STELLINGEN

BEHORENDE BIJ HET PROEFSCHRIFT

MEASUREMENT OF MINORITY CARRIER MOBILITY, CARRIER LIFETIME AND BANDGAP NARROWING IN HEAVILY DOPED SILICON

DOOR

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DELFt, 8 DECEMBER 1992.
1. Het meten van een redundante set parameters geeft direct informatie over hoe betrouwbaar de meetresultaten zijn.

_Dit proefschrift, hoofdstuk 3,4._

2. Bij lage temperaturen neemt de beweeglijkheid van minderheden niet toe met toenemende dotering zoals Swirhun et al. en Wang et al. beweren maar neemt gewoon af.

_S.E. Swirhun et al., IEDM Techn. Dig.,298, 1988_

_Dit proefschrift, hoofdstuk 5._

3. De beweeglijkheid van gaten als minderheden is bij lage doteringen beïnvloed door lager dan tot nu toe was aangenomen.

_Dit proefschrift, hoofdstuk 5._

4. De gemeten temperatuursafhankelijkheid van de beweeglijkheid van minderheden in zwaar gedoteerd silicium doet vermoeden dat begrippen als 'localised states' en 'mobility edge' niet belangrijk zijn voor het transport van minderheden.

5. Het gebruik van device simulatoren bij de extractie van parameters uit meetresultaten heeft als voordeel dat men niet beperkt is tot meetmethoden waarvoor analytische uitdrukkingen bestaan. Het blijft echter wel nodig om meetmethoden te gebruiken die zo veel mogelijk ongevoelig zijn voor andere dan de te meten parameters.

6. In de fysica van elementaire deeltjes wordt de theory gebruikt om in de hoeveelheid data de juiste event te vinden, in de device fysica, echter, wordt de meetdata zoals voor beweeglijkheid en bandgap narrowing juist gebruikt om het juiste theoretische model te selecteren.
7. Met het toenemend aantal functies en gebruikersmogelijkheden van consumentenapparatuur zou het aan te bevelen zijn als bij de beschrijving staat wat ze niet kunnen in plaats van wat ze wel kunnen.

8. In het wetenschappelijk onderzoek is het moeilijker om zich aan het voorgenoemde onderwerp te houden dan er van af te wijken.

9. Het is te hopen dat de energie die de regering steekt in energiebesparende en milieuvriendelijke maatregelen niet bijdraagt aan het broeikaseffect.
MEASUREMENT OF MINORITY CARRIER MOBILITY, CARRIER LIFETIME AND BANDGAP NARROWING IN HEAVILY DOPED SILICON
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Proefschrift

ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus, prof. drs. P.A. Schenck, in het openbaar te verdedigen ten overstaan van een commissie aangewezen door het College van Dekanen op dinsdag 8 december 1992 te 10.00 uur

door

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geboren te Amsterdam
elektrotechnisch ingenieur
Dit proefschrift is goedgekeurd door de promotors
prof. dr. M. Kleefstra
Bij de mjitte bliuwe

_Aan Lucy_
_Aan mijn ouders_
CONTENTS

1 Introduction ........................................... 1
  1.1 Purpose of this work ................................. 1
  1.2 Outline of this thesis ............................... 2

2 Review .................................................. 5
  2.1 Heavy doping effects ................................. 5
  2.2 Transport equations .................................. 6
  2.3 Transport parameters ............................... 9
  2.4 Measurement techniques for the transport parameters 10

3 Measurement method .................................. 15
  3.1 DC-transport parameters ............................ 16
    3.1.1 Principle of the method .......................... 16
    3.1.2 Modelling the device ............................ 17
    3.1.3 Extraction of the parameters: an inverse problem 18
    3.1.4 Implementation of the method ..................... 20
  3.2 AC-transport parameters ............................ 21
    3.2.1 Principle of the method .......................... 21
    3.2.2 Modelling the device ............................ 22
    3.2.3 Extraction of the parameters ..................... 25
  3.3 Error analysis ..................................... 28
    3.3.1 Measurement errors .............................. 30
    3.3.2 Model errors .................................... 31
    3.3.3 Numerical errors ................................ 34

4 Experimental results ................................ 39
  4.1 Fabrication of the devices .......................... 39
  4.2 Measurement of the DC-transport parameters ........... 43
    4.2.1 Design of the test structures .................... 43
    4.2.2 Measurements at room temperature ................ 45
CHAPTER 1

INTRODUCTION

1.1 Purpose of this work

Research on bipolar transistors is mainly focussed on two items. One is the reduction of the lateral dimensions of the transistor to enable very large scale integration (VLSI), and the second is increasing the gain or, even more importantly, maintaining the gain at very high frequencies. This last is a goal which is often summarised by only one figure of merit: the cutoff frequency $f_T$.

The lateral dimensions can be reduced by improved optical lithography and by self-aligned techniques and this also helps to increase $f_T$ because of the reduction in series resistances and depletion capacitances. To increase $f_T$ further, several compromises have to be made. For instance, when the basewidth is reduced to decrease the transit time of carriers through the base, the base resistance will increase and the Early effect also becomes more pronounced. To compensate for the increase in base resistance, the base doping can be increased. However, to maintain a reasonable current gain the doping concentration in the emitter, or more properly the Gummel number of the emitter, must also be increased. It appeared that at high doping concentrations the Gummel number can not easily be further increased by simply doping the emitter more heavily. Evidently, heavy doping affects the transport properties of the minority carriers and it was this observation that led to a considerable amount of research into these heavy doping effects.

Although in modern transistors the Gummel number is effectively increased by using polysilicon contacts, the transport of carriers in heavily doped regions still affects the performance of bipolar devices. For example, a Si-transistor used by Nanba et al. [71] with a reported $f_T$ of 64 GHz has an emitter top-dope of $10^{20}$ cm$^{-3}$ and a base topdope of $10^{18}$ cm$^{-3}$, in which case the heavy doping effects occur not only in the emitter but also start to occur in the base. Further, in SiGe-base heterojunction bipolar transistors, where the need
for a heavily doped emitter is less stringent because the heterojunction forms an extra energy barrier for the minority carriers injected in the emitter, it is necessary to dope the base heavily to lower its resistance.

In order to optimize the transistor performance, it is thus necessary to know the transport parameters of minority carriers in heavily doped regions accurately. Although considerable work has been done to characterize the minority carrier transport, very little measurement data is available from systematic, complete and accurate experiments. It is not even possible to model the transport as a function of temperature because of a almost complete lack of experimental data concerning the parameters as a function of temperature.

The main objective of this work is to present new experimental data. This is done, firstly, by developing an experimental method which can be used to determine the parameters characterizing the minority carrier transport and, secondly, by applying this method for the actual measurement of these transport parameters for several doping levels, not only at room temperature but for a whole range of temperatures. These measurements as a function of temperature are not only important for modelling purposes, but also help to provide deeper insight into the physical processes involved in the transport of minority carriers. This is possible since each scattering process, such as lattice scattering, impurity scattering or carrier-carrier scattering, has a different temperature dependence.

1.2 Outline of this thesis

In chapter 2 a short overview is given of the major effects that occur in heavily doped silicon, the transport equations that are necessary to describe the minority carrier transport correctly and, most importantly, the physical quantities which fully characterize the minority carrier transport. Some existing methods are also discussed by which these quantities or parameters can be measured.

In chapter 3 a new method is discussed by which a simultaneous determination of all the minority carrier transport parameters is possible and an elaborate error analysis is given. To test the measurement method, and to find the sensitivity of the parameters with respect to uncertainties in the measuring conditions, the whole measurement method is simulated with synthetic data.

How the actual measurements are performed, is shown in chapter 4. This description involves a discussion of the influence of parasitic elements on the measurements, and the accuracy of the measured parameters is estimated.

Chapter 5 gives a discussion of the results obtained. These results are sum-
marized by fitting them to analytical expressions and they are then compared with data obtained from the literature and some theoretical or semi-theoretical calculations.

Finally, in chapter 6, the important conclusions obtained from this work are repeated, together with some suggestions for future work.
CHAPTER 2

REVIEW

2.1 HEAVY DOPING EFFECTS

It is not the intention to give an exhaustive overview of the heavy doping effects occurring in silicon here. Some excellent reviews have been given by Abram et al. [3], Mertens et al. [66] and Kane [48], and in some standard textbooks from Bonch-Bruevich [9], Fistul [30] and Mott and Davis [69]. To summarize, the major effects that occur with high doping levels are (for simplicity, the effects are mentioned for N-type only):

- The majority and minority carrier mobility decreases with increasing doping level because of extra scattering mechanisms such as impurity scattering, electron-electron scattering and electron-hole scattering.

- The minority carrier lifetime decreases with increasing doping density due to more recombination traps. In addition, at high doping levels the Auger recombination process becomes dominant.

- The energy bandgap narrows with increasing dopant density. This is mainly caused by two effects. One effect is a rigid bandgap narrowing, that is the parabolic density of states is shifted with only a minor change in effective mass, due to electron-donor and electron-electron interactions. The second effect is the formation of band tails due to the disordered distribution of the impurities.

- Anderson [5] found that when the disorder in a system reaches a certain level localisation occurs. In fact, it is believed that localised states exist in the band tails. Transport can only take place by thermal activation to the conduction band or by hopping between the states. The fact that the electrons in the band tails are immobile leads to the concept of a 'mobility edge', which defines the boundary between the localised and the extended or free states.
• At already moderate doping levels the potentials of the impurities begin to overlap each other, which results in a broadening of the impurity level. The impurity levels can be described by a Gaussian distribution according to Morgan [68]. At a doping level of approximately $3 \cdot 10^{18}$ cm$^{-3}$ the impurity energy level merges with the conduction band, in which case all impurities at all temperatures are ionized. This means that silicon with a doping concentration below $3 \cdot 10^{18}$ cm$^{-3}$ and with a temperature close to zero Kelvin, can be considered as an insulator, since all electrons are bounded by impurities and thus there are no free carriers. For silicon with a doping concentration larger than $3 \cdot 10^{18}$ cm$^{-3}$, all electrons coming from the donors can be considered free and silicon behaves like a metal. This insulator-metal transition is called the Mott-transition [69].

2.2 Transport Equations

Considering all above heavy doping effects, it is important to know how these effects influence the device behaviour and what changes have to be made to model heavily doped devices correctly.

Of course, the Poisson equation and the continuity equations which are used in the drift-diffusion model are still valid since they are derived from first principles. However, in the general case of a position-dependent band structure drift by an electric field and diffusion by a concentration gradient are no longer the only transport mechanisms. Electrons and holes also experience a force due to a different electrochemical potential caused by the change in bandgap. Incorporating this mechanism effectively means that electrons and holes experience different fields, or 'quasi-electric fields' as Kroemer [57] called them.

Another problem is the calculation of the number of electrons and holes in heavily doped silicon. Figure 2.1 shows the density of states for heavily doped N-type silicon in equilibrium. Since the Fermi level lies in the conduction band, Fermi-Dirac statistics has to be used. However, to calculate the number of electrons as a function of the Fermi level the detailed structure of the density of states must be known. Such a calculation has been done, for example by Slotboom [90] and are especially important for describing the majority carrier transport.

Characterization of the transport of charged carriers becomes easier when only the minority carriers in quasi-neutral regions are considered. In this case the solutions of the Poisson equation and the continuity equation for the ma-
Figure 2.1: Realistic band diagram for heavily doped silicon ($N = 10^{19} \text{cm}^{-3}$) using the data from Slotboom [90]. The inset shows the distribution of the minority carriers in the valence band.

Majority carriers are trivial. Except at high-injection conditions, the quasi-Fermi level of the minority carriers (holes) lies far away from the valence band edge. So, to calculate the minority concentration, Maxwell-Boltzmann statistics suffices. Under this restriction, del Alamo [4] showed that the distribution of the density of states for, in this case the valence band, is of no importance. The minority concentration can simply be described by:

$$p = p_0 \exp \left( \frac{E_F - E_{FP}}{kT} \right)$$  \hspace{1cm} (2.1)

where $E_F$ and $E_{FP}$ are, respectively, the Fermi level of the electrons and the quasi-Fermi level of the holes. The quantity $p_0$, the minority carrier concentration in equilibrium, is defined by:

$$p_0 = \int_{-\infty}^{\infty} \rho_V(E) \exp \left( \frac{E - E_F}{kT} \right) dE$$  \hspace{1cm} (2.2)
where \( \rho_V \) denotes the density of states in the valence band. In equation 2.2, \( p_0 \) completely reflects the influence of the valence band structure. Further, because \( p_0 \) depends on \( E_F \), which in turn depends on the situation in the conduction band, \( p_0 \) also reflects the influence of the conduction band structure.

To complete the description of minority carrier transport in quasi-neutral regions, the following transport equations are used according to del Alamo [4] and Kleefstra [55]. The hole current density \( j_p \) is given by:

\[
j_p = p \mu_p \nabla E_F \tag{2.3}
\]

and the continuity equation by:

\[
\frac{\partial p}{\partial t} = - \frac{1}{q} \nabla \cdot j_p - \frac{p - p_0}{\tau_p} \tag{2.4}
\]

where \( \mu_p \) is the minority carrier mobility for holes and \( \tau_p \) is the minority carrier lifetime. For the simple case of homogeneous doping concentration, equation 2.3 substituted in 2.4 yields:

\[
\frac{1}{D_p} \frac{\partial p}{\partial t} = \Delta p - \frac{p - p_0}{L_p^2} \tag{2.5}
\]

which is the time-dependent diffusion equation where \( D_p \) denotes the diffusion coefficient and \( L_p \) the diffusion length. \( D_p \) relates with \( \mu_p \) through the Einstein relation \( (D = \frac{kT}{q} \mu) \) which still holds as long as Maxwell-Boltzmann statistics can be used. The diffusion length equals:

\[
L_p = \sqrt{D_p \tau_p} \tag{2.6}
\]

One comment has to be made about localised states and the mobility edge. The mobility edge does not influence the statistics (\( p_0 \)) but it does affect the transport of minority carriers since a number of minority carriers are frozen in their localised states. These localised states can be described properly by treating them as traps with a continuous energy distribution, as has been done by van Rijs [104] for amorphous silicon. More specifically, for the case of minority carriers in heavily doped silicon Pan et al. [79] extended equations 2.3 and 2.4 to model the effect of localised states and also allowed a non-zero mobility of carriers in the localised states. However, since the location of the mobility edge, or even its' existence in the valence band, is not clear, the mobility edge is not considered here.
2.3 Transport Parameters

Equations 2.1, 2.3 and 2.4 describe fully the transport of minority carriers in quasi-neutral regions. The only parameters which enter these equations are \( p_0 \), \( \mu_p \) (or \( D_p \)) and \( \tau_p \). These quantities are called the minority carrier transport parameters. For these parameters a distinction can be made between DC and AC-transport parameters. The DC-transport parameters fully determine the steady-state behaviour of a device; the diffusion length \( L_p \) is such a quantity. Moreover, in devices \( D_p \) and \( p_0 \) always enter the steady-state solution of the current by their product \( p_0 D_p \) and so \( p_0 D_p \) is called the second DC-transport parameter. The parameters \( \mu_p \) and \( \tau_p \), both containing the dimension time, are then called the AC-transport parameters.

Although, as mentioned in section 2.2, \( p_0 \) describes via equation 2.1, the statistics of minority carriers in quasi-neutral regions, this quantity is not easy to include in the complete set of the semiconductor device equations. As seen in practice, the effects of heavy doping on \( p_0 \) is, for modelling purposes, conveniently incorporated in the parameter called the bandgap narrowing. The bandgap narrowing \( \Delta E_g \) is then defined through the relation of the pn-product:

\[
p_0 N_D = n_{ie}^2 = n_{io}^2 \exp\left(\frac{\Delta E_g}{kT}\right)
\]

where \( N_D \) is the donor concentration, \( n_{ie} \) is the effective intrinsic carrier concentration and \( n_{io} \) is the intrinsic carrier concentration, which is defined as:

\[
n_{io}^2 = N_C N_V \exp\left(-\frac{E_g}{kT}\right)
\]

where \( N_C \) and \( N_V \) are, respectively, the effective density of states of the conduction and valence band, while \( E_g \) represents the bandgap in the low doped case.

It is important to stress that \( \Delta E_g \) is not equal to the real bandgap narrowing that takes place. In fact, \( \Delta E_g \), as defined in equation 2.7 also includes the following effects:

1. A different effective density of states: It is assumed that \( N_C \) and \( N_V \) are independent of impurity concentration. Since this is unlikely, the change in density of states is effectively ascribed to the bandgap narrowing.

2. Non-parabolic density of states: Equation 2.7 holds for a parabolic density of states, which is a good approximation in the low doped case. However, in the heavily doped case the density of states becomes non-parabolic. The effects of this non-parabolicity are thus incorporated in the bandgap narrowing.
3. Fermi-Dirac statistics: Since, for high doping levels, the Fermi level of the majority carriers joins the conduction band, Fermi-Dirac statistics has to be used. In equation 2.7 Maxwell-Boltzmann statistics is assumed.

4. Incomplete ionization: For \( N_D \) the total donor concentration is often taken instead of only the ionized part. Again, this erroneous assumption has been included in the bandgap narrowing.

The last two contributions are mostly incorporated in order to facilitate the calculations, although the bandgap narrowing can be corrected for this (when a parabolic density of states is taken). In this light, it is therefore better to speak of an 'apparent bandgap narrowing' \( \Delta E_g^{app} \), as pointed out by del Alamo et al. [24].

It has been shown that the parameters \( p_0 \) (or \( \Delta E_g^{app} \)), \( \mu_p \) (or \( D_p \)) and \( \tau_p \) are sufficient to characterize minority carrier transport in quasi-neutral regions. Since these parameters depend on the doping concentration and temperature, it is necessary to know these parameters for under these conditions. This thesis involves the measurement of the above transport parameters.

2.4 Measurement techniques for the transport parameters

There are many techniques for measuring the transport properties of minority carriers in semiconductors. They all have certain factors in common, such as the fact that, for example, in a silicon substrate excess minority carriers are generated and that the behaviour of the minority carriers is monitored in some way. Essentially, all these techniques differ in the methods used for the excitation and detection of the minority carriers.

A frequently used technique is a combination of the photoconductive (PC) and the photoelectromagnetic (PEM) method [59]. Here an excess of minority carriers in a sample is generated by using light, and the change in conductivity (PC-effect), or a current deflected by a magnetic field (PEM-effect) is monitored by contacting the sample. From steady-state measurements the lifetime can be determined when the mobility is already known. True lifetime measurements can be made with a time-resolved measurement. For example, monitoring the decay of excess minority carriers, due to recombination, after a light pulse can be performed by measuring the decay in photoconductivity [34]. However, since the above methods depend on monitoring the conductivity, these methods are only accurate for high-resistivity and thus low doped silicon. In addition, to detect a change in conductivity large amounts of minority carriers have to be generated and high-injection effects can disturb the measurements.
An alternative way of detecting a change in conductivity is by using the microwave reflection method, as done by Ogita [74] van Rijs et al. [108] and Otaredian [75]. The reflection of microwaves depends on the conductivity of the sample. Thus a decay of excess minority carriers after a light pulse can be followed by monitoring the reflection. Obviously, this method has the advantage that no contacts to the sample have to be made. However, the microwave reflection method suffers from the same disadvantages as mentioned for the PC-method, and thus can only be used for low doped silicon.

Other contactless measurement techniques are those that make use of optical generation and optical detection of carriers. For example, the decay of optical generated carriers can also be monitored by measuring the photoluminescence [28, 25], that is the radiation emitted with band-to-band recombination. Unfortunately, the luminescence efficiency is very low in silicon, which limits the application of this method to room temperatures [93] only. Another technique is the transient optical grating method [36]. Here a grating of excess carriers is generated by causing two coherent light pulses to interfere on the silicon surface. The generated carriers cause a slight change in the refractive index; this change in refraction, and thus the change of the grating intensity, is probed by a third light beam. With this technique it is possible, in principle, to measure both $\tau$ and $\mu$. However, because of the insensitivity of the method it can only be used at high injection levels. Extracting the band structure from absorption measurements [62, 109] or from photoluminescence (PL) spectra [111] also belong to this class of purely optical methods.

Another very useful method of detecting minority carriers is by using pn-junctions or, for low doping concentrations, Schottky contacts as collectors. These methods are often referred as ‘device methods’ since the samples have to be processed. The diffusion lengths can be determined from steady-state measurements when the collected current is measured at several distances between the source of excess carriers and the collecting area. Four ways in which this can be done are shown in Figure 2.2, where the arrows show the flow of minority carriers to the collector. The methods differ in the method of excitation. In the first example of Figure 2.2 a light beam (LBIC) is used to generate the minority carriers. An electron beam (EBIC), the second setup in Figure 2.2, has been used by Ioannou et al. [45] and Chi et al. [15]. Of course, a pn-junction when forward biased can also be used to inject carriers, which has been done by Wieder [115] and del Alamo et al. [24]. In the last setup of Figure 2.2, uniform illumination can be used. Here, the interconnect layer shields the light as was used by Wang et al. [113].

