation. The maximum force peak, F_{max}, can easily be selected from the force time history; its value is the joint result of the predetermined large crest and the random influence of the underlying process. In practice a time interval is set within which the maximum force peak is bound to be related to the wave crest of predefined height. In the simulations performed here a time range of 10 s is chosen (5 s before and 5 s after the occurrence of the implemented wave crest).

Using multiple constrained random simulations the building block of the method, i.e. the empirical distribution of maximum force associated with a crest of height A_{crest} at location X (distribution 1), can be determined with a relatively small simulation effort. As Eq. (2) requires a fine discretization of the crest distribution a large number of maximum force distributions associated with particular crest heights are needed. In practice a limited number of crest heights can be used as it has been demonstrated by Harland *et al.* [9] that it is possible to estimate the distribution for intermediate crest heights by interpolation.

The theory and mathematical operations of constraining a random process to include a wave crest of predefined height to occur at a predetermined time and location have been described by Taylor *et al.* [8]. The technique has already been successfully applied to the extreme quasi-static and dynamic response analysis of various fixed offshore structures [9,10].

It should be noted that there can be an influence of the location where the wave crests are implemented in the random wave signal in the procedure. This so-called 'constraint location' can be regarded to be the location X of the wave sensor. Note further that distribution 1 actually reflects the results from a 'wave-by-wave' force analysis. Therefore, the technique is suitable for the 'wave-by-wave' force analysis in this work and enables determination of the sensitivity to the 'wave sensor location'.

As discussed in Section 1, the wave sensor location cannot physically influence the results from a 'sea state-by-sea state' force analysis. In the application of the technique using constrained simulations however, large wave crests are embedded in a random series at the 'constraint location' and therefore the results from a 'sea state-by-sea state' force analysis might be influenced by this location. Therefore, in Section 5 results will be presented for the application of the constrained simulation technique on the basis of three different 'constraint locations': *CENTRE*, *EMI* and *MAREX*. If the results appear to be insensitive to the 'constraint location' this would be a rigorous test of the adopted technique to estimate the statistics of the extreme wave force.

3.2. Application of constrained simulations in this work

Table 3 gives the wave crest heights to which the random signals of surface elevation have been constrained for the three sea states studied in this work. For each wave crest height 100 constrained simulations of 128 s have been used to evaluate Eq. (1).

Finally, to verify the method using constrained simulations for determining the extreme wave force statistics, multiple 3 h of waves passing Tern have also been simulated using purely random simulations. This has been done for both the broad-side and end-on loading conditions in the multi-directional 1993[a] sea state. Each verification analysis used 100 simulations of 3 h length.

Table 3
Wave crest heights to which random signals of surface elevation are constrained for the three sea states considered

	'10 000 years sea state' (m)	Sea state 1993[a] (m)	'Monthly sea state' (m)
$A_{crest,1}$	10.0	6.0	3.5
$A_{crest,2}$	13.0	8.0	5.0
$A_{crest,3}$	16.0	10.0	6.0
$A_{crest,4}$	20.0	12.0	8.0
$A_{crest,5}$	25.0	15.0	10.0

