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NEARSHORE CIRCULATION

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1. INTRODUCTION

The topic of shelf circulation has been treated in the preceding chapter (Huyer(1)). Nearshore circulation, the topic of the present chapter, differs essentially from shelf circulation in scale and in the relevant driving forces, and therefore deserves a separate treatment.

Shelf circulation is driven primarily by wind- and tide-induced forces. It is laterally only weakly constrained so that the geostrophic (Coriolis) acceleration is manifest in the response. Nearshore circulation on the other hand is dominated by wave-induced forces associated with shallow-water wave breaking and is confined to a relatively narrow shore-bounded area. For brevity and for clarity of presentation, only wave-induced nearshore circulation is considered in this chapter, with zero mean flow far offshore.

The purpose of this chapter is to give a state-of-the-art review of the subject, rather than a presentation of recent research results. Emphasis is placed on the physics. Mathematical formulations of the most important relations are given, but solution techniques are only briefly referred to without analytical derivations or numerical algorithms.

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The outline is as follows. Section 2 gives a brief, qualitative indication of different types of motion in the nearshore zone. The nearshore wave field is dealt with in section 3. Section 4 focuses on the internal, vertical structure of the mean circulation in the reduced case of shore-normal motions. The more general circulation in two horizontal dimensions is described in a vertically-integrated sense in section 5. The chapter closes with a discussion of recent developments towards a three-dimensional modeling of the nearshore circulation.

2. TYPES AND SCALES OF MOTION

The flow pattern in the nearshore zone is dominated by wind-generated waves that evolve from deep water to the beach in processes of refraction, shoaling and breaking. As the waves propagate into shallow water, an increasing profile distortion occurs as a result of bound higher harmonics, whose relative intensity reaches a maximum in the vicinity of the location of initial breaking. In the breaker zone, a considerable part of the high-frequency kinetic energy is associated with breaking-induced turbulence.

Incident wave frequencies at exposed sea coasts are typically of the order of 0.1 Hz. However, spectral measurements through the surf zone identify a significant role for low-frequency motions or surf beat, frequencies less than about 0.05 Hz. Their relative importance is enhanced through the surf zone as a consequence of shoaling and breaking; they are arguably dominant during rough conditions (Guza and Thornton (2)). Both cross-shore and longshore propagation modes have been identified. Cross shore modes may derive from forced long waves that are bound to short wave groups (Longuet-Higgins and Stewart (3), Huntley and Kim (4)) or to long period variations in the break locations induced by incident short wave groups (Symonds et al (5)). Long wave motions radiate both seaward and shoreward from the breaker line as a consequence of this second mechanism. Longshore modes may be progressive or standing edge waves (Huntley et al (6), Holman and Bowen (7)).

The low-frequency motions in the nearshore zone referred to above are manifestations at difference frequencies of nonlinear interactions within a narrow-banded incident wave field. Such interactions also give rise to steady perturbations (spatial variations of the mean water level (MWL) and mean flows).

Spatial variations in the local MWL are most notable in the shore-normal direction where there is a close relationship between the shoaling-breaking evolution of the wave height and the setup of the MWL from the global still water level (SWL). The shoaling increase in wave height prior to breaking is accompanied by a set down of the local MWL and the subsequent decrease in wave height throughout the surf zone is accompanied (after some lag) by a rise in the MWL which evolves into a rather larger setup. In addition, there is a significant vertical mean-flow circulation in the on-offshore direction and a two layer structure can be identified. Incident progressive waves carry a small forward mass flux towards the beach in the trough-crest region, balanced by a small offshore flow or undertow beneath the trough.

Longshore currents of similar magnitude may be identified in the mean flow within the surf zone. Longshore nonuniformities together with the requirements of mass conservation may lead to nearshore cells of horizontal circulation. Return flows through the surf are narrow and intense (rip currents), as a result of vortex stretching (Arthur (8)), but the balance of the gyres beyond the surf zone and returning through the surf zone are broad and weak. Rip currents are regular features of most beaches and identify the existence of one or more circulation cells. The rips may be located by topographic constraints of the flow field such as groynes or headlands but are nonetheless observed on long straight beaches at near-normal incidence, suggesting an alternative origin in the nearshore hydrodynamics or perhaps morphology.

The dynamics of the vertical and horizontal nearshore circulation (mean flow) are governed primarily by the breaking incident waves; turbulence and low-frequency motions play a secondary role. Present-day nearshore circulation models do include turbulent momentum transfer, albeit in crude approximations, but the influence of low-frequency motions on the steady circulation is generally ignored. This is not correct because of the non-zero correlation between the low-frequency motions and the short-wave groups (see e.g. Goda (9), Dally and Dean (10)). However, there is presently no real alternative as a reliable prediction of the low-frequency nearshore motion is not yet available.

Although the fluctuating motions dominate in the nearshore zone, it is nonetheless convenient to adopt a Reynolds-style decomposition of the instantaneous flow variables into mean flow and fluctuating parts. We adopt a cartesian coordinate system located in the horizontal plane of the global SWL with horizontal axes x and y (or $x_\alpha = (x_1, x_2) = (x, y)$ in tensor notation or $\underline{x} = (x, y)$ in vector notation) and vertical axis z directed upwards. The α components of the horizontal velocity, for example, are written alpha

$$u_\alpha(\underline{x}, z, t) = \bar{u}_\alpha(\underline{x}, z) + \tilde{u}_\alpha(\underline{x}, z, t) \qquad (2.1)$$

tilde

where $\bar{u}_\alpha$ is the flow velocity averaged over a duration much longer than the wave groups, but still significantly shorter than any time scale associated with incident sea conditions. Such an averaging period will average over both the waves and the turbulence. Measured time scales in surf zone turbulence are significantly shorter than typical wave periods and it is further convenient to separate the fluctuating velocity $\tilde{u}_\alpha$ into turbulent (single prime superscript) and wave (double prime superscript) components, for example

$$\tilde{u}_\alpha(\underline{x},z,t) = u'_\alpha(\underline{x},z,t) + u''_\alpha(\underline{x},z,t) \qquad (2.2)$$

prime

double prime

The value of this separation is enhanced by the common lack of correlation between the wave and turbulent components.

3. NEARSHORE WAVE FIELD

Qualitative discussion.

This section gives a qualitative discussion of some aspects of wave models which are important for the calculation of nearshore circulation, prior to a quantitative presentation.