The same setups of Figure 2.2 can be used for the determination of the
Figure 2.2: Four experimental setups to determine the diffusion length from steady-state measurements of the collected current.
lifetime and mobility when a time-resolved measurement is done. Dziewior et al. [29] used a modulated light source to measure the mobility in heavily doped silicon. Ioannou [44] and Burk et al. [12] used electron beams. However, in heavily doped silicon, the diffusion length becomes very small (1-10 µm). Since it is difficult to make a light spot smaller than 1 µm, the spot cannot be considered as a point source, which makes the extraction of the transport parameters from the measurements using LBIC very difficult. The same holds for the electron beam, where the generation profile must be known accurately. The problem of the spot size can be solved by using the interconnect layer as a mask, as demonstrated by Wang et al. [113] for heavily doped silicon. However, the experimental setup is still complicated and measurements at low temperatures have not been presented. In addition, the quantity $p_0$ cannot be measured with optical excitation.

Conclusively, it can be remarked that pn-junctions as collectors offers the best way of detecting minority carriers. Monitoring the conductivity is very inaccurate for heavily doped silicon, while monitoring the photoluminescence is very insensitive and cannot be used at low temperatures. As for the excitation, a light or electron beam cannot be used in heavily doped silicon because of the small diffusion length. The use of optical excitation, for example with a mask, makes the experimental setup very complex. In addition, all above methods rely on the generation of excess carriers near the semiconductor surface and the effect of surface recombination must be considered carefully.
CHAPTER 3

MEASUREMENT METHOD

In the previous chapter, several methods were discussed to measure minority carrier transport parameters. Here, a new method [107, 105, 106] is presented which has certain advantages above existing methods. These advantages are:

- The complete set of transport parameters ($\mu_p$, $\tau_p$ and $p_0$) can be measured in the same material (processed silicon), under identical conditions and with the same type of devices.
- The effect of surface recombination on the extraction of the parameters is negligible small.
- The method enables the direct measurement of the minority carrier mobility which is much more accurate than obtaining the mobility from $L_p$ and $\tau_p$.
- The diffusion length $L_p$, the mobility $\mu_p$ and the lifetime $\tau_p$ are measured independently, which makes it possible to check the consistency in the values of the parameters.
- Since the measurement method is purely electrical, the experimental setup is relatively easy and low temperature measurements can be performed.

It was shown in the previous chapter that minority carrier transport parameters can be divided in DC-transport parameters ($L_p$ and $p_0D_p$) and AC-transport parameters ($\mu_p$ and $\tau_p$). The DC-transport parameters are sufficient to describe the steady-state behaviour of the minority carriers. This also implies that only these two parameters can be extracted from DC-measurements. Therefore, knowledge of one of the AC-transport parameters is necessary in order to decompose the DC-transport parameters into the three transport parameters. The AC-transport parameters can only be obtained from an additional time-dependent or AC-measurement.
In the measurement method presented here, it is also necessary to conduct DC-measurements as well as AC-measurements. Therefore this chapter is divided into two parts: one part covering the determination of the DC-transport parameters, while the other part explains the extraction of the AC-transport parameters. Finally, several error sources will be discussed, which become important in estimating the accuracy of the measurements.

3.1 DC-TRANSPORT PARAMETERS

3.1.1 PRINCIPLE OF THE METHOD

For the measurement of the DC-transport parameters the device shown schematically in Figure 3.1 is used. It consists of a lateral transistor with several collectors in a homogeneous, heavily doped substrate. The minority carriers, in this case holes, are injected into the substrate by the emitter and will flow by diffusion to the successive collectors. However, the larger the distance they have to travel, the more carriers will recombine before they reach a collector. Hence the decay of the currents measured from the collectors having increasing distances from the emitter, give information about the diffusion length \( L_p \). Thus, contrary to normal lateral transistors with one collector, as used by, for example, del Alamo et al. [23], only one device is needed to extract \( L_p \).

The second DC-transport parameter, the product \( p_0 D_p \), can in principle be extracted from the magnitude of the collector current, since the collector current can be described by [23]:

\[
I_C = -qA F_L p_0 D_p \frac{1}{L_p \sinh(W/L_p)} \left( \exp \left( \frac{qV}{kT} \right) - 1 \right)
\]  

(3.1)

where \( A \) is the collecting area, \( W \) is the basewidth and \( F_L \) is a geometrical factor. However, this extraction would normally be very inaccurate because of the not exactly known two-dimensional device geometry such as the junction depth. The geometrical factor \( F_L \) is therefore not precise. This problem is explained in in Figure 3.1 in more detail. The current of the first collector shown in Figure 3.1 is composed of two components. One component, in the region above the dashed line, flows parallel to the surface and is collected by the edge of the collector junction. The other component (below the dashed line) flows from the emitter in the bulk and is collected by the bottom of the collector.

The first component is highly undesirable because it depends strongly on the junction depth, out-diffusion of the junction-edges and on the unknown
Figure 3.1: Schematic view of the multi-collector structure. The arrows represent the flow of minority carriers from the emitter to the successive collectors.

recombination at the surface. The second component is less affected by these effects and only this component can be used to extract $p_0D_p$ accurately.

The advantage of the multi-collector device shown in Figure 3.1, is that only the first collector collects the unwanted lateral component, while the other collectors only collect the bulk component. Therefore, when the current of the first collector is not used, it is also possible to use this structure to extract the parameter $p_0D_p$.

3.1.2 Modelling the device

In the previous section it was qualitatively shown that with the multi-collector structure it is possible to gather information about the DC-transport parameters. However, to extract the DC-transport parameters from the collector currents quantitatively, it is necessary to describe the relation between the collector currents and these DC-transport parameters.

The currents at the collectors depend only on the transport of minority carriers in the base region. It is therefore only necessary to solve the diffusion equation in the quasi-neutral region of the base with the appropriate boundary conditions at the boundaries. For the homogeneous doped base this diffusion
equation, see equation 2.5, reads, for two dimensions:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{p - p_0}{\Gamma_p^2} = 0 \quad (3.2)$$

The base region can be considered as a semi-infinite layer bounded at the top by the depletion layers of the emitter $\Gamma_E$ and the collector $\Gamma_C$ junctions and by the semiconductor surface $\Gamma_S$, as shown in Figure 3.2.

At the boundaries the following boundary conditions apply:

$$\begin{align*}
\Gamma_E &: \ p = p_0 \exp\left(\frac{qV}{kT}\right) \\
\Gamma_C &: \ p = p_0 \\
\Gamma_S &: \ \mathbf{n} \cdot D_p \nabla p = s(p - p_0) \quad (3.3)
\end{align*}$$

where $V$ is the forward bias applied at the emitter and $\mathbf{n}$ is the unit vector normal to the semiconductor surface. The collectors are zero biased, and for the boundary condition at $\Gamma_S$ surface recombination is assumed with a surface recombination velocity $s$. At large distances from the emitter the hole concentration $p$ approaches the minority carrier concentration in equilibrium $p_0$.

Because the base is always heavily doped, the depletion layers are very thin (in the order of 20 nm), so that without much error the metallurgical junctions can be taken as the boundaries instead of the depletion layers. Finally, the current at the i-th collector can be calculated by:

$$I_i^C = -qD_p l \int_{\Gamma_e} \mathbf{n} \cdot \nabla p \, ds \quad (3.4)$$

where $l$ is the length of the collector.

It is evident that with the complicated geometrical structure and mixed boundary conditions, it is impossible to find an exact analytical solution of the equations 3.2-3.4. However, it is not necessary to use an analytical expression since the solution can be computed numerically.

3.1.3 Extraction of the Parameters: An Inverse Problem

To extract the DC-minority carrier transport parameters from the collector currents, it would be most convenient to have a direct formula which relates these parameters as a function of the collector currents.

Solving equations 3.2-3.4, however, results in an inverse expression which describes the collector currents as a function of the two DC-parameters. This is known as an inverse problem.

18
Figure 3.2: The geometrical structure of the two-dimensional diffusion problem. The ray $R$ can be considered to be infinitely large.

There are many examples of inverse problems. Ouwerling [77], for instance, determined, by inverse methods, two-dimensional doping profiles from capacitance versus voltage measurements. This method was first tested on one-dimensional profiles by van Rijs [103], in which case an approximated direct formula is also present. The deduction of the nature of force fields between two particles by scattering experiments [81] or the determination of oil-wells by seismic data [18] can also be treated as inverse problems.

A general way of treating inverse problems is to formulate a cost or object function and minimize this object function by optimization methods. As a cost function the well-known least squares error has been chosen, in which case the parameters can be found by minimizing the error $\xi$:

$$\xi = \frac{1}{N} \sum_{i=1}^{N} \left[ \log(I_m^i) - \log(I_c^i(L_p, p_0 D_p)) \right]^2$$  \hspace{1cm} (3.5)

where $N$ is the number of collectors, $I_m^i$ is the current measured at the $i$-th collector and $I_c$ is the collector current calculated for a given set of $L_p$ and $p_0 D_p$. Because the collector currents have equal relative errors, the logarithm
of the currents is taken.

One important question concerning the optimization is the independence of the two parameters. Although del Alamo showed that at most two parameters \((L_p \text{ and } p_0 D_p)\) can be extracted from the measurements, this condition is not enough to ensure the independence of the parameters. However, it appears that the parameter \(p_0 D_p\) always enters the expression for the collector current as a pre-factor, as can be seen in equation 3.1 for the one-dimensional case, while the diffusion length \(L_p\) enters the expression with a function of the basewidth \(W\). This means, that when the basewidth is varied, both parameters can be extracted simultaneously. Of course, this only holds when the distance from emitter to collector is greater than the diffusion length, otherwise the device is more or less transparent and the diffusion length is no longer important.

3.1.4 Implementation of the method

To calculate the collector currents \(I_{C}^{i}(L_p, p_0 D_p)\) the two-dimensional semiconductor device simulator PISCES-IIB [80] is used. This device simulator solves the complete set of device equations. The Poisson equation is, however, only important for calculating the depletion layers, which are very thin in heavily doped silicon, whereas consideration of the majority carriers is only important when series resistance or high-injection effects occur; a situation which will be carefully avoided in the measurements. Therefore, in essence, the equations which are solved by PISCES-IIB reduce to the diffusion equation mentioned for the minority carriers with the boundary conditions stated in equation 3.3.

The use of a device simulator such as PISCES-IIB facilitates the investigation of the effects of, for example, surface recombination and non-homogeneous doping at the junctions. In addition, it is very easy to change the geometry of the device.

As an optimization algorithm, Meyer and Roth's [67] method of Modified Damped Least Squares (MDLS) was chosen. This algorithm is a modification of the well-known Levenberg-Marquardt method [63]. The modification, which consists of a linear search in the calculated step direction when needed, results in a more robust optimization, as found by van Rijs [103] in the case of extracting doping profiles from CV-data. The optimization algorithm was implemented in the data processing package PROFILE/PROF2D [76, 78], which also controls the necessary communication with the device modelling program PISCES-IIB.
3.2 AC-transport parameters

3.2.1 Principle of the method

For the determination of the AC-transport parameters, which are the minority carrier mobility and lifetime, the device shown in Figure 3.3 is used. Just as for the device shown in the previous section to determine the DC-parameters, this device is also a lateral transistor except that in this case only one collector is sufficient. To obtain information about the minority carrier mobility and lifetime, the DC-collector current is not used, but rather the small-signal frequency response of the transistor. This frequency response is obtained by monitoring the AC-collector current $i_c$ for a small applied AC-voltage on the emitter, $v_{be}$, with the base connected to ground (common base configuration). This response is defined by:

$$Y_{21} = \left. \frac{\partial i_c}{\partial v_{be}} \right|_{v_{CE}=0}$$

(3.6)

which is called the transadmittance.

Measuring the current gain as a function of frequency is of no use because the base or emitter current also contains a component due to the injection of holes from the base into the emitter and hence also depends on the transport in the emitter region. Effectively, measuring the transadmittance means measuring the time delay or transit time of minority carriers through the base. It is well-known that the delay time $\tau_b$ [20] in a transparent base corresponds, for one-dimensional and homogeneous doped cases, to:

$$\tau_b = \frac{W_b^2}{2D_p}$$

(3.7)

where $W_b$ is the basewidth, and thus enables the measurement of $D_p$. Although a seemingly attractive method for obtaining $D_p$, this measurement using normal transistors with small basewidth is complicated because:

- The time delay in the base is not the only time constant. In fact, the total delay time also consists of contributions due to the stored charge in the neutral emitter and the transit time through emitter and collector depletion layers [20].

- The basewidth of a normal transistor is very thin. This means a small delay time, which is of course one of the main objectives of transistor design, but it also means that the measurements have to be performed at high frequencies.

21
• For a small basewidth, the base cannot be considered to be homogeneously doped since the contribution of the doping profile near the junctions is relatively large.

The device proposed in Figure 3.3 does not suffer from these difficulties because the basewidth is very large. This results in a slow transistor with a large base transit time which is dominant. Moreover, when the basewidth is larger than the diffusion length, it is also possible to extract the lifetime from the frequency response, as will be shown in the next sections.

![Schematic view of the lateral transistor used for the determination of the minority carrier mobility and lifetime.](image)

Figure 3.3: Schematic view of the lateral transistor used for the determination of the minority carrier mobility and lifetime.

3.2.2 Modelling the device

To describe quantitatively the time-dependent behaviour of the transistor, the time-dependent diffusion equation has to be used which, in two dimensions, reads:

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{p - p_0}{D_p} = \frac{1}{D_p} \frac{\partial p}{\partial t}
\]  

(3.8)

where \( p = p(x, t) \) and \( x = (x, y) \).

Since we are only interested in the small-signal response, the hole concentration can be split into a DC and a first harmonic AC-component, according to:

\[
p(x, t) = p(x) + \bar{p}(x, \omega) \exp(i\omega t)
\]  

(3.9)
where $\omega$ is the circular frequency. Substituting this in equation 3.8 yields, for the AC-component:

$$
\frac{\partial^2 \tilde{p}}{\partial x^2} + \frac{\partial^2 \tilde{p}}{\partial y^2} - \left( \frac{1}{L_p^*} + \frac{i \omega}{D_p} \right) \tilde{p} = 0
$$  \hspace{1cm} (3.10)

which can be simplified into:

$$
\frac{\partial^2 \tilde{p}}{\partial x^2} + \frac{\partial^2 \tilde{p}}{\partial y^2} - \frac{1}{(L_p^*)^2} \tilde{p} = 0
$$  \hspace{1cm} (3.11)

where $L_p^*$ is the complex diffusion length defined by:

$$
L_p^* = \frac{L_p}{\sqrt{1 + i \omega \tau_p}}
$$  \hspace{1cm} (3.12)

The boundary condition at the emitter-base depletion edge $\Gamma_E$ can be found by linearizing the boundary condition around the DC-bias voltage $V$. The collector is short-circuited while the semiconductor/oxide interface can be characterized by a surface recombination velocity $s$. For the boundary conditions this yields:

$$
\begin{align*}
\Gamma_E &: \quad \tilde{p} = p_0 \exp \left( \frac{qV}{kT} \right) \frac{q}{kT} v_{be} \\
\Gamma_C &: \quad \tilde{p} = 0 \\
\Gamma_S &: \quad \nu \cdot D_p \nabla \tilde{p} = s(p - p_0)
\end{align*}
$$  \hspace{1cm} (3.13)

Equations 3.11 and 3.13 describe fully the AC-minority carrier transport in the base. Analogous to the equations concerning the modelling of the DC-behaviour described in section 3.1.2, these equations cannot be solved analytically either. Therefore, to solve the equations numerically the two-dimensional device simulator CURRY [2] in the AC-analysis mode is used. Although CURRY solves the complete set of semiconductor device equations, it must be remembered that these equations effectively reduce to equations 3.11 and 3.13 for the device structure considered.

The absence of an analytical expression complicates the understanding how mobility and lifetime exactly relate with the frequency response. Nevertheless, some general remarks can still be made. Figure 3.4 shows the magnitude and phase of $Y_{21}$ which is typical for a lateral transistor with a basewidth larger than the diffusion length. To calculate the response the following values were used: $\mu_p = 136.5 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$, $\tau_p = 11.5 \text{ ns}$ and $L_p = 2 \mu\text{m}$. The basewidth was $3 \mu\text{m}$. This frequency response can be divided into three regions:
Figure 3.4: Typical frequency behaviour of the lateral transistor for a basewidth larger than the diffusion length.

- $\omega \ll \frac{1}{\tau_p}$: When $\omega \tau_p$ is much smaller than one, the complex diffusion length $L_p^*$ in equation 3.12 approaches the normal diffusion length $L_p$. In this region not much frequency dependence is expected which results, as can be seen in Figure 3.4, in an almost constant magnitude and zero phase shift of $Y_{21}$.

- $\omega \sim \frac{1}{\tau_p}$: When $\omega \tau_p$ approaches one, the complex diffusion length starts to deviate from $L_p$ and becomes effectively smaller. In this transition region, the magnitude of $Y_{21}$ starts to decay and a phase shift occurs.

- $\omega \gg \frac{1}{\tau_p}$: When $\omega \tau_p$ is much greater than one, the complex diffusion length approaches:

$$\lim_{\omega \tau_p \to \infty} L_p^* = \sqrt{\frac{D_p}{i\omega}} \quad (3.14)$$
in which case, the lifetime no longer enters the equations and the response only depends on $D_p$. For the one-dimensional case an analytical expression for the AC-diffusion equation can be obtained which, for the phase shift $\phi$ with $\phi = \arg(Y_{21})$ in this frequency range yields:

$$\phi = -W_b \sqrt{\frac{\omega}{2D_p}}$$  \hspace{1cm} (3.15)

which gives a straight line as a function of the square root of the frequency. The same holds approximately for the two-dimensional case, as can be seen in Figure 3.4.

3.2.3 Extraction of the Parameters

As in the case of the DC-parameters, the extraction of the AC-parameters from the frequency response can also be stated as an inverse problem. In principle, both the magnitude and the phase shift of $Y_{21}$ can be used for the extraction of the mobility and lifetime. However, since it is more convenient to use the phase shift, the parameters can be found by minimizing the object function:

$$\xi = \frac{1}{N} \sum_{i=1}^{N} [\phi_m(\omega_i) - \phi_c(\omega_i; \mu_p, \tau_p)]^2$$  \hspace{1cm} (3.16)

where $\phi_m$ is the measured phase shift at the frequencies $\omega_i$ and $\phi_c$ is the phase shift calculated using CURRY.

The independence and sensitivity of the parameters are important for their extraction. Obviously, when a parameter is not very sensitive, which means a large variation of the parameter does not change the least mean squares sum very much, that parameter cannot be determined very accurately. The independence and sensitivity of the parameters can be checked by investigating the error space near the minimum. For independent parameters only one definite minimum exists in the parameter space. However, when two parameters are fully dependent there is an infinite number of minima and the minimum is in fact no longer a point, but becomes a line. This means that one parameter is redundant and has to be removed. For less hard dependencies intermediate stages can be seen. Information about the minimum can be obtained from the Hessian matrix which describes the second derivatives of the error $\xi$ with respect to the parameters.

From the discussion in section 3.2.2, it is clear that the mobility and lifetime have distinct effects on the frequency response. That is, the lifetime defines the
corner frequency and the mobility determines the slope of the phase response for higher frequencies, which enables an independent extraction of the parameters when the frequency range considered is large enough. The question is, however, when can the frequency range be considered to be large enough.

To investigate the influence of the frequency range on the accuracy of the parameters, synthetic data are used. This means that measurement data, in this case the phase shift in $Y_{21}$, are generated by simulating the frequency response of a device assuming some values for mobility and lifetime. As synthetic data the response shown in Figure 3.4 is used, which was calculated for a lateral transistor with a basewidth of 3$\mu$m. To these data an error was added with a Gaussian distribution, with a standard deviation $\sigma$ of 0.2 degrees. The extraction scheme is then applied to these data and the mobility and lifetime are extracted as if they were not known. This is done for several frequency ranges. The results are shown in the following figures. Figure 3.5 shows the

![Diagram](image)

Figure 3.5: The 95.4 percent confidence region for a frequency range of 20 MHz.
minimum and a contour line of equal least squares error when the frequency response of Figure 3.4 is used up to 20 MHz. In this example the lifetime was 11.5 ns which gives a corner-frequency of 15 MHz. The contour value was chosen so that the ellipse agrees with the 95.4% confidence region. This means that it is 95.4% certain that the real values of the parameters lies within this region. The accuracy of the parameters can be found by the projection of the ellipse on the principal axes, which is also shown in Figure 3.5. It can be seen that both parameters can be obtained with a reasonable accuracy (respectively 3.3% and 5% for the mobility and lifetime).

When the frequency range is reduced to 15 MHz, the confidence region becomes more stretched, as shown in Figure 3.6. The uncertainty in the parameters, 7% for the mobility and 10% for the lifetime, becomes larger.

When the frequency range is reduced to only 10 MHz, thus lower than the corner-frequency, the situation changes dramatically. This is shown in
Figure 3.7: The 95.4 percent confidence region for a frequency range of 10 MHz.

Figure 3.7. The ellipse has spread out in one direction and the accuracy in the parameters becomes very low (mobility 20% and lifetime 27%). In fact the parameters become dependent and both parameters can no longer be extracted simultaneously.

3.3 Error analysis

The description of a measurement method is not complete without a discussion of the accuracy of the parameters obtained. Several sources of uncertainty can be distinguished, and three main types of error sources will be discussed here, namely:

1. accuracy of the measurement
2. model flaws
3. accuracy of the numerical solution of the model

The first source of errors is the inaccuracy of the measurement data, which can be composed of two kinds of errors:

1. random errors: Errors caused, for example, by noise.
2. systematic errors: Errors introduced, for example, by an inaccuracy in the temperature or a leakage current.