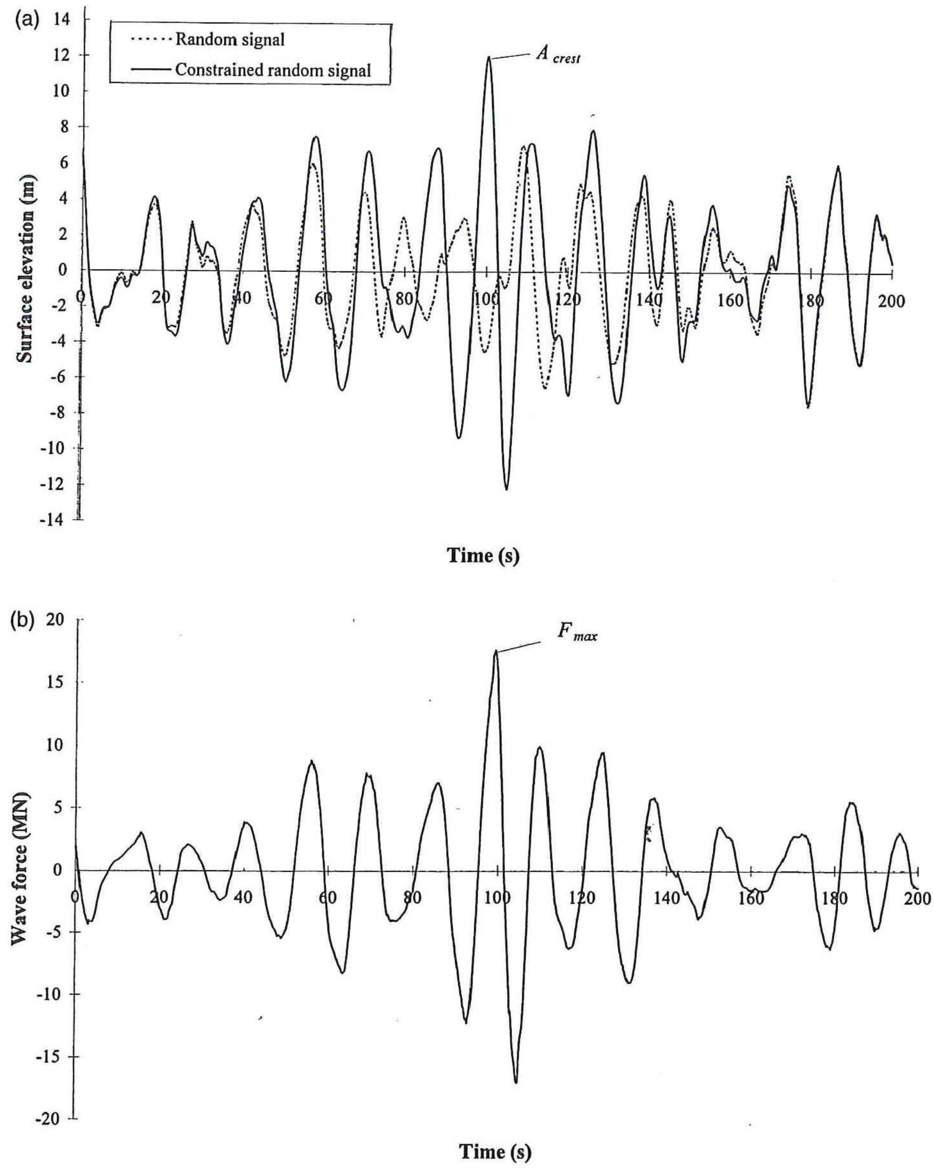

Fig. 2. (a) Two time series of a purely random simulation and a random simulation constrained to have a wave crest height of 12 m at 100 s. (b) Time series of the wave force corresponding to a random simulation constrained to have a wave crest height of 12 m at 100 s.

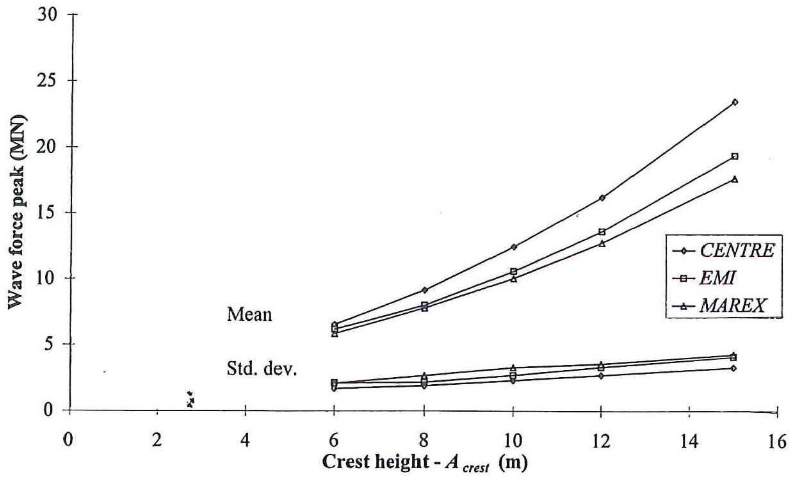

Fig. 3. Influence of the location of a wave sensor on the mean and standard deviation of the 'wave-by-wave' forces, for broad-side loading in a sea state with $H_S = 11.5$ m.

