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Sea wave information from bathymetric LIDAR

One-dimensional approach

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Executive Summary

Detailed knowledge of sea bottom topography in shallow seas is of vital importance for coastal zone management, coastal erosion, environmental development of eco systems, operational guidance of shipping traffic fisheries and offshore activities.

Recently airborne laser bathymetry systems have come available, measuring the bottom topography with the help of remote sensing techniques.

In such a system bathymetric information is recovered by comparing two signals: laser pulses reflected against the sea surface and against the sea bottom. Obviously the water surface plays a crucial role in the detection of both reflection signals. Knowledge of the water surface can enable us to correct for a bias in depth and bottom position due to the slant and spread of the laser beam, improving the performance of the bathymetry system. Fortunately, the received signal of the laser pulses, reflected on the water surface, contains much information, from which wave information can be extracted. What is needed is an analysis method to do this.

Besides improving the performance of the bathymetry system, recovered wave information is valuable in itself. Operating as a wave gauge, the system retains the features of speed and coverage. This makes it a flexible measuring device that can be used in remote areas. For instance, such a system has been used for underflights of the Shuttle Imaging Radar (SIR-B) mission (Walsh et al. 1987).

In this report we give the results of a simulation study in which we develop and test techniques to extract valuable wave information from the laser signal, reflected on the water surface. As this is a first inventory, we use a one-dimensional wave field description and disregard any source of noise. The analysis techniques we test are arranged according to the complexity of the data required: in the simplest case we only count reflected flares, in the most elaborate technique we assume a time signal of reflected laser intensities in addition to which also the distance of the airborne system to the surface has been
measured.

The results of this study show the potential of a LIDAR system. Simple stochastic analysis methods yield estimates for statistical wave information as the significant wave height. With the help of the most elaborate technique the water surface can be reconstructed and the wave spectrum can be measured accurately.

This study also indicates requirements that the LIDAR system has to fulfil if it is to be used as a wave gauge. These requirements mainly concern the sampling frequency of the reflection signal and the laser footprint, i.e. the area of the laser beam on the surface. From the tests it appears that these two quantities strongly determine the quality of the wave measurements and need to be tuned carefully.

Of course, one has to keep in mind the simplicity of the simulation used here. The conclusions reached in this report are indicative and not conclusive. A comparison with other wave measuring devices is therefore not opportune.

This study is carried out by dr. C. Calkoen and conducted by ir. G.J. Wensink of Delft Hydraulics on commission of Rijkswaterstaat Tidal Waters Division.
1. Introduction

Airborne bathymetric laser systems have a great potential in revealing accurate sea bottom depths. In its basic form the method used to measure the depth goes as follows. A laser pulse, emitted from the airborne system, partly reflects on the water surface and partly refracts into the water and reflects on the sea bottom. Ideally the airborne detector receives two reflected laser pulses and determines the distance between the water surface and the sea bottom from the time lapse between the two pulses.

The presence of waves on the water surface makes bathymetric measurements complex. One important consequence is that a laser pulse, reflected on the water surface, only reaches the receiver when the local surface slope is directed towards the receiver. Because of this the received laser reflection signal typically takes the form of flares, bursts of reflected intensities.

The wave field is a source of errors for the bathymetric measurements. What is measured is the distance between the sea bottom and the local water surface level, which varies due to the waves. The error, introduced this way, averages out but generates an uncertainty of the order of the mean wave height. The slope of the water surface determines the angle and thus the path length of the pulse under water. The error due to this effect not only adds to the uncertainty, but also may bias the depth measurements (for a more detailed discussion see Billard et al. 1986).

If the form of the water surface is known, then it is possible to correct the bathymetric measurements, thereby improving the performance of the system. An efficient way to obtain information on the wave surface is to analyse the time signal of the received laser pulse, reflected on the water surface. It is clear that this signal contains much information on the waves; the problem is how to extract that information. This problem is the subject of the present study: which analysis methods are available, which wave parameters can be retrieved and how reliable is the result.
Besides improving the performance of the laser bathymetry system, recovered wave data has a value in itself. Operating as a wave gauge, the LIDAR system can measure in remote areas and cover a large area. For these two applications, bathymetry system and wave gauge, different wave information is required. To correct bathymetric measurements one is only interested in the surface elevation and the slope at the point where a laser pulse refracts into the water. In the wave gauge mode one would preferably recover the wave energy spectrum.

This research was conducted using a model simulation approach. In a computer program we start with a wave energy spectrum, from which we simulate a representative wave surface. Next we use a simple reflection model to simulate the laser signal (reflected on the water surface), as it is observed from the aeroplane. This reflected laser signal is the basis on which we apply subsequently five different analysis methods. Since the wave spectrum, from which we started, is known, we can directly compare the outcome of an analysis method with the original wave data. Thus we can simply evaluate the reliability of the technique.

In section 2 we introduce the basic parameters needed in the simulation part of the computer program. Here we justify the use of a two-scale wave simulation and discuss the effect of a moving aeroplane. In section 3 we give a detailed description how the large "carrier" waves are simulated and in section 4 we give a simple model for the reflection of the laser pulses on the wave surface. In paragraph 5 the computer simulation program itself is described.

