FractalAnalyzer: A MATLAB Application for Multifractal Seismicity Analysis

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Online Material: MATLAB code and associated figures and tables describing the multifractal seismicity analysis method.

INTRODUCTION

Earthquakes are seismic phenomena caused by the sudden release of energy in the Earth's crust. Their effects range from ground shaking to faulting. Geological and geophysical studies, especially in light of plate tectonic theory have been used to explain the occurrence of earthquakes. Thus from the point of view of statistical fractals, earthquakes cannot be interpreted as random independent events (i.e., having Poisson distribution). Rather, it is observed that the events of the same sequence are clustered in time and space (Shlien and Toksoz, 1970; Vere-Jones, 1970; Smalley et al., 1987; De Natale et al., 1988; Roy and Mondal, 2012a,b).

Several past approaches studied the clustering effect of the earthquakes. The generalized Poisson model is one of the most commonly used tools to understand the distribution of the seismic events (Ripley, 1988; Stoyan and Stoyan, 1994). This model assumes that the Poisson distribution can be used to describe the uncorrelated clusters of events, whereas the number of events in each cluster is defined by the power law distribution. Some researchers have studied these seismicity clusters by considering the occurrence of an earthquake to be a stochastic self-exciting process (Vere-Jones, 1970; Ogata et al., 1982; Molchan and Kronrod, 2007). Another useful approach toward quantification of the clustering properties of an earthquake process is the multifractal analysis of the seismic events.

In the past, several authors have explained the regional seismicity clustering through fractal-like structures (Sadovskiy et al., 1985; Smalley et al., 1987), and these works suggest that multifractal analysis is an effective method for studying earthquake patterns. Moreover, this method provides a detailed explanation of the chaotic nature of distributions and geometry associated with the earthquake clustering phenomena (Bak and Tang, 1989; Huang and Turcotte, 1990; Chen et al., 1991; Kagan and Jackson, 1991; Godano and Caruso, 1995; Guo and Ogata, 1997; Telesca et al., 1999; Evison, 2001). The fractal dimensions needed for multifractal analysis can be calculated using different methods. The correlation integral approach calculates the correlation fractal dimension (Kagan and Knopoff, 1980; Grassberger and Procaccia, 1983a; Hirata, 1989). Generally, the fractal distributions or statistical scale invariance that exists in nature and in dynamic systems are found to be heterogeneous (Mandelbrot, 1989). Thus, a unique fractal dimension is not enough to understand the clustering of these events. In such cases, multifractal analysis is performed and the fractal dimensions are characterized by the general dimension \( D_q \) or the \( f(\alpha) \) spectrum (Hentschel and Procaccia, 1983; Halsey et al., 1986; Neuman, 2010). Moreover, multidimensional fractal study provides a quantitative measure of the spatial clustering (i.e., whether cluster exists within clusters), thereby giving information about the crustal deformation in space and time nucleation of events. Therefore, multidimensional will explain the state from one level to other level of cluster existence, when it saturates at higher value, it indicates that no further cluster exists within cluster. This spatial clustering result can be used to understand the seismicity of a region (Aki, 1981, 1984; King, 1983; Legrand et al., 1996; Nakaya and Hashimoto, 2002; Oncel and Wilson, 2002, 2004, 2006; Roy and Ram, 2006; Li and Xu, 2012).

This paper presents the application of the correlation integral method for the determination of the correlation dimension and the multifractal or generalized fractal dimension. These results are used further to determine the clustering of seismicity (Grassberger and Procaccia, 1983b; Roy and Ram, 2006; Roy and Padhi, 2007). Multifractal analysis also helps to interpret the heterogeneous fractal nature of seismicity. This heterogeneity is an indication of the complex stress pattern of the region (Legrand et al., 1996; Oncel and Wilson, 2004). For easy implementation of this method, we present FractalAnalyzer, a MATLAB-based (http://in.mathworks.com/support/compilers/R2010a/win32.html; last accessed August 2015) application that uses this algorithm to interpret the spatiotemporal cluster of earthquake occurrence. FractalAnalyzer is an interactive graphical user interface (GUI) developed in MATLAB R2010a. It is capable of using up to a total of 22 fractal dimensions to define a cluster of earthquake events. The interactive feature of this application provides the flexibility of choosing the number of events to be considered within each subset. Additionally, this application includes a graph-plotting module that can be used to plot various graphs associated with this algorithm. To demonstrate the capability and effectiveness of this application, we used it to analyze the clustering of seismicity before the 08 October 2005 Kashmir earthquake (\( M_w 7.7 \)).