4. Variability of the force peak associated with the largest wave occurring at a particular point

Before we discuss the results from the 'wave-by-wave' force analyses, first the hidden influence of the wave sensor in this type of analyses is commented on. To understand the role of the wave sensor location, an imaginary situation is taken. Suppose we have a single stick, instead of Tern, subjected to a harsh environment. Further, suppose that we are able to measure the wave force on the stick and the surface elevation at various locations for unlimited length of time under stationary conditions of the random, short-crested ocean surface. Let us consider a wave sensor which is positioned at, say, a distance of $\frac{1}{4}$ of a predominant wavelength upstream from the stick. Based on the time series of surface elevation and wave force the variability of the 'wave-by-wave' force can be determined where given a wave crest elevation at the wave sensor the associated peak force is selected from the time-series of forces on the stick. This peak force is set to occur within some time interval relative to the time instant of the incident wave crest elevation. Suppose a CoV of the force of 20% is found. Now it would only seem logical that this value will decrease if a similar analysis is performed, but then based on the time series of surface elevation measured at the stick, because the physical correlation between a large surface elevation and a large peak force is stronger. Imagine next that this analysis was based on a measured time series of surface elevation at more than, say, 1 km away from the stick (which is not unusual in offshore engineering practice!). The expected mean value of the peak force on the structure associated with a wave crest at that wave sensor will be small, since due to dispersion and wave spreading the time series of wave force will be completely uncorrelated with the time-series of the ocean surface. Please note that the mean value of the peak force will not converge to zero as one still relates wave peaks with force peaks in a 'wave-by-wave' analysis. The CoV will now be a reflection of the CoV of the wave force peak in that sea state without *any* knowledge of the wave which produced the force peak, except that the statistics of the environment at the locations of the wave and force sensors can be assumed to be identical. Consequently, this force CoV will have a much larger value than the value found in the first situation.

Next, the results from the numerical experiments of the 1993[a] sea state with $H_S = 11.5$ m will be considered. The mean wave direction is chosen such that Tern will be loaded onto its broad-side direction. For waves with crest heights of 6.0, 8.0, 10.0, 12.0 and 15.0 m respectively, a 'wave-by-wave' force analysis has been performed.

Fig. 3 shows the mean and the standard deviation of the horizontal force at the seabed for a 'wave-by-wave' analysis of the crests as measured at three locations: *CENTRE*, *EMI* and *MAREX*. As one would expect, the mean of the wave force peaks associated with a crest at *CENTRE* location is larger than for a crest at the *EMI* and *MAREX* locations. In contrast to the drop in the mean force peak as the wave sensor is moved away from the centre of the structure, the standard deviation of the wave force peaks increases with separation distance. This is even better illustrated in Fig. 4, where the mean values are normalised on the mean *CENTRE* values and the associated CoVs are given for the three locations. Now a considerable effect can be seen by moving the wave sensor away from the *CENTRE*. The figure shows that the normalised mean force peak for the locations *EMI* and *MAREX* decrease with increasing crest height. The effect of moving the sensor, for a wave crest of 15 m at *MAREX*, is a reduction in the mean of the wave force peak of up to 25%. Furthermore, the variability of wave force peaks given a crest height is large. In the region of the most probable crest height for the given sea state the CoV is still about 18% for the *CENTRE* location, rising to 30% for a wave crest at the *MAREX* sensor. These values are in good agreement with the values found in the analysis of real measured data as discussed in Section 1.

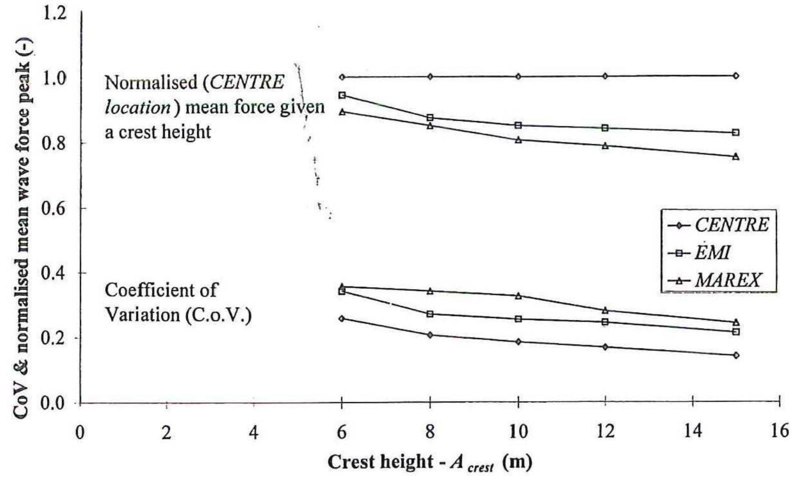

Fig. 4. Influence of the location of a wave sensor on the normalised mean and CoV of the 'wave-by-wave' forces, for broad-side loading in a sea state with $H_S = 11.5$ m.

Next the same analysis is performed with the same sea state but now with the mean wave direction rotated 90°, so Tern is loaded end-on. The normalised mean and the CoV of the wave force peaks associated with the maximum crest can again be determined (Fig. 5). The same trends are seen as before: the mean forces decrease and the standard deviations and CoVs increase when the wave sensor is moved from the *CENTRE* location.

