The Fusion of SAR Tomography and Stereo-SAR for 3D Absolute Scatterer Positioning

Sina Montazeri
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On cover:
The two-dimensional TomoSAR point cloud of city of Berlin, colour-coded based on elevation, after the fusion of results from an ascending and a descending track. (processed by Tomo-GENESIS of DLR)
The Fusion of SAR Tomography and Stereo-SAR for 3D Absolute Scatterer Positioning

A thesis submitted to the Delft University of Technology in partial fulfilment of the requirements for the degree of

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by

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Abstract

For decades, spaceborne Synthetic Aperture Radar (SAR) interferometry has evolved into a widely used geodetic method for three-dimensional mapping and analysing the geophysical processes of the surface of the earth. Nevertheless, effects such as atmospheric disturbances, temporal and geometric decorrelation are considered as the main degrading factors for the quality of Interferometric Synthetic Aperture Radar (InSAR) products. By overcoming the major deficiencies of the conventional InSAR, Persistent Scatterer Interferometry (PSI) proved the applicability of the interferometric methods also in monitoring of urban areas by restricting the analysis only on the time-coherent scatterers. In particular, the launch of modern SAR sensors, such as the German TerraSAR-X, characterized with very high spatial resolution and short revisit times has enhanced the capability of PSI in mapping and deformation monitoring of urban areas. However, PSI considers only a single dominant scatterer in each resolution cell which is not a valid assumption in urban areas due to the prevalent occurrences of layover phenomenon. This gives the motivation to exploit the most advanced SAR interferometric method, namely tomographic SAR inversion (TomoSAR) including SAR Tomography and differential SAR Tomography. TomoSAR coupled with very high resolution TerraSAR-X images produces the most detailed multi-dimensional maps of urban areas by distinguishing among multiple scatterers within a resolution cell. Nevertheless in TomoSAR, similar to conventional InSAR and PSI, elevation and deformation rates are estimated with respect to a previously chosen reference point which makes them relative 3D estimates.

One unique feature of TerraSAR-X is the precise orbit determination and high geometrical localization accuracy of the sensor. After compensating for the most prominent geodynamics and atmospheric error sources, the absolute two-dimensional (range and azimuth) positions of targets such as corner reflectors and persistent scatterers can be estimated to centimetre-level accuracy, a method called SAR Imaging Geodesy. Moreover, using two or more SAR observations acquired from different satellite orbits, their absolute 3D positions can be retrieved by means of Stereo-SAR.

In this thesis a framework is proposed that fuses the SAR imaging geodesy and TomoSAR approaches to obtain absolute 3D positions of a large amount of natural scatterers. The method is further utilized in order to automatically fuse multi-track TomoSAR point clouds. The methodology is applied on four Very High Resolution (VHR) TerraSAR-X spotlight image stacks acquired from the city of Berlin. The horizontal and vertical localization accuracy of the obtained fused point cloud is analysed by comparing them with a highly accurate geo-localized Digital Surface Model (DSM) of Berlin obtained from aerial laser scanning. The fusion of absolute TomoSAR point clouds obtained from cross-heading tracks also allows for the decomposition of the Line of Sight (LOS) motion observations into the real 3D motion vectors. As the final contribution of this study, the 3D motion decomposition of seasonal deformation observations is carried out on two test sites located in the city of Berlin.
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<th>Definition</th>
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<tr>
<td>APS</td>
<td>Atmospheric Phase Screen</td>
</tr>
<tr>
<td>CS</td>
<td>Compressive Sensing</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DIA</td>
<td>Detection, Identification and Adaptation</td>
</tr>
<tr>
<td>DLR</td>
<td>German Aerospace Center</td>
</tr>
<tr>
<td>DSM</td>
<td>Digital Surface Model</td>
</tr>
<tr>
<td>GENESIS</td>
<td>GENEric System for Interferometric SAR</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite Systems</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>IAS</td>
<td>Image Analysis Software</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>IERS</td>
<td>International Earth Rotation Service</td>
</tr>
<tr>
<td>IGS</td>
<td>International GNSS Service</td>
</tr>
<tr>
<td>InSAR</td>
<td>Interferometric Synthetic Aperture Radar</td>
</tr>
<tr>
<td>IP</td>
<td>Ionospheric Piercing Point</td>
</tr>
<tr>
<td>LOS</td>
<td>Line of Sight</td>
</tr>
<tr>
<td>PRF</td>
<td>Pulse Repetition Frequency</td>
</tr>
<tr>
<td>PSI</td>
<td>Persistent Scatterer Interferometry</td>
</tr>
<tr>
<td>PTA</td>
<td>Point Target Analysis</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Squared Error</td>
</tr>
<tr>
<td>RSF</td>
<td>Range Sampling Frequency</td>
</tr>
<tr>
<td>SAR</td>
<td>Synthetic Aperture Radar</td>
</tr>
<tr>
<td>SLM</td>
<td>Single Layer Model</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>TEC</td>
<td>Total Electron Content</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
</tr>
<tr>
<td>VHR</td>
<td>Very High Resolution</td>
</tr>
<tr>
<td>VMF1</td>
<td>Vienna Mapping Function 1</td>
</tr>
<tr>
<td>vTEC</td>
<td>vertical Total Electron Content</td>
</tr>
<tr>
<td>ZPD</td>
<td>Zenith Path Delay</td>
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Chapter 1

Introduction

This chapter describes the necessary background and the motivation of conducting this research. The chapter also introduces the investigated test site as well as the available datasets used for the study. It is finalized with outlining the contents of the remaining chapters.

1.1 Background

Long term deformation monitoring in urban areas is of great importance since it leads to early detection of possible subsidence or uplift of individual buildings. Gradual subsidence may eventually cause the building to collapse and consequently risks the lives of the inhabitants. This gives the motivation to exploit gap-less regular spaced time series of datasets which also provide suitable coverage of the area (Soergel, 2010). These characteristics best describe the products of Synthetic Aperture Radar (SAR) technique and its more advanced interferometric extensions. SAR is the only remote sensing technique that is able to provide centimetre and millimetre deformation rates from space with large scale coverage. Furthermore, its ability to work in any kind of weather conditions and operation without any external source of illumination makes it one of the most popular techniques among its counterparts (Bamler and Hartl, 1998, Hanssen, 2001).

The launch of high resolution SAR systems such as the German TerraSAR-X has paved the way for analysing the deformation patterns of urban areas in small scale. TerraSAR-X Very High Resolution (VHR) spotlight acquisitions with an azimuth and range resolution of 1.1 m and 0.6 m, respectively provide detailed maps of individual buildings (Bamler et al., 2009, Zhu, 2011). In Fig. 1.1 an example of such images taken from the central railway station of Berlin is shown. The great number of bright points with high phase stability, the so called persistent scatterers, can be clearly seen which indicates the possibility of obtaining the motion history of individual buildings in a very detailed sense.

Due to the side-looking geometry of SAR systems, image distortions are introduced which handicap the interpretation of single SAR images (Hanssen, 2001, Méric et al., 2009, Rees, 2012). One of these effects, namely layover, causes the scatterers having the same slant-range coordinates to be mapped into one azimuth-range pixel. This phenomena mostly occurs in high rise urban areas where scatterers from the ground and the facade (sometimes also from the roof) of buildings are located in the same pixel due to the steep slopes of buildings toward the sensor (Fornaro et al., 2005, Reigber and Moreira, 2000).

Since SAR images provide only a projection of the real 3D space into a 2D intensity map of the area, exploiting multi-temporal interferometric SAR techniques such as InSAR, Persistent Scatterer Interferometry (PSI) and SAR Tomography are necessary to derive information about the geophysical processes of the investigated scenes (Zhu, 2011). InSAR gets access to
Chapter 1. Introduction

Figure 1.1: TerraSAR-X VHR spotlight image of the Berlin central station illustrating the capability of modern SAR sensors for urban mapping in great details.

the third dimension, namely elevation (perpendicular to range-azimuth plane), by exploiting the phase differences of two complex valued acquisitions (Bamler and Hartl, 1998, Hanssen, 2001). However, because of limitations imposed on this method due to geometrical and temporal decorrelation plus atmospheric disturbances, strict conditions should be met to achieve high accuracies (Hanssen, 2001, Samiei-Esfahany, 2008). To overcome these limitations PSI was developed by Ferretti et al. (2001). The method makes use of stacks of interferometric data sets taken from the same scene, in different times, for the purpose of long-term deformation monitoring by only analysing the persistent scatterers. These scatterers are the ones which have stable phase behaviour over long periods and can often be identified visually as very bright points in SAR images. However, PSI assumes only a single dominant scatterer in the center of each pixel which is not a valid assumption in urban areas due to layover effect (Bamler et al., 2009, Zhu, 2011).

SAR tomography (TomoSAR) extends the synthetic aperture principal in the elevation direction by using stacks of SAR images taken from different orbital positions (similar to PSI) (Zhu, 2008, Zhu and Bamler, 2010b). This multi-baseline method is an extension of conventional InSAR since it benefits from more than two acquisitions to reconstruct the full reflectivity profile of each range-azimuth pixel along the elevation direction (Fornaro et al., 2005, Reigber and Moreira, 2000, Zhu, 2011). Similar to differential InSAR, TomoSAR can be extended to a differential form by taking into account the motion of each scatterer in the pixel (Lombardini, 2005). Differential TomoSAR (D-TomoSAR) with layover separation capabilities can provide the most detailed 4D (space-time) representation of single buildings when coupled with VHR spotlight TerraSAR-X datasets (Zhu, 2011). However, TomoSAR suffers from poor 3D localisation accuracy in the order of 1 m and more importantly, similar to conventional InSAR and PSI, the elevation and deformation rates are estimated with respect to a previously chosen reference point which makes them relative estimates (Gernhardt, 2012, Zhu and Shahzad, 2013).
Apart from the mapping and deformation monitoring capabilities of mentioned SAR techniques, recently it was demonstrated that modern SAR sensors, such as TerraSAR-X, can achieve unprecedented pixel localization accuracy in the order of 11 to 14 millimetres if the most prominent errors affecting the quality of the range and azimuth measurements are compensated for, see (Balss et al., 2012, 2011, Cong et al., 2012, Eineder et al., 2011, Schubert et al., 2012). The technique to achieve this magnitude of accuracy is termed SAR imaging geodesy after Eineder et al. (2011).

The scope of this thesis is to propose a framework that fuses the SAR imaging geodesy and TomoSAR approaches to obtain absolute 3D positions of a large amount of scatterers. Moreover, it is demonstrated that the TomoSAR results obtained from different viewing geometries can be merged together to improve the scatterer density of the investigated scenes and consequently allow for a very detailed deformation monitoring of individual buildings in urban areas.

The remainder of this chapter is organized as follows. In Section 1.2 the problem and the motivation of conducting this research is expanded and finally Section 1.5 gives an overview of the organization of this thesis.

### 1.2 Problem Statement

Despite of its superiority in providing detailed 3D or 4D maps similar to conventional InSAR and PSI, TomoSAR also suffers from several shortcomings. In the following these weaknesses are stated and the procedure to overcome them forms the principal motivation for carrying out this thesis.

- The 3D geocoded point cloud obtained from TomoSAR shows certain coordinate offsets with respect to the true positions of the scatterers. The reason is the unknown height of the reference point chosen during the TomoSAR processing with respect to which the elevation and deformation values are estimated. In (Gernhardt, 2012), it is shown that this offset is propagated to all of the samples in the point cloud, and it results in coordinate shifts in both horizontal and vertical dimensions.

- Due to the side-looking viewing geometry of SAR sensors, the final TomoSAR point cloud is only capable to map the area which could be illuminated during the acquisition. Therefore, considering urban areas, it is only possible to capture fractions of buildings while it is desirable to enhance the scatterer density on each building for comprehensive mapping and deformation analysis.

- The TomoSAR results of different datasets acquired from variable acquisition geometries cannot be directly fused together to produce a shadow-free point cloud. This is mainly due to the selection of different reference points, with unknown 3D coordinates, during TomoSAR processing. Therefore, in addition to the coordinate offsets of the scatterers relative to their true positions, different point clouds also show offsets with respect to each other.

- Differential TomoSAR, similar to differential InSAR and PSI, is only capable to provide displacement vectors projected on to the radar Line of Sight (LOS). This happens due to the side-looking geometry of the SAR instrument and pose difficulties for interpretation of deformation patterns (Hanssen, 2001).

Based on the mentioned problems, the following questions can be formulated:
• How is it possible to compensate for the coordinate shifts between the geocoded TomoSAR point clouds and their respective true positions without using an external data source to produce 3D absolute TomoSAR point clouds?

• How is it possible to improve the information coverage of the TomoSAR point clouds by fusing the results from different tracks?

• How is it possible to decompose the LOS displacement measurements of differential SAR tomography into the real 3D motion vectors by using SAR image stacks acquired from different viewing geometries?

These questions can be merged together to form the overall research question of this study:

*How to fuse the capabilities of SAR tomography and SAR Imaging geodesy to produce an absolute fused TomoSAR point cloud for detailed mapping and deformation monitoring of individual buildings?*

### 1.3 Test Site and Data Availability

In this thesis, the investigated test site includes the central area of the city of Berlin, Germany. The available dataset consists of four stacks of TerraSAR-X very high resolution spotlight images acquired with a bandwidth of 300 MHz and therefore a slant-range resolution of 60 cm and an azimuth resolution of 1.1 m.

In terms of viewing geometry two stacks are acquired from ascending tracks and two are acquired from descending tracks. Ascending and descending images were acquired at approximately 16:50 UTC and around 05:20 UTC, respectively. The details about the system parameters and stack properties are summarized in Table 1.1. From now on, each stack will be referred to with the corresponding beam number (e.g. Beam 57). Fig. 1.2 shows the mean scene coverage of each individual stack overlaid on the optical image of Berlin.

Since Berlin is regularly monitored by TerraSAR-X, a large number of images is available for each stack ranging from 102 to 138. The acquisition period is from February 2008 until March 2013 for all the stacks with mostly a regular time interval of 11 days between each image except for the rare occurrence of temporal gaps. The stacks consist of non-corregistered complex images.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Incidence Angle ($\theta_{inc}$)</th>
<th>Heading Angle ($\alpha_h$)</th>
<th>Track Type</th>
<th>Nr. of Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>57</td>
<td>41.9°</td>
<td>350.3°</td>
<td>Ascending</td>
<td>102</td>
</tr>
<tr>
<td>85</td>
<td>51.1°</td>
<td>352°</td>
<td>Ascending</td>
<td>111</td>
</tr>
<tr>
<td>42</td>
<td>36.1°</td>
<td>190.6°</td>
<td>Descending</td>
<td>109</td>
</tr>
<tr>
<td>99</td>
<td>54.7°</td>
<td>187.2°</td>
<td>Descending</td>
<td>138</td>
</tr>
</tbody>
</table>

**Table 1.1:** Acquisition parameters of each stack including the average incidence angle, the flying direction or azimuth, the track type and the number of available images.

In addition to the TerraSAR-X datasets, a point cloud of the test area obtained from aerial laser scanning is available. This dataset is kindly provided by Land Berlin, promoted by the Europäischer Fonds für Regionale Entwicklung (EFRE). This dataset is used to construct a Digital Surface Model (DSM), which serves as a reference for the localization accuracy analysis of TomoSAR point clouds. The point cloud corresponding to a small area close to the *Reichstagsgebäude*, Berlin is visualized in Fig. 1.3.
Chapter 1. Introduction

Figure 1.2: The optical image of the city of Berlin (Google Earth™). Rectangles mark the coverage of the four TerraSAR-X data stacks.

Figure 1.3: LiDAR point cloud of a small area in Berlin. The high point localization accuracy in the order of 10 cm makes the point cloud an excellent source to serve as a reliable reference surface model for accuracy analysis of the retrieved TomoSAR point clouds.
1.4 Research Methodology

In order to answer the research question outlined in Section 1.2, different approaches are used which are briefly explained in this section. The complete work-flow of this research can be seen in Fig. 1.4.

Initially, tens of opportunistic point scatterers visible from multiple stacks of SAR data are manually selected. The range and azimuth time coordinates of the scatterers are extracted from the focused SAR images. The errors are modelled and their impacts are eliminated from the timing measurements. At a subsequent step, the corrected range and azimuth observables, available from multiple geometries, are participated in an iterative process to estimate the absolute three-dimensional coordinates of the scatterer using the Stereo-SAR software.

Interferometric stacking is carried out for each stack of SAR data in order to co-register all of the images onto an optimum master scene. The atmospheric phase contribution is eliminated from each InSAR stack by performing TomoSAR pre-processing. During this step, for all of the stacks an identical reference point, with absolutely known 3D coordinates, is considered. The TomoSAR processing is done separately for each stack. The point clouds obtained from multiple geometries are absolutely fused together to provide a shadow-free point cloud. The scatterer localization of the fused point cloud is compared with a surface model built from LiDAR point cloud based on extracting the facade lines of buildings.

Figure 1.4: The work-flow of the fusion of TomoSAR and Stereo-SAR concepts. The white ellipses show the input (output) of each stage. The rectangles represent the processes that are coloured based on the used software.
Chapter 1. Introduction

The availability of point clouds from different geometries allows for decomposition of the one-dimensional motion estimates onto the original 3D motion vector. This is carried out for two test sites by a straight-forward algorithm based on least-squares adjustment.

This chapter is continued with the outlining the organization of this thesis in Section 1.5.

1.5 Outline

This thesis is organized in five chapters. The current chapter gives the necessary background and describes the motivation of conducting this research as well as introducing the investigated test site and the available datasets. Chapter 2 is dedicated to a detailed description of the SAR imaging geodesy concept and introduces the novel technique of Stereo-SAR developed by Gisinger et al. (2013). The chapter is finalized with reporting the first results of precise 3D localization of natural scatterers in the city of Berlin. Chapter 3 starts with describing the basics of SAR tomography. The chapter is continued with a mathematical description of some spectral estimators applicable to tomographic SAR inversion as well as introducing the German Aerospace Center (DLR)’s tomographic SAR processing system (Tomo-GENESIS). The chapter further describes the methodology for absolute fusion of multi-track TomoSAR point clouds and is concluded with reporting the experimental results over the city of Berlin as well as analysing the localization accuracy of the fused point cloud with respect to a reference Digital Surface Model (DSM). Chapter 4 deals with exhaustive treatment of decomposition of the LOS motion estimates of differential TomoSAR in the Cartesian East, North and Up. The decomposition strategy is described with considering acquisitions from more than two different geometries and eventually the decomposition results are reported for two test sites in the city of Berlin. Finally, the concluding remarks and recommendations for possible future work are given in Chapter 5.
Chapter 2

3D Scatterer Localization with TerraSAR-X

This chapter introduces the technique of SAR Imaging Geodesy proposed by Eineder et al. (2011) and Stereo-SAR software developed by Gisinger et al. (2013). It consists of an overview of the most prominent error sources affecting the range and azimuth measurements of SAR sensors followed by the correction methods applied in Stereo-SAR for 3D scatterer positioning. At the last section of the chapter, the first results of natural scatterer localization using the Stereo-SAR software is reported.

2.1 Introduction

In the field of satellite remote sensing, the SAR technique has gained high popularity because of its capability to image the Earth surface for a wide range of applications. Additionally, by exploiting two or more SAR acquisitions from the same scene, advanced interferometric techniques are able to measure relative displacements of the surface of the Earth in the millimetre regimes (Ferretti et al., 2001). Nevertheless, the inherent ability of SAR systems for providing good geo-location accuracy has been ignored until recently (Eineder et al., 2011).

Pixel localization accuracy or geometric accuracy of a SAR system is regarded as the degree of capability of the sensor to assign a pixel in the image to its corresponding geographic position on the surface of the earth (Schubert et al., 2012). Recent studies have shown that the German SAR satellite, TerraSAR-X, has the potential to localize ideal point scatterers (radar corner reflectors) in the image with centimetre-level accuracies (Balss et al., 2012, Eineder et al., 2011, Schubert et al., 2012) and, that using the most effective compensation of all error sources, this accuracy can be improved to sub-centimetre regimes (Balss et al., 2012, 2013). The method of correcting SAR measurements is termed SAR Imaging Geodesy after Eineder et al. (2011). The core element of all mentioned experiments is to compare the image positions of ideal point targets, such as corner reflectors, to surveyed coordinates obtained by dual-frequency high-end Global Navigation Satellite Systems (GNSS) receivers or terrestrial geodetic survey. In all of the methods attempts were made to exclude all the factors contributing to position errors. Eineder et al. (2011) and Schubert et al. (2012) demonstrated that a significant improvement can be achieved by compensating for the error sources affecting both range and azimuth observations of the SAR instrument. In azimuth direction, a constant azimuth timing shift and earth motion components were reported as the driving factors of error while in the range direction, the signal path delay caused by propagation in atmosphere has the most prominent impact followed by smaller disturbances of geodynamics effect (Balss et al., 2012, 2013, Eineder et al., 2011, Schubert et al., 2012). The subtle differences in the reported absolute pixel localization accuracies of TerraSAR-X by these studies are mainly related to the applied methods for mitigation of error.
sources. In this study, the corrections are applied using the DLR in-house software developed by Balss et al. (2013) which employs mostly GNSS-based correction approaches for atmospheric effects and International Earth Rotation Service (IERS) conventions for geodynamics effects.

One application that can benefit from the improved pixel localization accuracy of TerraSAR-X is the absolute localization of prominent scatterers available in the SAR images. By combining corrected range and azimuth observations from two or more orbits, stereo-SAR approaches can be exploited to reconstruct the scatterers in 3D (Balss et al., 2013, Gisinger et al., 2013). The first experiments using stereo SAR for this purpose were carried out on a perfectly stable 1.5 m corner reflector installed in Wettzell, Germany (Balss et al., 2013). The results obtained from this experience illustrate the high potential of TerraSAR-X for absolute 3D localization and give the motivation to apply this method for natural scatterers in urban areas.