In section 6 to 11 we introduce, test and evaluate several analysis methods to extract wave information from the reflected laser signal. These methods are:

- flare statistics (section 6)
- clipped signal approximation (section 7)
- reconstructed slope signal method (section 8)
- pulse run time statistics (section 9)
- reconstructed surface elevation method (section 10)
- mean width of the flares (section 11)

The flare statistics method has been studied in the literature (see Prokhorov 1987). The clipped signal approximation is borrowed from signal theory and adapted for use in this field. The other methods are developed for this study.
Finally, in paragraph 12 the conclusions and recommendations are written down.
2. Review of relevant parameters

To simulate the reflection of laser light on the surface of the sea we consider the following situation:
An aeroplane flies above the sea surface with a velocity \(v\).
Every \(\Delta t\) second it emits a laser pulse under an angle \(\psi\).
The reflected intensity of \(N\) subsequent pulses is recorded.

In this simulation we restrict ourselves to a one-dimensional analysis
and therefore we assume that the waves are represented by a one-
dimensional energy spectrum \(E(f)\). We further assume that all waves head
to the same direction (this is realistic for wind sea). The aeroplane
may fly upwind or downwind.

To generate a random wave surface from the spectrum \(E(f)\) we use a
non-deterministic spectral amplitude model. For a time series of the
surface elevation \(\eta\) at \(x=0\) we have

\[
\eta(0,t) = \text{Re} \left\{ \sum_{n=1}^{N/2} c_n \exp(-i\omega_n t+i\phi_n) \right\}.
\]

(1)

Here \(\phi_n\) is a random phase, \(c_n\) is a frequency bin amplitude proportional
to \(\sqrt{E(f_n)}\), and \(\omega_n = 2\pi f_n\). Formula (1) can be used as the initial
condition for the wave equation to find the surface elevation at all \(x\)
(using a linear approximation):

\[
\eta(x,t) = \text{Re} \left\{ \sum_{n=1}^{N/2} c_n \exp(ik_n x - i\omega_n t + i\phi_n) \right\},
\]

(2)

with

\[
\omega_n^2 = gk_n \tanh(k_n h),
\]

(3)
g being the gravitational acceleration and \(h\) the local depth.

An important parameter for the simulation is the length of the laser
footprint LF. Since the reflected intensity of the laser pulse is averaged over the footprint, we use the following notation

\[
<F(x,t)> = \frac{1}{LF} \int_{x-1/2LF}^{x+1/2LF} F(x',t) \, dx'
\]  \hspace{1cm} (4)

for the mean value of \(F(x,t)\). It is natural to make a distinction in this averaging between long waves, that vary hardly over the footprint and waves with a length much smaller than LF, which average out and only increase the noise level. Such a distinction results in a two-scale model. We need to make a choice where the dividing line between long and short waves lies. To that end we introduce limiting wave number \(k_f\):

\[
k_f = 1 / LF,
\]  \hspace{1cm} (5)

and a matching \(\omega_f\) and \(f_f\). Equation (2) can now be split into a long and a short part: \(\eta(x,t) = \eta_l(x,t) + \eta_s(x,t)\). In \(\eta_l\) the summation over \(n\) runs from 1 to \(f\) and in \(\eta_s\) from \(f+1\) to \(N\). In a two-scale model the actual choice for the partition between the long and the short scale should not be sensitive. In later sections we shall see that the analysis of the lidar signals is very sensitive for the value of \(k_f\). This means that in practice another choice, say \(k_f = 2 / LF\), may give better results. However, such a change is not important for for the methodology and the conclusions of this report. What is important in the simplified model that we employ, is the physical effect that waves, shorter than the footprint cannot be measured with the lidar system.

We use the separation in long and short waves to approximate the reflectivity of the water surface, averaged over the footprint. The specular reflection of the laser pulse is a function of \(\theta(x,t) - \psi_1\), the angle of the water surface minus the angle of the laser beam. Since the water surface angles are small, we may separate a long-wave and a short-wave part: \(\theta(x,t) = \theta_l(x,t) + \theta_s(x,t)\). The long-wave angle \(\theta_l\) is the average tilt of the water surface in the footprint, while \(\theta_s\) represents a modulation of this tilted surface. The mean value of the reflexivity \(R < R [\theta(x,t) - \psi_1] >\) can be approximated as \(R [\theta_l(x,t) - (\psi_1 - \theta_l(x,t))] >\), with \(\theta_l\) a constant for the averaging and \(\psi_1 - \theta_l\) the effective incidence angle. A consequence of this approximation is that the modulation of the lidar signal, the received intensity flares, only depends on \(\theta_l\). The effects of the short oscillations can be represented in the reflection
model. For that reason only the long waves will be simulated explicitly in the model. The wave spectrum will be cut off at $\omega_r$.

In the simulation we have not yet included the velocity $v$ of the aeroplane. This can easily be resolved with the help of a Galileo transformation. Suppose that the plane starts at $t=0$ at the point $x=0$. Then the aeroplane 'sees' the waves $\eta(vt,t)$. From eq. 2 we find

$$\eta(vt,t) = \text{Re} \sum_{n=1}^J c_n \exp(-i(\omega_n v - k_n x) t + i\phi_n),$$

(6)

taking the summation only over the long waves. Thus we can use a simple time series with Doppler-shifted frequencies.

The strategy we adopt in the simulation follows directly from the discussion above. We start from a (JONSWAP) spectrum and cut it off at the limiting footprint frequency $\omega_r$. From this spectrum, transformed to the rest frame of the aeroplane we simulate the long waves as a time series with step $\Delta t$ and record the angle of the surface. At each simulated point the reflected intensity is computed with the help of a simple model for the reflectivity. The resulting lidar signal or flare signal will be the basis for the analysis methods in the sections 6–12.