5. Variability of the largest force in a particular sea state

5.1. 'Sea state-by-sea state' variability

Next the variability of the extreme force in a certain period is investigated without considering an association with a large crest occurring. Firstly, the waves are set to attack Tern broad-side. Fig. 6 and Table 4 give the extreme wave forces in the 1993[a] sea state as obtained using both constrained and fully random simulations. The first observation is that the statistics of the extreme wave force can be adequately determined using constrained simulations. The second observation is that in the application of the constrained simulation technique, the distribution of the extreme wave force is hardly influenced by the location of the wave sensor, i.e. where the predetermined wave crests have been embedded in the simulations. It should be realised that the distributions of the extreme wave force obtained using either simulation techniques are empirical and not exact.

Within the accuracy of the calculations, these results are identical. This is a reassuring demonstration that the method is numerically robust as it is physically impossible for the extreme wave force on a structure to be influenced by the location where the waves are measured. The small differences are due to differences in the accuracy of the results which is further discussed in Appendix A. In principle, one can state that moving the wave sensor away from the *CENTRE* in constrained simulations will decrease the computational efficiency of the constrained simulations which will

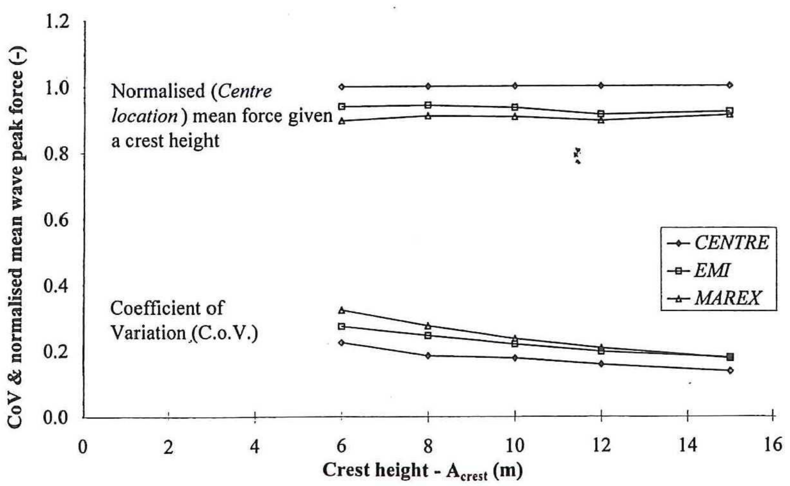

Fig. 5. Influence of the location of a wave sensor on the normalised mean and CoV of the 'wave-by-wave' forces, based on end-on loading in a sea state with $H_S = 11.5$ m.

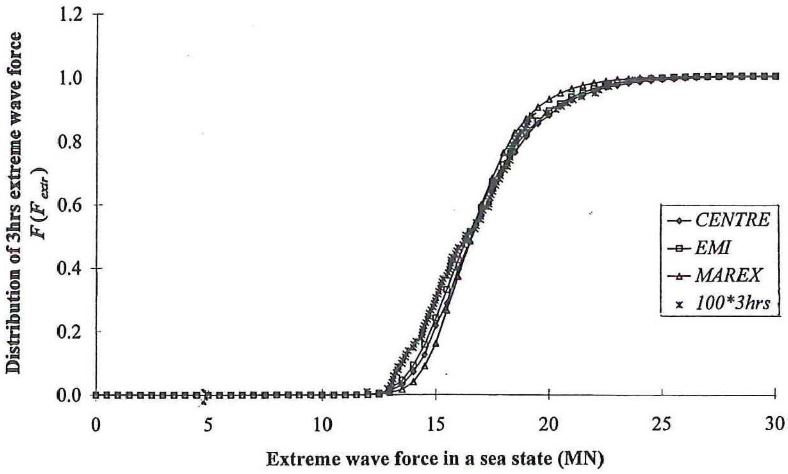

Fig. 6. Influence of the location of a wave sensor on the distribution of the extreme wave force in a broad-side sea state with $H_S = 11.5$ m.

eventually degenerate into fully random time domain simulations.

The most important observation is that the variability of the extreme wave force given the 1993[a] sea state, which is approx. 14%, is considerably smaller than the variability of the wave force given the most probable crest height within the sea state to occur at the chosen location of the wave sensor (varying between 18–30%, see the previous section).

Next, the same analyses have been performed for a sea state with waves end-on to Tern. As Fig. 7 and Table 5 show, the 'wave-by-wave' variability is again integrated out for the three locations of the wave sensor.

The mean values of the extreme end-on wave force are considerably smaller than for broad-side loading, the reason being that the structural steel is now distributed over a longer distance in the downstream mean wave direction. As before, the extreme wave force CoV given a sea state is smaller than the CoV of the wave force peak associated with the most probable maximum wave crest in the sea state occurring at a sensor (ranging from 17% to 22% dependent on sensor location, see Fig. 5).