The detailed explanation of procedure to achieve absolute 3D geo-location of natural scatterers is addressed in this chapter. Before going into details of the methodology a quick overview of SAR measurement principle is outlined in Section 2.2. The first step of the method which relies on determining the azimuth and range time coordinates of desired scatterers from focused complex SAR images is described in Section 2.3. The measured radar times are affected by different errors which have to be eliminated in order to achieve absolute azimuth and range observations. An overview of the disturbance factors and correction methods are addressed in Section 2.5 and Section 2.6. With the combination of corrected SAR measurements collected from different orbits, Stereo-SAR is exploited to estimate the 3D absolute position of selected natural scatterers. The brief explanation of the basics of Stereo-SAR is given in Section 2.7. Finally, the first absolute 3D fix results of regular point scatterers using the Stereo-SAR software is addressed in Section 2.8.

2.2 SAR Measurement Principle

A single pixel in a focused SAR image is characterized by two time coordinates. In the along track direction, the time relative to the time of the closest approach defines the azimuth coordinate (Cumming and Wong, 2005). In the across track direction, the difference in the time travel of the transmitted and the received chirp at the zero-Doppler location of the target characterizes the range coordinate (Cumming and Wong, 2005). Azimuth and range time coordinates can be converted to distances by multiplying with the platform velocity and the velocity of light, respectively. Therefore, a completely geometrical coordinate system is formed in which the range geometry corresponds to a 3D sphere and the azimuth geometry is described by the zero-Doppler plane (Gisinger et al., 2013). Consequently, the Range-Doppler equation system is expressed as (Cumming and Wong, 2005):

\[ |X_s - X_t| - c \cdot t_\rho = 0 \]

\[ \frac{\dot{X}_s(X_t - X_s)}{|X_s||X_t - X_s|} = 0, \]

where \(X_s\) denotes the satellite position vector, \(X_t\) is the vector defining the target location, \(\dot{X}_s\) is the satellite velocity vector, \(c\) is the speed of light in vacuum and \(t_\rho\) is the range time. Eq. (2.1a) defines the geometric distance of the satellite and the target which would be equal to the travel time difference of the transmitted pulse and its echo multiplied with speed of light if no delay was presented in \(t_\rho\). Eq. (2.1b) is the property of zero-Doppler plane which requires the velocity vector of the satellite to be perpendicular to the vector between the satellite and the target (Cumming and Wong, 2005). This equation is different from the range equation in a sense that
the azimuth time \( t_a \) is not explicitly present in the equation and it should be introduced via an orbit model. This issue is treated in more details in Section 2.7. For now, it is important to note that the azimuth time observable, comparable to \( t_\rho \), is also affected by disturbance factors which leads the azimuth equation to be not exactly equal to zero (Balss et al., 2012, 2013, Eineder et al., 2011, Schubert et al., 2012).

The ultimate aim of this chapter, is to solve the system of equations outlined in Eq. (2.1) for obtaining the 3D position of strong scatterers in radar images. This is achievable in two independent steps:

1. the range and azimuth time observables of the scatterers should be extracted from the SAR images and be corrected to achieve absolute measurements. This requires the detailed modelling of all the perturbations factors which affect Eq. (2.1).

2. the 3D unknown target coordinates \( X_t \) can be estimated if, at least, a second set of observations from the same or a different orbit is available.

Therefore, the work-flow implemented in the Stereo-SAR software is introduced and illustrated in Fig. 2.1. The steps from loading the input files until performing Point Target Analysis to retrieve the azimuth and range timings are the topic of Section 2.3. Introduction to the error sources is given in Section 2.4 and their methods of mitigation are expanded in Section 2.5 and Section 2.6. This is followed by explaining Stereo-SAR in Section 2.7. Finally the results on experimental natural scatterers are reported in Section 2.8.

**Figure 2.1:** Processing scheme of the 3D scatterer localization using TerraSAR-X.
2.3 Azimuth and Range Time Retrieval

In order to precisely obtain 3D positions of certain scatterers, the first step is to accurately measure the azimuth and range time coordinates of such scatterers in focused complex SAR images. This is carried out by exploiting fundamental signal processing procedures which are performed with DLR’s Image Analysis Software (IAS). This section is dedicated to explaining the measurement method which is performed on identical scatterers from several TerraSAR-X images taken from two different orbits.

Each pixel in the focused complex TerraSAR-X image is characterized by a unique azimuth and range time tag. The information about radar timing is adopted from level 1b product annotations which are provided in XML format complementary to the complex image (Eineder et al., 2008). In these scene specific files, the first azimuth and range sample times are annotated as well as Pulse Repetition Frequency (PRF) and Range Sampling Frequency (RSF). Knowledge of the acquisition times and pixel spacings provided by PRF and RSF results to measuring the radar coordinates at pixel level. Therefore, refinements should be applied in order to achieve sub-pixel accuracy.

2.3.1 Point Target Analysis

The impulse response of SAR is the output of the system when an impulse from an isolated point target such as a corner reflector is applied at the input (Cumming and Wong, 2005). In processed SAR images, perfect point targets are recognized with a distinctive two-dimensional sinc pattern which is induced from the super-position of two perpendicular sinc signals that are formed independently from each other based on the SAR antenna pattern and Fraunhofer diffraction phenomenon in azimuth and range directions, respectively (Hanssen, 2001, Rees, 2012). Analysis of such responses provide valuable information about the SAR image quality. This information is mainly useful for radiometric and geometric calibration as well as obtaining the resolution of the SAR system (Cumming and Wong, 2005). Thus for certain purposes the peak position of the point target response should be measured with sub-pixel accuracy. The process that facilitates such measurements is known as Point Target Analysis (PTA). In the following, the method implemented in IAS of DLR is explained in details. It is worth mentioning that the basic principle behind PTA is the same for all implementations and that they only vary depending on the aimed accuracy (Cumming and Wong, 2005).

PTA is performed in the following steps (Balss et al., 2012, Cumming and Wong, 2005):

1. A measurement patch of size $32 \times 32$ centred on the desired scatterer is extracted from the focused complex SAR image. Generally speaking, The patch size can vary from $16 \times 16$ to $64 \times 64$.

2. A two-dimensional Discrete Fourier Transform (DFT) is taken from the $32 \times 32$ patch selected in the previous step.

3. The spectrum of the point target is expanded by spectral zero-padding with factor of 32. It can be shown that zero-padding with higher factors is not necessary since the resolving capability cannot be further enhanced, according to Balss et al. (2012).

4. A two-dimensional Inverse Discrete Fourier Transform (IDFT) is applied on the zero-padded spectrum. At this stage, an expanded version of the point target is obtained which facilitates the detection of the integer line and pixel values of the point target response peak.
5. A small window of size $3 \times 3$ is selected from the oversampled patch centred on the detected peak position.

6. In order to refine the measurement into sub-pixel accuracy a 2-D parabola interpolation is applied based on the derived small patch. Since a parabola can be uniquely defined with 3 points, a window of size $3 \times 3$ is chosen in previous step. The idea is to perform a convolution between the oversampled signal and a pre-defined parabola known as an interpolation kernel in both azimuth and range direction. This process leads to reconstructing the signal even in values between the sampled ones. Consequently, the peak position of the desired point target can be measured with accuracies far less than a pixel. In this step interpolation is preferred upon pure oversampling due to its flexibility and computational efficiency (Balss et al., 2012, Cumming and Wong, 2005).

![Simulation of PTA](image)

**Figure 2.2:** Simulation of PTA. In order to enhance the sensitivity to $1/10000$ pixel, after spectral zero-padding a 2D paraboloid kernel interpolation is performed on the oversampled patch. The figure is not in scale.

The idea of PTA is visualized applying simple simulations in Fig. 2.2. It is clearly observed that after zero-padding in the second step, the peak position becomes sharper. According to Balss et al. (2012) the accuracy of this method with an oversampling factor of 32 is $1/10000$ pixel. Since high resolution spotlight TerraSAR-X images are used, the expected accuracy is translated into $1/10000 \times 1.1[m] = 0.11[mm]$ in azimuth direction and $1/10000 \times 0.6[m] = 0.06[mm]$ in range direction. However external factors such as atmospheric delays and orbit inaccuracies limit the overall retrieval accuracy which is in order of couple of centimetres. Obviously, the measurement accuracy decreases for lower resolution products such as spotlight and stripmap images since the peak response appears much blurred in comparison with VHR spotlight images.

### 2.3.2 An Example of Point Target Analysis by IAS

The IAS software is developed mainly for visualisation of complex and detected SAR images as well as image statistics calculation. Moreover, operations such as multi-looking, re-sampling, sharpening and point target analysis can be applied on input images. In this subsection, an
example of point target extraction and performing point target analysis employing IAS is illustrated. The schematic description of the procedure can be seen in Fig. 2.3.

![Figure 2.3: An example of Point Target analysis performed by DLR’s Image Analysis Software.](image)

At the first step, the complex SAR image as well as corresponding product annotation files are loaded. In order to be restricted to the identified target, smaller patches are extracted from the image. Samples of these patches are visualized in windows number one and two in Fig. 2.3 which are adopted from a high resolution spotlight image of the city of Berlin, Germany. The sought target is visible as a bright dot in the center of the red square in windows one and two. After applying PTA, the peak position of the target response is analysed in both azimuth and range spectra. These spectra are shown in window number three in Fig. 2.3. Window four visualizes the target response in the azimuth-range plane. It is noticeable that the target response does not represent a perfect sinc because it is a natural scatterer and not an ideal point target such as a corner-reflector. Finally, window number 5 gives information about the estimated position of the peak with 1/10000 pixel accuracy. These sub-pixel measurements of radar coordinates (line,pixel) are then converted to radar timings employing straightforward calculations based on the product annotation files (Balss et al., 2012, Eineder et al., 2008). Eventually, the azimuth timing is expressed in Coordinated Universal Time (UTC) and the range timing is reported in milliseconds which are denoted by $t$ and $\tau$, respectively.
2.4 Error Sources

The measured radar time coordinates obtained from PTA are affected by error sources which deviate the actual measurements from the absolute ones. The most prominent error sources consist of atmospheric propagation delays and geodynamics effects. The atmospheric delays are related to the propagation of the X-band SAR signal in ionosphere and troposphere which degrade the accuracy of the range measurements by approximately 2 to 6 centimetres and 2.4 to 4 meters, respectively based on the meteorological condition of the investigated area (Balss et al., 2013). The geodynamics phenomena affect both range and azimuth measurements and can be divided to solid earth tides, plate tectonics, pole tides, atmospheric pressure loading and ocean tidal loading. The largest magnitude of error is caused by solid earth tides which can reach almost 40 centimetres dependent on the latitude of the site. The other effects lead to position changes in the millimetres or sub-millimetres ranges (King et al., 2010). Apart from the mentioned perturbations, in azimuth timings, the requirement for achieving high localization accuracy necessitates usage of an updated azimuth calibration constant which is obtained after complete modelling of all the error effects. Also, in range observations the instrument electronic delays should be taken into account. Fig. 2.4 summarizes the error sources and the inaccuracies in the orbit determination of the SAR system.

Based on the aforementioned discussion, the measured range and azimuth time coordinates extracted from a SAR image can be written as follows if no correction is applied:

\[
\begin{align*}
\text{rg} &= 2 \cdot \frac{R}{c} + \delta t_{tropo} + \delta t_{iono} + \delta t_{rg, \text{geo}} + \tau_{el} - t_{r0} \ [s] \\
\text{az} &= t + \delta t_{az, \text{geo}} + \tau_{az} - t_{az0} \ [s],
\end{align*}
\]

(2.2)

with

- \(\text{rg}\) the measured range time
- \(R\) the Euclidean distance from satellite to the target
- \(c\) speed of light in vacuum
- \(\delta t_{tropo}\) tropospheric delay \([s]\)
- \(\delta t_{iono}\) ionospheric delay \([s]\)
- \(\delta t_{rg, \text{geo}}\) geodynamic effects on range timing \([s]\)
- \(\delta t_{az, \text{geo}}\) geodynamic effects on azimuth timing \([s]\)
- \(\tau_{el}\) instrument range delay \([s]\)
- \(t_{r0}\) first range sample time
- \(t\) acquisition time
- \(t_{az0}\) first azimuth sample time
- \(\tau_{az}\) constant azimuth time shift.

Initial range and azimuth calibration constants \((\tau_{el}, \tau_{az})\) are stated in the product annotation files. However, since the initial aimed geo-location accuracy of TerraSAR-X was within sub-meter range (Fritz and Eineder, 2009) these constants should be modified by taking into account the instantaneous atmospheric condition and geodynamics effects of the imaged scene. This issue is not expanded here. The calibration constants used in this study are adjusted based on the experiments carried out in (Balss et al., 2013). The other error sources outlined in Eq. (2.2) and Fig. 2.4 are described in detail in the following sections of this chapter.
2.5 Atmospheric Signal Path Delays

TerraSAR-X operates at the altitude of 514 km with a carrier frequency of 9.65 GHz (Buckreuss et al., 2003). Therefore, generated SAR signals are delayed due to propagation in different layers of atmosphere. The atmosphere, concerning TerraSAR-X, similar to GNSS signals, can be categorized into two layers namely ionosphere and troposphere. In this section, the concept of signal path delays due to the travel of TerraSAR-X signals in the atmosphere are addressed and GNSS-based methods on mitigating these effects to achieve very accurate range measurements are explained. It is shown that due to the well-established GNSS geodetic networks and the fact that GNSS and TerraSAR-X operate in the GHz frequency domain, delay estimates obtained from GNSS can be used to correct TerraSAR-X range measurements (Eineder et al., 2011).

Atmospheric signal path delays are mainly due to the refraction phenomenon which changes the speed and the direction of the signal. Consequently, a change in speed causes a change in the travel time of the signal. This can be best explained using the Fermat’s principle which states that a signal follows the path resulting in shortest travel time (Misra and Enge, 2011, Verhagen et al., 2012). Therefore, the excess delay caused by the propagation of the signal in the atmosphere can be modelled by expressing the difference between the measured range and the perfect ray path which has the refractive index of 1 \((n = 1)\):

\[
\Delta \tau = \frac{1}{c} \left[ \int n(l) \, dl - \int \, dl \right]
\]  

where \(l\) represents the path length, \(c\) denotes the speed of light in the vacuum, \(n(l)\) is the refractive index defined as the ratio of \(c\) and the speed of signal in the current medium and \(\Delta \tau\) describes the excess delay in seconds. The delay can be readily converted to distance by multiplying both sides of Eq. (2.3) with \(c\):

\[
\Delta \rho = \Delta \tau \cdot c = \int [n(l) - 1] \, dl
\]  

where \(\Delta \rho\) describes the signal path delay in meters.
2.5.1 Ionospheric delay

The ionosphere is the part of atmosphere which starts from 50 km above the surface of the earth and is extended to a height of approximately 1200 km (Misra and Enge, 2011). This region contains charged particles which are produced due to the ionization process caused by solar radiation (Verhagen et al., 2012). Since the behaviour of ionosphere is highly variable both spatially and temporally, accurate models are required to be able to explain its behaviour (Misra and Enge, 2011).

As is mentioned in Section 2.5, the signal path delay imposed on TerraSAR-X signals is caused by changes in the refractive index. Since the ionosphere is dispersive for TerraSAR-X signals, the refractive index depends on the frequency of the signal and is characterized by the number of electrons along the travel path of the signal (Misra and Enge, 2011, Verhagen et al., 2012). This gives us the motivation to introduce the term Total Electron Content (TEC) which describes the number of electrons within a tube of $1 \text{ m}^2$ and is mathematically described as the integral of the electron density $n_e(l)$ along the signal’s path (Misra and Enge, 2011):

$$ TEC = \int n_e(l) \, dl. \tag{2.5} $$

On the other hand, the refractive index of the ionospheric group delay, which is of interest for SAR image signals, can be written as (Misra and Enge, 2011, Verhagen et al., 2012):

$$ n_g = 1 + \frac{40.3 n_e}{f^2} \tag{2.6} $$

where $f$ is the carrier frequency of the signal. Following Eq. (2.4), Eq. (2.5) and Eq. (2.6) signal path delay induced from propagation of the signal in ionosphere in meters is calculated as follows:

$$ \Delta \rho_{\text{iono}} = \int [n_g(l) - 1] \, dl = \int \frac{40.3 n_e(l)}{f^2} = \frac{40.3}{f^2} \cdot \text{TEC}. \tag{2.7} $$

Eq. (2.7) describes the signal delay in the direction of the path. However, TEC values are usually reported in the vertical direction, referred to as the vertical Total Electron Content (vTEC). In this way computing the signal path delay requires the usage of a mapping function $F(z)$ which projects the estimated delay in the zenith direction into the desired slant range direction. Therefore, the delay is calculated as (Verhagen et al., 2012):

$$ \Delta \rho_{\text{iono}} = \frac{40.3}{f^2} \cdot \text{vTEC} \cdot F(z). \tag{2.8} $$

From Eq. (2.8), it is inferred that in order to be able to estimate the delay and remove its effect from SAR slant range measurements, correct determination of vTEC at corresponding location and time and applying a proper mapping function are essential. These issues are addressed in more details in Subsection 2.5.2 which focuses on estimation of vTEC values based on permanent GNSS receivers. Therefore it is worth mentioning that the methodology is only applicable if at least one permanent GNSS receiver is located close to the investigated scene.

2.5.2 GNSS-based Ionospheric Delay Estimation

Since GNSS signals suffer from the same propagation error sources as TerraSAR-X, they can serve as a valuable tool for mitigating these effects from SAR range measurements. This can be done by first estimating the vTEC values based on the GNSS observations and later map these estimates into the radar slant range direction by taking into account the instrument carrier
frequency and the geometry related to individual scenes (Balss et al., 2013, Gisinger, 2012).

The task of vTEC estimation is carried out by forming the so called geometry-free GNSS linear combinations (Misra and Enge, 2011). These equations are established by taking the difference of two GNSS pseudo-range observations obtained from different carrier frequencies. Therefore, using dual-frequency GNSS receivers, the linear combination is written as (Verhagen et al., 2012):

$$\rho_{R,L1}^S - \rho_{R,L2}^S = \left(1 - \frac{f_{L1}^2}{f_{L2}^2}\right) \cdot \frac{40.3 \times 10^{16}}{f_{L1}^2} \cdot \text{vTEC} \cdot F(z^S) + c \cdot (\Delta b^S - \Delta b_R).$$ \hspace{1cm} (2.9)

The left side of Eq. (2.9) denotes the GNSS pseudo-range observations of the same satellite and receiver. The only expression that is not previously defined is the one from the end of the right side which stands for satellite and receiver clock biases. These biases are precisely measured and preserved for each satellite-receiver combination of certain permanent GNSS receivers which constitute the International GNSS Service (IGS) network (Verhagen et al., 2012). The network consists of more than 400 globally distributed permanent stations which provide GNSS-based products in a daily basis (Dow et al., 2009). Therefore, with the knowledge of the clock biases and introducing the geometry of the observation obtained from satellite ephemerides available for IGS stations, the calculation of vTEC can be conveniently carried out.

Besides estimating the vTEC, an appropriate mapping function should be employed to project the vTEC values according to the geometry of the satellites. One of the most practical and well-established mapping functions is the so called Single Layer Model (SLM) (Misra and Enge, 2011) shown in Fig. 2.5a. The model assumes that all free electrons are contained in a shell of infinitesimal thickness at an altitude of about 450 km which corresponds to the maximum electron density (Misra and Enge, 2011). The Ionospheric Piercing Point (IP) is defined as the point of intersection of the line of sight with the assumed spherical shell at height $H$ which denotes the mean ionospheric height. If we name the zenith angles of the satellite at the receiver position and at IP with $z$ and $z'$, respectively, by simple geometry one can relate vTEC to TEC at zenith angle $z$ as (Misra and Enge, 2011):

$$\text{TEC}(z) = \frac{1}{\sqrt{1 - \sin^2(z')}} \cdot \text{vTEC} \quad \text{with} \quad \sin(z') = \frac{R}{R+H} \sin(z) \hspace{1cm} (2.10)$$

where $R$ stands for the average radius of the earth. The realization of the SLM mapping function with different zenith values is illustrated in Fig. 2.5b. It can be seen that the role of obliquity factor $F(z')$ becomes more prominent for the satellites observed at higher zenith angles (lower elevation angles).

### 2.5.3 Ionospheric Correction for TerraSAR-X Range Measurements

In Subsection 2.5.2, the GNSS-based method for ionospheric path delay estimation was outlined. This method can be effectively used for the correction of TerraSAR-X range measurements if the acquired scene is located sufficiently close to one or several permanent GNSS receivers. This subsection is dedicated to describing the process of correcting the SAR range observables using local GNSS measurements.

Depending on the constellation of GNSS satellites, normally there are up to 10 satellites visible during one epoch of SAR observation (Balss et al., 2013). Since the observation geometry of each
satellite is unique, there is one value of vTEC available for each satellite-receiver combination. At an initial step, the vTEC values are interpolated to the time that the target of interest is recorded in the SAR image. Since it is mostly common to register the radar time coordinates to the time of closest approach, the vTEC values are interpolated to the zero-Doppler time characterizing the desired target. Furthermore, an interpolation should be applied in order to calculate the vTEC at the correct geographic location of the desired target. This is performed by fitting a 2D plane in the form of $a\phi + b\lambda + c$ to the vTEC values, where $\phi$ and $\lambda$ denote the latitude and longitude, respectively, in order to interpolate the value for areas in the vicinity of the GNSS receiver (Gisinger, 2012). The result of the two above-mentioned operations is the vTEC value at the actual time and location of the desired target corresponding to the SAR satellite ionospheric piercing point. The last step is projecting the estimated vTEC into the SAR slant range direction. This is done by applying the SLM mapping function outlined in Subsection 2.5.2.

The required inputs are the information about the geometry, the exact position of the satellite and the carrier frequency of the SAR instrument. The state vector of the satellite is annotated in the product annotation files corresponding to each image with the sampling frequency of 10 s (Fritz and Eineder, 2009) employing the accurate science orbits (Yoon et al., 2009). The approximate incidence angle is also stated in the annotation file. Therefore, by having the orbit of the SAR satellite and the incidence angle one can easily obtain ionospheric delay from Eq. (2.8).