The second source of errors is uncertainties introduced by flaws in the model. In our case, this occurs, for example, when the structure which is simulated does not correspond to the real physical structure of the device. Such model flaws are caused by:

1. unknown surface recombination: Surface recombination at the semiconductor/oxide interface is very process dependent. Recent measurements performed by King [49, 50] shows that the surface recombination velocity can vary between 100 cm s\(^{-1}\) for a phosphorus concentration of \(10^{17}\) cm\(^{-3}\) and \(10^{4}\) cm s\(^{-1}\) for a concentration of \(10^{20}\) cm\(^{-3}\).

2. uncertainty in the device geometry: The device geometry is not well-known because the junction depth and out-diffusion of the P\(^{++}\) implant forming the emitter and the collectors depend very much on the processing conditions.

3. model simplifications: In the device model it is assumed that the parameters are constant throughout the whole base region. However, this is not true near the junctions where the doping level is non-homogeneous.

The last type of error, which is often neglected because analytical expressions are normally used, is the error caused by the numerical solution of the model equations. This error can be divided into the following contributions:

1. discretisation: To calculate the collector currents the diffusion equation has to be discretised on a two-dimensional grid. Obviously, a too coarse grid will not represent the continuous nature of carrier diffusion and some error will be introduced.

2. simulation area: The physical device can be seen as a semi-infinite layer. However, it is only possible to simulate a finite region. This limited simulation area can cause some errors.
Of course the contribution of these two errors can be decreased by simulating a large area with a dense grid. The question is, however, how large the area must be and how dense the grid must be in order to reach an acceptable error with a practical number of nodes. In the following sections, more attention will be paid to these aspects.

3.3.1 MEASUREMENT ERRORS

Each measurement is subjected to random errors. Clearly, these statistical errors will influence the accuracy of the parameters obtained from minimizing the object function:

$$
    \xi = \frac{1}{N} \sum_{i=1}^{N} \left[ y_m(x_i) - y_c(x_i; \vec{p}) \right]^2
$$

(3.17)

which is the least squares error as used in equations 3.5 and 3.16 where $y_m$ is the measured data and $y_c$ is the model which describes the data with the parameter set $\vec{p}$.

It is known [6, 26] that after optimization the variances of the parameters can be obtained by:

$$
    var(p_k) = 2\sigma^2 H_{kk}^{-1}
$$

(3.18)

where $\sigma$ is the standard deviation of the measurement data and and $H_{kk}^{-1}$ are the diagonal elements of the inverse of the Hessian matrix. This Hessian matrix contains information about the curvature of the least squares error sum near the minimum. Calculating the variances of the parameters with this equation only holds, however, when the error distribution of the measurements is Gaussian-shaped (also known as a normal error-distribution), and when the model describes the real world perfectly [82].

In practice, both constraints are difficult to fulfil. Generally, the error distribution of the measurements is not known or non-normal. For example, in our case the measurement of the collector currents at low currents is subject to discretisation error because of the limited bit-representation of the current. This kind of error has a non-normal distribution. The model also only describes a part of the real world and thus does not represent the measurement results exactly. Nevertheless, in order to say something about the accuracy of the parameters, the variance of the measurement data is often estimated by the least squares error in the minimum according to:

$$
    \sigma^2 \approx \xi_{\text{min}}
$$

(3.19)
in which case the variances of the parameters can be estimated by [1]:

\[ \text{var}(p_k) = 2\xi_{\text{min}}H_{kk}^{-1} \] (3.20)

Knowledge about the variances of the parameters is not very useful without knowledge about the error-distribution. When the measurement data have a normal error, the error distribution of the parameters can be described by a Student distribution. In this case one can calculate confidence intervals, intervals for which one is \(100 \cdot \phi\) percent confident that the true value of the parameter exists within this interval, as follows:

\[ p_k = p_k^0 \pm \sqrt{\text{var}(p_k)} t(1 - \phi, N - M) \] (3.21)

where \(t\) is the Student distribution, \(N\) is the number of data points and \(M\) is the number of parameters.

In principle, the error caused by a random measurement error can easily be detected and can be reduced by performing a lot of measurements, thus effectively averaging out the random nature of the measurement. Systematic measurements errors cannot be detected and reduced so easily. In fact, systematic errors and flaws in the model have the same effect; they both cause the minimum of the objective function to be higher than one would expect from the standard deviation \(\sigma\) of the measurement data.

### 3.3.2 Model Errors

To investigate the effects of the model flaws on the accuracy of the parameters, the whole measurement method can be simulated using synthetic data. This means that real measured data are not used, but that the data (the collector currents) are generated by simulating the device. This is very convenient, because now the geometry of the 'measuring' device is known exactly, as are the values of the extracted parameters.

In order to make the results obtained here useful, the same device structure as used for the measurement of the DC-transport parameters, is used to calculate the synthetic data. This structure is shown in Figure 4.2 and has seven collectors.

To show the effect of surface recombination on the DC-transport parameters, the following numerical experiments were performed. First, sets of synthetic data were generated with \(L_p = 10\mu m\) and \(p_0 D_p = 1200 \text{ cm}^{-1}\text{s}^{-1}\), which are common values for heavily doped silicon, and for surface recombination velocities varying from zero to \(10^5 \text{ cm} \text{s}^{-1}\). Then the parameters were extracted from the synthetic data with the optimization scheme, using the
Figure 3.8: Deviation of the diffusion length as a function of surface recombination velocity.

same device geometry but with zero surface recombination velocity only, thus introducing an error into the model of the device. These experiments were also repeated for a hole diffusion length of 5 and 15 μm.

The results of these experiments are shown in Figures 3.8 and 3.9. Figure 3.8 shows the deviation of the extracted diffusion length and Figure 3.9 the deviation of $p_0 D_p$ for several values of surface recombination velocity. The markers represent the calculated values. The lines are the best fit curves, assuming a linear relation between the deviation and the surface recombination. For the solid lines information obtained from all seven collectors except the first collector were used. For the dashed line the first collector was also used. The results clearly show that, ignoring the first collector, the error introduced by the uncertainty in the surface recombination velocity remains very small and only becomes significant for velocities above $10^8$ cm s$^{-1}$. As was expected,
the extracted parameters are more sensitive to the surface recombination when the first collector is also used. In fact, the current of the first collector assuming no surface recombination deviates more than 30% from the current calculated with a surface recombination velocity of $10^4$ cm s$^{-1}$. The deviation in the parameters remains, however, limited because of the compensating effect of the other six collectors.

The influence of the uncertainty in the junction depth on the parameters was also investigated by generating synthetic data using a device geometry with a junction depth of, successively, 0.2, 0.4 and 0.6 µm. To extract the parameters a device model was used with a junction depth of only 0.4 µm. The results are shown in Figures 3.10 and 3.11. When the first collector is not used (solid lines) the uncertainty introduced into the parameters is negligible (less than 1%). However, when the first collector is included in the data, the deviations become unacceptably large, which again shows the importance of
not incorporating the data from the first collector.

Similar results were obtained, but are not shown, when the out-diffusion of the junctions was varied in order to investigate the influence of its' uncertainty on the parameters.

3.3.3 NUMERICAL ERRORS

To determine the relation between the parameters and the measurements, the model, consisting of differential equations, has to be solved numerically. It is important that the numerical procedure does not contribute additional errors to the parameters. In practice, this means that two questions have to be answered; namely, how fine the grid density must be and how large the simulation area must be. These questions are particularly important for the multi-collector structure because a large structure has to be simulated here.

To investigate the required density of the discretisation grid in the sub-
Figure 3.11: Deviation of $p_0D_p$ as a function of junction depth.

A device is simulated with one very small emitter, which can be considered as a point source, and with one large collector. In this case the geometry of the device is very simple and an analytical solution can be derived. Simulating the device with different grid densities, with different diffusion lengths and comparing these results with the analytical solutions, yields a simple rule. For an accurate solution (less than one percent error) of the diffusion equation in the bulk of the substrate, a grid density of 5 nodes per diffusion length suffices. Of course, to represent the shape of the junction edges accurately a much denser grid must be applied here.

A limited simulation area can also cause a deviation in the calculated currents. Figure 3.12 shows an example of a simulation area with a restricted substrate thickness. The result of the boundary condition applied at the bottom can be pictured by mirroring the simulation structure. This results in a parasitic device, shown in Figure 3.12 with dashed lines. This means that
Figure 3.12: The effect of a limited substrate thickness can be described by a parasitic device (dashed lines).

there are now two sources, the main emitter and a parasitic emitter, which contribute to the collector currents. Obviously, the thicker the simulated substrate, the larger the distance between the parasitic emitter and the collectors, and thus the smaller the influence of the boundary condition. It can also be reasoned that the last collector will suffer the most from the parasitic emitter; this is because the first collectors are closer to the main emitter than to the parasitic emitter compared with the last collector. Simulations show that to reduce the deviation of the last collector to one percent, the thickness of the substrate must be at least 3 à 4 diffusion lengths thick.

So far, the effects of the error sources have been discussed with respect to the measurement of the DC-transport parameters. Similar results were also obtained concerning the measurement of the AC-transport parameters, but are not given here. However, for the AC-measurements, attention has to be paid to two additional aspects. One aspect is the frequency range, as already demonstrated in section 3.2.3, and the other aspect concerns the simulation of the frequency response of the lateral transistor. The objective is to simulate the intrinsic transistor behaviour, the transistor without series resistances and other parasitic elements. Simulating the whole structure with CURRY means however that the extrinsic transistor is simulated. Most notably, the effect
of the base resistance can be seen in the simulated frequency response. To eliminate this effect the base contact is moved very close to the emitter, thus effectively reducing the base resistance.
CHAPTER 4

EXPERIMENTAL RESULTS

To measure the minority carrier transport parameters as a function of doping with the method discussed in the previous section, homogeneous silicon is needed with different high doping concentrations. The use of heavily doped epilayers on a substrate as a starting material has the advantage that a large variation in doping concentrations can be easily achieved. However, the measurement method used here essentially relies on the semi-infinite nature of the layer that is to be characterized. The heavily doped silicon layer must therefore be at least 4 diffusion lengths thick in order to reduce the influence of the underlying substrate. This means that for diffusion lengths in the order of 10 μm, which is common for doping concentrations around $10^{18}$ cm$^{-3}$, the epitaxial layer must be at least 40 μm thick! Of course, in principle a thinner layer can be used, say 2 diffusion lengths thick, but to correct the data for the influence of the substrate the layer thickness must be accurately known, so that a new variable is introduced which must be considered.

To circumvent this, heavily doped substrates are used as a starting material with, however, the disadvantage that one is dependent on doping concentrations that are commercially available.

The method used here is a device method; hence the substrates have to be processed. In this chapter the design and fabrication of the devices as well as the measurements performed on these devices will be discussed.

4.1 FABRICATION OF THE DEVICES

To measure the transport properties for holes as well as for electrons, both N-type (antimony doped) and P-type (boron doped) wafers are used. These substrates, 4 inches in diameter, were mainly obtained from the suppliers Wacker and Aurel. More details about the specifications of the substrates are listed in
Table 4.1.
The processing of the devices in the substrates was done at Philips Research Laboratories Eindhoven with a stripped version of the BICMOS 2.0 process, which is a 1.5μm process. The basic process steps are sketched in Table 4.2. To make the devices six masks are sufficient, namely:

<table>
<thead>
<tr>
<th>N type</th>
<th>P type</th>
</tr>
</thead>
<tbody>
<tr>
<td>supplier : Wacker</td>
<td>supplier : Aurel</td>
</tr>
<tr>
<td>type : N-type (Sb)</td>
<td>type : P+CZ (B)</td>
</tr>
<tr>
<td>diameter : 100 mm</td>
<td>diameter : 100 mm</td>
</tr>
<tr>
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<tr>
<td>resistivity : 50-90 mΩcm</td>
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<tr>
<td>orientation : &lt; 100&gt;</td>
<td></td>
</tr>
<tr>
<td>thickness : 525 μm</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Specifications of the substrates.

- **SN-mask**: For making the base-contacts (N⁺⁺ in an N-type substrate) an arsenic implantation has been used together with the SN-mask.
- **SPP-mask**: The SPP-mask was used to mask the borium (BF₂) implantation to fabricate the P⁺⁺ regions which are used as emitters and collectors.
- **IN-mask**: This mask defines the first aluminium interconnect layer.
- **CO-mask**: The contact holes between IN and silicon are defined with this mask.
- **INS-mask**: The INS-mask is used for the second interconnect layer.
- **COS-mask**: The COS-mask is used for the contact holes between IN and INS.

For the P-type substrates, the two implantations for the SN and SPP-masks are reversed.
Table 4.2: Main steps of the process flow used to fabricate the devices.

To determine the doping concentration of the substrates, a four-probe resistor test structure was designed to measure the resistivity of the wafer. In this test structure the four contacts to the substrate are arranged in a square with a spacing of 130μm between them. Further details of the test structures for resistivity measurements can be found in Buehler et al. [11]. From the measured resistance, the resistivity of the wafer can be calculated using a correction factor [11]. To check the measurement, some resistivity data obtained from these test structures were compared with resistivity measurements done by a mechanical collinear four-point probe PROMETRIX Versaprobe VP10. The results agreed within a few percentages.

To get an idea of the homogeneity of the substrates, the resistivity was measured with the resistor test structure; the measurements, as with other structures, being repeated 21× throughout the wafer. Figure 4.1 shows the results of one particular antimony-doped wafer. The dashed boxes shown in the middle of Figure 4.1 denote the PCM-structures. For this wafer, the minimum
Figure 4.1: *Deviation in resistivity from the mean resistivity (14.07 mΩcm) along a wafer.*

<table>
<thead>
<tr>
<th></th>
<th>+1.1%</th>
<th>+0.1%</th>
<th>-1.9%</th>
<th>-0.6%</th>
<th>+1.4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.1%</td>
<td></td>
<td>-3.8%</td>
<td>-4.5%</td>
<td>-1.4%</td>
<td>+2.1%</td>
</tr>
<tr>
<td>0.00%</td>
<td>-2.5%</td>
<td></td>
<td>-2.7%</td>
<td>+1.5%</td>
<td></td>
</tr>
<tr>
<td>+2.8%</td>
<td>+0.3%</td>
<td>-0.6%</td>
<td>+0.2%</td>
<td>+2.1%</td>
<td></td>
</tr>
<tr>
<td>+3.1%</td>
<td></td>
<td></td>
<td>+4.7%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and maximum measured resistivities are, respectively, 13.44 and 14.73 mΩcm. The values shown in Figure 4.1 represent the deviation in percent from the mean value (14.07 mΩcm). The inhomogeneity in the resistivity is about 5% with a large deviation in the center of the substrate, where the resistivity is the lowest. Wafers with a larger inhomogeneity were not used; moreover from the substrates used, only devices outside the center were selected. In general, the boron-doped wafers showed a better uniformity of about 3%.

The impurity concentration of the antimony-doped (N-type) wafers were found from the resistivity data using the conversion formula of Thurber et al. [97, 99] which, for the substrate considered in Figure 4.1, yields an average doping level of $2.55 \times 10^{18}$ cm$^{-3}$. Although this formula was obtained for phosphorus-doped silicon, the resistivity data of Irvin [46] composed of several impurity types, show no differences in resistivities for different impurities for the doping concentrations considered here. For the determination of the doping level of the boron-doped (P-type) wafers, the data of Thurber et al. [98, 99] was used. Since the resistivity is not always measured close to the devices which are used for the measurement of the minority carrier transport parameters, the uncertainty in the doping concentration is caused by the inhomogeneity of the wafer. Therefore the doping concentration is known to an accuracy of about
5% and 3% for N-type and P-type wafers respectively.

4.2 Measurement of the DC-transport parameters

4.2.1 Design of the test structures

For the doping range considered, the diffusion length can vary between 1 and 20 μm. To cope with this large range, two essentially different structures were designed. Figure 4.2 shows a device especially optimized for diffusion lengths larger than 5 μm. The layout of the device is shown in Figure 4.3. Here, for clarity, only the SPP-mask is shown, which defines the emitter and collector regions. The contact holes, base contact and the interconnect are not shown. Special design considerations are:

- Seven collectors are used. The first five collectors have a mask width of 3μm, the minimum allowed dimension, while the last two collectors are larger (7.2μm) so that they are able to collect a larger current.

- The emitter is 20μm wide, so that for most values of the diffusion length the emitter can be considered as a half-infinite source.

- The basewidth, defined by the shortest distance between the emitter and the first collector, is 3μm. A smaller distance, which has the advantage that the influence of surface recombination becomes smaller, would lead to an undesirably large current at the first collector.

- The gap between the collectors should be as small as possible. For an estimated lateral diffusion of 0.3μm this gap is about 0.6μm.

- A second emitter is placed at the other side so that the results can be confirmed by repeating the measurement in opposite direction.

This device has been optimized by extensive simulations for optimal measurement of the transport parameters. The influence of the device geometry for this device on the accuracy of the parameters was shown in Section 3.3.

To avoid three dimensional effects, and to collect large currents, the collectors were made sufficiently long (200μm). Another measure to avoid three-dimensional effects consists of using a longer emitter with two dummy collectors, see Figure 4.3, in which case at the ends of the collectors the current also flows straight to these collectors. The effect of these measures is that the (effective) lengths of the collectors are well defined. This is important for the determination of the second DC-transport parameter, $p_0D_p$, which is proportional to the collector currents. The length of the emitters and collectors are also made twice as long (400μm) to check the effectiveness of these measures.
Figure 4.2: *Cross-section of the device used especially for the measurement of the DC-transport parameters for diffusion lengths greater than 5μm.*

Figure 4.3: *Layout of the device. Only the emitter and collector regions are shown.*
The second design, used for smaller diffusion lengths, has a smaller collector width (1.6\mu m), which was made possible by specifying the contact holes only at the ends of the collectors.

4.2.2 MEASUREMENTS AT ROOM TEMPERATURE

To perform the measurements on the DC-structures, the devices were mounted in a DIL-24 package, but on-chip measurements were also performed. For the measurements an HP4145B parameter analyzer was used. Figure 4.4 shows a typical Gummel plot of the device shown in Figure 4.2 for a substrate with a doping level of 2.55 \cdot 10^{18} \text{ cm}^{-3} (N\text{-type}). The collector-base voltages were kept at zero. As can be seen from Figure 4.4, the currents, especially the collector

Figure 4.4: Typical Gummel plot measured on the multi-collector device with collector length of 200\mu m.
currents, consist for a large part of an ideal region with a slope of $\frac{q}{kT}$. The collector currents deviate at high voltages only, due to series resistance effects. High injection effects are not likely to occur because of the high doping concentrations. The first collector current shows a proportionately large current in comparison with the other collector currents. This is due to the fact that the first collector collects both a lateral and a bulk component, while the others only collect the bulk component, as explained in Section 3.1.1. The fifth and the sixth collector currents almost coincide because of the larger width of the sixth collector as can be seen in Figure 4.2. The seventh collector current is not shown here since this collector exhibited a large current, which was probably due to a short-circuit between the substrate and the collector contact.

For the extraction of the two DC-transport parameters, although knowledge of the collector currents for one bias point is in principle sufficient, it is notably advantageous to make use of all the information in the Gummel plot for two reasons. Firstly, using more measurement points effectively reduces the random error of the measurement and, secondly, the exact temperature can be extracted from the Gummel plot. Since $p_0$, and therefore $p_0D_p$, are very temperature dependent, it is important to know at what temperature the measurement has been performed.

Therefore, the measurements were described with an analytical model which consists of an ideal diode with a lumped series resistance. The ideal diode current of the emitter $I_E$ and collectors $I_C$ can be described by:

$$I_{E,C} = I_s \left( \exp \left( \frac{qV'_{EB}}{kT} \right) - 1 \right)$$  \hspace{1cm} (4.1)

where $I_s$ is the saturation current, $T$ is the temperature and $V'_{EB}$ is the internal voltage drop over the emitter-base junction. The total voltage drop is given by:

$$V_{EB} = V'_{EB} + I_E R_s$$  \hspace{1cm} (4.2)

where $R_s$ is the series resistance. The ohmic resistance in the emitter and in the base both contribute equally to the series resistance because the base current is almost equal to the emitter current. The results of the model fitted on the collectors are shown in Table 4.3. Here a series resistance of 2.39 $\Omega$, obtained from fitting the model on the emitter current, was used. The accuracy of the calculated saturation currents is about 1% while the uncertainty in the temperature is about $\pm 0.15$K. This uncertainty in the temperature is also reflected in the different temperatures obtained from the collectors, as shown in Table 4.3. Ideally, the temperatures should have been the same since the device heating effects, for which checks were made, play no role.
<table>
<thead>
<tr>
<th>$I_s$ ($10^{-20}$ A)</th>
<th>$I_C^1$</th>
<th>$I_C^2$</th>
<th>$I_C^3$</th>
<th>$I_C^4$</th>
<th>$I_C^5$</th>
<th>$I_C^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>298.76</td>
<td>165.4</td>
<td>23.49</td>
<td>9.638</td>
<td>4.772</td>
<td>2.541</td>
<td>2.309</td>
</tr>
</tbody>
</table>

Table 4.3: Results of fits of equations 4.1 and 4.2 on data shown in Figure 4.4.