5.2. *Influence of severity of sea state on 'sea state-by-sea state' variability*

The variability of the extreme wave force has also been estimated for a much more severe and a much milder sea state: the '10 000 years' and 'monthly' sea states from Table 2. The more severe one is chosen to be drag dominant whereas the second should be inertia dominant. The constrained simulation methodology has again been adopted with the crests conditioned in space at the *CENTRE* location of Tern. An overview of the results for broad-side and end-on loading is given in Table 6.

The extreme wave force CoV ranges from 9% for the end-on 'monthly sea state' to 16% for the very severe drag dominant '10 000 years' sea state.

5.3. *Source of extreme wave force variability*

As the measurements of the extreme wave force on Tern in a real sea state suggested, the variability of the extreme wave force may be (partly) due to the uncertainty in the extreme wave crest to occur (see Table 1). Using this assumption it is possible to give expressions for the variability of the extreme wave force within a single 3-h sea state. The basis for this is the distribution of the height of the largest wave crest in a sea state. The distribution of the largest wave crest in a sea state with a constant significant wave height, H_s, and a total number of waves, N, is simply obtained by powering up the Rayleigh distribution, $R(H)$, of the wave heights: $F(H_{extr}|H_s, N) = [R(H|H_s)]^N$.

The CoV of the extreme wave force can now be evaluated assuming a fixed relation between wave force and wave height. In this way the CoV has been determined for two extreme cases, i.e. entirely drag or entirely inertia dominated. The results in Table 7 suggest that the CoV of the extreme drag force is twice the CoV of both the extreme

Table 4
Mean and CoV of extreme wave force in a broad-side sea state with $H_S = 11.5$ m

	Constrained random simulations			Fully random simulations
	CENTRE	*EMI*	*MAREX*	
Mean (MN)	17.1	16.8	16.8	16.6
St.dev. (MN)	2.4	2.4	2.0	2.4
CoV	0.14	0.14	0.12	0.14

Table 5
Mean and CoV of extreme wave force in an end-on sea state with $H_S = 11.5$ m

	Constrained random simulations			Fully random simulations
	CENTRE	*EMI*	*MAREX*	
Mean (MN)	13.1	13.3	12.7	12.8
St.dev. (MN)	1.8	2.1	1.8	1.7
CoV	0.14	0.16	0.14	0.13

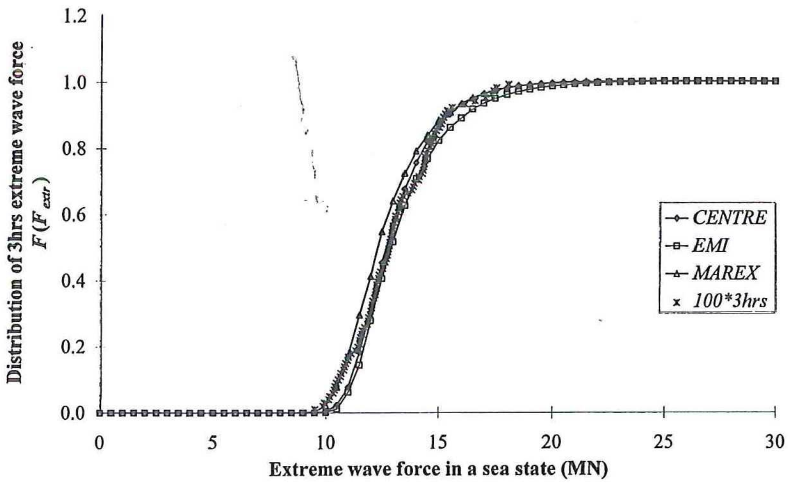

Fig. 7. Influence of the location of a wave sensor on the distribution of the extreme wave force in an end-on sea state with $H_S = 11.5$ m.

inertia force and the extreme wave height, all given the same sea state.

Comparison of these CoVs with those given in Table 6 shows that the assumption that the extreme wave force statistics are controlled by the uncertainty in the height of the largest wave crest is a very reasonable one.

It is surprising that the minimum possible CoVs of the extreme wave force in a sea state in Table 7 are so close to the CoVs deduced from simulations of the situation at Tern (Table 6) and are fully in line with those derived from actual measurements (Table 1). The influences of directional spreading and dispersion, which are very pertinent to the variability of the force peaks in a 'wave-by-wave' analysis given the largest wave crest occurring at a particular point, are apparently integrated out in the analysis of the variability of the largest force in a period for a 'sea state-by-sea state' analysis. This implies that the largest force in a period can occur at a completely different time instant than the occurrence of the largest wave crest in the same period.

