Force network ensemble for the triangular lattice: A tale of tiles

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In granular materials, contact forces between neighboring grains are organized in disordered force networks. At present, it is unclear whether or not theories based on statistical mechanics can correctly predict the statistics of these force networks.

The force network ensemble is a convenient model system for studying force networks. This ensemble comprises all sets of noncohesive contact forces on a fixed underlying contact network and a fixed load, i.e., global stress tensor, for which all grains are in static force and torque balance. Each force network is assigned an equal a priori probability. The corresponding phase space can be sampled using the Monte Carlo method. Figure 1(a) shows a typical force network for the frictionless triangular lattice. Each force network has a complementary representation known as a reciprocal tiling [Fig. 1(b)] in which each grain maps to a particular tile and each contact force to a tile face. The face is oriented at a rotation to the contact force $f$ with a length proportional to $f$. Due to local force balance, the tile faces form closed loops and all tiles fit together without any gaps. The local pressure of a single grain is equal to its tile perimeter. Fixing the global stress tensor leads to a conservation of the total tile area. The animation shows the sampling of force networks in the force network ensemble. (c) Theoretical (dashed line) and numerical (solid line) local pressure probability distribution $\rho(p)$ for the frictionless triangular lattice (enhanced online).[URL: http://dx.doi.org/10.1063/1.3207833]

In Ref. 2, we derived an analytical expression for the distribution of the local pressure on individual grains, by maximizing the entropy while conserving $\langle \hat{a} \rangle$ and $A$. Figure 1(c) shows that this theoretical prediction is in excellent agreement with the numerical result from the force network ensemble. Due to these constraints, large local stresses obey Gaussian statistics, in sharp contrast to the common belief that exponential force statistics are characteristic for granular materials. This observation is robust to changes in contact network (including disordered networks) and finite friction coefficient.


FIG. 1. (Color) (a) Typical force network for a frictionless triangular lattice. Line thickness is proportional to contact force. All grains are in static force balance. Periodic boundaries are used. (b) Reciprocal representation of (a). Tile edges correspond to contact forces (rotated by $\pi/2$) with lengths proportional to their magnitude. Due to Newton’s laws, the tile faces form closed loops and all tiles fit together without any gaps. The local pressure of a single grain is equal to its tile perimeter. Fixing the global stress tensor leads to a conservation of the total tile area. The animation shows the sampling of force networks in the force network ensemble. (c) Theoretical (dashed line) and numerical (solid line) local pressure probability distribution $\rho(p)$ for the frictionless triangular lattice (enhanced online).[URL: http://dx.doi.org/10.1063/1.3207833]