From the analysis so far we can already find a suitable value for the sample time $\Delta t$. The Nyquist frequency $f_c$, associated with $\Delta t$, is equal to:

$$f_c = \frac{1}{2 \Delta t}.$$  

(7)

This is the highest frequency that can be observed from the time series. On the other hand, the largest wave number that can be measured, because of the finite length of the laser footprint, is of the order of $\pi/L$. Its angular frequency, observed from the aeroplane, is approximately equal to $\pi v/L$, according to eq. 6. Equating its frequency $v/(2L)$ to the Nyquist frequency we find as a suitable sample time

$$\Delta t = \frac{LF}{v}.$$  

(8)

Sampling with a shorter time step gives information on frequency bins
which cannot be measured properly with the used laser technique. Sampling with a longer time step misses a measurable part of the spectrum.
3. simulating the carrier waves

The large waves \((k \leq 1/L)\) are simulated with the help of the Fourier series

\[
\eta_m = \text{Re} \sum_{n=1}^{N/2} c_n e^{i \Omega n t_m},
\]

with \(\eta_m\) the surface elevation \((m = 1, 2, \ldots, N)\) and \(t_m = m \Delta t\). The angular frequencies \(\Omega_n\) are given by: \(\Omega_n = (2\pi n)/(N \Delta t)\). The complex amplitudes \(c_n\) are connected to the frequency spectrum, but to frequencies in the rest frame of the sea. So for a given \(\Omega_n\) we first have to solve

\[
\omega_n - v_k = \Omega_n
\]

in which \(\omega_n\) is related to \(k_n\) as given in eq. 3. Negative values of \(v\) apply for upwind flights, positive values for downwind flights.

The actual computation of a bin amplitude from the power spectrum depends on the (non-standardized) normalization factor in the Fourier transform. In our computer simulation program we adhere to the form

\[
c_n = \frac{1}{N} \sum_{m=0}^{N-1} \eta_m e^{2\pi i mn/N}.
\]

Then we have the relation with the spectrum \(E:\)

\[
E(0) \Delta f_o = :c_0^2 = 0
\]

\[
E(f_k) \Delta f_k = :c_k^2 + \sum_{N-k}^k :c_{-k}^2\quad k=1,2,\ldots(N/2-1)
\]

\[
E(f_{N/2}) = :c_{N/2}^2
\]

with \(f = \omega/2\pi\). The frequency increment, \(\Delta f_k = f_{k+1} - f_k\), in the rest frame is related to the frequency increment in the frame, moving with the aeroplane, \(\Delta F_k = (\Omega_{k+1} - \Omega_k)/2\pi = 1/(N \Delta t)\), as

\[
\Delta f_k = \frac{d\omega}{d\Omega} \quad \Delta F_k = \frac{c_g}{c_g + v} \frac{1}{N \Delta t}.
\]

In this equation \(c_g = d\omega/dk\) is the wave group velocity. We assume that the mean water level is normalized to zero, so that we can take \(E(0) = 0\).
0. As a check of eq. 11 we note that the total energy is conserved as it should be:

\[ \int_0^C E(f) df \approx \sum_{n=0}^{N/2} E_n \Delta f = \sum_{n=0}^{N-1} \left| c_n \right|^2 = \frac{1}{N} \sum_{n=0}^{N-1} \left| \eta_n \right|^2 \approx \frac{1}{T} \int_0^T \left| \eta(t) \right|^2 dt \]

Assuming that \( c_n = c_{N-n} \); equation 11 allows us to find the average bin amplitude from the power spectrum. However, as noted by Tucker (1984), applying this mean value directly in eq. 9 would give wrong higher order statistics (wave groups etc.). This is because \( c_n \) has a Rayleigh distribution with rms value equal to itself. To correct for this the simulation program has the option to compute the average value of \( c_n \), give it a random Gaussian deviate and finally a random complex phase. In the first applications here we will not use that option in order to make a comparison between the original and a reconstructed spectrum easier.

As described in section 2, for laser reflection simulation the water surface slopes are even more important than the surface elevation. The slopes can be computed as:

\[ \frac{\partial}{\partial x} \eta_m = \text{Re} \sum_{n=1}^{N/2} i c_n k_n e^{i\Omega t} \tag{12} \]

The pair \( \eta_m, \partial \eta_m/\partial x \) can be found efficiently if we compute

\[ \zeta_m = \sum_{n=0}^{N-1} d_n e^{i\Omega t} \tag{13} \]

with

\[ d_0 = 0 \tag{14a} \]
\[ d_n = c_n \frac{(1 - k_n)}{n} \quad (n < N/2) \tag{14b} \]
\[ d_{N-n} = c_n \frac{(1 + k_n)}{n} \quad (n < N/2) \tag{14c} \]
\[ d_{N/2} = c_{N/2} \frac{(1 + k_{N/2})}{N/2} \tag{14d} \]

with the help of a fast Fourier transform. Then we have:
\[ \eta_m = \text{Re} \zeta_m \]  
(15a)

\[ \frac{\partial}{\partial x} \eta_m = \text{Im} \zeta_m \]  
(15b)

The surface elevation is important for analysis method in which the run time of the laser pulse is supposed to be known. Also a later stage of the analysis the surface reflectivity may depend on the phase of the carrier waves.