A further illustration of the remarkable observation about the origin of the extreme wave force variability is given in Fig. 8. In this figure the distributions of the extreme force normalised on their median value are given for the three sea states broad-side to Tern. Furthermore, in this figure the normalised distributions for extreme force have also been plotted assuming a fixed relationship between force and wave height.

If we look at the normalised distribution of the extreme force for the drag dominant '10 000 yr sea state' we observe an excellent agreement with the approximated distribution which is obtained by assuming a squared relation between wave force and wave height. Furthermore, for the sea states with a decreasing influence of drag wave loading the normalised extreme force distributions converge to the approximated distribution for a purely linear relation between force and wave height.

6. Discussion of results

From the results obtained with the numerical experiments it can be concluded that the location of the wave sensor relative to the structure does indeed have a significant influence on the variability of the force peak which is associated with the largest wave crest at the sensor. In contrast, the variability of the largest wave force in a sea state is not influenced by the location of the wave sensor. Consequently, one should be careful in the assessment of a 'wave-by-wave' force analysis of measured time series of surface elevation and associated forces.

In Section 5, it was demonstrated that for sea state based force statistics a large proportion of the randomness of the wave field is integrated out. Obviously, this observation may be associated with the set-up of the numerical experiments and its limitation in modelling the real problem. However, the fact that the minimum possible CoVs of the extreme wave force in a sea state given in Table 6 are so close to the actual CoVs deduced

Table 6
Extreme wave force statistics for three sea states

	'10 000 years sea state'		Sea state 1993[a]		'Monthly sea state'	
	Broad	End	Broad	End	Broad	End
Mean (MN)	46.0	40.1	17.1	13.1	6.5	4.9
St.dev. (MN)	7.5	6.5	2.4	1.8	0.7	0.4
CoV	0.16	0.16	0.14	0.14	0.11	0.09

Table 7
Estimated CoVs of the extreme wave crest and the extreme wave force within a sea state

Variable	Relation	CoV
H	$H = H$	0.085
F_I	$F_I \approx H$	0.085
F_D	$F_D \approx H^2$	0.170

H, Wave height; F_I, Inertia force; F_D, Drag force.

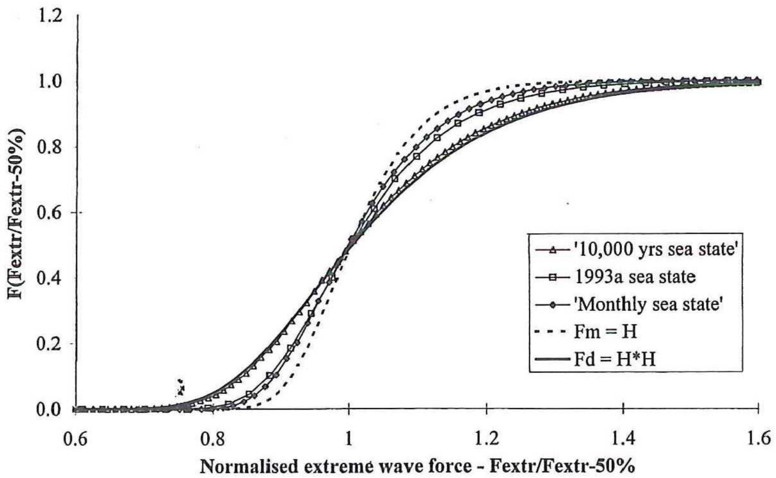

Fig. 8. The distribution of the extreme force normalised on the median value, $F_{extr\text{-}50\%}$, for three different (broad-side) sea states and two theoretical cases assuming a fixed relation between wave force and wave height.

from Tern measurements (Table 1) demonstrates that the observation is not unrealistic. Note that in the analysis of measured data no modelling was involved. The results therefore suggest that other sources of randomness (e.g. varying values of C_D and C_M for each element in the structure, assuming this variability to be uncorrelated) could also integrate out for the sea state based statistics. The role of the structure in the analysis of the extreme force due to (severe) waves may therefore be considered as 'averaging out' many types of variability that occur in nature.