Eventually, the corrections are calculated for each target at the corresponding acquisition time and location and a single value is reported.

There is still one remaining issue to be tackled in order to compensate for the delay properly. Since the TerraSAR-X satellites operate in the altitude of 514 km, they are partly in the ionosphere while the corrections obtained from aforementioned analyses assume that the signal travels through the entire ionosphere. Thus, a weighting factor should be applied to exclude the top-side vTEC contribution. Based on the analysis described in (Balss et al., 2012), the weighting factor of 75 % provides reasonable results and is used in the SAR imaging geodesy software. The more detailed treatment of this issue is addressed in (Gisinger, 2012).

### 2.5.4 Tropospheric delay

Apart from the ionosphere, TerraSAR-X signals are also delayed by the lower part of the atmosphere which is composed of gases and water vapour. This region is called the troposphere which starts from the surface of the earth and is extended to a height of almost 50 km (Misra and Enge, 2011). The troposphere is a non-dispersive medium for X-band signals which means that the refractive index is not dependent on the carrier frequency of the generated signal. The tropospheric signal path delay can be decomposed to a dry and a wet delay from which the latter
is more unstable but cause less disturbance than the former (Misra and Enge, 2011). The magnitude of absolute tropospheric delay in SAR range measurements varies between 2.4 to 4 meters depending on the variation of water vapour and the amount of dry gases (Balss et al., 2013). Apart from the absolute delay, which can be considered as a constant bias for the whole imaged scene, a spatial dependent contribution is also present in the tropospheric delay (Hanssen, 2001).

Similar to ionospheric delay, tropospheric delay is caused by the variations in the refractive index. Since the refractive index of the propagated wave in troposphere is very close to unity ($\approx 1.0003$), we define refractivity as $N = (n - 1) \times 10^6$ and express it as the sum of the refractivities of dry gasses and the water vapour (Misra and Enge, 2011):

$$N = N_d + N_w,$$

where $N_d$ and $N_w$ are called the dry and wet refractivities, respectively. Similar to the derivation of ionospheric path delay, the tropospheric delay in meters is expressed as follows (Misra and Enge, 2011, Verhagen et al., 2012):

$$\Delta \rho_{trop} = 10^{-6} \int N(l) \, dl = 10^{-6} \int [N_d(l) + N_w(l)] \, dl = \tilde{T}_d + \tilde{T}_w$$

(2.12)

where $\tilde{T}_d$ and $\tilde{T}_w$ denote the dry and wet delays in the slant direction, respectively. Based on Eq. (2.12), the total absolute delay can be estimated by modelling the dry and wet refractivity profiles which are dominated by temperature and the partial pressures of dry gasses and water vapour in a specific parcel of air. Empirically, these models are defined as (Misra and Enge, 2011):

$$N_d = 77.64 \cdot \frac{P}{T}$$

$$N_w = 3.73 \times 10^5 \cdot \frac{e}{T^2}$$

(2.13)

where $T$ is temperature in kelvin and $P$ and $e$ are total and water vapour pressures, both in millibars. Therefore, insight about pressure, humidity and temperature along the propagation path of a signal can be used to determine the refractivity profile and consequently the slant tropospheric delay at desired locations.

Eq. (2.12) can be re-formulated in terms of Zenith Path Delay (ZPD) by introducing different mapping functions for the dry and wet parts. In this context, mapping functions, similar to ionospheric delay estimation case, depend mainly on the zenith angle of the satellites ($z^s$). Therefore, Eq. (2.12) is re-written as (Misra and Enge, 2011, Verhagen et al., 2012):

$$\tilde{T} = \tilde{T}_{z,d} \cdot m_d(z^s) + \tilde{T}_{z,w} \cdot m_w(z^s)$$

(2.14)

where $\tilde{T}_{z,d}$ and $\tilde{T}_{z,w}$ are the dry and wet zenith path delays with $m_d$ and $m_w$ as the corresponding mapping functions.

The discussion above only takes into account the absolute tropospheric delay which, as is mentioned earlier, can be considered as a fixed contribution and consequently can be subtracted from the entire image. Concerning interferograms, another component is also present in the tropospheric signal which is variable dependent on the position of the target in the imaged scene. Therefore, in terms of the total refractivity, for a single pixel we can write (Hanssen, 2001):

$$N(x, y, h) = \bar{N}(h_c) + \Delta N(x, y, h),$$

(2.15)
where $\bar{N}(h_c)$ is the mean refractivity for all values at height $h_c$ within the interferogram area and the variable $\Delta N(x, y, h)$ is the lateral variation of the refractivity. Eq. (2.15) can be used to describe the absolute and the relative tropospheric delay by taking into account the proper mapping function and integrating the refractivities along the path of generated SAR signal.

It is important to note that in this study the spatial variability of the tropospheric delay is not considered and only the absolute delay is evaluated for the point of interest. Therefore, the estimation of tropospheric delay, comparable to ionospheric delay, is narrowed down to two steps namely evaluation of ZPD at desired locations and selecting the most appropriate mapping functions to project the dry and wet delays into the slant range directions.

### 2.5.5 GNSS-based Tropospheric Delay Estimation

The tropospheric delay cannot be estimated independently for each GNSS satellite since introducing them as unknown parameters will cause the system of equations to be under-determined (Van der Marel, 2013). Therefore, absolute delay estimation is only feasible in regional and global GNSS networks where ionospheric-free linear combinations are exploited in double-differenced measurements after fixing the receiver coordinates (Van der Marel, 2013). Because of these issues, tropospheric delay in a specific site is generally estimated every one or two hours by combining observations from visible satellites with the precise point positioning technique. This is how the measurement of tropospheric ZPD is carried out for permanent GNSS stations in IGS network and offered to users. In order to enhance the quality of the delay estimates, the following steps are further taken into account (Beutler et al., 1996). At first, the hydrostatic delay is modelled by knowledge of the atmospheric parameters relevant to the specific site. The modelling accuracy is not very crucial since this a-priori value will be updated with new delay estimates. At the second step, the hourly total ZPD is estimated along with other desired parameters. The third part takes into account the azimuth asymmetry and horizontal gradients induced from chaotic behaviour of the atmosphere. The latter, depends on the time, elevation angle and heading angle of the satellite and is well-explained in (Beutler et al., 1996, Kleijer, 2004).

Similar to the ionospheric delay estimation, the tropospheric ZPD needs to be projected onto the slant range direction of the satellite by applying appropriate mapping functions. The simplest mapping function used for small incidence angles is equal to $\frac{1}{\cos z}$ where $z$ denotes the zenith angle. However, for most precise applications the continued fraction in terms of $\cos z$ is commonly used (Kouba, 2008):

$$F(z, a, b, c) = \frac{1 + \frac{a}{\cos z + \frac{b}{\cos z + \frac{c}{\cos z + \frac{a}{\cos z + \frac{b}{\cos z + \frac{c}{\cos z + \frac{a}{\cos z + \frac{b}{\cos z + \frac{c}{\cos z + \frac{a}{\cos z + \frac{b}{\cos z + \frac{c}{\cos z + \frac{a}{\cos z + \frac{b}{\cos z + c}}}}}}}}}}}}}}{\cos z + \frac{a}{\cos z + \frac{b}{\cos z + \frac{c}{\cos z + \frac{a}{\cos z + \frac{b}{\cos z + \frac{c}{\cos z + \frac{a}{\cos z + \frac{b}{\cos z + \frac{c}{\cos z + \frac{a}{\cos z + \frac{b}{\cos z + c}}}}}}}}}}},$$

(2.16)

where the coefficients $a$, $b$ and $c$ are small constants and differ for dry and wet delays. A large number of mapping functions has been developed based on Eq. (2.16), which only vary in terms of determining the dry and wet coefficients. In the imaging geodesy software package, the gridded Vienna Mapping Function 1 (VMF1) was used for both wet and dry delays in accordance with (Balss et al., 2013). It is important to note that based on the experiments outlined in (Gisinger, 2012), assuming a total ZPD of 2.3 m yields a deviation of about 1 millimetres and 10 millimetres for the cosine mapping function $\frac{1}{\cos z}$ with $z = 34^\circ$ and $z = 55^\circ$, respectively compared to the VMF1. Nevertheless the gridded VMF1 was chosen for the software due to the flexibility of retrieving the mentioned coefficients and also its higher accuracy (Gisinger et al., 2013). For the gridded VMF1 the mapping function coefficients $(a_h, a_w)$ as well as the dry and wet ZPDs $(z_d, z_w)$ are available for four daily epochs in a grid of $2.0^\circ \times 2.5^\circ$ (Kouba, 2008). The
ZPD values correspond to the grid mean height and the coefficients are given for zero elevation. Therefore, proper mapping should be done for height transfer. Using the given data and experimental formulas expressed in (Kouba, 2008) one can interpolate the ZPD values for any desired test site taking into account the corresponding temperature and pressure. The details about VMF1 is not addressed any further in here and the interested reader is referred to (Kouba, 2008).

### 2.5.6 Tropospheric Correction for TerraSAR-X Range Measurements

Based on the explanations given in Subsection 2.5.5, if permanent GNSS receivers are sufficiently close to the desired locations, the receiver ZPD estimates can be used to correct the range measurements of TerraSAR-X. According to (Gisinger et al., 2013), if at least one permanent GNSS receiver is located in a distance of 50 km relative to the desired area, suitable tropospheric corrections are possible. It is worth mentioning that by considering the sparse GNSS networks, it is not possible to completely eliminate the tropospheric delay within an entire scene mapped in a SAR image due to the high spatial variability of the atmosphere, especially, the amount of water vapour. Nevertheless, the IGS ZPD values are offered to users with a temporal frequency of 5 minutes (Byram et al., 2001) and are the basis for tropospheric corrections in the imaging geodesy method.

For the corrections, all the visible GNSS satellite observations are taken into account to evaluate the ZPD at the receiver location. In order to interpolate the ZPD value on the desired target location and time, the gridded VMF1 mapping function explained in Subsection 2.5.5 is used for both dry and wet delays while the pressure and the temperature in the area are obtained from global pressure and temperature models. Therefore with having all the coefficients in Eq. (2.16) and the zenith angle from the product annotation files one can define the mapping function. Furthermore, a height transfer from the station position to the desired scatterer height is performed following the implementations in (Kouba, 2008). With all the information at hand, the total tropospheric delay can be estimated via Eq. (2.14).

Another possibility for estimation of tropospheric correction is using the 3D numerical weather model data as is proposed in (Cong et al., 2012). Calculation of corrections with this method has the advantage of world-wide coverage in comparison with the GNSS-based ZPD estimation which is restricted to areas located close to permanent GNSS receivers. This topic is not further expanded here and the interested reader is referred to Cong et al. (2012).

### 2.6 Geodynamic Effects

Geodynamic effects concern phenomena which deform the surface of the earth. As a result, the position of a certain point on the ground varies with different magnitudes in different time scales with respect to the International Terrestrial Reference System (ITRS) and the local coordinate system. In order to improve the accuracy of range and azimuth measurements to centimetre level, these site-specific effects should be identified and their respective impacts should be removed from the SAR measurements (Balss et al., 2012, Eineder et al., 2011). This section introduces some of the sources for aforementioned anomalies, known also as earth motion components, and will further give insights on how to model and mitigate their impacts from SAR measurements.
2.6.1 Different Geophysical Signals and Correction methods

In this subsection, the geodynamic effects which are included in the imaging geodesy software to correct the SAR range and azimuth observations are described. These impacts can be grouped into displacements caused by tidal motions, such as solid earth tides, ocean loading, pole tides and tidal atmospheric pressure loading. Another important issue while dealing with time-series of measurements is the effect of continental drift which causes gradual position changes of a point in the order of couple of centimetres per year. Table 2.1 gives an overview about the magnitude of deformations caused by geodynamic effects.

<table>
<thead>
<tr>
<th>Geodynamic effect</th>
<th>Magnitude [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Radial</td>
</tr>
<tr>
<td>Solid Earth tides</td>
<td>&lt; 40</td>
</tr>
<tr>
<td>Ocean loading</td>
<td>&lt; 10</td>
</tr>
<tr>
<td>Atmospheric pressure loading</td>
<td>&lt; 2</td>
</tr>
<tr>
<td>Pole tides</td>
<td>&lt; 2.5</td>
</tr>
<tr>
<td>Continental drift</td>
<td>&lt; 5</td>
</tr>
</tbody>
</table>

Table 2.1: An overview of the geodynamic effects and their corresponding magnitudes in the vertical and the horizontal dimensions. The numbers denote the upper limit of possible deformation magnitude.

In the following, each effect is introduced and the corresponding correction model is described based on the more detailed explanations given in (Petit and Luzum, 2010) and (King et al., 2010).

Solid Earth Tides

The solid Earth tides are induced from gravitational forces of the moon and the sun and cause Earth surface deformation with the highest magnitude comparing to the other mentioned effects. The tidal waves which are responsible for the effect can be categorized based on the period of occurrence from which the waves with semi-diurnal and the diurnal periods are considered as the most important ones with higher deformation magnitudes (Petit and Luzum, 2010). The displacement due to the tidal effects can reach up to 40 centimetres in the radial direction and approximately 5 centimetres in the horizontal direction dependent on the position of the point with respect to the Sun and the Moon and accelerating forces of the Sun and The Moon (Petit and Luzum, 2010). Therefore, computation of the tidal deformation for a particular time and location requires knowledge of two components. First, up-to-date ephemerides of the Moon and the Sun are needed. Second, the Earth’s response to the tidal potential must be modelled which includes the realization of the Love and Shida numbers considering the latitude and tidal-frequency dependency of the waves (King et al., 2010). The tidal potential for a point can be modelled with a spherical harmonic function and since the largest contribution comes from the second-order term of the harmonic, the calculation of the displacement vector is restricted to the degree 2 tides and is given by (Petit and Luzum, 2010):

\[
\Delta \vec{r} = \sum_{j=2}^{3} \frac{GM_j R_j^4}{GM_E R_j^3} \left\{ h_2 \hat{r} \left( \frac{3(\hat{R}_j \cdot \hat{r})^2 - 1}{2} \right) + 3l_2 (\hat{R}_j \cdot \hat{r}) [\hat{R}_j - (\hat{R}_j \cdot \hat{r}) \hat{r}] \right\},
\]

(2.17)

with
\[ GM_j \] the gravitational parameter for the moon \((j = 2)\) or the sun \((j = 3)\),
\[ GM_E \] the gravitational parameter for the Earth,
\[ \hat{R}_j, R_j \] unit vector from the geocenter to the Moon of the Sun and its magnitude,
\[ R_e \] Earth’s equatorial radius,
\[ \hat{r}, r \] unit vector from the geocenter to the point and its magnitude,
\[ h_2 \] nominal degree 2 Love number,
\[ l_2 \] nominal degree 2 Shida number.

Based on Eq. (2.17) for calculating the displacement caused by solid Earth tides the exact location of the Moon and the Sun should be known. These are available from the NASA mouse routines (Anton, 1996). Another issue is using the most recent Love and Shida numbers which takes into account the latitude of the site and also the frequency of the modelled signals. With this considerations Eq. (2.17) can be evaluated in order to report the displacement vector of the desired point in the local East, North and Up coordinates.

**Ocean Loading**

Ocean tides cause a temporal variation of the ocean mass distribution and the associated load on the crust and produce time-varying deformations (Petit and Luzum, 2010). Consequently, the level of water column below a certain point on the ground will change and this leads to radial displacements up to 10 centimetres (Petit and Luzum, 2010, Tuttas, 2010). The horizontal displacements are in much smaller extent below 1 centimetres for continental sites (Eineder et al., 2011). Similar to the solid Earth tides, the ocean loading effect can be modelled by an expansion into a set of tidal harmonics (Petit and Luzum, 2010). However, unlike the solid earth tides, the response of the oceans strongly depends on the local and regional fluid flow.

For modelling ocean loading effects, the contribution of 11 tidal waves, with different frequencies, is considered. Therefore, the total displacement caused by this effect can be obtained by summing the impacts of all the harmonics (Petit and Luzum, 2010). Assuming \( \Delta c \) as a displacement component (East, North, Up) at a particular site at time \( t \), one can write (Petit and Luzum, 2010):

\[
\Delta c = \sum_{j=1}^{11} A_{c,j} \cos \left( \chi_j(t) - \phi_{c,j} \right),
\]

where \( A_{c,j} \) and \( \phi_{c,j} \) are the amplitude and the phase of the harmonic \( j \), respectively and \( \chi_j(t) \) denotes the astronomical argument of the corresponding wave (Petit and Luzum, 2010). In the imaging geodesy software, the ocean loading corrections are applied based on the routines explained in the IERS-2010 conventions which evaluates the phase lag and amplitude of each of the 11 tidal waves for the corresponding geographic coordinates (Petit and Luzum, 2010).

**Atmospheric Pressure Loading**

Redistribution of air masses due to atmospheric circulation causes loading deformation of the Earth’s surface, which can be as large as 20 millimetres for the vertical component and 3 millimetres for horizontal components (Petrov and Boy, 2004). In order to quantify these types of deformation, accurate knowledge of the air pressure in the desired location is necessary. This periodic motion is mostly governed by the diurnal S1 and the semi-diurnal S2 tidal harmonics and is modelled with the RP03 model described in (Petit and Luzum, 2010). The spatial resolution of the input surface pressure grid is 1.125°. Therefore according to (Petit and Luzum, 2010), at any geographic location, at any time, the tidal deformation, expressed in terms of
East, North and Up components, is the sum of \(d(u, e, n)_{S1}\) and \(d(u, e, n)_{S2}\) defined as (Petit and Luzum, 2010):

\[
\begin{align*}
    d(u, e, n)_{S1} &= A_{d1}(u, e, n) \cdot \cos(\omega_1 T) + B_{d1}(u, e, n) \cdot \sin(\omega_1 T) \\
    d(u, e, n)_{S2} &= A_{d2}(u, e, n) \cdot \cos(\omega_2 T) + B_{d2}(u, e, n) \cdot \sin(\omega_2 T),
\end{align*}
\]

(2.19)

where \(A_{d1}, B_{d1}, A_{d2}\) and \(B_{d2}\) are the surface displacement coefficients expressed in the same length unit as the deformation components, \(T\) is UT1 in days and \(\omega_1\) and \(\omega_2\) are the frequencies of the S1 and S2 atmospheric tides.

**Pole Tides**

The deformation of the Earth induced by the changes in the Earth’s rotational axis due to polar motion is called pole tides. The perturbations to the Earth’s rotation axis primarily occur at periods of 433 days (called the Chandler wobble) and annual (Vanicek and Krakiwsky, 1986). The position displacements are in the magnitudes of 25 millimetres in the vertical direction and 7 millimetres in the horizontal direction, respectively. Based on the centrifugal potential caused by the Earth’s rotation mentioned in (Petit and Luzum, 2010), the radial displacement \(S_r\) and the horizontal displacements \(S_\theta\) and \(S_\lambda\) of a point, with geographical coordinates of \(\phi = \frac{\pi}{2} - \theta\) and \(\lambda\), are obtained in millimetres as follows (Petit and Luzum, 2010):

\[
\begin{align*}
    S_r &= -33 \sin(2\theta) (m_1 \cos \lambda + m_2 \sin \lambda) \\
    S_\theta &= -9 \cos(2\theta) (m_1 \cos \lambda + m_2 \sin \lambda) \\
    S_\lambda &= +9 \cos \theta (m_1 \sin \lambda - m_2 \cos \lambda)
\end{align*}
\]

(2.20)

with \(m_1\) and \(m_2\) describing the time-dependent offset of the instantaneous rotation pole from the mean in arc seconds. The \(m_1\) and \(m_2\) variables can be obtained from the IERS-2010 convention which updates the values on an annual basis. Therefore, displacements outlined in Eq. (2.20) are evaluated for the desired geographical coordinates and then are transformed to the local East, North and Up components.

**Continental Drift**

If we consider a time series of acquisitions in course of a few years, the position of the target will change due to the plate tectonics. This affects both the vertical and horizontal coordinate components and the magnitude can reach up to a couple of centimetres per year. The correction of this effect includes the GNSS coordinate time-series analysis for the permanent receivers belonging to the IGb08 GNSS network. The horizontal and vertical drift rates of these stations are archived after removal of linear and periodic trends, discontinuities due to probable earthquake or receiver problems and proper noise characterization of the station’s coordinates. Therefore, the final coordinates after removal of all spurious effects are fixed with respect to a specific epoch and is reported in the local coordinate system.

### 2.6.2 Geodynamic correction for SAR Measurements

All of the corrections described in Subsection 2.6.1 were computed for the desired points in local East, North and Up coordinates at the corresponding times of the data-takes. Eventually, these corrections are transformed to radar azimuth and range time coordinates by a two-step transformation process (Balss et al., 2013, Eineder et al., 2011, Tuttas, 2010):

1. Displacement in the radar LOS direction is calculated by projecting the coordinate components using the SAR incidence angle \((\theta)\) and the angle between the north and the ground.
track of the beam ($\beta$):

$$\delta_{LOS} = -\delta_{Up} + \delta_{East} \sin \theta \cos \beta - \delta_{North} \sin \theta \sin \beta.$$  \hfill (2.21)

2. The displacement value obtained in previous step is divided by the speed of light and multiplied with 2 to express the correction in radar time coordinates.