In the set up of the program we assume that the aeroplane always flies faster than the (free) gravity waves. Taking for the plane velocity the value of \( v = 200 \, \text{m/s} \), this means that the contribution from waves with a periodicity of more than 130 seconds (wave length of 25 km) should be negligible. This seems realistic for applications in the north sea.
4. simple models for the reflectivity

In the two-scale model we have set up, the reflection of the laser pulse on the waves is a function of the slope of the large waves and a reflectivity, caused by the roughness of the surface. We assume that this roughness is given by the high-frequency part of the wave spectrum. Suppose that \( \eta(x) \) represents a wave surface, from which all long waves (\( k < k_r \)) are filtered out. Then the intensity reflected by the surface at point \( x \), averaged over the footprint, as a function of incidence angle \( \phi \) is equal to \( R(\phi) \) with

\[
R(\phi) = \langle S(\phi_n(x); \Delta \phi) \rangle. \tag{16}
\]

In this equation the average \( \langle ... \rangle \) is defined in eq. 4, the slit function \( S \) is given by

\[
S(\phi; \Delta \phi) = 1 \quad (|\phi| < \Delta \phi/2) \tag{17}
\]
\[
= 0 \quad \text{else}
\]

and \( \phi_n = \arctan(\partial \eta(x)/\partial x) + \pi/2 \). The requested ensemble average of \( R(\phi) \) can be approximated by the surface average, which can subsequently be tabulated.

This seems a direct approach to compute the reflectivity. However, there are some problems. Transforming the frequency spectrum into a wave number spectrum, is not a straightforward procedure for the small waves due to higher order wave and current effects. Also the small waves may be (partly) localized on the crests of the long waves, (wind effect) so that a frequency spectrum may be insufficient.

To circumvent these problems and to keep the model simple, we use experimental statistical results of the slope distribution of the waves. Measurements indicate that the reflectivity can well be parameterized as (Prokhorov 1987)

\[
R(\phi) = C \exp \left( - \frac{\tan^2(\phi)}{2\sigma_1^2} \right). \tag{18}
\]
The width of the Gaussian distribution $\sigma_1$, which represents the surface roughness, is the only relevant parameter left. The normalization factor C is taken equal to one, since we are not interested in absolute but only in relative intensities. The distribution given in eq. 18 is measured for the whole spectrum, and not only the small waves. We assume none the less that the form of eq. 18 also applies for the small scale roughness of the surface.

The final point to address in this section is how to add the surface slopes caused by the carrier waves. In section 2 we found that the effective angle to be used in the reflectivity table or function is

$$\phi_{\text{eff}} = \arctan(\delta \eta(x)/\delta x) + \pi/2 - \psi_1,$$  \hspace{1cm} (19)

with $\eta(x)$ the surface generated by the long waves only and $\psi_1$ the angle of the laser beam. Using elementary algebra we find that

$$\tan(\phi_{\text{eff}}) = \frac{\delta \eta(x)/\delta x + \cotan(\psi_1)}{1 + \delta \eta(x)/\delta x \cdot \cotan(\psi_1)}.$$  \hspace{1cm} (20)

This formula provides a simple way to use the time series of the simulated wave slopes in order to compute a time series for the reflected intensity.
5. The computer simulation

The ideas, given above, have been assembled in a computer program, in order to simulate the laser backscatter signal. Input parameters for the Lidar measuring device in the program are: the velocity of the aeroplane \( v \), the footprint length \( LF \), the time between two laser pulses, the number of laser pulses used and the laser angle. For the spectrum of the sea a JONSWAP spectrum is used (see Hasselmann et al. 1973):

\[
F(f) = \alpha' f^{-5} \exp\left(-\frac{5}{4} \left(\frac{f}{f_p}\right)^{-4}\exp\left(1g - \frac{(f - f_p)^2}{2\sigma_f^2}\right)\right),
\]

(21)

with \( f = \omega/2\pi \). The parameters \( \alpha' = \alpha g^2(2\pi)^{-4} \), \( f_p \) (the peak frequency) and \( 1g = \log(\gamma) \) can be given to the program. Other input parameters are: \( h \), the local depth of the sea and \( \sigma_s \), the width in the scattering model. The JONSWAP spectrum is considered to give a suitable description of (young) waves. As this spectrum was measured on the North Sea, it is applicable for simulations in that area.

The computer program first computes the wave spectrum \( F(\omega) \) according to eq. 21 up to \( \omega_f \), the footprint frequency. Next \( \Phi(\Omega) \), the spectrum, as observed from the aeroplane, is computed as:

\[
\Phi(\Omega) \ d\Omega = F(\omega) \ dw,
\]

(22)

with \( \Omega \) given in eq. 10. To the spectrum \( \Phi(\Omega) \) random phases are added in order to simulate the sea surface and slopes. On the time series for the slopes the reflection model of section 4 is applied. The resultant LIDAR backscatter signal is then used all following analysis techniques. As a demonstration we give for the input parameters as given in table 1 the spectra \( F(\omega) \) and \( \Phi(\Omega) \) in plot 1-2 and the time series for the simulated surface, slope and laser reflection in plot 3. In the time series only the first 0.2 second of a total length of about 2 seconds is shown.