THEORY AND ALGORITHM

Fractals

Fractal dimension is a characteristic index of a fractal object or set (Hirabayashi et al., 1992). As per Mandelbrot (1977), a
fractal is defined as a set for which Hausdorff–Besicovitch dimension strictly exceeds the topological dimension. The Hausdorff–Besicovitch dimension is not easy to calculate for fractals in the real world. There are several ways of defining the fractal dimensions (Takayasu, 1990). For example, the similarity dimension \( D_s \) is defined for an exactly self-similar or a smaller portion of distributions and has exactly the same kind of distribution in a larger portion set as

\[
D_s = \frac{\log b}{\log a},
\]

in which \( a \) is the linear size and \( b \) is the number of similar daughters. \( D_s \) has a more theoretical than practical base. Scale invariance distribution, which is not exactly self-similar in all scales, can be quantified with the capacity dimension \( D_c \), defined as

\[
D_c = \frac{\log N(r)}{\log(1/r)},
\]

in which \( N(r) \) is the smallest number of coverings of the set with size \( r \). \( D_c \) has the practical advantage of extracting information about a system in a statistical sense. Further statistical extraction of the scale invariance distribution is done with the help of the information dimension \( D_I \), which is based on probability distribution and defined as

\[
D_I = \lim_{r \to 0} \frac{\sum P_i(r) \log P_i(r)}{\log r},
\]

In equation (3), \( P_i(r) \) is the probability for a point in the sample space to belong to the \( i \)th box with size \( r \). Correlation dimension \( D_C \) depends on the correlation integral \( C(r) \) and the relation can be expressed as follows:

\[
C(r) \approx r^{D_C-d},
\]

in which \( d \) is the spatial dimension. \( C(r) \) is calculated from \( N \) using the following relation:

\[
C(r) = \lim_{N \to \infty} \frac{1}{N^2}.
\]

in which \( N \) is the number of pairs of points in the sample space for which distance is less than \( r \). In general, \( D_1 \geq D_I \geq D_c \). The relationship holds true for homogeneous fractals, and the inequality is valid for heterogeneous multifractals (Hirabayashi et al., 1992). They are applied to extract information about distributions of various theoretical and physical situations. A unique fingerprint for each of the dimension to be obtained will have its own significance as a multifractal object, which demands an infinite hierarchy of fractal dimensions, known as the generalized fractal dimensions \( D_q \). This can be expressed as

\[
D_q = \lim_{r \to 0} \frac{\log \{\sum [P_i(r)]^q\}}{\log r},
\]

in which \( D_q \) exhibits a nontrivial scaling behavior for different values of \( q \), and \( P_i(r) \) is the probability of an event lying within a square box of dimension \( r \). The generalized dimension \( D_q \) is defined for all real values of \( q \) and is a monotonically decreasing function.

Mandelbro (1989) showed that lower- and upper-limiting dimensions, \( D_{w0} \) and \( D_w \), respectively, exist that are related to the different regions of the set. \( D_{w0} \) and \( D_w \) correspond to the regions in which the measures are most dilute and most dense, respectively, and this phenomena is termed multifractality. Usually, the multiplicative cascades of the random processes generate multifractal structures, whereas the additive processes produce simple fractures (Bunde et al., 1990). The correlation dimension \( D_c \) thus obtained is

\[
D_c = \lim_{q \to 2} D_q = D_2.
\]

In two dimensions, the values of \( D_q \) approaching a value of 2 signify a uniform coverage of the plane. We use the spherical triangular method to calculate the distance between two epicenters (Oncel and Wilson, 2002; Mandal et al., 2005; Roy and Nath, 2007; Roy and Mondal, 2009).