To obtain the final data that will be used for the extraction of the DC-transport parameters, the average of the temperatures of the six collector currents was taken; after which equation 4.1 was fitted again, but with the average temperature now. With the obtained consistent saturation currents, and with the average temperature, the collector currents were calculated for a certain bias (usually 0.7 V) with an estimated accuracy of 0.1%. In the calculation of the accuracy of $I_s$ and $T$, the accuracy of the measured currents is important. Figure 4.5 shows the deviations in the measured and fitted current of the second collector as a function of emitter-base voltage. The solid

![Graph showing the difference in measured and fitted current](image)

Figure 4.5: Difference in measured and fitted current of the first collector after optimization.
line is only used to connect the points. The standard deviation $\sigma$ of this set of data is about 0.0015. Translating this to a relative error for $I$ yields a value of 0.35%, which lies within the accuracy specifications of the parameter analyzer. The pattern, which can be recognized in Figure 4.5, resembles the change to different measurement ranges of the parameter analyzer.

4.2.3 Extraction

For the extraction of the parameters, the device is simulated and compared with the measurement in an optimization loop, as explained in Chapter 3. In order to do this, several assumptions have to be made. Firstly, the device geometry must be known. SUPREM-III [42] simulations indicate a junction depth of about 0.4 $\mu$m and this junction depth does not vary much for different substrate concentrations. In section 3.3.2 it was shown that the extracted parameters are still accurate if the junction depth is not 0.4 $\mu$m but appears to be 0.6 $\mu$m or 0.2 $\mu$m. Secondly, the lateral diffusion of the emitter and collector regions is taken to be about 0.3 $\mu$m and, lastly, no surface recombination at the silicon-oxide interface is assumed. The latter assumption is valid since it was shown in section 3.3.2 that even a surface recombination velocity of $10^5$ cm s$^{-1}$ in the device only leads to a 1% error in the diffusion length.

The optimization can be started when some initial values of the DC-parameters are estimated. To illustrate the optimization process, Figure 4.6a shows the steps taken in parameter space starting from: $L_p = 21.4\mu$m and $p_0 D_p = 487$ cm$^{-1}$s$^{-1}$. The final parameters, after 5 iterations, become: $L_p = 13.6\mu$m and $p_0 D_p = 782$ cm$^{-1}$s$^{-1}$ for a substrate doping of $2.55 \cdot 10^{18}$ cm$^{-3}$.

Figure 4.6b shows the subsequent least squares error $\xi$, defined in equation 3.5, for each iteration. For most optimizations done only 4 to 5 iterations suffice to reach the acquired solution. Each iteration uses about two function evaluations whereas each function evaluation, which consists of a full numerical simulation of the device, requires about 2 min. CPU time on a Convex C240 machine.

Judging from the estimated standard deviation $\sigma$ of the collector currents, the minimum least squares error could be as low as $5 \cdot 10^{-7}$. Since the least squares error $\xi_{\text{min}}$ reaches $3 \cdot 10^{-6}$, this suggests that the accuracy in the parameters is not limited by the statistical error in the measurement but by a systematic measurement error or, more likely, by an imperfection in the model describing the collector currents. Nevertheless, the uncertainty in the parameters using equation 3.21 for a 95.4% confidence interval, is quite low. This is shown in Table 4.4. The results are also shown for a substrate with a
Figure 4.6: (a) Steps taken in parameters space to illustrate the optimization process for a typical example. (b) The least squares error $\xi$ as a function of iterations.

<table>
<thead>
<tr>
<th>$N$ (cm$^{-3}$)</th>
<th>$T$ (K)</th>
<th>$L_p$ (µm)</th>
<th>$p_0D_p$ (cm$^{-1}$s$^{-1}$)</th>
<th>$\xi_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.90$\cdot$10$^{18}$</td>
<td>299.79</td>
<td>18.7 $\pm$ 1.5 (8.0%)</td>
<td>1223 $\pm$ 81 (6.6%)</td>
<td>4.10$^{-5}$</td>
</tr>
<tr>
<td>2.55$\cdot$10$^{18}$</td>
<td>298.72</td>
<td>13.6 $\pm$ 0.2 (1.4%)</td>
<td>781.9 $\pm$ 14 (1.8%)</td>
<td>3.10$^{-5}$</td>
</tr>
</tbody>
</table>

Table 4.4: Optimization results for two substrate doping concentrations.

doping level of 1.90 $\cdot$ 10$^{18}$ cm$^{-3}$. Here the accuracy is much less because the fit was worse ($\xi_{\text{min}} = 4 \cdot 10^{-5}$).

In Figure 4.7 the currents after the optimization are shown for the two substrate concentrations; the markers represent the measured currents for the successively numbered collectors. For the sake of clarity, the simulated currents for the optimized parameters are denoted by the broken lines, which should not be construed as suggesting any relation between the collector numbers. As can be seen, the simulated current of the first collector, which was not used in the optimization, is somewhat larger than the measured current. This indicates that either the junction depth or the lateral diffusion have been over estimated in the model, or that surface recombination plays a role.
4.3 Measurement of the AC-Transport Parameters

4.3.1 Design of the Test Structures

In the previous chapter it was shown that the parameters $\mu_p$ and $\tau_p$ can be extracted simultaneously from AC-measurements on lateral transistors, provided that the basewidth is not much smaller than the diffusion length. Apart from other advantages, a large basewidth has the additional advantage that the transit time of carriers through the base becomes relatively large. This means that to extract the parameters the maximum frequency does not need to be very high. In practice, the necessary frequency range can change from 2 MHz to 200 MHz, depending on the lifetime $\tau_p$.

A complicating factor, however, is that the collector efficiency is very low (typically $I_C/I_E = 10^{-2}$). This means that the collector current, which must be measured as a function of frequency, is rather small. Hence crosstalk, regardless of the underlying mechanism causing it, can become important for relatively low frequencies if it is not properly suppressed. For an accurate extraction of the parameters, the measured phase shift must also be accurate.
to at least 0.5 degrees. Therefore, any combination of parasitic elements, even if their corner frequency lies a few times above the measured frequency range, could cause a serious deviation in the phase shift.

It is clear from this discussion that, although the frequencies remain comfortably low, the measurement still requires precautions that are usually only important for high frequency measurements. Therefore, special lateral transistors were designed for the measurement of the AC-transport parameters instead of using the multi-collector structures used for DC-measurements. Figure 4.8 shows such an AC-structure. Special design considerations, concluded from basic reasoning and device simulations, are:

- For an optimal measurement the basewidth must be approximately one diffusion length long. A short basewidth would require high frequency measurements and the extraction of the lifetime in the base, being almost transparent, would become difficult. A long basewidth would cause an unacceptable low collector efficiency. To cope with a large variation in diffusion lengths, structures were designed with various basewidths.

- At most two collectors are used, where the first collector collects the lateral current component. Structures with a basewidth larger than 10μm, used for measurements on substrates with a large diffusion length, only have one collector.

- The collector width is as small as possible (3μm) to ensure a small depletion capacitance.

- The emitter is as small as possible in order to increase the current gain.

- The base contact is positioned close to the emitter to reduce the base resistance.

A layout is shown in Figure 4.9. This layout is used for devices which are designed to measure small lifetimes (< 100ns) and, hence, requires high frequency measurements. The bondpads (80×80μm²) are as small as possible to reduce their capacitance. The interconnect was kept as short as possible to reduce the series inductance of the connections. The length of the transistor is about 100μm. For devices with a large basewidth (> 10μm), which are used for long lifetimes, the demands governing the layout are less stringent and several lateral transistors having different basewidths were combined.

4.3.2 Measurements at room temperature

For the extraction of the AC-parameters, the Y_{21} parameter is needed, which was defined in equation 3.6. To measure this parameter directly, the frequency
Figure 4.8: Cross-section of the device used for the measurement of the AC-transport parameters.

Figure 4.9: Layout of the device optimized for high frequency measurements especially used for measurements on substrates with small diffusion length. The dark box in the middle represents the actual transistor.
response of the transistor in the common-base configuration can be measured with the collector short-circuited as indicated in Figure 4.10. However, short-circuited conditions are difficult to realize for high frequencies. Therefore, for the high frequency characterization of transistors, the scattering or S-parameters are commonly measured, in which case the transistor is terminated with the characteristic impedance of the measurement system (50 \( \Omega \)). These S-parameters can afterwards be transformed into the desired Y-parameters. For instance, \( Y_{21} \) can be obtained by:

\[
Y_{21} = \frac{1}{Z_o (S_{11} + 1)(S_{22} + 1) - S_{12}S_{12}} \tag{4.3}
\]

where \( Z_o \) is the characteristic impedance of the system.

For the measurement of the S-parameters, the HP4145 Network Analyzer (10 Hz-500 MHz) was used and an HP41951-69001 impedance test kit provided the necessary bias on the emitter. The devices were mounted on a SOT-172/D1 microwave transistor package. The measurements on these packaged transistors were done with a transistor test fixture (a Frequency Source Inc. Model 70254). For the \( S_{11} \) and \( S_{22} \) measurement a full calibration (short, open and 50 \( \Omega \) load) was performed with accurate calibration standards. To calibrate the \( S_{21} \) measurement only a through-calibration was done. The isolation-calibration, usually done to compensate for the crosstalk caused by parasitic capacitances of the test fixture from input to output, was not necessary.

Finally, to avoid distortion, the oscillator level was kept low (10 mV) while most of the measurements were performed with a bias-current of 0.5 mA.
For this current, the input-impedance of the transistor is approximately 50 Ω, which gives a good impedance matching to the system. It was carefully checked that this current would not necessitate such a high emitter-base voltage that series resistance effects would dominate.

4.3.3 INTERPRETATION OF THE MEASUREMENT RESULTS

Before the measured frequency response $Y_{21}$ will be used as input for the optimisation routine to extract the AC-transport parameters $\mu_p$ and $\tau_p$, it is important to check whether this is indeed the required frequency response of the intrinsic transistor. As indicated earlier, any extra phase shift introduced by parasitic elements could be devastating for the final extraction of the parameters.

It is therefore important not to focus only on the $Y_{21}$ response of the lateral transistor, but on the frequency behaviour of the entire device. In order to do this, the input and output behaviour of the transistor are modelled first, and then the frequency response of the whole transistor (intrinsic transistor plus environment) is considered. The results that are shown here correspond to measurements done on a lateral transistor with a basewidth of 15μm and a substrate doping level of $2.55 \cdot 10^{18}$ cm$^{-3}$. The diffusion length, which already has been extracted, is 13.6μm.

Modelling of the input circuit

After calibration of the measurement system, the calibration plane lies very close to the input lead of the packaged transistor. To set up an equivalent circuit after the calibration plane, it is only necessary to take into account the connection network to the test device and the input network of the test device itself. The connection network to the transistor consist of three parts, namely a small part of the SOT-172 package lead, the bond wires and the metal interconnect on the chip. These three parts can be considered as transmission lines. However, because the distance between the calibration plane and the intrinsic transistor is very small when compared with the wavelength of the applied signals, it is sufficient to model this connection network by a few simple lumped elements. Since the transistor is a low impedance device, only the inductance of these three parts is important and not the capacitance. This is indicated in Figure 4.11, where the equivalent input circuit of the transistor is shown. Here, $L_e$ and $L_b$ describe the inductance of the leads to the emitter and to the base respectively. The rule-of-thumb value for the inductance of a

54
bonding wire is about 0.4 nH mm\(^{-1}\) (for example, see [37]) where the wires are approximately 1 mm long.

The value for the inductance of the interconnect on the chip needs a little more consideration. This interconnect can be seen as a microstrip line on a Si-SiO\(_2\) layer and its properties, concerning signal propagation, have been investigated [39] for their importance in high-speed integrated circuits. In general, three propagation modes can be distinguished, depending on the frequency and on the substrate conductivity. For high resistivity substrates and high frequencies, the substrate can be considered as a dielectric and a dielectric wave mode appears. For very low resistivities and high frequencies, the substrate is almost a perfect conductor and the wave is in the skin-effect mode. For moderate resistivities and frequencies, which is applicable here, the resistivity is low enough to conduct charges so that the capacitance of the stripline is fixed by the oxide-thickness. The resistivity is, however, too high to have any significant skin effect in the substrate, in which case the inductance is fixed by the substrate thickness. This means that compared with the other modes, this mode combines a large capacitance with a large inductance. The resulting mode is called a slow wave mode, since the velocity of the signal can be 15-30 times slower than the speed of light [39]. In fact, Lawton et al. [58]

55
utilise this mode to design a small delay line for a solid-state reference waveform filter. Using the data provided by Hasegawa et al. [39], the inductance of the interconnect would be 1 nH mm \(^{-1}\). For the interconnect used, with a length of 100\(\mu\)m, this yields an inductance of 0.1 nH. A complicating factor is that the stripline, because of the Si-substrate, is not lossless. According to Hasegawa et al., this loss can be modelled with a series resistance which has a quadratic frequency dependence. However, for the frequencies considered, this does not seem to be important.

The total inductance \(L_e + L_b\) would come to 1 nH. The measured inductance, using a short-circuited test structure, was 0.8 nH, which is comparable with characterizations done on other microwave bipolar transistor packages [38]. The capacitance \(C_{pad}\) shown in Figure 4.11 is largely due to the bondpad, for which a value of 0.1 pF was measured.

To conclude the equivalent network from the calibration plane to the intrinsic transistor, two series resistances \(R_e\) and \(R_b\) of respectively the emitter and the base can be identified, which basically consist of a contact resistance and a sheet resistance. The total resistance \((R_e + R_b)\) is obtained from the Gummel plot, as done in Section 4.2.2. The value, changing from device to device, is on average 5 \(\Omega\).

Before setting up an equivalent circuit for the input impedance of the intrinsic transistor, it should be noted that the base is not transparent so that, for a typical current gain of about \(I_E/I_C = 0.01\), the base current is almost equal to the emitter current. This means that the existence of a collector does not influence the emitter current. Hence, to model the input impedance only the emitter-base diode lying in a semi-infinite substrate needs to be considered. In Figure 4.11 the following elements contributing to the input impedance are shown:

- The depletion or space-charge capacitance \(C_T\) of the emitter-base junction.
- The differential resistance \(r_e\) and the diffusion capacitance \(C_d\) resulting from the minority carriers in the emitter.
- The impedance \(Z_b\) caused by the minority carriers in the base region.

The emitters of the processed devices are transparent. In this case the small-signal impedance due to the minority carriers in the emitter can be described by a differential resistance \(r_e\) and a diffusion capacitance \(C_d\), as is usually done in the equivalent circuits of bipolar transistors. The resistivity \(r_e\) is given by:

\[
r_e = \frac{kT}{qI_B^{\text{min}}} \tag{4.4}
\]
where $I_E^{\text{inj}}$ is that part of the emitter current which results from the injection of electrons from the base into the emitter (for an N-type substrate). The diffusion capacitance $C_d$ is proportional to the excess minority charge in the emitter and can be described by:

$$C_d = C_o \left( \exp\left(\frac{qV_{EB}}{kT}\right) - 1 \right) \quad (4.5)$$

where $C_o$ is a capacitance dependent on mobility and minority carrier concentration in equilibrium in the emitter. The same equations cannot be used to describe the AC-behaviour of the excess minority carriers in the base. Here the quasi-static approximation is no longer valid and a full AC-analysis must be done. Solving equations 3.10-3.12 for the one dimensional and homogeneous case, and substituting the DC-current $I_E^{\text{inj}}$ caused by the injection of holes in the substrate, gives for the impedance:

$$Z_b = \frac{kT}{qI_E^{\text{inj}}} \frac{1}{\sqrt{1 + i\omega\tau_p}} \quad (4.6)$$

which thus depends on the lifetime $\tau_p$, as indicated. In fact, Bilotti [8] and Overstraeten et al. [101] used this expression to extract the lifetime from impedance measurements on (long) diodes. Johansson et al. [47] even attempted to measure the lifetime in SiGe-layers where frequency measurements up to 5 GHz were necessary. Clearly, the frequency behaviour of the impedance $Z_b$ cannot be described with a few standard elements but, to be more precise, a transmission line model is needed. This is shown in Figure 4.12, where the capacitance $C_d$ represents the charge storage, $r_d$ represents the diffusion process and $\tau_r$ represents the recombination.

![Figure 4.12: Transmission line model describing the diffusion process.](image)

To apply the equivalent circuit on the measurements, the amplitude and phase of the input impedance are shown in Figure 4.13 and 4.14. Here the
lines represent the measurements up to 500 MHz on a lateral transistor with a substrate doping level of $2.55 \cdot 10^{18}$ cm$^{-3}$ for three bias currents. The markers represent the calculated input impedance using the equivalent circuit shown in Figure 4.11. The values used are listed in Table 4.5; the capacitance $C_{pad}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_e + L_b$</td>
<td>1nH</td>
</tr>
<tr>
<td>$R_e + R_b$</td>
<td>5Ω</td>
</tr>
<tr>
<td>$C_T$</td>
<td>8pF</td>
</tr>
<tr>
<td>$C_d$</td>
<td>-</td>
</tr>
<tr>
<td>$I_{E}^{pinj}$</td>
<td>0.9$I_E$</td>
</tr>
<tr>
<td>$I_{E}^{pinj}$</td>
<td>0.1$I_E$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>370ns</td>
</tr>
</tbody>
</table>

**Table 4.5:** Values for the elements as used in Figure 4.11.

was neglected. Since the injected current in the emitter $I_{E}^{pinj}$ is much larger than the injected current in the base $I_{B}^{pinj}$ (measurements indicate a factor of 10 difference), the impedance at low frequencies is dominated by the emitter region. The stored charge in the base is, however, much larger than in the emitter so that at higher frequencies the impedance decreases due to the minority carriers in the base. To calculate $Z_b$, equation 4.6 is not used but $Z_b$ is simulated with CURRY since this problem is essentially two dimensional (the diffusion length is larger than the width of the emitter). For lower bias current the depletion capacitance $C_T$ becomes important. At high frequencies some deviations between the measurement and the equivalent circuit occur. The fit at these higher frequencies can be made perfect when a frequency dependent series resistance is assumed; this could be the frequency dependent resistance of the metalisation mentioned earlier. However, a more reasonable explanation would be current crowding effects in the emitter stripes. At high frequencies, where the input impedance of the emitter becomes small, the series resistance distributed in the emitter stripe becomes important and part of the emitter stripe does not contribute to the input impedance.

For an accurate measurement of $Y_{21}$ of the intrinsic transistor, and particularly the phase of $Y_{21}$, the phase of the input voltage $v_{Ve}$ at the emitter of the intrinsic transistor must be known. From Figure 4.11 it is clear that the phase of $v_{Ve}$ can deviates from with the phase of the applied voltage $v_{be}$. The inductances will introduce some differences in phase, but only at high frequencies. The series resistances, together with the capacitive behaviour of the transistor,
Figure 4.13: Magnitude of the input impedance for three bias currents $I_E = 0.01 \, mA, 0.5 \, mA$ and $1.0 \, mA$. The solid lines are the measured values and the markers are the simulated values using the equivalent circuit of Figure 4.11.

Figure 4.14: Phase of the input impedance for three bias currents $I_E = 0.01 \, mA, 0.5 \, mA$ and $1.0 \, mA$. 
will, however, also introduce a phase difference at lower frequencies. With the equivalent circuit shown in Figure 4.11 and using the values given in Table 4.5, a phase error of 0.6° already exists at 10 MHz, which has a serious effect on the extraction of the mobility and lifetime. The phase error can be reduced by decreasing the bias current, which lowers the diffusion capacitance. The output level of the transistor will, however, be reduced at the same time and crosstalk will become more important at lower frequencies, as will be shown later.

Modelling of the output circuit

Modelling the output network of the test device can be done in the same manner as the input network. Figure 4.15 shows the equivalent circuit describing

![Diagram](image)

**intrinsic transistor**

Figure 4.15: Small-signal, lumped, equivalent circuit of the output stage of the lateral transistor.

the impedance of the collector junction and the connection network up to the calibration plane. The collector base junction, being reversed biased, is modelled properly by a depletion capacitance $C_{cb}$ and a collector resistance $r_{cb}$. These two elements can clearly be recognized in Figures 4.16 and 4.17, which show the output impedance measured at different reverse bias voltages. At zero voltage the impedance can be explained by a capacitance of 4.6 pF.
At -4 and -5 V the depletion capacitance reduces to approximately 2.1 pF, but the collector resistance $r_{cb}$ decreases rapidly. This is caused by avalanche breakdown in the collector depletion layer.

The current source in Figure 4.15 represents the current caused by minority carriers that travel through the base and are collected by the collector. It is this current that is of interest because it 'contains' information about the mobility and lifetime of carriers in the base. Because of the current source nature, the series resistances $R_c$ and $R_b$ and the inductances $L_c$ and $L_b$ have no effect when this current is detected in the load $Z_o$, which is the characteristic impedance (50 Ω) of the measurement system.

The collector depletion capacitance $C_{cb}$, which shunts the current source, does however affect the measured response. The resistance $r_{cb}$ can here readily be neglected for $V_{CB}=0$ V. However, the capacitance $C_{cb}$ forms with $Z_o$ an RC-time which rapidly distorts the response after its corner frequency. In the designs of the lateral transistors the $C_{cb}$ is therefore kept small.