Apart from the laser reflection time series, which we can simulate as given in section 4, the laser bathymetry system can also measure the run time of a pulse, reflecting on the water surface and coming back to the aeroplane. From this the distance between the aeroplane and the water.
surface can be computed. Subtracting the mean distance we get the local surface elevation. However, we do not know the water surface at any point but only at points with a suitable surface slope. This is because the laser pulse must be able to get back to the receiver on the aeroplane, otherwise it cannot be measured. In section 9 and 10 we make use of this additional information.
6. Flare statistics

In this first analysis method only the statistics of the reflection flares is used. Assuming a gaussian distribution for the surface elevation and slope it can be shown that \( E(\alpha) \), the mean frequency of a crossing by the slope of the level \( \alpha \), is equal to:

\[
E(\alpha) = \frac{1}{\pi} \left( \frac{\mu_4}{\mu_2} \right)^{1/2} \exp\left(-\alpha^2/2\nu_2^2\right).
\]  

(see e.g. Cramer and Leadbetter 1967). The moments \( \mu \) in eq. 23 are defined as

\[
\mu_n = \int \Omega^n \Phi(\Omega) \, d\Omega
\]

\[
\nu_n = \int k^n F(\omega) \, d\omega
\]

\[
\nu_2 \propto \nu^{-2} \mu_2.
\]

We need mixed moments, \( \mu \) and \( \nu \), because for \( E(0) \) the standard analysis applies, but for non-zero angle the laser reflections are determined by the slope of the surface, i.e. the spatial derivative of the surface elevation and not by the temporal derivative. Taking the moments gives practical problems. For the JONSWAP spectrum the tail of \( F(\omega) \) is proportional to \( \omega^{-5} \) (see eq 21). Transforming this according to eq. 22 to the frame, moving with the aeroplane, we find that \( \Phi(\Omega) \) is proportional to \( \Omega^{-3} \) for large \( \Omega \) values. This means that \( \mu_2 \) would be logarithmically divergent and \( \mu_4 \) quadratically, if we had no cut off in the spectrum because of the finite laser footprint (see equation 5). Although this cures the infinity, the moments \( \mu_2 \) and especially \( \mu_4 \) are very sensitive for the choice of the footprint length. A useful parameter that can be constructed from the three moments mentioned is:

\[
e^2 = \frac{\mu_0 \mu_4 - \mu_2^2}{\mu_0 \mu_4}
\]

which is a measure for the width of the peak of the spectrum. For a narrowly peaked spectrum \( \epsilon \) is very small and if the signal is nearly white noise, then \( \epsilon \) is approximately equal to one. In tests with the
computer simulation it appeared that by doubling the length of the footprint the value of $\epsilon$ could drop from 0.8 to 0.2.

Measuring $E(\alpha)$ for a number of $\alpha$'s allows us to estimate $\mu_2$ and $\mu_4$, using equation 23. These values are mainly useful in combination with analysis techniques as given in the next sections. If we assume that $\epsilon$ is small (footprint length sufficiently large), we can estimate the total variance (energy) $\mu_o$ as $\mu_o = \mu_2^2 / \mu_4$. This is however a rather rough estimate.

In plot 4 we compare the simulated flare frequency as a function of the laser angle with the stochastic prediction (23). The moments occurring in eq. 23 have been computed using the input JONSWAP spectrum.
7. clipped signal approximation

In this section we try to reconstruct the wave spectrum of the sea by taking the time when a flare occurs into account. First we consider the situation that the laser beam points perpendicular to the mean water surface. A received flare then denotes a zero crossing of the (unknown) surface slope signal. If the slope signal is processed by an amplifier with infinite gain and clipped at the level of ±1, we obtain a block signal with a level jump at the zero crossings of the slope signal. These clipped signals have been studied in literature (see e.g. Lawson and Uhlenbeck 1950). Assuming that the original signal has a gaussian distribution it can be shown that

\[ R(\tau) = R(0) \sin\left(\frac{\pi}{2} r(\tau)\right) \]  \hspace{1cm} (26)

with \( R(\tau) \) the auto correlation function of the original signal and \( r(\tau) \) the auto correlation function of the clipped signal.

To estimate the wave spectrum from the laser reflection signal we thus construct a block signal with amplitude ±1, which changes sign at the peak of a flare. The auto correlation function of the block signal is multiplied by a factor of \( \pi/2 \), and of that value the sine is taken. Fourier transforming the resulting signal we obtain an unscaled estimate for the wave spectrum. The proper height of the spectrum can not be retrieved in this method.

In the plots 5 we show the results of this method for the simulated surface as given in section 5. The resulting spectrum varies wildly (grass), and somewhat overestimates the high frequencies. In plot 6 we show the average of ten spectra, which looks much smoother. We note that spectra, measured with any technique, only are smooth when averaged. At very low frequencies also peaks occur. The reason for this is that in this section we actually estimate the slope spectrum. Transforming that into the wave spectrum means a division by the wave number squared. At low frequencies small errors blow up. Negative values of the spectrum, which should be impossible, are caused by the statistical nature of eq. 26; for a finite length signal small errors may occur. These errors are most prominent at low frequencies, because less of those waves are
contained in the measured signal. Subsequently, the errors at low frequencies are blown up.

From tests with this method we found some restrictions for the sampling frequency of the laser reflection signal. To high a frequency causes errors because too few significant waves are contained in the measured signal. Also the resolution of the spectrum is reduced. If the sampling frequency is too low, then more noise is added to the spectrum. In practice, a satisfactory balance was reached by containing about ten significant waves in the total time series. For deep water waves this means that a suitable sampling frequency is given by:

\[
f_{\text{samp}} = 1 / \Delta t = (10 / g) 2\pi n v f_p^2
\]  

(27a)

with \( g \) the gravitational constant, \( n \) the number of samples taken, \( v \) the velocity of the aeroplane and \( f_p \) the peak frequency.