**Correlation Dimension**

The fractal correlation dimension is derived from the correlation integral, which is a cumulative correlation function that measures the fraction of points in 2D space and is defined as

\[
C(r) = \frac{2}{N(N-1)} \sum_{j=1}^{N} \sum_{i=j+1}^{N} H(r - r_{ij}),
\]

in which \( N \) is the number of pairs that can be formed from a given cluster of seismic events (for 50 events, the \( N \) window will be \( 50 \times 2 \), which equals 1225), \( r \) is the length scale, \( r_{ij} \) is the distance between any two points of the cluster set obtained using the spherical triangle method, and \( H \) is the Heaviside step function. Grassberger and Procaccia (1983a) defined \( N \) as the total number of events that form the set. We consider it to be the set of all possible unique vector pairs, with two events taken each time. \( C(r) \) is proportional to the number of vector pairs of the fractal set having length less than \( r \). Figure 1 shows a graph of \( \log C(r) \) versus \( \log r \) at different stages of an example fracture process. Based on regression analysis, we determine the equation of the line that fits the distribution. The slope of this line gives the fractal dimension \( D_c \) of the system. The large-scale deviations from linear dependence are associated with the finite size of samples, whereas the small-scale deviations are due to the boundary effects of data.

**Generalized Dimension**

Multifractal dimension parameter \( D_q \) represents the complicated fractal structure or multiscaling nature of the seismic
events. In this work, we use $D_q$ to analyze the seismicity clustering toward the multiscale notion. Some of the most common methods for calculating $D_q$ are the fixed-mass method, the fixed radius method, and the box-counting method (Grassberger and Procaccia, 1983a; Halsey et al., 1986; Mandelbrot, 1989). Pawelzik and Schuster (1987) extended the Grassberger–Procaccia method for the recovery of dimension from a time series. The related formulae are given as

$$\log C_q(r) = D_q \log r (r \to 0)$$

and

$$C_q(r) = \left\{ \frac{1}{N} \sum_{j=1}^{N} \left[ \frac{1}{N} \sum_{i=j}^{N} H(r - |X_i - X_j|) \right]^{q-1} \right\}^{1/(q-1)},$$

in which $C_q(r)$ is the qth-order correlation integral. The above formulation is implemented in FractalAnalyzer to process the datasets. $D_q$ is the slope of the straight line fitted on the data of log $r$ versus log $C_q(r)$ graph using the linear regression method. For a different value of $q$, we get a different $D_q$. The curve of $q - D_q$ is termed the $D_q$ spectrum.

**Multifractal Computation Module**

MCM contains functionalities for performing correlation-integral-based multifractal analysis and saving the results in an organized format. Figure S1 shows a screenshot of this module, providing information about the format of the input file. The input file required for MCM is a fixed seven-column format without any headers. The file extension is .dat. The seven columns are year (yyyy), month (mm), day (dd), hours, minutes, latitude, and longitude. Once the input file is supplied, this module provides the flexibility of choosing the number of data points to be kept in each cluster for $D_q$ computation. Figure 2 shows a flowchart describing the process of computing the $D_q$ values from the provided input. The main steps for multifractal analysis are described in the following sections.

**Input Data**

As mentioned above, FractalAnalyzer is very specific with the format of the input data. The data needs to be arranged in a fixed seven-column format without any headers and the file name should contain a .dat extension.

**Choose the Window Size**

The data need to be divided into several segments based on the length of the window. The multifractal dimension is then calculated for each of the segments independently. Further, to understand the stress accumulation, the fractal dimension values from all the segments are grouped together and on a temporal scale. The length of the window should neither be very large nor too small. The optimum length of the window is 50–75 events.
If the number of events within each segment exceeds 100, it is expected that the shorter patterns will be missed out (Roy and Mondal, 2012a). For windows with few events, it is difficult to obtain a reliable linear fit for the graph; in this case, statistical invalidity would be prominent due to lack of data.

Determine r-Values
The distance ($r_{ij}$) between any two points of a set is calculated as stated above and operated with $r$, defining the space containing a fraction of points. The result is characterized with the Heaviside function and summed over all the possible pairs of points that lie within a defined window. The MATLAB code for determination of $r$-values is contained within the distance.m file, and it forms the integral part of the FractalAnalyzer package.