*De-embedding of the intrinsic transistor response*

Now that the input and output circuits have been modelled in detail, and now that the elements that can cause an error in the phase shift have been identified, it is necessary to develop a scheme to de-embed the response of the intrinsic transistor from the measured extrinsic transistor. In order to do this, the scheme shown in Figure 4.18 is used. Here the small-signal behaviour of the intrinsic transistor is described by four Y-parameters. The impedances $Z_e$, $Z_b$ and $Z_c$ represent the series impedances, which includes the series resistances and series inductances of respectively the emitter, base and collector, discussed earlier, according to:

\[
\begin{align*}
Z_e &= R_e + i\omega L_e \\
Z_b &= R_b + i\omega L_b \\
Z_c &= R_c + i\omega L_c.
\end{align*}
\]  

(4.7)

To complete the scheme, the parasitic capacitance $C_p$ is added to describe the capacitive crosstalk from input to output.

Starting from the intrinsic Y-parameters, the extrinsic Y-parameters at the terminals can be calculated. To avoid unnecessarily lengthy expressions some simplifications can be made by assuming:

- The inverse operation of the lateral transistor, described by $Y_{12}$, can be considered to be zero.
Figure 4.16: Magnitude of the output impedance for three collector voltages $V_{cb} = 0V, -4V, -5V$.

Figure 4.17: Phase of the output impedance for three collector voltages $V_{cb} = 0V, -4V, -5V$. 

62
Figure 4.18: Scheme used to de-embed the intrinsic from the extrinsic behaviour.

- The current gain is so low that the emitter current and the base current are the same.
- The output impedance of the intrinsic transistor is high and thus the $Y_{22}$ is low, so that $Z_b$ and $Z_c$ do not really contribute to the output impedance observed at the terminals.

The results for the extrinsic $Y$-parameters are then:

\[
Y_{11}^{ex} = \frac{Y_{11}}{1 + Y_{11}(Z_e + Z_b)} \quad (4.8)
\]
\[
Y_{22}^{ex} = Y_{22} \quad (4.9)
\]
\[
Y_{21}^{ex} = \frac{Y_{21}}{1 + Y_{11}(Z_e + Z_b)} - \frac{Y_{11}Y_{22}Z_b}{1 + Y_{11}(Z_e + Z_b)} + i\omega C_p \quad (4.10)
\]

As can be readily seen from equation 4.10, the response of the extrinsic transistor is composed of three terms. The first term is caused by the actual response of the intrinsic transistor $Y_{21}$. The other two terms are due to the extrinsic
elements and must be considered as undesirable crosstalk. The first crosstalk effect is caused by the base resistance, whereas the other effect comes from the parasitic capacitance $C_p$. The crosstalk can clearly be seen in Figures 4.19 and 4.20, where the solid lines indicate the measured response. At low frequencies, the magnitude of response decreases and the phase increases negatively, which is the expected and desired frequency behaviour of the intrinsic transistor. At a certain frequency of about 10 MHz the response starts to increase again and the crosstalk becomes dominant. This crosstalk is not caused by $C_p$ since this capacitance, independently measured on an empty test structure, is very low ($C_p \approx 1 \text{ fF}$), but the crosstalk can indeed be explained by the other mechanism due to the base resistance.

To calculate the intrinsic behaviour from the measured extrinsic behaviour, the equations 4.8-4.10 must be solved. This yields, when the term of $C_p$ is neglected:

$$Y_{11} = \frac{Y_{11}^{ex}}{1 - Y_{11}^{ex}(Z_e + Z_b)}$$  \hspace{1cm} (4.11)

$$Y_{22} = Y_{22}^{ex}$$  \hspace{1cm} (4.12)

$$Y_{21} = \frac{Y_{21}^{ex}}{1 - Y_{11}^{ex}(Z_e + Z_b)} + \frac{Y_{11}^{ex}Y_{22}^{ex}Z_b}{1 - Y_{11}^{ex}(Z_e + Z_b)}$$  \hspace{1cm} (4.13)

so that knowledge of $Z_e$ and $Z_b$ is required to accomplish the de-embedding.

For the frequencies considered, the small inductances can be neglected which leaves $R_e$ and $R_b$. The sum of these resistances, for the device in the example 5 $\Omega$, was already known from the extraction of the series resistance from the Gummel plot. Generally, the methods used to measure the base resistance $R_b$ of transistors alone are not applicable here because of the non-transparent base. The base resistance can, however, be obtained from the crosstalk itself. When the crosstalk becomes dominant, the other two terms in equation 4.10 can be disregarded, which yields:

$$Y_{21}^{cross} = -\frac{Y_{11}Y_{22}Z_b}{1 + Y_{11}(Z_e + Z_b)}$$  \hspace{1cm} (4.14)

Substituting equations 4.8 and 4.9 into this equation gives:

$$Y_{21}^{cross} = -Y_{11}^{ex}Y_{22}^{ex}Z_b$$  \hspace{1cm} (4.15)

so that the base resistance can simply be found by:

$$Z_b \approx R_b = -\frac{Y_{21}^{cross}}{Y_{11}^{ex}Y_{22}^{ex}}$$  \hspace{1cm} (4.16)
Figure 4.19: Magnitude of $Y_{21}$. The solid line represents the measured response and the dashed line represents the de-embedded response of the intrinsic transistor.

Figure 4.20: Phase of $Y_{21}$. The solid line represents the measured response and the dashed line represents the de-embedded response of the intrinsic transistor.
Results of the measurement of $R_b$ are shown in Figure 4.21, where the calculated $R_b$ remains constant for at least a decade. A value of 2.5 $\Omega$, the dotted line in Figure 4.21, seems a reasonable choice. As a result the emitter series resistance, $R_e$, also becomes 2.5 $\Omega$.

With equation 4.13, and with the information obtained about the series resistances, the intrinsic response can be calculated. The results are the dotted lines shown in Figures 4.19 and 4.20. The response of the intrinsic transistor is seen much more clearly now. In Figure 4.20 it can be seen that even before crosstalk becomes dominant its’ influence is noticeable at much lower frequencies. This clearly demonstrates that the de-embedding can not only be used to correct the response, and hence increase the usable frequency range, but it can also be used to determine which frequency range can be taken reliably, without the corrections.

4.3.4 EXTRACTION

To extract the parameters $\mu_p$ and $\tau_p$, the measured and de-embedded phase of $Y_{21}$ is compared with the simulated response of the lateral transistor using the device simulator CURRY. This comparison is done with the object function defined in equation 3.16, which has to be minimized to find the most probable values of the parameters. The measured data consists of about 150 frequency
points. To simulate the device for one frequency point with CURRY takes on average 40 seconds CPU-time on the Convex C240. Therefore the number of data points has to be kept low and 12 frequency points is seen as a reasonable compromise between computer time and an accurate description of the features in the frequency response. To make full use of all the measured data, the data were first smoothed by performing a polynomial fit, after which the 12 data points were selected for the optimization.

A typical example of an optimization is shown in Figure 4.22. Here the solid line indicates the measurements, which was also depicted in Figure 4.20. Although de-embedded with the scheme described in Section 4.3.3, the corrections remained small since the phase shifts up to 2 MHz were used only. The open dots show the simulated phase shifts before optimization with estimated values of the parameters ($\mu_p=190 \, \text{cm}^2\text{V}^{-1}\text{s}^{-1}$ and $\tau_p=250 \, \text{ns}$). The solid dots show the results of four iterations with the obtained parameter values: $\mu_p = 179.1 \, \text{cm}^2\text{V}^{-1}\text{s}^{-1}$ and $\tau_p = 371 \, \text{ns}$. About 11 function evaluations were needed in total, which took approximately 80 min. on the Convex machine.

The standard deviation $\sigma$ of the measured phase is about 0.08°, although it depends strongly on the magnitude of the measured response. The minimum obtainable error $\xi$ of the object function would then be: $\xi_{\text{min}} = \sigma^2 = 0.0064$. The final error $\xi_{\text{min}}$, that was reached in the optimization was 0.036. It must

![Figure 4.22](image-url)

Figure 4.22: Measured and simulated phase shift before and after optimization.
therefore again be concluded that the measurement error does not limit the accuracy of the parameters, but rather the incompleteness of the model does. The 95.4% confidence interval, obtained from the statistics, yields an uncertainty of 1.6% and 4.8% for mobility and lifetime respectively. From the parameters, a diffusion length can be calculated, which gives: $L_p = 13.1 \pm 0.4 \mu m$ (3.2%). This diffusion length was already independently measured, see section 4.2.3, with the multi-collector structure. This value was: $L_p = 13.6 \pm 0.2 \mu m$ (1.4%). Before concluding that these diffusion lengths are consistent with each other, it must be noted that the error in the mobility and lifetime are correlated, so that the accuracy of the diffusion length cannot be obtained by simply adding both errors. However, the consistency of the independently measured diffusion lengths can be clearly seen in Figure 4.23. Here the dark ellipsoid represents

![Figure 4.23: Check of consistency between the two independently measurements. The ellipse denotes the 95.4% confidence region of the measured mobility and lifetime, which are correlated. The lightly shaded region denotes the possible combinations of mobility and lifetime for the measured diffusion length.](image)

the 95.4% confidence region of the measured mobility and lifetime. The lightly shaded curved region denotes the possible combinations of mobilities and lifetimes covered by the measured diffusion length. The regions coincide and thus the measurements are consistent. The independent measurement of the diffusion length, which can be considered as a redundant measurement, is a very

68
powerful tool for checking the results obtained.

Another check of the acquired parameters is made by repeating the measurement on different devices. The example given above concerned the results obtained from a device with an as-drawn basewidth of 15μm. Measurements were also done on devices with basewidths of 9 and 12μm. These measurements are shown in Figure 4.24. As expected, the devices show much less phase shift because of the smaller basewidth. For these devices and for frequencies above the corner-frequency, the phase is no longer a linear function with the square root of frequency. This is caused by the fact that the basewidths are smaller than the diffusion lengths, so that the base becomes more or less transparent. The solid dots in Figure 4.24 represent the simulated phase after optimization. The obtained parameters are listed in Table 4.6. The values correspond quite well. Especially the mobility data are very consistent. The lifetime results show larger deviations, which can be explained by the fact that the devices are not situated close together and, thus, the doping levels are not necessarily the same. A variation in the doping of 5% gives a difference in the lifetime of 10%, when a quadratic dependence of lifetime with donor concentration is assumed.

Figure 4.24: Measured, solid lines, and simulated phase shifts after optimization, solid dots, for three devices with an as-drawn basewidth of 9, 12 and 15 μm.
\[ \begin{array}{|c|c|c|c|} \hline W_b (\mu m) & \mu_p (cm^2 V^{-1} s^{-1}) & \tau_p (ns) & L_p (\mu m) \\ \hline 9 & 178.5 \pm 2.5 (1.4\%) & 344 \pm 12 (3.6\%) & 12.6 \pm 0.3 (2.5\%) \\ \hline 12 & 180.6 \pm 1.8 (1.0\%) & 404 \pm 13 (3.2\%) & 13.7 \pm 0.3 (2.1\%) \\ \hline 15 & 179.1 \pm 2.9 (1.6\%) & 371 \pm 18 (4.8\%) & 13.1 \pm 0.4 (3.2\%) \\ \hline \end{array} \]

Table 4.6: Results of measurements on three different devices on the same substrate \((N=2.55 \times 10^{18} \text{ cm}^{-3})\).

4.4 LOW TEMPERATURE DC AND AC-MEASUREMENTS

The measurements shown in the previous sections were performed at room temperature. The measurements were, however, also done at lower temperatures down to 77 K. The experimental setup and some results will be discussed in this section.

4.4.1 MEASUREMENT SETUP FOR LOWER TEMPERATURES

To be able to measure at lower temperatures, a cryostat is used which is depicted schematically in Figure 4.25. A tube, with the device under test at the bottom, is placed in an isolated can of liquid nitrogen. To control the temperatures, heating resistors are placed in good thermal contact with the test device. The temperature was measured with a PT100 element and, combined with a temperature controller, the temperature during the experiments remained stable within 0.05 K. The accuracy of the temperature extracted from the Gummel plots of devices was about 0.2 K.

For the DC-measurements, which are relatively simple, there are no special requirements. For the AC-measurements, however, special measures were taken to ensure reliable measurements. These measures were:

- The cables to the device consist of semi-rigid coax with Teflon as the dielectric to ensure good temperature stability.
- A transistor fixture was designed to connect the devices, mounted on SOT172 packages, to the coax cables. This fixture can be seen as a small version of the fixture used at room temperatures and also contains the temperature sensor and the heating resistors.
- Calibrations were performed with prepared SOT172 packages. For the load calibration, two SMD-resistors of 100Ω were used in parallel.
For frequencies below 300 MHz the inductance and capacitance of these resistors can still be ignored.

To check the measurement system, high frequency measurements were done at room temperature and the results agreed satisfactorily with the results already presented which were obtained from the setup discussed in Section 4.3.2.

4.4.2 Measurements

To extract $L_p$ and $p_0 D_p$ as a function of temperature, the collector currents were measured from a multi-collector lateral transistor at several temperatures. Figure 4.26 shows a typical result of the measurements at low temperatures. For comparison, the currents measured at $T = 300$ K are included. The collector currents show a large ideal region except at high forward bias where series resistance effects become important. The emitter current, however, exhibits a large non-ideal component. This is seen more clearly in Figure 4.27 where the emitter current is shown for temperatures between 222 and 86 K. This non-ideal component can be identified as a forward biased tunnelling current. Del Alamo et al. [22] has investigated this component at room temperature and found that for doping concentrations above $5 \cdot 10^{18}$ cm$^{-3}$ this tunnelling current...
Figure 4.26: Gummel plot measured on the multi-collector device for two temperatures.

affects the forward characteristics. Clearly, at lower temperatures, the current also becomes important for lower concentrations. Hurkx et al. [43] modelled this tunnelling current in a new recombination model. The tunnelling current is believed to be caused by tunnelling via traps in the energy bandgap. The fact that this current is very dependent from device to device seems to confirm this fact. It was found that devices with a smaller emitter area showed a lower tunnelling current, which suggest that the traps could be induced by stress caused by a large emitter area.

To determine the mobility and lifetime as a function of temperature, the frequency response was measured. Additional measurements were performed to obtained the series resistances of the emitter $R_e$ and base $R_b$ in order to correct the frequency response if necessary. Here it was found that $R_b$ is
Figure 4.27: Emitter currents for the following temperatures: $T = 86, 100, 125, 150, 175, 200$ and $222$ K.
temperature dependent while $R_x$ is almost temperature independent, a fact which was also mentioned by Satake et al. [84]. Figure 4.28 shows the de-embedded measured phase shift at 100 K compared with the phase shift at 273 K for a device with a basewidth of 15μm and a doping level of $2.55 \times 10^{18}$ cm$^{-3}$. The slopes are not very different. Remembering that the diffusion coefficient determines the slope (equation 3.15) suggests that the diffusion coefficient is not very temperature dependent. The mobility, on the other hand, obtained by using the Einstein relation then shows a large temperature dependence, as will be seen in the next chapter where the results are discussed.

Figure 4.28: Measured phase shift at two different temperatures.
CHAPTER 5

DISCUSSION OF THE RESULTS

In the previous section the measurement method was demonstrated with measurements done on a few substrates, which describes what typical measures were taken to obtain the transport parameters correctly. These measurements are, however, a fraction of the total number of experiments conducted. While some experiments were done to confirm the results, other measurements were performed on substrates with other doping concentrations (N and P-type) and at different temperatures.

Here, an overview will be given of the results obtained and they will be compared with available data from literature. In addition, the validity of the current device models will be discussed. The quantitative results of this work are summarized in Appendix A.

5.1 MINORITY CARRIER MOBILITY

The minority carrier mobility is a somewhat neglected and underestimated quantity. Up to 1979, it was assumed without question that the minority carrier mobility is similar to the majority carrier mobility. Even today, well-known device simulators like PISCES-IIIB still make no distinction between minority and majority carrier mobility.

Of course, for low doping concentrations, where the majority and minority mobility is only limited by lattice scattering, there is no physical reason to suppose that they should be different. This is also confirmed by measurements on lightly doped silicon. For example Canali et al. [13] measured the minority hole mobility by the TOF technique and obtained a value of 450 cm²V⁻¹s⁻¹. Müller et al. [70] used a photocurrent technique to measure a value of 500 cm²V⁻¹s⁻¹ and recently Gasquet et al. [33] obtained a value of 510 cm²V⁻¹s⁻¹. For the majority mobility of holes a value of 470.5 cm²V⁻¹s⁻¹ is commonly
accepted [64].

However, for higher doping densities (N>10^{17} \text{ cm}^{-3}), where the majority mobility decreases due to impurity scattering, the minority and majority mobility need not to be the same. The data of Prince [83], who extracted the minority carrier mobilities for several low doping concentrations from a Haynes-Shockley experiment, gave a first indication that the minority carrier mobility is possibly much lower than the majority mobility, at least when the data of Prince are extrapolated to higher doping concentrations.

Dziewior and Silber [29] were the first to measure the minority carrier mobility at higher concentrations. They determined the hole mobility in N-type silicon and stated that this parameter must "be even slightly higher than the corresponding majority-carrier hole mobility" [29]. The minority carrier hole mobility figures obtained by Dziewior and Silber, as well as all other available data for dopant concentrations above 10^{17} \text{ cm}^{-3}, are given in Figure 5.1. The results of our work are also included in the Figure. In addition, to compare these results with the majority hole mobility, the mobility model for holes of Masetti et al. [64] is shown as a solid line. After Dziewior and Silber, Mertens et al. [65] reported mobilities that were somewhat lower than the majority mobility. Neugroschel and Lindholm [72] even claimed to have found evidence for a much lower mobility. They obtained a hole mobility for the minority carriers that was one order of magnitude smaller than for the majority carriers (\mu_p \approx 6 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} for N \approx 10^{20} \text{ cm}^{-3}; not shown in Figure 5.1).

In contradiction with these results, del Alamo et al. [24] measured mobilities that were approximately two times higher than the majority mobility; a result which has been also theoretically predicted by Bennet [7]. From a review of existing mobility data, del Alamo only selected the mobilities that were found to be reliable and that were obtained independently without making any assumptions concerning the other transport parameters. One single point of Burk and de la Torre [12], see Figure 5.1, belongs to this class. The dashed line in Figure 5.1 was the best empirical fit obtained by del Alamo [24].

Despite the thorough research of del Alamo, it was still not clear to some authors whether the minority carrier mobility is higher than, equal to, or much lower than the majority carrier mobility. For doping concentrations above 10^{18} \text{ cm}^{-3}, however, the findings of Wang et al. [113], as well as the findings of this work, are consistent with the results of del Alamo, which undoubtedly shows that the minority hole mobility is indeed larger than the majority hole mobility for these concentrations.

For concentrations below 10^{18} \text{ cm}^{-3}, the picture is, however, not yet clear. In this region only the data of Dziewior and Silber [29] exist and these data
Figure 5.1: Minority carrier hole mobility versus doping concentration. The solid line represents the majority hole mobility in P-type silicon according to Masetti et al. [64], the dashed line the model of del Alamo et al. [24] for the minority hole mobility and the dotted line the unified mobility model of Klaassen [51, 52].
scatter a lot. One point is even much larger than the maximum achievable mobility limited by lattice scattering only. Del Alamo did not measure the mobility for these doping concentrations since the diffusion length becomes very large (> 50μm), in which case the epilayer could not be made thick enough to eliminate the effect of the substrate [21]. However, the method in this work utilizes the substrate for the determination of the transport parameters. The substrate, being 525 μm thick, can then still be considered as a semi-infinite layer. In Figure 5.1 a mobility value is included which is measured in a substrate with a donor concentration of 9.55 · 10^16 cm⁻³. This mobility deviates strongly from the results of Dziewior and Silber. To validate this measured mobility, some comments will be made concerning the extraction of the mobility. Figure 5.2 shows the measured and optimized phase shifts for the substrate with a donor concentration of 9.55 · 10^16 cm⁻³. For these AC-

![Graph](image)

Figure 5.2: Measured phase shift for a device with a doping concentration of 9.55 · 10^16 cm⁻³. The solid lines indicates the measurements and the markers the simulation results (see text).

measurements, the multi-collector structure shown in Figure 4.2 is used. The 400μm long version of this structure was used to avoid 3D-effects since DC-
diffusion length measurements indicated a diffusion length of approximately 150 \( \mu \text{m} \). The distances from emitter to collectors are all much shorter than the diffusion length and the lateral transistor can be considered to be transparent. This means that only the mobility can be extracted and not the lifetime. The phase shift from the third collector was used in the optimization and because of the long carrier lifetime, it was sufficient to measure up to 1 MHz. Next, with the extracted mobility (278.7 cm\(^2\)V\(^{-1}\)s\(^{-1}\)), the phase shift of the seventh collector was simulated, which corresponds perfectly with the measured response, as can be seen in Figure 5.2. On the other hand, the open dots in Figure 5.2 represent the simulated phase shift at the third collector when a mobility according to the model of del Alamo is used. Clearly, such a mobility disagrees with the measurements.