2.7 Stereo-SAR for 3D Scatterer Localization

In Section 2.2, the measurement principle of a typical SAR system is addressed. It is seen that in case of no squint angle the range-Doppler equations can be expressed as Eq. (2.1). Whereas the range observable $t_r$ is present explicitly in the range equation, the azimuth timing observation $t_a$ has to be introduced by an orbit model which is the realization of the actual position of the satellite along the orbit (Gisinger et al., 2013). This model can be established via a 6th order polynomial (Gisinger et al., 2013) which relates the position of the satellite ($x_s, y_s, z_s$) and their derivatives ($\dot{x}_s, \dot{y}_s, \dot{z}_s$) to the time observations using the $a, b$ and $c$ coefficients as follows (Gisinger et al., 2013):

$$\begin{align*}
    x_s &= a_0 + a_1 t_a + a_2 t_a^2 + \ldots + a_6 t_a^6 \\
    y_s &= b_0 + b_1 t_a + b_2 t_a^2 + \ldots + b_6 t_a^6 \\
    z_s &= c_0 + c_1 t_a + c_2 t_a^2 + \ldots + c_6 t_a^6 \\
    \dot{x}_s &= a_1 + 2 a_2 t_a + \ldots + 6 a_6 t_a^5 \\
    \dot{y}_s &= b_1 + 2 b_2 t_a + \ldots + 6 b_6 t_a^5 \\
    \dot{z}_s &= c_1 + 2 c_2 t_a + \ldots + 6 c_6 t_a^5.
\end{align*}$$  \hfill (2.22)

By substituting the state vector of the satellite and its corresponding velocity elements obtained from the orbit model in the range-Doppler equations of Eq. (2.1), all the possibilities for the unknown target $X_t$ will be located on a circle oriented perpendicular to the satellite orbit. Therefore, another set of observations for the same target, but with different acquisition geometry, can be exploited to mark the 3D position of the target via estimating the intersection point of the two circles (Gisinger et al., 2013). The estimation procedure is carried out by a non-linear least squares adjustment.

The first problem is non-linearity of the equations which is commonly treated with Taylor expansion series. The second one originates from the fact that no explicit relation between the observations and unknowns are available. This means that both observables and unknowns are gathered in the equation and the first attempt should be on separating these two components. The linearisation and de-coupling is achieved by the following set-up which also leads to form the functional model of the problem (Gisinger et al., 2013):

$$\mathbf{Jx} + \mathbf{B}^T \mathbf{e} + \mathbf{w} = \mathbf{0}.$$  \hfill (2.23)

Assuming $m$ the number of timing observations and $n$ the number of unknowns (3D coordinates of the target), the terms in Eq. (2.23) are defined as follows:
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\[ \mathbf{B}_{m \times m}^T \text{ Jacobian of Eq. (2.1) w.r.t the observations} \]
\[ \mathbf{e}_{m \times 1} \text{ Residual vector of observations } (d_{\rho}, d_{\tau}) \]
\[ \mathbf{J}_{m \times n} \text{ Jacobian of Eq. (2.1) w.r.t the unknowns} \]
\[ \mathbf{x}_{n \times 1} \text{ Vector of unknowns} \]
\[ \mathbf{w}_{m \times 1} \text{ Vector of inconsistency of the system}. \]

The problem outlined in Eq. (2.23) requires an inversion in order to compute the unknown vector of \( \mathbf{x} \). Let us simplify the equation once more. Since there is no correlation between \( t_{\rho} \) and \( t_{\tau} \) of a scatterer in one acquisition and also no correlation between the timings of different data-takes, inverse of matrix \( \mathbf{B}^T \) can be readily computed. Multiplying the system of equation with \( (\mathbf{B}^T)^{-1} \), Eq. (2.23) can be re-written as (Gisinger et al., 2013):

\[ \mathbf{y} + \mathbf{e} = \mathbf{Ax} \]  \hspace{1cm} (2.24)

with

\[ \mathbf{y} = (\mathbf{B}^T)^{-1}\mathbf{w} \]
\[ \mathbf{A} = - (\mathbf{B}^T)^{-1}\mathbf{J}. \]

Finally, the unknown vector of 3D coordinates can be estimated with regular weighted least squares:

\[ \hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1} \mathbf{A}^T\mathbf{W}\mathbf{y} \]  \hspace{1cm} (2.25)

where \( \mathbf{W}_{m \times m} \) is the weight matrix of observations and is defined as the inverse of the variance-covariance matrix of the observations and \( (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1} \) is the \( n \times n \) variance-covariance matrix of the estimates which reports the precision of the 3D coordinate estimates.

2.8 Results

2.8.1 Motivation

As a characteristic of all interferometric SAR techniques, the height and deformation updates are estimated relative to a reference point. Although special care is taken to choose the point in areas which are not affected by any deformation, this cannot be fully guaranteed and leads to complication in the interpretation of the final results. Moreover, the exact 3D position of the reference point is not known. Therefore, it is more likely that the final geo-coded results will show certain offsets with respect to their true positions. The latter can also be problematic when two or more sets of results obtained from different tracks need to be fused in order to improve the point density and enhance the information content. In this case, a lack of knowledge about the exact height of the reference point leads to inconsistencies between the point clouds (Gernhardt et al., 2012).

The theoretical aspects of 3D localization of point scatterers with TerraSAR-X were addressed in the earlier sections of this chapter. In this section, the methodology is applied on a number of natural point targets and the results are reported. Eventually, the best candidate is selected to further be exploited as the reference point for the subsequent TomoSAR processing of multiple stacks of high resolution spotlight TerraSAR-X data over the city of Berlin, See Chapter 3.
2.8.2 Reference Point Candidates

Based on the motivation given in Subsection 2.8.1, the targets on which 3D stereo SAR reconstruction will be performed should have certain characteristics. At an initial step 8 point targets are considered which fulfil the following criteria:

- The target should be located in an isolated area.
- The target should be visible through the entire stack of images.
- The target should be visible, at least, in two stacks acquired with different geometries.
- The target should be located close to ground surface (≈ 0 elevation) within a stable area, i.e. no deformation.

The first condition should be satisfied in order to minimize the impact of interference caused by other targets response on the aimed target. This is met by visual inspection of the amplitude image and the corresponding optical image of the scene to identify isolated targets.

The second condition is concerned with phase stability of the target in the interferometric stack and it is dealt with by calculation of the normalized amplitude dispersion index (Ferretti et al., 2001) or more conveniently applying thresholds on the mean amplitude images to select points with high Signal-to-Noise Ratio (SNR).

The third condition is vital from the radargrammetry point of view. Although in optical imagery selection of identical targets are commonly carried out with well-established algorithms such as Scale Invariant Feature Transform (SIFT) and KLT proposed respectively by Lowe (1999) and Lucas et al. (1981), in SAR images this cannot be done due to the existence of speckle (Dellinger et al., 2012). For this reason and also considering the low number of candidates, in this study, identical targets were selected manually by visual investigation of mean amplitude images of different stacks.

The fourth condition is mostly related to the next part of this study which is TomoSAR processing of multiple stacks. Therefore, the reference point candidates were considered by comparing the optical and SAR images of the scene and choosing point-shaped targets located close to the ground surface.

The outcome of the above-mentioned procedure is eight point scatterers chosen from the central area of the city of Berlin from TerraSAR-X very high resolution spotlight images. The scatterers are from three different types categorized based on the combination of geometry used for the 3D positioning, namely, Ascending-Ascending, Descending-Descending and Ascending-Descending. The time coordinates of the scatterers are retrieved from a number of SAR images by PTA outlined in Subsection 2.3.1. Table 2.2 gives an overview of data-take configurations, the time period within which the time coordinates were measured and the number of images used for PTA.

2.8.3 Chosen Reference Point

Among the point candidates reported in Table 2.2, the one with the highest quality, i.e. lowest 3D coordinates standard deviation should be selected as the reference point. The stability of the results depends on the geometry of the observation, the number of observations and the SNR of the targets. The geometrical configuration is the most important factor as for Ascending-Descending geometries the intersection occurs at almost a 90° angle providing a well-conditioned system of equations. This effect can be clearly seen in Fig. 2.7b where a very large
Table 2.2: Data-takes configuration for the selected natural point scatterers. The effective number of data-takes for $P_{AD1}$, $P_{AD2}$ and $P_{A1}$ were later decreased because of the reason outlined in Subsection 2.8.5.

<table>
<thead>
<tr>
<th>Scatterer</th>
<th>Geometry</th>
<th>Period</th>
<th># Data-takes</th>
<th>Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{AD1}$</td>
<td>Ascending-Descending</td>
<td>2008−2011</td>
<td>33</td>
<td>42, 57</td>
</tr>
<tr>
<td>$P_{AD2}$</td>
<td>Ascending-Descending</td>
<td>2008−2011</td>
<td>30</td>
<td>42, 57</td>
</tr>
<tr>
<td>$P_{A1}$</td>
<td>Ascending-Descending</td>
<td>2008−2011</td>
<td>22</td>
<td>57, 85</td>
</tr>
<tr>
<td>$P_{A2}$</td>
<td>Ascending-Descending</td>
<td>2010−2012</td>
<td>9</td>
<td>57, 85</td>
</tr>
<tr>
<td>$P_{A3}$</td>
<td>Ascending-Descending</td>
<td>2010−2012</td>
<td>6</td>
<td>57, 85</td>
</tr>
<tr>
<td>$P_{D1}$</td>
<td>Descending-Descending</td>
<td>2010−2011</td>
<td>11</td>
<td>42, 99</td>
</tr>
<tr>
<td>$P_{D2}$</td>
<td>Descending-Descending</td>
<td>2010−2011</td>
<td>8</td>
<td>42, 99</td>
</tr>
<tr>
<td>$P_{D3}$</td>
<td>Descending-Descending</td>
<td>2010−2011</td>
<td>11</td>
<td>42, 99</td>
</tr>
</tbody>
</table>

baseline between the ascending and descending acquisitions is achievable. On the other hand, Ascending-Ascending or Descending-Descending configuration (Fig. 2.7a) results into a nearly ill-posed system because of the separation between the satellites occurred with a rather small baseline compared to the cross-heading configuration. In order to support the aforementioned discussion, the coordinate standard deviations of the scatterers introduced in Table 2.2 are plotted in Fig. 2.6. These values are reported for each coordinate component and are among the outputs of the Stereo-SAR software. The horizontal axis consist of the names of the scatterers with subscripts denoting the geometry used for the 3D positioning. The vertical axis describes the standard deviation values ranging from 1 to 9 centimetres demonstrating the high potential of this method for precise 3D scatterer localization. The first conclusion is that the precision in $X$ direction is higher than the other two components. This is mainly the effect of geometry configuration as it is also visible in Fig. 2.7. Another information is that the last two scatterers, i.e. the ones from cross-heading geometries have low values of standard deviations. This again emphasizes the effect of geometry configuration for 3D positioning. It is worth mentioning that restricting the quality control of estimates solely based on the standard deviations may not be a reliable criterion. This is mainly due to the presence of covariances between the coordinate stochastics. Therefore, a more meaningful discussion can be based on the error ellipsoid which is obtained by transformation of the posterior variance-covariance matrix of the estimates to the uncorrelated diagonal matrix of the Eigen values based Eigen-vector decomposition. In this case, the diagonal elements of the decomposed matrix mark the stochastics in the inherent SAR coordinate system. The mentioned approach was carried out on the variance-covariance matrix of each point and the behaviour of the results were consistent with Fig. 2.6.

Based on the above discussion, the matter of configuration leads to discarding the point targets identified from the same-heading tracks narrowing the selection between $P_{AD1}$ and $P_{AD2}$. The other two criteria mentioned earlier (number of observations and SNR) cannot be helpful in this case since they are almost identical for both points. Therefore, $P_{AD1}$ was selected as the reference point since it has a slightly better precision and it was also visible in all the four stacks. Fig. 2.8 shows the selected target in the mean amplitude images of one ascending and one descending TerraSAR-X spotlight images. The target is assumed to be the base of a lamp-pole in a pedestrian area near the Berlin central station.

### 2.8.4 Corrections

As it is mentioned in Section 2.4, the time measurements obtained from SAR images are affected by systematic error effects. In this study, the ionospheric and tropospheric delays were estimated based on the initial vTEC and ZPD values obtained from a permanent IGS GNSS
Chapter 2. 3D Scatterer Localization with TerraSAR-X

Figure 2.6: The estimated coordinate stochastics for three components obtained from the Stereo-SAR software. The importance of geometry configuration on 3D positioning can be clearly seen as the scatterers obtained from cross-heading tracks are more stably localized.

Figure 2.7: Different geometries taken into account for 3D scatterer reconstruction in Berlin receiver located in Potsdam approximately 30 km away from the central area of Berlin. For geodynamics effects, formulas adapted from the IERS-2010 convention were used. Besides from these, the azimuth and range calibration constants of TerraSAR-X satellites which account for the cable delays and all the effects that were not modelled during the initial calibration should be modified in accordance with our requirements. It is important to note that all the errors were projected into the range and azimuth directions and their effect were removed from the measurements. Each of the aforementioned issues are briefly explained in the following and the
Ionospheric Corrections

The ionospheric correction is followed from the methodology outlined in Subsection 2.5.2 and is only applied on the range measurements. The closest permanent IGS receiver is located in city of Potsdam approximately 30 km away from Berlin in south-western direction. The corrections are estimated for each time measurement of the scatterer. Fig. 2.9 shows the ionospheric delay estimates for \( P_{\text{AD1}} \). The black dots mark the observations from the ascending track (beam 57) which are acquired approximately at 16:50 UTC. The red dots stand for the descending acquisitions (beam 42) occurred at 5:25 UTC. The difference in the acquisition times and also the geometry of the satellites explains the higher delay values for ascending observations. In general the values vary between 1 to approximately 6 centimetres. These magnitude of errors were expected due to operation of TerraSAR-X satellites in X-band frequency domain.

Tropospheric Corrections

Similar to the ionospheric corrections, tropospheric corrections are estimated based on the Potsdam IGS receiver and are only applied on the range measurements. It is important to note that the precision of tropospheric delay estimates could be enhanced if the receiver was located closer to the investigated area. The corrections applied on \( P_{\text{AD1}} \) related to each of the data-takes can be seen in Fig. 2.10. As it was expected, the tropospheric delays are the most prominent error sources for the range observations ranging from approximately 2.9 to 3.3 meters. The plot also illustrates that the correction values for ascending observations are higher than the descending ones. This is due to the fact that ascending observations are acquired in the mid afternoon while the descending ones are acquired in early morning. Therefore, due to higher temperature and consequently more water vapour amounts, higher correction values correspond to the ascending observations. Also the seasonal variation of the corrections show that higher error values correspond to warmer seasons while in the winter the magnitude of errors drops dramatically.
Figure 2.9: Ionospheric delay estimated for P_{AD1}. The horizontal axis describes the date of the data-takes. The vertical axis represents the ionospheric corrections in meters. Ascending and descending acquisitions are distinguished by color black and red, respectively.

Figure 2.10: Tropospheric delay estimated for P_{AD1}. The horizontal axis describes the date of the data-takes. The vertical axis represents the tropospheric corrections in meters. Black dots stand for ascending observations and red dots mark the descending ones.

Geodynamic Corrections

The geodynamic corrections are applied in order to compensate for the position changes of the point in the period of data-takes. They are used to correct both azimuth and range measurements. Fig. 2.11 and Fig. 2.12 illustrate the correction applied on each of the time measurements of P_{AD1} in range and azimuth coordinates, respectively. In Fig. 2.11 Distinguishing between the ascending and descending observations let us see the smooth behaviour of the geodynamics effects from one to another observations. Sudden jumps occur due to non-continuous time measurements as the scatterer was not visible in certain acquisition times. The corrections in range
varies from -8 to almost 15 centimetres. In Fig. 2.12, the same smooth progression of the corrections are visible. The range of corrections are from -6 to 6 centimetres where the sign of the corrections are related to different acquisition geometries.

\[ \text{Figure 2.11: Geodynamic corrections estimated for } P_{\text{AD1}} \text{ in the range direction. The black dots represents the observations from the ascending geometry while red ones stand for descending observations. By separating the ascending and descending observations, the smooth behaviour of geodynamic effects are apparent.} \]

\[ \text{Figure 2.12: Geodynamic corrections estimated for } P_{\text{AD1}} \text{ in the azimuth direction. Ascending and descending observations can be separated by sign of the corrections. The magnitude of errors are smaller comparing to range corrections.} \]

**Calibration Constants**

The calibration constants include certain azimuth and range time shifts which are stated in the L1B product annotation files of each scene. Since the initially aimed pixel localization accuracy of TerraSAR-X was in the meter level, these constants should be modified based on proper
atmospheric and geodynamics error modelling in order to achieve centimetre accuracies. In this study, these values were adjusted based on the experiments carried out in Wettzell, Germany (Balss et al., 2013). Table 2.3 shows the default annotated values in the left column and the updated values in the right column.

<table>
<thead>
<tr>
<th>Default</th>
<th>Updated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range [m]</td>
<td>Azimuth [m]</td>
</tr>
<tr>
<td>0</td>
<td>-0.0861</td>
</tr>
</tbody>
</table>

Table 2.3: TerraSAR-X geometrical calibration values. The default values refer to initial geometrical calibration stated in the annotation files and the updated values are based on Wettzell geolocation experiment (Gisinger et al., 2013).

2.8.5 Stereo-SAR Results

As is mentioned in Section 2.7, by combination of corrected azimuth and range timings of a scatterer from two or more different orbits and knowledge about satellite state vector from the product annotation files, a precise scatterer localization is achievable. In this subsection the 3D position of $P_{AD1}$ (Fig. 2.8) obtained from stereo-SAR and considerations to improve the statistics of the results are reported.

Initial Results

For the initial result, 33 radar timing observations in a four year period from 2008 to 2011 were used for 3D processing (Table 2.2). After applying the corrections and adjusting the calibration constants based on Subsection 2.8.4, the precision of the final 3D solution was in the order of 1 meter. This large variability can be better explained by Fig. 2.13 which shows the azimuth and range observation residual behaviour. One can clearly recognize the large jump occurred around March 2010 and a smaller jump on June 2011 in both residual graphs. According to (Balss et al., 2011) the SAR processor of TerraSAR-X satellites was updated in exactly these two dates. Therefore, the graph implies the improvements of geo-location capabilities of TerraSAR-X satellites.

Besides from the jumps, azimuth residuals provide another piece of information. Least squares adjustment tries to solve for the parameters of interest by minimizing the squared sum of the residuals. Thus normally residuals fluctuate very close to zero. However, this is not the case for the azimuth residuals and certain biases with different magnitudes are visible before and after the processor update in March 2010. This bias is introduced due to opposite movements of the satellites in the ascending and descending tracks. Therefore, the circles that are obtained as a result of solving the zero-Doppler equation (Eq. (2.1)) for the scatterer should be shifted towards each other until they intersect and the orthogonality condition can be met. This can be achieved by modifying the azimuth observables by the mean value of the azimuth residuals.

Modified Results

After investigation of the initial results reported above two major modifications were performed in order to improve the precision of the estimates and decrease the residual bias. At the first step, all the time observations belonging to the period before March 2010 were discarded as the results from the new processor provides less residual offset. Furthermore, the last five observations were also discarded because of inconsistent behaviour relative to other acquisitions. Secondly, the azimuth time observations of the period after March 2010 were modified with the
mean value of azimuth residuals assuming that the other error effects were properly removed. Therefore, the azimuth calibration constant was changed to -0.3098 m computed as an average of the residuals shown in Fig. 2.13. The range calibration constant remained unchanged. With these adjustments the 3D coordinate calculation was repeated. Fig. 2.14 illustrates the azimuth and range residual behaviour of $P_{AD1}$ after the modifications. It can be clearly seen that the residuals magnitude is considerably improved in comparison with the initial results. The range residuals act more precise comparing to azimuth residuals with variations less than 5 centimetre. The approximately 30 cm bias is not visible in the azimuth residuals due to the adjustment of the calibration constant based on the average value of the azimuth residuals related to the initial results.

Based on the discussion above, the final 3D coordinate estimate of $P_{AD1}$ with the ascending-descending geometry and with 17 corrected azimuth and range observations using TerraSAR-X is:

$$X = 3783630.014 \pm 0.010 \text{ m}$$
$$Y = 899055.0040 \pm 0.010 \text{ m}$$
$$Z = 5038487.589 \pm 0.011 \text{ m}.$$ 

It is important to note that since the satellite state vectors adopted from the product annotation files are defined in the ITRF 2008 reference frame, the 3D coordinates are automatically calculated in this reference frame and no transformation is required.

Besides the coordinate estimates, the respective precision in each coordinate component is also reported. It can be clearly seen that regarding the stochastics, the result is remarkably stable with precision of about 1 cm in each component. However, no information can be obtained from
Figure 2.14: Azimuth and range observation residuals of $P_{AD1}$ after restricting the period to after March 2010 and adjusting the azimuth calibration constant. The azimuth residuals have a peak to peak variation of 20 cm where the range residuals are not higher than 5 cm (Gisinger et al., 2013).

The absolute accuracy of the coordinates. The coordinate results may have certain offsets with respect to the true position which can be caused due to non-proper tropospheric correction, not modifying the range calibration constant and, most probably, not selecting the exact identical points from ascending and descending image stacks. The latter can be explained by the finite dimension of the lamp pole, say 20 cm, which was not modelled during the processing.
Chapter 3

SAR Tomography

This chapter reviews the basic principles of SAR tomography. An overview of some important spectral estimators applicable to TomoSAR is given as well as the detailed description of the processing steps implemented in the Tomographic SAR processing system of DLR. The methodology for fusion of absolute multi-track TomoSAR point clouds is introduced by selecting an identical reference point. The results of TomoSAR processing and point cloud fusion are reported as well as analysing the localization accuracy of the fused point cloud with respect to the reference Digital Surface Model (DSM) of the city of Berlin.

3.1 Introduction

SAR sensors provide 2D images of the scene reflectivity based on reconstructing the scatterers response in the azimuth and the range directions (Hanssen, 2001). However, the imaged area is three dimensional and a single SAR image, due to the side-looking geometry of the sensor, is a projection of the 3D reflectivity onto the azimuth-range plane (Fornaro et al., 2005). Access to the third dimension, namely elevation or cross-range, is possible by applying InSAR. Nonetheless, InSAR is capable of providing an approximate estimate of the digital elevation model since it only uses two acquisitions to exploit the phase difference. This gives the motivation to use more advanced methods for retrieving the full reflectivity profile of an azimuth-range pixel along the elevation direction and consequently resolve different scattering mechanisms within a single pixel (Fornaro et al., 2005, Zhu and Bamler, 2010b). This method is called SAR tomography (TomoSAR).