These results have implications for the determination of the extreme wave force on a structure and its uncertainty. In design guidelines like e.g. API [2], the extreme force is determined as the force which is associated with the largest wave height in a period. As demonstrated it is not only impractical to determine the characteristic value of variability of the force given this wave height due to its dependency on the location of the wave environment, it is also irrelevant to link the extreme force to an extreme wave crest (or wave height). The only thing that really matters is the uncertainty in the extreme force in a particular sea state. This would imply that in the design or re-assessment of an offshore structure, a sea state based extreme force analysis should be performed for which extensive techniques like time domain simulation could be used. However, a much more simplified technique may be developed based on the observation that the uncertainty of the largest force in a period is essentially dominated by the uncertainty of the largest wave height in a period. The problem then degenerates into the determination of a relationship between wave force and wave crest (or height) which should give a satisfactory estimate of e.g. the mean extreme wave force in a period. Note that this relationship will be different when a different extreme wave force problem is analysed, e.g. due to a change in environmental conditions or in structural configuration or dimensions. Such a relationship might then be obtained by running waves of different height through the model. Presently we are looking at how such a relationship could be determined using standard engineering models.

Finally, the demonstration that the global force in a sea state is independent of the sensor (constraint) location is a rigorous test of the constrained random time domain simulations methodology and its implementation in NIRWANA. Further, it has been demonstrated that the statistics of the extreme wave force can be predicted using constrained simulations. A reduction in simulation time of greater than a factor of 15 compared to fully random time domain simulations for the same accuracy has already been established. We consider that further reduction in simulation time should be possible but this has not been the objective of the present work.

7. Conclusion and recommendations

7.1. Conclusions

(i) The location of the wave sensor has a significant influence on the statistics of the wave force peaks that are associated with a crest elevation at the wave sensor ('wave-by-wave'). A longer distance between the wave sensor and the structure (weaker relationship) decreases the mean value and increases the standard deviation and CoV of the force peaks that are associated with wave crests of a specified size and location of occurrence. There can be no influence of the location of the wave sensor on the statistics of the extreme wave force in a (spread) sea ('sea state-by-sea state'), provided only that both the wave sensor and the structure are in the same wave field.

(ii) It has been shown that the variability of the extreme wave force in a given sea state is significantly smaller than the variability of the wave force peak associated with the most probable extreme crest elevation in that sea state at a predetermined location. This is in agreement with observations from measurements of wave forces on Tern.

(iii) It has been demonstrated that the technique based on constrained random time domain simulations can estimate the distribution of the largest wave force in a multi-directional sea state with a reduced amount of computing time.

(iv) When constrained random time domain simulations are used, the centre of a structure should be taken as the location of the conditioning point for the embedded wave crest elevation. At this location the probability distribution of the extreme wave force in a given sea state is obtained with the smallest degree of statistical uncertainty for a given amount of simulation effort. When the sensor is moved away from the centre of the structure, the computational efficiency decreases and eventually the constrained simulations reduce to simple 'brute-force' random time domain simulations.

(v) The CoV of the extreme wave force on a structure in a given sea state is dominated by the (inevitable) statistical variability in the size of the largest wave crest in the sea state. Thus, for an inertia dominated structure, the extreme wave force CoV is comparable to the extreme wave crest height CoV, whereas for a drag dominated structure it is twice as large, due to the 'velocity-squared' term in the Morison equation.

7.2. Recommendations for further work

(i) The influence of other random variables (e.g. varying C_D and C_M values) in the extreme wave force analyses using constrained simulations should be taken into account as well. We expect that their influence will be integrated out. Just using mean values for these variables should suffice, assuming that C_D, and C_M values are not correlated with the position of members in the time varying wave field.

(ii) Engineering type approaches for the estimation of the extreme wave force in a spread sea state are based on uni-directional design waves. These methods should be extended to include correction (reduction) factors for the extreme force in a spread sea. Extreme wave force analyses for platforms with different dimensions, steepness of waves and spreading variance of the sea state should be performed to increase insight in the importance of these modifications.

(iii) Guidelines for the successful application of the method of constrained simulations to determine the extreme structural response given a sea state should be developed with a strong focus on accuracy of the results and efficiency of the simulation.

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Appendix A Accuracy of extreme wave force distribution

In this appendix, the accuracy of the extreme wave force statistics is considered based on the use of the constrained simulation technique. This technique is based on convolution of maximum force distributions associated with a crest at a point over a range of crest heights. The conditional maximum force distributions are determined from (many) constrained simulations of a certain length, i.e. they are derived in an empirical manner. For further processing these distributions are interpolated and smoothed. As the Rayleigh distribution for wave crest heights is also smooth, the extreme wave force distribution thus obtained is always a smooth function. This may give the false impression that the answer is exact, especially when compared to empirical results obtained from fully random time domain simulations. However, as noted above, the method is based on empirical conditional maximum force distributions for a range of wave crest heights. Any uncertainty within these distributions is transferred directly into the final result, which means that the finally obtained extreme wave force distribution is only approximate.