Additional support for a lower mobility at lower concentrations is given by the unified mobility model of Klaassen [51, 52]. With this mobility model it is possible to describe both the majority and the minority mobility. Such a model is not the first one; Shigyo et al. [88], for example, also presented a model which incorporates the majority mobility as well as the minority carrier mobility in one model. However, the model of Shigyo et al. is only a blend between the empirical models of Alamo et al. (minority carriers) and Masetti et al. (majority carriers) and has, therefore, no predicting qualities. The model of Klaassen uses only the empirical expression for the majority carrier mobility and arrives, through physical reasoning, at a relation for the minority carrier mobility as a function of doping concentration. With this model it is thus possible to predict the behaviour of the minority carriers and it also provides a first, almost theoretical, model for the minority mobility for a whole range of concentrations. Other theoretical calculations concerning the minority carrier mobility, for example those of Bennet [7], are only valid in a narrow doping range \((2 \cdot 10^{19} \text{ to } 8 \cdot 10^{19} \text{ cm}^{-3})\). The model of Klaassen is also included in Figure 5.1 (dotted line). The latter also predicts a much lower mobility at the lower concentration than previously thought. Since the model of Klaassen describes the measured values presented in this work with enough accuracy, no attempt has been made to describe these measurement results of this work with a new empirical expression.

For the minority electron mobility in heavily doped P-type silicon the situation is much less confusing. This is partly due to the fact that very little experimental data exist. This is shown in Figure 5.3. Dziewior and Silber [29] applied the same method as used for the measurement of the minority hole mobility for the determination of the minority electron mobility. Swirhun [93]
Figure 5.3: Minority carrier electron mobility versus doping concentration. The solid line represents the majority mobility of electrons in N-type silicon according to Masetti et al. [64] and the dashed line the model of Swirhun et al. [96] for the minority electron mobility.
measured the mobility on the same type of devices as del Alamo; as a consequence, the values for the mobility at lower concentrations are not very reliable since the diffusion length becomes very large. The single point obtained from this work corresponds nicely with the data of Dziewior and Silber. In Figure 5.3 the majority mobility according to Masetti et al. [64] is also included, the solid line, as well as the best empirical fit obtained by Swirhun [96] for the minority electron mobility. As can be seen here, the minority carrier mobility approaches the majority mobility for lower doping concentrations, which was not the case for the minority hole mobility according to del Alamo et al. [24].

Temperature dependence

Except for very low doped silicon, no experimental data is available in the literature for the temperature dependence of the minority carrier mobility. Swirhun et al. [95] and Wang et al. [114] presented minority carrier mobility data as a function of temperature, but they only measured the diffusion length as a function of temperature. From an assumed temperature dependence of the lifetime, the temperature dependence of the mobility was calculated. Both Swirhun et al. and Wang et al. found an anomalous behaviour of the mobility. For temperatures below room temperature the calculated mobility increases strongly with an increasing doping concentration; thus adding more scattering centers would give a higher mobility!

To clarify this matter, mobility measurements have been done as a function of temperature. These measurements could be performed relatively easy, since the method of this work is purely electrical. The method used by Wang et al. [114] necessitates a light source, whereas Swirhun et al. [95] use optical excitation and optical detection of the photoluminescence radiation. In the latter case, Swirhun commented [93] that the photoluminescence radiation, necessary to measure the lifetime, was too weak to measure at lower temperatures.

In order to check the measurements, not only AC-measurements (for the extraction of mobility and lifetime), but also DC-measurements (for the extraction of diffusion length) have been conducted. From the measured mobility and lifetime, the diffusion length can be calculated which can be compared with the diffusion length independently measured. The results for two doping concentrations (1.90 \cdot 10^{18} \text{ and } 2.55 \cdot 10^{18} \text{ cm}^{-3}, \text{ N-type}) are shown in Figure 5.4. The deviations between the diffusion lengths are always less than 10%. As can be seen, the deviation is systematic and is probably due to 3D-effects. For the DC-structures special measures were taken to eliminate side-effects, as described in section 4.2.1. This was, however, not done for the
Figure 5.4: Measured hole diffusion length as a function of the temperature for two doping concentrations. The thick markers represent the diffusion length obtained from the DC-measurements and the thin markers the calculated diffusion length from the measured mobility and lifetime.
AC-structures. The lengths of these structures, being 100 μm long to limit the capacitances, cannot be considered as infinitely long when compared with the diffusion lengths measured here. If this is the case, since the absolute value of the complex diffusion length:

$$L_p^* = \frac{L_p}{\sqrt{1 + i\omega\tau_p}}$$

(5.1)
becomes shorter for higher frequencies, it can be argued that the side-effect only becomes important for low frequencies. This means that the lifetime, which is determined form the low frequency response, suffered mostly from this deviation. Nevertheless, the agreement between DC and AC-diffusion length is sufficiently good and no corrections have been made.

The measured minority hole mobility as a function of temperature is shown in Figure 5.5. For comparison, the majority hole mobility (solid lines) for the two doping concentrations as a function of temperature based on the work of Li [60, 61] is also shown. A surprising result is that the minority carrier mobility is strongly dependent on temperature, while the majority carrier mobility for these doping concentrations shows only a weak dependence. At 77 K the minority carrier mobility is almost 6 times higher than the corresponding majority mobility. Obviously, assuming the minority carrier mobility to be equal to the majority carrier mobility seriously under-estimates the performance of bipolar transistors at low temperatures. In Figure 5.5 the only known model of the minority carrier mobility as a function of temperature is also included (dotted lines). Klaassen extended his mobility model [51, 52] for room temperature to other temperatures [53]. The agreement with our measurements is quite good.

To compare our results with the 'measured' mobility data of Wang et al. [114], the obtained values are plotted in Figure 5.6 for two selected temperatures of 100 K and 300 K. The solid lines are an empirical fit on the date of Wang et al. for the two temperatures. The derived mobility of Wang et al. disagrees with the results of this work. Our measured values around the doping concentration $2 \cdot 10^{18}$ cm$^{-3}$ are much higher and decrease with increasing doping concentration. As also indicated by Klaassen [53], Wang et al. used an incorrect temperature dependence of the lifetime. At low doping concentrations and at low temperatures, the Shockley-Read-Hall recombination comes into play (see also section 5.2), in which case the lifetime is smaller than that assumed by Wang et al.. This results in a under-estimated mobility. At high doping concentrations, where Auger recombination prevails, the temperature dependence of the Auger coefficient is under-estimated and a larger lifetime.
Figure 5.5: Measured minority carrier hole mobility versus temperature. The solid lines represent the majority hole mobility in P-type silicon according to the results of Li [60, 61] and the dotted lines are the minority carrier mobility when the mobility model of Klaassen [59] is used.
exists than is assumed by Wang et al.. As a consequence, the mobility is overestimated at the higher concentrations. Therefore it can be concluded that the anomalous behaviour of the mobility does not exist and thus the mobility remains a decreasing function of doping concentration.

To explain the different behaviour of the minority carrier mobility compared to the majority mobility, it must be noted that in general three scattering processes contribute to the minority carrier mobility:

- Lattice scattering
- Impurity scattering
- Electron-hole scattering.

Lattice scattering is a well understood mechanism and can easily be measured separately in low doped silicon. The lattice scattering limited mobility $\mu^L$ can
be described by a simple power law which reads:

$$\mu^L = \mu_0 \left( \frac{T}{300} \right)^{-\alpha} \quad (5.2)$$

where $\mu_0$ is the mobility at $T=300$ K. For the hole mobility (minority and majority) some common accepted values are: $\mu_0 = 470.5 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ whereas $\alpha$ lies between 2.2 and 2.4 [85].

The majority mobility due to impurity scattering can conveniently be calculated using the analytical formulae of Conwell and Weisskopf [17] and of Brooks and Herring [10, 41]; both expressions are based on the Born-approximation. It appears that such an approximation is valid for attractive impurities potentials, such as seen by the majority carriers, but not for repulsive impurity potentials, as in the case for minority carriers. To calculate the impurity scattering rate for the minority carriers, a more refined technique, the partial-wave phase shift method, must be used. Theoretical calculations by Bennet [7], using this technique, show that repulsive potentials scatter less effectively than attractive potentials, which results in a higher minority carrier mobility, as was also found in the experiments. The results of Bennet are only valid in the narrow doping range of $2 \cdot 10^{19}$ to $8 \cdot 10^{19} \text{ cm}^{-3}$. Extending the partial-wave phase shift calculations at lower doping concentrations poses a problem. At low concentrations the screening of the impurities becomes less and the impurity potentials may overlap. Here special measures must be taken to avoid third body interaction and to account only for the nearest scattering center. Conwell and Weisskopf bypassed this problem by defining a minimum scattering angle. This cannot be done easily when using the partial-wave phase shift analysis, in which case it is difficult to perform accurate calculations to theoretically explain the minority carrier mobility in detail.

Electron-hole scattering as the third scattering process is obviously only important for the minority carriers since the minority carriers 'experience' the influence of a lot of majority carriers. Fletcher [31] applied the results of Chapman and Cowling [14], who compared the mutual diffusion of two groups of gaseous particles to the case of electrons and holes in silicon. The expression for the electron-hole mobility $\mu^{eh}$ of Fletcher, slightly adapted by Choo [16], reads:

$$\mu^{eh} = \frac{2 \cdot 10^{17}T^{3/2}}{\sqrt{(pn)}} \left[ \ln(1 + 8.28 \cdot 10^8 T^2 (pn)^{-1/3}) \right]^{-1} \quad (5.3)$$

Dorkel and Leturcq [27] used this expression and found agreement with the results of Krausse [56] and Dannhäuser [19], who measured $\mu_n + \mu_p$ as a function
of the injected carriers in a pin-diode. Measurements from such a pin-diode provides strong evidence for the presence of electron-hole scattering. This fact was also recognized by Klaassen [53], who treated this problem by considering the majority carriers as moving scattering centers. Klaassen also found agreement with the results of Krausse and Dannhäuser by assuming that heating of the device occurs place due to high currents.

To get insight in the relative contributions of the three scattering processes (lattice scattering, impurity scattering and electron-hole scattering) to the mobility, the model of Klaassen is used. Figure 5.7 shows the results, compared with the measurement of the mobility at the doping concentration of $2.55 \cdot 10^{18}$ cm$^{-3}$. According to the model of Klaassen, the three processes are comparable at room temperature, but at lower temperatures the electron-hole scattering dominates. It can be seen that in the model of Klaassen, the mobility due to electron-hole scattering is a decreasing function of temperature. Equation 5.3 predicts, however, a mobility due to electron-hole scattering which increases with temperature. Clearly, such behaviour does not fit our measurement results.

5.2 **Minority carrier lifetime**

The minority carrier lifetime is probably the most studied minority carrier parameter and a lot of measurement data are available in the literature (for a review see, for example, Tyagi and van Overstraeten [100]). Unfortunately these data are not very consistent with each other. It was del Alamo [24] who showed that a large part of the lifetime data were not obtained by true lifetime measurements. Often, only the diffusion length was measured, so that knowledge of the minority carrier mobility was necessary to deduce the lifetime. Since the minority carrier mobility was not well known, the majority carrier mobility was used, which may result in large deviations.

Figure 5.8 shows some measurements reported in the literature of the minority carrier hole lifetime as a function of doping concentration. Only those data have been selected which were obtained by direct and independent measurement of the lifetime on uniformly doped silicon. Dziewior and Schmid [28] and Swirhun et al. [94] extracted the lifetime from photoluminescence decay measurements. The single point of Burk and de la Torre [12] and the data of Wang et al. [113] were obtained from, respectively an AC EBIC and an AC LBIC technique. The hole lifetime data from this work are also included. As can be seen in Figure 5.8 the doping dependence is quite consistent, although the absolute value of the lifetime can vary by a factor of two. The lifetime data
Figure 5.7: Decomposition of the mobility in a part due to lattice scattering, impurity scattering and electron-hole scattering according to the model of Klaassen [53]. The markers represent the measurements for a donor concentration of $2.55 \cdot 10^{18}$ cm$^{-3}$, and the solid line the combined mobility of all parts.
Figure 5.8: Minority carrier lifetime of holes versus doping concentration. The solid line represents the model of del Alamo et al. [25], the dashed line the model of Klaassen [53] and the dotted line is from this work.
of this work, which matches perfectly with the data of Wang et al., appear to be rather large compared to the others; an explanation for this will be given later.

To model the lifetime as a function of doping concentration, it is useful to look at the physical origin of the recombination processes. Basically two distinct mechanisms contribute to the recombination in heavily doped silicon:

1. Shockley Read Hall (SRH) recombination
2. Auger recombination

Direct band-to-band recombination of an electron-hole pair with the emission of a photon is excluded here since silicon is an indirect bandgap semiconductor, which makes this mechanism highly improbable.

The first type of recombination is the well-known Shockley Read Hall (SRH) recombination via deep level traps in the middle of the forbidden gap. SRH-type recombination through shallow states near the band edges is less efficient and can be ignored [100]. The SRH recombination is described by:

\[
R_{SRH} = \frac{np - n_i^2}{\tau_{pSRH}(n + n_1) + \tau_{nSRH}(p + p_1)}
\]  \hspace{1cm} (5.4)

where \(n\) and \(p\) are respectively the electron and hole concentration and \(n_1, p_1\) are equal to \(n_i\) when the trap level coincides with the centre of the bandgap. For N-type silicon and with low level injection, this equation reduces to:

\[
R_{SRH} = \frac{p - p_0}{\tau_{pSRH}}
\]  \hspace{1cm} (5.5)

where \(\tau_{pSRH}\) is the minority carrier lifetime due to SRH-recombination, which can be expressed by:

\[
\tau_{pSRH} = \frac{1}{\nu_{th}\sigma N_i}
\]  \hspace{1cm} (5.6)

where \(\nu_{th}\) is equal to the thermal velocity of the carriers, \(\sigma\) is the capture cross-section of the traps, and \(N_i\) denotes the trap density. When the trap density remains constant with increasing doping concentration, the lifetime is also independent of the doping concentration. This is indeed true for low doping levels. For moderate doping levels \((10^{16} - 10^{18}\ \text{cm}^{-3})\), however, a linear dependence with the doping concentration is observed. To account for this effect, it is believed that for higher doping concentrations more defects are
generated during the processing [32]. Effectively, the minority carrier lifetime can in this case be described by:

$$\tau_p^{-1} = \tau_o^{-1} + C_{SRH} N_D$$  \hspace{1cm} (5.7)

The second recombination mechanism is known as Auger recombination. Here, three possible recombination processes can be identified: band-to-band Auger recombination, photon-assisted band-to-band Auger recombination and trap-assisted Auger recombination. Which mechanism dominates in heavily doped silicon is however not fully understood. The first two mechanisms can be described by:

$$R_{AUG} = (C_{nAUG}n + C_{pAUG}p)(np - n_i^2)$$  \hspace{1cm} (5.8)

where $C_{nAUG}$ and $C_{pAUG}$ are the Auger coefficients for holes and electrons respectively. Trap-assisted Auger recombination can also be described by equation 5.8 when one assumes that the trap density increases linearly with the doping concentration. For P-type silicon and low level injection, equation 5.8 can be simplified, which yields:

$$R_{AUG} = C_{nAUG}n^2(p - p_0)$$  \hspace{1cm} (5.9)

The factor $C_{nAUG}n^2$ can be seen as the inverse of the Auger recombination lifetime $\tau_{AUG}$, in which case the lifetime shows a quadratic dependence with the (majority) carrier concentration.

Combining the SRH and Auger recombination, the minority hole lifetime as a function of concentration can be described by:

$$\tau_p^{-1} = \tau_o^{-1} + C_{SRH} N_D + C_{nAUG}n^2$$  \hspace{1cm} (5.10)

For the Auger coefficient $C_{nAUG}$ a lot of values exist; the most known and widely used value, however, is $2.8 \cdot 10^{-31}$ cm$^6$S$^{-1}$, which has been obtained by Dziewior and Schmid [28]. In Table 5.1 the coefficients are listed obtained by Dziewior and Schmid [28], del Alamo et al. [25] and Klaassen [53]. Also the results of this work is included in Table 5.1. The parameters $\gamma$ and $\delta$ describe the temperature dependence and will be discussed later. For comparison, the models of del Alamo et al. and Klaassen are depicted in Figure 5.8. Obviously, the differences in parameters come from fitting the lifetime model on different subsets of available lifetime data and even with the measured lifetime of this work no ”best” model of the hole lifetime for room temperature can be chosen.

Figure 5.9 depicts the situation for the electron lifetime in P-type silicon. Here, the data of Dziewior and Schmid [28], the data of Swirhun [93, 96] and the
Figure 5.9: Minority carrier lifetime of electrons versus doping concentration.
<table>
<thead>
<tr>
<th></th>
<th>$\tau_0$</th>
<th>$C_{SRH}$</th>
<th>$C_{AUG}$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ms)</td>
<td>($10^{-13}$ cm$^3$s$^{-1}$)</td>
<td>($10^{-31}$ cm$^6$s$^{-1}$)</td>
<td></td>
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<tr>
<td>Dziewior and Schmid</td>
<td>-</td>
<td>-</td>
<td>2.8</td>
<td>-</td>
<td>0.14</td>
</tr>
<tr>
<td>de Alamo et al.</td>
<td>-</td>
<td>7.8</td>
<td>1.9</td>
<td>-</td>
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<tr>
<td>Klaassen</td>
<td>2.5</td>
<td>11.76</td>
<td>2.8</td>
<td>0.57</td>
<td>0.72</td>
</tr>
<tr>
<td>This work</td>
<td>-</td>
<td>0.543</td>
<td>4.1</td>
<td>1.72</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 5.1: Parameters for the lifetime model for holes. The parameters $\gamma$ and $\delta$ describe the temperature dependence.

single point obtained from this work are shown. As can be seen, the agreement between the data is far less satisfactory than for the hole lifetime. The lifetime values reported by Dziewior and Schmid are the most widely used ones. For concentrations below $10^{19}$ cm$^{-3}$ the lifetimes of Swirhun start to deviate from the lifetimes of Dziewior and Schmid, although both sets of data are obtained from the same measurement method (decay of photoluminescence).

The measured lifetime from this work is even somewhat lower than that of Swirhun, and is approximately eights times lower than that of Dziewior and Schmid. The lifetime measured in this work is considered, however, to be quite accurate for the following reasons:

- The simultaneously measured electron mobility is consistent with the measurement results of Dziewior and Silber, as was shown in section 5.1.
- The diffusion length data obtained from DC-measurements on the multi-collector structures, is in perfect agreement with the diffusion length calculated from the independently measured mobility and lifetime. Both diffusion lengths were $3.87 \, \mu$m.
- It was previously shown that surface recombination plays a very minor role due to the structure of the devices.

Swirhun explained his results by the fact that his measurements were done, for the first time, on processed material. Certainly this is also true in our case. However, more experimental work needs to be done to clarify these results completely.

**Temperature dependence**

So far the minority carrier lifetime at room temperature has been discussed. From a theoretical point of view, and for modelling purposes, knowledge about
the temperature dependence of the lifetime is also important. From the temperature dependence, it can be deduced which mechanism dominates, since calculations show that lifetime due to SRH recombination increases with temperature, whereas lifetime due to Auger recombination decreases with temperature. Modelling the temperature with a simple power law [53] results in an extended version of equation 5.10:

\[
\tau_p^{-1} = \left( \tau_0^{-1} + C_{SRH} N_D \right) \left( \frac{300}{T} \right)^7 + C_{nAUG} n^2 \left( \frac{T}{300} \right)^6
\] (5.11)

Despite the large amount of data available concerning the lifetime at room temperature, very little measurement data is available on the temperature dependence of the carrier lifetime. Dziewior and Schmid [28] measured the temperature dependence, but only at three temperatures (77, 300 and 400 K). They found a very weak temperature dependence of the Auger coefficient. A fit done by Selberherr [86] on their data reveals a power dependence of \( T^{0.14} \). Svantesson and Nilsson [92] measured the temperature dependence of the Auger coefficient in highly excited undoped silicon. For the ambipolar Auger coefficient \( (C_{nAUG} + C_{pAUG}) \) they obtained a power dependence of \( T^{0.6} \). Klaassen [53] used the diffusion length data as a function of temperature of Wang et al. [114] and, with his proposed minority carrier mobility model, he extracted the coefficients for the temperature dependence of the lifetime. For the Auger coefficient a \( T^{0.72} \) dependence was found (see also Table 5.1).