TomoSAR is a multi-baseline technique which uses several SAR images, acquired from slightly different orbital positions, to reconstruct the reflectivity function of each azimuth-range pixel along the elevation direction and thus obtains focused 3D SAR images (Zhu and Bamler, 2010b). It is regarded as a spectral estimation problem as the complex-valued SAR measurements of an image stack, for a specific resolution cell, are actually the sampled Fourier transform of the reflectivity function in the elevation direction. Irregular sampling in the elevation direction as well as consideration of motion-induced phase differences causes the TomoSAR system model to be highly ill-conditioned (Zhu, 2008, 2011, Zhu and Bamler, 2010b). Various regularized spectral estimators have been applied for tomographic inversion for which a complete list can be found in (Zhu, 2011).

One of the most important applications of TomoSAR is 3D mapping and deformation monitoring of urban areas in which the occurrence of layover phenomena is quite prevalent. Experiments using this method on ERS data are reported in (Fornaro et al., 2005) and (Fornaro and Serafino, 2006) which demonstrate the strength of the method in multiple scatterer resolving capability for medium resolution data. In (Zhu, 2011, Zhu and Bamler, 2010a,b), TomoSAR processing was...
applied on TerraSAR-X VHR spotlight data with different approaches regarding the inversion procedure in order to achieve higher super-resolution capabilities in the elevation direction.

This chapter is continued with a description of the basic principles, the imaging geometry, and an extension of the imaging model of SAR tomography to Differential TomoSAR (D-TomoSAR) in Section 3.2. The most recently developed TomoSAR spectral estimators are explained in Section 3.3. In Section 3.4 the DLR’s tomographic SAR processing system (Tomo-GENESIS) developed by Zhu et al. (2013) is introduced and different processing steps are explained. Section 3.5 describes the methodology to fuse absolute 3D TomoSAR point clouds obtained from different viewing geometries. In Section 3.6 the experimental results of TomoSAR processing of four image stacks over the city of Berlin is reported and finally the chapter is closed with a localization accuracy analysis of the absolute fused TomoSAR point cloud relative to a reference DSM of Berlin that is discussed in Section 3.7.

3.2 Basic Principles & Imaging Model

SAR tomography aims at reconstructing the reflectivity profile along elevation for each azimuth-range pixel by extending the synthetic aperture principal into the elevation direction (Reigber and Moreira, 2000). This method uses several SAR images taken from different orbital positions with slightly different viewing angles which results in 3D focused SAR images (Zhu and Bamler, 2010b). The simplified imaging model of TomoSAR is visualized in Fig. 3.1 where $x$ denotes the flying direction of the satellite which is orthogonal out of the plane, $r$ represents the range coordinates, coordinate $s$ refers to the elevation and $\Delta b$ shows the baseline in the elevation direction (i.e. the elevation aperture length).

![Figure 3.1: The imaging geometry of TomoSAR (Zhu and Bamler, 2010b)](image)

Based on the imaging geometry illustrated in Fig. 3.1 if we consider $N$ acquisitions taken from slightly different orbital positions and consider one of them as the master, the $n^{th}$ acquisition with elevation baseline of $b_n$ with respect to the master will have the complex valued measurement $g_n$ for an azimuth-range pixel which is expressed as (Zhu, 2011):
Chapter 3. SAR Tomography

\[ g_n = \int_{\Delta s} \gamma(s) \exp(-j2\pi \xi_n s) ds \]  
(3.1)

where \( \Delta s \) is the elevation extent of illuminated objects of the scene, \( \gamma(s) \) is the reflectivity profile along elevation \( s \), and \( \xi_n = \frac{-2b_n}{\lambda r} \) is the elevation frequency. It is inferred from Eq. (3.1) that the focused complex valued measurement \( g_n \) is the Fourier Transform of the reflectivity function along elevation \( \gamma(s) \) at position \( \xi_n \). This can be explained based on the Fraunhofer diffraction phenomena (Rees, 2012). The mathematical derivation of Eq. (3.1) can be found in (Zhu, 2008).

Eq. (3.1) can be approximated by discretizing \( \gamma(s) \) within its extent \( \Delta s \) by considering \( L \) discrete elevation indices. Therefore we have (Zhu and Bamler, 2010b):

\[ g_n = \delta_s \sum_{l=1}^{L} \gamma(s_l) \exp(-j2\pi \xi_n s_l) \]  
(3.2)

where \( \gamma(s_l) \) describes the discrete reflectivity function along elevation \( s_l \) and \( \delta_s = \frac{\Delta s}{L-1} \) is the discretization interval. Including possible noise, the functional model of the problem can be written as:

\[ g = R\gamma + \epsilon \]  
(3.3)

where \( g \) is the complex valued measurement vector with size \( N \times 1 \), \( R \) is an \( N \times L \) sensing matrix with \( R_{nl} = \exp(-j2\pi \xi_n s_l) \), \( \gamma \) is the discrete reflectivity vector of size \( L \times 1 \) and \( \epsilon \) is the \( N \times 1 \) noise vector which can be caused by thermal noise, temporal decorrelation, phase errors due to atmospheric delay and ignored deformation (Zhu and Bamler, 2010b).

TomoSAR can be extended to differential form by taking into account the motion of scatterers in the azimuth-range pixel (Lombardini, 2005). Consequently, Eq. (3.1) is extended to (Zhu and Bamler, 2010b):

\[ g_n = \int_{\Delta s} \gamma(s) \exp(-j2\pi(\xi_n s + \eta_n V(s))) ds \]  
(3.4)

where \( \eta_n = \frac{2v}{\lambda} \) is the velocity frequency and \( V(s) \) is the deformation LOS velocity along elevation. Eq. (3.4) can be rewritten as (Zhu, 2011, Zhu and Bamler, 2010b):

\[ g_n = \int_{\Delta v} \int_{\Delta s} \gamma(s) \delta(v - V(s)) \exp(-j2\pi(\xi_n s + \eta_n v)) ds dv \]  
(3.5)

where \( \Delta v \) is the motion parameter extent. With Eq. (3.5) along with the complex reflectivity and elevation of different scatterers, the linear velocity can be estimated. This further implies that if two scatterers cannot be resolved due to the low resolution in elevation direction, they can be separated by their different motion during the period of acquisitions (Zhu and Bamler, 2010b).

As is mentioned in Section 3.1 due to irregular sampling of the elevation aperture, TomoSAR can be considered as an ill-conditioned spectral estimation problem (Zhu, 2011). In the following section, some of the regularized spectral estimation strategies related to SAR tomographic inversion, especially using VHR TerraSAR-X spotlight data, are discussed.
3.3 Spectral Estimators of SAR Tomography

As is briefly mentioned in Section 3.2, the system of equations describing the TomoSAR imaging model is under-determined due to irregular sample positions. This necessitates the utilization of regularized parameter estimation methods for retrieving the reflectivity profile of each azimuth-range pixel along the elevation direction (Fornaro et al., 2005). In this section, some of the well-known spectral estimators relevant to SAR tomography are described. An exhaustive list of TomoSAR spectral estimation strategies can be found in (Zhu, 2011).

3.3.1 Non-linear Least Squares

The functional model of TomoSAR outlined in Eq. (3.2) is linear in complex amplitude $\gamma$ and non-linear with respect to elevation $s$. Therefore, the under-determined system model can be reduced to an over-determined problem by separating the amplitude and the elevation and can be written as (Zhu, 2011):

$$g = R(s) \gamma(s) + \epsilon$$

(3.6)

where the $N \times K$ matrix $R(s)$ is only a function of the unknown elevations of the scatterers $s$ while $K$ denotes the assuming number of scatterers in one azimuth-range pixel. Consequently, the optimum complex amplitude $\hat{\gamma}(\hat{s})$ is estimated by minimizing the sum of the squared residuals (Teunissen et al., 2005, Zhu, 2011):

$$\hat{\gamma}(\hat{s}) = \arg \min \left\{ \| g - R(s)\gamma(s) \|^2 \right\}.$$  

(3.7)

In Zhu (2008), this method was implemented followed by a subsequent model selection in order to estimate the reflectivity profile, number and elevation of each scatterer in one pixel.

The non-linear least squares method is computationally expensive as a multi-dimensional search, based on the assumed number of scatterers and motion parameters, is required. Therefore, it is not suitable for operational processing of big areas, although theoretically it provides the best estimate for our application (Zhu, 2011).

3.3.2 Singular Value Decomposition (SVD)

Singular Value Decomposition (SVD) is a regularization tool which is beneficial in case of dealing with ill-conditioned systems. Utilizing SVD relevant to tomographic SAR inversion is described in (Fornaro et al., 2005). However, the method uses truncated SVD which discards the singular values smaller than a pre-defined threshold. Another inversion method based on SVD was proposed in (Zhu and Bamler, 2010b) which is used in this study.

From Eq. (3.2), consider an estimate of $\gamma$ that is obtained from the pseudoinverse $R^\dagger$. Using SVD we can write (Zhu and Bamler, 2010b):

$$\hat{\gamma} = R^\dagger g = \sum_{n=1}^{N} \sigma_n^{-1}(u_n^T g) v_n$$  

(3.8)

where $\sigma_n$ are the non-negative singular values of $R$ and the vectors $u_n$ and $v_n$ are the left and right singular vectors of $R$, respectively. It is evident that the noise will be amplified with smaller singular values. Therefore, regularization can be applied by allocating weights to the singular values according to their magnitudes. The application of the mentioned weighting to our problem which is known as Tikhonov regularization yields the following (Zhu, 2011):
\[
\hat{\gamma} = (R^T C_{\epsilon\epsilon}^{-1} R + C_{\gamma\gamma}^{-1})^{-1} R^T C_{\epsilon\epsilon}^{-1} g = \sum_{i=0}^{N} \frac{\sigma_i}{\sigma_i^2 + \sigma_i^2}(u_i^T g)v_i,
\]

if we consider the covariance matrix of observations as \(C_{\epsilon\epsilon} = \sigma^2 \cdot \mathbf{I}\) and a-priori covariance of the estimates as \(C_{\gamma\gamma} = \mathbf{I}\). As is demonstrated in (Zhu and Bamler, 2010b), this method is computationally efficient and is not sensitive to irregular sampling.

The result of the SVD regularization is a continuous reflectivity profile in the elevation direction for each azimuth-range pixel. The estimation process is continued with maximum detection and model order selection in order to estimate the number of scatterers in one resolution cell and later the information is used for amplitude, phase and motion retrieval of each scatterer.

The SVD method, known as SVD-Wiener (Zhu, 2008) has negligible super-resolution capability in the elevation direction. In Zhu (2011), it is shown that this algorithm is not able to distinguish two scatterers closer than \(\delta_s = 0.8 \rho_s\) where \(\rho_s\) is the elevation resolution. In the following, the recent tomographic SAR inversion algorithm, with high super-resolution capability will be briefly described.

3.3.3 The SL1MMER Algorithm

In (Zhu and Bamler, 2010a) the possibility of tomographic SAR inversion using a Compressive Sensing (CS)-based approach is introduced. CS is a relatively new technique in the field of signal processing that allows sparse signal reconstruction (Baraniuk, 2007). If we consider a finite one-dimensional discrete-time signal \(x\) defined based on the orthogonal basis of \(\Psi\), the signal \(x\) is \(K\)-sparse if the projection coefficient vector \(s = \Psi x\) has only \(K\) non-zero elements (Baraniuk, 2007, Zhu and Bamler, 2010a). The \(x\) and \(s\) vectors are equal representations of the signal in the time and \(\Psi\) domains, respectively (Baraniuk, 2007). The sparsity is the main pre-requisite for applying CS to a problem.

In Section 3.2 it is introduced that the initial objective of TomoSAR is retrieval of the reflectivity signal \(\gamma\) along elevation. For VHR TerraSAR-X data, \(\gamma\) can be considered a sparse signal due to the following contributions (Zhu and Bamler, 2010a):

- Weak diffuse scattering from vertical and horizontal surfaces such as roads and building walls. These backscattered signals are of much smaller extent than the elevation resolution. Therefore, they are recorded as delta functions along elevation. In Fig. 3.2 these reflections are shown in blue.

- Strong backscattered signals from metallic structures and dihedral or trihedral reflections. These are the so called persistent scatterers and the density of them is large using VHR images. In Fig. 3.2 they are shown with red colour.

- Returns from volumetric scatterers such as vegetation which decorrelates in time and are considered as noise. The green backscattered signal in Fig. 3.2.

Therefore, due to the tight orbital tube of TerraSAR-X satellites and if volumetric scatterers are treated as noise, the signal in the elevation direction is sparse. This means we can use the CS concept for reconstructing this signal. To make the CS-based approach more robust and overcome the limitations of it, which are biased reflectivity estimates and outliers, model order selection and linear parameter estimation strategies are added to the conventional algorithm (Zhu, 2011, Zhu and Bamler, 2012). The algorithm is called SL1MMER which is pronounced slimmer and stands for Scale down by L1 norm Minimization, Model Selection and Estimation.
Reconstruction. The SL1MMER algorithm is shown schematically in Fig. 3.3 and the three steps of it are briefly explained as follows (Zhu, 2011, Zhu and Bamler, 2012):

1. The sparse estimate of reflectivity along the elevation $\gamma$ is given by a L2 norm minimization of residual of Eq. (3.2) accompanied by a L1 norm regularization of $\gamma$ which is written as:

$$\hat{\gamma} = \arg \min \left\{ \| g - R\gamma \|_2^2 + \lambda_k \| \gamma \|_1 \right\}. \quad (3.10)$$

However, due to outliers and the fact that L1 norm introduces amplitude biases and Eq. (3.10) should be improved.

2. The number of scatterers in one azimuth-range pixel is estimated using model order selection techniques. Model complexity is described by the number of unknown parameters. In our case, we want to estimate the phase, amplitude and elevation of each scatterer. Therefore, if we assume $K$ to be the number of scatterers, the complexity is defined by $3K$. Since we know that the signal is at most four sparse, taking into account that the number of mapped scatterers in one resolution cell is not higher than four, we start the number of scatterers from zero to four and for each of them we evaluate the equation in the second block of Fig. 3.3. The optimum number of scatterers $\hat{K}$ then corresponds to the lowest value obtained from model order selection that demonstrates which model describes the situation better. For more information about the applicability of model order selection in TomoSAR, the reader is referred to (Zhu, 2008, Zhu and Bamler, 2010b, 2012).

3. In the last step, the under-determined system of equation described in Eq. (3.2), is substituted with another equation with a design matrix which only has the columns defining the sparse signal. Therefore, the size of the design matrix shrinks from $N \times L$ to $N \times \hat{K}$ and the system becomes over-determined. The final sparse reflectivity profile is obtained from Eq. (3.11).

$$\hat{\gamma}(\hat{s}) = (R^H(\hat{s})R(\hat{s}))^{-1}R^H(\hat{s})g \quad (3.11)$$
where $\mathbf{R}(\mathbf{s})$ refers to the design matrix describing the sparse signal and $\mathbf{R}^H$ is the conjugate transpose of $\mathbf{R}$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{flowchart.png}
\caption{Flowchart of the SL1MMER algorithm (Zhu, 2011).}
\end{figure}

With these three steps, the SL1MMER algorithm will provide robust estimates of the number of scatterers per azimuth-range pixel, amplitude, phase, elevation and possible deformation in LOS direction of each scatterer.

The SL1MMER algorithm is a robust spectral estimator relevant to TomoSAR with very high super-resolution capability in the elevation direction. However, this method is computationally expensive and is better to be integrated with other conventional spectral estimation strategies, for instance, SVD-Wiener.

3.4 DLR’s Tomographic SAR Processing System (Tomo-GENESIS)

This section introduces the DLR’s tomographic SAR processing system (Tomo-GENEric System for Interferometric SAR (GENESIS)) developed by Zhu et al. (2013). The following contents are based on the more detailed explanations given in Zhu et al. (2013) and its references.

The Tomo-GENESIS consists of three main procedures (see Fig. 3.4) namely, TomoSAR pre-processing including Atmospheric Phase Screen (APS) estimation and removal, tomographic reconstruction and fusion of TomoSAR point clouds obtained from different viewing geometries. The input for the system is the co-registered InSAR stacks which, in this study, are prepared using the DLR’s in-house PSI-GENESIS. In this section, each procedure is treated in details. Subsection 3.4.1 describes the InSAR stacking, Subsection 3.4.2 explains the pre-processing, Subsection 3.4.3 describes the tomographic reconstruction and Subsection 3.4.5 briefly discusses the point cloud fusion algorithm developed by Gernhardt et al. (2012).

3.4.1 Interferometric Stacking

Interferometric stacking refers to a multi-step operation that is used to prepare the focused complex SAR images to be initiated in multi-pass SAR interferometry. The problem arises from the fact that images from the same scene are acquired from slightly different geometries. Interferometric stacking is applied to co-register all images in a stack to a single master scene so that
Figure 3.4: Tomo-GENESIS processing chain (Zhu et al., 2013). The processing starts with co-registered complex SAR images and produces 4D (space-time) fused TomoSAR point clouds.

a specific pixel in all images correspond to the same location.

Interferometric stacking mainly consists of the following steps (Hanssen, 2001):

- Master selection - selecting the master image is a critical choice since it highly influences the quality of the final outputs, for instance, the interferograms and the deformation maps.
- Co-registration - includes computing the translation vectors between master and slave images on sub-pixel level.
- Re-sampling and interpolation - consists of reconstructing the continuous signal from its samples in the slave image and further re-sampling of the reconstructed signal in locations of master image.
- Spectral range and azimuth filtering - to reduce the impact of noise before interferogram formation, spectral filtering in both dimensions are applied in order to discard the non-overlapping parts of the signal in the spectral domain.
- Interferogram formation - after performing the steps outlined above, the interferograms are formed by a pixel-wise complex multiplication of the master and the slave image.

In this subsection, the stacking procedure in a way that is implemented in PSI-GENESIS of the DLR is described after (Adam et al., 2003, Kampes, 2006). This procedure is performed for four stacks of VHR spotlight images of TerraSAR-X as a requirement for TomoSAR processing. It is important to note that from the steps outlined above only the first one i.e. the master selection
step is explained in details, since it is not explicitly documented yet. The geometrical PS-based co-registration and the re-sampling steps are fully treated in (Adam et al., 2003). The fourth step, namely, the spectral filtering, is not applied since for determining the offsets between the slaves and the master image only the point scatterers are considered (Kampes, 2006). Finally, the interferogram formation is well-described in InSAR references such as (Bamler and Hartl, 1998, Hanssen, 2001).

The master SAR image is selected based on maximizing the theoretical stack coherence of the interferometric stack (Kampes, 2006). The stack coherence depends on three different parameters, namely, perpendicular, temporal and Doppler baselines of the acquired images and is defined as follows (modified after (Kampes, 2006)):

$$\gamma^m = \frac{1}{K} \sum_{k=0}^{K} g(B_{k,m}^\perp, A) \times g(T_{k,m}^t, H) \times g(f_{DC}^{k,m}, S), \quad (3.12)$$

where

$$g(x,c) = \begin{cases} 1 - \frac{|x|}{c} & \text{if } |x| < c \\ 0 & \text{otherwise} \end{cases}.$$

In Eq. (3.12), superscripts $m$ and $k$ denote the master and slave acquisitions, respectively. $K+1$ is the total number of images in the stack, $B_{k,m}^\perp$ is the perpendicular baseline between the master $m$ and the slave $k$ expressed in meters, $T_{k,m}^t$ is the temporal baseline in days and $f_{DC}^{k,m}$ is the mean Doppler centroid frequency difference or the Doppler baseline in Hertz (Kampes, 2006). The values of the function $g(x,c)$ are computed applying the second equation in which $c$ stands for a critical baseline, for which total decorrelation is expected for targets with a distributed scattering mechanism (Kampes, 2006). Furthermore, $A, H$ and $S$ are numerical values dependent on the type of the sensor. These values can be modified based on wavelength, look-angle and other sensor-specific properties (Kampes, 2006).

For each acquisition in the image stack, the theoretical stack coherence function (Eq. (3.12)) is evaluated. It is evident that $\gamma^m$ is higher if the master image is selected more centrally in time due to smaller temporal baseline values. However, the stack coherence value drops dramatically for large perpendicular or Doppler baselines. In PSI-GENESIS, after the calculation of the stack coherence, the acquisitions which correspond to the first eight highest $\gamma^m$ values are selected as best possible candidates for master selection. Usually these images are acquired close to the center of the acquisition period and the distribution of perpendicular and the Doppler baselines are approximately symmetric with respect to them. Furthermore, for these eight candidates, another criterion is taken into account which is the season of the acquired image (Adam et al., 2003). Based on this, the atmospheric delay, mainly caused by the propagation of the SAR signal in the troposphere, is simulated and acts as another useful criterion for choosing the optimum master scene. The tropospheric delay estimation algorithm for this purpose is described in details in (Adam et al., 2011). Meteorological information, namely the normal temperature, the partial water vapour pressure and the total pressure, at the time of the radar acquisition are first obtained from numerical weather data and then converted to dry and wet refractivities (Adam et al., 2011). Tropospheric dry and wet delays on satellite range measurements are reported in millimetres and the master candidates for which the variation of the total delay is smaller are preferred upon those with higher atmospheric variability (Adam et al., 2011).

Among the suggested eight candidates, the best candidate is to be selected by the user with the help of the so-called mini-stacking. The mini-stacking consists of building eight small interferometric stacks with different master scenes. It includes down-sampling of the master candidates,
applies co-registration and re-sampling of the slaves onto the master scene, generates interferograms between seven slaves and the master and eventually estimates coherence with respect to the master scene. An example of the mini-stacking can be seen in Fig. 3.5 and Fig. 3.6 from which the former visualizes the differential interferograms between each master and the slaves and the latter demonstrates the corresponding coherence estimates. The mentioned information are reported to the user as well as the predicted dry and wet delay estimates at the time of the master acquisition. Therefore, by visual inspection of each of the eight mini-stacks results, containing the interferograms and the coherence estimates and taking into account the atmospheric delay variability at the exact time of the master acquisition, the user can decide to select the optimum master scene.