Random or deterministic wave models. Randomness is an essential property of wind-generated waves and should be included if realistic results are to be obtained for the circulation induced by wind waves. Calculated results for monochromatic, unidirectional incident waves often contain rapid and large spatial variations in energy density (e.g. in refraction calculations) or energy dissipation-rate (e.g. near the so-called breaker line), which lead to spurious results in the calculated wave-induced circulation. This contrasts strongly with the more realistic smooth and weak modulations in the results of random-wave models, with a distribution of the wave energy over a continuum (in theory) or a multitude (in numerical models) of frequencies and directions.

Propagation. Nearshore wave propagation is characterized primarily by depth-induced shoaling, refraction and breaking. Effects of wave-induced currents usually are of secondary importance, which is not to say that they are always negligible. They can be significant in regions of strong velocity shear as in rip-current systems. A more fundamental reason for their importance is the fact that with mutual interactions, the nearshore wave-current system may be unstable against longshore perturbations (Dalrymple and Lozano

(11), Miller and Barcilon (12)). For these reasons, the current influence on the waves is taken into account in the quantitative formulations given below.

Diffraction smoothes amplitude variations such as may arise as a result of shoaling and refraction, or in the vicinity of obstacles such as breakwaters and headlands. In the latter category, the amplitude gradients are strong (significant variations within a wave-length) and diffraction should be modeled. In situations without obstacles, the influence of diffraction on the nearshore wave-field is generally negligible. This is understandable in view of the relatively smooth variations in the wave-field which result from the finite spectral bandwidth of the incident-wave spectrum.

Energy input and dissipation. The single most important source/sink of wave energy in the nearshore zone is the dissipation due to shallow-water wave breaking. Other processes such as local wind input or boundary-layer dissipation are relatively insignificant because of the relatively short propagation distances involved. Weak non-linear wave-wave interactions are conservative over the entire spectrum, but not locally in the spectral domain. They are an order of magnitude stronger in shallow water (triad resonant interactions) than in deep water (quadruplet resonant interactions). In shallow water breaking waves, the interactions are no longer weak. No theoretical formulation of these interactions is available.

Quantitative formulation.

In this paragraph, a quantitative formulation is given of the most relevant physical processes mentioned above; diffraction is excluded here since that is dealt with in detail in the chapter on "wave transformation" in the present volume (Liu (13)).

In the following, the depth h and current velocity $\underline{U}$ in the domain of computation are supposed to be known a priori. The influence of the wave field on the mean depth and the current velocity can be taken into account by simultaneous (iterative) integration of the wave propagation equations with the mass and momentum equations for the mean flow, which are described in section 5 of this chapter.

Ray Kinematics. For waves on a variable current, the Doppler shift between absolute frequency ω and intrinsic frequency σ should be taken into account: omega sigma

$$\sigma = (gk \tanh kh)^{1/2} = \omega - k_{\alpha} U_{\alpha} \tag{3.1}$$

where k is the magnitude of the wave number vector $\underline{k}$.

The kinematics of the propagation are described by the ray equations

$$\frac{dx_{\alpha}}{dt} = C_{g\alpha} + U_{\alpha} \tag{3.2}$$

in which $C_{g\alpha} = \partial\sigma/\partial k_{\alpha}$,

$$\frac{dk_{\alpha}}{dt} = \frac{\partial k_{\alpha}}{\partial t} + \frac{\partial k_{\alpha}}{\partial x_{\beta}}\frac{dx_{\beta}}{dt} = -\frac{\partial\sigma}{\partial h}\frac{\partial h}{\partial x_{\alpha}} - k_{\beta}\frac{\partial U_{\beta}}{\partial x_{\alpha}} \tag{3.3}$$

beta

and, for time-invariant depth and current,

$$\frac{d\omega}{dt} = \frac{\partial\omega}{\partial t} + \frac{\partial\omega}{\partial x_{\alpha}}\frac{dx_{\alpha}}{dt} = 0 \tag{3.4}$$

See Whitham (14), Phillips (15) or Mei (16) for a full treatment.

Spectral Wave Action Balance. Neglecting diffraction, the dynamics of waves in an area with non-uniform depth and current velocity are conveniently described on the basis of the principle of conservation of wave action, formulated in terms of the spectral action density in wave number space, which (to lowest order) equals the spectral density of the wave energy, divided by the intrinsic frequency (Bretherton and Garrett (17), Hasselmann et al (18)). In a non-dissipative wave-current system, and neglecting nonlinear wave-wave interactions, the spectral action density $N(\underline{k};\underline{x},t)$ is conserved. In a more general formulation, a source term $S(\underline{k};\underline{x},t)$ accounts for the net rate of transfer of wave action to the component $\underline{k}(\underline{x},t)$:

$$\frac{\partial N}{\partial t}+\frac{\partial N}{\partial x_\alpha}\frac{dx_\alpha}{dt}+\frac{\partial N}{\partial k_\alpha}\frac{dk_\alpha}{dt}=S \tag{3.5}$$

This balance equation can be transformed from wave-number space to the space of frequencies ω and directions θ, using the dispersion Equation 3.1, but these equations are not reproduced here. theta

In the nearshore zone, the dominant contribution to the source term S is due to wave breaking. Although the overall energy dissipation rate associated with random-wave breaking in shallow-water can be calculated realistically (Battjes and Janssen (19), Thornton and Guza (20), Battjes and Stive (21)), its spectral distribution is not known. Yamaguchi (22), who utilizes a two-dimensional spectral action density balance in (ω,θ), approaches the problem of wave breaking through the use a saturation level for the equilibrium range given by Kitaigorodskii et al (23).

The results presented by Yamaguchi are realistic for the wave field as well as for the induced currents. Nevertheless, some reservation is appropriate. The spectral saturation model given by Kitaigorodskii et al applies to an actively wind-driven wave field where the high-frequency spectral tail is in equilibrium between wind input, nonlinear transfer and dissipation; this dissipation is mainly due to white capping. The sea state is supposed to be spatially homogeneous, or at most varying so slowly that it can be regarded as quasi-homogeneous. This contrasts strongly with the situation in the nearshore zone, where depth gradients dominate the wave evolution. Also, shallow-water wave breaking on a sloping bottom is rather different from white capping in a wind-driven, homogeneous sea state. Altogether, the relevance of this approach to the nearshore zone is questionable at best for wind-driven waves. For swell, this approach does not apply at all.