In this work, for the first time, a detailed temperature dependence of the hole lifetime has been measured. This is done for two doping concentrations \( (1.9 \cdot 10^{18} \) and \( 2.55 \cdot 10^{18} \) cm\(^{-3}\). Figure 5.10 gives the results. Clearly, two distinct recombination processes can be seen. For low temperatures the SRH-recombination dominates, for which the lifetime increases with temperature, while at higher temperatures the Auger recombination becomes important. The solid lines indicates a simultaneous fit of equation 5.11 on the measured lifetime for both concentrations. The obtained parameters are listed in Table 5.1. As mentioned above, the extracted temperature dependence of the Auger coefficient \( (T^{0.44}) \) lies in the range of reported values. The larger temperature dependence of \( C_{SRH} \) \( (T^{-1.72}) \) is consistent with the finding that the SRH recombination is more temperature sensitive than Auger recombination. For the curve shown in Figure 5.10 with a doping level of \( 1.9 \cdot 10^{18} \) cm\(^{-3}\), the lifetime is separated into an SRH-part \( (\tau_{SRH}) \) and an Auger-part \( (\tau_{AUG}) \). It can clearly be seen that at room temperature the SRH-lifetime is much larger than the Auger-lifetime. This small contribution of the SRH recombination, and remembering that the SRH recombination depends very much on the number of defects \( N_t \) being very process and material dependent, this can explain why the
Figure 5.10: Hole minority carrier lifetime as a function of temperature.
measured lifetime at room temperature of this work is somewhat larger than some of the data shown in Figure 5.8. The dotted line in Figure 5.8 shows the results when the lifetime is extrapolated for other concentrations using the model of equation 5.11 with the coefficients found. At low concentrations the lifetime is over-estimated because of the lower SRH recombination found. At higher concentrations, the lifetime is somewhat under-estimated since the extracted Auger coefficient, $4.1 \cdot 10^{-31} \text{ cm}^6\text{s}^{-1}$ is larger than the usual one. The larger Auger coefficient is, however, consistent with the results of Sinton and Swanson [89], who also found a larger Auger coefficient for low carrier concentrations. This supports the idea that the Auger coefficient is doping dependent; a fact also recognized by Swirhun [93] and Wang et al. [114]. Swirhun found, for example, that the best fit on his data was not a quadratic function (equation 5.9) of carrier concentration but a function equal to:

$$\tau_{Aug}^{-1} \propto n^{1.65} \quad (5.12)$$

whereas Wang found a dependence of:

$$\tau_{Aug}^{-1} \propto n^{1.867} \quad (5.13)$$

Swirhun explained his results by the presence of two competitive Auger recombination mechanisms: direct band-to-band Auger recombination and photon-assisted Auger recombination. From a theoretical analysis Haug [40] found that both processes have an n^2 dependence in non-degenerate silicon (thus for concentrations below $3 \cdot 10^{18} \text{ cm}^{-3}$), but for degenerate silicon, where Fermi-Dirac statistics must be applied, the direct band-to-band Auger recombination changes from a quadratic dependence to a linear dependence. This was the basic evidence that for heavily doped N-type silicon with an observed quadratic doping dependence, photon-assisted Auger recombination is the dominant mechanism [100, 102]. Swirhun explained his doping dependence, however, by the fact that both processes could be comparable in magnitude. This indicates that for low concentrations the Auger coefficient is due to both processes, and hence a large coefficient exists, while for larger concentrations the direct band-to-band Auger recombination becomes less important, which results in a smaller Auger coefficient. This is in agreement with the results from this work. The temperature measurements of this work show that the larger Auger coefficient is indeed due to the Auger recombination process and not to the SRH recombination process.
5.3 Bandgap Narrowing

Bandgap narrowing has, as a heavy doping effect, the largest influence on the DC-device characteristics of bipolar transistors compared to the minority carrier mobility and lifetime. It is therefore not surprising that an enormous collection of data exists in the literature concerning bandgap narrowing. Unfortunately, these data show a lot of scatter. For the bandgap narrowing data which were inferred from device measurements, del Alamo et al. [24] showed that the scatter can largely be explained by the fact that $\Delta E_g$ is not the measurement quantity, but the product $p_0D_p$ (for N-type silicon). Again, as was also the case for lifetime data extracted from diffusion length measurements, the minority carrier mobility plays a crucial role for an accurate extraction of the bandgap narrowing.

The fundamental nature of the parameter $p_0D_p$ provides a good measure by which to compare our results with those of others. In Figure 5.11 the data of del Alamo et al. [23] are shown alongside with the data presented in this work. Since the data have been measured at different temperatures and $p_0$ is very temperature dependent, a comparison is facilitated by dividing $p_0D_p$ by the square of the intrinsic concentration $n_{io}$, which removes most of the temperature dependence. The data agree well with each other. Figure 5.11 also demonstrates the net result of all the heavy doping effects important for the forward current in transparent regions. Since the current density is here equal to:

$$J = \frac{q p_0 D_p}{W} \left( \exp \left( \frac{qV}{kT} \right) - 1 \right)$$

(5.14)

it is proportional to $p_0D_p$. At heavy doping levels $p_0D_p$ decreases less rapidly and increasing the doping concentration becomes a less efficient parameter to decrease the current density.

Figure 5.12 reflects the results when $p_0$ is extracted from $p_0D_p$ for known minority carrier mobility. For the calculation of $p_0$ in this work, our own measured mobility is used. For the data of del Alamo et al. the mobility was not measured separately below a concentration of $1.4 \cdot 10^{19}$ cm$^{-3}$, therefore the mobility model of Klaassen was used, which, as was shown in section 5.1, describes the mobility data of this work well enough. The solid line represents the normalised $p_0$ when no bandgap narrowing is assumed. It can be seen that even for our low doped value a small deviation still occurs due to bandgap narrowing, while for a doping concentration of $10^{20}$ cm$^{-3}$ the deviation increases by a factor of one hundred between $p_0$ with and without bandgap narrowing.
Figure 5.11: The quantity $p_0 D_p$ versus doping concentration.
Figure 5.12: The equilibrium hole density $p_0$ versus doping concentration. The solid lines indicate the results when no bandgap narrowing is assumed.
Most measurement data are, however, not presented in the true physical quantity \( p_0 \), but in \( \Delta E_g^{app} \) - the apparent bandgap narrowing. This apparent bandgap narrowing is related to \( p_0 \) through equation 2.7 which yields:

\[
\Delta E_g^{app} = kT \ln \left( \frac{p_0 ND}{n_{io}^2} \right)
\]

(5.15)

in which case the intrinsic carrier concentration in equilibrium \( n_{io} \) is necessary to calculate \( \Delta E_g^{app} \).

Figure 5.13 shows the most important bandgap narrowing results obtained thus far from electrical measurements. The solid line represents the best fit on the first set of bandgap narrowing data obtained by Slotboom and de Graaff [91] on P-type silicon. The dashed line describes the results obtained by del Alamo et al. [24, 25, 23] on N-type silicon. Swirhun applied the same method of del Alamo on P-type silicon and got the same results as Slotboom and de Graaff (not shown in the figure). Since the data obtained for N and P-type silicon are very different, it was believed that not the same bandgap narrowing occurs in N and P-type silicon. However, these results cannot be blindly compared since:

- In the extraction of \( \Delta E_g^{app} \), Slotboom and de Graaff assumed the minority carrier mobility to be equal to the majority mobility since no experimental data on the minority carrier mobility existed at the time. Swirhun et al., however, used the available data of the minority carrier mobility to extract his bandgap narrowing results.

- Slotboom and de Graaff, del Alamo et al. and Swirhun et al. used different values for \( n_{io} \) (respectively \( 1.19 \cdot 10^{10}, 1.45 \cdot 10^{10}, 1.45 \cdot 10^{10} \) cm\(^{-3}\) at 300 K) for their calculations.

Several authors recognized these facts. King [49], for example, corrected the data of Slotboom and de Graaff and found agreement with his measurement results for P-type silicon, but he found no agreement with the results of Swirhun et al., and no effort was done to revise the data of del Alamo et al.. Shigyo et al. [87] corrected the data of del Alamo et al. using a \( n_{io} \) suggested by Green [35] (\( n_{io} = 1.08 \cdot 10^{10} \) cm\(^{-3}\)) and found agreement between King's data for P-type silicon and the data of del Alamo et al. for N-type silicon.

In a more elaborate study, Klaassen et al. [54] corrected the data of Slotboom and de Graaff (P-type), del Alamo et al. (N-type) and other data. The data of del Alamo was revised with the new minority carrier mobility model, which yields large corrections for low concentrations. These corrected data are
Figure 5.13: The apparent bandgap narrowing versus doping concentration. The solid line represents the model of Slotboom and de Graaff [91] for P-type silicon, the dashed line the model of del Alamo [24, 25, 23] for N-type silicon and the chained-dashed line the unified model of Klaassen [54].
shown in Figure 5.13. One point of Swirhun et al. (P-type), for which sufficient information was given by Swirhun to correct it, is also included. Klaassen found that all these data can be described by one model, the dotted line in the Figure, irrespective of the type of impurity.

In Figure 5.13 the data of this work is also included. To be consistent, the same \( n_{i0} \) is used as was used by Klaassen et al. [54], which is:

\[
\begin{align*}
\frac{n_{i0}^2}{T^3} &= 9.61 \cdot 10^{32} \exp\left(\frac{-q1.206}{kT}\right)(5.16)
\end{align*}
\]

This equation gives: \( n_{i0} = 1.19 \cdot 10^{10} \text{ cm}^{-3} \) at \( T=300 \text{ K} \). When the value of Green [35] is used (\( n_{i0} = 1.08 \cdot 10^{10} \text{ cm}^{-3} \)), all the bandgap narrowing data becomes 5 meV larger. The first three points of this work correspond to N-type silicon, whereas the point at \( 7.07 \cdot 10^{18} \text{ cm}^{-3} \) was measured in P-type silicon. These measurements indeed confirm that within the experimental accuracy (approx. 5 meV) no difference can be found between the apparent bandgap narrowing in N-type and P-type silicon.

Such a conclusion was also arrived at by Wagner [110], who measured the bandgap narrowing by PL spectroscopy for N-type (P) and P-type (B) silicon. In a subsequent paper, however, Wagner et al. [112] found a somewhat larger bandgap narrowing for Ga-doped silicon than for B-doped silicon. This indicates that bandgap narrowing is indeed dependent of the chemical nature of the impurity, but with almost negligible differences.

Temperature dependence

It is important for modelling purposes to know whether the bandgap narrowing is dependent on temperature. Some authors already assumed a bandgap narrowing independent of temperature. Slotboom and de Graaff [91], for example, used this assumption in their second method of reference [91] in order to extract the bandgap narrowing from the measured collector currents at several temperatures. Neugroschel et al. [73] used the same method. However, no experimental proof from electrical device measurements exists to support this assumption. This is entirely due to the fact that no experimental data existed on the minority carrier mobility as a function of temperature, which is essential for the extraction of the bandgap narrowing. Swirhun [93] did some temperature dependent measurements and found a large temperature dependence of the bandgap narrowing, but he only measured the diffusion length as a function of temperature and guessed the temperature dependence of the mobility by assuming some temperature dependence of the lifetime.
In this work the temperature dependence of the minority hole mobility and the quantity $p_0 D_p$ have been measured. This enables us, for the first time, to determine the temperature dependence of the apparent bandgap narrowing.

Results of the extracted $p_0$ as a function of temperature are shown in Figure 5.14. In the temperature range of 100 K to 300 K, $p_0$ changes from $10^{-40}$ to $10^3$ cm$^{-3}$. However, these values must not be taken too literally, since a value of $10^{-6}$ cm$^{-3}$ already means that only one hole can be found in a cubic centimetre of silicon.

Converting the measured $p_0$ into an apparent bandgap narrowing, using equation 5.15, is a little more complicated since it requires the detailed knowledge of the temperature dependence of the intrinsic concentration $n_{io}$. As mentioned by Green [35], direct measurements of $n_{io}$ only exist for temperatures above 350 K. At these temperatures, low doped silicon becomes intrinsic and $n_{io}$ can be extracted from the measured conductivity which is equal to:

\[ \sigma = q(\mu_n + \mu_p)n_{io} \]  \hspace{1cm} (5.17)

To obtain $n_{io}$ for lower temperatures, however, one must return to the well-known relation:

\[ n_{io} = N_C N_V \exp\left(-\frac{E_g}{kT}\right) \]  \hspace{1cm} (5.18)

where $N_C$ and $N_V$ are defined by:

\[ N_C = 2(2\pi m^*_d kT/h^2)^{3/2} \]  \hspace{1cm} (5.19)

\[ N_V = 2(2\pi m^*_a kT/h^2)^{3/2} \]  \hspace{1cm} (5.20)

and fill in the independently measured temperature dependence of the bandgap $E_g$ and the effective masses $m^*_d$ and $m^*_a$.

Figure 5.15 shows the apparent bandgap narrowing as a function of temperature. To calculate the apparent bandgap narrowing, the following bandgap as a function of temperature is used:

\[ E_g(T) = 1.17 - \frac{4.73 \cdot 10^{-4} T^2}{T + 636} \]  \hspace{1cm} (5.21)

which describes the bandgap data mentioned by Green [35] sufficiently accurate. In the calculation, the temperature dependences of the effective masses are ignored so that $n_{io}$ can be expressed as:

\[ n_{io}^2 = N_{C300} N_{V300} \left(\frac{T}{300}\right)^3 \exp\left(\frac{E_g(T)}{kT}\right) \]  \hspace{1cm} (5.22)
Figure 5.14: Measured hole concentration in equilibrium $p_0$ as a function of temperature.
Figure 5.15: The apparent bandgap narrowing versus temperature, ignoring the temperature dependence of the effective masses.
where $N_{C300} = 2.86 \cdot 10^{19}$ and $N_{V300} = 3.10 \cdot 10^{19}$ cm$^{-3}$, as suggested by Green [35]. As can be seen in Figure 5.15, the bandgap narrowing can be considered to be almost temperature independent. Note that, since a different $n_{io}$ is used to calculate $\Delta E_g^{app}$, the values are shifted 5 meV with respect to the values shown in Figure 5.13.

Figure 5.16 shows the results when equations 5.18, 5.19 and 5.20 are used together with equation 5.21 and the temperature dependence of the effective masses, as described by Green [35]. Here, the bandgap narrowing becomes even more constant at higher temperatures, and at lower temperatures a slight increase in bandgap narrowing can be seen. However, considering the experimental errors one can conclude that the bandgap narrowing measured here can be taken as a constant. In this calculation, complete ionisation of the donor impurities was assumed. For the doping concentrations considered, this is not true (at room temperature, approximately 90% ionisation). However, apart from a small rigid shift of the bandgap narrowing, the corrections to the temperature dependence are negligibly small and the conclusion does not change. This conclusion is also supported by optical measurements, which yield a direct measurement of the bandgap narrowing. Wagner [110] measured the bandgap narrowing at 20 K and 300 K and also found negligible differences.
Figure 5.16: The apparent bandgap narrowing versus temperature with the temperature dependence of the effective masses taken into account.
CHAPTER 6

CONCLUSIONS

6.1 CONTRIBUTIONS

The main contributions of this work to the field of measurement methods for the determination of minority carrier transport parameters in heavily doped silicon are:

1. A new method is presented which enables the independent measurement of the diffusion length, the minority carrier mobility (or the diffusion coefficient), the minority carrier lifetime and the minority carrier concentration in equilibrium (or the apparent bandgap narrowing).

2. From DC-measurements on the multi-collector lateral transistors the diffusion length and the product of the minority carrier concentration and diffusion coefficient can be obtained. The measurement is very tolerant to the uncertainty in surface recombination and in the geometry of the devices.

3. From AC-measurements on lateral transistors, the minority carrier mobility and lifetime can be obtained. Due to a large basewidth of approximately one diffusion length, the base transit time is large, in which case measurements can be performed at relatively low frequencies.

4. The measurement of a redundant set of parameters, the diffusion length and the minority carrier mobility and lifetime, is a powerful tool to check whether the parameters are consistent.

The main contributions of this work with respect to measurement results are:

1. Measurements are performed on N and P-type silicon. For the first time, the minority transport parameters are also determined as a function of temperature (77 K - 300 K). This is done for two doping concentrations ($1.90 \cdot 10^{18}$ and $2.55 \cdot 10^{18}$ cm$^{-3}$, N-type silicon).
2. The measurements of the minority carrier mobility for concentrations above $10^{18} \text{ cm}^{-3}$ are consistent with data from the literature. The mobility is higher than the majority carrier mobility. However, the obtained hole mobility at a concentration of $9.55 \cdot 10^{16} \text{ cm}^{-3}$ is in disagreement with the results of Dziewior and Silber. The minority mobility is found to be almost equal to the majority mobility.

3. The measured minority hole mobility shows a strong temperature dependence and increases with decreasing temperature. This is in contrast with the majority carrier mobility, which is almost constant with temperature for the measured doping concentrations.

4. The measured temperature dependence of the mobility indicates that the mobility remains a decreasing function of doping concentration and not an increasing function as concluded by Wang et al. and Swirhun et al. for low temperatures.

5. The Auger coefficient extracted from minority hole lifetime measurements is somewhat larger than the values generally adopted for higher concentrations. This suggests that the Auger coefficient is a function of doping concentration.

6. The measured lifetime as a function of temperature shows a $T^{0.44}$ dependence for the Auger coefficient and a $T^{-1.72}$ dependence for the SRH-lifetime.

7. Determination of the apparent bandgap narrowing at room temperature in N and P-type silicon yields the same bandgap narrowing, irrespective of the impurity type.

8. Determination of the apparent bandgap narrowing as a function of temperature indicates that the bandgap narrowing is independent of temperature.

6.2 Recommendations for future work

Suggestions for future work are:

1. More experimental work concerning the measurement of the minority carrier mobility as a function of temperature for a whole range of doping concentrations is necessary. Such measurements should however not be restricted to silicon because other materials, such as SiGe-alloys, become increasingly important.
2. From a theoretical point of view it is interesting to measure the mobility at very low temperatures (< 10 K), in which case the Anderson localization and the related mobility edge could affect the mobility.

3. Thorough theoretical calculations of scattering processes are necessary to understand the behaviour of the minority carrier mobility.
APPENDIX A

SUMMARY OF MEASUREMENT RESULTS

In this appendix the results shown and discussed in chapter 5 are summarized. The first section displays the results of the measurements performed at room temperature and the second section at temperatures from 77 K until 300 K.

A.1 MEASUREMENTS AT ROOM TEMPERATURE

The measured minority carrier mobility and lifetime of this work are shown in Table A.1. Here, also the diffusion length computed from the mobility and lifetime is given. Table A.2 depicts the results from the DC-measurements

<table>
<thead>
<tr>
<th>N  (cm(^{-3}))</th>
<th>T (K)</th>
<th>(\mu) (cm(^2)V(^{-1})s(^{-1}))</th>
<th>(\tau) (ns)</th>
<th>L(AC) ((\mu)m)</th>
<th>batch/wafer</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.55 (\cdot 10^{18}) (Sb)</td>
<td>299.4</td>
<td>278</td>
<td></td>
<td></td>
<td>E14724 F/#3</td>
</tr>
<tr>
<td>1.90 (\cdot 10^{18}) (Sb)</td>
<td>299.0</td>
<td>193</td>
<td>605</td>
<td>17.4</td>
<td>E11511 BD/#6</td>
</tr>
<tr>
<td>2.55 (\cdot 10^{18}) (Sb)</td>
<td>297.0</td>
<td>179</td>
<td>371</td>
<td>13.1</td>
<td>E11511 BD/#2</td>
</tr>
<tr>
<td>7.07 (\cdot 10^{18}) (B)</td>
<td>296.29</td>
<td>230</td>
<td>25.2</td>
<td>3.87</td>
<td>E11511 BD/#20</td>
</tr>
</tbody>
</table>

Table A.1: Results of AC-measurements done at room temperature.

at room temperature. The listed apparent bandgap narrowing \(\Delta E_g^{app}\) has been computed from the DC-transport parameters using the mobility data of Table A.1 and a \(n_{io}\) of 1.19\(\cdot 10^{10}\) cm\(^{-3}\) at 300K where \(n_{io}\) has been corrected for the differences in temperatures.
<table>
<thead>
<tr>
<th>N (cm⁻³)</th>
<th>T (K)</th>
<th>L(DC) (μm)</th>
<th>pₒD_p,nₒD_n (cm⁻¹s⁻¹)</th>
<th>ΔE_{g}^{app} (meV)</th>
<th>batch/wafer</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.55 ·10¹⁶ (Sb)</td>
<td>299.4</td>
<td>18.66</td>
<td>1.32 · 10⁴</td>
<td>6.66</td>
<td>E14724 F/#3</td>
</tr>
<tr>
<td>1.90 ·10¹⁸ (Sb)</td>
<td>299.79</td>
<td>13.56</td>
<td>1.22 · 10³</td>
<td>31.8</td>
<td>E11511 BD/#6</td>
</tr>
<tr>
<td>2.55 ·10¹⁸ (Sb)</td>
<td>298.72</td>
<td>3.87</td>
<td>7.82 · 10²</td>
<td>34.1</td>
<td>E11511 BD/#2</td>
</tr>
<tr>
<td>7.07 ·10¹⁸ (B)</td>
<td>296.29</td>
<td>5.43 · 10²</td>
<td>55.2</td>
<td></td>
<td>E11511 BD/#20</td>
</tr>
</tbody>
</table>

Table A.2: Results of DC-measurements done at room temperature.

### A.2 Measurements at low temperatures

Table A.3 lists the results of the AC-transport parameters for a doping concentration of 1.90·10¹⁸ cm⁻³ and Table A.4 gives the results for a doping concentration of 2.55·10¹⁸ cm⁻³.

<table>
<thead>
<tr>
<th>Batch/Wafer: E11511 BD, #6</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-type (Sb)</td>
</tr>
<tr>
<td>N = 1.90 ·10¹⁸ cm⁻³</td>
</tr>
<tr>
<td>T (K)</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>77.0</td>
</tr>
<tr>
<td>99.7</td>
</tr>
<tr>
<td>123.5</td>
</tr>
<tr>
<td>147.7</td>
</tr>
<tr>
<td>172.5</td>
</tr>
<tr>
<td>197.3</td>
</tr>
<tr>
<td>222.3</td>
</tr>
<tr>
<td>247.5</td>
</tr>
<tr>
<td>273.2</td>
</tr>
<tr>
<td>299.0</td>
</tr>
</tbody>
</table>

Table A.3: Results of AC-measurements at several temperatures.