Figure 3.5: The differential interferograms made with six different master scenes selected based on stack coherence. The visual inspection of the interferograms along with the assessment of coherence estimates guide the user for choosing the best master scene.

Figure 3.6: The coherence map evaluated for six different master scenes selected based on stack coherence. Investigating the coherence estimates accompanied by analysis of the differential interferograms allows the user to select the optimum master scene.
After selection of the master scene for each stack, the slave images are co-registered onto the master using the method described in (Adam et al., 2003). The final output of the interferometric stacking is the differential interferograms. These are the interferograms formed between each slave and the master image for which the topographic phase is removed. The topographic phase is usually modelled by using an external Digital Elevation Model (DEM) of the investigated scene. Furthermore, the calibrated amplitude image of each slave image is also stored.

3.4.2 TomoSAR Pre-processing

TomoSAR pre-processing includes estimation and removal of the APS from the stack of co-registered complex SAR images (Zhu et al., 2013). APS is regarded as atmospheric inhomogeneities that are superimposed on each SAR image in an interferometric stack and affect the eventual deformation estimate quality (Ferretti et al., 2001).

In this subsection the steps required prior to application of the TomoSAR are described. The APS estimation and removal is carried out using the method outlined in (Zhu et al., 2013). This method is a modification of the original PSI technique developed by Ferretti et al. (2001) and is customized after the enhanced spatial differences method described in (Fornaro et al., 2009). To be able to use this method, we assume that the APS is spatially correlated but highly uncorrelated from one image to another (Hanssen et al., 2005, Zhu et al., 2013). In the following, each step of the pre-processing is described and, where necessary, intermediate results are reported for beam57 of the Berlin stacks.

Down-sampling the SAR images

The output of the interferometric stacking outlined in Subsection 3.4.1 are the differential interferograms made between the master and slave orbits as well as the calibrated amplitude images of each slave image. In order to reduce the computational time, it is beneficial to exploit multi-looked data, especially, when processing large areas such as entire cities or very high resolution data. Therefore, the entire stack including both the amplitude and phase images were multi-looked by factors of 5 and 10 in azimuth and range, respectively. The next steps of the pre-processing which are described in the following are applied on the low resolution data.

Coherence Estimation & Sparse Network Creation

The coherence is generally used as a measure for the interferometric phase accuracy and is defined as follows (Hanssen, 2001):

$$\hat{\gamma} = \frac{|\sum_{n=1}^{N} y_1^{(n)} y_2^{* (n)}|}{\sqrt{\sum_{n=1}^{N} |y_1^{(n)}|^2 \sum_{n=1}^{N} |y_2^{(n)}|^2}}, \quad 0 \leq \hat{\gamma} \leq 1 \quad (3.13)$$

where $\hat{\gamma}$ is the estimated magnitude of the complex correlation coefficient and is estimated via ensemble averaging with $N$ as the total number of samples. $y_1$ and $y_2$ are the complex values of pixel $n$ from images 1 and 2 and * is the complex conjugate operator. $\hat{\gamma}$ is available for each pixel and higher values indicate less noisy and more accurate phase content.

Relevant to the pre-processing, for each pixel in one slave image, the coherence is estimated with respect to the corresponding pixel in other slave images by Eq. (3.13). It is important to note that all the possible pairs are considered. Thus assuming $m$ as the number of images in one stack, for each pixel in the multi-looked data, a symmetric matrix of size $m \times m$ is constructed with 1 as the diagonal elements. The final coherence value corresponding to one pixel is achieved by averaging the upper or lower triangle values in the matrix while ignoring the diagonal elements.
It is important to note that with this approach the estimated coherence may be biased due to APS and non-homogeneous areas. The estimated temporal coherence for the ascending stack of beam 57 can be seen in Fig. 3.7.

![Figure 3.7: Temporal coherence estimated based on 102 multi-looked complex images. Since the investigated scene is an urban area, great number of pixels with high coherence values can be observed. The black parts are vegetated areas.](image)

The estimated temporal coherence is further utilized as a quality measure for selecting the most stable points to build a sparse grid through the entire scene. This is done by setting a threshold for the coherence values and selecting the highest one among an arbitrary number of pixels. In this implementation, a very first coarse sparse grid is built with selecting the best pixel among every 200 pixels. Then, a redundant network is built based on the pixels of the sparse grid. The density of the network increases by including the best pixels in every 50 and later 10 pixels. The result is a very dense network spanned throughout the entire investigated area where best pixels are triangulated. This network is further revised by discarding the pairs with arcs higher than a specific length (Zhu et al., 2013). The final network is the basis of the spatial differences method which is explained in the following.

**Spatial Differences & Topography Integration**

As is mentioned above, the pixel pairs with arcs shorter than a specific length are selected and connected to build the final redundant network throughout the entire area (Zhu et al., 2013). Following the assumption that APS is spatially slowly varying, the spatial differences made between the pixel pairs is assumed to be APS free (Fornaro et al., 2009, Zhu et al., 2013). In order to clarify the process we start with introducing the functional model of phase in one of the differential interferograms made between acquisition $k$ and $l$ considering both linear and seasonal deformation models (Ferretti et al., 2001, Fornaro et al., 2009, Kampes, 2006):

$$
\phi^{k,l} = -\frac{4\pi}{\lambda} \left( \frac{\Delta B^k_{\perp}}{R \sin \theta} \cdot \delta h + v \cdot \Delta t^{k,l} + a \sin (2\pi \cdot \Delta t^{k,l}) \right) + \phi_a^{k,l} + \phi_o^{k,l} + \phi_n^{k,l} + N \cdot 2\pi
$$

where $\lambda$ is the radar wavelength, $\Delta B^k_{\perp} = B^k_{\perp} - B^l_{\perp}$ is the relative perpendicular baseline between data-takes $k$ and $l$, $R$ and $\theta$ are the target range and incidence angle, respectively, $\delta h$ is the topography residual remaining after removal of topographic phase from an external DEM, $v$ is the linear deformation rate appears due to the temporal baseline $\Delta t^{k,l}$ between acquisitions $k$.
and \( l, a \) is the amplitude of possible seasonal deformation and \( N \) denotes the integer ambiguity term. The remaining phase terms are disturbance factors caused by possible orbit inaccuracies \( \phi^{k,l}_o \), atmospheric delays \( \phi^{k,l}_a \) and other noise contributions such as thermal noise, scattering variations, etc \( \phi^{k,l}_n \).

In each differential interferogram, the phase observations (Eq. (3.14)) from a pair of sufficiently close pixels are used to form the spatial differences. The phase contributions from atmosphere and orbital errors are reduced in this manner. Therefore, considering the pixels \( i, j \) and \( p, q \) and ignoring the superscripts for easier readability, the resulted spatial difference in differential interferogram made between takes \( k \) and \( l \) is written as:

\[
\Delta \phi = \phi_{i,j} - \phi_{p,q} = -\frac{4\pi}{\lambda} \left( \frac{\Delta B_\perp}{R \cdot \sin \theta} \cdot \nabla h + \Delta v \cdot \Delta t + \Delta a \cdot \sin (2\pi \cdot \Delta t) \right) + \Delta \phi_n \tag{3.15}
\]

where \( \nabla h = \delta h_{i,j} - \delta h_{p,q} \), \( \Delta v \) and \( \Delta a \) are the relative height difference, relative linear deformation rate and relative amplitude of seasonal deformations between pixels \( i, j \) and \( p, q \), respectively and \( \Delta \phi_n \) is the remaining noise contribution.

With the observation equations at hand, the target parameters (\( \nabla h, \Delta v, \Delta a \)), considering only a single dominant scatterer in each pixel, are estimated for each arc by a search in the three dimensional space using maximizing the ensemble coherence (periodogram) (Kampes, 2006). Later the differential topography estimates are integrated globally which results the topography residual at each pixel of the network. At the final step, the topographic phase contribution obtained from the previous step is removed from each original interferogram.

### APS Estimation and Removal

After removal of topographic phase contribution, the residual phase consists only of deformation signals, APS and scattering noise. The residual phase is firstly unwrapped in both azimuth and range directions. After the unwrapping, the deformation signals and noise should be eliminated in order to only remain with APS phase contribution. Thus, a spatial Gaussian low-pass filter is applied on the unwrapped residual phase to average out the noise followed by a temporal high-pass filter to eliminate the deformation signals. This is done for each interferogram in the stack and the result is APS for each image of the stack only on the network points. A global polynomial fit is further used for interpolating the APS values on network into the entire image. At the final step, the low resolution APS is up-sampled into the original size and removed from each image in the stack.

### 3.4.3 Tomographic Processing

Tomographic processing is applied on the APS-free image stack for retrieval of elevation and motion parameters of multiple scatterers in each azimuth-range pixel. The basics of SAR tomography are already covered in Section 3.2 and an overview of some relevant spectral estimators is given in Section 3.3. In this subsection, different steps used in Tomo-GENESIS are briefly explained in the following. The steps build an operational system which consists of Time Warp method to estimate multi-component motion parameters proposed by Zhu and Bamler (2011), PSI as a limited case of SAR tomography for its computational efficiency, Maximum Detection or SVD-Wiener (Zhu and Bamler, 2010b) for multiple scatterer discrimination and SL1MMER algorithm for its high super-resolution capabilities in the elevation direction (Zhu and Bamler, 2012).

**Time Warp Method**
Conventional D-TomoSAR proposed by Lombardini (2005) assumes the linear motion of each scatterer in the resolution cell. However, dealing with X-band data of urban areas, non-linear motion parameters, such as periodic seasonal motion, should also be taken into account which does not fit in the framework of spectral estimation (Zhu, 2011). This type of deformation most likely occurs because of thermal dilation of buildings and due to small order of magnitudes might not be visible in L or C-band SAR data. In (Zhu and Bamler, 2011), a generalized time warp method is proposed in order to allow for multicomponent non-linear motion retrieval of multiple scatterers in the azimuth-range pixel.

The functional model of D-TomoSAR by taking into account the linear motion parameters for multiple scatterers in the resolution cell is outlined in Eq. (3.4). Therefore, by taking into account the seasonal deformation with period of one year, Eq. (3.4) can be rewritten as:

\[ g_n = \int \frac{\Delta s}{\Delta s} \gamma(s) \exp(-j2\pi(\xi_n s + \eta_n V(s) + \omega_n a(s))) \, ds, \quad (3.16) \]

where \( w_n = \frac{2 \sin(2\pi(t_n-t_0))}{\lambda} \) in which \( t_0 \) is the initial phase offset and \( a(s) \) is the amplitude of seasonal deformation along the elevation. Similar to Eq. (3.5), Eq. (3.16) can be extended to the following form:

\[ g_n = \int \int \int \frac{\Delta s}{\Delta s} \Delta v \Delta a \gamma(s) \delta(v - V(s), a - a(s)) \exp(-j2\pi(\xi_n s + \eta_n v + \omega_n a)) \, ds \, dv \, da, \quad (3.17) \]

where \( \Delta a \) is the seasonal motion parameter extent.

Since the base function of seasonal motion is chosen as a sinusoidal signal, Eq. (3.17) defines a non-linear problem. The time warp method is based on rearranging the image acquisition times in order to imitate a linear motion trend (Zhu and Bamler, 2011). If we consider \( M \) as the user-defined motion model order, which is chosen based on prior knowledge of the physical processes relevant to the investigated area, the generalized time warp method rewrites the D-TomoSAR system model to an \( M+1 \) dimensional spectral estimation problem (Zhu and Bamler, 2011). Consequently, the motion estimation is possible for all complex motion models.

If we assume only a single component motion model, i.e. \( M = 1 \), Eq. (3.17) can be generalized to the following equation (Zhu and Bamler, 2011):

\[ g_n = \int \int \frac{\Delta s}{\Delta s} \gamma(s) \delta(p_1 - p_1(s)) \exp(-j2\pi(\xi_n s + \eta_{1,n} p_1)) \, ds \, dp_1, \quad (3.18) \]

where \( p_1(s) \) is the motion coefficient to be estimated and \( \eta_{1,n} = \frac{2\tau_{1,n}(t_n)}{\lambda} \) is the temporal frequency as a function of an assumed motion base function \( \tau_{1,n}(t_n) \). Eq. (3.18) converts the non-linear problem of D-TomoSAR model outlined in Eq. (3.17) to a linear 2D spectral estimation problem that makes all spectral estimators applicable (Zhu and Bamler, 2011). This concept can be generalized with assuming multicomponent motion models (\( M > 1 \)) and rewrites Eq. (3.18) to the following (Zhu and Bamler, 2011):

\[ g_n = \int \int \ldots \int \frac{\Delta s}{\Delta s} \gamma(s) \delta(p_1 - p_1(s), \ldots, p_M - p_M(s)) \cdot \exp(-j2\pi(\xi_n s + \eta_{1,n} p_1 + \ldots + \eta_{M,n} p_M)) \times ds dp_1 \ldots dp_M, \quad n = 1, \ldots, N. \quad (3.19) \]
In Eq. (3.19), \( \eta_{m,n} = 2\tau_m(t_n)/\lambda \) is the \( m \)th temporal frequency component at \( t_n \). The equation defines an \( M+1 \) dimensional Fourier transform of \( \gamma(s) \delta(p_1 - p_1(s), \ldots, p_M - p_M(s)) \) at locations defined by corresponding temporal frequencies. Eq. (3.19) allows for multicomponent non-linear motion estimation for D-TomoSAR.

**PSI**

PSI assumes only a single dominant scatterer in the resolution cell and allows for elevation and deformation (linear and non-linear) retrieval of each azimuth-range pixel (Ferretti et al., 2001). The assumption of one scatterer makes the method computationally effective, especially, for large scale urban monitoring.

**SVD-Wiener Algorithm**

The principles of this method are outlined in Subsection 3.3.2. The SVD-Wiener is a non-parametric spectral estimation method used in order to retrieve the reflectivity profile of each azimuth-range pixel in the elevation direction. Relevant to tomographic reconstruction, this method is associated with peak detection and model order selection to estimate the number, elevation and motion parameters of multiple scatterers in the resolution cell (Zhu and Bamler, 2010b). This method is computationally efficient but has negligible super-resolution capability. Therefore it is recommended for medium resolution applications (Zhu et al., 2013).

**SL1MMER Algorithm**

The SL1MMER algorithm is proposed by Zhu (2011) and is already described in Subsection 3.3.3. The method provides robust estimates with very high elevation resolution (Zhu and Bamler, 2012). With SL1MMER algorithm, the most detailed 4D urban mapping is achievable. However, this method is computationally very expensive and it is only recommended for monitoring small test sites including individual high rise buildings.

**Integrated Approach**

In Wang et al. (2012) an integrated approach is proposed for operational TomoSAR processing. The approach consists of combining all of the mentioned steps in order to make the TomoSAR processing applicable for large scale urban monitoring. Therefore, firstly PSI is applied for retrieval of elevation and motion parameters of each pixel due to its computational efficiency. As it is unlikely to have pixels with more than two scatterers in the resolution cell, the considered number of scatterers are limited to zero, one and two and a pixel classification is carried out based on model order selection and amplitude dispersion index (Wang et al., 2012). At the subsequent step, the detected pixels with having more than one scatterer are firstly processed by SVD-Wiener. Then pixels which include double scatterers with elevation distance smaller than a threshold are passed to be processed with the SL1MMER algorithm. It is important to note, for reducing the computational time even further, the elevation and motion estimates obtained from PSI are used as a prior knowledge to restrict the search dimension for SL1MMER algorithm.

Eventually, the aim of the Tomo-GENESIS is to reconstruct the reflectivity profile of each pixel along the elevation direction and further exploit this profile to obtain the following parameters:

1. The number of scatterers in the resolution cell
2. The elevation of the scatterers
3. The reflectivity of the scatterers
4. The linear deformation rate and amplitude of seasonal motion of the scatterers.

3.4.4 Geocoding

The Gocoding procedure is not part of the Tomo-GENESIS. However, the output point clouds obtained from the system have to be geocoded before being able to apply the point cloud fusion. The geocoding is carried out with the PSI-GENESIS of DLR.

In addition to the inherent azimuth and range coordinates of each target in the SAR image, the elevation is also estimated from TomoSAR processing followed by a conversion to topographic height based on the local incidence angle. The final height of each scatterer is therefore obtained from the used DEM plus the height update. A further coordinate transformation of the radar coordinates (azimuth, range, height) to a geodetic coordinate system is referred to as geocoding. The geocoding can be performed with respect to a global ellipsoid such as WGS84 or a best fit local ellipsoid (Hanssen, 2001). Therefore, the result is the 3D coordinates of each target expressed in latitude ($\phi$), longitude ($\lambda$) and height above the reference ellipsoid ($H$).

For calculating the geodetic coordinates of each scatterer the precise orbit and the range and azimuth timing informations as well as the estimated height are necessary (Gernhardt, 2012). A circle is defined by its center and its radius where the former is assumed to be the precise orbit position at the time of the master acquisition and the latter is the slant range distance to the desired scatterer (Gernhardt, 2012). Finally, the intersection of the mentioned circle and the ellipsoidal height of the scatterer leads to the 3D coordinates of the scatterer in the geodetic reference system.

One of the critical factors degrading the accuracy of geocoding is uncertainties in height of the reference point (Gernhardt et al., 2012). Since the height values calculated from TomoSAR (or PSI and InSAR) are estimated relative to this point, any bias in the height of the reference point directly propagates to all the final heights (Gernhardt, 2012). According to Fig. 3.8, this bias also leads to horizontal offsets between the geocoded point and its true position due to using the fixed range ($R_s$) for geocoding. This systematic error can be compensated for, if the ellipsoidal height of the reference point is accurately known from an external source.

![Figure 3.8: Horizontal and vertical offsets after geocoding (Gernhardt, 2012). If the ellipsoidal height of the reference point is not accurately known due to inaccurate DEM information, the final geocoded points will show horizontal ($\Delta xy$) and vertical ($\Delta z$) offsets with respect to their true positions.](image-url)
3.4.5 Point Cloud Fusion

The side-looking geometry of SAR sensors only allows for mapping the illuminated side of buildings in urban areas. Therefore, in order to enhance the mapping and deformation monitoring capability of SAR, a fusion of TomoSAR point clouds obtained from different viewing geometries is performed. In (Gernhardt et al., 2012) a method for fusion of the multi-track PSI results is proposed based on a least-squares matching scheme minimizing the distances between assumed identical points of two point clouds. The method tends to estimate the offset between the identical points in the elevation direction which is caused due to selecting different reference points during the processing (Gernhardt, 2012).

Another method of point cloud fusion is proposed in (Wang and Zhu, 2013). The method is a feature-based fusion algorithm which is based on detecting and matching building contour end points and aligning flat roofs in the two point clouds.

In Section 3.5, it is demonstrated that this step is not necessary if an identical reference point is chosen for the stacks acquired from different viewing geometries.

3.5 Absolute Fusion of Multi-track TomoSAR Point Clouds

In Chapter 2, precise 3D localization of persistent scatterers is reported depending on the high geometric accuracy of the TerraSAR-X sensor as well as exhaustive compensation of error sources. In Section 3.2 and Section 3.4 the principles and processing steps for producing detailed 4D (space-time) maps applying D-TomoSAR are described, respectively.

In this section, the approach to merge the two mentioned concepts are introduced in order to produce absolute 3D TomoSAR point clouds. This is achieved by selecting an identical reference point for all the four stacks while the absolute 3D position of this point is available from the Stereo-SAR method outlined in Section 2.7. Thus, it will be demonstrated that each of the four absolute 3D point clouds can be automatically fused due to the selection of an identical reference point.

3.5.1 Strategy for Absolute Point Fusion

Due to the slant viewing geometry of SAR satellites, only a small section of individual buildings can be captured from each flight track. With availability of datasets from both ascending and descending orbits with different geometry configurations, one can merge the datasets for obtaining a complete representation of the investigated areas. However, the geocoded point clouds cannot be simply combined because of the reason outlined in Subsection 3.4.4 i.e. uncertainties in height of the reference points, see also (Gernhardt, 2012).

The mentioned problem can be solved by choosing an identical reference point for each of the stacks available from different viewing geometries. Therefore, with this consideration, it is expected that the point clouds will be automatically fused without further manipulation. However this is not true. The reason lies in the geocoding process. Since each of the point clouds are geocoded separately based on the non-corrected range and azimuth timing information, the coordinates of the geocoded reference point shows certain offsets with respect to the absolute 3D position of the point obtained from Stereo-SAR. The deviations are consistent with the magnitudes of the most prominent errors affecting the range and azimuth time observations outlined in Section 2.8.
In our case, for all of the stacks an identical reference point is chosen based on the discussions given in Section 2.8 and later the correction is achieved by calculating the difference between the geocoded reference point of each stack and the absolute coordinate of this point obtained from Stereo-SAR. The correction value for each stack is then added to all points in the corresponding point cloud. This leads to producing four absolute 3D TomoSAR point clouds which are automatically fused. The work-flow of the correction for only a single stack is illustrated in Fig. 3.9.

![Diagram](image)

**Figure 3.9:** The work-flow of absolute 3D localization of TomoSAR point clouds. The corrections obtained from the differences between the geocoded reference point and the absolute coordinates from Stereo-SAR is added to all of the points.

### 3.6 Results

In this section, the TomoSAR results using four stacks of TerraSAR-X VHR spotlight images over the city of Berlin are reported. Subsection 3.6.1 presents detailed elevation and deformation maps of Berlin obtained from D-TomoSAR processing using Tomo-GENESIS of DLR. The 3D point clouds are geocoded by PSI-GENESIS and an example of them is also brought in this subsection. In Subsection 3.6.2, the results from absolute fusion of TomoSAR point clouds are shown.

#### 3.6.1 TomoSAR Processing

The input for the TomoSAR processing are the APS-free Single Look Complex (SLC) images that are co-registered to the optimum master scene (see Subsection 3.4.2). The images has a coverage of approximately 5km × 10km over the city of Berlin as is outlined in Section 1.4. The spatial-temporal baseline distribution of the data-takes for the four processed stacks can be seen in Fig. 3.10.
Following the criteria outlined in Section 2.8 and the discussion brought in Subsection 3.5.1, a common reference point has been chosen for the four stacks. The point is assumed to be the base of a street lamp close to the central station of Berlin and is approximately located in the center of the investigated area.