Narrow Band Approximations. A practical drawback of a fully two-dimensional spectral formulation is the considerable computational effort required. An approximation, which in many cases is capable of yielding realistic results, is to assume a narrow spectral distribution of wave action over the frequencies, so that the frequency-variation of parameters in the wave propagation model such as wave number and group velocity can be neglected. The effects of a small but nonzero frequency bandwidth are then taken into account analytically through the theoretical statistical properties of the wave field in time (to lowest order Gaussian instantaneous values and Rayleigh amplitudes and wave heights). This lumping in frequency therefore reduces the computation effort without oversimplifying to a monochromatic formulation. However, a similar reduction in the directions (to a unidirectional system) is not feasible in general because, in contrast with the lumping in frequency, results of a computation for a single incident direction cannot be generalized analytically so as to represent the effects of a small but finite angular bandwidth, since these depend on the depth (and current) field in the problem at hand. Moreover, in case of irregular bottom topography, unidirectional wave results soon lead to spatial distributions with unrealistically rapid variations, at least within the refraction approximation (neglecting diffraction). This is because the lateral amplitude gradient in a wave field propagating over irregular bottom topography is much stronger than the longitudinal one. A slight variation in angle of incidence can then lead to significant local amplitude variations.

A spectral wave action formulation for waves in shallow water, which is lumped in frequency but not in the directions, is given by Holthuijsen and Booij (24) (see Holthuijsen et al (25) for a more detailed description). The breaking-wave energy dissipation of Battjes and Janssen (19) has been incorporated in the model, using some ad hoc hypotheses about its spectral distribution. The model has been verified using laboratory data (Dingemans et al (26)) with good results as far as the prediction of the wave energy is concerned; the wave frequencies are not as well predicted. The model is in operational use for studies of nearshore circulation and coastal morphology (de Vriend and Ribberink (27)).

A further simplification is obtained if the narrow-banded incident wave action and energy are not only lumped into a single frequency (e.g. the peak frequency ω_p) but also into a single direction. This approach is feasible provided the bottom topography in the study area is not too irregular (see the discussion above). It is consistent with the use of a lumped, non-spectral representation of the dissipation due to random-wave breaking. The action balance equation for this case, assuming stationary conditions, reduces to

$$\frac{\partial}{\partial x_\alpha}\left(\left(C_{g\alpha}+U_\alpha\right)\frac{E''}{\sigma}\right)=-\frac{D}{\sigma} \tag{3.6}$$

in which E'' is the wave energy and D is the average rate of wave energy dissipation, both per unit horizontal area. Battjes and Janssen (19) present the following expression for D, based on a bore model for a shallow-water breaking wave:

$$D=\frac{\alpha}{8\pi}Q_b\rho g\bar{\sigma}_p H_m^2 \tag{3.7}$$

The coefficient α is of order one; σ_p is the peak intrinsic frequency; Q_b is the local fraction of breaking waves, which is calculated in the model as a function of the ratio of the (unknown) rms wave height to H_m, which represents a local depth-limited maximum wave height for non-breaking waves. In this formulation, the dissipation rate depends on the local rms wave height through Q_b.

The dissipation model represented by Equation 3.7 has been extensively calibrated (with respect to an empirical coefficient in H_m, assuming $\alpha = 1$) and verified with laboratory data and field data by Battjes and Stive (21). It appears to give realistic predictions of the rms wave height decay in random breaking waves, with relative errors typically less than 10%. The same applies to the formulation presented by Thornton and Guza (20), which is based on the same principles. These verifications were for conditions of essentially one-dimensional propagation. However, the formulation is not restricted thereto. Its implementation in two-dimensional models has likewise given good prediction of the rms wave heights, as noted above (Dingemans et al (26)).

Amplitude Domain Models. Goda (9) and Dally and Dean (10) present models for the calculation of random-wave transformation in the coastal zone utilizing the probability density function (pdf) of individual ("zero-crossing") wave heights p(H) (Goda) or the joint pdf of wave height and period p(H,T) (Dally and Dean). In these models, the incident-wave pdf is discretized and each element is transformed, including shoaling, breaking and post-breaking decay, as if it represented a periodic wave train. The results are recombined at each point of prediction so as to obtain a transformed pdf. These individual-wave approaches allow in principle the inclusion of certain nonlinear effects, e.g. in the shoaling (Goda), which are not included in the spectral formulations presented above. The formulations as originally presented are for one-dimensional (shore-normal) propagation only. It seems possible in principle to generalize them to two-dimensional propagation. This will require application of the pdf of directions θ of individual waves. Isobe(28) gives results for $p(H,\theta)$, the joint distribution of height and direction; no theoretical results seem to be available for $p(H,T,\theta)$, the joint distribution of height, period and directions of individual waves.

The individual-wave methods allow the estimation of the local wave height probability distribution, rather than merely the rms height or the mean wave energy. However, for the calculation of the wave-induced nearshore circulation, the mean wave energy dissipation rate is the primary wave-field property which is required (see section 5). It appears that this information can be obtained with sufficient accuracy from the semi-lumped spectral models or even the fully lumped models such as described above, which require far less computational effort.

4. VERTICAL CIRCULATION

Progressive waves carry a small mass transport of order E''/C towards the beach, where $C=\sigma/k$ is the phase speed of the wave. From an Eulerian viewpoint, this mass transport is concentrated between the trough and crest elevations. There can be no net mass flux through the beach and the wave-induced mass transport above the trough is largely balanced by a reverse flow or undertow below the trough. A similar vertical structure is present in the local on-offshore momentum balance, where the dominant terms are the vertically uniform hydrostatic pressure gradient arising from the setup $\bar{\eta}$ of the MWL from the global SWL, and the vertically nonuniform wave-induced momentum flux, the major part of which is concentrated in the trough-crest region.

Analysis of the vertical circulation has been based on the classical Reynolds equations, generalized to include separate wave and turbulent fluctuations. Under stationary conditions and longshore uniformity, the mass conservation equation is

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z} = 0 \tag{4.1}$$

and the x-momentum equation is

$$\frac{\partial}{\partial x}\left(\bar{u}^2\right) + \frac{\partial}{\partial z}(\bar{u}\bar{w}) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial x}\left(\bar{\tau}_{xx} - \rho\overline{u'^2} - \rho\overline{u''^2}\right)$$

$$+\frac{1}{\rho}\frac{\partial}{\partial z}\left(\bar{\tau}_{xz} - \rho\overline{u'w'} - \rho\overline{u''w''}\right) \tag{4.2}$$

rho
tau

The z-momentum equation additionally includes the gravitational acceleration.

The natural length scales in the horizontal and vertical directions are $1/k$ and h respectively. As kh is generally small in the nearshore zone, boundary layer arguments are commonly utilized. This involves an understanding that the flow pattern is slowly varying in the horizontal but potentially rapidly varying in the vertical. Further, distinctly different physical processes are observed to be active at the bed (an oscillatory boundary layer over a mobile bed) and at the free surface (wave-induced mass transport and wave breaking). It is accordingly convenient to envisage a three layer structure in the vertical, a narrow bottom boundary layer, a wider middle layer extending to the wave trough level and an upper layer extending from the trough to the crest. These layers are coupled by mass and momentum transfer across the interfaces, the elevations of which are slowly-varying with horizontal position. In principle, each of these layers can be considered independently provided that the appropriate interfacial boundary conditions can be specified.