Calculating the Correlation Integrals
FractalAnalyzer uses a correlation integral approach to determine the multifractal correlation dimension for a given sequence of seismic events. The cumulative correlation function is used to measure the distribution as shown by equation (8) (Grassberger and Procaccia, 1983a; Roy and Ram, 2006). Here, $N$ is used as $(N^2 C_2)$ for each window, as stated above in the Correlation Dimension section. Using the $r$-values and the Heaviside function value, the corresponding correlation integrals are calculated.

Regression Analysis for $D_q$
The correlation integral ($C_q(r)$) values are obtained, these data values are plotted with respect to $r$. Both, $C_q(r)$ and $r$ are plotted on a logarithmic scale. $D_q$ is obtained from a log–log plot corresponding to the linear portion as stated previously in the Generalized Dimension section. A minimum of 51% continuous points should be chosen to perform proper multifractal analysis, otherwise we would have unreliable estimate (Oncel and Wilson, 2002).

FractalAnalyzer uses the plotting functions of MATLAB to fit the points on a line, such that the fitness function or regression coefficient has a value > 0.99. In the region of active seismicity, the minimum values of 0.98 are also acceptable (Oncel and Wilson, 2002). If the misfit is more, then the results obtained from multifractal analysis will be ambiguous. For further development purposes, the subroutine distance.m can be used.

Grapher Module
GM is the secondary module of FractalAnalyzer, which provides the flexibility to view the several plots associated with the multifractal analysis procedure. This helps in immediate analysis of the results without the need to switch to any additional software. The module allows viewing the following plots: log $r$–log $C(r)$, $D_q - q$, and $D_q - t$ (Fig. 3b).

log $r$–log $C(r)$ Plot
This is a plot of the cumulative correlation function $C(r)$ versus the maximum allowable separation between two points $r$. $C(r)$ is defined as described in the Correlation Dimension section, mainly obtained with the controlling parameter $r$. As discussed in the Correlation Dimension section, when $C(r)$ and $r$ are plotted on logarithmic scales to obtain $D_q$, they show a linear relationship with slight deviations. Significant deviation from the above said value in the Regression Analysis for $D_q$ section will lead to ambiguity, which is shown in Tables S1 and S2.

Once MCM is executed, the entire study dataset is divided into several sets with $n$ number of events contained in each set. MCM allows the user to change the value of $n$ (default 100) as per the requirements, but it should not be < 30. The data (log $r$ and log $C(r)$) are saved in the folder corresponding to each cluster. For example, the output for the third set of 50 events for $D_q - 2$ is saved in log_files_50_3\correlation/Cqr_2.dat. This data file can be loaded in GM to view the plot.
Further, this data file can be example, the output for the third cluster of 50 events is saved .dqq extension in the folder corresponding to each cluster. For

The MATLAB code for this subroutine is located in FractalAnalyzer

The nature of slopes of the linear fits for the log $C_q(r) - \log r$ plots obtained for all the 20 consecutive time windows prior to the 8 October 2005 earthquake of $M_w$ 7.7 appears to be of similar nature. Hence, we show only one such window for demonstration purpose (Fig. S3). The range of $r$ for which the corresponding plot of log $C_q(r)$ versus $\log r$ is a straight line is an indication of the range over which a fractal model holds true. $D_q$ can be obtained from this linear portion of the

\[ D_q - t \text{ Plot} \]

The plot of $D_q$ versus time ($t$) is studied to identify the region that leads to low generalized fractal dimension ($D_q$) values. Intense clustering leads to low $D_q$ values. As the events get clustered, their limit tends toward a point, that is, they are approaching dimension 0. Hirata et al. (1987) demonstrated this with rock sample microfracturing in a laboratory experiment. Thus, the $D_q$ versus $t$ plot helps analyze the stress state of the regional crust. This can help in better hazard mitigation of the study region and its adjoining areas.

When FractalAnalyzer is run in MCM mode, the $D_q$ value is computed for each cluster of seismic events belonging to the study region and is grouped in a single file in the DQ folder with .dqv extension. For example, for $q = 2$, the output will be saved in DQ/DQ_2.dqv. The MATLAB code for this functionality is saved in computeDC.m.