Finally Table A.5 and A.6 show the results obtained from low temperature DC-measurements for the same two doping concentrations. Again the bandgap narrowing is computed from the AC-transport parameters using the mobility data of Table A.3 and A.4. For the temperature dependence of the in-
Batch/Wafer: E11511 BD, #2
N-type (Sb)
N = 2.55 \cdot 10^{18} \text{ cm}^{-3}

<table>
<thead>
<tr>
<th>T (K)</th>
<th>D_p (cm^2s^{-1})</th>
<th>\mu_p (cm^2V^{-1}s^{-1})</th>
<th>\tau_p (ns)</th>
<th>L_p(AC) (\mu m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>77.0</td>
<td>5.25</td>
<td>791</td>
<td>359</td>
<td>13.7</td>
</tr>
<tr>
<td>100.0</td>
<td>5.44</td>
<td>632</td>
<td>366</td>
<td>14.1</td>
</tr>
<tr>
<td>123.5</td>
<td>5.56</td>
<td>522</td>
<td>393</td>
<td>14.8</td>
</tr>
<tr>
<td>147.7</td>
<td>6.54</td>
<td>436</td>
<td>403</td>
<td>14.9</td>
</tr>
<tr>
<td>172.5</td>
<td>5.44</td>
<td>366</td>
<td>412</td>
<td>15.0</td>
</tr>
<tr>
<td>197.3</td>
<td>5.26</td>
<td>309</td>
<td>401</td>
<td>14.5</td>
</tr>
<tr>
<td>222.0</td>
<td>5.14</td>
<td>269</td>
<td>411</td>
<td>14.5</td>
</tr>
<tr>
<td>247.5</td>
<td>5.95</td>
<td>232</td>
<td>391</td>
<td>13.9</td>
</tr>
<tr>
<td>273.0</td>
<td>5.74</td>
<td>201</td>
<td>370</td>
<td>13.2</td>
</tr>
<tr>
<td>297.0</td>
<td>4.58</td>
<td>179</td>
<td>371</td>
<td>13.1</td>
</tr>
</tbody>
</table>

Table A.4: Results of AC-measurements at several temperatures.

Intrinsic carrier concentration Equations 5.18-5.21 have been used together with
the temperature dependent effective mass according to Green [35]
### Table A.5: Results of DC-measurements at several temperatures.

<table>
<thead>
<tr>
<th>T (K)</th>
<th>L_p(DC) (µm)</th>
<th>p_cD_p (cm⁻¹s⁻¹)</th>
<th>Δ E_q^{app} (meV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>85.06</td>
<td>17.53</td>
<td>9.58⋅10⁻⁴⁸</td>
<td>42</td>
</tr>
<tr>
<td>100.89</td>
<td>18.94</td>
<td>5.88⋅10⁻³⁷</td>
<td></td>
</tr>
<tr>
<td>123.35</td>
<td>19.86</td>
<td>2.04⋅10⁻²⁶</td>
<td></td>
</tr>
<tr>
<td>148.49</td>
<td>21.25</td>
<td>2.87⋅10⁻¹⁸</td>
<td></td>
</tr>
<tr>
<td>174.10</td>
<td>21.06</td>
<td>2.23⋅10⁻¹²</td>
<td></td>
</tr>
<tr>
<td>197.99</td>
<td>21.18</td>
<td>3.36⋅10⁻⁸</td>
<td></td>
</tr>
<tr>
<td>224.26</td>
<td>20.76</td>
<td>1.35⋅10⁻⁴</td>
<td></td>
</tr>
<tr>
<td>299.79</td>
<td>18.66</td>
<td>1.22⋅10³</td>
<td></td>
</tr>
</tbody>
</table>

### Table A.6: Results of DC-measurements at several temperatures.

<table>
<thead>
<tr>
<th>T (K)</th>
<th>L_p(DC) (µm)</th>
<th>p_cD_p (cm⁻¹s⁻¹)</th>
<th>Δ E_q^{app} (meV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>92.8</td>
<td>15.06</td>
<td>2.29⋅10⁻⁴²</td>
<td></td>
</tr>
<tr>
<td>99.59</td>
<td>15.19</td>
<td>7.85⋅10⁻³⁸</td>
<td></td>
</tr>
<tr>
<td>123.50</td>
<td>15.40</td>
<td>2.03⋅10⁻²⁶</td>
<td></td>
</tr>
<tr>
<td>147.43</td>
<td>15.63</td>
<td>1.21⋅10⁻¹⁸</td>
<td></td>
</tr>
<tr>
<td>172.65</td>
<td>15.57</td>
<td>1.06⋅10⁻¹²</td>
<td></td>
</tr>
<tr>
<td>197.83</td>
<td>15.46</td>
<td>2.46⋅10⁻⁸</td>
<td></td>
</tr>
<tr>
<td>223.08</td>
<td>15.28</td>
<td>8.07⋅10⁻⁵</td>
<td></td>
</tr>
<tr>
<td>248.43</td>
<td>14.97</td>
<td>4.49⋅10⁻²</td>
<td></td>
</tr>
<tr>
<td>298.72</td>
<td>13.56</td>
<td>7.82⋅10²</td>
<td></td>
</tr>
<tr>
<td>329.0</td>
<td>13.39</td>
<td>5.02⋅10⁴</td>
<td></td>
</tr>
</tbody>
</table>
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124


125
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{cb}$</td>
<td>collector-base depletion capacitance</td>
<td>F</td>
</tr>
<tr>
<td>$C_d$</td>
<td>diffusion capacitance of emitter</td>
<td>F</td>
</tr>
<tr>
<td>$C_T$</td>
<td>emitter-base depletion capacitance</td>
<td>F</td>
</tr>
<tr>
<td>$D_p$</td>
<td>hole diffusion coefficient</td>
<td>cm$^2$s$^{-1}$</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_T$</td>
<td>cut off frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>$E$</td>
<td>energy</td>
<td>eV</td>
</tr>
<tr>
<td>$E_F$</td>
<td>Fermi level</td>
<td>eV</td>
</tr>
<tr>
<td>$E_{FP}$</td>
<td>Quasi Fermi level of holes</td>
<td>eV</td>
</tr>
<tr>
<td>$E_g$</td>
<td>energy bandgap</td>
<td>eV</td>
</tr>
<tr>
<td>$H$</td>
<td>Hessian matrix</td>
<td></td>
</tr>
<tr>
<td>$i_c$</td>
<td>small signal collector current</td>
<td>A</td>
</tr>
<tr>
<td>$I_C$</td>
<td>collector current</td>
<td>A</td>
</tr>
<tr>
<td>$I_E$</td>
<td>emitter current</td>
<td>A</td>
</tr>
<tr>
<td>$I_{E_{pinj}}$</td>
<td>current injected in the emitter</td>
<td>A</td>
</tr>
<tr>
<td>$I_{E_{pinj}}$</td>
<td>current injected in the base</td>
<td>A</td>
</tr>
<tr>
<td>$I_m$</td>
<td>measured collector current of the i-th collector</td>
<td>A</td>
</tr>
<tr>
<td>$I_{c}^i$</td>
<td>calculated collector current of the i-th collector</td>
<td>A</td>
</tr>
<tr>
<td>$I_S$</td>
<td>saturation current</td>
<td>A</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann's constant</td>
<td>JK$^{-1}$</td>
</tr>
<tr>
<td>$L$</td>
<td>inductance</td>
<td>H</td>
</tr>
<tr>
<td>$L_p$</td>
<td>hole diffusion length</td>
<td>m</td>
</tr>
<tr>
<td>$L_p^*$</td>
<td>complex hole diffusion length</td>
<td>m</td>
</tr>
<tr>
<td>$p$</td>
<td>parameter vector</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>hole concentration</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>small-signal amplitude of hole concentration</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$p_0$</td>
<td>minority carrier hole concentration in equilibrium</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n$</td>
<td>electron concentration</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{ie}$</td>
<td>effective intrinsic carrier concentration</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
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<td>intrinsic carrier concentration</td>
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</tr>
<tr>
<td>$N$</td>
<td>number of measurements</td>
<td></td>
</tr>
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<td>$N_A$</td>
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<td>cm$^{-3}$</td>
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<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>(N_D)</td>
<td>donor concentration</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(N_C)</td>
<td>effective density of states of the conduction band</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(N_V)</td>
<td>effective density of states of the valence band</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(M)</td>
<td>number of parameters</td>
<td></td>
</tr>
<tr>
<td>(q)</td>
<td>magnitude of electronic charge</td>
<td>C</td>
</tr>
<tr>
<td>(r_{cb})</td>
<td>collector base resistance</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(r_e)</td>
<td>differential resistance of emitter</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(R_s)</td>
<td>serie resistance</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(R_e)</td>
<td>emitter serie resistance</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(R_b)</td>
<td>base serie resistance</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(s)</td>
<td>surface recombination velocity</td>
<td>cm (s^{-1})</td>
</tr>
<tr>
<td>(S)</td>
<td>scattering parameter</td>
<td></td>
</tr>
<tr>
<td>(t)</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>(T)</td>
<td>absolute temperature</td>
<td>K</td>
</tr>
<tr>
<td>(v_{cb})</td>
<td>collector base AC-voltage</td>
<td>V</td>
</tr>
<tr>
<td>(v_{eb})</td>
<td>emitter base AC-voltage</td>
<td>V</td>
</tr>
<tr>
<td>(V)</td>
<td>voltage</td>
<td>V</td>
</tr>
<tr>
<td>(V_{EB})</td>
<td>emitter base bias voltage</td>
<td>V</td>
</tr>
<tr>
<td>(W_b)</td>
<td>base width</td>
<td>m</td>
</tr>
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<td>(x)</td>
<td>lateral space coordinate</td>
<td>m</td>
</tr>
<tr>
<td>(y)</td>
<td>vertical space coordinate</td>
<td>m</td>
</tr>
<tr>
<td>(y_m)</td>
<td>measurement values in general</td>
<td>S</td>
</tr>
<tr>
<td>(Y_{21})</td>
<td>transadmittance</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(Z_b)</td>
<td>impedance of base</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(Z_o)</td>
<td>characteristic impedance</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>dielectric permittivity</td>
<td>(Fm^{-1})</td>
</tr>
<tr>
<td>(\mu_p)</td>
<td>(minority) hole mobility</td>
<td>cm(^2)(V)(^{-1})(s)(^{-1})</td>
</tr>
<tr>
<td>(\Psi)</td>
<td>electrostatic potential</td>
<td>V</td>
</tr>
<tr>
<td>(\phi_p)</td>
<td>hole quasi-Fermi potential</td>
<td>V</td>
</tr>
<tr>
<td>(\phi_n)</td>
<td>electron quasi-Fermi potential</td>
<td>V</td>
</tr>
<tr>
<td>(\rho)</td>
<td>bulk resistivity</td>
<td>(\Omega cm)</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>standard deviation of measurements</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>(\tau_b)</td>
<td>base transit time</td>
<td>s</td>
</tr>
<tr>
<td>(\tau_p)</td>
<td>hole lifetime</td>
<td>s</td>
</tr>
<tr>
<td>(\omega)</td>
<td>circular frequency</td>
<td>radians</td>
</tr>
<tr>
<td>(\xi)</td>
<td>least means square error</td>
<td>1</td>
</tr>
</tbody>
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127
SUMMARY

With further scale integration and increasing operating frequencies high doping levels in semiconductor devices become more and more important. However, high doping concentrations degrade the transport properties of minority carriers as is important for devices such as bipolar transistors. The degradation can be seen in the minority carrier mobility which decreases due to extra scattering mechanisms such as impurity scattering and carrier-carrier scattering. The minority carrier lifetime becomes shorter due to Auger recombination and due to the presence of impurities a narrowing of the energy bandgap occurs.

To describe and predict the behaviour of bipolar transistors it is thus important to know the transport parameters, which are the minority carrier mobility, the carrier lifetime and the apparent bandgap narrowing as a function of doping concentration. Despite considerable experimental work many questions remain unanswered and some results even conflict with each other. In addition, no experimental data of the transport parameters exist as a function of temperature. Therefore, this thesis deals with the measurement of the transport parameters not only as function of doping concentration but also as a function of temperature.

To measure the transport parameters a new measurement method is used which has some advantages above existing methods. The values of the parameters obtained by this method are not affected by the presence of surface recombination. In addition, this method facilitates the measurement of all the transport parameters in the same substrate with the same type of test structures, and, finally, it is possible to measure the mobility, lifetime and the diffusion length independently. The latter is important to check whether the data obtained are consistent.

The presented method consists of two parts. The first part concerns the extraction of the diffusion length and the mobility times the minority carrier concentration in equilibrium (a quantity describing more fundamentally the effect of bandgap narrowing). These parameters are extracted from steady-state measurements on lateral transistors with several collectors in a heavily doped substrate. The second part of the measurement method deals with the extraction of the minority carrier mobility and lifetime. This is accomplished by measuring the frequency response and in particular the phase shift of the transadmittance of lateral transistors. Extraction of the parameters from the measurements is an inverse problem. Therefore, to obtain the parameters optimization techniques were used in combination with device simulators.
The measurement results indicate that the minority hole mobility at higher doping concentrations is approximately two times higher than the majority hole mobility as was found also by other workers. However, the hole mobility at lower concentrations ($\approx 10^{17} \text{ cm}^{-3}$) equals the majority mobility in which case the minority hole mobility was always over estimated in this doping range. The minority hole mobility for two doping concentrations has been measured for the first time as a function of temperature (77 K until 300 K). The results show a large increase in mobility with decreasing temperature. Calculations using the mobility model of Klaassen suggest that electron-hole scattering is the dominant scattering mechanism especially at low temperatures.

The measured minority carrier lifetime data agree with data from literature. The lifetime obtained as a function of temperature shows two distinct recombination mechanisms, Shockley-Read-Hall recombination at low temperatures and Auger recombination at high temperatures. Results indicate that the Auger lifetime behaves less than quadratic with doping concentration.

The apparent bandgap narrowing determined in both N and P-type silicon suggests that bandgap narrowing is not dependent of doping type. This result is in contradiction with the well-known models, the model of Slotboom and de Graaff for P-type silicon and the model of del Alamo for N-type silicon. However, the results of this work agree with the model of Klaassen, Slotboom and de Graaff who reviewed the bandgap narrowing data from literature. For the first time the apparent bandgap narrowing has been determined as a function of temperature by electrical measurements. From 100 K to 300 K the apparent bandgap narrowing show very little or no temperature dependence.
SAMENVATTING

Zware doteringen in halfgeleiderstructuren worden steeds belangrijker naar mate de structuren kleiner worden en de werkfrequenties hoger liggen. Het blijkt dat zware dotering de transport van minderheidsladingsdragers nadelig beïnvloedt, wat belangrijk kan zijn voor de prestaties van bijvoorbeeld bipolaire transistoren. Zo wordt de beweeglijkheid van minderheden kleiner door extra verstrooingsmechanismen zoals verstrooing aan doteringsatomen en aan meerderheidsladingsdragers. De levensduur van minderheden wordt korter door Augerrekombinatie en er treedt een vernauwing van de verboden band op.

Om het gedrag van bipolaire transistoren te kunnen beschrijven is het dus belangrijk om de transportparameters, namelijk de beweeglijkheid van minderheden, de levensduur en de bandvernaauwing als functie van de dotering te weten. Ondanks vele inspanningen in de literatuur blijven veel vragen nog onbeantwoord en enkele resultaten spreken elkaar zelfs tegen. Er bestaan ook geen metingen van de transportparameters als functie van de temperatuur. Dit proefschrift gaat dan ook over het meten van de transportparameters als functie van dotering en temperatuur.

Voor het meten van de transportparameters wordt een nieuwe methode gebruikt die een aantal voordelen heeft boven bestaande meetmethoden. Zo worden de meetresultaten niet of nauwelijks beïnvloed door oppervlakte rekombinatie, zodat alleen de bulk eigenschappen gemeten worden en met deze methode is het mogelijk om alle transporteigenschappen te meten in hetzelfde substraat met dezelfde type meetstructuren. Ook kan met deze methode niet alleen de beweeglijkheid en levensduur gemeten worden maar ook onafhankelijk daarvan de diffusiellengte. Dit laatste zorgt ervoor dat gecontroleerd kan worden of de meetresultaten consistent zijn.

De nieuwe meetmethode bestaat uit twee gedeelten. Het eerste gedeelte bestaat uit het bepalen van de diffusiellengte en het produkt van de beweeglijkheid met de evenwichtsconcentratie van minderheden (een parameter die het effect van bandvernaauwing fundamenteel beter beschrijft) uit steady-state metingen aan laterale transistoren met meerdere collectoren. Het tweede gedeelte omvat het bepalen van de beweeglijkheid en levensduur uit de frequentieverdracht van laterale transistoren. Extractie van de parameters uit de metingen is een invers probleem. Om de parameters uit de metingen te extraheren wordt gebruik gemaakt van optimaliseringstechnieken in combinatie met device simulatoren.
De meetresultaten geven aan dat voor hoge doteringsconcentraties de beweeglijkheid van gaten als minderheden ongeveer twee keer hoger is dan de beweeglijkheid van gaten als meerderheden. Dit was al eerder door anderen gevonden. Echter de gevonden beweeglijkheid voor lagere concentraties ($\approx 10^{17}$ cm$^{-3}$) is beduidend lager dan was aangenomen in de literatuur. De beweeglijkheid van minderheden is hier ongeveer gelijk aan de beweeglijkheid van meerderheden. Voor het eerst is de beweeglijkheid van minderheden gemeten als functie van temperatuur (77 K tot 300 K). Het blijkt dat de beweeglijkheid van minderheden sterk toeneemt bij lagere temperaturen terwijl de beweeglijkheid van meerderheden niet of nauwelijks van de temperatuur afhangt. Berekeningen met het model van Klaassen tonen aan dat, vooral bij lagere temperaturen, elektron-gat verstrooiing het dominante verstrooiingsmechanisme is.

De gemeten levensduur van gaten en elektronen komen goed overeen met de gegevens uit de literatuur. In de gemeten temperatuursafhankelijkheid van de levensduur zijn duidelijk twee rekombinatieprocessen te zien. Het eerste proces is ten gevolge van Shockley-Read-Hall rekombinatie en is voor de gemeten doteringsconcentraties dominant bij lage temperaturen. Het tweede proces is Augerrekombinatie en is belangrijk bij hoge temperaturen. Resultaten doen vermoeden dat Augerrekombinatie zich minder dan kwadratisch gedraagt als functie van de dotering.

De meetresultaten betreffende bandvernauwing geven aan dat de bandvernauwing onafhankelijk is van het doteringstype (N of P-type). Dit resultaat is in tegenspraak met het bekende model van Slotboom en de Graaff voor P-type silicium en met het model van del Alamo voor N-type silicium. Wanneer echter de gegevens van bandvernauwing uit de literatuur wordt herzien zoals gedaan is door Klaassen, Slotboom en de Graaff, dan blijkt onderdaad de bandvernauwing onafhankelijk te zijn van het type dotering. In dit werk is de bandvernauwing voor het eerst gemeten als functie van de temperatuur. Van 100 K tot 300 K verandert de bandvernauwing niet of nauwelijks met de temperatuur.
Biography

Fred van Rijs was born in Amsterdam, the Netherlands, on April 2, 1965. He graduated in 1983 from the atheneum of the Jan van Scorel College in Alkmaar and studied Electrical Engineering at the Delft University of Technology, Delft, the Netherlands, from which he graduated in 1987 (cum laude).

In December 1987, he became a research assistant at the Electrical Materials Laboratory of the same university, where he has worked on the measurement of heavy doping effects in silicon.

In June 1992 he joined Philips Research Laboratories, Eindhoven, the Netherlands, where he is engaged in the research on advanced semiconductor devices.
ACKNOWLEDGEMENTS

The research presented in this thesis, partly financed by IOP under contract IC-DEL88.032, could not have been carried out without the help of many people.

First of all I want to thank Gertjan Ouwerling the for the many discussions concerning optimization topics and for his program PROFILE, which made it possible to implement the optimization scheme used in this work quite easy. I am also deeply indebted to Jan-Chris Staalenburg for his technical assistance in building up the measurement setup.

Further, I am much obliged to Philips Research Laboratories for the fabrication of the devices. In particular I want to thank dr. D.B.M. Klaassen for his support in organizing the fabrication and for the interesting discussions concerning heavy doping effects and dr. W. van der Wel for the processing of the devices.

Furthermore, I wish to thank W. van der Vlist of DIMES for the careful packaging of the devices, Philips Nijmegen and ir. M.P.J. Versleijen for the high frequency packages, ir. A. Heringa for letting me use the device simulator CURRY and R. van Wijk for the manufacturing of the transistor test-fixture. Also the discussions with dr. J. Tauritz and ing. L. de Vreede concerning high frequency matters are much appreciated.

In addition, I want to thank the staff and students of the Electrical Materials Laboratory and in particular dr. W. Crans and ir. M.J. Swanenberg. It was a pleasure to work in this group.

Finally, I wish to thank my wife Lucy and my parents for their encouragement and moral support.