Unfortunately, the accuracy of this distribution is unknown; only by repeating the analysis for the same conditions but with different random seeds more information can be obtained about its accuracy. As an illustration, the accuracy of the extreme wave force distribution will be determined for the 1993[a] sea state which loads Tern in a broad-side direction. Special attention will be given to the influence of the location of the conditioning point of the embedded wave crest elevation.

For each of the five crest heights given in Table 3, 100 constrained simulations were used. Next the same analyses were repeated four times so eventually five different analyses have been performed for the same set-up of the computations but with different random seeds. We thus have performed in total 500 constrained simulations per crest height. We can then also consider the results as obtained from 10 extreme wave force analyses but now using 50 constrained simulations per crest height. As each of these cases produces a distribution of the extreme wave force with one mean, μ_{Fextr}, and one standard deviation, σ_{Fextr}, 5 (or 10) individual analyses would produce 5 (or 10) combinations of the mean and standard deviation of the extreme wave force. Now a simple statistical analysis can be performed to get more information about the 'accuracy' of these values. This information is contained in the sample mean and standard deviation of both μ_{Fextr} and

Table 8
Accuracy analysis of the extreme wave force statistics as obtained using constrained simulations with their conditioning point at *CENTRE*

Parameter	Constrained random simulations (MN)		Fully random simulations (MN)
	'5*100'	'10*50'	(100*3 h)
$mean(\mu_{\text{Fextr}})$	17.0	17.0	16.6
$stdev(\mu_{\text{Fextr}})$	0.2	0.5	0.2
$mean(\sigma_{\text{Fextr}})$	2.4	2.1	2.4

Table 9
Accuracy analysis of the extreme wave force statistics as obtained using constrained simulations with their conditioning point at *EMI*

Parameter	Constrained random simulations (MN)		Fully random simulations (MN)
	'5*100'	'10*50'	(100*3 h)
$mean(\mu_{\text{Fextr}})$	16.7	17.0	16.6
$stdev(\mu_{\text{Fextr}})$	0.7	1.1	0.2
$mean(\sigma_{\text{Fextr}})$	2.2	1.9	2.4

Table 10
Accuracy analysis of the extreme wave force statistics as obtained using constrained simulations with their conditioning point at *MAREX*

Parameter	Constrained random simulations (MN)		Fully random simulations (MN)
	'5*100'	'10*50'	(100*3 h)
$mean(\mu_{\text{Fextr}})$	16.9	17.3	16.6
$stdev(\mu_{\text{Fextr}})$	0.4	1.1	0.2
$mean(\sigma_{\text{Fextr}})$	2.2	2.2	2.4

σ_{Fextr}, e.g.:

$$mean(\mu_{\text{Fextr}}) = \frac{1}{N}\sum_{i=1}^{N} \mu_{\text{Fextr}_i}$$

$$stdev(\mu_{\text{Fextr}}) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\mu_{\text{Fextr}_i} - mean(\mu_{\text{Fextr}})\right)^2}$$

Similarly the $mean(\sigma_{\text{Fextr}})$ and $stdev(\sigma_{\text{Fextr}})$ can be determined. In this appendix, the standard deviation of the set of standard deviations, $stdev(\sigma_{\text{Fextr}})$, will not be given. The $stdev(\mu_{\text{Fextr}})$ will be used as a measure of accuracy as it can be easily compared with the well known standard error in the mean as obtained from empirical results, i.e. fully random simulations.

Appendix A.1 Location of wave sensor and 'accuracy'

The results from the accuracy analyses, as described above, are given for the extreme wave force distribution based on 5*100 and 10*50 constrained simulations per crest height which will be referred to as '5*100' and '10*50'. The influence of the location of the wave sensor can be demonstrated nicely with the parameter $stdev(\mu_{\text{Fextr}})$ in Tables 8, 9 and 10 for the *CENTRE*, *EMI* and *MAREX* location, respectively.

It can be concluded from the tables that the extreme wave force statistics are not influenced by the location because all values for the $mean(\mu_{\text{Fextr}})$ for every location cannot be distinguished statistically from each other. This would suggest that for all locations the same statistics of the extreme wave force could be obtained provided that a very large simulation effort was applied. The results of the constrained simulations also correspond very well with the results obtained from 100 fully random simulations of 3 h each.

Furthermore, it can be seen that the accuracy of the obtained extreme wave force distribution is influenced by the location of the constraint on surface elevation: the further away the sensor the larger the value of $stdev(\mu_{\text{Fextr}})$ for a given level of computational effort. Generally, the most accurate results are obtained for a given simulation effort when the wave crests are constrained at the *CENTRE* of a structure.

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