Analyses of the middle and bottom layers in the nearshore zone (Svendsen (29), Dally and Dean (30), Stive and Wind (31), Stive and de Vriend (32), Svendsen et al (33)) have much in common with analyses of viscous effects on non-breaking wave propagation (Longuet-Higgins (34), Craik (35)). The basis is frequently a reduced form of the x-momentum equation, namely

$$0 = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial x}\left(-\rho\overline{u'^2} - \rho\overline{u''^2}\right) + \frac{\partial}{\partial z}\left(\nu_t \frac{\partial \overline{u}}{\partial z}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(-\rho\overline{u''w''}\right) \qquad (4.3)$$

nu

The omission of the convective acceleration and viscous shear terms follows from the turbulent boundary layer analogy; these terms are generally small and are rarely significant. The turbulent normal stress $-\rho\overline{u''^2}$ is zero for non-breaking waves.

The major assumption in Equation 4.3 is the Reynolds stress closure hypothesis for the turbulent shear stress $-\rho\overline{u''w''}$. In common with other turbulent shear flows, the turbulent shear stress is related directly to the mean flow through the eddy viscosity approach. The nature of the present flow however is rather different from classical turbulent shear flows, where the turbulent eddy viscosity ν_t would be defined by established zero, one or two equation turbulence models (Rodi (36)). Firstly, the mean flow is typically an order of magnitude smaller than the amplitude of the fluctuating wave motion. Secondly, turbulent vorticity is primarily generated by gravitational instability at the free surface rather than by viscous shear at the bed. Zero equation turbulence models, in which the adopted velocity and length scales of the turbulence are related to the wave motion (Stive and Wind (31), Svendsen et al (33)) have found most favor in the present context.

The time-averaged pressure $\overline{p}$ is available from integration of the z-momentum equation from elevation z to the crest. In a manner consistent with the boundary layer analogy, the leading terms in the vertical pressure profile are

$$\overline{p}(z;x) = \rho g\overline{(\eta - z)} - \rho\overline{w'^2} - \rho\overline{w''^2} \qquad (4.4)$$

eta

where x variation is implicit in all terms. The gravitational term is hydrostatic, $\rho g(\overline{\eta}-z)$, below the trough, such that the x-momentum equation for the middle and bottom layers becomes

$$0=-g\frac{\partial\overline{\eta}}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial x}\left(-\rho\overline{u'^2}-\rho\overline{u''^2}+\rho\overline{w'^2}+\rho\overline{w''^2}\right)+\frac{\partial}{\partial z}\left(\nu_t\frac{\partial\overline{u}}{\partial z}\right)+\frac{1}{\rho}\frac{\partial}{\partial z}\left(-\rho\overline{u''w''}\right) \quad (4.5)$$

Both terms involving horizontal gradients have little significant vertical structure in the middle layer. The setup term has none by definition and steady wave theory predicts a uniform distribution for the total wave-induced apparent normal stress term $-\rho(\overline{u''^2}-\overline{w''^2})$ below the trough. Measurements of turbulent apparent (i.e. Reynolds) normal stresses in the nearshore zone (e.g. Nadaoka and Kondoh (37)) do show some variation with depth but they are an order of magnitude smaller than the corresponding wave-induced apparent stresses (Sobey and Thieke (38)). The final term in Equation 4.5, the wave-induced analogue of the turbulent shear stress term, is certainly significant in the bottom boundary layer but it is assumed not to be influential in the middle layer. Steady wave theory in fact predicts that $-\rho\overline{u''w''}$ is zero in the middle layer. These assumptions reduce Equation 4.5 for the middle layer to an equation of the form

$$0=F(x)+\frac{\partial}{\partial z}\left(\nu_t\frac{\partial\overline{u}}{\partial z}\right) \quad (4.6)$$

where there is z variation only in the turbulent shear stress term. For fixed x, this is a second order ordinary differential equation in $\overline{u}(z)$ which may be solved by classical analytical or numerical means, depending on the z variation of the eddy viscosity and the boundary conditions. Mathematical solutions of the reduced equations for the middle layer (Svendsen (29), Dally and Dean (30), Stive and Wind (31)) follow directly from specification of the boundary conditions and the solution is seen to be crucially dependent on these boundary conditions. There is a strong empirical element in presently available estimates of these boundary conditions and the value of possible solutions for the middle layer must be similarly judged.

It is appropriate at this stage to reflect on the mathematical character of the problem as presently posed. It is assumed that the local wave field is given, so that the wave-induced apparent stresses and the trough elevation are known. It is further assumed that the bottom boundary layer is thin and that the lower interfacial elevation can be taken as the bed elevation. Given an appropriate turbulence closure model to define the eddy viscosity, there remain three unknowns, namely the horizontal velocity $\bar{u}(z;x)$, the vertical velocity $\bar{w}(z;x)$ and the local MWL $\bar{\eta}(x)$. There are two equations, Equations 4.1 and 4.5 (or 4.6), of which the momentum equation is second order so that three boundary conditions must be specified to uniquely define the solution. A common implicit assumption (Svendsen (29), Dally and Dean (30), Stive and Wind (31), Svendsen et al (33)) has been the consideration of the setup gradient as a given quantity, prior to the solution of the momentum equation. In fact the setup gradient must be determined from the depth-averaged momentum equation (Longuet-Higgins and Stewart (39)) or from simultaneous solution of coupled middle and upper layers. The three necessary conditions, of which two are associated with the momentum equation, must be drawn from mass and momentum transfer across the lower and upper interfaces. Interfacial boundary conditions in terms of the velocity component across the interface $\bar{q}_n$ together with the velocity component along the interface $\bar{q}_s$ and the shear stress along the interface $\bar{\tau}_s$ are appropriate.

The boundary conditions at the trough interface are clearly crucial to the analysis as they represent the influence of the trough-crest region that is predominantly responsible for driving the vertical circulation. There is a strong velocity shear across this trough level so that conditions in terms of interfacial shear stress and normal velocity would seem most appropriate. These interfacial conditions provide the coupling with the upper layer; they appear as boundary conditions for the local conservation equations and as explicit terms in the integral conservation equations for the both layers. The integral mass conservation equation for the upper layer is

$$\frac{d}{dx}\int_{\eta_{tr}}^{\eta_c} \rho\bar{u}\,dz - \rho\bar{q}_n = 0 \qquad (4.7)$$

The integral mass flux in the upper layer for non-breaking waves is of order E''/C but this flux is enhanced in breaking waves conditions (Svendsen (40)) by the "surface roller", a volume of water carried forward by the breaking wave at the phase speed. This leads to an estimate of the mass flux across the trough interface of the form

$$\rho\bar{q}_n = \frac{d}{dx}\left(\left(1+\Delta_1\right)\frac{E''}{C}\right) \qquad (4.8)$$ (u.c.)Delta

Svendsen gives an empirical estimate of Δ_1, the fractional increase in the total mass flux in the trough-crest region associated with the surface roller.