**APPLICATION**

To test the capability and effectiveness of the FractalAnalyzer application and to test efficiency of the correlation integral method, we use of two sets of seismic events, one before the strong 8 October 2005 Kashmir earthquake ($M_w$ 7.7), and the other immediately after it.

**Data**

For study purpose, we use the earthquake datasets for the region bounded by 32° and 36° N latitudes and 71° and 75° E longitudes. A total dataset of ~1300 events (1974–2012) is obtained from the U.S. Geological Survey (USGS) Preliminary Determined Epicenter database (body-wave magnitude $m_b$ ≥ 3.5).

The whole data are further divided into two sections: the events before the 2005 Kashmir earthquake and the events after it. The two subsets are arranged in a seven-column format so as to match the input file criteria of FractalAnalyzer.

**Results**

FractalAnalyzer is used to study the multifractal distribution of all events with $m_b$ ≥ 3.5 that occurred in the northwest Himalayan region during 1974–2012. From the $D_q - t$ plot, it is observed that the $D_q$ value fluctuates with time (Fig. 3a,b). $D_q$ values are plotted against mean time of each 50-event window for consecutive periods to study the variation of spatial-generalized fractal dimension with time. The correlation integral approach is used for the entire analysis as stated above in earlier section.

**Generalized Fractal Dimension**

The $D_q$ value is computed for $q = 2, 3, 4, ... , 22$. For values $> 20$, $D_q$ is found to approach zero and is thus restricted to 22. Thus, the validation of a fractal system can be done using the $D_q - q$ plot to see the trend of monotonically decreasing $D_q$ with $q$, as stated in the Fractals section.

Again, in the case in which the MCM is executed, $D_q$ value is computed for $q = 2, 3, 4, ... , 22$ and is saved in a file with .dqq extension in the folder corresponding to each cluster. For example, the output for the third cluster of 50 events is saved inlog_files_50_3\DQVsQ.dqq. Further, this data file can be loaded directly in GM mode to view the plot. © Figure S4 shows FractalAnalyzer in GM mode. An interesting point is observed: the value of $D_q$ decreases monotonically with increase in the value of $q$ and approaches 0 for higher $q$ values. To use this function as a subroutine for further development purpose, we suggest using the computeDQVsQ.m function. This file contains the MATLAB implementation for plotting the $D_q$ values with respect to $q$.
The $D_q$ values obtained for different values of $q$ are shown in figure S4, and the curve is known as the $D_q$ spectrum. It characterizes the multifractal or heterogeneous fractal for the spatial distribution of events. The multifractal nature suggests the events form subclusters within a cluster of the fractal structure.

The decrease and then increase in the value of $D_q$ with respect to the spatial distribution of seismic events indicates clustering and dispersion in multifractal structure. In figures S5 and 3b, the variation of $D_q$ with respect to time has a nature similar to those of the results published in the past (Hirata et al., 1987; Nakaya and Hashimoto, 2002; Roy and Nath, 2007; Roy and Padhi, 2007). The fluctuation is due to strain accumulation and liberation around the stressed zone.

Correlation Fractal Dimension

$D_c$ can be obtained from the linear portion of the log $C(r)$ versus log $r$ plot (Fig. 1). Figure S6 shows events prior to the 8 October 2005 Kashmir earthquake lying within the study area. Low $D_c$ values of 1.28 and 0.92 are observed for windows with mean time 2003.25 and 2005.083 years, respectively. The clustering of the aftershocks of the 2005 Kashmir earthquake is analyzed for the study area, and the low $D_c$ values can be observed in figure S7. The technique of using 50 events for each window provides a precursor for the 2005 Kashmir earthquake (Roy and Nath, 2007; Roy and Padhi, 2007). Here, we define precursor as relative low $D_c$ value for intermediate size events set, which excludes aftershocks. This precursor is an indicator of strain accumulation (Hirata et al., 1987; Nakaya and Hashimoto, 2002). Again we may state that clustering of intermediate size seismicity helps to identify crustal deformation on a regional scale (Oncel and Wilson, 2006), which possibly contributes to large earthquake preparation in self-organized mode (Bak and Tang, 1989; Bak et al., 1994; Al-Kindy and Main, 2003). Even Jaume and Sykes (1999) state that combined observational and simulation evidence indicates the period of increased moment release in moderate earthquake signals establishes long-wavelength correlation in the regional stress field. However, we would like to add that the above precursors cannot be classified as classical, because an exhaustive search for such precursors was not carried out and we do not know how common such an extreme $D_c$ value is or how good $D_c$ would be at predicting individual earthquakes.