A similar dilemma occurs when we choose the laser footprint length LF. If we take LF too small, then we have an abundance of detected flares, but the time resolution for when each flare occurs is poor. If we take LF too large, then we can accurately measure when a flare occurs and its width, but we have measured too few flares to compute a reliable spectrum with. In our tests we found a satisfactory balance for

\[
f_{\text{samp}} = (30 - 100) E(0).
\]  

(27b)

The flare frequency \( E(0) \) (eq. 23) strongly depends on the laser footprint. So once the sampling frequency has been fixed, LF can be tuned until equation 27b is satisfied.

The analysis up to here assumed that the angle \( \alpha \) between the laser beam and the normal to the mean water surface was zero or very small. The extension of equation 26 for finite, but not too large \( \alpha \) is

\[
R(\tau) = R(0) \left[ \sin(\frac{\pi}{2} r(\tau)) - \left( \frac{\alpha}{\sigma} \right)^2 \left\{ 1 - \sin(\frac{\pi}{2} r(\tau)) \right\} \right]
\]  

(28)

\((\alpha/\sigma)^2 \approx 1\). Measuring the spectrum with non-zero angle \( \alpha \) has some disadvantages. We need to know the width \( \sigma \) of the slope distribution \((\sigma^2 = \mu_2, \) as defined in eq. 24). There are fewer flares available (see
section 6) and thus less information. Moreover, equation 28 is much more sensitive for deviations from gaussian statistics then equation 26. Therefore, it is advisable to take the angle $\alpha$ equal to zero (the laser beam perpendicular to the mean sea surface) when measuring the wave spectrum.

Finally we note that clipped signal approximation needs relatively much computer power (three fourier transforms per spectrum), which may be inhibiting for real time applications.
8. reconstructed slope signal method

In this method we use not only the location of the flares but also the form of the flares. Here again we consider the case that the laser beam is perpendicular to the mean water surface. The main idea in this method is that the width of the flare is proportional to the width of the crest or troughs of the waves, and thus proportional to the inverse of the derivative of the slope signal. At the zero crossing of the slope signal, which occurs at the peak of a flare, we can estimate the derivative of the signal by integrating the volume under the flare and inverting that number. Giving the derivatives at any two sequential zero crossings an opposite sign, we can estimate the form of the slope signal with the help of cubic spline functions. As in the method of section 7 we do not have enough information to reconstruct the actual height of the slope signal, we can only reconstruct the form.

In plot 7 we compare the reconstructed with the original slope signal (the reconstructed signal has been scaled to the same height as the original signal). Apart from occurring sign changes, the reconstructed slope signal looks quite similar to the original. Also the spectrum in plot 7, obtained by fourier transforming the reconstructed slope signal, looks much better than the result of the clipped signal approximation.

The same method can be used when the laser beam is not perpendicular to the water surface, i.e. when non-zero level crossings of the slope signal are detected. However, the method becomes less reliable for larger angles.

The reconstructed slope signal method is much more efficient than the clipped signal approximation, as we need only one fourier transform per spectrum. We can further reduce the necessary computer capacity by resampling the reconstructed slope signal. If we have measured 2000 samples, then the method in this section produces a spectrum with 1000 bins, of which about 10 bins contain valuable information. Resampling the reconstructed slope signal to 64 samples yields a spectrum with 32 bins with again 10 valuable bins. All high frequency bins with zero content have been omitted this way. Resampling reduces in this example the time, necessary to compute the fourier transform, by more
than a factor 100. Applying the method of this section, real time processing of laser reflections should be possible with even simple computers.
9 pulse run time statistics

Up to now we assumed that the measuring aeroplane was equipped with detectors for the reflected laser pulses and a clock to measure when the pulses were received. In this and the next section we shall assume that also the run time of the laser pulse, i.e. the time difference between reception and emission, can be measured. From the run time we can compute the height of the water surface, on which the pulse reflected. The probability distribution of maxima is given by

\[ P(\eta) = \frac{1}{\sqrt{2\pi}} \left\{ \epsilon \exp \left( -\frac{\eta^2}{2\epsilon^2} \right) + \sqrt{1-\epsilon^2} \eta \exp\left( -\frac{1}{2} \eta^2 \right) \int_{-\infty}^{\eta\sqrt{1-\epsilon^2/\epsilon}} \exp\left( -\frac{1}{2} y^2 \right) \, dy \right\} \] (29)

(see Rice 1954), with \( \eta^2 = \zeta^2/\mu_\varnothing \), \( \zeta \) the surface elevation and \( \epsilon \) a measure for the width of the spectral peak defined in equation 25. As noted in section 6, the value of \( \mu_4 \) strongly depends on the footprint length of the laser beam. Therefore in actual measurements values of \( \epsilon \approx 0 \) (narrow spectrum, large footprint, \( P(\eta) \) approaches a rayleigh distribution), \( \epsilon = 1 \) (broad spectrum, small footprint, \( P(\eta) \) approaches a gaussian distribution) and everything in between can occur.