Similar layer integration of the x-momentum equation (Phillips (15)) leads to an estimate of the interfacial shear stress of the form

$$\bar{\tau}_s = \left(1+\Delta_2\right)\frac{D}{C} \qquad (4.9)$$

where D is again the total energy dissipation rate in the breaking wave and the Δ_2 fractional increase or decrease represents that part of the breaking wave dissipation that is located below the trough, together with any contribution that might be associated with the enhanced mass flux in the trough-crest region (Stive and Wind (31)) and with viscous attenuation (Craik (35)). For irrotational, non-breaking waves, both $\bar{q}_n$ and $\bar{\tau}_s$ at the trough interface are zero.

Shear is not strong by definition at the top of the bottom boundary layer and a $\bar{q}_s$ condition is appropriate there. In the classical boundary layer manner, an outer irrotational flow in the middle layer would be decoupled from the boundary layer flow and the rotational flow in the bottom boundary layer would in turn be driven by the outer flow. This is not entirely the case in the present situation as the outer flow is rotational and is not decoupled from the boundary layer flow. Classical boundary layer solutions have nonetheless been a popular basis for definition of the bottom boundary condition, in particular the solution for the oscillatory laminar boundary layer on a rigid horizontal bed under progressive Airy waves (Longuet-Higgins (34), Hunt and Johns (41)). This solution predicts that the thickness of the bottom boundary layer is indeed narrow. At second order in the Stokes expansion, it also predicts the existence of a finite mean flow velocity at the edge of the boundary layer, which is given by

$$\bar{u}(z \cong -h) = \frac{3\omega}{4k} \frac{(ka)^2}{(\sinh^2 kh)} \tag{4.10}$$

This is termed the Eulerian streaming or induced streaming velocity. This mean flow velocity is directed in the direction of wave propagation; it is conveniently independent of viscosity and provides an equally convenient bottom boundary condition for the flow in the middle layer. The convenience of this Eulerian streaming as a bottom boundary condition has perhaps been an inducement to overlook the reality of the flow in the bottom boundary layer under progressive waves approaching a beach. For a turbulent boundary layer under second order Stokes waves and Stokes first definition of phase speed, the Eulerian streaming is generally smaller in magnitude than the above estimate and follows the direction of wave propagation only in deep water; it passes through zero and reverses in direction in shallower water (Trowbridge et al (42)). When the additional complexities of a mobile sloping bed and a return current are also introduced, it is evident that both the magnitude and the direction of the Eulerian streaming velocity is far from being resolved, and that its use as a lower boundary condition on $\bar{u}$ in the middle layer is not justified. Coupled layer models (e.g. Svendsen et al (33)) provide a more rational basis for definition of the interfacial velocity.

As a closing comment, it is noted that the flow in the middle layer has received far more attention in the literature than the flows in the upper and lower layers. This is perhaps counterproductive in view of the fact that major physical interest in the vertical circulation should center on the surface layer (with regard to mass transport and wave breaking) and on the bottom layer (with regard to sediment transport and boundary shear).

5. HORIZONTAL CIRCULATION

Discussions of horizontal circulation patterns in the classical context of the long wave equations (astronomical tides, storm tides, fresh water flood waves) focus extensively on depth-averaged flow velocities and this approach is again useful for wave-induced horizontal motions in the nearshore zone. (Note however that there was little value in a depth-averaged analysis of the vertical circulation where the depth-averaged flow is zero.) The driving force for depth-averaged motions in the nearshore zone is provided by spatial gradients in the time- and depth-averaged excess momentum flux associated with the fluctuating motion (waves + turbulence). Cross-shore gradients are largely balanced by pressure forces which result in set up (or set down) of the

MWL and no current. Such is not the case in the longshore direction where any excess momentum flux will potentially drive a current.

Conservation Equations. The time and depth averaged stationary circulation is described by the time and depth averaged conservation equations (Phillips (15)) for mass

$$\frac{\partial}{\partial x_\alpha}\left(U_\alpha(h+\bar{\eta})\right)=0 \tag{5.1}$$

for momentum

$$\frac{\partial}{\partial x_\beta}\left(U_\alpha U_\beta\right)=-g\frac{\partial\bar{\eta}}{\partial x_\alpha}-\frac{1}{\rho(h+\bar{\eta})}\frac{\partial S_{\alpha\beta}}{\partial x_\beta}-\frac{\bar{\tau}_{b\alpha}}{\rho(h+\bar{\eta})} \tag{5.2}$$

and for fluctuating energy

$$\frac{\partial}{\partial x_\alpha}\left(U_\alpha E+F_\alpha\right)+S_{\alpha\beta}\frac{\partial U_\beta}{\partial x_\alpha}=-\epsilon+U_\alpha\bar{\tau}_{b\alpha} \tag{5.3}$$

epsilo

where the $S_{\alpha\beta}$ are the excess momentum fluxes or radiation stresses associated with the fluctuating motion, the F_α are the vector components of the fluctuating energy flux and ϵ is the rate of energy dissipation per unit area by molecular viscosity.

The relative simplicity of these conservation equations and the similarity to the long wave equations is achieved through the definition of the depth-averaged velocity U_α in terms of the total mass flux, such that

$$\rho(h+\bar{\eta})U_\alpha=\rho\overline{\int_{-h}^{\eta}u_\alpha dz}=\int_{-h}^{\eta_c}\bar{u}_\alpha dz \tag{5.4}$$

where $\eta(t)$ is the instantaneous water level and η_c is the crest level. Current, as distinct from the depth-averaged velocity U_α, is identified with the time-averaged flow velocity $\bar{u}_\alpha$ below the trough level. Between the trough and the crest level η_c, the current interacts with the mass flux associated with the wave in a continuous manner to define a mean flow $\bar{u}_\alpha(z)$ profile that peaks near the MWL. There may accordingly be potential value in a two layer approach with an interface at the trough level and layer-averaged rather than depth-averaged conservation equations (Thieke and Sobey (43)). The following discussion will nonetheless focus on the depth-averaged equations, although the general principles are equally applicable to layer-averaged equations.