Discussions

The main Himalayan thrust behavior within the western syntaxis of the range is poorly understood. Some of the surface ruptures have been mapped. The exact locations of active faults are ambiguous, and a few paleoseismological studies have also been completed. Tectonic processes generally activate the fault system, in which strain accumulation yields highly stressed zone. The rupture may nucleate from those stressed zones, accounting for most of the high-frequency seismic energy radiation and eventually causing a large earthquake (fig. S8). The stressed zones control the distribution of earthquakes over a fault zone that triggers repeated earthquakes, as controlled by fault surface heterogeneities. Such a stress trigger has been reported for the adjoining Himalayan zone by, for example, the coloumb stress transfer approach (Gupta et al., 2015). Interestingly, these zones possess different physical states and properties and hence can remain difficult to map by standard geophysical techniques. Imaging this intriguing nature of the subelements of the megathrusts is a challenge for geophysicists.

Analysis reveals significant variation in the multifractal properties of seismicity between the tectonic subdivisions of the area under study. Differences between $D_2$ and $D_{22}$ (as shown in fig. 3a,b) are related to the differences in the tendency for seismicity to be clustered or dispersed at different scales. Hence, the differences between the multifractal dimensions $D_2$ and $D_{22}$ are interpreted to result from fratral heterogeneity between regional and local scales (Oncel and Wilson, 2006). Changes between the fractal dimension $D_2$ and multifractal ($q=2–22$) measures illustrate the sensitivity of the multifractal characterization changes in the local complexity. The large difference between $D_2$ and $D_{22}$ implicates the presence of significant fractal heterogeneity within the hypocenter distribution of shallow seismicity. This is due to the differences in fault complexity at local scales (i.e., $q=15,16,...,22$). With the help of multifractal analysis, the fractal properties of complex fault systems can be more suitably characterized.

In addition to the precursor for the 2005 Kashmir earthquake, several other low-$D_c$-value zones exist. Some of these low $D_c$ values can be explained by the occurrence of events of the order of magnitude of 6. The rest of these are the precursors to the 2005 Kashmir earthquake, as mentioned in table S1.

Conclusions

FractalAnalyzer is a simple, versatile tool for analyzing a well-constrained catalog for seismically active regions to demonstrate a quantitative representation of seismicity. Here, we could have better spatiotemporal understanding of seismicity and its implication on physical understanding. FractalAnalyzer is also capable of identifying the seismicity clustering based on low $D_c$ or $D_q$ values. To demonstrate the effectiveness of this tool, we used it to study the 2005 Kashmir earthquake. The relative clustering is considered to be the set of all possible unique vector pairs formed by two elements each time from the fractal set. The modified conventional correlation integral approach has given added benefit in this multifractal analysis of the intermediate-size events prior to the major 8 October 2005 Kashmir earthquake. Consequently these patterns are totally in agreement, which may serve as a precursor for this major earthquake. Further, we use this tool to study the aftershocks. The results presented here demonstrate the capability of the multifractal method, as well as the FractalAnalyzer application. Moreover, the multifractal analysis approach gave a view of heterogeneity of the crust leading to such complex seismicity. The implementation of this method in a graphical interface for user-friendly interactive environment makes FractalAnalyzer a strong tool for studying the earthquake patterns. To be specific, FractalAnalyzer and its importance can be judged as a viable technique, given...
quantitative spatiotemporal distributions for numerical warning rather than earthquake prediction. Thus, this reproducible numerical precursor prior to major earthquakes might help in improving hazard mitigation and, therefore, also in disaster management for other seismically active regions having past event episodes. Thus FractalAnalyzer may play a key role in analyzing strong earthquake preparation within a short span by using a well-constrained catalog of seismically active zones.

DATA AND RESOURCES


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