In this section we again consider the case that the laser beam is perpendicular to the mean surface level. For the laser reflection we then are not only interested in peaks but also in troughs. The probability distribution for the surface elevation, normalized with the mean squared elevation, with the side condition that the surface slope is equal to zero, is then given by:

\[ Q(\eta) = 0.5 \ast \left\{ P(\eta) + P(-\eta) \right\} \] (30)

With the help of this probability function we can define the following moments:

\[ Q_n = \int_{-\infty}^{\infty} \eta^n Q(\eta) \, d\eta. \] (31)

All odd moments are equal to zero and for the first three even moments we find:
\[ Q_0 = 1 \]
\[ Q_2 = 2 - \varepsilon^2 \]
\[ Q_4 = 8 - 4 \varepsilon^2 - \varepsilon^4 \]

With the lidar system we can measure the variance \( \zeta_0^2 \) and the mean fourth power \( \zeta_0^4 \) of the surface elements with zero slope. According to equation 32 we can compute the variance (total energy) of the waves as:

\[
\zeta^2 = (2 - \varepsilon^2)^{-1} \zeta_0^2 \tag{32}
\]

\[
\varepsilon^2 = \frac{2\delta}{1 + \delta + \sqrt{1-\delta}}
\]

with \( \delta = \zeta_0^4 / (\zeta_0^2)^2 - 2 \). From equation 32 we see that the variance, as measured from the aeroplane can over-estimate the real variance by a factor of two, for narrow spectra.

In the figures 8 we have plotted the probability distribution, simulated for three values of the laser footprint. The change from a double rayleigh distribution for large footprints to a gaussian distribution for short footprints is clearly visible. For these cases we have also estimated the total variance according to equation 32. We found values which were about 10% too low, which was caused by an underestimation of \( \varepsilon^2 \). In further tests it appeared that the estimation of the variance is quite stable, but the estimation of \( \varepsilon^2 \) varies significantly.
10. reconstructed surface elevation method

This method is very similar to the one of section 8. Here we assume that at the peak of a flare we have measured the surface elevation by means of the pulse run time. So every time that the slope of the water surface is equal to zero, we know at which time it occurs and what the surface elevation is. We can interpolate these points with the help of quadratic splines, making the derivatives at the knots equal to zero (or equal to some fixed value if the laser beam is not perpendicular to the mean water surface). Applying a fourier transform to the reconstructed surface elevation yields the wave spectrum. This method has a number of advantages over the reconstructed slope signal method. The spectrum is now properly scaled and more accurate, especially at low frequencies. The method of this section is as efficient as the reconstructed slope signal method and can be expected to run in real time.

In plot 9 we compare the reconstructed surface elevation with the original simulation. The difference between the two is hardly noticeable. Also the spectrum, computed from the reconstructed surface elevation, looks remarkably similar to the spectrum we started from.
11. mean width of the flares

The mean width of the flares, $\sigma_f$, is proportional to $\sigma_i$, the width in the reflectivity model in section 4 and inversely proportional to velocity of the aeroplane and the width of the distribution of slope derivatives, defined as the square root of $\mu_4$. The value of $\sigma_f$ can be measured operationally by integrating the flare intensity and dividing the result by the number of flares times the flare peak intensity. Alternatively, the flare width can be found directly by measuring the time difference between the up and down crossing of a specified intensity level.

If the value of $\mu_4$ has been measured with some other method, we can estimate the value of $\sigma_i$. We expect that this value, which is a measure for the roughness of the water surface, is very sensitive for the local wind field. Modelling the dependence of $\sigma_i$ on the wind field is too complicated for this project an the dependence is probably best determined experimentally.
12 conclusions and recommendations

In this report we have tested several analysis methods to extract relevant wave data from lidar reflection data, using a simplified model to simulate a wave surface and the laser reflection on it. We have tried to keep the simulation model as simple as possible, whilst retaining a proper description of the essential underlying physical processes. Simplifications in the model are:

- a 1-dimensional model
- gaussian probability distribution for the waves
- linear wave theory
- no wave-current interaction
- noise or less than perfect measurements not considered
- a simplified reflection model
- a rough division into long and short waves

To interpret the results of the simulation we need to keep in mind that the wave information, extracted from the laser reflection signal, serves a double purpose. To improve the performance of the bathymetry system we are mainly interested in surface slope and elevation at the flare points. Statistical slope data gives information on mean bias and accuracy of measured bottom depth. A slope time series enables us to correct every single measurement for the local water surface slope. If in addition the surface elevation is known the bias due to the wave heights can be corrected. When the LIDAR system is operating as a wave gauge one is interested in the significant wave height, the peak frequency and, if available, the energy spectrum.

Summarizing the results of the techniques of the sections 6 - 11:
- flare statistics (section 6)
  This is the simplest method under consideration. Here we only need to count the number of reflected laser flares per unit of time. Doing this for several different angles between the laser beam and the nadir allows us to measure the wave slope distribution and estimate the significant wave height.
- clipped signal approximation (section 7)
  For this method we need a time series of the reflected laser signal.
From this we can find the spectrum of the wave slopes. This can be transformed into a wave energy spectrum, but the energy scale of that spectrum is unknown. Also the transformation blows up errors at low frequencies. Comparing with later techniques this method is rather rough: we need more measurements to get a reliable result.

- reconstructed slope signal method (section 8)
  This method needs the same input as the last method. By making use of the form of the laser flares we get more accurate results. Moreover, the reconstructed wave slope signal can be used to correct bathymetric measurements.

- pulse run time statistics (section 9)
  Here we assume that we can measure the distance between the aeroplane and the water surface by measuring the time that a laser pulse needs to reflect on the water and get back to the receiver. This method estimates the total wave energy from the total variance of the measured distance.