For the purposes of a detailed consideration of the complete conservation equations, it is necessary to identify the separate turbulent (single prime superscript) and wave (double prime superscript) components of all terms associated with the fluctuating motion. The mass conservation equation (Equation 5.1) remains unchanged, the momentum equations become

$$\frac{\partial}{\partial x_\beta}\left(U_\alpha U_\beta\right) = -g\frac{\partial\bar{\eta}}{\partial x_\alpha} - \frac{1}{\rho(h+\bar{\eta})}\frac{\partial S'_{\alpha\beta}}{\partial x_\beta} - \frac{1}{\rho(h+\bar{\eta})}\frac{\partial S''_{\alpha\beta}}{\partial x_\beta} - \frac{\bar{\tau}_{b\alpha}}{\rho(h+\bar{\eta})} \qquad (5.5)$$

and the fluctuating energy equation becomes

$$\frac{\partial}{\partial x_\beta}\left(U_\alpha E' + U_\alpha E'' + F'_\alpha + F''_\alpha\right) + S'_{\alpha\beta}\frac{\partial U_\beta}{\partial x_\alpha} + S''_{\alpha\beta}\frac{\partial U_\beta}{\partial x_\alpha} = -\epsilon + U_\alpha\bar{\tau}_{b\alpha} \qquad (5.6)$$

It is convenient further to separate the wave and turbulent components of the fluctuating energy equation, which requires the introduction of a wave-turbulent interaction term D representing dissipation of wave energy to turbulence in the breaking process and simultaneously (Battjes (44)) production of turbulent energy from the same breaking process. The turbulent energy equation becomes

$$\frac{\partial}{\partial x_\alpha}\left(U_\alpha E' + F'_\alpha\right) + S'_{\alpha\beta}\frac{\partial U_\beta}{\partial x_\alpha} - D = -\epsilon + U_\alpha\bar{\tau}_{b\alpha} \qquad (5.7)$$

and the wave energy equation

$$\frac{\partial}{\partial x_\alpha}\left(U_\alpha E'' + F''_\alpha\right) + S''_{\alpha\beta}\frac{\partial U_\beta}{\partial x_\alpha} = -D \tag{5.8}$$

It is clear from the above conservation equations that wave-current interactions, turbulence-current interactions and wave-turbulence interactions all have potential relevance to the prediction of the horizontal circulation. As a consequence of these interactions, the wave and current fields must in principle be computed simultaneously. For clarity of presentation, the equations for the wave field including the influence of a current have been dealt with separately in section 3 of this chapter. (The wave action Equation 3.6 is in fact equivalent to the wave energy Equation 5.8 if the kinematic wave equations are also taken into account.)

Longshore Currents. The broad features of the longshore current are particularly simple in the case of longshore uniformity (Bowen (45), Longuet-Higgins (46)). The longshore (x_2 or y) momentum balance reduces to

$$0 = -\frac{\partial S'_{yx}}{\partial x} - \frac{\partial S''_{yx}}{\partial x} - \bar{\tau}_{by} \tag{5.9}$$

The onshore gradient of the excess onshore flux of longshore momentum is significant only within the surf zone. This forcing is balanced by the longshore component of the bed shear. Simple models of the forcing in terms of breaking wave dissipation and of the bed shear in terms of the longshore velocity lead to predictive equations for $U_2(x_1)$ or $V(x)$ that increase from zero at the beach to a maximum value somewhere shoreward of the breaker line, decreasing back to zero further seaward.

Two-Dimensional Circulation. While the long uniform beach schematization is a useful artifice to introduce the broad features of the longshore current, the general horizontal circulation requires consideration of the complete conservation equations. All spatial gradients of the wave-induced excess momentum flux tensor $S''_{\alpha\beta}$ potentially contribute to forcing. Headlands and coastal structures such as groynes and jetties additionally impose no-flow constraints and locally tangential flow along the structure, in either the on- or offshore direction. In this general situation, the requirements of mass conservation will lead to nearshore cells of horizontal circulation. Vortex stretching by increasing depth (Arthur (8)) generally leads to a narrowing and intensification of the offshore flow and the reverse pattern for the onshore flow. $S''_{\alpha\beta}$

Circulation cells are also a common feature of the nearshore flow pattern on apparently long straight beaches under normal or near-normal wave incidence (Sonu (47)). The broad features of rip current formation and the associated counter-rotating gyres was initially demonstrated by Bowen (48). For a normally-incident wave field with a small spatially-periodic longshore variation in wave height, longshore currents were generated within the surf zone from each longshore maximum to each longshore minimum in the wave height. An offshore (rip) current was located at each wave height minimum and each gyre was completed by a weak counter-current outside the surf zone and a broad return flow at each wave height maximum. Just what initiates the longshore spatial periodicity is uncertain. Perhaps there is some fundamental instability in the nearshore hydrodynamics or even morphodynamics (Battjes (49)).

Wave Forcing. While all gradients of $S''_{\alpha\beta}$ contribute to the forcing, it follows from Kelvin's circulation theorem that only the rotational part will contribute to the generation of currents (Longuet-Higgins (50)). The wave field beyond the surf zone is well described by irrotational flow theory where the circulation and hence the current is zero. Dissipation from wave breaking and bottom friction will lead to vorticity $S''_{\alpha\beta}$

generation, finite circulation and to the generation of currents. A useful distinction can accordingly be made between the rotational and irrotational parts of the forcing:

$$\frac{\partial S''_{\alpha\beta}}{\partial x_\beta} = \frac{\partial S''_{\alpha\beta}}{\partial x_\beta}\Big|_{rot} + \frac{\partial S''_{\alpha\beta}}{\partial x_\beta}\Big|_{irrot} \tag{5.10}$$

The irrotational part will not drive currents and must be balanced by the set-up term (pressure gradients) in the momentum equation, such that

$$0 = -g\frac{\partial\bar{\eta}}{\partial x_\alpha} - \frac{1}{\rho(h+\bar{\eta})}\frac{\partial S''_{\alpha\beta}}{\partial x_\beta}\Big|_{irrot} \tag{5.11}$$

The residual momentum equation is then

$$\frac{\partial}{\partial x_\beta}\left(U_\alpha U_\beta\right) = -\frac{1}{\rho(h+\bar{\eta})}\frac{\partial S'_{\alpha\beta}}{\partial x_\beta} - \frac{1}{\rho(h+\bar{\eta})}\frac{\partial S''_{\alpha\beta}}{\partial x_\beta}\Big|_{rot} - \frac{\bar{\tau}_{b\alpha}}{\rho(h+\bar{\eta})} \tag{5.12}$$