![Graphs showing temporal and spatial baselines for different beams.](image)

**Figure 3.10:** Spatial-temporal distribution of the data-takes visualized for each of the stack which demonstrates the large number and irregular distribution of the acquisitions.

Dependent on the defined system model of D-TomoSAR outlined in Section 3.2, the result of the processing includes retrieving the number of scatterers in each azimuth-resolution cell and estimating amplitude, phase, elevation, linear deformation velocity and amplitude of seasonal motion of each detected scatterers. In Fig. 3.11, the elevation estimates of one ascending (beam 57) and one descending (beam 42) stack with respect to the identical reference point are visualized.

The long acquisition periods of the Berlin stacks provide a possibility to also estimate the motion parameters of each scatterer. Fig. 3.12 illustrate the deformation map of the area for beams 42 and 57. The linear velocity is estimated in the LOS direction of the satellites and therefore direct interpretation and comparison between the stacks is not possible. However, the figures give an indication of what part of the city might be in danger of slow subsidence. From Fig. 3.12, it can be seen that most of the area in the city does not undergo a significant linear deformation. The central railway station at the center of the image shows a subsidence of almost 2 millimetres in the LOS direction of descending track, beam 42.

Another relevant motion parameter, specially in case of urban monitoring, is the amplitude of the seasonal deformation which is introduced in Section 3.2. As stated in Zhu (2011), the origin of this type of motion is related to thermal expansion of steel construction. TomoSAR estimates the amplitude of the seasonal deformation by taking into account the initial phase offset with respect to a reference sinusoidal signal fitted to the monthly average temperature of the area. The positive and negative values are available with respect to the reference point.
Figure 3.11: Elevation estimates of the two out of four stacks (top: beam 42, bottom: beam 57) using an identical reference point. The estimates range between -50 to 150 meters as it is represented by the colorbar.
Figure 3.12: The linear deformation rates estimated for beams 42 and 57. The colorbar represents the linear velocity in $\text{mm year}^{-1}$. Most of the area have been quite stable during the acquisition period (2008 to 2013). Some parts of the city show small magnitudes of deformation in the order of couple of millimeters.
which is assumed to be stable. The positive sign implies a movement of the object towards the sensor in the LOS direction while the negative sign indicate a movement away from the sensor. Fig. 3.13 shows the amplitude of seasonal deformation estimates for the city of Berlin from stacks 42 and 57. It can be inferred from the image that the city is not significantly influenced by the seasonal deformation as the amplitude is approximately zero for most of the scatterers. However, interesting patterns are seen from the central railway station and the Eisenbahn bridge indicated with red ellipses. Deformation analysis on these two test sites is treated in more details in Chapter 4 where the displacement vector is reconstructed in three dimensions.

Geocoding

The results of TomoSAR processing over the city of Berlin are reported in Subsection 3.6.1. All of the four point clouds are geocoded by the PSI-GENESIS of DLR (Adam et al., 2003). The geocoded TomoSAR point cloud obtained from descending stack 42 is visualized in Fig. 3.14. The horizontal and vertical axes denotes the easting and northing in UTM projection, respectively. The absolute height values are colorcoded with respect to the global ellipsoid of WGS84. The height difference is approximately 40 meters where blue points represent lower height values.

3.6.2 Absolute 3D Fused TomoSAR Point cloud

According to Subsection 3.5.1, the coordinates offsets between the geocoded reference point and its absolute 3D position obtained from Stereo-SAR can be used to correct the geocoded point clouds. These corrections have been applied to each point cloud separately. Fig. 3.15 illustrates the fusion of two ascending and two descending absolute TomoSAR point clouds in 2D over the city of Berlin. The coordinates are expressed in UTM coordinate system. It is seen that the point clouds are reasonably overlaid on each other after applying the corrections.

In order to confirm that the corrections are necessary prior to merging the point clouds, a small test site including the Federal Intelligence Service building is chosen to compare the fusion results before and after applying the coordinate corrections. Fig. 3.16 shows the optical image of the building. In the red ellipse is the building section that is investigated in Fig. 3.17. In Fig. 3.17, the result from the ascending stacks are visualized in blue and the descending point clouds are shown in red. In the non-corrected fusion (left), the black arrow represents the shift available between the same heading tracks. Moreover, the black ellipse shows that the result from descending stacks (red) intersects with the building fraction captured from ascending stacks (blue). These two parts should be connected to each other by a single end-point which is the case for the fusion after applying the correction (right). Therefore, applying the correction will lead to appropriate connection of building end-points and results into eliminating the shift between same-heading tracks providing the absolute fused TomoSAR point cloud.

Apart from the improvement in the localization accuracy of the point clouds which is also evaluated later in this chapter, a dramatic increase in the number of scatterers are visible after fusing the results. This allows for detail mapping of individual structures in the cities. In Fig. 3.18, the increment in number of scatterers can be seen by comparing single stack TomoSAR results of a small test site and the results after the fusion of all the stacks. As it is visible each TomoSAR point cloud only captures a fraction of the buildings due to the slant viewing geometry. By merging all the absolute point clouds the full structure of the building is reconstructed. The result of fusion for this small test site demonstrates that the number of scatterers are almost four times of the scatterers from a single TomoSAR stack (see Fig. 3.18).

This section is finalized by 2D and 3D visualizations of the fused point cloud of the central urban area of Berlin illustrated in Fig. 3.19. The absolute point clouds are plotted in the UTM
Figure 3.13: Amplitude of seasonal deformation estimated for stacks 42 and 57. The colorbar represent the amplitudes in millimetre. The area is free from seasonal deformation except the central railway station and the Eisenbahn bridge from which the former undergoes the seasonal deformation with magnitudes up to 20 mm.
Figure 3.14: Geocoded TomoSAR point clouds of beam 42 in UTM projection. The horizontal and vertical axes denote the easting and northing in UTM projection, respectively. The absolute height values are color coded ranging between 70 to 110 meters where blue represents lower heights and red shows the higher ones.

Figure 3.15: The fusion result of two ascending and two descending tracks over the city of Berlin. The point clouds are absolutely localized after correcting the geocoded coordinates of the reference point and are automatically fused.
Chapter 3. *SAR Tomography*

Figure 3.16: The optical image of Federal Intelligence Building (Google Earth™). The part in the red ellipse is investigated later to compare the fusion results before and after applying the coordinate corrections.

Figure 3.17: Comparison between the fusion results before (left) and after (right) applying the reference point coordinate correction. The result from the ascending stacks are visualized in blue and the descending point clouds are shown in red. The non-corrected fusion (left) includes certain offsets between the results from same-heading tracks (the black arrow) and wrong intersection of different building fractions captured from cross-heading tracks (the back ellipse).

The coordinate system and the height of each scatterer is colorcoded with respect to WGS84 ellipsoid. The coverage of the test site is approximately 10km × 5km and the number of captured scatterers by TomoSAR is 63 million. It is clearly seen that the fusion of TomoSAR point clouds obtained from different geometries allow for a detail 3D mapping of the city.
Figure 3.18: The increase in the number of scatterers after the fusion. From left to right, point clouds of a small area of city of Berlin from beams 42, 57, 85 and 99, respectively. The total number of scatterers in each single beam varies between 8800 and 11400 and only fractions of buildings are visible. However, after the fusion the total number of scatterers reach more than 40000 and full structure of the building is reconstructed.
Figure 3.19: Automatically fused 3D absolute positioned TomoSAR point clouds in 3D (top) and 2D (bottom). The absolute height values are color coded and range between 70 meters to 110 meters. Clearly, the fusion of multi-track point clouds allow for a very detailed representation of the city where most of the structures can be easily recognized.
3.7 Localization Accuracy Analysis for the Fused Point Cloud

The localization accuracy of the fused TomoSAR point cloud is assessed by comparing the results with an accurate Digital Surface Model (DSM) of the city of Berlin calculated from a point cloud obtained from aerial laser scanning. This allows for a quantitative analysis on the positioning accuracy of the point cloud. Since a point-wise comparison is not possible between the two point clouds, strategies based on comparing a subset of samples from two groups are presented in this section.

The LiDAR data, in general, are characterized with large amount of data points and very high absolute geo-localization accuracy. From the LiDAR point cloud, a DSM is calculated which is served as the reference surface against the fused TomoSAR point cloud.

In order to check the overall accuracy of the point cloud, Fig. 3.20 shows the fused TomoSAR point cloud overplotted onto the LiDAR DSM for two areas in the city of Berlin. The heights of TomoSAR point cloud are colorcoded with red values stand for higher heights. The DSM is plotted in gray for better visualization purposes. The quite good fit of the TomoSAR point cloud on the DSM can be clearly seen. This implies that the fusion of the stacks was carried out successfully as no large deviations are visible between the fused point cloud and the DSM.

In order to validate the results illustrated in Fig. 3.20, the accuracy analysis is carried out for horizontal and vertical directions separately as are reported in the following.

3.7.1 Horizontal Accuracy Analysis

The optimum way to assess the accuracy of the TomoSAR point clouds is a point-wise comparison with respect to the LiDAR point cloud. However, this is not feasible as the LiDAR sensors map the surface with a zero degree looking angle while SAR sensors capture the scene from a side-looking geometry. The difference in acquisition geometry leads to different mapping of the same objects and therefore complicates the comparison.

In this study, the horizontal assessment relies on calculating the line-offset between the extracted building facades in the TomoSAR point cloud and the reference DSM. The building facades are extracted from the DSM by fitting a line based on the top view extent of the building. For the TomoSAR results, the scatterer density of the point cloud is used to extract building facades. The scatterer density is estimated in adaptive windows varying dependent on the orientation of the facades (Zhu et al., 2013). Furthermore, the assumption is made that areas with high scatterer densities represent the building facades due to side-looking geometry of SAR satellites. With this assumption, a line is fitted to the scatterer density results where the higher density estimates get more weight in the estimation of line parameters. In both, DSM and TomoSAR, cases the line is fitted by least squares. At the final stage, the offset between the line gives an indication about the horizontal deviation of the TomoSAR point cloud with respect to the reference DSM.

The mentioned strategy has been tested on three test sites. The test sites are different based on their distances relative to the reference point. In the following, only the result from one of the test sites is reported which is located in a moderate distance from the reference point.

Fig. 3.21 illustrates the test site located approximately in the distance of 1.5 km with respect to the reference point. In the left the fused TomoSAR point cloud and on the right the DSM calculated from the LiDAR point cloud can be seen. The heights are colorcoded in both plots.
Figure 3.20: Fused TomoSAR point cloud overplotted on the reference DSM. It is seen that the scatterers are fitted to the corresponding building parts on the DSM as no large deviations are visible from the plot.
with maximum difference of 27 meters between the lowest and highest points. The black rectangle shows the investigated facade.

![Figure 3.21: Fused TomoSAR point cloud and LiDAR DSM of the investigated building in Berlin. The heights are color-coded in both plots. The black rectangle shows the investigated building facade.](image)

The extraction of facade from the fused TomoSAR point cloud relies on the estimation of the scatterer density (Zhu et al., 2013). Fig. 3.22 visualizes the scatterer density estimated in $5m \times 5m$ windows adapted with the orientation of the facades (see (Zhu et al., 2013)) after applying a threshold of $2 \, m^2$. The threshold is set to exclude the scatterers from the ground and roof surfaces. It can be clearly seen that the number of scatterers is higher on the building facades. Moreover, in the black rectangle the investigated facade is shown. A weighted least squares is applied in order to fit a line to the scatterers belonging to the facade. A window similar to the one visualized in Fig. 3.22 is selected around the points while the pixels with higher scatterer densities participate in the estimation with higher weights.

The results of the line-fitting procedure is shown in Fig. 3.23 in the UTM coordinate system. In Fig. 3.23a, the assumed facade points are visualized as blue dots and the red line represents the extracted facade in the horizontal plane from the DSM. Fig. 3.23b shows the same facade in black extracted from the fused TomoSAR point cloud color-coded based on the scatterer density. Fig. 3.23c shows the extracted facade from the DSM (in red) and the one from TomoSAR point clouds (in green). It can be clearly seen that the facades are reasonably parallel to each other. Therefore, the offset between two parallel lines is easily calculated by subtracting the constants of each line. This results into a difference of approximately 42 centimetres in the UTM coordinate system. This is also confirmed by the histogram of the differences of the TomoSAR point clouds with respect to the extracted facade line from the DSM which is visualized in Fig. 3.23d. It is important to note that this value is higher for test sites on the border of the investigated scene due to relatively high distance with respect to the reference point (Gernhardt, 2012). The mentioned strategy has been carried out for a building on the border of the image and provide a distance of 1.2 meters.
Figure 3.22: Scatterer density map of the investigated area. The colorbar indicates the number of points in 1 × 1 m² windows. Higher point densities can be seen on the building facades. The investigated facade is marked with a black rectangle.

3.7.2 Vertical Accuracy Analysis

Since a point-wise comparison between the fused TomoSAR point cloud and the LiDAR DSM is not possible for the reason outlined in Subsection 3.7.1, the vertical localization accuracy should also be analysed in a subset of points taken from the point cloud and the reference surface.

The appropriate method to carry out this analysis is to select a building in the area with a flat regular-shaped roof. In this case, for the fused TomoSAR point cloud and also the LiDAR DSM, an identical patch can be selected containing point samples from the roof. Inside the patch common statistic measures, such as the mean and the standard deviation, can indicate the quality of the TomoSAR point cloud and the DSM. Moreover, the vertical accuracy of the TomoSAR points can be compared to the reference DSM by calculating the Root Mean Squared Error (RMSE) as follows:

\[ d = \sqrt{\frac{\sum_{i=1}^{N} (h_T - h_D)^2}{N}}, \quad (3.20) \]

where \( d \) is the RMSE, \( N \) is the number of samples in the window, \( h_T \) is the TomoSAR height values of the investigated points and \( h_D \) is the point heights from the DSM.

The mentioned approach requires the careful selection of a building with perfectly flat roof. However, locating such a building is not a trivial task in Berlin due to the special architectural style of European cities. Nevertheless, in order to give an indication about the vertical accuracy of the TomoSAR point cloud, a building with a moderately flat roof has been chosen. The RMSE value calculated inside an identical window from the point cloud and the surface was approximately three meters. It is important to note that despite the fact that the building was chosen close to the reference point, the estimated RMSE value is most likely not valid due to the following reasons:

1. The investigated building roof is not completely flat. This fact is confirmed by calculating the standard deviation of the height values inside the roof patch from the DSM which shows a magnitude of approximately 1.6 meters.

2. The fused TomoSAR point cloud is not outlier-free. Several outliers were removed from the point cloud by simple filtering based on the histogram of the height values. However,
Figure 3.23: Extracted building facades from DSM and fused TomoSAR point clouds in the horizontal plane and the corresponding histogram of differences. The comparison between the extracted facade lines leads to a difference of almost 42 centimetres for the test site at a moderate distance relative to the reference point.
reliable outlier elimination was not possible due to the fact that no information about the quality of the TomoSAR point cloud was available at the time of the study.

Based on the discussion above, it is inferred that without the availability of perfectly flat-shaped areas and also information about the quality of the TomoSAR point clouds, statements about the vertical accuracy of the point cloud is not sufficiently reliable.
Chapter 4

Motion Decomposition

This chapter introduces the problem of LOS motion decomposition of InSAR methods. The functional and stochastic model of reconstructing the real 3D displacement vector from the LOS deformation measurements are described. The methodology to carry out motion decomposition with availability of LOS deformation measurements from more than two viewing geometries is depicted. The chapter finishes with applying the mentioned method on two test sites in the city of Berlin.

4.1 Introduction

One of the limitations of SAR interferometric techniques, including SAR tomography, is that they are only capable to provide displacement vectors projected onto the radar LOS. This happens due to the fact that SAR sensors cannot distinguish between targets located at the same distance from the sensor but with different angles and poses difficulties for the interpretation of the deformation patterns (Hanssen, 2001). Therefore, decomposition of LOS motion estimates to the original three dimensional deformation vector is highly desirable in order to enhance insights about the deformation events in the investigated areas.

The common method to tackle the aforementioned problem is using a combination of deformation estimates from an ascending and a descending track where two out of three components can be reconstructed (Hanssen, 2001). For retrieving the third component a prior knowledge about the characteristics of the displacement is necessary (Hanssen, 2001, Samiei-Esfahany et al., 2009). If LOS deformation estimates from more than two suitable geometry configurations are available, then it is possible to reconstruct the 3D displacement vector in the local coordinate system (Wright et al., 2004). However, since the LOS deformation estimates obtained from different viewing geometries do not necessarily originate from the same object, a strategy should be introduced to overcome this problem.

This chapter is organized as follows. In Section 4.2, the projection of the 3D displacement vector onto the radar LOS is described and is extended with an explanation of the methodology to perform 3D motion decomposition of the LOS motion by using deformation measurements from more than two viewing geometries. Section 4.3 discusses the effect of available geometry configurations, obtained from two pairs of cross-heading tracks, on the final motion estimates in each direction. In Section 4.4, the proposed strategy for motion decomposition is described and finally Section 4.5 reports the results of the seasonal motion decomposition performed on two test sites within the city of Berlin.
4.2 Radar Line of Sight Deformation

The deformation measurement of SAR techniques $d_{LOS}$ is the projection of the original 3D displacement vector $\vec{d}$ with components $d_e$, $d_n$, and $d_u$ in East, North, and Up direction, respectively, into the LOS direction. Considering an incidence angle of $\theta_{inc}$ and a satellite orbit with heading angle $\alpha_h$, we can write: (Hanssen, 2001)

$$d_{LOS} = d_u \cos(\theta_{inc}) - d_{ALD} \sin(\theta_{inc})$$

(4.1)

where $d_{ALD}$ includes the projection of $d_n$ and $d_e$ on the azimuth look direction (ALD), that is perpendicular to the satellite flying direction and therefore is expressed as:

$$d_{ALD} = d_e \cos(\alpha_h) - d_n \sin(\alpha_h).$$

(4.2)

Fig. 4.1 depicts the aforementioned projection where (A) describes the projection of the components in the North and East direction onto the azimuth look direction and (B) illustrates a 3D sketch representing the projection of the Up-component onto the radar LOS.

![Figure 4.1](image)

**Figure 4.1:** Projection of the original displacement vector $\vec{d}$ with components $(d_n, d_e, d_u)$ onto the radar LOS direction (Hanssen, 2001). (A) Projection of $d_n$ and $d_e$ onto the azimuth look direction (ALD) depicted in the horizontal plane. (B) Projection of $d_u$ and $d_{ALD}$ onto the LOS direction illustrated in 3D space.

4.2.1 3D Displacement Reconstruction

If Eq. (4.2) is substituted in Eq. (4.1), the explicit relation between deformation measurement $d_{LOS}$ and the displacement components $(d_n, d_e, d_u)$ for a single pixel can be written as (Hanssen, 2001):  

$$d_{LOS} = d_u \cos(\theta_{inc}) - d_e \cos(\alpha_h) \sin(\theta_{inc}) + d_n \sin(\alpha_h) \sin(\theta_{inc}).$$

(4.3)

Considering the near polar orbit of TerraSAR-X, for instance, with heading angle of 190.6° and an incidence angle of 36.1°, the sensitivity decomposition of LOS deformation is $[0.8, 0.58, -0.1] [d_u, d_e, d_n]^T$. Therefore, it can be seen that the observations are most sensitive to the deformation in the vertical direction and least sensitive to deformation in the north direction. This fact should not be falsely interpreted as ignoring the deformation component $d_n$ as was carried out in Gernhardt...
Chapter 4. Motion Decomposition

(2012). From Eq. (4.2), converting \( d_{ALD} \) to \( d_e \) while ignoring \( d_n \) results in the bias \( \Delta d_e \) in the eastern component which is expressed as:

\[
\Delta d_e = d_n \cdot \tan(\alpha_h).
\]  

(4.4)

Although the magnitude of error in Eq. (4.4) is very close to zero, it demonstrates that for accurate motion decomposition all of the deformation components should be taken into account.

Based on the above discussion, the goal is to retrieve \( d_n, d_e \) and \( d_u \) from \( d_{LOS} \). In a more concise form the functional model can be written as:

\[
y = Ax
\]

(4.5)

where \( y \) denotes the observation vector containing the \( d_{LOS} \) measurements of a single point from different geometries, \( A \) represents the design matrix of the problem and \( x \) forms the three dimensional unknown vector. Assuming that the investigated target is visible from three or more different geometries, the real 3D displacement vector can be reconstructed by solving for the weighted least squares solution as follows:

\[
\hat{x} = (A^T P A)^{-1} A^T P y
\]

(4.6)

where \( P \) is the weight matrix of the observations and \( \hat{x} \) includes the estimated 3D displacement vector. Apart from the estimates, their corresponding precisions are reflected by the variance-covariance matrix of the estimates \( Q_{\hat{x}\hat{x}} \) as:

\[
Q_{\hat{x}\hat{x}} = (A^T P A)^{-1}.
\]

(4.7)

4.2.2 Stochastic Model for Motion Decomposition

As it is apparent from the name, components of the weight matrix of observations \( P \) describe the weight of the corresponding observation participated in estimation of the solution. The solution with minimum variance is desirable and is achieved only if the weight matrix is defined equal to \( Q_{yy}^{-1} \) where \( Q_{yy} \) is the variance-covariance matrix of observations (Teunissen et al., 2005). The diagonal elements of \( Q_{yy} \) defines the variances and the off-diagonal elements represent the covariance values.

In most of the cases, the available information is not sufficient in defining the proper \( Q_{yy} \) which leads to assuming that the errors in the observations are independent and have equal standard deviations \( \sigma \). With this hypothesis, all observations contribute to estimate the solution with equal weight and the weight matrix is expressed as:

\[
P_{m \times m} = \frac{1}{\sigma^2} \cdot I_{m \times m}
\]

(4.8)

where \( m \) denotes the number of observations and \( I \) is identity matrix with the same size as \( P \). Choosing the weight matrix based on Eq. (4.8) can be problematic because not all the measurements, in our case LOS deformations, have the same precision.