- reconstructed surface elevation method (section 10)
  For this method in addition to the laser reflection signal we need to know the run time of the laser pulse during the flares. This enables us to reconstruct the water surface, yielding the information needed to correct bottom depth measurements. The wave spectra, retrieved with this method, are accurate and properly scaled.

- mean width of the flares (section 11)
  This is not a fully developed analysis method but a suggestion how to compute the mean width of the flares from a time signal. We expect that this quantity is closely connected to the local wind field. What the exact relation looks like is beyond the scope of this study.

The first method has been studied previously (see e.g. Prokhorov 1987), the second has been adopted from signal theory (Rice 1954) and adapted to this application. The remaining analysis techniques have been developed for this study.

In this study it turned out that the quality of the retrieved wave data depend strongly on the number of samples n (pulses) used, the sampling frequency $f_s$ and the laser footprint LF.

- For the number of samples a value of the order of 1000 seems sufficient to obtain a spectrum (5000 - 10000 when the clipped signal approximation is used).

- A suitable compromise between the resolution of the peak frequency of
the spectrum and the noise level can be reached by containing about 10 significant waves (waves with frequency equal to the peak frequency) in the total time series. This means for deep water waves:

\[ f_s \approx \frac{10}{g} \pi n \sqrt{f_p^2} \]  

(eq. 27a)

- The laser footprint length strongly influences the flare frequency \( f_{fl} \). Best results were found when

\[ f_s \approx (30 - 100) f_{fl} \]  

(eq. 27b)

So when the sampling frequency is fixed the footprint can be tuned until this relation is obeyed. The above relation does not mean that only 1-3 % of the emitted pulses reaches the receiver. Since a flare has a finite width this percentage is larger (of the order of 10 % in the simulations).

Concerning the configuration of the LIDAR system: for wave measurements a nadir pointing system would be preferable because:
- a slanting beam overlooks part of the waves
- to process data from a not nadir pointing system the width of the wave slope distribution needs to be known (this is not true for the reconstruction methods of section 8 and 10).

Recommendations for further investigations:

At the beginning of this section we made a list of important features, not included in the present simple model.

The most important extension of the model seems to be at this moment a two-dimensional description of the waves and the addition of noise.

It would be helpful to have experimental data available, laboratory measurements or real flight data, because:
- to test the analysis methods, presented in this report, in situ and refine them when necessary, experimental data are needed.
- the analysis of the influence of noise can be so complex, that an inventory of what kind of noise in nature actually occurs can save much effort.
- a technique to estimate the local wind field from the width of the laser flares can optimally be determined by empirical means.
13. references

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Tucker, M.J., T.G. Challenor and D.J.T. Carter;
    Appl. Ocean Res. 6 (1984) 118
TABLE 1: simulation input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>1.5</td>
<td>length of footprint (m)</td>
</tr>
<tr>
<td>0.001</td>
<td>time between two wave measurements (s)</td>
</tr>
<tr>
<td>50.0</td>
<td>local depth of the sea</td>
</tr>
<tr>
<td>200.0</td>
<td>velocity of the aeroplane</td>
</tr>
<tr>
<td>2048</td>
<td>the number of (simulated) wave measurements</td>
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</table>

input values for spectrum generation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>3.0e-4</td>
<td>alfa * g**2 / (2*pi)**4</td>
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<tr>
<td>1.2</td>
<td>log(gamma)</td>
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<tr>
<td>0.3</td>
<td>the peak frequency of the spectrum</td>
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</table>

reflectivity model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>0.0</td>
<td>off nadir laser angle</td>
</tr>
<tr>
<td>0.01</td>
<td>the Gaussian width in the scattering model</td>
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Table 2: moments of the spectrum (section 6)

<table>
<thead>
<tr>
<th>Moment</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.01791</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>123.1</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>1.076 $10^6$</td>
</tr>
<tr>
<td>$\epsilon^2$</td>
<td>0.2136</td>
</tr>
</tbody>
</table>
Spectrum, as used in the simulation [JONSWAP]

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FIG. 1
Spectrum, as observed from the aeroplane

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FIG. 2
Simulated: surface elevation
wave slope
reflected flares

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FIG. 3
flare statistics

--- theoretical flare frequency
* simulated flare frequency

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clipped signal approximation

- upper: block signal
- lower: spectrum from clipped signal app.
- --- original spectrum

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FIG. 5
clipped signal approximation
(average of 10 time series)

---

av. spectrum from clipped signal appr.
original spectrum

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FIG. 6
reconstructed slope signal method

[Graphs showing time-domain and frequency-domain analysis of reconstructed slope signal.]

upper: reconstructed slope
lower: spectrum from rec. slope method
----- original spectrum

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pulse run time statistics

LF = 1.0 m

[Graph showing surface elevation distribution for LF = 1.0 m]

LF = 1.5 m

[Graph showing surface elevation distribution for LF = 1.5 m]

LF = 2.0 m

[Graph showing surface elevation distribution for LF = 2.0 m]

surface elevation distribution:
LF = 1.0 m
LF = 1.5 m
LF = 2.0 m

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FIG. 8
reconstructed surface elevation method

![Graph showing elevation over time](image1)

![Graph showing spectral density vs. frequency](image2)

**Legend:**
- **Upper:** reconstructed surface
- **Lower:** spectrum from rec. surface method
- **--- Original spectrum**

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FIG. 9
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