The setup field would be predicted from the Equation 5.11 and the horizontal circulation from Equation 5.12. The hydrostatic balance involving the setup term is a significant part of the complete momentum balance and its decoupling from the overall momentum balance has considerable numerical advantage for the computation of the current field. Under assumptions of no current, no diffraction and Airy wave theory (Longuet-Higgins (50), Dingemans et al (51)), the irrotational and rotational parts of the forcing are respectively

$$\frac{\partial S''_{\alpha\beta}}{\partial x_\beta}\Big|_{irrot} = (h+\bar{\eta})\frac{\partial}{\partial x_\alpha}\left(\frac{1}{h+\bar{\eta}}\left(\frac{C_g}{C}-\frac{1}{2}\right)E''\right) \tag{5.13}$$

$$\frac{\partial S''_{\alpha\beta}}{\partial x_\beta}\Big|_{rot} = -\frac{D}{\sigma}k_\alpha \tag{5.14}$$

Dingemans et al (51) show further that diffraction effects have little influence on the validity of Equations 5.13 and 5.14 but that ambient currents may be influential. In a nonlinear approximation, a similar separation is possible in principle but the appropriate quantitative expressions have yet to be determined.

Simultaneous computation of both the setup and current field of course remains a possibility but it requires some care in discrete numerical models. The radiation stresses $S''_{\alpha\beta}$ $S''_{\alpha\beta}$ and hence $\partial S''_{\alpha\beta}/\partial x_\beta$ may be determined from the computed wave $\partial S''_{\alpha\beta}/\partial x_\beta$ field. The discrete numerical differentiation may result in spurious discontinuities in the forcing and hence in the computed response. In its present form, Equation 5.14 avoids the numerical differentiation and provides a direct estimate of the rotational part of the wave-induced forcing.

Turbulence Modeling. The inevitable "Reynolds stress closure" problem here relates specifically to the turbulent momentum transport terms $S'_{\alpha\beta}$ in the momentum and fluctuating energy $S'_{\alpha\beta}$ equations and to the bed shear stress $\bar{\tau}_{b\alpha}$ in the momentum equations. The relative success of turbulence modeling for turbulent shear flows may eventually translate to the nearshore circulation and some progress has already been made. Modeling of $S'_{\alpha\beta}$ in the momentum equation and in the turbulent energy equation has generally adopted the depth-averaged eddy viscosity approach (Rodi (36)) where

$$S'_{\alpha\beta} = \rho\nu_t(h+\bar{\eta})\left(\frac{\partial U_\alpha}{\partial x_\beta}+\frac{\partial U_\beta}{\partial x_\alpha}\right)-\frac{2}{3}E'\delta_{\alpha\beta} \qquad (5.15)$$ i.e. delta

together with zero, one or two equation turbulence models for the eddy viscosity ν_t. Note that eddy viscosity here relates to the horizontal transfer of horizontal momentum rather than the vertical transfer of horizontal momentum as it does in the section on vertical circulation.

In zero equation models, ν_t is related to characteristic velocity and length scales of the turbulence. Turbulence generated in the bottom boundary layer is represented by the $\bar{\tau}_{b\alpha}$ term. Here we focus on turbulence generated in wave breaking, where an appropriate velocity scale (Battjes (44)) is $(D/\rho)^{1/3}$, leading to the following estimate of the turbulent eddy viscosity:

$$\nu_t = M\left(\frac{D}{\rho}\right)^{1/3} L \tag{5.16}$$

where M is a constant of order one. An appropriate length scale L might be either the wave height H or the water depth h (strictly h + $\bar{\eta}$). H and h are of the same order in the surf zone, but H may be a more convenient choice to accommodate conditions outside the surf zone. Alternative zero equation models have been used by Bowen (48), Ebersole and Dalrymple (52), Watanabe (53) and Wu and Liu (54).

In one equation models, the characteristic velocity scale is defined as the square root of the depth-averaged turbulent kinetic energy $k = E'/\rho(h+\bar{\eta})$. E' is determined from the turbulent energy equation, Equation 5.7, which nonetheless remains dominated by the wave energy dissipation. Such one equation models have been used by Visser (55) and O'Connor and Yoo (56).

In two equation or $k-\epsilon$ models, the eddy viscosity is related to k^2/ϵ where k is again determined from the turbulent energy equation and ϵ from a separate flux equation for ϵ. Two equation models have been used by Wind and Vreugdenhil (57) and Yoo and O'Connor (58). Such models rely very heavily on experience in turbulent shear flows, whereas zero and one equation models are more closely related to the wave breaking process which so clearly dominates turbulent mixing in the nearshore zone.

Bed Shear Stress. The final part of the Reynolds stress closure problem is the representation of the bed shear stress $\bar{\tau}_{ba}$. It is clear from Equation 5.5 that the bed shear is a crucial element in the prediction of the horizontal circulation, as it is principally through this term that the depth-averaged current appears in the momentum balance. The turbulent boundary layer beneath combined wave and current flow and the bed shear stress that is a consequence of this flow has been the subject of considerable attention but remains an unsolved problem. The reasons are abundantly clear from the complexity of the flow pattern, especially the contrasting time, length and velocity scales of the separate components where they exist alone. The time scales are very long (essentially steady) for currents and quite short (of order 10 s) for waves. The length scales, represented by the boundary layer thicknesses, are almost the full depth of water for currents and quite thin (of order cms) for waves. The velocity scales are small (< C/10) for the currents and somewhat larger (of order C) for the waves. Interaction of such contrasting scales results in a complex flow. Additional problems involve directional variations and especially a mobile bed.

Consideration of this boundary layer structure and the associated Reynolds stress closure leads to the familiar zero, one and two equation turbulence models which seek to determine the turbulent eddy viscosity for vertical turbulent momentum transfer, as briefly considered in the section on the vertical circulation. The present focus on depth-integrated conservation equations however directs attention away from the vertical structure towards a depth integrated closure model in the manner of the Chezy model for boundary shear in steady open channel flow or the Darcy-Weisbach model in steady turbulent pipe flow. Such integral closure models are typically written in quadratic form for the instantaneous shear stress as

$$\tau_{b\alpha} = \frac{f}{8}\rho|Q|Q_{\alpha} \tag{5.17}$$

where Q_a is an integral measure of the instantaneous flow velocity in the combined wave-current boundary layer and f is the Darcy-Weisbach friction-factor. It is clear that the friction factor must represent all influences not directly represented by Q and that a change in the definition of Q must be reflected by a change in the friction factor. A common definition of the integral measure of the combined flow is the vector sum of the depth-averaged current and the wave velocity predicted at the bed by the inviscid Airy wave theory, i.e.

$$Q_{\alpha} = U_{\alpha} + amp\left(u_b''\right)_{\alpha} \cos \omega t \tag{5.18}$$

where the amp() function is the amplitude of the oscillatory velocity. Time averaging of Equation 5.17 leads to an estimate of the time-averaged bed shear stress (Longuet-Higgins (46), Liu and Dalrymple (59)). The computational convenience of this model is deceptive however as it merely transfers the closure problem to the estimation of f. There is an expectation that the friction factor can be represented in the manner of the Moody diagram for pipe flow, as attempted initially by Jonsson (60) for waves alone. The subsequent literature is extensive, for example Bijker (61), Bakker and Van Doorn (62), Grant and Madsen (63), Christoffersen and Jonsson (64), Visser (65), O'Connor and Yoo (66) and Davies et al (67); see also Sleath (68). The general trends have been established in that the influence of the wave motion on the current has rough equivalence to the response of the current to a significant increase in the bed roughness. The details are far from complete however, especially considering the sensitivity of the longshore current to the bed shear. This would appear to be the weakest link in predictive models of the horizontal circulation.