Another alternative to define \( Q_{yy} \) and consequently \( P \) is to take into account the inconsistencies between observations and estimates. In this study, the phase residuals denote the difference between the measured and the modelled deformation values which are the output of TomoSAR processing. Therefore, the standard deviation of the observations can be defined based on them. However, this has one disadvantage. For instance, if a linear deformation model is assumed, there is a possibility that targets exist which are stable during the beginning of the acquisition period and suddenly they are exposed to deformations with large magnitudes. Since these points
will have a high variability with respect to the assumed model, their corresponding weight value will be low and this will lead to critical loss of information.

Unfortunately, at the time of this study, the information about the phase residuals are not available from the TomoSAR processing. Therefore, another criterion has been used for defining the weight matrix i.e. for each scatterer the weighting is done based on the corresponding amplitude estimates obtained from tomographic processing (see Section 3.2). The idea is that scatterers with higher amplitude values are participated in the least squares solution with higher weights. This assumption is based on the fact that bright point scatters have most likely more phase stability than the others which make them more dependable for the estimation process.

### 4.3 Current Configuration & Geometry Assessment

In this study, LOS deformation estimates from four different acquisition geometries are available (see Table 4.1). The average incidence angle $\theta_{inc}$ and the satellite heading angle $\alpha_h$ are available from the product annotation files of each beam. All acquisitions are from right-looking geometry. The incidence angle variability from $36^\circ$ to $55^\circ$ available from a combination of two ascending and two descending tracks provides a suitable geometrical configuration for solving the problem which will be evaluated in the following.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Incidence Angle ($\theta_{inc}$)</th>
<th>Heading Angle ($\alpha_h$)</th>
<th>Track Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>57</td>
<td>41.9°</td>
<td>350.3°</td>
<td>Ascending</td>
</tr>
<tr>
<td>85</td>
<td>51.1°</td>
<td>352°</td>
<td>Ascending</td>
</tr>
<tr>
<td>42</td>
<td>36.1°</td>
<td>190.6°</td>
<td>Descending</td>
</tr>
<tr>
<td>99</td>
<td>54.7°</td>
<td>187.2°</td>
<td>Descending</td>
</tr>
</tbody>
</table>

Table 4.1: Different viewing geometries used for decomposition of the LOS into real 3D displacement vector. The information are adopted from the product annotation files of each beam.

As is mentioned in Subsection 4.2.1, in addition to solving for the components of the 3D deformation vector, Eq. (4.6) provides information about the precision of the estimated components denoted by $Q_{\hat{x}\hat{x}}$. If we consider independent observations, the diagonal elements provide the variance of the estimated deformation in each component and off-diagonal elements are zero due to uncorrelated component errors. If we consider that the errors have equal standard deviations, $\sigma$, we can write:

$$Q_{\hat{x}\hat{x}} = \sigma^2 \cdot (A^T A)^{-1}.$$  \hspace{1cm} (4.9)

In Eq. (4.9) if we set $\sigma = 1$ then the square root of the diagonal elements provide a criterion for evaluating the effect of geometry in terms of the relative measurement error (Wright et al., 2004). This is commonly used in assessing the geometry of Global Positioning System (GPS) measurements and is known as Dilution of Precision (DOP) (Misra and Enge, 2011).

Based on mean values of $\theta_{inc}$ and $\alpha_h$ correspond to each viewing geometry outlined in Table 4.1, the design matrix $A$ can be constructed, for only one observation, by taking the derivative of Eq. (4.3) with respect to the unknown vector of $(d_u, d_e, d_n)$. Subsequently, by setting $\sigma = 1$ and substituting $A$ in Eq. (4.9) the $3 \times 3$ matrix is obtained:

$$\begin{bmatrix}
43.339 & -0.826 & 277.76 \\
-0.826 & 0.512 & -5.426 \\
277.76 & -5.426 & 1801.661
\end{bmatrix}.$$
The square root of the diagonal elements (6.583, 0.715, 42.445) describe the effect of geometry on the Up, East and North components from left to right, respectively. The non-diagonal elements shows the correlation between the components. The low DOP values indicate the strength of the geometrical configuration of the observations while the high DOP values show the weak geometry and inability of the available configuration in resolving the motion component in the corresponding direction. Therefore, it can be seen that the geometrical configuration consist of two pair of cross-heading tracks using TerraSAR-X is most suitable for retrieving the east-west deformation component. On the other hand the geometry is quite insufficient for reconstructing the north-south component. However, based on the correlation values in the matrix and also the discussion presented in Subsection 4.2.1, the north-south component should not be ignored as this approach leads into propagation of errors in the final motion estimates of both Eastern-Western and vertical direction.

4.4 Strategy in Motion Decomposition

The decomposition of the LOS motion estimates onto the horizontal and vertical directions are carried out on the absolute TomoSAR point clouds obtained after geocoding and correcting the coordinate offsets with respect to the reference point (see Subsection 3.5.1). Due to different geometry configurations of the tracks, the occurrence of identical scatterers in different point clouds is quite rare.

One strategy would be to interpolate the LOS deformation values of the fused TomoSAR point cloud on a regular-spaced 2D grid. In this case, a continuous field of deformation is obtained and the LOS deformation of each point on the grid can be decomposed to its real 3D displacement vector by assuming that the point is visible from all of the four viewing geometries. While this approach is quite practical in non-urban areas, due to identical origin of deformation phenomena, in cities special care should be taken into account as it is possible that the nature of the deformation can be quite different from one area to the other. One example might be an interpolated deformation value on the grid that is calculated from points on a facade of a building and ground points.

Due to the limitations mentioned above, the estimation of 3D motion vector from the LOS deformation is done in spatial windows in which the scatterers from all of the four stacks are available. The exact approach is explained in the following. One of the absolute point clouds is chosen as the reference. For each scatterer in the reference point cloud, a cube of size $4 \, m \times 4 \, m \times 4 \, m$ is defined centred on the scatterer. Since the point clouds are geocoded, the cube in one stack corresponds to the exact location in other stacks. Within the cube, it is checked that at least one scatterer from each stack is present. This is done mainly to preserve the optimum geometry for the least squares estimation since it influences the precision of estimates directly. Then with using the heading angle correspond to the point cloud of each beam, the local incidence angle of each scatterer and LOS deformation observations obtained from TomoSAR processing (see Section 3.6) the least squares estimation is carried out to calculate the unknown 3D displacement vector of the central scatterer. The estimation allocates the weights to deformation observations based on the relative distance of the scatterer to the central point of the cube and also takes into account the estimated amplitude of each scatterer which is obtained from TomoSAR processing. In this case, it is assumed that points with higher amplitude values are more stable and should be participated in the estimation with higher weights. The final output of the entire procedure is the deformation estimates in the east-west, vertical and north-south directions for each scatterer and their corresponding precisions.
4.5 Results

The decomposition of the LOS motion onto the real 3D displacement vector is only restricted to the test sites indicated with red ellipses in Fig. 3.13. The reason is that there are only these two areas which show prominent seasonal LOS deformation. The first investigated test site is the central railway station of Berlin (see Fig. 4.2) and the second one is situated at the north west of the central station which includes a railway bridge (see Fig. 4.3).

![Figure 4.2: Berlin central railway station. Left: Front view, Right: Top view (Google Earth™).](image1)

![Figure 4.3: Second test site: a railway bridge located at the north-west of the Berlin central station (Google Earth™).](image2)

4.5.1 Test Site: The Berlin Central Railway Station

The motion decomposition results for the Berlin central railway station as well as the distribution of standard deviation of the estimates are illustrated in Fig. 4.4. The amplitude of seasonal deformation estimates are expressed in millimetres. As is mentioned in Section 3.2, the seasonal deformation is estimated with applying a phase shift of the master acquisition with respect to
a reference sinusoidal signal fitted to the temperature changes within one year. Therefore, negative amplitude implies smaller magnitude of seasonal motion in the LOS direction.

In Fig. 4.4a, the pattern of seasonal deformation in the east-western direction can be seen. The deformation in this direction can be reliably reconstructed due to sufficient geometry configurations of two pair of cross-heading tracks. This is also inferred from Fig. 4.4b where the majority of the estimates have precisions better that 1 millimetre. The halls of the station go under heavy seasonal deformation with magnitudes of approximately 10 millimetres. In addition to the main parts and halls of the station, the rail tracks also show signs of deformation with lower amplitudes of around $-2$ to $-6$ millimetres. The reason for the deformation is related to the construction material of the building and the tracks. The building is mainly built of steel and glass which show significant expansion and contraction during summer and winter times, respectively. From the image, it is concluded that several other buildings close to the rail tracks are also affected by seasonal deformation, however with medium or lower amplitudes.

Fig. 4.4c and Fig. 4.4d show the deformation in the vertical direction and the corresponding precision distribution of the estimates, respectively. As is mentioned in Section 4.3, based on the DOP value, the geometry configuration effect is approximately 6 times worse for this direction in comparison to east-western component. This is also shown by the histogram of the standard deviations. Nevertheless, the periodic subsidence of the central stations main parts can be observed from Fig. 4.4c with magnitudes as large as 15 millimetres. This can be inferred that the main parts, in contrast with the halls and the rail tracks, are mostly influenced by the seasonal deformation in the vertical direction.

Finally, Fig. 4.4e and Fig. 4.4f report the seasonal deformation results in the north-south direction and the corresponding precision distribution, respectively. Due to the insufficient geometry configuration and the near polar orbit of TerraSAR-X satellites, the standard deviations of the estimates are quite large in this direction up to approximately 20 centimetre. Therefore, proper interpretation is not possible. However, it is important to note that this component should not be ignored in the functional model since the error will leak into the deformation estimates in the other two directions (see Subsection 4.2.1).

4.5.2 Test Site: Railway Bridge

Fig. 4.5 shows the deformation estimates in east-western, vertical and north-southern for the second test site in the left column as well as the corresponding histogram of precision values in the right column.

In Fig. 4.5a, several areas can be identified which are affected by the seasonal deformation in east-west direction. The most notable ones are the rail tracks at the right side of the image and the building located at the left side of the investigated area. The rail tracks show noticeable deformation with magnitudes up to $-10$ mm which means the expansion of the track during the summer time and the contraction in the winter cause a length change of 20 mm. Another, interesting pattern is seen from the building in the left side of the image which is quite similar to the one observed from the central railway station. It is seen that the building undergoes both expansion and contraction at the same time with magnitudes as high as 10 mm. There are also other areas in the image which are influenced by seasonal deformation with amplitudes up to 4 mm.

Fig. 4.5c and Fig. 4.5e shows the deformation estimates in the vertical and north-south directions together with their corresponding precision histograms in Fig. 4.5d and Fig. 4.5f. The standard deviations were expected to be high because of geometry configuration. Nevertheless, the rail
tracks which previously were identified to be highly affected by seasonal deformation in east-west direction are also influenced by the periodic deformation in the vertical direction as it is seen with blue colors in Fig. 4.5c. It is also seen that the building located close to the railways, undergoes an uplift of approximately -15 mm as the negative sign represents the movement of the object towards the sensor. The deformation might be related to the shape and material of the roof as is also discussed in Gernhardt (2012). Unfortunately, the deformation estimates in the north-south direction are not reliable enough for interpreting the results. However, this component should be considered in the functional model in order to avoid biased estimation in the other two components.
Figure 4.4: Estimated seasonal deformation in east-west, vertical and north-south direction for Berlin central railway station. The amplitude of the seasonal deformation is color-coded in millimetres. The right column describes the distribution of standard deviation of the estimates.
Figure 4.5: Estimated seasonal deformation in east-west, vertical and north-south direction for test site two, the railway bridge. The amplitude of the seasonal deformation is color-coded in millimetres. The right column describes the distribution of standard deviation of the estimates.
Chapter 5

Conclusions & Recommendations

This chapter is dedicated to outlining the summary of the carried out research. The chapter is continued with the concluding remarks and is finally closed by reporting potential improvements for future research.

5.1 Summary

In this thesis a framework was presented for the purpose of detailed mapping and deformation monitoring of individual buildings in urban areas using spaceborne SAR tomography. The method fuses the Stereo-SAR and SAR tomography concepts to obtain absolute 3D positions of a large amount of natural scatterers. Initially, tens of opportunistic point scatterers visible from multiple stacks of VHR spotlight TerraSAR-X image stacks were selected. It was seen that the Stereo-SAR method is capable of retrieving the absolute 3D coordinates of the natural scatterers with precisions ranging from 2 to 10 centimetres with exhaustive correction of SAR measurements and further solving the range-Doppler system of equations.

The TomoSAR processing was carried out on four stacks of TerraSAR-X VHR spotlight images over the city of Berlin, among them two from ascending orbits and two from descending. The elevation and deformation parameters of each stack were estimated by TomoSAR with respect to an identical reference point whose 3D coordinates are known from Stereo-SAR. It was demonstrated that the geocoded multi-track TomoSAR point clouds can be automatically fused by applying the coordinate offset between the geocoded and the known coordinates of the reference point. To assess the horizontal and vertical localization accuracy, the fused TomoSAR point cloud was compared to a DSM obtained from airborne LiDAR.

The availability of large image stacks with a period of almost five years allowed for analysing the linear and seasonal deformation of individual buildings in the city of Berlin. The results of the TomoSAR processing revealed deformation patterns in some areas of Berlin. As the SAR instruments are only capable of retrieving the projection of the original 3D deformation vector onto the LOS direction, motion decomposition was performed on the TomoSAR observed LOS seasonal deformation estimates for two test sites in Berlin.

The chapter is continued with concluding remarks of this thesis in Section 5.2 and is finalized with possible recommendations for future research in Section 5.3.

5.2 Conclusions

The carried out research can be divided into several categories that were combined together in order to be able to answer the following research question:
5.2.1 SAR Imaging Geodesy

The SAR Imaging Geodesy method aims at correcting the SAR azimuth and range time measurements to achieve centimetre-level pixel localization accuracies. The corrections are applied based on the closest permanent GNSS receiver. The most prominent error sources consist of atmospheric delays and geodynamics effects. The corrections were applied on natural scatterers in the city of Berlin using the SAR Imaging Geodesy software. It was revealed that errors up to 3.3 meters, 7 centimetres and 15 centimetres related to tropospheric delay, ionospheric delay and geodynamics effects, respectively affect the radar range measurements. Azimuth observations are only affected by errors caused by geodynamics effects in the order of 7 centimetres. Therefore, this recently developed method allows for absolute SAR measurements by compensating the most important errors.

5.2.2 Stereo-SAR for 3D Scatterer Reconstruction

The Stereo-SAR method retrieves the absolute 3D coordinates of corner reflectors and persistent scatterers with TerraSAR-X. The method combines the corrected SAR range and azimuth timings and solves the range-Doppler equations for scatterers visible at least from two different flight tracks. Apart from the corrections, it uses the modified range and azimuth calibration constants required for centimetre pixel localization. In this study, the Stereo-SAR software was used in order to estimate the absolute 3D coordinates of several natural scatterers in the city of Berlin. From the same heading tracks the scatterers originating from building facades are retrieved with 3D coordinates precisions ranging from 7 to 10 centimetres. For the cross-heading tracks, due to the optimal geometry configuration, the scatterers are localized with precisions below 3 centimetres which are assumed to be the base of lamp poles. It is important to note that this precision was achieved without taking into account the finite dimensions of the lamp pole. Therefore, no information can be obtained about the absolute accuracy of the coordinates as certain biases might exist in the results. Based on the discussion in Subsection 2.8.5, it is concluded that the focused SAR images should consistently generated by one accurate SAR processor in order to achieve high precisions. It has to be emphasized that these precisions were achieved based on total number of 10 to 12 acquisitions and therefore higher precisions are expected with larger number of observations.

5.2.3 TomoSAR processing

The objective of TomoSAR processing is to reconstruct the reflectivity profile of each azimuth-range pixel along the elevation and further retrieve the number, amplitude, elevation and motion parameters of multiple scatterers inside the resolution cell. The D-TomoSAR was carried out on four stack of VHR TerraSAR-X spotlight images over the city of Berlin using the TomoGENESIS of the DLR. The results demonstrate the high capability of TomoSAR processing for layover separation in urban areas where both scatterers from the ground and from the roof or facade of the buildings are retrieved. From the elevation map, a very detailed representation of each individual building can be observed. TomoSAR point cloud of each stack consists of 15 to 17 millions point scatterers which demonstrates the scatterer density of 300000 to 340000 per squared kilometres considering the area of the investigated scene to be 50 $km^2$. This leads to approximately ten times higher scatterer density comparing to PSI methods which assume a
single scatterer in each resolution cell.

The linear deformation map obtained from TomoSAR processing revealed LOS subsidence with velocities ranging from 2 to 8 mm/year in several areas of Berlin. Furthermore, it was seen by using X-band data, the seasonal deformation induced from thermal dilation of steel construction should be taken into account when monitoring urban areas. Two sites in the city of Berlin consist of the Berlin central railway station and a railway bridge located at the north-west of the central station undergo significant seasonal deformation with LOS movements of up to 20 millimetres between summer and winter. The reason lies in the construction material which mostly consist of steel for both test sites. Apart for these areas, most of the structures in the city are influenced by seasonal deformation with lower magnitudes of approximately 5 millimetres. Therefore, this again emphasize the importance of modelling non-linear motions in urban areas using VHR SAR data.

5.2.4 Absolute Fusion of Multi-track TomoSAR Point Clouds

The absolute fusion of TomoSAR point clouds obtained from two pair of cross-heading tracks was demonstrated by choosing a common reference point for all of the stacks while the absolute 3D coordinate of the reference point is known from Stereo-SAR. It was seen that the point clouds could not be simply merged together due to differences between known and geocoded reference point coordinates. The error is originated from the geocoding process in which the non-corrected range and azimuth timings of the master scenes are used. The magnitudes of differences confirm this statement as they are consistent with the numeric values of corrections outlined in Subsection 2.8.5. After the compensation of the coordinate shifts, the first absolute 3D TomoSAR point cloud with 63 million points covering an area of 10 km × 5 km over the city of Berlin was obtained. The point cloud fusion led to a shadow-free point cloud where full structure of individual buildings were mapped in a very detailed sense.

To validate the absolute position accuracy of the resulting fused TomoSAR point cloud, a reference DSM was calculated from a point cloud obtained from aerial Laser scanning. The horizontal accuracy analysis was based on calculating the distance between facade lines extracted from TomoSAR point cloud and the DSM for several buildings. For one test site located in the border of the imaged scene, the error was approximately 1.2 meters while for a building situated at the distance of 1.5 km of the reference point, the difference was almost 42 centimetres in the UTM coordinate system. This was expected since due to the error propagation rule, by increasing distance from the reference point the error will be higher. The magnitude of errors emphasize the importance of SAR measurements correction in order to achieve high accuracies. Moreover, the vertical localization accuracy was assessed for a single building located close to the reference point which demonstrated a RMSE value of approximately 3 meters with respect to the DSM. However, it was discussed that the RMSE value is not reliable due to non-perfectly flat roof of the investigated building. Finally a visualization assessment was carried out based on TomoSAR point clouds overlaid on the LiDAR DSM. The quite good fit of the TomoSAR point cloud on the DSM implied that the fusion of the stacks was carried out successfully as no large deviations were visible between the fused point cloud and the DSM.

5.2.5 Motion Decomposition

With availability of SAR data from ascending and descending tracks, the LOS seasonal deformation estimates obtained from D-TomoSAR were decomposed onto the real 3D displacement vector in the local coordinates. The study has shown that although the sensitivity of TerraSAR-X measurements with respect to the north-southern deformation component is very low, ignoring
the component leads to the bias estimation of the motion in the east-western direction. Considering the near polar orbit of TerraSAR-X of the four available stacks, the error ranges from 12% to 18% of the deformation magnitude in the north-southern direction. Therefore, for the motion decomposition all of the components should be taken into account when defining the functional model.

The geometrical configuration effect of using two pair of cross-heading track for 3D motion vector reconstruction was analysed based on the DOP values. The geometry is most suitable for retrieving the component in the east-western direction while it is quite insufficient for estimating the motion in north-western direction.

The east-western seasonal deformations were reliably estimated for majority of scatterers with precisions better than one millimetres. The two mentioned test sites show amplitudes of seasonal motion up to 10 millimetres. For the other two components the insufficient geometry configuration did not allow for reliable interpretations. However, it was demonstrated that using data from cross-heading tracks allows for a discrimination between deformations in different components. This is desirable for meaningful interpretation of the deformation pattern for individual buildings if datasets acquired from better geometries are available.

5.3 Recommendations for Future Research

The strength of the proposed method can be summarized as: producing absolute 3D point cloud, dramatic increase in the number of scatterers after absolute point cloud fusion and discrimination among motion components. However, the method can be improved by considering the following:

- Establish an automated procedure for identification and measuring the radar timings of identical scatterers visible in two or more image stacks to be initiated in Stereo-SAR processing. Regarding the identification, the modified version of the well-known SIFT algorithm, known as SAR-SIFT, can be used to identify identical targets in the SAR images (Dellinger et al., 2012). Another possibility for the identification of the points is to explore the regular pattern attributed to building facades (for same-heading orbits) and lamp poles (for cross-heading orbits).

- The Stereo-SAR method can be used to transform an entire TomoSAR point cloud from the radar coordinate system (azimuth, range, elevation) into the Cartesian geocentric coordinate system (X, Y, Z). By applying the method to retrieve the absolute 3D coordinates of dozens of scatterers distributed in the investigated scene, a seven-parameter Helmert transformation can be used to obtain the entire point cloud in the Cartesian geocentric system. In this case the conventional geocoding is not required.

- Automatic outlier detection and elimination can be carried out on the results obtained from TomoSAR processing. Instead of simple filtering, the Detection, Identification and Adaptation (DIA) algorithm (Teunissen et al., 2005) can be a substitute for reliable and fast blunder elimination.

- The absolute deformation estimates can be obtained from InSAR methods by cooperating the GPS measurements.
Bibliography