6. THREE DIMENSIONAL CIRCULATION.

Separate consideration of the horizontal and vertical circulation has demonstrated the existence of dynamically significant mean flows in all three spatial dimensions. While the above separation is convenient for analysis purposes, it filters potentially significant motions from detailed consideration. Three-dimensional models of the nearshore circulation are a recent development and the present discussion attempts merely to hint at the direction of current research.

Two approaches are possible, the first being a natural extension of vertical circulation analyses. This fully three-dimensional approach seeks direct numerical solutions of the Reynolds equations in three spatial dimensions, suitably generalized to include wave-induced apparent stresses. The second approach is a natural extension of the horizontal circulation analyses and seeks to establish both the depth-averaged flow and the vertical structure that was ignored in horizontal circulation analyses. This approach, which can be classified as quasi-three-dimensional, utilizes the fact that the horizontal length scale in the nearshore circulation is far greater than the vertical one. A brief outline of the quasi-three-dimensional approach is given below. The rather lengthy mathematical expressions which are needed for a complete problem statement and analysis (see e.g. de Vriend and Stive (69), Svendsen and Lorenz (70)) are not reproduced here but are represented in highly schematic form.

The three-dimensional circulation is mathematically described by the extension of the local conservation equations for mass (Equation 4.1) and momentum (Equation 4.2) to three dimensions. In the quasi-three-dimensional approach, the contribution of the vertical turbulent transfer of horizontal momentum ($\rho\overline{u'w'}$) is the dominant term in the mean horizontal momentum balance. Modeling this with a turbulent eddy viscosity model in terms of the vertical gradient of mean horizontal velocity, the local balance of horizontal momentum (e.g. Equation 4.2 generalized to two horizontal dimensions $\underline{x}$) can symbolically be written as

$$\frac{\partial}{\partial z}\left(-\rho\overline{u'_\alpha w'}\right)=\frac{\partial}{\partial z}\left(\nu_t\frac{\partial\bar{u}_\alpha}{\partial z}\right)=R_\alpha(\underline{x},z) \tag{6.1}$$

R_α represents the sum of the remaining terms; it can be separated into a depth-averaged part, $F_\alpha(\underline{x})$ say, and a vertically varying part $G_\alpha(\underline{x},z)$ whose depth-averaged part is zero by definition. The same applies to the left hand side of Equation 6.1. In this manner, a so-called primary velocity $\bar{u}_{p\alpha}$ can be defined as the solution of

$$\frac{\partial}{\partial z}\left(\nu_t\frac{\partial\bar{u}_{p\alpha}}{\partial z}\right)=F_\alpha(\underline{x}) \tag{6.2}$$

supplemented with the mass balance and the vertical momentum balance. Likewise, a so-called secondary velocity $\bar{u}_s$ can be defined as the solution of

$$\frac{\partial}{\partial z}\left(\nu_t\frac{\partial\bar{u}_{s\alpha}}{\partial z}\right)=G_\alpha(\underline{x},z) \tag{6.3}$$

Equation 6.2 is a generalization to two horizontal dimensions of Equation 4.6, which is restricted to circulations in a shore-normal vertical plane. In this formulation, the primary velocity is associated with the depth-averaged equations (although it has a vertical structure) and the secondary velocity is associated with the residual equations.

Major sources of secondary flow are the vertical non-uniformities of wave-induced fluxes of mass and momentum and of advection of mean flow momentum. In addition, wind shear stress and Coriolis accelerations can have some influence. The resultant secondary velocity vector will generally vary with elevation both in magnitude and direction. Swirling flows are expected and observed (de Vriend and Stive (69), Svendsen and Lorenz (70)). Much of the detailed analysis parallels the developments of horizontal and vertical circulation models and does not need separate discussion here.

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LIST OF NOTATION

a	Wave amplitude
C	Wave phase speed
C_g	Wave group speed
D	Rate of dissipation of wave energy
E	Energy per unit area of fluctuating motion
F	Energy flux of fluctuating motion
f	Friction factor
g	Gravitational acceleration
H	Wave height
H_m	Depth-limited maximum wave height
h	Water depth to SWL datum
k	Wave number, or
	Depth-averaged turbulent kinetic energy
L	Length scale of turbulence
$N(\underline{k})$	Spectral action density in wave number space
p	Pressure
Q	Instantaneous velocity scale
Q_b	Local fraction of breaking waves
q	Velocity component at interface
S	Radiation stress, or
	Source term in spectral action balance equation

T	Wave period
t	Time
U	Depth- and time-averaged velocity
u	Local horizontal velocity
w	Local vertical velocity
x	Horizontal coordinate
y	Horizontal coordinate
z	Vertical coordinate
α	Coefficient
Δ	Fraction
δ	Symmetric unit tensor
ϵ	Rate of energy dissipation by viscosity
η	Water surface elevation from global SWL
η_c	Crest elevation
η_{tr}	Trough elevation
θ	Direction
ν_t	Turbulent eddy viscosity
ρ	Mass density of sea water
σ	Wave angular frequency relative to current (Intrinsic frequency)
τ	Shear stress

ω	Wave angular frequency (Absolute frequency)

Subscripts

b	Bed
n	Direction normal to interface
p	Spectral peak, or
	Primary flow
s	Direction along interface, or
	Secondary flow
x	x direction
y	y direction
z	z direction
α	Tensor subscript = 1 or 2
β	Tensor subscript = 1 or 2

Superscripts

$'$	Turbulent fluctuation, e.g. u'
$''$	Wave fluctuation, e.g. u''

Miscellaneous

tilde	Fluctuation about time-averaged value, e.g. $\tilde{u}$
overbar	Time-averaged value, e.g. $\bar{u}$
underline	Vector quantity, e.g. $\underline{x}$

SUBJECT INDEX